TRANSACTIONS

OF THE

American Institute of Electrical Engineers

Vol. 48 JANUARY, 1929 No. 1

TABLE OF CONTENTS

Sphere-Gap and Point-Gap Arc-Over Voltage, by Joseph S. Carroll and Bradley Cozzens 1
Discussion 9
High-Voltage Gaseous-Conductor Lamps, by F. O. McMillan and E. C. Starr 11
Discussion 17
The Automatic Substation and Its Relation to the Electric Distribution System, by S. J. Lisberger 19
Discussion 30
The Electrolytic Zinc Plant of the Sullivan Mining Company, by Ellery R. Fosdick 33
Great Northern Railway Electrification in the Cascades, by Edward L. Moreland and Ralph D. Booth 40
Discussion 50
Power Supply for Railway Signals and Automatic Train Control, by C. F. King, Jr 53
Discussion 57
Automatic Mercury Arc Power Rectifier, by L. J. Turley 58
Discussion 65
Movements of Overhead Line Conductors during Short Circuits, by Wm. S. Peterson and H. J. McCracken, Jr 67
Discussion 90
Residual Voltages and Currents in Power Systems, by L. J. Corbett 92
Discussion 99
Power-Line Carrier Telephony, by L. F. Fuller and W. A. Tolson 102
Problems in Power-Line Carrier Telephony, by W. V. Wolfe and J. D. Sarros 107
Discussion 113
Carrier Telephone System for Short Toll Circuits, by H. S. Black, M. L. Almquist, and L. M. Ilgenfritz 117
Discussion 139
Lightning Arrester Problems, by A. L. Atherton 141
Discussion 144
High-Voltage Phenomena in Thunderstorms, by Marcel A. Lissman 146
Discussion 152
Heat Flow from Underground Electric Power Cables, by Neil P. Bailey 156
Discussion 163
Corrosion of Cable Sheath in Creosoted Wood Conduit, by R. M. Burns and B. A. Freed 166
Discussion 169
System Stability as a Design Problem, by R. H. Park and E. H. Bancker 170
Discussion 193
Calculation of the Capacitance between Two Wires of a Three-Conductor Cable, by Y. W. Lee 195
Radio Acoustic Position Finding in Hydrography, by Jerry H. Service 198
Economies in Central Power Service as Illustrated by the Duke Power System, by W. S. Lee 203
Discussion 212
Southeastern Power & Light System, by A. T. Hutchins 206
Discussion 212
Carrier-Current and Supervisory Control on Alabama Power Company System, by W. I. Woodcock and E. W. Robinson 214
Discussion 236
The Transmission of High-Frequency Currents for Communication over Existing Power Networks, by C. A. Boddie and R. C. Curtis 227
Discussion 236
Impulse Flashover of Insulators, by J. J. Torok and W. Ramberg 239
Discussion 254
A Study of High-Voltage Flashovers, by Joseph T. Lusignan, Jr 246
Discussion 254
Electrification of the Textile Industry in the Southeast, by Edwin M. Clapp and A. G. Stanford 261
Discussion 266
Power Limit Tests on Southeastern Power and Light Company's System, by S. Murray Jones and Robert Treat 268
Discussion 281
Some Photoelectric and Glow-Discharge Devices, by J. V. Breisky and E. O. Erickson 283

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PREFACE

This is Number 1 of Volume 48 of the Quarterly TRANSACTIONS of the A. I. E. E. It consists of the papers and discussions from the 1928 Pacific Coast Convention (Spokane, Washington) and the Regional Meeting of the Southern District (Atlanta, Georgia).

In addition to the papers at the above meetings, two papers, *Radio Acoustic Finding in Hydrography*, by Mr. Jerry H. Service, and *The Calculation of the Capacitance between Two Wires of a Three-Conductor Cable*, by Mr. Y. W. Lee, are included in this number.

For convenient reference, an index of authors and discussors has been included in this number.

TRANSACTIONS

OF THE

American Institute of Electrical Engineers

Vol. 48 APRIL, 1929 No. 2

TABLE OF CONTENTS

Anomalous Conduction as a Cause of Dielectric Absorption, by J. B. Whitehead and R. H. Marvin ... 299
Discussion ... 314
Corona Ellipses, by Vladimir Karapetoff ... 317
Discussion ... 326
Flux Linkages and Electromagnetic Induction in Closed Circuits, by L. V. Bewley ... 327
Discussion ... 337
Progress in High-Tension Underground Cable Research and Development, by G. B. Shanklin and G. M. J. Mackay ... 338
Discussion ... 367
Some Problems in High-Voltage Cable Development, by E. W. Davis and W. N. Eddy ... 373
Discussion ... 379
Ionization Studies in Paper-Insulated Cables—II, by C. L. Dawes, H. H. Reichard, and P. H. Humphries ... 382
Discussion ... 395
Reduction of Sheath Losses in Single-Conductor Cables, by Herman Halperin and K. W. Miller ... 399
Discussion ... 414
Losses in Armored Single-Conductor Lead-Covered A-C. Cables, by O. R. Schurig, H. P. Kuehni, and F. H. Buller ... 417
Discussion ... 434
Lightning, by F. W. Peek, Jr ... 436
Discussion ... 468
Theoretical and Field Investigations of Lightning, by C. L. Fortescue, A. L. Atherton, and J. H. Cox ... 449
Discussion ... 468
1927 Lightning Experience on the 132-Kv. Transmission Lines of the American Gas & Electric Company, by Philip Sporn ... 480
Discussion ... 492
Power Transmission and Distribution Committee Report, by H. L. Wallau, Chairman ... 487
Discussion ... 492
Power Factor and Dielectric Constant in Viscous Dielectrics, by Donald W. Kitchin ... 495
Discussion ... 504
A Graphical Theory of Traveling Electric Waves between Parallel Conductors, by V. Karapetoff ... 508
Theory of the Deion Circuit Breaker, by J. Slepian ... 523
Discussion ... 545
The Structural Development of the Deion Circuit Breaker up to 15,000 Volts, by R. C. Dickinson ... 528
Discussion ... 545
Field Tests of the Deion Circuit Breaker, by B. G. Jamieson ... 535
Discussion ... 545
Automatic Reclosing High-Speed Circuit Breaker Feeder Equipment for D-C. Railway Service, by A. E. Anderson ... 554
Discussion ... 562
Uses of Radio as an Aid to Air Navigation, by J. H. Dellinger ... 563
Discussion ... 567
The Predominating Influence of Moisture and Electrolytic Material upon Textiles as Insulators, by R. R. Williams and E. J. Murphy ... 568
Discussion ... 581
Purified Textile Insulation for Telephone Central Office Wiring, by H. H. Glenn and E. B. Wood ... 576
Discussion ... 581
Vector Presentation of Broad-Band Wave Filters, by R. F. Mallina and O. Knackmuss ... 582
Discussion ... 595
The Condenser Motor, by Benjamin F. Bailey ... 596
Discussion ... 629
The Fundamental Theory of the Capacitor Motor, by H. C. Specht ... 607
Discussion ... 629
The Revolving Field Theory of the Capacitor Motor, by Wayne J. Morrill ... 614
Discussion ... 629
Line-Start Induction Motors, by C. J. Koch ... 633
Discussion ... 641
No-Load Induction Motor Core Losses, by I. Spooner and C. W. Kincaid ... 645
Discussion ... 654
Insulation Tests of Electrical Machinery before and after Being Placed in Service, by C. M. Gilt and B. L. Barns ... 656
Discussion ... 661
Influence of Temperature on Large Commutator Operation, by F. T. Hague and G. W. Penney ... 666
Discussion ... 675

PUBLISHED QUARTERLY BY THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
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PREFACE

This is Number 2 of Volume 48 of the A. I. E. E. Quarterly TRANSACTIONS. In it appears the majority of the papers and discussions presented at the Winter Convention held in New York from January 28 to February 1, 1929. The remaining papers, together with those from the Regional Meeting held in Cincinnati during March, will be published in the July Quarterly.

TRANSACTIONS

OF THE

American Institute of Electrical Engineers

Vol. 48 JULY, 1929 No. 3

TABLE OF CONTENTS

Effect of Transient Voltages on Power Transformer Design, by K. K. Palueff ... 681
Discussion ... 701
Transient Analysis of A-C. Machinery, by Yu H. Ku ... 707
Discussion ... 714
Two-Reaction Theory of Synchronous Machines—Part I, by R. H. Park ... 716
Discussion ... 727
Magnetic Analysis, by Raymond L. Sanford ... 731
Discussion ... 736
Electrical Instruments Used in the Measurement of Flow, by W. H. Pratt ... 737
Discussion ... 741
Use of the Oscillograph for Measuring Non-Electrical Quantities, by D. F. Miner and W. B. Batten ... 742
Discussion ... 745
Study of Noises in Electrical Apparatus, by T. Spooner and J. P. Foltz ... 747
Discussion ... 752
Electrical Aids to Navigation, by Robert H. Marriott ... 753
Discussion ... 765
Telemetering, by C. H. Linder, C. E. Stewart, H. B. Rex, and A. S. Fitzgerald ... 766
Discussion ... 773
Totalizing of Electric System Loads, by P. M. Lincoln ... 775
Discussion ... 781
A New High Accuracy Current Transformer, by M. S. Wilson ... 783
Discussion ... 788
132-Kv. Shielded Potentiometer for Determining the Accuracy of Potential Transformers, by C. T. Weller ... 790
Discussion ... 801
A Precision Regulator for Alternating Voltage, by H. M. Stoller and J. R. Power ... 808
Discussion ... 811
Bessel Functions for A-C. Problems, by Herbert Bristol Dwight ... 812
Some Notes on Transformer Design, by E. H. Waldo ... 821
Recent Developments in Telephone Construction Practises, by B. S. Wagner and A. C. Burroway ... 836
Discussion ... 848
Lighting of Airways and Airports, by H. E. Mahan ... 849
Discussion ... 854
Iron Losses in Turbine Generators, by C. M. Laffoon and J. F. Calvert ... 856
Discussion ... 871
The Fabrication of Large Rotating Machinery, by H. V. Putman ... 873
Discussion ... 877
Electrical Equipment of Bar Plate and Hot Strip Mills, by J. B. Ink ... 878
Discussion ... 883
A New Type of Hot Cathode Oscillograph, by R. H. George ... 884
Discussion ... 890
Fused Arcing Horns and Grading Rings, by Philip Stewart ... 891
Discussion ... 893
Operating Experience with the Low-Voltage A-C. Network in Cincinnati, by F. E. Pinckard ... 896
Discussion ... 901
Quick-Response Generator Voltage Regulator Field Tests Made with Oscillograph, by E. J. Burnham, J. R. North, and I. R. Dohr ... 903
Discussion ... 909
Street Railway Power Economics on the Cincinnati System, by J. A. Noertker ... 912
Discussion ... 924
Automatic Mercury Rectifier Substations in Chicago, by A. M. Garrett ... 925
Discussion ... 927
Arc Welding of Steel Buildings and Bridges, by Frank P. McKibben ... 929
Discussion ... 938
Oscillographs for Recording Transient Phenomena, by W. A. Marrison ... 939
Discussion ... 947
Bushing-Type Current Transformers for Metering, by A. Boyajian and W. F. Skeats ... 949
Discussion ... 951
Cathode Ray Oscillograph Study of Artificial Lightning Surges on the Turners Falls Transmission Line, by K. B. McEachron and V. E. Goodwin ... 953
Discussion ... 964
Developments in Network Systems and Equipment, by T. J. Brosnan and Ralph Kelly ... 966
Discussion ... 975
Standard Voltage A-C. Network, by John Oram ... 977
Automatic Reclosing of High-Voltage Circuits, by E. W. Robinson and S. J. Spurgeon ... 981
Discussion ... 987
Bare Wire Overhead Distribution Practise, by M. C. Miller ... 988
Discussion ... 989

(*Continued on next page.*)

PUBLISHED QUARTERLY BY THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
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TABLE OF CONTENTS (*continued*)

Interconnection in the Southwest, by Geo. A. Mills 991
Discussion 993
Electrification of Oil Pipe Lines in the Southwest, by D. H. Levy 995
Discussion 997
Lightning Studies of Transformers by the Cathode Ray Oscillograph, by F. F. Brand and K. K. Palueff 998
Discussion 1008
Flying Field and Airway Lighting, by H. R. Ogden 1013
Electrical Features of the Kansas City New Water Works, by Albert L. Maillard 1020
Meeting Long Distance Telephone Problems, by H. R. Fritz and H. P. Lawther, Jr 1024
Discussion 1029
Telephone Transmission Networks, Types and Problems of Design, by T. E. Shea and C. E. Lane 1031
Discussion 1044
Telephone Circuits for Program Transmission, by F. A. Cowan 1045
Traveling Waves Due to Lightning, by L. V. Bewley 1050
Report of the Board of Directors for the Fiscal Year Ending April 30, 1929 1065

PREFACE

This is Number 3 of Volume 48 of the Quarterly TRANSACTIONS of the American Institute of Electrical Engineers. It contains those papers presented at the Winter Convention that were not included in the April Quarterly, as well as the papers given at the Regional Meetings held in Cincinnati, Ohio, March 20-23, and at Dallas, Texas, May 7-9, 1929.

Three miscellaneous contributions appear in this number: *Bessel Functions for A-C. Problems*, by Herbert Bristol Dwight, *Some Notes on Transformer Design*, by E. H. Waldo, and *Traveling Waves Due to Lightning*, by L. V. Bewley.

The issue concludes with the Annual Report of the Board of Directors for the fiscal year ending April 30, 1929.

TRANSACTIONS

OF THE

American Institute of Electrical Engineers

Vol. 48 OCTOBER, 1929 No. 4

TABLE OF CONTENTS

"Synchronized at the Load," A Symposium . . 1079-1107

I. A Fundamental Plan of Power Supply, by A. H. Kehoe . . . 1080

II. Calculations of System Performance, by S. B. Griscom . . . 1083

III. System Tests and Operating Connections, by H. R. Searing and G. R. Milne . . . 1093
Discussion . . . 1100

Automatic Transformer Substations of the Edison Electric Illuminating Company of Boston, by William W. Edson . . . 1108
Discussion . . . 1112

Rehabilitation of Steam Power Plants, by C. F. Hirshfeld . . . 1114
Discussion . . . 1120

Application of Induction Regulators to Distribution Networks, by E. R. Wolfert and T. J. Brosnan . . . 1123

The Electrification of the Mexican Railway, by J. B. Cox . . . 1130
Discussion . . . 1139

Contact Wire Wear on Electric Railroads, by I. T. Landhy . . . 1142
Discussion . . . 1145

An Electrified Railway Substation of the Pensylvania Railroad, by J. V. B. Duer . . . 1147
Discussion . . . 1149

D-C. Railway Substation for the Chicago Terminal Electrification, Illinois Central Railroad, by A. M. Garrett . . . 1150
Discussion . . . 1155

Master Reference System for Telephone Transmission, by W. H. Martin and C. H. G. Gray . . . 1157
Discussion . . . 1166

Electrical Wave Analyzers for Power and Telephone Systems, by R. G. McCurdy and P. W. Blye . . . 1167
Discussion . . . 1175

A New Automatic Synchronizer, by F. H. Gulliksen . . . 1178

High-Frequency Portable Tools and Equipment, by C. B. Coates . . . 1189
Discussion . . . 1194

Electric Heating Elements, Some Fundamentals Used in Their Design, by Edwin Fleischmann . . . 1196
Discussion . . . 1204

Safe Loading of Oil-Immersed Transformers, by E. T. Norris . . . 1206
Discussion . . . 1212

Induction Motor Operation with Non-Sinusoidal Impressed Voltages, by L. A. Doggett and E. R. Queer . . . 1217
Discussion . . . 1220

Outdoor Hydrogen-Ventilated Synchronous Condensers, by Robert W. Wieseman . . . 1221
Discussion . . . 1226

Short-Circuit Torque in Synchronous Machines without Damper Windings, by G. W. Penney . . . 1230
Discussion . . . 1240

Analytical Determination of Magnetic Fields, Simple Cases of Conductors in Slots, by B. L. Robertson and I. A. Terry . . . 1242
Discussion . . . 1260

Shielding and Guarding Electrical Apparatus Used in Measurements—General Principles, by Harvey L. Curtis . . . 1263
Discussion . . . 1269

Some Problems in Dielectric-Loss Measurements, by C. L. Dawes, P. L. Hoover, and R. H. Reichard . . . 1271
Discussion . . . 1280

Shielding in High-Frequency Measurements, by John G. Ferguson . . . 1286
Discussion . . . 1292

Guarding and Shielding for Dielectric Loss, Measurements on Short Lengths of High-Tension Power Cable, by E. H. Salter . . . 1294
Discussion . . . 1299

Precautions against Stray Magnetic Fields in Measurements with Large Alternating Currents, by Francis B. Silsbee . . . 1301
Discussion . . . 1305

Magnetic Shielding (Shielding of Magnetic Instruments from Steady Stray Fields), by S. L. Gokhale . . . 1307
Discussion . . . 1319

Automatic Stations Committee Report, by W. H. Millan, Chairman . . . 1322
Discussion . . . 1327

Electrical Communication Committee Report, by H. W. Drake, Chairman . . . 1329

Education Committee Report, by Edward Bennett, Chairman . . . 1334
Discussion . . . 1334

Electrical Machinery Committee Report, by W. J. Foster, Chairman . . . 1336
Discussion . . . 1352

Electric Welding Committee Report, by A. M. Candy, Chairman . . . 1354
Discussion . . . 1357

Electrochemistry and Electrometallurgy Committee Report, by G. W. Vinal, Chairman . . . 1359
Discussion . . . 1362

Electrophysics Committee Report, by V. Karapetoff, Chairman . . . 1363

(*Continued on next page.*)

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TABLE OF CONTENTS (*continued*)

General Power Applications Committee Report, by J. F. Gaskill, Chairman 1373
Instruments and Measurements Committee Report, by E. S. Lee, Chairman 1385
Discussion 1390
Iron and Steel Production Committee Report, by M. M. Fowler, Chairman 1392
Production and Application of Light Committee Report, by B. E. Shackelford, Chairman 1394
Discussion 1408
Applications to Marine Work Committee Report, by W. E. Thau, Chairman 1409
Applications to Mining Work Committee Report, by Carl Lee, Chairman 1410
Power Generation Committee Report, by F. A. Allner, Chairman 1413
Discussion 1438
Transmission and Distribution Committee Report, by H. R. Woodrow, Chairman 1439
Discussion 1446
Protective Devices Committee Report, by E. A. Hester, Chairman 1448
Research Committee Report, by F. W. Peek, Chairman 1453
Discussion 1456
Transportation Committee Report, by W. M. Vandersluis, Chairman 1457
The Engineer, Practical Idealist, by R. F. Schuchardt, President's Address 1460
Current Transformer Excitation under Transit Conditions, by D. E. Marshall and P. O. Langguth 1464

PREFACE

This number, which concludes Volume 48 of the Quarterly TRANSACTIONS of the American Institute of Electrical Engineers, contains the papers and committee reports presented at the 1929 Summer Convention held at Swampscott, Massachusetts, June 24-28, and *The Engineer, Practical Idealist*, the address delivered by President Schuchardt at that convention.

Excitation of Current Transformers under Transient Conditions, by D. E. Marshall and P. O. Langguth, a paper read at the Cincinnati Regional Meeting but received too late to appear in the July Quarterly, is published here.

An index of authors and a subject index to Volume 48 are included in this issue.

SUBJECT INDEX

PAPERS, DISCUSSIONS AND REPORTS

Subject Classifications

American Institute of Electrical Engineers Affairs 1475

Cables 1475
Central Station Systems 1475
Circuit Breakers and Switches 1475
Communication 1475
Commutation 1475
Condensers 1476
Conductors 1476
Control Devices 1476
Corona 1476

Dielectric Phenomena 1476
Distribution 1476

Education 1476
Electric Circuit Theory 1476
Electrochemistry 1476
Electrophysics 1476

Generators, Alternating 1476

Heating 1476

Industrial Applications of Power 1476
Insulation 1476
Insulators 1476
Interconnected Systems 1476

Lamps 1476
Lighting 1476

Lightning Arresters 1476
Loss and Efficiency Measurements 1477

Magnetic Phenomena 1477
Manufacturing Methods 1477
Marine Applications 1477
Measuring Instruments 1477
Measuring Methods 1477
Mining Applications 1477
Motors, Alternating Current 1477

Power Factor 1477
Power Stations 1477
Protective Devices 1477

Radio Phenomena 1477
Railways 1477
Rectifiers 1478
Research 1478

Substations 1478
Synchronizing Equipment 1478

Technical Committee Reports 1478
Telephone 1478
Transformers 1478
Transients 1478
Transmission Lines (Electrical Characteristics) 1478
Transmission Lines (General) 1478
Transmission Lines (Mechanical Features) 1478

Welding 1478

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS AFFAIRS

See also Technical Committee Reports

Engineer, Practical Idealist, The, *Schuchardt, R. F.*, President's Address, (October) 1460
Report of the Board of Directors for the Fiscal Year Ending April 30, 1929 (July) 1065

CABLES

Calculation of the Capacitance between Two Wires of a Three-Conductor Cable, *Lee, Y. W.* (January) 195
Corrosion of Cable Sheath in Creosoted Wood Conduit, *Burns, R. M.*, and *Freed, B. A.* (January) 166
Guarding and Shielding for Dielectric Loss Measurements on Short Lengths of High-Tension Power Cables, *Salter, E. H.* (October) 1294
Heat Flow from Underground Electric Power Cables, *Bailey, Neil P.* (January) 156
Ionization Studies in Paper-Insulated Cables, II, *Dawes, C. L., Reichard, H. H., and Humphries, P. H.* (April) 382
Losses in Armored Single-Conductor Lead-Covered A-C. Cables, *Schurig, O. R., Kuehni, H. P., and Buller, F. H.* (April) 417
Progress in High-Tension Underground Cable Research and Development, *Shanklin, G. B., and Mackay, G. M. J.* (April) 338
Reduction of Sheath Losses in Single-Conductor Cables, *Halperin, Herman, and Miller, K. W.* (April) 399
Some Problems in High-Voltage Cable Development, *Davis, E. W., and Eddy, W. N.* (April) 373

CENTRAL STATION SYSTEMS

Automatic Stations Committee Report (October) 1322
Economies in Central Power Service as Illustrated by the Duke Power System, *Lee, W. S.* (January) 203
Power Limit Tests on Southeastern Power and Light Company's System, *Jones, S. Murray, and Treat, Robert* (January) 268
Southeastern Power & Light System, *Hutchins, A. T.* (January) 206
System Stability as a Design Problem, *Park, R. H. and Bancker, E. H.* (January) 170

CIRCUIT BREAKERS AND SWITCHES

Automatic Reclosing High-Speed Circuit Breaker Feeder Equipment for D-C. Railway Service, *Anderson, A. E.* (April) 554
Automatic Reclosing of High-Voltage Circuits, *Robinson, E. W., and Spurgeon, S. J.* (July) 981
Field Tests of the Deion Circuit Breaker, *Jamieson, B. G.* (April) 535
High-Voltage Low-Current Fuses and Switches, *Wilkins, Roy*, September JOURNAL 692
Structural Development of the Deion Circuit Breaker up to 15,000 Volts, The, *Dickinson, R. C. and Baker, B. P.* (April) 528
Theory of the Deion Circuit Breaker, *Slepian, J.* (April) 523

COMMUNICATION

See also Telephone and Radio Phenomena

Carrier-Current and Supervisory Control on Alabama Power Company System, *Woodcock, W. I., and Robinson, E. W.* (January) 214
Carrier Telephone Systems for Short Toll Circuits, *Black, H. S., Almquist, M. L., and Ilgenfritz, L. M.* (January) 117
Electrical Communication Committee Report (October) 1329
Meeting Long Distance Telephone Problems, *Fritz, H. R., and Lawther, H. P. Jr.* (July) 1024
Power-Line Carrier Telephony, *Fuller, L. F., and Tolson, W. A.* (January) 102
Problems in Power-Line Carrier Telephony, *Wolfe, W. V., and Sarros, J. D.* (January) 107
Telephone Circuits for Program Transmission, *Cowan, F. A.* (July) 1045
Telephone Transmission Networks, Types and Problems of Design, *Shea, T. E., and Lane, C. E.*, (July) 1031
Transmission of High-Frequency Currents for Communication over Existing Power Networks, The, *Boddie, C. A., and Curtis, R. C.* (January) 227

COMMUTATION

See Motors, Alternating Current

Influence of Temperature on Large Commutator Operation, *Hague, F. T., and Penney, G. W.* (April) 666

CONDENSERS

Outdoor Hydrogen-Ventilated Synchronous Condensers, *Wieseman, Robert W.* (October) 1221

CONDUCTORS

See Electric Circuit Theory

Forces on Magnetically Shielded Conductors, *Morecroft, J. H., and Turner, Alva*, January JOURNAL 25

CONTROL DEVICES

See also Industrial Applications of Power, Motors

Application of Induction Regulators to Distribution Networks, *Wolfert, E. R., and Brosnan, T. J.* (October) 1123
Carrier-Current and Supervisory Control on Alabama Power Company System, *Woodcock, W. I., and Robinson, E. W.* (January) 214
Power Supply for Railway Signals and Automatic Train Control, *King, C. F., Jr.* (January) 53
Telemetering, *Linder, C. H., Stewart, C. E., Rex, H. B., and Fitzgerald, A. S.* (July) 766

CORONA

See also Dielectric Phenomena

Corona Ellipses, *Karapetoff, Vladimir* (April) 317

DIELECTRIC PHENOMENA

See also Corona

Anomalous Conduction as a Cause of Dielectric Absorption, *Whitehead, J. B., and Marvin, R. H.* (April) 299
Ionization Studies in Paper-Insulated Cables, II, *Dawes, C. L., Reichard, H. H., and Humphries, P. H.* (April) 382
Power Factor and Dielectric Constant in Viscous Dielectrics, *Kitchin, Donald W.* (April) 495
Predominating Influence of Moisture and Electrolytic Material upon Textiles as Insulators, The, *Williams, R. R., and Murphy, E. J.* (April) 568
Purified Textile Insulation for Telephone Central Office Wiring, *Glenn, H. H., and Wood, E. B.* (April) 576
Some Problems in Dielectric-Loss Measurements, *Dawes, C. L., Hoover, P. L., and Reichard, H. H.* (October) 1271

DISTRIBUTION

See also Substations, Transmission Lines

Application of Induction Regulators to Distribution Networks, *Wolfert, E. R., and Brosnan, T. J.* (October) 1123
Automatic Substation and Its Relation to the Electric Distribution System, The, *Lisberger, S. J.* (January) 19
Automatic Transformer Substations of the Edison Illuminating Co. of Boston, *Edson, William W.* (October) 1108
Bare Wire Overhead Distribution Practise, *Miller, M. C.* (July) 988
Developments in Network Systems and Equipment, *Brosnan, T. J., and Kelly, Ralph* (July) 966
Operating Experience with the Low-Voltage A-C. Network in Cincinnati, *Pinckard, F. E.* (July) 896
Standard Voltage A-C. Network, *Oram, John* (July) 977
Transmission and Distribution Committee Report (October) 1439

EDUCATION

Education Committee Report (October) 1334

ELECTRIC CIRCUIT THEORY

See also Power Factor, Transients

Calculation of the Capacitance between Two Wires of a Three-Conductor Cable, *Lee, Y. W.* (January) 195
Flux Linkages and Electromagnetic Induction in Closed Circuits, *Bewley, L. V.* (April) 327
Graphical Theory of Traveling Electric Waves between Parallel Conductors, A, *Karapetoff, V.* (April) 508
Residual Voltages and Currents in Power Systems, *Corbett, L. J.* (January) 92
Short-Circuit Torque in Synchronous Machines without Damper Windings, *Penney, G. W.* (October) 1230
System Stability as a Design Problem, *Park, R. H., and Bancker, E. H.* (January) 170

ELECTROCHEMISTRY

Electrochemistry and Electrometallurgy Committee Report (October) 1359

ELECTROPHYSICS

Electrophysics Committee Report (October) 1363

GENERATORS, ALTERNATING

See also Loss and Efficiency Measurements

Iron Losses in Turbine Generators, *Laffoon, C. M., and Calvert, J. F.*, (July) 856
Separation of Stray Load Losses in A-C. Generators, *Holmes, M. C.*, March JOURNAL 224

HEATING

Electric Heating Elements, *Fleischmann, Edwin* (October) 1196
Heat Flow from Underground Electric Power Cables, *Bailey, Neil P.* (January) 156

INDUSTRIAL APPLICATIONS OF POWER

See also Control Devices, Lamps, Mining Applications

A-C. Elevator Motors of the Squirrel-Cage Type (Abridged), *Dreese, E. E.*, January JOURNAL 32
Electrical Equipment of Bar Plate and Hot Strip Mills, *Ink, J. B.* (July) 878
Electrical Features of the Kansas City New Water Works *Maillard, Albert L.* (July) 1020
Electrification of Oil Pipe Lines in the Southwest, *Levy, D. H.* (July) 995
Electrification of the Textile Industry in the Southeast, *Clapp, Edwin M., and Stanford, A. G.* (January) 261
Electrolytic Zinc Plant of the Sullivan Mining Company, The, *Fosdick, Ellery R.* (January) 33
General Power Applications Committee Report (October) 1373
High-Frequency Portable Tools and Equipment, *Coates, C. B.* (October) 1189
Motor Drives for Cards and Roving Frames, (Abridged), *Reding, H. W.*, December 1928 JOURNAL 880
Variable Speed Spinning of Cotton Yarns, *Untersee, E. A.*, November 1928 JOURNAL 821

INSULATION

See also Dielectric Phenomena

Insulation Tests of Electrical Machinery before and after Being Placed in Service, *Gilt, C. M., and Barns, B. L.* (April) 656
Predominating Influence of Moisture and Electrolytic Material upon Textiles as Insulators, The, *Williams, R. R., and Murphy, E. J.* (April) 568
Purified Textile Insulation for Telephone Central Office Wiring, *Glenn, H. H., and Wood, E. B.* (April) 576

INSULATORS

See Transmission Lines (General)

Fused Arcing Horns and Grading Rings, *Stewart, P.* (July) 891
Impulse Flashover of Insulators, *Torok, J. J., and Ramberg, W.* (January) 239
Radio Interference from Line Insulators, *Van Atta, E., and White, E. L.*, September JOURNAL 682
Study of High-Voltage Flashovers, A, *Lusignan, Joseph T., Jr.* (January) 246

INTERCONNECTED SYSTEMS

See Central Station Systems

Interconnection in the Southwest, *Mills, Geo. A.* (July) .. 991

LAMPS

See also Lighting

High-Voltage Gaseous-Conductor Lamps, *McMillan, F. O., and Starr, E. C.* (January) 11

LIGHTING

Flying Field and Airway Lighting, *Ogden, H. R.* (July) ... 1013
High-Voltage Gaseous-Conductor Lamps, *McMillan, F. O., and Starr, E. C.* (January) 11
Lighting of Airways and Airports, *Mahan, H. E.* (July) .. 849
Production and Application of Light Committee Report (October) 1394
Some Photoelectric and Glow-Discharge Devices, *Breisky, J. V., and Erickson, E. O.* (January) 283

LIGHTNING ARRESTERS

High-Voltage Phenomena in Thunderstorms, *Lissman, Marcel A.* (January) 146
Lightning, *Peek, F. W., Jr.* (April) 436
Lightning Arrester Problems, *Atherton, A. L.* (January) .. 141
1927 Lightning Experience on the 132-Kv. Transmission Lines of the American Gas & Electric Company, *Sporn, Philip* (April) 480
Theoretical and Field Investigations of Lightning, *Fortescue, C. L., Atherton, A. L., and Cox, J. H.* (April) 449
Traveling Waves Due to Lightning, *Bewley, L. V.* (July) .. 1050

LOSS AND EFFICIENCY MEASUREMENTS

Guarding and Shielding for Dielectric Loss Measurements on Short Lengths of High-Tension Power Cable, *Salter, E. H.* (October) 1294
Iron Losses in Turbine Generators, *Laffoon, C. M., and Calvert, J. F.* (July) 856
Separation of Stray Load Losses in A-C. Generators, *Holmes, M. C.*, March JOURNAL 224
Some Problems in Dielectric-Loss Measurements, *Dawes, C. L., Hoover, P. L., and Reichard, H. H.* (October) 1271
Study of Noises in Electrical Apparatus, *Spooner, T., and Foltz, J. P.*, (July) 747

MAGNETIC PHENOMENA

Analytical Determination of Magnetic Fields, *Robertson, B. L., and Terry, J. A.* (October) 1242
Forces on Magnetically Shielded Conductors, *Morecroft, J. H., and Turner, Alva*, January JOURNAL 25
Magnetic Analysis, *Sanford, Raymond L.* (July) 731
Magnetic Shielding, *Gokhale, S. L.* (October) 1307
Precautions against Stray Magnetic Fields in Measurements with Large Alternating Currents, *Silsbee, Francis B.* (October) 1301

MANUFACTURING METHODS

Fabrication of Large Rotating Machinery, The, *Putman, H. V.* (July) 873

MARINE APPLICATIONS

Applications to Marine Work Committee Report (October) 1409
Electrical Aids to Navigation, *Marriott, Robert H.* (July) 753

MEASURING INSTRUMENTS

See also Loss and Efficiency Measurements, Measuring Methods

Instruments and Measurements Committee Report (October) 1385
Interconnected Integraph, The, *Glover, R. E., and Plumb, H. H.*, September JOURNAL 695
New Type of Hot Cathode Oscillograph, A, *George, R. H.* (July) 884
132-Kv. Shielded Potentiometer for Determining the Accuracy of Potential Transformers, *Weller, C. T.* (July) 790
Speed Indicator and Frequency Meter, *Greibach, E. H.*, August JOURNAL 633

MEASURING METHODS

See also Loss and Efficiency Measurements, Measuring Instruments

Bushing-Type Current Transformers for Metering, *Boyajian, A., and Skeats, W. F.* (July) 949
Cathode Ray Oscillograph Study of Artificial Lightning Surges on the Turners Falls Transmission Line, *McEachron, K. B., and Goodwin, V. E.* (July) 953
Electrical Instruments Used in the Measurement of Flow, *Pratt, W. H.* (July) 737
Graphic Solution of A-C. Transmission Line Problems, The, *Denton, F. M.*, January JOURNAL 49
Guarding and Shielding for Dielectric Loss Measurements on Short Lengths of High-Tension Power Cable, *Salter, E. H.* (October) 1294
Lightning Studies of Transformers by the Cathode Ray Oscillograph, *Brand, F. F., and Palueff, K. K.* (July) 998
Magnetic Shielding, *Gokhale, S. L.* (October) 1307
Oscillographs for Recording Transient Phenomena, *Marrison, W. A.* (July) 939
Precautions against Stray Magnetic Fields in Measurements with Large Alternating Currents, *Silsbee, Francis B.* (October) 1301
Quick-Response Generator Voltage Regulator Field Tests Made with Oscillograph, *Burnham, E. J., North, J. R., and Dohr, I. R.* (July) 903
Shielding and Guarding Electrical Apparatus Used in Measurements—General Principles, *Curtis, Harvey, L.* (October) 1263
Shielding in High-Frequency Measurements, *Ferguson, John G.* (October) 1286
Sphere-Gap and Point-Gap Arc-Over Voltage, *Carroll, Joseph S., and Cozzens, Bradley* (January) 1
Totalizing of Electric System Loads, *Lincoln P. M.* (July) 775
Use of the Oscillograph for Measuring Non-Electrical Quantities, *Miner, D. F., and Batten, W. B.* (July) 742

MINING APPLICATIONS

Applications to Mining Work Committee Report (October) 1410
Electrolytic Zinc Plant of the Sullivan Mining Company, The, *Fosdick, Ellery R.* (January) 33

MOTORS, ALTERNATING CURRENT

See also Control Devices, Industrial Applications of Power, Loss and Efficiency Measurements, Transients

A-C. Elevator Motors of the Squirrel-Cage Type (Abridged), *Dreese, E. E.*, January JOURNAL 32
Condenser Motor, The, *Bailey, B. F.* (April) 596
Fundamental Theory of the Capacitor Motor, The, *Specht, H. C.* (April) 607
Induction Motor Operation with Non-Sinusoidal Impressed Voltages, *Doggett, L. A., and Queer, E. R.* (October) 1217
Line-Start Induction Motors, *Koch, C. J.* (April) 633
No-Load Induction Motor Core Losses, *Spooner, I., and Kincaid, C. W.* (April) 645
Revolving Field Theory of the Capacitor Motor, The, *Morrill, Wayne J.* (April) 614

POWER FACTOR

Power Factor and Dielectric Constant in Viscous Dielectrics, *Kitchin, Donald W.* (April) 495

POWER STATIONS

See Central Stations Systems, Power Factor

Calculations of System Performance, *Griscom, S. B.* (October) 1083
Carrier-Current and Supervisory Control on Alabama Power Company System, *Woodcock, W. I., and Robinson, E. W.* (January) 214
Economies in Central Power Service as Illustrated by the Duke Power System, *Lee, W. S.* (January) 203
Fundamental Plan of Power Supply, A, *Kehoe, A. H.* (October) 1080
Power Generation Committee Report (October) 1413
Power Limit Tests on Southeastern Power and Light Company's System, *Jones, S. Murray, and Treat, Robert* (January) 268
Rehabilitation of Steam Power Plants, *Hirshfeld, C. F.* (October) 1114
Southeastern Power & Light System, *Hutchins, A. T.* (January) 206
System Tests and Operating Connections, *Searing, H. R. and Milne, G. R.* (October) 1093
Transmission of High-Frequency Currents for Communication over Existing Networks, The, *Boddie, C. A., and Curtis, R. C.* (January) 1

PROTECTIVE DEVICES

See also Lightning Arresters

Protective Devices Committee Report (October) 1448

RADIO PHENOMENA

Radio Acoustic Position Finding in Hydrography, *Service, Jerry H.* (January) 198
Radio Inteference from Line Insulators, *Van Atta, E., and White, E. L.*, September JOURNAL 682
Uses of Radio as an Aid to Air Navigation, *Dellinger, J. H.* (April) 563
Vector Presentation of Broad-Band Wave Filters, *Mallina, R. F., and Knackmuss, O.* (April) 582

RAILWAYS

Automatic Reclosing High-Speed Circuit Breaker Feeder Equipment for D-C. Railway Service, *Anderson, A. E.* (April) 554
Contact Wire Wear on Electric Railroads, *Landhy, I. T* (October) 1142
D-C Railway Substations for the Chicago Terminal Electrification, *Garrett, A. M.* (October) 1150
Electrification of the Mexican Railway, The, *Cox, J. B.* (October) 1130
Electrified Railway Substation of the Pennsylvania Railroad, An, *Duer, J. V. B.* (October) 1147
Great Northern Railway Electrification in the Cascades, *Moreland, Edward L., and Booth, Ralph D.* (January) 40
Power Supply for Railway Signals and Automatic Train Control, *King, C. F., Jr.* (January) 53
Railway Train Signal Practise, *Gault, P. M.*, June JOURNAL 457

Street Railway Power Economics on the Cincinnati System, *Noertker, J. A.* (July) 912
Transportation Committee Report (October) 1457

RECTIFIERS

Automatic Mercury Arc Power Rectifier, *Turley, L. J.* (January) 58
Automatic Mercury Rectifier Substations in Chicago, *Garrett, A. M.* (July) 925

RESEARCH

Research Committee Report (October) 1453

SUBSTATIONS

Automatic Mercury Rectifier Substations in Chicago, *Garrett, A. M.* (July) 925
Automatic Substation and Its Relation to the Electric Distribution System, The, *Lisberger, S. J.* (January) 19
Automatic Transformer Substations of the Edison Illuminating Co. of Boston, *Edson, William W.* (October) 1108
D-C. Railway Substations for the Chicago Terminal Electrification, *Garrett, A. M.* (October) 1150
Electrified Railway Substation of the Pennsylvania Railroad, An, *Duer, J. V. B.* (October) 1147

SYNCHRONIZING EQUIPMENT

Calculations of System Performance, *Griscom, S. B.* (October) 1083
Fundamental Plan of Power Supply, A, *Kehoe, A. H.* (October) 1080
New Automatic Synchronizer, A, *Gulliksen, F. H.* (October) 1178
"Synchronized at the Load," A Symposium, *Kehoe, A. H., Griscom, S. B., Searing, H. R., and Milne, G. R.* (October) 1079
System Tests and Operating Connections, *Searing, H. R. and Milne, G. R* (October) 1093
Two-Reaction Theory of Synchronous Machines, I, *Park, R. H.* (July) 716

TECHNICAL COMMITTEE REPORTS

Applications to Marine Work Committee Report (October) 1409
Applications to Mining Work Committee Report (October) 1410
Automatic Stations Committee Report (October) 1322
Education Committee Report (October) 1334
Electric Welding Committee Report (October) 1354
Electrical Communication Committee Report (October) 1329
Electrical Machinery Committee Report (October) 1336
Electrochemistry and Electrometallurgy Committee Report (October) 1359
Electrophysics Committee Report (October) 1363
General Power Applications Committee Report (October) 1373
Instruments and Measurements Committee Report (October) 1385
Iron and Steel Production Committee Report (October) 1392
Power Generation Committee Report (October) 1413
Production and Application of Light Committee Report (October) 1394
Protective Devices Committee Report (October) 1448
Research Committee Report (October) 1453
Transmission and Distribution Committee Report (April) 487, (October) 1439
Transportation Committee Report (October) 1457

TELEPHONE

Carrier-Current and Supervisory Control on Alabama Power Company System, *Woodcock, W. I., and Robinson, E. W.* (January) 214
Carrier Telephone Systems for Short Toll Circuits, *Black, H. S., Almquist, M. L., and Ilgenfritz, L. M.* (January) 117
Decibel, The Name for the Transmission Unit, *Martin, W. H.*, March JOURNAL 223
Electrical Wave Analyzers for Power and Telephone Systems, *McCurdy, R. G., and Blye, P. W.* (October) 1167
Master Reference System for Telephone Transmission, *Martin, W. H., and Gray, C. H. G.* (October) 1157
Meeting Long Distance Telephone Problems, *Fritz, H. R. and Lawther, H. P., Jr.* (July) 1024
Power-Line Carrier Telephony, *Fuller, L. F., and Tolson, W. A.* (January) 102
Precision Regulator for Alternating Voltage, A., *Stoller, H. M., and Power, J. R.* (July) 808
Problems in Power-Line Carrier Telephony, *Wolfe, W. V. and Sarros, J. D.* (January) 107
Purified Textile Insulation for Telephone Central Office Wiring, *Glenn, H. H., and Wood, E. B.* (April) 576
Recent Developments in Telephone Construction Practises, *Wagner, B. S., and Burroway, A. C.* (July) 836
Telephone Circuits for Program Transmission, *Cowan, F. A.* (July) 1045
Telephone Transmission Networks, Types and Problems of Design, *Shea, T. E., and Lane, C. E.* (July) 1031
Transmission of High-Frequency Currents for Communication over Existing Power Networks, The, *Boddie, C. A., and Curtis, R. C.* (January) 227

TRANSFORMERS

Bushing-Type Current Transformers for Metering, *Boyajian, A. and Skeats, W. F.* (July) 949
Current Transformer Excitation under Transient Conditions, *Marshall, D. E., and Langguth, P. O.* (October) 1464
Effect of Transient Voltages on Power Transformer Design, *Palueff, K. K.* (July) 681
Lightning Studies of Transformers by the Cathode Ray Oscillograph, *Brand, F. F., and Palueff, K. K.* (July) 998
New High Accuracy Current Transformer, A, *Wilson, M. S.* (July) 783
132-Kv. Shielded Potentiometer for Determining the Accuracy of Potential Transformers, *Weller, C. T.* (July) 790
Safe Loading of Oil-Immersed Transformers, *Norris, E. T.* (October) 1206
Some Notes on Transformer Design, *Waldo, E. H.* (July) 821

TRANSIENTS

See also Electric Circuit Theory

Cathode Ray Oscillograph Study of Artificial Lightning Surges on the Turners Falls Transmission Line, *McEachron, K. B., and Goodwin, V. E.* (July) 953
Current Transformer Excitation under Transient Conditions, *Marshall, D. E., and Langguth, P. O.* (October) 1464
Effect of Transient Voltages on Power Transformer Design, *Palueff, K. K.* (July) 681
Fused Arcing Horns and Grading Rings, *Stewart, Philip*, (July) 891
High-Voltage Phenomena in Thunderstorms, *Lissman, Marcel, A.* (January) 146
Lightning, *Peek, F. W., Jr.* (April) 436
Lightning Arrester Problems, *Atherton, A. L.* (January) 141
Lightning Studies of Transformers by the Cathode Ray Oscillograph, *Brand, F. F., and Palueff, K. K.* (July) 998
1927 Lightning Experience on the 132-Kv. Transmission Lines of the American Gas & Electric Company, *Sporn, Philip* (April) 480
Oscillographs for Recording Transient Phenomena, *Marrison, W. A.* (July) 939
Theoretical and Field Investigations of Lightning, *Fortescue, C. L., Atherton, A. L., and Cox, J. H.* (April) 449
Transient Analysis of A-C. Machinery, *Ku, Yu H.* (July) 707
Traveling Waves Due to Lightning, *Bewley, L. V.* (July) 1050

TRANSMISSION LINES

(Electrical Characteristics)

Bessel Functions for A-C. Problems, *Dwight, Herbert Bristol* (July) 812
Cathode Ray Oscillograph Study of Artificial Lightning Surges on the Turners Falls Transmission Line, *McEachron, K. B., and Goodwin, V. E.* (July) 953
Graphic Solution of A-C. Transmission Line Problems, The, *Denton, F. M.*, January JOURNAL 49
System Stability as a Design Problem, *Park, R. H., and Bancker, E. H.* (January) 170

TRANSMISSION LINES

(General)

Transmission and Distribution Committee Report (April) 487, (October) 1439

TRANSMISSION LINES

(Mechanical Features)

Movements of Overhead Line Conductors during Short Circuits, *Peterson, Wm. S., and McCracken, H. J., Jr.* (January) 67

WELDING

Arc Welding of Steel Buildings and Bridges, *McKibben, Frank P.* (July) 929
Electric Welding Committee Report (October) 1354

INDEX OF AUTHORS

A

Adams, C. A., *Discussion* (April) 642, 655
Adler, L. E., *Discussion* (July) 1030
Alger, P. L., *Discussion* (April) 630, 641, 654, 663
Allner, F. A., *Committee Report* (October) 1413
Almquist, M. L., L. M. Ilgenfritz, and H. S. Black, *Paper* (January) 117
Almquist, M. L., *Discussion* (January) 139
Ambelang, C. E., *Discussion* (April) 471, 493
Anderson, A. E., *Paper* (April) 554
Anderson, A. E., *Discussion* (April) 562
Anderson, C. S., *Discussion* (October) 1145
Anderson, L. E., *Discussion* (January) 236
Antoniono, Caesar, *Discussion* (January) 32
Atherton, A. L., *Paper* (January) 141
Atherton, A. L., J. H. Cox, and C. L. Fortescue, *Paper* (April) 449
Atherton, A. L., *Discussion* (January) 145
Atkinson, R. W., *Discussion* (April) 315, 370

B

Bailey, B. F., *Paper* (April) 596
Bailey, B. F., *Discussion* (April) 632
Bailey, J. C., *Discussion* (July) 924
Bailey, Neil P., *Paper* (January) 156
Bailey, Neil P., *Discussion* (January) 164
Baker, B. P. and R. C. Dickinson, *Paper* (April) 528
Baker, B. P., *Discussion* (April) 552
Baker, C. E., *Discussion* (January) 65
Ball, T. F., *Discussion* (January) 254
Bancker, E. H. and R. H. Park, *Paper* (January) 170
Bancker, E. H., *Discussion* (January) 194
Barns, B. L. and C. M. Gilt, *Paper* (April) 656
Barns, B. L., *Discussion* (April) 664
Baston, C. E., *Discussion* (January) 50
Bates, J. W., *Discussion* (July) 883
Batten, W. B. and D. F. Miner, *Paper* (July) 742
Batten, W. B., *Discussion* (July) 746
Bauer, P. S., *Discussion* (October) 1292
Beaver, J. L., *Discussion* (July) 871
Beck, Edward, *Discussion* (April) 472, July 964, 1009
Behrend, B. A., *Discussion* (October) 1227, 1241
Bennett, Edward, *Committee Report* (October) 1334
Bewley, L. V., *Paper* (April) 327, (July) 1050
Bewley, L. V., *Discussion* (April) 337
Black, H. S., M. L. Almquist, and L. M. Ilgenfritz, *Paper* (January) 117
Blye, P. W. and R. G. McCurdy, *Paper* (October) 1167
Blye, P. W., *Discussion* (October) 1176
Boddie, C. A. and R. C. Curtis, *Paper* (January) 227
Boddie, C. A., *Discussion* (January) 114, 238
Boeck, C. F., *Discussion* (January) 236
Booth, Ralph D. and Edward L. Moreland, *Paper* (January) 40
Boothby, C. R., *Discussion* (April) 629
Borden, Perry A., *Discussion* (July) 746, 765
Bosch, L. I., *Discussion* (July) 877, 901
Boyajian, A. and W. F. Skeats, *Paper* (July) 949
Boyajian, A., *Discussion* (July) 951
Brand, F. F. and K. K. Palueff, *Paper* (July) 998
Brand, F. F., *Discussion* (July) 701, (October) 1212
Breisky, J. V. and E. O. Erickson, *Paper* (January) 283
Brooks, H. B., *Discussion* (July) 781
Brosnan, T. J. and Ralph Kelly, *Paper* (July) 966
Brosnan, T. J. and E. R. Wolfert, *Paper* (October) 1123
Brosnan, T. J., *Discussion* (July) 976
Brown, C. G., *Discussion* (July) 782
Brown, H. W., *Discussion* (October) 1228
Brownell, F. A., *Discussion* (April) 367
Buck, N. E., *Discussion* (April) 371
Buller, F. H., O. R. Schurig, and H. P. Kuehni, *Paper* (April) 417
Burnham, E. J., J. R. North, and I. R. Dohr, *Paper* (July) 903
Burnham, E. J., *Discussion* (July) 911
Burns, R. M. and B. A. Freed, *Paper* (January) 166
Burns, R. M. and B. A. Freed, *Discussion* (January) 169
Burroway, A. C. and B. S. Wagner, *Paper* (July) 836
Byles, F. A., *Discussion* (July) 811

C

Caldwell, F. C., *Discussion* (July) 854, 938, (October) 1334, 1358, 1408, 1456
Calvert, J. F. and C. M. Laffoon, *Paper* (July) 856
Calvert, J. F., *Discussion* (July) 871
Campbell, I. S., *Discussion* (July) 938
Candy, A. M., *Committee Report* (October) 1354
Carey, C. E., *Discussion* (January) 30, 51, 57, 144
Carothers, R. M., *Discussion* (July) 909
Carpenter, H. V., *Discussion* (January) 9
Carroll, Joseph S. and Bradley Cozzens, *Paper* (January) 1
Carroll, J. S., *Discussion* (January) 10, 17, 154
Chase, P. H., *Discussion* (April) 545
Chilberg, E. E., *Discussion* (October) 1101
Chubbuck, L. B., *Discussion* (April) 550
Cirella, L. E., *Discussion* (October) 1280
Clapp, Edwin M. and A. G. Stanford, *Paper* (January) 261
Clark, F. M., *Discussion* (April) 380, 504
Clark, J. C., *Discussion* (October) 1391
Clark, N. C., *Discussion* (January) 17, 99
Clark, O. S., *Discussion* (July) 894
Coates, C. B., *Paper* (October) 1189
Coates, C. B., *Discussion* (October) 1195
Cohen, Louis, *Discussion* (July) 714
Coleman, J. O., *Discussion* (January) 100, (October) 1175
Cone, D. I., *Discussion* (January) 17, 99
Cooper, A. B., *Discussion* (October) 1215
Copley, A. W., *Discussion* (January) 99
Corbett, L. J., *Paper* (January) 92
Corbett, L. J., *Discussion* (January) 9, 57, 91, 100, (October) 1175
Corney, C. A., *Discussion* (April) 549
Covey, S. B., *Discussion* (July) 1044
Cowan, F. A., *Paper* (July) 1045
Cox, J. B., *Paper* (October) 1130
Cox, J. B., *Discussion* (October) 1140, 1146
Cox, J. H., C. L. Fortescue, and A. L. Atherton, *Paper* (April) 449
Cox, J. H., *Discussion* (January) 255, (April) 469, 477, 492, (July) 1009
Cozzens, Bradley and Joseph S. Carroll, *Paper* (January) 1
Cozzens, Bradley, *Discussion* (January) 144, 154
Crawford, M. T., *Discussion* (January) 30, (April) 367, (October) 1103
Creager, P. S., *Discussion* (April) 414, 632
Cross, H. C., *Discussion* (October) 1149
Curtis, Harvey L., *Paper* (October) 1263
Curtis, H. L., *Discussion* (October) 1270
Curtis, R. C. and C. A. Boddie, *Paper* (January) 227

D

Dagnall, C. H., *Discussion* (April) 595
Dann, W. M., *Discussion* (July) 704, (October) 1213
Davis, E. W. and W. N. Eddy, *Paper* (April) 373
Davis, E. W. and W. N. Eddy, *Discussion* (April) 368, 395, 434
Davis, E. W., *Discussion* (April) 380, (October) 1299
Dawes, C. L., P. L. Hoover, and H. H. Reichard, *Paper* (October) 1271
Dawes, C. L., H. H. Reichard, and P. H. Humphries, *Paper* (April) 382, 397
Dawes, C. L., *Discussion* (April) 675, (October) 1270, 1284, 1292, 1300
Dawes, C. L. and P. H. Humphries, *Discussion* (April) 369, 379
Dawson, W. F., *Discussion* (April) 663, (October) 1212
Dean, S. M., *Discussion* (April) 546
De Camp, S. M. *Discussion* (July) 990
deForest, A. V., *Discussion* (July) 736, 746
Dellinger, J. H., *Paper* (April) 563
Del Mar, Wm. A., *Discussion* (April) 371, 380, (October) 1446
Denton, F. M., *Paper*, January JOURNAL 49
D'Eu Stachio, G., *Discussion* (January) 169
Diamond, Harry, *Discussion* (April) 567
Dickinson, R. C. and B. P. Baker, *Paper* (April) 528
Doggett, L. A. and E. R. Queer, *Paper* (October) 1217
Doggett, L. A., *Discussion* (July) 871, (October) 1220, 1227
Dohr, I. R., E. J. Burnham, and J. R. North, *Paper* (July) 903
Don Carlos, H. C., *Discussion* (July) 894
Dovjikov, A., *Discussion* (January) 281
Drake, H. W., *Committee Report* (October) 1329

Drake, H. W., *Discussion* (April) 567
Dreese, E. E., *Paper*, January JOURNAL 32
Dreyer, W. C., *Discussion* (July) 997
Duer, J. V. B., *Paper* (October) 1147
Dunbar, J. R., *Discussion* (April) 661
Duryea, L. M., *Discussion* (October) 1408
Dwight, Herbert Bristol, *Paper* (July) 812

E

Easton, R. M., *Discussion* (July) 938
Eaton, J. R., *Discussion* (April) 472
Eddy, W. N. and E. W. Davis, *Paper* (April) 373
Eddy, W. N. and E. W. Davis, *Discussion* (April) 368, 395, 434
Edson, William W., *Paper* (October) 1108
Edson, W. W., *Discussion* (October) 1113
Elden, L. L., *Discussion* (April) 642
Erickson, E. O. and J. V. Breisky, *Paper* (January) 283
Eshelby, A. W., *Discussion* (January) 52
Evans, R. D., *Discussion* (October) 1104
Everitt, W. L., *Discussion* (July) 890, 965

F

Fechheimer, C. J., *Discussion* (October) 1226, 1240, 1352
Ferguson, John G., *Paper* (October) 1286
Ferguson, J. G., *Discussion* (October) 1292
Ficklenburg, Charles, *Discussion* (October) 1335
Field, A. B., *Discussion* (October) 1241
Fields, E. S., *Discussion* (October) 1103, 1120
Fitzgerald, A. S., C. H. Linder, C. E. Stewart, and H. B. Rex, *Paper* (July) 766
Fleischmann, Edwin, *Paper* (October) 1196
Fleischmann, Edwin, *Discussion* (October) 1205, 1362
Floyd, G. D., *Discussion* (April) 470, (July) 746
Foltz, J. P. and T. Spooner, *Paper* (July) 747
Foltz, J. P., *Discussion* (July) 752
Fondiller, William, *Discussion* (April) 581
Fortescue, C. L., A. L. Atherton, and J. H. Cox, *Paper* (April) 449
Fortescue, C. L., *Discussion* (April) 476, (July) 894, 964
Fosdick, Ellery R., *Paper* (January) 33
Foster, W. J., *Committee Report* (October) 1336
Foster, W. J., *Discussion* (October) 1121, 1352
Fowler, M. M., *Committee Report* (October) 1392
Fox, J. W., *Discussion* (January) 266
Franklin, C. W., *Discussion* (April) 642
Franklin, E. F., *Discussion* (October) 1240
Freed, B. A. and R. M. Burns, *Paper* (January) 166
Freed, B. A. and R. M. Burns, *Discussion* (January) 169
Fritz, H. R. and H. P. Lawther, Jr., *Paper* (July) 1024
Fritz, H. R., *Discussion* (July) 1030
Fuller, L. F. and W. A. Tolson, *Paper* (January) 102
Fuller, L. F., *Discussion* (January) 115

G

Gamble, L. R., *Discussion* (January) 31, 91
Gardner, M. F., *Discussion* (July) 714
Garrett, A. M., *Paper* (July) 925, (October) 1150
Garrett, A. M., *Discussion* (July) 927, (October) 1156
Garrett, P. B., *Discussion* (January) 31
Gaskill, J. F., *Committee Report*, (October) 1373
Gáti, Béla, *Discussion* (April) 581, (July) 745, 781
Gault, P. M., *Paper*, June JOURNAL 457
Gay, F. W., *Discussion* (October) 1446
George, R. H., *Paper* (July) 884
George, R. H., *Discussion* (July) 890
Gibbs, J. B., *Discussion* (July) 788
Gilt, C. M. and B. L. Barns, *Paper* (April) 656
Gilt, C. M., *Discussion* (April) 664, (October) 1105, 1228
Glenn, H. H. and E. B. Wood, *Paper* (April) 576
Glover, R. E. and H. H. Plumb, *Paper*, September JOURNAL 695
Gokhale, S. L., *Paper* (October) 1307
Gokhale, S. L., *Discussion* (October) 1305, 1320
Goodwin, V. E. and K. B. McEachron, *Paper* (July) 953
Graf, F. G., *Discussion* (July) 746
Graves, H. C., Jr., *Discussion* (July) 901
Gray, C. H. G. and W. H. Martin, *Paper* (October) 1157
Greibach, E. H., *Paper*, August JOURNAL 633
Griscom, S. B., *Paper* (October) 1083
Griscom, S. B., *Discussion* (October) 1106
Gullkisen, F. H., *Paper* (October) 1178

H

Hague, F. T. and G. W. Penney, *Paper* (April) 666
Halperin, Herman and K. W. Miller, *Paper* (April) 399
Halperin, Herman, *Discussion* (April) 315, 368, 470
Halperin, Herman and K. W. Miller, *Discussion* (April) 415
Hamill, S. M., Jr., *Discussion* (July) 894, 938, 964
Hand, A. R., *Discussion* (July) 802
Hanker, F. C., *Discussion* (April) 548, (July) 909, (October) 1105
Harding, C. F., *Discussion* (April) 470
Harding, W. T., *Discussion* (July) 855
Harrell, F. E., *Discussion* (April) 641
Heinz, W. B., *Discussion* (July) 728
Hellenthal, J., *Discussion* (January) 31
Hemphill, L. F. and E. G. Michelsen, *Discussion* (April) 630
Henderson, E. W., *Discussion* (October) 1220
Henderson, J. S., *Discussion* (April) 661
Hester, E. A., *Committee Report* (October) 1448
Hickernell, L. F., *Discussion* (July) 704
Hill, C. F., *Discussion* (April) 316, 506
Hilliard, J. D., *Discussion* (April) 548
Hirshfeld, C. F., *Paper* (October) 1114
Holmes, M. C., *Paper*, March JOURNAL 224
Hoover, P. L., H. H. Reichard, and C. L. Dawes, *Paper* (October) 1271
Huggins, L. G., *Discussion* (January) 236
Humphries, P. H., H. H. Reichard, and C. L. Dawes, *Paper* (April) 382
Humphries, P. H., *Discussion* (October) 1281
Humphries, P. H. and C. L. Dawes, *Discussion* (April) 369, 379
Humphries, P. H., H. H. Reichard, and C. L. Dawes, *Discussion* (April) 397
Hunt, L. F., *Discussion* (January) 91, 193
Hutchins, A. T., *Paper* (January) 206, 212, 213

I

Ilgenfritz, L. M., H. S. Black, and M. L. Almquist, *Paper* (January) 117
Ilgenfritz, L. M., *Discussion* (January) 113
Ink, J. B., *Paper* (July) 878
Ink, J. B., *Discussion* (July) 883

J

Jackson, D. C., *Discussion* (October) 1139, 1145, 1155
Jacoby, S. C., *Discussion* (July) 773
Jamieson, B. G., *Paper* (April) 535
Jacques, C. A., *Discussion* (July) 848
Johnson, E. E., *Discussion* (April) 337
Johnson, Frank E., *Discussion* (July) 975
Johnson, J. H., *Discussion* (January) 169
Jollyman, J. P., *Discussion* (January) 113
Jones, B. W., *Discussion* (April) 632
Jones, S. Murray and Robert Treat, *Paper* (January) 268
Jones, S. M., *Discussion* (April) 493
Junke, P. B., *Discussion* (October) 1438

K

Karapetoff, V., *Committee Report* (October) 1363
Karapetoff, V., *Paper* (April) 317, 508
Kates, W. S. and E. D. Tanzer, *Discussion* (January) 258
Kehoe, A. H., *Paper* (October) 1080
Kehoe, A. H., *Discussion* (April) 550, (October) 1107
Kelly, Ralph and T. J. Brosnan, *Paper* (July) 966
Kennedy, G. F., *Discussion* (April) 505
Kilgore, L. A., *Discussion* (July) 871
Kincaid, C. W. and T. Spooner, *Paper* (April) 645
Kincaid, C. W., *Discussion* (April) 641, 655
King, C. F., Jr., *Paper* (January) 53
King, C. F., *Discussion* (January) 57
King, W. V., *Discussion* (April) 396
Kinnard, I. F., *Discussion* (July) 741, 788
Kistler, R. E., *Discussion* (January) 52, 100
Kitchin, Donald W., *Paper* (April) 495
Kitchin, D. W., *Discussion* (April) 371, 506
Knackmuss, O. and R. F. Mallina, *Paper* (April) 582
Koch, C. J., *Paper* (April) 633
Koch, C. J., *Discussion* (April) 643
Kouwenhoven, W. B., *Discussion* (April) 395, (July) 736, 746, 801, (October) 1269
Ku, Yu H., *Paper* (July) 707
Kuehni, H. P., F. H. Buller, and O. R. Schurig, *Paper* (April) 417

L

Laffoon, C. M. and J. F. Calvert, *Paper* (July) 856
Lamson, H. W., *Discussion* (October) 1281
Landhy, I. T., *Paper* (October) 1142

Landhy, I. T., *Discussion* (October) 1146
Lane, C. E. and T. E. Shea, *Paper* (July) 1031
Langguth, P. O. and D. E. Marshall, *Paper* (October) 1464
Langguth, P. O., *Discussion* (July) 951
LaPierre, C. W., *Discussion* (July) 805
Lawther, H. P., Jr. and H. R. Fritz, *Paper* (July) 1024
Lawton, F. L., *Discussion* (July) 781
Lebenbaum, Paul, *Discussion* (January) 50
Lee, A. W., Jr., *Discussion* (July) 854
Lee, Carl, *Committee Report*, (October) 1410
Lee, E. S., *Committee Report* (October) 1385
Lee, E. S., *Discussion* (April) 370, 471, 676, (July) 765, (October) 1391
Lee, W. S., *Paper* (January) 203
Lee, W. S., *Discussion* (January) 212, (October) 1121
Lee, Y. W., *Paper* (January) 195
Levy, D. H., *Paper* (July) 995
Levy, D. H., *Discussion* (July) 997
Levy, R. D., *Discussion* (January) 163
Lewis, H. C., *Discussion* (April) 414
Lichtenberg, Chester, *Discussion* (January) 31, 65, 562, (July) 773, (October) 1327
Lincoln, J. C., *Discussion* (April) 641, (October) 1357, 1362
Lincoln, J. F., *Discussion* (July) 877, 938
Lincoln, P. M., *Paper* (July) 775
Lincoln, P. M., *Discussion* (July) 782
Linder, C. H., C. E. Stewart, H. B. Rex, and A. S. Fitzgerald, *Paper* (July) 766
Lippelt, Hans, *Discussion* (October) 1319
Lisberger, S. J., *Paper* (January) 19
Lissman, Marcel A., *Paper* (January) 146
Lissman, M. A., *Discussion* (January) 154
Lloyd, M. G. and James S. Mahan, *Discussion* (April) 493
Loew, E. A., *Discussion* (January) 17
Longley, F. R., *Discussion* (January) 281
Louis, H. C., *Discussion* (April) 630
Lusignan, Joseph T., Jr., *Paper* (January) 246
Lusignan, J. T., Jr., *Discussion* (January) 255, 260
Lyon, W. J., *Discussion* (July) 727

M

MacCutcheon, A. M., *Discussion* (April) 642, 676, (October) 1194
MacDonald, M. M., *Discussion* (July) 938
Mackay, G. M. J. and G. B. Shanklin, *Paper* (April) 338
Mackay, G. M. J., *Discussion* (April) 314, 371
MacMillan, C., *Discussion* (July) 727
MacNeill, J. B., *Discussion* (April) 549
Mahan, H. E., *Paper* (July) 849
Mahan, H. E., *Discussion* (July) 855
Mahan, James S. and M. G. Lloyd, *Discussion* (April) 493
Mahood, E. T., *Discussion* (July) 1030
Maillard, Albert L., *Paper* (July) 1020
Mallina, R. F. and O. Knackmuss, *Paper* (April) 582
Manbeck, P. D., *Discussion* (July) 854
Marriott, Robert H., *Paper* (July) 753
Marrison, W. A., *Discussion* (July) 939
Marshall, D. E. and P. O. Langguth, *Paper* (October) 1464
Marshall, S. W., Jr., *Discussion* (July) 1008
Martin, W. H., March JOURNAL 223
Martin, W. H., and C. H. G. Gray, *Paper* (October) 1157
Marvin, R. H. and J. B. Whitehead, *Paper* (April) 299
Marvin, R. H., *Discussion* (April) 316
Mason, R. C., *Discussion* (April) 546
McAuley, P. H., *Discussion* (April) 469
McCoy, R. L., *Discussion* (July) 893
McCracken, H. J., Jr. and Wm. S. Peterson, *Paper* (January) 67
McCurdy, R. G. and P. W. Blye, *Paper* (October) 1167
McCurdy, R. G., *Discussion* (January) 237, (October) 1220
McEachron, K. B. and V. E. Goodwin, *Paper* (July) 953
McEachron, K. B., *Discussion* (April) 468, 549, (July) 965
McKenzie, D. A., *Discussion* (April) 662, (July) 781
McKibben, Frank P., *Paper* (July) 929
McKibben, F. P., *Discussion* (July) 938
McMillan, F. O. and E. C. Starr, *Paper* (January) 11
McMillan, F. O., *Discussion* (January) 17, 144
McNairy, J. W., *Discussion* (April) 547
Mead, C. A., *Discussion* (July) 746
Melvin, H. L., *Discussion* (April) 473
Mendenhall, H. E., *Discussion* (January) 153
Michelsen, E. G. and L. F. Hemphill, *Discussion* (April) 630
Mier, C. W., *Discussion* (July) 1029
Millan, W. H., *Committee Report* (October) 1322
Miller, K. W. and Herman Halperin, *Paper* (April) 399
Miller, K. W. and Herman Halperin, *Discussion* (April) 415
Miller, M. C., *Paper* (July) 988
Miller, M. C., *Discussion* (July) 990
Mills, Geo. A., *Paper* (July) 991
Mills, G. A., *Discussion* (July) 989, 994
Milne, G. R. and H. B. Searing, *Paper* (October) 1093
Milne, G. R., *Discussion* (October) 1100
Miner, D. F. and W. B. Batten, *Paper* (July) 742
Mitchell, W. E., *Discussion* (January) 212
Monteith, A. C., *Discussion* (July) 909
Montsinger, V. M., *Discussion* (April) 664, (July) 703, (October) 1213
Moore, A. D., *Discussion* (October) 1260
Morecroft, J. H. and Alva Turner, *Paper*, January JOURNAL 25
Moreland, Edward L. and Ralph D. Booth, *Paper* (January) 40
Moreland, E. L., *Discussion* (October) 1145
Morgan, H. R., *Discussion* (January) 51
Morgan, P. D. and S. Whitehead, *Discussion* (April) 414
Morrill, W. J., *Paper* (April) 614
Morrill, W. J., *Discussion* (April) 632
Morrow, L. W. W., *Discussion* (October) 1122
Mowbray, F. E. H., *Discussion* (April) 662
Murphy, E. J. and R. R. Williams, *Paper* (April) 568
Murphy, E. J., *Discussion* (April) 581

N

Naef, Otto, *Paper* (July) 910
Naef, Otto, *Discussion* (July) 927
Naudain, W. A., *Discussion* (April) 643
Neumann, L. J., *Discussion* (January) 114
Newbury, F. D., *Discussion* (April) 663, 676, (October) 1214, 1227
Noertker, J. A., *Paper* (July) 912
Noertker, J. A., *Discussion* (July) 924, 927
Norinder, Harald, *Discussion* (April) 474
Norris, E. T., *Paper* (October) 1206
Norris, E. T., *Discussion* (October) 1215
North, J. R., I. R. Dohr, and E. J. Burnham, *Paper* (July) 903
North, J. R., *Discussion* (July) 911
Northmore, E. R., *Discussion* (January) 169
Nott, G. E., *Discussion* (January) 66
Nyman, A., *Discussion* (April) 315, 505

O

Oehler, A. G., *Discussion* (April) 631
Ogden, H. R., *Paper* (July) 1013
Ogden, H. S., *Discussion* (January) 51
Oliver, J. M., *Discussion* (January) 213, 266
Oram, John, *Paper* (July) 977
Ormondroyd, J., *Discussion* (October) 1204
Osborne, C. P., *Discussion* (January) 17

P

Palueff, K. K., *Paper* (July) 681
Palueff, K. K. and F. F. Brand, *Paper* (July) 998
Palueff, K. K., *Discussion* (July) 705, 1011
Park, R. H., *Paper* (July) 716
Park, R. H. and E. H. Bancker, *Paper* (January) 170
Park, R. H., *Discussion* (January) 193, (April) 550, (July) 729
Payne, E. B., *Discussion* (April) 326
Pearce, E. F., *Discussion* (January) 266
Pearson, E. F., *Discussion* (January) 57, 90
Peek, F. W., *Committee Report*, (October) 1453
Peek, F. W., *Paper* (April) 436
Peek, F. W., *Discussion* (April) 477, (July) 703, 1010
Penney, G. W., *Paper* (October 1230
Penney, G. W. and F. T. Hague, *Paper* (April) 666
Penney, G. W., *Discussion* (April) 676, (October) 1241
Perry, L. L., *Discussion* (October) 1226
Peters, J. F., *Discussion* (July) 701, 1008
Peterson, A. J. A., *Discussion* (October) 1112
Peterson, T. F., *Discussion* (April) 369, 396, (October) 1269, 1283
Peterson, Wm. S. and H. J. McCracken, *Paper* (January) 67
Peterson, W. S., *Discussion* (January) 91
Pinckard, F. E., *Paper* (July) 896
Pinckard, F. E., *Discussion* (July) 902
Plumb, H. H. and R. E. Glover, *Paper*, September JOURNAL 695
Plumb, H. T., *Discussion* (January) 115
Potter, W. B., *Discussion* (October) 1140, 1146, 1156
Power, J. R. and H. M. Stoller, *Paper* (July) 808

Power, J. R., *Discussion* (July) 811
Pratt, A. C., *Discussion* (January) 91
Pratt, W. H., *Paper* (July) 737
Proebstel, D. W., *Discussion* (January) 65, 169
Putman, H. V., *Paper* (July) 873
Putman, H. V., *Discussion* (July) 877

Q

Queer, E. R. and L. A. Doggett, *Paper* (October) 1217
Quermann, G. H., *Discussion* (July) 1029

R

Race, H. H., *Discussion* (April) 315
Ramberg, W. and J. J. Torok, *Paper* (January) 239
Ramey, B. B., *Discussion* (October) 1195
Reding, H. W., *Paper*, December JOURNAL, 1928 880
Reeve, K. A., *Discussion* (October) 1260
Reichard, H. H., C. L. Dawes, and P. L. Hoover, *Paper* (October) 1271
Reichard, H. H., P. H. Humphries, and C. L. Dawes, *Paper* (April) 382
Reichard, H. H., P. H. Humphries, and C. L. Dawes, *Discussion* (April) 397
Reukema, L. E., *Discussion* (January) 256
Rex, H. B., A. S. Fitzgerald, C. H. Linder, and C. E. Stewart, *Paper* (July) 766
Richter, H., *Discussion* (October) 1104
Robertson, B. L., and I. A. Terry, *Paper* (October) 1242
Robertson, B. L., *Discussion* (October) 1220, 1261
Robinson, E. W. and S. J. Spurgeon, *Paper* (July) 981
Robinson, E. W. and W. I. Woodcock, *Paper* (January) 214
Robinson, G. D., *Discussion* (July) 765
Roper, D. W., *Discussion* (July) 1009, (October) 1446
Rosch, S. J., *Discussion* (April) 581, (October) 1299
Rose, J. E., *Discussion* (July) 989
Rowland, D. H., *Discussion* (January) 255
Rusher, M. S., *Discussion* (July) 745
Rutherford, P. H., *Discussion* (April) 631
Ryan, H. J., *Discussion* (January) 152

S

Salter, E. H., *Paper* (October) 1294
Salter, E. H., *Discussion* (October) 1300
Sanford, Raymond L., *Paper* (July) 731
Sarros, J. D. and W. V. Wolfe, *Paper* (January) 107
Sarros, J. D., *Discussion* (January) 115
Schlink, F. J., *Discussion* (July) 746
Schoepf, T. H., *Discussion* (July) 924, 927
Schuchardt, R. F., *President's Address*, (October) 1460
Schurig, O. R., H. P. Kuehni, and F. H. Buller, *Paper* (April) 417
Schurig, O. R., *Discussion* (April) 434
Scott, A. C., *Discussion* (July) 990
Searing, H. R. and G. R. Milne, *Paper* (October) 1093
Searing, H. R., *Discussion* (April) 414
Service, Jerry H., *Paper* (January) 198
Sessions, A. P., *Discussion* (January) 32
Shackelford, B. E., *Committee Report* (October) 1394
Shand, E. B., *Discussion* (July) 848
Shanklin, G. B. and G. M. J. Mackay, *Paper* (April) 338
Shea, T. E. and C. E. Lane, *Paper* (July) 1031
Silsbee, Francis B., *Paper* (October) 1301
Silsbee, F. B., *Discussion* (July) 805, (October) 1305
Silverman, J. B., *Discussion* (July) 877
Skeats, W. F. and A. Boyajian, *Paper* (July) 949
Skinner, C. E., *Discussion* (April) 664
Sleeper, H. P., *Discussion* (October) 1328
Slepian, J., *Paper* (April) 523
Slepian, Joseph, *Discussion* (January) 154, (April) 550
Slichter, W. I., *Discussion* (April) 631, 676
Smith, B. M., *Discussion* (July) 736
Smith, J. J., *Discussion* (October) 1175, 1260
Snyder, J. H., *Discussion* (October) 1195
Soderberg, C. Richard, *Discussion* (April) 675
Sorensen, R. W., *Discussion* (January) 9, 144, 153
Specht, H. C., *Paper* (April) 607
Specht, H. C., *Discussion* (April) 631, 632, 641, (July) 727, 752
Spooner, T. and J. P. Foltz, *Paper* (July) 747
Spooner, T. and C. W. Kincaid, *Paper* (April) 645
Sporn, Philip, *Paper* (April) 480, (October) 1226
Sporn, Philip, *Discussion* (April) 473, 494, (July) 702, (October) 1113, 1122
Spurck, R. M., *Discussion* (April) 548
Spurgeon, S. J. and E. W. Robinson, *Paper* (July) 981
Stanford, A. G. and Edwin M. Clapp, *Paper* (January) 261
Starr, E. C. and F. O. McMillan, *Paper* (January) 11
Starr, E. C., *Discussion* (January) 9
Stein, I. M., *Discussion* (October) 1269
Stevens, T. S., *Discussion* (January) 57
Stewart, C. E., H. B. Rex, A. S. Fitzgerald, and C. H. Linder, *Paper* (July) 766
Stewart, C. E., *Discussion* (January) 237, (July) 774
Stewart, Philip, *Paper* (July) 891
Stewart, Philip, *Discussion* (July) 895
Stoller, H. M. and J. R. Power, *Paper* (July) 808
Stone, E. C., *Discussion* (October) 1102
Storer, N. W., *Discussion* (October) 1146
Sultzer, Morton, *Discussion* (July) 947
Summers, I. H., *Discussion* (July) 728
Swallow, George, *Discussion* (July) 927

T

Tanzer, E. D. and W. S. Kates, *Discussion* (January) 258
Tapscott, R. H., *Discussion* (October) 1102
Ten Brocck, R. L., *Discussion* (July) 803
Terry, I. A. and B. L. Robertson, *Paper* (October) 1242
Terry, I. A., *Discussion* (October) 1228, 1261
Thau, W. E., *Committee Report*, (October) 1409
Thieman, V. A., *Discussion* (October) 1283
Thomas, E. R., *Discussion* (April) 314, 414
Thomas, J. B., *Discussion* (July) 989, 993
Thomas, P. H., *Discussion* (April) 472, 550
Thompson, G., *Discussion* (July) 804
Tolson, W. A. and L. F. Fuller, *Paper* (January) 102
Torok, J. J. and W. Ramberg, *Paper* (January) 239
Torok, J. J., *Discussion* (January) 259, (April) 469
Treat, Robert and S. Murray Jones, *Paper* (January) 268
Treat, Robert, *Discussion* (January) 282
Turley, L. J., *Paper* (January) 58
Turley, L. J., *Discussion* (January) 66
Turner, Alva and J. H. Morecroft, *Paper*, January JOURNAL 25

U

Untersee, E. A., *Paper*, November JOURNAL 1928 821

V

Van Atta, E. and E. L. White, *Paper*, September JOURNAL 682
Vandersluis, W. M., *Committee Report*, (October) 1457
Viele, S. M., *Discussion* (April) 546

W

Wade, M. L., *Discussion* (January) 50, 66
Wagner, B. S. and A. C. Burroway, *Paper* (July) 836
Wagner, B. S., *Discussion* (July) 848
Waldo, E. H., *Paper* (July) 821
Wallau, H. L., *Committee Report*, (April) 487
Wallau, H. L., *Discussion* (April) 473
Weinstein, H. L., *Discussion* (July) 909
Weller, C. T., *Paper* (July) 790
Weller, C. T., *Discussion* (July) 806, 951, (October) 1282, 1305
Wells, A. R., *Discussion* (July) 782
Wensley, R. J., *Discussion* (July) 987, 993
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Sphere-Gap and Point-Gap Arc-Over Voltage

As Determined by Direct Measurement

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Synopsis.—This is a report of some of the work done at the Ryan High Voltage Laboratory, Stanford University, during the school year of 1927-28. A new method of measuring high voltages is described, in which the current through a water resistor is recorded on an oscillograph. Over a million volts to ground was measured with an accuracy believed to be better than 2 per cent.

A calibration of the meter sphere gap was determined for voltages up to 1100 kv. and the arc-over voltages for point gaps were determined for distances up to 30 ft. Point to plane arc-over voltages were also measured.

Included in the report are some tests made in connection with the shielding of the water resistor determining to what extent capacity currents were causing error.

Sphere gaps were also used in connection with the determination of the arc-over voltages of the point gaps to determine whether there was any high frequency, set up by the heavy corona streamers from the point, that could not be recorded by the oscillograph.

The procedure of taking an oscillogram of the corona current from a pointed electrode a million volts above ground is described.

The practicability of the use of the meter sphere gap for voltage measurement is discussed.

* * * * *

Introduction

At the time of this investigation, the methods of direct high voltage measurement (for voltages in excess of 200 kv.) were limited to the sphere gap, point gap, dividing condenser as used with electrometer or cathode ray tube, and direct measurement of the current flow in a condenser of known capacity. The latter method had been used as a basis for the calibration of the sphere gap of 50-cm. spheres for voltages up to 500 kv.[3] The calibration of the 100-cm. sphere gap was for the most part theoretical, based on electrostatics and values determined on spheres of a lesser diameter.[4] The values of arc-over voltage for points gap for voltages in excess of 900 kv. was a matter of extrapolation of sphere gap comparison. Neither the sphere gap of 100-cm. spheres nor the points gap had been calibrated by a direct method of voltage measurement for voltages in excess of 550 kv. It was known by those familiar with the use of these gaps that their action was not positively established. The object of this investigation was thus to establish the calibration for the meter sphere gap and points gap for the higher voltages, based upon values obtained by direct measurement within a limit of error of 2 per cent.

Method of Voltage Measurement

In the absolute measurement of high voltage it is necessary to use fundamental methods. One of the simplest is that involving Ohm's Law, in which the voltage is given by determining the current flowing through a known resistance. The question then arises as to the availability of a resistance that can be connected to a million-volt circuit. The value of the resistance is determined by the amount of current that is required in the measuring circuit. This can only be determined after the kind of meter has been chosen.

In order to choose the meter it is necessary to decide in what terms the voltage will be required: average, effective, or crest value. In high voltage measurements one is concerned largely with arc-over voltages and since arc-overs are dependent to a considerable degree upon crest value, it is necessary to know either the crest value or the wave form. The latter is usually as much of an unknown as the magnitude of the voltage. Therefore, it is a logical procedure to record the current through the high resistance on an ordinary oscillograph, which tells all that it is necessary to know about the voltage wave. This then fixes the current value, for reasonable accuracy in measurement, somewhere between 20 milliamperes and 100 milliamperes. For practically all of the measurements reported in this paper, a current of 50 milliamperes was used. With 50 milliamperes and one million volts a 20-megohm resistor is required capable of dissipating 50 kw. if operated continuously. At this point the question arises as to the necessity for continuous operation when the oscillograph is used. If the voltage can be raised in the proper manner for the test and the oscillograph cut in at the instant the voltage is to be determined and then the voltage turned off the resistance, continuous operation is not necessary. A glass tube filled with distilled water makes a suitable resistor when operated in the above manner. The length of the tube must be great enough so that the applied voltage will not arc it over from end to end, and the tube must have a diameter such that there is sufficient water to prevent excessive rise in temperature during the application of the voltage. Fortunately the specific heat of water is high. For all of the work involving the use of the water resistor, the voltage was raised from zero to full value

1. Assistant Professor of Electrical Engineering, Stanford University.

2. Research Engineer, Bureau of Power and Light, City of Los Angeles.

3. L. W. Chubb and C. Fortescue, *Calibration of the Sphere Gap Voltmeter*, A. I. E. E., Vol. XXXII, 1913, p. 739.

4. F. W. Peek, "Dielectric Phenomena," McGraw-Hill Book Co.

Presented at the Pacific Coast Convention of the A. I. E. E., Spokane, Wash., Aug. 28-31, 1928.

28-128

in one second. This is where a motor generator set as a source of power adapts itself very well to such work.

Change in Resistance as a Result of Heating. The voltage is applied to the test specimen by first closing in the main generator oil switch connecting the transformer set to the generator bus. Then with the correct voltage on the generator exciter, the generator field switch is closed in. The voltage is thus built up rapidly without surges. The heat that is generated in the water resistor during the one second that the voltage is building up based on a uniform rise in voltage from zero to 1000 kv.[5] (effective value) may be determined as follows:

Let H = amount of heat in calories
R = resistance of the resistor
I = current through the resistor
K = gram calories per watt sec. = 0.24

$$k = \frac{\text{Final amps.}}{\text{Total time}} = \frac{0.05}{1} = 0.05$$

$$dH = K I^2 R\, dt$$
$$I = k t$$
$$dH = K k^2 t^2 R\, dt$$
$$H = K k^2 R \int_{t=0}^{t=1} t^2\, dt$$
$$H = \frac{K k^2 R t^3}{3}$$
$$H = \frac{0.24 \times 0.05^2 \times 20 \times 10^6 \times 1}{3} = 4000$$

calories

The diameter of the glass tube used was 0.4 in. and the length was 16 ft. The tube was in three sections coupled with rubber tubing. The tube contains 383 cu. cm. of water, the temperature of which would be raised approximately 10 deg. cent. This rise in temperature causes a decrease in resistance which must not be overlooked. The approximate change in resistance can be determined from the formula:[6]

$$R_t = \frac{40 \times R_{20}}{20 + t}$$

t = Temperature deg. cent.
R_t = Res. at any temp. between 0 deg. and 100 deg. cent.
R_{20} = Res. at 20 deg. cent.

In the work herein described this change in resistance was seldom greater than 15 per cent. This decrease in resistance due to a change in temperature requires that the resistance measurement of the water resistor be made immediately after the oscillogram is obtained.

Operations Necessary for Voltage Measurement. The actual procedure for the measurement of arc-over voltage may be followed from Fig. 1. First the generator oil switch and main generator field switch are closed in. The cam operated contactor is then started which first closes contact A closing the Magnetic Switch and shorting out all of the external resistance in the generator field. The voltage of the set is thus built up with the increase in generator field current and after approximately one second the oscillograph shutter is opened by the closing of contact B. However, just before this takes place the closing of contact C boosts the voltage on the incandescent light used in the oscillograph. The voltage of the generator exciter is set by trial so that arc-over occurs sometime within ten cycles after the shutter is opened. A quick acting relay R_1 closes the shutter in less than two cycles after arc-over. The generator oil switch is opened in six cycles by the action of the overload relays. Immediately after the arc-over,

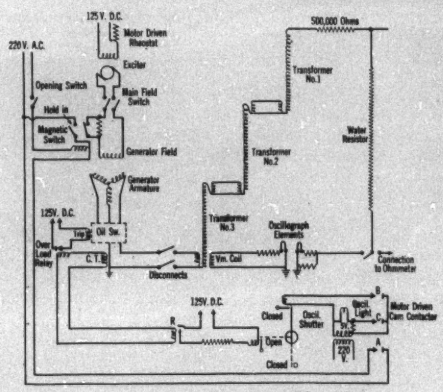

FIG. 1—SIMPLIFIED DIAGRAM OF MAIN CIRCUIT

the disconnects are opened preparatory to determining the resistance of the water tube resistor.

Method of Resistance Measurement. The resistance of the column can be determined within 20 seconds after arc-over. The slight radiation occurring in this time is negligible and is similar in magnitude to the additional heating caused by the voltage impressed during arc-over. Due to the high resistance used in the high voltage leads, the potential impressed upon the water resistor is small during arc-over, being less than one per cent of the voltage before arc-over. To measure the resistance of the water resistor, the switch S_1, Fig. 2, is thrown from position a to position b changing the water resistor connection from the oscillograph element to the high potential side of a 1000-volt d-c circuit. In position b the other contact of S_1 closes the 1000-volt circuit by operating a magnetic switch. Switch S_2 is thrown to position b connecting the galvanometer to a flexible lead which had been connected to the top of the water column. The galvanometer

5. This will vary somewhat from a straight line relation depending upon the degree of saturation of the generator field one second after the voltage is applied.

6. From tests by Applequest and McKenny, M. I. T. 1912. Pender Handbook (Edition 1922), p. 1356.

deflection is then read. Switch S_1 is then thrown to position a which connects the galvanometer in a circuit to which the 1000 volts is applied and having a known equivalent resistance approximately equal to the resistance of the water resistor. The galvanometer reading for this connection is also obtained. The resistance of the water resistor is then determined by the relation (Gal. Def. with Equiv. Res.)/(Gal. Def. with Water Res.) × Equiv. Res. = resistance of water

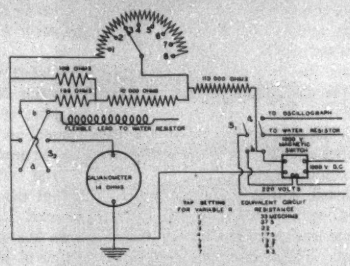

FIG. 2—CIRCUIT DIAGRAM FOR OHMETER

resistor. Following each arc-over, sufficient water is drained from the glass tube so that all of the heated water is replaced by fresh cool water from the gallon glass contained at the top.

Method of Oscillograph Calibration. The oscillograph element is calibrated by recording on the same film on which the record is taken, a current from the sine wave generator having approximately the same crest as the current through the water resistor. The effective value of this current is read on a milliammeter. Fig. 3 is a half section of film showing three of five records taken on the same film and also the calibration for this set of records. Each record contains two

FIG. 2—TYPICAL OSCILLOGRAM AS OBTAINED FOR VOLTAGE DETERMINATION

waves; one is the current through the water resistor and the other is the voltage as given by the voltmeter coil of the grounded transformer. The amplitude of the cycle preceding arc-over is the value used in the determination of the arc-over voltage. The total amplitude of the record (from positive crest to negative crest) is measured in all cases, being of the order of three inches. The ratio of the amplitude of the record to the amplitude of the calibrating wave multiplied by the effective value of the sine wave current used in calibration is the effective value of a sine wave current having the same crest as the current through the water resistor. This current value multiplied by the determined resistance of the water resistor is the total effective (sine wave equivalent) voltage computed from crest values.

TESTS AND DETERMINATIONS

During tests at the Ryan High Voltage Laboratory, it had been observed that at the time of arc-over of a 20-foot gap between points, the voltmeter coil in the ground unit of the chain connected set of six transformers indicated an effective value of terminal voltage 33 per cent less than that which was expected for the gap breakdown, based on the assumption that the value of 10 kv. per inch for points gaps at short separations was approximately correct for the longer gap lengths. It has been assumed previously in the explanation for the low voltmeter coil value for arc-over voltage, that the heavy corona streamers accompanying the breakdown of the gap caused high frequency or surges to be superimposed on the 60-cycle voltage. However, the oscillograms showed that the arc-over of the 20-foot gap was purely a 60-cycle breakdown and that the crest

FIG. 3—OSCILLOGRAM SHOWING WAVE FORM OF TRANSFORMER SET AT 1,000,000 VOLTS TO GROUND

A—Current through water resistor connected to high-voltage terminal
B—Voltage wave from voltmeter coil of grounded transformer

value of the voltmeter coil value of terminal voltage was but slightly different from that as determined by the water resistor and oscillograph connected to the high voltage terminal. The 33 per cent discrepancy mentioned above was further reduced by the fact that the crest value of the voltage of the voltmeter coil was greater than the $\sqrt{2}$ times the effective value. The distortion of the voltage wave of the transformer set and generator is caused by harmonics introduced in the load current by the many corona brushes and multiplied as a result of the reactance through which they flow. As can be seen from Fig. 4 the wave form of the voltmeter coil voltage is practically identical with that of the current through the water resistor, which is connected to the terminal of the transformer set.

Test for High Frequency. After some of the arc-over voltages had been determined by the above methods, it was decided to use sphere gaps as a check, lest there were some high-frequency voltage that could not be

recorded by the vibrator element of the oscillograph. The spheres available were two 100-cm. spheres and two 50-cm. spheres. It was decided to make up two gaps, one for each side of the transformer set, in order to keep a symmetrical set-up, since for all previous tests a water column had been used on both sides to preserve the balance of load.

The 50-cm. spheres were mounted so that their centers were about 4 feet from the concrete floor. Above these the 100-cm. spheres were suspended from the roof trusses by a string of 30 insulator units, Fig. 5. This is not a standard sphere-gap set-up. However, it was determined that the voltage values as given by the

FIG. 4—SPHERE-GAPS AS USED TO CHECK POINT-GAP VOLTAGE

theoretical curve for breakdown of a gap between 100-cm. spheres for separations up to 100 cm. would be within 10 per cent of the values for breakdown of the gap here used. For this set-up, the smallness of the lower sphere is partially offset by the proximity of the 50-cm. sphere to ground. This value was based on measurements that showed the charging current to the high potential sphere to be twice that to the grounded sphere for a 50-cm. gap between 50-cm. spheres.

These sphere gaps were used in connection with various point gap settings. For each gap setting the spheres were adjusted so that the arc-overs were about equally divided between the point gaps and the sphere gaps. The voltage was then determined by means of the water resistor and oscillograph. The following table shows the results for three gap settings.

Separation of points	Separation of spheres	Arc-over voltage*		
		Point gap (kv. to gv.)	Sphere gap	Voltages for corresponding separation of 100-cm. spheres
16 ft.	70. cm.	854 kv.	861 kv.	880 kv.
20 ft.	83.5 cm.	942 kv.	945 kv.	946 kv.
22 ft.	102.5 cm.	982 kv.	988 kv.	1020 kv.

*Each voltage value is the average of 3 observations.

The sphere gap was not used in connection with the point gaps in excess of 24 feet due to the fact that the limit of this type of gap had been reached. The results as given in the above table would indicate that the oscillograph was recording correctly the voltage that was impressed on the respective gaps. Since the normal rating of the transformer set is 2100 kv., the point gap calibration was continued up to a distance of 26 feet and later to 30 feet. Fig. 12 shows the arc following breakdown of a 30-foot gap between points. For this gap the transformer set is operating slightly above rated voltage, and as a consequence, the voltage was in every case raised rapidly to arc-over.

When the two sets of sphere gaps were being used with the 22-foot point gap, a very interesting thing was observed. During a preliminary test one of the 50-cm. spheres and its mounting were removed, leaving an 8-foot clearance from the 100-cm. sphere to ground. With the other sphere gap set at 100 cm. the voltage was raised, and it was surprising to find that the one 100-cm. sphere arced the 8-foot to ground. This distance was more than twice that between the other spheres. The voltage was applied a number of times with the same result. The 50-cm. sphere that had been taken out was replaced and the gap adjusted for 110-cm. The voltage was then applied and the 100-cm. gap arced over. This performance was repeated for the sphere gap on the other side of the set with a similar result. Calculations do not conform to this observed condition, and with what other information is available, at the present time, it is difficult to account for this action.

Test of Symmetry of Transformer Set. To be sure that there was no error in the assumption that the two sides of the transformer set were symmetrical with respect to ground, simultaneous oscillograms were taken of the current through the water resistors on each side of the set. Determinations from these oscillograms and the water column resistances gave the voltages to ground on the two sides of the set as 915 kv. and 930 kv. This difference is within the limit of the measurements.

Shielding of the Water Resistor. The water resistor was partially shielded by a 3-ft. horizontal circular disk at the top. (See Fig. 5.) For voltage measurements the shielding need not be nearly as perfect as for wattmeter work.[7] The charging current entering or leaving the resistance, because of imperfect shielding, may affect the phase angle for low power factor wattmeter measurements, but such current must be of relatively large value before it affects greatly the actual magnitude of the current through the high resistance. In order not to depend too much on such assumptions as to the correctness of shielding, simultaneous oscillograms were taken at the top and at the bottom of the water resistor. The oscillograph at the top was a

7. *Some Features and Improvements on the High-Voltage Wattmeter*, J. S. Carroll, A. I. E. E. TRANS., Vol. XLIV, 1925, p. 1010.

small two-element oscillograph, operated with a small motor. Both the oscillograph and the motor were adapted to be operated from a 6-volt storage battery. These were all enclosed in a wire screen cage which shielded them from corona effects and external electrostatic fields, (see Fig. 6). The turning on and off of

Fig. 6—Water Resistor and Oscillograph in Shielding Cage as used for Measurements at High Potential

the small incandescent light in the oscillograph at the proper instant was used instead of a shutter. This operation and also the motor control were accomplished by means of switches operated by linen threads controlled by relays. The results of this test were as follows:

Record No.	Current at top of resistor	Current at bottom of resistor	Percentage of difference
1	52.7 milliamperes	51.7 milliamperes	1.9 per cent
2	49.1 milliamperes	48.2 milliamperes	1.8 per cent

The difference in these two currents is within experimental error, so it was decided that no appreciable error was introduced into our results on account of improper shielding. On account of the dimensions involved, the capacitance of this water resistor to the ground or to the high voltage shield and connections is so small that even though there is some distortion in the electrostatic field, the current flow caused by this distortion is extremely small.

Another test that was made on the shielding was in connection with the use of the sphere gap of 100-cm. spheres. With the gap set at 100-cm., the arc-over voltage was measured with a water column resistance of 30 megohms and also with 10 megohms. Since there was nothing changed except the resistance of the water resistor, the capacitance current due to improper shielding remained practically constant. If this charging current were appreciable, the voltage as measured with the 10-megohm resistor would be noticeably different from that measured with the 30-megohm resistor. The average of 10 records in each case gave 994 kv. when the 30-megohm resistor was used and 1004 kv. when the 10-megohm resistor was used. This again shows that the resistor was shielded sufficiently for voltage measurements.

Discussion of Results

Points Gap. The full-line curve of Fig. 7 shows the arc-over voltage between two points for distances from 8 feet to 30 feet for the neutral grounded condition in which both points are at high potential. The points were 5/8-inch brass rods tapered to a point in a distance of 3 inches. The height of the gap from the floor was approximately 30 feet. The plotted points are averages of from 5 to 20 observations. The data from which these points were plotted showed a maximum difference in readings of less than 10 per cent of the mean value. No correction was made for air density or humidity for any of the point-gap curves for such a correction was

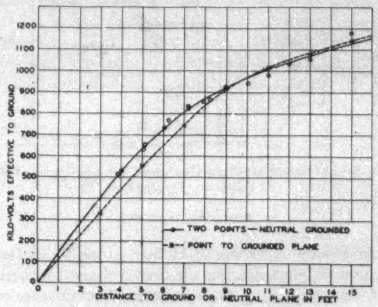

Fig. 7—Spark-Over Voltage for Point Gaps

Full line, two points, neutral grounded
Broken line, point to grounded plane

seldom greater than 2 per cent when calculated by present accepted methods.[4]

Arc-over voltages were determined for a point to plane set-up for distances from 3 feet to 15 feet. The point was mounted vertically above the floor, a nine-foot square section of which was covered with a sheet iron plate. The results of observations on this set-up are plotted in the dashed line of Fig. 7. The maximum variation in arc-over voltage for this set-up was seldom in excess of 5 per cent of the mean value. Plotted points are the average of four or more observations for the given gap setting.

The present practise in transmission line design is to use a dry insulator flashover that is approximately

three times the line to line voltage[8] for all voltages in excess of 165 kv. The change in the slope of the curve of point to plane arc-over (which is practically equivalent to the line to tower arc-over), with increase in voltage in excess of 900 kv., shows that the added tower clearance required will undoubtedly be a limiting factor in high-voltage transmission.

The curves of Fig. 7 show that the arc-over voltage for a point to grounded plane is practically equal to the voltage to neutral between two points at high potential when the distance arced over between the points is twice that of the point to plane.

Fig. 8 shows the effect of ground on the arc-over voltage of a vertical point-gap 9 feet in length as the distance from the lower point to ground is changed. Each observation is plotted separately. The apparent dropping of the curve as the gap is raised from the

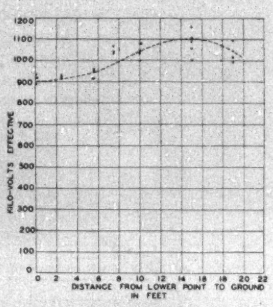

Fig. 8—Effect of Ground Proximity on Arc-Over of 9 Ft. Vertical Point-Gap

Lower point grounded

ground plane in excess of 15 feet is due to the lower or ground point nearing the high voltage connections and thus increasing the voltage gradient on the lower point, and also due to the fact that the upper point has reached a position with respect to the ground and sides of the building where a further increase in the distance to the floor has little effect upon the field intensity in the vicinity of the point.

100-Cm. Spheres. Due to some inherent surface property, which is as yet undetermined, the action of the 100-cm. aluminum spheres is rendered rather erratic where the separation between the spheres is in excess of 75 per cent of their diameter, and for smaller separations this erratic action may come in to a less degree. Observations were taken on both the 100-cm. spheres and 50-cm. spheres and in practically every case, many of the arc-overs were observed to be much lower than the theoretical voltage for the gap setting. It must be remembered, however, that this action is greatly increased when the separation between the spheres approaches the sphere diameter. A typical set of readings is given in the following table for values of the Hendrix voltmeter coil voltage for 50 successive arc-overs of the 100-cm. spheres spaced 100 cm. These were the first values following cleaning and polishing of the spheres and are characteristic of many similar sets obtained during these tests.

Arc-over No.	Voltmeter reading	Arc-over No.	Voltmeter reading
1	19	31	30
2	19	32	29
3	24	33	31
4	24	34	30.6
5	24	35	30.6
6	24.5	36	30.6
7	24.5	37	29.5
8	24.5	38	27
9	26	39	28
10	24	40	No vm reading
11	26	41	
12	25	43	
13	26	44	
14	26	45	29.5
15	24	46	30.7
16	27	47	30.8
17	27	48	31
18	Voltmeter was	49	31
19	not read	50	31
20			
21			
22			
23			
24			
25			
26	30		
27	28		
28	27		
29	30.6		
30	29.6		

From the above values it can be seen readily that at least 50 applications of voltage were necessary to get the spheres in condition. This fact by itself is no small item when it comes to wear and tear on high-voltage transformer test sets. Such a conditioning was found to be necessary to a greater or less degree if the spheres remained unused for more than 6 to 8 hours. This condition makes the voltage as determined by the sphere gap somewhat uncertain unless particular precaution is taken to condition the spheres. It can be said that the arc-over voltage for a given sphere gap setting will not exceed a definite value, which is very close to the theoretical value as given by Mr. F. W. Peek, but the arc-over voltage may be as low as 50 per cent of this value.

The use of the sphere gap for determining insulator flashover voltage has many disadvantages. A sphere gap cannot be used to measure the exact voltage at which an insulator string will arc over but can only be set so that it will arc over about equally with the insulator string. This means that the higher voltages that would have been impressed across the insulator string will arc the spheres due to the fact that the spheres will limit the voltage that may be applied. The character of these higher voltage arc-overs of insulators should be known even though the voltage applied at the time of

8. F. W. Peek, *General Electric Review*, Feb. 1922, Vol. 25, p. 114.

arc-over may be only 5 per cent in excess of that which is observed with the spheres in the circuit as a measuring gap.

In Fig. 9 the full line shows the calibration curve of the meter sphere gap that was determined by the water-resistor-oscillograph method of voltage measurement. The values plotted are averages of the records taken. The broken lines show the values as calculated by Mr. F. W. Peek. The difference between these curves

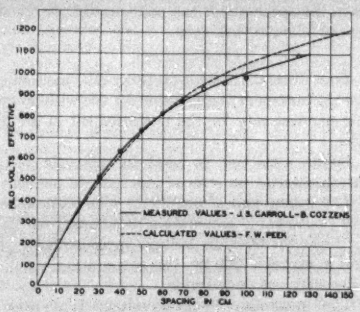

Fig. 9—Theoretical and Measured Values of Sphere-Gap Sparkover Voltage for 100-Cm. Spheres

The plotted points are averages of the measured values

is practically negligible for sphere separations up to 70 cm. For separations greater than 70 cm. the difference increases, but at 125 cm. it is only 5 per cent. The maximum values obtained agree in all cases with the theoretical values of Mr. Peek as mentioned in an above section. The slight difference in the curves in the range between 10 cm. and 60 cm. separation is due to a slight correction necessary for the increased clearance to ground as used by the authors.

After the calibration of the meter sphere gap was completed this gap was paralleled with the gap of 50-cm. spheres. The meter spheres were set at 30 cm. and the the 50-cm. spheres adjusted so that the arc-overs were equally divided between the two gaps. This gap setting on the 50-cm. spheres was 55 cm. According to this measurement the arc-over voltage for the 50-cm. spheres at this spacing is 8 per cent below that calculated by Mr. Peek. Time would not permit of a complete calibration of the 50-cm. sphere gap.

In all cases where records were taken the arc-over current of the gap was less than two amperes and the duration of the arc was limited to six cycles. This value of current and duration of arc caused no noticeable pitting of the meter aluminum spheres. These spheres were arced over more than 1000 times during the tests reported in this paper.

Corona Current from a Point

During the process of taking these records the question arose as to the amount of current that was drawn through the 500,000-ohm protective resistance that was connected in the high voltage circuit at the terminal of the line transformer. This current was the combination of the current through the water resistor, charging current to the connections, and the corona current from the point and connections. For the measurement of this current a shielded two-element oscillograph was used, the one element being connected in the water resistor circuit and the other used to record the total current from the load end of the protective resistance. The crest value of this total current was found to be 154 milliamperes when the voltage of the point to ground was 1,000,000 volts.

While this set-up was in place it was decided to obtain a record of the corona current from one of the points of the points gap a few cycles before the arc-over of a 24-ft. gap. The lead from the oscillograph element was completely shielded up to the point. (See detail of Fig. 10.) This arrangement eliminated all charging current and permitted the measurement of only the corona streamers from the point. The shielded cage set-up was similar to that shown in Fig. 6. The operation was as follows: The generator oil switch and main field switches were closed and then the oscillograph

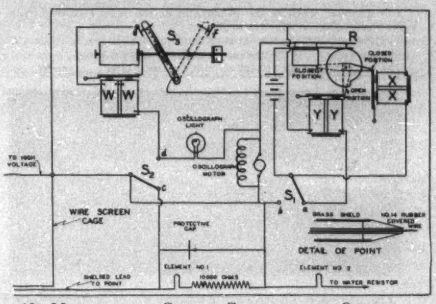

Fig. 10—Measuring and Control Circuit used in Obtaining Record of Corona Current from a Point at 1,000,000 Volts

motor was started by pulling a string that changed the position of the switch S_1 from a to b. The auxiliary field switch was then closed shorting out the field resistance and the voltage began to build up. Approximately one-half second later the motor operated cam closed a contact that operated a trip that pulled a string and changed the position of switch S_2 from c to d. This lighted the incandescent lamp in the oscillograph and at the same time energized the magnets $W\,W$. This released the switch S_3 from position e and was delayed in reaching position f by means of a kodak timer. This time delay was approximately 0.25 sec. When the contact at f is made the oscillograph shutter is released to the open position by the magnet $X\,X$. The current through the water resistor and the corona current from the point were then recorded for approximately 14 cycles before the arc-over of the gap. When the gap arced over, the overload relay opened the generator oil switch and when this switch had reached

the end of its travel a contact was closed that operated a relay which pulled a string changing the position of switch S_3 from position c to d. This energized the magnets $Y\,Y$ and closed the oscillograph shutter. When the shutter is in the closed position the cam on the shutter arm opens the contacts at R which opens the battery circuit, thus stopping the motor and turning off the light. The time required for this operation delays the closing of the shutter after arc-over sufficiently to record a zero line on the film.

The necessity for turning on the oscillograph light a quarter of a second before the shutter was opened was due to the fact that as the filament heats up there is a

FIG. 11—CORONA CURRENT FROM A POINT

The irregular wave is the oscillogram of the corona current, from a point 1,040,000 volts effective to ground

The crest value of the current at (a) is 28 milliamperes

The smooth wave is the current through the water resistor

The crest value of the voltage at (b) is 1,470,000 volts

FIG. 12—ARC FOLLOWING BREAKDOWN OF A 30-FT. GAP BETWEEN POINTS, 2,380,000 VOLTS EFFECTIVE

slight change in its position which causes a considerable shift of the spot on the film, in some cases amounting to almost as much as a quarter of an inch. Heating the light up to full brilliancy before the shutter is opened and keeping the shutter open after arc-over until the zero line is taken eliminates any possible shift of the record or the zero line due to this cause. This magnetically controlled shutter which was used on the small two-element oscillograph was designed and built by the authors.

In order to protect the sensitive oscillograph element (No. 1), which is connected to the point, from being burned out with the arc-over current of 1.5 amperes, the 10,000 ohm resistance was connected in series with the element and a gap shunted across the two. This gap consisted of a blunt point separated from a plate by the thickness of a cigarette paper. This proved ample protection for the oscillograph element because a great number of arc-overs was recorded with absolutely no injury to the element.

Fig. 11 is one of the records taken in this manner. The smooth wave is the current through the water resistor and the irregular wave is the corona current from the point. It is interesting to note that no two cycles of the latter have the same form. The apparent difference in phase angle of this current and voltage wave is due no doubt to a space charge effect. The current reverses before the voltage reaches zero because of a reverse field that is set up by the charge that remains in the space about the point until it is reversed during the following half cycle.

CONCLUSIONS

1. The Water Resistor-Oscillograph method for voltage measurement is adaptable for use with 60-cycle transformer test sets for the measurement of arc-over voltages.

2. Points gaps can be used for 60-cycle voltage measurements with an error of less than 10 per cent.

3. The sphere gap of one-meter aluminum spheres requires considerable preparation preliminary to using it as a basis for 60-cycle voltage measurement. For voltages in excess of 900 kv. this preparation is such as to render their use undesirable.

4. The maximum 60-cycle voltage that may be impressed on a gap between one-meter spheres is given by the theoretical value for sphere gap breakdown as determined by Mr. F. W. Peek.

5. The breakdown of a gap between meter spheres may be as low as 50 per cent of the theoretical breakdown voltage depending upon the previous treatment of the spheres.

6. The rapid decrease in kv. per inch necessary for the breakdown of gaps in excess of 9 ft. to ground establishes an approximate economic limit of high voltage for power transmission based on present design practise.

ACKNOWLEDGMENTS

This report would not be complete without calling to the attention of those not already acquainted with Dr. Ryan, the extent to which his past experience aids in such an undertaking.

The authors wish to mention Dr. J. T. Lusignan, whose assistance was of much value.

EQUIPMENT USED

2,100,000-volt set consisting of six 350-kv., 350-kv-a. transformers built by the General Electric Company, Power was supplied to the transformers through a General Electric Co. motor generator set,

4000-volt, 550-hp. Synchronous Motor

2300-volt, 1800-kv-a. Sine Wave Generator.

The nine-element oscillograph and the two-element "osiso" were made by the Westinghouse Electric and Manufacturing Company.

The power circuit analyzer used in some of the wave analysis was built by the American Telephone and Telegraph Company.

The two one-meter aluminum spheres were made for Stanford University by the Southern California Edison Company.

The sphere gap of the two 50-cm. spheres was made by the General Electric Company.

Discussion

E. C. Starr: The method used by Messrs. Carroll and Cozzens for voltage measurement is fundamental and direct, not being dependent upon transformer ratios, etc., but is subject to considerable error unless proper precautions are taken. Water-tube resistances can change very rapidly, particularly under changing conditions of current flow, and the charging current of a long water tube, part of which is operating at potentials of 1,000,000 volts and more, is of considerable magnitude. It seems, however, that the methods employed should give the proper resistance values and that the shielding devices should reduce the charging currents in the water-tube multiplier to a negligible quantity. These charging currents are, of course, in quadrature with the resistance currents and are not directly additive.

One possible source of error apparently was not investigated. Work with the cathode ray oscillograph has shown that the long, snapping streamers accompanying the corona discharge from electrodes at large spacings set up relatively large transient currents of extremely short duration in the supply circuit. These currents induce (in supply circuit) transient voltages which may have a considerable influence upon the apparent arcing voltage, particularly of sphere gaps. The ordinary Duddell oscillograph is not fast enough to record transients of this character.

It was found in this investigation that the rupturing gradients of extremely long point-gaps were much lower than for gaps of 10 ft. or less. It has been accepted rather generally that this gradient for all gaps greater than 4 or 5 ft. in length is about 100,000 volts (effective 60 cycles) per foot. I should like to suggest as a probable explanation of the observed discrepancy that the relative nearness of grounded objects and high-voltage conductors, transformers, etc., to the electrodes of the gaps at the extreme spacings created much greater flux concentration at the electrodes than would exist if the gaps were isolated in space. Hence, arc-over took place at lower apparent gradients at these extreme spacings than at shorter spacings when the ratio of gap clearance to gap length was much greater. Clearances of about 30 ft. were obtained for these tests but it is observed that the longest gaps investigated were nearly 30 ft. in length.

It is very difficult to obtain clearances of two or three times the gap length when we are dealing with gaps of 20 or 30 ft. but it is necessary that such clearances be obtained if the apparent rupturing gradients for the long spacings are to be of the same order of magnitude as for the short spacings. The Ryan Laboratory at Stanford University has clearances as great as those of any present indoor laboratory and the authors made an effort to use all available clearance, but the results seem to indicate that the proximity effect was entering to a pronounced degree.

The difficulties experienced with the 100-cm. sphere gap are interesting. A slight deviation of the actual from the calculated spark-over curve at the higher values might be expected, inasmuch as the calculated curve is based upon rupturing gradients as determined by much smaller spheres. As a measuring device, however, the gap should not be erratic, providing all conditions of operation are normal. Such conditions obtain when, for any normal spacing and voltage, the electrostatic field between the spheres is much more intense than between either sphere and surrounding objects. I have seen a 100-cm. sphere gap function very erratically at wide spacings when located in a region of strong extraneous fields. Arcs would occasionally start between the nearest surface of one sphere and a point nearly 90 deg. from the nearest surface of the other sphere. However, when these spheres were removed to a region of large clearances and small extraneous fields, their operation was perfectly normal.

When a sphere gap is set at spacings approaching the sphere diameter and the clearance to ground or other conductors is not materially greater than the point-gap arcing distance, the dielectric field between the spheres may be so distorted as to cause the arc-overs to be very erratic in voltage and character. The difficulties experienced by the authors with their 100-cm. sphere-gap operating at potentials of the order of 1,000,000 volts may have been of this nature. There should be nothing inherently wrong with the 100-cm. sphere-gap. The difficulty lies in obtaining sufficient isolation to eliminate the appearance, at the extreme voltages, of disturbing fields.

Reference was made to a condition in which a 100-cm. sphere was suspended above a 50-cm. sphere located a short distance above the floor. It was observed that arcs actually took place between the upper sphere and the floor at lower voltages than were required to arc the gap with the lower sphere in place. The weakest type of gap is that of a point to the concave side of a hemispherical surface. The next weakest is that of a point to a plane. With the lower sphere removed the condition obtaining is that of a sphere to a plane and, as Mr. Cozzens brought out, at the larger spacings a sphere approaches a point in effect. This gap would be relatively weak if the field were distorted by near-by projections on the floor. However, when the lower sphere is in place a certain symmetry of field is forced and the gap might actually become stronger.

In dealing with extremely high voltages the proximity effect becomes very important. Corona streamers of considerable length shoot out from conductors and electrodes and field distortion may produce gradients of nearly a rupturing value simultaneously in several directions.

The authors state that it was necessary to "warm up" the 100-cm. sphere-gap by taking about 50 arc-overs before the performance could be relied upon. I should like to know if, after these preliminary readings were taken, the gap was still somewhat erratic, or if it became fairly constant in its performance.

H. V. Carpenter: I should like to ask the authors in regard to the points where they measured the corona. Were the two points symmetrical, or was the corona measured from one point to something else?

I think we have all felt there were questions regarding the sphere-gap. Here we find a move in the direction of getting a substitute instrument which, while it is somewhat cumbersome and requires developing of photographic film, perhaps is still in a sense a direct-reading, single-observation instrument, which is something very desirable in measuring high voltages.

I believe the authors should give more data regarding the clearances to show as definitely as possible just where the limitations come, due to the distances from surrounding objects and the character of those surrounding objects.

L. J. Corbett: A year or two ago Dr. Reukema of the University of California presented the results of some researches which he had made in regard to the operation of sphere-gaps under the action of ultraviolet light, I think the effect was that the ultraviolet light caused the initial ionization of the air and made the spark-over value more definite. He did not get such non-uniform results as appear to have been obtained by Mr. Carroll. I merely call attention to this as a possibility for improvement in our methods of research in such fields.

R. W. Sorensen: For a long time we were quite satisfied, when a suitable electrostatic voltmeter was not available, to measure voltages with a spark-gap, utilizing sharp sewing needles as points. I still think for the practical purpose of insulation testing this means of measuring voltage was within certain limits very satisfactory. In the course of the increase in voltage the sharp needle gap was found at the higher voltages to be difficult to use in the duplication of results, and we had the development of the sphere gap.

For certain ranges of voltage, say up to 500,000 or 700,000 volts, the sphere-gap has shown itself to be a very satisfactory device, and should not be condemned without very careful consideration for use at voltages even higher. With higher voltages, however, we must be exceedingly careful about the location of the spheres and the objects surrounding them.

These authors have suggested that for the high voltages there are some advantages in the needle-gap over the sphere-gap. In some work I have done we have found it very convenient to use a needle-gap having blunt needles. With such needles we can duplicate test results fairly well, and we feel that we have a fair degree of accuracy by the use of such needles, an accuracy of say 5 or 10 per cent. It is true that as the voltage is brought up there is a large amount of corona around the needles before arc-over. This corona action, however, has a tendency to equalize the field and give proper distribution before arc-over, and does not, therefore, indicate a much lower arc-over potential than would be the case with spheres if the spheres are not very large. It would seem to me, therefore, that we should not too hastily condemn the sphere-gap and the work which was involved in establishing the sphere-gap as a standard for measuring of voltage; but rather, we must say that changing conditions have made it necessary to develop an additional system of voltage measurement which permits reasonable accuracy with less effort than can be accomplished by means of the sphere-gap.

In dealing with voltages which produce these long arcing distances it seems inevitable that there will be erratic arc-overs. The X-ray tube which Doctors Lauritsen and Bennett have recently been operating with one million volts in our laboratory, has shown in its operation that we can have very erratic flash-overs, but also they have shown that by properly distributing the field by means of shields each in the shape of a torus the air between terminals can be stressed very highly without arc-over. The X-ray tube to which I refer has a length of four times the glass cylinders commonly used on gasoline measuring tanks. Between the ends of the glass column built up of these four cylinders these men have applied more than a million volts without arc-over. In the early experiments, the torus rings which are placed between the sections of the glass column as built up were connected to definite voltage points with the expectation of improving arc-over voltage. We found, however, that connecting the rings thus was not as satisfactory as leaving the intermediate rings to float free, each to assume a natural potential in the field. The reason for the erratic arc-overs is very difficult to determine, though it is probable that some of them are due to gas pockets near the surface of the spheres from which the gas is easily pulled out. Such an explanation could account for Professor Carroll's having to arc-over his sphere-gaps about fifty times before they are stable in the sense of giving results which are easily duplicated, because after the gases have been driven out the arcing voltage would no longer be influenced by elimination of these gases from the sphere electrodes. This latter phenomenon is very common in the study of arc-over distances between electrodes in a high vacuum, and it may be that something similar takes place in air.

J. S. Carroll: One of the points that Mr. Cozzens emphasized at the beginning of the presentation of this paper was that the calibrations given in the paper are not put forth with the intention of having them accepted as standards. A standard calibration for a point-gap would be an extremely difficult task. Because of the large separation of the points at these voltages, clearances affect the arc-over voltages to a considerable degree. This is shown in Fig. 8 of the paper. Also I believe that the relatively large clearance in case of the 8-ft. to 12-ft. point-gap is somewhat responsible for the high average voltage gradient and the relatively small clearance for the 30-ft. gap would tend to decrease the average voltage gradient. However, it must be pointed out that these clearances would not seriously affect the point-to-plane arc-over voltages.

I am glad that Mr. Starr related to us some of his experiences in working with high voltages. We find that these erratic sphere discharges are not a peculiarity of the high-voltage laboratory at Stanford. Many who have witnessed the performance of the meter sphere-gap do not seem to have any considerable amount of confidence in it for voltages above 700 kv.

The question has been asked by several persons as to just how long the meter spheres remain conditioned after cleating them up with about 50 arc-overs. If the spheres had been left unused for half an hour we would arc them over a few times before taking an observation. Even with that precaution we would get a very low value of voltage every once in a while that had to be discarded. If the spheres stood idle over night it usually required about the 50 arc-overs to recondition them.

It might appear presumptuous on our part to assume that our error of observation was less than 1 per cent but it is interesting to note that the maximum observed values of arc-over voltage for each gap setting lie on Mr. Peek's curve, none of them is above it. Our values are averages of from four to eight observations. We, ourselves, are satisfied that the sphere-gap was not arcing over at its maximum voltage every time. We thought the information in the observed data was of sufficient importance to be published and we find we were justified in this conclusion by the numerous questions that have been asked concerning the nature of it.

In regard to Mr. Corbett's question concerning the effect of increasing the initial ionization about the spheres previous to arc-over by some such means as using ultraviolet light. That was tried with the 50-cm. spheres but with a 400-watt quartz-mercury are placed within about 5 ft. of the spheres we were unable to detect any difference due to its presence.

In answer to Dean Carpenter's question as to the nature of the set-up for the taking of the corona current oscillograms, I wish to state that this corona current was from one of the points of a 26-ft. point-gap in which the two points were at high potential. Sometimes we want to compare this current with a current from a point to a grounded plane.

Several people have made suggestions as to some of the things that might affect the arc-over voltage of the sphere-gap. The matter of clearance is an item but I do not think it is a very large factor if clearances of at least four diameters are maintained. Furthermore this should produce a constant error and would not in any way account for the erratic performance of the gap. When the spheres would arc-over at the side instead of directly between them there was no tendency for the arc to confine itself to any particular side. The variation of the ionization of the air about the gap I believe is a relatively small factor, because the normal ionization of the atmosphere is completely disturbed by the presence of the corona brushes from the high-voltage connections, and, as I see it, the arcing over of the spheres would not clear up the air about the spheres. From the experience we have had with the sphere-gap we are practically convinced that the erratic behavior of the spheres is due to the surface condition, the exact nature of which it is difficult to conceive.

As stated in the paper the erratic performance of the sphere gap increases with the size of the sphere and with the increase in spacing. It is a difficult matter to compare the performance of gaps between say 50-cm. spheres with gaps between 100-cm. spheres.

We have used the sphere gaps only in a vertical position so that we cannot say anything as to their behavior if used in the horizontal position. The only time I would expect the spheres to be used in this latter position would be when both spheres are at high voltage. Time did not permit us to mount the spheres and make this test.

Professor McMillan asked what effect the material of the spheres would have on the performance of the gap. We cannot say definitely. The largest copper spheres we have are only 25 cm. in diameter, and with those we know that, to begin with, at least the first three arc-overs are quite likely to be low.

The question in regard to the use of water columns for practical metering is not quite clear. Water columns are very convenient and fairly accurate for laboratory measurements but are not suitable for connecting to a power line for commercial metering.

High-Voltage Gaseous-Conductor Lamps

Electrical Characteristics of the High-Voltage Neon Type

BY F. O. McMILLAN[1] AND E. C. STARR[2]

Member, A. I. E. E. Non-member

Synopsis.—Gaseous-conductor lamps are finding a wide field of application in advertising and display lighting.

A brief discussion of the theory of the high-voltage type is included.

The electrical characteristics of typical high-voltage lamps were investigated by the use of indicating instruments, Duddell and cathode ray oscillographs.

Characteristic curves, data, and oscillograms are given.

Power-factor improvement was studied and data covering two methods of correction are included.

Introduction

THE field of application of the gaseous-conductor lamp is expanding very rapidly. In the form of neon signs, which are growing in popularity, it will probably soon include a large percentage of the total electrical sign load of the more progressive communities.

The investigation covered in this paper was undertaken in order to determine the various electrical characteristics of typical commercial neon sign lamps of the cold-electrode high-voltage a-c. type. Means of correcting the normally low, lagging power factor were also investigated.

There are several other types of gaseous-conductor lamps including the low-voltage hot-cathode type, the negative-glow type, the electrodeless induction type, and the television type. These are not considered in this paper.

Theory of Gaseous-Conductor Lamps

The usual form of high-voltage gaseous-conductor lamp consists of a length of glass tubing containing gas at low pressure and equipped with sealed-in electrodes. The gas pressure is adjusted to a value that gives the lowest possible ionizing potential consistent with good tube life. Visible radiation accompanies the ionization by collision phenomena created when a sufficiently high potential difference is maintained between the two electrodes.

Various colors of light are obtained by the use of different gases and gas mixtures in conjunction with different colors of tubing. Twelve or more shades and colors have been obtained by this means.

A characteristic of all gaseous conductors is that the voltage drop between electrodes after the discharge has been initiated is practically independent of the current flow. Consequently it is necessary, when supplying energy to lamps using gaseous conductors, to provide a circuit having current limiting characteristics because as soon as the discharge starts, the lamp gas becomes, in effect a conductor, the resistance of which is an inverse function of the current.

Power Supply for Gaseous-Conductor Lamps

Because of its ease of voltage transformation and because any space charge effect within the tube is obliterated each succeeding half cycle, alternating current provides the most convenient power source for gaseous-discharge lamps particularly of the high-voltage type. Since the resistance characteristic of the tube is an inverse function of the current flow, the high-voltage transformer used to supply the tube is practically short circuited as soon as the discharge starts. It is necessary then to employ a transformer of high impedance to limit the current flow This impedance must be principally reactance since if resistance were used an excessive amount of power loss would be incurred.

When lighted to approximately the brilliancy employed in the usual neon sign, a 15-mm. neon tube about 11-ft. long requires an input of approximately 50 watts. This power is consumed very largely at the electrodes (due to space charge effect and electron extraction from the electrodes). A tube of half the above length lighted to the same unit brilliancy would require much more than one-half of 50 watts.

Experimental Investigation

Electrical Characteristics. Several specimens of neon and neon-mercury tubing such as used in commercial neon signs and several neon transformers of the type commonly used for sign lighting were obtained for test. The tubes were each 142 in. in length and 15 mm. (0.59 in.) in diameter. The transformers were rated at 430 volt-amperes 110/12,000-14,000 volts.

Practically all of the data contained in this paper were taken with the same transformer. Slight differences in the construction of different transformers even of the same type and manufacture cause data taken under the same conditions of load and voltage but with different transformers to vary as much as 15 per cent. The data given represent average, typical conditions.

For simplicity in designation throughout the tabulated data and on the curves and hereafter in this paper

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the neon gas filled lamps have been designated as "Neon" and the lamps filled with a mixture of neon, argon, and mercury have been designated as "Mercury."

The curves of Figs. 1, 2, and 3 give the characteristics of series combinations of two neon tubes, two mercury tubes, and one neon and one mercury tube. Two sections of tubing involve the use of four electrodes in

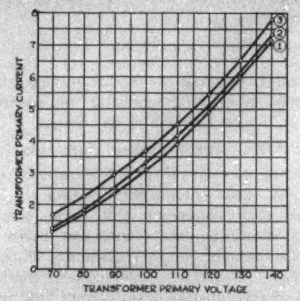


FIG. 1—VOLTAGE-CURRENT CHARACTERISTICS FOR TWO TUBES IN SERIES

1. Two mercury tubes in series
2. One mercury and one neon tube in series
3. Two neon tubes in series

series and constitute an average practical condition. It will be noted that the mercury tubes are highest in point of power consumption and power factor while the neon tubes are lowest in these respects. The neon

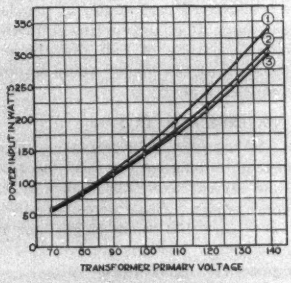


FIG. 2—VOLTAGE-POWER CHARACTERISTICS FOR TWO TUBES IN SERIES

1. Two mercury tubes in series
2. One mercury and one neon tube in series
3. Two neon tubes in series

tubes, however, draw the highest current and the mercury tubes the lowest.

The power factor at normal voltage (110 volts primary) ranges from 35 per cent for the neon tube combination to 45 per cent for the mercury tube combination and is lagging. As explained before, the resistance characteristic of gaseous conductors is such that a transformer supplying a shunt load of such conductors must possess a high internal reactance in

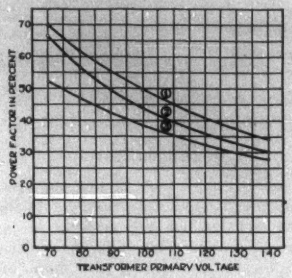


FIG. 3—VOLTAGE-POWER FACTOR CHARACTERISTICS FOR TWO TUBES IN SERIES

1. Two mercury tubes in series
2. One mercury and one neon tube in series
3. Two neon tubes in series

order to limit the load current. A transformer of this character when virtually short circuited draws a current having a large lagging component which increases in magnitude as the resistance of the secondary load decreases. The effective resistance of the mercury tubes is made higher than that of the neon tubes in order to

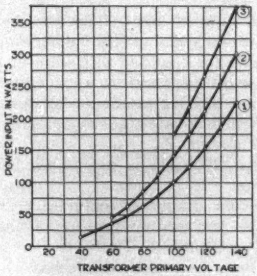


FIG. 4—VOLTAGE-POWER CHARACTERISTICS FOR ONE, TWO, AND THREE NEON TUBES IN SERIES

1. One neon tube
2. Two neon tubes in series
3. Three neon tubes in series

generate sufficient heat to vaporize the metallic mercury. Hence, it follows that a transformer loaded with these mercury tubes should consume more power at a higher power factor and lower current than when loaded with a similar number of neon tubes. The data and curves bear out this conclusion.

The characteristics of series combinations of one, two, and three neon tubes are given in Figs. 4 and 5. Since

the effective resistance of the transformer load depends upon the number of electrodes and length of tubing in series, it would be expected from the previous discussion that the power factor and power consumption would increase and the current decrease as the number

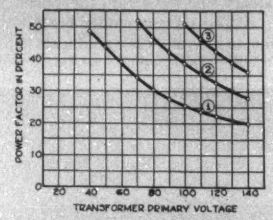


FIG. 5—VOLTAGE-POWER FACTOR CHARACTERISTICS FOR ONE, TWO, AND THREE NEON TUBES IN SERIES

1. One neon tube
2. Two neon tubes in series
3. Three neon tubes in series

of tubes in series is increased. The data and curves bear this out.

In the commercial neon transformers the necessary high impedance is obtained by providing a magnetic

Load—One neon tube

Load—Two neon tubes in series

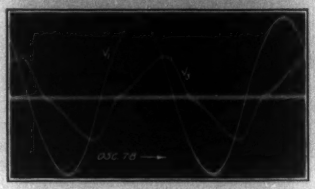
FIG. 6—LOW-VOLTAGE INPUT TO LOADED NEON TRANSFORMER

V_1 = Voltage 110 r. m. s.
V_2 = Current

shunt on the core between the primary and secondary windings. This shunt increases the leakage reactance of the transformer very greatly. With the transformer operating at 110 volts primary supplying a load of three neon tubes in series the primary current increased from 4.28 amperes with magnetic shunt to 12.8 amperes without magnetic shunt. The power consumption increased from 217 watts to 794 watts. The power factor increased from 0.461 to 0.563. This increased power input to the neon lamps resulting from the removal of the magnetic shunt from the transformer gave a very great increase in brilliancy of illumination. However, this condition is impractical because the volt-ampere load on the transformer is 3.3 times the normal rating.

Oscillograms of voltage and current were taken on the low-voltage and also on the high-voltage side of a loaded neon transformer. It was necessary to use a potential transformer to obtain the high-voltage wave forms. This potential transformer was a duplicate of the step-up

Load—Three neon tubes in series



Load—Three mercury tubes in series

FIG. 7—LOW-VOLTAGE INPUT TO LOADED NEON TRANSFORMER

V_1 = Voltage 110 volts r. m. s.
V_2 = Current

transformer and had an appreciable effect upon the performance. However, cyclograms (Fig. 11) taken by means of the low-voltage cathode ray oscillograph which required no appreciable power (approximately 0.5 watts) showed the general characteristics to be unaffected.

The current and voltage input to a transformer supplying first, one neon tube and then two neon tubes in series at normal voltage is shown in Fig. 6. The currents are typical low-saturation magnetizing current waves and for the two-tube case the current is smaller and more nearly in phase with the voltage than for the one-tube case.

Fig. 7 shows the input voltage and current waves first for a load of three neon tubes in series and then

for three mercury tubes in series. The current is smaller and more nearly in phase in the second case.

Fig. 8 was taken under the same conditions of load and voltage as Fig. 7 but in this case the phase angle was reduced to zero by placing static capacitance in parallel with the low-voltage side of the transformer. It will be noted that more corrective volt-amperes were required for the neon than for the mercury combination.

The output voltage and current waves of a loaded transformer are shown in Fig. 9. In the upper oscillogram (OSC. 83), that taken with neon tubes, the very pronounced oscillation in the current wave is due to

Load—Three neon tubes in series
Corrective volt-amperes 418
Corrective capacitance 91.6 microfarads

Load—three mercury tubes in series
Corrective volt-amperes 182
Corrective capacitance 39.8 microfarads

Fig. 8—Low-Voltage Input to Neon Transformer with Power-Factor Correction

V_1 = Voltage 110 volts r. m. s.
V_2 = Current

two factors. The initial part showing little decrement is produced by the impedance drop in the transformer drawing the terminal voltage down so low following each breakdown that the discharge stops only to be reestablished again as soon as the voltage recovers to the ionization potential. The latter part is the free oscillation of the circuit, that is between the capacitance, inductance, and resistance of the transformer and its load. This free oscillation takes place each half cycle after the high voltage has increased to a value high enough to sustain continuous ionization.

In the lower oscillogram of Fig. 9 (OSC. 86), that taken with mercury tubes, the current is lower, oscillations are much less pronounced, and sustained voltage is higher than in the neon-tube case. All of these effects are produced by the increased resistance of the mercury tubes.

The voltage waves are interesting. In both oscillograms, at the start of each half cycle, the voltage rises from zero at a rapid rate following the normal open-circuit secondary sine wave until the ionizing potential is reached. At this voltage sudden breakdown occurs, current begins to flow, and the circuit is set in oscillation.

Load—Two neon tubes in series

Load—Two mercury tubes in series

Fig. 9—High-Voltage Input to Neon Tubes

V_1 = Voltage of two tubes
V_2 = Tube current

This oscillation continues without decrement, as previously explained, until the voltage wave increases to a sufficient value to maintain continuous ionization. At this point the free oscillation of the circuit begins

Fig. 10—High-Voltage Input to One Neon Tube

V_1 = Tube voltage
V_2 = Tube current

and is damped out by the resistance of the tubes and transformer. Then as the voltage attempts to rise still further, the ionization increases, a greater current flows, a greater transformer impedance drop results, and the terminal voltage remains practically constant until the normal voltage wave decreases to this constant value. At this instant ionization ceases, the voltage

passes on through zero, increases in the reverse direction until the ionization potential is again reached, and the above procedure is repeated.

When the transformer is supplying only one section of neon tube the steady-state output voltage is very low, approaching a short-circuit condition. This is clearly shown in Fig. 10. The current is practically a sine wave in shape and is almost equal in value to the short-circuit current of the transformer.

The load voltage of a transformer supplying first one

Load—One neon tube

Load—Three neon tubes

Fig. 11—Cathode Ray Cyclograms of Neon Tube Voltages

and then three sections of neon tube as recorded by the low-voltage cathode ray oscillograph is shown in Fig. 11. The waves were drawn out horizontally by a sinusoidal deflector voltage which gives an abscissa scale that is not uniform but follows the sine law. The ordinates are proportional to instantaneous secondary voltage. The original negatives show the oscillations very plainly. These cyclograms were not affected by the measuring circuit and are truly representative of the phenomena.

The performance of the tubes when carrying abnormally high currents was studied. Power was supplied by means of a distribution transformer rated at 15 kv-a., 11,500/115-230 volts, 2.4 per cent impedance. Power consumption values as great as 21.9 kw. were obtained with three neon tubes in series. Currents as great as 7.5 amperes (about 240 times normal) were passed through the tubes. The accompanying illumination was very intense and the tubes heated up rapidly. The oscillograph showed the same type of transformer output characteristics as those obtained with the small transformers (Fig. 9). The voltage drop across the tubes was obtained by potential transformer and voltmeter and was found to be approximately 3000 volts for three neon tubes in series carrying 7.5 amperes. Under normal conditions of operation, with approximately 0.031 amperes in the tubes, the voltage drop as determined by sphere-gap and cathode ray oscillograph was substantially the same value.

When a heavy current was passed through the mercury tubes they glowed brilliantly for a few seconds and then began to increase in resistance very rapidly. The power was taken off and after the tube had cooled voltage was again impressed upon them. Full voltage of about 12,000 volts effective was not now sufficient to start complete ionization and produce any appreciable current flow. Probably a sufficient quantity of the gas in the tubes had been driven into the electrodes and glass walls by the violent bombardment to reduce the pressure below the value at which complete ionization would occur at the available voltage.

Power-Factor Correction

The resistance characteristic of gaseous-conductor lamps makes it necessary that they receive their power supply, when operating on alternating current, from a source of high impedance. This is accomplished most satisfactorily from the standpoint of power consumption and current requirement by employing high-voltage transformers having abnormally high leakage reactances. Such a load combination will necessarily draw lagging currents from the supply system.

Correction on Low-Voltage Side of Transformer by Use of Static Condensers

By the use of static condensers shunted across the low-voltage side of the transformer it is possible to correct the power factor to any desired value. In Table I are given the corrective volt-amperes and the corresponding condenser capacitance required to correct the power factors of the various combinations of tubes to 80, 90, and 100 per cent. It will be noted that a fairly large amount of capacitance is necessary in almost every case and that for any one case the necessary capacitance increases very rapidly as the desired power factor approaches unity.

Correction on High-Voltage Side of Transformer

It is possible to improve the power factor of a given combination of transformer and tubes by inserting a high resistance in the high-voltage circuit. The over-all

TABLE I
LOW-VOLTAGE POWER-FACTOR CORRECTION
110.0 Volts, 60 Cycles

Tubes		Power factor (lagging)		Low-voltage correction	
Type	Number in series	Uncorrected	Corrected	Volt-amperes (leading)	Condenser capacitance (microfarads)
Neon	1	0.234	0.800	419.	91.8
"	1	0.234	0.900	452.	99.0
"	1	0.234	1.000	511.	112.1
Mercury	1	0.264	0.800	390.	85.4
"	1	0.264	0.900	426.	93.3
"	1	0.264	1.000	491.	107.5
Neon	2	0.350	0.800	335.	73.3
"	2	0.350	0.900	382.	83.6
"	2	0.350	1.000	466.	102.0
Mercury	2	0.450	0.800	239.	52.3
"	2	0.450	0.900	291.	63.7
"	2	0.450	1.000	385.	84.3
1 −Hg +1 −Ne	2	0.394	0.800	287.	62.8
" "	2	0.394	0.900	336.	73.5
" "	2	0.394	1.000	423.	92.6
Neon	3	0.461	0.800	255.	55.8
"	3	0.461	0.900	313.	68.5
"	3	0.461	1.000	418.	91.6
Mercury	3	0.621	0.800	74.	16.2
"	3	0.621	0.900	113.	24.7
"	3	0.621	1.000	182.	39.8

power consumption is not greatly affected but the illumination is reduced and energy is wasted in the resistance.

Interesting results can be obtained by employing a series condenser in the high-voltage circuit. Table II gives the results of a series of measurements using three neon tubes and various values of series capacitance. The power factor remains high, over a wide range. The power requirement increases with increase in capacitance up to a certain point and then decreases. The tube brilliancy is in this case apparently roughly proportional to the power consumption. For very low values of series capacitance the tubes are unstable at normal voltage because the condensers charge up quickly during each half cycle through the resistance of the tubes and block off the current flow. As a result the illumination persists for only a fraction of each half cycle.

TABLE II
LOW-VOLTAGE CONDITIONS AS AFFECTED BY HIGH-VOLTAGE SERIES CONDENSER
3 Neon Tubes in Series, 110 Volts, 60 Cycles

Low voltage			High-voltage series condenser (microfarads)
Amperes	Watts	Power factor	
1.87	204.	0.990	0.00715
2.21	241.	0.992	0.00813
2.99	329.	1.000	0.01080
3.48	383.	1.000	0.01250
5.29	578.	0.993	0.01730
8.34	862.	0.940	0.02210
10.10	740.	0.664	0.02470
6.76	384.	0.516	0.03260

Table III gives the results of a group of measurements made on various tube combinations with three finite values of series capacitance and with the normal condition which corresponds to infinite series capacitance, for each combination. The three capacitance values were so chosen that they gave power consumption values in the case of the neon tubes approximately equal to, 20 per cent greater, and 20 per cent less than normal. It will be observed that the power factor is slightly over corrected in every case.

Resonant conditions are produced by the series condensers and consequently voltages of abnormal magnitude appear across the high-voltage winding of the transformer and across the condenser. Table IV shows the crest voltages appearing across the various parts of the high-voltage circuit for the different condenser and tube combinations. These voltages were measured by means of a one-inch sphere-gap. The transformer voltages are greatly increased by the series condenser but it will be noted that practically no change occurs in the tube voltage for a given number of tubes of one type.

TABLE III
POWER-FACTOR CORRECTION WITH HIGH-VOLTAGE SERIES CONDENSER
110.0 Volts, 60 Cycles

Tubes		High-voltage series condenser (microfarads)	Low voltage		
Type	Number in series		Current (amperes)	Power (watts)	Power factor
Neon	1	0.00553	1.22	113.	0.842 (Leading)
"	1	0.00652	1.53	138.	0.821 "
"	1	0.00776	1.82	164.	0.820 "
"	1	∞	4.78	123.	0.234 (Lagging)
Mercury	1	0.00553	1.30	126.	0.881 (Leading)
"	1	0.00652	1.56	148.	0.860 "
"	1	0.00776	1.97	183.	0.843 "
"	1	∞	4.62	134.	0.264 (Lagging)
Neon	2	0.00553	1.45	147.	0.924 (Leading)
"	2	0.00652	1.76	177.	0.912 "
"	2	0.00776	2.12	212.	0.910 "
"	2	∞	4.53	174.	0.350 (Lagging)
Mercury	2	0.00553	1.75	185.	0.958 (Leading)
"	2	0.00652	2.03	210.	0.942 "
"	2	0.00776	2.54	264.	0.943 "
"	2	∞	3.92	194.	0.450 (Lagging)
1 −Hg +1 −Ne	2	0.00652	1.91	193.	0.920 (Leading)
1 −Hg +1 −Ne	2	∞	4.18	181.	0.394 (Lagging)
Neon	3	0.00553	1.64	175.	0.972 (Leading)
"	3	0.00652	2.01	210.	0.950 "
"	3	0.00776	2.44	254.	0.948 "
"	3	∞	4.28	217.	0.461 (Lagging)
Mercury	3	0.00553	2.04	220.	0.982 (Leading)
"	3	0.00652	2.46	264.	0.977 "
"	3	0.00776	2.99	321.	0.976 "
"	3	∞	2.11	144.	0.621 (Lagging)

If the series condenser correction were to be employed in practise it would be necessary to insulate the trans-

TABLE IV
CREST VOLTAGES*
110 Volts, 60 Cycles

Tubes		High-voltage series condenser (microfarads)	Crest voltage in kilovolts		
Type	Number in series		Transformer	Series condenser	Tubes
Neon	1	0.00553	....	....	....
"	1	0.00652	23.9	21.8	4.81
"	1	0.00776	....	....	....
"	1	∞	3.82	0	3.82
Mercury	1	0.00553	....	....	....
"	1	0.00652	24.6	21.8	4.25
"	1	0.00776	....	....	....
"	1	∞	3.32	0	3.32
Neon	2	0.00553	27.3	20.5	9.76
"	2	0.00652	25.6	20.8	9.76
"	2	0.00776	27.3.	22.2	9.76
"	2	∞	9.76	0	9.76
Mercury	2	0.00553	....	....	....
"	2	0.00652	26.6	21.2	10.2
"	2	0.00776	....	....	....
"	2	∞	8.13	0	8.13
Neon	3	0.00553	25.6	19.1	13.3
"	3	0.00652	27.7	19.1	13.3
"	3	0.00776	29.0	21.2	13.3
"	3	∞	15.3	0	15.3
Mercury	3	0.00553	27.3	18.7	14.3
"	3	0.00652	28.0	19.1	12.0
"	3	0.00776	29.0	20.5	14.3
"	3	∞	12.3	0	12.3

*Crest voltages were measured by means of a one-inch sphere-gap.

formers for approximately twice the high-voltage potentials that at present prevail.

Conclusions

1. The inherent characteristics of gaseous-conductor lamps are such that when supplied from an a-c. source the transformer must have a high leakage reactance.

2. Because of the combined characteristics of the lamps and supply transformers, the currents drawn have a large lagging component.

3. Power factor is improved by increasing the number of tubes in series or by increasing their resistance.

4. The probable future almost universal use of gaseous-conductor sign lamps will necessitate the use of power-factor correction or the application of special power rates if the use of the high-voltage type lamp is continued.

5. Power-factor correction can be accomplished by the use of parallel capacitance on the low-voltage side of the transformer, or series capacitance in the high-voltage circuit.

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Discussion

E. A. Loew: I should like to ask the authors whether they have made any careful study to determine what is the most economical ratio of length of tube to applied voltage, for tubes of a given size. In other words, is there an optimum impressed voltage per unit length of tube, other factors remaining constant? It seems to me there is such a relation.

The power factor of an air-blown arc is a function of the voltage. The same is true of the arcs here discussed as shown in Fig. 5 of the paper. If the curves of Fig. 5 were extended to the point where the arc became unstable, my suspicion is that this limiting condition would be found at a power factor of about 71 per cent. At any rate, such is the case with power arcs whose limiting stable operation is found at a power factor of 0.707, as may readily be shown.

The question naturally arises,— is it not possible, and if it is possible, does not good economy demand, that neon tubes and other similar tubes used for sign illumination, be so designed that the voltage applied is properly proportioned to the length and cross section of the tubes, in order that, under normal operating conditions, the power factor will closely approach the maximum value for stable operation?

J. S. Carroll: I should like to enquire as to the possibility of using a third winding on the transformer to which a condenser could be connected for power-factor correction. This would eliminate some of the difficulties encountered with the series condenser and would no doubt be less expensive than using condensers on the low-voltage primary.

N. C. Clark: I wonder if Mr. Starr can tell us how the resistance can be changed by changing the shape or the size of the electrodes at the end.

C. P. Osborne: I am vitally interested in this subject from an operating man's standpoint. Here is a case where the customer is reducing his electric bill by using a small amount of energy and getting a great amount of illumination, but creating a very poor power factor on the system furnishing the service. Why should this additional burden be borne by the power companies when the manufacturer of the sign can install condenser capacity in the signs sufficient to give a reasonable power factor? It is time for the power companies to take a stand in this matter before it is too late.

D. I. Cone: The oscillograms presented with this paper show that in the process of correcting the power factor the action of the tube as a generator of high-frequency harmonics of the fundamental load current has been accentuated. I therefore suggest that in the further study of power-factor corrective apparatus, means shall be included to insure that these harmonics are not fed back into the main power system to aggravate the problems of inductive coordination.

It should be practicable so to arrange the condensers and the inductances in the transformer network as to serve both purposes.

F. O. McMillan: Professor Loew asks if it is not possible to operate high-voltage gaseous-conductor lamps at approximately 70 per cent power factor by either adjusting the tube length to the voltage or the voltage to the tube length. His contention is that from theoretical considerations the maximum power factor is obtained when the resistance and reactance voltage drops in the circuit are equal and the power factor is cosine of 45 deg. This condition is obtained and can be taken advantage of when the reactance and resistance values in the circuit can be adjusted and maintained in a fixed relationship. However, in high-voltage gaseous-conductor lamps the ionization is not sustained from half cycle to half cycle; hence, it is necessary to apply a high ionizing potential each half cycle. This ionizing potential, determined by cathode ray oscillograph and sphere-gap measurements, for the tubes tested was approximately 4.5 times the practically constant voltage required to sustain the current discharge during the remainder of the half cycle after the ionization is established. This condition then requires that approxi-

mately 78 per cent of the transformer voltage must be consumed by the leakage reactance of the supply transformer. The maximum possible power factor under these conditions is approximately 62.5 per cent. By actual experiment it was found that the neon tubes began to flicker when the voltage was reduced to a value that gave a power factor of 50 per cent. This is shown by Fig. 5 of the paper in which the curves are discontinued at the flicker value of voltage or the point at which the voltage fails to reach the ionization potential consistently.

The gaseous-conductor lamp then, in its behavior, differs from the power arc because of the fact that ionization is not sustained from half cycle to half cycle during the reversal of current due to the absence of incandescent metallic vapor and hot electrodes capable of electron emission which are always present in the power arc.

In reply to Professor Carroll's question regarding the use of a third winding on the supply transformer for correcting the power factor, we did not do any experimental work using such a transformer. However, such a winding on the primary winding side of the magnetic shunt, designed for a fairly high voltage, would make it possible to use a condenser of smaller capacitance than would be required directly connected to the primary winding and possibly might prove more economical.

In reply to Mr. Clark's inquiry regarding the influence of electrode size and shape upon gaseous-conductor lamp resistance, all of our tests were made with one size and shape of electrode (the type at present used in commercial signs); hence, we have no data on that subject.

Mr. Cone calls attention to the harmonics in the current waves of the two oscillograms shown in Fig. 8 of the paper and states that the neon tube is apparently a generator of high-frequency harmonics and may cause trouble from inductive interference with communication circuits. The high-frequency harmonic current which appears in the current of these two oscillograms is due to a rather bad slot harmonic in the voltage wave of the supply alternator. A filter was used between the alternator and the neon transformer but there was sufficient residual harmonic voltage passing through the filter to cause the observed current harmonic when the high values of corrective capacitance were used.

The Automatic Substation and its Relation to the Electric Distribution System

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INTRODUCTION

SOME 15 years ago in a substation supplying power to an interurban electric railway system, it was demonstrated that electro-mechanical devices could be made to operate the apparatus in the substation so successfully that human attendance might not be necessary. Since that time, the development of these automatic devices has been extremely rapid and, as a result of the progress in this branch of the art, the automatic substation of today has reached a high state of development. The application of these devices to perform and supplant the duties of the human attendant, has been so fascinating that more attention has been called to the automatic substation itself, than to the change that the automatic substation has brought to the distribution system, of which it is an integral part. It might not be amiss to state that the general impression prevails that the automatic substation has its best application in those locations where it will supplant manual attendance; it is true that this reduction in labor expense may be the predominating factor for selecting automatic equipment versus an attended station, but there are other conditions that may affect this choice.

It must be borne in mind that the substation is a definite part of the electric distribution system and that the location of the substation to the distribution network has a most important bearing on the cost of the distribution system. This is particularly important in street railway systems and in metropolitan areas where the load is dense and where better regulation and better service are required as compared to rural communities. In considering the economics of the distribution system, the cost of the substation, the overhead or underground network, the labor of operation and maintenance, the quality of service and the regulation, must all receive their due amount of consideration, as the total costs (capital, operation, and maintenance) are after all the costs that must finally be reckoned with.

It is the desire of the author to point out some of the advantages to be gained by the use of the automatic substation and to discuss in a general way some of the factors which play a most important part in the distribution of the electrical energy to a community. It is, of course, impossible in such an article to do more than discuss the general situation, since there are so many local conditions that affect any solution. Each installation can, therefore, be correctly solved only by a study of the prevailing conditions in the particular territory under consideration.

The automatic substation has been found ideally adapted to the following types of installation.

1. A supply of d-c. energy to urban, metropolitan, and interurban street railway systems.
2. The rural substation supplying only alternating current.
3. On consumer's premises where a special substation is necessary and where the customer's requirements may call for either or both alternating or direct current.
4. In supplying energy to a 220-volt d-c. Edison network in a metropolitan community of not excessive size.
5. In metropolitan areas where one or more types of service may be supplied from the same substation.

The latter may involve a combination of several of the above types. There are perhaps several other classes that might be added, such as automatic operation of synchronous condensers at isolated points and such as a semi-automatic installation in a large station. This condition assumes that the station has a large number of units and that switching conditions require an attendant. In such a station, it is probable that complete automatic operation is not advisable, yet partial automatic operation may reduce operating expense and give better service than a complete manually operated station.

In the study of the distribution system, of which the automatic substation may be a part, the first consideration that enters the study is that of the control or type of automatic equipment that is to be installed and here again the automatic substation may be subdivided into several classes, the most important of which are:

1. Complete or partial supervisory control from an adjacent station or location.
2. Indication (without supervision) at a distant point as to the performance of the apparatus in the automatic station.
3. The complete automatic substation without control or indication at any adjacent station or location.

Distant indication or control is obtained today largely by means of a wire circuit, but it is to be noted that the "carrier" system will undoubtedly be used successfully in the future.

In making a selection of any one of the above types, it is necessary to have a full understanding, not only of the operating performance involved in the particular situation, but also a full knowledge of how the several types of automatic installations perform under varying conditions. Those organizations or individuals, who

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28-114

have operated only manual substations when considering the application of the automatic substation, always have in mind what might happen with automatic equipment and due to inexperience with automatic equipment, it is likely that they will either overdo the installation of automatic devices or select an equipment which is much more expensive and complicated than conditions justify. It must not be forgotten that even in manually operated stations things do happen, that the operators do make mistakes, and the mere fact that there is always a man there, does not always insure as good operation as with automatic devices.

It is today generally recognized that for certain installations the automatic equipment can be made more foolproof than manually operated and that in certain types of installations, the automatic equipment will restore service faster than the attended equipment. It is further known that the automatic devices will protect the apparatus better than the keenest operator and will start and stop the machines, lock out the feeders, etc., with more accuracy and certainty than when the similar operations are performed by the hand of man. It is therefore necessary to understand clearly what automatic equipment will do before making a selection of the type of control desired.

Assume for a moment a medium size system, wherein one or more automatic stations are equipped with supervisory control, the control point being an adjacent substation. If such a system is to be operated on the orders of a central load dispatcher and if the control station must obtain orders from the load dispatcher before carrying out any switching operations in the automatic substation, it is probable that better service will develop if the automatic substation be of the fully automatic type rather than supervisory, as in times of trouble or if faults develop, the fully automatic substation will eliminate the defective equipment and will restore service faster than if supervised. If the control station for the automatic be one where the operator is likely to be very busy, in the event of system trouble, then the automatic station will deliver better service if it be not of the supervisory control type. However, there is always a feeling on the part of the operating man, in which feeling he is justified, that some one should know what is going on in the automatic substation, as apparatus may lock out and nothing will be known of it, either until the customer complains or until the routine inspection takes place. Such operation may cause a serious impairment in the service. Hence, a combination of automatic and indication at some distant point may be found to be the ideal solution, as the equipment will take care of itself during times of trouble on the system or locally and at the same time some one knows whether the station is functioning satisfactorily or not. If there are to be several substations and if they are not too far apart, the remote indications becomes all the more necessary, as with several substations the investment in the indicating equipment will be justified in view of the possibility of reduced inspection and maintenance costs.

The above statements have reference largely to substations in the metropolitan areas. It is not believed that such installations are necessary in rural substations located some distance from the nearest community, where said indications would be uneconomical. The selection of a particular type for the purpose at hand, must consider not only the question of what type of service it is desired to give, but also the economics of the situation, and it likewise involves a question of judgment.

Supply of Power to Urban and Interurban Railway Systems

In installations of this type, the automatic substation finds its ideal application. Certainly in both metropolitan and interurban communities there is the opportunity for a large reduction in the amount of feeder copper necessary, with an attendant improvement in voltage regulation if the substations can be economically spaced and located. In interurban systems, the substation capacity is not large and in many cases there may be only one equipment necessary and in the majority of cases rarely over two equipments in any one substation installation, all of which tends to simplify both the installation and operation, as well as reduce the capital cost. Where more than two machine equipments are necessary, the present development of the automatic apparatus is somewhat complicated owing to the number of combinations introduced by the addition of the third (or fourth) machines.

If conditions are such that more than two machines are necessary, it is possible that some form of supervisory control equipment may be simpler and less costly in both installation and operation. This statement is particularly true if remote control indication is desired, since the control or indicating circuit would be existent and the addition of supervisory control would, therefore, not be a costly addition. It must not be overlooked that the installed cost of the automatic equipment for a railway machine with a reasonable number of feeders, represents an expenditure of approximately 30 per cent of the cost of the machine itself and that the cost of this equipment does not materially change with the size of the machine, if sizes between 500- and 2000-kv-a. units are considered. It is therefore desirable on any given system to use as large equipments as are economically possible, not only to reduce the number of units, the size of the building, the number of switching elements, etc., but in order to obtain simpler operation. In selecting the number of substations and the size of the units, the relation of the location of the substation to the railway system must of course be considered, and, likewise, the cost of the power supply to the substation will enter the economic problem. All of these factors must be properly balanced for the best solution.

In the automatic substation supplying power to a railway system, load resistors, high-speed circuit breakers, temperature relays, and similar equipment provide better protection to the machine and to the equipment than that equipment ordinarily installed where it is manually attended and where such devices are not absolutely necessary. The use of the high-speed circuit breaker and its reclosing equipment gives better service than that manually operated, as these equipments automatically reclose the circuit the instant that the trouble disappears, a thing not accomplished with a manual attendant. When system faults occur and several substations are shut down, the automatic equipments invariably restore service faster than those manually operated. From my observation, automatic operation is certainly to be preferred to manual for this type of service.

From a service standpoint, even in the substation where there may be an attendant, the automatic equipment is preferred, even though it may not be economically justified, in view of the fact that some attendance at the station is necessary for other purposes. In those substations in metropolitan communities where the load is very heavy and more than three equipments are necessary, it is doubtful if any form of automatic substation will be economical, although where traffic is very dense, some automatic substations can very probably be located as an economical means of feeding the distribution network under peak load conditions, provided, of course, that the cost of real estate and the power supply extension does not make such a station uneconomical.

The above statements apply entirely to motor-generator and rotary equipments supplying power to railway systems. It is entirely possible that the mercury arc rectifier in the future may change some of the conditions that have been outlined above. There is no question but that the rectifier has some material advantages over rotating apparatus and it is entirely possible that some of the difficulties will be ironed out in the future. This is a development that must be watched and one that may permit the economical feeding of a railway network from a larger number of separate substation units than is now regarded as advisable.

Rural Substations, Isolated Consumers, and Special Synchronous Condenser Installations

These installations may be classed as being of the same type, in so far as the application of automatic devices are concerned. They are exceedingly simple and offer no special problem.

The rural substation prior to the development of the automatic art was unattended in the majority of cases. Fuses and oil circuit breakers gave some protection to the lines and equipment. If a circuit was interrupted, it remained out of service until the consumers complained or until inspection revealed the trouble. The introduction of the automatic devices has materially bettered the service with only a slight increase in cost. The outages, due to line troubles, have been materially decreased by the automatic reclosing breaker as troubles often clear themselves on the first opening and service is resumed as soon as the breaker is reclosed.

To both the isolated consumer and rural substation it is possible to give a most reliable type of service if two supply lines feed the substation, as the present day automatic devices effect a quick transfer of the load from one circuit to the other in the event of interruption and will even transfer the load back to the preferred circuit if desired. To those classes of consumers that require an isolated d-c. supply, such as mines and similar specialized industries, the development of automatic equipment has made it possible to supply these industries with most satisfactory service at a cost that would have been otherwise prohibitive. The automatic equipment in this type of installation will certainly render better service and the apparatus will be better protected than with manual operation, bearing in mind that the type of operator normally obtainable for this class of service is not particularly intelligent and the type of service required in many cases could hardly be supplied without some supervision.

The application of the automatic principle to the isolated synchronous condenser station located at the most advantageous point on the system for power-factor and voltage correction has made such installations economically justifiable since the substation does not require any attendant or housing facilities for him. Proper location for this type of installation means a reduction in the capital investment that would otherwise be necessary, either to provide proper regulation or suitable power-factor correction elsewhere on the system.

The type of equipment now available for this installation will perform its duties as well as the human attendant, if not better.

In all of the above types of substations, with the possible exception of the synchronous condenser station, supervisory control or remote control indication is hardly needed or justified, and therefore a full automatic substation finds its ideal application in these cases. In so far as the synchronous condenser station is concerned, the only advantage of remote control indication will be to inform the distant control station of what is happening in the substation. The question as to what expense is justified in having this added feature is one that can be answered only after a study of the local conditions that pertain.

With the most satisfactory performance of the automatic substation, it is now possible to locate such facilities at the most advantageous point on the distribution system. This means a reduction in the length of feeders, better regulation, and, particularly in rural communities, cuts down the length of individual feeders so that in the event of line trouble a large

territory is not out of service at one time. Such rural substations occupy only a small amount of ground space, and, as the apparatus can be housed either in steel cells or be made completely outdoor, such construction is not expensive, it being possible to build such a substation, including land and regulators, for a cost not to exceed $15.00 per kv-a. if the load is of the order of 3000 kw. and the supply and distribution voltages 11 and 4 kv. respectively. Such installations materially reduce the cost of the secondary network and certainly provide better service than if a large territory were supplied from one substation feeding a very large area.

Supply of Energy to a 220-Volt D-C. Edison Network

Many communities in this country with a population of more than 400,000 people have in the past been furnished with 220-volt d-c. Edison service in the congested areas. Such service was originally supplied for all purposes in this congested area, but of late there has been a tendency to reduce the area in which this type of service has been supplied and to supply in its stead alternating current. Some utilities have even gone so far as to supplant the Edison service entirely with a-c. supply. However, as there are many buildings that have large installations of d-c. equipment, it is necessary in such cases to supply some d-c. service or else to provide some conversion apparatus located on the consumers' premises to give the service required. Whether this conversion apparatus should be supplied in duplicate is a question that again must be determined by knowing local conditions.

Why should the d-c. supply be abandoned in favor of an a-c. supply? Many of the d-c. systems were originally built with storage battery standby. As the load grew, it became necessary either to increase the existing station with its battery facilities or else to establish other stations, some of which may have had stand-by batteries. With one or more substations, the network, which is largely underground, was tied together with junction boxes and fuses and the system operated as a network. If there were no batteries or if the batteries were not large enough to carry the system even at reduced voltage during times of system trouble, there was always the difficulty as to how service would be restored on the Edison network during times of system trouble. Such a d-c. network requires a large amount of copper and a larger number of ducts. The capital investment and the operating expense for such a system, particularly if batteries were required, are greater than for an a-c. system of equal capacity.

With these general factors in mind, it is not difficult to see why the a-c. system is to be preferred. Yet on the other hand there is nothing simpler to operate than a d-c. Edison network, and maintenance costs for the underground system are not high. If a new system were to be built today, it is doubtful if the d-c. system would be installed, and yet where such systems are installed with the changed conditions brought about by the automatic substation it is likewise doubtful if an a-c. supply should be selected to supplant the existing d-c. system. With the newer types of motor-generators with load limiting and similar characteristics and with the automatic devices now available and in service, and if the d-c. system can be fed from a number of substations with not more than two units per substation, a very economical and high quality of service can be rendered: the d-c. system can be operated as a network with a material reduction in the length of feeders, number of ducts, and consequent savings in losses and investment, and without storage batteries,—the automatic equipment restoring service most satisfactorily in the event of trouble. It is not intended to imply that automatic equipment in a system of this type is the equal of manual stations with storage batteries, nor is it intended to imply that, in extremely large metropolitan areas where the density of load is very heavy, the automatic substation can supplant the manual substation entirely, as the number and size of the machine equipments necessary in a given substation may require such a complicated and costly outlay that the installation would not be justified, but where the load is not too dense and where a number of substations can be ideally located, and particularly if a main and auxiliary bus are not necessary, the automatic equipment as applied to this form of supply is certainly worth careful consideration as against a change to an a-c. supply with its attendant difficulties. An a-c. supply to deliver service equal to that of direct current under such conditions should be operated as a network. Also, such a type of supply means large transformer vaults, with some problems of ventilation, network protectors, etc. Certain consumers will still demand some d-c. supply, for which there must be some form of conversion apparatus.

It is not always possible, owing to crowded street conditions, to find ample space for the necessary a-c. facilities, and when one considers the growing density of load, with the street obstructions, the cost of equipment necessary to provide an a-c. network and the changes in the service and metering equipment that are necessary, to say nothing of the write-off involved in abandoning the existing system, the newer application of the automatic substation with its d-c. network is worthy of careful consideration. It is to be particularly noted that the cost of real estate, extension of supply feeders, and the possible future density of the territory will require a careful consideration of this problem, and it is entirely possible that even though the service be overlapping in a given area, some direct current, rather than a complete change, will be the economical solution.

If such d-c. automatic substations are used, there again enters the question of the type of station—whether it shall be full automatic or supervisory, or a combination of the two. As the distances between

substations will not be great and as there will be rotating apparatus installed, and as the service demanded must be of an excellent character, it is probable that some form of supervisory control will be most advisable. Such a control system will be essential if there be more than two machines in a given substation, as the complications introduced by the addition of the third machine do not make for a simple layout. In any event, the cost of either full supervisory or partial control will hardly be a controlling factor in the solution of this problem, as the added expense of such equipment will be very nominal, particularly where there are several substations; and as there will be a reduction in inspection and maintenance expense, a supervisory system of some type will most probably be warranted.

With such a control system, it is reasonable to suppose that even in the large metropolitan area with dense loads that a small number of automatic substations may find an ideal application in the economic solution of the supply of the territory in question.

Automatic Substations in Metropolitan Areas

In large metropolitan areas the use of the automatic substation vs. the attended station requires more careful consideration than in any other situation, as there are many things that affect the problem, such as the several classes of service that may have to be supplied out of one station, the density of load, the cost of the supply lines to the substation, the cost of real estate and building, and the question of fire risk.

If the utility is to supply several classes of service, such as power to a street railway system, d-c. Edison service for a limited area, a-c. service for industrial purposes, and if there is street lighting, all to be supplied out of the same station, it is most probable that the variety of service and the amount of apparatus that must be installed can best be served from an attended substation. With such an installation, the application of partial automatic operation, such as reclosing feeders on the a-c. system, automatic operation of all station auxiliaries, automatic starting of motor-generators or rotaries for the railway system, will be justified as it will reduce the number of required attendants and at the same time better service will be given.

However, outside of the congested areas, such as in the industrial and residential territories where only railway and a-c. service are demanded, it is quite possible that the automatic substation will be best suited for meeting the additional load growth.

It may be found that the existing substation facilities cannot be readily increased, due to limited real estate, particularly where the adjoining property is covered with substantial structures and where the cost of reconstruction of the existing facilities to provide increased capacity will be quite high. In such cases, particularly in the older parts of the city, there will be crowded street conditions where the cost of obtaining additional output facilities will likewise be expensive. Under such conditions the automatic substation as an adjunct to the existing station may be the ideal solution for meeting the additional growth, particularly if the amount of load on the new substation will be a minimum of 4000 to 5000 kv-a.

In such territory, it is most probable that the supply lines to the new substation must be underground, and as the cost of such underground supply lines is a large percentage of the cost of the additional facilities, sufficient load must be available if an economical installation is to be had.

Where the generating capacity is of such an order that the use of oil switches of high rupture capacity is necessary, it will be found that switching costs at the substation are also a factor in the cost of the total development. In order to reduce the cost per kv-a. of switching and underground lines, supply cables must be of such a size that they will carry loads of approximately 7000 kv-a., this assuming that the voltage of the supply system is approximately 12,000 volts, and in addition to this it is possible to eliminate the switching at the substation end by treating the cable and transformer bank at the substation as a unit, thus not only saving the cost of switches, switchboard, etc., but also the building space required for them. The automatic station lends itself particularly well to this type of design if a spare cable and transformer bank unit are installed, the banks feeding the secondary bus through oil switches, the secondary bus in turn being arranged in units with a given number of feeders per bank; the secondary bus sections so arranged that they can be tied together through oil switches. In the event of a failure of a cable or bank unit, the main secondary oil switches on the bank open, the secondary bus tie switches close, and the other bank takes the load. Such an installation can be made with two or more banks and cable units, it being necessary that the bank and cable unit have some spare capacity in order to take the load of the adjoining section.

Such an installation requires no switches on the high-voltage side, the minimum number of relays, is without the complications of high-tension switching, and it is most probable that it would work cheaper than any other installation if the costs of switching and building are of serious moment in any other type of design. Such an installation will function faster in times of trouble than could be done by any manual attendant, and it is probable that such an installation, if the feeders were equipped with reclosing devices, would hardly need any form of supervisory or remote indication except to ease the mind of the operating man.

If the conditions of service require that railway load be supplied in addition to alternating current, it is possible that some high-tension switching will be necessary, particularly if the railway load requires such capacity that the converting apparatus cannot be economically fed from the low-voltage bus, and for the reason that there is always some diversity between the

railway load and the general lighting and power load. The most economical installation will be had by supplying both loads from the same supply cables. In such an installation it is most probable that some form of supervisory equipment will be advisable, particularly if the number of feeders for the a-c. system is large and if it is found necessary to keep a check on what is going on in the railway part of the installation.

There is a variety of arrangements that can be had which will deliver a high degree of service without supervisory control, but supervisory control should not be regarded as prohibitive in cost, for if duct line space is readily available and the substation located approximately 3½ mi. from the control point, the supervisory system, including the control cable, should not cost more than $1.00 per kw., if the substation is of the order of 10,000 kv-a.; this is certainly not an unreasonable sum.

In the choice of any system of control, it should be realized that the full automatic apparatus will clear the trouble and restore service faster than can be done with supervisory equipment or with a manual attendant, and it will relieve the distant operator from any duties, particularly if that operator has other duties that will require his attention during time of system trouble; and furthermore if the system be operated under dispatcher's orders whereby the operator must get orders from the dispatcher before closing the line, certainly the full automatic is to be preferred to the supervisory control for the reasons as given above. It seems best, therefore, that the system be one in which the apparatus functions automatically without human attendance, and that when necessary, as a check and in order to save inspection trips, etc., that supervisory or remote control indication be superimposed in order that some knowledge may be had on how the apparatus is functioning.

There is, I think, one further advantage in the full automatic apparatus in that trouble as a rule can be more easily located than if the equipment be controlled from a supervisory system, for if trouble on a machine or line develops, the protective devices will open the circuit, perhaps lock it out, but at any event will usually clear the fault. If the station must be visited to ascertain the cause of trouble, the fault should be more readily located with the full automatic type as there are more relays which have a definite function to perform, and if the inspector is familiar with the apparatus it is often much quicker to locate the trouble by observing the position of the key relays rather than have to run down the trouble step by step which is necessary when the supervisory system is used, as this system uses less relays and thereby more nearly approaches the conditions that pertain when the station is manually attended.

In passing, it might be well to mention the subject of street lighting in connection with automatic substations. While it is entirely possible to control constant current transformers by time switches, supervisory control, etc., there are certain disadvantages in the use of these transformers such as the required building space, the large number of outgoing wires, and the inconvenience caused when a series circuit opens, as it is likely to cover quite a street area. Automatic apparatus can be had which will open and close the circuit normally and protect the circuit in times of trouble. This may require the use of quite a few auxiliary devices.

The experience of the writer on the system with which he is connected has demonstrated that there is a material saving and improved service in the use of the pole type constant-current transformer, said transformer being fed from the multiple 4-kv. circuit (which circuit is combined in part with the regular feeder system) and said transformers feeding small groups of series circuits. These transformers are usually about 20 to 30 kv-a. capacity each.

The primary street lighting circuits are controlled either by time-clock or supervisory control, or they can be controlled from a distant point by carrier at very nominal expense.

The multiple circuit occupies a substation space equivalent only to one feeder position. This circuit can supply 800 lamps, and if some cascading is used the number can be increased to 1000 without difficulty.

It may be possible to eliminate entirely the street lighting equipment from an automatic substation by supplying said lighting from other substations. This is particularly true if "carrier" control is to be used. The only point to be brought out in the above discussion is that the the street lighting installation is extremely simple and offers no special problem in the design of the substation.

Remote Load Indication and Other Metering Equipments

If the distribution network is fed by a number of substations and if the feeders are or can be tied together through sectionalizing switches, it is often desirable to know the load that the several units in the automatic substations are carrying; this is all the more important if the station is supervisory controlled, as the control station must be in a position to know when to put in and take out the supply units. The necessity for remote load indication equipment depends, in a large measure, upon the detail design of the system. If, for example, the cable and transformer bank are a unit and this unit feeds a group of low-voltage feeders, a fair idea of the load can be had from the reading of the ammeters at the feeding point, and if all the supply cables emanate from the same station, remote metering is hardly necessary. On the other hand, if there are several units fed from the same supply cable, load conditions are not so readily known. During times of system disturbance, a substation may have to take a heavy overload and it is often desirable to know just what is going on in the automatic substation under such

conditions, as it is not always possible to send an inspector to the substation when the system is upset.

If there are communication wires between the control point and the substation for supervisory control, the added expense of remote load indication equipment is not excessive if a large number of readings is not desired. The cable type for direct indication of remote load indication is not satisfactory for distances in excess of approximately 2½ mi.

Remote load indication equipment in the present state of development cannot be regarded as entirely satisfactory, although it can be reasonably expected that improvements will be made in time. It would, therefore, appear advisable at present to install only such equipment as is felt will be absolutely necessary. Most stations have a fairly well defined load characteristic and after this is determined a fair amount of information can be had from the supply line readings if the operator is carefully instructed.

There does appear to be some room for improvement in the lower price recording meters for automatic substation work. As the station will be visited only at periodic intervals, it is necessary to have recording instruments that will register, with a reasonable degree of accuracy, the desired record for at least seven days. If charts must be changed any oftener, it becomes a burden. If the regulators are equipped with indicators that register the maximum boost positions, a fair idea can be had of feeder performance. During the peak period of the year, it may be advisable to take a few readings rather than go to the expense of equipping every feeder with a recording ammeter.

The entire instrument equipment should receive careful attention as there normally is no one present to read the instruments and only those instruments that have to be read at intervals or are used in test should be installed.

There is a definite need for a reliable thermometer that will register the maximum temperature on apparatus such as transformers and regulators, the present curve drawing types being too large and expensive.

Substation Design

The design of the automatic substation is somewhat different from the attended substation. The most important elements to be noted are:

1. The design of the building if apparatus must be housed, or design of the structures if it is to be outdoor.
2. Ventilation in the case of housed equipment.
3. Fire risk.
4. Economical arrangement of the apparatus, particularly with relation to the building structure, and with the interconnected devices.
5. Arrangement and grouping of the apparatus to facilitate quick inspection, and location of faults.

Building Design. If the substation is to be located in a residential area, special treatment must be accorded the building to make it harmonize with the surrounding property. It must be architecturally attractive, should very probably be not over two stories high, and must be of such construction that if the apparatus located therein be noisy, such noise must be confined to the building. Where only static apparatus is housed, such as transformers and induction regulators, noise is not much of a problem, but where rotating machines are used, the elimination of the noise requires much more care in building design.

In a noiseproof station, it will be necessary to provide some form of double walls; that is, an outside wall with intervening air space between it and the inside wall, the latter being of plaster or similar material. In place of the double wall, it is possible to use sound absorbing materials such as celotex and plaster, or in lieu thereof, hollow tile with a felt and plaster covering. There are various materials that can be effectively employed so as to eliminate entirely the noise going through the wall. It will be found advisable to eliminate all windows and skylights and reduce the number of doors to a minimum. If the doors must be of sufficient size to admit large machines, it will be difficult to make this door noiseproof unless special precautions are taken. It will be found advantageous to use a heavy door of wood construction and then to place a removable covering over this door, such covering being placed on the inside of the station, and built of soundproof materials. It must be bolted tight against the main door frame. The smaller entrance door should either be through a vestibule having two doors, or there must be two doors of a construction similar to the main door. All cracks around the doors must be absolutely tight as small cracks offer a passageway for a large amount of sound.

The foundations of the machines and the supports for contactors should be kept separate from the main building structure. It will be found advisable to mount such apparatus as induction regulators on cork pads.

If the transformers be located outside of the building and there are no motor-generators or rotaries in the building, it will probably be unnecessary to provide forced ventilation. As a matter of fact, in cold weather it may be necessary to provide some method of heating the building. However, motor-generators, etc., will require forced ventilation of the building.

The air entrances and exits must be properly baffled, otherwise they will become passages for the exit of noise. It is probable that in a residential area, all wires leading to and from the station will have to be underground for at least a limited distance. If the station contains several classes of apparatus located in separate rooms, it may be necessary to build only one part noiseproof, the static apparatus not causing sufficient noise to require the extra precaution and expense necessary with rotating machinery.

If real estate conditions permit sufficient width of building to allow for easy movement of machines, and

if they do not exceed 1500 kw. in size, it will be found advisable to build such building without a crane. This not only reduces the height of the building, but materially reduces the cost.

If the station is to be located in an industrial or rural area, the above precautions may not be necessary and it may even be desirable in the case of static apparatus that all of such apparatus be entirely outdoor type, thus eliminating the building. However, some modification of the conventional outdoor design may be found advantageous, such as the use of steel cells to house oil switches, instrument transformers, disconnecting switches, etc. As the substation will be unattended, therefore, not under the care of an operator, everything should be done to eliminate the possibility of outside interference, such as kite strings, birds, small boys, etc. Where outdoor structures with exposed conductors and switches are used, greater clearances are necessary, as faults have developed from birds and animals grounding the conductors to the structure. These conditions require clearances which sometimes produce a rather large structure. Furthermore, some form of housing is necessary for instruments, relays, etc.

A very economical design has been worked out, wherein all wires are brought to the substation underground, such wires terminating in a steel cell which houses the oil switch, instrument transformers, disconnecting switches, switchboard instruments, etc. All leads to the transformers and regulators are wiped in. This type of design has a decided advantage in that there is not an exposed wire in the installation. Indoor equipment (which is cheaper than outdoor and incidentally more easily obtained) can be used. The oil switch, instruments, control relays, etc., are grouped close together, requiring extremely short interconnecting leads, thus reducing expense. Bypass switches (air type) are provided in the cell to allow for inspection of the oil switch. Such a station supplied at 12 kv. and distributing energy at 4 kv., if of 3000 kv-a. and above, can be built at a cost of $15.00 per kw., which price includes three single-phase induction regulators per feeder. (For complete details of such a station, see article by R. C. Powell in *Electric World*, Feb. 26, 1927, Vol. 89, p. 449.)

Ventilation. If ventilation is required, it presents no difficult problem; it is only an added expense. It will be found advisable to install some type of air-cleaning equipment in order to reduce the quantity of dust and dirt brought into the station. Such equipments are now obtainable in the form of dry filters so arranged that said units can be quickly cleaned without taking the apparatus apart, in fact can be made automatic cleaning. The installation will be found justified by the reduced cleaning and maintenance that would otherwise be necessary.

These filters cannot be relied upon to decrease passsage of noise, and if noise is to be eliminated it will be necessary to provide baffles or mufflers, both in entering and leaving air passages. The type of ventilating installation may require the use of either forced or exhaust fans, or both. If the substation is under supervisory control and if the load has a definite daily characteristic, the fans can be controlled from the supervising station after the temperature characteristics of the station are known. The use of thermostats for starting and stopping fans is very satisfactory and quite reliable, and low in first cost. The layout of the ventilating systems requires some careful thought and study; several schemes can be used, that is, sufficient air can be discharged into the room to keep the temperature of the room in the vicinity of the machines at the proper point, or the duct system can be so arranged that air is dicharged into a pit under the machines or from ports at the side of and adjacent to the machines. It is often difficult to provide a duct system that is reasonable in cost, due to space limitations, and under such conditions it might be advisable to discharge a larger amount of air into the room and do the cooling a little less efficiently.

Two installations, designed by the writer, which have been in operation for more than two years, have shown the effectiveness of discharging the air into the room in the vicinity of the machine, rather than using a costly duct system to convey the air under the machine. The standard machine of today draws in the air at the ends and in a direction parallel to the shaft. Therefore, if air enters the pit under the machine, it must find its way to the ends of the machine; hence the effectiveness of the air discharged into the room in the vicinity of the machine. If there is a number of induction regulators installed with barrier partitions, only sufficient air is required to keep the air moving to prevent dead pockets and hot spots.

If the transformers are located inside of the building, some ventilation will be required, the amount and type depending largely upon the size of the transformers and their location.

Double walls or similar construction to prevent noise introduces an effective insulation which decreases the radiation of heat through the building walls. Due consideration must be taken of this condition when designing the ventilating system, as well as the fact that the automatic substation is usually smaller than an attended substation.

The building should be of fireproof type.

Fire Risk. The question of fire risk has received perhaps the least attention and yet it is one of the most important considerations. While electric fires are few in number, they do occur, and if it happens in an unattended station, considerable damage may result before anything is known of the accident.

It is therefore most desirable to locate transformers out of doors, if this can be done. It may be necessary to house them in a separate building or part of the substation; it is entirely probable that they can be located outdoors in such a way as not to be unsightly, partic-

ularly if they are protected by a wall or screen of proper design. In the event such transformers are located outside, it is advisable to have all connecting leads wiped in rather than have exposed bushings, see Fig. 1.

Oil switches should be in compartments with barriers

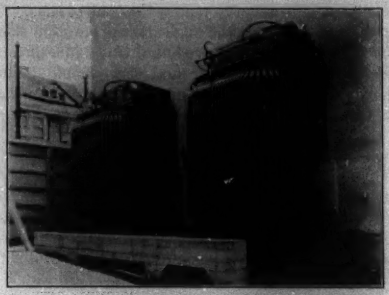

FIG. 1—THREE-PHASE, 11-KV., 1500-KW. TRANSFORMERS

Note the wiped-in high-voltage leads. Secondary leads are housed in a copper box. These leads enter the building in a protected box

arranged to prevent the spread of fire and the flow of oil in the event a tank bursts, (Fig. 2). Induction regulators cases should be of strong construction, preferably the boiler plate type, and there should be proper barriers between each group, a small pit or oil dam being provided for each set. If there are high-tension switches, it will be advisable to locate them in a part of the building isolated from the other apparatus, and should there be any large number of switches or regulators, it may be advisable to erect partitions which will provide a number of small rooms instead of one large open space. Such partitions are not difficult or costly to install. The location of the apparatus with reference to adjoining apparatus, wiring on the switchboard, etc., should receive most careful consideration.

If a fire should occur, it is of course most necessary to have the ventilating fans stop. This can be done by locating fusible links at critical parts of the room (which parts would be the first to get hot) the links so arranged that they will open the control circuit of the ventilating fan; and if the substation be supervisory or the indicating type, the opening of such circuits could be registered at the control point.

In a railway station, there is always the possibility of the bus becoming grounded and the machines will then feed into the short circuit until the load limiting resistors commence to function through the operation of the temperature relays. This short circuit might start a serious fire if the apparatus is not properly spaced. It is possible to insulate the switchboard and its supports and then ground the frame through a grounding relay, which will trip all d-c. feeders and machines, thus clearing the bus. This type of protection, while very effective against the bus grounding, may cause a total interruption if some of the auxiliary devices ground, particularly if the grounding relay is not set high enough. It is a question of the choice of two evils.

As there is always the chance of a regulator breaking down and of oil being thrown out of the case, this might result in a serious fire. As a fault in the regulator usually goes to ground, it is possible to protect, in part, against this hazard by insulating the regulator cases from the ground and then grounding the cases through a grounding relay. In the event of a failure of the regulator, the operation of the grounding relay locks out the circuit so that the reclosing devices do not allow the feeder switch to be thrown in again. This device is very reasonable in cost and it has been found to be an excellent protection.

If the substation be a large one, the question of fire risk should receive careful attention. It is the one case, perhaps, where the human attendant may be relied upon better than the automatic devices, unless it is proposed to equip the station with some form of fire protection which will shut off the sources of supply and flood the station or room with a gas that will quench the fire. Such installations are possible, but perhaps not advisable from an operating or cost standpoint.

Arrangement of Apparatus with Relation to the Building Structure, and Grouping to Facilitate Quick Inspection. The designer of the station must consider both of the

FIG. 2—11-KV. SWITCHES IN STEEL COMPARTMENTS

A simple roll-out type of switch is used requiring less space than the truck type. Control connections are made through flexible couplings

above features when laying out the station, as it is essential that an economical arrangement be had and at the same time it is equally essential that the apparatus be so grouped that trouble can be readily located if and when it develops. There are really two separate ideas,

which are, in actuality, combined in the final design, but each must be borne in mind when planning the job.

The additional auxiliary apparatus necessary in an automatic substation requires more room than that ordinarily required in the attended station, and as there are many devices which will interconnect and interlock, a careful study of the physical layout is necessary to produce an economical installation. In railway substations in particular, the switchboard room required is several times that of the non-automatic; the heavy duty resistors, high-speed circuit breakers, circuit interrupters, etc., also require additional space and as these various pieces of apparatus must be interconnected, it is essential that the grouping be carefully studied.

The transformers, machines, and the auxiliary devices will cost a certain sum and this cost is a constant irrespective of the arrangement of the apparatus. If the location of the various units is not studied carefully,

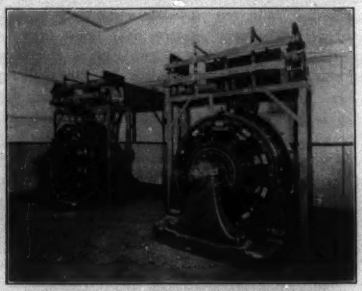

Fig. 3—Two 1500-Kw. 600-Volt Converters

Note short leads from the machine to the two high-speed circuit breakers on negative side. The metal boxes overhead enclose the secondary leads from transformer

it is surprising what sums can be spent for interconnecting cables, buses, conduits, and control leads; it will be found that the building costs will also be higher if these features are not well worked out. Where rotary converters are used, the placing of the transformers outside of the main building, as previously mentioned, has distinct advantages in reducing the cost of the building and ventilation. Such transformers should be located as close to the machines as possible, in order to reduce the length of the secondary leads. The high-speed circuit breakers when on the negative side of the machine should be as close to the machine as possible and adjacent to their complementary resistors; the machine and breakers can be treated as a unit because all this equipment must be out of service if it is desired to do any work on any part of the unit. If these devices are not located close to the machine, long runs of heavy cable and control leads are necessary. An example of how this apparatus may be grouped and made a part of the unit is shown in Fig. 3.

As the load limiting resistors are quite large and are likely to get quite hot, they should be located well overhead where the heat is easily dissipated and yet as close to the bus as possible, in order to reduce the heavy copper runs necessary; there are devices that are interlocked with the feeder breakers and they should likewise be close to them to reduce the control runs. The contactors used with the load limiting resistors should be located so that the arcs will not travel in a direction where damage may occur and they should be well above the floor line so as to prevent the arc's blowing in the face of a passer-by. Where heavy arcs are likely to occur, extra spacing and barriers should be provided to confine them effectually. Where fuses are used on 600-volt potential or control circuits, the cartridge should be enclosed in a non-conducting fire resisting box; this is particularly necessary on the back of switchboards where the failure of a fuse to function properly is likely to cause serious destruction to the adjacent wires. The detail wiring of the railway switchboard should be given most careful consideration, as there is a large number of wires carrying several voltages, both alternating current and direct current, on the back of the board and it is essential that the spacings be ample, lest a fault on one communicate trouble to the others, which may render all the protective apparatus inoperative.

The standard size of railway boards where high-speed breakers or circuit interrupters are used, has been a 90 in. high by 20 in. wide panel, on which panel it has been customary to mount all of the apparatus except the high-speed breaker, this device being mounted on a supporting frame at the top and to the rear of the face of the panel. It is necessary to have two disconnecting switches for isolating the breaker or interrupter to do maintenance work. Such an arrangement takes but little space, but has a decided disadvantage of crowding too much apparatus on one panel, allowing insufficient spacing for relays, fuses, etc. and, furthermore, producing a very congested mass of wiring on the back of the panel. Severe burnouts have taken place, particularly when a 600-volt piece of apparatus, such as a relay, goes bad. The congested condition on the back of the switchboard is worthy of attention, as there are control wires of several voltage classes, heavy busses, and interconnecting leads, all crowded into a very close space, rendering it somewhat hazardous to do maintenance work and resulting in rather bad burnouts if faults occur. If the panel is widened, it makes the switchboard rather long, yet it does not materially assist in relieving the congested wiring condition on the back of the board.

In order to overcome these objectionable features, it may be found advisable to arrange the switchboard in two sections, back to back, the machine relays and instrument equipments, the feeder equipments such as the relays, and circuit interrupters being on the front of the board, and on the board to the rear, there are placed

the disconnecting switches and such relay and metering equipment as it is not essential to have on the front of the board, (Fig. 4). This arrangement has several decided advantages. It makes it possible to reduce the length of the building, but more important, it allows the placing of the heavy bus in a position well above the other apparatus, thus making it easy to connect the bus to the breakers or circuit interrupters and the disconnecting switches, and at the same time leaves the backs of the boards clear of heavy copper, which, under the

FIG. 4—FRONT OF SWITCHBOARD

Contains 11-kv. control panels, automatic equipment for two machine control, and four feeder circuit breakers. Note the load-limiting resistors on the gallery

older type of installation, makes it extremely difficult to get at or change incidental wiring on the rear of the panels.

The interconnecting and interlocking control leads, of which there are quite a few, can be carried in metal troughs or conduits between the two boards, thus allowing for easy interconnection to the several devices. It will be found advisable where there are several classes of control wires, to carry each voltage class of wires in separate conduits rather than mingle the several classes in one conduit, as this makes it very easy to shoot trouble and further provides some additional safety in case one of the wires grounds. A general view of such a type of installation is shown in Fig. 5.

These are details that seem hardly worthy of mention, yet if a detailed examination of some installations is made, it will be found that the above features are most important in obtaining economy in the first cost, and safety and speed in maintenance work. It should be borne in mind that where there is no attendant in the station, it is not always known just what happens and when the inspector arrives on the job, it is essential that the arrangement be such that he can readily find the trouble without having to resort largely to a cut-and-try method of isolating the component parts of the several circuits.

It is essential that careful attention be paid to the seemingly small details, for experience shows that more faults occur from the small apparatus than from the major parts of the equipment. Many relays and auxiliary devices are necessary, particularly with d-c. apparatus, and these relays should be carefully marked and lettered. Of course, the operating man must have a thorough knowledge of the equipment as the position of the relays will often be a guide in informing him as to just what piece of apparatus has functioned and much delay in rectifying trouble can be saved if he will observe the position of the relay, that is, if it is out of its normal position, such as being closed when it should be open or vice versa. Particular care should be taken with the lettering and symbol marking of the switchboard wiring and equipment.

CONCLUSION

The automatic substation has been developed to a high degree of perfection. It can be relied upon to deliver an excellent type of service, particularly if proper precautions are taken in the design and installation: in fact, in certain cases it will deliver better service than that given from an attended station. Careful inspection and maintenance will prevent much trouble.

FIG. 5—VIEW BETWEEN FRONT AND REAR SWITCHBOARDS

Note the absence of heavy copper runs on back of panels. Metal trough overhead is divided into three compartments with removable sections on sides. Note ease with which the interconnecting leads are taken out of this box to various control panels. Barriers on left are between the d-c. feeder panels

This development has provided the means for locating the supply station at those points in the distribution network where previously such a station would not have been possible, and as a result large economies in the distribution network can often be effected by the application of the automatic substation, but to make this possible it is essential that the substation investment should not overbalance the saving to be effected in the distribution system. The cost of the supply lines,

real estate, and buildings often determines the economic solution, as the cost of much of the apparatus is almost a constant no matter where it is located on the system.

The type of control should receive most careful consideration, particularly if the station is not of extraordinarily large capacity.

With careful design, proper spacings, and isolation of apparatus, the fire risk can be reduced to a minimum.

Further development of the apparatus and control systems and careful attention to the schematic design of unattended stations and its connected system will produce, in the future, even greater economies (with improved service) in the distribution of electrical energy, than are apparent today.

Discussion

M. T. Crawford: The Puget Sound Power & Light Co. provides automatic substations not only in city residential areas, but in a number of suburban and rural districts. In the city, the substations are of the full automatic type and the amount of load handled warrants the installation of the best and most modern equipment available.

A different economic problem presents itself, however, in the case of a substation to supply a suburban or rural community where a load of only a few hundred kilowatts is involved. If this load is largely suburban homes and farms, it may represent three to five thousand consumers. The high degree of electric range saturation on our company's system, the large number of water heaters, electric brooders, and other domestic appliances, has brought us to face a situation where an interruption of thirty minutes may cause great public inconvenience and impair both public relations and sales. It has, therefore, become rapidly very important to safeguard against interruptions on this class of service, and yet at the same time the highest economy is essential.

Our present practise for this service usually consists of an installation of three transformers up to 200 kw. maximum size each, on a two- or four-pole structure, transforming from the 13-kv. local transmission line down to 2 kv. or 4 kv. for distribution purposes. These voltages are in the more closely built areas where farms consist of only a few acres each of rich soil, or in suburban residential sections near the city. In the outlying regions, where the load is more scattered, high distribution voltages are employed.

We have equipped a number of these pole-type transforming stations with automatic switching on both primary and secondary side. On the 13-kv. side, two pole-top sectionalizing switches of standard type are installed, connecting the transformer bank to either of two 13-kv. supply lines. Automatic operation is provided which switches the load over from the preferred to the auxiliary supply line in case of a power failure on the preferred line.

On the 2-kv. side, induction regulators are installed for voltage regulation and crossarm-mounted oil switches are provided for outgoing feeders and equipped with automatic reclosing devices. The automatic equipment on these installations is of a comparatively inexpensive type and the general class of construction is also of a very modest character. The total cost runs around $12,000 for a 600-kw. installation with three regulated lighting feeders and one unregulated power feeder, including automatic equipment. It is true that the cost per kw. is no less than a high-class full automatic substation in the city where built for large capacity; however, this cost per kw. would be very much higher if a high class installation were put in for only 600 kw. In other words, for a modest total investment, a substation can be built in suburban territory which will have automatic features for protecting the service and which will be entirely adequate for a period of five to ten years while territory is developing. If it assumes urban proportion at the end of that time, a permanent high-class substation can be built and the less expensive equipment readily moved farther out to newer territory.

The problem of a-c. or d-c. service in congested areas of large cities has to be settled on a basis of the economics of the local situation, as the author points out. In Seattle the older d-c. system was paralleled by an a-c. network with interleaved feeders and automatic network switch protection. This was done eight years ago, and although the d-c. system has been kept up with full battery protection, and for a number of years past no effort has been made to require customers to take a-c. service rather than d-c., nevertheless the a-c. load has grown very rapidly and the d-c. load has just held its own, due to customer preferences. A certain amount of d-c. service is required, however, to meet public needs fully. The automatic substation described by the author, installed in moderate sized units on customer premises, admirably meets this need in new territory where no older d-c. system exists.

The need for remote load-indication equipment is very acute in some situations where there is no well defined load characteristic at various hours of the day, such as trunk-line railway electrifications. We hope soon to have equipment perfected for trial installation which will give a totalized indication at one point of three large blocks of power at different locations, the maximum single distance being 120 mi. We now have satisfactory carrier telephony over this route via 110-kv. lines, and there seems to be no reason why carrier impulse indications cannot be transmitted for remote metering purposes.

C. E. Carey: I had hoped the paper would bring out the economic relation of the number of small automatic substations and inherent low feeder losses as compared to the large manually operated substation and high feeder losses. Somewhere between these extremes there is an economic balance and that economic balance bears a very important relation to the distribution system.

The automatic substation places in the hands of the operating men a system of high-voltage transmission and low-voltage distribution. It places in their hands a system of a-c. transmission and d-c. distribution which is quite similar to that used for a-c. systems. The automatic substation is as reliable as any piece of apparatus we have, whether it is a-c. or d-c. This fact has been recognized on the large suburban and interurban electrifications which have made use of the automatic substation which not only reduced the operating cost but also increased reliability of service.

The problem of applying automatic substations to a distribution system resolves itself into an economic study. The economic study is the same as any other engineering economic problem, namely the balancing of the fixed charges against the operating expenses. Restating that problem means to find the lowest annual operating expense for a given combination. This would be my attack and method of applying automatic stations to any distribution system.

I should like to discuss the question of fire hazard mentioned in Mr. Lisberger's paper. One of the developments to reduce this hazard is the inertaire transformer, in which an inert gas has replaced the oxygen in and above the oil.

Mr. Lisberger states that induction regulators should be made with boiler-plate tanks so as to confine any explosion within the induction regulator in its case. This does not seem to me to be the true answer. When you consider the enormous energy released at the time of a secondary explosion you find that the thickness of the boiler-plate tank must approach more nearly an armor plate. We have provided regulator tanks with corrugations so designed that they will expand in case of explosion. Actual tests by the use of controlled bombs have shown that the

corrugated tank will withstand explosions which rip open the boiler-plate tanks.

L. R. Gamble: In reading Mr. Lisberger's paper, I noted particularly his discussion of the use of the automatic d-c. substations for supplanting the present tendency to use a-c. networks. It seems that it is always the question of economics which must be considered in the final analysis, although the public must not be forgotten. Their demands must be supplied.

I feel that in a few years from now direct current will be quite a relic. People are demanding more and more the a-c. service, and why? It is a question of maintenance with them because the a-c. motor does not require nearly the maintenance that a d-c. motor requires, and as a result, the demand is for a-c. service and the power companies are going to supply it.

Analyses made in Spokane, and on other systems would indicate that the annual charges for an a-c. network run from 15 to 25 per cent lower than those for a d-c. network. With these points in view, it seems that the direct current will be a thing of the past in a few more years.

Chester Litchtenberg: A vision of important electric system changes is disclosed in this paper. A radical step in the design of distribution systems is discussed.

One feature makes it difficult to establish broad standards for automatic-station application at this time. It is the embryonic state of the automatic station art. Combined with this is a period of rapid development. For example, remote load-indicating equipment is just now being developed to a point where successful application is practicable. This extension of the automatic station art is sure to make for a broader field together with a more intensive use of the equipment.

The recent development of less expensive curve-drawing instruments is just being announced. These will make it possible to record information on circuit conditions as they occur in automatic stations. It will also make possible less frequent visits to these stations for the purposes of obtaining records. Moreover, they will permit less voluminous records to be kept. These records too will be more accurate than those usually available.

Automatic switching equipment with its extensions into supervisory control and telemetering now forms the nucleus for radical revisions in the art of generating, transmitting, and distributing electric power. As these new developments become more familiar to electrical engineers, greater economies in operation will result.

P. B. Garrett: Mr. Lisberger has drawn certain general conclusions which do not seem entirely justified in view of the fact that the experience in question has been limited in some cases to certain types of equipment only, and does not cover all such equipments available for the applications outlined.

Mr. Lisberger states that remote load-indication equipment in the present state of development cannot be regarded as entirely satisfactory. It should be noted that a number of large operating companies in other sections of the country have experienced very satisfactory operation of remote metering equipments of other types over a period of several years.

Experience and tests have demonstrated the ability of the corrugated welded steel tank of induction feeder regulators to withstand shock from internal explosions. Mr. Lisberger's paper would tend to indicate greater strength in the boiler-plate type.

As Mr. Lisberger states in his paper, location of the high-speed breakers on the negative side of the machine does make it very desirable to locate them as close to the machines as possible. However, when the high-speed breaker is inserted in the positive lead, which is equally as desirable, it is often found convenient to mount it above and to the rear of the switchboard. No extra lengths of cable and control leads are necessary in this case since the positive cable is normally carried to the switchboard.

Mr. Lisberger lays considerable stress on the matter of speed in maintenance work. It should be noted that with the type of equipment which allows for annunciation of any or all causes of shutdown, it is not necessary to resort to a cut-and-try method of isolating the component parts of the several circuits. Due to the location of certain relays, such as grid thermal relays, and also due to the necessity of removing the covers of certain types of relays to observe their physical position, much delay in rectifying trouble can be eliminated by the use of annunciators grouped in a single relay, which tell the cause of shutdown at a glance.

Mr. Lisberger's outline of the application of distributed automatic substations to supply an existing Edison network offers interesting possibilities of improved service. Such a scheme eliminates the cost of a changeover to an a-c. network but does not result in a saving in operating expenses comparable to the a-c. network. Independent studies made by several operating companies agree that the best type of a-c. network is not more than half as expensive in total annual charges as a corresponding d-c. network.

A-c. network systems are being operated, installed, or planned in a large number of cities, large and small, in all parts of the country. In such a system, all of the secondary mains in the area are tied solidly together to form a grid of conductors to which all customers are connected. This grid is fed by a considerable number of transformer banks which are connected to several independent primary feeders and located in vaults or manholes throughout the area. This renders the space requirement of the individual transformer vault comparatively small. The dependability of the Edison d-c. network and the economy and efficiency of the a-c. method of distribution using high-voltage feeders and transformer banks at the points of utilization are provided by the a-c. low-voltage network.

J. Hellenthal: I should like to touch on the automatic substation as applied to congested load centers. For our city loads we use fully automatic substations which give fully automatic control of supply lines, full protection for the transformer banks, and fully automatic protection to the feeders. We have found in five years' experience that the automatic substations are very economical from the standpoints of design and operation. In the design of the station we have to take into consideration various factors, the most economical grouping of equipment, the proper construction of the building to forestall fire hazards and to obtain minimum insurance rates.

We find that in the a-c. automatic stations, we get considerable variation in costs per kv-a. In the costs we include not only our buildings, our equipment, but also our land and the feeder run to the cable terminal pole. In our original designs we ran into costs that went up to $28 and $50 per kv-a. installed capacity. By giving considerably more study to our designs, we have reduced these costs dependent upon the size of the stations. For instance we have one station with the present installed capacity of 9000 kv-a., with sufficient building and structural facilities to double that capacity. With the present capacity our total cost is $25.09 per kv-a. When this station has been developed to its ultimate capacity our cost will have been reduced to $12.04 per kv-a.

This station is a modern fireproof building with decorative features, in the heart of a residential area, with ample shrubbery and everything about it to make it a very attractive building.

In our other designs we have gone to a more compact station, built upon a unit principle, where so many feet of building will provide for a certain number of incoming lines, transformers, and feeders. As our demands increase we add another unit to the building. This type of station we have developed to 6000 kv-a. installed capacity for $18.55 per kv-a. When increased to 9000 kv-a., which is the ultimate capacity of the one unit, this cost will be reduced to $16.74. We feel this is a very representative cost and is very economical.

We maintain a rigid inspection of our substations. At present we have five full automatic substations with a total kv-a. capacity of about 32,000, with provision for more than double this on the existing premises. In addition we have quite a number of industrial stations that are semi-automatic.

We have an inspector on duty at all times. He is provided with a car. The inspector must visit each station on each shift, giving twenty-four hours' service. This facilitates precluding any misoperation, and gives an opportunity to check up on his records to see what feeders and transformers are doing, and assists in much quicker restoration of service after a feeder has been locked out.

A. P. Sessions: Mr. Kellogg touched on automatic notification of switches tripping out in automatic stations. In the Southern California Edison Company we have found a very effective way of doing this by the use of our telephone lines, using the telegraph-type relay.

We have what we call switching centers or grouped supervision of our stations. These outlying automatic substations will report to the switching center. While the only indications we get is that something needs attention, it is always within a few miles of a switching center from which an electrician may be sent as soon as the switches trip out the third time. It is a very cheap way of getting results, of course, assuming you already have the telephone lines.

Caesar Antoniono: (communicated after adjournment) There is one phase of the automatic substation application on urban and interurban railway systems that should be more fully emphasized, an item of major importance in that class of service and that is the possible mitigation of electrolysis by the proper spacing and location of the substations as suggested, thus reducing the capital investment which would be necessary under certain conditions to prevent damage to underground structures.

The Electrolytic Zinc Plant of the Sullivan Mining Company

BY ELLERY R. FOSDICK[1]

Associate, A. I. E. E.

Synopsis.—This paper describes the recently constructed electrolytic zinc plant of the Sullivan Mining Company which is located near Kellogg, Idaho. The first unit which has just been completed has a capacity of 50 tons of slab zinc per day. Provision has been made for two additional units which will bring the capacity up to 150 tons per day.

The "Tainton-Pring" electrolytic process, which uses strong acid for leaching and high current density for electrolysis, is employed in this plant.

The metallurgical process and the physical layout of the plant are briefly described, after which follows a general discussion of the electrical features.

General

THE Sullivan Mining Company is owned jointly by the Bunker Hill Sullivan Mining and Concentrating Company and the Hecla Mining Company. It owns the Star Mine located near Mullan, Idaho. This mine is connected, by means of an 8900-ft. cross-cut, with the 2000-ft. level of the Hecla Mining Company's shaft. A complex silver, lead, zinc ore is mined in the Star ground, trammed through the cross-cut, raised through the Hecla shaft, and is then shipped to a concentrating mill at Kellogg. The zinc concentrate from this mill is then taken a distance of one mile to the electrolytic zinc plant. Zinc concentrate from sources other than the Star Mine will also be treated, this being a custom plant which will afford an outlet for the refractory and complex zinc ores of the Coeur d'Alene district. Construction on the first unit was started in August, 1926 and practically completed by August, 1928.

The erection of this plant is the result of much experimental work in developing a method of treatment for these ores. The plant has a nominal capacity of 50 tons of slab zinc per day but provision has been made for additional units to increase the capacity to about 150 tons daily.

Metallurgy

The Sullivan Plant employs the "Tainton-Pring" electrolytic process, which is based on the use of strong acid in leaching and of high current density in the electrolysis. The acid strength is about 28 per cent and current density about 100 amperes per sq. ft.

Concentrate. Briefly outlined the process is as follows: Zinc concentrate is roasted and the roasted calcine is magnetically separated into two parts, (1) a magnetic portion, containing most of the iron and the zinc ferrite which is formed in roasting the ore, and (2) the non-magnetic part, containing practically all zinc oxide.

Leaching. The magnetic portion containing the zinc which is not readily soluble is added to the hot, strong 28 per cent acid which attacks and dissolves the zinc compounds. The oxide portion is then added to this pulp to neutralize the unconsumed acid and to precipitate iron and other impurities which have also been dissolved by the acid.

Purification. The pulp is then filtered. After being dried the residue, containing the lead and silver which was in the original zinc concentrate, is shipped to the Bunker Hill smelter for the recovery of these metals. The zinc solution is agitated with metallic zinc to precipitate cadmium, copper, and cobalt. It is then filtered again after which the neutral zinc sulphate solution goes to the electrolytic cells.

Electrolysis. The electrolytic cells operate at an acid range between 22 and 28 per cent sulphuric acid. When the acid reaches the upper limit, a portion of the solution is withdrawn from the cell solution circuit and a corresponding volume of neutral zinc sulphate solution is added. This reduces the acid strength to the lower limit. The strong acid which is withdrawn is used in the leaching of the calcine as previously described. The process is thus cyclic.

The heat developed in the electrolysis raises the temperature of the electrolyte. This therefore is circulated through coolers consisting of lead-lined boxes in which are placed lead pipe coils through which cold water is flowing. The flow of warm electrolyte is counter current to that of the cooling water in the coils. The electrolyte after being cooled returns to the cells in continuous circuit.

Arrangement of Generator Building, Cell Room, and Substation

Geographical Location. The site of the new electrolytic zinc plant is in a canyon known as Government Gulch near Kellogg, Idaho. The buildings of the electrolytic zinc plant are placed upon the east slope of the gulch. See Figs. 1, 2, and 3 for the general layout. This is an advantageous location, for most of the solutions can flow by gravity from the higher buildings down through the various processes to the cell room, thus reducing the number of solution pumps to a minimum. A further advantage of the location

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is taken by placing the stack upon a saddle of the east side of the gulch about 200 ft. above the highest building of the plant.

Buildings. The main building in which the metallurgical processes are carried on is 216 ft. long and 305 ft. wide. The generator building is 57 ft. wide and 54 ft. long and the electrolytic cell building is 57 ft. wide and 271 ft. long.

Fig. 1—General View of Electrolytic Zinc Plant

All footings and substructures are constructed of concrete designed for 3000 lb. strength. The coarse aggregate used consisted of tailings from the Bunker Hill & Sullivan Mill.

The buildings are steel frame structures with the walls made of wood girt studs on which was stretched wire netting. Concrete was then sprayed on the netting against forms placed on the outside of the wall, in this way forming concrete panels and concrete covered studs. The result is a type of wall called gunnite.

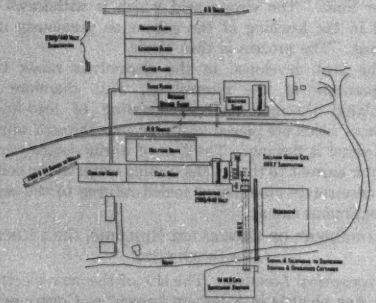

Fig. 2—General Layout of Electrolytic Zinc Plant

The roofs of the buildings were built up of 2 by 4 in. timbers placed on edge and side by side over the steel trusses. Waterproof roofing material was laid upon this surface.

Fully 25 per cent of the wall space of all the buildings is occupied by windows. The large amount of window space together with white ceilings gives very satisfactory daylight working conditions. Artificial illumination with an average intensity of 6 foot candles is provided in both the generator building and the cell room, while in other parts of the plant it is of somewhat lower intensity but is ample for the work to be done.

Substations. The generator building is located at the south end of the cell room, and immediately adjacent to this building is the Sullivan Mining Company's 110-kv. outdoor substation. Opposite the Sullivan Mining Company's substation on the west side of the gulch is the Washington Water Power Company's 110-kv. outdoor switching station.

Fig. 3—Another View of Electrolytic Zinc Plant Showing Power Plant

Equipment. In the generator building there is a main floor and a basement. The synchronous motor-generator set bases are resting on the main floor. Machine foundations extend down to footings below the basement floor. The walls of the machine foundations have openings for ventilating louvers.

Fig. 4—Synchronous Motor-Generator Sets

The main control board is at right angles to the shafts of the two machines, and is located on the main floor. See Fig. 4.

A 2300-volt three-phase bus and switching structure is located in the basement directly below the main control board. Here the synchronous motor-generator, starting, running, and compensator oil circuit breakers are mounted with their accessory equipment.

On the opposite side of the basement, and in line with the copper busses from the generators to the

electrolytic cells in the cell room, is the 500-volt d-c. switching equipment.

All conduit runs were kept in the concrete forming the floors and walls of the generator building. It was desired to keep the basement free from exposed conduit because of the large amount of bus copper to go in, and also to reduce to a minimum any windage losses in ventilation. Numerous junction boxes and pull boxes were provided to facilitate future changes and extensions of station control wiring.

Fig. 5—110-Kv. Outdoor Transformers

The air washer and blower equipment for the motor-generator sets is at one end of the basement placed beside, and discharging toward Unit No. 1 foundation.

Each generator and each synchronous motor has a motor driven remote controlled field rheostat mounted on the side of the machine foundation.

Direct current for oil circuit breaker operation, signals, and emergency lighting in the cell room and generator room is supplied by a 60-cell Exide type E-11 storage battery located in a separate room in the basement of the generator building.

The generator room is equipped with a 25-ton Whiting crane having the travels operated manually by means of endless chains but with the hoist motor driven.

Substations

Power Supply. Two transmission lines feed the Washington Water Power Company's switching station, one being a 110-kv. transmission line from the East Side Substation at Spokane and the other a 60-kv. line from the Post Falls Generating Plant. The 60-kv. line is connected through a 15,000-kv-a. transformer bank to the 110-kv. line. Continuity of service is further insured by a third line operating at 110 kv. extending from this switching station through the larger substations of the Coeur d'Alene Mining District and joining at a point near Burke, Idaho, the 110-kv. line from the Montana Power Company's Thompson Falls Plant.

Substation Equipment. The Sullivan Mining Company's Substation has a 10,000-kv-a. transformer bank protected by an oxide film lightning arrester and a 110-kv. oil circuit breaker. The transformers are 110k-v. to 2300-volt, oil insulated, self-cooled, delta-delta connected. See Fig. 5.

Power Cables. Twenty-three hundred volt three-phase power enters the generator building through four 750,000-cir. mil, three-phase lead-covered cables connected in parallel. These cables connect to the 2300-volt indoor switching structure to which the two synchronous motors and five 2300-volt plant feeders are connected through oil switches. See Fig. 6.

Plant Feeders. Three of the 2300-volt plant feeders terminate in smaller outdoor substations where the voltage is stepped down by transformers for 440-volt, three-phase power and 110-volt lighting. One of the 2300-volt feeders supplies power to the 2300-volt, three-phase well pump motors located at the wells in the Coeur d'Alene River Valley about one mile away, the other feeder supplies 2300-volt, three-phase power to the pumps for circulating the electrolytic cell solution.

Synchronous Motor-Generator Sets

Choice of Generating Equipment. After carefully considering the demand to be made upon them and the reliability of the source of power, two three-unit synchronous motor-generator sets were chosen to supply direct current to the electrolytic cells. Although their efficiency is somewhat lower it was thought best to use synchronous motor-generator sets instead of booster synchronous converters. The following are some of the advantages that favor synchronous motor-generator sets for this particular installation.

Fig. 6—2300-Volt Indoor Switching Structure

1. They are more stable, not being affected to such a great extent by surges in the long transmission lines.
2. It is easier to attain automatic regulation with them for they require fewer automatic features.
3. They have a wider range of voltage.
4. They will correct the power factor of the total plant load.

5. Only one 110-kv. transformer bank and one 110-kv. oil circuit breaker with the accessory equipment are needed. A synchronous converter producing 500 volts d-c. would not operate at a voltage on the alternating current side which would be suitable for power distribution about the plant. It would, therefore, be necessary to have a second 110-kv. transformer bank for this service when using synchronous converters.

Synchronous Motor-Generator Sets. The synchronous motor-generator sets are of General Electric Company manufacture and supply the current for each electrolytic cell unit of 150 cells. The normal output is 8000 amperes d-c. at 500 volts. The synchronous motor of the generating unit is rated at 4400 kv-a., 2300 volts, speed 514 rev. per min. On each end of the motor shaft is a 4000-ampere, 500-volt shunt wound d-c. generator. A direct connected exciter supplies field current for both the synchronous motor and d-c. generators.

The following tables give the efficiencies of the synchronous motor-generator sets and transformers:

Power factor	Synchronous motor-generators: Full load	¾ load	½ load	Per cent maximum current load for one min.
1.0	92	91.3	89.0	150

110-kv. transformers

Power factor	Full load	¾ load	½ load
1.0	99.0	99.05	98.95

Including transformer losses, the synchronous motor-generator sets have an efficiency of 91.0 per cent when operating at full load.

The motors are started on low voltage obtained from a compensator and after getting up to speed are connected to the 2300-volt bus. A voltage relay gives the desired sequence interlock between the starting and running oil circuit breakers.

D-c. Busses The copper busses connecting the generators to the electrolytic cells are quite long. The current flowing in them is approximately 8000 amperes and does not vary more than 4 per cent up or down. It was, therefore, necessary to study heating losses carefully in these busses. As a result of these studies, two curves were plotted, one using cost per year for each foot of bus with a 20-year life, 10 per cent scrap value, and 6 per cent interest on investment, and the other using cost of power loss per year for each foot of bus based upon the cost of power when the machine and transformer losses were included. Each of these cost curves was plotted cost against cross-section of the bus in square inches. The intersection of the two curves indicated quite clearly the most economical size of bus to use, which is 15 sq. in. cross-section for 8000 amperes or a current density of 533 amperes per sq. in.

Direct current is carried from each generator to the circuit opening devices by five bars ¼ by 6 in. copper. In each generator positive bus is a 6000-ampere, 650-volt d-c. air circuit breaker. Each negative bus has two 3000-ampere, 500-volt knife switches connected in parallel. After passing through the circuit opening devices, the two negative busses and the two positive busses each combine to form a bus of ten bars of ¼ by 6 in. copper. These carry the 8000 amperes of direct current to the electrolytic cells in the cell room. The total amount of bus copper used for carrying the current to the electrolytic cells weighs 61,700 lb.

When the plant is expanded to five units of generating equipment a transfer bus will be used. Unit No. 5 will be for stand-by service and can be used in place of any unit by means of the transfer bus.

Automatic Regulation. The cyclic variation in the conductivity of the electrolyte, taken in conjunction with periodic shut-downs of electrical apparatus for cleaning, oiling, and brush attendance, made it imperative to adopt some means for obtaining constant

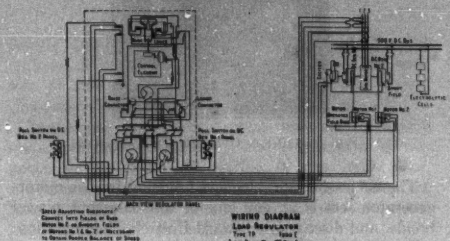

FIG. 7—REAR VIEW OF SWITCHBOARD AND WIRING DIAGRAM

kw. input to the synchronous motors in order to maintain the desired load factor. Automatic regulating equipment for this purpose was furnished by the General Electric Company, and regulation is obtained in the following manner:

Current transformers and potential transformers in the 2300-volt power cables to the synchronous motors of the motor-generator sets (see Fig. 7 for wiring diagram) have their secondaries connected to the control element, or contact making wattmeter of the automatic regulator which is mounted on a control panel of the main switchboard. This control element through a timing device, closes one of two pairs of contacts at regular intervals, or does not permit either of them to close, until some fluctuation in load causes a change in power taken by the synchronous motor. When a pair of contacts close they complete a circuit, by means of an auxiliary contactor, through the field rheostat head driving motors of both generators. The driving motor circuits closed, cause the driving motors to rotate the field rheostat brusharms. This regulates the field current so as to change the generator loads, either

increasing or decreasing them as needed, and so changes the power taken by the synchronous motor.

Machine Ventilation. In considering the ventilation of the synchronous motor-generator sets, the usual problem of washing and cooling air was encountered and, in addition to this, it was necessary to make sure that no air was sent through the machines without first going through the air washers.

The gases that leave the stack will have a large amount of sulphur dioxide in them; being readily soluble this combines with atmospheric moisture to form sulphurous acid. Even though the top of the stack is in a favorable air current and some 500 ft. above the generator room, it may happen that some of this sulphur dioxide and sulphurous acid will be present in the air taken in for machine ventilation. There is also the ventilating air from the cell room which at times may contain a mixture of dilute sulphuric acid and zinc sulphate solution, which may also be in the air supplied to the machines. Hence, it was thought advisable to wash the air thoroughly to remove both of these and any sulphurous acid that may be present.

A semi-enclosed system of machine ventilation was installed. The generator building itself is used as the enclosure around the machines. Enough cool air is drawn in from outside the building to keep the temperature of the air being drawn in through the washers down to approximately 65 deg. fahr. Air is drawn through the air washers, located in the basement, and is discharged into the basement by blowers. This washed and cooled air passes through louvered openings into the machine foundations and is taken up by the machines, circulated through them, and discharged into the generator room. From here it is drawn down again into the washers, mixing with the desired amount of cool air from the outside to maintain the correct temperature, and the cycle is repeated. The inside of the building is under a static pressure of about ⅛ in. in the generator room and ⅝ in. in the basement so that any leakage is out of the building rather than into it, which eliminates the possibility of any acid gas fumes around the machines and switchboards to injure insulation or tarnish metal fittings.

The temperature of the air flowing into the air washers is automatically maintained by means of thermostatically controlled louvers, a double set being used, one set opening into the building from the air washer intakes, and the other set opening to the outside of the building. These are mechanically interlocked so that as one opens, the other closes. When cool air is being drawn from the outside, a similar amount of heated air will be discharged through louvers at the top of the building.

The air for machine ventilation is circulated by two Buffalo Forge No. 14, single inlet, duplex conoidal fans. These are three-quarter housing type, top discharge with a capacity of 71,430 cu. ft. per min. at 192 rev. per min. against ½ in. static head. Each fan is driven by a 25-hp., 440-volt, three-phase induction motor. The cooling air is washed by two Buffalo Forge carrier generator coolers each having a capacity 72,000 cu. ft. per min. Thirty gallons of fresh water per minute is continuously replacing the water in each air washer sump to prevent building up the sulphuric acid content.

The generator building will be heated in winter by air discharged from the machines. There is a possibility that there will be enough surplus heated air to assist in heating the cell room located adjacent to the generator building.

Electrolytic Cells

Cell Construction. Each electrolytic cell contains 10 aluminum cathodes and 20 lead anodes. The 10 aluminum cathodes are approximately 1 ft. 8¾ in. by 3 ft. 4¾ in. by $^{5}/_{32}$ in. thick, and are suspended in the electrolyte contained in a lead-lined wooden box. Each cathode has two lead alloy anodes, one arranged on each side, and each one spaced ½ in. from the cathode.

Fig. 8—Electrolytic Cells

The anodes which are slightly smaller than the cathodes are approximately ¼ in. thick. Each cell group consists of 150 cells with the electrical circuit connected in series. The present installation consists of two such cell groups. Fig. 8 shows the cells before the anodes and cathodes were installed.

Anodes and Cathodes. The lead alloy anodes are made in the form of flat grids, being perforated with square holes. The aluminum cathodes are smooth, flat sheets.

Two anodes are used with each cathode to:

1. Facilitate the flow of electrolyte between the anodes and cathodes.

During electrolysis the circulating electrolyte enters the space between the anode and cathode through the holes in the anode. By using two anodes for each cathode it is possible to have a space between the two anodes of the two adjacent cathodes from which is taken the supply of electrolyte that is circulated between the anode and cathode. Thus, the zinc content of the solution at all places between the anode and cathode is nearly uniform.

2. Reduce the current density in the anodes and so reduce electrical losses resulting in higher cell efficiency.

3. Reduce gassing at each anode. More gassing takes place at the anodes than at the cathodes. Excessive gasing at the anodes will result in gas pockets which reduce the amount of surface of the anodes which are exposed to the electrolyte and so lower the cell efficiency.

Electrolyte. Purified zinc sulphate solution is fed to the cells and recirculated until approximately 90 per cent of the zinc has been extracted from it. The purified solution going to the cells contains over 20 per cent zinc. The liquor returned to the agitator tanks after the zinc is removed contains approximately 28 per cent acid. The cell solution is circulated continuously and at intervals a portion is removed to the acid storage tanks.

There is parallel flow of electrolyte to the cells: that is, each individual cell of the series is fed from a common supply and the effluent electrolyte from the cells is collected in a common sump from which it is pumped back to the supply tank, thus completing the circuit of flow. The electrolysis is continuous so that the electrolyte is gradually depleted of its zinc content and increased in acidity. The process, in respect to the electrolyte, is cyclic in its operation but continuous in respect to the electrolysis.

The decomposition voltage of zinc sulphate is 2.36 volts, but the minimum allowable voltage per cell is 3.1 volts in order that zinc may be positively deposited on the cathodes. Each cell acts, naturally, as a storage battery, so that if short circuited, there is a counter e.m.f. of 2.7 volts per cell. There is, of course, a very heavy instantaneous current flow which falls rapidly to zero.

The potential drop per cell depends upon several factors such as conductivity of the electrolyte and the spacing and composition of the electrodes. Due to the fact that several of these variables may be subject to change in practise or upon further developments of the process, there is a possible range of 3.4 volts to 3.7 volts per cell with an average of 3.55 volts. These conditions may be met by varying the number of cells in series, and provisions have been made for this in selecting 150 as the number of cells per circuit.

Current Density. The 150 cells of any one unit are connected electrically in series. The ten cathodes in each cell are connected in parallel and divide the 8000-ampere current flow equally among them. Hence, there are 800 amperes flowing in each cathode, and the current flow at each surface of the cathode is 400 amperes. The current density at each surface of the aluminum cathodes is therefore slightly less than 100 amperes per sq. ft.

Current density in the cells is limited by two factors:

1. It may be as high as the cell will permit without excessive circulating and recurrent cooling of the solution before returning it to the agitator tanks.

2. It is limited by the necessity of keeping down power consumption.

Cell Efficiency. Calculations on zinc tonnage and other factors are based upon 85 per cent current efficiency. Under normal operating conditions the efficiency is considerably in excess of this, but at times it may fall to 80 per cent due to impure solutions.

Stripping Zinc Deposit. The electrolytic zinc deposited on the aluminum cathodes can be stripped at varying intervals of 12, 18, or 24 hr. The stripping is done without interrupting the circuit. A cathode is disconnected from its clips by being raised and suspended above the cell for drainage of acid electrolyte. The zinc is then stripped from the aluminum cathode and the latter is replaced in its copper clips. A group of five or ten cells in a unit of 150 cells can be short circuited at one time by a copper bar carried on an overhead track. After the short circuiting bar is in place across the header bars these cells may be cleaned and repaired before being again placed in service.

Telephone, Relays, and Alarms

Telephones. The generating room has a telephone connected with the Washington Water Power Company's transmission telephone system, so that in case of failure of power the attendant at the plant may communicate with the power company's system operator at once. There are also local telephone connections with the operators' residences, the office, and various parts of the plant, as well as with the city telephone system.

Relay Protection. The "running" oil circuit breakers for the synchronous motors and the 2300-volt plant feeder oil circuit breakers are tripped by both undervoltage and overload. The 6000-ampere d-c. air circuit breakers in the cell circuits trip from both overload and reverse current. The 110-kv. oil circuit breaker protecting the transformer bank trips from both overload and differential relays. The tripping of any oil circuit breaker due to overload, or of any air circuit breaker due to overload or reverse current, rings the alarm bell to attract the operator's attention. Signal lights mounted with the control switches on the control switchboard indicate which switch has operated.

Alarms. The Washington Water Power Company's substation normally receives power from two transmission lines, one 110-kv. and one 60-kv. line. The 110-kv. line is connected through an oil circuit breaker to the 110-kv. bus and the 60-kv. line through a transformer bank and oil circuit breaker to the 110-kv. bus. An auxiliary switch on each of these oil circuit breakers is connected by means of an overhead line to a signal in the Sullivan Mining Company's generator building so that upon power failure the operator will know if one of these oil circuit breakers opened.

Cottrell Precipitator

General Arrangement. The gases from the roasting furnaces pass through a balloon type flue running up

the hillside to the Cottrell precipitator located at the base of the stack. After passing through the precipitator these gases pass up the stack and out into the air currents.

The Cottrell precipitator is of the plate type consisting of ten sections divided into two groups of five sections each. Two high-voltage d-c. busses run the length of the Cottrell Precipitator Building. A suitable switching arrangement makes it possible to connect any section to either bus. The first four sections will normally operate at a higher voltage to give higher precipitation efficiency.

Equipment. The high-voltage direct current is supplied from two synchronous motor driven disk type rectifiers. Each rectifier disk gets high voltage alternating current from a 15 kv-a. single-phase transformer which has a 400-volt primary with reduced voltage taps and a 60,000-volt secondary. A variable grid resistor in series with the primary of the transformer makes it possible to adjust the voltage between taps.

Dust Recovered. The gases that pass through the Cottrell precipitator contain daily about 13.5 tons of dust or about 10 per cent of the concentrate fed to the furnaces. This dust consists mainly of zinc, lead, and silver compounds. The Cottrell precipitator removes most of this dust from the gases.

Individual Motor Drives

Mechanical power throughout the plant is furnished by 440-volt, three-phase motors. One means of attaining flexibility of plant operation was to install individual motor drives wherever practicable.

There are 156, 440-volt, three-phase motors connected to apparatus in the plant, the size of which range from ½ hp. to 150 hp.

Safety fused cabinets located at convenient places in the plant provide distribution points for power to the motors. To make them safe for the men to operate, all small motors are controlled by push button stations which operate magnetic contactors that are protected by fused safety switches. The larger motors are controlled by hand starting compensators which are protected by fused safety switches.

Conclusion

Throughout the design of the plant every effort was made to select and install equipment to facilitate flexibility of plant operation. This is particularly desirable in a plant of this type so that refinements in the metallurgical processes can be added without radical changes to the initial installation.

The writer wishes to acknowledge the helpful suggestions of Messrs. A. C. Stevenson, James B. Fisken, and L. R. Gamble, and the assistance of Mr. W. G. Woolf in the preparation of this paper.

Great Northern Railway Electrification in the Cascades

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Synopsis.—The paper gives first a brief history of the series of improvements to better operating conditions over the Cascade Range which have been undertaken by the Great Northern Railway since the line was put in operation in 1893, and refers briefly to the three-phase electrification of the Cascade Tunnel put in operation in 1909.

The paper next gives (Part II) a description and the principal operating characteristics of the motor-generator type of locomotives now in service on the electrified section from Skykomish (Washington) to Cascade Tunnel Station, and to be used through the new tunnel and extension of electrification to Wenatchee when work now in progress is completed.

Part III gives a description of the electric transmission and distribution system along the railroad line with particular emphasis on the relay protection designed to guard the system so far as practicable against interruption of power supply at the locomotives.

Part IV describes the three-point power supply and includes a description of the frequency converter stations at Skykomish and Wenatchee for converting the 110,000-volt, 60-cycle, three-phase power purchased at these points to 25-cycle, single-phase power required for the traction service.

* * * * *

Part I.—Introduction

THE changes now in process on the Cascade Division of the Great Northern Railway in the State of Washington are the latest steps in a series of successive important changes which have been made since this part of the railroad was put in full operation in 1893, all of which have looked toward improvement of operating conditions on the difficult mountain section over the Cascade range. The present changes include the construction of the new tunnel from Scenic to Berne (7.79 mi. long, the longest railroad tunnel in America), improvement of curves at many points between Berne and Winton, the construction of the Chumstick cut-off from Winton to Peshastin, and completion of electrification from Skykomish to Wenatchee.

The first operation of the railroad across the Cascades was by means of a switch-back which worked its way over "the hill" through Stevens Pass. The grades on the switch-back were so heavy that only a few freight cars could be handled at a time, and snow conditions in the Pass during the winter rendered operation of the railroad extremely difficult.

The first major improvement in these conditions was the construction of the present Cascade tunnel completed in 1901 from Tye Station at the west to Cascade Tunnel Station at the east, a distance of 2.63 mi. This Cascade tunnel (which is still in use but will be abandoned when the new tunnel is put in operation) eliminated the switch-back operations, greatly improved grade conditions, and avoided the section of deepest snows in Stevens Pass. It still left the long winding climb up the mountain with heavy grades and sharp curves and several miles of line where it has been necessary to protect the railroad from slides of snow and rock by construction of heavy snow sheds.

The next major improvement in this section was the electrification of this tunnel in 1909, which was undertaken to eliminate the serious smoke and gas conditions which had developed with increased traffic, and to improve operating conditions for heavy freight trains though the tunnel. This electrification was planned and supervised by Dr. Cary T. Hutchinson, as consulting engineer, and was a three-phase, 6600-volt system with two trolley wires, the rails being used as the third conductor. The locomotives were equipped with three-phase induction motors and could be operated at two speeds, *i. e.*, at approximately 15 mi. per hr. on passenger trains and light freight trains and (by Cascade connection of the motors) at approximately 7½ mi. per hr. with heavy freight trains. The steam locomotives were not removed from the trains but were pulled through the tunnel along with the trains. The greater part of the smoke was eliminated as the steam engines did not work in the tunnel. Electric power for operation of the system was supplied from a small hydroelectric generating station built and operated by the Great Northern Railway at Tumwater, in Tumwater Canyon on the Wenatchee River, about 30 mi. from the tunnel but on the line of the railroad. Transmission was at 33,000 volts, three-phase, over a double circuit line on wood poles, with a step-down substation at Cascade Tunnel Station, delivering 6600 volts, three-phase, to the trolleys. This is the only example we have had in this country of three-phase railroad electrification. It was in operation nearly eighteen years, and fulfilled the limited requirements for which it was intended.

The freight service east bound carries tonnage materially heavier than the west bound. With the former steam operation, except for the electric service in the old Cascade tunnel, heavy oil-burning Mikado steam

1. Jackson & Moreland, Consulting Engineers, Boston, Mass.

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locomotives brought the east-bound freight trains from the Interbay terminal yard near Seattle to Skykomish, the trains being made up to about 2500 tons trailing load. Skykomish is near the foot of the heavier grades on the western slope of the Cascade mountains. At Skykomish two additional locomotives (heavy Mallet type) were cut into the train as swing and pusher engines for the pull up the grades to Tye at the western portal of the tunnel. At Tye the steam helpers were replaced by the three-phase electric engines for the pull through the tunnel. This operation from Skykomish to Cascade Tunnel Station at the east end of the tunnel required a normal running time slightly in excess of 5 hr. for the 25 mi. of route, including the time of taking water at Scenic and the delay for cutting locomotives in and out of the train.

In 1925 the officials of the Great Northern Railway decided to take advantage of additional power available from the Tumwater power plant to improve operating conditions by electrifying the heavy grade section on the west slope of the Cascades from Skykomish to Tye, and utilizing electric helpers on this section. This is the ruling grade against east-bound main traffic on the entire system. On account of the complication of a double trolley and the operating limitation imposed by constant-speed operation inherent with the three-phase system, that system was not adapted for extension, and a single-phase, 11,000-volt trolley system with locomotives of the a-c.—d-c. motor-generator type was selected by the railroad officials as suitable to utilize the available facilities and to give the required service which included operation of both heavy freight and passenger trains. This scheme employs d-c. traction motors supplied with d-c. power by means of a motor-generator set carried on the locomotive. This system went into operation in February, 1927, over the 25 mi. from Skykomish to Cascade Tunnel Station, the old three-phase electrification through the tunnel between Tye and Cascade Tunnel Station being then abandoned.

For the service from Skykomish to Cascade Tunnel Station four electric locomotives were provided, descriptions of which are given later. The method of operation, on east-bound traffic, is to pull the trains into Skykomish by steam power, and there, at the foot of the heavy grade section, couple on one or more electric locomotives to pull the trains up the grade and through the tunnel. On account of the relatively short length of the present electrified zone, the steam locomotives are not taken off the trains but are run on through, assisting in pulling up the grade to Tye, but shutting off while passing through the tunnel from Tye to Cascade Tunnel Station. This method of operation has cut approximately three and one-quarter hours off the normal running time of freight trains between Skykomish and Cascade Tunnel Station.

The electric locomotives are also used to take trains down the grade west-bound from Cascade Tunnel Station to Skykomish, utilizing regenerative braking.

Comparatively soon after this improvement was conceived it was decided to improve operating conditions still further by construction of a new tunnel, now nearing completion. This tunnel lowers the summit 500 ft., shortens the route approximately 7¾ mi., eliminates over 1900 degrees of curvature, cuts down the length of the heavy grade approach, and avoids substantially all of the dangerous snow-slide area, thus

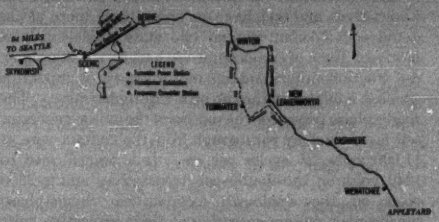

Fig. 1—Map of Electrified System of Great Northern Railway in the Cascade Mountains

eliminating 6 mi. of snow sheds. As part of this improvement it was decided (besides carrying the electrification through the new tunnel) to extend the electrification eastward from the portal of the new tunnel at Berne to Wenatchee, the eastern end of the Cascade Division, and to make extensive improvements in line and grade between Berne and Peshastin, including construction of the Chumstick cut-off which will replace the present line through Tumwater Canyon with its heavy grades and sharp curves. Fig. 1 shows an outline map of the old and the new line, and Fig. 2 shows a profile of the new line.

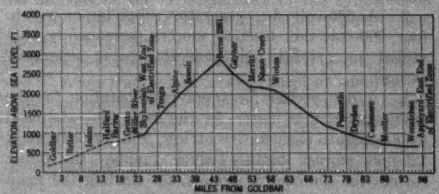

Fig. 2—Profile of Great Northern Railway in the Cascade Mountains after Completion of Present Line Changes

When these changes are completed the electrification will extend from Skykomish, Wash., at the foot of the heavy grade section west of the Cascades (84 mi. by rail from Seattle), to Wenatchee, Wash., a total distance, via the new tunnel, of approximately 72 mi., and all through trains will be operated with electric traction between these points. Five additional locomotive units are now on order for this service.

The remainder of this paper deals with the characteristics of the locomotives, the electric distribution system, and sources of electric power supply.

Part II.—Electric Motive Power

The motive power for the electrification from Skykomish to Wenatchee will consist entirely of motor-generator locomotives. The railway company now has in use two double-cab locomotives and two single-cab locomotives, and they have on order three more of the double-cab locomotives and two more single-cab locomotives. Each of the locomotives has geared, axle-slung, d-c. traction motors and has motor-generator apparatus which is mounted in the cab on a cab underframe. The single-phase power for the motors of the motor-generator sets is obtained by pantograph from the contact wire at 11,000 volts, 25 cycles per sec. It is transformed to 1240 volts on the double-cab locomotives and to 2300 volts on the single-cab locomotives. Single-phase synchronous motors operating at these respective voltages drive d-c. generators, which supply the d-c. energy to the traction motors.

The motor-generator set in each cab of the double-cab locomotives comprises a four-unit set. The four several units of this motor-generator set consist of:

1. A single-phase synchrorous motor operating at 1240 volts, the armature being equipped with an auxiliary phase winding in order to provide three-phase power for starting induction blower motors. One terminal of this auxiliary phase winding is connected to an intermediate point of the secondary winding of the main cab transformer. The other terminal of this winding, together with two terminals from the secondary winding of the transformer form a three-phase system for the power supplied to the blower motors.

2. A 600-volt d-c. generator supplying power to four 600-volt d-c. traction motors permanently connected in parallel, this generator being separately excited.

3. A d-c. exciter for supplying excitation for the main synchronous motor, for the main generator, and for a separate regenerative exciter, and for charging a 125-volt storage battery which supplies the direct current for the control circuits, for starting the main motor generator set, and for driving the air compressors.

4. A 10-volt regenerative exciter, which is used for separately exciting the traction motors during regenerative braking, and also for the low excitation used for high speed motoring.

These four units are mounted on the same shaft. The motor and generator of this motor-generator set are ventilated by a fan which is mounted on the shaft between the motor and generator armatures, the air being sucked in through the two electrical machines and discharged through the fan into air ducts which lead into openings in the floor and roof. This motor-generator set is started by means of the main d-c. generator operated substantially as a series d-c. motor supplied with power from the 125-volt storage battery. When brought up to approximately one-third speed the synchronous motor is thrown on the line at reduced voltage as an induction motor, by which means it is accelerated to approximately 80 per cent of full speed before the excitation is applied. Normal voltage is applied after the motor is in synchronism. Air blast transformers are provided in each of the cabs to transform the current collected by the pantograph to the voltage required by the synchronous motor, and a blower driven by a three-phase induction motor supplies air for ventilating the transformer, this machine being provided with the necessary current as heretofore described for the blower motors. The speed of the locomotive is chiefly controlled by varying the voltage of the main d-c. generator by adjustment of its field current by adjustable resistance controlled through contactors. Higher speeds may be obtained by putting the traction motors on to separate excitation, under which conditions the field strength can be reduced approximately 50 per cent.

The motor-generator set on each of the single-cab locomotives comprises five units: 1, A synchronous motor having full three-phase armature windings but operated as a single-phase motor for driving the motor-generator set, the armature at the same time acting as a phase balancer to provide three-phase power for driving induction motors on the blowers for the traction motors. 2 and 3, Two 750-volt d-c. generators connected in series and supplying current to the three groups of traction motors. 4, A 65-volt d-c. exciter for providing current to excite the synchronous motor, the two main generators, and the regenerative exciter, and to provide control current and storage battery charging current. 5, A d-c. generator used for separately exciting the traction motors for regenerative braking and also used as a single-phase series a-c. motor for starting the motor-generator set, in the latter function being connected to a special winding on the main locomotive transformer. The complete set is made up of synchronous motor and main exciter on one shaft, the two main d-c. generators with a blower between them and the exciter for regenerative braking on another shaft, the two being coupled end to end through a flexible coupling. The main locomotive transformer is arranged for air blast, two blowers being provided, each being driven by a single-phase repulsion commutator type motor to supply the ventilating air.

The speed of the locomotive is chiefly controlled by varying the voltage of the d-c. generators by adjusting the field current by adjustable resistance controlled through contactors, but the traction motors are equipped with field shunting means, with two operating positions, which permit operation down to 50 per cent of field strength.

Each cab of the double-cab locomotives is equipped with four 600-volt d-c. traction motors, axle-slung,

permanently connected in parallel. Each motor has a one-hour rating of 540 hp. with forced ventilation, and drives one driving axle of the locomotive through a flexible or cushion type gear mounted on the hub of each driving wheel. These locomotives were the first type purchased for operation on the electrification from Skykomish to Tye and thence through the old Cascade Tunnel to Cascade Tunnel Station, and were designed primarily for operation as common motive power equipment for both passenger and freight service. As now operated each of these locomotives consists of two cabs operated as a unit, and is of the $1-D-1+1-D-1$ type.

Each single-cab locomotive is equipped with six 750-volt motors, axle-slung, each driving one driving axle and geared to its axle through cushion-type twin gears. These motors are permanently connected with two in series and the pairs in parallel. Each motor has a one-hour rating of 550 hp. when ventilated with 3500 cu. ft. of air per minute.

These locomotives were purchased after the decision to build the new tunnel and to extend the electrification eastward, and are designed for higher maximum operating speeds as the specification placed more emphasis on their use as passenger locomotives. Each of these locomotives is of the articulated $1-C+C-1$ type.

A summary tabulation of the mechanical and electrical characteristics of these two types of locomotives is presented in Table 1.

TABLE I
LOCOMOTIVE CHARACTERISTICS

	General Electric Co. Single Cab Locomotive	Westinghouse Elec. & Mfg. Co. Double Cab Locomotives
Number of locomotives	4	5
Mechanical		
Classification	1–C+C–1	1–D–1+1–D–1
Total weight each locomotive	518,000 lb.	715,000 lb.
Weight on drivers	409,800 lb.	549,600 lb.
Weight per driving axle	68,300 lb.	68,700 lb.
Maximum rigid wheel base	15 ft. 4 in.	16 ft. 9 in.
Total wheel base	58 ft. 8 in.	78 ft. 11 in.
Height over pantograph locked down	15 ft. 3 in.	15 ft. 10 in.
Electrical		
Number of motors	6	8
Horsepower 1 hr. rating	550 per motor 3300 per locomotive	541 per motor 4330 per locomotive
Tractive effort 1 hr. rating	67,200 lb.	112,500 lb.
Speed full field 1 hr. rating	18.2 mi. per hr.	14.4 mi. per hr.
Horsepower cont. rating	3,000	3,600
Tractive effort cont. rating	60,500 lb.	88,500 lb.
Speed full field cont. rating	18.6 mi. per hr.	15.5 mi. per hr.
Voltage synchronous motor	2,300	1,240
Voltage d-c. generators	750 each, two in series	600
Voltage on traction motors	1500 volts on two motors permanently in series	600 volts all motors in parallel
Capacity of motor-generator set continuous	1-2500 kw.	2-1500 kw. each

Locomotive inspection and maintenance for the four electric locomotives now in service between Skykomish and Cascade Tunnel Station are at present handled at the steam locomotive repair shops at Skykomish. A new electric locomotive shop with complete facilities for this work is now under construction at Appleyard (Wenatchee) and will be available for service upon completion of the present extension. All work on the electric locomotives will be handled in the new shop.

Part III.—Electric Distribution System

Contact System. The initial installation of this Great Northern electrification covers some 25 route miles and consists of 25 mi. of main line electrification plus approximately 6 mi. of electrified sidings and yards. This installation runs from Skykomish to the old Cascade Tunnel yard. The extension now being put into service connects with the old line at Scenic and extends over the new route as shown in the map, Fig. 1, to Appleyard, which is a large freight yard just east of Wenatchee. These sections of the railroad consist entirely of single main track construction. Yards at Appleyard and at Skykomish and passing tracks at Scenic, Berne, Winton, Cashmere, and Wenatchee are included in the electrification.

The catenary system over all tracks provides the required current carrying capacity without the use of additional feeders except across the Tongo loop in the existing construction between Skykomish and Scenic. At this point light construction is used around the loop and a heavy feeder ties across the two ends of the loop, the feeder being carried on poles used for the transmission line which also cuts across the loop. The design of the distribution system provides for a maximum regulation of approximately 10 per cent under normal traffic conditions with a trolley feeder section open at a substation. This regulation was found to be desirable on account of the characteristics of the synchronous motors employed for driving the motor-generator sets on the locomotives.

Wood pole and bracket construction is used in general for all single track construction, except on some curves where cross-span construction is used, and multi-track construction is of the cross-span type except where guyed structures could not be employed, and at these points wooden poles with expanded metal trusses are used.

The catenary system employed for this electrification consists of a combination of inclined catenary on the curves and simple catenary with loop hangers on the tangents. A composite main messenger of high tensile strength and a single bronze contact wire are used in all sections except through the New Cascade Tunnel where a copper messenger and a copper contact wire are permissible because of the lower temperature ranges and consequent lower stringing tensions. Bronze contact wires on the system in general were justified, because of high strength rather than for wearing qualities, since the traffic over the line is relatively light. The hangers employed in curve construction are the solid

type hanger clamped rigidly to the messenger and trolley; the hangers used in tangent construction are of the simple, light weight, loop design, and both types are completely non-ferrous. Non-ferrous cross-span wire is employed in the yards at Appleyard, where considerable steam locomotive operation will continue. Steel cross-span wire construction is used on sidings and passing tracks in those sections of the line where

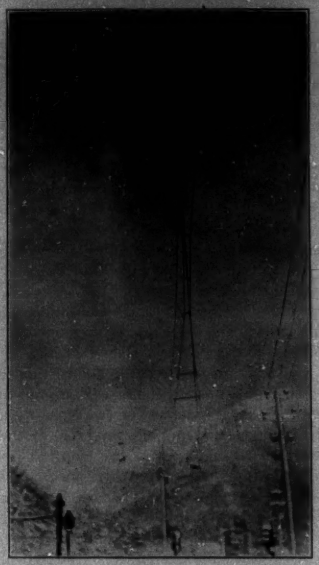

Fig. 3—Catenary Construction

operations will be completely electrified, and also at the present terminal at Skykomish since at the time Skykomish yard was installed it was assumed that electrification would probably go on to Gold Bar in a relatively short time.

The main messenger in the heavy grade sections of the main line track consists of nineteen strands of number 7 wire. The inner seven wires are of calsun bronze in the existing sections of the electrification and of red brass in the newer sections. The outer twelve wires in each case are of hard drawn copper. The trolley wire in the existing section of the main line is 4/0 "Hitenso C" bronze wire of 55 per cent conductivity and in the newer sections is 4/0 "Hitenso A" bronze wire of 80 per cent conductivity. This gives a conductivity of 390,000 cir. mils equivalent in those sections now electrified and 456,000 cir. mils in the newer sections. Siding construction consists of a 3/8 in. calsun bronze messenger and a 3/0 "Hitenso C" trolley wire.

The catenary system on the main line has a normal tension of 4000 lb. in the messenger and 2500 lb. in the 4/0 trolley wire, with a maximum tension under Class B (N. E. L. A.) loading of 7000 lb. in the messenger and 4000 lb. in the 4/0 trolley wire.

The tangent track construction, as noted before, is of the side bracket type, employing two 1½ by 2½ by ¼ in. galvanized steel angle bars placed back to back as a mast arm and a 5/8 in. hog rod. Pin insulators of 45,000 volts normal voltage are used for the messenger with this type of construction on all tangent tracks. Three suspension insulator disks in series are used on curves and on multiple track construction and in the new long tunnel, although four disks in certain particularly severe locations in the old construction were found necessary.

The line design in general is based upon a normal spacing of poles on tangent track of 150 ft. This spacing could not be adhered to rigorously, since the line consists of many curves with short lengths of tangent track between, and in order to obtain equipoise span construction it was necessary to shorten the spacing to adapt it to the requisite number of spans of uniform or equipoise length. Hanger spacings of 15 ft. were employed, the shortest hanger being located 7½ ft. from the low point of the span, accommodating the resultant spacing at the point of support in order to use standard hanger lengths rather than to have a number of special hangers. On curve construction, as previously noted, inclined catenary was used, made up with solid hangers. In order to obtain a line without cusps in the contact wire, and to preserve a high factor of safety in pantograph clearances by maintaining an angle between the hanger rods and horizontal of 22

Fig. 4—Catenary Construction at Tongo Loop

deg. or better, it was determined that 4 deg. was the maximum curve on which inclined catenary should be used without pull-offs, and that all curves of shorter radius should be made by utilizing an inclined catenary system of 4 deg. curvature pulled over by means of pull-offs. A maximum deviation of 9 in., from the center line of the track, and a maximum middle ordinate

distance of 15 in. were determined upon for normal temperatures. With these limitations it was found that 150-foot span lengths could be maintained on tangents and on all curves up to and including 6 deg. Curves of 7 deg. and 8 deg. were designed up to a maximum span length of 120 ft.; 9 deg. curves up to 105 ft.; and 10 deg. curves up to 90 ft.

Jumpers between the main line messenger and contact wires are inserted every 500 ft. in the tangent sections (where loop hangers are employed), but no jumpers are used on curves, since the hanger design provides for a solid connection from the messenger to the contact wire at these points.

Contact line construction in general consists of messenger sections of approximately 10,000 ft. in length in all sections of the line except through the long tunnel. The messenger is dead-ended at each end of these sections and is by the type of construction, insulated from the adjacent messenger sections. Jumpers are employed between the messengers of the various sections except at transformer substations where the two sections of messenger are tied together through the 11,500-volt bus in the substations. The yard construction at Skykomish and Appleyard is so designed that the yard is fed separately from the transformer substation at each of these locations.

Fig. 5—Typical Rail Bond Installation

Bonding. Gasweld bonds have been used throughout the entire electrification. The initial installation employs one 2/0 double strand U type Gasweld bond per rail joint. The new installation, at present under construction, will employ two No. 2 Gasweld single strand bonds per rail joint. All special work is bonded with 4/0 bonds. Impedance bonds are installed adjacent to each substation location. No negative return wires are used except at the Tonga loop cut-off, previously referred to.

Relay Protection. Of the arrangements for relay protection for the electrification circuits, perhaps the most interesting feature is the arrangement adapted for protection of the transformer substations. The protection which is required is against faults in the substation itself, faults on the 11,500-volt contact wire system, and faults on the 44,000-volt transmission system respectively.

For the clearing of faults occurring in a transformer substation differential current protection is used, the relays being energized by bushing type current transformers in the incoming 44,000-volt transmission line oil circuit breakers and in the outgoing 11,500-volt trolley feeder circuit breakers. In the event of a fault in the substation these relays actuate the circuit breakers and serve to trip out both the incoming and outgoing circuit breakers, and thereby isolate the substation. This meets in a simple and economical way the necessity for protecting the outside system from the influence of a fault arising within a transformer substation, and at the same time isolates the substation so that the fault will not be propagated within the substation itself.

Protection for faults on the contact wire system and its feeders is provided by one directional impedance type relay in each such feeder. The reason for using an impedance type relay for this service arises from the relatively high reactance of the contact system, which is sufficient to limit the minimum fault current to a value that is too close to the maximum load current safely to insure discriminatory action with an ordinary directional over-current relay with a constant restraining torque. In the impedance relay the restraining torque decreases with the additional drop of substation voltage, which is caused by the relatively low power factor of the fault current compared with the operating load currents. The result is that the margin available for discrimination between a fault current and a load current when this relay is used is considerably increased by the change in restraining torque due to change in voltage.

It is also possible to have a fault of sufficiently high resistance on the contact system so that it will not cause sufficient current flow at the transformer substation to actuate the overload relays although the current is in excess of the continuous rating of the transformers. It would therefore be possible for such a fault to hold on and eventually overheat the transformer. To guard against this possibility it is only necessary to arrange that the contact making thermometers with which the transformers are equipped, located inside the transformer cases, in addition to ringing an alarm bell, will also close the trip circuit and cut the substation out of service, opening both the 44,000-volt and the 11,500-volt breakers, if the temperature of the transformer exceeds a predetermined safe limit.

By these two arrangements we secure adequate protection for faults on the contact system with a minimum of cost.

The protection for the 44,000-volt incoming lines at the transformer substations imposes rather exacting requirements upon the relay arrangement. Each 44,000-volt line is tapped, through a circuit breaker, into a common bus at each of the transformer substations. Consequently, a fault consisting of a ground or of a wire to wire short-circuit occurring on one of

these circuits between any two substations should require the circuit breakers at the transformer substations to operate successively in the order of the progression of the substations away from the fault. For example, a fault on Circuit Number 1 between Winton and Tumwater requires the successive operation of circuit breakers at Winton, Berne, Scenic, and Skykomish, in the order given, before the fault is cleared from the system. The relative positions of these transformer substations along the line are shown in the diagram of the transmission and distribution system, Fig. 6. The relays at Berne should not begin their controlled cycle of operation until the circuit breaker on the faulty line at Winton has opened, since prior to this time there is nothing at Berne to indicate to the relay devices which of the two 44,000-volt circuits is in fault. The same is true successively at Scenic with respect to Berne and at Skykomish with respect to Scenic. These successive operations make it necessary to have the operating cycle of these relays at each transformer substation as short as possible so that a faulty transmission circuit may be cleared from the system promptly. The conditions also require that either of the two transmission circuits shall be free to be opened by sectionalizing switches between any of the transformer substations without interfering with the operation of the protective relays. In order to provide protective arrangements to insure rapid isolation of a faulty section of transmission line, which are not affected by unbalance between the load currents carried by the two transmission circuits, two relays are used in each transformer substation to take care respectively of short circuits and ground faults, one (marked relay P in Fig. 7) being a directional over-current relay actuated by heavy unbalance between the line currents in the two incoming transmission lines and the other (marked relay Q) being a directional over-current relay actuated by slight unbalance between currents in the two transmission lines associated with flow of current in the neutral ground connection of a substation transformer. Fig. 7 shows diagrammatically the connection utilized for these two relays. In order to minimize the investment required in the protective system, it is desirable to energize the relays from bushing type current transformers, for which there are only four available locations in the terminals of the two 44,000-volt oil circuit breakers. This is accomplished by the circuit arrangement shown in Fig. 7 already referred to. It will be observed that the current flowing in the current coil of relay P is proportional to the difference between the currents in the conductors A and C (*i. e.*, the unbalance between the line currents), whereas the current flowing in the current coil of relay Q is proportional to the difference between A–B and C–D (*i. e.*, the unbalance of residual current in the two circuits). The relay P is given a sufficiently high current setting so that it will not be actuated by the unbalance of load current resulting from an open circuit in one of the transmission circuits. It will, however, operate on the heavy unbalance resulting from an accidental contact or short circuit between the two conductors of one circuit. The current in the coils of relay Q is always equal to zero

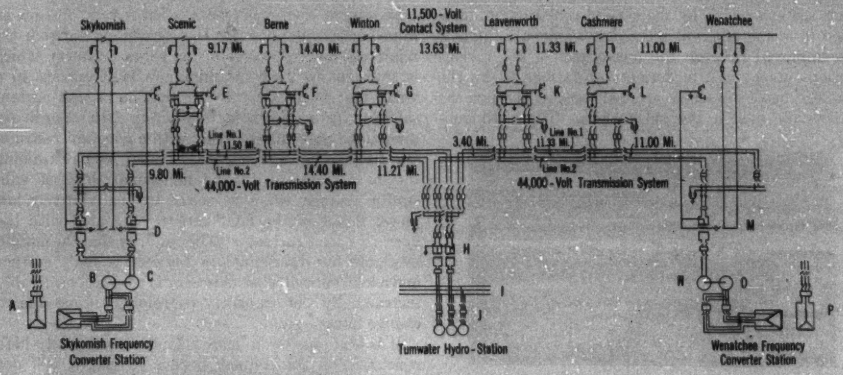

FIG. 6—GENERAL WIRING DIAGRAM OF ELECTRIFICATION CIRCUITS

A. Three 2750 kv-a. transformers, 60 ~ 110/13.2 kv.
B. Motor 13.2 kv. 60 ~, 3 ϕ, 8250 kv-a.
C. Generator 13.2 kv., 25 ~, 1 ϕ, 7500 kv-a.
D. Two transformers 7500/5000 kv-a., 44/13.2/11.5 kv.,
E. Two transformers 2500 kv-a. each, 44/115 kv., 25 ~
F. Two transformers 2500 kv-a. each, 44/115 kv., 25 ~
G. Two transformers 1500 kv-a. each, 44/11.5 kv., 25 ~
H. Two transformers 3750 kv-a. each, 6600/44,000 volt, 1 ϕ, 25 ~
I. 6600-volt bus
J. Three generators 2500 kv-a. each, 6600 volt, 3 ϕ, 25 ~
K. Two transformers 1500 kv-a. each, 44/11.5 kv., 25 ~
L. One transformer 2500 kv-a., 44/11.5 kv., 25 ~
M. One transformer 7500/5000 kv-a., 44/11.5 kv., 25 ~
N. Generator 13.2 kv., 25 ~, 1 ϕ, 7500 kv-a.
O. Motor 13.2 kv., 60 ~, 3 ϕ, 8250 kv-a.
P. Three transformers 2750 kv-a. each, 60 ~, 110/13.2 kv.

unless there is a ground in the transmission circuit. Therefore, this relay is unaffected by any degree of load unbalance and can be given a current setting sufficiently small so that the relay will respond in the minimum of time to the smallest ground fault which is likely to occur. As indicated in Fig. 7, the potential coil of relay *P* is energized from a potential transformer on the 11,500-volt trolley feeder bus and the corresponding coil on relay *Q* is energized from parallel current transformers in the neutral ground connections of the substation transformers, so that the directional elements of the relays will discriminate in such a way as to trip the circuit breaker of the faulty circuit.

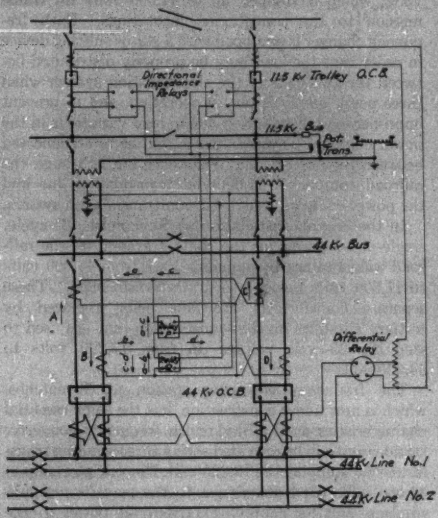

FIG. 7—TRANSFORMER SUBSTATION RELAYING—SCHEMATIC DIAGRAM WITH TRIPPING CIRCUITS OMITTED

Note: All disconnecting switches are normally closed
Current transformers in transformer neutral leads are instrument type
All other current transformers are bushing type
Contact making thermometer in each transformer is arranged to trip all our circuit breakers in case of excess transformer temperature

This combination of relays satisfies the requirements of the system, namely, to procure high sensitivity and quick action for ground faults whether of large or small magnitude, in combination with freedom from actuation occurring due to open circuits, and these results are accomplished with economical expenditure for equipment. No special auxiliary switches are required in the relay trip circuits, and the only additional instrument transformers compared with the number that would have to be used for straight directional overload relay arrangements are the current transformers placed in the neutral ground connection of each station transformer. Auxiliary locking relays are provided to prevent opening of the circuit breaker on the good line as a result of rebound of the directional element after a circuit breaker on a faulty line has opened.

The protection associated with the 44,000-volt transmission lines and the 11,500-volt contact system at the frequency converter stations and with the 44,000-volt transmission lines at the Tumwater generating station is of similar character to that at the transformer substations. For the remainder of the equipment in the circuits in the converter stations, the protective devices introduced for faults within or without the station are of the usual differential and overload character that is commonly associated with motor-generator stations and high potential transformers.

A somewhat special type of relay protection is provided at the two frequency converter stations for the 13,200-volt signal power feeder which runs between the two. This 60-cycle, single-phase, 13,200-volt circuit is connected directly to the 13,200-volt, 60-cycle, auxiliary power bus at each frequency converter station. The circuit supplies power for operation of the signal system, for lighting the new long Cascade Tunnel, and for miscellaneous purposes.

There is a number of sectionalizing switches in this signal power feeder circuit, and it is intended that one of them shall always be open, thereby providing for independent supply of power to each section from one frequency converter station. There is a possibility of this circuit being closed so as to connect through from converter station to converter station, in which case there might be an excessive flow of current over the circuit which is of No. 6 copper wire. In order to provide against the contingency of an excessive flow of current the circuit breaker controlling this circuit at each converter station is actuated by a directional over-current relay of the usual type, except that the trip circuits of the over-current element and the directional element are connected to operate independently. The operation of the over-current element causes the circuit breaker to trip on the occurrence of excess current in either direction and the operation of the directional element causes the circuit breaker to trip whenever there is any flow of power towards the bus.

PART IV.—ELECTRIC POWER SUPPLY

The power requirements for electric service in the section from Skykomish to Appleyard are materially in excess of the capacity available from the present Tumwater plant, and a sufficient independent power supply appeared to be more costly than purchased power from existing commercial hydroelectric power properties. A plan was therefore worked out by the Great Northern Railway officials with the Puget Sound Power & Light Company whereby the power company took over the operation of the Tumwater power station and contracted to sell to the railroad all of the electrical service required for operation of the railroad west of Wenatchee. To meet the requirements

of the electrification now in operation between Skykomish and Cascade Tunnel Station the power company supplies 25-cycle, single-phase power to the railroad company's step-up substation at Tumwater, and also supplies 60-cycle, three-phase, 110,000-volt power to the railway company's high-tension transmission line at Gold Bar for transmission to a frequency converter station at Skykomish, the western end of the present electrification. To meet the added requirements imposed by extension of the electrification to Appleyard (Wenatchee), power will also be delivered by the power company at Wenatchee, at 60 cycles, three-phase, 110,000 volts, where it will be converted to 25-cycle, single-phase power for transmission to the transformer substations. Under the terms of the power contract the demand for billing purposes is determined from the simultaneous demands at the three points of delivery.

Fig. 6, previously referred to, shows in diagrammatic outline the circuit and substation arrangements for the completed electrification from Skykomish to Appleyard.

Purchase of power under the contract arrangement is cheaper than the cost would have been for developing an independent source of supply of sufficient capacity to meet the maximum requirements of the system, and also gives materially greater assurance of continuity of service than would have been possible from an individual power station.

The plan developed gives three points of supply, namely, at Skykomish, Tumwater, and Wenatchee, with double circuit, single-phase, 44,000-volt transmission lines connecting the three sources of supply and serving the intervening transformer substations along the railroad route. With this arrangement failure of any one source of supply or failure of both 44,000-volt transmission circuits at any one point cannot cause serious interruption of railroad service.

The two frequency converter stations, at Skykomish and Wenatchee, are supplied with 60-cycle current from transmission lines feeding from two separate points in the power company's system, the line supplying Skykomish coming from the Beverly Park substation, and the line supplying Wenatchee coming from the White River generating station, so that the probability of simultaneous failure of high-tension power supply at Skykomish and Wenatchee is minimized. An interconnection between the Puget Sound system at Wenatchee and the Washington Water Power Company's new hydroelectric plant at Lake Chelan is now under construction, which will still further add to the assurance of continuity of power supply.

Three point supply also makes possible materially better voltage regulation on the 25-cycle system of the railroad than would have been possible with single point supply, and also makes possible economies in design of the 25-cycle transmission system serving the transformer substations.

Purchase of power from a large commercial power system also has a distinct advantage over independent generation by the railroad company in that it offers an adequate power load for absorbing the power produced during regenerative braking when there are not sufficient electric power demands on the railroad system itself to absorb this power, and consequently it obviates the necessity of providing a complicated system of rheostats available for absorbing excess regenerated power.

At the Wenatchee frequency converter station power is received at 110,000 volts, three-phase, 60 cycles, and is stepped down to 13,200 volts, three-phase, passed through a 7500-kv-a. frequency converter, which converts the power to 25 cycles, single-phase, at 13,200 volts, which is stepped up to 44,000 volts for transmission to the transformer substations. Each frequency changer is equipped with a stator shifting device in order that the load may be properly distributed between the two ends of the electrification system when three point supply is put in operation, and to prevent improper load distribution due to load variations in the power company's system, which will be interconnected between Skykomish and Wenatchee both through the railroad company's 44,000-volt transmission line and the power company's high-tension transmission system.

In the original installation at Skykomish, 25-cycle, single-phase power for the contact system on the railroad was obtained by stepping down from 44,000 volts to 11,500 volts through separate transformers. These separate transformers are now being replaced by tertiary windings on the transformers that are used to step the 25-cycle voltage up from 13,200 volts to 44,000 volts.

The frequency converter station at Wenatchee, which is now under construction, has the same essential characteristics as the Skykomish frequency converter station, and will be provided with a three-winding transformer for stepping the 13,200-volt, 25-cycle power up to 44,000 volts for transmission to the transformer substations, and for supplying 11,500-volt power to the contact system at Wenatchee.

The type of construction employed at the frequency converter stations is as simple and inexpensive as is consistent with the reliability requirements. The step-down equipment on the 60-cycle side of each frequency converter station is of the conventional single bus outdoor substation type as shown in the illustration of the step-down substation at Skykomish given in Fig. 8. The 25-cycle step-up substation is of a similar type of construction except that rigid bus design has been used throughout the substation. This construction is shown in Fig. 9. The frequency converter at Skykomish is installed in a building adequate for the future installation of an additional unit, and the frequency converter station at Wenatchee is so planned that it can be readily extended to provide for installation of a second frequency converter unit in that station if it should ever be required. The indoor construction consists of 13,200-volt switch gear, with no low-tension

bus work other than that necessary for obtaining auxiliary power in the substation. The layout for future construction contemplates, in the event of installing a second frequency converter at either station, that the frequency converters will not be paralleled directly on low-voltage busses at any point, that the motors will be paralleled only on the 110,000-volt, 60-cycle bus and the 25-cycle generators will be paralleled only on the 44,000-volt bus, or as necessitated on the 11,500-volt traction bus through the tertiary windings of the three-winding transformers.

Fig. 8—Skykomish Frequency-Converter Station
60 ~, 110,000/13,200 volt substation

The use of three-winding transformers at the frequency converter stations, which serve also as transformer substations for supply of traction current to the trolley circuit, permits economies in the investment and also material savings in the operating expense of the system by eliminating a large part of the core loss which would be incurred by the installation of separate transformers for supplying the traction service.

The transmission system connecting the transformer substations along the line with the frequency converter stations and the Tumwater power station consists of a double circuit, single-phase, 4/0, 44,000-volt transmission line running from Skykomish to Tumwater and from Tumwater to Wenatchee, and tapped into the various transformer stations between these points. From Skykomish to Scenic, from Berne to Winton, and from Leavenworth to Wenatchee, these circuits are carried on the wood poles which support the contact system. The section of the transmission line from Scenic to Cascade Tunnel Station runs directly over the mountain, and is of the H frame, flat type of construction. The section from Cascade Tunnel Station to Berne is wood pole construction running adjacent to the old railway right-of-way. At Berne the line ties in with the trolley construction. The section from Winton to Leavenworth is wood pole, two circuit construction, located on the old right-of-way as far as the eastern end of the Tumwater Canyon, at which point the line leaves the right-of-way and ties across to the new section of line in the Chumstick Valley adjacent to the new Leavenworth transformer substation. The line in general is insulated with 60,000-volt insulators having a dry arc-over voltage of 170,000 volts. The line is operated with the mid-point of all 44,000-volt transformers grounded, giving a normal working potential to ground of 22,000 volts.

The transformer substations are fed from both of the transmission circuits through oil circuit breakers, and the relaying on the transmission system is so arranged that a fault to ground or from conductor to conductor on any one circuit will remove the circuit complete from Skykomish to Tumwater or Tumwater to Wenatchee, as the case may be, and will open the substation breakers connected to the faulty line, leaving the operation of the substations unimpaired by any single line fault.

Fig. 9—Skykomish Frequency-Converter Station
25 ~, 13,200/44,000/11,500 volt substation

The transformer substations along the route are in general of the two-unit type, and consist of outdoor equipment with a high-voltage, single-phase, 44,000-volt bus, and a low-voltage, single-phase, single-conductor, 11,500-volt bus. The substation is normally operated with the 44,000-volt bus between transformers closed and normally supplied from both circuits. The transformers in each substation in normal operation are tied solidly to the 11,500-volt bus, with the sectionalizing switch in the center of the bus closed. The two

transformers at each substation (where two transformers are installed) are hence normally operated as a single unit. The relaying necessary for this method of operation is described in a previous section. This method of operation as compared with operation with the 11,500-volt bus sectionalized permits a more effective utilization of the transformer capacity, since each transformer is available for service to locomotives on either side of the air section break in the contact conductor.

The location of communication circuits is such as to permit operation with the 11,500-volt bus closed. This method of operation would not be permissible with oil circuit breakers of the standard design if the trolley system of the railroad were closely paralleled by communication circuits, since fault currents resulting from short circuit would thus be permitted to extend through more than one trolley section and would produce correspondingly increased disturbance in the communication circuit.

This paper sets forth the characteristics of the electrical portions of the group of notable improvements that are being carried out by the Great Northern Railway on its Cascade Division as enumerated in the early part of the paper. The plans for the improvements and supervision of the execution have been carried out by the company's engineering department, of which Colonel F. Mears is Assistant Chief Engineer in charge of the work on the west end of the system.

The detailed plans and specifications for the electrification were prepared by the railroad company's engineering staff in cooperation with the consulting engineers, and the construction work on the extension of the electrification is, with a few minor exceptions, being carried out by the company's own forces. Mr. D. M. Burckett and Mr. J. E. Hawe are the principal assistants of Colonel Mears in direct charge of the electrification work. The construction in connection with the electrification is being carried out under their direction and supervision.

Discussion

Paul Lebenbaum: In discussing a paper such as this, it must continually be kept in mind that the men who are responsible for the operation of a steam railroad and face its many problems are not electrical engineers; they are men who have grown up with the steam locomotive, which, in the last ten years, they have so improved as to enable it almost to keep pace with the tractive power possibilities, though not the speed, of the electric locomotive. Therein lie both the strength and the weakness of the electric locomotive. A steam machine good for large tractive powers at slow speeds is at the same time so counterbalanced and has the necessary horsepower to enable it to run at speeds on the level of from 50 to 55 mi. per hour, so that the same locomotive, with helpers on a mountain grade, can take a tonnage train in level or rolling territory at speeds that the present designs of mountain-type electric locomotives cannot attain.

The Great Northern electrification is of course a compromise of what may be considered best in both the a-c. and the d-c. systems. An 11,000-volt, single-phase trolley, avoiding the necessity for heavy feeders; a synchronous motor on the locomotive, assuring unity power factor, and the use of d-c. traction motors, with an almost infinite number of speeds through exciter control of the d-c. generator, make for a comparatively simple design. Whether it is the system that will be adopted as the standard remains to be seen. I doubt it, because in an art in which changes are made as fast as in the electrical field, especially in our fundamental conceptions, I cannot feel that the present is the right time to think of standardization on any system.

M. L. Wade: Railway electrification is a subject of extreme interest to residents in the Canadian Rockies. Our problems are rather similar to both the Chicago, Milwaukee & St. Paul and the Great Northern, but for the present these are restricted pretty well to the consideration of electrifying the Rogers Pass tunnel, known as the Connaught Tunnel on the Canadian Pacific Railway. This tunnel pierces Mount Sir Donald and is approximately five miles in length from Glacier, at the west portal, to Donald, at the east end. The second unit of the contemplated electrification would probably be on the heavy grades known as the Field Hill, involving two spiral tunnels. For some years the railway company has made a study of the Columbia River near Beavermouth with this electrification in view. Our rapid mountain streams carry large quantities of debris and silt, offer very little natural pondage, and present a problem of combined engineering difficulties and high development cost—a problem which has to be attacked with much deliberation.

The salient feature of the Great Northern cascade electrification is, to my mind, the development of the 11,000-volt single-phase trolley system and the comparatively simple motor-generator equipment carried in the locomotives. It is to be hoped that the future may see this arrangement worked out satisfactorily at 60 cycles in order to get away from the cost of frequency-changer stations along 60-cycle transmission lines, tending to bring the electric locomotive into closer competition with steam on hauls of lower traffic density.

A point raised in a prior discussion was that of antipathy or lack of interest on the part of certain railway officials. To this might be added an observation on the general attitude of railroaders toward electrification. They don't like it. It is going to be a matter of long-time education to get them to like it. The general attitude of the average engineer, fireman, and trainman is this,—"The more tonnage a locomotive can handle, the fewer are the required train crews to handle a given traffic, the more lay-off's there are going to be, and the less money we can make."

C. E. Baston: The motor-generator locomotive was first put into operation, on the Detroit, Toledo & Ironton, operating over 17 miles of single track. This locomotive was designed by engineers of the Westinghouse Company, the mechanical parts being built by the railroad company and the electrical equipment being furnished by the Westinghouse Company.

The control of the Westinghouse locomotive, as you know, is by field control of the main generator. There were some problems in the development of this locomotive that required considerable thought in solution, understanding that this locomotive must be operated not by a substation operator but by an old steam engineer. Of course something had to be made that could be handled with very little introductory training. Automatic synchronizing of motors of 2000 kw. or more on a moving locomotive is quite a problem. It must be done with as few relays as possible, and those relays must not be delicate in construction.

Our aim has been to force the engine men to be in the equipment compartment watching the motor-generator set as it comes up the speed. For that reason the starting of the motor generator is not done from the operating compartment.

There are two types of motor-generator locomotives on the Great Northern, a single-cab unit built by the General Electric Company which is used mainly for passenger service and built for high speeds, and a two-cab locomotive built by the Westinghouse Company for freight service.

As the paper brings out, previous to electrification freight trains weighing about 2500 tons were operated by a Mikado at the head end, a Mallet about two-thirds of the way back, and another Mallet at the rear behind the caboose. At the present time there is placed an electric locomotive and a Mikado type steam engine at the head end and an electric locomotive about two-thirds from the head end of a train of 3500 tons. Originally the speed of trains with steam operation was around 6 to 9 mi. per hr. At the present time trains are handled at 15 mi. an hr., but due to the power situation the demand is limited by restricting the speed to 10 or 11 mi. per hr.

Originally there was a delay at Scenic of from ½ to 1½ hr. to take water on three steam locomotives, a further delay at Tye for cutting off two of the Mallets and adding two of the old three-phase locomotives. I might say that we have been operating for approximately 18 months with two freight locomotives and two passenger locomotives, handling practically all of the traffic over the hill, which requires the power to be in service almost all of the time. We have never yet had to pull a train through the tunnel with the steam engine, due to failure of the electric locomotive. We have had one or two failures of power supply. In that case we had to use steam engines.

A particular point to note about this type of locomotive is its extreme flexibility. The main speed handle controlling the generator field has 29 positions, every one of which is a running position. When the 29th position is reached a closed-circuit transition is made without change in tractive effort, to a connection whereby the traction motors are separately excited from a special field exciter which is driven by the main motor-generator set. The fields of the traction motors are then weakened by decreasing the field excitation of the field exciter, thus further increasing the speed of the locomotive in a number of very easy steps. The locomotive is in this way made to deliver its rated hp. output almost to its maximum speed of 45 mi. per hr.

The separately excited connections are used for regeneration and the method of transition used makes it possible to change these connections easily at any speed either motoring or regenerating and also to change back to the series connection when motoring, without loss of power.

After maximum field strength has been applied to the traction motors in regeneration the main generator voltage may be reduced, making it possible to exert full retarding tractive effort down to 3 or 4 mi. per hr.

In spite of the fact that a large part of the present electrified section is through snow sheds and tunnels giving rise to a slippery rail, the parallel connection of the traction motors enables the locomotive to cling to the rails even under adverse conditions. In case slipping does occur it is only necessary to move the control lever back a few notches to allow the wheels to grip the rail again, resulting in very little loss in speed.

C. E. Carey: I think it is very interesting to go back of the present installation and determine why the motor generator type of locomotive was chosen. Prior to the Great Northern electrification there was only one installation ahead of it, namely the Ford electrification, and this was largely in the experimental stages.

When we consider any electrification we must study it on the basis of the type of service which that railway renders, that is, whether it is a service of heavy trains, slow speed over heavy grades, or whether it is a service requiring high speeds, heavy trains, and over light grades. We must also determine whether or not the service requirements are a combination of these two.

The requirements for the locomotives for the first class of service are high tractive effort, low speed, and flexible operation. The requirements of the locomotives for the second class of service are high speed, high capacity, and high tractive effort with a tremendous amount of power under the locomotive.

A study of the Great Northern system shows that in the Cascades we have very heavy grades which require high tractive effort and slow-speed operation, while between the Columbia River and the Rocky Mountains the country is practically flat, and in the Rocky Mountains again we have heavy grades. So in considering the electrification of the Great Northern a system was chosen which would serve locomotives of both classes.

The motor-generator type of locomotive is one using a system of variable voltage control for the traction motors. Such a system of control has numerous running and regenerating points and is therefore the logical type of control to apply to a locomotive for low speed and heavy hauling operation. The other type of locomotive which requires high speed, high capacity, and high power can only be met with a single-phase locomotive; therefore in the electrification of the Great Northern the 25-cycle single-phase electrification was adopted which provided a system of power supply for both types of locomotives, which ultimately would be required.

I like to look at this sytem of electrification from the angle that it provides a form of energy which puts the work horse and the race horse at their best advantage.

H. R. Morgan: I should like to ask several questions on matters pertinent to this electrification. First, what is the capacity for which the system is designed? We know the Great Northern is looking forward to the future as we did, and we should like to know what they consider the ultimate capacity of their newly electrified section to be. Second, what is the approximate power consumption per gross ton-mile? Under the existing operating conditions, it is highly improbable that the Great Northern has succeeded in getting the highest efficiency out of their system. But they must have some figure and it would be interesting to know what their experience has been to date. Third and last, what is the overall efficiency of their electrified system? We have made quite a few calculations on that ourselves, comparing their a-c./d-c. system to our straight 3000-volt d-c. system, and we cannot agree with them that their system is the most efficient.

H. S. Ogden: The authors have given a rather short discussion on the General Electric locomotive in the paper, but perhaps a more complete description would be appreciated.

The locomotive is a 260-ton 2500-kw. multiple-unit machine of the motor-generator type. It is mounted on the 1 − C + C + 1 type of running gear. Each of the two trucks carries three motors known as the G. E. 290 that rate approximately 500 hp. The locomotive is 73 ft. 9 in. over knuckles, has a height of 15 ft. 3 in. with the pantograph locked down, and a width of 11 ft. ¾ in.

It has a safe speed of 45 mi. per hr. and a maximum emergency speed of 50 mi. per hr.

On each end of the locomotive is an operating compartment, and back of each of these is an enclosed compartment containing the necessary contactors, reversers, switches, etc. The motor-generator set, of course, takes up most of the space in the cab; near the No. 2 end are located the main transformer and the air compressors. Underneath the motor-generator set coupling is located the traction-motor blower set that supplies cooling air for the traction motors.

The motor-generator set is a five-unit four-bearing set that operates at 750 rev. per min., having a 3500-hp. single-phase synchronous motor that drives the two 1250-kw. 750-volt d-c. generators. Overhung on the No. 1 end of the shaft is the combination single-phase series starting motor, and regenerative exciter. The other end of the shaft mounts the main 65-volt exciter that supplies power for charging the batteries, operating the control, exciting all the machines, and for the lights.

On both sides of the regenerative exciter are located the two high-speed d-c. circuit breakers for protecting the traction motors and the d-c. generators.

The main transformer is a 3285-kv-a. air-blast transformer, 11,000/2300 volts, with a small 150-volt tertiary winding for operating the starting motor. This transformer is cooled by two

6¼-hp., 540-volt induction-repulsion motors that drive single-runner fans.

In order to compensate for conditions of low trolley voltage there are two normal running taps that may be used to supply the synchronous motor. Changing from one tap to the other is accomplished under load automatically, this being controlled by means of a sensitive potential relay.

Between the transformer and the No. 2 end contactor compartment are located the two 100-cu. ft. air compressors. These are driven directly from the main transformer from a low-voltage tap. These deliver air for the air-brake system, and for the air-operated control apparatus.

The control equipment of the locomotive is made up of air-operated contactors, switches, and reversers. These take care of the traction-motor and synchronous-motor circuits. There is no oil switch. Protection is afforded by the track substation breakers and a pantograph-lowering relay that in case of trouble in the main transformer will lower the pantographs and lock them down after the substation breakers have opened the circuit. There are also quite a few magnetic contactors on the locomotive, notably those in the main regenerative exciter field, compressor, and motor circuits.

The engine can be controlled from either of the two operating compartments which are identical. The control is centered at each engine man's position, that is, on the right-hand side of the locomotive. Here is located a two-story master controller, the straight and automatic air-brake valves, the control of the motor-generator set and its auxiliaries, along with the necessary gages, meters, sanders, etc. These are conveniently situated with respect to the engine man. The master controller as noted above is a two-story affair, the bottom containing the drum that controls the main generator field and the upper the drum that controls the regenerative exciter field. The former drum is known as the speed drum as it has direct control on the speed of the engine whether regenerating or motoring. The latter drum is known as the braking drum, as it has direct control upon the braking effort of the locomotive by controlling the field strength of the traction motors. There is also a forward and reverse lever. All of the above levers are mechanically interlocked against false and improper operations.

In starting the locomotive it is only necessary to push the motor-generator starting button. This operates contactors that connect the starting motor to the 150-volt tertiary winding of the main transformer. When the set reaches approximately 80 per cent speed a potential relay connected across the terminals of the main exciter picks up, connecting the stator of the synchronous motor to the 1800-volt tap on the main transformer. Then both the synchronous motor and the starting motor pull the set up to speed. As soon as it locks in step the synchronizing relay, which determines synchronism by becoming de-energized when the pulsations in the field-discharge resistor circuit die out, applies excitation. This operation starts a magnetic time-delay relay functioning that allows about five seconds to elapse before disconnecting the synchronous motor from the 1800-volt tap and connecting it on to the 2300-volt tap corresponding to normal operating voltage. The purpose of the time delay is to allow sufficient interval for the field to build up. The locomotive is then ready for service.

Locomotive movement is controlled by varying the field strength of the main generators. This is varied by means of the master controller that operates the small magnetic contactors which operate on the resistance in the field circuit. Weak-field motoring and the consequent higher speed is obtained by shunting the traction-motor fields by a special connection of the balancing resistors. The fields are reduced in two steps to 50 per cent of their normal values in order to obtain the maximum speed of the locomotive.

The traction motors are made to regenerate by separately exciting them by means of operating the starting motor as a d-c. generator. This combination machine has been mentioned above. Current is circulated in the proper direction through the traction-motor fields and balancing resistors and is directly under control of the operator who varies it at will by operating the braking handle of the master controller. The braking drum operates magnetic contactors in the field circuit of the exciter.

This type of locomotive has very flexible control under all operating conditions and it is possible to regenerate practically to a standstill without trouble.

A. W. Eshelby: Mr. Morgan has raised the question of the overall efficiency of the Milwaukee 3000-volt d-c. system as compared with the Great Northern. Inasmuch as there are so many parasites on the power supply it is very difficult as yet to get any definite overall efficiency figures for actual service. When the new tunnel is completed and the present construction load is eliminated from the power system, it will be possible to answer his question satisfactorily. It must be remembered that the tunnel supply has much to do with determining the cost of power.

One feature that should be stressed in discussing the motor-generator type of locomotive in comparison with the d-c. locomotive using series traction motors, is the perfect control that the operator has over his locomotive-wheel slippage. With the straight parallel motor connection used on the Westinghouse motor-generator locomotive, it is possible to slip wheels at any speed desired by the operator and should undesired slipping occur, due to poor track conditions, it is only necessary to back off a notch or two, until slipping stops.

With the series connection used on all d-c. locomotives, and some motor-generator locomotives, when slipping occurs it is necessary to throw off all power and start over again. When this happens in a heavy train with several locomotives, if any one locomotive throws off power to stop slipping, the slipping is usually transferred to the other locomotives. As a result of this the practise on such locomotives is to allow the slipping to continue until the track condition changes rather than throw it over on the other locomotives.

R. E. Kistler: (communicated after adjournment) In a-c. electrifications such as the Great Northern, the problem of inductive interference with paralleling signal lines is an important one. In order to limit these effects the Great Northern moved their open-wire telegraph lines from their right-of-way to adjacent highways. Some tests were made of the induction on signal wires on the new and old lines. These test data indicate that the induced voltage to ground at 1500 ft. separation was about 4 volts per mile per 100 amperes of trolley current or about 30 per cent of the induction at 50-ft. separation. Tests through the old tunnel where the signal circuits are in submarine cable with the armor grounded gave values comparable with those obtained in open wire at 1500 ft. separation. In order to limit induction on the telegraph lines through the new tunnel it is planned to carry the lines in cable placed in iron pipe.

Power Supply for Railway Signals
and Automatic Train Control

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Synopsis.—This paper deals, in a general way, with the several systems of automatic block signaling, particularly the a-c. system, and the apparatus commonly used to insure continuity of power supply. It covers briefly the older phases of the subject and more specifically the later applications.

The most modern system of continuous inductive train control, the code system, has required the development of frequency-converting apparatus for this use. Within the past three years, a number of frequency-converting automatic substations has been installed by the railway companies. The class of apparatus used in these stations is discussed, including some details of installations on the Pennsylvania and Long Island Railroads.

AUTOMATIC block signaling on railroads has reached a high degree of perfection. Within the last few years train accidents due to failures of signals to operate properly have been practically unknown. Developments were started many years ago but perhaps the greatest impetus toward developing and applying the automatic signals came in 1922 when the Interstate Commerce Commission ordered 45 railroads each to equip one locomotive division with automatic train control or train stop. In 1924 a second order was issued calling upon 41 of the original 45 carriers to equip a second locomotive division. These two orders covered approximately 15,000 track mi. and 6300 locomotives. All of this work has been completed.

The experience gained in this widespread use of the automatic system has been very valuable and has led to the development of the modern reliable signal equipment.

The most important early development was the adoption of the closed-track-circuit principle. In this system absence of current in the track causes indication of the low-speed or stop signal. This makes for safety and this principle is the basis of all modern systems.

For a-c. signaling the track circuit consists of the rails, rail bonds, track relay, and track transformer. For d-c. signaling, primary cells or storage cells take the place of the track transformers.

There are also two combination systems using both alternating current and direct current known respectively as the a-c. floating-battery system and the a-c. primary-battery system. These systems utilize direct current for the track circuit under all conditions. Normally, alternating current is used for the lamps which give the position-light and color-light signals. If the alternating current fails the lamps are connected quickly to a battery by means of an automatic change-over switch or "power-off" relay. There is, in effect, an automatic substation of the order of 150-volt-amperes capacity at each signal location.

The fundamentals of these systems and the fundamentals of the d-c. system are such as to provide excellent insurance against signal outages due to power failures, if the batteries are properly maintained.

A-c. signaling owes its development to the application of automatic signaling on d-c. electric-traction systems and to difficulties encountered on steam railways caused by interference with d-c. track circuits by stray currents from nearby d-c. circuits.

For the operation of a-c. signals the practise is to provide a power line paralleling the railway. At each signal location the line voltage is stepped down to the proper voltage for the track circuit, relays, lamps, and semaphore mechanisms.

Power for operating the early a-c. signal systems was provided by means of steam-engine or turbine-driven alternators operated by the railways. The switching was performed manually and two attendants were always on duty, one to fire the boilers and one in charge of the engine room. When synchronizing the alternators, the fireman manipulated the throttle and the engineer operated the switchboard. These power plants were located along the right-of-way, consideration being given to the division of load among stations and the possibility of locating the signal apparatus in plants, such as shop power plants, where the labor was already available.

At the present time railway companies realize the advantages of purchasing power from central station companies with the result that the isolated plants are rapidly being replaced by unattended automatic substations supplied from power company lines.

It is well known that the cost of central station electric power has declined during the past decade. Public utility lines have been extended so that power can be purchased in almost any locality. Outages of considerable duration are exceptional. At the same time the cost of generating power in isolated plants has increased.

Automatic switchgear may be obtained to perform practically any function that can be performed manually and to do it more reliably and more quickly. Switchboard operators may, therefore, be eliminated.

To provide a degree of reliability for a-c. signaling

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28-119

comparable with the reliability of systems using batteries, it is necessary to provide two sources of power supply for each section of signal line. These sources of supply are usually at opposite ends of the line and the switchgear provides for one of the sources to be feeding at all times and for the other to be connected quickly to the line upon failure of the normal or preferred source. Protective relays are provided which make it impossible for both sources to be connected to the line at the same time, also low-voltage protection to disconnect the signal line from the source of supply in the event of loss of voltage on the supply, and over-current relays to disconnect the signal line from the supply in the event of heavy overloads on the signal line.

The signal power supply on the Florida East Coast Railway between Jacksonville and Miami is an excellent example of this type of installation. Over the 366 route miles between Jacksonville and Miami, eight automatic substations are located. Between Jacksonville and Daytona Beach the voltage is 4400, and between Daytona Beach and Miami the signal line voltage is 2200.

A large majority of the present automatic train control and train stop installations are of the intermittent inductive type or the continuous inductive type. The intermittent inductive type is effective only at wayside signal locations. No power is required for the wayside elements, or inductors as they are called. The continuous inductive type provides continuous cab-signal indications. It is effective regardless of the location of the train in the block. It makes use of a track circuit superimposed on the wayside-signal track circuit.

The code system is the latest development of the continuous inductive system. In this system the receiving circuits on the locomotives are "tuned" so that they will respond only to energy supplied at a certain frequency. To eliminate the possibility of interference from commercial power circuits a nominal frequency of 100 cycles has been widely selected as the most favorable frequency for the track-circuit current.

Railways using the code system are the Pennsylvania, the Long Island, the West Jersey and Seashore, the Central of New Jersey, the Reading, the New York, New Haven & Hartford, and the Delaware, Lackawanna and Western. Some of these present interesting track-current conditions. The Long Island has direct current for propulsion, 25-cycle current for wayside signals, and 100-cycle current for automatic train stop. The Pennsylvania between Altoona, Pa. and Harrisburg, Pa. has 60 cycles for wayside signals and 100 cycles for train stop, and on the Pan Handle Division, Pittsburgh, Pa. to Newark, Ohio, 100 cycles for both signals and train stop. The D. L. & W. uses direct current for wayside signals and 100 cycles for train stop between East Buffalo, N. Y. and Elmira, N. Y., and 100 cycles for both wayside signals and train stop between Elmira and Johnson City, N. Y.

Rotating apparatus must be provided to change from 25-cycle, 60-cycle, or d-c. primary supply to 100-cycle output. The machines may consist of d-c. motors, synchronous motors, single-phase or polyphase induction motors driving synchronous alternators, or of induction motors driving induction generators.

With the exception of very unusual conditions (as on the Long Island which will be described later) the choice is usually between (a) squirrel-cage induction motors driving synchronous generators with direct-connected exciters and (b) squirrel-cage motors driving induction generators.

Induction generators cost less than synchronous generators. Part of the power output is obtained by transformer action between the stator and rotor of the generator. With a 60 to 100-cycle conversion, 60 per cent of the power output is obtained by transformer action and the driving motor need be large enough to supply only the remaining 40 per cent plus the friction and windage. There are characteristics, however, such as low-power factor, changes in secondary voltage in direct proportion to changes in primary voltage, and inability to adjust the voltage over a wide range by means of a rheostat as can be done with a synchronous generator, that probably will limit the induction type to the smaller ratings and to applications where the high-frequency track circuit is used for train control only.

One of the divisions ordered by the Interstate Commerce Commission to be equipped with automatic train control was the Pan Handle Division of the Pennsylvania Railroad. This division extends from Pittsburgh, Pa. west to Newark, Ohio. It consists of 158 route miles and 375 track miles over which there are operated an average of 233 trains a day.

For this division the code system of continuous inductive train stop with "forestaller" was adopted. Four locomotive cab-signal indications were provided, the three fundamental signals, *viz.*, "proceed," "proceed with caution," and "stop," and a fourth signal indicating that the approach to the next block must be made at restricted speed. To "forestall" an automatic brake application each change in the cab signal indicating a less favorable condition ahead, must be acknowledged by the engineman within six seconds. The acknowledgment is made by momentarily reversing a conveniently located switch and then returning it to normal.

At the same time the automatic train stop was installed, manually operated and d-c. semaphore signals were replaced with a-c. position-light signals.

To provide 100-cycle energy for the wayside signals and automatic train stop, eight automatic substation equipments were installed. The accompanying diagram, Fig. 1, indicates the relative locations of the equipments.

At Newark, Coshocton, and Dennison 60-cycle energy is purchased from the Ohio Power Company and the equipments are located in Ohio Power Company substations. At Weirton Junction and McDonald, power

is purchased from the West Penn Power Company, the equipment at McDonald being located in a substation of the power company. At Pittsburgh, power is purchased from the Duquesne Light Company. Substations were built along the right-of-way to house the equipments at Pittsburgh and Weirton Junction. The railroad company has a large freight locomotive terminal at Scully and the facilities include a power plant with generating apparatus. This provided an excellent location for one of the signal-power apparatus units.

At all of the above mentioned stations the frequency-

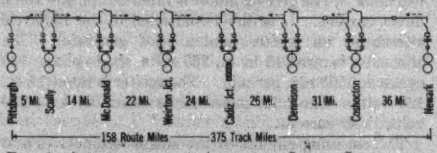

Fig. 1—Arrangement of Automatic Signal Substations Pan Handle Division, Pennsylvania Railroad System

converting apparatus consists of a 75-hp., 2200-volt, three-phase, 60-cycle squirrel-cage induction motor direct-connected to a generator rated 60 kv-a. 80 per cent power factor, 220 volts, 100 cycles at 1200 rev. per min. single-phase with a direct-connected exciter. The motors were designed to carry full load with normal voltage impressed on their terminals with a slip of not more than two per cent of synchronous speed. This insures an output frequency of not less than 98 cycles.

An exception to the rule regarding the purchase of current from public utility companies was encountered at Cadiz Junction, where satisfactory service was not available. Therefore, a gasoline-engine-driven generator was installed. The engine develops 100 brake hp. at 1000 rev. per min. The generator is rated 50 kv-a., 80 per cent power factor, 220 volts, single-phase, 100 cycles, 12 poles. A 15-hp., 100-volt starting motor, operating from a storage battery, cranks the engine and brings it up to speed before the engine takes the load. The motor is located between the engine and the generator. The exciter is direct-connected to the other end of the generator shaft.

Several schemes of operation are possible with the apparatus provided for the Pan Handle Division. The scheme that was decided upon as offering the most advantages from both the economic and operating points of view was for all stations, except Cadiz Junction and Pittsburgh, to be in service under normal conditions. Newark feeds east to Coshocton; Coshocton feeds east to Dennison and Dennison feeds east to Cadiz Junction. Weirton Junction feeds west to Cadiz Junction, McDonald west to Weirton Junction. Scully feeds west to McDonald and east to Pittsburgh. At all stations the 100-cycle voltage is stepped up to 6600 volts for transmission, a pole line having been built the length of the division for the special purpose of taking care of this single-phase power line.

If the voltage fails on the line between Newark and Coshocton the contactor on the west feeder panel at Coshocton instantly closes and restores voltage. In like manner a voltage failure between Coshocton and Dennison provides the impulse for instantly closing the signal-line contactor on the west feeder panel at Dennison. The gasoline engine generator set at Cadiz Junction is used only for emergency service. It is, like the motor-generator sets, fully automatic in operation. Starting impulse is received in the event of a voltage failure on the signal line between Cadiz Junction and Dennison or Cadiz Junction and Weirton Junction. It starts and takes the load in less than six seconds. A line-voltage failure between Weirton Junction and McDonald causes the contactor on the east feeder panel at Weirton Junction to close instantly. A failure at Scully causes the east feeder contactor at McDonald to close and provides the starting impulse for the emergency motor-generator set at Pittsburgh. A clapper-operated oil switch connects the motor to the 65 per cent tap of an auto-transformer for starting and a solenoid-operated oil switch (the solenoid being energized from the exciter) connects the motor to full voltage. Starting in this manner it requires, depending principally upon bearing temperatures and voltages of the primary supply, from five to six seconds to restore voltage to the signal line. The motors are started through auto-transformers. They were designed, however, for full voltage starting under which condition the time required to restore voltage on the signal lines could be reduced somewhat.

Transfer of load from any station to an adjacent station may be made in less than one second. For example, to transfer the load on the Newark Station to Coshocton it is necessary only to depress a button on the feeder panel at Newark causing the signal-line contactor to open. The loss of voltage on the signal line, as previously stated, provides the closing impulse for the west feeder contactor at Coshocton.

In the event of signal-power line trouble it is important to segregate the defective section as quickly as possible. On the Pan Handle Division a pole-mounted oil circuit breaker is located approximately midway between each pair of adjacent substations. These breakers are set to open instantly on short circuit. If a short circuit occurs on the side of the breaker farthest away from the station feeding the section, the breaker opens instantly and sectionalizes the line. If the trouble occurs between the breaker and the station feeding the line, the line contactor on the feeder panel opens after an appreciable time depending upon the setting of the controlling relay. The loss of voltage provides the closing impulse for the corresponding feeder contactor in the adjacent station. The instant this contactor closes the oil circuit breaker opens and sectionalizes the line. At the west end of the division,

where the traffic is relatively light as compared with the east end, there is a possibility of 18 mi., or half the distance between Newark and Coshocton, being out of service due to line trouble. Eastward the distances are shortened until, between Scully and Pittsburgh, there is a possibility of 2.5 mi. being automatically segregated because of line trouble.

Manually operated, two-way disconnecting switches are located one at each signal location for further sectionalizing the line. The signals average approximately one per mile east of Dennison and one for every five to ten thousand feet west of Dennison.

Additional flexibility is obtained by the use of a two-pole, manually remote-controlled, gang-operated pole-top disconnecting switch at each substation which provides means for tying together any two adjacent sections of the power line.

With generating apparatus, such as is used on the Pennsylvania, supplying current for both signals and train control, automatic voltage regulators are used. Overvoltage does not make for a dangerous condition in so far as the train control is concerned but it does provide a dangerous condition in connection with the operation of the wayside signals. This is because overvoltage may provide so much current in the track circuit that sufficient current will not be shunted by trains to deenergize the track relays. If automatic voltage regulators are not properly maintained the contacts may, in time, stick, which will result in excessive voltage. Provision should be made for either automatically removing the generator from service or automatically inserting a fixed resistor in the exciter field sufficient to reduce the voltage to a safe operating value.

Provision is made for opening the signal-line contactors on short circuit. If the station is an end station, such as Newark or Pittsburgh, and, therefore, has only one line contactor, the motor-generator set will shut down and will be automatically locked out of service if the contactor opens due to short circuit. If the station has two feeders, both line contactors must be opened before the machine will be automatically locked out of service. Overloads which if of short duration will do no damage but, if sustained for a considerable time, will result in trouble, cause thermal relays to function and remove the motor-generator set from service.

Other features, such as reverse-phase protection for the motors, reclosing of line contactors a definite number of times after opening on short circuit, before finally locking open, and annunciator relays to indicate to the maintenance man the protective relay that has functioned, are incorporated in this class of substation apparatus if warranted.

Another modern installation of automatic signals is located on a branch of the Long Island Railroad. This installation is interesting on account of one unusual feature as already mentioned. The unusual feature consists of a d-c. driving motor in the motor-generator set instead of the more usual a-c. motor. It was not possible here to use an a-c. motor because the frequency of the main a-c. supply varies so much that the signal frequency would vary beyond the permissible limits of 96 and 100 cycles.

In this installation the Long Island Railroad has automatic train stop between Jamaica and Babylon, a distance of 27.6 mi. An alternator for providing 100 cycles is located in each of the substations which house the electric-traction synchronous converters at Babylon, Lynbrook, and Jamaica. Lynbrook is 8.3 mi. east of Jamaica and 19.3 mi. west of Babylon. The motor-generator sets at Babylon and Lynbrook are four-unit machines. The driving motor is rated 20 hp., 650 volts, direct current. It is direct-connected to the 100-cycle alternator, an exciter, and a pilot generator. The alternator is rated 15 kv-a., 220 volts, single-phase, 100 cycles at 1500 rev. per min. The exciter is rated 1 kw., 125 volts. The pilot generator is rated 0.5 kw., 250 volts direct current.

The receiving circuits of the steam locomotives and multiple-unit cars are tuned for best results at 98 cycles. The efficiency of reception decreases as the frequency changes in either direction from normal, the permissible limits of frequency being 96 and 100 cycles. The motors are operated from the 650-volt d-c. busses which also supply energy for the multiple-unit cars. As the bus voltage may vary between 450 and 700 a speed regulator had to be provided. The pilot generator operating in conjunction with a simple and sturdy vibrating regulator accomplishes the desired results.

The regulator consists, principally, of a stationary coil and magnetic circuit and a movable coil in series with the stationary coil. The movable coil is suspended on a knife edge in the magnetic field. When the voltage of the pilot generator is 250, which it is at normal speed, the movable coil is in mid-position and the contact on the arm attached to the coil is midway between two stationary contacts. If the speed of the motor increases, the voltage of the pilot generator increases and the movable coil takes such a position as to bring together the contact on its arm and one of the stationary contacts. This strengthens the motor field by shunting the field rheostat through a fixed resistor. If the speed falls below normal a spring pulls the movable arm in the opposite direction and draws together the movable arm contact and the other stationary contact, which weakens the motor field by shunting it through a fixed resistor and the speed increases. The contacts are made of high-grade graphite, 1⅜ in. in diameter. They will not stick and they have long life.

Under normal operating conditions the alternator at Lynbrook feeds east to Babylon and west to Jamaica. The 100-cycle voltage is stepped up to 2200 volts for transmission. The alternators at Babylon and Jamaica are for emergency service only. The switching in all stations is performed manually. As attendants are present to supervise the electric-traction synchronous converters and as all stations receive their primary

25-cycle energy from the same source, little benefit would be obtained from automatic switchgear.

Conclusion

Automatic block signals are established firmly as necessities on those railways where they are installed and where operations are planned so as to take advantage of the safety and dispatch which the signals provide. Relatively simple, automatic substations used in conjunction with the a-c. systems, when the frequency of the signal energy is the same as the frequency of the primary supply, will be employed in increasing numbers as signaling systems are extended.

The future of the frequency-converting substation is not as certain. The 100-cycle current is primarily for train-control operation. All of the installations ordered by the Interstate Commerce Commission have been completed. Whether additional orders will be issued and whether the carriers on their own initiative, will make additional installations are open questions. In any event, the frequency-changing motor-generator sets in ratings of 1.5 kv-a. to 60 kv-a. and the accompanying switchgear, that have been installed in the past three years, are interesting additions to the steadily increasing electrical facilities of our railway transportation systems.

Discussion

E. F. Pearson: The statement is made in this paper that transfer of load from any station to an adjacent station may be made in less than one second.

In order to perform this operation it appears that three or four relay operations and two contactor operations are necessary. If we add together the time increments necessary for the various relaying and contactor operations it would appear that the operation would take considerably longer time than one second, and I, therefore, question the accuracy of this statement.

L. J. Corbett: In the early days of the automatic train-control systems, when the Interstate Commerce Commission issued its order on the matter, some of us in the inductive-co-ordination work saw a cloud on the horizon. The early systems developed used the commercial frequencies of 25 and 60 cycles, and it was found that when power lines of such frequencies paralleled railway lines equipped with these systems, induction from the power lines could sometimes cause wrong signals in the cabs of the locomotives. Such a condition would have added another obstruction to the placing of power lines along railways, and the right-of-way man would have had a more difficult time in all later constructions.

Fortunately, however, due to the agitation by men who saw the induction difficulty, and also to the cooperation of the railroads and the signal manufacturers, who departed slightly from their earlier competitive policy in producing the control systems, there was introduced that additional link,—the transformation from the commercial frequencies of 25 and 60 cycles to one which insured freedom from inductive interference. The 100-cycle control systems now standard will not be adversely affected by any harmonics of 25 or 60 cycles. This was a big step in this development, and we watch with interest its continued progress.

C. E. Carey: Mr. Pearson questioned the length of time required to restore voltage to the line. The time required to depress the button and the time for the feeder contactor to open are not included in the "less than one second" required to transfer the load from any station to an adjacent station. The loss of voltage on the line, following the opening of the feeder contactor in the station dropping the load, causes a low-voltage relay in the station at the other end of one of the adjacent sections to close its contacts. This closes the circuit to the operating coil of a feeder-contactor control relay, which relay functions to close the circuit through the operating coil of the feeder contactor. The feeder contactor closes and voltage is restored to the line. By as careful timing as possible, with a split-second stop watch, it was found to require 0.9 sec. to restore voltage.

T. S. Stevens: (communicated after adjournment) I am not at all certain that the first paragraph is true. The Union Pacific has been equipped with automatic signals long before 1922. It is probable that the largest mileage of automatic signals was installed after this year, but I do not remember that the development was dependent on the train control order by the Interstate Commerce Commission. In fact I know of several instances where the installation of automatic signals was postponed until the policy of the Interstate Commerce Commission had been established.

The description of the track circuit does not bring out the essential feature of a closed track circuit. The absence of current in a track circuit is an abnormal condition. Ordinarily the circuit is closed continuously. The display of a stop signal is caused by the current being shunted away from a relay through the wheels of a train.

Frequency changers have been in service on the Santa Fe for about twenty years. They are used to obtain a 25-cycle current from a 60-cycle commercial supply in order to take care of the higher power factor of the 25-cycle current used on track circuits which results in a longer length of track circuit than could be obtained by the use of 60 cycles. It is probably true that with the development of all a-c. relays the efficiency of these devices has been increased to such an extent that if the installations were made today it would not be deemed necessary to provide 25-cycle current.

C. F. King: In answer to the communication from Mr. T. S. Stevens received after adjournment of the meeting, I wish to call attention to the fact that during the five-year period January 1, 1923 to January 1, 1928 there were installed 17,379 route-miles of automatic block signaling on the railroads of the United States and Canada. Probably 3 per cent of this mileage was on Canadian roads. This was, by far, the most active five-year period of automatic-signaling installations in the history of our country. This period of great activity occurred immediately after the automatic-train-control orders of the Interstate Commerce Commission were issued and it does not appear unreasonable to conclude that these orders were not entirely in the background when consideration was being given to installations of automatic signals on much of the above mileage. The figure of 17,379 was obtained from "Railway Signaling," issue of January 1928.

The purpose of the statement that "absence of current in the track causes indication of the low-speed or stop signal" was to point out that the fundamental principle of automatic signaling systems, *viz.*, the closed track circuit, protects against abnormal conditions such as "absence of current in the track" which may be the result of a broken rail. The paper was prepared to deal particularly with the power supply apparatus and the discussion of the signals was purposely made as brief as possible. Paragraph three probably was made too brief.

Mr. Stevens states that frequency changers have been in service on the Santa Fe for about 20 years to obtain 25-cycle current for track circuits from a primary 60-cycle supply. It is not claimed that rotating apparatus of this class is, essentially, new. It is the frequency, *viz.*, 100 cycles, which attaches the principal interest, if any, to the frequency changers discussed in this paper.

Automatic Mercury Arc Power Rectifier

Substation on the Los Angeles Railway

BY L. J. TURLEY[1]

Associate, A. I. E. E

Synopsis.—*The development of the commercial mercury arc rectifier is outlined briefly in this paper and there are given the advantages which led to its adoption in a double-unit automatic station recently installed by the Los Angeles Railway. A report is made of the very satisfactory operation which has resulted. A description of the station is given and methods of operation and maintenance are presented.*

* * * * * *

INTRODUCTION

ON reviewing the various epochs of the historical development of the mercury arc rectifier, commencing with the experimental work of Peter Cooper Hewitt in 1902 and covering a period of twenty years, one is at first amazed at the slow rate of progress of American manufacturers toward perfecting such an equipment as compared with that usually observed in the development of similar important modern productions. For ten years prior to the war, the manufacturers seemed to confine their research to the theory and problem of proper vacuum maintenance, vacuum seals, and temperature allowances, apparently without reaching a complete and satisfactory solution. Naturally, a general state of apathy existed during the war; while with the few years immediately following, the reluctance to concentrate their efforts toward a final solution, sufficient to start production, may be attributed largely to the enormous demand for immediate delivery of rotating apparatus required to meet the general expansion and growth of the various electric industries. Sound engineering expediency seemed to give way to the exigency arising from aggressive commercial expansion.

Meanwhile, the European manufacturers developed a type of rectifier that followed the established principles common to early American designs, and gave sufficient service satisfaction in the foreign field to invite its introduction into this country for an apparent and ready market. Simultaneously, the enterprising spirit of American manufacturers was prompted to activity in finishing its original design, to the extent that production was started, followed by a rigid field service test to meet all preliminary requirements, and resulting in the creation of a pronounced interest and demand in the field of prospective users.

Although the basic structure of present day rectifiers shows no radical or marked advance over early developments, still a variety of problems arose, not only with respect to the design, but involving the protection and operation of this type of apparatus. These broadened the field of exploration for considerable added and novel improvements—especially with the auxiliary equipment. The speed of all this accomplishment has been reflected in a final solution as embodied in the present type of rectifier now under discussion. Thus, during the last three years, we have witnessed many pioneer installations of European and American makes of equipment, scattered on diversified railway loads, that have shown a rapid growth in popularity and have produced a considerable degree of confidence, due to the rectifier's general simplicity, compactness, and reliablity.

As greater familiarity with this equipment developed, many other important factors in its installation and operation appealed to the operating engineer. A study of conditions that entered into a problem for a specific class of application, and the results from its short time operation, placed this rectifier in a favorable position and demonstrated its relative advantages in its supplementing synchronous converters, where such application involved load centers of low load factor and heavy momentary peaks.

Finally, its insensitiveness to short circuits; its adaptability to full automatic control; its comparative noiselessness in operation; its ability to carry sustained short time overload; its absence of moving parts, excepting small auxiliaries; and the elimination of the attending troublesome features commonly experienced with the operation of large rotating machines, all substantiate the claims advanced by its proponents and constitute such valuable and economical assets as obviously to insure for the rectifier a most favorable recognition by the more enterprising engineers of the railway industry.

There are a few apparent disadvantages. The penalty of considerably lower power factor may be serious with some installations, but this is more than offset by a high and favorable all-day efficiency factor, when based on over-all conditions, in comparison with a synchronous converter installation of similar rating and operating under similar load requirements. The complaints from sections, of interference with communicating circuits, are comparatively few and are fast disappearing. The characteristic high harmonic voltages, as generated by a rectifier, are subject to correction at very little expense. Suitable and satisfactory mitigation of ad-

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28-125

verse noise units have been applied through the co-operative measures instituted by utilities and manufacturers, to the complete elimination of all such contentions. Surely, the 10 to 15 per cent higher first cost over synchronous converters would seem to be justifiable only to cover a certain amount of development cost common to all new introductions. Usually, quantity productions soon absorb and wipe out the excess in such overhead costs. How soon this lower price is to be realized by the users of this equipment would seem to depend upon the rate with which the manufacturers expand their design and facilities to offer larger ampere capacity units for active competition throughout the entire range of synchronous converter applications. The various economies resulting from the operation of rectifiers will then accrue to the operating companies in amounts of substantial proportions.

The numerous papers on this subject published during the last few years have dealt mainly with the theoretical principles applied to or developed from the initial and subsequent design of the mercury arc steel rectifier of relatively small capacity. Only recently we have been given information on actual performance of a few installations of 1000 kw. capacity. Throughout this outline we will draw deductions from comparisons with the synchronous converter. We will not touch upon the fundamental theories of this type of apparatus, nor attempt an explanation of its physical phenomena, but present important factors that led up to the decision in favor of this type of equipment, and give added proof of the rectifier's satisfactorily meeting a railway load requirement, as gained from our experience with a short time operation of the double-unit automatic substation recently installed by the Los Angeles Railway in Inglewood, near the City of Los Angeles.

Fig. 1—Inglewood Substation

General Description of Building and Equipment

Situated in a beautifully parked ground, the exterior of this steel framed, reinforced concrete building, with its amber colored glass windows, large arched doorway, heavy tiled roof, and double belfry towers, presents a striking similarity to a chapel of typical Spanish architecture. It was purposely designed as such, to meet the architectural harmonies of adjacent buildings. Fig. 1 shows exterior view of the building. The blind windows and small entrance door, suitably fitted, prevent the entrance of dust, except as it may be carried through the regular air channels, which are provided with air filters. No extra expense was incurred toward sound-proofing against what little noise or hum might develop from the operation of the totally inclosed apparatus.

The interior of the building is artificially lighted, effectively air cooled and has plastered walls painted white. The equipment consists of a double 500-kw. unit complete with separate three-phase power transformers, interphase transformers, reactors, rectifier tanks, vacuum pumps and switchboard, the protection and operation of which are under full automatic control. The feeder panels are similar to the standard automatic high-speed circuit breaker equipment and control scheme as installed in other automatic substations on this railway property. On the main floor level, on either side of the entrance, is located the transformer compartment of each unit, while in the main room a rectifier has been placed to each side of the aisle-way, with switchboard facing same at the rear of building. Above are consecutively arranged on a double gallery and in switching rooms, the double 15-kv. high-tension entrance lines, lightning arresters, busses, and oil circuit breakers. Figs. 2 and 2A give an interior view of the station, showing position of rectifier and switchboard section.

Certain layout of equipment in building might require greater or even less space allowance for the rectifier; but with this application the width of building was determined largely by the length of switchboard and height by state law clearances, while the building length was due to the 15-kv. equipment and bus line, all of which would have been the same area and length whether for 500- or 1000-kw. double-unit installation of the synchronous converters or rectifiers.

Application

The decision to apply rectifier equipment with automatic control resulted after considerable study, the analysis being carried out in some detail, an abstract of which may be of some interest.

The entire length of railway line to be fed from this station, extends over six miles beyond the intermediate section of the railway system. Fig. 3 shows map layout. Formerly this line received its power from the small 25 year old manual station, consisting of three small units and located within 2000 ft. of the site of the new substation. Due to other developments, both old building and equipment had to be abandoned and removed. Except for special events, the car service normally rendered had a peak headway of four min., graded to 25 min. for lighter load periods. The low

average energy consumption and the resultant low station load factor, coupled with intervals of 30 to 60 min. between load demands during early morning hours, which would create a possible frequency of starting and stopping of station equipment, were essential factors of operation that gave the rectifier more favorable comparison in applying its electrical characteristics as to efficiency, readiness to serve, and simplicity in starting.

Fig. 2—Switchboard for Double Unit and Three Feeders

Further, there was a commendable interest on the part of the railway company in assisting the industry in pioneering this new introduction with a practical application. Provisions have been made to keep necessary records on results of operation, so as to verify all claims as before set forth and those claims that are expected to be proved as to less frequent inspection

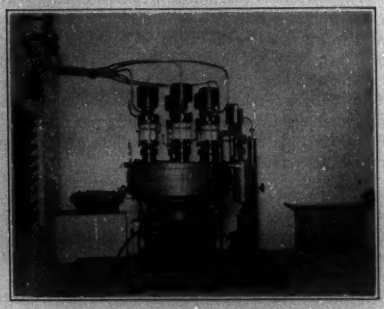


Fig. 2a—Number 1 Unit, 500-Kw. Rectifier

requirements, longer periods between overhauling and lower maintenance charges per 1000 kw. installed capacity in comparison with the several automatic synchronous-converter stations of the railway company.

Doubtless a single 500-kw. unit would have been ample in capacity with a slight marginal allowance for future growth; however, in the event of this single unit's failure, normal railway service to this extended line could not have been economically rendered from the next closest substation. The investment in a double unit was considered advisable and followed the liberal policy of the railway company toward guaranteeing the greatest possible safeguard for continuity of power supply and car operation to this important section of its system.

The experience obtained from many years of operation with automatic control applied to synchronous-converter railway substations, naturally led to choosing an automatic application for this rectifier station. It is simple, effective, and surprisingly reliable, and has become generally adopted. Those chief factors that

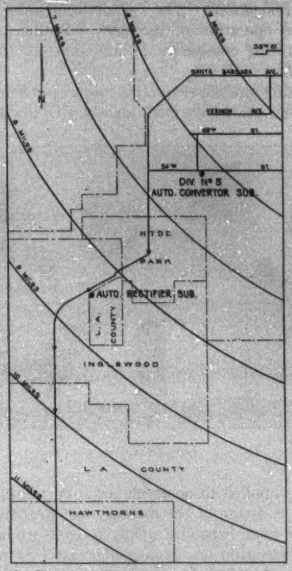


Fig. 3—Trolley Line and Substation Location

are usually presented in favor of automatic control—improved labor conditions, improved service continuity, and reduced operating expense—may be considered as the salient factors derived from operation of rectifier equipment.

The skepticism and doubts witnessed in the early part of full automatic control applications are being repeated with respect to the rectifier introduction and performance. Time alone should efface all such uncertainties.

Weights

It is interesting to note from Table I, which gives approximate weights of various apparatus, including 8000 lb. for interphase transformer and d-c. reactor, the

total weight of the rectifier complete installation is only 400 lb. greater than for synchronous-converter units. The rectifier itself weighs one-third that of the converter and did not require any special foundation, except as given by a six inch cement flooring.

TABLE I
COMPARATIVE WEIGHTS

	Converter	Rectifier
Machine	15,000	4,500
Main transformer	14,850	19,500
Starting panel	1,500	....
Interphase transformer	....	4,700
D-c. reactor	....	3,600
Switching equipment	27,000	26,750
Total	58,350	58,750

Rating

With the exception of their momentary load ability, the rating of these 500-kw. unit rectifiers is equal to that of synchronous converters, since they are based upon the same standard method of rating such equipment for railway application. Table II shows this comparison.

TABLE II
COMPARATIVE RATING

	Synchronous converter	Rectifier
Continuous rating—amperes	833	833
Two hour rating—per cent load	150	150
Two hour rating—amperes	1250	1250
One minute rating—per cent load	300	200
One minute rating—amperes	2500	1667

Although a converter would ordinarily carry its two-hour overload rating of 150 per cent load continuously, yet the sustained high temperature would naturally shorten the life of the insulation. On the other hand, the rectifier, exclusive of transformers, could be expected to operate continuously at its two-hour overload rate without shortening its life.

The momentary load of one minute duration allows the converter 300 per cent load and the rectifier 200 per cent load. This difference is attributed to the rapidity with which the maximum allowable temperature rise affects operation. The converter operates through a wide range of temperature allowances. Due to its thermal capacity, a 300 per cent load superimposed for one minute upon its continuous load temperature does not raise the temperature of the hot spots of the machine fast enough to do injury before the load has been reduced. The rectifier works with temperatures of a very narrow range, has low thermal capacity, and is affected by low as well as high temperature. A one-minute load of 300 per cent creates such a rapid rise of temperature that the heat affects the performance, and might damage the anode tips.

The instantaneous overload of a converter is limited usually by its commutating capacity, and when such limits are reached there is a possibility of damage to the machine; locking it out of service, especially if a flashover prevails. The rectifier is not so affected by instantaneous overloads, even of short circuit proportions. Under certain conditions the rectifier may occasionally "arc-back," which results in practically no damage; at least not enough to prevent it going back into operation within a few seconds.

Efficiency

A large amount of the power used by the auxiliary apparatus is foreign to a synchronous-converter installation. The segregation of this power for the six-phase unit at Inglewood is given by Table III.

TABLE III
POWER FOR AUXILIARIES

	Watts
Six, 150-watt heaters for anodes	900
Three, 2000-watt heaters for tank	6000
One, 500-watt heater for condensation pump	500
One vacuum pump, complete	700
One arc striking set, complete	1000
Total	9100

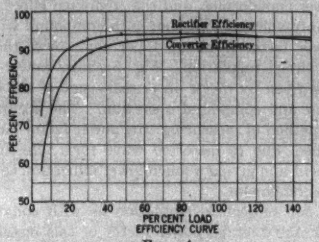


Fig. 4

Under ordinary operation, the 6000-watt tank heaters are seldom in use; and to a slight extent only are the anode heaters and vacuum pump load used. These facts require a comparison of efficiency to be made on over-all conditions. The efficiency of a rectifier itself is practically constant for all loads and considerably higher than with a converter. The drooping characteristic in the efficiency curve for a rectifier unit is largely attributed to the effect of the power transformer. The manufacturer's curve on efficiency, as estimated for this installation, is given by Fig. 4. A certain number of check points is plotted on this curve, as actually obtained by using properly tested a-c. and d-c. watthour meters, which power includes consumption by all auxiliary apparatus. On an average 24 hr. load of 200 kw., each rectifier unit in this substation has an efficiency of 94.0 per cent against 90.0 per cent for a converter of corresponding size. At a lower load factor than at Inglewood station, a greater difference in efficiency would be obtained, which would favor the application for interurban railways.

Power Factor

Although the specific power factor of the rectifier itself has a fairly high value, when measured on the primary of the power transformer, it ranges from 90 per cent to 92 per cent for ordinary loads, reaching as low as 75 per cent at 10 per cent light load. The operating power factor for Inglewood, during the normal 18 hr. range of load, remains about 86 per cent. It is lower

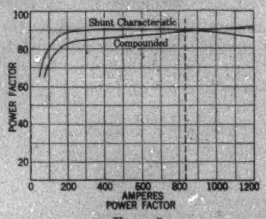


Fig. 5

than the manufacturer's estimate, doubtless due to the compounding features, the drooping characteristic of the power transformer, and the combined effect of the a-c. supply voltage. In fact, with compound winding, the power factor is about 4 per cent lower than for shunt voltage regulation. Fig. 5 shows power-factor curve as given by meter on primary side of the circuit for shunt and compound operation. The accuracy of such a method of measuring true power factor has been questioned, due to distorted wave form affecting the instrument and giving a much lower indication than the wattless component.

It is not subject to regulation as with converters; however, the average power factor of this installation is about the same as on portions of the railway system using induction motor-generator sets. Since the railway company has no penalty in contract with the power company on purchase of power, based on kilovolt-ampere demand factor, a lower power factor than that obtained with synchronous converter is not so important an objection, especially with a substation of such small capacity.

Regulation and Voltage Characteristics

Although the voltage drop in the rectifier proper is nearly constant for all ordinary loads, yet the shunt characteristics of the combined unit of rectifier and transformer is claimed to give about six per cent voltage regulation at full load. This variation was considered with less favor for this particular double-unit application of the Los Angeles Railway since the company's policy is one of maintaining a fairly high uniform trolley voltage along the route of car travel to the outlying districts.

Voltage curve, (Fig. 6) includes regulation for both shunts and compound voltages with this installation. The compound features of the combined unit give a 2 per cent regulation. The a-c. supply voltage is 3 per cent above normal and results in a 3 per cent higher d-c. voltage than originally calculated. To guard against such conditions, it would seem advisable to have one, if not two, higher voltage taps above normal operating voltage, where formerly it has been the custom to place all taps below normal.

The additional investment which compounding entails was considered more desirable to meet requirements than a similar investment in feeder copper with its doubtful result of voltage uniformity at the trolley. The periods of high peak, 730 volts at no-load, for which the interphase transformer is solely responsible, were of such short duration that these were not detected at the feeder outlet of the station, doubtless due to a voltage reduction to normal immediately as a small load develops, at the instant of automatic closing the rectifier d-c. main line contactor.

Telephone System Interference

Tests were conducted for inductive interference on the various communicating circuits that parallel the trolley line for several miles in both directions from the substation. Perfect satisfaction was expressed by the telephone company engineers on the freedom from noise developed and it was surprising to note there was less interference with rectifier in operation than the noise experienced from operation of the motor-generator sets in the adjoining manual station.

This was accomplished by connecting between station bus and rail ground a filter system tuned to resonance at the characteristically pronounced frequencies developed by the rectifier equipment. This filter comprises four networks, each consisting of an air reactor and condenser. Under a test with the resonant shunt on the circuit, the harmonic voltages were reduced to approximately one-fifth the value without the shunt. These frequencies are rated at 300-600-900-1200 cycles.

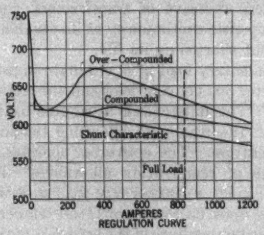


Fig. 6

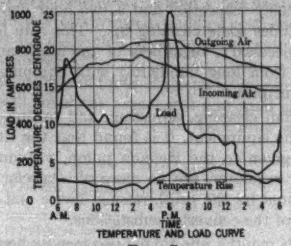
Ventilation

The Inglewood Station is provided with forced air for transformer cooling and general station ventilation. Cool air enters above roof line through the two belfrey towers, down separate shafts, to filters located at rear of each transformer unit; thence from transformer room through louvres to machine room, from which it is discharged by one motor-operated exhaust blower,

located near roof. The blower operates continuously, except as controlled by under-speed relays, or whenever both units are shut down. The 11,000 cu. ft. per min. give a complete change of air in the building at the rate of ten times per hour. During periods of hottest summer days, at 40 per cent load factor, the maximum temperature rise of transformer oil by thermometer is 27 deg. cent.; while general room temperature averages 3 deg. cent. rise above outside temperature. At other enclosed 1000-kw. substation using converters, a temperature rise of 7 deg. cent. is the general average. Radiation of heat from rectifier tank, heaters, and accessories, is calculated at 25 per cent of total kilowatt loss equivalent, and does not raise the room temperature over 1 deg. cent. following maximum load periods. Fig. 7 shows air temperatures with 24-hr. load curve.

Water Cooling System

Tap city water is used, averaging 20 deg. cent. with discharge of waste water into cesspool. The fixed charges on investment covering a closed circulating system or any other method of cooling and circulating water were found to be unreasonable compared to the



TEMPERATURE AND LOAD CURVE

Fig. 7

annual cost of wasting water as used in this case. A water meter installed on each rectifier intake line gives means of checking the rate of flow for various temperatures. The total 24-hr. water consumption equals 825 cu. ft., giving a rate of 0.9 gal. per min. per 100 amperes, which is considerably higher than the manufacturer's estimate, but could be reduced without fear of over heating.

The cooling water for the mercury condensation pump flows continuously and has been adjusted by hand valve to a rate of 1/5 gal. per min., to obtain slightly under 30 deg. cent. temperature on discharge. A water jacket of considerable volume surrounds the vacuum chamber of the rectifier. It is fundamentally necessary to keep the temperature of the jacket within certain limits, because the mercury vapor pressure is controlled more by this water temperature than by anything else. The flow of tank cooling water is regulated by an automatic thermostat operated valve. It is set to open at a value of 33-35 deg. cent. and to close at 30-32 deg. cent. During the period that the temperature of the outgoing water is lower than 30 deg. cent. the operating valve is closed automatically; which is the case when the tank is under extremely light load, or is shut down.

The jacket heaters are adjusted to come on automatically, to maintain the water above 25 deg. cent. temperature. A check for a three day period on the frequency of the jacket heaters to come on with an idle unit, showed an average per 24 hr. of 5 times on, at an average of 6 min. per interval.

The heat developed at the anode tip is carried upward to the anode radiators through the efficient action of mercury cooling and condensing near the top of the anode chamber. It is vital to the performance that a certain fairly high temperature be maintained on the anodes to prevent mercury condensing on the tips. Under ordinary load with this station, the anode heaters are not in use, being frequently on and off during the extremely light load periods under 150 amperes.

Vacuum Control Operation

High vacuum is of prime importance in the operation of the rectifier. Satisfactory operation depends upon the maintenance and checking of the degree of vacuum. Under normal conditions, the rectifier operates at approximately 0.1 of a micron—being 0.0001 millimeter of mercury pressure (absolute) or 1/7,600,000 of an atmosphere. In order to obtain this very low pressure a pumping system is used, involving a roughing pump or oil pump, and a fine pump or mercury condensation pump, the two operating in series. The rough pump is a motor-driven mechanical pump operating under oil, and will obtain a pressure by itself of 5-40 microns. In order to obtain the finest degree of vacuum to 0.1 micron or less, the mercury condensation pump is operated in series with the roughing pump, which will obtain a theoretically perfect vacuum and pump against a back pressure of 3000-4000 microns. An electric heater delivers heat to a boiler holding about 70 cu. cm. of mercury, which sends a stream of mercury vapor up through the condensation pump, through a pipe toward the incoming side of the pump. This action of boiling, condensing, and returning to the boiler of the mercury is a ceaseless cycle, so that the gases are constantly being brought down with the vapor and then forced out of the pump into a line to a receiving tank located between the condensation pump and the oil rotary pump. The rough pump draws the gas away from the receiving tank and forces it into the atmosphere. There is just enough heat applied to the mercury in the mercury pump so that all is condensed and returned within the pump itself. The pipe line which leads from the rectifier to the mercury pump is surrounded by water, and any mercury which enters this pipe never reaches the mercury pump, but is condensed and falls back into the rectifier chamber. After the high vacuum has been obtained the automatic vacuum protection shuts the oil rotary pump down, and this degree of vacuum is maintained for long periods requiring the operation of the pump at intervals of one or two hours a day only to

a possible one hour in several days. The mercury condensation pump is kept in operation at all times and pumps into the receiving tank what gases develop and what seepage there is in the rectifier.

The automatic control features contain a few special and interesting means of protection. Several devices have been included with the vacuum system for suitable protection in case of vacuum failure. In addition, they function to control the starting and stopping of the oil rotary pump. A thermocouple vacuum gage, a millivolt meter, and a mercury contacting device perform the following operations:

1. Start oil rotary pump, if vacuum falls to 2.5 microns.
2. Lock the station out, if vacuum falls to 15 microns.
3. If station has been locked out and vacuum of 1 micron is again obtained, the oil rotary pump stops after a time delay, and the station is allowed to go back on the line.
4. If the oil rotary pump has started, due to vacuum falling to 2.5 microns, when the vacuum returns to one micron the pump is shut down after a time delay.

With a rectifier, reverse current can occur only at the instant of the arc back or a short circuit of the electrode. If this occurs, a high-speed breaker, connected for such reversed direction of current, will trip-out, disconnecting the unit from load and reclose within a few seconds. Three consecutive openings of this breaker locks out the unit. There have been no arc backs with the Inglewood Substation rectifiers.

There are conditions in operation which develop a high-voltage kick and which might do injury to a transformer windings. Protection against this possibility is obtained through absorption by connecting a group of condensers with resistors across one leg of each wye of the power transformer low-tension windings and with the interphase transformer; also, as added protection through short circuiting, a pellet type of arrester parallels each of these groups.

The arc drop is influenced by the state of the vacuum. The exciting arc is established the instant of closing the circuit and is maintained during the entire load period. The ½-kw. exciter generator has a load of 7 amperes at 65 volts.

TABLE IV
COMPARISON OF DOUBLE-UNIT CONVERTER AND RECTIFIER AUTOMATIC DEVICES

Device	Converter	Rectifier
Main current parts	15	7
Protective relays	29	28
Auxiliary devices	36	45
Total	80	80

The number of control devices, switches, etc., required for the complete two-unit rectifier, has been compared, as given by Table IV, with those required for a synchronous-converter installation.

For the rectifier there are one-half the number of main current carrying parts, practically the same number of protection features, with a slightly greater number of auxiliary devices, totaling an equal number for both types of units.

Inspection and Maintenance

Regardless of an equal number of control devices and switches, as given by Table IV, which consists of a few special pieces of apparatus and devices of delicate construction, the frequency or general routine inspection for this substation has been conservatively established on an interval of every two weeks. A daily visit of one hour is made to take necessary meter readings, check general state of small rotating equipment, and force the operation of the vacuum pump against its automatic control, so as to assure a faster rate of seasoning-in of the vacuum tank. This practise is not absolutely necessary, but it is our method of giving this new type of equipment all the attention deserved. This inspection rate is considered ample and commensurate with the service various parts are called upon to render. The automatic converter substations of this company require weekly inspection and a two hour daily visit, which includes some cleaning up. This rate for converter is necessary, regardless of the importance and assurance from long familiarity with equipment and control principles.

Aside from cleaning, adjusting, and testing regular parts of the equipment, it is found that the time and attention given to the vacuum pump, vacuum relay and water regulator, are no more of a special nature than required by the brush-raising device and master controller of the converter substations.

Every six months, and taking two men six hours, it is planned to clean the mercury in the anode heating chamber and replace all gaskets. This does not break the vacuum in the tank.

Doubt has been expressed only too often, that maintenance cost of rectifier and its auxiliaries, especially that cost incidental to the vacuum pump, would not be less than the items required to cover actual labor and cost of material for repairs to synchronous converter and its coordinated equipment. In fact, overhauling of the rectifier has been heralded as taking longer than necessary work on the synchronous converter. These doubts and claims have not come from operating engineers using the rectifier equipment.

The vacuum pump and its appurtenances are rigidly and sturdily constructed and should give less trouble than the small time-aged motor-generator set used for exciter purposes. All the necessary and essential spare parts for the rectifier cost less than the average carried for six converter stations of this company. Since these rectifiers had proper care at the initial drying-out period and have developed no sign of trouble during operation, the most that may be expected in the way of maintenance attention would be that following a two year

period of operation, at which time the total expense of labor and material for overhauling all anodes, tank, vacuum seals, vacuum pump, and necessary time to dry out, should be far less than the usual attention to converters in grinding commutators, attention to new brush fit, changing oil and checking bearing alinement, together with the occasional reinsulating of windings.

Considering that there are no intermediate starting or running a-c. contactors, brush raising devices, no checking of polarity and synchronizing requirements, no massive moving parts and oiled bearings subject to excessive strains of frequent starting, with no carbon and copper dust developing to be scattered throughout the station with high velocity of circulating air, it is obvious there should be no surprise at the present claim and at the eventual proof of considerably lower cost for inspection, cleaning, and maintenance of rectifier equipment.

Savings

As originally planned, the Inglewood Substation was to have two 500-kw. converter sets. The change to rectifiers was made on the consideration of various economies as enumerated. Table V has been prepared showing estimated savings in building cost, energy loss, and labor for inspection, in comparison with the synchronous converter.

The net savings in favor of the rectifier amounting to $660.00 will cover the excess cost and fixed charges of the rectifier in approximately six years, after which it is estimated the total annual saving of $2500.00 will carry 6 per cent interest charges on 50 per cent of the entire investment.

TABLE V
ESTIMATED SAVINGS

Excess cost of rectifier equipment over converter as per contract	$7,450.00	
Annual fixed charges at 12 per cent		$894.00
Excess cost of converter installation for foundations, air ducts, air equipment, sound proofing, etc.	3,540.00	
Annual fixed charges at 12 per cent		425.00
Difference in costs	3,910.00	469.00
Estimated yearly labor savings for rectifier	604.00	
Estimated yearly power savings for rectifier	525.00	
Total estimated savings for rectifier		1,129.00
Total net annual savings for rectifier		660.00

Conclusion

To sum it all up, what could be more simple in effecting such a method of conversion of power than to have, at the instant of load demand, a moment to check proper a-c. conditions, state of vacuum, temperature of tank and anodes, to be followed by closing of the oil switch, striking the arc and closing of d-c. line contactor—all automatically accomplished in a time elapse of 6 sec.? The average converter gets under load in 40 sec.

I trust this paper presents sufficient information of a practical nature on the Los Angeles Railway installation, to serve as a guide to those hesitating as to the choice of this type of equipment.

Discussion

C. E. Baker: Fig. 5 of Mr. Turley's paper on the Automatic Mercury-Arc Power Rectifier shows the power factor of the rectifier as measured by the usual power-factor indicator at the primary of the rectifier transformer. The paper suggests that the power factor of the rectifier may be a serious disadvantage in some installations.

In the case of rotating apparatus, the power factor is largely due to phase displacement of the fundamental wave, and the power producer is very much interested in the power factor of the connected apparatus, as it materially affects the amount of equipment required to deliver a given amount of power.

The primary current wave produced by the rectifier is somewhat distorted from the sine wave, due to the fact that the rectifier itself draws substantially square waves of anode current. There are then several harmonics present in the primary current wave. The indicated power factor of the rectifier is really the average power factor of all the harmonics and the fundamental. Actually there is very little phase displacement of the fundamental, but there may be considerable phase displacement of some of the harmonics. Therefore, the average power factor may be relatively low, while the power factor of the fundamental is practically unity.

Unless there is a complete understanding of the difference between rectified and rotating apparatus in this respect, a direct comparison between the power factor of the two classes of apparatus may be misleading, and is not justified.

D. W. Proebstel: In the paper there is a statement, the intent of which is to justify 10 or 15 per cent higher cost. I think it might have been well if this statement had been qualified by a comment relative to the voltage rating of the rectifier, knowing that the costs for increased capacity do not increase very rapidly with increased voltage rating. For instance, we have two of these rectifiers in operation at Portland, Oregon, with a rating of 750-kw. each at 1400 volts where the costs were considerably lower than a similar capacity in synchronous converters would have been.

There is one other statement in the paper that I should like to criticize. It is the one on telephone-system interference, namely, "Under a test with the resonant shunt on the circuit, the harmonic voltages were reduced to approximately one-fifth the value without the shunt. These frequencies are rated at 300-600-900-1200 cycles." I think this should have been qualified by the statement that such apparatus produces even harmonics such as the sixth, twelfth, eighteenth, and twenty-fourth harmonics.

Again referring to our pieces of apparatus which operate at 60 cycles, the frequencies of harmonics are 360, 720, 1080, and 1440. The telephone people in Portland were not slow in reminding us of the fact that these pieces of apparatus were giving them interference. An analysis was made of these harmonics, after which the General Electric Company equipped the pieces of apparatus with resonant shunts for the lower harmonics, which have very satisfactorily mitigated the telephone interference.

Chester Lichtenberg: Automatic mercury-arc power rectifier substations are relatively new for American street railway service. Mr. Turley's paper presents the results of operating experience in Los Angeles. It enables one to form a comparison between automatic stations containing mercury-arc power rectifiers and automatic stations containing the conventional synchronous converters.

Mr. Turley's paper may be considered as the second of a series. Mr. Antoniono's paper presented at Chicago in 1927 gave experience with mercury-arc power rectifiers in automatic stations on an interurban railway carrying important traffic. Mr. Turley's paper gives operating experience with similar equipment carrying important loads in urban territory.

One interesting feature of the comparison is the number of

devices required by synchronous-converter and rectifier automatic stations. The number of main current-carrying parts is more than twice as great in the converter stations as in the rectifier stations. The number of protective devices is approximately the same. The number of auxiliary devices, however, is 25 per cent greater in the rectifier station than in the converter station. This condition, while it obtains at the present time, is sure to change as the operation of mercury-arc power rectifiers becomes more familiar. The synchronous-converter equipment has now been reduced to almost the fewest number of parts for successful railway service. The rectifier equipment is in the process of development. Each year witnesses modifications in the design which reduce the number of parts. It is safe to predict, therefore, that when the rectifier equipments will have been in production as long as the converter equipments have now been in service, the number of parts required for a rectifier automatic station will be reduced probably by 25 per cent.

The inspection and maintenance data given by Mr. Turley are of great importance since he has had experience not only with synchronous converters and motor generators in manually operated stations, but he has also had wide experience with synchronous converters in automatic stations.

Likewise, the estimated savings for the operation of an automatic station using mercury-arc power rectifiers are based on broad experience. Consequently, it should be of intense interest to all executives responsible for the operation of 600-volt railway systems.

G. E. Nott: At the time the Inglewood installation was ready for service, certain tests were conducted to determine the effect of the new installation upon telephone service. These tests were conducted cooperatively by the Los Angeles Railway and the local telephone company. The results were very satisfactory.

As mentioned in the paper, exposures between the railway lines and the telephone circuits throughout the territory served by both utilities are rather extensive. The exposures which were used for the tests and which were relatively close to the new substation, were somewhat less extensive, but they were considered to be fairly indicative of conditions as they existed.

As an indication of the effect obtained through the use of resonant shunts, a comparison of the voltage telephone interference factors, as measured at the d-c. bus, may perhaps be of interest. In this instance, it appeared that the addition of the shunts reduced the telephone interference factors to approximately 13 per cent of the value which obtained without the shunts. It was also demonstrated from similar measurements that the use of the shunts resulted in a voltage telephone interference factor equal to approximately 60 per cent of that which was produced by the existing rotating equipment. The latter equipment had, however, been in use over a considerable period of years and the telephone interference factor was somewhat higher than that generally obtained from similar equipment of more recent manufacture.

It is gratifying to note that the results of the tests which were conducted, indicated that, through the use of a small amount of properly designed auxiliary equipment, such as is employed at Inglewood, serious interference with telephone circuits can be avoided under normal conditions. It also appears that, from the interference standpoint, such an installation may be compared favorably to rotating equipment of modern design.

M. D. Wade: Several references have been made to telephone interferences, and I should like to hear something in the final discussion about radio interference.

L. J. Turley: There seems to be no trouble with radio reception from the rectifier installation that we have at Inglewood; in fact, I doubt very much if there could be.

With reference to the question that was raised on the 10 to 15 per cent higher cost, I appreciate the fact that my study did not go into the higher voltage of 1500 volts. Therefore I was not versed in what the relative cost would be between that and the 600-volt price, but I do realize that the higher voltage leads into a lower cost, in comparison with similar equipment of the synchronous converters.

Movements of Overhead Line Conductors
During Short Circuits*

BY Wm. S. PETERSON[1] AND H. J. McCRACKEN, Jr.[1]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—When overhead lines carry large short-circuit currents, the resulting magnetic forces on the conductors are such as to cause the cables to be repelled to greater distances than the usual spacings. The elementary principles involved are stated. A single-phase short circuit is shown to be the one producing the greatest movements. A theoretical calculation of the forces and actual experience showed the necessity of making tests to study the problem. A description of the tests indicates how photographic records of the movements of relatively large cables were obtained. The results of over 330 tests are shown by pictures and curves. The effects of a variation in each of the five principal variables, cable size, span length, spacing, tension, and current are discussed. A mathematical expression is derived by means of which the maximum movement of conductors in a horizontal plane can be approximately determined. It has been found possible to set up a miniature test with small wire that very closely duplicates to a small scale the movements of a larger conductor. The results indicate that it is necessary for moderately large systems to take account of these short-circuit forces in the design of their overhead systems. There is a large field for research on this subject and more work should be done.

Object

THE object of this paper is to present the results of investigations recently made on overhead line conductors in order to determine the movements of those conductors when they are carrying the heavy short-circuit currents that exist in large electric distributing systems. It will give the essential data from over 330 tests that were made by the Los Angeles Bureau of Power and Light, on full sized conductors, at various spacings, spans, and tensions that might be used in practise.

Reasons for the Investigation

Just as the growth of electric systems has brought the circuit breaker problem, so has it brought the problem of conductor movements. In past years power systems have been relatively small and the short-circuit currents were not large enough to seriously disturb overhead lines, so very little attention has been given to this problem.

The first case of trouble on the lines of the Los Angeles Bureau of Power and Light that gave proof that overhead line movements are such as to cause serious trouble, was on a 33-kv. line leaving the receiving station where the 110-kv. lines from the Aqueduct power plants terminate. The 33-kv. line is of 3/0 copper cable, on approximately 4.5 ft. spacing. The span lengths are approximately 400 ft. near the station, as the line is carried on some lower crossarms mounted on the same towers as the transmission line. These spans are longer than what would normally be used on a 33-kv. pole line. When a short circuit occurred about one-half mile from the station, the cables in some of the longer spans, between the point of trouble and the station, wrapped around each other so as to remain tangled. Obviously, such movements were due to magnetic forces set up by the large short-circuit currents. The same action has been suspected in distribution primaries, when spans have been burned in two.

If we grant the possibility of such a range of movement, it can be shown that certain so called faulty relay actions, that sometimes happen, might be the result of conductors touching each other and relaying out a section of line that should have stayed in. This would be especially true where a single line loops through several stations, and several line sections carry the same short-circuit current. Another example would be where one line interferes with an adjacent circuit.

In striving to give high class service, a power system then has the economic problem, of which is the best way to build so that trouble from conductor swinging will be avoided. To determine whether the answer is longer crossarms, more poles, heavier conductors, limitation of currents by reactors, limitation of system size, or underground distribution, is a tremendously important problem, especially to systems supplying power to rapidly growing metropolitan areas, such as that represented by Los Angeles. In order to design not only the lines, but the entire system to supply satisfactorily the future load of this city, it was essential that this problem be studied rather completely and that the engineering be based on precise and accurate data.

Investigation of engineering literature revealed that very little work, if any, had been done on this subject. The study of magnetic forces on busbars and disconnect switches apparently constitutes the nearest approach to the problem at hand. Preliminary mathematical studies of the problem indicated that the solutions were so involved that it would be necessary to make tests on full sized conductors in order to have exact data to apply to the economic design of lines.

Elementary Statement of the Problem and Fundamental Theory

Although the subject is new in its application to overhead lines, the fundamental principle is an old and familiar one; namely, that two wires carrying currents

1. Both of the Bureau of Power and Light of the City of Los Angeles, California.

A. I. E. E. Initial Prize Paper.

Presented at Section Meeting of the A. I. E. E., Los Angeles, Calif., Dec. 6, 1927, and Pacific Coast Convention of the A. I. E. E., Spokane, Wash., Aug. 28-31, 1928.

in opposite directions are repelled from each other. The more convenient conception is the one where a conductor has a force exerted on it, due to its carrying a current in the magnetic field set up by the other conductor. The fundamental formula expressing the force of repulsion between two long straight conductors is as follows:[2]

$$F' = 2.04\, i_1\, i_2\, (L/D)\, 10^{-8} \qquad (1)$$

where F' is the instantaneous force in kilograms, i_1 and i_2 are the instantaneous values of current flowing in the conductors in amperes, L is the length of the conductors in any units, and D is the distance between the conductors in the same units. If currents varying as a sine wave are assumed, then the instantaneous values of force vary as a sine wave of double frequency, similar to that for instantaneous power in a-c. circuits. For given currents, the maximum value of force is greatest when the two currents are in phase or 180 deg. out of phase, the latter being the condition that exists in a single-phase short circuit.

Since a conductor has an appreciable mass, its movement is more influenced by the average of these impulses than by their instantaneous values. It can be shown that the average value of force, in lb. per ft. of conductor, is given, for a single-phase circuit, by the following formula:[3]

$$F = \frac{0.045\, I^2}{10^6\, D} \qquad (2)$$

where I is the effective value of current in each of the conductors, D is the distance between conductors in feet, and F is the force per ft.

It is interesting to note that for 15,000 amperes, and a separation of 4 ft., the force is approximately 2.5 lb. per ft.

In the case of three-phase short circuits, we find that each wire is acted upon by a force due to its own current and the resultant field set up by the currents in the other two wires. In general, the conductors tend to move away from each other and the forces are smaller than for the same effective values with single-phase current. In the case of a flat arrangement of the three conductors, the forces are such on a balanced three-phase short circuit, as to cause the two outward conductors to move outward, while the center conductor tries to move back and forth at double the circuit frequency, which means that it practically stands still. The worst condition for this arrangement of conductors, is for a single-phase short circuit to occur between the center and one of the outer conductors, thus causing these conductors to be violently repelled from each other. Since the single-phase consideration is the one leading to the most unfavorable condition, the remainder of the discussion will be limited to that particular phase of the problem.

2. See "Standard Handbook for Electrical Engineers," (1922) Sect. 2, par. 41.

3. See Appendix A

Reference to Fig. 1 will help in the explanation of the mechanics involved in the motion of the conductors. It is a cross-section of two horizontally spaced conductors, taken at the center of the span. The initial position of the conductors is shown by the small circles. At some interval of time after the start of the short circuit, the conductors will have reached some position indicated by the solid lines. At any such instant, it is seen that the conductor is acted upon by three forces; first, the constant force, W, which represents the weight of the conductor; second, the force F, due to the current, which is in this case a horizontal force and becomes less as the conductors move further apart; and third, the force T_R, due to the tension in the cable, and which may be assumed radial. The unbalanced resultant force accelerates and stretches the conductor, thus increasing T_R. The velocity of the conductor carries it beyond the point of static equilibrium of the three forces until it is stopped by the tension developed. It then snaps back until stopped by the repulsion due to the current, and starts outward again. The problem is complicated

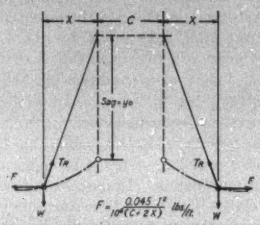


Fig. 1—Cross-Section Showing Forces Acting on Horizontally Spaced Conductors

by the fact that the high current heats up the conductor, adding to its length and decreasing the tension, and also by the fact that each portion of the cable in half a span length is a different distance from the other cable and has a different value of magnetic force, so the cable is no longer strictly following a catenary curve.

For any given type of spacing we find that the problem contains six independent variables, namely: wire size, span length, sag (or tension), spacing, current, and the length of time the current is flowing. The resulting mathematical complexity of the problem made advisable full scale tests to determine the laws of action.

Description of Tests

The large number of variables made a complete experimental determination of values impossible. Therefore a set of standard conditions was established and then each variable was investigated with respect to this standard point. Excepting for a few special tests, the variable, time, expressing duration of current, was eliminated by leaving the current on long enough for the cable to reach the end of its first throw. The standard condition was 3/0 cable, 150 ft. span, 12,000

lb. per sq. in. tension, 4 ft. spacing and the highest value of current obtainable from the equipment available for the test. In addition to the single variable variation with respect to the above point, it was planned to obtain a few scattered points representing other possible combinations of the variables.

The cable sizes tested were No. 2, 3/0, 500,000 cir. mils, and 4/0 weather-proof. The spans used were 150, 300, 450, and 600 ft. The stringing tensions used

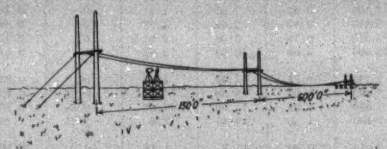


Fig. 2—General Arrangement of Test Spans and Cameras

were 12,000, 18,000, and 24,000 lb. per sq. in., approximately. The spacings were 2, 4, 8, and 12 ft. Currents varied from 4000 to about 13,000 amperes.

In making the tests two spans were set up and the movements of each were measured simultaneously to save time. Fig. 2 gives a general idea of the arrangement of the spans. Two dead-end H-frame structures were set up 750 ft. apart. At an intermediate point was placed another similar structure. By moving this latter pair of poles, the span lengths were varied. The cable was dead-ended through suspension type insulators and clamped to pin type insulators at the intermediate poles. At one end of the line the two cables were connected through an oil switch to the low-voltage side of the transformers supplying power for the test. The far ends were short circuited. To obtain the highest currents a jumper was placed across the cables at the end of the first span.

The record of the movement of the wire was made photographically. In order to do this, an automobile headlight bulb was fastened by means of an insulated clamp to the cables at the center of each span. The power for the light was supplied through flexible leads from a storage battery. In addition, some reference lights, a fixed distance apart and mounted truly horizontal were used. The cameras used to record the movements of the lights were mounted as nearly as possible on a horizontal line from the lights so no corrections for angularity would have to be made in scaling distances from the pictures. In order to have a record of the time for the cable to reach any point a disk with one or two holes in it was revolved in front of the lens by means of a synchronous motor. Thus a dotted line record of the movements of the wire was obtained.

By running the tests after 1:00 a. m. a considerable portion of the power generating capacity of the Los Angeles Bureau of Power and Light system could be used. The night work also facilitated the photography. The wiring diagram, Fig. 3, shows the equipment and connections used for the test. The portion labeled 33-kv. bus represents two wires of a three-phase 33-kv. line, to which was connected the single-phase bank of eight 1000-kv-a., 33,000/2400 volt transformers which furnished the power for the test loop. The low-voltage circuit of this bank is shown as a full line diagram to indicate the details more completely. By means of the disconnect switches in this circuit, the number of transformers could be varied and therefore a variation in current could be obtained. The circuit was closed through the solenoid operated oil circuit breaker in the 2400-volt circuit and opened by a relay operating the 33-kv. breaker. An oscillograph element was connected to a shunt in the secondary circuit of a 3000/5 ampere current transformer.

In addition to the main circuit, it was necessary to have auxiliary circuits for supplying power to the oscillograph, synchronous motors, lights on the cables, etc. A telephone circuit was used to correlate properly the work of the men operating the cameras, oscillograph, and oil circuit breakers.

Results of Tests

Horizontal Spacing. In presenting the results, only a few of the many pictures that were taken can be

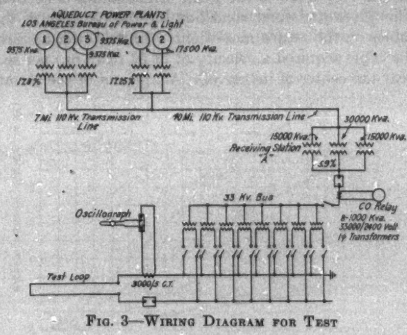


Fig. 3—Wiring Diagram for Test

shown. These will be grouped so as to have three or more together, so comparisons can be made as the variations are pointed out. Tables I and II give data pertaining to the various pictures that will be shown. Complete data obtained from all the tests are given in Appendix B.

Fig. 4 shows the movements of a 3/0 copper cable on a 600-ft. span for three different tests. These pictures show the variation in the deflection for different values of current. The distance between the large dots where the cable started from is 4 ft. (1.22 m.), so there is an indication that the cable moved 12.8 ft. (3.91 m.) from its initial position with a current of only 7430 amperes. The time to reach the point of maximum deflection in

TABLE I
RESULTS OF TESTS
CONDUCTOR MOVEMENTS WITH CABLES HORIZONTALLY SPACED

							Current		Maximum deflection data			
Fig. no.	Test no.	Cable Size	Span ft.	Tension lb. per sq. in.	Sag ft.	Spacing ft.	Amperes	Duration sec.	Radius ft.	Horiz. throw-ft.	Vert. movement-ft.	Time sec.
4	1B	3/0	600	12,800	13.5	4	5,470	1.21	16.8	8.65	−0.91	1.21
"	2B	"	"	"	"	"	6,150	1.01	17.9	10.8	−0.84	1.19
"	5B	"	"	12,300	14.1	"	7,430	1 +	19.4	12.8	−0.51	1.16
7	2A	3/0	150	12,800	0.85	4	6,550	1.01	1.51	1.24	0	
"	3A	"	"	"	"	"	7,100	0.52	1.65	1.41	0	0.33
"	7A	"	"	12,300	0.88	"	8,340	0.87	2.08	1.87	0	0.34
"	8A	"	"	"	"	"	11,420	0.75	3.04	2.91	0	0.33
"	10A	"	"	"	"	"	13,500	0.69	3.63	3.52	0	0.31
11	3A	3/0	150	12,800	0.85	4	7,100	0.52	1.65	1.41	0	0.33
"	133A	"	300	14,000	3.1	"	7,200	0.95	5.56	4.50	−0.16	0.60
"	133B	"	450	12,800	7.6	"	7,030	0.95	11.00	7.67	−0.29	0.86
"	5B	"	600	12,300	14.1	"	7,430	1 +	19.4	12.8	−0.51	1.16
13	13B	3/0	600	24,000	7.2	4	7,030	0.99	11.5	8.14	−0.7	0.88
"	11B	"	"	18,000	9.6	"	7,140	0.92	15.2	11.0	−0.91	1.04
"	5B	"	"	12,300	14.1	"	7,430	1 +	19.4	12.8	−0.51	1.16
24	70A	4/0W	150	16,900	0.8	1.5	13,300	1.05	3.90	3.82	0	0.26
"	71A	"	"	13,500	1.0	"	13,400	1.03	3.98	3.86	0	0.26
"	72A	"	"	10,400	1.3	"	13,320	1.05	4.24	4.07	0.16	0.27
"	76A	"	"	5,900	2.3	"	13,450	1.07	5.62	5.38	0.65	0.31

Amperes given is average current during time it takes for cable to reach maximum deflection.
4/0W designates 4/0 weather-proof cable, weighing 0.8 lb. per ft.

Fig. 4 is approximately 1.2 sec. The current was interrupted very close to the point of maximum deflection. This gives the worst condition for the tangling of the cables on the return movement. In the lower picture, the cable acquired a velocity of 18 ft. (5.49 m.) per sec. near the center of its travel. At the end of the throw, the sag distance was increased to 19.4 ft. (5.91 m.), or nearly 6 ft. (1.83 m.) greater than the initial sag of 13.5 ft. (4.12 m.).

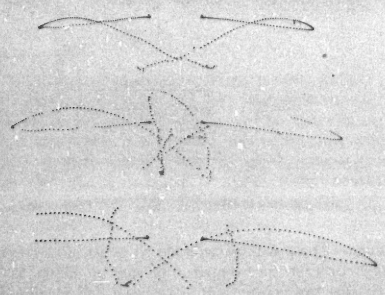


FIG. 4—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT CURRENTS WITH 3/0 CABLE, 4 FT. HORIZONTAL SPACING, 600 FT. SPAN, 12,800 LB. PER SQ. IN. TENSION, 13.5 FT. SAG

Top, 5470 amperes; middle, 6150 amperes; bottom, 7430 amperes

Approximately the same general characteristics of movement were observed for spans of 450 and 300 ft., excepting that the maximum deflection and the time to reach that point were reduced.

The results that were obtained with 3/0 cable on a 150-ft. span are indicated in Fig. 7. This length of span is quite typical of the spans that are used in city line construction. The time required for the cable to reach the maximum deflection is very short for this group, being of the order of ¼ to ⅓ of a sec. The current remained on from ½ to 1 sec.

FIG. 5—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT CURRENTS WITH 3/0 CABLE, 4 FT. HORIZONTAL SPACING, 450 FT. SPAN, 12,800 LB. PER SQ. IN. TENSION, 7.6 FT. SAG

Top, 5400 amperes; middle, 6410 amperes; bottom, 7030 amperes

In the top picture, it can be observed that the cable was thrown out twice by the current. The camera shutter was closed about the same time that the current went off, so none of the subsequent oscillations of the cable towards the center are shown.

In the next picture underneath is shown the condition where the current went off real quickly, and while the cable was on its way back. This resulted in the elliptical paths shown. In the next picture, the cable had started on its second trip out, when the current went off, thus producing a smaller oscillation. If the current could be interrupted at precisely the right point, considerable of the oscillation, wherein the cables approach each other, could be avoided. In the next two pictures, this was accomplished, but not premeditatedly.

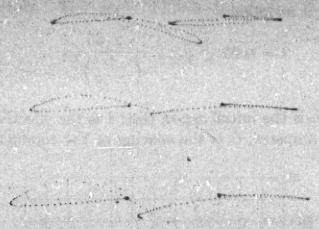


FIG. 6—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT CURRENTS WITH 3/0 CABLE, 4 FT. HORIZONTAL SPACING, 300 FT. SPAN, 14,000 LB. PER SQ. IN. TENSION, 3.1 FT. SAG

Top, 5450 amperes; middle, 6500 amperes; bottom, 7200 amperes

In this short span, it can be observed that the initial movements are horizontal. The maximum outward throw in the lower picture is over 3.5 ft. (1.07 m.).

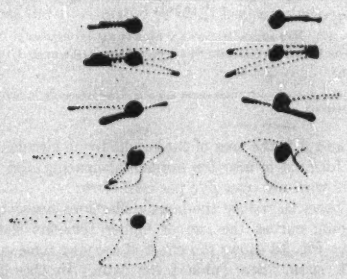


FIG. 7—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT CURRENTS WITH 3/0 CABLE, 4 FT. HORIZONTAL SPACING, 150 FT. SPAN APPROXIMATELY 12,800 LB. PER SQ. IN. TENSION, 0.85 FT. SAG

Top, 6550 amperes; 2nd, 7100 amperes; 3rd, 8340 amperes; 4th, 11,420 amperes; bottom, 13,500 amperes

The maximum velocity reached is approximately 20 ft. (6.1 m.) per sec.

Due to the fact that in the great majority of cases the movement of the conductors was so nearly horizontal, it was possible to develop a formula by which could be calculated approximately the maximum deflection and the sag of the conductors at this point. The results of such calculations and the points obtained by test are shown in Fig. 9. Here is shown the variation of maximum deflection with current. The data obtained from Fig. 4 and Fig. 7 are plotted together with that from other tests.

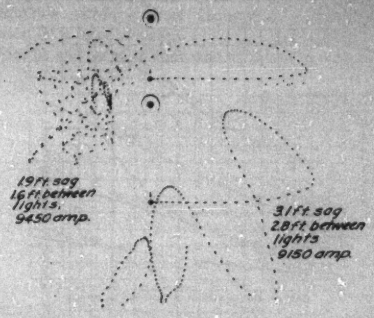


FIG. 8—CONDUCTOR MOVEMENTS AS OBTAINED IN A PRELIMINARY TEST ON 3/0 CABLE, 4 FT. HORIZONTAL SPACING, 160 FT. SPAN

Reference light is 0.3 ft. below line of sight between conductor supports

It will be observed that the calculated curve fits the points quite closely except that it falls a little short on the 600-ft. span curve. The extension of the results by means of the curves, derived from a purely theoretical basis and fitting the data as closely as they do, gives one confidence in their use in extrapolating to high current values. From the curves it is observed that the cables can move to unbelievable distances under the influence of 15,000 to 17,000 amperes. The derivation of the formulas will be taken up later.

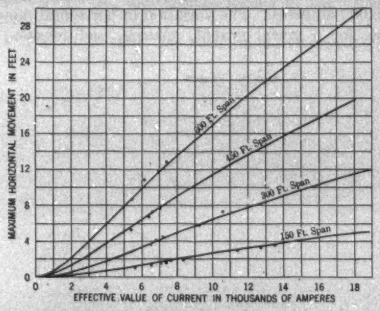


FIG. 9—RELATION BETWEEN MAXIMUM HORIZONTAL MOVEMENT OF CONDUCTORS AND CURRENT FOR VARIOUS SPANS

3/0 copper cable, 4 ft. horizontal spacing, approximately 12,500 lb. per sq. in. tension

0—Test points—calculated curve

From the way these curves lie with respect to each other, it is possible to judge the effect of span length. A better comparison is given in Fig. 11, which shows the movements of a 3/0 copper cable, on 4 ft. spacing, with 12,000 lb. per sq. in. tension, and a current of approximately 7000 amperes, while the span is varied from 150 to 600 ft. The same data are shown by the curve for 7000 amperes shown in Fig. 12. The upper curve shows the movements that take place when the current is 15,000 amperes. The variation in horizontal movement, for a given current, spacing, and tension, is approximately as the square root of the span length cubed, that is span to the 3/2 power.

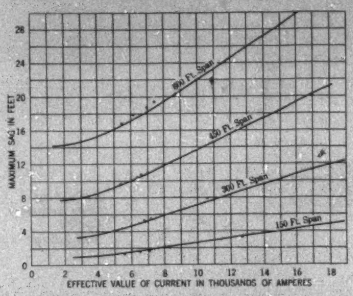


Fig. 10—Relation Between Maximum Radial Sag of Conductors and Current for Various Spans

3/0 copper cable, 4 ft. horizontal spacing, approximately 12,500 lb. per sq. in. tension
0—Test points—calculated curve

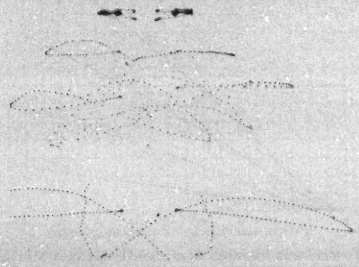


Fig. 11—Comparison of Conductor Movements for Different Spans, with 3/0 Cable, 4 Ft. Horizontal Spacing, Approximately 12,800 Lb. per Sq. In. Tension, Approximately 7000 Amperes

Top 150 ft. span, 0.85 ft. sag; 2nd, 300 ft. span, 3.1 ft. sag; 3rd, 450 ft. span, 7.0 ft. sag; bottom 600 ft. span, 14.1 ft. sag

It is to be noted that the time required to reach the maximum deflection varies very closely as the span length for a given tension. It has been observed that for the horizontal spacings and currents used in all the tests considered here, the time to reach the maximum throw point can be roughly determined by the formula:

$$t = \frac{\sqrt{\text{Sag}}}{3} \tag{3}$$

where the sag is expressed in feet. A more accurate empirical formula that takes care of the variation in the period of copper cable due to wire size, current, and spacing is as follows:

$$t = 0.25\sqrt{y_o} + \frac{2.65}{\left(\frac{I^2}{A\ C}\right)^{2/3}} \tag{3.1}$$

where y_o is the initial sag in feet; I is the effective current in amperes; C is the spacing of the conductors in feet; and A is the area of the conductor in circular mils. This formula is also for horizontal spacing and gives results with not over five per cent error.

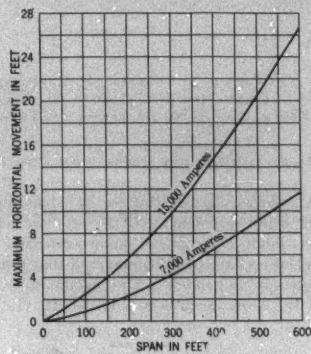


Fig. 12—Relation Between Maximum Horizontal Movement of Conductors and Span for Two Different Current Values

3/0 copper cable, 4 ft. horizontal spacing, approximately 12,500 lb. per sq. in. tension

In order to reduce the large deflections noted in the foregoing curves the use of higher tensions suggests itself. Fig. 13 shows the effect of varying tension on a 600-ft. span. See Table I for data. In this figure, doubling the tension has decreased the movement from nearly 13 ft. (3.96 m.) to a little over 8 ft. The tendency for the path of the conductor to rise rather than droop is typical of low tensions.

The variation in movement as the tension is changed is shown to better advantage in Fig. 15, where the test values for various span lengths are plotted. These curves indicate that the movement is varying approximately inversely as the square root of the tension. The movement is therefore not cut down as rapidly as the sag, which is varying inversely as the tension. Very high values of tension cannot be used because of other

loadings imposed on the cable by wind and ice in addition to that produced by the magnetic forces.

The effect of spacing is shown in curve form in Fig. 18. The curves are drawn for a current of 7000 amperes. Although it can be observed that spacing reduces the movement of the individual conductor, there is not a great change in the total space occupied by the swinging conductors. The great difference between the curve

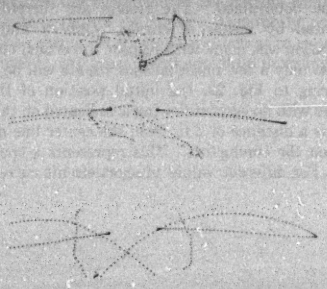


FIG. 13—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT TENSIONS, WITH 3/0 CABLE, 4 FT. HORIZONTAL SPACING, 600 FT. SPAN, AND APPROXIMATELY 7000 AMPERES

Top, 24,000 lb. per sq. in., 7.2 ft. sag; middle, 18,000 lb. per sq. in., 9.6 ft. sag; bottom, 12,300 lb. per sq. in., 14.1 ft. sag

for the 150 ft. span and the others indicates the importance of using short spans.

Fig. 22 indicates how the use of heavy conductors will decrease the movements. Fig. 24 is also an example of

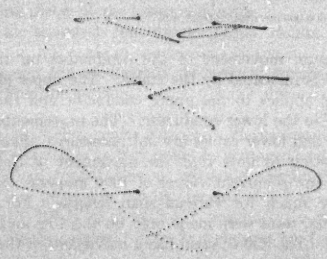


FIG. 14—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT TENSIONS, WITH 3/0 CABLE, 4 FT. HORIZONTAL SPACING, 300 FT. SPAN AND APPROXIMATELY 7000 AMPERES

Top, 20,600 lb. per sq. in., 2.1 ft. sag; middle, 14,000 lb. per sq. in., 3.1 ft. sag; bottom, 6370 lb. per sq. in., 6.8 ft. sag

a heavy conductor, but is also interesting because it is representative of conditions on a feeder circuit. It is a picture of 4/0 triple braid weather-proof cable, on a 150 ft. span, with a spacing of 1.5 ft. The current in each case was approximately 13,300 amperes. The variation in the movements as the tension is changed are shown. It can be observed that with the highest tension, which is larger than what would normally be used, the movement is approximately 4 ft. (1.22 m.) or 2.5 times the spacing. This condition is readily obtainable within a short distance of a distributing station on a large system.

Fig. 25 will serve to show the variation in movement

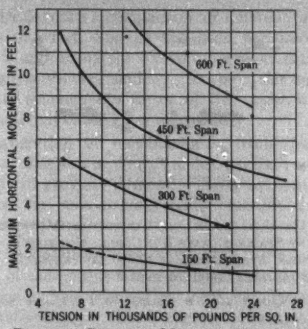


FIG. 15—RELATION BETWEEN MAXIMUM HORIZONTAL MOVEMENT OF CONDUCTORS AND TENSION FOR VARIOUS SPANS

3/0 copper cable, 4 ft. horizontal spacing, approximately 7000 amperes

with current for the tensions shown in the previous figure.

In Fig. 27 an effort has been made to consolidate on

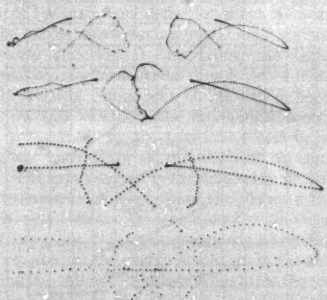


FIG. 16—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT SPACINGS, WITH 3/0 CABLE, HORIZONTAL SPACING, 600 FT. SPAN, APPROXIMATELY 12,500 LB. PER SQ. IN. TENSION AND APPROXIMATELY 7000 AMPERES

Top, 12 ft. spacing; 2nd, 8 ft. spacing; 3rd, 4 ft. spacing; bottom, 2 ft spacing

one sheet a considerable amount of information on the performance of one particular conductor at a given spacing and a given tension. It shows the location of the maximum point of throw both vertically and horizontally for any current and span length.

The origin of coordinates at the upper left hand corner is on the line of sight between supports of the conductor. The distance down indicates the sag below that point at the center of the span. Distances to the right indicate the extent of movements from the initial

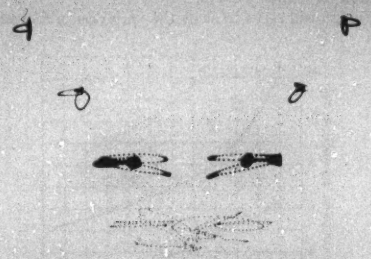


Fig. 17—Comparison of Conductor Movements for Different Spacings, with 3/0 Cable, Horizontal Spacing, 150 Ft. Span, Approximately 12,500 Lb. per Sq. In. Tension and Approximately 7000 Amperes

Top, 12 ft. spacing; 2nd, 8 ft. spacing; 3rd, 4 ft. spacing; bottom, 2 ft. spacing

position. For example, the initial sag for a 300 ft. span is indicated on the vertical axis a distance of 3.3 ft. down, corresponding to the assumed tension of 12,800 lb. per sq. in. This is the position of the cable

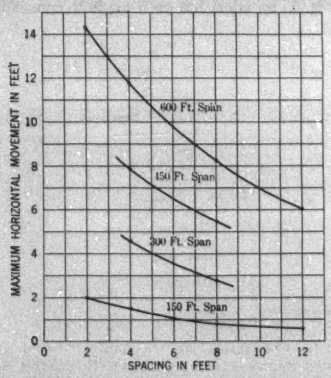


Fig. 18—Relation Between Maximum Horizontal Movement of Conductors and Spacing for Various Spans

3/0 copper cable, approximately 12,300 lb. per sq. in. tension and 7000 amperes

for zero current. For various currents the cable will be propelled to the right. For a current of 5000 amperes the cable reaches the point shown by the intersection of the 300-ft. span line and the 5000-ampere line. The location of the point shows both its vertical and horizontal position with respect to the center about which it swings. The curves are drawn from combined test and calculated data with intermediate values approximated. Due to the test data available, the tensions used are not precisely alike throughout the whole range of values.

Results with Vertical Spacing. In making tests on cables arranged above one another, it is found that the movements are primarily vertical, which leads to a confusion of dots, so that only the maximum points can be determined. The results are therefore best presented by means of curves. Only a limited investigation was made with respect to vertical construction, so only a few miscellaneous results will be shown. Referring to Fig. 29, the initial position of the conductors with no current flowing is indicated on the vertical axis a distance of 2 ft. from the center line midway between the conductors. This represents a spacing of 4 ft. For different values of short-circuit current, the

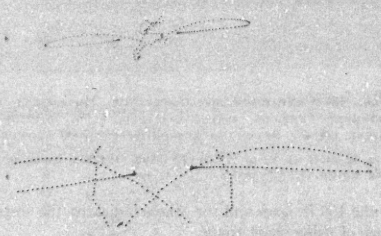


Fig. 19—Comparison of Conductor Movements for Different Conductor Sizes, with 4 Ft. Horizontal Spacing; 600 Ft. Span; 12,300 Lb. per Sq. In. Tension, 14.1 Ft. Sag and Approximately 7000 Amperes

Top, 500,000 cir. mils; bottom, 3/0

maximum movement of the conductors on the first throw is indicated by the curve. The upper group of curves applies to the upper conductor and the lower group to the lower conductor. The movements of the upper and lower cables are not necessarily alike. It is interesting to note that the movement of the upper cable, up to current values of 7000 amperes, does not change much for various span lengths. For higher currents, tests were made only on the 150- and 300-ft. spans. The 300-ft. span has a movement of the upper conductor that seems to be almost erratic and makes a large change in movement for a small change in current.

In Fig. 30, a study is made of this extreme movement of the 300-ft. span. A picture was taken with just a slight offset from the vertical so the dots would not confuse each other. A plot of the movements of the two conductors was made with respect to time. One pair of curves shows the movements for 5600 amperes and the other pair shows the movements for 10,800 amperes. It is to be noted that as the current increases, the time to reach the maximum point increases for the upper conductor, and decreases for the lower conductor. For

the high current values, the lower conductor reaches its maximum sooner than the upper one, and as the tension of the conductor pulls it upward, the increasing magnetic force due to the decreasing distance between the cables, causes the upper conductor to have an increased upward acceleration after it had begun to lose velocity. There is probably some critical value of current at which this phenomenon occurs.

Tests on the 450 and 600 ft. spans with 3/0 cable have indicated that the movement of the bottom conductor is greater than that of the upper. Apparently, on long spans, the heating effects alone are such as to cause the conductors to sag great distances, thus dropping both

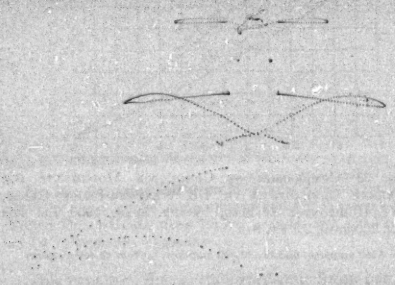


Fig. 20—Comparison of Conductor Movements for Different Conductor Sizes, with 4 Ft. Horizontal Spacing, 600 Ft. Span, 12,300 Lb. per Sq. In. Tension, 14.1 Ft. Sag

Top, 500,000 cir. mils, 5870 amperes; middle, 3/0 cable, 5470 amperes; bottom, No. 2 wire, 4810 amperes

conductors to a lower point, and possibly decreasing the upward throw of the top conductor and adding to the downward throw of the lower conductor.

Fig. 31 shows a few miscellaneous results with 500,000-cir. mil cable in the vertical arrangement.

In general the movements for vertical construction are less than for horizontal, except at the higher current values.

Results with Triangular Spacing. Pictures were also taken of two cables as they might be arranged as part of three cables constituting a triangular arrangement. Fig. 33 is a picture of the movements of a 3/0 copper cable, on a 300 ft. span with 4 ft. spacing, at a 30 deg. angle with the vertical. The four pictures are for four different current values. Table II gives the data pertaining to this figure.

The same effect, wherein the upper wire takes a long flight at the higher current values, is to be observed in this arrangement, just as was the case for vertically spaced cables. The maximum throw in the lower right hand picture is a little over 11 ft. for the upper conductor.

In order to make a comparison for different angles or amounts of offset, Fig. 34 has been prepared. The current is approximately 10,500 amperes in each case. The tensions are very similar. As the cables approach horizontal spacing, the throw of the upper wire decreases and that of the lower increases.

No tabulation of movements is made for these particular tests as it is difficult to decide what part of the motion is of greatest interest. The great bulk of information for such arrangements probably must be obtained from direct tests.

Mathematical Considerations

Most of the mathematical analysis that the authors have done on this problem has been with reference to horizontal spacing. This is slightly more favorable to work with than the other cases due to the symmetry of the movements. Efforts were made to calculate the path of the cables. An equation was found that was of the correct form, but it would be very difficult to determine the various coefficients in terms of the physical dimensions. This effort was abandoned.

It became evident, however, that the principal thing of interest in connection with horizontal spacing was the location of the maximum deflection point. As in most cases the cables move out approximately horizontally and reverse direction at this point, the horizontal component of velocity at such a time is zero, and in most cases the vertical component of velocity is negligibly small. On this basis, all the mechanical energy imparted to the cable by the magnetic forces during its motion has been absorbed by the stretching of the cable.[4] The input energy is the integral or summation of the instantaneous force times the distance through which it acts. From the equation for the magnetic force at any point, this input energy can be determined. This is equated to the energy stored up in the cable by stretching it. Allowance must be made for the increase

TABLE II
CONDUCTOR MOVEMENTS WITH CABLES TRIANGULARLY SPACED
(See pictures for actual movements)

Fig. no.	Test no.	Cable size	Span ft.	Tension lb. per sq. in.	Sag ft.	Spacing ft.	Amperes	Duration of current sec.	Offset ft.
33	179A	3/0	300	10,300	4.2	4	5570	0.96	2.0
"	181A	"	"	"	"	"	7040	0.98	"
"	183A	"	"	"	"	"	9120	0.87	"
"	184A	"	"	"	"	"	10500	0.73	"
34	177A	3/0	300	11,400	3.8	4	10800	0.69	1.0
"	184A	"	"	10,300	4.2	"	10500	0.73	2.0
"	190A	"	"	13,100	3.3	"	10350	0.77	3.0

4. See Appendix C.

in temperature of the cable due to the high currents. With this method of procedure, and one slight empirical correction, the following formula is derived, for copper cable:

$$\frac{0.0447\, y_0 \dfrac{I^2}{A} \log_e \dfrac{C + 1.5\, X_m}{C}}{K_0 \left(R_m^2 - y_0^2\right)} = \frac{18 \times 10^6\, K_0}{T_1^2} \left(\frac{R_m^2 - y_0^2}{y_0}\right) + 2 - 5.13 \times 10^6\, \frac{I^2 t}{A^2\, T_1} \tag{4}$$

where A, is the area of the conductor in circular mils;

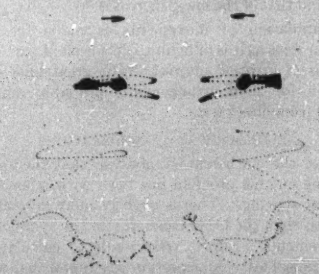


FIG. 21—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT CONDUCTOR SIZES, WITH 4 FT. HORIZONTAL SPACING, 150 FT. SPAN, APPROXIMATELY 12,800 LB. PER SQ. IN. TENSION, 0.85 FT. SAG

Top, 500,000 cir. mils, 7300 amperes; middle, 3/0 cable, 7100 amperes; bottom, No. 2 wire, 6020 amperes

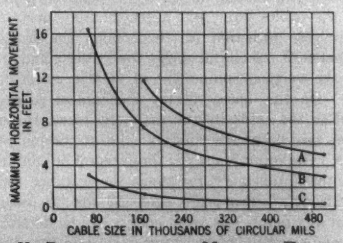


FIG. 22—RELATION BETWEEN MAXIMUM HORIZONTAL MOVEMENT OF CONDUCTORS AND WIRE SIZE

Four ft. horizontal spacing, approximately 12,500 lb. per sq. in. tension
A—7000 amperes, 600 ft. span. B—4800 amperes, 600 ft. span. C—7000 amperes, 150 ft. span

C, is the initial distance between the cables in feet; I, is the effective value of the current in amperes in each cable (single phase); K_0 is the ratio of total weight per foot of cable plus any weather-proof or other covering it might have, to the weight of the bare cable; t, is the time in seconds required to reach the maximum deflection, and can usually be taken as $\sqrt{y_0/3}$; T_1, is the initial stringing tension in pounds per square inch; y_0, is the initial sag at the center of the span, in feet; R_m, is the maximum radial sag at the point of maximum throw; and X_m, is the maximum horizontal movement in feet. The formula applies to copper cable, with a modulus of elasticity of 14,000,000, and of 97.3 per cent conductivity.

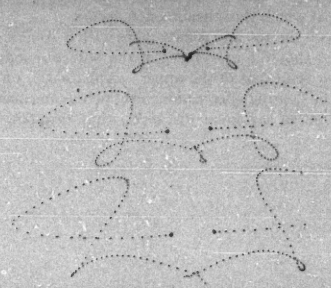


FIG. 23—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT CURRENTS WITH 4/0 WEATHER-PROOF CABLE, 1.5 FT. HORIZONTAL SPACING, 150 FT. SPAN, 5900 LB. PER SQ. IN. TENSION, 2.3 FT. SAG

Top, 8150 amperes; middle, 10,890 amperes; bottom 13,450 amperes

This formula is deficient in one respect, namely, that both R_m and X_m are unknown, so there is an infinite number of combinations that would satisfy the equa-

FIG. 24—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT TENSIONS WITH 4/0 WEATHER-PROOF CABLE, 1.5 FT. HORIZONTAL SPACING, 150 FT. SPAN, APPROXIMATELY 13,300 AMPERES

Top, 16,900 lb. per sq. in., 0.8 ft. sag; 2nd, 13,500 lb. per sq. in., 1.0 ft. sag; 3rd, 10,400 lb. per sq. in., 1.3 ft. sag; bottom, 5900 lb. per sq. in., 2.3 ft. sag

tion. However, it was observed that most of the tests showed the movement to be nearly horizontal. This establishes a relation between R_m and X_m that makes it possible to eliminate R_m and obtain the equation in the following form:

$$\frac{0.0447\, y_0 \dfrac{I^2}{A} \log_e \dfrac{C + 1.5\, X_m}{C}}{K_0 X_m^2} = \frac{18 \times 10^6\, K_0\, X_m^2}{T_1^2\, y_0} + 2 - 5.13 \times 10^6 \frac{I^2 t}{A^2 T_1} \qquad (5)$$

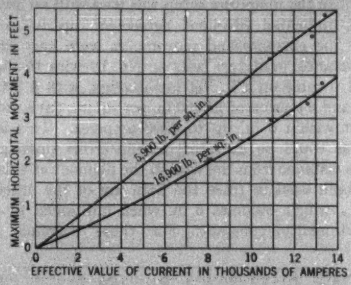


FIG. 25—RELATION BETWEEN MAXIMUM HORIZONTAL MOVEMENT OF CONDUCTORS AND CURRENT FOR 4/0 WEATHER-PROOF TRIPLE BRAID CABLE WITH 1.5 FT. SPACING AND 150 FT. SPAN AT TWO DIFFERENT TENSIONS

This equation, due to its logarithmic form, has to be solved for X_m by trial. It is this equation that was used to make the calculated curves shown in Fig. 9. This equation will not give exact values for all cases, but in general it gives a conservative answer, that is, larger values of movement rather than smaller. For large spacings, and for small wire sizes, that is, where the magnetic forces are relatively small and where the heating effects are relatively large, the conductors

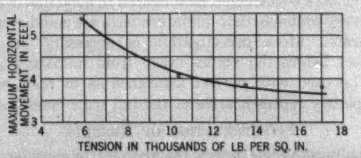


FIG. 26—RELATION BETWEEN MAXIMUM HORIZONTAL MOVEMENT OF CONDUCTORS AND TENSION FOR 4/0 WEATHER-PROOF TRIPLE BRAID CABLE WITH 1.5 FT. SPACING, 150 FT. SPAN, AND APPROXIMATELY 13,300 AMPERES

move on a drooping path. This means that the cable becomes stretched to a certain value of radial sag with a smaller value of horizontal movement. Consequently, such cables will not move as far horizontally as indicated by the equation. If the tensions are low, the cables have a tendency to rise rather than droop. In this case the vertical component of velocity remains a considerable amount as the point of maximum deflection is reached. As some percentage of the input energy is stored in velocity instead of elongation of the cable, the formula again gives a deflection that is large, as it assumes all the energy caused elongation.

In making calculations for very large current values, it was found necessary to use a more exact expression for taking care of temperature effects. In order to do this the last term of the equation, which term is:

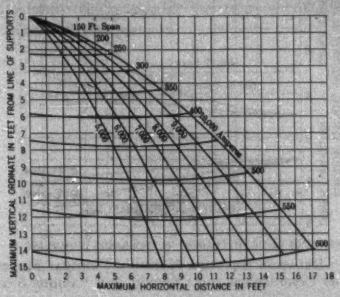


FIG. 27—COORDINATES OF POINT OF MAXIMUM DEFLECTION FOR VARIOUS CURRENTS AND SPANS FOR 3/0 COPPER CABLE, 4 FT. HORIZONTAL SPACING AND APPROXIMATELY 12,800 LB. PER SQ. IN. TENSION

$$5.13 \times 10^6 \frac{I^2 t}{A^2 T_1}$$

is replaced by:

$$\frac{63{,}400}{T_1} \left(e^{\frac{74.5\, I^2 t}{A^2}} - 1\right)$$

See Appendix C for more complete development.

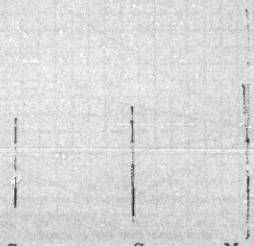


FIG. 28—COMPARISON OF CONDUCTOR MOVEMENTS FOR DIFFERENT CURRENTS WITH 3/0 CABLE, 4 FT. VERTICAL SPACING, 300 FT. SPAN, 12,400 LB. PER SQ. IN. TENSION, 3.5 FT. SAG

Left, 5550 amperes; center, 6990 amperes; right, 10,620 amperes

As yet no entirely satisfactory formula has been developed for calculating the movements of vertical conductors.

Miniature Tests. On the basis of these formulas it is possible to determine the proper dimensions of a mini-

ature test to reproduce the same conditions to a smaller scale. This should make possible the solution of any problems of the type discussed here, by making a small scale set up and taking an actual test, which can be performed probably as quickly as any calculations

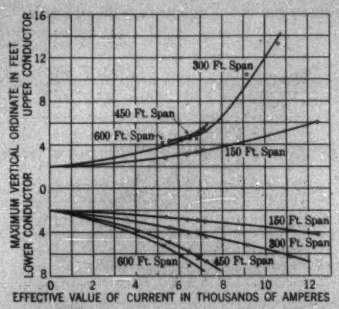


FIG. 29—RELATION BETWEEN MAXIMUM VERTICAL MOVEMENT OF CONDUCTORS AND CURRENT FOR VARIOUS SPANS

3/0 copper cable, 4 ft. vertical spacing, approximately 12,000 lb. per sq. in. tension

could be made. The miniature test to use is the one that will have each term of the equations mentioned above equal in value to the same term for the larger size being duplicated. To do this assume some scale multiplier n; that is, some number by which the small scale distances will be multiplied to equal the large test.

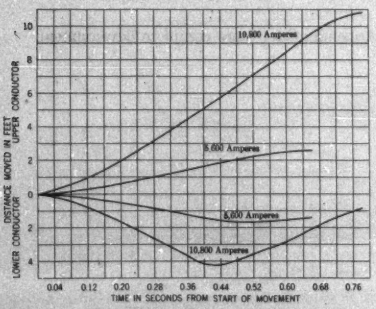


FIG. 30—RELATION BETWEEN MOVEMENT OF CONDUCTORS AND TIME FOR TWO DIFFERENT CURRENTS

3/0 copper cable, 4 ft. spacing with 1 ft. offset from true vertical, 300 ft. span, 11,400 lb. per sq. in. tension and 3.8 ft. sag

It can then be shown[5] that circular mils, spacing, sag, movement, and current all have the multiplier n. Unit tension and time have a multiplier $n^{1/2}$ and the multiplier for span length is $n^{3/4}$. Point-to-point calculations of conductors with vertical spacing have indicated that the same multipliers apply to vertical arrangements.

Miniature tests on horizontal spacing, but with the current read by means of an ammeter instead of an oscillograph, agree with the large scale tests being duplicated, within the accuracy of the ammeter reading. More recent tests on vertical arrangement, where the current in the miniature setup was read with an oscillograph, indicated excellent agreement up to points where the currents and movements were very large. For instance, miniature tests on No. 10 wire to duplicate 3/0 cable on 4 ft. vertical spacing, with a 300 ft. span, and a tension of 12,000 lb. per sq. in., gave results correct within 6 in. up to 8500 amperes. This is as close as could be expected, as the errors introduced by the distance from the wire to the filament of a flashlight bulb used to trace the path are of the order of ⅜ in., which becomes 6 in. when multiplied by the proportionality constant. For those currents and

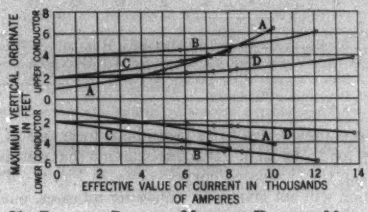


FIG. 31—RELATION BETWEEN MAXIMUM VERTICAL MOVEMENT OF CONDUCTORS AND CURRENT

500,000 cir. mil copper cable, vertical spacing, approximately 12,500 lb. per sq. in. tension

A—300 ft. span, 2 ft. spacing. B—300 ft. span, 8 ft. spacing. C—600 ft. span, 4 ft. spacing. D—150 ft. span, 4 ft. spacing

movements which could be tolerated in good line design, it is very probable that miniature tests are commercially accurate. More investigation is necessary on this subject before the values taken from miniature tests can be used with perfect assurance as to their being completely representative of the full scale condition.

INTERPRETATION OF RESULTS

In almost all line design, the main factor considered in determining the spacing between wires has been voltage. Only to a small extent has span length been considered as affecting the necessary spacing. From the data in this paper it seems that an additional factor, namely, short-circuit currents and the movements which they cause, must be considered. On large systems it will apparently be necessary to correlate the choice of reactors, spacing, span length, conductor size, and tension, so as to obtain the most economical line that will not be subject to trouble due to swinging conductors.

A detailed economic design cannot be worked out in the space allotted to this paper. However, a few of the most salient factors should be pointed out. From

5. See Appendix D.

the data it is evident that for the most practicable span lengths and tensions, the cables reach their maximum deflection in so short a time, approximately ⅛ of a sec., that with present day designs of oil circuit breakers, it is almost impossible to interrupt the current sufficiently fast to reduce the throw of the cables. This makes it necessary to accept as the first basis of design, that the current be considered as being on until the cable reaches its maximum position. Then in order to be safe it is desirable to assume that the current goes off at the maximum position, so as to produce the most unfavorable condition during the return of the conductors, that is, when there is no current opposing their motion while swinging back.

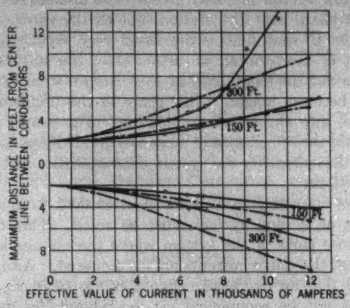


Fig. 32—Comparison of Movements of Conductors with Vertical and Horizontal Spacing for Variation in Current

3/0 copper cable, 4 ft. spacing, approximately 12,000 lb. per sq. in. tension, spans as indicated. Solid lines are vertical spacing. Dot and dash lines are horizontal spacing

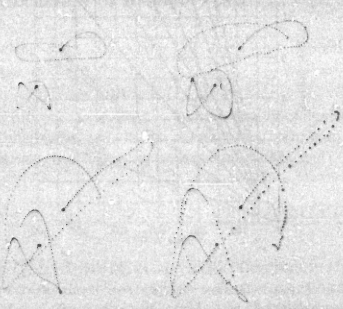


Fig. 33—Comparison of Conductor Movements for Different Currents with 3/0 Cable, 4 Ft. Angular Spacing (30 Deg.), 300 Ft. Span, 10,300 Lb. per Sq. In. Tension, 4.2 Ft. Sag

Upper left, 5570 amperes; upper right, 7040 amperes; lower left, 9120 amperes; lower right, 10,500 amperes

It may be that for some purposes it will be sufficient if the conductors do not interfere with some other conductor or circuit on their path outward. In accepting this basis of judgment, one would be assuming that the overlapping of the conductors as they snap back would do no harm. This is a possible criterion for a short, unimportant line that is not so connected that it supplies any other lines that may have short circuits and which should be relayed off without affecting the line in question.

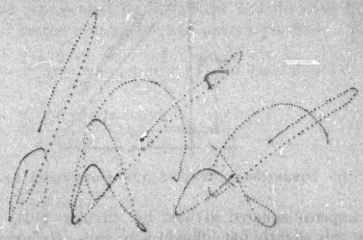


Fig. 34—Comparison of Conductor Movements for Different Angular Spacings (4 Ft. with Different Horizontal Offsets), 300 Ft. Span, Approximately 11,000 Lb. per Sq. In. Tension, 4 Ft. Sag, Approximately 10,500 Amperes

Left, 1 ft. offset, 14.5 deg.; center, 2 ft. offset, 30 deg.; right, 3 ft. offset, 48.6 deg.

A more generally useful basis is to attempt to design the line so that for the short-circuit current contemplated, the conductors will avoid meeting at the midpoint between the wires as they return. If the current is off when the cables are returning from their maximum deflection, they will in general swing back to such a point that the sag measured radially is the same as that at the maximum deflection. In general it is sufficiently accurate to assume that the conductors will swing as far towards the center as they swung outward beyond their initial position. If this assumption is made, a line would be so designed that with a short-circuit current flowing, it would not be deflected as much as half of the spacing between conductors.

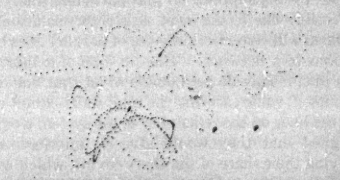


Fig. 35—Movement of 3/0 Cable, with 4 Ft., 30 Deg., Angular Spacing; 450 Ft. Span, 12,500 Lb. per Sq. In. Tension, 7.8 Ft. Sag, 7310 Amperes

To indicate what this means, reference is made to a set of design curves shown on Fig. 37, which curves were

calculated by means of Equation (5). These curves are for 3/0 conductor on horizontal spacing, with a stringing tension of 12,000 lb. per sq. in. For a given current they show the relation necessary between span and spacing to avoid having the conductor thrown outward more than one-half the spacing. Such curves can be prepared for any wire size and tension. It will be found that if the curves are prepared for various values

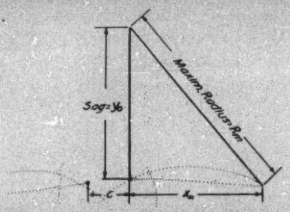


Fig. 36—Dimensional Symbols for Horizontal Spacing

of amperes squared divided by circular mils, they vary only slightly for different wire sizes. Referring to Fig. 37, and selecting some value of short-circuit current such as 16,000 amperes, it will be noticed that there is a choice of the use of 13 ft. spacing with a span of 245 ft.; 10 ft. spacing with a span of 195 ft.; 8 ft. spacing with a span of 160 ft.; 4 ft. spacing with a span of 90 ft.; or any other intermediate values. Which of these choices is the best to use is to be determined from a cost estimate, and any other pertinent requirements.

This one example probably indicates that lines on systems become more expensive as the size of the system increases, requiring larger spacings, shorter spans, and heavier copper, more or less independent of the size of the load being supplied on that particular line.

Possibly one of the most satisfactory solutions of this problem will come from the use of parallel lines between stations in a network. Then in case of a short circuit on a line, it may be permissible to let that line interfere with itself, while carrying the total short-circuit current, but the lines on the remainder of the system would be so designed that they would give no interference while carrying their share of the short circuit, which would be a much smaller value of current than that in the line which is in trouble. As the magnetic forces vary as the square of the current, conditions are readily improved by designing in this way.

Short-circuit forces also enter into the choice of an economic voltage for a line. If the line itself constitutes the principal reactance that limits the short circuit, then the higher the voltage of the line, the higher the short-circuit current, and the greater the difficulty would be with conductor movements. On the other hand, if the equipment supplying the line constitutes the principal reactance, then the currents are reduced by the use of high voltages.

When this latter condition exists, the high voltage would be favorable from the standpoint of conductor movements, but it is possible that the increased cost of higher voltage oil circuit breakers, transformers, and station space might over-balance the advantages to be derived from the higher voltage.

A low voltage line may give trouble and require very short spans near the station, but as the short-circuit current in such lines is decreased very rapidly as the point of short circuit is further from the station, the spans may be lengthened as the distance from the station becomes longer. This phase of the problem also enters into the economic solution. The proper solution for the dimensions of a line is a decidedly particular one, dependent on the system characteristics and local conditions.

The one thing that seems most evident is that the spans on a large system must be small. This refers to those parts of the system where the voltage is low enough that large currents are encountered. Also, it is likely that an overhead network of too large capacity would not be economical, as the advantages due to diversity, etc., would be lost in excessive line costs. This leads to the advocation of the idea that a central station company should have its total system consist of several nearly independent, relatively small capacity networks, practically complete in themselves; but

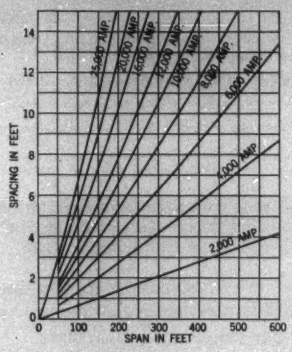


Fig. 37—Typical Design Curves for Horizontal Spacing

3/0 copper cable, 12,000 lb. per sq. in. tension.

For a given current, the curves show the relation between spacing and span when the conductors have a maximum deflection of one-half the spacing

loosely coupled, that is, coupled through high reactance to each other for standby service. This reduces the short-circuit currents to moderate values, and should lead to a lower cost system.

Conclusions

In closing, the following conclusions should be drawn from the results described in the paper:

1. It is necessary to take account of short-circuit forces in designing overhead lines.

2. Present mathematical and experimental data on the movements of overhead conductors during short circuits are very incomplete. A large field exists for research on this problem.

3. It would seem that the improvements in system and line design that can come from a study of this problem would justify various central station companies in extending and making public the research on this subject.

Acknowledgment is made to Messrs. M. O. Bolser, J. C. Albert, H. H. Cox, C. P. Garman, and R. Martindale, who, as members of the Los Angeles Bureau of Power and Light Technical Committee, made this investigation possible, and contributed many valuable suggestions.

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Appendix A

Calculation of Average Force per Foot of Conductor

Starting with Equation (1), with the definition of symbols as there given,

$$F' = 2.04\, i_1\, i_2\, (L/D)\, 10^{-6} \qquad (1)$$

If the force is expressed in pounds per foot instead of total kilograms, and if the currents are assumed to be sine waves, the following expression is obtained:

$$F'' = \frac{0.045 \times \sqrt{2} \times I_1 \sin(\omega t) \times \sqrt{2} \times I_2 \sin(\omega t + \phi)}{10^6 \times D}$$

where I_1 and I_2 are effective values of current in the two conductors, ω is the angular velocity in radians per second, t is the time in seconds, ϕ is the phase angle in radians between the currents in the two conductors, D is the distance between conductors in feet, and F'' is the force in pounds per foot of conductor. When F'' is positive the force is one of attraction. When ϕ is π or 180 deg., that is, when the currents are in opposite directions, the highest values of instantaneous force are obtained for given current values. This is the condition that is found in a single-phase circuit. For single phase, the currents in each wire are equal and opposite, so a simpler expression can be written from the above.

$$F_i = \frac{0.045 \times 2 \times I^2 \sin^2(\omega t)}{10^6 \times D}$$

where F_i is the force of repulsion between conductors in pounds per foot and I is the effective value of current in amperes.

This force varies from zero to a maximum at double the frequency of the circuit. The movements of the conductors can be assumed to be due to the average value of this force. This can be obtained by integrating the above expression for instantaneous force for the period represented by π radians and dividing by π. Let F be the average force in pounds per foot. Then:

$$F = \frac{2 \times 0.045\, I^2}{10^6 \times D} \times \frac{1}{\pi} \int_0^{\pi} \sin^2(\omega t)\, d(\omega t)$$

$$= \frac{2 \times 0.045\, I^2}{10^6 \times D} \times \frac{1}{\pi} \times \frac{\pi}{2}$$

$$F = \frac{0.045\, I^2}{10^6 \times D} \qquad (2)$$

Appendix B

Additional Data Giving Results of Tests Not Shown in Main Portion of Paper

Fig. 5 and Fig. 6 are shown here. These figures together with Fig. 4 and Fig. 7 will serve to indicate the effects of increasing current on the four different span lengths used in the series of tests.

Fig. 8 shows two of the first three pictures that were taken of full scale tests. These tests were preliminary to the more comprehensive tests that have been discussed in the paper. In this case the reference light is 0.3 ft. below the line of sight between supports. The movement shown in the lower picture is 4.9 ft., while that in the upper is 4.3. The results shown in this figure were the immediate reason for starting on the more complete research. The tensions used in this span were lower than those used in the remainder of the tests. The difference in the pattern is due to this fact. The pertinent data are given in the figure. The current values given in this figure are not entirely reliable.

Fig. 10 presents the same results as those in Fig. 9 except that they are replotted to show the variation in radial sag at the point of maximum sideways deflection. In this curve the exceedingly great increase in sag for long spans is to be noted. However, for per cent increase in sag, the shorter spans show the greater values.

Fig. 14 is a companion figure to Fig. 13 and shows the effect of tension on a 300-ft. span.

The effects of spacing can be noted in Fig. 16 and Fig. 17. For a given current, the path of the wire becomes more drooping as the larger spacings are used, which is due to the slower horizontal movements combined with the same increased sag due to heating.

Figs. 19, 20, and 21 were prepared to give a comparison of different conductor sizes. The current in the No. 2 wire in Fig. 21 was on an excessively long time.

Fig. 23 gives a comparison of movements due to different currents. It was the best example of symmetry that was found in the entire group of tests. The effects of increased tension for 4/0 weather-proof wire are indicated in the curve designated as Fig. 26.

Fig. 28 will give an idea of the appearance of the pictures taken of conductors with vertical spacing. The extreme movement indicated in going from 7000 to over 10,000 amperes should be noted.

In Fig. 32 is given a comparison of movements for horizontal and vertical movements, plotted in the same form as has been used in presenting the vertical movements.

An additional picture of the movements that take place with angular spacing is given in Fig. 35, where the results with a 450 ft. span are shown.

The remainder of the results can be obtained from a tabulation of all tests, which will follow.

The test numbers indicate the order in which the tests were made. The designations *A* and *B* refer to short and long spans respectively. The tension given is the tension in the cable at the time of the test, which is calculated from the measured sag, using the formula given in Section II, Paragraph 103, of the 1922 Edition of the "Standard Handbook for Electrical Engineers." The current given is the average of the effective values during the time it takes the cable to reach its maximum deflection. This is, in general, different for the two spans during the same test. The column headed "Duration" gives the time for which the current was on. The "Maximum Point" is the point at which the radial sag is a maximum and in general corresponds to the point at which the horizontal movement is a maximum. These values are given in the columns headed "Radius" and "Horizontal Throw." If this point is appreciably above or below the initial position, the amount is indicated in the column marked "Vertical Movement." The time required to reach this maximum point is recorded in the column headed "Time." Discrepancies in this column are due to the difficulty in counting the dots in some pictures.

A symbol has been used to describe the type of movement taking place. The first letter describes the direction of the movement on the way out; that is, *H*, means that the conductor moved horizontally; *D* means that it drooped or sagged downward on its way out, while *R* means that it rose above the horizontal. The amount of the drooping or rising can be judged by the values given under "Vertical Movement." The second letter describes the movement on the return. The letter *S* means that the cable started back along the same path that it came out on, whether that path is drooping, horizontal, or rising. The letter *O* means that in returning it rose above the outward path and returned over or above that path. The letter *U* means that it returned under the original path. The tabulation for horizontally spaced conductors follows immediately, in Table III.

The tabulation of results for vertical tests is similar to that for horizontal spacing except that the maximum movement on the first throw is given for both the upper and lower conductors. The time it takes to reach that point is also given where the confusion of dots is not such as to interfere with counting them and thus determining the time. The tabulation follows in Table IV.

Appendix C

Calculation of Maximum Movement of Overhead Conductors with Horizontal Spacing

Calculations Based on Energy Relations

In these calculations, no attempt will be made to determine the complete path of the conductor, but merely to calculate the maximum horizontal deflection on the first throw. As was indicated in the main portion of the paper under Mathematical Considerations, it will be assumed that all of the mechanical energy imparted to the conductor will be used in stretching it to its maximum sag, for the velocity energy and the energy used in raising the cable vertically are very small at this point. It will also be assumed that the point of maximum horizontal throw is the same as the point of maximum sag.

The following symbols will be used in this development:

A = Area of cross-section of conductor in circular mils.

α = Temperature coefficient of expansion, (= 0.000,009,6 per fahr. deg. for copper).

B = Area of cross-section of conductor in square inches.

C = Spacing between conductors in feet.

E = Modulus of elasticity, (14,000,000 for copper cables).

F = Magnetic force in pounds per foot of conductor.

I = Effective single-phase short-circuit current in each of two conductors in amperes.

K_0 = Initial loading ratio, *i. e.*, ratio of weight of cable loaded with weather-proof, or other covering, to weight of bare cable.

L = Length of span in feet.

m = Weight of cable one ft. long with 1 sq. in. cross-section, in pounds (= 3.85 for copper).

q = Factor by which the horizontal deflection at center of span is multiplied to obtain a distance which, when used in input energy equation, will give average energy per foot of cable.

R = Radius or sag in feet at center of span.

R_m = Radius or sag in feet at center of span at point of maximum deflection.

r = Resistance in ohms per foot.

t = Time in seconds required to reach maximum deflection.

TABLE III

RESULTS OF TESTS, SHOWING CONDUCTOR MOVEMENTS WITH CABLES HORIZONTALLY SPACED

Test	Cable Size	Span ft.	Tension lb./in.²	Sag ft.	Spacing ft	Current		Maximum point data				
						Amperes	Duration sec.	Radius ft.	Horiz. throw ft.	Vertical movement ft.	Time sec.	Type of movement
23A	3/0	150	12,500	0.87	2	7,600	0.92	2.51	2.34	0	0.30	HS
24A	"	"	"	"	"	7,450	0.99	2.4	2.23	0	0.30	"
25A	"	"	"	"	"	6,870	1.00	2.2	2.0	0	0.30	"
26A	"	"	"	"	"	5,880	1.36	1.79	1.54	0	0.32	"
27A	"	"	"	"	"	13,080	0.68	4.61	4.52	0	0.26	HO
1A	3/0	150	12,800	0.85	4	5,610	1.21	1.27	0.94	0		HU
2A	"	"	"	"	"	6,550	1.01	1.51	1.24	0		"
3A	"	"	"	"	"	7,100	0.52	1.65	1.41	0	0.33	"
4A	"	"	"	"	"	7,360	0.48	1.82	1.61	0	0.33	"
5A	3/0	150	12,300	0.88	4	7,650		1.99	1.77	0	0.37	HU
6A	"	"	"	"	"	7,380	0.91	1.82	1.58	0	0.37	HU
7A	"	"	"	"	"	8,340	0.87	2.08	1.87	0	0.34	HS
8A	"	"	"	"	"	11,420	0.75	3.04	2.91	0	0.33	HO
9A	"	"	"	"	"	12,720	0.71	3.32	3.20	0	0.31	HO
10A	"	"	"	"	"	13,500	0.69	3.63	3.52	0	0.31	HO
11A	3/0	150	18,000	0.6	4	7,660	0.92	1.35	1.21	0	0.31	HU
12A	"	"	"	"	"	13,380	0.70	2.92	2.86	0	0.27	HS
13A	3/0	150	24,000	0.45	4	7,480	0.99	0.99	0.88	0	0.29	HS
14A	"	"	"	"	"	13,200	0.71	2.11	2.00	0	0.29	HS
15A	3/0	150	12,800	0.85	8	6,960	1.00	1.20	0.85	−0.19	0.38	DU
16A	"	"	"	"	"	6,720	1.06	1.17	0.80	−0.19	0.38	DU
17A	"	"	"	"	"	6,360	1.12	1.10	0.70	−0.17	0.40	DU
18A	"	"	"	"	"	5,450	1.40	0.98	0.48	−0.1	0.40	DU
19A	"	"	"	"	"	12,780	0.70	2.56	2.42	−0.08	0.36	DS
20A	3/0	150	12,500	0.87	12	6,750	0.98	1.08	0.6			DU
21A	"	"	"	"	"	6,510	1.00	1.04	0.53			DU
22A	"	"	"	"	"	12,500	0.68	2.16	1.86	−0.20	0.34	DU
142A	3/0	300	6,370	6.80	4	5,380	0.99	7.91	4.30	0.16	0.74	RO
143A	"	"	"	"	"	6,400	0.97	8.43	5.46	0.40	0.70	RO
144A	"	"	"	"	"	6,820	0.97	8.7	5.95	0.46	0.70	RO
145A	"	"	"	"	"	7,010	0.93	8.8	6.10	0.46	0.70	RO
146A	"	"	"	"	"	9,010	0.87	10.2	8.23	0.77	0.68	RO
147A	"	"	"	"	"	10,450	0.69	11.4	9.80	0.99	0.60	RO
130A	3/0	300	14,000	3.1	4	5,450	0.87	4.35	2.86	−0.18	0.62	DO
131A	"	"	"	"	"	6,500	0.87	4.95	3.71	−0.18	0.62	DO
132A	"	"	"	"	"	6,670	0.85	5.28	4.14	−0.19	0.62	DO
133A	"	"	"	"	"	7,200	0.95	5.56	4.50	−0.16	0.60	DO
134A	"	"	"	"	"	9,260	0.89	6.66	5.90	0	0.56	HO
135A	"	"	"	"	"	10,580	0.69	7.86	7.23	0	0.56	HO
136A	3/0	300	20,600	2.1	4	7,280	1.08	4.06	3.36	−0.19	0.52	DU
137A	"	"	"	"	"	7,010	0.98	3.85	3.11	−0.17	0.50	DU
138A	"	"	"	"	"	6,540	0.98	3.59	2.79	−0.16	0.48	DU
139A	"	"	"	"	"	5,620	1.04	3.10	2.16	−0.13	0.54	DU
140A	"	"	"	"	"	9,340	1.14	5.10	4.65	0	0.46	HS
141A	"	"	"	"	"	10,800	0.72	6.22	5.85	0	0.44	HS
148A	3/0	300	21,600	2.0	4	5,560	1.01	3.04	2.15	−0.16		DU
149A	"	"	"	"	"	6,560	0.95	3.57	2.82	−0.18	0.50	DU
150A	"	"	"	"	"	6,990	0.97	3.83	3.14	−0.20	0.48	DU
151A	"	"	"	"	"	7,370	0.94	4.01	3.31	−0.20	0.48	DU
152A	"	"	"	"	"	9,250	0.88	5.14	4.68	−0.11	0.46	DS
153A	"	"	"	"	"	10,700	0.67	6.28	5.91	−0.12	0.46	DS
154A	3/0	300	12,700	3.4	8	5,170	1.05	4.15	1.68	−0.40		DU
155A	"	"	"	"	"	6,090	0.99	4.59	2.35	−0.54		DU
156A	"	"	"	"	"	6,520	0.97	4.79	2.62	−0.60		DU
157A	"	"	"	"	"	6,710	0.97	4.92	2.80	−0.65		DU
158A	"	"	"	"	"	8,690	0.95	5.78	4.22	−0.55		DS
159A	"	"	"	"	"	10,020	0.79	6.70	5.35	−0.62	0.62	DO
142B	3/0	450	6,170	15.8	4	5,300	0.99	17.28	8.46	0.71	1.16	RO
143B	"	"	"	"	"	6,260	0.97	17.90	10.45	1.25	1.12	RO
144B	"	"	"	"	"	6,600	0.97	18.3	11.43	1.52	1.12	RO
145B	"	"	"	"	"	6,870	0.93	18.35	11.90	1.85	1.08	RO
130B	3/0	450	12,800	7.6	4	5,400	0.87	9.60	5.31	−0.40	0.92	DO
131B	"	"	"	"	"	6,410	0.87	10.42	6.72	−0.38	0.88	DO
132B	"	"	"	"	"	6,630	0.85	10.77	7.28	−0.33	0.88	DO
133B	"	"	"	"	"	7,030	0.95	11.00	7.67	−0.29	0.86	DO

TABLE III—*Continued*

RESULTS OF TESTS, SHOWING CONDUCTOR MOVEMENTS WITH CABLES HORIZONTALLY SPACED

Test	Cable Size	Span ft.	Tension lb./in.^{2}	Sag ft.	Spacing ft	Current		Maximum point data				
						Amperes	Duration sec.	Radius ft.	Horiz. throw ft.	Vertical movement ft.	Time sec.	Type of movement
136B	3/0	450	21,600	4.5	4	7,100	1.08	7.81	5.99	−0.52	0.76	DU
137B	"	"	"	"	"	6,860	0.98	7.58	5.66	−0.52	0.80	DU
138B	"	"	"	"	"	6,450	0.98	7.30	5.22	−0.46	0.80	DU
139B	"	"	"	"	"	5,460	1.04	6.36	4.04	−0.42	0.80	DU
148B	3/0	450	27,000	3.6	4	5,530	1.01	5.31	3.56	−0.34	0.84	DU
149B	"	"	"	"	"	6,460	0.95	6.23	4.77	−0.40	0.76	DU
150B	"	"	"	"	"	6,870	0.97	6.52	5.10	−0.46	0.72	DU
151B	"	"	"	"	"	7,210	0.94	6.71	5.37	−0.43	0.72	DU
154B	3/0	450	13,500	7.2	8	5,100	1.05	8.84	3.44	−0.94	1.04	DU
155B	"	"	"	"	"	6,000	0.99	9.61	4.64	−1.21	1.00	DU
156B	"	"	"	"	"	6,420	0.97	9.86	5.10	−1.25	0.96	DU
157B	"	"	"	"	"	6,600	0.97	10.11	5.48	−1.32	0.96	DU
23B	3/0	600	12,500	13.8	2	7,040	0.92	19.37	14.3	0.75	0.98	RO
24B	"	"	"	"	"	6,820	0.99	18.68	13.7	1.14	1.00	RO
25B	"	"	"	"	"	6,370	1.00	18.1	12.8	1.00	1.04	RO
26B	"	"	"	"	"	5,520	1.36	17.5	10.75		1.04	RO
1B	3/0	600	12,800	13.5	4	5,470	1.21	16.8	8.65	−0.91	1.21	DO
2B	"	"	"	"	"	6,150	1.01	17.9	10.8	−0.84	1.19	DO
3B	"	"	"	"	"	6,930	0.53	18.2	10.5		1.07	DO
4B	"	"	"	"	"	7,150	0.49	16.95	9.18	−0.78		DO
5B	3/0	600	12,300	14.1	4	7,430	1. +	19.4	12.8	−0.51	1.16	DO
6B	"	"	"	"	"	6,990	0.91	18.75	11.75	−0.51	1.16	DO
11B	3/0	600	18,000	9.6	4	7,140	0.92	15.2	11.0	−0.91	1.04	DS
13B	3/0	600	24,000	7.2	4	7,030	0.99	11.5	8.14	−0.7	0.88	DU
15B	3/0	600	12,800	13.6	8	6,600	1.00	17.9	8.25	−2.38	1.12	DO
16B	"	"	"	"	"	6,510	1.06	17.8	8.0	−2.18	1.15	DO
17B	"	"	"	"	"	6,190	1.12	17.1	6.95	−2.03	1.16	DO
18B	"	"	"	"	"	5,340	1.40	16.35	5.78	−1.72	1.20	DO
20B	3/0	600	12,500	13.9	12	6,610	0.98	17.15	6.04	−2.18	1.16	DO
21B	"	"	"	"	"	6,360	1.00	17.05	5.78	−2.16	1.20	DO
58A	500M	150	12,000	0.9	2	7,620	0.91	1.41	1.09	0	0.34	HS
59A	"	"	"	"	"	8,400	0.98	1.54	1.25	0	0.34	HS
60A	"	"	"	"	"	8,900	0.95	2.11	1.91	0	0.34	HS
61A	"	"	"	"	"	12,680	1.05	2.59	2.45	0.10	0.32	RO
62A	"	"	"	"	"	13,910	1.03	2.90	2.77	0.10	0.32	RO
63A	500M	150	6,370	1.7	4	7,930	0.85	2.04	1.19	0.05	0.48	RO
64A	"	"	"	"	"	11,220	1.03	2.54	1.89	0.07	0.46	RO
65A	"	"	"	"	"	12,900	1.08	2.78	2.20	0.09	0.46	RO
66A	"	"	"	"	"	13,830	1.06	2.92	2.45	0.11	0.44	RO
32A	500M	150	13,500	0.8	4.3	6,020	1.21	0.89	0.39	0		HS
33A	"	"	"	"	"	7,300	0.95	0.97	0.56	0		HS
34A	"	"	"	"	"	7,920	0.87	1.04	0.66	0		HS
35A	"	"	"	"	"	8,570	0.77	1.07	0.72	0		HS
36A	"	"	"	"	"	13,580	0.69	1.82	1.64	0	0.34	HS
37A	500M	150	21,700	0.5	4.2	7,300	0.93	0.62	0.37	0		HS
38A	"	"	"	"	"	7,920	0.87	0.69	0.48	0		HS
39A	"	"	"	"	"	8,540	0.83	0.71	0.51	0		HS
40A	"	"	"	"	"	12,420	0.67	1.14	1.02	0	0.30	HS
41A	"	"	"	"	"	12,420	0.69	1.16	1.04	0		HS
42A	"	"	"	"	"	13,650	0.67	1.24	1.14	0		HS
43A	500M	150	27,100	0.4	4	7,440	0.93	0.52	0.33	0		HS
44A	"	"	"	"	"	8,100	1.01	0.55	0.38	0		HS
45A	"	"	"	"	"	8,690	0.93	0.59	0.44	0		HS
46A	"	"	"	"	"	12,830	1.07	1.01	0.93	0	0.26	HS
47A	"	"	"	"	"	13,610	1.07	1.12	1.05	0	0.26	HS
48A	500M	150	12,000	0.9	8	7,230	0.88	0.97	0.37	0		HS
49A	"	"	"	"	"	8,020	0.88	1.00	0.43	0		HS
50A	"	"	"	"	"	8,450	0.95	1.02	0.48	0		HS
51A	"	"	"	"	"	13,290	1.04	1.37	1.03	0	0.30	HS
52A	"	"	"	"	"	14,100	0.98	1.47	1.17	0		HS

TABLE III—*Continued*

RESULTS OF TESTS, SHOWING CONDUCTOR MOVEMENTS WITH CABLES HORIZONTALLY SPACED

						Current		Maximum point data				
Test	Cable Size	Span ft.	Tension lb./in.²	Sag ft.	Spacing ft	Amperes	Duration sec.	Radius ft.	Horiz. throw ft.	Vertical movement ft.	Time sec.	Type of movement
53A	500M	150	12,000	0.9	11.7	7,140	0.99		small	0		HS
54A	"	"	"	"	"	7,760	1.03	0.95	0.3	0		HS
55A	"	"	"	"	"	8,180	1.01	0.95	0.32	0		HS
56A	"	"	"	"	"	13,200	1.05	1.16	0.74	0		HS
57A	"	"	"	"	"	14,300	1.03	1.26	0.88	0		HS
58B	500M	600	12,400	14.0	2.1	7,100	0.91	15.29	5.9	0.5	1.12	RO
59B	"	"	"	"	"	7,770	0.98	15.68	8.33	0.58	1.12	RO
60B	"	"	"	"	"	8,250	0.95	15.8	8.9	0.75	1.12	RO
32B	500M	600	12,300	14.1	4.3	5,870	1.21	14.63	3.92	0	1.28	HO
33B	"	"	"	"	"	6,980	0.95	15.08	5.26	0	1.24	HO
34B	"	"	"	"	"	7,530	0.87	15.2	5.7	0	1.24	HO
35B	"	"	"	"	"	8,140	0.77	15.37	6.10	0	1.20	HO
37B	500M	600	18,700	9.2	4.2	6,990	0.93	9.9	3.7	0	1.04	HS
38B	"	"	"	"	"	7,550	0.87	10.22	4.49	0	1.00	HS
39B	"	"	"	"	"	8,190	0.83	10.39	4.85	0	1.00	HS
43B	500M	600	24,400	7.1	4.0	7,310	0.93	7.83	3.32	0	0.96	HS
44B	"	"	"	"	"	7,790	1.01	8.05	3.80	0	0.96	HS
45B	"	"	"	"	"	8,400	0.93	8.20	4.11	0	0.92	HS
48B	500M	600	12,400	14.0	8	7,160	0.88	14.65	3.65	−0.19	1.28	DO
49B	"	"	"	"	"	7,880	0.88	14.8	4.09	−0.25	1.28	DO
50B	"	"	"	"	"	8,210	0.95	14.92	4.47	−0.25	1.28	DO
53B	500M	600	12,600	13.7	11.4	7,070	0.99	14.3	2.83	−0.32		DO
54B	"	"	"	"	"	7,580	1.03	14.42	3.12	−0.36	1.32	DO
55B	"	"	"	"	"	7,940	1.01	14.57	3.48	−0.43	1.32	DO
106A	No. 2	150	12,000	0.9	4	4,650	1.06	2.23	1.78	−0.44	0.38	DU
107A	"	"	"	"	"	5,290	1.10	2.56	2.18	−0.44	0.37	DU
108A	"	"	"	"	"	5,370	1.06	2.58	2.21	−0.44	0.36	DU
109A	"	"	"	"	"	5,470	1.10	2.72	2.34	−0.49	0.36	DU
28A	No. 2	150	12,600	0.86	4	6,020	1.40	2.96	2.56	−0.6	0.42	DU
29A	"	"	"	"	"	7,760	0.50	3.81	3.64	−0.22	0.36	DU
30A	"	"	"	"	"	11,720	0.70	6.55	6.44	−0.33	0.32	DS
31A	"	"	"	"	"	4,790	2.60	2.77	2.19	−0.8	0.44	DU
102A	No. 2	150	13,500	0.8	4	7,820	0.71	3.74	3.61	−0.19	0.32	DS
103A	"	"	"	"	"	9,950	0.40	5.45	5.35	−0.24	0.32	DS
104A	"	"	"	"	"	10,600	0.42	6.24	6.16	−0.25	0.30	DS
105A	"	"	"	"	"	11,750	0.38	7.09	7.01	−0.25	0.29	DS
96A	No. 2	150	18,000	0.6	4	7,760	0.42	2.87	2.76	−0.19	0.30	DU
97A	"	"	"	"	"	9,850	0.40	4.25	4.16	−0.27	0.28	DU
98A	"	"	"	"	"	10,770	0.42	4.67	4.58	−0.33	0.27	DU
99A	"	"	"	"	"	11,750		5.28	5.20	−0.32	0.27	DU
100A	"	"	"	"	"	3,700		1.01	0.72	−0.11		DU
83A	No. 2	150	12,000	0.9	8	5,540	0.40	1.98	1.47	−0.42	0.42	DU
84A	"	"	"	"	"	7,690	0.44	3.14	2.74	−0.63	0.42	DU
85A	"	"	"	"	"	8,920	0.42	4.13	3.79	−0.74	0.42	DU
86A	"	"	"	"	"	9,820	0.42	4.75	4.42	−0.84	0.42	DU
87A	"	"	"	"	"	10,200	0.44	5.15	4.85	−0.84	0.42	DU
88A	"	"	"	"	"	10,500	0.42	5.44	5.16	−0.93	0.41	DU
89A	"	"	"	"	"	10,700	0.42	5.91	5.59	−1.05	0.41	DU
90A	"	"	"	"	"	10,810	0.42	6.75	6.48	−1.05	0.41	DU
91A	No. 2	150	18,000	0.6	8	7,580	0.44	2.04	1.83	−0.29	0.36	DU
92A	"	"	"	"	"	9,650	0.42	3.34	3.16	−0.46	0.38	DU
93A	"	"	"	"	"	10,500	0.40	4.15	3.99	−0.58	0.37	DU
94A	"	"	"	"	"	11,300	0.40	4.82	4.62	−0.79	0.38	DU
95A	"	"	"	"	"	3,620	0.63	1.01	0.48	−0.29		DU
77A	No. 2	150	12,000	0.9	11.6	4,490	0.44	2.16	1.01	−1.01	0.48	DU
78A	"	"	"	"	"	5,150	0.38	2.62	1.34	−1.2	0.44	DU
79A	"	"	"	"	"	5,350	0.42	3.08	1.68	−1.68	0.46	DU
80A	"	"	"	"	"	5,470	0.40	3.08	1.68	−1.68	0.46	DU
81A	"	"	"	"	"	5,280	0.44	1.76	1.12	−0.45	0.50	DU
82A	"	"	"	"	"	..	..	..	..	..	..	DU

TABLE III—*Continued*

RESULTS OF TESTS, SHOWING CONDUCTOR MOVEMENTS WITH CABLES HORIZONTALLY SPACED

Test	Cable Size	Span ft.	Tension lb./in.²	Sag ft.	Spacing ft.	Current		Maximum point data				
						Amperes	Duration sec.	Radius ft.	Horiz. throw ft.	Vertical movement ft.	Time sec.	Type of movement
106B	No. 2	600	12,000	14.5	4	4,350	1.06	24.4	13.58	−5.83	1.20	DU
107B	"	"	11,800	14.7	"	4,810	1.10	27.0	16.4	−6.82	1.20	DU
108B	"	"	11,500	15.1	"	4,900	1.06	27.7	17.1	−6.70	1.20	DU
109B	"	"	11,400	15.2	"	4,920	1.10	27.7	17.0	−6.75	1.20	DU
28B	No. 2	600	12,600	13.7	4	Hit ground						
31B	"	"	"	"	"	"	"					
100B	No. 2	600	18,800	9.2	4	3,600		14.9	7.81	−3.52	1.12	DU
101B	"	"	"	"	"	4,470	0.79	18.6	10.70	−5.99	1.12	DU
95B	No. 2	600	18,200	9.5	8	3,570	0.63	14.7	4.79	−4.41	1.12	DU
77B	No. 2	600	12,100	14.3	11.8	4,460	0.44	19.8	4.84	−4.96	1.16	DS
78B	"	"	"	"	"	5,150	0.38	20.4	5.59	−5.33	1.12	DS
79B	"	"	12,300	14.1	"	5,300	0.42	21.1	6.58	−5.95	1.12	DS
80B	"	"	12,400	14.0	"	5,470	0.40	21.0	6.51	−5.95	1.08	DS
73A	4/0 WP	150	5,900	2.3	1.5	8,150	0.99	3.78	3.23	0.35	0.36	RO
74A	"	"	"	"	"	10,890	1.05	4.71	4.35	0.49	0.32	RO
75A	"	"	"	"	"	12,820	1.11	5.19	4.89	0.54	0.31	RO
76A	"	"	"	"	"	13,450	1.07	5.62	5.38	0.65	0.31	RO
72A	4/0 WP	150	10,400	1.3	1.5	13,320	1.05	4.24	4.07	0.16	0.27	RO
71A	4/0 WP	150	13,500	1.0	1.5	13,400	1.03	3.98	3.86	0	0.26	HO
67A	4/0 WP	150	16,900	0.8	1.5	8,110	0.85	2.20	2.05	0	0.3	HO
68A	"	"	"	"	"	10,920	1.03	3.07	2.96	0	0.26	HO
69A	"	"	"	"	"	12,605	1.09	3.44	3.34	0	0.26	HO
70A	"	"	"	"	"	13,300	1.05	3.90	3.82	0	0.26	HO

NOTE: In the column headed "Cable Size," the symbol M means thousand circular mils, while WP means weather-proof.

θ = Temperature rise in degrees fahrenheit during time t.
T = Tension in cable in pounds per square inch.
T_1 = Initial tension in cable in pounds per square inch.
T_2 = Tension in cable at point of maximum deflection, in pounds per square inch.
λ = Length of cable along catenary in feet.
λ_0 = Unstressed length of cable at initial temperature.
λ_1 = Initial length of cable.
λ_2 = Length of cable when at maximum deflection.
X = Horizontal distance moved by conductor, in feet.
X_c = Horizontal distance moved by cable at center of span, in feet.
X_m = Maximum horizontal distance moved by cable at center of span, in feet.
y_0 = Initial sag at center of span in feet.

A sketch showing the principal dimensional symbols is given in Fig. 36.

From Equation (2), the magnetic force per foot of conductor, when each conductor has moved a distance X from its initial position, is:

$$F = \frac{0.045\, I^2}{10^6\,(2\,X + C)} \qquad (2.1)$$

The energy imparted to the conductor by this force for the entire span is given by

$$\text{Input energy (ft-lb.)} = L \int_0^{X_m} \frac{0.045\, I^2}{10^6\,(2\,q\,X_c + C)}\, d\,X_c \qquad (2.2)$$

The value q is to be evaluated later semi-empirically. The solution of the above integral gives:

$$\text{Input energy} = L\,\frac{0.045\, I^2}{10^6} \cdot \frac{1}{2} \log_e \left(\frac{C + 2\,q\,X_m}{C} \right) \qquad (2.3)$$

The energy stored in the conductor due to its stretching is given by the product of the average tension and its change in length.

Thus:

$$\text{Stored energy (ft-lb.)} = B\,\frac{T_1 + T_2}{2}\,(\lambda_2 - \lambda_1) \qquad (2.4)$$

The energy calculated by equation **2.3** and that calculated by equation **2.4** may be equated. This will be done after the various parts of equation **2.4** have been determined.

According to the parabolic sag formulas, the length of cable at any sag is given by

$$\lambda = L + \frac{8}{3\,L}\,R^2 \qquad (2.5)$$

("Std. Handbook for Elec. Eng.", page 966, 1922 Ed.)

Therefore the change in length is

$$\lambda_2 - \lambda_1 = \left(L + \frac{8}{3\,L}\,R_m^2\right) - \left(L + \frac{8}{3\,L}\,y_0^2\right)$$

$$= \frac{8}{3\,L}\,(R_m^2 - y_0^2) \qquad (2.6)$$

TABLE IV

RESULTS OF TESTS, SHOWING CONDUCTOR MOVEMENTS WITH CABLES VERTICALLY SPACED

Test	Cable size	Span ft.	Tension lb./in.²	Sag ft.	Spacing ft.	Current		Maximum movement data			
								Upper wire		Lower wire	
						Amperes	Duration sec.	Throw ft.	Time sec.	Throw ft.	Time sec.
110A	3/0	150	12,000	0.9	4	5,380	0.80	0.79		0.59	
111A	"	"	"	"	"	6,375	0.90	1.16		0.84	
112A	"	"	"	"	"	6,840	0.86	1.33		0.91	
113A	"	"	"	"	"	7,180	0.82	1.50		0.99	
114A	"	"	"	"	"	12,450	0.61	4.05		2.20	
115A	3/0	150	18,000	0.6	4	5,580	0.83	0.64		0.59	
116A	"	"	"	"	"	6,670	0.80	0.94		0.89	
117A	"	"	"	"	"	7,090	0.91	1.08		0.89	
118A	"	"	"	"	"	7,350	0.89	1.20		0.99	
119A	"	"	"	"	"	13,000	0.57	3.26		2.19	
200A	3/0	300	12,000	3.6	2	5,600		3.77	0.60	2.65	
201A	"	"	"	"	"	7,500		6.35	0.70	3.84	0.40
202A	"	"	"	"	"	11,000		11.25	0.64	5.75	0.36
197A	3/0	300	24,000	1.8	2	5,600		2.94	0.42	2.54	
198A	"	"	"	"	"	7,500		4.50	0.44	3.70	
199A	"	"	"	"	"	11,000		7.80	0.54	5.95	
160A	3/0	300	12,400	3.5	4	5,550	1.05	2.00		1.55	
161A	"	"	"	"	"	6,450	0.97	2.71		2.04	
162A	"	"	"	"	"	6,990	0.97	2.98		2.19	
163A	"	"	"	"	"	7,220	0.95	3.29		2.35	
164A	"	"	"	"	"	9,175	0.93	8.47	1.02	3.18	
165A	"	"	"	"	"	10,620	0.69	11.22	1.	4.20	
166A	3/0	300	17,300	2.5	4	5,440	0.82	2.09		1.55	
167A	"	"	"	"	"	6,480	0.77	2.97		2.04	
168A	"	"	"	"	"	6,880	0.79	3.46	0.62	2.19	
169A	3/0	300	17,300	2.5	4	7,140	0.79	3.80	0.66	2.32	
170A	"	"	"	"	"	9,200		6.30	0.68	3.16	
171A	"	"	"	"	"	10,470	0.73	8.77	0.74	3.90	
194A	3/0	300	18,000	2.4	8	5,300		0.99		1.36	
195A	"	"	"	"	"	6,800		1.78		2.09	
196A	"	"	"	"	"	10,000		3.46		5.14	
191A	3/0	300	21,600	2.0	8	5,300		0.90		1.29	
192A	"	"	"	"	"	6,800		1.52		2.05	
193A	"	"	"	"	"	10,000		3.50		3.62	
200B	3/0	450	12,000	8.1	2	5,600		3.50	0.72	3.50	0.56
201B	"	"	"	"	"	7,500		5.77	0.88	4.95	0.52
197B	3/0	450	24,400	4.0	2	5,600		4.81	0.64	3.96	
198B	"	"	"	"	"	7,500		7.60	0.72	5.79	0.56
160B	3/0	450	13,500	7.2	4	5,550	1.05	2.34		3.00	
161B	"	"	"	"	"	6,450	0.97	3.04		3.84	
162B	"	"	"	"	"	6,990	0.97	3.18	0.60	4.03	
163B	"	"	"	"	"	7,220	0.95	3.50		4.37	
166B	3/0	450	21,600	4.5	4	5,440	0.82	3.09		3.14	
167B	"	"	"	"	"	6,480	0.77	4.16		4.06	
168B	"	"	"	"	"	6,880	0.79	4.70		4.44	
169B	"	"	"	"	"	7,140	0.79	5.20		4.70	
194B	3/0	450	18,700	5.2	8	5,300		1.68		2.30	
195B	"	"	"	"	"	6,800		2.85		4.00	
191B	3/0	450	23,800	4.1	8	5,300		1.68		2.31	
192B	"	"	"	"	"	6,800		2.71		4.42	
110B	3/0	600	12,200	14.2	4	5,250	0.80	2.04		3.22	
111B	"	"	"	"	"	6,100	0.90	2.52	0.64	4.13	
112B	"	"	"	"	"	6,500	0.86	2.58	0.64	5.04	
113B	"	"	"	"	"	6,850	0.82	2.58		4.39	
115B	3/0	600	17,500	9.9	4	5,370	0.83	3.90		4.16	
116B	"	"	"	"	"	6,300	0.80	5.03		5.22	
117B	"	"	"	"	"	6,710	0.91	5.56	0.80	5.62	
118B	"	"	"	"	"	8,900	0.89	5.56	0.80	5.54	

TABLE IV—*Continued*

RESULTS OF TESTS, SHOWING CONDUCTOR MOVEMENTS WITH CABLES VERTICALLY SPACED

Test	Cable size	Span ft.	Tension lb./in.²	Sag ft.	Spacing ft.	Current		Maximum movement data			
								Upper wire		Lower wire	
						Amperes	Duration sec.	Throw ft.	Time sec.	Throw ft.	Time sec.
120A	500M	150	13,500	0.8	4	6,070	0.81	0.35		0.33	
121A	"	"	"	"	"	7,290	0.91	0.50		0.46	
122A	"	"	"	"	"	8,080	0.91	0.56		0.50	
123A	"	"	"	"	"	8,400	0.87	0.66		0.56	
124A	"	"	"	"	"	13,750	0.59	1.76		1.18	
125A	500M	150	18,000	0.6	4	5,900	0.86	0.28		0.28	
126A	"	"	"	"	"	7,250	1.00	0.39		0.39	
127A	"	"	"	"	"	7,880	0.95	0.46		0.42	
128A	"	"	"	"	"	8,400	0.96	0.51		0.51	
129A	"	"	"	"	"	13,750	0.59	1.30		1.02	
212A	500M	300	11,100	3.9	2	6,000		1.68		1.29	
213A	"	"	"	"	"	9,000		2.96	0.56	2.06	
214A	"	"	"	"	"	11,800		5.41	0.66	3.10	0.38
209A	500M	300	19,600	2.2	2	6,000		1.19		1.35	
210A	"	"	"	"	"	9,000		2.26		2.00	
211A	"	"	"	"	"	11,800		3.74		3.10	
206A	500M	300	12,700	3.4	8	5,400		0.45		0.45	
207A	"	"	"	"	"	7,700		0.88		0.84	
208A	"	"	"	"	"	11,000		2.05		1.72	
203A	500M	300	21,600	2.0	8	5,400		0.39		0.39	
204A	"	"	"	"	"	7,700		0.71		0.71	
205A	"	"	"	"	"	11,000		1.42		1.29	
212B	500M	450	12,300	7.9	2	6,000		2.30	0.76	1.95	0.56
213B	"	"	"	"	"	9,000		3.84	0.88	2.93	0.56
209B	500M	450	24,400	4.0	2	6,000		2.16	0.60	1.98	0.56
210B	"	"	"	"	"	9,000		3.66	0.60	3.15	0.56
206B	500M	450	12,300	7.9	8	5,400		0.89		0.47	
207B	"	"	"	"	"	7,700		1.22		1.22	
203B	500M	450	23,800	4.1	8	5,400		0.76		0.82	
204B	"	"	"	"	"	7,700		1.24		1.37	
120B	500M	600	12,700	13.7	4	5,840	0.81	1.42		1.78	
121B	"	"	"	"	"	7,040	0.91	2.02	0.80	2.02	
122B	"	"	"	"	"	7,800	0.91	2.13	0.80	2.82	
123B	"	"	"	"	"	8,050	0.87	2.32	0.80	2.52	
125B	500M	600	17,900	9.7	4	5,790	0.86	1.87		1.68	
126B	"	"	"	"	"	7,010	1.00	2.52		2.45	
127B	"	"	"	"	"	7,600	0.95	2.90		2.58	
128B	"	"	"	"	"	7,760	0.96	3.13		2.18	

The length of the cable when strung is

$$\lambda_1 = L + \frac{8}{3\,L}\,y_0^2 \qquad (2.7)$$

which is the unstressed length plus the stretch due to the tension developed. The stretch is $L\,T_1/E$, so the initial unstressed length can be written:

$$\lambda_0 = L + \frac{8}{3\,L}\,y_0^2 - \frac{T_1}{E}\,L \qquad (2.8)$$

The unstressed length after any temperature change θ can be taken as $\lambda_0 + L\,\alpha\,\theta$. T_2 may be calculated from the relation that the tension is $(E/L) \times$ stretch.

Therefore $$T_2 = \frac{E}{L}\,(\lambda_2 - \lambda_0 - L\,\alpha\,\theta) \qquad (2.9)$$

substituting the value for λ_2 as obtained from equation (2.5), and the value of λ_0 from equation (2.8), the following is obtained.

$$T_2 = \frac{E}{L}\left(L + \frac{8}{3\,L}\,R_m^2 - L - \frac{8}{3\,L}\,y_0^2 + \frac{T_1}{E}\,L - L\,\alpha\,\theta\right) \qquad (2.10)$$

$$T_2 = E\left[\frac{8}{3\,L^2}\,(R_m^2 - y_0^2) + \frac{T_1}{E} - \alpha\,\theta\right] \qquad (2.11)$$

Since the sag can be expressed as $y_0 = K_0\,m\,L^2/8\,T_1$ (see page 966 of "Standard Handbook for Elec. Eng.," 1922 Edition), then $8/3\,L^2$ may be written $K_0\,m/3\,y_0\,T_1$. The equation for T_2 may now be written in its final form

$$T_2 = E\left[\frac{K_0\,m}{3\,T_1}\left(\frac{R_m^2 - y_0^2}{y_0}\right) + \frac{T_1}{E} - \alpha\,\theta\right] \qquad (2.12)$$

The area of the conductor in square inches is converted to circular mils by

$$B = A/1.272 \times 10^6 \tag{2.13}$$

Substituting the values from equations, (2.13), (2.12), and (2.6) into equation (2.4), and equating that to equation (2.3), the following expression is obtained:

$$L \frac{0.045\, I^2}{10^6} \cdot \frac{1}{2} \log_e \frac{C + 2\, q\, X_m}{C}$$

$$= \frac{A}{1.272 \times 10^6} \cdot \frac{1}{2} \left[T_1 + \frac{E\, K_0\, m}{3\, T_1} \left(\frac{R_m^2 - y_0^2}{y_0} \right) \right.$$

$$\left. + T_1 - E\, \alpha\, \theta \right] \left[\frac{8}{3\, L} (R_m^2 - y_0^2) \right] \tag{2.14}$$

Dividing through by L and substituting $K_0\, m/3\, y_0\, T_1$ for $8/3\, L^2$ and rearranging terms, the following is obtained.

$$\frac{0.0572 \dfrac{I^2}{A} \log_e \dfrac{C + 2\, q\, X_m}{C}}{\dfrac{K_0\, m}{3} \left(\dfrac{R_m^2 - y_0^2}{y_0} \right)}$$

$$= \frac{E\, K_0\, m}{3\, T_1^2} \left(\frac{R_m^2 - y_0^2}{y_0} \right) + 2 - \frac{E}{T_1}\, \alpha\, \theta \tag{2.15}$$

In this equation the values for q and θ have yet to be determined. They will be determined for copper cables.

The resistance of 97.3 per cent conductivity copper cable at some temperature θ above 68 deg. fahr. is given by

$$r = (1 + 0.00212\, \theta)\, \frac{10.65}{A}$$

Let $d\,t$ and $d\,\theta$ be elementary increases in time and temperature, respectively. Then, taking the specific heat of copper as 0.093, and the weight of copper cable as being $3.09/10^6$ lb. per cir. mil ft., and 1 watt-hr. as 3.41 B. t. u.:

$$\frac{3.41}{3600} I^2 (1 + 0.00212\, \theta) \frac{10.65}{A}\, d\,t$$

$$= \frac{3.09\, A}{10^6}\, 0.093\, d\,\theta \tag{2.16}$$

Transposing, integrating, and writing the solution, the following expression is obtained:

$$t = 0.01342 \frac{A^2}{I^2} \log_e (1 + 0.00212\, \theta) \tag{2.17}$$

from which

$$\theta = 471.5\, (e^{74.5 \frac{I^2}{A^2} t} - 1) \tag{2.18}$$

and the last term of equation (2.15) becomes

$$\frac{14{,}000{,}000 \times 0.0000096\, \theta}{T_1} = \frac{63{,}400}{T_1} (e^{74.5 \frac{I^2}{A^2} t} - 1) \tag{2.19}$$

This expression is cumbersome to use, so a simpler expression that gives results that are within 5 per cent error from the correct value of this term was determined empirically. It can be used up to values of $I^2\, t/A^2$ of 0.004. The empirical expression is:

$$\frac{E\, \alpha\, \theta}{T_1} = 5.13 \times 10^6 \frac{I^2\, t}{A^2\, T_1} \tag{2.20}$$

The value of t still remains unknown. A study of the time to reach the point of maximum deflection was made and the purely empirical formulas given in equation (3) and (3.1) in the main part of the paper resulted. The term given in equation (2.20) is a correction term in the main formula, and even errors of ten per cent or more in it are usually unimportant.

The determination of the value of the term q will now be taken up. Referring to equation (2.3), it is evident that the energy input for a movement X is $(0.0225\, I^2/10^6) \log_e [\, (C + 2\, X)/C\,]$ per linear foot.

If we assume that a cable retains its parabolic form throughout its movement, then the horizontal projection is also a parabola, and the value of X at a distance l from the mid-point of the span is:

$$X = X_m - p\, l^2 \tag{2.21}$$

where p is a proportionality constant. The average input for the entire span is then:

Input per ft.

$$= \frac{0.0225\, I^2}{10^6} \int_0^{L/2} \frac{\log_e \dfrac{C + 2(X_m - p\, l^2)}{C}\, d\,l}{L/2} \tag{2.22}$$

It can be shown that the solution of equation (2.22) is:

$$\text{Input per ft.} = \frac{0.0225\, I^2}{10^6} \sqrt{\frac{1}{\dfrac{2\, X_m}{C + 2\, X_m}}}$$

$$\left[\log_e \left(\frac{1 + \sqrt{\dfrac{2\, X_m}{C + 2\, X_m}}}{1 - \sqrt{\dfrac{2\, X_m}{C + 2\, X_m}}} \right) - 2 \sqrt{\frac{2\, X_m}{C + 2\, X_m}} \right] \tag{2.23}$$

This value of energy when multiplied by L, can be equated to that expressed in equation (2.3). A direct solution for q is not simple. It can be observed that the value for q would vary for different values of X_m/C. It is found, however, that for a wide range of variation in this ratio, the value of q changes very slowly. For example, the following table shows the comparison:

X_m/C	q
10.0	0.57
1.0	0.62
0.1	0.66

These results indicate that about 1.25 would be a good value to use for $2q$ in equation (2.3). However, because of the fact that the loading due to the magnetic forces is greater near the supports, the cable departs from the catenary or parabola, and has greater sag near the supports than a true catenary. This means that the average energy input is increased above that calculated. The exact amount of this increase, and corresponding value of $2q$, is probably difficult to determine mathematically.

It has just been shown that it should be greater than 1.25, and of course it must be less than 2, since this is the value it would have if the cable hung in a rectangular form, that is, with a constant sag for its entire length. Obviously, the cable remains closer to being a catenary in form rather than a rectangle, so the above values should be closer to 1.25. The value need not be very precise, because, being a logarithmic term, it can be varied considerably without affecting the final result very much. It was found that the use of $2q = 1.5$ gave quite good agreement between calculated and experimental results, so this figure was chosen.

If the value in equation (2.20) and the above value for $2q$ is substituted in equation (2.15), and if m is taken as 3.85 and E as 14,000,000 (values for copper), then the following expression is obtained.

$$\frac{0.0447\, y_0 \dfrac{I^2}{A} \log_e \dfrac{C + 1.5\, X_m}{C}}{K_0 (R_m^2 - y_0^2)} = \frac{18 \times 10^6\, K_0}{T_1^2} \left(\frac{R_m^2 - y_0^2}{y_0} \right) + 2 - 5.13 \times 10^6 \frac{I^2 t}{A^2 T_1} \qquad (2.24)$$

Since R_m and X_m are both unknown, this equation cannot be solved directly. However, for the great majority of cases, the path of the cable is horizontal, or nearly so. Assuming horizontal movement, then:

$$R_m = \sqrt{X_m^2 + y_0^2} \qquad (2.25)$$

By substituting this in equation (2.24), the following usable form is obtained.

$$\frac{0.0447\, y_0 \dfrac{I^2}{A} \log_e \dfrac{C + 1.5\, X_m}{C}}{K_0 X_m^2} = \frac{18 \times 10^6\, K_0 X_m^2}{T_1^2\, y_0} + 2 - 5.13 \times 10^6 \frac{I^2 t}{A^2 T_1} \qquad (2.26)$$

Equation (2.26) is the same as equation (5) in the main part of the paper.

Appendix D.

Miniature Tests

As explained in the main part of the paper, the conditions of the miniature test should be so chosen that each of the terms in equation (5) will be the same as for the large scale condition being duplicated. This conclusion was reached because of the fact that equation (5) contains the constant term 2, and if one term is the same it is reasonable to suppose that all terms should be the same. Let n, n_i, n_t, and n_a be the multipliers of the full scale distances, current, tension, and area of cable, respectively, to obtain the equivalent small scale values, and substitute $\sqrt{y_0}/3$ for t. Then in order to keep the respective terms of equation (5), the same for both full scale and small scale values, the following relations must hold:

$$\frac{n\, n_i^2}{n^2\, n_a} = 1 \qquad \text{(From left hand term)}$$

$$\frac{n^2}{n_i^2\, n} = 1 \qquad \text{(From first right hand term)}$$

$$\frac{n_i^2 \sqrt{n}}{n_a^2\, n_t} = 1 \qquad \text{(From last term)}$$

The solution of these three simultaneous equations gives:

$$n_i = n_a = n$$
$$n_t = \sqrt{n}$$

Since $y_0 = K_0\, m\, L^2/8\, T_1$, $L = \sqrt{8\, T_1\, y_0/K_0\, m}$

The multiplier for span length, then, is:

$$\sqrt{n_t\, n} = \sqrt{n^{3/2}} = n^{3/4}$$

Discussion

E. F. Pearson: The Northwestern Electric Company in Portland has had several cases of heavy conductors on three-phase 11,000-volt lines slapping together while carrying short-circuit currents of from 5000 to 14,000 amperes. Our case seems to parallel closely some of the test cases as set up in the paper.

Since reading the paper I have been wondering just what will be the best economic solution of this problem as applied to our existing systems within city limits.

The paper recommends as a solution the spacing of poles at closer intervals, the use of heavier conductors, higher stringing tensions, the use of feeder reactors to limit the current, and increased spacing of conductors. In our city, and the same condition should apply in other large cities, the pole spacing in general is practically as low as city regulations will permit. Additional poles on residential and business streets are objectionable and I do not look for much relief from this source.

The use of conductors of much larger than 4/0, together with heavier stringing tensions, results in considerable increase in guying costs and in many cases it is impractical to install additional guys at dead ends due to protests from property owners. A certain amount of relief, however, may be obtained from this source, more particularly where present conductors are very light, such as changing from No. 4 to No. 1/0 copper.

Since the magnetic force between conductors is proportional to the square of the current, it seems to me that limiting of

current with reactance, either in the form of reactors or high-reactance transformers, offers the best economic solution.

Some relief can also be obtained through the increase of conductor spacing, but at considerable cost on existing lines.

One other possible solution for lines of 13 kv. or less in congested urban territories has occurred to me. Since the conductors are repelled from one another under short-circuit conditions, we might consider the installation of a semi-rigid string of small strain insulators between conductors at the center of existing spans.

This idea at first may seem far-fetched. However, this should, to a certain degree, act as a substitute for decreased pole spacing and thus minimize the conductor movement. The cost of installation would be considerable, but for 13 kv. and less, it should not cost over one-third of the value of an intermediary pole, particularly for the long poles necessary in congested districts.

Whatever plan we use to effect a complete solution on systems already existing in our city will cost a considerable amount of money and the one which will best effect the solution on an economic basis should be the one chosen.

L. R. Gamble: We have already checked up some of the lines on our system with the data presented in this paper, and find we are still safe with the higher voltage lines like 110,000 and 60,000, but we are getting dangerously close with our 13,000-volt lines.

A. C. Pratt: We have a great deal of rather heavy distribution at 2300 volts. The short-circuit values can reach probably 25,000 amperes with reactors in series, and on lines without reactors can reach 50,000 or 60,000 amperes. So far we have got along pretty well by the use of spreaders. For that low voltage our spreaders are exceedingly simple, being nothing but ordinary cross-arms and ordinary pins, the same as a pole structure without the pole. They hang in the middle of the span. The spans are about 110 ft., and this spreader cuts the effective span down to about 55 ft. Inasmuch as the maximum short circuit, of course, is considerably greater in the immediate vicinity of the substation, we have recently adopted the expedient of using weather-proof wire for, say, 500 ft. outside the substation, at the same time using large cables, say 500,000 or 600,000 cir. mils for this same distance. Beyond this point we split up into a number of smaller cables in order to reduce the inductive drop.

We have one case of a short 2300-volt tie between substations, which acts to tie the buses of the substations together, making, in effect, one station.

There are two circuits on this pole line. Each circuit is 633,000 cir. mils. The spreaders on those circuits are 50 ft. apart. We have never yet had a flash on these lines, although I have no doubt we reach 50,000 amperes.

L. F. Hunt: In designing one of our long spans of multi-circuit type over the Long Beach Channel the slapping of conductors together, due to short circuits, was considered. We took steps to calculate what effect the heavy short-circuit currents would have on that span. It was found that a single-phase short circuit, that is, one conductor short-circuited with the other conductor, gave the most severe condition. It was also found that the temperature rise in the conductor due to the heating of the current, caused considerable sag in the span, which would give considerable trouble if the spacing was not vertical. The spacing in these spans being vertical, the sag would cause a short circuit on the same circuit, therefore causing the same circuit breaker to trip out.

It was also found that by selecting the circuits so they could be relayed at an increased rate, the amount of swing of conductors would be reduced and furthermore that the heavier conductor would decrease the swing. Due to the fact that it took a very peculiar coincident short circuit to cause sufficient current to flow, and the short circuit had to be on one particular feeder to give any trouble, the standard conductor was used.

L. J. Corbett: I wish to add a word regarding the three-phase short circuit. It is my opinion that no greater swing of conductors would be experienced from full metallic short circuits on three-phase lines than from the single-phase short circuits considered in the paper. The frequencies in common use are 25 and 60 cycles per second. Because of their inertia the conductors will not get into motion soon enough to give any trouble from their displacement, due to the rapidly reversing forces which would exist when all three phases are involved. In the single-phase short circuit there is a succession of impulses, each tending to force the conductors apart, since the current in the two wires is flowing in opposite directions on both sides of the cycle, so the effect is cumulative.

I believe the authors have discussed the maximum condition by considering a single-phase short circuit between two of the conductors.

W. S. Peterson: I agree with Mr. Pearson that it is best, in general, to use reactors to cut down the currents and thus reduce the forces on the conductors, because you are then working with a factor that varies as the square. It is the same principle that makes the use of shorter spans valuable, because the movements vary approximately as the 3/2 power of the span, while the variation with other factors is at a lower power. This leads to the point that has been mentioned, that is, putting insulators at the center of the span, thus dealing with the next most effective thing, namely, span length.

I have seen the insulators so installed on the Montana Power Company system. I believe this method is of value and should be used at times. The only objection that I have ever heard against it, is that it is difficult to maintain the insulators. On the other hand, such work is infrequent on low-voltage circuits, and if the conductors are heavy enough so that some sort of a trolley can be used to transport the men out to the center of the span on the conductor, it should still be an effective method of limiting conductor movements.

Mr. Wade is correct in assuming that in some cases the movement is so extreme that the stresses are close to the elastic limit of the cable. Equation (**2.12**) in Appendix C of the complete paper can be used to calculate this maximum tension. In some cases, however, the elongation due to heating decreases the tension more than increased radial sag has increased it, so that the tension at the point of maximum deflection is not higher than the initial tension.

The three-phase short circuits, of course, open up a new field, possibly more complicated than the single-phase condition. In brief, in the three-phase condition, one is dealing with a conductor carrying a current in the magnetic field set up by currents in the other two conductors. These currents are out of phase, which makes the problem more complicated mathematically. I believe that, in three-phase investigations, the thing to do is to use the miniature test set-up, which should give results quicker and more accurately than a system of calculation.

In the case of the Long Beach Channel crossing mentioned by Mr. Hunt, I imagine the reduction in the time to trip the breakers might be a distinct help. I said in the paper that in general the circuit breakers could not be tripped fast enough to be of help. I was referring to spans of the order of 150 ft. in length where the time to reach the maximum point is one-quarter to one-third of a second. In the Long Beach Channel crossing, the spans are over 1000 ft. and the time to reach the maximum point is much longer. If one or two seconds should elapse before the maximum deflection is reached, the circuit breaker can be of use and the movement would be reduced by tripping more quickly.

The vertical construction of these lines is an advantage, as it would have a tendency to confine the interference to the circuit in trouble, unless resulting arcs are blown into adjacent circuits, or unless the movements should acquire horizontal components from the currents in the adjacent circuits.

Residual Voltages and Currents in Power Systems

BY L. J. CORBETT[1]
Member, A. I. E. E.

Synopsis.—In the problem of inductive coordination of power and communication systems, residual voltages and currents in the power lines are an important element, yet it is apparent that they are not in general well understood. Studies have been made and others are under way by well-equipped organizations, which will add much to the data concerning them. In this paper the general principles which apply are briefly reviewed and applied to typical power circuits and networks, and oscillograms are shown which verify the theory presented.

* * * * * *

Balanced and Residual Voltages and Currents

In the ideal three-phase circuit the vector sum of the voltages to ground of the phase wires is zero, and the vector sum of the currents in the three wires is also zero. In the practical three-phase circuit such is not the case. The vector sum of the voltages to ground of the phase wires is never quite zero, and the vector sum of the currents in the three wires is not zero. In inductive coordination studies it is customary to divide the voltages to ground and the currents into "balanced" and "residual" components.

The balanced voltages are those components of voltage to ground which are equal in value and so related in phase that their vector sum is zero.

The residual voltage is the vector sum of the voltages to ground of the phase wires. It is equivalent to a single-phase voltage impressed between the three wires in multiple and ground.

The balanced currents are those components of the currents in the wires whose vector sum is zero. They are confined to the wires of the circuit and their algebraic sum from instant to instant is also zero.

The residual current is the vector sum of the currents in the three phase wires. It is equivalent to a single-phase current in a circuit consisting of the three wires in multiple as one conductor and ground as the other.

In recent terminology the balanced components are those of positive and of reverse phase sequence, the residual being a zero-phase sequence component.

The effects of balanced voltages and currents can be neutralized to a large extent by the use of suitably coordinated transpositions in the power and telephone circuits. The effects of residual voltages and currents are cumulative along an exposure, bringing the potential of the telephone line and equipment to a relatively high value above ground. Induction from these components obviously is not affected by transpositions in the power circuit.

Generator Wave Form

The type of wave of voltage striven for in the design of a-c. generators is the pure sine wave. This is impossible of exact attainment, but a very close approximation is reached in modern designs. In the Standardization Rules of the Institute a limit is set for the Deviation Factor, a measure of the deviation of the actual open-circuit terminal voltage wave of a synchronous machine from a pure sine wave.

Actually certain harmonics are present in the generator wave in varying amounts. If the positive lobe of the wave is identical with the negative lobe, only odd harmonics are present. The tendency is for the higher harmonics to be damped out in the impedance or magnified slightly in the capacitance of the circuit to which the generator is connected. At times certain harmonics are carried through one or two transformations and can be detected from their inductive effects on paralleling communication circuits.

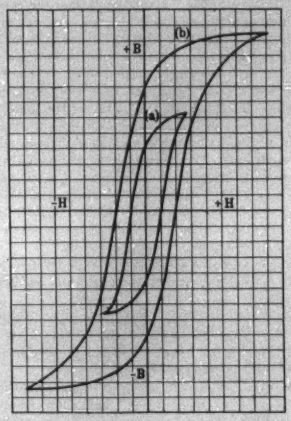

Fig. 1—Magnetization and Hysteresis Curves at Different States of Saturation

Transformer Exciting Current

The chief cause of higher harmonics in a power system is the exciting current of the transformers. Even if the generator wave is a pure sine wave of voltage, the

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28-121

exciting current demanded by the transformers will contain higher harmonics superposed on the fundamental wave if the circuit will permit their flow. If it will not, the deficiency shows up as higher harmonics in the voltage wave.

To recall the relations existing, Fig. 1 is presented showing the magnetization and hysteresis curve, Fig. 2 showing the peaked exciting current required by a sine

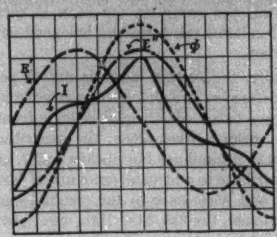

Fig. 2—Sine Wave of Voltage, Peaked Wave of Magnetizing Current

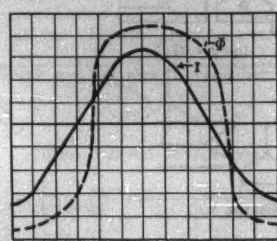

Fig. 3—Sine Wave of Current, Flat Wave of Flux

wave of voltage, and Fig. 3 showing the resulting wave of flux if the current is compelled to follow a sine wave, as for example in a current transformer with open circuited secondary coil.

If a single-phase transformer is connected to an a-c. source, the resulting exciting current is as shown in Fig. 2. If the iron of the transformer is worked at low density, the hysteresis curve being of the form indicated in Fig. 1A, the distortion of the current wave will be slight. If worked at high density, the hysteresis curve being of the form shown in Fig. 1B, the current wave will be considerably distorted. Note that the distorted wave of current *is necessary* in order to produce a sine wave of counter e. m. f. to oppose that applied, and that consequently the secondary e. m. f. wave is also a pure sine wave.

The exciting current of the single-phase transformer contains a pronounced third harmonic, an appreciable fifth, and higher harmonics in varying amounts. Working the iron at high saturation increases the magnitude of the harmonics, bringing out also some of the higher ones which are not noticeable at low densities.

Transformer Connections

When transformers are connected in banks on a three-phase system, certain connections permit the flow of harmonic currents and others do not. It is not intended here to repeat analyses of a large number of possible combinations, but to emphasize the relations existing on a few of the types commonly used. An elemental system supplied from one generator is shown in Fig. 4.

Generator

Present practise favors the Y connection for three-phase generators, with neutral grounded, either solidly or through a resistance. This limits the insulation strain on the windings definitely to the Y voltage. It is somewhat easier to obtain a good wave form with this than with the delta connection.

Step-Up Transformers

Practise also favors the delta connection for the primary of the step-up transformer bank. In this connection, a sine wave of voltage between conductors will result in a peaked wave of magnetizing current, the necessary third harmonic currents being in phase in each transformer and thus circling around in the closed delta with very low impedance. The secondary voltage will then be free of third harmonic components.

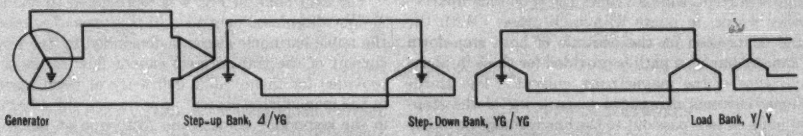

Fig. 4—System Supplied from One Generator

If other harmonics than the fundamental are present in the wave of applied voltage, the same considerations as noted regarding the third harmonic current apply to the third multiples of the other harmonics, such as the 9th, 15th, 21st, 27th, 33rd, etc., which will be represented in the delta current, as these will be required in the magnetizing current in order that the 3rd, 5th, 7th, 9th, 11th, etc., components of the primary voltage may be passed on faithfully to the secondary side.

The secondary or line side of the step-up transformer bank is usually connected Y with grounded neutral. As in the case of the generator, this adds to safety and reliability of operation by limiting the voltage from line to ground definitely to the Y voltage. Thus far on our typical system we have on the line a good wave form of voltage if we started with a good one at the generator.

Step-Down Transformers

Again practise favors the Y-grounded connection for the line side of the step-down bank, in this case the primary side. With practically a sine wave of voltage applied, these transformers demand the harmonics required in the magnetizing current in order to produce the same type of voltage wave. Each transformer therefore will draw from the line, if the connections permit, current of 3rd, 5th, 7th, 9th, 11th, etc. harmonics of the fundamental frequency.

Taking the 3rd harmonic as of major importance in this analysis, it will be recalled that in a three-phase system the third-harmonic components, either of voltage or current, will be 3 times 120 or 360 deg. apart—in other words, in phase with each other. With the ground connection on the neutrals of both step-down and step-up banks, a path is provided for these in-phase components of the magnetizing current. The third-harmonic currents demanded in each leg of the step-down bank cannot pass out to the line through the other two legs of the Y, but they can pass through the neutral connection and ground to the neutral of the step-up bank, and through the Y-connected secondary windings of that bank to the line, thus finding a closed path. The delta-connected primary windings of the step-up bank allow rather free circulation of any third harmonic currents necessary to offset those demanded by the step-down bank. The "triples" thus find in that bank a low-impedance portion of their path, the rest of it consisting of the loop formed by the three conductors in parallel and the ground, and the bank demanding them.

The third-harmonic currents thus form the chief portion of the residual current. Other important components are the 9th, 15th, 21st, etc. harmonics, if the 3rd, 5th, and 7th harmonics were present in the voltage wave. The term "triples" applies to all such components.

Those harmonics of exciting current whose designating number is not divisible by 3, such as the 5th, 7th, 11th, 13th, etc., are supplied over the line, and like the fundamental, the resultant for each is zero. They, with the fundamental, form the balanced components of the current.

Practically, the voltage wave is not of perfect sine form; transformers are not always exactly alike; load is not always balanced; unequal leakage from the conductors to ground takes place; the capacitances from conductors to ground are not always equal; all of these factors contribute to add small amounts of fundamental and other odd harmonics to the residual current, but those mentioned, the 3rd, 9th, 15th, etc., predominate.

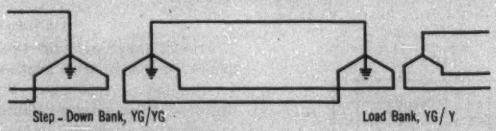

Fig. 5—Load Bank Connected YG/Y (Primary, Y-Grounded, Secondary, Y with Isolated Neutral)

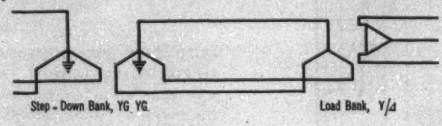

Fig. 6—Load Bank Connected Y/Δ (Primary, Y with Isolated Neutral, Secondary, Delta)

Load Bank

The load bank in Fig. 4 is represented as connected Y–Y with isolated neutral. In this type of connection, the triple-harmonic currents demanded by the exciting current of the transformers cannot flow as no path is provided for them. The deficiency of third harmonic in the magnetizing current appears as a third harmonic in the secondary voltage and distortion of the applied voltage wave. The same is true of the other harmonics which seek the neutral path,—the 9th, 15th, 21st, etc. As in the case of the currents, these harmonics combine into a residual voltage by which the isolated neutral differs from ground potential. This difference is superposed upon the various phase-wire voltages from ground, and there results a residual electrostatic field in the medium surrounding the conductor which varies with the frequency and resultant magnitude of these harmonics.

As in the case of the currents, those harmonics of voltage whose designating numbers are not divisible by

three,—such as the 5th, 7th, 11th, 13th, etc., appear on the different conductors, but their resultants are each zero.

If the load bank is connected Y–Y with the primary neutral grounded as in Fig. 5, the third-harmonic currents required for magnetization can come only from the step-down bank, through the conductors and ground return. In order to supply them, the step-down bank must draw upon the step-up bank with its delta-connected primary. Thus the load bank increases the residual current in the line back to the source which can furnish them.

If the secondary of the load bank is connected delta and the ground on the primary neutral is removed as shown in Fig. 6, no third-harmonic currents can flow in the primary, so the secondary voltage wave and the primary wave of back e. m. f. tend to become distorted. However, at the slightest appearance of a third-harmonic in the delta voltage, a third-harmonic current flows in the closed delta and supplies the deficiency in magnetization, so the voltage waves remain smooth except for the small amount of triple harmonic necessary to overcome the triple-harmonic impedance.

If the primary neutral is grounded, the secondary being delta-connected as shown in Fig. 7, nearly all the third-harmonic magnetizing current will be supplied by the secondary delta. Some, however, will be supplied to the primary over the line from the source through the step-down bank. The relative amount of third-harmonic current supplied by the secondary delta and the line will depend upon their relative impedances. The delta secondary path will, in general, have a very low impedance compared to the line path, so there will be very little residual current in or voltage on the line between the load bank and the step-down bank.

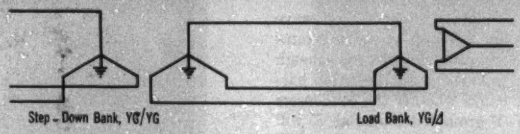

Fig. 7—Load Bank Connected $Y\,G/\Delta$ (Primary, Y-Grounded, Secondary, Delta)

Tertiary Windings

If separate windings are added to the transformers of the step-down bank and connected delta, most of the triple-harmonic currents required for the excitation of that bank and of the load bank will be supplied in the delta circulating current, so the line back to the step-up bank, (see Fig. 8) will be comparatively free of residual current. The division will depend upon the relative impedance. If the delta is of large kv-a. capacity with low impedance it will be more effective in this regard than if of small kv-a. capacity with high impedance, and as will be shown later, it may even be too large to be of maximum benefit.

Fig. 8—System Supplied from One Generator, Tertiary Delta in Step-Down Bank

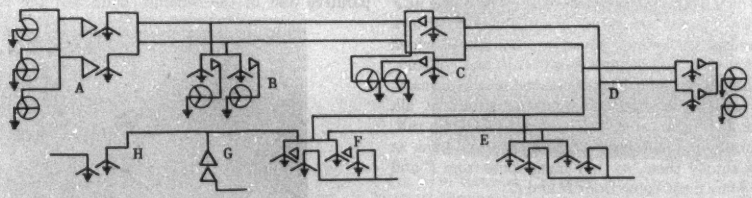

Fig. 9—Typical System Network

Power Networks

The usual power system is not so simple as the elemental one depicted in Figs. 4 to 8. Most systems have more than one power station, with more than one generator and transformer bank at each, feeding the system at different points and into multiple lines. When some of the transformers are connected Y-grounded on both sides, some with tertiary-delta windings, some Y-delta, some delta-delta or delta-Y, it is rather difficult to trace out fully the course of the residual currents, which is further complicated by variations due to system changes.

However, by analyzing the conditions from the funda-

mental principles just treated, at least the general features of any problem can be determined, and reasons deduced for any particular conditions encountered.

In Fig. 9 a more complex condition is presented. *A* and *B* are power plants not far apart feeding into twin transmission lines. At *C* is a station with step-down auto-transformer banks with delta-tertiary windings which are connected to synchronous condensers. Another power plant feeds in at *D*, and at *E*, *F*, *G*, and *H* are step-down stations serving communities and industries.

The delta-Y connected plants deliver a good wave form of voltage to the line. The Y-connected auto-transformers at *C*, if the tertiary delta is of low enough impedance, will furnish practically all the triple harmonics needed by those banks whether the condensers are on or not. The Y–Y grounded banks at *E* will require triples for their magnetizing current. They will draw them from the banks with deltas in them, at

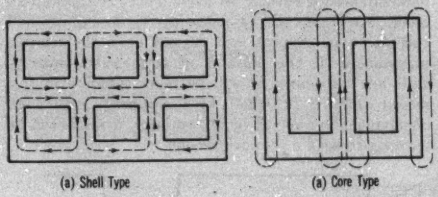

Fig. 10—Magnetic Circuits of Three-Phase Transformers

D and *C*, the relative amounts depending on the impedance of the paths to these sources. The banks at *F* with delta tertiaries will supply practically all their own triples unless the tertiary impedance is high, in which case part of them will be furnished from *D* and *C* also.

The bank at *G*, being delta-connected on the primary, will not affect the situation regarding triples at all. The bank at *H*, connected Y–Y with grounded neutrals, will require triple harmonics of current. The bank at *G* cannot supply them, so they must come from *F* and possibly some must come from *D* and *C*.

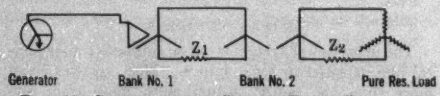

Fig. 11—Connections for Test of Triple Harmonic Currents to Load

If a power network is well supplied with banks having delta windings, the residual currents are reduced to low values. The most effective means for their suppression is to have the delta in the bank itself either as a delta connection on primary or secondary, or as a tertiary winding if it is a Y–Y bank. At a station where there is a number of banks, the use of such windings in one or two of the banks will serve for all practical purposes,—while the banks so equipped are in service.

Supplementary Y–Delta Banks for Suppressing Residuals

Some engineers have preferred to install supplementary banks of relatively small transformers connected Y-grounded on the primary and delta on the secondary

Fig. 12—Neutral Current Through Pure-Resistance Load When Voltage Wave Distorted.

Top curve is neutral current, 0.068 amperes. Middle: Y-voltage at load, 106 volts. Bottom: Load current, 1.67 amperes

side, where suppression of residual currents and voltages was required. Such a bank will do its share of the task of furnishing triple-harmonic components of magnetizing current, its effectiveness being in inverse ratio to its impedance compared to other available paths. In the individual case, the effectiveness desired and the comparative cost of the separate bank and the addition

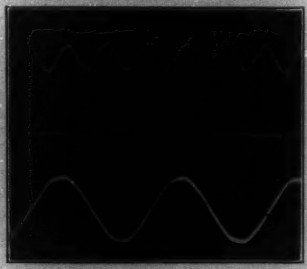

Fig. 13—Primary Neutral Isolated. Harmonics Supplied Through Non-Inductive Load and Secondary Neutral

Top curve is current in neutral from load, 0.205 amperes. Middle: Current in primary neutral 0 ampere (open). Bottom: Line voltage, 110 volts.

of tertiary windings on the main bank, including installation, will govern.

Three-Phase Transformers

A discussion of residual currents and voltages would

not be complete without mention of three-phase transformers. The shell type is quite similar in its characteristics to three separate transformers. In standard designs the middle phase is reversed in order to reduce the area of the iron magnetic circuit necessary between phases. Thus there is an independent magnetic circuit for each phase, and the triple-harmonic fluxes have short

Fig. 14—Primary Neutral Closed. Fundamental Only in Load Neutral

Top curve is current in neutral from load, 0.007 ampere. Middle: Current in primary neutral, 0.2 ampere. Bottom: Line voltage, 110 volts

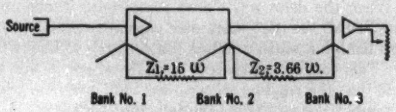

Fig. 15—Connections for Regulation of Residual Currents

Fig. 16—Neutral Currents with Delta Open

Top curve is neutral current, bank No. 2 to bank No. 3. Middle, neutral current, bank No. 1 to bank No. 2. Bottom, line voltage, 220 volts

and low-reluctance paths as shown in Fig. 10A. If the primary or secondary windings are connected delta they will furnish them, or if Y–Y with tertiary delta, part or all of the triple-harmonic fluxes required will be furnished by currents in the delta.

In the core type the magnetic flux of any one phase has its return path through that of the other two phases, so for the balanced components the resultant of the fluxes must be zero. However, any triple-harmonic fluxes due to the absence of triple-harmonic magnetizing currents are simultaneously in the same direction in

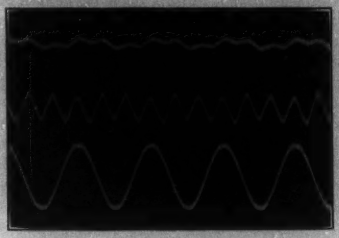

Fig. 17—Neutral Currents with Delta Closed Through Resistance

Top curve is neutral current, bank No. 2 to bank No. 3. Middle: Neutral current, bank No. 1 to bank No. 2. Bottom: Line voltage, 220 volts

each leg as shown by the arrows, Fig. 10B, so the magnetic circuit for them can be completed only through high-reluctance leakage paths through the air or case.

Laboratory Tests

In order to check some of the features outlined regarding banks of single-phase transformers, an interesting series of experiments was conducted in the labora-

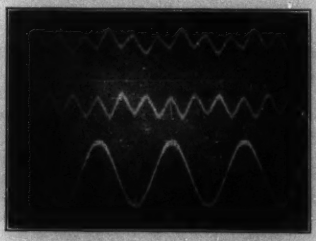

Fig. 18—Neutral Currents with Delta Closed Solid, Phase of Residual Current, Bank No. 2 to Bank No. 3 Reversed

Top curve is neutral current, bank No. 2 to bank No. 3. Middle. Neutral current, bank No. 1 to bank No. 2. Bottom: Line voltage, 220 volts

tory of the Pacific Gas and Electric Company at Emeryville, Calif., with three banks of 1-kw. transformers, using the 110-volt windings as primary and secondary and taping up the 2300-volt terminals, except that in some of the experiments the 2300-volt windings of one bank were used as a closed delta. By confining attention to the exciting currents the relative values of the harmonic currents were appreciably large. A number of conclusions was determined by the tests, which substantiated the theory presented.

A Y–Y grounded bank will not send out triple-har-

monic currents on the secondary side to pure-resistance loads if there is free circulation of the triple harmonics required on the primary side for excitation. If the latter are suppressed, causing triple harmonics of voltage on the secondary side, there wlll be triple-frequency components in the load current.

The connections for this test were as shown in Fig. 11. Bank No. 1 was connected delta-Y in order to eliminate any triple harmonics of the generator. The load consisted of a 200-watt lamp in each phase. Small air-core impedances were placed in the connections between neutrals to represent line impedance for the residual currents. With large values of impedance the wave of secondary voltage was distorted and a triple-harmonic current flowed in the neutral conductor between the load and the transformer bank serving it as shown in Fig. 12.

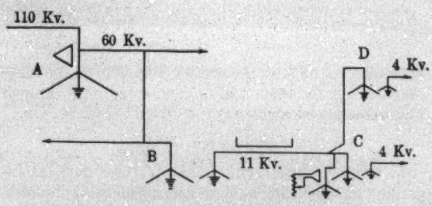

Fig. 19—Residual Currents Suppressed by Supplementary Bank

Still clearer results were obtained when the neutral was opened between banks and Z_2 reduced to zero by omitting the coil in the neutral between load and bank. In this case the triple harmonics demanded by the Y–Y bank were supplied through the secondary side and the load, as shown by the oscillogram of Fig. 13.

When the primary neutral of the Y–Y bank was closed through, (Z_1 and Z_2 both zero), the free flow of triple-harmonic currents was permitted on the primary side and the neutral from load to bank contained only fundamental-frequency current as shown in the oscillogram of Fig. 14.

Another matter determined was the action of a delta secondary (or tertiary) winding, in supplying a part only, all required, or an excess of triple-harmonic current over that required in the same bank. The connections for this test were as shown in Fig. 15, the transformers in the first two banks in this case being connected as auto-transformers, the delta in the first bank consisting of the 2300-volt windings.

With the delta of bank No. 3 completely open, the oscillograph was set so that one wave of each of the neutral currents and the 60-cycle line voltage were in phase as shown in Fig. 16.

By adjusting the resistance in the delta it was possible to reduce the neutral current to practically a fundamental-frequency wave. Owing to the impossibility of observing visually and taking an exposure at the same time with one oscillograph, Fig. 17 shows the neutral current between No. 2 and No. 3 banks with some triple-harmonic current superposed on a small fundamental-frequency wave, and the triple harmonics in the two neutrals are still in phase, *i. e.*, there are still triple-harmonic currents flowing from bank No. 2 to bank No. 3.

Fig. 20—Residual Current on Line, Delta of Supplementary Bank Open

Top curve is delta voltage. Middle: Residual current on line. Bottom: Reference or timing wave, 104 volts

When the delta was closed completely, it not only supplied all the triple-harmonic current required by its own bank but aided in filling the demands of bank No. 2. This can be noted from Fig. 18, where it will be seen that the residual current has completely reversed in phase between the second and third banks.

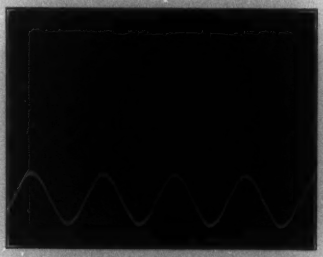

Fig. 21—Residual Current on Line, Delta of Supplementary Bank Closed

Top curve is delta current. Middle: Residual current on line. Bottom: Reference or timing wave, 104 volts

Field Tests

A very similar arrangement was set up in the field at a substation connected to a line which had given trouble from residual currents. The conditions were as indicated in Fig. 19. An auto-transformer bank at *A* contained a delta but at *B* the voltage was stepped down from 60,000 to 11,000 volts through a Y-Y bank with neutrals grounded. The 11,000-volt line supplies two stations also equipped with Y-Y banks transforming

from 11,000 to 4,000 volts. A parallel existed between B and C on the 11,000-volt line.

A supplementary bank of about 25 per cent of the capacity of those at C or D was installed at C, connected Y-grounded on the primary, and delta on the secondary side, with an adjustable resistance across one corner of the delta. With the delta open the triple-harmonic currents were supplied from A through bank B for the banks at C and D. It was possible to vary the resistance till the residual current in the line was approximately all fundamental, but due to system variations and lack of facilities to observe and make the exposure at the same time, no oscillogram of this condition was obtained. Figs. 20 and 21, however, show the neutral current with delta completely open and completely closed, compared with the same timing wave. A complete reversal can be noted, indicating that the supplementary bank, when the delta was closed, supplied the triple-harmonic currents required by its own primary windings, by the large bank at C and by the bank at D, and a portion of those required by the bank at B. This corresponds to the results obtained in the laboratory on the somewhat similar set-up shown in Fig. 15, in which the conditions were more easily controlled.

Conclusion

It appears evident that attention to fundamental principles of the electric circuit will enable the engineer to determine in general the course of residual currents in any given transmission system or network. Higher harmonics of currents are required in transformer magnetizing current, the suppression of which will give rise to higher harmonics of voltage. The higher harmonics follow definitely the ordinary law of alternating currents, namely, $I = E \div Z$, in each case.

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Discussion

N. C. Clark: I wonder if Mr. Corbett would tell us if he has done anything on higher harmonics. I know from oscillographic studies at the University of California that there is quite a large fifth harmonic introduced by the iron in transformers, sometimes amounting to as much as 40 per cent of the third. Of course the relations of three-phase circuits do not at present yield any obvious method of connection to neutralize or choke the fifth and seventh harmonics, but perhaps Mr. Corbett has done something on this or could suggest some method of suppressing them.

A. W. Copley: The test results indicated by Mr. Corbett for the circuits shown in Figs. 15 and 19 are of considerable interest from a practical standpoint. It has been generally accepted as good practise for star-star transformer banks to be equipped with a delta winding for the purpose of supplying the triple harmonics of magnetizing current required by the banks. There are, however, many transformer banks in service which are not provided with such a delta winding. When these banks have a grounded neutral the triple-harmonic current is obtained from other transformer banks which are connected star-delta. As the banks with the delta may be at a considerable distance from the star-star banks, the triple harmonic flows for a long distance in the ground and therefore gives rise to telephone interference.

Judging from Mr. Corbett's paper, the particular location of the star-delta bank supplying the triple harmonic seems to be quite important. Moreover, the impedance of the delta winding is also quite as important as the location of the bank. It is implied that, by properly proportioning the impedance of the delta winding and properly locating the star-delta bank, it is possible to reduce to a very considerable extent the magnitude of the triple harmonics which might flow in the ground at any particular location. The application of this idea can undoubtedly be successfully used in the prevention of the flow of triple harmonics along certain specific parallels. It is not, however, a general solution of inductive interference troubles from triple-harmonic currents.

D. I. Cone: Mr. Corbett has mentioned transformers and generators as causes of harmonics in power systems. A third important source is the electric arc, as in mercury-arc rectifiers.

To illustrate the importance of the factor of transformer saturation, it may be noted that a change of 1 per cent in the voltage on a transformer, as commonly rated, produces a change of from 5 to 15 per cent in the exciting-current harmonics. Thus a few per cent change in the voltage applied to the transformer has been observed to increase or decrease these harmonic currents by as much as one-half. In investigating a case of induction trouble, which increased at times of light load, it was found that the transformers responsible were being energized at a higher voltage at light load with corresponding increase of the triple harmonic residuals.

Under "Step-up Transformers" it is stated that the secondary voltage will be "free" of third-harmonic components where deltas of low impedance are provided. As a matter of practise, it is found that the star-star transformers give the maximum triples, while a low-impedance delta connection may reduce them to as little as 5 to 10 per cent. A technique was developed some years ago in connection with the California studies to which Mr. Corbett referred, by which approximate values can be estimated for the amounts of these triple harmonics. In studies that have been made, even on complicated systems of the kind shown in Fig. 9, a fair agreement between preliminary estimates and actual test results has resulted.

This method is described by Osborne in a discussion, (see page 2175, Trans. A. I. E. E., Vol. 34, 1915). Each transformer, as far as the exciting-current harmonics are concerned, is treated as an impedance in series with a generator, and with appropriate representation of leakage and line impedances.

The use of supplementary banks to absorb the triple harmonics of star-star banks has been explored considerably. In one case of a star-star bank with both neutrals grounded, and an exposure adjacent, the proposal was made to provide an auxiliary Y-delta bank to be connected in this way. The proposed bank was small, (about 20 per cent or less of the capacity of the Y-Y bank). However, it was pointed out that, having the Y-delta bank at the same station with the Y-Y bank, it was practicable to connect the neutrals of the two banks together, but isolated from ground. This procedure provided for the triples of the Y-Y bank and opened the circuit for triple harmonics along the exposure.

The proposal indicated in Fig. 19 of the paper, for controlling the triple-harmonic residuals in a particular exposure between

B and *C*, by means of a delta with resistance, has the effect of increasing the residual voltage and of causing increased flow of triple-harmonic residuals in other parts of the network. Such a scheme must be applied with caution to the particular case, because in improving one part, difficulty may be caused elsewhere. Observations of the induction also indicate that the adjustment is rather unstable so that variations of voltage and of switching arrangements in adjacent lines tend to upset the balance and increase the residual current.

R. E. Kistler: In Mr. Corbett's paper he has emphasized the importance of having the step-up banks connected delta-star. If they were connected star-star and grounded, the odd triple harmonics from the generator would circulate freely and would produce high inductive effects. Where, however, there are intermediate sections of transmission network connected directly to the generators having their neutral grounded, the capacitance to ground of the feeder network, arising either from long open-wire sections or cable sections, may provide sufficiently low impedance to permit the flow of enough residual current, due to the impressed residuals from the generators, to cause interference. In some such cases residual current or prominent harmonics thereof have been limited by the installation of the resistors or reactors or anti-resonant shunts in the generator neutral. In other cases, consideration has been given to the use of special design features in new generators to limit the triple harmonics in the phase-to-neutral voltage wave.

The oscillograms shown in Mr. Corbett's paper relative to the effects produced on the triples in the various portions of the setups shown on Figs. 15 and 19 clearly indicate the benefits to be derived by the use of star-delta banks. The results also show the importance of locating such star-delta banks properly so as to provide for reduction of the odd triples in the desired sections of the power network. The use of resistance in the delta winding of such banks is equivalent to the insertion of resistance in the neutral-to-ground connection on the primary side in so far as limiting the residual currents taken from the line by this transformer bank is concerned, with the exception that its own odd triple magnetizing currents, with the delta resistor, would of necessity then be supplied from the line in correspondingly increased proportions.

Roy Wilkins: I should like to point out in Mr. Corbett's paper a couple of the practical objections the operating engineer will meet when he applies such a system to residual-current correction.

Until 1910 exciting current on the average power transformer ran from 8 to 10 per cent. Some of the older ones that are still in service, of the older Stanley make that were built twenty years ago, run even higher than that at normal voltage. Unfortunately, those transformers are run many times at more than a normal saturation. They are, of course, the worst offenders toward putting out odd harmonics on the line.

The present transformers as manufactured run as low as 2 per cent of the full-load current. There have been cases in the last two or three years where banks as large as 50,000-kv-a. had less than 2 per cent. One case that I know of measured current at normal voltage of 1.4 per cent exciting current of the normal full-load current.

As you run up on the knee of the saturation curve, the percentage of exciting current increase for rise in voltage, and on many transformers a 10 per cent increase in voltage will increase the exciting current 100 per cent.

As you go up in exciting current, you also go up in the effect of the higher harmonics. If you get a heavy exciting current the effects of the seventeenth and twenty-first harmonics become apparent, and those harmonics are particularly detrimental for communication circuit. That was one point.

The second is that in applying a Y-delta bank, provision must a ways be made for transient troubles on the system. Whenever a Y-delta bank is put on a grounded system, the delta must be big enough to protect the bank itself. In other words, with one grounded phase that delta must be able to pass enough current to relay out the high-tension side.

One obvious method of correcting this is to open the delta, and in all cases that I know of where that has been tried, disastrous results were encountered. If you relay the delta open in a case of trouble on the high-tension side, you give rise to voltages in the transformer which probably will puncture the insulation.

One other thing it is well to keep in mind is that in all of these circuit connections we do not dispense with the harmonics of the primary wave. In other words, you only change from current to voltage or vice versa; connecting a bank up in a certain way does not eliminate the exciting current. It simply converts it from current to voltage. It then becomes a problem of deciding which is the most detrimental use for the bank to be put to.

J. O'R. Coleman: (communicated after adjournment) In this paper two very important points are brought out, which if given proper consideration in the planning of electrical supply could be taken care of with little or no inconvenience or expense. However, if these points are not properly considered initially the coordination problem may be difficult and expensive.

The transformer requires certain harmonics in the exciting current even though the impressed voltage is a sine wave. Fortunately, these harmonics are usually not excessive in transformers of good design operated at rated voltage. However, in order to obtain economy in construction the core is designed to operate at a high flux density, and as shown by Fig. 1 in the paper a relatively small increase in the flux density above the design value caused by increased impressed voltage above normal required a considerable increase in magnetizing force. This increase in magnetizing force results in a large increase in harmonics in the exciting current. Therefore, a relatively few large transformers on a system operating above normal voltage can materially increase the inductive influence of a whole system.

The paper not only shows how delta-connected windings can be used to limit the transmission of triple-harmonic exciting current over the lines but shows how and why the use of delta-connected windings may exaggerate the conditions that they are supposed to remedy.

A proper consideration of these two apparently small factors in both transmission and distribution will materially aid in the solution of a rather difficult and trying problem.

L. J. Corbett: Possibly if Mr. Clark had read the paper a little more thoroughly he would have noted my mention of the fifth and seventh harmonics. Those which are not triples are balanced exactly the same as the fundamental. The resultant of each becomes zero in any three-phase line. The residual is not absolutely free of them, however, because the value of the residual current is also influenced by leakage of insulators, by the difference in capacitance to ground of the conductors, and by things of that sort, which do bring in a little of the fifth and seventh and other odd harmonics, just as they do the fundamental. There is always some fundamental in the residual current. Because of the influence of the Y-Y and Y-delta banks on the triple harmonics, they were given the most attention in the paper.

I am glad Mr. Copley emphasized the point that it is not always best to have a delta in a transformer bank. Sometimes you have too much delta for your conditions, and you may get stray residual currents from other directions. It is, as has been pointed out by others, a matter for very careful consideration of every case.

Mr. Cone believes that the Y-Y banks are the worst offenders. I agree. He said further that the harmonics are reduced if the delta is present, but he never saw the delta which reduced them

to zero. I think the answer to that is in the reversals shown in the paper. There is a certain point at which the residual currents on the lines are reduced to zero, but due to changing system conditions they may flow back and forth. At a given instant for a certain setting they are zero. Then the slightest rise in voltage on the system throws them in one direction; the slightest lowering of voltage sends them in the other direction. This is due to the changing requirements for magnetization.

Mr. Cone referred to Mr. Osborne's discussion and to his method which is common among our telephone friends, in which these banks are called generators of residuals. To my mind, calling Y-Y bank a generator of residuals is like calling a hungry man a generator of a chicken dinner. It is a void; it is a vacancy; it is a suction which pulls these residual currents into the Y-Y bank. To consider such banks sources of residual currents seems to me a misnomer. This may be only a difference of language but it seems a real difference in thought, to the extent of 180 deg., as mentioned by Mr. Cone. Mr. Osborne's method is no doubt useful mathematically for certain purposes, but in this paper the endeavor was to present the physical facts.

Mr. Kistler stated that he would prefer to see the resistance inserted in the Y-ground rather than in the delta. I think there is a difference, because when a resistance is placed in the ground connection of the Y, there is an increase in triple-harmonic voltage as well as a reduction in triple-harmonic current. In using a delta you actually furnish some of the triple-harmonic current required for magnetizing, and you lessen the triple-harmonic voltage.

I was glad to have Mr. Wilkins call attention to the differences in values of harmonics with magnetization which were not fully emphasized in the paper. If we had run the saturation higher in our laboratory tests we would have introduced much higher values of harmonics and would have had a very much more distorted wave. I refer anyone interested to Mr. Frank's[1] excellent paper before the Institute in 1910. He used a number of transformers, singly and in banks, and took a number of oscillograph curves. He increased the excitation to as high as 200 per cent, which gave an extremely peaked wave of current, and brought in very large values of harmonics far up in the range.

1. John J. Frank, *Observation of Harmonics in Current and in Voltage Wave Shapes of Transformers*, A. I. E. E. Trans., Vol. XXIX, 1910, p. 809.

Power-Line Carrier Telephony

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Synopsis.—The inherent advantages of carrier-current telephony over high-voltage power transmission lines during the past five years have resulted in an unusually rapid development of this form of communication. To one who has not been constantly in touch with this rapid development, the necessity for the numerous types of carrier-current telephone equipment now available is not always apparent. This paper gives a brief outline of modern communication requirements in this field, the problems involved in providing this communication, and a brief outline of the different types of equipment now available to provide the required communication.

COMMUNICATION requirements may be classified according to their complexities as follows:

1. Requirements which may be fulfilled by the use of a single channel. A system meeting these requirements is analogous to a single telephone line connecting all stations on the system. The practical limit to which such a system may be extended is determined by the volume of traffic handled.

2. Requirements making necessary the use of several communication channels on the same transmission lines, to handle more traffic than can be satisfactorily cared for by a single channel.

FIG. 1—TRUNK-LINE CARRIER-CURRENT TELEPHONE TRANSMITTER—RECEIVER UNIT

3. Requirements making it advantageous to employ long "trunk" channels, each trunk station collecting traffic locally by private line extensions, or by secondary carrier channels from points not on the trunk channel.

4. Communication is often required between trunk or secondary carrier stations and isolated points, such as (a) patrolman's houses near the transmission line, (b) open air stations permanently located at isolated switching points, and (c) designated points for use of portable equipment.

In providing equipment to meet the communication requirements just outlined, many problems present themselves in connection with the operating requirements and the electrical characteristics of the transmission system.

The problems met in superimposing high-frequency currents on conductors carrying power at high potential may be divided under the two general headings of *Coupling* and *Transmission*. The problem of coupling high-frequency equipment to high-tension transmission lines was quite novel, and at the outset there was no experience available for guidance. Economic considerations naturally formed a large part of the problem, and are responsible solely for the development and design of several types of coupling capacitors, each type being best suited to a particular field of application.

The transmission of high frequencies over a power network involves very different problems from the transmission of low frequencies over the same network, or from the transmission of high frequencies over telephone conductors. It has been necessary, therefore, to develop a radically different technique for handling power-line communication problems.

The ideal line for the transmission of high frequencies would consist of a single high-voltage transmission line which has no directly-connected branch lines and which may be terminated at its ends in an impedance equivalent to its surge impedance. Such an ideal condition is very seldom realized or even very closely approached in actual practise. Branch lines produce two effects on a carrier communication channel; first, due to the fact that the branch line must be energized as well as the main transmission line a loss of some carrier energy is incurred; and second, a branch line gives rise to reflection. When the carrier-frequency voltage reaches the remote end of the branch line, it is reflected back toward the point of origin and arrives at that point with a phase relation (with respect to the original voltage) which will

1. Both of the General Electric Co., San Francisco, Calif.

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depend upon the length of the branch line and the frequency of transmission. This reflected wave may therefore either aid or oppose the original voltage. Fortunately, however, in most practical applications the two above effects are either too small to seriously affect communication, or may be avoided to a large extent by a proper selection of operating frequency. Where necessary, the effect of branch lines may be eliminated from the carrier circuit by means of high-frequency trap circuits.

Frequently, it becomes necessary to communicate over a transmission line which is broken permanently by a voltage transformation or frequency changer, or which may be opened by switches. In these cases, it is necessary to provide a means for transferring the carrier energy around the break in the power circuit.

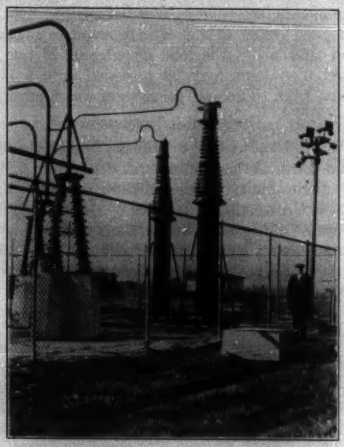

Fig. 2—Coupling Capacitors for Power-Line Carrier Telephone

The same problem is involved in transferring a communication channel from one transmission system to another.

In some cases, the communication distances involved are such that a power in excess of that supplied by the normal transmitter is required. This problem can usually be solved in either of two ways: by increasing the power of the transmitter until sufficient energy is received at the remote point, or by using carrier-current repeater equipment near the electrical center of the transmission system to raise the energy level of the received signals to their original value, and re-transmit them along the transmission system. The latter method has three distinct advantages; first, the total power required to transmit a signal over a long distance is less if a repeater is used than if the signal is transmitted directly; second, the repeater requires no abnormally large output from any of the transmitters and therefore does not increase interference between carrier channels nor greatly increase the probability of interference with broadcast receivers; third, the two remote communicating stations may be using entirely different carrier channels, the transfer in channels being made at the repeater.

It is evident from a consideration of these problems that an equipment designed to provide absolutely satisfactory service for transmission over telephone conductors might be entirely inadequate to provide communication over a power network.

The following description of the available carrier-current telephone equipments developed by the company with which the authors are connected will serve as an indication of the methods used in meeting the communication requirements and in solving the problems related thereto.

Among all types of apparatus complete intercommunication is provided, a single frequency band being employed for each two-way channel.

Primary Station Equipment

This equipment has been developed to meet the requirements on trunk channels. The necessary facilities for efficient and economical operation of trunk channels are provided in the following operating features:

1. Full automatic operation is provided from any operator's phone, the necessary procedure in placing or receiving a call being practically identical with that of a conventional wire-line automatic telephone system.

2. A maximum of ten two-wire telephone extension lines may be connected to each station equipment, each line having complete control of the carrier station and being selectively called from any carrier station on the same system. Provision may also be made to allow communication from other telephones connected with private branch exchanges, either manual or automatic.

3. Each incoming telephone line has exclusive and private use of the carrier equipment, and the conversation cannot be interrupted by a party on any other incoming line. An exception to this is that dispatchers' lines, or other designated lines, may be connected for preferred service, thus permitting the operators at designated telephones to interrupt a conversation on any other line and request the use of the carrier channel.

Parties on lines not connected for preferred service receive a "busy tone" if they attempt to place a call when the station equipment is in use.

4. Four-wire circuits may be employed if the noise level on a telephone line is so high as to make it unsuitable for a two-wire extension. When a four-wire circuit is used, the signal input to the line from the operator's telephone instrument may be raised to any desired level by means of simple amplifier equipment, and thus the ratio between signal and noise may be increased. The strength of the audio-frequency input to the carrier

set may be reduced by means of adjustments at the carrier set, so that when an amplifier is used at the operator's end of the line, satisfactory operation may be obtained, even though the noise level on the line is quite high. The service provided over a four-wire circuit is the same as for a two-wire circuit.

5. Provision is made for the application of "intersystem" attachments, to provide communication from the same station equipment over either of two separate carrier channels. Wire-line extensions from a station so equipped may receive or transmit calls selectively over either channel, the selection of channels being entirely automatic. While an operator is engaged in a conversation over one channel, he can receive calls over the other channel.

6. By means of a suitable attachment, high-tension telephone lines can be used for extension service, the operation secured on such extensions being the same as that for "quiet line" extensions.

7. Two primary equipments may be used together for repeating signals on very long channels, or for transferring signals from one transmission network to another. This service will be discussed under a later heading.

Secondary Station Equipment

Some communicating points do not require all the operating features provided in a primary equipment. A secondary station equipment has been developed to meet this condition economically. Four wires are required for connecting each telephone instrument to the set. When several telephone instruments are used, code ringing is employed. Full selective ringing may be provided by the use of one additional conductor between the set and each instrument.

The apparatus is designed to operate normally from 110-volt a-c. power supply, but may be made independent of the a-c. supply by the addition of a motor alternator operating from a storage battery. Where no a-c. supply is available, this storage battery may be charged from a gasoline-engine-driven generator.

Booth Equipment

This equipment is suitable for communication from isolated points where the communication distances are moderate. It is designed for operation either from dry cells or from a 110-volt a-c. supply. The equipment is mounted in a weatherproof case and when battery operation is used, may be installed on a tower or other convenient location exposed to the weather. This application is intended primarily for use by line patrolmen or service crews, and intermittent service only is provided. When this equipment is used with a-c. supply, full time operation is provided and it is suitable for installation in switching stations and line patrolmen's houses. The equipment includes a small 60-cell storage battery which provides emergency operation during failure of the a-c. supply. The communication provided by this equipment is simplex; that is, the operator employs a push-button on the hand set to change from "receiving" to "transmitting" position.

Portable Equipment

This equipment is designed to provide communication between any station equipment and fixed points along the transmission line within a radius of approximately 30 mi. It is a self-contained and extremely compact transmitter receiver unit, weighing approximately 15½ lb., and provided with shoulder straps to facilitate carrying. The communication provided is simplex.

Intersystem Equipment

This equipment is designed to operate as an attachment to the primary station equipment, permitting the operation of such an equipment on either of two communication channels. A load dispatcher may thus communicate selectively over either his own communication system or that of an interconnecting system. As an alternative, the load dispatchers of two interconnected systems may be provided with a channel separate from those used for system communication.

By-Passing Equipment

When it is necessary to transfer the carrier energy around a transformation, open disconnecting switch, or other discontinuity in the carrier circuit, a by-pass equipment consisting of a series of resonant circuits is employed. Since no vacuum tubes or moving parts are employed the equipment requires only periodic inspection and is therefore suitable for tower mounting at isolated or unattended points.

Carrier Current Repeater Equipment

In discussing the primary equipment, mention was made of the fact that two carrier sets may be used together to comprise a repeater assembly. Such an equipment may be installed to serve two distinct purposes: First, it may be used to raise the energy level on a carrier channel and therefore permit much greater communication distances; and second, it may be used to connect two otherwise independent communication channels and permit either completely independent operation of the two channels or complete intercommunication between stations on the two channels, as desired.

When operating as a normal repeater, the equipment functions as follows: The incoming modulated carrier is received and demodulated in the normal manner. The signal component is then amplified and impressed upon the transmitter operating over the outgoing channel. The apparatus is so designed that a signal which has been received and re-transmitted by the repeater cannot pass back through the repeater to the side from which it was originally received. This is accomplished by apparatus so designed that when one of the transmitters begins to function, the output circuit of its associated receiver is blocked. All tendency toward "feed-backs," which would result in oscillation

or "singing" of the repeater circuit, is thereby eliminated. The operations for repeating the signals are identical for both directions.

The repeater equipment is in the operative condition only while actually in use. It is started automatically from a calling station as follows:

An operator desiring to communicate through the repeater station first dials the "repeater exchange" number. This number serves to start the motor-generator set and place the repeater in operation. Any further numbers dialed by the calling operator are repeated over the outgoing channel. The ensuing conversation is similarly transferred from one channel to the other.

When the repeater is placed in operation by an incoming call, a time delay circuit is energized to maintain the apparatus in an operative condition for a definite time interval after the last signal is repeated. At the end of this time interval the repeater apparatus is automatically shut down, thus effecting a saving in operating cost and increasing the operating life of the apparatus to that of a normal station equipment. The time delay feature is provided to prevent the repeater shutting down during pauses in a conversation. The length of time delay is adjustable within wide limits to suit local requirements.

The operator at the repeater station may be provided with automatic selective control, permitting communication over any channel selectively, or over two channels simultaneously. He may monitor the conversations on either side of the repeater or passing through the repeater, or may enter into these conversations.

The operation of a repeater in connection with an intersystem attachment is the same as that of a normal repeater except that the "repeater exchange" number, in addition to placing the apparatus in service, selects the proper outgoing channel.

High-Tension Telephone Line Extension

As an extension to a primary station equipment, it is sometimes desired to use a high-tension telephone line; that is, a telephone line so situated in the proximity of a high-voltage transmission line that the extraneous voltages induced in the telephone line are relatively great. Such a line is unsuited for direct connection to a primary carrier set. In order to eliminate the effects of power frequency induction, a special form of extension equipment may be employed on any high-tension telephone line over which successful voice-frequency communication may be provided. This type of extension employs carrier-frequency transmission from the remote operator to the primary carrier set, and voice-frequency transmission from the set to the operator.

The use of a high-frequency carrier for transmission from the remote operator to the station apparatus makes it possible to eliminate power-frequency voltages from the input to the station transmitter, by means of filters which do not pass the low frequencies. The use of these filters also makes it possible to separate the incoming from the outgoing signal without the use of balancing networks such as are used with voice-frequency telephone repeaters. The amplification afforded by this type of extension apparatus is such that the length of telephone line that can be used is much greater than in the case of the usual two-wire extensions as designed for quiet lines.

The telephone line used in connection with a high-

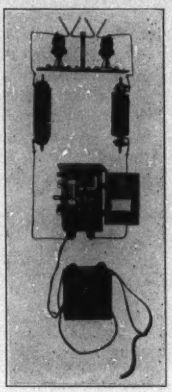

Fig. 3—Portable Carrier-Current Telephone

With connection box and protective equipment for coupling unit installation.

tension extension does not necessarily constitute a complete d-c. circuit, and can be used, therefore, with the normal insulating and drainage transformers and protective devices. The protection afforded the operator from voltages induced on the telephone line by power circuits or lightning is even better than that provided when employing such a telephone line in the normal manner.

Multiplex Carrier Channels

Because of their peculiar advantages, the demand for additional uses of carrier current channels is growing. These are for relaying sections for line, distant control of switches, telemetering, and frequency transmission. Fortunately, a pair of coupling capacitors may be used for the simultaneous transmission of more than one carrier frequency. Lines using capacitors for carrier telephony at the outstart are now being equipped with these later developments.

The number of additional channels it is possible to operate without interference is limited, and it is obviously most desirable to employ but one frequency per channel. The development of the single-frequency duplex-carrier telephone some years ago has been most helpful in this regard.

New Uses for Carrier Current

In general, a carrier-current channel can be made to accomplish what would otherwise require a separate pair of wires, without the investment and maintenance charges of the latter. When the distances involved are sufficient these factors result in distinct monetary and operating advantages in favor of the carrier channel.

New applications are arising daily. An interesting example is the case of a large pumping load supplied by one power system, but scattered over a large territory. These motors may be controlled at will by carrier current over the distribution system, thereby making it possible to reduce the peak and improve the load factor.

In another instance, it is desired to transmit the frequency of one generating station to another, and carrier current offers a ready solution. There are, no doubt, numerous other operating situations wherein carrier-current methods present the best engineering solution.

Problems in Power-Line Carrier Telephony

And Recent Developments to Meet Them

BY W. V. WOLFE* and J. D. SARROS*

Member, A. I. E. E. Non-member

Introduction

POWER-LINE carrier telephone systems have been in operation in this country for the past six years.

This form of communication may be used either as a primary means of telephone service for the transaction of dispatching and other business, or it may be employed chiefly as an emergency service to supplement facilities the power company already has at its disposal.

To date, several types of equipment have been designed to meet these needs. However, the growing complexity of power systems, involving a higher standard and more comprehensive communication service, indicates the desirability for a better type of carrier circuits and equipment. The purpose of this paper is to outline in general the difficulties to be met in applying high-frequency communication to power systems, and to describe a recent development in equipment for this class of service.

The Problem

The problem of providing high-frequency communication over power circuits must first be considered from the angle of the general purpose of this type of communication. Formerly, power-line carrier telephone circuits were considered primarily as channels for load dispatching business. While the primary use is for load dispatching work, there is also considerable demand for such circuits to be used for general business transactions. For load dispatching service only, the circuit may be an isolated one, providing communication from the dispatcher to one or more generating or sub-stations, but this may not meet load dispatching requirements and is rarely satisfactory for general business. It therefore becomes desirable to make the carrier circuit a coordinated part of the entire communication system of the power company.

Considering the general circuit requirements we find that these in general have not changed. Stable, reliable circuits of good quality and with low noise have been required from the start. One factor which perhaps has increased in importance is the requirement that the equipment itself be designed for sufficient flexibility to permit of adapting it to all the different conditions encountered on power circuits. Another factor which, while not previously overlooked, has risen to considerable prominence recently, is the necessity for multiple channel operation on any given power circuit without interference between channels.

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Character of the Transmission Medium Presented By a Power Network

In designing apparatus to provide communication service, the most important consideration is the character of the transmission medium to be employed for the communication channel.

Earlier investigations[1] showed the superiority of a full metallic circuit over a ground return circuit both from a standpoint of attenuation and stability as well as noise. Over the range from 50 to 150 kilocycles the attenuation of the power line without branch lines or connected apparatus has been found to be of the order of 0.05 to 0.1 $T\,U^2$ per mi. This loss is very low and indicates the possibility of securing excellent com-

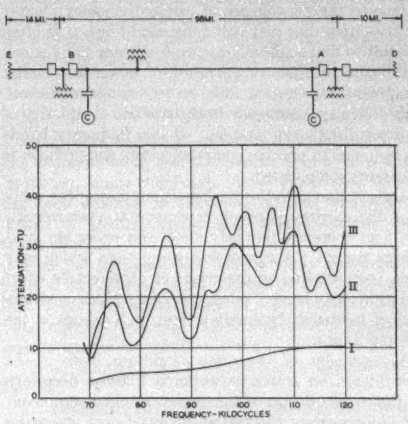

Fig. 1—Effect of Line Switching on the Attenuation Characteristic.

munication under such conditions. Unfortunately, power circuits are not ordinarily simple straightforward circuits but involve branch lines, closed loops, and other irregularities which radically change their character as a transmission medium.

As a concrete example of these facts there is shown on Fig. 1 a power network involving a 110-kv. line with carrier terminals located at A and B, a distance of about 98 mi. Fig. 1 also shows the attenuation frequency characteristics made on this line under three different conditions. Curve I gives the characteristic of the line $A\,B$ which was, for this test, entirely isolated

1. For references see Bibliography.

28-124

from the system except for a transformer bank 23 mi. from *B*. Curve II shows the characteristic of the same line with the 14 mi. circuit *B E* and its associated equipment connected. It is evident that the introduction of this circuit not only increases the average attenuation of the line *A B* but also introduces serious irregularities caused by reflection and absorption effects. Thus, in a 5000 cycle interval, there is, under this condition, as much as 10 *T U* variation in attenuation. Curve III shows the characteristic of this same section *A B* with both *B E* and a 10 mi. branch *A D* with its associated equipment connected in at the other end. Again we have an increase in average attenuation. The irregularities due to reflections are now so pronounced as to give a 20 *T U* variation in attenuation over a 5000 cycle interval. There may be no similarity between the last two conditions; that is, the peaks and troughs in attenuation may not occur at the same frequencies. Thus, we not only have *irregularities* in the high-frequency characteristics but *instability* as well.

Due to the dependence of carrier-frequency characteristics upon line and switching conditions, it is often difficult to find a suitable carrier-frequency band which will insure satisfactory operation of power-line carrier equipment for several line or switching conditions. This difficulty increases with the size and complexity of a power distribution system. If two frequency bands are required to provide communication, the problem is even more complicated.

A slight change in the line conditions may appreciably alter the carrier-frequency transmission characteristic and may therefore seriously affect the operation of a carrier system. A carrier-frequency band which may have been selected as the result of considerable effort and which may be very satisfactory for a given condition is often rendered absolutely useless by a change in the line conditions.

A knowledge of the noise conditions likely to be encountered on power networks is likewise necessary for the correct design of the carrier terminal equipment but unfortunately the factors involved here are many and of varied character. Among these sources of high-frequency noise may be mentioned corona and leaky insulators. The latter trouble may be due to faults in the insulators or to collection of dust or other foreign material on the insulators. A prolific source of noise has been found in the spitting or arcing which occurs between the power conductor and metal objects close to the conductor but not connected to it. For example, on a particular power system the carrier installation was started and completed during a relatively wet season and after the installation the circuit became excessively noisy. The source of this noise was traced to 12 lightning-arrester choke coils of the strain type. In each of these choke coils there was a number of iron bolts employed in the reenforcing of the coil but not in contact with the conductor. During the dry season there was a continuous spark from the closest point of the power conductor to each one of these bolts. Bonding these bolts to the conductor removed this noise entirely.

The presence of corona on a power circuit introduces a very interesting possibility from the standpoint of noise production. Considerable light is thrown on this subject by the researches on corona by Dr. Harris J. Ryan[3] and his associates. Dr. Ryan has disclosed the existence of a space charge surrounding a conductor in corona. This space charge results in an effective change, at periodic intervals, in the electrical diameter of the conductors. Effective conductor variations occurring as indicated by Dr. Ryan would cause the attenuation of the power circuit to vary and thereby produce modulation on the line itself. The modulation between the carrier frequencies as a function of this varying attenuation will result in noise after the carrier signal has been demodulated. If such is the case, the amount of noise observed in the voice frequency range would increase directly as the transmitter power in the carrier circuit was increased and it might therefore develop that the operation of a carrier system on such a power circuit could not be insured by the simple expedient of increasing the transmitting power.

It is evident from these observations that the successful operation of carrier communication over power lines does not depend alone on the development of the carrier apparatus. The transmission line is a very important link in the system and can vitiate the performance of otherwise perfect equipment.

Fundamental Circuit Considerations

Bearing in mind the service requirements and the character of the transmission medium, as briefly described, we may now consider the fundamental types of circuits available for high-frequency communication on power lines.

It is scarcely necessary to mention simplex or manual switching from the transmitting to the receiving condition as a method of operating these circuits since such a scheme is very cumbersome and does not permit coordination with other telephone circuits. Full duplex operation can be obtained by using two independent pairs of wires or circuits, one circuit for transmission in each direction. This method is highly satisfactory but in this case is uneconomical since it requires two independent circuits for a given conversation. To obtain duplex or automatic two-way operation when one circuit is used for both transmission and reception, means must be provided for separating the transmitted and received currents. This is generally done by using any one or a combination of three general methods:

1. Frequency separation.
2. Balance or bridge method of separation.
3. Use of voice controlled relay circuits.

Frequency separation involves the use of two frequencies, one for transmitting in each direction. Selection between the two is obtained by the use of

selective circuits or filters of high transmission loss to the unwanted frequencies and low loss to the desired frequencies.

Separation by the use of balance or bridge methods is extensively used in the communication art where the lines are electrically long and their impedances remain substantially constant. In general, the transmitting circuit may be connected across one diagonal of an a-c. bridge and the receiving circuit across the other. The transmission line is connected to one arm and the bridge standards or network to the other arm of the bridge. So long as the bridge standard, or network, matches the impedance of the line at each frequency used, no energy will be transferred from transmitter to receiver.

An automatic two-way circuit which in its operation is essentially a duplex circuit may be secured by the use of voice controlled relay circuits. These circuits may employ mechanical relays or vacuum tube circuits for their operation. In either event the transmitting circuit is rendered inoperative during the receiving interval and conversely the receiving circuit is inoperative during the transmitting interval.

Where the voice controlled relay circuits are used the same band of frequencies is normally employed both for transmitting and receiving although this is not a fundamental requirement of the system. The balance or bridge method of separation may also be employed on a single frequency circuit.

Perhaps the most important consideration is the type of high-frequency transmission to be employed. There are three commonly used systems:

1. Systems transmitting the carrier and both side-bands.
2. Systems transmitting the carrier and one side-band.
3. Systems transmitting one side-band only.

When two frequencies, such as a speech frequency and a carrier frequency, are properly introduced into the ordinary vacuum tube modulator, the products of modulation consist of the sum and difference of these two frequencies, called respectively the upper and lower side-band, and the carrier frequency, besides other modulation products which may be neglected in this discussion. The demodulator performs the same functions as the modulator but in reverse order; that is, it receives the two side-bands and the carrier and resolves them so that one of the products of demodulation is the original speech. The two side-bands are each of the same frequency width as the original speech. Furthermore, these side-bands are symmetrical in position with respect to the carrier frequency and each side-band by itself contains all the elements necessary to transmit the original speech.

Obviously, therefore, it is not necessary to transmit both side-bands with the carrier since the original voice frequencies may be faithfully reproduced by demodulation between the carrier and either the upper or the lower side-band. Furthermore, the carrier is a constant frequency whose amplitude need not vary with the amplitude of the voice. The carrier may be suppressed at the transmitting terminal after modulation has taken place, and be re-introduced at the receiving terminal for demodulation.

The system transmitting the carrier and both side-bands is that employed for broadcasting and prior to this time for all power-line carrier telephone circuits. This type of system requires a frequency band of approximately 5000 cycles in width for the transmission of speech having reasonably good quality.

A system transmitting the carrier and one side-band has essentially the same features as that transmitting the carrier and both side-bands except that the band width may be reduced to approximately 2500 cycles for the same quality.

The third system in which the side-band alone is transmitted requires supplying the carrier locally to the receiving circuit. This system also employs a frequency band of 2500 cycles for comparable quality.

Two general methods of coupling to the power line are available. These are antenna coupling and condenser coupling. The present tendency seems to be so largely toward the use of condenser coupling that it is scarcely necessary to discuss their respective merits.

Characteristics of the New Power-Line Carrier Telephone System

From consideration of the service requirements, the line characteristics, and the circuits available, a single frequency, single side-band, carrier suppressed system has been adopted as the most efficient and reliable means of getting satisfactory communication over power circuits.

With the foregoing as a basis we may now consider some of the improvements effected by this new power-line carrier telephone system as compared first with a two-frequency system, and second with a single-frequency carrier transmitted system.

Its advantages over a two-frequency system are as follows:

1. Less likely to be affected by the irregularities and instability of power-line high-frequency characteristics.
2. Transmission loss in each direction is the same.
3. Party line conversation practicable.

Carrier suppression and the use of a single side-band gives it the following additional advantages.

4. More easily adaptable to single-frequency operation.
5. Gives better quality through irregular line characteristics.
6. Ordinary line noise and noise due to any line modulation is more effectively subordinated.
7. Frequency conservation makes multi-channel operation easier.
8. Received speech level is less affected by line attenuation changes.
9. It has a greater transmitting range for the same power.

10. Lower transmission loss and greater selectivity secured in coupling to power line.

The question of the irregularity and instability of the line attenuation characteristic and its effect on the operation of any system has already been referred to. Although the use of the same band of frequencies for transmission and reception simplifies the problem, this advantage is enhanced by the use of a single side-band system since the required frequency band is a minimum. This is a distinct advantage because cases have been met where the peaks in the frequency spectrum were extremely sharp and only one frequency band offered good operation under all line conditions.

It is obvious that the transmission in each direction with any single-frequency system is the same.

Party line conversations are not practicable with a two-frequency system, although it permits "broadcasting" instructions to all terminals on the line. With a single-frequency system, instructions may be issued to all operators simultaneously, and a general discussion can be carried on between terminals.

Single side-band transmission simplifies the problem of obtaining automatic two-way operation by the use of voice operated vacuum-tube relay circuits since there is no output from the transmitter so long as there is no input into the microphone.

Single side-band transmission gives better quality through irregular line characteristics by virtue of the fact that the band width is narrow.

It is apparent that a system with such a high degree of selectivity will be less sensitive to interference either from line noises or from other carrier channels. Conversely, since a very narrow band is transmitted, the possibility of its interfering with other systems or channels is reduced to a minimum. There is the added advantage that interference from noise as a result of possible line modulation is reduced.

Since in single side-band transmission the frequency band is only half as wide as is required with double side-band transmission, the conservation of frequency spectrum resulting thereby facilitates multi-channel operation. Because of the use of balanced double demodulation and the fact that the transmitted band is reduced to a minimum, a very high order of selectivity is obtained.

It can be shown that the amplitude of the received signal is a function of the amplitudes of the carrier frequency and of the side-band frequency. For this reason, variations in the transmission loss may cause a greater change in the amplitude of the received speech of a carrier transmitted system than of a carrier suppressed system. For a carrier transmitted system, the amplitude of the received speech may vary at a greater rate than the changes in the transmission loss, since the amplitudes of both carrier and the side-band are correspondingly varied. For a carrier suppressed system, the amplitude of the received speech changes in the same ratio as the transmission loss, since only the amplitude of the side-band is affected and the amplitude of the locally supplied carrier remains constant.

Assuming the maximum of 100 per cent modulation for a system in which the carrier and both side-bands are transmitted, 50 per cent of the transmitted energy is associated with the carrier. For ordinary modulation we can say that 100 watts of single side-band energy is approximately equivalent to 500 watts where the carrier and both side-bands are transmitted. Thus, by suppressing the carrier and transmitting one side-band only, we increase the efficiency of the system by at least 400 per cent.

The fact that a narrow band is transmitted makes possible the use of relatively low capacity coupling condensers and consequently, a coupling filter of high efficiency and maximum selectivity can be designed. Additional selectivity through filters is provided at the input of the first demodulator, between the first and second demodulator, and at the output of the latter.

Description of Circuits

At the risk of repeating much that has already been written[4] we shall now discuss in general the fundamental principles underlying the operation of this new power-line carrier telephone equipment. The satisfactory performance of a single side-band carrier suppressed system depends to a great extent on the solution of the following problems:

1. The sufficient suppression of the carrier at the transmitting end.
2. The elimination of the undesired side-band.
3. The design of high stability oscillators.

From the standpoint of fundamentals, it is obvious that the incomplete suppression of the carrier and undesired side-band and their reception by the distant demodulator, will not cause distortion if the "carrier leak" of the transmitting station and the carrier supplied at the receiver are in synchronism. The oscillators used to supply high frequency for modulation and demodulation are of extremely high stability and have proved very satisfactory. However, synchronism cannot be realized in practise and measures are taken for suppressing to the desired degree the carrier and unwanted side-band in order to circumvent the slight discrepancy between the modulating and demodulating carriers. Balanced or push-pull modulators, as employed here, suppress the carrier to a considerable extent. Further attenuation is obtained by locating the frequency of the carrier at a point beyond the cut-off of the attenuation characteristic of the band pass filter used for eliminating the undesired side-band. Any carrier that may be transmitted under these conditions is further prevented from causing trouble by the use of balanced demodulators.

Another interesting phase in connection with this problem is the use of double modulation and demodulation. Although this scheme of modulating twice provides added selectivity, the reasons that lead directly

to its adoption are as follows. At power-line carrier telephone frequencies, that is, the band from 50 to 150 kc., the manufacture of suitable band pass filters to suppress the unwanted side-band is very difficult and correspondingly expensive. Assume, for instance, that a 2000-cycle tone modulates a 100-kc. carrier frequency. The resulting side-band frequencies are 98 kc. and 102 kc. It is difficult to build a selective circuit to select one of these frequencies to the complete exclusion of the other. The problem becomes still more difficult at the lower voice frequencies as, for instance, 400 cycles. In this case, the resulting side-band frequencies are 100.4 kc. and 99.6 kc. However, if double modulation is employed, the side-bands can be given a wide frequency separation, thereby greatly facilitating the selection of one side-band to the complete exclusion of the other and of the carrier frequency. Assume that a 2000-cycle tone modulates a 30-kc. carrier frequency. The resulting side-band frequencies will be 32.0 kc. and 28.0 kc. These can be separated by a commercial band pass filter. Further assume, that the lower side-band, 28.0 kc., is selected and modulates a 100-kc. carrier frequency. The resulting side-bands will be 128.0 kc. and 72.0 kc. Either one of these side-bands can be separated from the other by comparatively simple selective circuits. It is also obvious that the side-band frequency that is put on the line can be varied to suit a particular line by merely varying the second carrier frequency and leaving the first carrier frequency fixed. Thus, double modulation provides a convenient and simple means for obtaining the advantages of a single side-band suppressed carrier system at power-line carrier telephone frequencies.

It is not necessary to employ double demodulation, when double modulation is used. The original message may be obtained from the side-band by supplying the proper carrier frequency, which is equal to either the sum of or the difference between the two modulating frequencies, depending upon whether the upper or lower side-band is transmitted. For instance, in the case above, if the 72 kc. side-band is demodulated by a 70 kc. carrier frequency, the resulting side-band will be 2000 cycles, which is the original tone. Such an arrangement, however, requires at least 3 carrier-frequency generators at each terminal, two to supply the modulating frequencies and one to supply the demodulating frequency. If double demodulation is employed in a system using the same band of frequencies for transmission and reception, the same carrier frequencies may be used for both modulation and demodulation but in the reverse order.

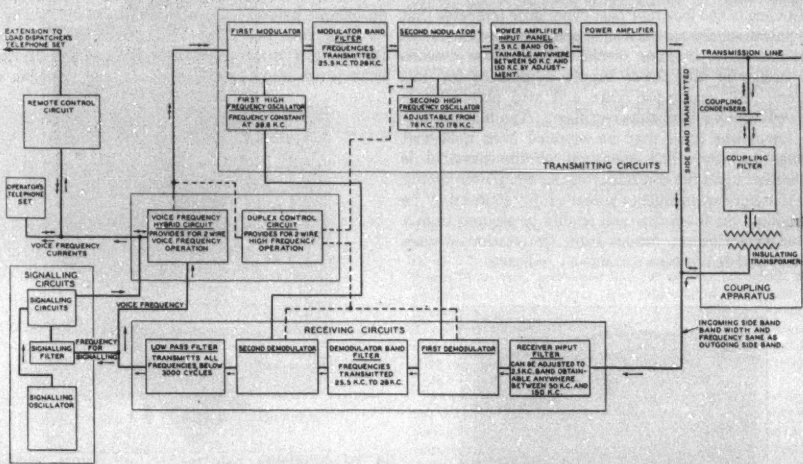

FIG. 2—BLOCK SCHEMATIC OF SINGLE-FREQUENCY DUPLEX POWER-LINE CARRIER TELEPHONE SYSTEM EMPLOYING SINGLE SIDE-BAND SUPPRESSED CARRIER TRANSMISSION

In the case of the system under consideration, duplex or automatic two-way operation is obtained by the use of voice operated vacuum tube "relay circuits." This is effected by rendering either the transmitter or receiver inoperative by causing the voice itself to control a heavy negative grid bias on the modulator and demodulator tubes. There is also provided an interlocking device which prevents incoming energy or room noise from rendering the transmitting circuit operative while receiving. Essentially this same scheme is employed in the transatlantic radio circuit.[5] It has also been in

use for years in long cable toll circuits where echo suppressors[8] have been found to be indispensable.

The block schematic of the system, shown in Fig. 2, indicates how voice or signaling currents are modulated twice, before being amplified and passed to the transmission line; also how the incoming high-frequency currents are demodulated twice before being passed to the voice-frequency line. This diagram indicates the use of the same oscillators for the modulators and demodulators. The duplex control circuit which provides for single-frequency operation, and the voice-frequency hybrid circuit which makes possible the extension of the communication service on a two wire basis to a distant point, are other features of this system. Two-wire remote control may be provided on these circuits.

In view of the fact that past experience indicates that the power supply has furnished one of the main sources of maintenance troubles, special attention was given to the design of the power supply equipment for this system. All power for plate and grid potentials is derived from vacuum-tube rectifiers. The filaments for the low-power tubes may be supplied from a 30-volt storage battery. The operation of the terminal is dependent upon the continuity of the a-c. power supply and therefore an emergency power supply source may be desirable. Such a source can readily be secured from a motor-generator set driven from the station storage battery and delivering a suitable a-c. voltage.

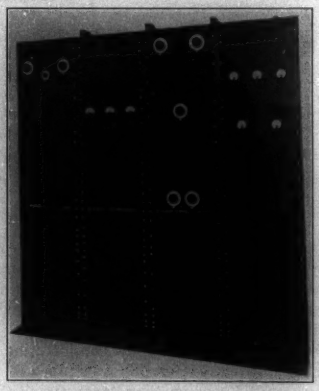

Fig. 3—Front View of Equipment

Fig. 3 shows a front view of the equipment. Switchboard type of mounting has been followed in that all apparatus with the exception of meters and controls is mounted on the back of the panels. The rear view, Fig. 4, shows the locker type of back cover that has been adopted for this equipment. Fig. 5 shows the 220-kv. condensers used to couple this equipment to the line. The box between the two bus towers near the foreground, houses the coupling filter. This filter may be adjusted to pass efficiently a frequency band of 2500 cycles anywhere in the region of 50 to 150 kc.

Initial Installation

Three terminals of this new equipment are now in operation on the lines of the Pacific Gas & Electric Company. They have provided a communication service over power circuits which have presented very serious communication difficulties to other apparatus

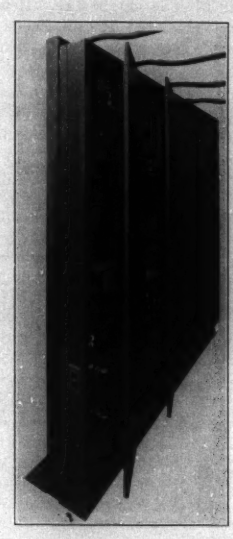

Fig. 4—Rear View Showing Locker Type of Back Covers

previously tried. The high-tension circuit over which this equipment operates, consists of the 202-mi., 220-kv. lines from Pit No. 1 Power House to Vaca Dixon Substation and of the 58-mi., 110-kv. lines from this point to the Claremont Substation in Berkeley. From Claremont to the load dispatcher in Oakland there is a five mile two-wire low frequency extension of underground cable with remote control at the dispatcher's office. The voice-frequency circuit may be extended from the dispatcher's office or any of the carrier terminals by connection to other two-wire telephone circuits.

There are three points of particular interest in this installation. First, the 58-mi. section is part of one of the largest 110-kv. networks in the country and potentially therefore subject to wide changes in its high-frequency characteristics. Second, the transfer of the

high-frequency energy from the 110-kv. line to the 220-kv. line, and vice versa, is accomplished through a by-pass or filter circuit without the use of a repeater. Third, this particular 220-kv. line presents an unusually difficult problem because of the large amount of high-frequency noise present. This noise is probably produced from two major sources—corona and static discharges between the line conductors and the corona shields.

Conclusion

The terminal equipment described in the foregoing discussion represents a distinct advance and employs the circuits and apparatus best suited for our present conception of the service requirements and our present knowledge of power-line conditions at carrier frequencies. It must be remembered, however, that power lines as at present constructed and operated present an uncertain medium for communication and that although this equipment overcomes to a large extent these difficulties, the effectiveness of power-line carrier

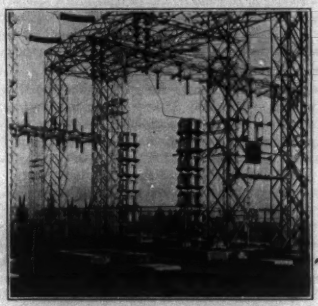

Fig. 5—220-Kv. Coupling Condensers

telephony would be considerably enhanced by an improvement in the transmission medium.

Some work has been done on methods for stabilizing power-line conditions at high frequencies but so far the results have been meager and the apparatus developed has not met with general approval. Considerable work remains to be done on this problem. Its satisfactory solution will materially increase the effectiveness of power-line carrier telephone circuits. These improvements, of course, will in no way detract from the superiority of the single-frequency, single side-band carrier suppressed system herein described.

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6. *Echo Suppressors for Long Telephone Circuits*, by A. B. Clark and R. C. Mathes, Trans. A. I. E. E., Vol. 44, 1925, p. 481.

Discussion

L. M. Ilgenfritz: It may be of interest to compare a few of the major problems arising in connection with the development of power-line carrier systems with those of carrier systems for telephone lines such as are described in the paper[1] presented at the recent Denver convention by Affel, Demarest, and Green, and the paper[2] presented at this sesssion by Black, Almquist, and myself.

The earlier developments in carrier systems used in the Bell System passed through several stages in which commercial trials were obtained of both the carrier-transmitted type and the carrier-suppressed type of system. The superiority of the carrier-suppressed systems was so marked that the carrier-transmitted type has become obsolete and is no longer manufactured.

In providing telephone service, it is of course essential to provide for ultimate growth of service and for that reason it is desirable to design such carrier systems so that a maximum number of channels can be ultimately provided on a pole line. The limitations in providing additional service on telephone lines are found in the characteristics of the line circuits themselves. Inter-system interference, controlled by transposition design, of course, limits the number of channels which can be placed on a pair of wires. The difficulty of laying out satisfactory transposition designs increases rapidly with frequency. The frequency space occupied by the channels is consequently at a premium. Therefore, the single-band type of system is the only one which has been considered for use on these telephone lines.

As power networks are interconnected more completely, and the demand for communication facilities by this carrier method increases, undoubtedly the type of system which makes a most economical use of the frequency spectrum and transmits a minimum of non-useful high-frequency current into the power network will be the most desirable for general use.

It may be noted that while this method of duplex or two-way operation, described in the paper, possesses certain advantages in connection with power-line carrier, this method is not so advantageously applicable to carrier systems on telephone lines.

In the latter case, the problem is one of obtaining a maximum carrier use of a telephone pole line as a whole, involving a large number of pairs, each pair carrying a number of channels. It turns out that the most satisfactory arrangement for reducing crosstalk between the various pairs of the pole line and of effecting a maximum total carrier development of the pole line, dictates the use of separate bands of frequencies for transmission in the two directions. For this reason, the present standard types of carrier systems used in the Bell System are lined up on the basis that transmission in the two directions is upon different frequency bands.

J. P. Jollyman: The Pacific Gas and Electric Company was led to install the carrier telephone system described by Messrs. Wolfe and Sarros by reason of the necessity of maintaining communication between the terminals mentioned and by the

1. *Carrier Systems on Long Distance Telephone Lines*, A. I. E. E. Quarterly Trans., Vol. 47, October, 1928, p. 1360.

2. *Carrier Telephone System for Short Toll Circuits*, see p. 117.

knowledge that the wire line is subject to interruption especially in the mountain section which follows the highway through a heavily timbered area subject to heavy snow. While the power line passes through the same region its greater mechanical strength and cleared right-of-way renders it practically free of interruption, hence the desire to use the power line for a communication channel.

The carrier system has been in regular operation for several months and has been very satisfactory. The quality of speech transmitted is excellent. Practically no maintenance has been required. No tuning of the oscillators has been required, although it had been anticipated that some tuning would be necessary. The circuit is in regular use. Wire-line extensions, especially on the power house end, are frequently used to enable the dispatcher to reach the other plants in the region. No difficulty is experienced in using a wire-line extension up to 70 mi. of copper circuit and during emergencies longer circuits have been used.

The performance of the carrier circuit has been observed under a number of special conditions of the power system. It is not interrupted by power-line switching or even by power-line trouble. Switching out one of the 220-kv. circuits results in a slight improvement in the carrier but switching out one of the 110-kv. circuits has little if any effect. The power system has been so arranged as to open the direct 49-mi. path for the carrier between Claremont and Vaca and substitute a path some 250 mi. in length. Under these conditions the carrier circuit continued to give satisfactory communication between these points. A number of other power system changes has been made with little effect on the carrier.

L. J. Neuman: Mr. Wolfe mentioned modulation of the carrier by the corona discharge. May we have a further word of explanation concerning this peculiar phenomenon? Also, I believe the term corona is used quite differently by various engineers and physicists. In order to clarify this discussion would you please define, Mr. Wolfe, what you mean by the term corona?

Roy Wilkins: I have been closely in touch with the carrier-current development on the Pacific Gas & Electric system during the past six years.

I should like to emphasize several points in connection with the paper by Wolfe and Sarros on the final installation by this company.

It must be remembered that the circuit chosen was a particularly difficult one as regards carrier communication. It was chosen in order to be sure that future requirements could be cared for with the same equipment.

As to the sets themselves, single-sideband carrier-suppressed sounds formidable to a power engineer, yet the physical equipment is the simplest mechanically that I have ever seen, including the original simplex.

Mechanically moving parts are reduced to a minimum; even the power supply is a tube rectifier. This has now given three months continuous service without maintenance or interruption.

The voice-operated blocking arrangement as described is carried out by grid-bias on the modulator tubes. It is operating in 0.002 sec. and is adjustable downward.

This is faster action than could be obtained by blocking the oscillator on account of the nature of the circuits.

Another point of interest is that the over-all frequency characteristic is as good as a great many, and better than some, leading makes of present-day broadcast receivers between 400 and 2900 cycles.

One of the troubles encountered was a great deal of noise. As pointed out, the system of double demodulation used gives a signal noise ratio of 15 to 1 as compared to a simple tuned detector.

The line modulation mentioned has been the occasion of much discussion and some study. Its effect on the carrier, however, will be a matter of speculation until specific quantitative measurements either establish or disprove it.

Its effect if present would not appear as 60 cycles, but as a noise covering a wide band.

The remedy for such conditions already applied to telephone toll lines and the now familiar transatlantic telephone channel was some form of single-sideband transmission using a very narrow channel.

Carrier-current communication is essentially a long-haul facility which will probably not supplant entirely the wire line on most power networks, but rather act as a separate means of communication between the more important points.

C. A. Boddie: The general ideas set forth in the first part of this paper are in close agreement with views which I have advanced actively from the time telephone communication over power lines was first undertaken. That a power line is a difficult medium for communication ought to have been apparent at the start. The first difficulty is due to noise arising from spitting insulators and the like, and the second is the ever changing character of the circuit due to switching.

I feel that the whole development of power-line communication has been seriously retarded by the failure to recognize the various factors as pointed out here, and by not making the necessary provision for them in commercial designs. No doubt the basic assumption that a power line is a simple circuit comparable to an ordinary telephone line, led to the development of equipment similar to that used on telephone lines, and having a normal output of communication energy of only 1 watt.

It now seems to be clearly recognized that the standards of communication required by a power company are much more rigid than those applying to an ordinary telephone circuit. The great importance and value of communication to a power company, together with the natural difficulties involved, offer to the engineer a problem of considerable magnitude.

The type of equipment proposed by the authors was developed in order to meet conditions such as encountered on the 220-kv. lines of the Pacific Gas and Electric Company. The authors state that the standard commercial equipment used in attempting communication over this line failed because of the natural difficulties presented by the power line itself, and by the inadequacy of the design. It is unsound, however, to assume that the failure of a particular design necessitates the abandonment of the entire scheme, and the development of new and totally different methods.

As a result of the difficulties encountered in applying their standard equipment to the particular 220-kv. line mentioned in the paper, it was assumed that there was something about a 220-kv. line in general, which made it impossible as a circuit for telephone communication. It was believed that corona was the immediate cause of the difficulty. The theory advanced was that the electrical diameter of the conductor varied with the power frequency, and therefore produced modulation of the communication current.

As a result of actual experience with this particular line and tests which I personally directed, it is my opinion that corona is not responsible for the difficulties encountered in communication over this line. A 220-kv. line is not inherently and fundamentally different from any other line. It is likely to be more noisy because of the higher voltage. The particular line in question was indeed very noisy, in fact much more noisy than there is any need of. This noise was found to be due to discharges between the corona shields and the supporting hardware. At night it was common to see the discharge take place on numerous insulators between the corona shield and hardware. This type of discharge sets up oscillations of sufficient magnitude to interfere seriously with communication currents. This source of noise can be avoided by making a firm electrical connection between the corona shield and the hardware.

In support of my opinion that communication is not made impossible by corona, I can state that thoroughly satisfactory communication of high quality was effected over this line by standard 250-watt equipment, and that this fact was witnessed by a number of persons interested in the subject. The quality of the talk and the ratio of noise to signal was such as to offer no opportunity for criticism.

In the design of equipment for power-line communication, I have always felt that simplicity, strength, and electrical stability were of utmost importance. For this reason I have always advocated the use of the simple system of modulation commonly used in broadcasting stations, and the use of ample power to overcome line noises. We have adhered strictly to the two-frequency type of equipment because of its simplicity and stability. Our experience is that this class of apparatus does not get the kind of maintenance commonly given to commercial telephone equipment. For this reason only the simplest and most stable circuits and adjustments are advisable.

Our experience with power lines agrees exactly with the general views of the authors in that we have found that power lines afford a more or less uncertain medium for communication. Instead of changing our communication apparatus we have attacked the cause and provided corrective means to the lines. As a result of progress in the art of stabilizing power lines, we can now reduce a complex circuit to a relatively simple circuit. Equipment for doing this is now available. Under these conditions the simple equipment using standard methods of modulation which has been available for years, is entirely satisfactory.

In Fig. 1 are given some test data on a 110-kv. line. Sufficient details are not given to check these data. The value of attenuation for the 98-mi. line as read from curve *I* is about 3 T U at 70 kilocycles and 10 T U at 120 kilocycles. The rate of increase of attenuation with the frequency does not seem to agree with the values obtained from the Bureau of Standards tables. The value of 3 T U is less than half that obtained in similar tests.

It is stated in the paper that, "the present tendency seems to be so largely toward the use of condenser coupling that it is scarcely necessary to discuss their respective merits." The present tendency toward the use of condensers rather than antennas is due largely to a technical misunderstanding regarding the electrical efficiency of the antenna type of coupling, which is commonly thought to be low. This view is in error as it can be shown that there is little practical difference in the over-all efficiency of either method. The selection of the type of coupling should be made with reference to mechanical rather than electrical considerations.

L. F. Fuller: I rather feel that the reduction in noise on the Pacific Gas & Electric System was effected very largely by the use of coupling capacitors. Whether the single side-band method caused a further material reduction or not, I do not know, but I think due credit should be given to the coupling capacitors for effecting a very substantial reduction as compared with the noise observed with the coupling wires.

All discussors of this paper, and Mr. Wolfe as well, have mentioned the desirability of stabilizing the attenuation characteristic of power lines. I want to point out that until such stabilizing is somewhat effected, we can hardly hope to crowd our narrow bands close together as we possibly would on a telephone line. We can hardly hope to take full advantage of the large number of channels which single side-band might permit, until the line itself, the transmitting medium, can be very materially improved.

Under "Initial Installation," the right-hand column, there is a sentence reading, "They have provided a communication service over power circuits which have presented very serious communication difficulties to other apparatus previously tried." The other apparatus, of course, was that of the General Electric and Westinghouse Companies.

In the interest of accuracy in the A. I. E. E. records, there is one fundamental point here that I should like to bring out. Mr. Wolfe pointed out that the actual setup involved the Claremont Substation, the Vaca Dixon Substation, and the Pit No. 1 Power House. One problem, of course, was by-passing or transferring signals around the transformers at Vaca. At the time that our apparatus was being tested, the Pacific Gas & Electric Company considered it desirable to couple on the line side of the oil breakers. However, when the equipment described by the authors was put into service, it was connected to the bus, and I believe it is now connected in that manner. That makes an entirely different problem. Mr. Jollyman mentioned in his discussion that connecting one 22-kv. line gave no detrimental result. I think that is obvious. You always have one good line or the other, unless both lines are off the bus, in which case you have lost the carrier. But the switching of the heavy transformers on and off the carrier circuit does not occur except as a balanced operation. If they switch on one or two of the three banks, they are affecting the carrier circuit on both sides. I think that should be borne in mind in considering this whole matter.

H. T. Plumb: The simple fundamental carrier communication, where carrier and sidebands are transmitted over a transmission line, has been found to be pretty good, and there isn't much noise.

I have had a little connection with carrier work that was done on the system of the Utah Power & Light Company. About three years ago they installed two equipments, one at either end of their 132,000-volt three-circuit transmission line. After operating very satisfactorily for about two years and, so far as I have been able to learn, without any interruption of that communication, they decided to install further equipments, and they now have six equipments scattered over about 300 mi. of high-voltage transmission lines. Those equipments are very satisfactory in their performance. They occasionally have trouble with the automatic selector of the telephone, or something of that sort, but they do not have any major troubles and the system is exceedingly simple.

If you refer to Fig. 2 in this paper you will begin to see complexities creep in, and it will give you the reason why it is necessary to have so many jacks on the front of the switchboard to seek out those troubles. But if you will simplify your equipment, you will not have to hunt trouble to the same extent.

J. D. Sarros: Mr. Neuman's questions on line modulation have been answered in part by other discussion but several points on this subject brought up by Mr. Boddie will be discussed later.

A sentence in our paper under "Initial Installation" has apparently given rise to a misunderstanding. We mentioned "very serious communication difficulties to other apparatus previously tried" but had reference only to our equipment of which we tried three distinct systems: first, a carrier-transmitted system; second, a carrier-suppressed system; and third a single-sideband system.

In connection with these trials, the following points are of interest.

About two years ago we installed terminals of our then standard carrier-transmitted equipment at Claremont and Vaca-Dixon substation and at Pit No. 1 Power House. We had no difficulty in providing a first-class communication circuit between Claremont and Vaca-Dixon. We found it impossible, however, to extend this service over the 220-kv. lines to Pit No. 1 Power House because of the excessive noise conditions encountered. Taking the 220-kv. circuit alone we found that the signal and noise levels were approximately equal at the receiving end. What was more interesting, however, was that raising the transmitting level from about 1 to 50 watts did not improve the signal-to-noise ratio. In other words, the noise increased as the transmitting level was raised.

Because of this interesting phenomenon, we were granted

permission by the power company to conduct further tests to discover the nature of this noise condition and devise means, if possible, for overcoming it. We built a special amplifier which was capable of delivering as much as 1100 watts of high-frequency energy into the antenna system. Our supposition that this was not ordinary line noise was confirmed. Increasing the transmitting level from 1 to 1100 watts did not improve the signal to noise ratio at the receiving end. For certain special set-ups of the power system, of course, conditions were not so serious. The noise, for instance, would be considerably lessened with one 220-kv. line disconnected because of the peculiarity of the antenna system, as will be discussed later.

The equipment was finally modified in the field into a single-sideband system with the result that a very satisfactory circuit was provided from Vaca-Dixon to Pit No. 1 Power House with the 220-kv. power system normal.

On the basis of this experimental demonstration and because of the distinct advantages, as pointed out in the paper, resulting from single-sideband transmission, it was decided to develop commercial equipment of this design for use in power-line carrier telephony. As Mr. Ilgenfritz has stated in his discussion, the superiority of the carrier-suppressed type of transmission in telephone practise is so marked that the carrier-transmitted type has been discontinued and is becoming obsolete.

Although line modulation due to corona has not yet had direct experimental proof, our tests, outlined above, indicate that corona modulation is very plausible and that single-sideband transmission will overcome the distortion and noise resulting from it. Even with no line modulation present, the use of a single sideband will give a 3 T U discrimination against ordinary line noise as compared with the ordinary carrier-transmitted system. There is also, as Mr. Wilkins points out in his discussion, additional discrimination because of the inherent selectivity in the single-sideband system. In our tests, higher transmitting levels when using a carrier transmitted system resulted in correspondingly higher noise levels at the receiving end. This and the fact that the use of a single sideband or the elimination of the carrier (a carrier-suppressed system was successfully tried at the same time) resulted in a satisfactory circuit, gave rise to the theory that corona is the cause of line modulation.

In a carrier-transmitted system, for ordinary modulation more than 50 per cent of the energy is concentrated in the carrier frequency. Assuming that corona is the cause of line modulation between the carrier frequency and the corona frequencies, it is easy to see that this carrier undergoes a decided distortion. Now, modulation is a function of the product of this carrier frequency and the modulating frequencies and therefore an increase in the carrier will increase this distortion and the consequent noise after the signal is demodulated. By eliminating this carrier from the line and restricting this distortion to the single sideband alone, it is obvious that the noise level is reduced since the carrier necessary for demodulation is a pure carrier supplied locally at the receiver. This, we believe, accounts for the fact that even with 1100 watts of transmitted carrier it was impossible to set up a commercial circuit.

As Dr. Fuller states, the use of coupling condensers, because of more efficient coupling, is responsible for a large reduction in the noise. It was impossible, in this case, because of the rotation of the power conductors as they approached the substation yard, to obtain efficient coupling by means of antenna wires. In any case, however, there is no question but the present tendency towards the use of condenser rather than antenna coupling is due to the greater efficiency, stability, flexibility, and better quality afforded by the use of concentrated capacity for coupling the terminal equipment to the power line.

As stated in the paper, our object is to provide the power people with stable, reliable communication circuits of good quality with a maximum of flexibility and a minimum of maintenance. For this reason Mr. Jollyman's discussion is both encouraging and gratifying, for it shows plainly that this equipment is meeting successfully each and every one of these specifications.

Carrier Telephone System for Short Toll Circuits

BY H. S. BLACK,[1] M. L. ALMQUIST,[2] and L. M. ILGENFRITZ[3]
Associate, A. I. E. E. Associate, A. I. E. E. Non-member

Synopsis.—This paper describes a new carrier telephone system which is designed for use in the telephone plant as a substitute for open-wire line construction for circuits of shorter lengths than have been economical heretofore. This system, which is known as the type D carrier telephone system, provides one additional talking circuit per pair of wires. The equipment is adapted to provide facilities quickly, and is capable of being moved readily from one location to another when temporary facilities are desired. The system employs a novel and original modulation circuit which is fully described.

Introduction

ONE of the important developments in telephone engineering during the last decade has been the practical application of high-frequency currents for the transmission of telephone and telegraph messages over circuits simultaneously carrying other traffic. For the transmission of speech by this method, the band comprising the frequency components in the voice of the speaker, which in the ordinary telephone circuit are transmitted by electrical currents of the same frequencies, is translated into a band of high-frequency currents. These high-frequency or "carrier" currents are above those transmitted by the ordinary voice-frequency channel, so may be sent over a pair of wires that is being used for the transmission of speech frequencies. At the receiving end, the bands of high-frequency currents are separated from each other and from the voice currents on the same pair of wires by electrical filters so that each delivers its message independently.

The first commercial installation of carrier telephone equipment was in 1918 between Baltimore and Pittsburgh. Engineering and installation of carrier systems have progressed steadily from this time on. In 1921 the status of the art was reported to the Institute by Messrs. Colpitts and Blackwell in a paper entitled *Carrier Current Telephony and Telegraphy*,[3] which gave the fundamentals of the carrier current art and described two carrier telephone systems which had been developed up to that time, the type *A* and the type *B*. This paper also included a bibliography of material which had appeared up to that date.

Further advances in the carrier telephone art resulted in the development of the type *C* carrier telephone system, which provides three telephone circuits or "channels" in addition to the voice-frequency facilities normally carried by a pair of wires. This system was described by Messrs. Affel, Demarest, and Green[4] at the recent June convention of the Institute in a paper entitled *Carrier Systems on Long Distance Telephone Lines*, and a bibliography was given of material which had appeared since the presentation of the paper by Messrs. Colpitts and Blackwell in 1921.

In the paper describing the type *C* carrier telephone system, it was shown how, by the development of this system, the field of use of carrier telephony on the longer circuits in the Bell System has been greatly extended. There has also appeared to be a considerable field of use for carrier telephony on the shorter open-wire toll circuits, that is, those ranging from about 50 miles in length upward. The technical requirements for a carrier system designed particularly for use on these shorter circuits are in some respects less severe than those for a system capable of operation over much greater distances. In view of the relatively large number of circuits of these shorter lengths in the telephone plant, it has been found desirable to develop a type of system particularly adapted to this field of application. The purpose of this paper is to describe this system, which is known as the type *D* carrier telephone system.

In order to provide a carrier system which would be sufficiently low in first cost and maintenance to effect economies when used in place of wire construction for these shorter circuits, special consideration has been given to reducing the amount of apparatus required to a minimum and at the same time providing a system of such stability that very little maintenance would be required. By making the system a single-channel system, that is, one which provides only one additional talking circuit per pair of wires, by employing comparatively low carrier frequencies, and by including several new design features it has been possible to realize this aim. The provision of only a single channel results in a simpler and less expensive design for the filters, the use of low carrier frequencies results in decreased line attenuation and reduces the cost of line transpositions and carrier loading which may be required, while the new design features are of such a nature as not only to simplify the apparatus but to increase the stability of the circuit as well. The complete equipment for each system terminal is assembled as a unit which is wired and tested at the factory before shipment, thereby reducing engineering and installation costs.

1. Systems Development Department, Bell Telephone Laboratories.

2. Department of Development and Research, American Telephone and Telegraph Company.

3. *Carrier Current Telephony and Telegraphy*, E. H. Colpitts and O. B. Blackwell, A. I. E. E. TRANS., Vol. 40, 1921, pp. 205-300.

Presented at Pacific Coast Convention of the A. I. E. E., Spokane, Wash., Aug. 28-31, 1928.

4. A.I.E.E. Quarterly TRANS., Vol. 47, No. 4, 1928, p. 1360.

General Arrangement and Functioning of System

The type *D* carrier telephone system is a single-channel system employing two separate carrier frequencies, one for each direction of transmission. It is a carrier suppressed system, that is, the carrier frequency itself is not transmitted from one terminal to the other but is generated locally at each terminal, and only a single sideband, which is all that is needed in order to transmit the speech, is passed over the circuit.

The frequency range occupied by the type *D* system is

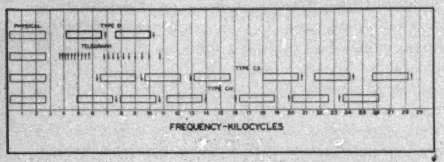

Fig. 1—Frequency Allocation of Type D System

indicated in Fig. 1. For transmission in one direction, the lower side-band of a carrier of 10.3 kilocycles is used, while for transmission in the opposite direction, the lower sideband of a carrier of 6.87 kilocycles is used. This frequency allocation was selected in order to obtain comparatively low carrier frequencies and at the same time to permit the system to be used on the same pole line with certain types of long-haul carrier systems.

The type *D* carrier telephone system has been provided in two general arrangements; the first employs certain basic equipment only and is suitable for operation over circuit lengths up to about 125 miles; the second employs the basic equipment together with terminal amplifiers and other additional equipment and is suitable for use on circuits up to about 200 miles in length. No provision is made for intermediate carrier repeaters as they are not required for the circuit lengths over which this system is designed to operate.

Basic Type D System. Fig. 2 shows in a general way the elementary features of the basic type *D* system. Referring to this figure, it will be noted that each terminal consists principally of a channel unit which comprises the modulator and demodulator and a hybrid coil to connect these two branches to a two-wire voice-frequency circuit, modulator and demodulator band filters to direct the carrier frequency currents into their proper paths, a line filter set consisting of high-pass and low-pass filters to provide separate paths for the voice and carrier currents which pass over the toll line, and a signaling unit to provide means for signaling over the carrier circuit. The channel unit also provides a demodulator output filter to suppress the high-frequency components in the output of the demodulator, and a network to balance the subscriber's line or other circuit to which the carrier system is connected.

The new electrical features which have been incorporated in this system consist principally of (1) the use of an arrangement whereby modulators and demodulators are self-oscillating, that is, the same vacuum tubes function both as oscillators and modulators or as oscillators and demodulators, thus economizing in the use of tubes and power, (2) the use of a new method of modulation and demodulation requiring the expenditure of a relatively small amount of plate battery power, (3) the elimination of grid batteries in the modulator and demodulator circuits by the use of grid current to produce grid biasing voltage, (4) the use of the modulator-oscillator circuit as a source of signaling current supply, and (5) the use of a ballast resistor to maintain the filament current within suitable limits. These new arrangements have resulted in economy in equipment as well as in a system of high stability. Economies also have been effected by the use of improved types of paper condensers in place of mica condensers in certain parts of the circuits, and by the use of unpotted coils where the requirements are such as to permit this to be done.

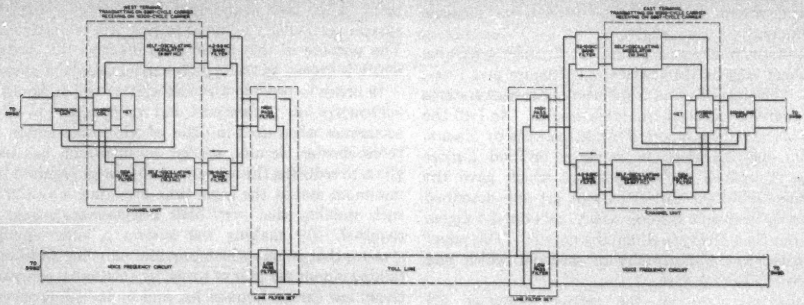

Fig. 2—Layout of Basic Type D System

The general circuit arrangement is illustrated in greater detail in Fig. 3, which shows a schematic

diagram of one terminal of the basic type *D* system. Carrier is produced by causing the tubes to oscillate in parallel, the frequency of the oscillations being controlled by the tuned circuit which couples the grid and plate circuits of the tubes. Speech currents from the switchboard pass into the hybrid coil, which functions both as hybrid coil and input transformer impressing the speech frequencies upon the grids of the modulator tubes. The application of these speech frequencies to the modulator, with the carrier frequency present, results in modulation, the principal products of which are the upper and lower side-bands. The desired grid biasing voltage is produced by the flow of rectified grid current through the resistance in the common lead of the grid circuits. By means of the balanced connection of the modulator output transformer, the carrier frequency is almost entirely suppressed and only a negligible amount appears in the output circuit. The output of the modulator passes to the modulator band filter where the upper sideband and other unwanted frequencies are suppressed and the lower sideband is permitted to pass on to the line through the high-pass line filter. The low-pass line filter prevents the high-frequency current from passing to a subscriber at the local terminal who may be talking on the voice-frequency line circuit on which the carrier is superimposed.

At the receiving terminal, the transmitted sideband passes through the high-pass line filter and the demodulator band filter to an input transformer which impresses it on the grids of the demodulator tubes. The application of these sideband frequencies to the demodulator, with the carrier frequency present, results in a reproduction of the voice frequencies applied at the sending end. Carrier and grid biasing voltage for the demodulator are obtained in the same manner as for the modulator. The various undesired high-frequency currents present in the output of the demodulator are suppressed by the low-pass filter in the output circuit. The voice currents are passed on to the hybrid coil and thence to the switchboard. The operation of these circuits is described in greater detail in the section on "Theory of Operation."

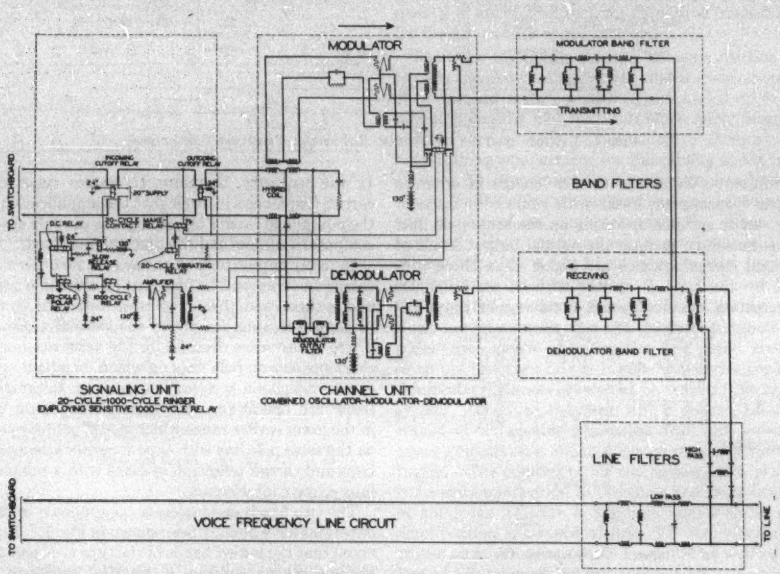

FIG. 3—SCHEMATIC OF TERMINAL OF BASIC TYPE D SYSTEM

The method provided for signaling over the carrier circuit is similar to that employed in the 1000-cycle signaling system used on wire circuits.[4] It differs in that signaling current is obtained by changing the connections of the modulator during the ringing period so that a separate source of 1000-cycle signaling current is not required. This arrangement is shown in Fig. 3. In order to signal over the carrier circuit, the modulator is unbalanced and the oscillator frequency reduced by 1000 cycles. The resulting high-frequency output is then interrupted at a 20-cycle rate. The receiving portion of the signaling circuit consists of an arrange-

ment designed to respond to 1000-cycle current interrupted at a 20-cycle rate. The operation of the signaling equipment is described in more detail later.

Type D System with Terminal Amplifiers. The arrangement employing terminal amplifiers is known as the type *D-A* system, and is used where it is desired to cover the range of circuit lengths from about 125 miles up to about 200 miles. In addition it is used in some cases on circuits of shorter lengths in order to equalize transmission levels with respect to those of other carrier systems operating on the same pole line. This is necessary because the normal output levels of long-haul carrier systems are higher than those provided by the type *D* system without an amplifier. Also, certain circuit layouts involving only type *D* systems may require level equalization by the use of the amplifier.

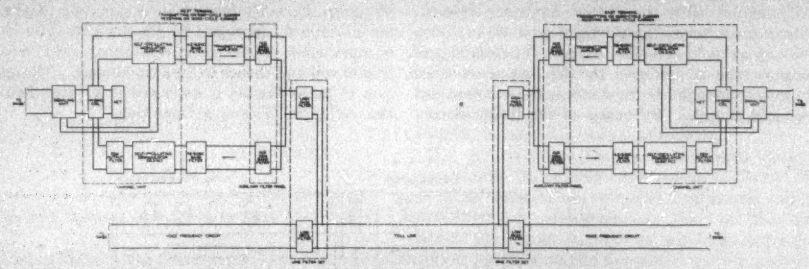

Fig. 4—Layout of Type D System Employing Terminal Amplifiers

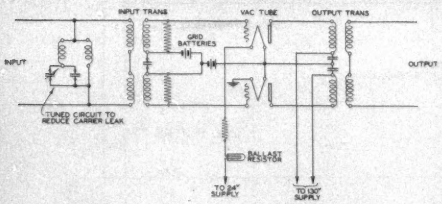

Fig. 5—Schematic of Amplifier Circuit

Fig. 4 shows in a general way the elementary features of the type *D-A* system. It will be noted that the arrangement is similar to that for the basic type *D* system, the principal difference being the addition of an amplifier and a change in the design and arrangement of the band filters to provide additional selectivity.

The general circuit arrangement of the amplifier is shown in Fig. 5. It is a single stage, push-pull amplifier of conventional design, and has two tubes.

Transmission Considerations

In setting the transmission requirements for the type *D* system, it was first necessary to view the existing plant to see that other circuits would not be adversely affected by the use of this new type of facility, and to insure that it would provide transmission comparable with the latest standards of circuit performance.

Line Factors. Since the system was to be designed to operate over relatively short distances, it was important that it should be as simple and inexpensive as possible. It was necessary, therefore, to employ relatively low carrier frequencies in order that the amplification which the equipment would be required to provide should be comparatively low and the line stability good.

In many applications of the system no other types of carrier facilities will be operated on the same pole line, and in these cases there is no close restriction on the use of carrier frequencies above the normal voice range. There is also some demand for the application of these short systems on pole lines on which long-haul (type *C*) carrier telephone systems[4] and carrier telegraph systems[3] are operating. Therefore, bands were chosen in the lower carrier range which would permit operation on the same pole line with type *C* carrier telephone systems and carrier telegraph systems with a minimum of inter-system interference.

The two bands employed for transmission in the two directions have already been shown in Fig. 1. It will be noted that the lowest bands of the type *C-N* and type *D* systems are transmitted in opposite directions. As a result, since carrier transpositions are designed primarily to reduce far-end rather than near-end crosstalk between pairs on which carrier systems operate, as was explained in the recent paper on long-haul systems,[4] it will not be possible, in general, to operate type *D* systems on the same pole line with type *C-N* systems. Satisfactory operation can be obtained, however, with type *C-S* carrier telephone systems and carrier telegraph systems.

The difference in transmitting level between a basic type *D* system and a type *C* or a carrier telegraph system is rather large. To avoid excessive crosstalk from carrier telegraph systems into type *D* systems operated on the same pole line, it is necessary to use the type *D-A*

system in order to obtain comparable levels. In certain unfavorable cases this will be necessary when type *C* systems are operated on the same line.

A comparatively inexpensive transposition system has been designed to permit the maximum number of type *D* systems to be operated on the same pole line. This system provides for the use of type *D* carrier systems on as many as four crossarms. When it is applied to lines on which no other type of carrier facility will be operated, it permits the use of basic type *D* systems on all pairs with the exception of the pole pairs. With slight modifications, it provides for the operation of two three-channel type *C-S* carrier telephone systems, one system on each of the outside

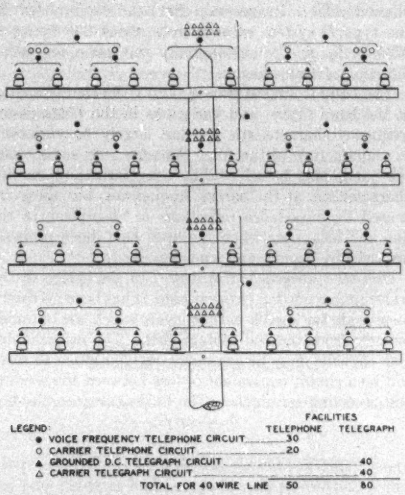

FIG. 6—AN ARRANGEMENT OF COMMUNICATION FACILITIES ON POLE LINE OF FOUR CROSSARMS

phantom groups of the top arm, and, in addition, either a carrier telegraph or a type *D-A* carrier telephone system on the other pair of each of these phantom groups. Type *D-A* systems may also be operated on all other pairs of the line except on the pole pairs. Carrier telegraph systems can generally be operated on one or more of the pole pairs. The former transposition arrangement for the basic system, therefore, permits the operation of a total of 16 type *D* systems on a four-arm lead, and the latter arrangement with the long-haul systems permits the operation of two three-channel carrier telephone systems, 14 type *D-A* systems, and, in favorable cases, four ten-channel carrier telegraph systems on the same number of pairs, in addition to the grounded d-c. telegraph and the telephone facilities ordinarily obtained. Fig. 6 illustrates the relative positions of these facilities on the four crossarms.

In many of the open-wire circuits there are frequently included short sections of cable often aggregating several miles in length. When a number of systems is to be operated on the same pole line, it is generally necessary to load these sections of cable to avoid impedance irregularities and consequent reflection cross-talk. Loading may also be desirable in order to reduce reflection and cable attenuation losses in the line when only one or two systems are installed. An economical loading system has been developed to supply the demand in connection with type *D* systems. This same loading system may also be used for carrier telegraph systems, inasmuch as the frequency range employed is about the same. This loading system employs a coil spacing of about 3000 feet, roughly three times that of the design used for type *C* systems.

The office cabling in the runs between the line protectors, the distributing frames, and the various pieces of equipment in the circuit frequently adds up to considerable lengths and may introduce fairly large impedance irregularities. Consequently, there is need for a loading arrangement for the cabling in the larger offices. An economical loading system has been designed for the office wiring consisting of a loading coil and a pair of adjustable condensers. When comparatively short entrance cables are involved, these office loading units can be adjusted to load the office and entrance cable together.

Over-all Transmission Equivalent. At the comparatively low frequencies used and the short distances over which the system operates, the line losses are considerably less than at the higher frequencies used and the greater distances covered by channels of the type *C* system. The type *D* system will, if desired, provide a circuit of no greater equivalent than the voice frequency circuit on a 104-mil open-wire pair of the same length. The equipment for the basic system provides sufficient amplification to overcome line losses in excess of about six TU[5] for a 100-mile line, with an extra amplification allowance of four TU to overcome entrance or intermediate cable losses. The wet weather loss of 100 miles of 104-mil pair, which is the type of line on which the system will be operated in most cases, is approximately 12 TU at 10 kilocycles, which is about the highest frequency transmitted. Therefore, an aggregate amplification of approximatly 10 TU is desired in the sending and receiving circuits if a six TU equivalent is to be maintained. In cases involving negligible lengths of cable, the four TU amplification allowance can be used to extend the range of operation.

The distribution of this amplification between the

5. "The Transmission Unit," by R. V. L. Hartley, *Electrical Communication*, Vol. 3, No. 1, July 1924, p. 34; *Transmission Unit and Telephone Transmission Reference System*, by W. H. Martin, TRANS. A. I. E. E., Vol. XLVIII, 1924, p. 797, *Bell System Tech. Jl.*, Vol. 3, July 1924, p. 400-408.

transmitting and receiving circuits depends upon the noise on the line and the relative levels of other systems. Approximately six TU of amplification is obtained in the transmitting circuit, and the remaining four TU is obtained from the receiving circuit. Adjustments are provided in both circuits for changing these amplifications over certain ranges.

The equivalents are adjusted at the time of installation of the system by means of combinations of resistance networks in the modulator output and demodulator input circuits. These combinations of networks

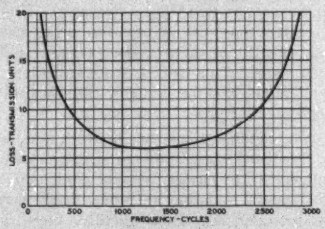

Fig. 7—Representative System Transmission Frequency Characteristic

provide adjustments such that the overall equivalent may be brought within approximately 0.5 TU of the value desired for the circuit.

Over-all Stability. It was considered very desirable to avoid the necessity for adjustments to compensate for changes in line loss and apparatus fluctuations either manually or automatically because of the cost of such control. The variations in line loss with weather at the frequencies employed by the type *D* system are much less than in the case of type *C* systems which operate at higher frequencies and over longer distances, and accordingly it appeared feasible to omit these adjustments, provided the equipment itself could be made sufficiently stable.

It was necessary, therefore, to insure a large degree of inherent stability in the design of the equipment so that apparatus for the adjustments of amplification, frequency, and filament current usually provided with carrier equipment might be omitted, thereby simplifying the system and reducing its cost.

The circuits have been so designed that the variations in the amplification and carrier frequency of the sending and receiving circuits with respect to normal fluctuations in power supply are negligible, and no compensating adjustments are required. The variations of transmitting and receiving amplification with plate battery fluctuations from 125 to 135 volts are not more than 0.1 TU. Variations in transmitting and receiving amplification with filament battery fluctuations from 20 to 28 volts are even less. Combined transmitting and receiving amplification changes with tube changes do not exceed approximately 1 TU. These are, however, of lesser importance as tubes are changed very infrequently.

The variations of high-frequency line equivalent are large in comparison with other variations in the system, although for the distances over which most of the systems operate these variations do not change the overall equivalent more than the usual maintenance limits. In the case of the systems approaching 200 miles in length, these limits may be slightly exceeded. However, the changes are apt to be of a seasonal character and may be corrected to some extent through the use of a pair of resistance networks controlled by a key.

Over-all Transmission Frequency Characteristics. Fig. 7 shows a representative transmission frequency characteristic. In general, the band transmitted by the type *D* system extends from about 200 cycles to 2700 cycles and is substantially the same for either direction of transmission.

The band is limited principally by the characteristics of the band filters, and variations in the transmission frequency characteristic are due largely to variations in manufacture of the filter elements. In some cases, the bands may be affected by irregularities in the line characteristic at the carrier frequencies, but these are caused by impedance reflections or absorption in the line and disappear when the lines have been properly prepared by loading and transposing.

Over-all System Load Capacity. In the design of the transmitting portion of the system, it has been necessary to provide for speech energy levels which are obtained directly from the toll switchboard. The demodulator load capacity may be somewhat less inasmuch as there will be a circuit equivalent or loss between the sending and receiving terminals. The load characteristic of a

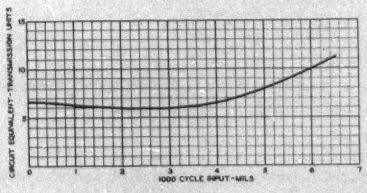

Fig. 8—Representative System Load Characteristic

representative system as measured with 1000-cycle current is shown in Fig. 8.

Impedance. It has been necessary to insure that the impedances of the equipment lie within satisfactory limits for a number of reasons. The impedance of the line filters has been made approximately the same as the average of the facilities over which the sytem will be operated. It is sufficiently uniform so that no serious impedance irregularities are introduced into the voice-frequency circuit on which the carrier system is super-

imposed. Any irregularities introduced by the filters would, of course, interfere with the balance obtainable at voice repeaters in the physical circuit at offices distant from the carrier terminals.

When a repeater in the physical circuit is located at a carrier terminal, the filter in the line is balanced by another filter in the network of the repeater, and the impedance of these filters must be similar in order that good balance may be obtained. For use in these cases, pairs of filters are selected at the factory to meet this requirement. This is considerably more economical than to manufacture each filter element to extremely high precision.

The high-frequency impedance of the carrier terminal is held to fairly close limits in order to avoid undesirable reflection effects, thereby increasing the effectiveness of the carrier transposition layouts.

Frequency Stability of Carriers. It is, of course, essential that the transmitting and receiving carrier frequencies for each direction of transmission maintain a reasonable degree of synchronism. The stability of

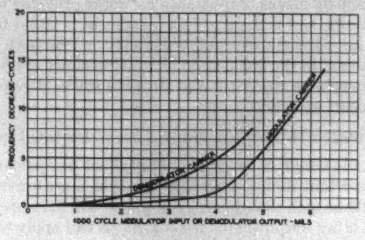

Fig. 9—Representative Variation of Carrier Frequency with Load

the transmitting and receiving carriers of this system has been made as high as practicable in order to eliminate the necessity for adjustments which otherwise would have to be made rather frequently. The plate and filament battery fluctuations have a negligible effect on the frequencies of these carriers. The frequency changes amount to less than one cycle for the normal voltage ranges in either case. The frequency changes due to tube replacements are somewhat larger, the maximum being of the order of two cycles, and may be in either direction.

The carrier frequency decreases slightly with input load as shown in Fig. 9. This is peculiar to the particular type of circuit employed in this system. The change in transmitting carrier frequency is of the order of two cycles for single-frequency input currents up to four milliamperes into the hybrid coil, while that of the receiving carrier frequency is of the order of three cycles for output currents up to three milliamperes from the hybrid coil. Both changes are in the same direction so that the difference remains small. The speech currents transmitted will generally be much smaller than those mentioned above.

The largest frequency variations are due to temperature changes. These are illustrated in Fig. 10. The curves in this figure represent two extremes taken from a series of four oscillator-modulator circuits and four oscillator-demodulator circuits. A study of the variations of office temperature has shown that such variations will not cause the carrier frequencies at the two

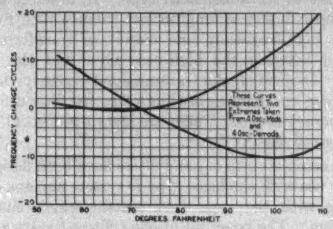

Fig. 10—Representative Variation of Carrier Frequency with Temperature

terminals to differ by more than about 20 cycles. Due to the comparatively low carrier frequencies employed, it has been possible to use relatively inexpensive paper condensers in the oscillator circuit and still obtain a satisfactory degree of stability.

Filters. A line filter set has been designed particu-

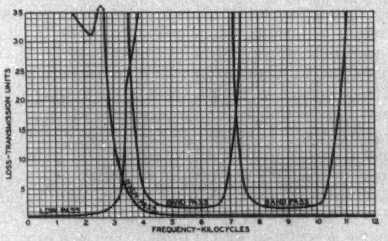

Fig. 11—Representative Filter Transmission Characteristics

larly for use with the type *D* system. This line filter set is connected into the circuit on the office side of the telegraph composite set. It will, of course, introduce some loss in the voice-frequency circuit on which the carrier is operated.

The low-pass line filter is designed to suppress frequencies above approximately 2800 cycles. The characteristic of this filter is shown in Fig. 11. In this figure it will be noted that at 1000 cycles the filter introduces a loss of about 0.5 TU, this loss increasing slowly with frequency up to the cutoff point. Since such a filter is put in the circuit at each end, the net

effect of the superposition of the carrier system is to introduce a loss of about one TU into the voice-frequency circuit.

The modulator and demodulator band filters have been designed to provide selectivity as economically as possible, that is, they offer selectivity only at those frequencies at which it is necessary in order to avoid interference originating either within the system or from external sources. For economic reasons these filters are of the unbalanced type. In order to insure the proper degree of longitudinal balance with respect to the high-frequency line circuit, a transformer is inserted between the high-pass filter and the band filters. In general, the transmission-frequency characteristics of the channels are limited by the filter attenuation characteristics. These are shown in Fig. 11 for a representative set of filters.

Two complete sets of band filters have been designed, one for use with the basic type *D* system and the other for use with the type *D-A* system which employs amplifiers. The latter set of band filters has been split so that part of the selectivity is furnished on each side of the amplifier in order to keep the unwanted frequencies produced in the modulator from overloading the amplifier, and at the same time to provide satisfactory impedance.

Amplifier. The amplifier has been designed to provide a high degree of flexibility. Either *O* or *L* type[6] tubes may be used, the grid potential and plate impedance of the amplifier being adjustable to accommodate either type. No changes in the amplifier apparatus other than certain minor changes in the connections are necessary in making this substitution. The *O* tubes are used only where maximum transmitting levels are required and *L* tubes are used for all other purposes.

The advantage in using *L* tubes is due to their lower power consumption and to arrangements which permit the grid batteries to be omitted and voltage drop in the filament circuit to be used to supply grid potential, thereby eliminating the cost of supplying and maintaining grid batteries. A total transmitting amplification of as much as 20 TU can be employed if the amplifier is equipped with *O* tubes. The load capacity with *L* tubes is about four TU less, but the available gain is about 3.5 TU greater.

The stability of the amplifier with power supply and tube changes is such that variations due to these causes are entirely negligible.

It will be noted in Fig. 5, the schematic of the amplifier, that an adjustable tuned circuit has been provided at the input. This is to suppress any carrier current which is present in the output of the modulator circuit so that the amplifier will not produce too much carrier current on the high-frequency line. Carrier current on the high-frequency line is objectionable from the standpoint of interference into dissimilar types of carrier systems.

Tests and Adjustments. The design and arrangement of the type *D* equipment are such that but little testing and adjustment are ordinarily required at the time of installation, and the high degree of stability in performance makes but little subsequent adjustment necessary. The performance of each type *D* terminal is tested before shipment from the factory. After the installation work has been completed it is necessary only to check the wiring and make initial adjustments before placing the system in service.

The initial adjustments made in the field consist of a synchronization of the carrier frequencies at the two terminals, and an adjustment to give the desired circuit equivalent. The adjustment of the circuit equivalent is made by means of resistance networks provided with the units, and is more or less permanent. Other tests and adjustments required are largely of the sort encountered in the installation and maintenance of ordinary telephone apparatus and are not such as to require frequent attention.

Equipment Features and Typical Installations

In the design of new types of telephone equipment, it is important that the arrangements which are provided should be such that the new equipment will fit in readily with other telephone plant, and that it will apply to as wide a variety of plant conditions as practicable with little change in form. Care has been taken in the development of type *D* carrier telephone equipment to meet these requirements.

Assembly arrangements have been provided to meet the different office conditions. These are adapted for association with the other terminal room equipment. The circuit arrangement of the system is such that it does not require changes in the various types of switchboards with which it may be associated. From an operating standpoint, the circuit introduces no feature requiring the operator to differentiate between it and any other circuit. In these respects, the system is similar to other types of carrier systems.

Equipment for Basic Type D System. The equipment comprising each terminal of the basic type *D* system is shown in Fig. 12. It consists of five principal units, namely, the signaling unit, channel unit, modulator band filter, demodulator band filter, and line filter set. These are associated with jacks in such a way as to permit such testing and patching as may be required.

The apparatus for each of these units is assembled on a panel 19 in. wide and of sufficient height to provide the required mounting space. These panels are

6. The approximate characteristics of the *L* and *O* tubes employed in this system are as follows:

Type	Filament voltage	Filament current	Plate potential	Average plate current	Grid potential	Voltage amplification factor	Output impedance
L	4.5 v.	.97 amp.	130 v.	8 mils.	− 9 v.	5.9	5500 ohms
O	4.5 v.	.97 amp.	130 v.	23 mils.	− 22 v.	2.5	2000 ohms

mounted on a rack and wired together, the leads which go to points outside of the bay being wired to a terminal block located at the top of the rack. At the time of installation, wiring is provided from this terminal block to a central point known as the intermediate distributing frame (designated *I. D. F.* in Fig. 12), where connection is made to the particular toll line desired and to the switchboard.

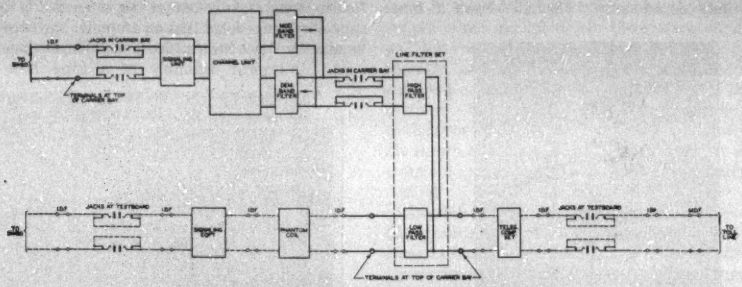

FIG. 12—SCHEMATIC OF TERMINAL OF BASIC TYPE D SYSTEM

Showing principal equipment units and method of connection to toll line

FIG. 13—TYPICAL ASSEMBLY OF TYPE D CARRIER TERMINAL ON FLOOR MOUNTED RACK

A typical assembly of a type *D* carrier terminal is shown in Fig. 13. This arrangement employs a floor mounted rack approximately seven feet high and includes all of the equipment comprising the carrier terminal with the exception of the power supply. The two box-like units near the bottom of the rack are the band filters. Directly above the band filters are the channel unit, signaling unit, adjusting unit, and jack panel, in the order named. Above the jack panel are the line filters and certain balancing equipment which is required when voice repeaters are used on the voice-frequency circuit on which the carrier system is superimposed, or on the circuit obtained by the carrier system. The adjusting unit comprises an arrangement to simplify adjustment of the 20-cycle polar relay used in the receiving part of the signaling circuit. This particular assembly arrangement is of advantage in temporary installations and in offices of low ceiling heights.

Where greater economy in floor space is desired, an assembly which provides the equipment for two type *D* carrier terminals on a rack 11½ feet high is used. The installation at Cedar Rapids, Ia., which consists of four type *D* terminals mounted in this manner is shown in Fig. 14. In this particular case, three bays instead of two were used as the installation consisted of three "east" terminals and one "west" terminal.

For each of these arrangements, the equipment is assembled on the rack and wired and tested at the factory as a complete unit, thereby reducing the time and work required for installation in the plant. The wiring required at installation consists only in cabling from the terminal block at the top of the bay to the distributing frame and to the sources of power supply.

The individual panels have been designed with a view to making the equipment as compact as possible. The modulator and demodulator apparatus, comprising the sending and receiving elements of the system, is mounted on one panel. This panel includes in one unit employing four vacuum tubes, the functions requiring two panels and six vacuum tubes in the long-haul carrier telephone equipment. The reduction in size

has been due in part to the omission of adjusting apparatus, such as potentiometers. Such adjustments as are required are provided by soldered connections. This arrangement has proved satisfactory as the circuit is sufficiently stable so that these adjustments need be changed only occasionally. Fig. 15 shows a front

Fig. 14—Installation of Type D Carrier Terminals on High Type Racks at Cedar Rapids, Ia.

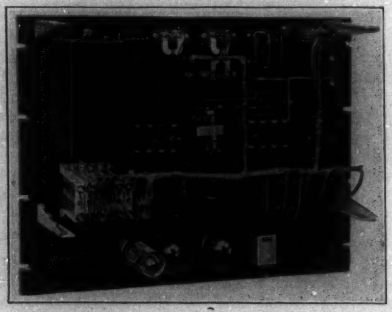

Fig. 15—Front View of Channel Unit with Cover Removed

view, and Fig. 16 a rear view, of the channel unit with the equipment covers removed, and illustrate the method of assembly and wiring used for this equipment. The equipment required for signaling is mounted on a separate panel, a front view of which is shown in Fig. 17. The filters are completely enclosed in copper containers which are sealed at the factory. This is done in order to provide sufficient shielding to permit the filters for different systems to be mounted in close proximity to each other. These containers are similar in form to the equipment covers which have been employed heretofore.

Equipment for Type D-A System. The equipment for the type *D-A* system is the same as for the basic type *D* system except that an amplifier and an auxiliary band filter have been added and different band filters used. A typical assembly of a type *D-A* carrier

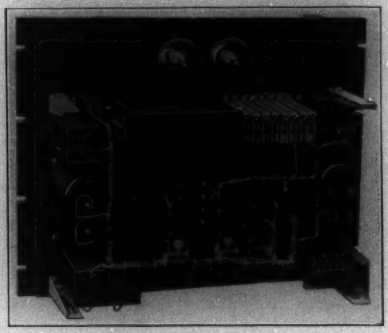

Fig. 16—Rear View of Channel Unit with Cover Removed

terminal is shown in Fig. 18. This arrangement includes on one rack all of the equipment comprising the carrier terminal with the exception of power supply. The amplifier is located above the jack panel, and the auxiliary band filter directly above the amplifier. A

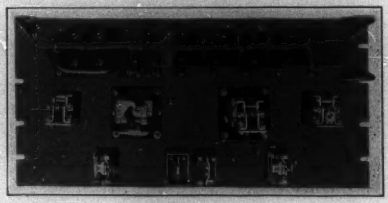

Fig. 17—Front View of Signaling Unit with Equipment Cover and Relay Covers Removed

front view of the amplifier panel is shown in Fig. 19. As in the case of the basic type *D* system, the equipment is assembled on the rack and wired and tested at the factory.

Where a number of type *D-A* systems is to be installed in one office and economy in floor space is important, a slightly different arrangement from that illustrated above may be used. The equipment for two systems, with the exception of the line filters and balancing equipment, may be mounted on an 11½-ft.

rack, and the line filters and balancing equipment for a number of systems mounted together on a separate rack.

Equipment Quantities and Space Required. From the description of the assembly arrangements, it will be seen that the total amount of equipment for the two terminals of a basic type *D* system, exclusive of power supply, involves about 14 panels which may be assembled in space equivalent to about one bay,[7] occupying

Fig. 18—Installation of Type D-A Carrier Terminal at Omaha, Neb.

a total floor space, including aisle space, of about six square feet. If the channel which the system provides were obtained by wire circuits, the only office equipment required in general would be the phantom repeating coils, signaling equipment, and in some cases a telephone repeater, involving on the average only a small part of one bay. Thus, the furnishing of facilities by means of carrier while reducing the amount of outside plant required, considerably increases the space requirements for the inside plant.

In the case of the type *D-A* system, the total equipment per system is about 30 per cent greater. The corresponding wire circuit in most cases would include a telephone repeater.

Power Supply. The total power required for the two terminals of the basic type *D* system is in the neighborhood of two amperes at 24 volts, and 40 milliamperes at 130 volts. This corresponds roughly to the amount of power consumed by four telephone repeaters employing tubes requiring one ampere of filament current. For the type *D-A* system, additional power is required for the amplifier. This amounts to one ampere of filament current and about 16 milliamperes at 130 volts for plate supply at each terminal when *L* type tubes are employed, and one ampere of filament current and about 46 milliamperes for plate supply when *O* type tubes are employed.

It is frequently desired to install this carrier equipment in small telephone offices where adequate power supply is not available. In these cases, small 24-volt and 130-volt power plants, which have been developed recently, are used. These power supply units are of sufficient capacity to take care of several of the carrier systems. Each is assembled on a floor-mounted rack of the type employed for the type *D* equipment assemble shown in Fig. 13, and is so designed that it may be mounted adjacent to the carrier equipment when desired.

Adaptability to Establishing Service Quickly. An outstanding advantage of the type *D* system results from the features of its design and arrangement whereby the equipment for each terminal may be furnished as a complete self-contained unit requiring relatively little engineering and installation work. It is thus useful in providing facilities quickly, and is capable of being moved readily from one location to another where temporary facilities are involved. This has already proved to be an advantage of some importance to the telephone companies. A brief description of typical cases of this sort may be of interest.

The use of type *D* equipment played a prominent part in restoring telephone service in the New England flood area last November. A number of terminal equipments was just being completed in the factory when the flood occurred. In view of the emergency conditions, release of the equipment for five systems was obtained from the telephone company to whom it had been assigned, and immediate shipment made. These

Fig. 19—Front View of Amplifier Panel with Cover Removed

systems were installed and placed in service within a period of about two weeks under conditions which would not have permitted line construction. The location of these systems and the manner in which they were used in providing through circuits is shown in Fig. 20.

Another instance in which the type *D* carrier telephone equipment served as a means of providing telephone facilities quickly under emergency conditions occurred last December. Due to a storm causing

7. A bay consists of two channel or I-beam uprights, ordinarily about 11½ ft. high, arranged to mount panels 19 in. wide and of varying height.

complete failure of the line between Syracuse and Watertown, New York, and partial failure of the circuits between Watertown and Utica, it was necessary to establish additional facilities on short notice. This was done by moving the Syracuse terminals of the two Syracuse-Watertown type *D* systems to Utica, thereby providing additional Watertown-Utica circuits which could be used to build up connections to Syracuse and other points. The time required for disconnecting the equipment at Syracuse, transporting it to Utica, and reinstalling it in that office amounted to about 32 hours.

There have also been cases in which type *D* equipment has been used to provide facilities required as a result of sudden growth in oil fields. One case of this kind occurred during the fall of 1926, when oil was discovered some 40 miles northeast of Amarillo, Texas. Within three weeks, oil companies and other enterprises had established temporary quarters in this locality, which was named Borger, and requested that telephone service be furnished. A temporary line providing one phantom group was built from Amarillo to this newly established town. It was evident that this would not hold the traffic and accordingly, plans were formulated to build a permanent lead. In the meantime, however, it was necessary to provide additional facilities and it was decided to install a type *D* carrier telephone system. Two weeks from the date of shipment of the equipment, an additional talking circuit was provided between Amarillo and Borger with this carrier system.

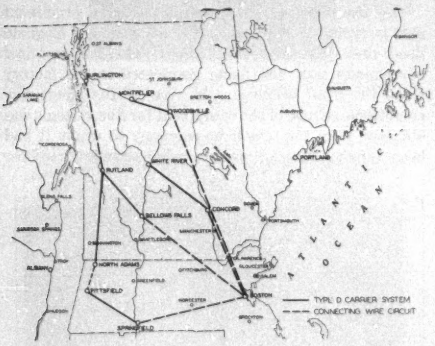

Fig. 20—Layout of Circuits Employing Type D Carrier Systems

Provided to meet emergency conditions in flood area in New England

After about 14 months, the oil activities at Borger were shifted to Panhandle, Texas. The toll traffic at Borger did not continue to justify the facilities that had been provided between this point and Amarillo. As traffic had increased between Pandhandle and Amarillo, Texas, the terminal of the type *D* carrier telephone system was removed from Borger and reinstalled in the Panhandle office, thus providing an additional circuit to this point at a very small additional expense.

Theory of Operation

It is believed that the principles governing the design of the system, particularly the modulator and demodulator units, because of their unique functioning, may be of interest to some readers. The following additional theoretical discussion and information is therefore offered.

Self-Oscillating Modulator. Aside from improved stability, important theoretical differences between this modulator and modulators employed previously in carrier systems are the combining of the functions of modulation and carrier generation, the replacement of grid biasing batteries by a grid resistance and capacity, and the use of a new method or type of modulation requiring the expenditure of only a very small amount of plate battery power.

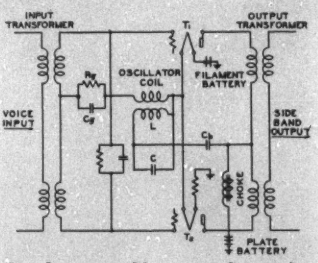

Fig. 21—Self-Oscillating Modulator. Carrier Suppressed

Fig. 21 is a schematic of a self-oscillating push-pull modulator of this type. It is designed to suppress the carrier from the output. The self-oscillating modulator of Fig. 21 may be separately excited by an oscillator as shown by Fig. 22. Measurements have demonstrated that if other conditions are unchanged, the modulator amplification and load carrying capacity of both circuits are substantially identical as indicated by Fig. 23.

The generation of carrier frequency oscillations is accomplished by causing the two tubes to oscillate in parallel, the potentials of both grids being alike and rising and falling together at a rate corresponding to the carrier frequency. The input and output transformers offer negligible impedance to the carrier frequency oscillations because the impedance of the transformer windings parallel-opposing is very small. This low impedance to the carrier is important in connection with the stability of the oscillator in that any variations in the leakage inductance and distributed capacities of the transformer windings with temperature or time are required to have a negligible effect upon the frequency of the oscillations. The operation of the self-oscillating modulator as an oscillator is shown by Fig. 24, the

oscillator of this figure being equivalent as regards the generation of oscillations to the self-oscillating modulator of Fig. 21.

The substantially constant grid biasing voltage applied to the grids of the oscillator tubes of Fig. 24 is obtained by rectifying in the grid circuit a portion of the

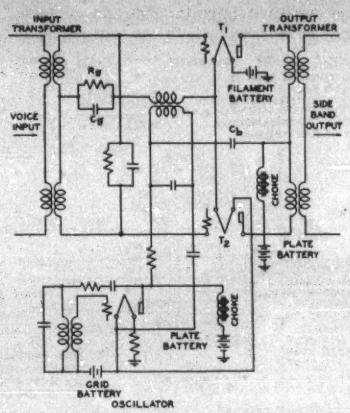

FIG. 22—SEPARATELY EXCITED MODULATOR. CARRIER SUPPRESSED

sinusoidal carrier wave applied to the grids. The values of grid resistance, R_g, and grid capacity, C_g, are proportioned so that the grid biasing voltage obtained in this manner approaches closely the peak voltage of the applied carrier wave. The unidirectional conducting

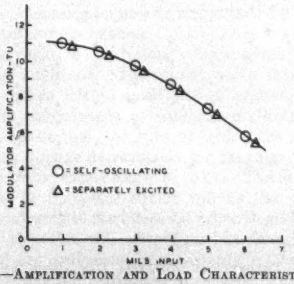

FIG. 23—AMPLIFICATION AND LOAD CHARACTERISTICS OF A SEPARATELY EXCITED MODULATOR COMPARED TO A SIMILAR SELF-OSCILLATING MODULATOR

path from the grid to the filament functions as a two-element rectifier in the usual way. The quantity of electricity stored by the condenser, C_g, remains substantially constant. Referring to Fig. 25, during the discharge interval, t_d, the total condenser charge is diminished exponentially by a relatively small amount, the extent of the reduction depending upon R_g and C_g. During the charging interval, t_c, the grid is positive with respect to the filament and the total quantity of electricity stored in the condenser C_g is slightly increased. The relatively small quantity of electricity, ΔQ, discharged by the grid condenser and dissipated by the grid resistance during the discharge interval must equal

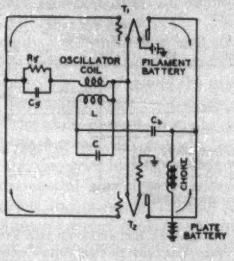

FIG. 24—EQUIVALENT OSCILLATOR CIRCUIT OF THE SELF-OSCILLATING MODULATOR OF FIG. 21

the differential increment of quantity accumulated by the condenser during the charging interval. The portion of the wave rectified may be made as small as

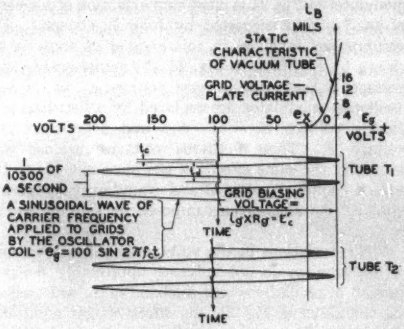

FIG. 25—RELATION OF THE TUBE CHARACTERISTIC TO THE GRID VOLTAGES IN THE OSCILLATOR OF EITHER FIG. 21 OR FIG. 24. NO SPEECH INPUT

desired by making either R_g or C_g or both R_g and C_g sufficiently large. The constants of the circuit have been proportioned so as to rectify only a small portion of the wave. Fig. 25 represents the relation of the static characteristic of the tubes to the applied grid voltages. The constant grid biasing voltage applied to each of the two oscillator tubes of Fig. 24 differ by an amount corresponding to the voltage drop across the filament of a single tube owing to the series

connection of the filaments and the use of a common grid biasing circuit. This together with the fact that the portion of the carrier wave rectified is exceedingly small explains why, with no speech input, the grid of only one tube, T_1, is driven positive.

For the self-oscillating modulator of Fig. 21, Fig. 26 represents to a close approximation the relation of the

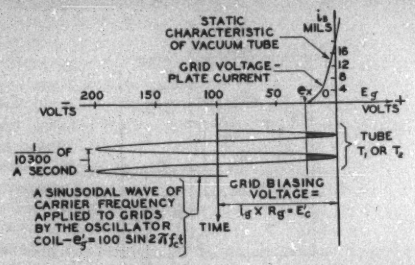

Fig. 26—Approximate Relation of the Tube Characteristic to the Grid Voltage in the Self-Oscillating Modulator of Fig. 21. No Speech Input

static characteristic of the tubes to the constant grid biasing voltage and the varying voltage of the carrier wave, each applied to the two grids in parallel, assuming no speech input. A convenient method for computing and studying the characteristics of the self-oscillating modulator of Fig. 21 is merely an extension of a scheme of analysis first suggested by John R. Carson.[8] The assumptions involved in this method of analysis are shown by Fig. 27. In Fig. 28, the actual grid biasing voltage, E_c', and applied grid voltage, e_g', of the self-oscillating modulator are replaced by a fictitious grid biasing voltage, $E_c = e_x - E_{dc}$, and a fictitious grid voltage, e_g. These fictitious voltages produce substantially the same effects in the plate circuits as the actually applied voltages and simultaneously permit fulfilling the assumptions imposed by this method of analysis.

While variations in the voltage amplification factor of the tube, μ, do not influence appreciably the production of oscillations and sideband in the self-oscillating modulator of Fig. 21, the effect of this additional variable might have been included in the analysis in a way similar to that suggested by Mr. F. B. Llewellyn.[9]

In considering the operation of the self-oscillating modulator as an oscillator, after the steady oscillations have been established, the attenuation around the closed oscillator circuit equals the amplification. Assume e_g' of Fig. 28 to be the steady-state voltage applied to the two grids in parallel. This voltage is substantially sinusoidal because the plate winding, L, of the oscillator coil is sharply tuned to the carrier or oscillator frequency. The grid biasing voltage, E_c', approaches the peak value of the alternating voltage applied to the grids. The grid voltages, E_c' and e_g', may be replaced by fictitious voltages, E_c and e_g, as

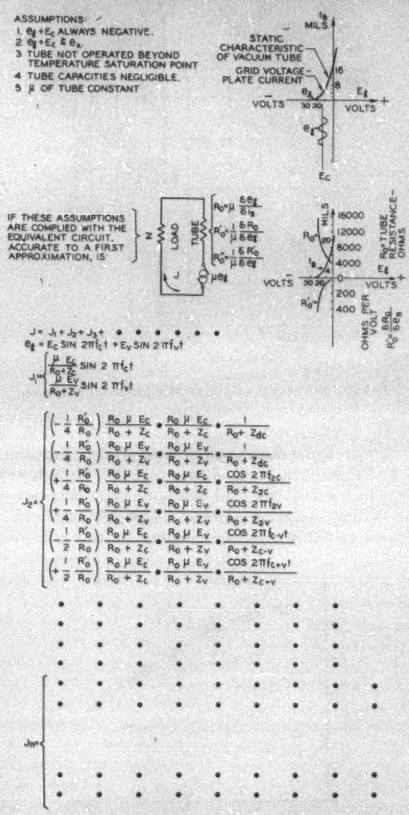

Fig. 27—Theoretical Analysis Developed by John R. Carson

previously explained. The alternating voltage, e_g, is amplified and distorted by the tubes and causes periodic pulses of current to flow in the plate circuit. The closed circuit $L\,C$ is only slightly damped and hence the current flowing in it induces in the secondary or high impedance winding of the oscillator coil a voltage in which the tuned or fundamental frequency predomi-

8. "Theory of Three-Electrode Vacuum Tube." *I. R. E. Proc.*, Vol. 7, April 1919, pp. 187-200.

"The Equivalent Circuit of the Vacuum Tube Modulator." *I. R. E. Proc.*, Vol. 9, June 1921, pp. 243-249.

"Notes on Theory of Modulation." *I. R. E. Proc.*, Vol. 10, February 1922, pp. 57-64.

9. "Operation of Thermionic Vacuum Tube Circuits," *The Bell System Technical Journal*, Vol. 5, No. 3, July 1926, pp. 433-462.

nates and which is the same as the assumed grid voltage, e_g'.

The conditions for sustained oscillations are somewhat different from those of the ordinary type of oscillator since there is the possibility of the oscillations becoming intermittent unless this is prevented by an appropriate design of the circuit. As the oscillations are built up from a condition of rest they tend to increase until checked by some change in the circuit which results from their presence. In the ordinary

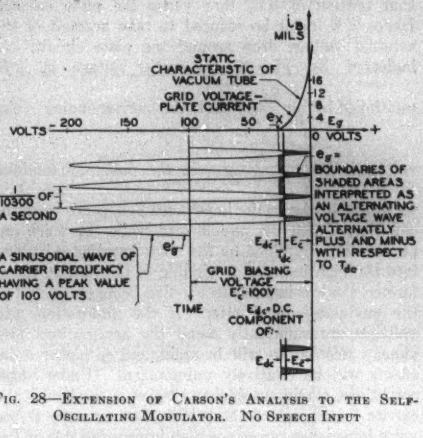

FIG. 28—EXTENSION OF CARSON'S ANALYSIS TO THE SELF-OSCILLATING MODULATOR. NO SPEECH INPUT

oscillator this change, which may be an increase in the resistance of the plate path, for example, occurs simultaneously with the change in amplitude. In this circuit the increasing amplitude also causes a charge to accumulate on the condenser, C_g, with an accompanying increase in negative grid biasing voltage which tends to reduce the amplitude. The rate at which this charge leaks off (when left to itself) is an important factor in the maintenance of oscillations. Referring to Fig. 25, suppose that the rate at which the negative grid biasing voltage decreases during the interval, t_d, is decreased by increasing either R_g or C_g. The next pulse of current in the plate circuit will be less than its predecessor owing to the increased negative grid biasing voltage. Hence it will contribute less energy to the support of the oscillations in the circuit, $L\,C$, with the result that they will tend to decrease, which in turn will decrease the plate pulses and the energy supply for the oscillations. If this resultant rate of decrease in the amplitude of the alternating voltage applied to the grids is greater than the rate of change of grid biasing voltage, the process will continue until the oscillations cease. (Once the amplitude has fallen so that the grid no longer becomes positive at the peaks, no further increments of charge are given to C_g and the grid biasing voltage follows the continuous exponential discharge of the condenser). The discharge of C_g through R_g continues until the grid voltage is such as to permit a flow of plate current sufficient to start oscillations. When these reach the value of those present before the assumed change in R_g and C_g, the initial conditions will have been restored and the process will be repeated.

The form of these intermittent oscillations is shown in Fig. 29, which is drawn from an oscillogram of the current through the condenser, C_g. However, the values of R_g and C_g at which the oscillations become intermittent can be increased by making the inductance of the choke coil in Fig. 24 large and the capacity of the condenser C_B reasonably small. At the end of a train of oscillations the plate current drops to zero and the negative grid biasing voltage exceeds the extinction voltage, e_x. As the grid biasing voltage follows the continuous exponential discharge of the condenser it slowly approaches and passes e_x, causing plate current to flow. The plate current continues to increase until after the start of the next train of high frequency oscillations. At the end of this train of oscillations the plate current drops again to zero. The frequency of the recurrent growth and decay of plate current corresponds to the relatively slow rate of occurrence of the spurts of oscillations. This low frequency current in the plate circuit is always present, if the circuit oscillates by spurts. If the plate circuit does not permit the presence of this low frequency current then this phenomenon can not take place. A large inductance choke coil and small by-pass condenser suppress the low frequency current necessary for this phenomenon by offering a high impedance to this current from the

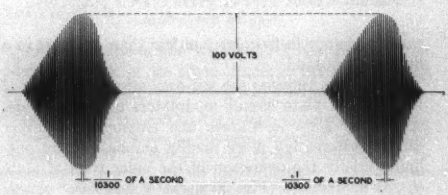

FIG. 29—OSCILLATION BY SPURTS

two plates of the two tubes in parallel to the filaments. From another point of view the effect of the choke coil and by-pass condenser may be considered as increasing the time constant of the oscillating circuit, $L\,C$.

A large inductance choke coil and a small by-pass condenser likewise stabilize the carrier oscillations with heavy speech loads, even to an extent that otherwise would be impracticable of attainment. As an illustration of the effectiveness of this expedient, the use of a 180-henry choke coil increases the level or load carrying capacity of the self-oscillating modulator as gaged by

articulation tests by more than six TU as compared to the performance with a choke coil having an inductance of 15 henrys.

A large inductance choke coil and a small by-pass condenser, serve indirectly to stabilize the frequency of the oscillations, since, by reducing the tendency of the circuit to oscillate by spurts, the grid resistance, R_g, and grid condenser, C_g, may be made larger than would otherwise be possible. Taking advantage of this will cause a reduction in the amount of energy dissipated in the grid circuit, thereby eliminating a number of variable factors which tend toward instability of the frequency of the oscillations. As an indication of the effectiveness of this scheme of control, if the condenser, C, in an oscillator similar to that of Fig. 24 is a high grade mica condenser having little energy dissipation, and the inductance, L, has but a slight amount of resistance, a change of plate battery voltage from 50 to 200 volts will

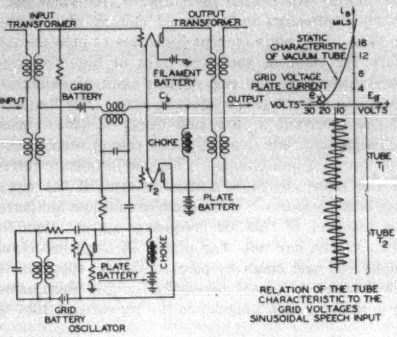

FIG. 30—CARSON TYPE OF PLATE CIRCUIT MODULATOR. CARRIER SUPPRESSED

cause a change in frequency of less than one part in a hundred thousand.

Differences between the self-oscillating modulator of Fig. 21 and plate circuit modulators of the type described by John R. Carson and employed in former carrier systems are more readily emphasized by outlining briefly the operation of the older type of plate circuit modulator. Fig. 30 shows a modulator of this type together with the relationship of the static characteristic of the tube to the grid voltages. This circuit complies with the assumptions listed in Fig. 27 and hence the equations of this figure are suitable for evaluating the side-band current to a first approximation. The use of two tubes in a push-pull relationship does not sensibly alter the application of the equations, the use of two tubes instead of one serving to suppress the carrier and certain undesired higher order modulation products, and allow the carrier output impedance to be placed in a part of the circuit which is separate from the voice and sideband output impedances. The presence of sideband or new-frequency currents in this type of modulator is due to the sweeping of the voice and carrier voltages over the curved characteristic of the tube. The lower the load or output impedance to the carrier and voice-frequency currents, the greater will be the proportion of the amplified grid voltages available for sweeping over the tube characteristic, and consequently, the greater will be the output of the resulting sideband and other currents of new frequencies. The reason for this is that the sideband is proportional to the carrier and voice voltages. The carrier voltage, for example, is not the amplified carrier voltage, μE_c, but that portion of μE_c that varies the plate current. Hence μE_c must be reduced to take account of the external voltage drop around the plate circuit. As indicated by Fig. 27 the carrier voltage is μE_c multiplied by $\frac{R_0}{R_0 + Z_c}$ and the corresponding voice voltage is $\mu E_v \frac{R_0}{R_0 + Z_v}$. In the push-pull modulator of Fig. 30 the carrier impedance Z_c depends upon the capacity, C_B, in the middle lead while the voice impedance is determined by the voice impedance looking into the high impedance winding of the output transformer, the low impedance winding being connected to the modulator band filter. If the alternating grid voltages are sufficiently small the assumptions previously referred to will be valid, and all higher order effects will be relatively unimportant. Under these conditions, maximum sideband power is obtained if the carrier impedance of the middle lead is zero, if the voice impedance across the high impedance side of the output transformer is zero, and if the impedance ratio of the output transformer is so adjusted as to make the load impedance connected to the tubes equal to the resistance of the tubes at frequencies corresponding to the sideband desired.

Attention is now directed toward certain properties of the Carson type of plate-circuit modulator which differ from corresponding properties of the self-oscillating modulator of Fig. 21. These properties are as follows:

1. The only grid voltages effective in varying the plate current are the impressed voice and carrier waves.
2. Modulation is present solely because of the curvature of the static characteristic of the tube.
3. The output impedances at frequencies corresponding to the voice, carrier, and side-band desired are for all practical purposes the only impedances effective in altering the sideband current, the impedance at all other frequencies being immaterial.

Keeping in mind the preceding explanation of the requirements for efficient production of new frequencies in the Carson type of plate-circuit modulator, an explanation of the reasons for the production of sideband in the self-oscillating modulator of Fig. 21 may be conveniently arrived at by analyzing only *that portion* of

the applied sinusoidal grid voltages that is *effective* in varying the plate current. Fig. 31 indicates the approximate relation of the static characteristic of the tube to the constant grid biasing voltage and sinusoidal carrier and voice voltages applied to the grids. As before, energy dissipation in the grid circuit is slight and for the purpose of analysis it may be assumed that neither grid is driven positive, the effect of grid current

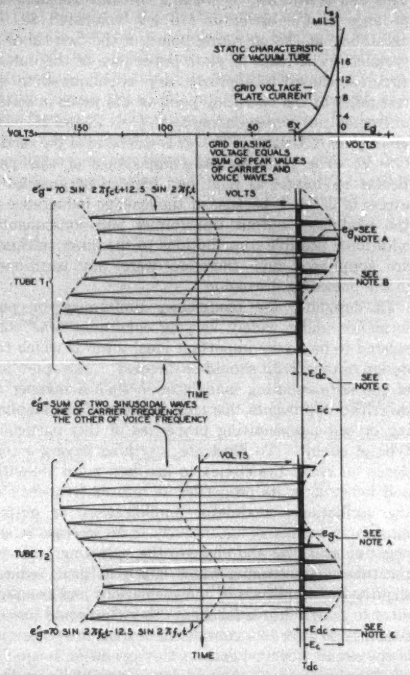

FIG. 31—APPROXIMATE RELATION OF THE TUBE CHARACTERISTIC TO THE GRID VOLTAGES IN THE SELF-OSCILLATING MODULATOR OF FIG. 21—SINUSOIDAL SPEECH INPUT

A. Boundaries of shaded areas interpreted as an alternating voltage wave, alternately plus and minus with respect to T_{dc}

B. The total instantaneous plate current is zero approximately 80 per cent of the time. If the output impedance is zero to all frequencies except the one sideband desired $(C - V)$, at which frequencies the load impedance should be a pure resistance—then the total instantaneous plate current is zero whenever the negative grid voltage exceeds approximately 25 volts

C. D-c. component of boundaries of shaded areas

on sideband production being negligible. Accordingly, the only alternating voltages existing in the grid circuit are the applied sinusoidal carrier and voice voltages, and furthermore, the constant grid biasing voltage closely equals the sum of the peak values of the voice and carrier voltages for the same reasons as were explained previously in describing the operation of the circuit as an oscillator with no speech input.[10] What happens in the plate circuit may be considered a result, the cause being a change in the grid voltage. Assuming that the extinction voltage, which is defined as the grid voltage for which the total instantaneous plate current is zero, is the static extinction voltage, e_x, Fig. 32 shows an indefinitely large number of new and fictitious sinusoidal grid voltages, which when taken together

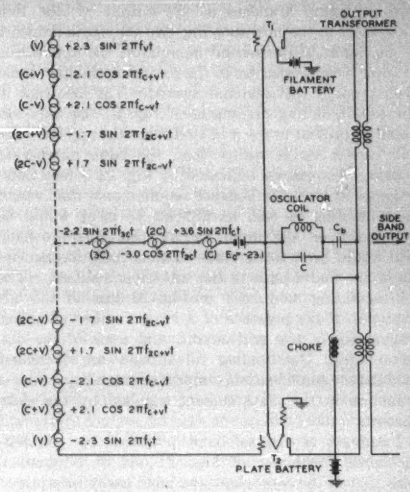

FIG. 32—A DIAGRAM OF VOLTAGES WHICH IF APPLIED TO THE GRIDS AS SHOWN PRODUCE THE SAME RESULT IN THE PLATE CIRCUIT AS THE CARRIER AND VOICE VOLTAGES IN THE MODULATOR OF FIG. 3

around a fictitious and smaller grid bias produce the identical result in the plate circuit as the two actually applied sinusoidal grid voltages and large grid bias.[11]

10. Whereas the grid biasing voltage equals the sum of the carrier and voice peaks, as explained, it should be understood clearly that the amplitude of the carrier oscillations will be unavoidably decreased as the speech increases. The presence of speech diminishes the amplitude of the oscillations by reducing that portion of the applied carrier voltage that is effective in varying the plate current.

11. The extinction voltage is a function of the plate or load impedance. In this particular modulator, however, the *ideal* towards which the design of the output impedance should be directed is an impedance that is zero to *all* frequencies except the one sideband desired, at which frequencies the load impedance should be a pure resistance, and, for maximum power transfer, match the tubes at the operating point. Restricting the load impedance in this manner and simultaneously applying a relatively huge carrier voltage to the grids in parallel, causes the static and dynamic characteristics to approach coincidence as the applied voice voltage approaches zero. Even with heavy speech inputs, the static and dynamic extinction voltages may be assumed alike to a degree consistent with the equations of Fig. 27, which in turn are accurate only to a first approximation.

The result attained is exactly the same as if the sideband was produced as the result of a special scheme of double modulation employing two balanced modulators successively. In accordance with this scheme, the carrier and its harmonics together with the voice would be applied conjugately to the first modulator to yield in the two conjugate branches of the output of this modulator higher frequencies together with the voice, carrier, and sideband frequencies. By connecting the two conjugate branches of the output of the first modulator to the corresponding two conjugate branches of the input of the second modulator, the application of these new frequencies to the second modulator would then produce the sideband desired. The fact that in the self-oscillating modulator of Fig. 21 the only frequencies present in the grid circuit are the applied voice and carrier waves rather than the large number of fictitious frequencies indicated by Fig. 32 is fortunate, because otherwise impedance requirements that would be conflicting as well as difficult to meet would be imposed upon the input circuit. Suitable provision also would have to be made so that these frequencies could not find a path to the subscriber's subset. The self-oscillating modulator of Fig. 21 has all the advantages of the presence of a large number of higher frequencies in the grid circuit, and none of the disadvantages. Outstanding advantages are increased modulator amplification, improved stability, and a reduction in the plate current supplied by the plate battery.

Inasmuch as the sideband produced by the self-oscillating modulator of Figs. 21 and 31 is identical with that of the equivalent and more easily interpreted circuit of Fig. 32, the latter will be employed to describe the steps involved. The lower sideband is the modulation product desired, and this product, in preference to all other frequencies, is selected by the modulator band filter and transferred with slight attenuation to the line. The lower sideband is the result of five contributions,—one amplification and four modulations. One contribution is the amplification of $C-V$ which is one of the voltages present in the grid circuit of Fig. 32. Only a portion of the total sideband is contributed by modulation of carrier and voice due to the application of C and V to the grids, this phenomenon being identical with that in the Carson type of plate-circuit modulator of Fig. 30. The three remaining contributions to the lower sideband are modulations due to the application to the grids of three pairs of voltages, namely: C and $2C-V$; $2C$ and $C+V$; $3C$ and $2C+V$. The operation in each case is analagous to the Carson type of plate circuit modulator of Fig. 30, the carrier being replaced by C, $2C$, and $3C$ and the voice by $2C-V$, $C+V$, and $2C+V$ respectively. Assuming zero impedance in the common branch of the output circuit at the frequencies C, $2C$, and $3C$ and zero impedance plate to plate at frequencies corresponding to V, $C+V$, $2C+V$, and $2C-V$, then all five contributions are exactly in phase, fortunately, and add directly. Each of the sideband components produced by the four simultaneous modulations may be evaluated by suitably altering the equations of Fig. 27.

In this system the impedance in the middle lead or common branch of the output circuit approaches zero at the frequencies of the carrier and its harmonics. This end is attained by making the anti-resonant impedance of the oscillator coil low compared to the resistance, at the operating point, of the two tubes in parallel. The plate to plate impedance of the output circuit is caused to approach zero as compared to the resistance at the operating point of the tubes in series, at frequencies corresponding to V, $C+V$, and $2C-V$, and a pure resistance to the lower sideband of the carrier $C-V$. The plate to plate impedance of this particular modulator is relatively high at frequencies in excess of $2C+V$ because of the leakage inductance of the modulator output transformer and consequently additional possible contributions to the lower sideband are suppressed and therefore were not mentioned in the preceding discussion.

In designing the modulator, uniformity of performance under widely varying conditions was considered to be a very important ideal towards which the design of the circuit should be directed. The operation of the self-oscillating modulator in such a manner as described approaches this ideal because of the stabilizing or self-compensating properties of this particular type of circuit. To illustrate, anything having a tendency to cause the circuit to oscillate more violently and hence alter its properties as regards frequency of the oscillations, modulator amplification, or output impedance results simultaneously in an increase in the negative grid bias and changes the operating point on the tube characteristic, which effect, in turn, reduces slightly the amplitude of the oscillations and compensates to a high degree for what otherwise would have a tendency to alter to a considerable extent the properties of the circuit as a modulator. Compensation is equally effective irrespective of what circuit variable is tending to alter the performance. An idea as to the effectiveness of this type of control may be had by referring to Fig. 33, which indicates the stability of the more important characteristics of the self-oscillating modulator and represents results obtained with commercial equipment.

Self-Oscillating Demodulator. The operation of the demodulator, a schematic of which is shown in Fig. 3, is analogous to that of the modulator, the essential difference being that the signal frequency applied differentially to the grids of the demodulator is the lower sideband $C-V$ instead of the voice V as in the modulator. The former modulates with the locally generated carrier to yield in the demodulator output circuit the voice wave originally applied to the modulator at the distant or sending terminal.

The operation of the demodulator differs from that of the modulator in that the output impedance instead of being zero to all frequencies except the voice, is zero to the carrier but high to harmonics of the carrier, and zero to $C + V$ and $C - V$ but high to all other frequencies. This procedure has been followed, since, compared to the modulator, the demodulator should have a higher amplification although the load carrying capacity may be less. As an aid in attaining this end the peak voltage of the oscillations applied to the grids is 35 volts for the demodulator as compared to 100 volts for the modulator. The output impedance relationships are obtained by bridging two series tuned circuits across the high impedance winding of the voice-frequency demodulator output coil. Each circuit is broadly tuned to the carrier frequency and hence provides a low impedance path plate to plate at frequencies corresponding to $C + V$ and $C - V$, thereby

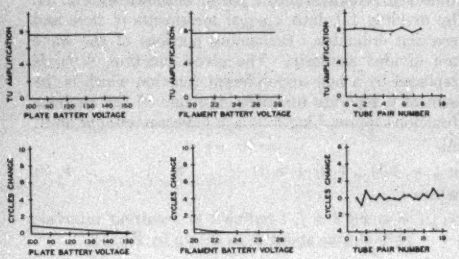

FIG. 33—STABILITY OF THE SELF-OSCILLATING MODULATOR

increasing the efficiency of the demodulator. These tuned circuits also provide a convenient means for connecting the plates to the oscillator coil and condenser.

Signaling. The method of signaling corresponds in general to voice-frequency signaling methods in current use on wire circuits with the exception that no separate source of 1000-cycle signaling current is required. Economy in power is obtained by the use of a sensitive alternating-current relay which, in effect, acts as a detector of 20-cycle modulation of 1000-cycle current. The sensitiveness of this relay permits a tube requiring only 60 milliamperes of current for filament operation to be used for amplifying the signaling current.

The signaling system is shown in schematic form in Fig. 3. Twenty-cycle ringing current from the toll switchboard is received by the signaling unit at the sending end, where it operates a relay chain. One relay in this chain connects an auxiliary condenser to the tuned circuit which controls the carrier frequency, thereby lowering the frequency of the carrier oscillations by 1000 cycles, and also unbalances the modulator by disconnecting plate battery from one of the tubes so that the carrier current is freely transmitted to the line. Simultaneously, another relay in this chain, the armature of which vibrates at a 20-cycle rate, short-circuits the output of the modulator twenty times a second. The net result of an outgoing 20-cycle signal from the switchboard is, therefore, to transmit modified carrier current interrupted at a 20-cycle rate over the line.

At the distant end the received carrier current differs by 1000 cycles from the demodulator carrier frequency, so it appears in the demodulator output as 1000-cycle current interrupted at a 20-cycle rate. This current after amplification operates a 1000-cycle relay. The input circuit of the amplifier is tuned to 1000 cycles to reduce the sensitivity to currents of other frequencies. The 1000-cycle relay follows the 20-cycle interruptions of the signaling current, that is, its armature vibrates at a 1000-cycle rate during the periods of approximately one-fortieth of a second that 1000-cycle current is applied, and remains at rest during the periods of approximately one-fortieth of a second that the current is interrupted. A set of contacts on this relay opens when the armature vibrates, so the net effect is that these contacts open and close at a 20-cycle rate.

A sensitive polar relay is associated with the contacts of the 1000-cycle relay in such a way that its armature vibrates at the rate at which the contacts on the 1000-cycle relay open and close. A direct-current relay and a 20-cycle tuned circuit consisting of a coil and condenser are associated with the contacts of this polar relay in such a way that when the armature of the polar relay is vibrating, the condenser in the tuned circuit alternately charges through one winding and discharges through a second winding of the direct-current relay. If the armature of the polar relay vibrates at a 20-cycle rate, the current flowing through the direct-current relay will be sufficient to cause it to operate. The tuning of the 20-cycle circuit is sufficiently sharp so that the direct-current relay will fail to operate if the armature of the polar relay vibrates at a rate differing from 20 cycles by more than a few cycles. This is an important factor in making the signaling equipment free from false operation from voice and other interfering currents.

When this direct-current relay has remained operated for approximately three-quarters of a second, it causes 20-cycle current to be sent to the switchboard, where it operates a relay which causes a visual signal to be given to the toll operator. This delay is controlled by a slow release relay, and is an additional safeguard to prevent false operation of the ringer, inasmuch as it requires that signaling current be applied for a specified length of time.

CONCLUSIONS

With the development of the type *D* carrier telephone system, the art of carrier telephony has been brought to the point where it can be used to advantage in providing telephone facilities for much shorter circuits than have been economical heretofore. Type *D* systems now in service are operating principally over distances of about

75 to 200 miles, although in certain cases where the conditions are favorable, systems as short as about 50 miles are in use. The exact distance beyond which it is more economical to employ this carrier system than to string additional wire, is, of course, dependent upon the conditions applying to each particular case. For greater distances, up to the limit of operation of the system, the economies effected by its use increase quite rapidly.

The performance of the type D system is comparable with that of a corresponding wire circuit. The form of the equipment is such that it is well adapted to the various office conditions, that it can be used to provide facilities quickly, and that it can be transferred readily from one location to another in case of emergency.

On June 1, 1928, approximately 125 type D carrier

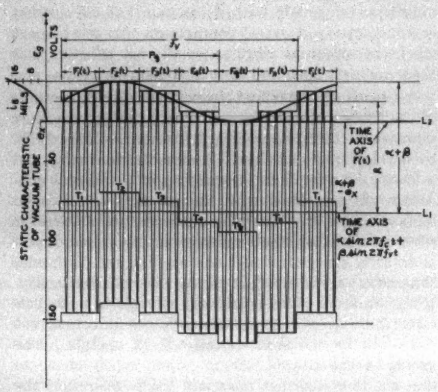

FIG. 34

telephone systems were in service and operating satisfactorily. It is expected that by the end of 1928 the number of systems in use will have increased to about 225. The growth in short open-wire toll circuits is such that there is a considerable field of application for this system.

Appendix

The mathematical treatment of the steps involved in the transition from Fig. 31 to Fig. 32 may not be evident, although this type of problem is frequently encountered in the theoretical analysis and design of vacuum tube circuits and analogous electrical systems. The problem may be stated as follows: *given,*—a complex wave which is the sum of two sinusoidal waves; *required,*—to express as a series of sine and cosine terms and a constant term, all having coefficients readily evaluated, that portion of the complex wave which lies above a line, parallel to the axis of the given wave and at a stipulated distance from it.[12]

$$\Phi(t) = \alpha \sin 2\pi f_e t + \beta \sin 2\pi f_v t^{13} \qquad (1)$$

$\Phi(t)$ is the given complex wave. In addition the simplifying, although not essential, assumption is made that f_e and f_v are incommensurable.[14] Referring to Fig. 34, that portion of $\Phi(t)$ which is to be analyzed is denoted $F(t)$. $F(t)$ lies above line, L_2, which is parallel to line, L_1, the time axis of $\Phi(t)$ and at a distance $(\alpha + \beta - e_x)$ above it. Line, L_2, is taken as the time axis about which $F(t)$ is to be expanded. Fig. 34 is drawn for the special case in which $2\beta = e_x$; this does not in any way interfere with the generality of the equations.

The envelope of $F(t)$, Fig. 34, is a sinusoidal wave of period $1/f_v$, drawn about a line above, parallel to, and displaced from L_1 by an amount $(\alpha + \beta)$. This sinusoidal wave commencing with $t = 0$ and over an interval of time $1/f_v$ is divided into n parts, as shown by Fig. 34, by dividing $1/f_v$ into n equal increments of time and erecting ordinates. Remaining portions of the wave are divided similarly. The given function, $\Phi(t)$, is replaced by a new and different function which is the sum of n separate functions, the sum of the separate functions approaching $\Phi(t)$ as n increases without limit. As

$$n \doteq \infty, \Phi(t) \doteq \Phi_1(t) + \Phi_2(t) + \ldots \Phi_q(t) + \ldots \Phi_n(t)$$

where

$\Phi_1(t) = \alpha \sin 2\pi f_e t$ redrawn at recurrent intervals of time about T_1 as shown by Fig. 34

$\Phi_2(t) = \alpha \sin 2\pi f_e t$ redrawn at recurrent intervals of time about T_2 as shown by Fig. 34

. . . . . .

. . . . . .

$\Phi_q(t) = \alpha \sin 2\pi f_e t$ redrawn at recurrent intervals of time about T_q as shown by Fig. 34

. . . . . .

. . . . . .

$\Phi_n(t) = \alpha \sin 2\pi f_e t$ redrawn at recurrent intervals of time about T_n as shown by Fig. 34

and q denotes the system 1, 2, 3, ... n.

12. Theoretically, it is possible to write the solution of this problem as either a Fourier's Series or Integral depending upon whether or not the frequencies are commensurable. It is impracticable, however, to compute the coefficients of the various terms in the expansion of $\Phi(t)$ by direct use of the Fourier formulas because each coefficient is itself expressed as a sum of a very large number of terms.

13. If the waves are commensurable the effect of phase displacement between the given sinusoidal waves may be taken into account by replacing (1) by $[\alpha \sin(2\pi f_e t + \Psi_1) + \beta \sin(2\pi f_v t + \Psi_2)]$ and modifying certain of the subsequent equations.

14. This assumption simplifies the results sought by reducing greatly the number of terms of interest in the series and by making all terms independent of the particular numbers selected to represent the frequencies denoted f_e and f_v.

As $\Phi(t)$ defines $F(t)$ so by analogy $\Phi_1(t)$ defines $F_1(t)$; $\Phi_2(t)$, $F_2(t)$; . . . . $\Phi_q(t)$, $F_q(t)$ . . . . $\Phi_n(t)$, $F_n(t)$.

$$\text{As} \quad n \doteq \infty \quad F(t) \doteq \sum_{q=1}^{q=\infty} F_q(t) \tag{2}$$

Consider the function $F_1(t)$, Fig. 34. Let S_1, Fig. 35, denote the Fourier's series which would represent the shaded peaks of a wave of frequency f_c recurring from $t = -\infty$ to $t = +\infty$ at an amplitude equal to the amplitude of the peaks during the time intervals corresponding to T_1. Let U_1, Fig. 35, be the function which has the value unity during the time intervals

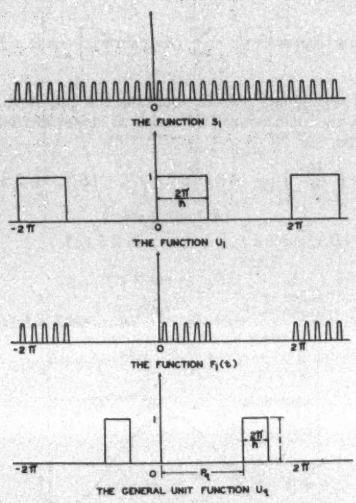

FIG. 35

corresponding to T_1, Fig. 34, and zero all remaining times.

$F_1(t) = S_1 U_1$ and as $n \doteq \infty$

$$F(t) \doteq F_1(t) + F_2(t) + \ldots F_q(t) + \ldots F_n(t)$$
$$= S_1 U_1 + S_2 U_2 + \ldots S_q U_q + \ldots S_n U_n$$

Representing an approximation to the general unit function, U_q, of Fig. 35[15] by the Fourier's series:

$$U_q = \frac{1}{n} + \sum_{k=1}^{k=K} \frac{1}{k\pi}\left[\sin\frac{2\pi k}{n}\cos(2\pi f_{kv} t - k p_q) + \left(1 - \cos\frac{2\pi k}{n}\right)\sin(2\pi f_{kv} t - k p_q)\right] \tag{4}$$

where K is an arbitrary number which will eventually be made infinite.

Letting n increase without limit and replacing $\sin\frac{2\pi k}{n}$ by $\frac{2\pi k}{n}$ and $\cos\frac{2\pi k}{n}$ by 1, we obtain:

$$U_q = \frac{1}{n} + \frac{2}{n}\sum_{k=1}^{k=K}(\cos k p_q \cos 2\pi f_{kv} t + \sin k p_q \sin 2\pi f_{kv} t)^{16} \tag{5}$$

Representing S_q by the Fourier's series:

$$S_q = \left(\frac{B_0}{2}\right)_q + (A_1)_q \sin 2\pi f_c t - (B_2)_q \cos 2\pi f_{2c} t - (A_3)_q \sin 2\pi f_{3c} t + \ldots \tag{6}$$

The expression for $F(t)$ desired is obtained by multiplying Equations (5) and (6) term by term and noting terms of the form[17]

$$\left.\begin{aligned} &\sum_{q=1}^{q=n}(B_m)_q \cos k p_q = 0 \\ &\sum_{q=1}^{q=n}(A_m)_q \cos k p_q = 0 \end{aligned}\right\} \text{for } k \text{ odd}$$

and

$$\left.\begin{aligned} &\sum_{q=1}^{q=n}(B_m)_q \sin k p_q = 0 \\ &\sum_{q=1}^{q=n}(A_m)_q \sin k p_q = 0 \end{aligned}\right\} \text{for } k \text{ even}$$

Hence

$$F(t) = \left[\frac{1}{n}\sum_{q=1}^{q=n}\left(\frac{B_0}{2}\right)_q\right] + \left[\frac{1}{n}\sum_{q=1}^{q=n}(A_1)_q\right]\sin 2\pi f_c t - \left[\frac{1}{n}\sum_{q=1}^{q=n}(B_2)_q\right]\cos 2\pi f_{2c} t$$

15. In this and certain subsequent figures it has been found convenient to express the abscissa in terms of radians.

16. Although the limiting form of this series is divergent it can be interpreted so as to be a valid representation of an infinitely narrow unit function.

17. The particular symmetry of the coefficients, some of which are shown in Fig. 38, is a factor in causing these terms to vanish.

$$-\left[\frac{1}{n}\sum_{q=1}^{q=n}(A_3)_q\right]\sin 2\pi f_{3c}t + \ldots\ldots$$

$$+2\left[\frac{1}{n}\sum_{q=1}^{q=n}\left(\frac{B_0}{2}\right)_q \sin p_q\right]\sin 2\pi f_v t$$

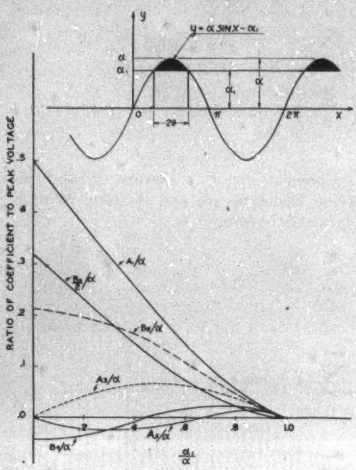

FIG. 36

$$+2\left[\frac{1}{n}\sum_{q=1}^{q=n}\left(\frac{B_0}{2}\right)_q \cos 2p_q\right]\cos 2\pi f_{2v}t$$

$$+\ldots\ldots$$

$$+\left[\frac{1}{n}\sum_{q=1}^{q=n}(A_1)_q \sin p_q\right]$$

$$(\cos 2\pi f_{c-v}t - \cos 2\pi f_{c+v}t)$$

$$+\left[\frac{1}{n}\sum_{q=1}^{q=n}(B_2)_q \sin p_q\right]$$

$$(\sin 2\pi f_{2c-v}t - \sin 2\pi f_{2c+v}t)$$

$$+\left[\frac{1}{n}\sum_{q=1}^{q=n}(A_3)_q \sin p_q\right]$$

$$(-\cos 2\pi f_{3c-v}t + \cos 2\pi f_{3c+v}t) + \ldots$$

$$+\left[\frac{1}{n}\sum_{q=1}^{q=n}(A_1)_q \cos 2p_q\right]$$

$$(-\sin 2\pi f_{c-2v}t + \sin 2\pi f_{c+2v}t) + \ldots$$

$$+\ldots\ldots\ldots\ldots \tag{7}$$

All the coefficients of this series are arithmetical averages, taken over one cycle of the sinusoidal wave of frequency f_v. Each coefficient may be evaluated either analytically by integration or graphically by plotting the coefficient and determining the average value of the graph.

Illustrative Example. To illustrate the method, the operations which were performed to obtain a graphical solution in a particular case will be listed. For the sake of brevity, only the term of frequency $c - v$ will be considered, other terms being obtained in an analogous manner.

This term is $\left[\frac{1}{n}\sum_{q=1}^{q=n}(A_1)_q \sin p_q\right]\cos 2\pi f_{c-v}t.$

As a preliminary step in obtaining the graphical solutions, the coefficients of the series, S_q, are evaluated and plotted, Fig. 36

$$S_q = \left(\frac{B_0}{2}\right)_q + (A_1)_q \sin 2\pi f_c t - (B_2)_q \cos 2\pi f_{2c}t$$

$$-(A_3)_q \sin 2\pi f_{3c}t$$

$$+(B_4)_q \cos 2\pi f_{4c}t + (A_5)_q \sin 2\pi f_{5c}t \ldots\ldots$$

$$\ldots + (-1)^{\frac{3m+1}{2}}(B_{m-1})_q \cos 2\pi f_{(m-1)c}t$$

$$+(-1)^{\frac{3m+1}{2}}(A_m)_q \sin 2\pi f_{mc}t$$

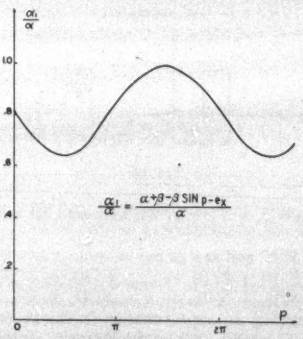

FIG. 37

$$\frac{B_0}{2} = \frac{\alpha}{\pi}\left[\sin\theta - \frac{\alpha_1}{\alpha}\theta\right]$$

$$A_1 = \frac{\alpha}{\pi}\left[\theta - \frac{2\alpha_1}{\alpha}\sin\theta + \frac{1}{2}\sin 2\theta\right]$$

$$B_2 = \frac{\alpha}{\pi}\left[\sin\theta - \frac{\alpha_1}{\alpha}\sin 2\theta + \frac{1}{3}\sin 3\theta\right]$$

$$A_3 = \frac{\alpha}{\pi}\left[\frac{1}{2}\sin 2\,\theta - \frac{2}{3}\frac{\alpha_1}{\alpha}\sin 3\,\theta + \frac{1}{4}\sin 4\,\theta\right]$$

$$B_4 = \frac{\alpha}{\pi}\left[\frac{1}{3}\sin 3\,\theta - \frac{2}{4}\frac{\alpha_1}{\alpha}\sin 4\,\theta + \frac{1}{5}\sin 5\,\theta\right]$$

$$A_5 = \frac{\alpha}{\pi}\left[\frac{1}{4}\sin 4\,\theta - \frac{2}{5}\frac{\alpha_1}{\alpha}\sin 5\,\theta + \frac{1}{6}\sin 6\,\theta\right]$$

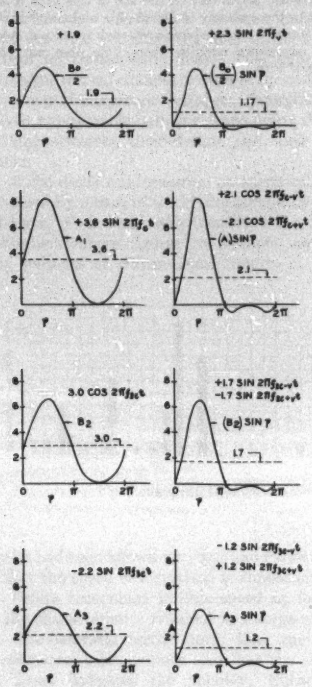

FIG. 38

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$$B_{m-1} = (-1)^{\frac{3m+1}{2}}\frac{\alpha}{\pi}\left[\frac{1}{m-2}\sin(m-2)\,\theta - \frac{2\,\alpha_1}{\alpha(m-1)}\sin(m-1)\,\theta + \frac{1}{m}\sin m\,\theta\right]$$

$$A_m = (-1)^{\frac{3m+1}{2}}\frac{\alpha}{\pi}\left[\frac{1}{m-1}\sin(m-1)\,\theta - \frac{2}{\alpha}\frac{\alpha_1}{m}\sin m\,\theta + \frac{1}{m+1}\sin(m+1)\,\theta\right]$$

. . . . . .

. . . . . .

where m is odd and > 1.

As may be seen by referring to Fig. 36, the first six of these coefficients are plotted as a function of $\frac{\alpha_1}{\alpha}$ where $\frac{\alpha_1}{\alpha} = \cos\theta$.

From the equation $\frac{\alpha_1}{\alpha} = \frac{\alpha+\beta-e_x}{\alpha} - \frac{\beta}{\alpha}\sin p$, derived from the geometrical relationships of Fig. 34, $\frac{\alpha_1}{\alpha}$ is plotted as a function of p from $p = 0$ to $p = 2\,\pi$, Fig. 37. This and other functions of p which are plotted and shown by figures are periodic.

For each value of p may be determined the corresponding value of $\frac{\alpha_1}{\alpha}$, Fig. 37, and hence A_1, Fig. 36. Proceeding in this manner ($A_1 \sin p$), Fig. 38, may be plotted as a function of p. The average ordinate of this graph is the coefficient sought, namely

$$\left[\frac{1}{n}\sum_{q=1}^{q=n}(A_1)_q \sin p_q\right].$$

In Fig. 31, $\alpha = 70$, $\beta = 12.5$, and $e_x = 25$ volts. Employing these values of the constants, the coefficients of the terms shown in Fig. 32 may be evaluated as described. Fig. 38 shows graphs, the average values of which equal these various coefficients.

Discussion

M. L. Almquist: There are certain points in connection with the development of the equipment described in this paper which may be of interest.

In many cases, the short toll circuits to which the type D carrier telephone system is applicable connect small towns with the larger toll centers, so that there is a considerable use for this equipment in small offices. In these small offices it is particularly desirable that the installation and maintenance of the equipment should be as simple as possible. Due to its high inherent stability and to the features of its design whereby it is provided as a complete self-contained unit, the type D equipment is well adapted to these requirements.

With the type D system, it has been found practicable to eliminate the daily adjustments necessary with carrier systems operating over greater distances. The few adjustments which are required are made by means of soldered connections and are of a semi-permanent nature. These arrangements simplify the maintenance and are an important factor in making the use of the type D system economical, particularly in cases where the equipment is located in small outlying offices.

The office equipment for a wire circuit of a length corresponding to that of the circuit provided by a type *D* carrier system is comparatively simple and is of such a nature that the tests required to insure proper installation consist merely in checking the wiring and seeing that the various relays are adjusted properly. It has been found practicable, by assembling the complete equipment for the carrier system on a rack and testing it as an operating unit at the factory, to reduce the amount of testing at the time of installation to an item comparable with that required with the simpler equipment used with the corresponding wire circuit. This results in an appreciable saving as it avoids the necessity of shipping expensive testing equipment to the job and employing installers trained in its use. This arrangement also reduces the amount of wiring to be done at the time of installation.

My purpose in bringing out these points is to emphasize how, in the design of the type *D* system, simplifications have been included which particularly adapt it to its field of application. The use of carrier equipment has been extended so that it now proves economical on shorter circuits than heretofore, and in addition is well adapted to use in the small offices.

R. E. Kistler: It may be of interest to note the application of type *D* systems on the Pacific Coast. By the end of this year we will have 37 type *D* systems in operation in Washington, Oregon, and California, giving a total of 3173 channel-miles which you will note is about 15 per cent of the total for the Bell System. The systems are being used on circuits ranging from 35 to 150 mi. in length with an average of 85 mi. Next year we are adding 23 systems with a total of 1500 channel-miles. One important advantage in the use of this type of system in addition to the economy of installation and simplicity of maintenance is that by maintaining a few such systems in stock we can provide emergency circuits where time does not permit the stringing of wire.

Lightning Arrester Problems

New Light from Klydonograph Studies

BY A. L. ATHERTON*

Member, A. I. E. E.

Synopsis.—This paper outlines the present status of lightning arrester application on transmission lines and suggests future investigations to determine the most effective design and method of application. Information from klydonograph tests is presented to show that lightning arresters are needed on both high-voltage and low-voltage lines. It is suggested that present practise in both design and application has limitations which may be overcome with the result of better protection which will be economically justified.

MUCH valuable information has been gathered in the last few years on the over-voltages which occur on transmission lines. This information has led to some radical changes in ideas of the magnitude and frequency of dangerous voltages. Simultaneously it has given a reasonable basis for determining the need for lightning arresters, their design, and their proper application.

Most of the definite information on surge values has been obtained by means of the klydonograph developed in 1923 by J. F. Peters,[1] who conceived the idea of using the Lichtenberg figures to measure surge and lightning voltages of transmission circuits in service.

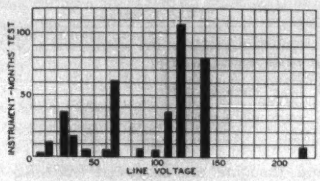

FIG. 1—DISTRIBUTION OF LIGHTNING INVESTIGATIONS BY LINE VOLTAGE

The figures had been known for years and Pedersen[2] had shown that the speed of formation is almost incredibly rapid. Peters recognized in this speed of formation one of the characteristics required in a surge-recording device. He learned, furthermore, that the size of figures varies with the voltage and that the character of the figure indicates the polarity. Subsequently he conceived and developed the klydonograph.

The device has now been in rather widespread use for four years. Much information has been gathered and this information has been made public and discussed freely.[3]

MEASUREMENTS OF LIGHTNING VOLTAGES

Klydonographs have been used in 27 investigations intended to give information as to lightning voltages. The total of instrument-months' records is 381. The distribution by voltage of line is indicated by Fig. 1, which shows that attention has been directed mainly at the higher voltages. The results are summarized in Fig. 2.

PERFORMANCE OF ARRESTERS

There have been 16 tests to record and measure the performance of lightning arresters as indicated by discharge current values, with a total of 61 instrument-months' record.

In these installations, record has been made of the voltage from line to ground at the arrester terminals

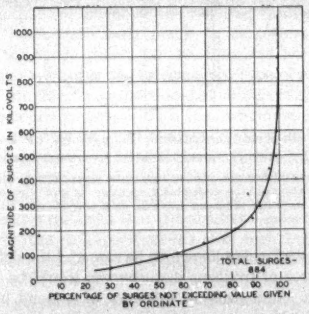

FIG. 2—SUMMARY OF LIGHTNING VOLTAGE RECORDS

and of the current flow through either one phase of the arrester or through all three phases combined. About the only fact that stands out definitely is that, based on double line voltage, or 3.5 "times normal" as the lowest surge value requiring operation of the arrester, the operation is what would be expected.

MEASUREMENT OF SWITCHING SURGES

In the tests to determine lightning voltages, records have been secured as to switching surges. The results

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1. "The Klydonograph," *Electrical World*, April 19, 1924.
2. "The Electric Spark," P. O. Pedersen, *Ann. der Physik*, Vol. 71 (1923) pp. 317-376.
3. *Klydonograph Surge Investigations*, J. H. Cox, P. H. McAuley, L. G. Huggins, A. I. E. E. TRANS., Vol. XLVI, pp. 315-329.

Presented at Pacific Coast Convention of the A. I. E. E., Spokane, Wash., Aug. 28-31, 1928.

28-118

show that no voltages are extremely high, very few are high enough to be dangerous at all, and that these are of such short duration as to make them of no serious importance.

Dangerous Over-Voltages Occur Infrequently

Perhaps the most outstanding of the changes in ideas indicated by these results is that dangerous voltages occur only infrequently. Before these data were available, the general idea was that there might be several hundred dangerous voltages in a season at any point on the system, and that a lightning arrester, to be effective, should operate as many times. This idea was doubtless mostly the outgrowth of observation of those early forms of arresters which were adjusted after installation so that they would spark over as often as the attendant thought proper. A little consideration of probabilities, however, will show this conception to be wrong, and that the klydonograph findings referred to above were inevitable. Fig. 3 illustrates this.

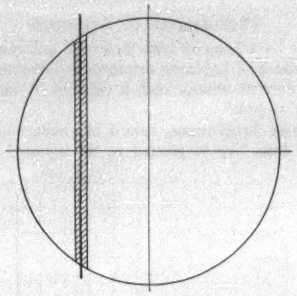

Fig. 3—Sketch Illustrating Probability of Dangerous Circuit Voltages Due to Lightning

The outer circle represents the area under observation by the Weather Bureau Station and the shaded band represents an area ¼ mi. each side of the line over which the cloud discharge must take place to give rise to a dangerous voltage.

Direct strokes are known to be rare and the only possible source of frequent disturbances would be induced charges. The circle of influence of a cloud has been shown by Norinder[4] and Peek[5] to be of hardly more than one-fourth mile radius. If we assume a generous average of 20 discharges in the clouds during a storm under the radius of perhaps 100 sq. mi. within the observation of the Bureau Station, the probability of any discharge taking place in a location to influence any one circuit to a dangerous extent is possibly one in three. This will be reduced further by the variation in induced voltage even for cloud discharges directly overhead, and for any one point on the line by the decrease in voltage due to resistance and corona losses, the effects of lightning arresters and line flashover, etc., as the wave travels. Bureau records show, for the Middle Atlantic States for example, an average of not more than 50 thunderstorm days per year. Then the number of dangerous voltages per year at any point should not be expected to average more than 5 or 10.

Limitation Imposed by Line Insulation

A second point of significance is the almost controlling part played by line insulation in determining the results secured, and this seems likely to have the greatest influence on lightning arrester practise. Again, considering 3.5 times normal or double line-voltage value as the minimum surge requiring arrester operation, two-thirds of all the dangerous voltages caused line flashover. Ninety per cent of the voltages above seven times normal (four times line-to-line value) caused flashover. This is shown by Table I, which makes a comparison between (a) lightning surges requiring arrester operation which caused flashover and (b) those which did not cause flashover, based on average number of surges per station per year.

TABLE I
TIMES NORMAL

	3.5 to 4.9	5 to 6.9	7 to 9.9	Over 10
Lightning surges which caused flashover	1.46	0.75	1.18	0.51
Lightning surges which did not cause flashover	1.11	0.64	0.2	0.0
Per cent which did not cause flashover	43.3	45.6	14.4	0

Several thoughts are suggested by these figures. If it is granted that the voltage transmitted along the circuit after a flashover is so brief and of such small energy content as to be reduced to a safe value by transmission over a short distance and by the various electrostatic capacities within the station, then the protection afforded by lightning arresters is limited to one-half of those surges within the range of 3.5 and 6.9 times normal and one-tenth of the higher surges. Any tendency toward increasing line insulation to reduce flashover naturally increases the dependence on and need for lightning arresters.

Moreover, in concentrating them at the stations, we may not be using our arresters to the best advantage. This practise certainly limits the real benefits to approximately one-third of all the possible cases of need; and to make matters still worse, it concentrates this remaining sphere of benefit in the least necessary cases,—those which exceed line voltage by the smallest amount.

Present practise is based on the thought that a lightning arrester can do no good to any apparatus more than a few hundred feet away, and that any project to protect the line or to reduce line-insulator flashover by protective devices is hopeless. This is not necessarily true. Records over a relatively long period of time

4. "Electric Thunderstorm Field Researches," Dr. H. Norinder, *Electrical World*, February 2, 1924.

5. *Lightning, A Study of Lightning Rods and Cages, with Special Reference to the Protection of Oil Tanks*, F. W. Peek, Jr., A. I. E. E. Trans., Vol. XLV, 1926, pp. 1131-1144.

have shown in one case at least that on a long line, flashover is noticeably reduced for a distance of approximately two miles on each side of the lightning arresters. To determine the benefit empirically one or two installations have been made of arresters a mile or so apart along a line. To be sure, if such a scheme is to be at all practical, the arresters must be radically different from those we now know; at least for high-voltage service. But arresters for such use may be of relatively small capacity and can be designed to absorb and dissipate the energy of the surge so that transmission along the line is practically prevented and protection is automatically rendered to the stations.

Thus it might develop that by directing our protective efforts to the line rather than to the stations, we could add to the present benefits some reduction in line-insulator flashover with little or no increase in over-all cost.

The demonstrated limitation by line insulation suggests that the station and line insulation must be coordinated to insure so far as possible that troubles occur where the damage will be least.

Switching Surges

The fact that the switching tests have shown that dangerous voltages are not to be expected from switching operations makes a rather radical change in our conception of the function of lightning arresters. It had previously been felt that lightning arresters are required for protection against both lightning voltages and those voltages arising from switching. Arresters have been recommended and used in some cases for the protection against such voltages when actual lightning voltages are not anticipated or are so infrequent as to fail to justify the installation of any protective equipment.

All arresters have been made with special provision for easy paths between lines to take care of such voltages. This use of four elements connected in "multiplex", or the similar use of an interconnection between phases at the neutral point of the arresters, introduces some disadvantages of reduced factor of safety against abnormal conditions, and is now known to be unjustified, since the compensating advantage of shorter paths between lines is not required. Since this information has been secured, general practise in lightning-arrester design has changed and three independent elements connected from line to ground are now used. Since at least 90 per cent of all arrester failures are known to be due to some overvoltage condition, and since this change increases the factor of safety against overvoltages, it is to be expected that this is a very tangible benefit of the tests.

Indications as to the Need for Arresters

With first thought, the facts above set forth might seem to indicate that the need for protection has been over-stressed and that the use of arresters might be reduced or discontinued with safety. This, however, is not a safe conclusion as there are several cases on record in which the addition of lightning arresters has made reductions ranging from 80 per cent to 95 per cent in lightning failures of apparatus for entire systems. These are all at relatively low voltages (below 33 kv.), since the number of installations at high voltage on any one system is too small to yield a dependable average.

If it may be assumed that these cases establish the justification of arresters at the lower voltages, the klydonograph results add something to the conception as to the need for arresters at the higher voltages. First, it is established that dangerously high voltages do occur even on the high-voltage lines. Second, the fact that line insulation sets the upper limit of surge voltage reduces the low-voltage and high-voltage problems to close similarity for the most severe case. Third, so far as the data indicate, dangerous voltages occur as often on high-voltage as on low-voltage lines.

Because of the lack of klydonograph data at the lower voltages, no final conclusion may justly be drawn, but the indications thus far are that arresters are needed as much at high voltages as at low voltages.

Arresters Operate as per Laboratory Results

The tests on lightning-arrester operation have demonstrated one point which previously, in one sense, has been inferred only. In the laboratory it is not possible to make complete tests on high-voltage arresters at high current owing to limitations in the amount of energy readily available and complications introduced into the measuring circuit by the large mechanical extent of the required testing equipment. Tests have been made to demonstrate that within the range which can be measured accurately, the characteristics of the sectionalized type of lightning arrester are directly proportional in voltage to the number of sections in series. This is as would be expected and the industry has continued making and using very high-voltage arresters beyond the range which has been definitely tested, on the basis that this relation holds throughout. Our assurance of this assumption is increased by the facts that the arresters have operated whenever the voltage recorded was high enough to warrant an operation and that the current values have corresponded closely with what would be expected from the voltages and the characteristics of the line.

The Present Problems

If these tentative conclusions are correct, then, although we may justify lightning arresters as now made and used, present practise limits their possible usefulness to a small fraction of the cases of real need. There is grave doubt as to whether the field of usefulness can practicably be extended to cover all the need, but this doubt is far from being a certainty. There can be no justification for failing to attack the problem even though the chances of success may seem slight, since the possible gain is so great and since no alternative has proved to be effective.

Two possible approaches suggest themselves: (1)

extension of empirical trials in service, and (2) determination of the basic data necessary to evaluate the possibilities. Both, it seems, should be tried.

The extension of the trials of arresters at intervals along the line has in its favor that it will begin to secure some results immediately and that it will stimulate the design and development effort necessary to make the scheme practicable. Against this plan is the fact that there is some expense involved and that it is far from certain that it leads to the final solution.

The approach by study of conditions deserves favor in that it will ultimately point to the correct solution. Against this are the facts that it will be very expensive and that it will require some years to make the tests, after which schemes for connection must be developed.

As valuable as the klydonograph has been, it appears that it is not going to give us all we need to solve the lightning problem. Voltage-time relations for lightning voltages must be determined and it may be necessary to eliminate the flashover limitation by over-insulation of some test circuit before the final answer is secured.

The cathode ray oscillograph opens the way to begin this phase of the problem; in fact, the work is already under way on a small scale. But very much broader tests are required. Data should be secured simultaneously in many locations distributed over that part of the country where transmission is required and where lightning troubles are likely. The efforts should be coordinated and the results gathered together for recording, analysis, and dissemination. This work cannot be left to one unit of the industry or it will not be done. It is a problem for the whole industry and requires the coordinated effort of operators and manufacturers alike to solve it.

Discussion

R. W. Sorensen: This paper again calls to mind the need for a plan which some of us for a long time have been trying to present to industrial engineers and executives interested in lightning protection, namely, the idea that the lightning problem is so big as to present possibility of solution only when undertaken as a group problem by a considerable number of investigators who together can command large resources for covering the expense, and a large number of ideas for attacking the problem.

I am quite in accord with Mr. Atherton's statement that switching disturbances can for the most part be neglected. In this connection, however, may I present a few observations made in experimenting with switches during the last few years. They are as follows: The quicker the circuit is interrupted the less the resulting disturbance; in other words, the interruption of a circuit within the first half cycle after the switch contacts are open does not produce a very high-voltage surge, as is the case if the arc between contacts is maintained for a period of several cycles.

Another source of high-voltage disturbance, arcing grounds, has been thoroughly studied, and solutions for the difficulty are now very well known. Thus we have left the real lightning problem for our chief attention in transmission and distribution lines.

There is also another phase of lightning protection to be considered, namely, protection against fire in oil reservoirs, arsenals, and structures of various kinds. In studying means for such protection we have found that it seems very difficult to predict exactly how lightning will strike because there are so many variables involved as to make accounting for all of them almost impossible. In other words, the amount of protection which can be provided by lightning rods must in final analysis be a matter of probability obtained from statistical data. We can, however, with the information now at hand, apply protection which will care for a large amount of the lightning hazard to which oil reservoirs or other structures may be subject. Having accomplished this result there is, of course, a natural tendency for industries to lose interest in the determination of the character of lightning phenomena. From a scientific point of view, however, it is obvious that best results cannot be obtained until we know more completely the true nature of lightning, and it is on this basis that I again wish to urge a comprehensive program, undertaken in part by small groups, with all groups cooperating toward the solution of the problem in the large.

C. E. Carey: I should like to ask Mr. Atherton two questions: 1, from his studies what fundamental principle has he evolved for lightning protection; 2, what records, if any, has he secured to date of transient voltages due to lightning?

F. O. McMillan: Mr. Atherton has called attention to the fact that lightning research is now in progress by several independent organizations without a coordinated cooperative program. This condition results in a duplication of effort and unnecessary expenditure of research funds, and does not produce anything like the results that could reasonably be expected from a well organized and directed joint program. A plan such as Mr. Atherton proposes should be subscribed to and supported by all of those interested in the problem of lightning protection.

As electrical engineers we may be somewhat prone to think that only the electrical industry is concerned in the lightning-protection problem, but such is far from being the fact. The protection of oil-storage reservoirs is of vital importance to the petroleum industry. Aviation interests are in need of more information about thunder storm and lightning phenomena. The United States Forest Service and various state and private timber interests have many lightning problems, three of which are very important. First, the protection of forest look-out houses on high mountains having bad lightning exposures; second, the protection of telephone lines to maintain reliable and safe communication; and third, the very serious problem of preventing or reducing the damage from lightning-caused forest fires.

This year the Oregon State Engineering Experiment Station is conducting an investigation of the protection of look-out houses and forest telephone lines from lightning. This research work is being done in cooperation with the United States Forest Service, the Oregon State Board of Forestry, and the Oregon State Forest Fire association. The importance of lightning protection for look-out houses and telephone lines in exposed locations is emphasized by the fact that several lookouts have been killed or injured by lightning while on duty.

Bradley Cozzens: Dr. Magnussen, in his address to the Spokane convention, stated that the rate of formation of Lichtenberg figures was from 7 to 10 cm. per microsecond.

In a recent article by C. L. Fortescue (*Journal of Electricity*, June 4, 1928, p. 293) it is stated that the rate of formation of an arc along an insulator string is of the order of 12 ft. per microsecond (366 cm. per microsecond). This gives the rate of flashover of an insulator string as from 36.6 to 52.3 times the rate of formation of klydonograph figures.

The length of insulator strings on transmission lines should limit the voltage that may be impressed upon the lines, for any specific rate of increase in voltage, to a very definite value. This maximum value should be approximately proportional to the length of the strings involved. Various types of shields and horns are here disregarded. It is well known, and Mr. Atherton has stated, that the voltage as indicated by klydonograph

records obtained on low-voltage lines, and those as obtained on higher voltage lines with longer insulator strings, do not agree with the possible limit of voltage as imposed by the insulator string impulse arc-over voltage.

Where the insulator string is less than 40 in. long there may be an error in the klydonograph figures as a result of the difference in the speed of formation of the figure and the speed of string arc-over. This error will increase with a decrease in insulator-string length. This fact may be the explanation for the disagreement between the theoretical and indicated maximum induced voltage values.

A. L. Atherton: The question of local hazards was raised by Professor Sorensen. Each of these schemes has been shown by experience to lack something. A rather careful review of the possibilities indicates at the present time (and again I must admit that it is an indication and not a proof) that the proper way to design protective equipment is to make the voltage on the lightning protective device as high as is consistent with safety at the point where the arrester is connected. The practical application of this thought results in what is almost a reversal of ideas in lightning-arrester design.

Under the old idea our aim would be to produce an arrester which has a constant volt-ampere characteristic which lies only slightly above line voltage. Our aim under the present idea is to produce an arrester which has a similar characteristic but one which lies well above line voltage, probably between 2 and 2½ times line voltage. The idea is that within this range the higher the voltage the greater will be the amount of energy taken out of the lines, and the less likely will be the passing along or reflecting back of the transient on the system to cause damage at a second point. This change in attitude of mind toward the lightning-arrester design problem is not wholly formulated, but has a great deal of support.

Mr. Carey also asked as to the records secured in our Tennessee investigations. One good record of lightning voltage has been secured with the oscillograph. This was supported by parallel klydonograph records, verifying the crest value of voltage. The information secured is of interest and significance in that a first conception of the voltage rise and duration is secured. For instance, the voltage rose at a relatively uniform rate during a few microseconds to a crest value of 650 kv., and fell along a somewhat irregular line to zero, the total duration being approximately 50 microseconds.

On the other hand, the point of main significance is a complete demonstration that there are now available the necessary instruments to secure results of this sort with reliability. In other words, we have demonstrated that we are now in a position to determine the entire course of the transient voltage from beginning to end, registering every variation by the mere application of instruments already developed.

Another point of major importance is that only one opportunity for a record has been available during the duration of the test thus far approximately two months. This indicates the need for the use of a rather large number of instruments, probably well distributed over the country, if we are to get the necessary information as to the extreme and average or ordinary values of rate of voltage rise, maximum voltage, and the duration in a reasonable number of years.

These findings support the suggestion that coordinated effort is necessary.

Professor McMillan mentioned the study which is being made in connection with the protection of forests against lightning damage. He referred to the protection of oil tanks. It is interesting to note that the United States Navy Department and the Army Department are interested in the problem of thunder-storms from the standpoint of aviation. Several very serious accidents have occurred due to lightning storms. Some thought that some of the airplane failures have been due to lightning.

The fact that this kind of study is applicable to several industries, the fact that it is widespread, and the unfortunate fact that there is very little definite provision for coordination between these different studies, further support the necessity for setting up some kind of national guiding organization.

Mr. Cozzens has brought up an interesting point. We must be careful, in thinking of the data in regard to speed of either flashover or formation of the Lichtenberg figure, not to overlook the fact that the values are not precise and that they are not on the same basis. The speed of formation of the Lichtenberg figure is exponential. That is, the formation is very rapid at the start and gradually becomes less, so that even though it may take 0.1 microsecond to form a figure 1 cm. in radius as stated, a very large part of the same figure forms in a small fraction of 0.1 microsecond.

As to the basis of time, the Lichtenberg figure begins forming with the beginning of the voltage rise and in general can substantially keep pace with any surge which does not rise to its crest in a very small fraction of a microsecond. On the other hand, the flashover of the insulator does not begin until a considerable overvoltage value is reached, this value being of the order of the crest value of 60-cycle dry flashover. The time for insulator flashover begins at this point, when the Lichtenberg figure is quite well formed.

There is evidence that the accuracy of the klydonograph decreases somewhat when the wave front is made extremely steep, or in other words, when the voltage application is extremely short, but this inaccuracy is far from being as great as might be indicated by the figures Mr. Cozzens cited.

It is our laboratory experience that measurements made in parallel by the klydonograph and by sphere gaps usually result in not lower but rather higher values by the klydonograph.

High-Voltage Phenomena in Thunderstorms

BY MARCEL A. LISSMAN[1]
Associate, A. I. E. E.

Synopsis.—Lightning phenomena are analyzed in the light of laboratory experience with high-voltage phenomena in the atmosphere. Special emphasis is placed on the effect of space charges in producing high local stresses when mobilized through channels of high conductivity caused by high temperatures. A detailed analysis is included of the steps through which air at atmospheric pressure passes when, due to applied electric stresses, its electric properties change from those of a non-conductor to those of a highly conducting body,—perhaps many times as conducting as a metallic conductor at ordinary temperature,—and then again resumes its original non-conducting properties.

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THE electrical quantities involved in thunderstorms are of an order of magnitude different from those encountered in ordinary engineering experience. In the laboratory, sparks 20 ft. in length have been obtained with an applied potential of 2,000,000 volts, 60 cycles. Compared with a lightning flash several miles in length these results seem puny indeed. Yet the results obtained in the laboratory enable us to analyze the phenomena occurring in nature, and thus obtain a clearer mental picture of the mechanism involved in the formation of lightning flashes. Such a procedure should prove of great value in obtaining and interpreting evidence relative to lightning to be used in determining the most effective means of protection.

It is very desirable to lay particular stress upon the chief differences between electrical phenomena in thunderstorms and in engineering practise. We are ordinarily concerned with complete metallic circuits handling large quantities of electricity at constant and relatively low voltages for long intervals of time. On the other hand, in a thunderstorm the quantities of electricity present are small, in the form of isolated electric charges located on water drops forming a volume distribution of electricity. The masses of the water drops are so large compared to their charge that, for the duration of a lightning stroke, the distribution of electric charges remains fixed. The charged water particles are acted upon by the electric field and their slow motions give rise to displacement currents. The actual velocities attained by the charged particles are sufficiently low to enable the wind to do work upon them against the electric field and thus raise the potential to breakdown of the air, when a lightning flash takes place. The voltages present in thunderstorms may be extremely high, of the order of 100,000,000 volts, and are subject to wide and sudden variations. While the quantities of electricity in lightning flashes are small,—of the order of 20 coulombs—yet as the time during which they are mobilized is short, the current in a flash might rise to 10,000 amperes. The total energy released in a lightning flash is not large, but the rate of expenditure of this energy may be tremendous.

In order to understand the mechanism of formation of a lightning flash it is necessary to have in mind the behavior of air at atmospheric pressure when overstressed. For a simple configuration of the electrodes mathematical physicists give us expressions for the current in terms of the physical quantities involved. A spark is said to pass when the current, as determined by the expression, becomes infinite. When the electric field is at all complex, the mathematical solution becomes too involved. Needless to say, it would be quite out of the question to determine mathematically the particular path to be taken by a lightning flash even though the initial configuration of the field were known. This particular path depends upon a combination of numerous small causes, or guiding circumstances, so that it should be possible to show the general tendencies of the discharge by the use of statistical methods.

The conductivity of the atmosphere is due to the presence of ions which, when acted upon by an electric field, give rise to a displacement current. If it were not for these charged particles, air would be a perfect insulator. Ions are always present in the atmosphere, as they are formed spontaneously due to the action of traces of radio active matter, cosmic rays, etc. However, the conductivity which these sources of ions lend to the atmosphere is so small that it has no engineering significance. When the saturation current is not reached, that is, when the ions are not removed by the field as fast as they are formed, then the current through the air is proportional to the voltage gradient and to the cross-section of the path. In symbols, the relation is

$$I \alpha G . A \qquad (1)$$

where I is the current, G the voltage gradient, and A the cross-section of the path considered.

Relation (1) represents the behavior of air under low voltage stresses.

When the voltage is gradually raised, the saturation current is soon reached. The voltage can then be raised considerably without change of current. However, as soon as the voltage gradient at some point in the field reaches 76 kv. per inch, the current suddenly increases. This is due to the cumulative effect of collision ionization. Ions present in the field are then so accelerated between collisions as to possess sufficient kinetic energy to produce new ions when they collide with neutral

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28-120

atoms. The rate of formation of ions is thus enormously accelerated. The clouds of ions thus formed are referred to as space charges and greatly disturb the original electric field. The action of space charges upon the electric field is always such that it prevents the gradient from rising above the critical value of 76 kv. per inch where the stress is most intense. Outside the region of breakdown the gradient is increased above the value it would have for the same applied voltage if the space charge were not present. This is illustrated in Fig. 1, *a* and *b*. The body *A* is at a positive potential and *B* at a negative potential, the configuration being such that the electric field diverges from *A*. If the potential between *A* and *B* is raised, the gradient in the neighborhood of *A* soon reaches the critical value for air. Collision ionization then enormously increases the supply of ions in that region. The negative ions travel towards *A* and become discharged. The positive ions travel toward *B* and form a space charge. At 1*b* the applied potential is much greater than at 1*a*, yet the gradient in the neighborhood of *A* has not been increased. However, the gradient between the space charge and *B* is greatly increased, as shown by the additional lines of force originating on the space charge and terminating on negative charges at *B*.

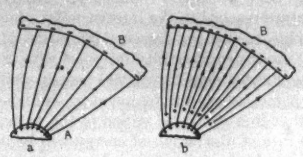

FIG. 1

When collision ionization takes place in a restricted region of a divergent field, the resulting phenomena are referred to as corona. Even though the initial electric field might have been symmetrical, the appearance of corona shows a strong tendency of the corona current toward concentration into preferred paths of restricted cross-section compared to the cross-section which seems available from mathematical considerations. This tendency results in the formation of corona streamers. It may be represented in symbols by the relation

$$g \propto A \,.\, F\,(i) \qquad (2)$$

or the conductance, g, varies as the cross-section of the path and some function of the current density, i. The function $F\,(i)$ is such that the increase in conductance resulting from an increase in current density more than offsets the decrease due to the reduction in cross-section.

Because in relation (2) the conductance of the path increases with a reduction in cross-section, the behavior of air under these conditions may be referred to as unstable, or abnormal, while the behavior of air when relation (1) holds may be called normal, as it is by far the most common in our experience.

The voltage which just causes corona to appear in the most stressed region of a divergent electric field is called the critical voltage. When this critical voltage is exceeded space charges are planted due to the action of the field upon the ions formed by collision in the corona region. The planting of these space charges can be referred to as being due to the flow of corona current through the overstressed air. The amount of corona current flowing depends upon the excess voltage applied above the critical voltage. It is evident that the corona current must be thus limited, because the space charge resulting from an excess of current would decrease the voltage gradient in the most stressed region of the field to such an extent that collision ionization would stop, thus effectively preventing a further increase in space charge.

The abnormal behavior of air when relation (2) holds can be illustrated in words as follows. Due to an increase in current density:

a. The SAME current is transmitted at a LOWER voltage gradient when the cross-section of the path is reduced;

b. For the SAME voltage gradient MORE current is transmitted through the air when the cross-section of the path is decreased; or

c. MORE current is transmitted at a LOWER voltage gradient when the cross-section of the path is reduced.

The formation of corona streamers is the first stage in the building up of preferred discharge paths of restricted cross-section. Although the exact mechanism of formation of corona streamers is not known, it is practically certain that it cannot be due to an increase in temperature causing a reduction in air density with a corresponding decrease in the critical gradient required for cumulative collision ionization. The various manifestations of corona, such as light, sound, and chemical changes, show that considerable ionic activity is present in corona streamers. Physicists give us plausible reasons why such intense ionic activity in corona streamers should have a tendency to lower the critical gradient of cumulative collision ionization. Some results obtained by them with the discharge tube suggested that the bluish light observed was not due directly to the recombination of ions. At the low pressures used, the velocity of the ions due to the electric field would be such that the Doppler effect should be observed. As this is not the case it has been concluded that the sources of light were neutral atoms not affected by the electric field, and consequently at rest. An explanation which has been proposed is that when ions of opposite sign recombine to form neutral atoms, *i. e.*, when a positive nucleus recaptures a lost electron, the quantum of energy radiated is in the form of very short wavelengths outside the visible spectrum. These rays are supposed to have a very limited range. They have the power to excite neutral atoms, that is, they furnish sufficient energy to a neutral atom to lift one

of its electrons from an inner stable orbit to a less stable outer orbit, where the electron possesses potential energy with respect to the nucleus. The electron upon falling back to a more stable orbit radiates this energy at a longer wave length which falls in the visible spectrum. It is this light emanating from neutral atoms which is observed.

With the above in mind, we must consider that the intense ionic activity in a corona streamer is due to the active recombination of ions constantly taking place. Due to such active recombination a considerable number of neutral atoms in the streamer is excited, that is, the electron has already been furnished part of the work required for complete ionization. For these excited atoms a collision of lesser magnitude, corresponding to a lower voltage gradient, will suffice to complete the work of ionization. Such a mechanism explains why the space charge can be mobilized through corona streamers at voltage gradients slightly lower than would be required in the surrounding space.

As the current density in a streamer further increases, temperature begins to play a more important part. In the final stage, a high degree of conductivity, perhaps

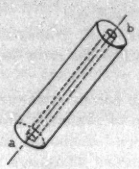

Fig. 2

greater than that of metallic conductors of the same cross-section, is imparted to the filamentary channels due to the very high temperature attained as the result of the large amount of energy released per unit volume in the conducting path. It is probable that when the temperature in the filament is sufficiently high, the violent thermal agitation of the gas suffices to keep up a supply of ions great enough to account for the high conductivity attained. This final stage in the condensation of the discharge into narrow filamentary channels lends itself particularly well to analysis and will be discussed here, but it must be remembered that the change from the initial corona streamer is gradual and not in distinct steps.

Let us assume that a discharge is being transmitted from a to b, Fig. 2, along a cylindrical path. Some heat will be generated by the passage of the discharge. At low temperatures the heat will be dissipated mostly by conduction or convection across the cylindrical boundary surface. We can reasonably assume that the conductance of the path varies in some manner with the temperature, T. This will be the case when the thermal agitation of the gas becomes so high that a certain proportion of the collisions will result in ionization. Theoretically, for any temperature above absolute zero, some velocities will be sufficiently high to cause ionization, but the probability of such velocities at low temperatures is vanishingly small. However, this probability increases rapidly with the temperature. The ions when free are acted upon by the electric field and subjected to a drift which is superimposed upon the random thermal agitation and is equivalent to a current. Thus the path has some of the properties of metallic conduction. The current will depend upon the proportion of free ions, and this in turn will depend upon the temperature. We can therefore write

$$g \alpha A \,.\, F\,(i, T) \qquad \text{(2a)}$$

or the conductance varies as the cross-section of the path and some function of the current density and temperature.

This relation is essentially unstable, because the temperature reached is caused by the energy loss per unit volume, and therefore depends on the current density, and the current density for a given voltage gradient depends on the conductivity which increases in some fashion with the temperature.

Coming back to the cylindrical path a–b, Fig. 2, the heat generated by the passage of the discharge must at first be dissipated mainly by conduction or convection through the cylindrical boundary. Therefore, the temperature reached in the interior will be higher than the average, so that the degree of ionization will also be higher. As the conductivity increases with the ionization, the current density will not remain uniform but will be higher where the temperature is higher. This means that in a restricted region of the path along its axis the rate of dissipation of energy will be increased, causing a further increase of temperature. This process continues until some limiting action takes place, so that finally the discharge will be concentrated in a narrow thread-like filament and will require a smaller voltage gradient than initially.

The limiting action referred to is not far to seek. When the temperature is very high, radiation has a greater influence upon cooling the filament than conduction. The radiation of a black body varies as the fourth power of the absolute temperature, and such a relation is more than ample to limit the path to a finite cross-section by limiting the temperature which the filament might attain.

Work done by physicists on exploded wires gives some idea of the characteristics of gaseous thermal dissociation filaments. The temperatures thus reached are the highest attainable. The ionization is so intense that the gaseous column is opaque to light because the free electrons impart to it some of the optical properties of metals, even though on account of the high temperatures reached the density of the gaseous column is very low.

In a thundercloud, positive and negative charges are distributed throughout great volumes of cloud in the form of large ions having low mobilities. It is con-

venient to represent the stress to which the intervening air is subjected by drawing lines from any suitable unit of positive charge to a similar unit of negative charge, as shown in Fig. 3. These lines are always at right angles to the equipotential surfaces, the traces of which are also shown in Fig. 3. The number of lines crossing a unit surface normal to them is a measure of the stress

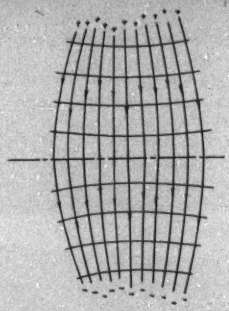

Fig. 3

at the point considered. Hence, the stress, or voltage gradient, is greatest where the lines of force are closest together. In Fig. 3 the lines are drawn from a positively charged portion of cloud to a negatively charged portion,—or the neutral plane might be considered to represent the ground surface, in which case the negative charge is an electrical image of the positive charge—a great help in computing the field between a charged body and the ground.

If in Fig. 3 we consider the neutral plane to be the ground surface, it is seen that the stress is smallest at the ground because the lines of force are farthest apart there. This will always be the case, as the lines of force must either originate or end on the charges in the cloud depending upon the polarity—a pure convention—but will then spread apart in such a manner that the permittance of the path between cloud and earth is maximum. Due to irregularities at the ground surface there will no doubt be high local stresses, but even if breakdown takes place, this as a rule will result only in the formation of a local space charge which will be sufficient to relieve the stress, because the volume of overstressed air is small. Breakdown will generally take place in the cloud due to the wind shifting the charges in such a manner that a large volume of air becomes overstressed. This is necessary in order that sufficient energy be available for the formation of filaments of high conductivity. These conducting paths then transmit the high stresses to their end points and lengthen by successive breakdown of the air.

The manner in which a filament of high conductivity transmits the stress to its end points will now be illustrated. Fig. 4a shows the lines of force in a parallel field, the voltage gradient being considerably below the critical value for air. Let the dotted line represent a channel of high conductivity caused by thermal dissociation. This channel contains both positive and negative ions in equal numbers so that their total effect upon the external field is zero. The ions continually form and recombine, the percentage of atoms ionized at a given temperature of the channel remaining constant. While the ions are free they are acted upon by the electric field, so that a small unidirectional shift is superposed upon their random thermal agitation, the positive ions shifting towards one end of the filament and the negative ions shifting towards the other end. This takes place along the whole length of the channel, and although each ion may shift only slightly, if their number is large the total effect might represent a very large current. Due to this shifting caused by the electric field, ions of one sign will accumulate at one end of the filament and ions of the opposite sign at the other end. Such an excess of ions of one sign at the ends of the filament alters the original field in a manner shown at 4b. It is seen that the gradient at each end of the filament has been considerably increased, while the gradient along the filament, which causes the shifting action, has been decreased.

If the energy initially stored in the dielectric is too small, the shifting action in the filament will cease as soon as the voltage gradient along the filament falls to a sufficiently low value. This will happen before the critical gradient for air has been reached at the end points. The conductivity of the filament is due to the high temperature caused by the passage of the current in a very restricted channel, and as soon as the current decreases the filament will cool and its resistance will increase. This will decrease the current still further, resulting in additional cooling and increase in resistance. This action will continue at an accelerated rate until all traces of the path have disappeared, just as if a switch had suddenly been opened.

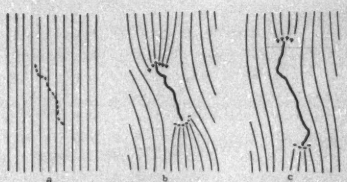

Fig. 4

On the other hand, if the energy stored in the dielectric is sufficiently high, then the critical gradient for air will be reached at the end points before the gradient along the channel has fallen to the point where the resulting current fails to maintain thermal dissociation in the channel. As soon as the critical gradient is reached at the end points, collision ionization takes place

and corona streamers form radiating outward from the point. As a rule, thermal dissociation will first become effective in one of these streamers. When this takes place large currents will flow along this streamer in the manner outlined above, resulting in a considerable reduction in the voltage gradient. The high gradient required to maintain the action along the other corona streamers having collapsed, the activity in them will stop. Fig. 4c illustrates the lengthened channel, showing already an accumulation of charges and increased gradients at the end points. The channel continues to grow in this manner as long as the gradients at its end points reach sufficiently high values to cause local breakdown of the air.

The conductivity reached in a channel where thermal dissociation takes place depends solely on the current density and resulting temperature, so that the gradient along the channel may fall to a value as low as a few hundred volts per foot, while the original gradient may be as high as 100,000 volts per foot. Thus, as soon as

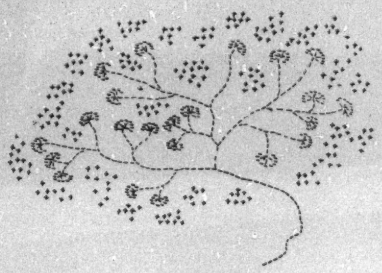

Fig. 5

the conducting channel reaches a length of several hundred feet, the energy available to cause breakdown at its end points becomes very large. The voltage gradient accumulating at the end points is limited by the breakdown strength of air which results in lengthening the channel. The voltage used up in this manner might reach five million volts. For a strong initial field, the rate of breakdown and lengthening of the filament increases.

The channel will continue to increase in length, branching out in the clouds wherever the necessary gradient is available, until the potential of the cloud has been so reduced that the current density in the channel can no longer be maintained, when the action stops in the manner outlined above. Fig. 5 illustrates a cloud discharged in this manner. The dotted lines show where the conducting filaments were located. It will be seen that the initial charges are still located in the cloud. Wherever the gradient was sufficiently high, ionization channels formed which planted space charges of opposite sign throughout the body of the cloud. The potential of the cloud has thus been considerably reduced. The gradient between opposite charges in the cloud is still quite high, and the charges gradually neutralize each other due to convection. However, the resultant gradient between cloud and earth has fallen to such a low value that it cannot permit the growth of, or even maintain, conducting channels.

The above analysis shows that any oscillatory discharge of the cloud is out of the question. Some authors have come to the conclusion that the current in a lightning flash rapidly alternates by assuming the cloud to be a conducting surface, forming a condenser with the earth, while the main flash formed a low resistance path between the oppositely charged plates of the condenser. When it is considered that the charges in the cloud reside on water particles of very low mobility isolated from one another, and that the final branching of the flash in the cloud must be composed of corona streamers requiring gradients of the order of 10,000 volts per inch, while collision ionization at their end points requires 76,000 volts per inch at normal air density, it becomes apparent that a very high resistance is at all times in series with the main discharge path, so that the discharge of the cloud cannot even proceed until ground potential has been reached.

In the cloud, conducting channels will branch out in various directions wherever the requisite gradient for their growth is available. Along the main flash, however, where the initial field is approximately parallel, the branching of the conducting channel requires that two corona streamers shall pass simultaneously through the same values of temperature and conductivity. Should the conductivity of one of the streamers become greater than that of the other, the voltage gradient will fall too rapidly to allow the other streamer to continue its growth and the air in it will quickly resume its original non-conducting properties.

While the probability of the branching of the main channel at any one local breakdown is small, photographs of lightning often show flashes in which such branching has occurred. Each branch then grows independently of the other. As a rule, one of the branches will connect to earth before the other and its brilliancy will greatly increase due to the great increase in current density and temperature caused by the discharge of the cloud. The electric field between cloud and earth will then fall to a value so low that it does no longer allow the other branch to continue its growth. The main flash will thus appear in the photograph to be much brighter, while the fainter branch apparently terminates in mid-air. However, there have been quite a few reliable observations of lightning in which both branches of the flash connected to earth at the same time, so that the discharge current divided itself equally between them and they attained the same brilliancy. There are even reliable reports of three such branches connecting to earth simultaneously. The above analysis shows how vanishingly small the probability of the latter event must be.

As a rule, the main branches discharging various portions of the cloud will not grow simultaneously, but instead will succeed each other in time, perhaps overlapping to some extent. At any event, the maxima of current in the various branches will not occur at the same time. When one of the main branches taps new portions of the cloud, the current in the main channel will increase, resulting in an increase in its temperature and in the amount of light it radiates. Thus, coincident with the growth of the large branches discharging the cloud, the main channel will radiate light of increased brilliancy. This is strikingly shown in photographs of lightning with a moving camera, the maxima of brilliancy appearing as parallel flashes on the photograph.

The main channel of a lightning flash forms a very effective radio antenna. The current in this channel rises rather slowly, reaching a maximum in perhaps 0.01 second. However, as the main current is the resultant of numerous local breakdowns in the cloud occurring more or less at random, it will not vary uniformly. There will be pulsations of current of large

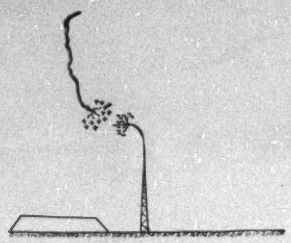

Fig. 6

magnitude corresponding to the formation of the main branches discharging the cloud. These can be heard directly in a telephone receiver connected to an antenna. Besides these large pulsations of current, there will be smaller ones corresponding to the growth of the smaller streamers, some of very short duration giving an impulse which will affect a radio antenna by causing it to swing at its own frequency.

It remains to indicate what bearing the above analysis may have on the possible application of a scale factor to laboratory experiments in lightning protection. Even the tallest lightning rods distort the general field of a thundercloud to such a slight extent, that they scarcely can have any influence on the instigation and guidance of a lightning flash. Whenever a lightning flash originates in the clouds and grows toward a lightning rod, during the period of growth the gradient at the rod increases, at first slowly, then more rapidly as the end of the conducting channel comes closer to the rod. When it approaches within less than 100 feet from the rod, the gradient at the rod might reach a value of the order of 300,000 volts per foot. This is illustrated in Fig. 6. The voltage gradient and volume of overstressed air at the rod then becomes sufficiently great to permit the formation of a streamer of high conductivity starting out from the rod and growing in the general direction in which the field is most intense. When this occurs, the gradient between the ends of the growing lightning flash and the streamer from the rod may reach values several times as great as the gradient in any other direction, and the two ends therefore grow towards each other until they ultimately connect. This explains why lightning hits elevated points which initially do not sufficiently distort the original field to guide the flash.

Similar considerations show why some lightning transients on transmission lines may have steep wave fronts of the order of a few microseconds. If a lightning discharge strikes very close to a transmission line, the voltage gradient at the point of nearest approach first increases slowly, reaches a maximum, and then suddenly collapses in the time required for the flash to bridge the last 100 feet or so of the gap. The latter time depends upon the driving power behind the flash, that is, the energy initially stored in the electric field of the thundercloud.

In the case of very small scale laboratory experiments, the volume of overstressed air around the grounded point is not sufficient to allow the formation of streamers of high conductivity, and the protective ratio obtained is likely to be too low. There is also the danger of having the scale of the experiment so small that the successive breakdown of air in an actual lightning flash is no longer simulated. As the cloud in laboratory experiments is replaced by a conducting surface, the manner of discharge of a thundercloud by means of streamers branching out throughout its volume, and the resulting gradual rise of the current in the main flash, pulsating to some extent, is never simulated in the laboratory.

To increase the effectiveness of a protective installation, it is desirable that the objects to be protected be given such a shape as to prevent breakdown of the air (as at sharp corners), under the intense stress just before the flash connects to ground, thus decreasing as much as possible the chance for the formation of conducting streamers. On the other hand, the protecting towers should be so located as to allow the formation of these conducting streamers at as low a stress as possible, to increase the likelihood of the flash connecting with them.

It must be kept in mind that the culmination of a lightning flash striking a particular object is the result of a particular combination of numerous small causes, or guiding circumstances, so that protection can seldom be made absolute. "Freaks" will occur when the combination of circumstances is particularly unfavorable to an exposed object, while at the same time favorable to a less exposed object, so that a streamer from the latter actually connects with the flash. The probability

of such an unfavorable event can perhaps be made vanishingly small by paying close attention to the contributing factors outlined above.

In the above analysis of the behavior of air in thunderstorms no distinction has been made between positive and negative clouds, because it is believed by the author that present high-voltage laboratory experience does not warrant such a distinction. Although the volumes of overstressed air in the laboratory are rather small, yet the voltage gradients attained are as large as those occurring in thunderstorms. Space charge studies show that negative ions do not necessarily travel farther than positive ions in these intense fields. For small stresses there are striking differences between positive and negative corona. However, when photographs of arrested spark discharges are obtained, showing the initial growth of streamers under sudden stresses, no striking differences can be observed between streamers growing out from either a positive or negative body.

Reliable field observations may show that there are striking differences between flashes from positive and negative clouds. Even so, it is believed that the qualitative analysis presented above, based on considerations which perhaps are too general, will enable observers to secure data giving us a better understanding of lightning phenomena.

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Discussion

H. J. Ryan: Study of "ultimates" has established the identity of the electric spark and of the lightning stroke. It would be meticulous to particularize wherein they differ in essential qualities. Surely they have this much in common: metallic or other conducting electrodes are unessential. They are discharges through conducting air channels of intense ionization and high conductivity that develop with extreme suddenness when the voltage gradient in the corona discharge channel exceeds a certain critical value. The channels develop high temperatures and intense thermal electromagnetic radiations that are both luminous and non-luminous.

In the thunderstorm the scale of operations is such that lightning occurs with facility between the opposing charges held in clouds or between clouds and earth. In the laboratory this facility of spark-discharge without metallic electrodes is rarely encountered. The best we have found so far was suggested by members of our present laboratory personnel. Two 500-watt gas-filled, incandescent lamps were mounted axially as the discharging electrodes of a 60-cycle high-voltage source, neutral grounded, with their tipless bulbs in face opposition, about one-fifth of their diameters apart. The action to be inaugurated was materially enhanced by the connection of an air-core condenser of

FIG. 1

almost any convenient, available size, in multiple with the lamp electrodes. As the voltage was raised the gas in the lamps was ionized and the resulting space charges were moved up and bound to the insides of the opposing faces of the lamp bulbs. As the voltage was further elevated, the air in the open space between the outer walls of the lamp bulbs was overstressed. Ions resulted in abundance that were rapidly separated by the electric field and driven to their respective bulb faces on the inner walls of which there were bound charges of opposite polarity. Then the charges thus placed on the outer walls of the bulbs were reversed by spark-conduction through the open air in the succession of ensuing alternating voltage crests. They were the snappy, blue-white, enormously quick discharges that we know to be sparks in which metallic electrodes have no direct function. Fig. 1 herewith is a reproduction of a photograph of a small series of such sparks.

When 60-cycle voltage at a value near to flashover is applied with the neutral of the source grounded, between pointed metal electrodes, 20 ft. apart, they are covered profusely with corona and at irregular intervals of a few seconds, sparks that terminate abruptly in space having a length of several feet are produced from each electrode. Each spark appears blue-white, with a snap. To the unaided eye it terminates abruptly. Through the telescope it is seen to terminate in a brush of a few straight,

highly divergent, short streamers. We believe these to be substantially a laboratory form of the spark-streamer attached to the lightning rod in Fig. 6 of the Lissman paper. Our laboratory personnel obtained photographs of some of these sparks with a quartz lens from which they scaled lengths. Fig. 2 herewith is a reproduction of a pen-and-ink tracing of one of them, made in an optical projector. They have not photographed well enough for direct reproduction.

To focus one's mental picture of lightning the author has recognized and specified three distinct stages of electrical conduction in thunderstorms:

1. Convection of ions.
2. Corona, visible as glow, brush, and streamers.
3. Lightning, the nearest laboratory form of which is the spark.

In specifying the cause of the transition from the corona to the lightning stage of conduction he has used the hypothetical term "thermal dissociation." The difficulty with this term is due to the fact that many will assume that the intense ionization constituting the advanced degree of electrical or physical dissociation that occurs in the spark or the stroke is thus specified by the author to be the result of high temperature. Now high temperature and thermal effects may differ greatly. Some physicists have shown that high temperature will not account for the intense

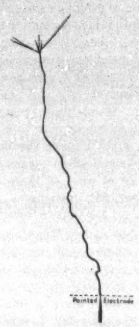

Fig. 2

ionizations present in the spark. Others appear to be finding that thermal radiations including their near relatives, the luminous or ultra-luminous radiations, do facilitate ionization. Thus the author selected the term *thermal dissociation* to specify the group of causes that bring about the intense ionization in the spark and in the lightning stroke.

The author's view that the numbers of positive and negative ions in the lightning stroke (or spark channel) at any one instant are equal, is also hypothetical. The view doubtless facilitates the approximation of the focus of the true picture for present practical purposes. Only subsequent study can determine more definitely this phase of the conditions present in the spark channel.

On the whole I am much indebted to the author for this paper because of the mental picture of lightning that it has established. It will help in our work of alert observation of the actual lightning occurring in thunderstorms and of its close relative, the spark, in the laboratory. For it is only through further, well directed observations and the analysis of the results obtained therefrom that the true nature of lightning will eventually become more abundantly known.

R. W. Sorensen: Mr. Lissman's paper again gives us opportunity to emphasize the need of research as to the character of lightning. In providing protection for oil reservoirs certain experiments have been conducted, all of them based largely on the assumption that properly erected lightning rods well grounded may be relied upon to protect a rather large area around the rod. The data led us to believe that under certain conditions we might always have protection from a rod over a distance of four times rod height. Further experiments, however, have certainly indicated that no such general law can be made. The oil companies in California have erected lightning rods or towers, the spacing of which is determined on the basis that a rod will protect a distance of about two and one-half times the rod height. Our laboratory work indicates that even this protection cannot be considered as absolute because often some condition arises which causes a lightning stroke to behave differently from that of many other strokes. In such cases rods do not protect for a distance of even two and one-half times their height.

I have observed a lightning storm in which for more than half an hour the number of strokes was in excess of 100 per minute. I do not know what portion of the strokes were between clouds and ground and what portion were from cloud to cloud. This observation confirms the fact that in many cases the accumulation of charge occurs at a very rapid rate; in fact, at a rate so great as to eliminate most certainly one proposal which has gained a great deal of attention; namely, that by the erection of a large number of small points, as for example a high fence of barbed wire, the charges might be dissipated so rapidly as to prevent lightning strokes. The observation of such a storm as this indicated that aviation will introduce lightning problems which at present are practically unknown to us.

H. E. Mendenhall: I believe some of the terms used in this paper are not in agreement with the usual practise. The author uses such terms as, "thermal ionization," and "thermal dissociation" in such a way as to confuse their true meanings.

By "thermal dissociation" I assume he refers to the process of ionization by collision which occurs at high temperatures as a result of collisions between very fast-moving ions and atoms.

By "thermal ionization" I believe he refers to the process of ionization by radiation known as the photoelectric process. If so, I feel that the expression "ionization by the photoelectric process" used in place of "thermal ionization" would be more expressive in this connection.

Bradley Cozzens: Many theories have been advanced as to the manner of formation of lightning. This very intensive paper is, for the most part, in line with the generally growing idea that lightning cloud voltages are not nearly so high as previously conceived. Lightning voltages have been estimated to be of the order of 300,000,000 volts, while very recent investigations indicate that the value may be of the order of 10,000,000 to 30,000,000 volts.

Going back to Dr. Steinmetz and his investigations with Mr. Peek, and the early studies of Dr. H. J. Ryan, on the electric strength of thin air films, it has been found that any gap up to 0.001 in. will require 0.32 kv. to rupture. To rupture gaps up to 0.07 in. requires voltage gradients up to 20 kv. per in., and for gaps from 0.07 in. up to 100 in. requires average voltage gradients across the gap of from 8 to 10 kv. per inch in order to rupture, depending upon the formation of the field. In a paper by Professor Carroll and myself (*Sphere-Gap and Points-Gap Arc-Over Voltage as Determined by Direct Measurement*, p. 1) it was found that this average value of voltage per inch necessary to break down gaps 180 in. in length between a point and plane had been reduced to 6 kv. per inch. It is not wished to convey the idea that these values will decrease indefinitely, but the general trend of all of the investigation so far conducted is for the average value of voltage per unit distance necessary for breakdown to decrease with the increase in gap length.

Mr. Lissman has pointed out the manner in which negative streamers may be formed from an electrode. In further explanation of the formation of streamers I should like to present the following theory. There is apparently a resistance, valve action between the cloud and the streamer, which we will call "entrance

resistance." Any streamer that may start from a section of the cloud will have inductance and resistance, and also capacity to the cloud and to earth. The resistance of the streamer will be low compared to the inductance for it is well known that the conductivity of an arc stream is as great as 1000 times that of metal conductors.

The cloud itself has capacity, but it is impossible to conceive of a current flow back into the cloud in the same manner in which it is drained from the cloud, which would be necessary were the cloud to be included in the oscillatory circuit. However, the streamer itself, being a low-resistance conductor, may support oscillation. It has long been known that high-frequency voltage will produce much longer streamers than a similar value of voltage continuously applied. Further, as the streamer increases in length, the inductance increases, and the frequency or natural period of the streamer is decreased. With this decrease in frequency, the energy radiated electro-magnetically per unit length is also decreased.

Mr. Lissman has called attention to the fact that ionization as caused by a spark in air persists for some time after the flow of current has ceased. The flow of current into a streamer will not be continuous and as a result, at the time the current ceases there may be considerable potential built up between sections of the cloud and the old streamer channel. Since this old streamer channel is the point of greatest potential difference, the cloud will again discharge into the streamer and will cause the streamer to progress a greater distance due to the small amount of energy necessary to establish the streamer over the previously ionized channel. One or more oscillations may occur within the streamer before the cloud will again discharge into it, finally causing it to reach the earth as a unidirectional pulse.

With 60-cycle applied voltage, streamers are formed on succeeding crests, and are often found to follow similar routes on each succeeding cycle. These streamers have been photographed extending 10 ft. from a pointed electrode.

A recent cathode-ray oscillogram as obtained by the General Electric Company on the lines of the Pennsylvania Power Company showed a voltage wave induced upon the line which started in an oscillatory manner, somewhat above 200 kv., and built up in a transient manner with superimposed oscillations, over a period of 5 microseconds, to a value of 1500 kv. At this point the voltage rose suddenly to 2500 kv., which is explained as an insulator flashover, but may have been a stroke direct or close to the line resulting in the insulator flashover.

The fact that streamers are formed and have such a high conductivity is similar to placing a conducting element close to the cloud, which places the greater portion of the cloud potential at the head of the streamer. The added impulses of voltage as a result of the successive discharges from the cloud into the streamer will account for the long lightning strokes as a result of the rather moderate value of cloud voltage.

Joseph Slepian: (communicated after adjournment) This paper I believe to be the best analysis yet published of the physical occurrences during a lightning flash. I have a few remarks to make which are mostly of a confirmatory nature.

I think it is unfortunate that Mr. Lissman emphasizes the particular voltage gradient of 76 kv. per inch as the one at which cumulative collision ionization begins. The erroneous impression is still widespread in engineering circles that air has a definite critical breakdown gradient of 76 kv. per inch independent of shape and separation of electrodes. This impression seems to be due to the false analogy with failure of materials under mechanical stress, and is quite at variance with experimental facts. Even for plane electrodes, the breakdown gradient varies with electrode separation and may be greater or less than 76 kv. per inch, and for electrode shapes such as points, the maximum gradient at breakdown may be very much greater than 76 kv. per inch.

The part played by space charges during the formation of the flash is very important and is well brought out by Mr. Lissman. Such space charges are neglected in the classical Townsend theory of breakdown between plane electrodes, but in this case their consideration is necessary for understanding of the phenomena. The part played by space charge in breakdown was also pointed out by Professor Loeb, (*Journ. Franklin Inst.*, 1928) and Mr. Torok has recently obtained some corroboratory evidence of the existence of these space charges at the Westinghouse High-Voltage Laboratory.

The development of streamers and conducting filaments as a consequence of high temperature and thermal ionization is an attractive idea, and one which I have used in trying to account for the very great speed of breakdown of spark gaps (*Electrical World*, April 14, 1928, p. 761). I make this thermal ionization come into the picture earlier than Mr. Lissman does, however, as my calculations indicated that sufficiently high temperatures would be reached in the ionizaton path of a single initiating electron. I think this view is supported by the extreme speed with which streamers are formed as in Torok's suppressed discharges.[1]

The explanation given of how very steep wave-front voltage may be induced in a transmission line by a lightning discharge which does not actually strike the line is important, and the need for such an explanation has been felt for some time, since the slow rate of propagation of streamers has been known. To make this explanation complete it is necessary to state that considerable current is flowing in the incompleted streamer. I am happy to say that this last had been experimentally confirmed previous to this paper by Mr. Torok at my suggestion, and we hope to publish these results soon.

J. S. Carroll: Mr. Mendenhall brought up the matter of terms. I am quite in agreement with him in that many of these matters of discussion depend upon the definition of terms. I think the term photoelectric effect comes in here all right, and it could be used in place of some of these others. But I would hesitate myself to alter Mr. Lissman's phraseology, because he has been in the work a little longer than I have.

I should like to emphasize Dr. Ryan's point on the growth of this streamer that we observed in the laboratory. The single streamer from the point appears like a wire extended out with the full voltage on the end and the corona forming out at the end. That is along the line of Mr. Lissman's idea of the lightning stroke. That is one reason why I agree with Mr. Cozzens in the belief that the lightning voltages are not as high as we used to estimate. If you have a cloud above a line at a potential, say, of 10,000,000 volts, then start the streamer down, the streamer is just like a wire pushed down through the cloud when you get near the line. In that case you are subjecting the line to the potential of the cloud almost directly. This is just a theory so far. As pointed out, the matter of electric fields under the clouds is an uncertain variable proposition.

M. A. Lissman: The preceding discussion has shown that it would be advisable to expand some of the concepts advanced in the paper. There, the statement was made that in a conducting streamer ions of both signs are practically equal in number, so that their total effect upon the electric field is zero. It seems to me impossible to escape such a conclusion.

Whatever may be the cause of the high conductivity in a streamer, we know that very large instantaneous currents flow urged by very small voltage gradients. Such a high conductivity necessitates a high concentration of ions. In a metallic conductor we know that the ions are confined with the boundaries of the conductor at ordinary temperature. Incidentally, we know also that within the conductor positive and negative ions offset each other, so that there are no high local electric stresses between adjacent portions of the conductor.

In the case of a conducting filament, we cannot assume that

1. *Surge Impulse Breakdown of Air*, J. J. Torok, A. I. E. E. Quarterly Trans., Vol. 47, April, 1928, p. 349.

the ions are confined within sharp boundaries. Also we must assume, for the present at least, that each ion can only be affected by the electric gradient prevailing where the ion is located. This gradient is the resultant of all surrounding electric charges. It is a vector quantity having both magnitude and direction. Some authors have published sketches of lightning paths of high conductivity composed only of ions of one sign. We know these paths to be very restricted in cross-section, and if we attempt to determine the gradient affecting these ions, we find that it must have a very high value and be directed radially outward. If such a filament could come into being, it would immediately explode, as we know of no force which could prevent the ions from moving in the direction of the electric gradient. We know that such filaments are stable, and that they have a core of high conductivity and restricted cross-section. We can assume that a sufficient number of ions of one sign move radially outward from the highly conducting core to offset the radial component of the electric field, thus shielding the core. There remains a small longitudinal component which causes enormously large currents to flow in the restricted filament. It does not seem possible to escape the conclusion that ions of both signs must be present in substantially equal numbers in this highly conducting core, thus preventing high local stresses between adjacent portions of the core.

The objection made by Dr. Slepian against the specific mention of a voltage gradient of 76 kv. per inch being required to produce cumulative collision ionization is well taken. It would have been better to introduce some constant K, specifying that in common with most of our engineering constants it has the habit of varying with the mean free path of the molecules, and perhaps as the result of a number of other causes.

In the present paper the author endeavored to outline that if we assume a sufficiently restricted region of the field, any ordinary time interval which we might use as a unit, such as a microsecond, is long compared to that required to produce collision ionization, and that for a given mean free path the instantaneous gradient is the only factor affecting cumulative ionization. This gradient is the resultant of all surrounding electric charges, but is especially affected by those in the immediate vicinity, which came into being as the result of cumulative collision ionization taking place in the adjacent portions of the field.

When we ordinarily speak of a gradient causing collision ionization for a certain configuration of the electrodes, we mean a hypothetical gradient obtained by dividing the applied voltage by the distance between the electrodes when these are plane, or a mathematical gradient disregarding space charges when they are not. We may consider this applied voltage causing breakdown to be the statistical summation of numerous local breakdowns. We know that it depends to a large extent upon the manner in which space charges are mobilized in the field, and therefore, upon the shape, separation of the electrodes, and the waveform of the stress applied. In the case of large point gaps we are forcibly reminded that conditions in the gap are far from uniform at breakdown. Even in the case of small sphere gaps where we might easily imagine this to be the case, the work recently published by Mr. Torok has shown that it is not so. If we had means of determining the gradient in a restricted portion of the field, the resultant of the distribution of the surrounding electric charges, it is possible that we would discover that cumulative collision ionization becomes effective at a definite electric gradient.

The author cannot entertain at present the idea advanced by Mr. Cozzens of an oscillating streamer whose frequency decreases as it grows in length. It is granted that a streamer has high conductivity and must have inductance. Yet to obtain satisfactory oscillations we must also have capacity in order to transform the kinetic energy of the magnetic field into the potential energy of the electric field. In the present paper the author endeavored to show that before any reasonable amount of electricity could be stored at the end of a streamer, this streamer had to branch out many times with increasing resistance in the branches, and that the final charges are deposited through the process of collision ionization. This introduces sufficient series resistance to cause critical damping. There can be no energy stored by a convection current of ions, as by definition of ion mobility we assume that the energy made available by the potential through which they fall is immediately transformed into other forms of energy.

I would suggest to Mr. Cozzens that he assume a convenient electric charge to accumulate at the end of a streamer without branching, and then compute the resultant electric field. The values of gradient thus obtained should prove enlightening.

However, the experimental evidence upon which this theory is based is quite interesting. When the development of a 60-cycle flashover is photographed by means of a quartz lens on a rotating film, it is found that the power arc does not immediately follow the enormously rapid condenser discharge. Rather, several condenser discharges succeed each other at a definite frequency, following the same path.

I am tempted to give a widely different explanation from that advanced by Mr. Cozzens. There is a certain amount of energy stored as a displacement current between the transformer terminals, and between these and the ground. This energy stored in the electric field is immediately available when breakdown culminates into a condenser discharge, as the series inductance is quite low. The resultant temperature in the streamer is very high, but as the filament is still very restricted, it cools rapidly. As soon as the condenser is discharged, the current in the streamer dies down and it begins to cool. The transformer terminals are now discharged, and due to the large value of leakage reactance in the secondary, it takes an appreciable time to restore the charging current to the terminals. During this charging time the filament has cooled sufficiently so that the succeeding discharge, although following the same path, is not yet a power arc, but another condenser discharge. This takes place several times over each crest. In the meantime heat diffuses radially outward from the filament, and finally the volume of heated air is sufficient to prevent rapid cooling and true arc conduction is initiated.

Thus, a condenser discharge is caused by a streamer lengthening by successive breakdowns due to a very high gradient at its growing end, resulting in a high average gradient, while the arc discharge has a low average gradient. The arc does not grow by successive breakdown, but imitates metallic conduction in that a sufficient supply of ions is available to carry the current.

Heat Flow from Underground Electric Power Cables

BY NEIL P. BAILEY[1]

Non-member

Synopsis.—The heat resistance of the soil surrounding underground cable ducts is usually large, and the path of the heat through the soil is not simple, especially when there are several adjacent ducts. This heat resistance and the path of the heat flow may be determined graphically, but except in the simpler cases, this method becomes very difficult.

The heat resistance of any one of several ducts can be calculated theoretically by making assumptions of unknown magnitudes. To check these calculations by testing actual installations would be slow and expensive. Advantage may be taken of the similarity between the flow of heat and the flow of electricity to set up and test the electrical equivalent of any duct system. In this way the theoretical results can be checked readily with a minimum outlay of equipment.

Results obtained by this method show the theoretical calculations to be accurate even for close spacing and also give a picture of the actual path of the heat from a group of several ducts. Thus, the heat resistance of a duct may be found quite accurately if the specific heat resistance of the soil is known.

Object

SINCE the current carrying capacity of an underground electric power cable is usually limited by heating, it is desirable to understand the manner in which this heat is dissipated. It must obviously pass through the cable insulation and sheath and across the air-space to the duct. Up to this point the path of the heat is quite definite and it is not affected by surrounding cables. However, after the heat leaves the duct, its path is not so obvious, and the heat flow from one duct is affected by that from adjacent ones.

It is the object of this paper to present experimental verification of theoretical methods of calculating the heat resistance of the earth surrounding underground cable ducts.

Symbols

Q = Rate of heat flow per unit length of duct
T = Temperature
r = Duct radius
p = Any radial distance from the center of a duct
S = Specific heat resistance
C = Specific heat capacity
t = Time
d = Depth of a duct below the ground surface
l = Spacing between duct centers
R = Heat resistance per unit length of duct.

Direction of Heat Flow

The study of the heat flow from the ducts is facilitated by considering them as new cylindrical heat sources. Also, it is usually permissible to assume that the ducts are buried directly in the soil, since the heat resistance of the materials in which the ducts are commonly set is very close to that of most soils. (See Appendix B.)

In the case of a duct whose length is great compared with its diameter, it is evident that the heat will flow out radially into the soil. However, since the surface of the ground is within a few feet on one side, and there is an infinite extent of earth on the other, the heat will not flow out uniformly in all radial directions.

An idea of this heat distribution may be obtained by studying the heat flow from a duct surrounded by an infinite extent of soil in all radial directions.

Consider an annular ring of soil of specific heat resistance S and thickness $d\,p$ at a distance p from the center of the duct, of radius r. The heat resistance of this ring per unit length is given by,

$$d\,R = \frac{S\,d\,p}{2\,\pi\,p} \qquad (1)$$

Since the soil extends an infinite distance in all radial directions, the total heat resistance per unit length of duct is,

$$R = \int_r^{00} \frac{S\,d\,p}{2\,\pi\,p} = \frac{S}{2\,\pi} \log_e \frac{00}{r} = 00 \qquad (2)$$

From this, it is evident that in the case of an actual duct, no heat will flow downward after the transient period is over, for to do so it would have to flow into an infinite extent of soil.

The objection might be raised that since the heat capacity of the earth is infinite, steady state conditions will never be realized. However, in Appendix A, the heat capacity of the soil is considered and the transient period is shown to be over in a relatively short time.

Graphical Method of Calculating Heat Resistance

Knowing that all of the heat from a linear underground heat source flows to the surface of the earth, it is possible to solve for the heat resistance graphically. As shown by Fig. 1, the ground surface may be considered as an equi-temperature surface. Other equi-temperature surfaces may then be drawn in arbitrarily.

Lines of heat flow must cross these surfaces at right angles, for otherwise heat would be flowing without an

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28-113

accompanying temperature drop. If the rectangles between equi-temperature surfaces and lines of flow are squares, ($m = n$ in Fig. 1) and if the plot is for a unit length of source, the resistance of each square is numerically equal to the specific resistance of the soil. The resistance of each tube may be found from the number of squares in series, and the total resistance from the number of tubes in parallel.

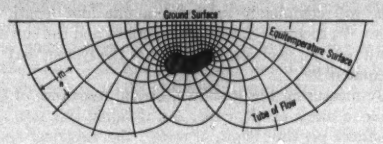

FIG. 1—GRAPHICAL METHOD OF CALCULATING THE HEAT RESISTANCE FOR ANY LINEAR UNDERGROUND HEAT SOURCE

This field plot is found by the method of trial and error, but only one solution exists. Lines of flow and equi-temperature surfaces must be drawn in and adjusted until the correct solution is obtained. In the example shown, the resistance of each square is S, the specific heat resistance of the soil. There are eight squares per tube, giving a resistance per tube of $8\,S$. There are 29 tubes in parallel, giving a total resistance per unit length of source of $8\,S/29$.

The beauty of this method is that it is independent of the cross-section of the source, as indicated in Fig. 1. The chief difficulty is that it becomes increasingly tedious as the number of ducts is increased.

THEORETICAL EQUATIONS FOR CYLINDRICAL SOURCES

Theoretical expressions for the heat resistance of underground ducts may be derived as follows:

The heat flow from a line source to the earth's surface

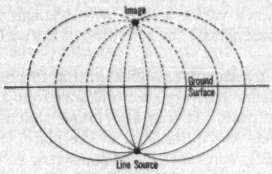

FIG. 2—GROUND SURFACE REPLACED BY THE IMAGE OF A LINE SOURCE

may be represented by the lower portion of Fig. 2 which is a line source from which the rate of heat flow is $+\,Q$, per unit length of source. If the earth's surface is considered as a zero temperature plane, this same field of flow may be realized by replacing the ground surface by an image of the line source at an equal distance above the surface. If the heat flow from the image is $-\,Q$ and the system is placed in the soil, the plane previously occupied by the surface of the ground will remain a zero temperature surface, and the heat flow from the original line source to this surface will be the same as before (see Fig. 2).

Making use of the principle of superposition, the temperature difference between any point and the zero temperature surface is that which the point would have due to the heat flow $+\,Q$ leaving the line charge radially and undistorted, plus that due to the heat $-\,Q$ flowing radially from the image.

Consider the point A in Fig. 3. When only the line source with a flow per unit length of $+\,Q$ is acting, the heat flows radially. The temperature drop across an annular ring of thickness $d\,p$ at a distance p from the source is,

$$\frac{Q\,S}{2\,\pi\,p}\,d\,p$$

The temperature difference between A and the zero temperature surface is,

$$\Delta\,T_1 = \int_{d_1}^{h} \frac{Q\,S\,d\,p}{2\,\pi\,p} = \frac{Q\,S}{2\,\pi}\log_e\frac{h}{d_1} \qquad (3)$$

In a similar manner, the difference in temperature

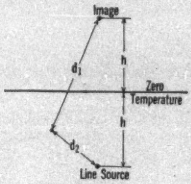

FIG. 3

between A and the zero temperature surface due to the flow $-\,Q$ from the image acting alone is,

$$\Delta\,T_2 = \int_{d_2}^{h} \frac{-\,Q\,S}{2\,\pi\,p}\,d\,p = \frac{-\,Q\,S}{2\,\pi}\log_e\frac{h}{d_2} \qquad (4)$$

Thus, the temperature difference for both acting simultaneously is the sum of these two, or

$$\Delta\,T = \frac{Q\,S}{2\,\pi}\left[\log_e\frac{h}{d_1} - \log_e\frac{h}{d_2}\right]$$

$$= \frac{Q\,S}{2\,\pi}\log_e\frac{d_2}{d_1} \qquad (5)$$

So far, only a line source has been considered, and the actual case is that of a cylindrical heat source. By neglecting distortion, the line source may be considered as being at the center of the duct, and the duct surface will be approximately an equi-temperature surface. This assumption could be avoided by further refinement, but it was not found to be necessary.

Upon this basis, the temperature of the duct surface above the temperature of the ground surface for a duct of radius r buried at a depth d becomes,

$$\Delta T = \frac{Q S}{2 \pi} \log_e \frac{2 d}{r} \tag{6}$$

The resistance R per unit length of duct is,

$$R = \frac{\Delta T}{Q} = \frac{S}{2 \pi} \log_e \frac{2 d}{r} \tag{7}$$

In a similar manner, the heat resistance per unit length for any duct arrangement may be calculated. The temperature of any duct above that of the ground surface is the sum of the temperature differences due to the heat flow of each duct and image acting alone. Appendix C gives the theoretical equations for three and six ducts.

Since in these theoretical equations all effects of distortion are neglected, it is the chief aim of this paper to investigate the accuracy of such equations.

Experimental Verification of Theoretical Equations by Electrical Analogy

The laws governing the flow of heat are the same as those governing the flow of electricity, and the only difference between the equations of the two is one of units. The equations for the flow of heat through the soil are exactly similar to the equations for the flow of d-c. electricity through a conducting solution.

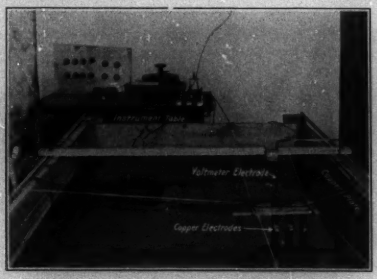

Fig. 4—Electrical Test Tank

Advantage was taken of this fact to check experimentally the theoretical equations previously derived. To run a comprehensive set of tests on actual cable installations would be a slow and expensive procedure, while the equivalent electrical circuits can be set up and tested very readily.

The heat system is represented in the following manner. A unit length of the duct and its surrounding soil may be replaced by a rectangular wooden box containing ordinary tap water. (See Fig. 4.) Since the heat all flows to the surface of the earth, the ground surface may be replaced by a copper plate along one vertical side of the box, the remainder of the box being waxed to insure its insulating qualities.

A length of duct is represented by copper tubing set vertically in the water. The temperature difference between the cable and the earth's surface may be replaced by a d-c. voltage between the copper plate and the copper tubing, and the flow of electricity replaces the heat flow.

If the electrical system is constructed with the ratio of depth of electrode from the plate to radius of electrode which corresponds to the ratio of depth of duct to radius of duct, an electrode in one foot depth of water in the tank will completely represent a one foot length of duct. If, then, a d-c. voltage is applied and the current measured, the electrical resistance of the system may be calculated.

By measuring the specific electrical resistance of the water in ohms per ft. cube, and dividing this into the resistance per unit depth of water, the electrical resistance coefficient (R/S), (ratio of resistance per unit length to specific resistance), may be found.

Now, since the flow of heat through the soil is governed by the same laws that govern the flow of electricity through the solution, and since the units of R/S are independent of system considered, the R/S for the heat system is exactly the same as the R/S for the equivalent electrical system. In this way it is possible to obtain an experimental check on the theoretical equations previously derived.

In running these tests it was necessary to take certain precautions to insure accurate results. In the first place, in order that the flow of electricity through the water would correspond to the flow of heat through the soil, it was necessary to limit the current density to such values that there would be no disturbing convection in the water due to heating and violent gassing at the electrodes. This limit was readily found by observation.

In the second place, it was necessary to devise some means of eliminating the error due to surface voltage drop between the electrodes and the water. This film resistance varies rapidly with changes in current density, gassing, and the condition of the surface of the electrode.

Careful investigations indicated that this film voltage drop all occurs in a very short distance from the surface of the electrode. It was also found that even when the electrode was quite close to the plate, the voltage was essentially constant around concentric surfaces for a distance of from 1/10 to 2/10 of an in. from the smallest electrode used. In addition, it was found that the surface voltage drop at the plate was entirely negligible due to the large area.

In the light of the foregoing investigations, it was decided to measure voltage between the plate and a point 1/10 of an in. in a radial direction from the surface of the electrode under consideration, and compensate for this by considering the radius of the electrode to be 1/10 in. greater. Thus, all readings were independent of the surface voltage drop which varied from 2 to 10 volts depending on conditions. In this manner, an

electrode 5/10 of an in. in diameter became 7/10 of an in. in diameter in the results.

The voltage was measured at 1/10 in. radially from the electrode by using a fine copper wire which protruded a short distance out of a small glass tube. The end of the tube was surrounded by a ring of 1/10 in. radius. Thus, when the ring on the rod was held in contact with the electrode surface, the copper wire voltmeter terminal was exactly 1/10 in. from the surface of the electrode. The fact that the voltmeter electrode projected only a short distance out of the glass tube made it possible to read the voltage at any depth in the water. In this way it was found that the flow was uniform at all depths for a given point.

This exploring electrode was mounted on a bar which extended across the tank, and which was mounted on longitudinal ways. Thus, the bar could be moved along the tank, and the electrode could be moved along the bar. By calibrating the longitudinal ways and the bar in feet, tenths, and hundreds, measuring the center

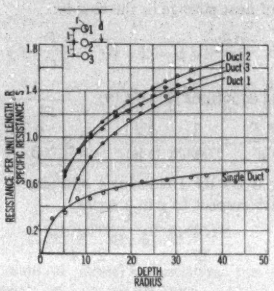

FIG. 5—RESISTANCE COEFFICIENT (R/S) FOR A SINGLE DUCT AND FOR THREE DUCTS, SPACED VERTICALLY AS SHOWN FOR $l/r = 4$

Theoretical curves and experimental points shown

of coordinates from the center of the plate, the voltage at any point in the tank could be recorded directly in its rectangular coordinates.

Further investigation showed that the voltage readings given by a high-resistance voltmeter checked within one per cent of the readings given by a potentiometer, thus making it possible to speed up the work by using a high-resistance, direct-reading voltmeter instead of a potentiometer. A low-resistance voltmeter could not be used because the current drawn by it would be great enough to cause the exploring electrode to act as an auxiliary current electrode, thus distorting the flow.

As a final precaution, any electrical potential due to battery action was eliminated by having no metal in the water except high grade copper. Any possibility of ground current through the box and floor was eliminated by using a generator and lines which were carefully insulated from the ground.

All specific electrical resistance measurements were made by means of a carefully calibrated, bridge type, Leeds-Northrup cell. Determinations were made at the beginning and end of each test, and were made at the temperature corresponding to the temperature of the water during the test.

The electrical equivalent of one, three, and six ducts was set up and tested, and the values of a resistance coefficient R/S were found for various values of d/r.

Those experimental points are shown plotted in Fig. 5 and Fig. 6, together with the theoretical curves for the same conditions as derived in Appendix C. The experimental data are given in Appendix D.

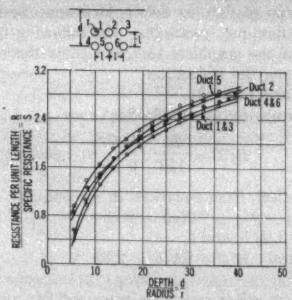

FIG. 6—RESISTANCE COEFFICIENT (R/S) FOR SIX CABLES SPACED HORIZONTALLY IN TWO ROWS

Theoretical curves and experimental points shown for $l/r = 4$

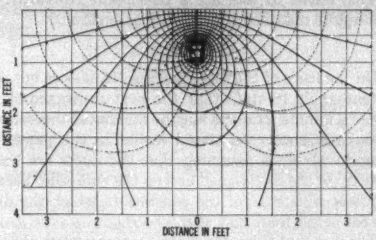

FIG. 7—FLOW FROM SIX ELECTRODES

Equivoltage surfaces—full lines
Lines of flow—dotted lines

As shown by the curves, the theoretical equations and experimental points check quite satisfactorily, showing that the distortion caused by close spacing of the electrodes does not cause appreciable error in the theoretical equations.

VERIFICATION OF GRAPHICAL METHOD

In order to check the graphical method and to study the field of flow from six ducts, the equivalent electrical circuit was built up and every second equi-voltage surface was plotted from data obtained by means of the exploring voltmeter electrode. The lines of flow were then sketched in at right angles with the equi-voltage surfaces so that the intersected areas were approximate squares. This is shown in Fig. 7. It

might be of interest to note the manner in which the flow is forced to leave the lower four electrodes and the large area between them through which practically nothing seemed to flow.

For this set-up, six 0.36-in. radius electrodes were spaced 2.16 in. on centers, the center row 9.6 in. from the plate in 1.5 ft. of water having a specific electrical resistance of 123 ohms per ft. cube. For a current of 0.15 amperes per electrode, the voltage of the two top electrodes was 24 volts, and 28 volts for each of the other four.

The lines of flow as drawn give approximately six tubes of flow per electrode. The following is a comparison of the graphical test, and theoretical methods of obtaining the resistance.

Electrodes 1 and 2:

Graphical—$R/S = 12/6 = 2.0$

Test $\quad R/S = \frac{24}{123} \cdot \frac{1.5}{0.15} = 1.96$

Theoretical, from equations in Appendix C,

$$R/S = 2.03$$

Cables 3, 4, 5, and 6,

Graphical $\quad R/S = 14/6 = 2.33$

Test, $\quad R/S = \frac{28}{0.15} \cdot \frac{1.5}{123} = 2.27$

Theoretical, $R/S = 2.25$

This seems to indicate a reasonable consistency in the results obtained by the various methods.

Conclusions

On the basis of the foregoing results, it does not seem unreasonable to state that the heat resistance of any one of a number of ducts may be calculated theoretically with confidence. As stated earlier in the paper, the error involved in assuming that the ducts are buried directly in the soil is small and on the safe side, because the heat resistance of the concrete in which the ducts are set is less than that of most soils.

The fact that the cables lie in contact with the bottom of the ducts may cause the temperature of the bottoms of the ducts to be higher than that of the tops, and thus violate the assumption that the ducts are equi-temperature cylinders. However, it is difficult to believe that this could cause serious distortion.

Throughout the discussion it has been assumed that the ground surface is an equi-temperature plane. This is not strictly true, since it will be at a higher temperature directly over the duct than it is farther away. This effect is quite small except when the ducts are extremely close to the surface.

The expression for temperature of any duct of a group having different values of heat flow may be found by using a system of images in a manner exactly analogous to that used when the ducts dissipate the same amount of heat. However, the temperature of any one duct can be found only by knowing the heat flow from each duct, and the solution will be more involved.

Enough data exist to determine the specific heat resistance of most soils encountered so it seems entirely justifiable to calculate the heat resistance for the ordinary arrangements of underground power cables by the theoretical method.

Appendix A

Transient Heat Flow from a Long Cylinder in an Infinite Extent of Soil

The effect of the heat capacity of the soil may be best studied by considering the transient heat flow from a cylinder of radius r buried in an infinite extent of soil having a specific heat resistance S and heat capacity C.

If the heat flow per unit length of cylinder at any radial distance p and time t is Q, the temperature drop across an annular ring of radius p and thickness $d\,p$ is given by:

$$\frac{\partial T}{\partial p}\, d\,p = \frac{Q\,S}{2\,\pi\,p}\, d\,p \tag{8}$$

The rate of heat storage in the ring is:

$$\frac{\partial Q}{\partial p}\, d\,p = 2\,\pi\,p\,C\,\frac{\partial T}{\partial t}\, d\,p \tag{9}$$

Combining equations (8) and (9)

$$\frac{\partial^2 T}{\partial p^2} + \frac{1}{p}\,\frac{\partial T}{\partial p} - C\,S\,\frac{\partial T}{\partial t} = 0 \tag{10}$$

Replacing $\partial/\partial t$ by Heaviside's operator P and integrating with respect to p in a series,

$$T = a_0 \left[1 + \frac{S\,C\,P\,p^2}{2^2} + \frac{(S\,C\,P)^2\,p^4}{2^2 \,.\, 4^2} + \frac{(S\,C\,P)^3\,p^6}{2^2 \,.\, 4^2 \,.\, 6^2} \text{ etc.} \right] \tag{11}$$

Making use of the terminal conditions at the cable where $p = r$ and $T = T_0\,1$, equation (11) becomes:

$$T = T_0\,1 \left[\frac{1 + \frac{S\,C\,P\,p^2}{2^2} + \frac{(S\,C\,P)^2\,p^4}{2^2 \,.\, 4^2} + \frac{(S\,C\,P)^3\,p^6}{2^2 \,.\, 4^2 \,.\, 6^2} \text{ etc.}}{1 + \frac{S\,C\,P\,r^2}{2^2} + \frac{(S\,C\,P)\,r^4}{2^2 \,.\, 4^2} + \frac{(S\,C\,P)^3\,r^6}{2^2 \,.\, 4^2 \,.\, 6^2} \text{ etc.}} \right] \tag{12}$$

In Heaviside's notation, $T_0\,1$ represents a square wave of temperature applied at time $t = 0$ and remaining constant. It is the same as saying that at time $t = 0$ the temperature of the cable is raised T_0 degrees above the initial temperature of the surrounding earth. A square wave of heat flow $Q_0\,1$ would more nearly represent the true case of a cable, but the solution for that gives rise to an indeterminant solution.

From equations (8) and (12):

$$Q = \frac{2\,\pi\,p}{S}\,\frac{\partial T}{\partial p} = \frac{2\,\pi\,T_0\,1}{S}$$

$$\left[\frac{\dfrac{S\,C\,P\,p^2}{2}+\dfrac{(S\,C\,P)^2\,p^4}{2^2\,.\,4}+\dfrac{(S\,C\,P)^3\,p^6}{2^2\,.\,4^2\,.\,6}\text{ etc.}}{1+\dfrac{S\,C\,P\,r^2}{2^2}+\dfrac{(S\,C\,P)^2\,r^4}{2^2\,.\,4^2}+\dfrac{(S\,C\,P)^3\,r^6}{2^2\,.\,4^2\,.\,6^2}\text{ etc}}\right] \tag{13}$$

This is in Heaviside's form, $Q = \dfrac{2\,\pi\,T_0\,1}{S}\,\dfrac{Y\,(P)}{Z\,(P)}$ (14)

And the solution is:

$$Q = \frac{2\,\pi\,T_0}{S}\left[\frac{Y\,(0)}{Z\,(0)}+\sum\frac{Y\,(P)\;e^{Pt}}{P\,\dfrac{d\,Z\,(P)}{d\,P}}\right] \tag{15}$$

The summation being evaluated for each root of $Z\,(P) = 0$. $Y\,(0)$ and $Z\,(0)$ are found by substituting $P = 0$ in $Y\,(P)$ and $Z\,(P)$.

$$\frac{Y\,(0)}{Z\,(0)} = \frac{0}{1} = 0 \tag{16}$$

$$Y\,(P) = \frac{S\,C\,P\,r^2}{2}+\frac{(S\;C\;P)\;r^4}{2^2\,.\,4}+\frac{(S\;C\;P)^3\,r^6}{2^2\,.\,4^2\,.\,6}\text{ etc.} \tag{17}$$

$$Z\,(P) = 1+\frac{S\,C\,P\,p^2}{2^2}+\frac{(S\;C\;P)^2\,p^4}{2^2\,.\,4^2}+\frac{(S\;C\;P)^3\,p^6}{2^2\,.\,4^2\,.\,6^2}\text{ etc.} \tag{18}$$

$$P\,\frac{d\,Z\,(P)}{d\,P} = \frac{S\,C\,P\,p^2}{2^2}+\frac{(S\;C\;P)^2\,p^4}{2^2\,.\,4\,.\,2}+\frac{(S\,C\,P)^3\,p^6}{2^2\,.\,4^2\,.\,6\,.\,2}\text{ etc.} \tag{19}$$

From a table of Bessel's functions, the roots of $Z\,(P) = 0$ are:

$$P_1 = -\frac{5.78}{S\;C\;p^2} \qquad P_5 = -\frac{223}{S\,C\,p^2}$$

$$P_2 = -\frac{30.5}{S\,C\,p^2}, \qquad P_6 = -\frac{326}{S\,C\,p^2}$$

$$P_3 = -\frac{75.0}{S\,C\,p^2} \qquad P_7 = -\frac{450}{S\,C\,p^2}$$

$$P_4 = -\frac{103.8}{S\,C\,p^2} \qquad P_8 = -\frac{591}{S\,C\,p^2}$$

Since only the heat flow at the cable is desired, equation (5), will be evaluated directly for $p = r$ where $Q = Q_0$.

For $p = r$ $\quad \dfrac{Y\,(P)}{P\,\dfrac{d\,Z\,(P)}{d\,P}} = 2$ for all values of P.

Equation (15) becomes:

$$Q_0 = \frac{4\,\pi\,T_0}{S}\left[\,e^{-\frac{5.78\,t}{S\,C\,r^2}}+e^{-\frac{30.5\,t}{S\,C\,r^2}}+e^{-\frac{75.0\,t}{S\,C\,r^2}}+e^{-\frac{103.8\,t}{S\,C\,r^2}},\ \text{etc.}\,\right] \tag{20}$$

Equation (20) gives the rate of heat flow Q_0 per unit length of cable of radius r surrounded by an infinite extent of soil, at any time t after a square wave of temperature T_0 is applied.

Since this equation has no steady state term, it follows that in the case of cables buried underneath the surface of the earth, no heat flows downward into the earth after the transient period is over. Thus, all the heat must then be dissipated at the surface of the ground, even though the heat capacity of the earth is infinite.

Evaluating equation (20) for a duct system equivalent to a 1½ ft. diameter cylinder in an infinite extent of average soil will give an idea of the length of the transient period.

Using:

S = 6.5 deg. cent., per watt per ft. cube
C = 12 watt hr. per cu. ft. per deg. cent.
r = 3/4 ft.

Equation (13) becomes:

$$Q_0 = 1.92\,T_0\left[e^{-.13t}+e^{-.68t}-e^{-1.89t}+e^{-2.34t}+\text{etc.}\right] \tag{21}$$

Solving equation (21) shows that the heat flow drops very rapidly and has practically died out in 24 hr.

Appendix B

The following values for the resistance of various materials may be convenient in connection with this problem:

Material	Resistance in deg. cent. per watt per ft. cube	Authority
Stone concrete	3.5	Physics & Chemistry Handbook
Brick, common red	5.2	Physics & Chemistry Handbook
Dry, clean yellow sand	11.1	G. B. Shanklin
Dry, clean white sand (fine)	13.2	A. E. Kennelly
Dry, yellow sandy soil	12.1	G. B. Shanklin
Clean yellow builders sand—2 per cent moisture	9.2	G. B. Shanklin
Clean yellow builders sand—4 per cent moisture	6.8	G. B. Shanklin
Clean yellow builders sand—6 per cent moisture	4.8	G. B. Shanklin
Clean yellow builders sand—8 per cent moisture	3.4	G. B. Shanklin
Yellow sand clay soil—2 per cent moisture	11.6	G. B. Shanklin
Yellow sand clay soil—4 per cent moisture	11.2	G. B. Shanklin
Yellow sand clay soil—6 per cent moisture	9.7	G. B. Shanklin
Yellow sand clay soil—8 per cent moisture	7.4	G. B. Shanklin
Yellow sand clay soil—10 per cent moisture	5.4	G. B. Shanklin
Yellow sand clay soil—12 per cent moisture	3.7	G. B. Shanklin
Yellow sand clay soil—14 per cent moisture	2.4	G. B. Shanklin

Appendix C

Heat Resistance for More Than One Duct

As previously discussed, it is possible to derive an expression for the heat resistance of any one of a number of ducts by replacing the earth's surface by a system of images and applying the principle of superposition.

Consider three ducts spaced in the vertical a distance l between centers, and the center duct a distance d

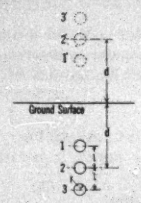

Fig. 8—Three Ducts and Their Images

below the ground surface, each duct having a radius r and a rate of heat flow per unit length of Q. Referring to Fig. 8, the temperature rise of duct No. 1 due to its heat flow acting alone and undistorted is,

$$\Delta T_1 = \frac{Q\,S}{2\,\pi} \int_r^d \frac{d\,p}{p} = \frac{Q\,S}{2\,\pi} \log_e \frac{(d-l)}{r} \qquad (22)$$

The temperature rise of duct 1 due to the heat flow $-Q$ of the image of No. 1 acting alone,

$$\Delta T_1^1 = -\frac{Q\,S}{2\,\pi} \log_e \frac{(d-l)}{2\,(d-l)} \qquad (23)$$

The temperature rise due to duct 2 acting alone,

$$\Delta T_2 = \frac{Q\,S}{2\,\pi} \log_e \frac{d}{l} \qquad (24)$$

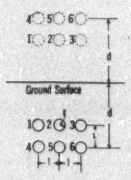

Fig. 9—Six Horizontally Spaced Ducts and Their Images

Due to the image of duct 2,

$$\Delta T_2^1 = -\frac{Q\,S}{2\,\pi} \log_e \frac{d}{2\,d-l} \qquad (25)$$

Due to duct 3,

$$\Delta T_3 = \frac{Q\,S}{2\,\pi} \log_e \frac{d+l}{2\,l} \qquad (26)$$

Due to the image of duct 3,

$$\Delta T_3^1 = -\frac{Q\,S}{2\,\pi} \log_e \frac{(d+l)}{2\,d} \qquad (27)$$

The total temperature rise of duct 1 becomes the sum of the above, or

$$\Delta T = \frac{Q\,S}{2\,\pi} \log_e \left[\frac{(d-l)}{r} \frac{(2)\,(d-l)}{(d-l)} \frac{(d)}{(l)} \frac{(2\,d-l)}{(d)} \frac{(d+l)\,(2\,d)}{(2-l)\,(d+l)} \right] \qquad (28)$$

or,

$$R_1 = \frac{\Delta\,T}{Q} = \frac{S}{2\,\pi} \log_e \left[\frac{(2\,d)}{r} \frac{(d-l)\,(2\,d-l)}{l^2} \right] \qquad (29)$$

In a similar manner for cable No. 2

$$R_2 = \frac{S}{2\,\pi} \log_e \left[\frac{2\,d}{r} \frac{(2\,d)^2 - l^2}{l^2} \right] \qquad (30)$$

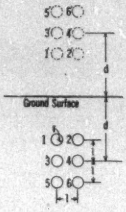

Fig. 10—Six Vertically Spaced Ducts and Their Images

and, $$R_3 = \frac{S}{2\,\pi} \log_e \left[\frac{2\,d}{r} \frac{(2\,d-l)\,(d+l)}{l^2} \right] \qquad (31)$$

For six ducts spaced as shown in Fig. 9,

$$R_1 = R_3 = \frac{S}{2\,\pi} \log_e \left[\frac{(d-l)}{r} \frac{\sqrt{4\,(d-l)^2 + l^2}\,\sqrt{4\,(d-l)^2 + 4\,l^2}}{\sqrt{10}\,l^2} \right] \text{plus}$$

$$\frac{S}{2\,\pi} \log_e \left[\frac{(2\,d-l)\,\sqrt{(2\,d-l)^2 + l^2}\,\sqrt{(2\,d-l)^2 + 4\,l^2}}{l^3} \right] \qquad (32)$$

$$R_2 = \frac{S}{2\,\pi} \log_e \left[\frac{(d-l)}{r} \frac{[(2\,d-2\,l)^2 + l^2]\,[(2\,d-l)^2 + l^2]\,(2\,d-l)}{l^5} \right] \qquad (33)$$

$$R_4 = R_6 = \frac{S}{2\,\pi} \log_e$$

$$\left[\frac{d}{r}\,\frac{\sqrt{4\,(d)^2+l^2}\,\sqrt{4\,d^2+4\,l^2}\,(2\,d-l)}{\sqrt{10}\,l^3}\right] \text{plus}$$

$$\frac{S}{2\,\pi}\log_e\left[\frac{\sqrt{(2\,d-l)^2+l^2}\,\sqrt{(2\,d-l)^2+4\,l^2}}{l^2}\right] \quad (34)$$

$$R_5 = \frac{S}{2\,\pi}\log_e$$

$$\left[\frac{d}{r}\,\frac{[(2\,d-l)^2+l^2]\,[2\,d-l]\,[4\,d^2+l^2]}{l^5}\right] \quad (35)$$

For six cables spaced as shown in Fig. 10,

$$R_1 = R_2 = \frac{S}{2\,\pi}\log_e$$

$$\left[\frac{2\,d}{r}\,\frac{(d-l)\,(2\,d-l)\,\sqrt{4\,(d-l)^2+l^2}\;\sqrt{(2\,d-l)^2+l^2}\,\sqrt{4\,d^2+l^2}}{\sqrt{10}\,l^5}\right] \quad (36)$$

$$R_3 = R_4 = \frac{S}{2\,\pi}\log_e$$

$$\left[\frac{d}{r}\,\frac{(2\,d-l)\,(2\,d+l)\,\sqrt{4\,d^2+l^2}\;\sqrt{(2\,d-l)^2+l^2}\,\sqrt{(2\,d+l)^2/+l^2}}{\sqrt{10}\,l^5}\right] \quad (37)$$

$$R_5 = R_6 = \frac{S}{2\,\pi}\log_e$$

$$\left[\frac{2\,d}{r}\,\frac{\sqrt{4\,d^2+l^2}\,(2\,d+l)\;\sqrt{(2\,d+l)^2+l^2}\,[d+l]\,\sqrt{4\,(d+l)^2+l^2}}{\sqrt{10}\,l^5}\right] \quad (38)$$

Appendix D

Experimental Data

DATA FOR A SINGLE DUCT PLOTTED IN FIG 5

Conditions: 1.5 ft. depth of water, 0.8 amperes per electrode, specific electrical resistance of water S = 136.3 ohms per ft. cube. Effective radius of electrode r 0.36 in.

TABLE I

d (inches)	d/r	Volts	R for 1.5 ft.	R for 1 ft.	R/S
1	2.78	20.7	25.9	38.8	0.285
2	5.66	26.7	32.2	48.3	0.355
3	8.35	31.0	38.8	58.2	0.427
4	11.12	34.5	43.2	64.8	0.476
5	13.90	36.7	45.9	68.8	0.505
6	16.68	40.0	50.0	75.0	0.550
8	22.25	44.0	55.0	82.3	0.605
10	27.80	45.8	57.4	86.1	0.632
12	33.35	48.0	60.0	90.1	0.660

TABLE I—*Continued*

Conditions: 1.5 ft. depth of water, 0.475 amperes per electrode, S = 136.0 ohms per ft. cube, effective electrode radius r of 0.136 in.

d (inches)	d/r	Volts	R for 1.5 ft.	R for 1 ft.	R/S
5	36.8	28.5	60.0	90.0	0.662
7	51.5	31.0	65.8	97.8	0.720
9	66.2	33.25	70.0	105.0	0.772

DATA FOR THREE DUCTS SPACED VERTICALLY PLOTTED IN FIG. 5

Conditions: Depth of water 1.5 ft., effective electrode radius r = 0.36 in., spacing of electrodes l = 1.44 in., l/r = 4, S = 117 ohms per ft. cube. Amperes per electrode = 0.25.

TABLE II

		No. 1		No. 2		No. 3	
d	d/r	Volts	R/S	Volts	R/S	Volts	R/S
2	5.55	7.0	0.359	13	0.666	13.5	0.692
3	8.34	12.5	0.641	17.25	0.885	17.0	0.872
4	11.1	16.25	0.833	20.25	1.038	19.5	1.000
5	13.9	18.5	0.948	22.0	1.128	21.0	1.076
6	16.7	20.25	1.038	23.75	1.215	23.0	1.180
7	19.45	22.5	1.153	25.5	1.305	24.0	1.230
8	22.2	23.5	1.205	26.5	1.36	25.0	1.280
9	25.0	25.0	1.281	28.0	1.435	26.5	1.360
10	27.8	26.5	1.358	29.0	1.485	28.0	1.435
11	30.5	27.0	1.382	30.0	1.538	28.5	1.460
12	33.3	27.75	1.42	31.0	1.59	29.5	1.515

DATA FOR SIX DUCTS IN TWO HORIZONTAL ROWS PLOTTED IN FIG. 6

Conditions: 1.5 ft. of water, r = 0.36 in., l = 1.44 in., l/r = 4 S = 118.5 ohms per ft. cube, amperes per electrode = 0.15.

TABLE III

		Nos. 1 & 3		No. 2		Nos. 4 & 6		No. 5	
d	d/r	Volts	R/S	Volts	R/S	Volts	R/S	Volts	R/S
2	5.55	6.1	0.515	7.4	0.625	10.3	0.87	11.3	0.955
3	8.35	12.3	1.038	13.8	1.165	14.7	1.24	16.5	1.39
4	11.1	15.6	1.315	17.7	1.495	17.7	1.495	19.5	1.645
5	13.9	18.9	1.515	20.8	1.76	20.8	1.76	22.8	1.935
6	16.7	21.5	1.815	23.2	1.96	23.1	1.95	24.4	2.06
7	19.45	23.1	1.950	25.0	2.11	24.8	2.095	26.5	2.24
8	22.2	25.1	2.120	27.1	2.29	26.4	2.23	27.3	2.30
9	25.0	26.3	2.220	28.1	2.37	27.4	2.32	29.1	2.46
10	27.8	27.9	2.355	30.0	2.43	29.1	2.45	31.4	2.65
11	30.55	29.2	2.46	31.2	2.63	29.5	2.49	32.0	2.70
12	33.3	30.4	2.57	32.4	2.73	31.5	2.66	33.5	2.83
13	36.1	31.3	2.64	33.3	2.81	32.8	2.77	34.3	2.89

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Discussion

R. D. Levy: (communicated after adjournment) I should like to take exception to parts of Mr. Bailey's paper. Mr. Bailey shows that there can be no heat flow from a source, into a medium of infinite dimensions surrounding the source, except during a transient period. He also gives a mathematical demonstration to show that the law for this transient flow variation with time is the sum of a series of decaying exponentials. His relation, as applied to ordinary duct banks, makes it appear that the heat flow into the medium, where the medium is soil, will be negligible after the first few hours. On the basis of the assumption that the transient flow into the earth is of short duration, and may therefore be neglected, the following relation is derived—

$$\Delta T = \frac{Q S}{2 \pi} \log \frac{2 d}{r}$$

ΔT is the temperature rise of the duct bank surface over earth surface.

This relation may be reduced to an absurdity by making Q zero. Then the temperature of the duct becomes equal to earth surface temperature. We know that under this condition the temperature of the duct is the same as the temperature of ambient earth at duct level.

The correctness of Mr. Bailey's mathematics, where he derives the relation for the decay of heat flow, is questionable. The series which he gives as the solution of the Bessel equation, (10), involves but one arbitrary constant. Inasmuch as this is a differential equation of the second order, it should have two arbitrary constants in its complete solution. The complete solution is,

$$T = (A + B \log C S P p)\left(1 + \frac{C S P p^2}{2^2} + \frac{(C S P)^2 p^4}{2^2 . 4^2} + \ldots\ldots\right)$$

$$+ B\left(1 - \frac{(C S P)^2 p^4}{2^2 . 4^2} \frac{1}{2} - \frac{(C S P)^2 p^6}{2^2 . 4^2 . 6^2}\left(\frac{1}{2} + \frac{1}{3}\right) - \ldots\right)$$

The constants, A and B, must be determined by two known conditions, namely, that when p is r, T is $T_e 1$, and that when p is infinite, T is zero. Mr. Bailey's solution considers B zero. I have been unable to determine these constants. However, an examination of the results obtained by neglecting B, will show its fallacy. The author assumes a cylindrical duct imbedded in earth of infinite extent, raised to a temperature of T_e at time zero and maintained at that temperature forever after. His derived expression for Q, the heat flow from the cylinder, is shown in his Equation (20). For the special case where S is 6.5, C is 12, r is ¾ ft., and T_e is 10 deg. cent., temperature of undisturbed earth assumed 0 deg. cent., at the end of 20 hours, Q is 1.47 watts by this relation. The resistance to heat flow from the cylinder surface to an imaginary cylindrical surface of 10 feet radius is, from Equation (2),

$$R = \frac{S}{2 \pi} \log_e \frac{10}{r}$$

For the assumed constants, this resistance is 2.7 deg. cent. per watt. If we assume that all of the 1.47 watts pass through the 10-ft. cylinder or in other words, that none of it goes into storage within the 10-ft. cylinder, we can make a guess at the temperature drop from the duct cylinder to the outside of the 10-ft. cylinder which will be greater than the truth. The actual drop, then, will be less than,

$$2.7 \times 1.47 = 3.97 \text{ deg. cent.}$$

The temperature at the outside of the 10-ft. cylinder will be at least 6.03 deg. cent., and the temperature everywhere within the 10-ft. cylinder will be greater than 6.03 deg. cent. If we multiply 6.03 by 12 (heat capacity of earth) and by the volume of the earth affected, we get 22,700 watthours stored in the 10-ft. cylinder of earth. Of course, this is low since the average temperature is higher than 6.03 deg. cent. If we integrate we get 25,400 watt-hours, which is somewhat more accurate. Now this is only a portion of the heat which has emanated from the duct cylinder in 20 hr., since heat has also been stored in the earth outside of the 10-ft. cylinder which we have been considering. However, if we go back to the author's original expression for heat flow, and integrate directly to find the total heat which has come out of the duct cylinder, we get,

$$Q = 19.2 \int_0^{20} (\epsilon^{-.13t} + \epsilon^{-.68t} + \epsilon^{-1.69t} + \ldots) \, dt = 220 \text{ watt-hours}$$

Thus, by integrating Mr. Bailey's relation, we get 220 watthours emitted by the duct cylinder, and by reasoning from the temperature drops computed from the relation, we get in excess of 25,400 watt-hours. Something must be wrong with the relation. Furthermore, the integration process indicates that only 231 watt-hours will be emitted in an infinite time, under the conditions which we have assumed.

We know, of course, that some heat from the duct bank does go down, else the duct bank could never be cooler than street level temperature. I suppose that a correct treatment of the problem of the duct cylinder in an infinite extent of earth would show a transient heat flow of long enough duration to be consistent with observed facts.

A fairly accurate and partially rational expression for duct surface rise over ground level temperature, may be derived by making a simplifying assumption, which is true for all practical purposes. The assumption is, that an equipotential temperature plane exists about 15 ft. below duct level. This plane may be assumed to be at a temperature unaffected by duct heat and by seasonal temperature variations at the earth's surface. The derived relation is not very sensitive to the assumed depth. That is to say, that 10, 15, or 20 ft. may be used with but little effect on the computed results.

Q is the watts loss dissipated from the duct bank, per trench-foot.

S is the soil heat resistivity in deg. cent. per watt, for a foot cube,

h is the height of the duct bank in feet,

w is the width of the duct bank in feet,

d is the depth of the bank below the earth surface in feet (to the center of duct),

T_c is the temperature of the assumed plane. This may be taken as 15 deg. cent. very nearly,

T_s is the temperature of the surface earth. For this, use monthly averages of the air temperatures,—deg. cent.

T_d is the average temperature of the duct bank surface, in deg. cent.

Let $$A = \log_e \frac{2 h + 2 w - \pi h + 4 \pi d}{2 h + 2 w}$$

$$B = \log_e \frac{2 h + 2 w - \pi h + 60 \pi}{2 h + 2 w}$$

Then $$T_d = \frac{\frac{Q S}{2 \pi} A B + T_c A + T_s B}{A + B}$$

The results obtained with this relation, while not precise, agree fairly well with test data.

N. P. Bailey: This summer conditions at Chelan were such that we could find out some of the temperatures which actually exist. We had an opportunity to run a test of short duration which indicated how in practise the temperatures follow cyclic variations.

The accompanying curves show the results of this test. Each phase current was carried by two 2000-cir. mil lead cables, surrounded by rather coarse, loose sand, which is dry practically all the hot part of the season, the moisture level being considerably lower.

When these cables were installed, arrangements were made to obtain the temperatures during operating conditions. Resistance temperature elements were placed on the outside of the cable lead and the outside of the duct, with the anticipation that those cables would be watched under operation.

At the outset of the test, the load was increased from 1000 amperes to 1600 amperes per phase and held constant during the rest of the time.

The top curve shows the temperature of the middle cable gradually increasing, having a rather sudden increase at first and at the end of the period increasing almost as rapidly as ever. The next curve represents the temperature on the outside of the fiber duct, also increasing, the temperature difference being

constant practically all the time, which would be expected, since there was very little chance for heat storage in the duct.

The curve which represents the temperature on top of the concrete structure directly over the center of the cable, rises gradually, and seems to have very much the same rate of increase. In fact, after 12 a. m., within the accuracy of the measurements, there is very little change in the temperature difference between the duct and the concrete, showing that very little heat was being stored in the concrete itself after that time.

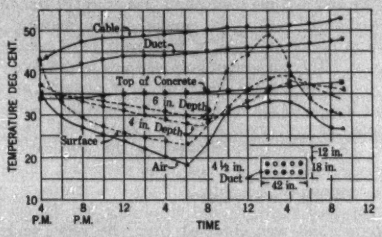

FIG. 1—TEMPERATURE TESTS ON CABLES AT CHELAN

There were two 2000-in. mil cables per phase. The load per phase was 1600 amperes during the test. It had been held at 1000 amperes for a day previous to the test.

The air temperature taken in the shade starts in at 4 p. m. at a temperature in excess of 35 deg. cent., dropping down almost as a straight line until about 6 a. m., when the minimum temperature is obtained, then rising rather rapidly, and falling off again in this same way. That is a typical cycle of the weather there for about four months out of the year. The surface temperature, as would be expected during the night at least, is higher than the air, showing that heat is flowing from the surface; 6 in. down being hotter yet, and the concrete being still hotter than the 6-in. level, showing a very definite temperature gradient for the flow of heat upward.

However, after 8 or 9 a. m. the surface temperature starts up. About 8 o'clock, it became hotter than the 4-in. depth and the 6-in. depth, going up almost to the temperature of the cable, around 4 in the afternoon, then dropping off in the evening. From 8 a. m. until 5 p. m. the surface is hotter than it is down 4 in. deep, which shows definitely that the heat is flowing downward for that first 4 in. From 9 a. m. to 6 p. m. the 4-in. depth was hotter than the 6-in. depth, showing that during that period which started in a little later than the other, the heat was flowing from the 4-in. position downward toward the 6-in. Then for a short period in the afternoon the 6-in. depth became still hotter than the top of the concrete, which showed definitely that heat was flowing toward the concrete. The curves show the successive time lags between the times when the different peaks are reached; the peak of the surface temperature at 2 p. m., the peak on the 4-in. depth at 3:30 p. m., etc. In a rather rough way is shown also the damping of the waves.

We were not able to carry this test to its logical conclusion, but it is hoped that when the opportunity arises we will be able to continue the test for a long enough time to determine more about the cycle of the temperatures. The test was run a little longer than 30 hr. We had obtained data at 1000-ampere load for a day or so before.

In tests run by the Los Angeles Bureau of Light and Power, and which I imagine were for much deeper cables than these because they were on a distribution system, they had curves of temperature which continued to rise for 160 hr., and were still rising at a very substantial rate. So in considering the length of heat transient, it is a different proposition from the electrical transient.

The first point in the discussion of Mr. Levy has already been answered in my closing comments concerning cyclic temperature changes. The question on the accuracy of the transient equations is well taken as in the paper one of the terms of the Bessel function is neglected.

Corrosion of Cable Sheath in Creosoted Wood Conduit

BY R. M. BURNS[1] and B. A. FREED[2]
Non-member Non-member

FOLLOWING the successful use of conduit of creosoted yellow pine in the eastern and southern part of this country, similar duct made of Douglas fir was introduced on the Pacific Coast in 1911. Manufacture was undertaken in the Puget Sound territory; and extensive use of the duct has since been made for both main and subsidiary communication subways in all parts of Washington, Oregon, and California, excepting Southern California where its field has been limited by economical considerations to lateral or subsidiary construction. This conduit was also considered satisfactory, until the latter part of 1923, when corrosion developed on some newly placed telephone cables, and the attending circumstances were such as to indicate that the duct itself might be responsible. It is with the identification of this corrosion and the determination and application of methods of allaying it that this paper is concerned.

Discovery of corrosion presented a problem of some magnitude in so far as the application of palliative measures was concerned. Between 1911, when wood conduit was first introduced on the Pacific Coast, and 1926, when its manufacture for the Bell System was discontinued because of its effect upon the sheath of the cables, approximately 7,250,000 duct ft. were placed as telephone subway; and cables had been pulled into about one-third of it. This problem was simplified to some extent, however, by the fact that the cables had not been affected in all duct runs. There were also approximately 130,000 duct ft. of conduit in the store yards, the salvage of which was, of course, very desirable.

The conduit is made of 4½-in. by 4½-in. sections of timber having a three-in. bore, and mortise and tennon ends. The practise has been to machine it while green, as a rule within five days after the felling of the timber, to overcome the losses which would otherwise occur due to splitting of the duct during the boring process. Until about 1915 the conduit machinery and treating plant were at the same location. Since this date, however, it has been necessary to transport the conduit by barge, from the mill at which it is machined, across Puget Sound to the treating plant. This and other movements of the conduit, which occur between machining and the time it enters the retorts of the treating plant, are made as expeditiously as possible to obviate the checking in thin sections of timber during air seasoning. To avoid this every effort has been made to place the conduit in the retorts within four or five days after it has been machined. Thus, the entire process of manufacture including the felling of the timber and its treatment rarely has exceeded ten days.

Within the period that conduit of this material has been in use, two types of treatment have been employed. The purpose of both of these treatments was to obtain an absorption of 15 lb. of creosote oil per cubic foot of timber by what is known as the "full cell" process, as contrasted with the "empty cell" process, but the movements followed in the application of pressure, heat, and oil have been somewhat different. The creosote oil used with both types of treatment was an imported grade of dead oil of coal tar, or coal tar creosote, and was obtained from the same sources. It has consisted wholly of distillates of gas tar produced by the destructive distillation of bituminous coal, either in the manufacture of coal gas, or in manufacture of coke by the by-product process. So far as is known, the only variation in composition shown by the creosote during the period was in the tar acid content. This has fluctuated to some extent but has never exceeded 10 per cent.

The original process seasoned the wood by boiling it in creosote oil. This operation was followed by application of sufficient temperature and pressure to permit the forcing of the required amount of oil into the duct. In 1916 this method of treatment was superseded by the "vacuum" process which had been developed a short time previously, primarily for use on bridge stringers, etc., the effect upon the fibre strength being decidedly less detrimental. The principal advantages of the new process, as applied to conduit, were that it minimized the losses in treatment from checking and distortion, and presumably resulted in obtaining a better spread of the preservative. Of the latter point, however, there is considerable doubt. With the vacuum process, the wood is immersed in creosote oil and subjected to a minimum vacuum of 20 in. of mercury and to temperatures ranging between 190 deg. fahr. and 200 deg. fahr. until the moisture content is reduced to approximately the fibre saturation point, after which pressure is applied until the timber has absorbed the required amount of oil. At the end of the pressure period the retorts are emptied of preservative, and a vacuum of 20 in. of mercury created for about two hours, under an average temperature of 160 deg. fahr. The entire treating process consumes about 20 hr. on an average, and is of somewhat longer duration than the earlier process.

In the light of more recent experience it has become apparent that cable failures due to corrosion by creo-

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28-116

soted wood conduits occurred prior to 1923, but were at the time attributed to other causes. The first case which can be ascribed with any assurance to creosoted wood occurred at Yakima, Washington in 1921. Even as late as 1925, however, it was not certain that the trouble was associated with the conduit. The fact that creosoted wood had not been known previously to cause corrosion, that there had been no change in the wood or creosote used in the manufacture of conduit, that other users of creosoted wood conduit had not reported trouble, all tended to absolve the conduit. The change in the method of creosoting introduced in 1916 was not such as to arouse suspicion, and moreover so far as was known, corrosion did not follow immediately after the change in process. When late in 1923 serious cases of corrosion occurred first in Seattle and then in San Francisco and elsewhere on the Coast it became evident that the action was not due to the soil as was suspected at first. A study of soil samples from the points of failure also had given no indication of unusual acidity. It was recognized that the trouble could not be due to electrolysis since its occurrence bore no relationship to the potential condition of the cable with respect to earth and since perforation of the sheath occurred on the top or sides of the cable which were not in contact with the duct, and then only in duct runs which were free from water and well drained. Corrosion was never experienced on cables which were submerged in water continually or for a part of the year. On the other hand, the action was facilitated by the condensation of moisture in the form of droplets on the cable. That climatic conditions, particularly temperature and humidity, were important is seen in the fact that in the region of Seattle, corrosion was rapid only in the Spring and Fall, whereas in San Francisco it occurred at much the same rate the year around. The alloy used for cable sheath in recent years consisted of 99 per cent lead and 1 per cent antimony, and the question arose as to whether the sheath suffering corrosion was of normal composition. Chemical and microscopic analysis, however, showed it to be of the composition specified and not abnormally susceptible to corrosion.

Corrosion of the type experienced manifests itself by the formation of a white encasement around the globules of moisture condensed on the sheath. In the early stages a mottled effect is produced, but as the attack progresses, the surface of the cable, particularly the top and sides, becomes heavily coated with a white encrustation of lead carbonate or basic carbonate, as is shown in Fig. 1. Chemical analysis of this product gave an indication also of the presence of volatile organic acids. Beneath the carbonate coating the sheath was pitted corresponding to the original globules of moisture. It was perforation of the sheath by these pits which led to failure of the cables. In one instance an 18-in. section of sheath displayed seven perforations.

The persistence of corrosive conditions with time has been a matter of great interest. Observations have indicated that the corrosive qualities of the conduit become depreciated with time and are influenced to some extent by conditions of ventilation, its exposure to the weather before installation in the ground, etc. A recent analysis of the corrosion failures in one locality has shown that about 70 per cent of the failures occurred on cables which had been under ground for less than 15 months, and furthermore, that about 90 per cent of the failures occurred in conduit which at the time the cables were placed in it had been in the ground for less than 13 months. In mitigation of these facts it should be pointed out that possibly sufficient time had not elapsed for many cases of the less active corrosion to reveal themselves and further, that in most cases the conduit was laid for immediate occupancy and service conditions have not been such as to warrant the later installation of cables, the performance of which would indicate the persistence of corrosive conditions.

Fig. 1—White Carbonate Formed on Lead Cable Sheath

The volatile nature of the corroding agent was evident from the fact that the corrosive action in many cases extended into the manhole beyond the mouth of the ducts. This was particularly noticeable in manholes situated on hillsides where the action was visible on cables leading into the manhole from the lower side, but was not discernible at the mouth of the ducts on the side leading up-hill.

A careful consideration of the facts disclosed by field observations led to the conclusion that the most likely cause of corrosion was to be sought in the high proportion of carbon dioxide and other acid vapors in combination with humidity changes in the duct atmosphere. It had been shown previously that the characteristic attack could be imitated in the laboratory by exposing pieces of cable sheath bearing drops of water to moist air containing carbon dioxide and a trace of acetic acid vapor. A more quantitative method of determining the rate of corrosion so produced was developed later and consisted in the measurement of the rate of increase in electrical resistance of extruded lead wires which resulted when these wires were reduced in cross-section by the progress of corrosion. This method was particularly useful later in testing the effectiveness of various palliative treatments.[3]

3. The development of this method of measuring corrosion rates was due to W. A. Hyde and W. E. Campbell of the Bell Telephone Laboratories and will be described in more detail in a later paper.

Having concluded that the corrosion was definitely associated with vapors present in the duct it became desirable to investigate the composition of duct air where corrosion was actually in progress and for this purpose a portable apparatus was developed. It consisted essentially in an aspirator, a trap for freezing out volatile vapors, a thermal conductivity cell with a Wheatstone bridge arrangement for measuring carbon dioxide, and wet and dry bulb thermometers for determining the humidity. The aspirating device was a small mechanical pump and was driven by a motor which was operated on a six-volt storage battery. Capillary flow meters were used to measure the flow of air from the duct. The most suitable rate of air-flow was found to be from 9 to 11 cu. in. per min. The temperature of the vapor trap was maintained near −110 deg. fahr. by surrounding it with solid carbon dioxide. An auxiliary set of vapor traps made it possible to study the air from four ducts simultaneously. The apparatus was mounted generally on a truck which, during a test, was parked beside a manhole. Glass tubes led from the apparatus to the ducts. The conductivity cell was calibrated for the range 0.1 to 20.0 per cent carbon dioxide and the values could be read to 0.05 per cent. The substances obtained in the vapor trap were analyzed in the laboratory for tar acids and acids of the acetic type and tests were made for aldehydes, sulphur compounds, nitrates, and chlorides.

After preliminary work with this apparatus in New Jersey had shown it to be suitable for the purpose, it was taken to the Pacific Coast and used at Seattle, Tacoma, Portland, San Francisco, Oakland, and Los Angeles. In the 25 runs which were made there were included both corrosive and non-corrosive creosoted wood ducts, tile ducts, and fibre ducts. Some of the ducts contained cables and others had never been occupied. In one case the apparatus was set up in a central office cable vault and operated continuously for three days on a bank of corrosive ducts which led into that office. Most of the wood conduit studied had been placed since 1920 but in two cases it dated from 1914. Finally runs were made at the creosoting plant which had manufactured the conduit, vapors being collected from the air in the work-storage and condensate tanks.

The results of these tests showed that small amounts of a volatile organic acid, probably largely acetic acid, were present in the air in creosoted wood ducts which had been installed in recent years, and that in general, the higher concentrations of acid were found in the more corrosive sections. Although the actual concentration of acid calculated as acetic acid did not exceed 0.1 mg. per cu. ft. of air, previous laboratory investigations had shown that acid concentrations of this magnitude were definitely corrosive to cable sheath. The runs made at the creosoting plant revealed the presence of still greater concentrations of acid in storage tank air, the values being from two to four times greater than those found in conduit air.

Tar acids and naphthalene were collected in considerable quantities from the creosoted wood ducts, but neither of these substances, nor carbon dioxide, within the limits found, 0.1 per cent to 6.0 per cent, bore any relationship to the corrosive action. Nor did these substances show in laboratory tests, the marked corrosive action displayed by traces of acetic acid.

In the meantime, additional laboratory tests had shown that green Douglas fir wood contains considerably larger quantities of volatile acid than does southern pine or western hemlock and that the corrosive qualities of sawdust samples of these woods are related to their acid contents.

It had become evident in view of these observations that corrosion was due to acids derived from the conduit and therefore, from a knowledge of wood chemistry, principally acetic acid. Hence, the mechanism of the corrosion is the same as that involved in the

Fig. 2—Apparatus For Supplying Ammonia to Neutralize Creosoted Wood Duct

production of "white lead." This reaction, from the classical point of view, consists in the formation of lead acetate, by the action of acetic acid and oxygen on lead, and the conversion of this product into a basic lead carbonate with the liberation of acetic acid which then repeats the process cyclically. Essentially the process is dependent upon the maintenance of a suitably high concentration of hydrogen ions and the presence of oxygen and carbon dioxide.

The determination of the cause of corrosion was in one sense only preliminary to the important problem of finding a means of saving the cables which were in the process of corroding and to devise a method of treatment which would destroy the corrosiveness of idle ducts prior to their occupancy by cables. In this study comprehensive tests were carried on both in the field and in the laboratory. A large number of materials, including petrolatum, greases, oils, soap solution, water glass, pyridine, ammonium carbonate, ammonium dichromate, ammonia gas, etc., were tried out. Heavy petrolatum applied to cables upon installation gave some promise, but examination of the coated cables later revealed marked evidences of corrosion in areas where the grease had been rubbed off during the installation of the cable. Eventually the most effective

palliative treatment was found to be ammonia gas which neutralizes the acetic acid forming ammonium acetate, a non-volatile salt.

A suitable apparatus was developed for supplying ammonia mixed with air to cable ducts.[4] This equipment consisted of a large capacity blower operated by a gasoline engine, a mixing tank, the necessary connections, and an outlet through a manifold for distributing the air mixture to the ducts. A tank of liquid ammonia was connected to the mixing tank through a flow meter. The system was mounted on a truck, as is shown in Fig. 2. The air-ammonia mixture found most suitable contained about 2 per cent ammonia, and this was used. The greater part of the affected creosoted wood conduit in the plant has now been given the ammonia treatment with the result that trouble due to corrosion has practically ceased. There is still some question as to the necessity of repeating the treatment, but the evidence indicates that a single application will suffice in perhaps all but the most severely corrosive ducts.

The final phase of the problem, not yet completed, has had to do with the determination of the source and process of formation of the corrosive acid in creosoted wood ducts. An extensive laboratory investigation has shown that Douglas fir is markedly more acidic than pine, which is used for conduit in other sections of the country, and that the acid content is increased by raising the temperature or increasing the duration of heat treatment. Douglas fir is particularly difficult to penetrate with creosote and the relatively longer duration of treatment which has been employed in an attempt to improve penetration has tended probably to increase the acid content of the product. There is evidence that the acid is associated with the untreated part of the duct and that the creosoted portion of the wood is not corrosive to cable sheath.

Discussion

D. W. Proebstel: An interesting case of lead-sheath corrosion occurred in Portland a few years ago in a section where waste slab wood and sawdust had formerly been dumped. This wood had later been covered with gravel. It was found necessary to install temporarily some 11-kv. cables in this section and these were laid in a fir-wood box which was buried in the sand. After they had been in operation a short while, we observed that the cable sheaths were badly corroded on the sides and top. The corrosion pits in some cases were so deep that they were practically through the sheath. The crystals on that sheath were white, what we would ordinarily call white lead.

We began to look for the reason for the phenomenon. Knowing the history, we concluded that fermentation of the old wood caused acetic acid gases to pass up through the cable and coming in contact with moisture condensation on the cable this started the chemical reactions that produced the corrosion.

There is a description of this case in the proceedings of the March 1927 meeting of the Northwestern Electric Light and Power Association Technical Session.

G. D'Eu Stachio: It is well known that the acid contents of wood vary with the age of the trees, the season of cutting, and the nature of the soil in which the trees are grown. As only some of the conduits mentioned in the paper gave trouble (corrosive effect), it seems possible that investigation of the above mentioned variable may show the cause for the irregularity of the trouble and means of prevention by selection of the wood.

Our experience is that concentration of 0.1 mg. of acetic acid per cu. ft. of air is practically not active on lead in the absence of moisture but quite active in moist air. The condensation of moisture in the conduit increases the concentration of acid in the condensed water which accounts for the corrosion occurring at the upper part or on the sides of the cable sheath where the drops of water are formed, and never at the bottom of the cables or in cables submerged in water.

Referring to the air-ammonia treatment, it seems that the treatment may need to be repeated at intervals as the authors point out that the wood is quite difficult to penetrate and that the acid is contained in the untreated inner parts of the wood. Thus much of the acid can hardly be reached by air-ammonia treatment, and may later be leached out and again cause trouble.

E. R. Northmore: We had several cable failures in an old duct line located in an alley and could find no effects of electrolysis, but the cables had the same appearance and were damaged similarly to the ones described by the authors of this paper. We found that this alley had been used by a large number of truck horses for years and the fiber ducts had absorbed moisture. This had formed acetic acid and the lead was attacked as in the old Dutch process used in making white lead.

I should like to ask the author if he had ever experienced similar trouble with fiber ducts; while we have had no trouble of this character with fiber ducts installed in the last few years, our trouble was all in ducts 25 to 30 years old.

Also, how often will it be necessary to use this ammonia treatment?

J. H. Johnson: I should like to ask the author if he made tests for electrolysis, or tests on the local current form, because besides the flow of current from outside sources, there may be concentration due to the chemicals adjacent to the lead. The forces may be generated through difference in temperature. So I am wondering if any tests were made to see if there were local currents in that sheath caused by difference in conditions.

R. M. Burns and B. A. Freed: The case of corrosion described by Mr. Proebstel is of considerable interest and was due, we believe, to acetic acid originally present in the raw fir wood and did not depend necessarily upon a fermentation process.

Referring to Mr. D'Eu Stachio's discussion we may say that we have observed a variation in the acid content of wood samples from different trees of both fir and hemlock, but the magnitude of this variation for a given species is not sufficient, in our opinion, to furnish a basis for the selection of non-corrosive specimens. As to the effectiveness of the ammonia treatment, it has been found that a second treatment is necessary in a small per cent of the cases where the acid content was exceptionally high. We do not believe that neutralization of the acid is accomplished by ammonia penetrating the wood, but rather by absorption of the acid vapors by the ammonia solution. The moisture in the duct and on the cable is converted into an ammonia solution at the time of the ammonia treatment and tests have shown that ammonia persists in the ducts in some cases for more than five months. In other words, treatment with ammonia neutralizes the readily accessible acid in the duct and on the cable and forms an ammonia solution which then absorbs acid vapors which may come from the wood during a period of several months. The acid remaining in the wood after that period does not appear to furnish dangerous concentrations of acid vapor to the duct air.

Replying to Mr. Northmore's inquiry, we may say that our field inspection did not disclose any cases of corrosion in fiber conduit, though reports have been received that traces of corrosion have been found in fiber duct placed in early years.

4. This apparatus was devised by E. M. Honan and E. G. D. Paterson of the Bell Telephone Laboratories and will be the subject of a separate paper.

System Stability as a Design Problem

BY R. H. PARK* and E. H. BANCKER*
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—Under modern conditions of operation the ability of the component machines of a power system to hold in step during system disturbances has become of major importance in the choice of system layout and machine design. This situation has greatly stimulated activity in the study of the problems involved. This paper reviews the general problem in the light of recent analytical and experimental studies made by the authors. The subject matter of the paper is grouped according to the following headings:

Part I reviews the factors affecting stability, including generator short-circuit ratio, voltage regulators, excitation systems, neutral impedance, governors, amortisseurs, intermediate condensers.

Part II gives methods of calculating stability, under the following heads: preliminary calculations, idealized case of two machines, general case of two machines, extension to include more than two machines, simplified method of calculating tie lines.

The appendixes include an example illustrating the method of calculation in a particular case.

* * * * *

ORIGINALLY the maximum power output of synchronous apparatus was fixed by the maximum current which the machines could supply without excessive heating, while the maximum power which could be transmitted economically over a transmission line was determined by the allowable losses in the line.

However, as progress was made in the design of electrical machinery and as the voltages at which power was transmitted were increased, it was found that the line and machine reactances must be held low enough to avoid the possibility of pull-out or instability, whether occasioned simply by excessive loads or by system disturbances.

Practically this situation was met principally by designing generators with high enough short-circuit ratio, and lines with low enough reactance to meet ordinary requirements. At the same time, however, the latent economic advantages in view have greatly stimulated interest in the development of means auxiliary to these, by which increases in stability of operation can be economically effected.

Further, there has been an active demand for more accurate methods of calculating stability, in order to make possible the design of systems on a rational basis. The present paper presents first, a brief review of the principal factors affecting the stability of power systems; and second, a summary of some of the methods of calculation which the authors have employed in the study of practical systems.

Part I

Review of Factors Affecting Stability

a. Generator Short-circuit Ratio. In general the maximum power that can be delivered by a synchronous generator depends upon the character of the load circuit to which the generator is connected. To illustrate, suppose that a turbo alternator is supplying power for an induction motor load connected directly to its terminals, and suppose that as the induction motor load is gradually increased the generator terminal voltage is maintained substantially constant by occasional manual adjustment of the generator's exciter field rheostat. Further, suppose in the interests of simplicity and definiteness that it is provisionally agreed that the effects of saturation may be neglected in both machines. In this case, neglecting armature resistance the system may be represented by the equivalent electrical circuit of Fig. 1.

where e = generator terminal voltage
E = generator nominal voltage
R = induction motor rotor resistance
x_m = induction motor magnetizing reactance
x_s = generator synchronous reactance
x = induction motor rotor reactance
s = induction motor slip

from which the relation of slip and mechanical torque may be found.

In general the mechanical load of the motor will

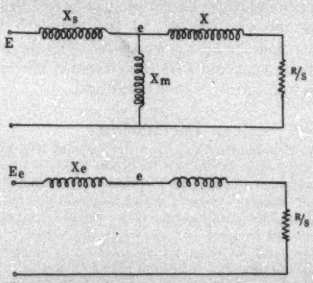

Fig. 1—Equivalent Circuit of Synchronous Generator Supplying an Induction Motor Load

depend upon its slip, but for any given speed-torque characteristic of the shaft load, there will be some definite value of load, slip, and excitation. If the generator excitation is held constant while the slip is increased due to a momentary increase in the motor load torque, the system will be stable and will return to a position of equilibrium, if $\frac{dT}{ds} > \frac{dL}{ds}$, where L = induction motor

*Both of General Electric Co., Schenectady, N. Y.

Presented at the Pacific Coast Convention of the A. I. E. E., Spokane, Wash., August 28-31, 1928.

28-129

load torque; T = torque exerted by induction motor.

Thus, Point A, Fig. 2, shows a condition of stable operation and Point B one of unstable operation.

Actually the motor load will depend upon the motor speed rather than merely upon its slip with respect to the generator. As the generator has a load-speed characteristic of its own it would be necessary in general to give consideration to it. If the induction

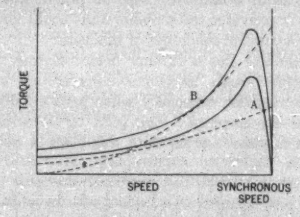

FIG. 2—TORQUE SPEED CURVES OF INDUCTION MOTOR AND LOAD

——— Motor characteristic
- - - Load characteristic

motor is supplying a constant torque load, instability will occur with $\frac{d\,T}{d\,s} = 0$. For equal machines, choosing $x = 0.15$, $x_m = 3$ in per unit* of machine reactance, the torque at instability is found to vary in this case with generator per unit reactance, as shown in curve A, Fig. 3.

On the other hand if the load torque is proportional to the square of the speed—*i. e.*, to $(1 - s)^2$, as would occur substantially with a fan or centrifugal pump load, it is impossible to find any condition under which $\frac{\alpha\,T}{d\,s} < \frac{d\,L}{d\,s}$, *i. e.*, the motor cannot be unstable. However, there is a maximum torque which can be carried in this case. This is shown in Curve B of Fig. 3.

It will be seen that in general the generator synchronous reactance must not exceed approximately 1.20 for stability at full-load. In practise the effects of saturation and of any shunt load will be to permit a higher value of machine reactance with stability.

Were the induction motor replaced by a synchronous motor there would be with equal excitation and neglecting salient poles,

$$T = \frac{E^2 \sin\theta}{2\,x_0}$$

where θ = displacement angle of machine rotors.

*Fraction of normal: Thus 3 per unit = 300 per cent; reference (5).

Instability occurs at $\frac{d\,T}{d\,\theta} = 0$, *i. e.*, at $\theta = 90$ deg. for which $E = \sqrt{2}\,e$, so that with normal terminal voltage ($e = 1.0$)

$$T_{max.} = \frac{1}{x_0}$$

Thus, in this case the generator and motor per unit synchronous reactances must not exceed unity for stability at full load, while if the generator and motor are separated by a transmission line it is necessary that their reactances be less than unity for stability under the same conditions. Actually, of course, as mentioned in the previous instance, the effects of saturation and of impedance load are to reduce the requirements as to low synchronous reactance, or what is substantially the same thing, high short-circuit ratio. Nevertheless, even considering this circumstance the fact remains that machines which are to be operated with manual control of excitation require in general a short-circuit ratio in the neighborhood of unity,† especially if ability to operate through severe system disturbances is desired, while if long distance transmission over high-voltage lines is involved a short-circuit ratio considerably higher than unity may be found advantageous.

When synchronous machines were first being built

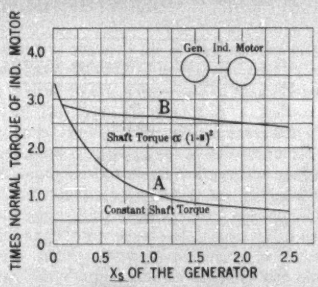

FIG. 3—CALCULATED VALUES OF MAXIMUM AND CRITICAL TORQUE OF INDUCTION MOTOR CONNECTED TO A SYNCHRONOUS GENERATOR, AS A FUNCTION OF GENERATOR, SYNCHRONOUS REACTANCE, (X_s)

this limitation was not felt, since the requirements for inherent voltage regulation necessitated construction with high short-circuit ratio. The advent of the Tirrill regulator largely disposed of this necessity. This circumstance, together with the development of improved designs permitting greater current loading of the armature, has established a considerable inducement toward the use of machines of low ratio both from the standpoint of reduction in first cost and improvement in efficiency. On the other hand power transmission developments have tended to demand a high ratio.

†Bibliography 11.

b. Voltage Regulators. If the excitation voltage can be continuously varied in proper phase relation to the machine terminal voltage it is possible to operate above the limit of stability with hand control. Recent tests have shown that when an appropriate type of regulator is employed a large gain in stability is available

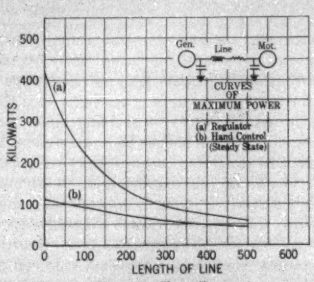

FIG. 4—TEST DATA

Steady load which can be carried over model line. (exciter speed less than 250 volts/sec.)

both under steady and transient conditions. Thus, Fig. 4 shows the gain in steady-state stability due to such a regulator as a function of the length of the transmission circuit. In the case of system disturbances the regu-

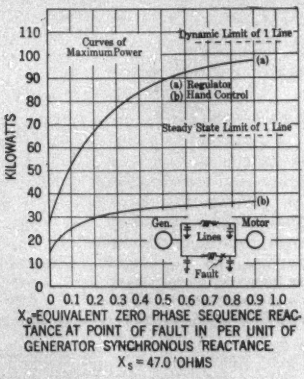

FIG. 5—TEST DATA

Power which can be carried through a line to ground fault on one of two parallel 250-mi. model lines. Fault cleared at both ends of line in 0.6 sec. Fault on generator bus corresponds approximately to $X_0 = 0.10$; fault at other points of the line correponds to $X_0 > .1$

(Exciter build-up speed of 225-250 volts per sec.; exciter ceiling 320 volts; exciter operating voltage 70 volts)

lator has an additional function to perform; *viz.*, to rapidly effect a substantial increase in the exciter voltage, thus reducing the decrease of field flux linkages which would otherwise occur, or actually causing them to increase. In practise the gain to be anticipated in stability through faults depends both upon its ability to effect rapid increases in excitation and upon its ability to resume "balancing" after the initial phase of the disturbance is past.

The magnitude of the gain to be anticipated in practise is illustrated by Figs. 4 and 5.

Besides its effect in increasing the stability of machines of normal design the regulator should make possible the utilization of machines of lower short-circuit ratio than would otherwise be practicable for by its action it provides the stability that could not otherwise be obtained in machines of this type.

The regulator has a further field. It may also be applied with benefit to synchronous motors and frequency converters, especially when severe load surges are to be encountered. The gain to be anticipated should be especially great with this class of apparatus

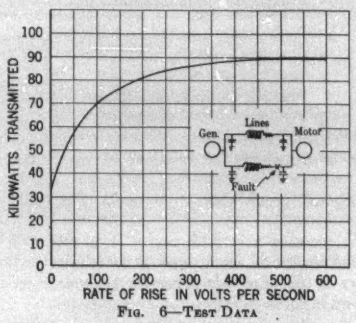

FIG. 6—TEST DATA

Showing the effect of speed of excitation on the transient stability of the model transmission system (ceiling voltage 320 volts; operating voltage 70 volts)

since it operates substantially from an infinite bus; *i. e.*, there is no transmission line.

c. Excitation Systems. The gains in stability which can be effected by a voltage regulator of a given type depend upon the rate of voltage build-up of the exciter, and upon its operating and ceiling voltages.

The available data indicates that if normal ceiling voltage is available, a moderate speed of excitation, say for large machines 150 to 200 volts per sec.,* is sufficient in the usual case to prevent pulling out of step on other than the first swing of the machine rotors. In general the gain due to a higher speed of build-up than this is small in comparison with the gain obtainable with the speed of excitation referred to, since with these higher speeds the time available for modifying the rotor flux linkages is small (of the order of ¾ sec.). Thus careful step-by-step calculations were made in the case of three representative systems in which water-wheel generators feed load centers over transmission lines of respectively 60, 120, and 250 mi. The results showed that the

*For 250-volt fields. One-half as much for 125-volt fields.

power which can be carried without pull-out on the first swing of the machine rotors, after the occurrence of a line to ground fault on the generator bus, is only from four to six per cent more with excitation sufficient to maintain constant rotor linkages* than can be so carried with a speed of excitation sufficient to prevent pull-out on the second swing, *i. e.*, about 150 to 200 volts per sec.* This conclusion is borne out by the test data in Fig. 6 which shows the gain in transient stability as a function of exciter speed for a line to ground fault on one of two 250-mi. lines.

As these data were taken with small machines the excitation speeds shown should be multiplied by a factor less than unity in order to pro-rate them to correspondence with large machines. This factor probably ranges from 0.5 to 0.6.

On the other hand, in some cases, and especially in connection with synchronous condenser stations, it may be desirable to secure additional stabilizing action by building up the machine excitations to a very high value within a fraction of a second. To achieve this result requires a new type of exciter; one with a high ceiling voltage, and with a correspondingly high speed of build-up.

d. Neutral Impedance. The generally admitted

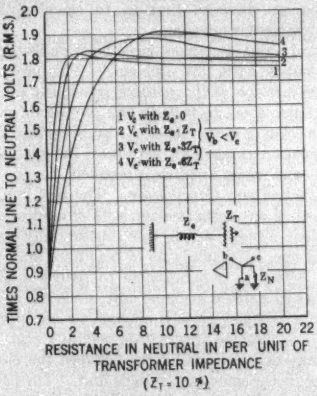

FIG. 7—CALCULATED LINE TO GROUND VOLTAGES ON THE OPEN PHASES WITH A LINE TO GROUND FAULT, AS A FUNCTION OF RESISTANCE IN THE NEUTRAL

circumstance that the majority of transmission line lightning flashovers establish arcs from one line to ground, and thus that this type of fault is of primary importance in determining the transient stability of transmission systems, suggests the desirability of using current limiting resistors or reactors in the high-voltage transformer neutrals, or as an equivalent grounding the neutral, and designing the transformer for high impedance to ground current.

Obviously any means for reducing the magnitude of the system fault current is beneficial from a stability standpoint. In general, however, it is desirable that enough reactive current be permitted to flow to allow

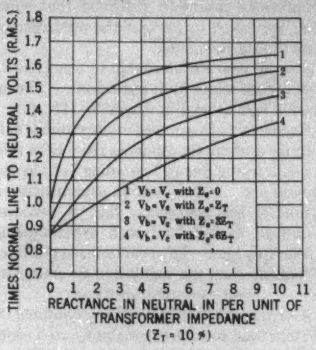

FIG. 8—CALCULATED LINE TO GROUND VOLTAGES ON THE OPEN PHASES WITH A LINE TO GROUND FAULT, AS A FUNCTION OF REACTANCE IN THE NEUTRAL

rapid selective relaying by ground current relays, and to avoid the possibility of arcing grounds. Consideration is invited to the greater use of reactors instead of the usual resistors. In principle, a reactor in the neutral is no more likely to cause excessive system voltages than a grounding transformer is, that is, precautions against excessive voltages are not necessary unless the reactor approaches the dimensions of a Petersen coil. Also even with a Petersen coil there does not appear any source of danger which cannot be overcome with relative simplicity. In most cases, however, very large gains in stability are obtainable with reactors which do not even approach to the Petersen coil value, *i. e.*, to a value such that the reactor current equals the fault charging current.

The principal advantage of reactors as compared with resistors are their greater reliability, usually lower cost, and the fact that in case of line to ground fault they tend to increase the phase to ground voltage on the clear phases to a much lower extent. This last factor which is illustrated in Figs. 7 and 8 is of great importance in connection with duty on lightning arresters. In order to obtain an appreciable gain in stability the resistance must be considerably greater than the system reactance viewed from the point of fault. Referring to figure this means that the voltages to be anticipated from a clear phase to ground are necessarily $\sqrt{3}$ or more times normal voltage.

*Equivalent to about 600 volts per sec. with a normal ceiling exciter.

With a reactor this is not the case, and in fact a marked gain in stability can be obtained with a value of reactance such that clear phase to ground voltage is say less than 30 per cent more than normal.

Although the use of grounding reactors of high enough reactance to approach the Petersen coil value has not been recommended generally, it is felt that in a large number of cases the use of Petersen coils may nevertheless be desirable, provided that suitable precautions are taken to prevent the occurrence of excess voltages during switching.

In general, it is also desirable to check for the possibility of corona under fault conditions. The method of calculation is discussed in Appendix III.

e. Governors. In a considerable number of instances momentary system disturbances have been followed by prolonged hunting due to "pumping" of the steam turbine governors. This condition usually arises when the regulation of the governors, that is the speed range from no-load to full load with synchronizing spring fixed, is unduly low. To avoid hunting of this character it is desirable that the regulation be four per cent or above.

f. Amortisseurs. In general any damping influences in a system tend to improve stability, and especially in connection with systems involving a number of independent machines each of which is capable of swinging more or less independently of the others. Further, when high resistance lines are involved, stable operation of laminated pole machines without damping windings may be impossible under some load conditions. A consideration of these factors suggests the general desirability of equipping water-wheel generators with low resistance amortisseur windings.

g. Intermediate Condensers. The use of synchronous condensers or of synchronous condensers in combination with shunt reactors is sometimes preferable to the construction of additional overhead line. In such cases, however, it is probably desirable that use be made of a balancing type of voltage regulator, as the stabilizing influence of the condenser is greatly increased in this way. Thus, shop tests on a model 500-mi. line show the following results:

Gain in steady power limit over hand control value with no condenser

a. With regulator on generator and motor but no condenser 21 per cent,

b. With intermediate condenser equal in size to generator and motor but hand control on all machines 25 per cent,

c. With intermediate condenser and regulator on all machines 99 per cent.

When the regulator is used throughout, the gain affected by the condenser is $199 \div 121 - 100 = 64$ per cent. When hand control is employed, the gain due to the condenser is only 25 per cent.

While the effect of short circuits and other disturbances will, in general, be such as to reduce the power which can be carried with stability, the gain from an appropriate regulator will probably be about as under steady-load conditions.

In order to reduce to a minimum the adverse effect on stability of short circuits and similar disturbances, the use of a high ceiling or superexciter would appear to be, in general, desirable. The exciters for the condensers of the Plymouth Meeting Station of the Philadelphia Electric Company with a ceiling of 950 volts and an exciter speed of 6400 volts per sec. illustrates this principle. The calculated gain in stabilizing influence is such that three condensers with the exciter are found to be equivalent as regards transient stability to five with ordinary speeds of excitation.

Part II

Methods of Calculating Stability

Originally the stability of power systems was calculated principally in terms of the maximum power which could be carried under steady-load conditions. In general, knowledge of the steady power limit of a system is valuable, since it indicates the margin of stability available under normal conditions, and thus, when coupled with an experience factor, gives a measure of the power which can be carried through disturbances. However, a much more direct and accurate method of approach to the problem is to calculate the effect of disturbances directly, that is, to calculate stability under transient conditions.

Several valuable papers have been published in the past, dealing with this problem.* The present paper provides a review of the methods previously established and extends them to include any number of machines and the effect of salient poles. Also, simplified methods of calculations have been developed for application in special cases where their use is justified.

a. Preliminary Calculations. The first step in the treatment of any actual system is its simplification to an equivalent system which will substantially represent the real one, particularly as to reactance and flywheel effect. Often it is permissible to reduce a group of machines to a single machine unit, since the individual parts are coupled closely enough to be so treated. Thus, it is assumed that within such a group, though there may be local oscillations between the parts, the net effect is the same as the single unit. The process of simplification thus consists in the replacement of part of the system by a unit machine, either synchronous or induction, which behaves with respect to observations made on the connecting lines in substantially the same manner as it is expected that the part of the system would.

In order to select intelligently the equivalent machines, it is first necessary to choose substantially correct values of reactance for the units of the real system. The larger units such as generators, condensers, and possibly large synchronous motors may be

*Bibliography 3, 12, 13.

correctly represented by their actual values if known. Attention is called to the fact that the value of transient reactance composed of stator and field leakage is to be used and not the subtransient which includes the effect of amortisseurs, etc. The latter has frequently been confused with transient reactance since it is used in determining the instantaneous current rush during a short circuit. The effects which cause it to differ from transient reactance, disappear so rapidly that in any study of conditions obtaining more than a few cycles after the occurrence of a transient, the real transient reactance should be used. It is most convenient to combine the smaller loads into substation loads. Thus each station would have one resistance load, one induction motor load, and one synchronous load, each equal to the total load of that particular character supplied from that particular station. Tables I to III show the average per unit reactances applying to the different classes of apparatus and the values given in it may be used for all apparatus whose actual values are unknown.

These reactances are based on kilovolt-ampere rating so that when the load is known in kilowatts, allowance must be made for the difference. If data are available regarding the connected kilovolt-ampere of load per kilowatt, they should be utilized in making this correction, but if not, certain assumptions may be made which will give approximately correct results. The induction motor load is assumed to be composed of a number of 75 per cent loaded motors which would have 0.89 power factor at full load and enough unloaded motors to bring the total power factor down to 0.7. From these premises it may be shown that the kilovolt-ampere rating of the induction motor load is 2.75 times the kilowatt load. The synchronous motors are assumed at half load and their rating in kilovolt-ampere is accordingly 2 times kw. at rated power factor.

Having assigned reactance to all of the apparatus, the positive, negative, and zero-phase sequence reactances of the receiver system viewed from the point of interconnection are found in the usual manner, either by computation or from the d-c. calculating table. The current in any circuit is obtained from the loads and terminal voltages. When the currents are known the machine field flux linkages and excitation may be found from the m. m. f. diagram. Fig. 9 shows the most general form of such a diagram. The figure is adapted from that shown in the paper *Synchronous Machines II* by Messrs. Doherty and Nickle, by introducing terminal linkages and rotor excitation, and discarding nominal voltages. Rotor linkages and the linkages inside transient reactance are also shown. The latter is represented physically by the initial terminal voltages which would be obtained if the armature were suddenly open circuited. It is desirable to work with linkages instead of voltage due to the fact that the machine torque can be expressed in terms of linkages and displacement angle, while if voltages are used the speed must be considered.

To represent the real system, the replacing machines should have the same inertia effect as the groups which they represent. This is readily found as the sum of the inertia constants of all the rotating apparatus in the group. Where the $W R^2$ of a machine is known, the inertia constant M may be calculated from the equation

$$M^* = \frac{0.462\, W R^2 \left[\dfrac{\text{r. p. m.}}{1000}\right]^2}{\text{Base kw.}} \text{ (sec).}$$

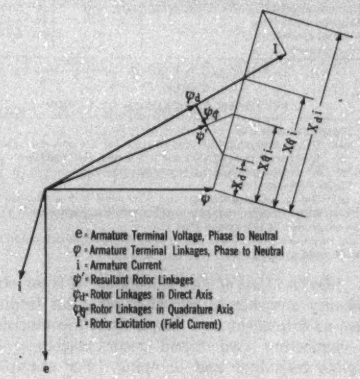

FIG. 9—GENERAL MAGNETOMOTIVE FORCE DIAGRAM

For those machines whose inertia is not known, representative values may be obtained from Table IV. Before adding the inertia constants, they must be calculated to a common kilowatt base and therefore the values given in the table are multiplied by the ratio of kilovolt-ampere rating to kilowatt base. In general it is preferable to use the same kilowatt base as the kilovolt-ampere base on which the reactances are calculated so that both kilowatt and kilovolt-ampere are expressed as per unit values of the same number.

TABLE I
SYNCHRONOUS REACTANCES

	In direct axis	In quadrature axis
Turbo alternators	110 per cent	110 per cent
Synchronous motors	100 per cent	60 per cent
Synchronous condensers	150 per cent	85 per cent
Water-wheel generators	100 per cent	65 per cent
Induction motors	300 per cent	

TABLE II
TRANSIENT REACTANCES

	In direct axis	In quadrature axis
Turbo alternators	18 per cent	18 per cent
Synchronous motors	30 per cent	60 per cent
Synchronous condensers	40 per cent	85 per cent
Water-wheel generators	30 per cent	65 per cent
Induction motors	15 per cent	

*M = 2 times stored energy divided by base power.

TABLE III

NEGATIVE PHASE SEQUENCE REACTANCES

Turbo alternators	14 per cent
Synchronous motors	22 per cent
Synchronous condensers	28 per cent
Water-wheel generators	40 per cent
Induction motors	15 per cent

TABLE IV

INERTIA AND OPEN CIRCUIT TIME CONSTANTS

	M = inertia constant	T_0 = open-cir. time constant
Turbo alternators	16.0	5—8
Synchronous motors	4.5	2—4
Synchronous condensers	3.0	5—7
Water-wheel generators	6.0	4—6
Induction motors	1.0	

TABLE V

ZERO-PHASE REACTANCE OF LINES†

1. Three-conductor line
   (a) Heavy non-magnetic ground wire use 2 x pos. seq. reactance
   (b) Light steel ground wire use 3½ x pos. seq. reactance
   (c) No ground wire use 3½ x pos. seq. reactance
2. Three-conductor lines
   (a) Heavy non-magnetic ground wire use 3 x pos. seq. reactance
   (b) Light steel ground wire use* 5 x pos. seq. reactance

*This value used in example in Part II.

†When more accurate figures are desired they may be calculated by the method given in Appendix II.

b. Idealized Case of Two Machines. It is sometimes sufficiently accurate to regard a generator feeding a system as equivalent to a simple system comprising a line connecting two round rotor machines, with negligible resistance and damping. For example, a

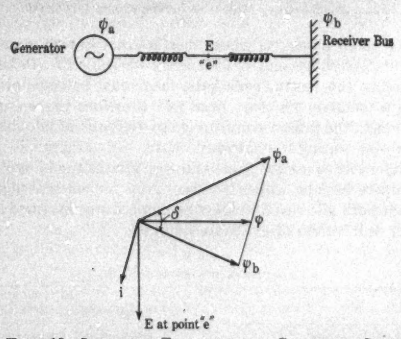

FIG. 10—SIMPLIFIED EQUIVALENT OF GENERATOR SUPPLYING A SYSTEM THROUGH A LINE WITH VECTOR DIAGRAM OF CURRENT, VOLTAGE, AND LINKAGES

generator supplying a system through a line may be reduced to the simple system shown in Fig. 10.

Further, if only the first swing is under consideration, or if more than the first swing is to be considered but a suitable regulator and exciter are available, on the basis of these premises it is approximately correct to assume that the machine field circuit flux linkages remain constant. The initial conditions are determined from a vector diagram as shown in Fig. 10.

The power transfer at any angle is $T = \dfrac{\psi_a \psi_b}{x} \sin \delta$ where ψ_a and ψ_b are the flux linkages of the equivalent machines, x is the total transient reactance between the linkages, and δ the angle between the rotors. The maximum power occurs at $\delta = 90$ deg. and is $T_m = \dfrac{\psi_a \psi_b}{x}$. Fig. 11 shows the power angle or, more strictly, torque angle characteristic.

Consider then a system having the characteristic shown in Fig. 11, and operating at an initial angle δ_0 corresponding to a torque T_0 and suddenly impress a mechanical torque T_1 upon the motor. The acceleration at any time is proportional

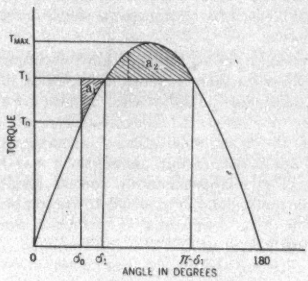

FIG. 11—TORQUE ANGLE DIAGRAM SHOWING EFFECT OF SUDDENLY APPLIED LOAD

to $T_1 - \dfrac{\psi_a \psi_b}{x} \sin \delta$ and the work done upon the rotor in giving it a velocity is equal to the integral of this quantity with respect to δ, which is the area between T_1 and the torque angle curve. By the time the angle has increased to δ_1 the rotor has acquired a velocity proportional to the work done upon it and will therefore overtravel until it gives up as much work as it received, or until the area above T_1 equals that below.† If too high a torque is impressed, the area above can never equal that below. The rotor swings until the electrical torque is less than the mechanical torque and the machine falls out of step.

There is then a maximum that may be so applied without loss of synchronism, this maximum varying with the previous steady load T_0. The curve in Fig. 12 shows the maximum additional load which may be thrown on with stability maintained as a function of initial load, all values being expressed in per unit of the

†See Bibliography 12, 15 and 19.

maximum steady-state power. In using this curve, it should be remembered that it holds true only when one machine has infinite inertia. In the case of two finite inertia machines with sudden application of torque to one end only, and a gradual rise in torque at the other, as the speed drops, the maximum is higher than is shown on the curve and is dependent upon the rate of rise of torque and the relative inertias.

When one of a number of parallel circuits is opened, the circuit reactance is increased and the maximum

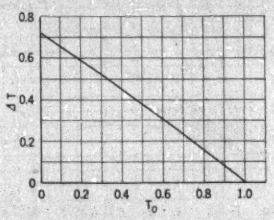

FIG. 12—TRANSMISSION LINE CONNECTING TWO ZERO REACTANCE MACHINES

Effect of a suddenly applied load
T_0 = Intial torque
ΔT = Maximum sudden increase in torque with stability maintained
Maximum sudden increase in load with stability as a function of initial load

steady-state power reduced. Thus, if the torque angle characteristic of a circuit with two lines in service is given by curve A in Fig. 13, and one line is suddenly opened, it becomes curve* B. If the system has an

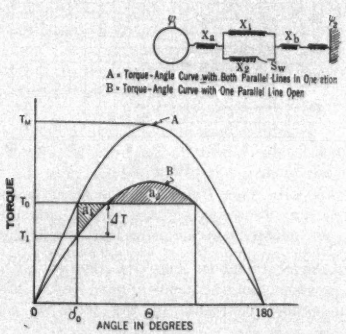

FIG. 13—EFFECT OF OPENING A PARALLEL LINE

initial load of T_0 and is at an angle δ_0 the torque will immediately change to T_1. The effect is the same as though the system had been operating with curve B characteristic and an initial load T_1, and the mechanical load, were suddenly increased at both ends to T_0. Applying the reasoning outlined above regarding a suddenly applied load and the work area, a curve such as Fig. 14 may be plotted showing the maximum load that may safely be carried if a circuit is to be opened.

When a change in torque occurs, the rotors of the machines must move to new angular positions and they will accelerate in proportion to the net torque acting upon them and inversely as their inertias. The total angle between two machines having inertia constants M_1 and M_2 is the same at any instant as the angle between two machines having constants infinity and

$M = \dfrac{M_1 M_2}{M_1 + M_2}$ respectively. On this basis

$$t = \sqrt{\frac{M x}{4 \pi f \psi_a \cdot \psi_b}} F\ (\delta, T_1/T_m, T_0/T_m)$$

where t = time in seconds
f = frequency

$F\left(\delta, \dfrac{T_1}{T_m}, \dfrac{T_0}{T_m}\right)$ is shown in Figs. 15, 16, 17, and 18.

In its action upon the positive phase sequence net-

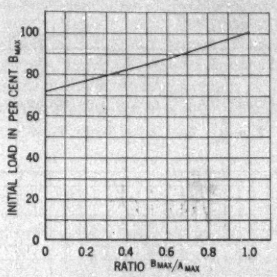

FIG. 14—MAXIMUM INITIAL LOAD WITH STABILITY ON OPENING A PARALLEL LINE

work, a short circuit is the same as connecting a three-phase shunt reactor at the point of fault.† If the short circuit is three-phase, the reactance of the shunt is zero. If line to line, it is the total negative phase sequence reactance of the system as viewed from the point of fault, and if line to neutral, it is the sum of the negative and zero-phase sequence reactances to the fault.

Under the assumed conditions of negligible resistance and no damping, there is neither any negative nor zero-phase sequence power and it is merely necessary to calculate the positive phase sequence power. The positive phase sequence diagram for the system shown in Fig. 10 becomes as shown in Fig. 19, for a line to neutral short circuit. An equivalent reactance may be obtained by paralleling the shunt with the reactance of the system from that end to the point of fault and changing the flux linkages of the machine with which

*See Appendix VI.

†See Appendix (VII).

it was combined to $\dfrac{x_0 + x_2}{x_0 + x_2 + x_b}$ times its former value. If it is combined with the receiver, the system becomes as shown in Fig. 19.

The torque angle characteristic is then

$$T = \frac{\psi_a \psi_b}{x_a + x_b + \dfrac{x_a x_b}{x_0 + x_2}} \sin \delta$$

where x_a = positive phase sequence reactance from generator to point of fault.

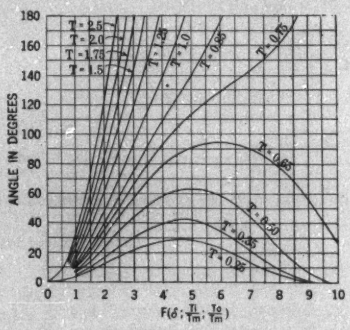

FIG. 15—ANGLE TIME CURVES FOR SEVERAL VALUES OF $T = \frac{T_1}{T_m}$, $T_0 = 0.0$ OF TOTAL TORQUE

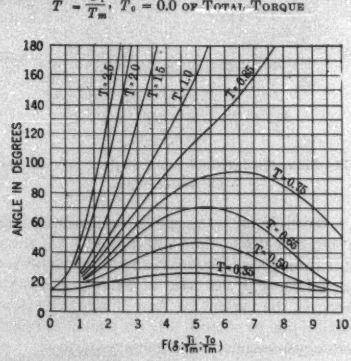

FIG. 16—ANGLE TIME CURVES FOR SEVERAL VALUES OF $T = \frac{T_1}{T_m}$, $T_0 = 0.25$ OF TOTAL TORQUE

x_b = positive phase sequence reactance from receiver to point of fault.

x_2 = total negative phase sequence reactance of system to point of fault.

x_0 = total zero-phase sequence reactance of system to point of fault

This characteristic is shown as the curve B in Fig. 20 Curve A is that obtaining prior to the short circuit.

Let the initial load and angle be T_0 and δ_0. At the instant of short circuit, the angle would remain unchanged and the electrical load would drop to T_1, the shaft torques, however, remaining constant at T_0. Again the effect is the same as though the system had been operating on the lower characteristic at a load T_1 and that the shaft torques had suddenly been raised to

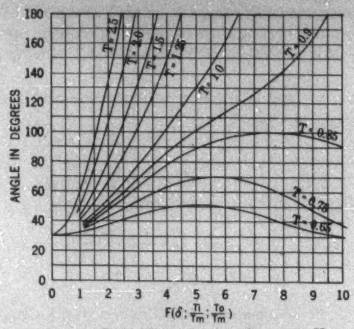

FIG. 17—ANGLE TIME CURVES FOR SEVERAL VALUES OF $T = \frac{T_1}{T_m}$, $T_0 = 0.50$ OF TOTAL TORQUE

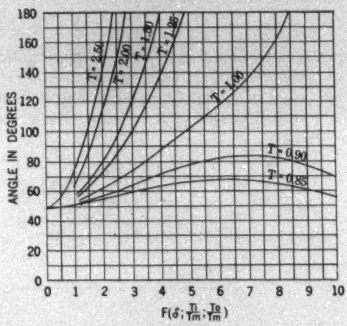

FIG. 18—ANGLE TIME CURVES FOR SEVERAL VALUES OF $T = \frac{T_1}{T_m}$, $T_0 = 0.75$ OF TOTAL TORQUE

T_0 and the treatment is, therefore, identical with that given previously for a suddenly applied load. Insertion of the proper constants in the term to convert $F\left(\delta, \dfrac{T_1}{T_m}, \dfrac{T_0}{T_m}\right)$ seconds enables the rotor angle displacement to be found at any time t.

The purpose in finding the angle is to determine the behavior following the clearing of the short circuit and the angle should, therefore, be found at the time at which it is expected the circuit breaker will interrupt the fault current.

If the fault has been on a feeder, the opening of the circuit breaker does not change the system reactance and the circuit again returns to its original condition and torque angle characteristic. During the period of the short circuit, the rotors acquired a velocity and a new angle. Referring again to Fig. 20, the energy which has been imparted to the rotors during the fault time is represented by the shaded area a_1. At the angle δ_1, the short circuit was cleared and the electrical

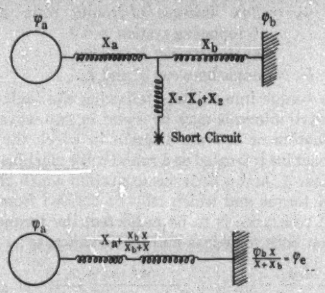

FIG. 19—METHOD OF TREATING A LINE TO GROUND FAULT

torque immediately rose from x to y. The rotors will continue to swing until the shaded area a_2 is equal to a_1, or in other words, until the stored energy has been given up. If the area a_2 does not equal a_1, it indicates that at an angle $\pi - \delta_0$, which is the maximum to which the rotors may separate and still transmit the required power, the rotors still have a velocity apart and will fall out of step. The criterion of stability is, therefore, the relative size of areas a_1 and a_2, or of the rectangle $\delta\, e\, f\, g$ and the area enclosed within the heavy line. The area of the rectangle is $T_0\,(\pi - 2\,\delta_0)$.

The area of the irregular curve is $= T_m\,\{(1 + r_1)\cos\delta_0 + (1 - r_1)\cos\delta_1\}$.

δ_1 = angle at which circuit breaker opened.

r_1 = ratio of the peak of torque angle curve with short circuit on, to the peak of the initial curve.

If the area of the rectangle is larger than that of the irregular curve, the system will be unstable.

When a short circuit takes place on one of the interconnecting lines of the system, its isolation causes a return not to the original condition, but to some third one. Assume a short circuit to have occurred at the receiver end of one of the tie lines of Fig. 13 and that it has been cleared. Up to the time of the opening of the breaker there is no difference between such a fault and one on a feeder off the receiver bus, but following its disconnection, the torque angle characteristic is given by $T = \dfrac{\psi_a\,\psi_b}{x_a + x_1 + x_b}$ from Fig. 13, and shown by the intermediate curve C of Fig. 21. Following the same procedure as for a feeder fault, the areas which determine stability are

Area of rectangle $= T_0\,(\delta_2 - \delta_0)$

Area of irregular curve $= T_m\,\{r_1\,(\cos\delta_0 - \cos\delta_1) + r_2\,(\cos\delta_1 - \cos\delta_2)\}$

where $\delta_2 = \pi - \text{arc}\sin\dfrac{\sin\delta_0}{r_2}$

r_2 = ratio of peak of torque curve following clearance of fault to peak of initial curve.

From the configuration of the areas, it is obvious that more load may be safely carried during short circuits if the fault is rapidly isolated. An example of the method of calculation outlined is given in the Appendix VII.

c. General Case of Two Machines. When it is necessary to take into account other features such as salient pole characteristics, flux decrement, voltage regulators, resistance of lines, governors, and amortisseurs, recourse must be had to a step-by-step method.*

The procedure in a study of two machines involving all of the characteristics noted is as follows. The process of simplification is carried out as for the case of two round rotor machines of constant flux linkages. From known voltages, loads, and current or power factor prior to the short circuit, calculate the magnitude and angle from some reference voltage of the flux linkages inside of the transient reactance of the equivalent machine replacing the system. To find the flux

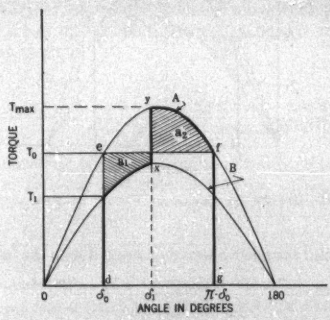

FIG. 20—EFFECT OF CLEARING FAULT ON FEEDER

linkages of the salient pole machine it is first necessary to find the rotor position with respect to the reference voltage V and this done by adding vectorally to V the quadrature reactance drop between it and the rotor. The quadrature reactance involved consists of the quadrature reactance of the machine plus any additional external reactance between the machine terminals and the point in the system at which the voltage V exists. The voltage so determined does not repre-

*A time interval of 1/20 of a sec. has been satisfactory in most cases.

sent any physical quantity, but its angle gives the rotor angle correctly. The angle between the current and the rotor position is next found and i_d is obtained from

$i_d = I \sin$ (angle between current and rotor)

On the assumption, which is correct for a salient pole machine that $x_q' = x_q$

The rotor linkages are

$\psi' = V \cos$ (angle between V and rotor) $+ i_d\, x_d'$

where x_d' is the transient reactance between V and the rotor winding.

The excitation m. m. f. is calculated from

$$I = \psi' + i_d\,(x_d - x_d')^*$$

Determine the reactance of the shunt simulating the fault for the type of short circuit considered in the manner previously described. The system is then as indicated in the upper Fig. 19, except that x_a should be the reactance from the fault to the machine terminals. By paralleling the shunt and round rotor machine, it becomes the lower Fig. 19.

At the instant of short circuit ψ', ψ_b remain constant;

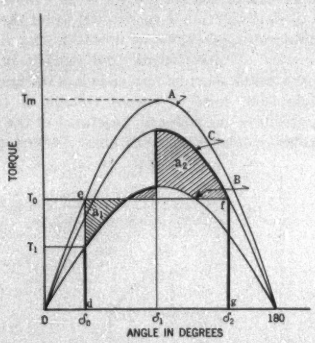

FIG. 21—EFFECT OF CLEARING FAULT ON PARALLEL LINE

and δ and the new values of i_d and ψ due to the fault are

$$i_d = \frac{(x_q + x_e)\ (\psi' - \psi_e \cos \delta_e) - r_e\, \psi_e \sin \delta_e}{r_e^2 + (x_d' + x_e)\ (x_q + x_e)}\dagger$$

$$I = \psi' + i_d\,(x_d - x_d')$$

Where x_q = quadrature transient reactance of machine alone

x_d' = direct transient reactance

x_d = direct synchronous reactance

$$r_e + j\,x_e = Z_e = Z_a + \frac{Z_b\, Z_s}{Z_b + Z_s}$$

Z_a = impedance from salient pole machine to point of fault

Z_b = impedance of equivalent round rotor machine to point of fault

Z_s = impedance of shunt at point of fault

ψ' = flux linkages of salient pole machine

$\psi_e = \psi_b \dfrac{Z_s}{Z_b + Z_s}$ flux linkages of machine replacing short and system

ψ_b = flux linkages of round rotor machine replacing system

δ_e = angle between ψ' and ψ_e

δ = angle between ψ' and ψ_b.

The torque immediately following the fault and at successive intervals may be found in two ways, either by equation or a circle diagram in which the salient pole machine is treated as a round rotor machine having reactance x_q and a fictitious excitation which gives the correct torque and which may be derived from ψ' and ψ_e. If resistance is to be neglected, the torque is the same on both machines and the equation is

$$T = \frac{I\,\psi_e}{x_d + x_e} \sin \delta_e + \frac{\psi_e^2\,(x_d - x_q)}{2\,(x_d + x_e)\ (x_q + x_e)} \sin 2\,\delta_e$$

The first term contains the variables I and δ_e and the second term δ_e only. The easiest method of finding the power at each interval is to plot each term as a function of δ_e or δ which differs from δ_e by a constant angle, with I as unity. The second term may then be read directly from the curve for the known angle, while the first term merely requires the multiplication of the curve reading by the value of I at the time.

When resistance is to be included, the torque equations contain a large number of terms and would necessitate much tedious arithmetic at each step, so the circle diagram method is preferable in spite of the increased preliminary work to prepare it.

Using x_q for the salient pole machine and the other circuit constants as in Fig. 19, the general circuit constants are found.‡ From these, torque circles are constructed.

The equation of the receiver torque circles is

$$(T_2 + \psi_b^2\, l)^2 + (Q_2 + \psi_b^2\, m)^2 = (n\ O\ Q\ \psi_b)^2$$

and of the generator circles is

$$\left(\frac{T_1^2}{O\,Q^2} - l'\right)^2 + \left(\frac{Q_1}{O\,Q^2} - m'\right)^2 = \left(n\,\frac{\psi_b}{O\,Q}\right)^2$$

These are families of circles having centers $-\ \psi_b^2 l$, $-\ \psi_b^2\, m$ and l', m' respectively and radii $n\ O\ Q\ \psi_b$ and $n\,\dfrac{\psi_b}{O\,Q}$ respectively. A number of circles are drawn for various values of $O\,Q$. The terminology used is

*For a round rotor machine both axes should be considered. However, as the decrement in the quadrature axis is about 4 times that in the direct axis, this refinement is not always essential.

†Bibliography 3 and 5.

‡See table on p. 144 "Electrical Characteristics of Transmission Circuits," by William Nesbit.

T_2 = torque received
Q_2 = reactive kilovolt-ampere received
T_1 = torque generated
Q_1 = reactive kilovolt-ampere generated
OQ = fictitious excitation of equivalent round rotor machine

$$l = \frac{a_1 R_0 + a_2 x_0}{R_0^2 + x_0^2}$$

$$m = \frac{a_1 x_0 - a_2 R_0}{R_0^2 + x_0^2}$$

$$n = \frac{1}{\sqrt{R_0^2 + x_0^2}}$$

$$l' = \frac{d_1 R_0 + d_2 x_0}{R_0^2 + x_0^2}$$

$$m' = \frac{d_1 x_0 - d_2 R_0}{R_0^2 + x_0^2}$$

where the general circuit constants are

$A_0 = a_1 + j\,a_2$
$B_0 = R_0 + j\,x_0$
$D_0 = d_1 + j\,d_2$

To complete the diagram, the reference axes for the displacement angles are laid off through the centers of the circles at a clockwise angle from the vertical whose tan is R_0/x_0. The line of zero angle is downward in the generator circles and rotor angles are measured counter-clockwise from it. In the receiver circles, it is upward, and angles are measured clockwise from it. For a given OQ and δ the per unit receiver torque is read directly, but the number read from the generator circle is multiplied by OQ^2 to get the per unit torque generated.

The fictitious excitation OQ is obtained from the real excitation I from $OQ = I - i_d\,(x_d - x_q)$. For the instant following the short circuit i_d has already been calculated so OQ is readily found but it is unnecessary to get i_d at later intervals for other purposes. Therefore, the determination of OQ is simplified if a curve is plotted of its variation with δ. $OQ = k\,I + k_1 \cos\delta_e + k_2 \sin\delta_e$.

Since δ_e differs from δ by a constant angle, the sum of the last two terms may be plotted as a function of δ. At any later time, the first term is obtained from I and the sum of the other two from the curve.

The torque acting to retard or accelerate the rotor is the difference between the mechanical and electrical torques, both of which are complex functions of time and involve governor characteristics, damping action, flux linkages, and angles. The degrees change in angle of each machine during the interval immediately following the short circuit is given by

$$\Delta\delta = \left(1 - \frac{F\,\Delta t}{4\,M}\right)\frac{360\,f\,\Delta t^2}{2\,M}\,(T_0 - G_0)$$

in which Δt = time interval in seconds
M = inertia constant of machine
F = per unit damping torque per unit speed change
f = frequency in cycles per second
T_0 = electrical torque at instant after fault
G_0 = mechanical torque at instant of fault

The change of angle of both machines is added to or subtracted from the original angle between the rotors and the angle at $\Delta t = 1$ is found. Before calculating the power at this interval a new value of I must be determined from the field decrement, excitation system, and change in armature reaction. A per unit slip-ring volt *vs.* time curves should be plotted from regulator and excitation data and the average value of slip-ring volts during the interval picked out. I will tend to die away from its initial value to that which would be sustained by the slip-ring volts (E) along an exponential curve having a time constant of

$$T_f = T_{f0}\,\frac{x_d' + x_e}{x_d + x_e}$$

where T_{f0} is the time constant of the field with the stator circuit open. For the short intervals used, it will have a value of $I_{B(n-1)} = E + \left(1 - \frac{\Delta t}{T_f}\right)(I - E)$ at the end if i_d did not change. However, i_d does change with rotor position and to determine its effect it is assumed that the rotors suddenly move through the angles $\Delta\delta$ at the end of the period. The new value of I is then given by

$$I_{A(n-1)} = I_{B(n-1)} + \frac{\psi_e\,(x_d - x_d')\,\Delta\delta\,[\,(x_q + x_e)\sin\delta_{eA} - r_e\cos\delta_{eA}]}{r_e^2 + (x_d' + x_e)\,(x_q + x_e)}$$

where subscript B refers to conditions before rotor angle changed and A applies after the change. Here also a curve of the term

$$\frac{\psi_e\,(x_d - x_d')\,[\,(x_q + x_e)\sin\delta_e - r_e\cos\delta_e)\,]}{r_e^2 + (x_d' + x_e)\,(x_q + x_e)}$$

of δ_e makes the simplest method of using the equation. To $I_{B(n-1)}$ is added the difference between the curve readings for δ_{eB} and δ_{eA}. OQ is again calculated from $I_{A(n-1)}$ and the torques from the circle diagram.

For any other than the first period the angular change is computed as follows. The average per unit change in speed during the preceding interval from $(n-2)$ to to $(n-1)$ is $S_{n-\frac{3}{2}} = \dfrac{\Delta\delta_{n-1}}{360\,f\,\Delta t}$

where $\Delta\delta_{n-1}$ is the degree change between $(n-2)$ and $(n-1)$.

The average gate or throttle opening during the same interval cannot be determined so the value at $(n-2)$ must be used. There is

$$G_{n-1} = G_{n-2} + \Delta t\, A\, (G_{n-2}, S_{n-\frac{1}{2}})$$

in which A is a function of G and S and will have to be obtained from the designer of the governor.

$T_{(n-1)}$ is found from the circle diagram or the torque curve

$$\alpha_{(n-1)} = \frac{360 f\, (\Delta t)^2}{M} (T_{(n-1)} - G_{(n-1)})$$

$$\Delta \delta_n = \left(1 - \frac{F\, \Delta t}{M}\right) \Delta \delta_{(n-1)} + \left(1 - \frac{F\, \Delta t}{2\, M}\right) \alpha_{(n-1)}$$

This process is then repeated as many times as desired.

d. Extension to Include More Than Two Machines. When the system connections are such that two machines will not correctly represent real conditions and three or more must be taken into consideration, the principle of super-position is employed to calculate the currents and torques. The system should be condensed and simplified as much as possible by the combination of equipment into equivalent machines and the replacement of mesh connections by star connections.

The initial flux linkages and angles are determined as described in the section dealing with two machines and also the value of impedances to represent the short circuit. When the connections are complex and resistance is considered, the work of calculating the angle time curve may be shortened by an approximation which consists of putting a single resistor in the fault impedance which would initially have the same losses as the total of the positive, negative, and zero-phase sequence losses if the divisions of current were influenced by reactance only. The value of this resistance is obtained for each of the phase sequence networks by allowing unit current to flow into the fault, finding its division in all the circuits if they had reactance only and finding the total $I^2 R$ losses in them. The resistance which would have the same loss at the point of fault would be the sum of the $I^2 R$ losses divided by the unit I^2 assumed. The value to be placed in the fault impedance is the sum of the three resistors applying to the three-phase sequence networks plus any that may be assumed for the earth and fault itself.

The flux linkages and angles remain unchanged at the instant of short circuit, and the conditions immediately subsequent to the fault are determined by impressing on the circuit each of the machine linkages taken one at a time and considering all the other machines as shunt reactors.*

The current in each branch is the vector sum of the currents caused by the individual application of the flux linkages. From the currents and linkages, the electrical torque may be calculated and from them the angular change of each machine during the time interval. The excitation amperes are obtained by $I = \psi' + i_d\,(x_d - x_d')$ for each machine and the change in flux linkage if the angle were constant is $\Delta \psi' = \frac{1}{T_{f0}} (E_{n-1/2} - I_{n-1})\, \Delta t$ where T_{f0} is the open circuit time constant, $E_{n-1/2}$ is the average per unit exciter volts during the interval $(n-1)$ to n and I_{n-1} is the per unit field current at instant $(n-1)$. There is $\psi_n' = \psi_{n-1}' + \Delta \psi'$. Now let the angles change by the previously calculated amount, impress the individual linkages and find the branch currents, field amperes, and torques and repeat for as many steps as desired.

When the short circuit is removed the circuit constants again change and the problem is treated in just the same manner except that the initial velocities must be included.

e. Simplified Method of Calculating Tie Line. When both of the systems are very large, relative to the power limit of the line alone, an approximate answer may be obtained as follows, without the necessity of the detailed step-by-step calculations.

The maximum power which can be transmitted over a line of this type is approximately

$$V^2 \frac{\sqrt{x^2 + r^2} - r}{x^2 + r^2}$$

where r = line resistance
x = line reactance + the transient reactance of each system viewed from its end of the line†
V = line-to-line voltage

We wish to determine, however, the maximum amount of average power which can be sent over the line. To do so, it is essential that some method be available for determining the magnitude of the load surges to be anticipated.

The value of these surges may be calculated from the formula

$$\Delta p = \frac{\dfrac{\Delta f}{f_0}}{\dfrac{\alpha_1}{c_1} + \dfrac{\alpha_2}{c_2}}$$

where f_0 = normal frequency
Δf = the momentary differences in frequency which would exist if the systems were not tied together.
α_1 and α_2 = the average governor regulations of each system.
c_1 and c_2 = the corresponding generating capacities.

The quantity Δf may either be determined from a comparison of frequency charts, or, if it is imagined due to sudden loss of a specific load or generating capacity in either system, may be calculated. For example, the sudden loss of a load ΔL_1, in system 1, would increase

*In general, the reactance of each machine will be different in each of its axes.

†Measured by the short-circuit current fed from the system for a three-phase fault at the junction of the line and system.

Δf over what it otherwise would be by an amount

$$\frac{\alpha_1 \Delta L_1}{c_1}.$$

Under normal conditions of operation quite distinct from emergency conditions, a somewhat unsystematic variation of frequency in a system of about ¼ cycles in 60, is to be anticipated even in the best regulated of systems. This means that at best $\frac{\Delta f}{f_0} = \frac{1}{120}$. With normal four per cent governors, this gives the result

$$\Delta p_{min} = \text{about} \frac{0.2\, c_1\, c_2}{c_1 + c_2}.$$

Thus, if both systems are of approximately equal size, the minimum fluctuations to be anticipated are

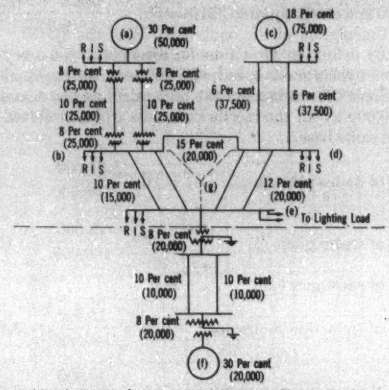

FIG. 22—ASSUMED SYSTEM AND PROPOSED EXTENSION USED AS A BASIS FOR THE SAMPLE CALCULATIONS

about 10 per cent of the generating capacity of either system, while if one system is much smaller than the other, they may be 20 per cent of the generating capacity of the smaller systems.

One peculiarity of the type of tie line under consideration is the fact that in general the wattmeters show a more or less steady pulsation of power at the natural frequency, determined by the inertias of the system and the synchronizing power of the line. The magnitude of pulsation seems to be of the order of one-half per cent of the generating capacity of the smaller systems, while the pulsation in itself may not be of significance from the standpoint of pull-out, because of its small magnitude, it can cause inconvenience to customers tapped off near the middle of the line due to the variation of brilliancy of lamps concurrent with the regular pulsation of voltage.

ACKNOWLEDGMENTS

The authors wish to express their appreciation of the assistance in the preparation of the paper and many valuable suggestions of Mr. J. B. McClure. They wish also to thank Messrs. R. C. Buell and Alan Howard for their assistance in calculations.

Appendix I

EXAMPLE TO DEMONSTRATE THE APPLICATION OF THE CONSTANT LINKAGE METHOD TO A PROPOSED EXTENSION—BASED ON DERIVATIONS IN THE APPENDIX

In Fig. 22 is shown a fictitious system of generating plants, together with transformer banks and connecting transmission lines. The portion of the system from *e* up constitutes an existing system, and that from *e* down represents a proposed extension to a new hydro station.

Apparatus to be Considered. The generating capacities are as follows

a. 50,000-kv-a. hydro station (existing)
c. 75,000-kv-a. steam station (existing)
f. 20,000-kv-a. hydro station (proposed)

Load centers (1), (2), (3), (4), and (5) are each assumed to consist of:

5000 kw. resistance load
5000 kw. at 0.7 power-factor lag induction motor load
5000 kw. at 0.8 power-factor lead, synchronous motor load.

Hence, each load center is supplying 15,000 kw. at 0.99 power factor lagging or 15,100 kv-a.

In order to assign equivalent reactances and inertia constants to the rotating apparatus of the various loads, the installed kilovolt-ampere capacities of the machines must be determined. In the event no specific data are available, it is reasonable to assume, and very close to operating conditions, that the synchronous motors are operating at half load 0.8 power-factor lead, but with field excitation to give full load 1.0 power factor and the installed kilovolt-ampere capacity will be twice the kilowatt load. Assuming that the 0.7 power-factor lagging induction motor load is composed of

I, motors at no load (synchronous impedance 300 per cent)

II, motors at ¾ load, the power factor of these motors being 0.89 lag at full load,

the installed kilovolt-ampere capacity of induction motors is 2¾ times the kilowatt load. On this basis, each load center has 13,800-kv-a. induction motors and 10,000-kv-a. synchronous motors, and with this knowledge the proper values of reactances and inertia constants may be chosen from the tables and referred to the assigned base.

Reduction of System to Equivalent Series Reactance. Transient reactances are used for positive phase impedances of rotating machines, except that synchronous reactance is used for the positive reactance of an induction motor.

Negative sequence reactances are used for negative phase impedances of rotating machines except that transient reactance is used for the negative reactance of an induction motor.

See Table V for zero-phase reactances of lines.

In the following calculations, all values of reactance are referring to *100,000-kv-a. base*, while in Fig. 22 the per cent values are given on the base indicated by the number in parenthesis.

Positive Sequence Reactance

$$\text{Generator (a)} = 30 \times \frac{100{,}000}{50{,}000} = 60 \text{ per cent on}$$

100,000 base

In parallel with load (3)

$$\text{Induction motor } 300 \times \frac{100{,}000}{13{,}800} = 2180 \text{ per cent}$$

$$\text{Synchronous motor } 30 \times \frac{100{,}000}{10{,}000} = 300 \text{ per cent}$$

From this the equivalent reactance of generator (a) and load is 48.7 per cent in series with

$$\text{Transformers } \tfrac{1}{2} \times 8 \times \frac{100{,}000}{25{,}000} = 16.0 \text{ per cent}$$

$$\text{Lines } \tfrac{1}{2} \times 10 \times \frac{100{,}000}{25{,}000} = 20.0 \text{ per cent}$$

$$\text{Transformers } \tfrac{1}{2} \times 8 \times \frac{100{,}000}{25{,}000} = \begin{array}{r} 16.0 \text{ per cent} \\ 100.7 \text{ per cent} \end{array}$$

in parallel with load (4)

2180 per cent
300 per cent

= 73.0 per cent which is equivalent positive reactance from point *b* to generator (a).

Generator (c)

$$= 18 \times \frac{100{,}000}{75{,}000} = 24.0 \text{ per cent in parallel with load.}$$

Induction motors

$$= 300 \times \frac{100{,}000}{13{,}800} = 2180.0 \text{ per cent}$$

Synchronous motors

$$= 30 \times \frac{100{,}000}{10{,}000} = 300 \text{ per cent}$$

From this the equivalent reactance of generator (c) and load
is 22 per cent in series with lines
of 8 per cent gives a total of 30 per cent in parallel with

Synchronous motors	300 per cent
Induction motors	2180 per cent

= 27.2 per cent which is the equivalent positive reactance from point *d* to generator (c).

Transforming the mesh circuit *b*, *d*, *e* into the equivalent star as indicated by the dotted lines to *g*

b–g = 75 × 67 ÷ 202 = 24.8 per cent
d–g = 75 × 60 ÷ 202 = 22.3 per cent
g–e = 67 × 60 ÷ 203 = 19.9 per cent
Reactance a–g = 73.0 + 24.8 = 97.8 per cent
Reactance c–g = 27.2 + 22.3 = 49.5 per cent

which equal in parallel . . . . . 32.8 per cent
which has in series g–e giving a total of . . 52.7 per cent
with load in parallel of induction motor . 2180 per cent
synchronous motor . 300 per cent

giving a total of 44 per cent for total equivalent positive reactance of the main system.

The positive reactance of the proposed extension is equal to

$$\text{Generator (f) of } 30 \times \frac{100{,}000}{20{,}000} = 150$$

Transformers	8 × 5 =	40
2 lines in parallel	10 × 5 =	50
Transformers	8 × 5 =	40
		280 total,

from which, total positive series reactance between the two systems is equal to 324 = x_1.

Calculation of the equivalent M of the system

By definition M = time for machine to come to rest from synchronous speed with full load torque applied.

Since the reactances have been calculated on a base of 100,000 kv-a., the inertia constants will be referred to the same base.

$$M \text{ of hydro plant (a)} = 6 \times \frac{50{,}000}{100{,}000} = 3.0$$

$$M \text{ of steam plant (c)} = 16 \times \frac{75{,}000}{100{,}000} = 12.0$$

$$M \text{ of resistance load} = 0$$

$$M \text{ of induction motor load} = 1.10 \times \frac{13{,}800}{100{,}000} = 0.152$$

$$M \text{ of synchronous motor load} = 4.5 \times \frac{10{,}000}{100{,}000} = 0.450$$

M_2 = total M of main system
$= 3.0 + 12.0 + 5 \times 0.152 + 5 \times 0.450 = 17.8$

$$M_1 = M \text{ of extension} = 6 \times \frac{20{,}000}{100{,}000} = 1.2$$

M_0 = equivalent M which is assigned to the extension in order that the relative oscillation will be similar while the main system is assumed to be infinitely stable.

$$\frac{1}{M_0} = \frac{1}{M_1} + \frac{1}{M_2} \therefore M_0 = \frac{1.2 \times 17.8}{19.0} = 1.13$$

Initial Conditions. The torque at any relative rotor position is given by $\frac{\psi_a \psi_b}{x} \sin \delta$ where ψ_a and ψ_b are the flux linkage voltages or voltages behind transient reactance. It is, therefore, necessary to determine these flux voltages for various loads which may be carried while a terminal voltage is maintained constant

at some point in the system which is chosen here as point e.

Assuming a voltage $E = 1.0 + j\,0.0$ at e and a load of 10,000 kw. at 0.99 power-factor lag, which is the power factor of the assumed system load

$$\psi_b = 1.0 + j\,0.0 - j\,.44\left[\frac{10{,}000}{100{,}000} - j\,\frac{1423}{100{,}000}\right]$$

$$= 0.995\,\underline{/\,2.5^\circ}$$

$$\psi_a = 1.0 + j\,0.0 + j\,2.8\left[\frac{10{,}000}{100{,}000} - j\,\frac{1423}{100{,}000}\right]$$

$$= 1.073\,\underline{/\,15.1^\circ}$$

Under this load condition, which results in the above flux voltages, the maximum instantaneous torque which can be developed at synchronous speed is equal to $\frac{1.073 \times 0.995}{324.0} \times 100 = 0.33$ times the base kilovolt-ampere = 33,000 synchronous kilowatts.

In Fig. 23 a curve is plotted of maximum torque for various values of load on the machine. These loads

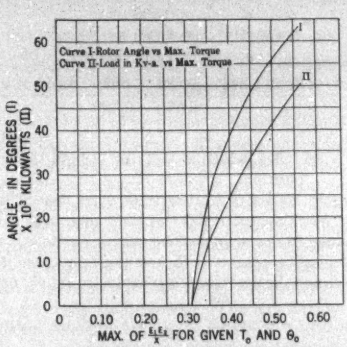

FIG. 23—CURVES OF MAXIMUM SYNCHRONOUS TORQUE AS A FUNCTION OF INITIAL LOAD AND INITIAL ANGLE

determine the flux voltages ψ_a and ψ_b and also the angle δ at which the machine must be operating to supply that load.

Maximum Load which May be Suddenly Applied. The machine is delivering 10,000 kw. which is 0.303 of the maximum and has, therefore, an internal angle of $\delta = \sin^{-1} 0.303 = 17.6$ degrees.

From the curve in Fig. 12 the maximum additional load which may be thrown on and have stability maintained is $0.508 \times 33{,}000 = 16{,}800$ kw., making a total load of 26,800 kw.

Opening a Parallel Line. With both parallel lines carrying power, the total equivalent positive sequence series reactance is 324 per cent, but with one line open and power going over the remaining one only, the series reactance is equal to 374 per cent. It is seen then that with one line only carrying power, the maximum torque is less than with both lines in. The ratio of

$$\frac{\text{max. torque one line}}{\text{max. torque two lines in parallel}} = 0.865.$$

From curve 14 it is noted that the system could have been carrying 0.833 of the original maximum torque. From curve 23 the maximum torque, due to the rotor being at an angle $\theta = \sin 0.833 = 56.5$ deg. and with a corresponding excitation to maintain a voltage E at e under this load condition is 49,500 kw. The initial load which could be safely carried while one line is opened is $0.833 \times 49{,}500 = 41{,}100$ kw.

Line to Ground Short Circuit Maintained on the High-Tension Side of the Transformer Bank at the Generator End. The negative phase sequence equivalent reactance of the main system as calculated is 27.7 per cent, and the total negative sequence reactance of the system as viewed from the point of short circuit is $27.7 + 40 + 50$ in parallel with $200 + 40 = 79.0$ per cent.

The zero sequence reactance of the system viewed from the point of short circuit consists of 40 per cent in parallel with

40 per cent + 5 × 50 per cent = 35.2 per cent

x_a = pos. reactance from gen. to short circuit = 190 per cent

x_b = pos. reactance from main system to short circuit = 134

$x = x_2 + x_0 = 79.0 + 35.2 = 114.2$

The equivalent series reactance now is equal to

$$x_a + x_b + \frac{x_a x_b}{x} = 545 \text{ per cent}$$

Hence, with the short maintained, the maximum torque is 0.595 of the original. Under these conditions from curves (Fig. 14) and (Fig. 23) the maximum initial load that can safely be carried when the short circuit is applied and maintained is

$$0.517 \times 36{,}500 = 18{,}900 \text{ kw.}$$

Short Circuit Maintained on the High-Tension Side of the Transformer Bank at the System End.

x_a = positive reactance from generator to short circuit = 240 per cent

x_b = positive reactance from main system to short circuit = 8 per cent

$x = x_2 + x_0 = 55.0 + 35.2 = 90.2$ per cent

The equivalent series reactance is equal to 547 per cent and from this the ratio of torque angle curves with or without the short circuit is 0.590. Under these conditions from curves (Fig. 14) and (Fig. 23) the maximum initial load than can safely be carried when the short circuit is applied and maintained is

$$0.512 \times 36{,}300 = 18{,}600 \text{ kw.}$$

Short circuit maintained in the middle of the line section on one conductor only.

$x_a = 215$ $\quad x_0 = 114$

$x_b = 109$ $\quad x_2 = 81.5$

$x = x_0 + x_2 + x_{1e}$ $\quad x_{1e} = 12.5$ which must be added as a result of the short occurring out on the line.

The equivalent series reactance $= 324 + \dfrac{215 \times 109}{208}$ $= 437$ per cent and from this the ratio of the peaks of the torque angle curves is $\dfrac{324}{437} = 0.742$. Under these conditions, from curves (Fig. 14) and (Fig. 23) the maximum initial load which can be safely carried when the short circuit is applied and maintained is

$$0.68 \times 41{,}300 = 28{,}000 \text{ synchronous kw.}$$

The curve in Fig. 24 shows how the maximum power which may be safely carried varies with the location of the line to ground short circuit on one of two parallel lines and Fig. 25 shows the variation in zero-phase sequence reactance as a function of the point of fault along the line.

Effect of Speed of Switching on Maximum Load which can be Carried throughout the Disturbance Caused by a Short Circuit on an Interconnecting Line. In this example, the short circuit is assumed to occur on the high-tension side of the transformer bank at the main system end of the extension. In this case, if the system has a maximum torque of 1.0 before the short circuit, the maximum torque with the short circuit will be 0.59, and after the short circuit has been cleared by opening the parallel line, the maximum is 0.865. These conditions give the more general case of dealing with three torque angle curves to obtain the solution, while

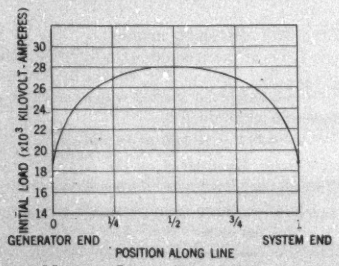

FIG. 24—MAXIMUM INITIAL LOAD WHICH MAY BE CARRIED THROUGH SUSTAINED LINE TO GROUND FAULT AS A FUNCTION OF POSITION OF FAULT

if the short circuit occurred on a spur line as indicated coming from the interconnecting bus, the system would return to the initial torque angle curve after the short circuit had been cleared.

With an initial torque of 0.75 and a corresponding angle $\delta_0 = 48.8$ deg., the angle δ_1 is solved for as the angle at which the breaker must open to maintain stability.

δ_2 is the maximum angle to which the machine can swing with stability on curve C

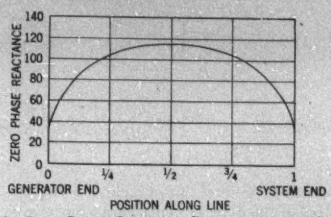

FIG. 25—ZERO-PHASE SEQUENCE REACTANCE AS A FUNCTION OF POSITION OF LINE TO GROUND FAULT

$$\delta_2 = \pi - \sin^{-1}\frac{0.75}{0.865} = \pi - 60 = 120 \text{ deg.}$$

$$\begin{aligned} \sin\delta_0 &= 0.75 \\ \cos\delta_0 &= 0.662 \\ \cos\delta_2 &= -0.500 \\ r_1 &= 0.590 \\ r_2 &= 0.865 \\ \delta_2 - \delta_0 &= 1.242 \text{ radians} \end{aligned}$$

$$\cos\delta_1 = \frac{0.75 \times 1.242 - 0.590 \times 662 - 0.866 \times 0.500}{0.865 - 0.590} = 0.404$$

$\delta_1 = 66.1$ deg. which is 17.3 deg. behind the initial position.

It is still necessary to determine what time it takes for the machine to travel this 17.3 deg. and this is determined from the angle time curves.

The initial torque is 0.75 and the equivalent total torque which the machine is advancing to supply on the "B" torque angle curve is $\dfrac{0.75}{0.59} = 1.27$ of its maximum which is still 0.75 of the initial maximum. From the angle time curves, Fig. 18, it is seen that the machine will have advanced to $\delta = 66.1$ when

$$\text{Time in seconds} = \frac{F\left(\delta, \dfrac{T_1}{T_m}, \dfrac{T_0}{T_m}\right) = 1.66}{\sqrt{\dfrac{4\pi f \psi_1 \psi_2}{M_0 x}}}$$

From curve (Fig. 23) the initial conditions give a maximum torque $= \dfrac{\psi_a \psi_b}{x}$ of 44,500 kw. $= 0.445$ of the maximum which is taken as unity so that

$$t = \frac{1.66}{\sqrt{\dfrac{4\pi \times 60 \times 0.445}{1.13}}} = 0.097 \text{ secs.}$$

Several points have been obtained in a similar manner from which the curve in Fig. 26 was plotted. It is seen that the system could safely have been carrying 18,500 kw. and be stable with the short circuit maintained, and also that unless the speed of switching is of the order of

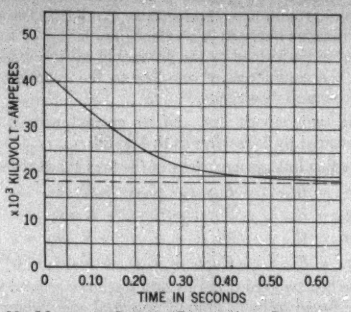

FIG. 26—MAXIMUM INITIAL LOAD WITH STABILITY AS A FUNCTION OF SPEED OF CLEARING FAULT

a ¼ of a sec. or less, the initial safe power limit is not increased appreciably, in this case.

Appendix II

CALCULATION OF ZERO-PHASE SEQUENCE REACTANCE

Single Circuit Reactance. The zero-phase sequence reactance of a transmission line is the reactance of all the conductors in parallel with a ground return. This reactance is referred to the current in one phase only, that is, to one-third of the total ground current. Let the conductors be termed a, b, and c, the ground wire d, and their images a' b' c' and d'. The distance to the image depends on soil conditions. Tests indicate that this is generally 1000-5000 ft. The flux linkages with conductor a per unit length of line are

$$\psi_a = 2\, I_a \log \frac{a\,a'}{r_a} + 2\, I_b \log \frac{a\,b'}{a\,b} + 2\, I_c \log \frac{a\,c'}{a\,c} + 2\, I_d \log \frac{a\,d'}{a\,d}$$

For the zero-phase network

$I_a = I_b = I_c = I_0$ If the conductors are completely transposed.

$I_d = -\,m\, I_0$

$K_{aa} = 2 \log \dfrac{a\,a'}{r_a}$ = coefficient of self inductance of conductor a.

$K_{ab} = 2 \log \dfrac{a\,b'}{a\,b}$ etc., coefficients of mutual inductance of conductors a and b, etc.

The equations for the flux linkages about the several conductors may now be written.

$$\psi_a = I_0\,(K_{aa} + K_{ab} + K_{ac} - m\,K_{ad})$$
$$\psi_b = I_0\,(K_{ab} + K_{bb} + K_{bc} - m\,K_{bd})$$
$$\psi_c = I_0\,(K_{ac} + K_{bc} + K_{cc} - m\,K_{cd})$$
$$\psi_d = I_0\,(K_{ad} + K_{bd} + K_{cd} - m\,K_{dd})$$
$$\psi_0 = \frac{\psi_a + \psi_b + \psi_c}{3}$$

Let $K_{aa} + 2\,K_{ab} + 2\,K_{ac} + K_{bb} + 2\,K_{bc} + K_{cc} = K_{00}$

Then $K_{00}\, I_0 - m\,(K_{ad} + K_{bd} + K_{cd})\, I_0 = 3\,\psi_0$

$$\psi_d = (K_{ad} + K_{bd} + K_{cd} - m\,K_{dd})\, I_0$$

Neglecting resistance* we may say that $\psi_d = 0$ because the ground wires are connected to each tower.

$$m = \frac{(K_{ad} + K_{bd} + K_{cd})}{(K_{dd})}\, I_0$$

$$K_{00} - \frac{(K_{ad} + K_{bd} + K_{cd})^2}{K_{dd}} = \frac{3\,\psi_0}{I_0} = 3\,L_0$$

This gives the inductance in abhenrys per cm.

$$\frac{30.48 \times 5280}{10^9}\, 2\,\pi\, f\, L_0 = x_0 \text{ in ohms per mile.}$$

The current returning in the ground wire is found from the equation

$$I_{gw} = \frac{(K_{ad} + K_{bd} + K_{cd})}{K_{dd}}\, I_0$$

Twin Circuit Reactance. In general the zero-phase sequence reactance of a system, viewed from a point of fault, is low in the neighborhood of any grounded neutral

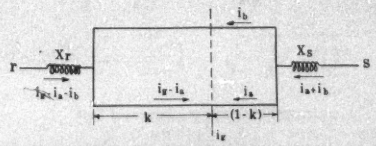

FIG. 27—DISTRIBUTION OF ZERO-PHASE SEQUENCE CURRENTS IN TWO PARALLEL LINES

transformer banks and tends to increase as the distance of the fault from such grounding points increases.

The distribution of zero phase reactance, along two or more parallel lines, is in general as in Fig. 25. This variation may be scaled off roughly from a knowledge of the zero phase ohms per mile of one line alone, and of several lines in parallel.

Referring to Fig. 27, for a fault distant k from the r end of the line in per unit of the total length of line there will be no flow of zero-phase sequence current in any line but that on which the fault occurs if

*A non magnetic wire is assumed. Steel ground wires apparently have little effect on zero-phase sequence reactance.

$$k = \frac{x_{0r}}{x_{0r} + x_{0s}}$$

Where x_{0r} = zero phase sequence reactance of terminal apparatus at the r end

x_{0s} = zero phase sequence reactance of terminal apparatus at the s end

Since in this case the mutual effect of the current returning to r will be proportional to

$$\frac{k}{x_{0r} + k\,x_{0s}} = \frac{1}{x_{0r} + x_{0s} + x_{0s}}$$

Where x_{0s} = zero phase sequence reactance of 1 line.

While the mutual effect due to the current returning to s will be equal and opposite and therefore the voltage at each terminal will be equal; *i. e.*,

$$\frac{x_{0r}}{x_{0r} + k\,x_{0s}} = \frac{x_{0r} + x_{0s}}{x_{0r} + x_{0s} + x_{0s}} = \frac{x_{0s}}{x_{0s} + (1 - k)\,x_{0s}}$$

The reactance at this point is readily found to be

$$x_{0k} = \frac{x_{0r}\,x_{0s}\,(x_{0r} + x_{0s} + x_{0s})}{(x_{0r} + x_{0s})^2}$$

On the other hand for a short at the r end of the line it is equal to the r end terminal apparatus reactance in parallel with the zero phase reactance of all the lines in parallel plus the s end terminal reactance. Thus, the reactance may be determined at three points and it is often sufficient to approximate the intermediate points. In general, however, a more accurate calculation may be desirable.

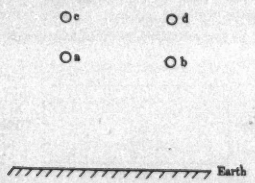

FIG. 28—EQUIVALENT CONDUCTOR ARRANGEMENT OF TWIN CIRCUIT LINE WITH TWO GROUND WIRES

The problem of performing such a calculation is much simplified if the n_1 conductors of each line and the n_2 ground wires over each line are each replaced by equivalent single conductors. If there are sufficient transpositions* this may be correctly accomplished by taking the n^2 root of the product of 0.779 times the conductor radii times the distances between the center of the conductor and all the other conductors times the same terms for each of the other conductors, *i. e.*, for a three conductor line

$$r_0 = \sqrt[9]{0.779^3 \times r^3 \times s_{12}^2 \times s_{13}^2 \times s_{23}^2}.*$$

*Ground wires of course cannot be transposed but there is still no error if there are two ground wires symmetrically placed. In the general case the error is small.

If there are two or more ground wires, they are similarly treated and the radius of the equivalent ground wire determined.

The effective distance between the equivalent ground wire and equivalent line is given by the $n_1\,n_2$ root of the $n_1\,n_2$ terms in the product of all the distances between line and ground wire conductors. Similarly, the effective distance between equivalent lines is the n_1^2 root of the n_1^2 terms formed by the product of all the distances from the conductors of one line to the conductors of the other and vice versa.

The twin circuit line and its ground wire is then represented by four wires as in Fig. 28.

The flux linkage equations of the four conductors are:

$$\psi_a = K_{aa}\,i_a + K_{ab}\,i_b + K_{ac}\,i_c + K_{ad}\,i_d$$
$$\psi_b = K_{ab}\,i_a + K_{bb}\,i_b + K_{bc}\,i_c + K_{bd}\,i_d$$
$$\psi_c = K_{ac}\,i_a + K_{bc}\,i_b + K_{cc}\,i_c + K_{cd}\,i_d = 0$$
$$\psi_d = K_{ad}\,i_a + K_{bd}\,i_b + K_{cd}\,i_c + K_{dd}\,i_d = 0$$

The last two are equal to zero since the ground wires are assumed to be at ground potential everywhere and no voltage drop is assumed in the earth. Solving them for i_c and i_d and recalling that for symmetrical lines

$$K_{cc} = K_{dd}$$
$$K_{ac} = K_{bd}$$
$$K_{ad} = K_{bc}$$

There is

$$i_c = -\frac{(K_{ac}\,K_{cc} - K_{ad}\,K_{cd})\,i_a + (K_{ad}\,K_{cc} - K_{cd}\,K_{ac})\,i_b}{K_{cc}^2 - K_{cd}^2}$$
$$= -\,(J_{ac}\,i_a + J_{bc}\,i_b)$$
$$i_d = -\frac{(K_{ad}\,K_{cc} - K_{ac}\,K_{cd})\,i_a + (K_{ac}\,K_{cc} - K_{cd}\,K_{ad})\,i_b}{K_{cc}^2 - K_{cd}^2}$$
$$= -\,(J_{ad}\,i_a + J_{bd}\,i_b)$$
$$i_d = -\,(J_{bc}\,i_a + J_{ac}\,i_b)$$
$$\psi_a = (K_{aa} - K_{ac}\,J_{ac} - K_{ad}\,J_{bc})\,i_a + (K_{ab} - K_{ac}\,J_{bc} - K_{ad}\,J_{ac})\,i_b$$
$$= K_{aa}'\,i_a + K_{ab}'\,i_b$$
$$\psi_b = K_{ab}'\,i_a + K_{aa}'\,i_b$$

Then if k is the distance from the r end of the line in per unit of the total line length, and if

V_A = total voltage induced from line A to ground

V_B = total voltage induced from line B to ground

there is

$$V_A = c\,(K_{aa}'\,i_a + K_{ab}'\,i_b - k\,K_{aa}'\,i_g)$$
$$V_B = c\,(K_{ab}'\,i_a + K_{aa}'\,i_b - k\,K_{ab}'\,i_g)$$
$$V_A - V_B = (K_{aa}' - K_{ab}')(i_a - i_b) - k\,i_g(K_{aa}' - K_{ab}') = 0$$
$$i_a - i_b = k\,i_g$$
$$i_b = i_a - k\,i_g$$

where $c = \dfrac{30.48 \times 5280}{10^9}\,2\,\pi\,f \times$ (length of line in miles)

$$3\,V_0 = x_{r0}\,(i_g - i_a - i_b) + c\,3\,k\,[K_{aa}'\,(i_g - i_a) - K_{ab}'\,i_b]$$
$$= x_{s0}\,(i_{sa} + i_{ba}) + c\,3\,(1-k)\,(K_{aa}'\,i_a + K_{ab}'\,i_b)$$
$$3\,V_0 = x_{r0}\,[(1+k)\,i_g - 2\,i_a] + c\,3\,k\,[(K_{aa}' + k\,K_{ab}')\,i_g - (K_{aa}' + K_{ab}')\,i_a]$$
$$3\,V_0 = x_{s0}\,(2\,i_a - k\,i_g) + c\,3\,(1-k)\,[(K_{aa}' + K_{ab}')\,i_{a0} - k\,K_{ab}'\,i_g]$$
$$= c\,[K_{1a}\,i_a + K_{gr}\,i_g]$$
$$= c\,[K_{1b}\,i_a + K_{gs}\,i_g]$$
$$i_a = \frac{K_{gs} - K_{gr}}{K_{1a} - K_{1b}}\,i_g$$
$$3\,V_0 = \frac{K_{1a}\,K_{gs} - K_{1b}\,K_{gr}}{K_{1a} - K_{1b}}\,i_g = x_0\,i_g$$
$$x_0 = \frac{K_{1a}\,K_{gs} - K_{1b}\,K_{gr}}{K_{1a} - K_{1b}}$$

where,

$$K_{1a} = -2\,x_{r0} - c\,3\,k\left[(K_{aa} + K_{ab} - \frac{(K_{ac} + K_{ad})^2}{K_{cc} + K_{cd}}\right]$$

$$K_{gr} = x_{r0}\,(1+k) + c\,3\,k\left[(K_{aa} + k\,K_{ab}) - \frac{K_{cc}\,(K_{ac}^2 + K_{ad}^2 + 2\,k\,K_{ac}\,K_{ad})}{K_{cc}^2 - K_{cd}^2} + \frac{K_{cd}\,(k\,K_{ac}^2 + k\,K_{ad}^2 + 2\,K_{ac}\,K_{ad})}{K_{cc}^2 - K_{cd}^2}\right]$$

$$K_{1b} = 2\,x_s + c\,3\,(1-k)\left[(K_{aa} + K_{ab} - \frac{(K_{ac} + K_{ad})^2}{K_{cc} + K_{cd}}\right]$$

$$K_{gs} = -k\,x_s - 3\,c\,(1-k)\,k\left[K_{ab} + \frac{K_{cd}(K_{cd}(K_{ac}^2 + K_{ad}^2) - 2K_{ac}K_{ad}\,K_{cc})}{K_{cc}^2 - K_{cd}^2}\right]$$

x_0 = zero phase sequence reactance to the point of fault.

Appendix III

Corona Limits Under Unbalanced Conditions

The corona limit of a line under unbalanced conditions may be found in terms of its corona limit under balanced conditions, and from the maximum ratio of unbalanced to balanced charging current of any wire.

For example, with normal line-to-line voltage maintained, it was found in a specific case that with one wire grounded the charging currents on the ungrounded wires were 1.50 times their value under normal conditions. This indicated, then, that the maximum line to line voltage to avoid corona under the unbalanced condition was 1/1.50 times the corresponding value under normal condition of operation.

In most cases the ratio referred to is less than 1.50, as shown by the table below.

	Volts kv.	Ratio
Conductors—795,000 cir. mils A. C. S. R. 1.14 in. diam. 25.5 ft. flat spacing two ground wires 184,000 cir. mils A. C. S. R. 0.707 in. diam. spaced 25.5 ft. horizontal 17 ft. above conductors	220	1.50
Conductors same as above but no ground wires	220	1.43
Conductors as above but spaced 32.1 ft. delta. No ground wires	220	1.4
Conductors 4/0 copper 0.528 in. diam. Spaced 6.3 ft. flat. No ground wires	66	1.27

Appendix IV

Angle—Time Curves

Considering the case of a suddenly applied load to a definite reactance machine connected to an infinite bus by a line, the "per unit" equation of motion is

$$\frac{M}{2\,\pi\,f} \times \frac{d^2\,\delta}{d\,t^2} = T_1 - \frac{\psi_a\,\psi_b}{x}\sin\delta$$

where from Fig. 11

T_1 = new total torque

$\frac{\psi_a\,\psi_b}{x}\sin\delta$ = machine torque at angle δ

Integrating with respect to δ and separating the variables gives

$$t = \int_{\delta_0}^{\delta} \frac{d\,\delta}{\sqrt{\frac{4\,\pi\,f}{M}\left[T_1\,\delta + \frac{\psi_a\,\psi_b}{x}\cos\delta - \left(T_1\,\delta_0 + \frac{\psi_a\,\psi_b}{x}\cos\delta_0\right)\right]}}$$

but $T_1 = \frac{\psi_a\,\psi_b}{x}\sin\delta_1$

so that $F\left(\delta, \frac{T_1}{T_m}, \frac{T_0}{T_m}\right) = t\sqrt{\frac{4\,\pi\,f}{M}\,\frac{\psi_a\,\psi_b}{x}}$

$$= \int_{\delta_0}^{\delta} \frac{d\,\delta}{\sqrt{\sin\delta_1\,(\delta - \delta_0) + (\cos\delta - \cos\delta_0)}}$$

The angle—time curves in Figs. 15, 16, 17, and 18 were plotted for several values of T_1 with

T_0 = 0; 0.25; 0.50, and 0.75 respectively.

These curves apply to any machine where all values are expressed in a common base as "per unit"; time being obtained from the abscissas by dividing by

$$\sqrt{\frac{4\,\pi\,f}{M}\,\frac{\psi_a\,\psi_b}{x}}$$

Appendix V

Maximum Additional Load Which May be Suddenly Applied with a Given Initial Load

Referring to Fig. 11 where

T_0 = initial torque with the corresponding initial angle δ_0

T_1 = new total torque resulting from additional load and δ_1 the corresponding angle

For stability the area $a_2 > a_1$ and the stability limit occurs when $a_2 = a_1$ or when the area of the rectangle is equal to the area under the curve so that

$$\frac{\psi_a \psi_b}{x} \sin \delta_1 (\pi - \delta_1 - \delta_0) = \frac{\psi_a \psi_b}{x} \int_{\delta_0}^{\pi - \delta_1} \sin \delta \, d \, \delta$$

from which

$$(\pi - \delta_0 - \delta_1) \sin \delta_1 = \cos \delta_0 + \cos \delta_1$$

The curve in Fig. 12 was plotted from a graphical solution of the above equation for various values of δ_0. The equation of this curve is approximately

$$\Delta T = 0.75 (T_m - T_0)$$

where

ΔT = maximum safe additional load which may be suddenly applied

T_m = maximum torque

T_0 = initial torque.

Appendix VI

Maximum Initial Load with Stability on Opening a Parallel Line

Referring to Fig. 13, curve A is the torque-angle characteristic of the system with both lines in parallel and curve B with one line open. If the system is initially carrying a load of T_0 and one line is opened the problem resolves itself into the sudden application of a load ΔT with an initial load of T_1. The relation between these is given by the following:

$$\frac{T_1}{T_0} = \frac{T_1}{T_1 + \Delta T} = r \text{ from which}$$

$$\Delta T r = T_1 (1 - r)$$

For values of r between 1.0 and 0 the curve in Fig. 12 was used to determine values of T_1 and ΔT to give stability. The corresponding T_0 was then expressed in terms of the maximum of curve B. These results are plotted in Fig. 14, and the approximate equation of this curve is

$$T_0 = T_m (0.7 + 0.3 r)$$

where T_0 = initial torque

T_m = max. torque with one line (curve B)

r = ratio B_{max}/A_{max}.

Appendix VII

The Effect of Clearing a Line to Ground Short Circuit on an Interconnecting Line

Consider the system in Fig. 13 operating initially at a load T_0 and angle δ_0. The torque angle characteristic as shown by curve A Fig. 21 is $T = \frac{\psi_a \psi_b}{x} \sin \delta$ where x is the total series reactance. With the short circuit the torque angle characteristic as shown by curve B Fig. 21 is $T = \frac{\psi_a \psi_b}{x} \sin \delta$ where x is the equivalent series reactance $= x_a + x_b + \frac{x_a x_b}{x}$ (see Fig. 19). With the short circuit cleared by opening one of the parallel lines the torque angle characteristic as shown by curve C Fig. 21 is $T = \frac{\psi_a \psi_b}{x} \sin \delta$ where x is the new total series reactance. δ_1 is the position angle of the machine where the fault is cleared and δ_2 is the maximum angle to which the machine can swing on curve C with stability and an initial load T_0.

Stability will result if $a_2 > a_1$ and the limit of stability occurs when the area of the rectangle equals the area under the irregular curve given by

$$\sin \delta_0 [\pi - \delta_0 - (\pi - \delta_2)] = r_1 \int_{\delta_0}^{\delta_1} \sin \delta \, d \, \delta + r_2 \int_{\delta_1}^{\delta_2} \sin \delta \, d \, \delta$$

from which

$$(\delta_2 - \delta_0) \sin \delta_0 = r_1 [-\cos \delta_1 + \cos \delta_0] + r^2 [-\cos \delta_2 + \cos \delta_1]$$

where $r_1 = \text{ratio } \frac{B_{max}}{A_{max}}$

$r_2 = \text{ratio } \frac{C_{max}}{A_{max}}$

No attempt has been made to present results which would be of general use due to the difficulty in choosing general conditions with so many variables.

Appendix VIII

Time of Breaker Opening After Line to Ground Fault Occurs on a Parallel Line Versus Maximum Load which May be Carried Through the Disturbance

Possibly the simplest method of attacking this problem is to assume an initial load T_0; that a line to ground fault occurs causing the system to operate on a new torque angle characteristic as shown in curve B, Fig. 21, with an initial load of T_1, and a new applied load T_0; to determine the angle δ_1 at which the breaker must open to maintain stability by allowing the system to operate on a new torque angle characteristic (curve C) with the limit of swing δ_2.

The stability limit occurs when $a_2 = a_1$ and from appendix VII

$$\cos \delta_1 = \frac{\sin \delta_0 (\delta_2 - \delta_0) - r_1 \cos \delta_0 + r_2 \cos \delta_2}{r_2 - r_1}$$

δ_0 = initial angle of rotor for T_0

δ_1 = angle of rotor when breaker opens

$$\delta_2 = \pi - \sin^{-1} \frac{T_0}{r_2}$$

For various initial loads the maximum value of Curve A may be determined. A similar curve for the example in Appendix I is shown in Fig. 23. From the angle time curves in Figs. 15, 16, 17, and 18 by interpolation, the time may be found that the machine requires to advance to the angle δ_1 for the assumed initial and equivalent applied torque.

initial torque $\sin \delta_0 = T_1$ on Curve B

total torque $T_1 = T_1/r_1$ in per unit of B_{max}.

$$\text{time} = \frac{F\left(d, \frac{T_1}{T_{max}}, \frac{T_0}{T_m}\right)}{\sqrt{\frac{4\pi f \psi_a \psi_b}{M x}}}$$

This has been carried through completely in the example given in Appendix I from which the curve in Fig. 26 was plotted.

Appendix IX

Derivations of Effect of Rotations on Field Current

As given in the paper, there is,

$$I_A = \psi_d' + i\, d_A\, (x\, d - x\, d')$$
$$I_B = \psi_d' + i\, d_B\, (x\, d - x\, d')$$
$$I_A = I_B + (x\, d - x\, d')\, (i\, d_A - i\, d_B)$$

but

$$i\, d_A = \frac{(x\, q + x\, e)\, (\psi' - \psi_e \cos \delta_{eA}) - r\, e\, \psi_e \sin \delta\, e_A}{r\, e^2 + (x\, d' + x\, e)\, (x\, q + x\, e)}$$

$$i\, d_B = \frac{(x\, q + x\, e)\, (\psi' - \psi_e \cos \delta e_B) - r\, e\, \psi_e \sin \delta\, e_B}{r\, e^2 + (x\, d' + x\, e)\, (x\, q + x\, e)}$$

$$i\, d_A - i\, d_B = \frac{\psi_e[(x\, q + x_e)(\cos \delta\, e_B - \cos \delta\, e_A) + r\, e(\sin \delta\, e_B - \sin \delta\, e_A)]}{r_e^2 + (x\, d' + x_e)\, (x\, q + x_e)}$$

but $\cos \delta\, e_A - \cos \delta\, e_B = -\, [\sin \delta\, e_B]\, \Delta\, \delta$

$\sin \delta\, e_A - \sin \delta\, e_B = [\cos \delta\, e_B]\, \Delta\, \delta$

thus,

$$I_A = I_B + \frac{\psi_e\, \Delta\, \delta\, (x\, d - x\, d')}{r_e^2 + (x\, d' + x\, e)\, (x\, q + x\, e)}\, [\,(x\, q + x_e)\, \sin \delta\, e_B - r\, e \cos \delta\, e_B]$$

Appendix X

Derivation of Formula for Voltages to Ground on the Open Phases of a $\Delta - Y$ Transformer Bank with a Line to Ground Fault and Impedances in the Neutral

Referring to the diagram in Fig. 7, assuming an infinite bus, as shown maintaining balanced voltages and using the method of symmetrical components.

$$I_1 = I_2 = I_0 = \frac{I_g}{3} = \frac{E_1}{Z_1 + Z_2 + Z_0}$$

where

$$Z_1 = Z_e + Z_t$$
$$Z_2 = Z_e + Z_t$$
$$Z_0 = Z_t + 3\, Z_n$$

But

$$V_a = V_1 + V_2 + V_0$$
$$V_b = a^2\, V_1 + a\, V_2 + V_0$$
$$V_c = a\, V_1 + a^2\, V_2 + V_0$$

Also

$$V_1 = E_1 - Z_1\, \frac{I_g}{3}$$

$$V_2 = 0 - Z_2\, \frac{I_g}{3}$$

$$V_0 = 0 - Z_0\, \frac{I_g}{3}$$

Then with the short circuit applied

$$V_a = 0$$

$$V_b = E_1 \left[a^2 - \frac{a^2\, Z_1 + a\, Z_2 + Z_0}{Z_1 + Z_2 + Z_0} \right]$$

$$= E_1 \left[\frac{\sqrt{3}\, Z_2\, /\overline{90} + \sqrt{3}\, Z_0\, /\overline{150}}{Z_1 + Z_2 + Z_0} \right]$$

$$= \frac{-\, j\sqrt{3} Z_2 - j\, \frac{\sqrt{3}}{2}\, Z_0 - 1.5\, Z_0}{Z_1 + Z_2 + Z_0}$$

in per unit of normal line to neutral volts.

Similarly

$$V_c = E_1 \left[a - \frac{a\, Z_1 + a^2\, Z_2 + Z_0}{Z_1 + Z_2 + Z_0} \right]$$

$$= \frac{+\, j\, \sqrt{3}\, Z_2 + j\, \frac{\sqrt{3}}{2}\, Z_0 - 1.5\, Z_0}{Z_1 + Z_2 + Z_0}$$ in per unit of normal line to neutral volts.

These values have been calculated neglecting resistance of the lines and transformer for various values of Z_e in per unit of transformer reactance. Figs. 7 and 8 give line to ground volts in per unit of normal line to neutral volts on the open phases as a function of resistance and reactance respectively in the neutral in per unit of transformer reactance.

Appendix XI

Derivation of Formula for Torque of Induction Motor Directly Connected to a Round Rotor Synchronous Machine

Referring to Fig. 1 E is excitation voltage

e is terminal voltage

Then $E_e = \frac{x_e}{x_s}\, E$ which is equivalent excitation voltage and

$$x_e = \frac{x_m x_s}{x_m + x_s}$$

For this equivalent circuit the equation for torque is

$$T = \frac{E_e^2 \dfrac{R}{s}}{(x_e + x)^2 + \left(\dfrac{R}{s}\right)^2}$$

(I) Derivation of formula for maximum torque of an induction motor with constant shaft load.

Instability occurs when $\dfrac{d\,T}{d\,s} = 0$ and maximum torque therefore occurs when $\dfrac{R}{S} = x_e + x$ and the current lags E_e by 45 deg. which gives $T_{max} = \dfrac{E_e^2}{2(x_e + x)}$ but it is desirable to express this in terms of terminal voltage which is required to be held constant.

From the vector diagram of this equivalent circuit the in-phase component of current with respect to E_e is

$$\frac{E_e}{2\,(x_e + x)}$$

and

$$e^2 = E_e^2\left[1 - \frac{x_e}{2\,(x_e + x)}\right]^2 + E_e^2\left[\frac{x_e}{2\,(x_e + x)}\right]^2$$

from which $$T_{max} = \frac{e^2}{x_e + x + \dfrac{x^2}{x_e + x}}$$

Letting $x = 0.15$; $x_m = 3.0$; $e = 1.0$, the values of maximum torque with constant shaft load as a function of x_e have been calculated and plotted as shown in Fig. 3.

(II) Derivation of formula for critical torque of an induction motor with shaft load varying as $(1 - s)^2$

The critical torque as indicated by point B in Fig. 2 occurs when $\dfrac{d\,T}{d\,s} = \dfrac{d\,L}{d\,s}$ with $L = \dfrac{T\,(1 - s)^2}{(1 - s_c)^2}$ so that

$$\frac{d\,L}{d\,s} = -\frac{2\,T}{(1 - s_c)^2}\,(1 - s) = \frac{-2\,T}{(1 - s)} \text{ when } s_c = s$$

but $$T = \frac{\dfrac{R}{s}}{(x_e + x)^2 + (R/s)^2}$$

so that $\dfrac{d\,T}{d\,s}$

$$= \frac{\left[(x_e + x)^2 + \left(\dfrac{R}{s}\right)^2\right]\left(-\dfrac{R}{s^2}\right) - \left(\dfrac{R}{s}\right)\left[-2\dfrac{R^2}{s^3}\right]}{[(x_e + x)^2 + (R/s)^2]^2}$$

which is also equal to $$\frac{-2\,\dfrac{R}{s\,(1 - s)}}{(x_e + x)^2 + \left(\dfrac{R}{s}\right)^2}$$

giving

$$1 - \frac{2\,S}{1 - s} - \frac{2\left(\dfrac{R}{s}\right)^2}{(x + x_e)^2 + (R/s)^2} = 0$$

Solving the above expression for s_c which is the critical slip when $\dfrac{d\,T}{d\,s} = \dfrac{d\,L}{d\,s}$ for values of x_e and the corresponding values of x_e the calculated values of torque with unit terminal voltage are plotted in Fig. 3.

$$\text{using } T_{critical} = \frac{e^2\,\dfrac{R}{s_c}}{x^2 + \left(\dfrac{R}{s_c}\right)^2}$$

It must be borne in mind that in the above treatment the terminal voltage e is held constant for steady-state changes in load by adjustment of the generator field rheostat from time to time but that for the transient changes in load under consideration the excitation voltage E or the equivalent E_e remains constant.

Appendix XII

Step-by-Step Calculation of Angular Position

Consider any successive time intervals, $n - 2$, $n - \dfrac{3}{2}$, $n - 1$, $n - \dfrac{1}{2}$, n, etc. There will be, using the notation of Part II.

$$\delta_n = \delta_{n-1} + \Delta\,\delta_n$$

$$\Delta\,\delta_n = 360\,f\,\Delta\,t\,S_{n-\frac{1}{2}}$$

$$S_{n-\frac{1}{2}} = S_{n-\frac{3}{2}} + \Delta\,S_{n-1}$$

$$\Delta\,S_{n-1} = \frac{\Delta\,t}{M}\,[T_{n-1} - G_{n-1} - F\,S_{n-1}]$$

from the equation

$$M\,\frac{d\,S}{d\,t} = T - G - F\,S$$

Where T = per unit synchronizing torque
F = per unit damping torque
G = per unit shaft torque

$$S_{n-1} = \frac{S_{n-\frac{3}{2}} + S_{n-\frac{1}{2}}}{2}$$

$$S_{n-\frac{1}{2}} = S_{n-\frac{3}{2}} + \frac{\Delta t}{M}\left[T_{n-1} - G_{n-1} - F \cdot \frac{S_{n-\frac{3}{2}} + S_{n-\frac{1}{2}}}{2}\right]$$

$$\left(1 + \frac{F\,\Delta t}{2\,M}\right) S_{n-\frac{1}{2}} = \left(1 - \frac{F\,\Delta t}{2\,M}\right) S_{n-\frac{3}{2}} + \frac{\Delta t}{M}\,(T_{n-1} - G_{n-1})$$

if $\frac{F\,\Delta t}{2\,M}$ is small

$$S_{n-\frac{1}{2}} = \left(1 - \frac{F\,\Delta t}{M}\right) S_{n-\frac{3}{2}} + \left(1 - \frac{F\,\Delta t}{2\,M}\right) \frac{\Delta t}{M}\,(T_{n-1} - G_{n-1})$$

$$\Delta\,\delta_n = \left(1 - \frac{F\,\Delta t}{M}\right) \Delta\,\delta_{n-1} + \frac{360\,f\,(\Delta\,t)^2}{M}\left(1 - \frac{F\,\Delta t}{2\,M}\right)(T_{n-1} - G_{n-1})$$

for any interval except the first.

For the first interval

$$S_{n-\frac{1}{2}} = \frac{1}{2}\,\frac{\Delta t}{M}\left(T_{n-1} - G_{n-1} - F\,\frac{S_{n-\frac{1}{2}}}{2}\right)$$

$$S_{n-\frac{1}{2}} = \frac{1}{2}\,\frac{\Delta t}{M}\left(1 - \frac{F\,\Delta t}{4\,M}\right)(T_{n-1} - G_{n-1})$$

$$\Delta\,\delta_n = \frac{360\,f\,(\Delta\,t)^2}{2\,M}\left(1 - \frac{F\,\Delta t}{4\,M}\right)(T_{n-1} - G_{n-1})$$

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Discussion

L. F. Hunt: Southern California Edison Company has been making stability studies for several years. These studies have given information on what should be done to increase stability.

The first is to prevent flashovers. Some flashovers on the 220-kv. lines, when the lines are loaded, cause interruptions. Therefore very great pains are taken to prevent flashovers.

The second important thing is the clearing of faults. Faults cleared instantly will give little instability. It takes from 0.9 to 1.2 sec. to clear the faults on 220-kv. systems. In all the tests that were made the shortest time was 0.9 sec. to open switches at ends of the line. Most of that time is taken in tripping the oil switches. If oil switches could be made instantaneous instead of taking from 0.4 to 0.5 sec. to open an arc, instability of the system would be materially reduced.

Third, by adding parallel transmission lines, the section that is cut out of the transmission system due to a short circuit is a smaller part of the system; thus the stability will be increased.

Fourth, almost every time the system is out of step it is caused by the generators overspeeding. At one end of the system are the generators and at the other end the load similar to that shown in the paper. Usually the effect of a short circuit is equivalent to adding a very high resistance load to the system. This causes the governors on the system to pick up load. After the fault is cleared the governors are set for a higher load than the load then on the system. Therefore, the machines tend to speed up, causing a greater angular displacement between the sending and the receiving ends, ultimately causing an interruption. This out-of-step condition will be eliminated by slowing down the machines at the generator end at the instant when trouble occurs on the system.

Fifth, quick excitation on synchronous machines will help very materially, because, in case of trouble, it will allow more synchronizing power to pass between the sending end and the receiving end.

R. H. Park: Mr. Hunt has presented a valuable résumé of the results of the stability studies which have been made by the Southern California Edison Company. Of the conclusions presented, the decision in favor of high-speed governing is particularly interesting. Undoubtedly there is every reason for assuming that high-speed actuation of the waterwheel governing mechanism will tend to improve operating condit

during disturbances, both by reducing the tendency toward pulling out of step, and also by reducing the time required for again pulling into step after pull-out has occurred.

In addition to Mr. Hunt's written discussion, there has been considerable informal discussion which is not on record. The conclusions arrived at in this informal discussion may be briefly summarized as follows:

a. Neutral Reactors. The design of these reactors is necessarily a compromise between the improvement in stability which is obtained, the reduction in relaying current, and the increase in transformer neutral insulation. A preliminary study of these influences has indicated that the reactor may advantageously be designed for voltage under line-to-ground fault conditions of from 60 to 70 per cent of normal leg voltage. This will usually result in a reduction in shock to the system of about 50 per cent for line-to-ground faults and about 30 per cent for two-wire-to-ground faults.

b. Neutral Resistors. With neutral resistors, advantage may sometimes be taken of the braking torque, which results with this device due to the heavy power loss which it causes. While such a power loss reduces the shock on the generators, it increases the shock to the system. Therefore, in view of the action of the steam-turbine governors, its use may not do as much good as would be anticipated at first thought. Field data are urgently needed to determine this important point.

c. Amortisseurs. Unless neutral reactors are employed, much amortisseur action during the period when a fault is actually on the system cannot be expected, because of the equivalent series impedance introduced by the fault. However, it has been the experience of operating companies that the actual instant of instability may occur some time after the removal of the fault. The amortisseur should have a beneficial action in these cases. On the other hand, the amortisseur unavoidably has the effect of reducing the generators' negative-phase-sequence reactance. This means that it tends to increase the severity of the fault somewhat. For line-to-ground faults this effect is small, but for two-wire-to-ground faults, it is quite appreciable. Therefore, if a preponderance of two-wire-to-ground faults is anticipated, it would appear that the beneficial influences of an amortisseur will be reduced and in fact it might be disadvantageous. The final story as to the value of an amortisseur will only be arrived at after further study and field tests.

d. Synchronous Condensers. It would appear that some of our present stability difficulties are the result of the loading up of lines which has been made possible by the increasing use of synchronous condensers, because the condensers permit the attainment of satisfactory voltage conditions at either end of a line with loads much larger than could be employed were no condensers used. Also, apparently there is a further danger attending the use of condensers located remote from generating equipment, as has been borne out by some experiences on the Southern California Edison system. These experiences indicate that when there is a large loss of power in the fault, a considerable fraction of it may be contributed by the condensers, especially if the fault occurs near to them, with the result that they pull out of step and in fact actually slow down to a fraction of normal speed. Thus, the use of intermediate condensers in long lines may be undesirable when heavy power losses are anticipated under fault conditions.

E. H. Bancker: It is believed that some further elaboration of the circle diagram method employed would be desirable. Although this method of calculating torque is strictly applicable only to machines with equal reactance in any axis, it may be used for salient pole machines with correct results if an appropriate reactance and excitation are employed. The reactance to be used is the quadrature synchronous. The excitation is the actual value minus the $i_d\,(x_d - x_q)$ drop. As the excitation and rotor angles change the fictitious excitation changes. As stated in the paper the variation may be represented by the equation

$$O\,Q = k\,I + k_1 \cos s_e + k_2 \sin s_e$$

Through oversight, the values of the k terms were not given. They are,

$$k = 1 - \frac{(x_q + x_e)\,(x_d - x_q)}{r_e^2 + (x_d + x_e)\,(x_q + x_e)}$$

$$k_1 = \frac{\psi_e\,(x_q + x_e\,(x_d - x_q)}{r_e^2 + (x_d + x_e)\,(x_q + x_e)}$$

$$k_2 = \frac{\psi_e\,r_e\,(x_d - x_q)}{r_e^2 + (x_d + x_e)\,(x_q + x_e)}$$

Calculation of the Capacitance between Two Wires of a Three-Conductor Cable

BY Y. W. LEE*

Enrolled Student

Synopsis.—This paper develops a formula for calculating the capacitance between two wires of a three-conductor cable. A direct method of calculation is employed, based on the work of Doctor H. B. Dwight. This method gives accurate results independent of empirical or restricted premises.

* * * * *

IN the electrical problems of underground cables, a factor which is very difficult to determine is the "geometric factor" which, as the name implies, involves the dimensions of the cable. The geometric factor is very closely related to the capacitance of the cable; and as a matter of fact, it is a constant times the reciprocal of the capacitance.

In a three-conductor cable, there are nine geometric factors[1, 2], and of course as many capacitances resulting from the different connections of the four elements of the cable, namely, the three conductors and the sheath. The capacitance of the three wires against the sheath, and the capacitance between two wires are the most important ones as the other seven are expressible in terms of them.

Greater accuracy in the determination of the geometric factor is demanded in problems such as the calculations of the dielectric loss, the stress under three-phase voltage, the insulation resistance, the charging current, and so on.

The inverse method of determining the capacitance by first assuming the distribution of the charges, then calculating the equipotential lines, is good only in a few cases where these lines agree with the shapes of the conductors. This method is not applicable to the three-conductor cable. An attempt has been made to correct graphically the lines of flow to secure a more accurate result,[1] but this has not been extended to the case to be considered in this paper. All published formulas relating to the capacitance in a three-conductor cable, with the exception of Professor H. B. Dwight's, are either empirical or restricted.

Professor Dwight of the Massachusetts Institute of Technology presented in the TRANSACTIONS of the A. I. E. E., 1924,[3] a direct method for calculating the capacitance of conductors. The most important case he worked out was the capacitance of the three wires against the sheath in a three-conductor cable. The capacitance between two wires of the cable remained to be determined in order to complete the accurate determination of the geometric factors.

With the valuable assistance of Professor Dwight, the writer has derived a formula for the capacitance between two wires in a three-conductor cable. A check has been made against the experimental curves published by Mr. D. M. Simons.[4] The difference is small, being 3.4 per cent. This calculation was done as thesis work in the Electrical Engineering Department of the Massachusetts Institute of Technology.

METHOD OF CALCULATION

Consider a wire A of radius a and an infinitesimal wire B separated by a distance p. Assume A to carry a charge of Q per unit length, and B to carry $-Q$ per unit length.

From the symmetry of the arrangement, we can

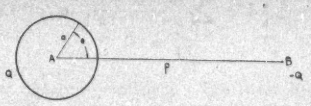

FIG. 1

assume that the charge density at any point (a, θ) is given by the equation

$$q(\theta) = H_0 + \sum_{n=1}^{n=\infty} H_n \operatorname{Cos} n\,\theta \qquad (1)$$

where H_0 and H_n are constants; and upon their determination, the equation becomes[3]

$$q(\theta) = \frac{Q}{2\,a\,\pi} + \frac{Q}{\pi\,a} \sum_{n=1}^{n=\infty} (a/p)^n \operatorname{Cos} n\,\theta \qquad (2)$$

Evidently, $\frac{Q}{\pi\,a} \sum_{n=1}^{n=\infty} (a/p)^n \operatorname{Cos} n\,\theta$ is the charge density at (a, θ) due to $-Q$ in B.

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1. *Cable Geometry and the Calculation of Current-Carrying Capacity*, by D. M. Simons, A. I. E. E. TRANS., 1923, p. 600.

2. "Alternating Currents," by Alex. Russell, Vol. 1, edition of 1914, p. 178.

3. *The Direct Method of Calculation of Capacitance of Conductors*, by H. B. Dwight, A. I. E. E. TRANS., 1924, p. 958.

4. "Calculation of the Electrical Problems of Transmission by Underground Cables," by D. M. Simons, *The Electric Journal*, 1925, p. 366.

28-132

Now when two wires A and C carrying Q and $-Q$ per unit length respectively are placed close to each other, we can calculate the charge density at any point on either wire by a method of successive approximations.

First, assume that the charges are uniformly distributed over the surfaces of the conductors. By Equation (2), an expression can be set up for the charge density at (a, θ) due to the charge in an element of area on C. Remember that the element of area is the same as the infinitesimal wire B in Fig. 1. To get the

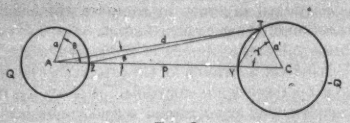

Fig. 2

charge density at (a, θ) due to the uniformly distributed charge on C, the expression is integrated over the surface of C. Or, mathematically,

$$q_1(\theta) = \int_{\gamma=0}^{\gamma=2\pi} \frac{Q\, a'\, d\,\gamma}{(a\,\pi)\,(2\,a'\,\pi)} \sum_{n=1}^{n=\infty} (a/d)^n \operatorname{Cos} n\,(\theta - \beta) \quad (3)$$

which is integrable since[5,6]

$$\frac{\operatorname{Cos} n\,\beta}{d^n} = \frac{1}{p^n}\left[1 + \sum_{k=1}^{k=\infty} \frac{(n+k-1)\,!}{(n-1)\,!\,k\,!}\,(a'/p)^k \operatorname{Cos} k\,\gamma\right] \quad (4)$$

and

$$\frac{\operatorname{Sin} n\,\beta}{d^n} = \frac{1}{p^n} \sum_{k=1}^{k=\infty} \frac{(n+k-1)\,!}{(n-1)\,!\,k\,!}\,(a'/p) \operatorname{Sin} k\,\gamma \quad (5)$$

Other conductors carrying charges can be taken into consideration in the same manner. This is a first approximation, and we shall call $q_1(\theta)$ the first additional charge density. In exactly the same way, $q_1(\gamma)$ can be obtained as the first additional charge density on C, and so on for other conductors that may be present.

In the present problem, there are six conductors to be considered as shown in Fig. 3. A', B', and C' are the images which replace the sheath. A and C' each carries Q per unit length, and C and A' each carries $-Q$ per unit length. B and B' have zero total charge.

A repetition of the process of integration replacing the uniform density by $q_1(\theta)$ and $q_1(\gamma)$ will give $q_2(\theta)$ and $q_2(\gamma)$, the second additional charge densities. When this is done, general expressions for the additional charge densities can be written. The sum of the additional charge densities on one wire and the charge density due to the total charge on itself, is the actual charge density on that wire.

The work done in carrying a unit charge from Z to Y (Fig. 2) against the total charge on C is

$$\int_{\gamma=0}^{\gamma=2\pi} 2\,a'\,q(\gamma) \log (Z\,T/Y\,T)\,d\,\gamma \quad (6)$$

where $q(\gamma)$ is the total charge density on C. The expression can be integrated because $\log (Z\,T/Y\,T)$ is expressible in terms of γ. For example,[7]

$$\log\left(\frac{Z\,T}{p-a}\right) = -\sum_{k=1}^{k=\infty} (1/k)\left(\frac{a'}{p-a}\right)^k \operatorname{Cos} k\,\gamma \quad (7)$$

Similarly, the work done against all other charges can be evaluated. When the total work is found, the capacitance can be determined immediately since

$$\text{Capacitance} = \frac{Q}{\text{Total work}} \quad (8)$$

The Formula

Let C be the capacitance between any two wires of a three-conductor cable.

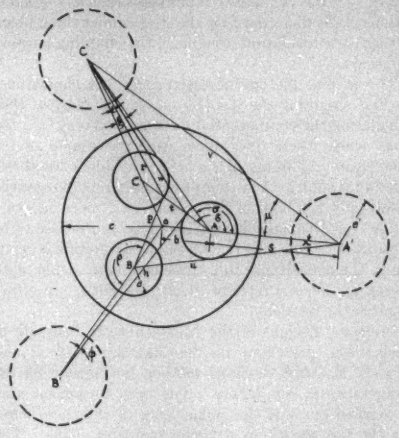

Fig. 3

$$\frac{1}{C} = -4 \sum_{m=1}^{m=\infty} (1/m)\left[\left\{\left(\frac{a}{t-a}\right)^m - 1\right\}\right.$$
$$\{M_m \operatorname{Cos} m\,\delta + M_m' \operatorname{Sin} m\,\delta\}$$
$$+ (a'/w)^m (N_m \operatorname{Cos} m\,\xi + N_m' \operatorname{Sin} m\,\xi)$$
$$- (a'/r)^m (N_m \operatorname{Cos} m\,\epsilon + N_m' \operatorname{Sin} m\,\epsilon)$$

5. "Theory of Infinite Series," by T. J. I'A Bromwich, pp. 158-159.

6. "An Integration Method of Deriving the A-c. Resistance and Inductance of Conductors," by H. L. Curtis, Scientific Papers No. 374 of the Bureau of Standards, Washington, D. C., Appendix 2.

7. *Ibid.* Appendix 1.

$$+2(a/p)^m M_m' \operatorname{Sin} m\,\rho + 2(a'/h)^m N_m' \operatorname{Sin} m\,\phi \Big]$$

$$- 4 \log \left(\frac{a}{t-a} \right) \left(\frac{w}{r} \right). \tag{9}$$

$$M_m = A_m + B_m + C_m + D_m + \ldots \tag{10}$$

$$M_m' = A_m' + B_m' + C_m' + D_m' + \ldots \tag{11}$$

$$N_m = F_m + G_m + H_m + I_m + \ldots \tag{12}$$

$$N_m' = F_m' + G_m' + H_m' + I_m' + \ldots \tag{13}$$

$$A_m = (a/s)^m + (a/t)^m \operatorname{Cos} m\,\delta - (a/u)^m \operatorname{Cos} m\,\sigma \tag{14}$$

$$A_m' = (a/t)^m \operatorname{Sin} m\,\delta - (a/u)^m \operatorname{Sin} m\,\sigma \tag{15}$$

$$B_m = \sum_{n=1}^{n=\infty} \frac{(m+n-1)!}{(m-1)!\,n!} \Big[- (a/s)^m (a'/s)^n F_n + (a/t)^{m+n} \{ A_n \operatorname{Cos}(m-n)\,\delta - 3 A_n' \operatorname{Sin}(m-n)\,\delta \} + (a/u)^m (a'/u)^n \{ F_n \operatorname{Cos}(m\,\sigma + n\,\lambda) + 3 F_n' \operatorname{Sin}(m\,\sigma + n\,\lambda) \} \Big] \tag{16}$$

$$B_m' = \sum_{n=1}^{n=\infty} \frac{(m+n-1)!}{(m-1)!\,n!} \Big[- (a/s)^m (a'/s)^n F_n' + (a/t)^{m+n} \{ A_n \operatorname{Sin}(m-n)\,\delta - A_n' \operatorname{Cos}(m-n)\,\delta \} + (a/u)^m (a'/u)^n \{ F^n \operatorname{Sin}(m\,\sigma + n\,\lambda) + F_n' \operatorname{Cos}(m\,\sigma + n\,\lambda) \} \Big] \tag{17}$$

$$F_m = - (a'/s)^m + (a'/u)^m \operatorname{Cos} m\,\lambda - (a'/v)^m \operatorname{Cos} m\,\mu \tag{18}$$

$$F_m' = (a'/u)^m \operatorname{Sin} m\,\lambda - (a'/v)^m \operatorname{Sin} m\,\mu \tag{19}$$

$$G_m = \sum_{n=1}^{n=\infty} \frac{(m+n-1)!}{(m-1)!\,n!} \Big[- (a'/s)^m (a/s)^n A_n + (a'/u)^m (a/u)^n \{ A_n \operatorname{Cos}(m\,\lambda + n\,\sigma) + 3 A_n' \operatorname{Sin}(m\,\lambda + n\,\sigma) \} + (a'/v)^{m+n} \{ F_n \operatorname{Cos}(m-n)\,\mu - 3 F_n' \operatorname{Sin}(m-n)\,\mu \} \Big] \tag{20}$$

$$G_m' = \sum_{n=1}^{n=\infty} \frac{(m+n-1)!}{(m-1)!\,n!} \Big[- (a'/s)^m (a/s)^n A_n' + (a'/u)^m (a/u)^n \{ A_n \operatorname{Sin}(m\,\lambda + n\,\sigma) + A_n' \operatorname{Cos}(m\,\lambda + n\,\sigma) \} + (a'/v^{m+n} \{ F_n \operatorname{Sin}(m-n)\,\mu - F_n' \operatorname{Cos}(m-n)\,\mu \} \Big] \tag{21}$$

For C_m, C_m', H_m, and H_m', use the expressions for B_m, B_m', G_m, and G_m' respectively, but change A_n, A_n', F_n, and F_n' to B_n, B_n', G_n, and G_n' respectively. A_n, A_n', B_n, B_n', F_n, F_n', G_n, and G_n' are defined by Equations (14), (15), (16), (17), (18), (19), (20), and (21) where m and n are interchanged.

D_m, D_m', I_m, I_m', and the subsequent terms are similarly defined.

For the images,[8]

$$a' = \frac{c^2 a}{b^2 - a^2} \qquad s + b = \frac{c^2 b}{b^2 - a^2} \tag{22}$$

$$\delta = 150^\circ \qquad \mu = 30^\circ$$

An Example

The geometric factor G_2 is defined as the geometric factor under three-phase operation.[1] It is related to the capacitance (C) between two wires of a three-conductor cable by the equation

$$G_2 = \frac{3\,k}{4\,C} \tag{23}$$

k is the dielectric coefficient. The letter G_2 for the geometric factor should not be confused with the quantities G_1, G_2, G_3, etc. used throughout this calculation.

Take $a = 1$, $b = \sqrt{3}$, and $c = 2 + \sqrt{3}$. Using Simons' notations,[4] this corresponds to $t/T = 1$, and $(T + t)/d = \frac{1}{2}$. The calculated G_2 is 2.031, and Atkinson's experimental value published by Simons is 2.10. The difference is 3.4 per cent.

8. "Elements of the Mathematical Theory of Electricity and Magnetism," by J. J. Thomson, fourth edition, pp. 149 and 176.

Radio Acoustic Position Finding in Hydrography

BY JERRY H. SERVICE*

Associate, A. I. E. E.

Synopsis.—In hydroelectric surveys it is difficult to fix the positions of the soundings when the survey ship is beyond the range of visibility of shore objects; the radio-acoustic method is designed to help in meeting this difficulty. Temporary shore stations are set up, each equipped with a microphone placed in the water in a known position and connected with an amplifier ashore through a cable; the amplifier is in turn connected through a relay to an automatic key driven by clockwork, which causes a radio transmitter to send out a characteristic signal whenever the microphone is disturbed.

When the position of the ship is wanted, a small bomb is fired in the water alongside the ship; the instant of the explosion is automatically recorded on a chronograph aboard the ship. The sound of the explosion disturbs the shore station microphone and the resulting characteristic radio signals of the shore stations are received on board the ship and also recorded on the chronograph. Thus the ship obtains the time of travel of sound in water from her position to each of the shore station microphones, which are in known positions. The speed of sound in sea water being known, the ship's position is thus fixed. The method has been used successfully in the U. S. Coast and Geodetic Survey with the ship 75 mi. or more away from the shore stations.

A device for quick plotting on the field sheet is described. The accuracy of the method is discussed.

Introduction

In hydrographic surveys, the fundamental objective is the production of accurate nautical charts for the guidance of mariners. Soundings in great number and well distributed over the area surveyed must be taken. These soundings must be accurate as to depth, but it is just as important that the positions of the soundings should be accurately known. That is to say, a survey ship while taking soundings must know her position accurately at all times.

So long as the ship is within the range of visibility of shore objects, natural or artificial, such as mountain peaks, tall chimneys, elevated tanks, prominent lone trees, or specially constructed signals, the problem is comparatively easy in the daytime and in clear weather. At regular short intervals, angles between the right and center, and center and left objects, are simultaneously measured with sextants by two observers on board the ship; these two angles can be set up on a three-arm protractor, or "station pointer," and the ship's position can be plotted within a minute of time.

But at night (except in those rare instances in which three well located lights can be identified), in foggy weather, or when the ship is beyond the range of visibility of shore objects, the problem becomes difficult. Dead reckoning, in which the ship's position is fixed by her distances run (measured by the patent log or the revolution counter) in the directions as given by her compass, with allowances for current and leeway (due to the ship being blown sidewise through the water by the wind), is always open to question. Formerly, when the ship had to stop for every sounding, dead reckoning was especially questionable; now, with the advent of echo sounding, by means of which the ship can run continuously at full speed, dead reckoning is much more satisfactory than it was. Astronomical fixes, even when taken at their best on stars at morning and evening twilight by several observers, have comparatively large probable errors and are limited to a few times per day, and then only when the sky is clear and the horizon is well defined.

If some means could be devised that would locate a survey ship accurately, at frequent intervals, day or night, clear weather or foggy, in rough weather or in smooth, and as far off shore as she might have occasion to work, it may be seen that such a method, if its cost were not excessive, would be very useful.

The British have such a method,[1,2] in which the pressing of a key on board the ship simultaneously detonates a small charge of explosive in the water near the ship and sends out a radio dash. A station on shore records on an Einthoven galvanometer photographic recorder, the time of arrival of the radio dash and also the times of arrival of the sound wave from the explosion at each of a number of hydrophones suitably disposed in known positions on the sea bed. What they get is the distance of the explosion from each hydrophone, and the bearing of the explosion from the mid-point of each pair of hydrophones. The use of the bearing lines in connection with the range lines makes considerable accuracy possible. The shore station transmits the position of the explosion to the ship by radio. Very little equipment is required on board the ship, so that the method is of use in navigation. It is stated that with a 9-oz. charge of explosive, locations can be given within a range of 40 mi. from the hydrophones.

In 1923 the U. S. Coast and Geodetic Survey, in collaboration with the Sound Laboratory of the U. S. Bureau of Standards, worked out a method of locating a survey ship by radio-acoustic methods.[3] While some important practical modifications in details have been made during the past five years, this method is still used with no changes in the general principle. Since 1925 it has been proved economical and successful in the waters off the Pacific Coast of the United States, especially in the waters off Washington and

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1. For all references see Bibliography.

Oregon. In one test run off the coast of Oregon, the Survey Ship *Guide* ran over 200 mi. directly off shore and obtained consistent positions during the entire distance; both the *Guide* and the *Pioneer* have successfully surveyed large areas with the positions of the soundings determined almost exclusively by the radio-acoustic method, the ship being as much as 75 to 80 nautical miles away from the shore station hydrophones during the progress of the work. Thus far, the method has not been used with success in the waters off the Atlantic Coast of the United States, although it has been the subject of experiments off the coasts of North Carolina and Florida.[4]

This American method differs from the British method chiefly in that the working out of position is done on board the ship, the shore station apparatus being essentially automatic so that the shore station personnel is required mainly to keep the equipment in good condition. By this means the ship's position at the instant of explosion becomes known within a very

Fig. 1—Radio-Acoustic Shore Station

few minutes after the explosion, which is very important in survey work.

Space does not permit us to trace the development of the American method. The reader can refer to the Publication[3] by Commander Heck, Dr. Eckhardt, and Mr. Keiser for the details of the original apparatus and procedure. The present method has been discussed by Commander W. E. Parker,[5] Chief of the Division of Hydrography and Topography of the Coast and Geodetic Survey, and by the author.[6] It will be treated in its essentials in this paper; a more detailed description can be found in the new Special Publication of the Coast and Geodetic Survey upon the subject, which is now in press.[11]

The Shore Stations

The Survey Ship having a given water area to survey sets up two shore stations, one near each inshore corner of the area. These stations are usually 30 to 40 mi. apart and are often in wild places at a considerable distance from any towns or settlements. Since they will be used at most only a few months, the shelters are of a temporary nature, constructed or set up in one or two days by a party sent ashore from the ship. Fig. 1 shows such a station on San Clemente Island off the coast of California.

The hydrophone, sealed into a watertight, wooden case, filled with sea water, is planted on the sea bottom at a distance varying from a few hundred yards to a mile or more offshore, depending upon the topography of the bottom, the presence or absence of projecting ledges, etc. The hydrophone is a single-button microphone with a rubber case; sometimes two or more hydrophones are used, mounted in the same wooden case and usually connected in parallel. The hydrophone (or set of hydrophones) is connected to the shore station through a single-conductor cable and the sea water ground.

The wooden case into which the hydrophone is sealed may be a keg or a specially constructed box of about the size of a keg. It was introduced by the writer as the result of a study of a paper by Brillié,[7] after a series of experiments by the writer and Chief Wireless Operator A. M. Vincent, of the Survey Ship *Guide*. These experiments showed that the wooden case passes sound from the explosion without appreciable attenuation, but eliminates a great many very troublesome strays that are probably due to the mechanical action of the water, bits of sand, shells, etc., upon the hydrophone.

The cable was designed by Lieutenant Commander Thomas J. Maher, U. S. Coast and Geodetic Survey, and the author, and consists of 19-strand tinned aircraft cable ⅛ in. in diameter) insulated with a ⅛-in. belt of 40 per cent to 60 per cent rubber. This cable has great tensile strength, its insulation resistance remains high after long immersion in sea water, and the insulation is very resistant to abrasion. The high electrical resistance of this cable is not a serious disadvantage. The cable is specially manufactured for the Survey. In some localities, as on rocky beaches, single-conductor deep-sea cable is used through the breakers. Usually it is found satisfactory to run the lighter cable through ¾-in. pipe from a point inside high-water line to a point outside low water line; for protection of the cable, the pipe is fitted with a fiber or wooden bushing and a piece of rubber hose at each end.

The essentials of the shore station equipment except for batteries, are shown schematically in Fig. 2. When the sound from the distant explosion reaches the hydrophone its resistance changes and an electrical impulse passes through the input winding of the first transformer of the amplifier. This impulse, amplified, passes through the line circuit of the commercial standard telegraph relay and as a result the local circuit of that relay is momentarily closed. The closing of the local circuit of the relay has two effects: (1) the radio sending key is short-circuited by the relay contacts and a dash is broadcasted by the station with a scarcely appreciable lag behind the arrival of the sound from the explosion at the hydrophone; and (2) the automatic

key is set into operation, which is driven by clockwork, and by means of a notched wheel causes the radio transmitter to broadcast the characteristic signal of the station, which may be three long dashes, or a dot and two dashes, or whatever signal may be desired.

There is nothing of special interest connected with the amplifier. The input impedance of the first transformer is matched approximately to the impedance of the hydrophone and cable. The line circuit impedance

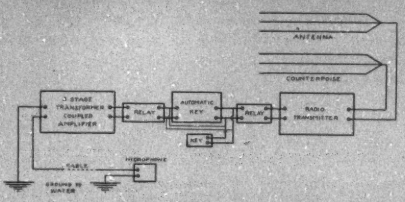

FIG. 2—ESSENTIALS OF RADIO-ACOUSTIC SHORE STATION APPARATUS—U. S. COAST AND GEODETIC SURVEY

of the relay is matched to the output impedance of the last tube of the amplifier. Usually, radiotrons UX201A are used in the first two stages, and a UX112 in the third stage. Suitable values for plate and grid potentials have been found by experiment.

The automatic key is that described by Heck, Eckhardt, and Keiser.[3] It consists of a pair of contacts connected in parallel with the radio sending key. These contacts are normally held open by a wiper resting upon the periphery of a notched wheel. This wheel is driven by a clockwork mechanism set in motion whenever the relay local circuit is closed; a device that stops the clockwork automatically when the wheel has made exactly one revolution is included.

The relay shown alongside the sending key is merely a pony telegraph relay included to insulate the sending key and the automatic key from the high plate potential of the transmitter; the transmitter is keyed in the plate circuit.

There is nothing particularly remarkable about the radio transmitter. It is simply a Hartley oscillator using two UX210 radiotron tubes in parallel and operating at 140 meters wavelength. Both stations operating in connection with a given ship must keep their transmitters adjusted to approximately the same frequency.

THE SHIP INSTALLATION

The essentials of the ship installation, exclusive of batteries, are shown schematically in Fig. 3. The hydrophone is mounted well below the water line in a tank of water in contact with the outer plating of the ship.

The amplifier preferably has the first transformer that is connected with the hydrophone distinct from the first transformer that is connected with the output of the radio receiver, in order that the transformer input impedances may be properly matched to the impedances of the circuits into which they are connected. Usually, two or three stages of amplification are sufficient either for the incoming radio signals or for the hydrophone circuit impulses.

The chronograph drives a ¾-in. tape at the rate of about one inch per second. The drive was modified by the author and consists of a 12-volt shunt motor driven by a 12-volt storage battery of large capacity; by this arrangement satisfactory speed regulation is obtained without a governor. The pen magnet winding that is connected with the amplifier has its impedance approximately matched to the output impedance of the last tube of the amplifier.

EXPLOSIVE CHARGES

The explosive charge is detonated by a fulminate blasting cap into which a short length of waterproof fuse is fitted. If the ship is within 15 or 20 mi. of each of the two stations, one of these blasting caps will usually send sound energy of sufficient amplitude to the hydrophones at both station to actuate the station apparatus. Beyond the range of a blasting cap, a friction top, pint-size, tin can filled with TNT into which the blasting cap is imbedded is usually sufficient. For the greatest distances, a bomb designed by the author is used. This bomb consists of a hollow cast iron sphere of 5-in. inside diameter and ½-in. wall thickness, filled with TNT; the blasting cap and fuse (as well as the TNT) are introduced through a small opening; the blasting cap is imbedded in the TNT at the center of the sphere. In these cast iron bombs, as

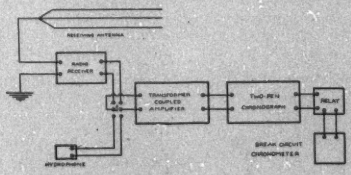

FIG. 3—ESSENTIALS OF RADIO-ACOUSTIC SHIP APPARATUS —U. S. COAST AND GEODETIC SURVEY

well as in the tin can bombs, the fuse opening is sealed with soft pitch or some similar material after the fuse is in place.

OPERATION OF THE SYSTEM

When a position is wanted, the switch S, (Fig. 3) is thrown to the hydrophone, the chronograph is started, the pens are seen to be functioning with the time pen making the second marks, and a bomb is ignited (by touching the end of the fuse to a hot wire) and thrown overboard. The ship need not modify either her course or speed for this procedure. The instant of explosion is recorded on the chronograph by means of the ship's hydrophone, the amplifier, and the amplifier pen

magnet. The switch S, (Fig. 3) is then thrown to the ship's radio receiver.

The sound energy flows out from the explosion and, after intervals, reaches the hydrophones at the two stations. When this sound reaches the hydrophone at a given station, that station, with scarcely appreciable lag, broadcasts a radio dash, identified immediately afterward by the characteristic signal of the station. Dash and signal are received by the ship and recorded on the chronograph tape. The time of travel of the radio dash from station to ship is negligible. Instrumental lags can be determined at the beginning of the season and then checked from time to time.

From the chronograph tape, the time of travel of sound energy from the explosion to the hydrophone at each station can be read off quickly to within one or two one-hundredths of a second. The positions of the station hydrophones have been located accurately and plotted upon the survey sheet when the hydrophones were planted.

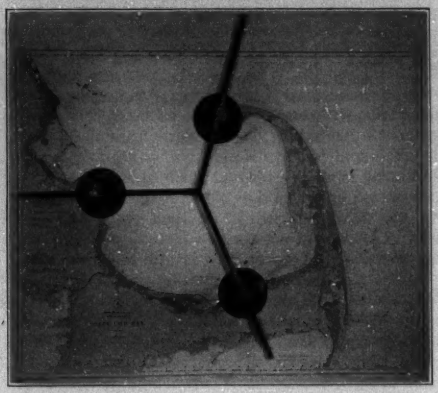

Fig. 4—Parkhurst—Service Radio-Acoustic Plotter

Speed of Sound in Sea Water

Since the distance from the explosion to the hydrophone at each station is required, in addition to the time of travel of sound energy to each station the speed of sound in sea water must be known. This speed is a function of the mean (from surface to bottom and over the range) temperature, pressure, and salinity of the water. From studies made within the past few years the speed of sound in sea water is quite well known under all conditions of temperature, pressure, and salinity likely to be encountered.[4,8,9] Also, the temperature and salinity conditions are determined during the season in order that correct sound speeds may be known for echo soundings. Hence, it may be assumed that the correct speed of sound for computing radio-acoustic ranges will be available.

Plotting the Positions

Since the positions of the station hydrophones are known and have been plotted upon the survey sheet, and the distances from the explosion to each hydrophone have been determined by the radio acoustic method, the problem is to find (upon the survey sheet) the vertex of a triangle when the base and the lengths of the other two sides are known. This problem would be easy of solution were it not for the distortion of the survey sheet and the necessity for economy of time. In the plotting of the final "smooth sheet," corrections must be made for distortion, and speed is not so essential; in the plotting of the preliminary work sheet or "boat sheet," distortion may usually be disregarded and speed is quite important.

For plotting on the boat sheet where distortion of the sheet may be disregarded, the plotting device designed by Douglas L. Parkhurst, Chief of Division of Instruments of the Coast Geodetic Survey, and the author, can be used to advantage. This plotting device is shown in Fig. 4, and consists of a pair of arms pivoted at a pricking device; with zero at the pivot, graduations are marked on the arms which represent the distances (on the scale of the hydrographic sheet) that sound will travel in the number of seconds represented by the graduation marks; of course, a mean value of the speed of sound in sea water must be used in laying out the graduations on these arms. An index head is tacked to the chart table through the survey sheet over the plotted position of each hydrophone. The arms slide in grooves on these index heads. Suppose then, that it is found that sound travels from the ship to hydrophone A in 44.82 seconds, and to hydrophone B in 39.19 seconds. The arm in the index head that is over the plotted position of A is moved in or out until graduation 44 is opposite 0.82 on the index head, and then clamped; likewise 39.19 is set up on the arm that slides in the index head at B. The pricker will then be over the position of the ship on the sheet, and the position can be pricked.

Three arms and three index heads are provided, but usually only two are used because there are generally but two stations.

Accuracy of the Method

The accuracy of the method is limited by the accuracy with which the speed of sound is known and by the angle at which the two distance lines intersect. If the physical conditions of the water are fairly well known, the speed, and therefore the two distances, will have an uncertainty not greater than 0.5 of 1 per cent, probably no greater than 0.3 of 1 per cent. That is to say, if the ship is about 50 mi. from the two hydrophones, she can determine her position from either with an uncertainty of the order of 300 meters. Then, of course, the accuracy of her position, determined from the two distances, will depend upon the angle of inter-

section of the two arcs about the respective hydrophones as centers.

Captain L. Tonta, Director of the International Hydrographic Bureau, has proposed[10] a rather brief method of computation of radio-acoustic positions that may prove of advantage for final plotting on the smooth sheets. This method eliminates error of position due to a small error in the value assumed for the speed of sound. The reader is referred to Captain Tonta's original paper for the details of this method.

In conclusion, the author wishes to express his appreciation to those of his superior officers and associates in the Coast and Geodetic Survey who have helped him in the preparation of this paper, and to C. L. Terrel, of the class of 1928 at the Ohio State University, who inked and lettered the diagrams.

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Economies in Central Power Service
as Illustrated by the Duke Power System

BY W. S. LEE[1]
Fellow, A. I. E. E.

Synopsis.—This paper invites the attention of the engineering profession to the advantages and economies which may be obtained by coordinating both hydro and steam power stations of various kinds, and electric transmission and distribution lines serving a large industrial territory in a unified and centrally controlled system. For illustration, a general description of the generating and distribution system of the Duke Power Company, operating in the Piedmont section of the Carolinas, is given.

EVERYONE who studies the statistics on electric power output in the United States will be amazed at the rapid growth of the electrical industry during the past decade and how, by the construction of larger and larger commercial electrical central stations and the merging of existing electrical properties, the increasing demand for electricity for lighting, power, and transportation was met.

Through this centralization, it was made possible to give uniform and dependable service to the customer, to maintain the same rates, and even lower them to domestic consumers in the face of increasing costs of labor and materials, to make the investments in properties of such character safe and attractive, to afford the support of an organization highly trained in the efficient planning, constructing, operating, and improving of utility systems, to make purchases in large quantities and to engage aggressively in building-up the electric light and power business. Other advantages are the grouping of hydro and steam power stations of various kinds to permit the full and economical utilization of the available natural resources of power.

The large generating plants deliver the electric energy to trunk transmission lines which feed the distributing systems of cities and towns and furnish power to the lines at industrial centers. In addition, interconnections of trunk lines of the transmission systems of large independent power companies operating in adjoining territories provide means for marketing wholesale power blocks from one utility to another, for interchange of power between systems and, in case of emergency, for stand-by service.

Due to their higher plant efficiencies and the lowering of construction costs, the number of large steam electric generating stations is rapidly increasing and water-powers of large size, which were considered uneconomical on account of the inability of marketing such large blocks of power within a reasonable period of time by local utilities, are being developed. Industrial plants abandon operation of their old inefficient power stations, and wasteful generating stations of small suburban electric properties are being shut down. An increasing number of manufacturing plants is being located away from the large centers of population, this trend towards decentralization being made possible by reason of the fact that high-tension transmission lines can be tapped at suitable locations away from metropolitan areas. Old inefficient water-power plants are being overhauled and existing hydro stations utilizing only a portion of the available water-power of the stream are being rebuilt and enlarged so as to lower the cost of the electric power generated.

A striking example showing the development of a local concern organized for the sale of electricity generated at a single water-power plant about 23 years ago to one of the largest central stations generating and distributing systems in the world is given by the Duke Power Company operating in the Piedmont section of the Carolinas. This company is now operating a number of hydro and steam electric plants having an aggregate installed capacity of 873,895 kv-a., and the power is fed into a transmission and distribution system consisting of approximately 4000 mi. of circuits, of which about 50 per cent are 100,000-volt steel tower lines. One hundred and sixty thriving industrial communities and many isolated cotton mills and factories are served by these lines, and in 1927 the total power generated and purchased by the Duke Power System amounted to 1,745,776,428 kw-hr. This compares with an average output of 19,000,000 kw-hr. of the old Catawba Station, which was the first plant operated and which was redesigned and reconstructed in 1925.

The generating system of the Duke Power Company may be subdivided into the following groups:

1. Hydro stations which use the fall and run of the river.
2. Hydro stations which use the fall and run of the river plus a small amount of storage taking care of the night and Sunday flow and, perhaps, of minor weekly fluctuations of the flow of the river.
3. Hydro stations which have a large storage capacity behind them and which are used as a valve, or outlet, to supply the deficiency in power of other stations during low-water seasons.
4. Steam plants designed to operate intermittently and during low-water periods, termed "stand-by plants."
5. Steam plants designed to operate twelve months of the year and capable of supplying a constant amount of power, termed "base plants."

Of special interest are the twelve hydro stations

1. Vice-president and Chief Engineer, Duke Power Co., Charlotte, N. C.

Presented at the Regional Meeting of Southern Dist. No. 4 of the A. I. E. E., Atlanta, Ga., Oct. 29-31, 1928.

28-130

located on the Catawba River the flow of which is almost completely regulated so that all these stations may be classed under Group 3. Beginning with the power plant farthest upstream, the names and installed capacities of these stations are as follows:

Name of hydro station on Catawba River	Rated capacity of station kv-a.
Bridgewater	25,000
Rhodhiss	21,875
Oxford	45,000
Lookout Shoals	23,400
Mountain Island	75,000
Catawba	75,000
Fishing Creek	37,500
Great Falls	30,000
Dearborn	56,250
Rocky Creek	30,000
Cedar Creek	56,250
Wateree	70,000
Total	555,275

The Bridgewater, Rhodhiss, and Oxford Stations, located in the upper region of the Catawba River, and the Catawba Station, located below the junction of the South Fork of the Catawba River, have large storage reservoirs behind them for the retention of the flood-waters and excess flow of the river.

The aggregate installed capacity of the six steam power plants is 244,313 kv-a., of which the Buck Station, using powdered fuel and having a rated capacity of 87,500 kv-a., is to be classed under Group 5. It should be added that the Duke Power Company now has under construction a large powdered fuel steam power station on the Catawba River near the Mountain Island hydro station, where an ample supply of condensing water is available. This station is located closest to the center of gravity of the entire load system and the initial installation will consist of two 60,000-kw. units, whereas the plans call for an ultimate installation of 480,000 kw.

The hydro stations to be classed under Groups 1 and 2 are operated in conjunction with the stand-by steam plants under Group 4, which arrangement will permit the most effective utilization of the stream flow under the conditions. The locations and rated capacities of these stations are as follows:

Name of hydro station	Name of river	Rated capacity of station kv-a.
Gaston Shoals	Broad	12,750
99 Island	Broad	22,500
Tuxedo	Green	6,250
Turner	Green	6,880
Portman Shoals	Seneca	9,500
Gregg Shoals	Savannah	2,250
Lake Lure	Rocky Broad	4,500
Saluda	Saluda	2,600
Idols	Yadkin	1,000
Plants of less than 1000 kv-a. capacity		6,077
Total		74,307

The steam power plants are located at strategical points of the system; combining Groups 4 and 5, they are:

Name of steam station	Rated capacity of station kv-a.
Buck	87,500
Eno	31,250
Greensboro	8,000
Greenville	8,000
Mount Holly	45,500
Tiger	37,500
Stations leased	26,563
Total	244,313

A summary giving the generating capacity of the Duke Power System, arranged by groups, discloses the following interesting facts:

Number of group	Total rated capacity of stations kv-a.	Per cent of total capacity of entire system
Hydro No. 1	7,777	0.89
Hydro No. 2	66,530	7.61
Hydro No. 3	555,275	63.54
Steam No. 4	156,813	17.95
Steam No. 5	87,500	10.01
All groups	873,895	100.00

If the steam power station of 150,000 kv-a. initial capacity now under construction is included, the ratio of steam to total generating capacity of the system will be increased from 27.96 per cent to 38.51 per cent.

To give an outline of the transmission and distribution system of the Duke Power Company, 100,000-volt double-circuit feeder lines from points of concentration of power generation tap the 100,000-volt double-circuit trunk lines running through the load centers of the Piedmont Section of the Carolinas. At these centers, the current is stepped down to 44,000 volts and delivered to the distribution system. The object of using the lower voltage is to reduce the cost of the substations serving the large power customers and public utility branches of the company; and wherever feasible, a generating station is tied-in directly to the 44,000-volt system to avoid double transformation.

At present approximately the following lines are in operation:

Voltage of circuit	Approximate length of circuit miles
100,000	2000
44,000	1900
13,200	100

The standard secondary delivery voltages are 575 and 2300.

Interconnection of the Duke Power Company's trunk lines is made for delivery of large blocks of power to it at the High Rock Station on the Yadkin River of the Tallassee Power Company, and a line is contemplated for connection with the hydro plant of the Lexington Water Power Company, now under construction on the Saluda River near Columbia, S. C. Interconnections

for interchange of power and stand-by service are made with the trunk lines of the South-Eastern Power Company and the Carolina Power & Light Company.

In the process of development of the Duke Power Company effort was made to maintain the entire system in good operating condition and to obtain the best results by making improvements on existing plants wherever necessary, even to the extent of abandoning old plants in the interest of fuller and more efficient utilization of the properties. This latter policy was followed up with the fullest success in the case of the old Catawba Station representing an investment of more than one million dollars.

As already mentioned, this hydro plant containing eight rope-driven generators of a total capacity of 8000 kv-a. was built 23 years ago. The normal head at this station was 23 ft. and the over-all efficiency of the plant was about 70 per cent. The storage capacity of the pond was practically negligible and to obtain the benefit of the night and Sunday flow, the plant had to be operated continuously. Furthermore, the available uniform flow at this station was considerably augmented due to the regulating effect of the large storage reservoirs, since built on the Upper Catawba River. The question therefore arose whether additional machinery should be installed and the old plant overhauled or the dam be raised so that the water would back to the tail-race of the company's Mountain Island Station, 25 mi. further upstream, thus creating another large storage reservoir, and the building of an entirely new power house. By the adoption of this latter scheme, not only a reservoir of approximately ten billion cubic feet available storage capacity was created, making an additional regulated flow available at the existing stations on the lower Catawba River with a combined head of 250 ft., but the installation of the new power station could be increased from 8000 kv-a. of the old station to 75,000 kv-a. due to the higher head of 70 ft. and full utilization of the regulated stream flow.

The map in Fig. 1 shows the outline of the states of North and South Carolina. There is shown by dots, each 10,000 spindles located within these two states. It will readily be noted that the industrial expansion follows the transmission lines of a central system.

Notwithstanding the great industrial development in the Carolinas during the last ten years, the prospects for further growth of the electric light and power business are very bright. The demand for power and light in the home, on the farm, and in industries is increasing steadily and new fields are being opened up constantly.

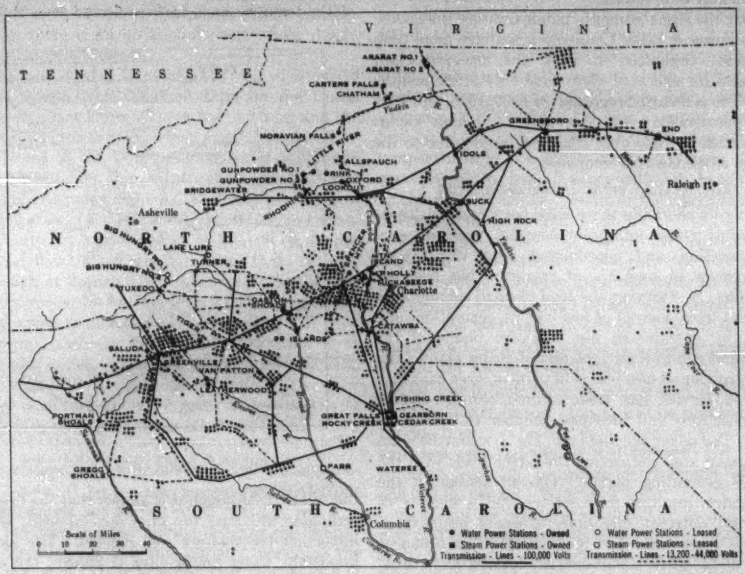

FIG. 1

In anticipation of this ever increasing demand for electric energy, the Duke Power Company is going ahead with a definite program of extensions and improvements to its system to meet future service requirements.

Discussion

For discussion of this paper see page 212.

Southeastern Power & Light System

Interesting Features of Development and Operation

A. T. HUTCHINS*
Non-Member

Synopsis.—The rapid growth of the Southeastern Power & Light system is noted. The long forecast of system load which has been made together with some other features of development are presented briefly. A short discussion of the routine of operating methods and importance of load dispatching together with interesting features of a new steam unit of 60,000-kw. capacity, and interesting features of design for a hydro plant at Jordan Dam rapidly nearing completion are included.

LARGE interconnected groups of public service units are receiving a great deal of attention at this time. The growth of the systems here in the Southeastern Section has been discussed at some length in a number of articles which have appeared during the past several months, so that it is impossible to discuss such problems without some repetition. Some of the articles have gone into detail in discussing important features of design and operation. We shall attempt to point out briefly a number of features which we think may be of interest.

Among the larger groups of public utilities, the Southeastern Power & Light Company is scarcely more than an infant. Organized in 1924, the last important changes in its structure were made only a few months past. The generation reported for 1925, 1926, and 1927 were approximately 1.2, 2.0, and 2.2 billion kw-hr. The increase in 1926 over 1925 was largely due to the addition of the Georgia companies.

Advantages from Interconnection

During its short life there have been realized for its subsidiaries many important benefits from physical interconnection and coordinated load dispatching. Improvement in economy of operation has been secured, first, by carrying the load of small communities over lines from sources of low cost of generation, and second, by coordinated use of hydro plants of the storage and run-of-river type and of steam plants, and by construction of new plants in units of large capacity. Improved service has been secured through better interconnections and increased number of sources of power. The accompanying map, Fig. 1, shows the main transmission lines connecting the larger loads with the principal generating plants. The advantage to the load districts resulting from service from a number of lines from different directions is easily appreciated when it is noted that severe meteorological disturbances do not often occur simultaneously over a wide area. Usually only a narrow zone is affected by wind storms of such severity as to disable a circuit so that it cannot be reclosed after tripping. Similarly storms accompanied by severe lightning usually travel over the country so that these disturbances are not simultaneous over wide areas. In general, the larger loads are supplied by lines and plants of such capacity that service is maintained even if more than one line is out of service. These facts explain the great improvement secured by building lines supplying the same area along routes separated from each other.

System Planning

Extensive surveys of load prospects in every section of the territory have been projected several years into

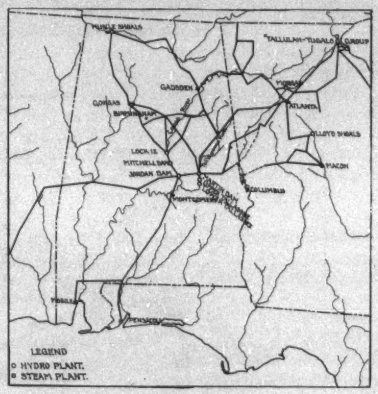

Fig. 1—Principal Lines and Plants of the Southeastern Power & Light Company's System

the future. Although it is known that the best of such estimates will miss the final development, some very important features are well defined. The location of important water power sites is known, the extent of coal fields and coal qualities is fairly well known, and the major trunk lines for the system have already been built. With this foundation, a study of load densities for the various sections of the system, extending ten or twelve years into the future, are of value principally to prevent any construction which would not fit with good economy into the final set-up. This being secured, the

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Presented at the Regional Meeting of the Southern District No. 4 of the A. I. E. E.

28-142

procedure resolves itself into making definite plans to care for load conditions two years in advance.

Hydro Plants

Engineers are familiar with the usual method of the evaluation of a stream at a point under consideration as a site of a hydroelectric plant using the entire history of stream flow, the power-per cent-time curves for the stream, and an estimate of the cost of construction, including transmission lines to the load.

More engineering judgment is required for the evaluation of local conditions, such items as foundations, protection for abutments in earth banks, and protection of downstream surface exposed to erosion by water spilled at high heads. We will discuss some details of design a little later.

In general, the hydraulic structures being built for this system have a relatively high factor of safety, due first to the point of view of the engineers, and, no doubt, partly to the comparatively short history of stream flows resulting in a greater uncertainty as to maximum flow to be expected.

Although most of the hydro plants follow conventional lines there is an occasional plant whose design includes features of interest.

The design of Jordan Dam which is located about 18 mi. down stream from Mitchell Dam on the Coosa River makes use of a variation in forebay level and the buttress type of power house which may be of interest.

Descriptions of the Thurlow back-water suppressor installed at Mitchell Dam have been published* and these will not be repeated here. It is only necessary to note that the use of the suppressor will maintain normal head on the plant with flows up to 170,000 cu. ft. per sec. at which flow the tail race is up about 15 ft.

The design at Jordan Dam takes advantage of this feature and raises the forebay level 7 ft. to compensate for tail-race rises at high flows.

The normal tail-race level at Mitchell Dam is at elevation 282 to 283. The spillway at Jordan is 1122 ft. long with 612 ft. open at elevation 283. The remainder is controlled by 17 gates whose crest is at elevation 290, each gate being 30 ft. long and 18 ft. deep.

With a stream flow of 52,000 cu. ft. per sec. approximately 17,000 cu. ft. per sec. may be used through the four 36,000-hp. wheels and 35,000 cu. ft. per sec. will flow through the open spillway section bringing the forebay level to the elevation 290 and actually increasing the head on the plant. It is expected that this seven-foot rise in the forebay level will compensate for the tail race rise up to stream flows of 75,000 cu. ft. per sec.

It is apparent that while 283 is the elevation for the tail race at Mitchell Dam and also the open spillway at Jordan, the backwater suppressor at Mitchell Dam will maintain full head against the rise of seven feet.

*"New Developments in Hydroelectric Power Plant Designs," by J. A. Sirnit, Amer. Soc. Mech. Eng. *Trans.*, Vol. 44, p. 361-'22.

In addition to this the open spillway avoids the necessity for operating spillway gates except for flows above 52,000 cu. ft. per sec.

A general idea of the buttress type power house is shown in Fig. 5. At each end of the building and between units, buttresses extend from the upstream face of the powerhouse to the solid masonry between draft tubes, thus carrying the greater part of the load on this section of the dam. Such design permits the location of units 40 ft. nearer the face of the dam, and shortens the penstocks by that distance.

Steam Plants

The position of new steam plants in the load diagram and the determination of the use factor will define the relation of increased investment, increased efficiency, and the resulting over-all economy with a known fuel cost.

The use of either 600-lb. or 1200-lb. pressure of steam with reheat cycles and the combined cycle of mercury vapor and steam has had wide publicity and plants using these cycles are increasing in number with a possible trend to neglect 600-lb. pressure and step from 400-lb. to 800-lb. or 1200-lb. pressure.

The most recent unit under construction on this system is designed to use 400-lb. pressure at the turbine throttle, a steam temperature of 725 deg. fahr., and a net economy of the plant for a year's output of approximately 16,000 B. t. u. per kw-hr. On account of high temperatures of circulating water at Gorgas, this unit was designed for best efficiency with 2-in. back pressure at the exhaust nozzle. Some of the essential features of this unit are as follows:

Turbo generator—60,000-kw., 90 power factor, 13,800-volt, single-shaft, 1800-rev. per min., closed generator cooling system using raw water for cooling.

Turbine—three valves 17-stage with 4 bleed points for heating the feed water to 360 deg. fahr. at 52,000 kw. load and a B. t. u. rate chargeable to the turbine—11,350 B. t. u. per kw-hr. at that load.

The unit is built for a maximum load of 66,667 kw. at unity power factor. Steam will be supplied to this turbine from two boilers, each rated at 3040 hp. and equipped with pulverized fuel burners and fans of sufficient capacity to secure a maximum rating of 450,000 lb. per hr. With each boiler there is installed a plate type air preheater of 30,600 sq. ft. and fin type side and rear walls with 3050 sq. ft. of surface. Each boiler is equipped with eight cylindrical burners for horizontal firing and has a furnace with cubical contents of 24,000 cu. ft. so that there will be a release of 18,000 B. t. u. per cu. ft. per hr. at normal rating, this being approximately 350,000 lb. of steam per boiler per hr. and the specified efficiency of boiler unit and furnace at that output is 83.7 per cent.

The two boilers will be supplied with pulverized coal through a central system from three 15-ton Raymond mills having 6 rolls each. These are arranged for mill

drying and the air exhaust from the system with the fines not thrown down in the cyclones goes directly to the furnace. This provides a system as free from escaping dust as is possible to secure. In fact, a great deal of attention to design is given to securing freedom from dust in all parts of the plant.

It may be interesting to note the reason for some of these features of design.

The unit arrangement is followed without spares, capacity at the hydro plants being available in case of a drop in load on any steam plant unit. The use of 400-lb. pressure at the throttle results in maximum drum pressure of 475-lb. gage, and this necessitated the use of 600-lb. valves and fittings. It was found that the increased capacity of the turbine and slight increase in efficiency more than compensated for the added cost of valves and fittings designed for 600-lb. pressure over those for 400-lb. pressure.

The use of air preheaters and a correspondingly high temperature of gases leaving the boiler surface with the high pressure used showed good economy in the use of the fourth bleed point for feed water heating. However, on account of crowding of piping and heaters and consequent difficulty in operation and maintenance, it is likely that the next unit will use only three bleed points for feed-water heating.

Lines and Tie-in Facilities

Lines are always designed for economy based on load and use factor but the necessity for carrying loads with one or more circuits out results in a system which is more or less flexible and which, under certain conditions, makes a maximum use of regulating equipment such as synchronous condensers or spare generators operated as condensers.

The conditions where design based on pure engineering warrants the use of higher voltages for transmission, high-speed excitation of generators, synchronous condensers, and tap changing transformers are receiving a great deal of attention from writers for the technical and trade press and will not be discussed here.

At this time 125-kv-a. capacity in synchronous condensers are installed at advantageous points on the system. One of these having a capacity of 15,000 kv-a., located at Lindale, Georgia, is equipped with a high-speed excitation system. A 90-mi. line from the U. S. Government plants on the Tennessee River to Gorgas, near Birmingham, has been in operation at 154 kv. for some time. A 133-mi. line from Martin Dam to the Atlanta district is spaced and insulated for the same voltage. Three of the newer hydro plants, the last of which will not be in service until next year, were designed for high speed excitation.

A capacity of 110,000 kv-a. in tap changing transformers for regulating voltage while carrying load are in use or purchased for installation at three of the generating plants. This location at the generating plants was determined by the fact that these particular plants are required by the allocation of load to operate part time as a generating plant and part time as substations.

In a system so extensive and having generating plants using stream flow varying so greatly from season to season or over a cycle of years the securing of high load factors on transmission lines is extremely difficult and increased losses, due to overloading under unusual conditions, must be accepted in place of the cost of additional copper and a lower use factor.

The combination of run-of-river and storage plants, served by the same lines, is, in general, advantageous, but extreme conditions arise under which it is not practicable to transmit with normal losses the output from these plants to the load.

Routine meter testing, checking of relays and protective equipment, system voltage surveys, and routine checking of equipment are carried out in a regular program. Daily reports show system performance setting out distribution of energy and its source at generating plants. Accompanying these is a statement of all unusual conditions, line interruptions, etc.

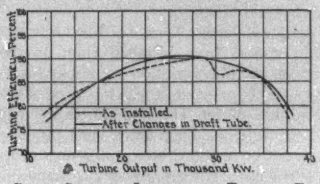

Fig. 2—Graph Showing Increase in Turbine Efficiency after Change in Draft Tube

Solid line shows efficiency after a hydraucone was installed

Reports from individual plants show the details of operations. Steam-plant reports include a heat balance for the day's operation, and hydro plant reports show a record of water used, stored, or passed over the spillway. At the hydro plants records of the operation of the wheels at the gate opening used and the discharge through the spillway gates are employed to check the stream-flow gages. At an earlier period the results from tests of models and the specifications of the manufacturers were used for rating of the wheels. For some time there has been in progress a series of tests which will include one unit of each of the various sizes and manufacture. To date these tests have been of high value.

An example of extreme value is shown in the accompanying graphs in Fig. 2. The broken line shows the characteristics of the hydraulic unit as first installed, and the solid line, the improvement made through a change in the draft tube by installing a hydraucone.

It is often possible to install at the time of testing a pair of piezometers in the penstock and scroll case so

arranged that the differential may be determined by the use of a manometer and the values plotted in terms of water flow and thus be available for check runs at any later date.

It is found that a careful audit of stream flow and water use, especially for flows less than the capacity of the plant, is of real value in securing maximum output or maximum time operation at gate openings of high efficiency. This, of course, implies a nice cooperation between plant operators and load dispatchers.

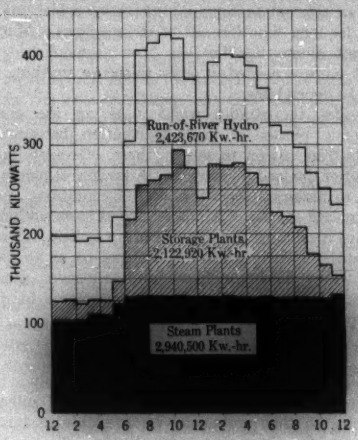

Fig. 3—Load Curve—Typical Day, Normal Dry Month

Load Dispatching

In a system so wide-spread, serving a considerable portion of three states and a part of fourth state which is interconnected, the work of load dispatching becomes extremely important. The load dispatchers must be responsible for coordinating the entire system, not only for daily and weekly runs but they must follow through for an entire year or even a cycle of two or three years on some well conceived program for coordinated use of hydro plants—both those having storage capacity and run-of-river plants—and the various steam plants, each having its own characteristics of economy and protection to a particular load.

In general, the run-of-river hydro plants are operated on the base load whenever the stream flow approaches the capacity of the plant. During the dry weather such plants are operated on a daily or weekly cycle to secure the maximum output in hours of peak load. On the other hand, those hydro plants supplied from storage reservoirs will be operated on a schedule based on stream-flow variations for a year or even more, and effort is being made to have full reservoirs available for the dry season of each year, including a year of low stream flow. This will permit replacing some of the loss of capacity at the run-of-river hydro plants resulting from extremely low stream flow.

The steam generation will vary widely from a minimum during wet weather seasons, when run-of-river plants are carrying full load, to a maximum during periods of lowest stream flow.

Typical plant operation during a wet and a dry day in a year having ordinary seasonal stream flows is shown on Figs. 3 and 4.

Fig. 3 shows conditions in October of 1927 when the storage plants existing at that time were being used for a considerable output to supplement the run-of-river plants at which the stream flows were low during that month. The steam plants operated throughout the twenty-four hours of the day together with run-of-river energy purchased from the government at Muscle Shoals, which plant was available only on a basis of constant output. The other run-of-river power was blocked into the daily peak together with most of the output from the storage plants.

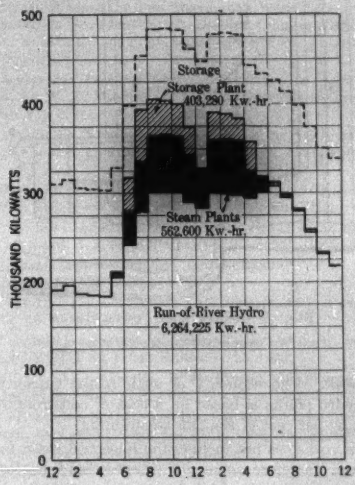

Fig. 4—Load Curve—Typical Day, Normal Wet Month

In a sharp contrast to this, Fig. 4 shows the daily load diagram for March of 1928 under conditions which may be termed normal in wet weather. Storage at this time was being built up, and the most efficient steam plants were being run for a half million kw-hr. daily, as shown, to carry the load with the output from the run-of-river hydro plants and a small output from the storage plants. Stream flow into storage above that used by the plants is plotted above the load curve.

It might be interesting to expand this study into week-end and yearly load diagrams, but the general scheme of operation is set out very nicely by this contrast.

The output from the storage hydro plants of the system in service in 1927, secured by a maximum drawdown, is approximately 230,000,000 kw-hr.

The use of Upper and Lower Tallassee plants, the first finished in June 1928 and the latter to be put into service in 1929, will increase this output through the use of water stored at Martin Dam by approximately 130,000,000 kw-hr. so that the output from stored water in 1930 will have nearly the same ratio to the total load at that time as shown in these load diagrams.

The maximum hourly load carried by the storage plants in 1927 is shown as 165,000 kw. with fairly long-hour use. Capacity available in 1930 will be double this amount with provision in the hydraulic structures for 80,000 kw. additional which may be installed when the proportion of steam generating capacity has greatly increased and the storage plants are pushed higher into

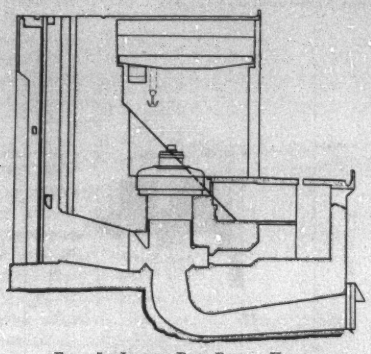

Fig. 5—Jordan Dam Power House

Transverse cross-section through center line of unit

the peak. The economy of such added capacity at the hydro plants is based on a cost of construction of about 50 per cent of that for steam units.

This cost does not include the lines and tie-in facilities and therefore some advantage would be lost if the peak loads to be carried were served over new circuits at a greater distance in the case of hydro units than for steam units. Additional units at existing hydro plants usually call for only an extension of building, a penstock and draft tube, the generating unit itself with the necessary transformer capacity and auxliaries. Additional capacity in steam plants costs the same per kw. as the original plant except (1) for combined use of certain facilities such as real estate, water supply, etc., (2) any saving which can be made by accepting lower efficiency on account of low use factor.

Returning to the subject of load dispatching, it is interesting to note that the schedule for storing and using water at those plants where a considerable stream regulation is possible must take into consideration the cost of steam generation at the various steam plants and an allocation of the load to the lowest cost plants for their maximum use for the required output from fuel, taking reasonable care to avoid a later loss of water from the storage plants. This is, indeed, a most difficult problem and has turned the attention of engineers in the various parts of the country to the study of recurring cycles in weather conditions or stream flow and an attempt at some interpretation which may be used for long time forecast of stream flows. Much of interest along this line has been accomplished, but much more is needed in order to secure anything that may be dependable.

Besides the studies which take up the general economic phases of load distribution between the plants from season to season, there, of course, fall to the lot of the load dispatchers the loading of plants and lines to maintain service under all conditions and to manipulate the use of equipment so as to secure proper voltage conditions through the entire system. To this is added the daily control of streams so as to secure relief from flood conditions and whenever possible obtain full heads at the run of river plants for peak load periods.

Seasonal Maintenance of Equipment

On a system where the only source of supply is from steam plants, there is necessarily provided sufficient capacity so that units may be available in turn as required for inspection. Practically all operating companies make it a practise to inspect the large turbines once every year. On such systems, it sometimes happens that the load demand approaches capacity so closely that the inspection must be made during seasons of low demand, thus making all equipment available for operation during the year's peak.

In contrast with this, it is interesting to note the schedule of inspection and maintenance possible where the plants have a fairly definite seasonal requirement. On this system turbo-generator units at the steam plants are scheduled for inspection during the wet months of the year, which are included in January to June. During this time, besides the regular dismantling and inspection, any pre-scheduled major repairs are also undertaken. Similarly the work on the hydraulic units at the run of river plants is undertaken during the dry months when only a portion of the installed equipment is needed to use the entire stream flow.

Repair of Wheels by Electric Welding

The largest single item of maintenance of hydraulic units is the welding of wheels to replace metal lost through pitting.

Very rapid pitting of cast iron runners at one of our

plants brought about conditions calling for extensive repairs by welding. Cast steel inserts of considerable size were used and the metal of the wheel surrounding these was built up to normal thickness by electric welding.

The results of this work have emphasized to our operators that repairs should not be delayed in the hope that the pitting will cease as a natural course of events. On the other hand, we are convinced that repair by welding should be undertaken just as soon as any considerable area has lost metal to such an extent that a layer of metal placed by electric welding can be used to good advantage.

There has been considerable discussion concerning the use of special metals for surfacing repair sections to reduce the rate of pitting. At this time, the results of the work done for us are not such that we can add anything definite to this item.

Besides such maintenance work, we find it necessary to overhaul completely the moving parts from the servo-motors to and including the wicket gates every twelve to fifteen years. This work includes the replacing of worn liners, bushings, rings, pins, welding up the shafts of wicket gates which may be worn, and, in general, bringing these parts of the machine practically to a new condition.

The schedule for maintenance of equipment at the storage plants may not be forecast as definitely as for the steam plants and run-of-river plants, since a high flow into the pond supplying such a plant may result in the use of these plants on the same schedule as a run-of-river plant at certain periods of the year, and, therefore, the maintenance schedule for these plants is set up only a few weeks in advance. By carrying on maintenance as far as practicable during the season of low generation at the steam plants a further benefit is secured, namely, the reduction in labor turn-over and a more advantageous use of the force of men necessarily continuously employed at the steam plants.

Modern steam-electric units should be available for use 85 per cent of the time. With spare units available to permit inspection and repair, the factor of availability, when required, should approach 100 per cent. By means of scheduling inspection as outlined above, we have obtained a factor of availability, when required, of 99 per cent for three years, and of 99½ per cent for the year 1927.

Hydraulic Structures

Throughout the history of the subsidiary companies of the Southeastern Power & Light Company there has never been any necessity for repairs to hydraulic structures at any of the major plants. The Upper Tallassee Plant, built in the nineties for the Emerson-McMillan Company, was destroyed by a flood in 1920, before being purchased by the Alabama Power Company. This dam was replaced in 1924, using the equipment at the old plant, and since that time has been raised so as to secure a 58-ft. head and a new plant has been built. The erosion of concrete from the spillway sections of any of the dams is thoroughly negligible, and at the oldest plant on the Coosa River, Lock 12, a survey of the river bed immediately below the dam shows excellent conditions.

In the foregoing paragraphs we have touched very briefly upon those features of system extension, operation, and maintenance which seem to us to present points of at least passing interest. A great deal of additional detail is of course at hand, but its presentation is scarcely warranted when so many of the subjects touched upon have been discussed at great length in various articles in the technical and trade press.

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Discussion

ECONOMIES IN CENTRAL POWER SERVICE

(LEE)

SOUTHEASTERN POWER & LIGHT COMPANY'S SYSTEM

(HUTCHINS)

ATLANTA, GA., OCTOBER 29, 1928

W. E. Mitchell: The system that Mr. Lee has built up, in the Duke Power Company, is very nearly ideal. He started out with a hydro system, and fortunately he was able to find load centers interspersed between his logical and economical hydro developments, which made it possible to shift load back and forth, and to get power to the market with the shortest transmission lines. He was working then, for this dollar economy which he has been telling you about. Then, as in other utility companies which have started with hydro developments, he found very soon that the so-called unlimited amounts of water became very much limited as compared to the market, and he was forced to go to some other form of power. With Mr. Lee, as generally in the Southeast, it has been coal. Some in other places will go to oil.

In the Southeastern Power and Light Company when we came to installing these steam plants, we must locate them on ample bodies of cooling water, and of course as close to the load as possible. This didn't happen by chance. It came through thorough study and weighing of load centers, transmission cost, and all other items. We haven't been quite as fortunate in the plant locations as Mr. Lee. A large part of our hydro development in the state of Georgia is up in the northern part, and we had to come 100 miles to get to the main load. In Alabama, it was in the southern part, and we had to carry the power north. But with the growth of those systems, exactly as in the growth of any system, we are finding that now as we build the steam plants we are getting a better distribution of plants with reference to load, which is going to make for more economic use of all our transmission lines.

We are also learning a lesson that Mr. Lee has explained, of the value of the base-load steam plants as plants for filling up the storage reservoirs. Studies very much more thorough have been made in the last few years, of our available water in the effort to get out the last kilowatt-hour. As an example, we started up steam plants in April this year to make certain of having storage reservoirs full on the first of August.

As we learn more about the river flows, as we develop readings over a greater number of years, I believe the time will come when we can forecast still more accurately our available water, and the cycles in which it is likely to come, which will permit of more complete use of it, and that will mean, of course, more economical power development.

In this section of the country which some may think is rather backward, the first real interconnection of a high-voltage type of high capacity was made, between Mr. Lee's system and the system of the Georgia Power Company. One of the next largest interconnections was between the Georgia Power Company and the Alabama Power Company.

We learned years ago the value of coordinated effort and of interlinking the systems of different companies and so have got the benefit of the diversity of these different systems. This, of course, was brought out most clearly in 1925, during the very severe drought when power was shifted from system to system so that actually power generated at Muscle Shoals was available up in North Carolina, some 600 miles away, keeping mills at work that would otherwise have been idle. That is one of the benefits that has come from this closer coordination between the systems. In the individual systems, we go still further. As the systems get larger, we can get more out of each kilowatt capacity that we install. We ought to be able, I believe, to install each kilowatt capacity at a lower dollar cost, and it ought to be a more efficient kilowatt-hour when we get it. I am talking of dollars, not thermal efficiency, because we often lose sight of that. In our business we ought to be interested in the final dollar cost, not thermal efficiency, because that is the thing that it has to be finally measured by. That will tell whether we are going to keep on lowering this cost per kilowatt-hour and developing more capacity per dollar of investment.

W. S. Lee: Mr. Hutchins' descriptions are very interesting. There is a peculiar coincidence, the steam station which is being built by his company and one very similar, being built by my company. The engineering staffs of both companies investigated the matter of type of station about the same time, independently; they came to pretty nearly the same conclusions, so I think I can congratulate him on the type of plant they are building. We are now constructing what we know as the River Bend Steam Station. Two 68,500-kv-a. units at 450-lb. boiler pressure, 725 deg. total temperature, 4-stage bleeding are being installed in this station.

It is interesting to know why both companies decided to use 450-lb. pressure. Preliminary designs and estimates were made by my staff for complete plant at 450-lb., 650 lb., and 1200-lb. pressure. Leaving out the steam generating equipment there was little difference in the total cost of the plant. Steam generating equipment for 450-lb. steam pressure costs approximately 17 per cent of the station. Steam generating equipment for 1200 lb. pressure costs approximately twice as much as for 450-lb. pressure; therefore, the cost of a station equipped with 1200-lb. steam pressure is about 17 per cent over a station equipped with 450 lb.

Out service is for 100 per cent load factor when the plant is operated, and during certain periods of the year there is a total shut-down. This increased cost of installation, with interest and depreciation, brought the cost up so that 1200-lb. pressure was not desirable for our use. Higher steam pressures, no doubt, will be used but they will only be used when the load factor and service can use them at a profit.

The Duke system is not operated normally sectionalized. The system usually works in parallel but in the event of any local disturbance or local storm, the operators immediately sectionalize it to keep disturbances from interfering with other parts of the system.

It is true that as the system gets larger we are forced to use much bigger switches that have a much larger rupturing capacity.

Referring to Mr. Hutchins' discussion of efficiency of water wheels, in our experience we find with a larger system and a great number of units, the operating department often operates more units than are necessary, endeavoring to get advantage of their synchronous effect on the line. During periods of high water there is no loss on this account, but in low water often units operate at a very low point of efficiency. We require the operating department to send record of the load, number of machines, and output to the engineering department promptly. They are computed by the engineering department and returned to the operating department showing the amount of energy loss due to inefficient load on the water wheels. It requires about three days to get these reports back to the operating department. This has resulted in a great saving in the efficiency on all of the stations.

A. T. Hutchins: In connection with Mr. Lee's comments, I might bring out the conditions existing at our hydro plants during periods where the flow of the river is less than the capacity of the plant. Under such conditions, we operate the units in service whenever possible at the gate opening of maximum efficiency. Units required for standby are run as synchronous motors with the wicket gates closed and the governor set to start opening the gates at about one cycle below common frequency. In this way we get very good standby service and avoid operating units at less than their best efficiency, (the point shown either

by test or by manufacturers specifications as the best points). Recently, we have done just as Mr. Lee outlined and are auditing the stream flow against the turbine hours at points of maximum efficiency.

We carry this through the entire year and show accumulated losses based upon maximum efficiency secured by operating wheels with the best gate opening. Furthermore we have sufficient storage capacity on the system now so that we can usually operate the steam plants also at maximum efficiency.

I want to bring out one more point. The map (Fig. 1 of the paper) does not show all of the properties of the Southeastern Power & Light Company. The outlying towns served by only one line may require operation of steam units at certain times in order to protect service and this is provided.

J. M. Oliver: I want to ask Mr. Hutchings what his available methods are for testing hydro units.

A. T. Hutchins: I can say that none of us knows what the limits are on conditions at a hydro plant for a fairly accurate test. We raised a question as to whether or not it would be of any use to attempt a test on these units with so small a volume ahead of the wicket gates on the machines. Professor Allen, the originator of the Allen Method, said that he thought it was possible, by putting the first stage of his apparatus right at the penstock gate grooves, to get sufficient volume so that he could get fair accuracy, but many of us questioned it.

Here in Georgia, they are carrying on now some tests of hydro units. Some of the work has produced what seemed to be very consistent results but their accuracy is certainly questionable.

Carrier-Current and Supervisory Control on Alabama Power Company System

By W. I. WOODCOCK[1] and E. W. ROBINSON[1]
Non-member Non-Member

Synopsis.—The experience of the Alabama Power Company with carrier-current telephony over power lines and supervisory control is outlined in the paper. A sketch is included of the early development of these systems, and this is followed by descriptions of the systems later installed throughout the Company's territory.

In an Appendix extracts are given from the general specifications for the equipment employed in the systems.

* * * * *

Early Carrier-Current and Supervisory Control Developments

No article on the subject of carrier-current and supervisory control on the Alabama Power Company's system would be complete without a brief résumé of early developments along these lines prior to the inception of the major program, which may be said to date from about 1925.

Prior to that time, the company had built up a communication system consisting of about 1400 mi. of telephone line, part of which consisted of separate pole lines on the same rights-of-way with the 110-kv. power lines, most of the remainder being carried on the same poles with 44-kv. and 22-kv. lines. These telephone lines connected various generating plants, primary substations, and local offices, and also had connection with many booths along the rights-of-way. They were also used, of course, for the connection of test sets in cases of line patrol.

Due to the long parallels and comparatively small separation, these telephone lines were usually quite noisy, noise levels often being as high as 500 to 600 noise units. These conditions obtained in spite of the application of standard methods employed by the communication companies to eliminate such conditions. To cope with these troubles, considerable experimenting had been done with the use of high-power telephone equipment, such as "Wonderphones," but such equipment only partially overcame the difficulties.

The communication system of the power company was used for load dispatching, commercial service, ordinary operation and maintenance activities, emergency patrols, etc. The company early had realized that its power service could be no better than its communication system. Consequently, as the system grew and traffic increased, the power company had been very much concerned with the possibilities of expanding the communication system to keep pace with the demands upon it. Recourse could not be had to such expedients as phantoming, etc., used by the signal companies, so that it had been necessary to double circuit many lines. This only partially met the requirements, in that the telephone circuits could usually be counted on to go in trouble with the parallel power circuits, so that they were out of commission just when they were needed most. Furthermore, early difficulties were aggravated as the system grew older and depreciated.

The conditions outlined and a survey of the future brought about a very early and intense interest in other means of extending and supplementing the communication system.

The first development that seemed partially to fulfil this requirement was space radio, so that as soon as this reached a reasonable degree of development, the power company was ready to try it, and in 1922 installed Station WSY at Birmingham, with 250 watts capacity. This was employed largely as an advertising proposition, but the real consideration back of it was for use in emergency load dispatching. Consequently, certain of the generating plants, important primary substations, and district offices, were equipped with receiving sets. This station had a very unusual record in the way of long distance transmission, but during the sleet and snow storms of 1922 and 1923, when practically all the communication lines were put out of commission, it was practically worthless for reaching the near-by generating plants, although dispatching orders broadcast at that time were picked up as far away as Central America. Consequently, it was very soon perceived that space radio was not the answer, and Station WSY was discontinued in 1925.

When carrier-current equipment reached a reasonable degree of development, the power company became interested in this, and in 1924 installed two 250-watt single-frequency simplex sets at stations about 60 mi. apart. It was soon found that this equipment was not sufficiently developed, and that the simplex operation was very unsatisfactory. Furthermore, many troubles in operation were encountered, so that these sets were discontinued in 1925.

The experience with the first two sets when they were working properly, however, gave some insight as to the possibilities of satisfactory carrier-current equipment,

1. Both of the Alabama Power Company.

Presented at the Regional Meeting of District No. 4, of the A. I. E. E., Atlanta, Ga., Oct. 29-31, 1928.

28-143

and the power company therefore participated in a further trial installation in 1925. Two duplex, double-frequency, low-power sets were installed at substations about 65 mi. apart. This installation was extremely successful, although it had its shortcomings and convinced the power company engineers that carrier current was due for very serious consideration in the future expansion of the communication system.

The foregoing review of the communication situation up to 1925 will serve as an introduction for the later discussions of carrier-current equipment on the Alabama Power Company's system; a similar review of supervisory-control developments is in order. Early the power company had realized the need for some form of supervisory control. It was realized that service could be greatly improved and operating expenses reduced if a satisfactory system could be developed. Therefore, some time prior to 1925, experiments were started with a supervisory control system operating over a telephone circuit and employing d-c. impulses as a means of control. It was hoped that such a scheme could be worked out to provide quick sectionalization and location of trouble on long transmission lines; also, that this scheme could be amplified and made use of in dispensing with attendants at various substations by putting them under the control of some central dispatcher.

The first installation to be made was on the Gadsden-Huntsville line, a sending station being installed at the Gadsden Steam Plant to operate a very inaccessible and highly important sectionalizing switch about 50 mi. from Gadsden. This first application was not very successful, due to lack of reliability of the telephone circuit, induction in the telephone circuit from the parallel 110-kv. line, lack of suitable sensitive relays, and, generally, to incomplete development of the idea. Later, the equipment was entirely revamped, and the sending station located at Huntsville, but again the attempt was not very successful. However, just as in the case of the carrier current equipment, this early installation served to visualize the enormously useful possibilities of such a control system.

1925 Expansion Program

Early in 1925 the Power Company was confronted with a very extensive program of expansion, which involved some major items of communication and supervisory control. For the purposes of this paper three of these will be mentioned and discussed at some length. In the first place, in North Alabama there was an acute need for additional communication between Magella substation (Birmingham), Huntsville, Sheffield, and Gorgas Steam Plant. These points were connected by a loop of 110-kv. lines, but only part of this loop had telephone facilities, and these were becoming very inadequate.

In Southwest Alabama two very long lines were projected, radiating from Lock 18 (Jordan Dam). One of these was to extend through Selma, Demopolis, Meridian, and Laurel, to Hattiesburg, with two possible branches beyond that point. The over-all length was to be about 325 mi. A second line, the route of which was not so definitely decided, was to extend from Lock 18 to Flomaton, with branches to Pensacola and Mobile. The total length of this line was to be in excess of 300 mi. These were to be highly reliable, single-circuit, 110-kv. lines, and were to traverse very lean, but potentially good, territory where existing loads were comparatively small and widely separated. Because of the distances involved and necessity for keeping the cost low, it was planned to use carrier current for primary communication, backed up by a secondary system of communication making use of two insulated overhead ground wires which formed a part of the typical construction proposed for these lines. In that these lines had to be highly reliable and provide great continuity of service, it was proposed to install supervisory-controlled, motor-operated, air break switches for sectionalizing at intervals of approximately 15 mi.

The first parts of this program to come up for active consideration were the North Alabama Loop and a portion of the Mississippi line extending from Lock 18 to Demopolis. Appendix A is an extract from the first set of general specifications covering communication and supervisory-control requirements for these and other projects which came up at the same time. Reference to the appendix will give a clear general conception of the typical carrier current and supervisory-control installations contemplated. These same general ideas have been kept in mind in developing carrier current and supervisory control on the Alabama Power Company's system since the first permanent installations. A study of the Alabama Power distribution system, as shown on the map, (Fig. 1), will afford a clear idea of the present extent of carrier-current and supervisory-control installations on the power company's system.

North Alabama Carrier-Current Project

Due to operating requirements, as well as physical location of properties and infrequent necessity of communication between north and south Alabama, north Alabama was considered as a separate project. Four stations were included in original plans and provision was made for installations at two additional locations when traffic demands warranted. The problem of interference had been given consideration and different types of apparatus for different channels appeared to be a solution. However, unforeseen troubles developed, which will be mentioned in detail later.

Stations under consideration were located roughly at the points of a quadrilateral as indicated in Fig. 1 (Index 1). Maximum distances involved were ap-

proximately 150 mi. with parallel circuits between all locations. Communication was required from chief dispatching point to all other stations for operation, and in addition, inter-office communication for commercial purposes was essential and necessitated wire line varying in length from 1½ to 4 mi. Carrier equipment was to be coupled to 110-kv. lines, forming a loop, constituted of a single circuit in certain sections and parallel circuits in others, and including a 44-kv. parallel for approximately 65 mi. (Magella to Anniston) on one branch, and approximately 50 mi. (Magella to Jasper) on the other branch of the loop circuit. Both 110-kv. and 44-kv. lines extended beyond the loop circuit into other territory. While all lines were normally tied in parallel, operating procedure demanded frequent changes, and any section of the line might be cut out at any time to meet a specific maintenance or operating problem.

Apparatus decided upon and purchased was of the double-frequency duplex type, having a nominal low power rating of one watt intended for regular operation, and high-power rating of 50 watts for use under abnormal line conditions or severe atmospheric disturbances. Antenna coupling was decided upon due to trouble previously experienced with early types of coupling condensers, and due to cheaper construction possible with antennas without sacrifice of efficiency, (as indicated from test installations). Space available at two locations permitted installation of equipment in substation operating rooms, at one other location construction of a separate carrier current house was necessary, while at the fourth location, a steam plant, equipment was installed on the top floor of the building and necessary control and telephone leads carried to the operating room—a distance of approximately 500 ft. As would be expected, costs of the installations varied widely. However, installed costs per set, exclusive of overhead, averaged $9394.00 per station. This total was split approximately as follows:

Antenna installation—labor and materials	$1200.00
Carrier-current apparatus	5570.00
Miscellaneous material and carrier-current apparatus installation costs	2624.00

Operating Results. The first two stations installed were at Magella and Sheffield, approximately 120 mi. apart, and operated satisfactorily on low power from station to station under normal line conditions. High-power range appeared adequate to meet routine switching conditions and operating set-ups. When an intermediate set was installed (Gorgas), trouble began to appear on terminal equipments due to difficulty in making adjustments to accomodate high-energy levels from near-by sets to avoid howling, and yet be sufficiently sensitive to hear distant station. Installation of the fourth set further complicated the operating problem by increasing maximum distance to 155 mi. and requiring more sensitive adjustments on terminal equipments. Received energy levels varied widely with switching on 110-kv. lines, and were so low as to make adjustments of relays extremely difficult over the range of line conditions encountered. It was also found that switching on parallel 44-kv. lines affected received energy levels sufficiently to cause ringing failures. At about this time a short tap line to Wilson Dam was constructed, which constituted a low impedance shunt on the low frequency (70 kilocycles) and necessitated the continuous use of high power for routine operation with approximately the same over-all results as previously obtained on low power. At a later date, a second tap line was constructed to Decatur substation, which effectively shunted the high frequency (104 kilocycles). Since equipment was already being operated on high power, and output could not be increased due to inherent limitations of equipment, trouble was practically overcome by a shift in frequency to 120 kilocycles.

When equipment was first installed, lines normally operated with loop circuit closed. Later it was found necessary to split the circuit at Sheffield. This change produced a marked reduction in received energy between terminal stations and required relay adjustments too close for reliability. It was found impossible to make relay adjustments sufficiently stable to pick up and hold during conversation, and yet release when conversation was finished, under varying line conditions encountered in daily operation. Adjustments could be made for a specific line condition, but any change resulted either in incompleted rings or in relays freezing-in and effectively putting equipment out of commission. The margin between normal space current of receiver tubes and received signal was so small that any relay adjustment sensitive enough to pick up signal was so close that a slight change in battery voltage and resultant change in plate current caused relays to pick up and freeze, thus making the equipment inoperative.

Interference. A few months after equipment on the North Alabama loop had been put into service, more powerful equipment was installed in Southwest Alabama with a set at the chief dispatching point, Magella. Immediately troubles due to interference were encountered. Even though frequency separation of equipments was more than 20 kilocycles, it was found that when one of the more powerful sets was in use all equipment on the North Alabama loop was held inoperative. In commercial equipment, elimination of interference between transmitter and receiver of a carrier-current installation is necessary for acceptable results. This feature on the equipment under consideration was obtained by means of cut-off high- and low-pass filters for separating transmitted and received energy. As a means of eliminating inter-channel interference selection of an operating frequency on interfering equipment between cut-off limits of high- and low-pass filters was considered and tried out. It was found, however, that capacity of filters was not

sufficient to block interfering energy and trouble was not materially lessened.

Prior to this time the problems of lack of volume, failures to signal, and howling on office extensions, had been taken up with manufacturers and remedial measures tried as suggested, without much success. Results indicated that equipment did not have enough power to work reliably over distances involved, taking into account the rather complicated network and tap lines. The new problem of interference was brought to the manufacturers' attention and a formal complaint made on performance of equipment. The manufacturer made a complete field study of the problem under normal operating and varying line conditions. Fig. 3 shows curves of received energy under two specific line conditions. In Curve No. 1 parallel circuits were available between stations under test, Curve No. 2 having been taken between the same stations over a single circuit. Minimum energy level for acceptable operation is indicated by dot and dash line. It will be noted that the energy levels were materially less over a single circuit, and that while curves have the same general form, there has been some shift in peaks with the frequency. A direct comparison of quality of talk cannot be made from curves, however, since the two branches of the parallel circuits were of different length and the maximum received energy includes an out-of-phase component, which introduces some distortion. From the signaling point of view alone, received energy is a true index of reliability of ringing and relay adjustment is much simplified with increased energy levels.

The final results of tests conclusively demonstrated that equipment was not suitable to meet operating conditions imposed by length of lines and complicated network, combined with switching met in routine operation. A satisfactory adjustment was made by the manufacturer, and equipment was maintained in service for the time being, on a station to station basis, largely as a standby.

At about the same time, one of the branch circuits was changed over to 154 kv. by means of auto-transformer step-ups effectively open-circuiting the line from a carrier-current standpoint. It has been found possible, however, to use a land line from Magella to Gorgas as a wire line extension, tying into carrier current equipment at the latter location, and carry on a fairly satisfactory conversation between Magella and Sheffield in case of emergency.

Experience with this equipment has shown the necessity of using more power when tying into a complicated power network as compared to commercial practise over straight-away lines. Means must be provided for using as narrow a frequency band as possible, not over 5 to 8 kilocycles maximum, because of necessity of multiple channels, and this band should tune rather sharply to minimize interference.

Southwest Alabama Carrier-Current Project—(Mississippi Line)

After general specifications for the Southwest Alabama project, as covered in detail under Appendix "A," had been worked out, the final set-up was somewhat changed to conform to later developments. The Mississippi line channel, as finally decided upon, contemplated communication over a maximum distance of approximately 325 mi., as indicated in Fig. 1 (Index

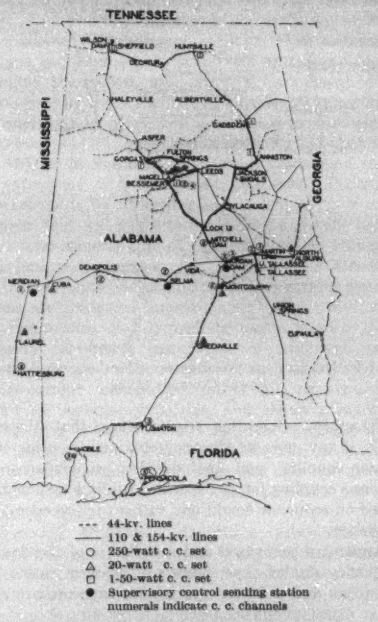

Fig. 1—Electric Distribution Lines in Alabama and Portion of Florida and Mississippi

2). Installation of ten sets was decided upon, seven of which were required to talk to all other sets of equal capacity, and three were of lower capacity, from which communication was required only to adjacent large sets, a maximum distance of approximately 55 mi.

Antenna coupling was decided upon, except for one location (Martin Dam), where physical location of lines and bus structures would have made necessary a double antenna and expensive construction due to an extremely long span over a river. At this location, coupling condensers were installed and connected to substation 110-kv. buses by a remote electrically controlled gang-operated change-over switch. Normally, both station buses are operated in parallel, but the change-over

feature of connecting condensers permits either bus to be isolated when necessary for maintenance or repairs without interfering with carrier-current operation. Condenser connections and change-over switching facilities are shown in Fig. 4. Condensers are of the mica dielectric type, and to date have proved entirely

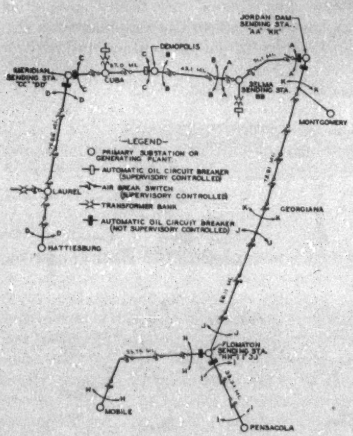

FIG. 2—DIAGRAM OF SUPERVISORY CONTROL ON ALABAMA POWER COMPANY LINES

satisfactory. Operating results show that there is little, if any, gain in efficiency obtained by using condenser coupling, and final decision on condenser or antenna coupling for any specific installation should be based on economic conditions, rather than on operating efficiency.

Apparatus purchased and installed is of the double frequency duplex type, having a 250-watt rating for large sets and a 20-watt rating for intermediate sets

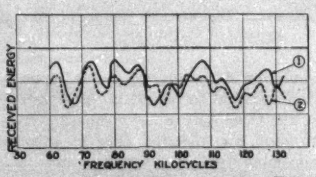

FIG. 3—RECEIVED ENERGY CURVES BETWEEN TWO CARRIER INSTALLATIONS UNDER CHANGED POWER LINE CONDITIONS

from which medium distance communication is desired. Transmitters of 250-watt equipments operate from 110-volt, 60-cycle, a-c. station service supply, and motor-generator sets operating off the station storage batteries provide a-c. supply in case of failure of a-c. station service. As originally installed, transmitter change-over switches were manually operated. Automatic change-over switches for transferring transmitters from a-c. station service to motor-generator sets were later installed to provide for full automatic operation. Twenty-watt sets operate direct from station batteries.

Distances between sets average, roughly, 50 mi., and equipment is in all cases installed in substations, several of which are non-attended. Communication demands, as on the North Alabama loop, call for dispatching and commercial service and require wire line extensions at six locations, varying in length from 2½ to 5 miles. Standard telephone spacing is employed on wire line extensions, however, since all line extensions are exposed to interference from power circuits on the

FIG. 4—CARRIER-CURRENT COUPLING CONDENSERS, WITH FUSES AND REMOTE ELECTRICAL CONTROLLED GANG OPERATED CHANGE-OVER SWITCHES AT MARTIN DAM

same pole lines, special care was exercised to obtain accurate balance and hold noise levels to as low a value as possible.

In all cases, except at Magella, full automatic control is desired from office extensions, and direct signaling is required. At Magella a telephone operator is on duty during office hours and switching to office extensions is done manually. A very special problem with respect to line extensions exists at this location. Four land line trunks connect Magella dispatching station with Birmingham office building, these lines being used for connecting any stations of a private automatic exchange to land lines radiating over the older portions of Alabama Power Company territory within approxi-

mately a 100- to 150-mi. radius. It was desired to use any one of these office trunk lines as a carrier current extension for any telephone connected to the private automatic exchange in the Birmingham office building. Since two-wire extensions are used at other installations, a very accurate hybrid coil balance is necessary to prevent feed-backs and howling. This problem of balance was satisfactorily solved by manufacturer's engineers and any private automatic exchange telephone in the Birmingham office building can be connected to carrier current equipment. Outgoing calls are placed with Magella operator in the same manner as a long distance call is given to a private branch exchange operator. Incoming calls are answered by the Magella

Fig. 5—250-Watt Carrier-Current Transmitter and Receiver with Two-Wire Tuning Unit at Martin Dam

operator, who, in turn, dials office telephone desired, using any idle trunk line.

Carrier-Current Installations. A typical carrier-current installation on the Mississippi line using antenna coupling, is shown in Fig. 5. A separate room is provided for carrier current equipment, which is also used for supervisory control sending equipment at several locations. Antenna drainage and protective equipment is mounted on a special structure. The same protective equipment is used at the condenser-coupled installation, mounting details of which are shown in Fig. 4.

Interphase coupling is used at all locations for transmitter, while at some locations the receiver is coupled phase-to-ground, using a third wire for receiving antenna. At other locations, phase-to-phase coupling is used to obtain combined transmission and reception on a phase-to-phase basis, using a special tuning unit as shown above transmitted in Fig. 5. Selection of receiving method was made in all cases by trial to obtain best results or to overcome some special problem.

During installation a number of unusual problems was encountered, a detailed discussion of which is beyond the scope of this paper. Among these may be mentioned balance of exposed wire line extensions, provision of a system busy and lockout signal except at dispatching points, adjustments to provide for transmission from end to end of system and at the same time communicate with adjacent sections. The solutions of these problems were worked out as the problems presented themselves, and operating equipment was turned over by the manufacturer's engineers in a working condition and taking care of all special requirements.

Approximate average installed cost of a 250-watt set was $10,500.00, exclusive of general overhead and wire line extensions; of a 20-watt set, $6,050.00. Per mile costs are apt to be misleading and no definite figure can be given that will apply to other than a specific location. The same is true of cost comparisons between land line and carrier-current installations. On the specific line under consideration, carrier-current installations are separated approximately 50 mi., and size and importance of intermediate towns is such that necessity for future installation of additional sets appears remote. Under conditions such as outlined, a carrier-current installation meets all requirements except for emergency patrols, and shows a marked saving over a land line. Complete operation and maintenance costs are not available, but incomplete figures indicate that the average cost will not exceed $500.00 per station per year, and probably will be materially less.

Operating Experience. Operating experience has shown the necessity of routine inspection and maintenance by a trained personnel at regular intervals. A special daily operating report form has been adopted which gives a record of performance and indicates units such as tubes or batteries that are nearing the end of their useful life. These report forms are forwarded to carrier-current maintenance men, and failing units are checked and replaced, if necessary, on the next regular routine inspection. Field forces keep the batteries filled, replace burned out tubes, and keep the equipment clean, but have definite instructions not to change any adjustments or make any modifications or additions either to carrier-current equipment or line extensions. Operating results have, in general, been quite satisfactory, and an over-all efficiency of 93.7 per cent has been obtained on 250-watt sets. Complete records are not available on intermediate 20-watt sets, but operating results have not been as good, probably due in large measure to a less completely worked out design.

Equipment, with exception of line extensions, has been free from trouble due to lightning or surges. Line

extension troubles due to above causes have been confined in all cases to circuits external to carrier-current equipment, and have consisted of blown fuses and destroyed vacuum bulb arresters. In several instances of direct contact between line extensions and power conductors (11-kv.), line protective equipment successfully cleared the trouble without damage to carrier current sets or extension telephones. However, line surges and lightning frequently cause incorrect or incomplete rings. During lightning storms, it is frequently necessary to dial a given station several times before a correct ring is obtained.

When the equipment was first put into service, considerable trouble was experienced due to burn-outs of 50-watt tubes used in speech amplifier and master oscillator circuits. These tubes were replaced by manufacturer with a later design that has given excellent results. To date, not a single tube has failed in over nine months' service. Life of all other tubes, with the exception of rectifier tubes on battery charging equipment, has been satisfactory. Rectifier tubes of the vacuum bulb type used for floating control (18-volt) and receiver filament (6-volt) batteries, have given life comparable to this type of equipment in continuous service, but not up to standard of life obtained from vacuum tubes in carrier-current equipment proper. Receiver plate supply is obtained from a rectifier across which is floated a dry type "B" battery to provide for continuous operation in case station service a-c. supply is interrupted. It was found that the regulation of the rectifier was such that voltage did not remain constant, but built up to approximately 50 per cent above the nominal "B" battery rating before a stable condition obtained. The consequent change in plate current caused failures in relay operation until it was found that this abnormal voltage was not a transient condition, but maintained for hours after a-c. supply was interrupted. Relay and bias battery adjustments were changed to conform to maximum stable plate supply voltage, and troubles from this source were eliminated. Considerable trouble was experienced on wire line extensions, due to inductive pick-up. Noise was not only transmitted, but destroyed the quality of outgoing talk. It was found necessary to re-transpose several line extensions, and in all cases keep lines absolutely clear from grounds to obtain satisfactory results. Other equipment troubles have been of a rather minor nature, such as sticking relays, blown fuses, loose connections, and failures of specific units, such as condensers and transformers.

Open Switches. Any change in a power system set-up is reflected in energy levels received at carrier current equipments and affects communication to some degree. It has been found that communication is possible in practically all cases, as long as there is a complete metallic circuit between sending and receiving stations, provided adjustments in equipment have been made to accommodate the worst, rather than normal, operating conditions. Communication across open switches on single circuit power lines cannot be made dependable without auxiliary by-pass equipment and a reduction in over-all efficiency. On several occasions, successful communication across an open breaker was obtained by retuning carrier current equipment. This, however, necessitated rather critical adjustments by a trained personnel, a condition that could not be obtained in normal operating procedure. Results obtained, therefore, cannot be considered commercial.

On a specific section of single circuit line (Meridian-Hattiesburg) on H-frame structures and having insulated overhead ground wires, (see Fig. 6, for construction), it has been found possible to signal and talk with a fair degree of reliability, without retuning, across as many as four open switches. Power conductors are

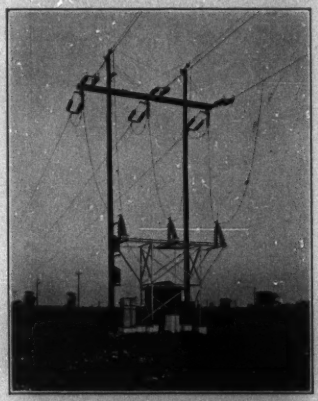

Fig. 6—Supervisory Controlled 110-Kv. Sectionalizing Switch between Selma and Demopolis

397,000-cir. mil aluminum. Insulated overhead ground wires in this section of line are composite steel-aluminum cables, having three strands of aluminum and four of steel (with a d-c. resistance of 3.4 ohms per mile), and are transposed at approximately 4-mi. intervals. These results are apparently due to direct coupling, rather than to distributed capacity between power conductors and insulated overhead wires acting as a by-pass condenser, since in all cases overhead wires are transposed at open switches referred to and at three intermediate points. From a carrier current standpoint, this is essentially a double circuit line, and results obtained do not modify preceding statement relative to communication across open switches on single circuit lines.

Prior to the use of carrier current over power con-

ductors, communication facilities of power companies were entirely separate and additions to or modifications of power lines were made without affecting the communication system. Installations of carrier-current equipment have entirely changed this condition, and complete information on changes in operating procedure, as well as line modifications or additions, must be transmitted to the communication engineers if maximum results are to be obtained. A short tap line may constitute a low impedance shunt, or a transformer bank installation may effectively open circuit a line, at carrier-current operating frequencies, and remedial measures must be taken if carrier current equipment is to remain operative. As a typical example, it is proposed to operate the Alabama Power system in the near future, split at Jordan Dam. This proposed operating change will necessitate a carrier current by-pass at Jordan Dam to tie the Mississippi channel through. Necessary materials are on order, so that when this change becomes effective communication will not be interrupted.

Southwest Alabama Carrier-Current Project—(Mobile Line)

Stations on the Mobile channel, as indicated in Fig. 1 (index 3) are, with one exception, fed by a single circuit 110-kv. line. Typical construction of this 110-kv. circuit is the same as used on the Mississippi line, except that insulated overhead wires on the entire line are composite steel-aluminum cables. This insulated overhead ground wire circuit is continuous, and the desirability of using it as a carrier current channel to avoid interruptions to communication from open sectionalizing switches, is readily evident. To determine the feasibility of using this circuit, a 60-mi. section of line was isolated and measurements of attenuation and line characteristics made. Results obtained[2] show this circuit to be commercially practicable, and decision was made to couple carrier current equipment direct to insulated overhead ground wires. Communication requirements are the same as on the North Alabama loop and Mississippi line channels, and include office line extensions at two locations, approximately 4½ and 12 mi. long respectively. Coupling is direct to the insulated overhead wires by means of 0.001 microfarad condensers having a 15,000-volt rating, and protective equipment is essentially the same as that used on the supervisory control system at receiving stations. All installations are now complete, but equipment has not yet been put into service. A typical installation of transmitter and receiver is shown in Fig. 5. Costs are not yet available, but it is believed that costs will be somewhat less than on the Mississippi line, since antenna construction was not necessary.

2. Telephone Communication over Power Lines by High-Frequency Currents, by C. A. Boddie.

Southwest Alabama Supervisory Control Project

The supervisory control installation as finally worked out for the Jordan Dam-Hattiesburg line was somewhat different from that outlined in specifications of Appendix A. In the first place, Jordan Dam was developed sooner than expected, so that the first control station was located there instead of at Mitchell Dam. Secondly, the line was terminated at Hattiesburg, instead of extending beyond that point. Otherwise, however, the final installation followed quite closely the original specifications. In discussing supervisory control on the Alabama Power system, this line was taken as the best example, since it is typical, and is the oldest and most complete of the supervisory control installations.

The Mississippi line is a 110-kv., single-circuit line. The supporting structures are creosoted pine H-frames, carrying, for the most part, 397,500 cir. mil steel reinforced, aluminum conductors, suspended by 8 unit suspension insulators. The power conductors are spaced 14 feet apart horizontally, and, about 9 feet above and between them, there are two ½-in. Siemans-Martin steel ground wires, carried on the projecting tops of the poles. These are insulated by 20-kv. pin type insulators or suitable strain insulators. The insulated overhead ground wires are transposed at intervals of approximately 4 mi. At the various substations they are carried in to the supervisory control and emergency telephone equipment through appropriate protective devices. At 2-mi. intervals drops extend down the poles, on 20-kv. insulators, from the insulated overhead ground wires and are terminated at suitable protective equipment so that patrolmen may use the insulated overhead ground wires as a communication circuit in emergencies. At approximately 15-mi. intervals, and at the substations, there are motor operated, supervisory controlled, air break sectionalizing switches. Also, at some of the substations,—Selma, Demopolis, and Cuba,—there are certain supervisory controlled oil circuit breakers.

Fig. 2 shows the various supervised switches on the Mississippi line. A short discussion will demonstrate the typical functions of these switches. Assume a case of line trouble just north of Laurel. The oil circuit breaker in the Laurel line at Meridian opens automatically, all others remaining closed as they are controlled by properly coordinated relay installations. This breaker would reclose twice automatically, and then lock out. The Meridian operator would then open the first sectionalizing switch south of Meridian by supervisory control, and test the line that far. Obviously, it would test good. He would then close the first switch and open the second switch by supervisory control, and get a second good test. However, when he closed the second switch and opened the third, (at Laurel), and tested, the test would be bad, and he would know the trouble was in the first 15 mi.

north of Laurel. All this would be done in a few minutes, and the maintenance crews would be promptly on their way to the trouble, with only 15 mi. of line to patrol instead of about 50 mi., as would be true without the supervisory controlled sectionalizing switches. Similar analyses will demonstrate the purposes of the other supervised and non-supervised switches and oil circuit breakers.

Supervisory control is usually considered to be extra remote electrical control, generally employing the ordinary dial and selector mechanisms of the automatic telephone, with some sort of electrical control circuit connecting the sending (dial) and receiving (selector) stations. There is also usually some form of answer-back signal to indicate the functioning of the controlled equipment. The control circuit may be a simple two-wire telephone circuit employing a-c. impulses; it may be a power circuit carrying superimposed carrier impulses; it may be space radio; or it may be some other form of circuit carrying some form of impulses.

The Alabama Power Company's specifications, Appendix A, brought forth two general proposals. The first was to use the power conductors to transmit carrier-current impulses to operate the supervisory equipment. This involved the use of carrier-current sets and antennas separate from those provided for communication. The second proposed the use of the insulated overhead ground wires to carry alternating-current impulses of medium frequency (450 to 600 cycles) which were to operate the supervisory controlled equipment and were to be obtained from small motor-generator sets operated by the station control batteries. These impulses were to be put on and taken off the insulated overhead ground wires at substations and sectionalizing switches by means of special insulating and drainage transformers. The middle points of the high-tension windings were to be grounded, thus providing ground connections for the insulated overhead ground wires. These transformers were to be so designed and protected as to provide low impedance paths to ground for high-frequencies from either ground wire, to insert comparatively high impedance between ground wires for frequencies of 600 cycles and less; they were to provide ample insulation between the ground wires and the supervisory control and emergency telephone equipment, and, finally, they were not to interfere with the use of the insulated overhead ground wires for audio communication. As to functioning, it was proposed that the operator at any control station might ascertain the position of any supervised switch or breaker in his control section. If open, he might close it, and if closed, he might open it. In the two latter cases he was to get an indication of correct and complete operation. Various safeguards were provided, the most interesting being the use of 450 cycles to set up the relays ready for operation, and 600 cycles actually to perform any operation. Actual operation of the switches as outlined in Appendix A was to be by means of motor mechanisms driven by the local 12-volt or 110-volt storage batteries.

The second proposal was adopted as being the most feasible, least expensive, least complicated, most reliable, and as involving the fewest untried items of equipment. Although many difficulties were encountered in designing, installing, and putting into operation this equipment, this choice was later proved to be amply justified.

A more complete description cannot be attempted within the scope of this paper. Some data follow which will be of interest and value in forming an opinion of this type of supervisory control equipment; and in passing, it might be said that every objective set forth in the original specifications had been attained,—even to the charging of the 12-volt control batteries by means of drainage from the insulated overhead ground wires.

The cost of the entire installation on the Mississippi line, exclusive of general overhead, and not including insulation of the overhead ground wires, amounted to about $38,000. The cost of the original supervisory equipment installed for a typical sectionalizing switch was $2100, and for a typical single line control station $2225. Equipment costs have since increased, and present figures would approximate $2700 and $3000 respectively.

All equipment is inspected and tested on a monthly schedule, and so far the operation and maintenance cost have been comparatively low, being practically confined to the time and expenses of the inspector.

The insulation of the overhead ground wires added about $50 per mile to the cost of the power line, exclusive of general overhead, but all of this is not chargeable to supervisory control, as it is also essential to the emergency communication system.

For the period from July 1927 to July 1928, all supervised equipment in the 90-mi. section from Hattiesburg to Meridian was given a monthly operating test with the power line energized, up to and including the motor mechanisms. In this period there was only one failure to function properly, and this was due to improper adjustments at one sectionalizing switch following installation of a new type insulating and drainage transformer which was being tried.

The Alabama section was not in full operation until 1928, but every part of it is now functioning and has been successfully operated on test.

The inspection and testing of all carrier current and supervisory control equipment for the Jordan Dam-Hattiesburg line takes less than half the time of two men.

Insulation of the overhead ground wires and their use for supervisory control and communication does not appear to have materially lessened their protective value, as line troubles have been very few and not of such nature as to be attributable to decreased protective value of the overhead ground wires. Momentary

interruptions, in spite of very bad lightning conditions, have been comparatively few, varying from a minimum of 0 to a maximum of 6 per month for the entire line.

The operation and maintenance of the supervisory control equipment is not extremely difficult, nor beyond the capacity of a first-class operator or maintenance man.

Conclusions

In view of experience on the Alabama Power Company system, the following conclusions as to carrier current communication may be drawn:

It is thoroughly practicable for power lines, but operates better on radial lines than networks. Short taps on lines used as carrier-current channels must be avoided or isolated.

For long distance, it is more reliable and economical than telephone lines.

It has not been satisfactorily developed to take care of patrol work, nor are the intermediate capacity sets thoroughly satisfactory, but this end seems probable of attainment.

Careful study of power circuit characteristics prior to installation of carrier current is essential.

It cannot be expected to operate consistently through transformers or open switches.

Antenna and condenser coupling are equally satisfactory, the choice depending on cost and physical conditions.

Careful, frequent, and competent inspection and reporting are essential to satisfactory operation.

Before embarking on a carrier current program, very careful consideration must be given to frequencies, etc., to provide for a sufficient number of carrier current channels.

There are certain improvements and modifications in equipment that should be made, notably—redesign for cheaper and easier installation; greater accessibility for inspections and repairs; greater factors of safety and wider operating margins; improved battery charging equipment and elimination of "B" and bias batteries if possible.

As to supervisory control on the Alabama Power Company's system, the following conclusions may be drawn:

It is of considerable value in decreasing operating costs and improving service by reducing length of interruptions.

Medium frequency a-c. impulses transmitted over insulated overhead ground wires have been proved entirely practical as a control medium.

Regular, frequent, and competent inspection and reporting are essential to satisfactory operation.

Except for simple forms of supervisory control over ordinary pilot circuits, there is still a great deal of room for development and improvement.

Appendix A—Extract from Specifications—Communication and Control—Alabama Power Company System

Part 1—North Alabama Project

General. This project contemplates provision for through communication facilities between Magella, Gorgas, Sheffield, and Huntsville. Carrier-current sets called for in these specifications will operate interphase over power conductors, using antenna coupling.

Circuits to be utilized in the above consist in general of H-frame structures, carrying three 110-kv. conductors, in a horizontal plane, these conductors being 250,000 cir. mils or 300,000-cir. mils copper, and spaced 14 ft. center to center. Between conductors and approximately 9 ft. above them, are uninsulated ½-in. galvanized steel ground wires. Antennas will be supported above or between conductors, and not over 10 ft. from same. Antennas will be 1500 to 2000 ft. in length.

No communication facilities are contemplated at present except for the main sets specified below, no provision being made for portable set communication. Operation through open disconnecting switches is not contemplated.

Gorgas Steam Plant. Equipment to be installed at this point will consist of one duplex, single or double frequency, selective ringing, carrier current set. This equipment will operate from station 110-volt a-c. service or in an emergency from a 125-volt station storage battery of ample capacity. Equipment will be installed adjacent to operator, and there will be no long connection between operator's phone and set. Equipment is to be complete with sending and receiving apparatus, operator's phone, ringing equipment, any recommended protective devices, motor-generator sets, and any other necessary accessories. No batteries or cable will be included.

Sheffield. Equipment to be installed at this point will, in general, be the same as that specified for Gorgas Steam Plant, except that set will be installed at a distance of approximately 500 ft. from the operator, and phone at set should be included.

Huntsville. Equipment to be installed at this point will be the same as that specified for Gorgas Steam Plant.

Magella. Equipment to be installed at this point will be the same as that specified for Gorgas Steam Plant.

Operation is not contemplated through open disconnecting switches, but equipment shall be satisfactory for emergency operation with at least one conductor grounded.

Part 2—Montgomery–Greenville Project

General. These specifications are intended to cover all communication equipment necessary for satisfactory operation of the Montgomery-Greenville 44-kv. line.

All equipment will be antenna coupled to the 44-kv. conductors. Intermediate capacity sets will be installed at Montgomery and Greenville, which must be satisfactory for through communication between these points. Portable sets will be used at other locations on the line, which must be suitable for communication with the nearest intermediate set. This line consists of single wood pole structures, carrying three No. 2 copper conductors, two of these being in the same horizontal plane, approximately 5 ft. apart, and the third one being between and approximately 4 ft. above them. Antennas will be of ⅜-in. galvanized steel, spaced below and outside the two lower conductors, and approximately 3 ft. from them. These antennas will be approximately 1500 feet long.

Montgomery. Equipment to be installed at Montgomery Gas Plant Substation will consist of one intermediate capacity duplex, single or double frequency, selective ringing, carrier-current set, capable of communication with Greenville under all conditions. Operation past open disconnects is not contemplated, but set shall be capable of satisfactory operation in emergency with one conductor grounded. This set will be installed approximately 4000 ft. from the operator, and control will be effected over a 4-conductor line consisting of two No. 14 twisted pairs (or 2-conductor, at option of manufacturer). Set will operate from a 24-volt storage battery of ample capacity, which is already installed. Coupling to 44-kv. lines at this point will be by means of condensers already provided by the Power Company. Proposal must include sending and receiving equipment, suitable for reception of code calls from Greenville, or vocal or code calls from portable sets. Proposal must also include ringing and telephone equipment for installation at the remote operator's station, and telephone equipment for installation adjacent to the set. There must also be included all motor-generator sets, protective equipment, and other accessories which may be deemed necessary. This set shall be suitable for manual connection at the operator's station to the existing 2-wire company telephone circuit extending to our Montgomery office.

Greenville. Equipment for this location shall be the same as that described for Montgomery, except that extension to Greenville office, from which point the set will be controlled, will be only 3000 ft. long. This set will be coupled to the line by means of a 3000-ft. antenna extending 1500 ft. on each side of the line disconnecting switch installed at Greenville.

Line Equipment. In addition to the foregoing locations, there will be 12 points along the line where antennas will be installed for use of portable sets. For these locations, there will be provided 7 portable carrier-current sets, capable of communication between portables for limited ranges, or between portables and the nearest intermediate set from any location on the line. Portable sets shall include transmitting and receiving equipment, and shall be simplex, single- or double-frequency, audible (or selective code) calling equipment. These sets must not exceed 15 lb. in weight, and dimensions shall be restricted as much as possible. Sets and cases shall be substantially constructed, must be weatherproof, and must be provided with straps for carrying in the hand or suspended over the shoulder. There shall also be provided suitable drainage and protective equipment for 13 locations, including the 12 antennas for portable sets and the Greenville antenna. This equipment must be compact, as simple as possible, weatherproof, and suitable for mounting on line structures out of doors. It should be noted that 44-kv. insulated platforms will be provided for the use of operators of the portable sets and that drops will be provided extending within 10 feet of the ground from the antenna conductors.

Part 3—Southwest Alabama Project

General. These specifications are intended to cover control and communication for two main circuits radiating from Lock 18, one of which will ultimately extend 305 mi. to Slidell, and the other 309 mi. to Moss Point. These circuits will be supplied from Mitchell Dam, so that the length from the source of supply will be approximately 325 mi. Magella, Mitchell Dam, Meridian, Hattiesburg, Andalusia, Mobile, Pensacola, and Moss Point will be important control centers, and ultimately communication will be necessary between Mitchell Dam and Magella, and Mitchell Dam and any point on either of the two circuits described, this to be accomplished by through communication or by the use of repeaters in the two lines.

In each of the circuits described, it is contemplated that sectionalizing switches under control of central control points will be installed approximately every 15 mi., and it will be necessary to provide control for these sectionalizing switches, and also control for certain oil circuit breakers.

For communication, it is proposed to provide interphase carrier current sets as detailed later, which will be antenna coupled to power conductors. The power lines will consist of wooden H-frame structures, on which the 110-kv. power conductors will be supported in a horizontal plane approximately 14 ft. apart. Between, and approximately 9 ft. above these, there will be two ½-in. galvanized steel ground wires, insulated for 25 kv. These ground wires will be sectionalized and used for control, as hereinafter described. Coupling antennas will be 1500 to 2000 ft. long, and will be supported above and between conductors, the distances from conductors not exceeding 10 ft. in any case. It should be noted that the ground wires on the line between Lock 18 and Mitchell Dam, and between Lock 18 and Union Springs, are not insulated as described above, but are both solidly grounded. On the newer sections of the line, where the ground wires are insulated, they will be transposed at each sectionalizing point, and at three locations between adjacent

sectionalizing points, so that transpositions will be provided at approximately 4 mi. intervals. Drainage equipment will be provided for these ground wires at sectionalizing points, or, in other words, at 15-mi. intervals. These ground wires will be used for emergency communication as described later, and for this purpose, drops for the use of patrolmen will be provided at sectionalizing points and at two mile intervals between sectionalizing points, these drops to be provided with suitable switching and protective facilities.

As previously stated, ordinary communication will be by means of antenna coupled carrier current sets, but it is not contemplated that these facilities will operate through open sectionalizing switches. When a section of line goes in trouble, the sectionalizing switches at control points at each end will be opened, and for the section in trouble communication will be established by means of ordinary telephone equipment over the insulated ground wires. Under these conditions, patrolmen engaged in locating or repairing trouble will communicate with the control stations at either end by means of the drops from the ground wires previously mentioned, and use of ordinary portable telephone sets. Under these conditions, through communication will be relayed vocally at each end of the section in trouble.

Control of sectionalizing and oil switches will be effected over the insulated ground wires previously mentioned, using carrier current or some recommended alternative scheme. The only exception is for the section between Lock 18 and Mitchell Dam, where a standard telephone circuit on separate poles will be provided, this to consist of No. 4 A. C. S. R. This circuit will take the place of the insulated ground wires on the newer sections of line for both emergency communication and control. It is possible that on the Moss Point circuit, recourse will be had to a No. 8 copper clad telephone circuit for similar purposes between Lock 18 and Union Springs, this circuit being in place already.

It should be noted that present facilities at Mitchell Dam and Magella include 7½- 50 watt, duplex, single frequency, selective ringing, General Electric carrier-current sets, operating interphase, which it is proposed to use, if possible, in the ultimate set-up.

Mitchell Dam. At this point it is proposed to use the present carrier current set with or without adaptations, as may be necessary, or if it is not economical to coordinate new equipment with present set, to replace the present set with one of sufficient capacity and suitable characteristics to communicate with Meridian, Andalusia, and Magella without repeaters, or with Slidell and Moss Point by the use of intermediate repeater sets. This set, or any new set which may be necessary, will operate from the 125-volt station storage battery, and will be located approximately 1000 ft. from the operator, the intervening connection being made by means of a multi-conductor lead covered cable. In case it is proposed to install a new set at this point, it must be duplex, single- or double-frequency, selective ringing, and suitable for interphase communication under conditions of operation and installation as set forth above, and must be complete with transmitting and receiving equipment, ringing equipment, telephone and ringing equipment for the operator's station, motor-generator sets, protective equipment, and all other necessary accessories.

The control equipment for Mitchell Dam will operate from the 125-volt station storage battery, and will be installed approximately 1000 ft. from the operator, and adjacent to the carrier current set previously mentioned. This equipment will be under control of the operator, the same as the carrier current set. Equipment shall include transmitting apparatus for control of two oil circuit breakers at Lock 18, one oil circuit breaker at Selma, and five sectionalizing switches. There shall also be included supervisory equipment for giving a visual (or satisfactory audible) return indication of the operation of the two oil circuit breakers at Lock 18, whether under control or automatic, and further, equipment shall be provided for checking the position of these breakers. The indicating equipment will be installed at the operator's station previously referred to. In addition to the transmitting and indicating equipment, proposal shall include all motor-generator sets, selector equipment, protective equipment, and other accessories necessary for the complete and satisfactory functioning of this system of control.

Lock 18. Control equipment to be supplied for this location will include supervisory control receiving and return signal equipment for two oil circuit breakers as previously described, and receiving equipment for one sectionalizing switch. This control will work in parallel with hand control provided at this station. Power supply for this equipment will be from a 24-volt, 120-ampere hour station storage battery, the voltage of which may vary between 18 and 30 volts. Proposal should include receiving and return signal equipment as described, also any protective equipment recommended and any motor-generator sets or other accessories necessary for the complete and satisfactory operation of this equipment, as outlined. The Power Company will furnish necessary storage batteries.

Selma. Carrier-current equipment for this location will consist of one duplex, single- or double-frequency, selective ringing, carrier-current set suitable for interphase operation with antenna coupling, and with sufficient capacity to insure satisfactory and reliable communication with Meridian and Magella. It should be noted, however, that operation through open disconnecting switches is not contemplated, but set should be capable of satisfactory communication, even with one conductor grounded. Power supply for this set will be from a 110-kv., 120-ampere hour station storage

battery, voltage of which may fluctuate between 90 and 130 volts. This set will be installed approximately 3½ mi. from the operator, and the intervening connection will be made by means of a 4-conductor, No. 14 twisted pair circuit (or 2-conductor at option of manufacturer). Proposal must include transmitting and receiving equipment, ringing equipment for the set and for the operator's station, telephone equipment at the set and at the operator's station, protective equipment, motor-generator sets, and any other accessories necessary to complete and satisfactory operation. The Power Company will furnish necessary storage batteries.

Control equipment will be provided at Selma for operation of the sectionalizing switches between Selma and Demopolis. This will include transmitting equipment for two sectionalizing switches as indicated on attached drawings. No provision will be made for checking position of these switches. Proposals shall include transmitting equipment, protective equipment, and any motor-generator sets, selector devices, or other accessories necessary to complete and satisfactory operation. Power supply will be from 110-volt, 120-ampere hour station storage battery, and operator will be located at the control set when it is necessary for same to be operated. Control equipment for this location will also include receiving equipment for operation of one oil circuit breaker and one sectionalizing switch from Mitchell Dam, but no provision shall be made for checking their position. This equipment will operate in parallel with the hand control provided at this station, and shall include all accessories necessary to complete and satisfactory operation.

Demopolis. A duplex, selective ringing, single or double frequency carrier current set for interphase operation, with antenna coupling, shall be provided for this location. This set will operate off of the 24-volt, 120-ampere hour station storage battery. The operator will be located approximately 3½ mi. from the set, and the intervening circuit will consist of a 4-conductor, No. 14 twisted pair circuit (or 2-conductor at option of manufacturer). Proposition shall include transmitting and receiving equipment, selective ringing equipment at set and at operator's station, telephone equipment at set and operator's station, motor-generator sets, and all other accessories necessary to complete and satisfactory operation of set. This set shall have sufficient range to communicate with Mitchell Dam and with Meridian under all conditions. Operation through open disconnecting switches is not contemplated, but set should be able to function satisfactorily, even with one conductor grounded. Power Company will furnish necessary storage batteries.

Line Equipment. For use along the line between Mitchell Dam and Demopolis, control equipment shall be provided for five sectionalizing switches. Power supply at each of these locations will be from 12-volt, 120-ampere hour storage batteries, to be provided by the Power Company, the voltage of which may fluctuate between 10 and 15 volts. These equipments shall include receiving apparatus, and any protective equipment which may be recommended. The sectionalizing switches which these devices will control will be motor operated, and will require approximately 20 seconds to open or close.

Drainage and protective equipment shall be provided for the insulated overhead ground wire previously described, for eight locations, this equipment to be ample for protecting these conductors and the control equipment connected to them, from excessive voltages incidental to line failures of any sort. These equipments, in general, shall include an iron core choke, a spark-gap, and a quick acting relay inserted in the connection from the middle of the choke to ground, which will act to provide a direct path to ground for both conductors in case of an excessive discharge through the choke to ground. These devices shall also be such that they will not interfere with the use of these conductors for telephone communication in emergencies when the adjacent power circuit is "dead." This equipment shall be compact, weatherproof, and suitable for mounting on standard line structures.

Equipment shall be provided for charging 12-volt batteries at five locations as previously referred to, utilizing drainage from the insulated overhead ground wire or from separate antennas strung parallel to the conductors. This equipment should be suitable for a continuous (or trickle) charge, and should include any necessary safety or protective equipment.

All of the above drainage and protective equipment should be specified with the idea of taking care of any ordinary line faults, and any conditions which are apt to be encountered incidental to the ordinary operation of the power circuit, or to any ordinary faults which may develop in same. All equipment must be sturdy and suitable for outdoor installation. It should be noted that in case of short circuits from conductor to ground, the ground currents will probably vary between 500 and 1000 amperes, but may ultimately go as high as 2500 amperes for the portion of the lines adjacent to Lock 18.

Control equipment,—receiving or return signal,—shall be of such nature as not to depend on vacuum tube equipment requiring continuously lighted filaments for correct operation, except that such devices may be used in the visual return signal equipment at Mitchell Dam.

In the case of the above equipment, all items shall be suitable for complete and satisfactory operation under the conditions outlined and described, and shall include all accessories necessary for their operation and protection, and for reasonable safety of operators.

Discussion

For discussion of this paper see page 236.

The Transmission of High-Frequency Currents for Communication over Existing Power Networks

BY C. A. BODDIE[1] and R. C. CURTIS[1]
Associate, A. I. E. E. Non-member

Synopsis.—The use of high-voltage power lines for carrier-current communication presents a number of difficulties not met in the use of simple communication circuits. This paper tells how these difficulties may be overcome without resorting to complicated transmitting and receiving equipment. The advantages of employing tuned choke coils are described. These chokes are used to isolate the communication channel from the remainder of the power system which gives in effect a simple communication circuit between communicating points.

* * * * *

HIGH-voltage power lines are constructed with such strength that they are in general superior to any communication circuit except underground cable. Experience has demonstrated that power lines withstand storms, sleet, and floods long after all other circuits are carried away. In view of the vital necessity of communication to a power company, it is but natural that the power line should be used as a communication circuit because of its mechanical superiority. Except for short distances, it also offers an economic superiority.

The method of utilizing a power line as a telephone circuit is to superimpose high-frequency currents on the power conductors. These currents are transmitted over the line as ordinary alternating currents. They are produced and received by equipment similar to the usual space radio apparatus.

In considering the power line as a communication circuit it is immediately apparent that such a circuit differs from the usual telephone line. The principal difference is that the line is operated at high voltage. This gives rise to more or less noise due to spitting insulators and similar effects. In addition the line is not a simple circuit connecting the transmitter and receiver. In practise a power line is usually part of an extensive high-voltage network with loops, taps, and spurs. Such a network is not a constant and stable system from a communication point of view because of more or less continuous changes due to switching. In general, every time a switch is opened or closed in any part of the system, it makes a change in the communication circuit.

These factors are now generally recognized although at the time this type of communication was first undertaken, the importance of some of them was not fully appreciated. In some cases it was found that the natural changes in line characteristics due to switching were so great that a satisfactory communication circuit could not be obtained.

Efforts have been made to solve this problem by modifying the communication apparatus. Modifications tending to reduce the number of frequencies per channel and the introduction of special systems of modulation have greatly increased the amount and complexity of the equipment.

By its very nature power line communication equipment should be kept just as simple and reliable as possible. Instead of complicating the equipment it is proposed to correct the trouble at its source by stabilizing the power line. The method is to insert high-frequency resistances into the power line at such points as are necessary to block off detrimental circuits and provide a clear circuit for the channel desired.

Apparatus for this purpose has been developed which has proved simple and effective. This apparatus consists of an inductance coil very similar to those ordinarily used for lightning arrester work together with the necessary tuning equipment. These tuned circuits do not absorb energy at the power frequencies nor in any way disturb the power system.

In order to appreciate the problem presented by a power network some of the properties of transmission lines at high frequencies may be noted.

A line of great length does not act like a large capacity or a large inductance but rather as a pure resistance of approximately 800 to 850 ohms for the average power line construction. The impedance of a short line open at the end varies over a wide range.

A representative type of construction is shown in Fig. 1 with the high-frequency currents superimposed between conductors "1" and "3." The attenuation ratio, that is the ratio of the current or voltage received at the distant end to that impressed at the sending end for the type of construction assumed, is shown in Fig. 2. It is interesting to note that the losses in a simple circuit are not serious. For example, the current delivered by a line 100 mi. long is 40 per cent of that transmitted. A 200-mi. line delivers 16 per cent and a 300-mi. line about 6 per cent of the transmitted current.

In Fig. 3 is shown a type of power system in which the branch lines are all of considerable length. The impedance of such branch lines when long is approximately the characteristic impedance. Suppose it is desired to provide a communication channel between *A* and *F*, it is seen by inspection that 50 per cent of the

1. Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

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28-139

current fed into the system at A is transmitted towards B. The part transmitted in the opposite direction is lost since it contributes nothing to the desired channel A–F.

When a wave reaches a junction of two long lines a division of current takes place such that ⅓ of the original current is reflected back towards the sending end

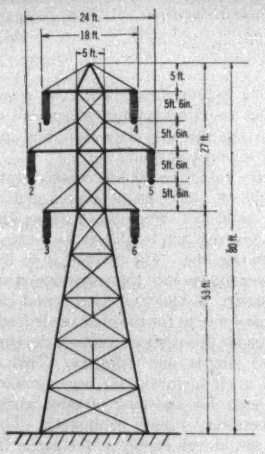

Fig. 1—Typical Line Construction

while ⅔ is transmitted down each branch line, the total being ⅓ greater than the original.

Hence, the wave transmitted from A reaching the junction point B divides, ⅔ of the current being

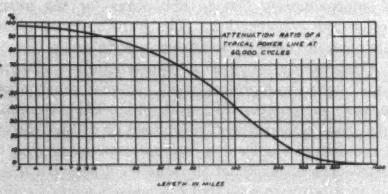

Fig. 2—Attenuation Ratio of a Typical Power Line at 60,000 Cycles

transmitted toward C, the rest being lost. Neglecting the transmission loss in the line, ⅓ of the original is transmitted towards C. At C a similar division takes place leaving 2/9 to be transmitted to E. At E ⅔ of the arriving current that is 4/27 of the original is transmitted into the pair of lines towards F. Since only one of these lines continues to F, ½ of this or 2/27 is transmitted to F as useful current.

The net effect of the subdivision of current at the junction points is the same as the attenuation of slightly less than 300 mi. of simple circuit, as seen from Fig. 2. In addition there is the loss due to the normal attenuation in the line.

Consider the case shown in Fig. 4 of a long line with a 4-mi. spur line connected on at B. The impedance of the 4-mi. spur line at frequencies commonly used is shown in Fig. 5. The impedance of the spur line at 58.1 kilocycles is so low that less than 4.5 per cent of the current arriving at B is transmitted down the line towards the receiving apparatus at C. The effect of such a spur line is equivalent to the loss incurred in a line of over 300 mi. in length. It should be noted that

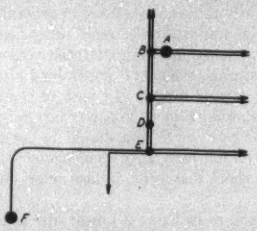

Fig. 3—Power System with Branch Lines

at certain frequencies the effect of a single short spur line is greater than that due to the many subdivisions shown in Fig. 3.

At other frequencies the impedance of the spur line is very high and at these points has little or no effect on the transmission of energy from A to C. The frequencies at which these points of maximum and minimum transmission occur are a function of the length of the spur line. If a number of such spur lines of varying

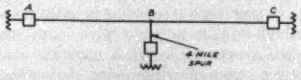

Fig. 4—Power System with a Spur Line

length is connected to the main line it may be impossible to find a single frequency that will be satisfactory for communication.

A set of resonant line chokes may be installed in the spur line at the point where it connects to the main line. The minimum effective impedance of the spur line can thus be raised and the diversion of energy reduced to any desired amount.

Another condition often encountered is that of two or more parallel paths of slightly different lengths At certain frequencies the current arriving by one path is exactly out of phase with the current arriving by the other. If the paths are of nearly equal length the amplitudes of the arriving currents will be very nearly equal. The two will then combine in such a way as nearly to cancel each other, leaving only a very small resultant to act on the receiver.

This condition may be corrected by inserting a set of resonant line chokes in one of the lines to prevent the flow of current by that path. Sometimes this procedure is objectionable because either of the lines may be opened during the normal operation of the system.

By inserting reactances at one or more points along

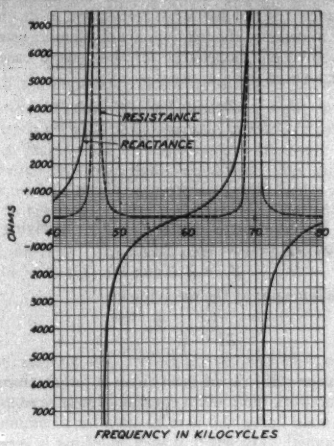

FIG. 5—RESISTANCE AND REACTANCE OF A FOUR-MILE SPUR LINE

the line the phase of the currents arriving at the receiving end can be shifted so that the resultant acting on the receiver is an appreciable portion of that delivered to the receiving end of the line. The construction of these phase shifting reactors is similar to the resonant line chokes. The tuning of the circuit is such as to present a reactance of the required value. In this way use is made of both the available paths. The opening or failure of one of the lines does not materially affect the communication channel.

In Fig. 6 is shown another condition met in practise in which F is energized either by way of C and D or by E. The loop is not maintained closed in normal operation of the power system. The communication channel is from A to F.

If the loop is opened at C, D, or E the effect is that of two spur lines connected across the communication channel. If the loop is opened at B or F the effect is that of one spur line. It is necessary to provide for communication over either circuit so that high impedance line chokes cannot be used at B and F to isolate one circuit.

The corrective measures in this case consist of installing at B and F in both circuits, resonant line chokes adjusted to an impedance such that a considerable amount of current can pass through the circuit to F. At the same time the impedance is high enough to prevent an excessive amount of current flowing when the circuit becomes part of a spur.

The impedance of some types of transformers is quite low at high frequencies. Other types have been encountered in which the impedance varies with the instantaneous values of the power current giving the effect of modulation on the high-frequency currents. These conditions may be remedied by the use of the resonant line choke to prevent any appreciable amount of high-frequency energy being absorbed in the undesirable circuits.

Power lines are now being studied very carefully before any attempt is made to install power line telephone equipment. Actual tests have demonstrated the validity of the commonly accepted theory of lines for high frequencies. By applying accepted analytical methods, the important features of the line characteristics may be determined. If these are found to be unsatisfactory, corrective impedances are inserted at such points as will give the desired results.

THE PROPAGATION OF HIGH-FREQUENCY CURRENTS ON POWER LINES

The fundamental properties of lines at high frequencies may be best understood by first considering a line of infinite length, since no energy is reflected back toward the sending end to combine and interfere with the advancing wave. The amplitude of the voltage vector is progressively reduced as it travels away from the sending end. The magnitude of the voltage vector at a point x is expressed by

$$E_x = E_0\, \epsilon^{-\alpha x} \tag{1}$$

where α is the attenuation constant.

The phase position of the voltage vector is shifted backwards as the distance from the sending end increases. The position is given by the equation:

$$E_x = E_0\, \epsilon^{-j\beta x} \tag{2}$$

where β is the wavelength constant. It is the number of radians per mile which the voltage vector is shifted backwards. This action is shown graphically in Fig. 7.

The complete expression for the voltage at any point x along an infinite line is then,

$$E_x = E_0\, \epsilon^{-x(\alpha + j\beta)} = E_0\, \epsilon^{-Px} \tag{3}$$

where P is the propagation constant and is equal to $(\alpha + j\,\beta)$.

FIG. 6—POWER SYSTEM WITH LOOP

The propagation constant may also be expressed in terms of the fundamental line constants per unit length.

$$P = \sqrt{R + j\,\omega L}\,\sqrt{G + j\,\omega C} \tag{4}$$

where R is the resistance in ohms.
L the inductance in henrys.
G the leakage conductance in mhos
C the capacity in farads.

The vector impedance of an infinite line measured at the sending end is known as the characteristic impedance and may be demonstrated to be,

$$Z_0 = \frac{\sqrt{R + j\,\omega L}}{\sqrt{G + j\,\omega C}} \tag{5}$$

For a power line the R and G terms at high frequencies are negligible as compared to the reactive components. The characteristic impedance is therefore a pure resistance,

$$Z_0 = \sqrt{\frac{L}{C}} = 276 \log_{10} \frac{2\,D}{d} \tag{6}$$

where D is the distance between conductors and d the diameter of the conductor.

The values of α and β are more readily calculated from the following:

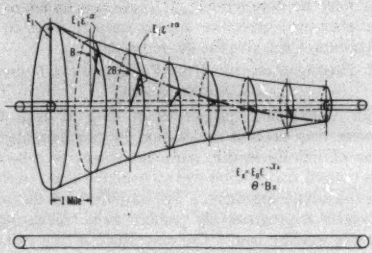

FIG. 7—DIAGRAM SHOWING THE PHASE AND MAGNITUDE OF POTENTIAL AT VARIOUS POINTS ALONG A LINE

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \tag{7}$$

As stated above the leakage for a power line is negligible. The second term of equation (7) drops out and the expression becomes,

$$\alpha = \frac{R}{2}\,\frac{1}{Z_0} \tag{8}$$

where

$$Z_0 = \sqrt{\frac{L}{C}} \tag{9}$$

The value of the wavelength constant can be obtained from the relation:

$$\beta = \frac{2\,\pi\,f}{V} = \frac{2\,\pi\,f}{186{,}000} \tag{10}$$

The velocity of propagation is nearly equal to the velocity of light since the resistance and leakage conductance are necessarily small as compared to the reactive components. The value of the wavelength constant obtained from equation (4) for the propagation constant by neglecting R and G is

$$\beta = 2\,\pi\,f\,\sqrt{L.\,C.} \tag{11}$$

Substituting this value in the equation for velocity

$$V = \frac{2\,\pi\,f}{\beta} = \frac{1}{\sqrt{L\,C}} = 186{,}000 \text{ mi. per sec.} \tag{12}$$

The impedance of an infinite line measured at the sending end is the characteristic impedance as defined by equation (5). This is also the impedance of a finite line of any length terminated in its characteristic impedance. If the line is not terminated in its characteristic impedance the relations are much more complex. The sending end impedance for the general case of a line terminated in an impedance Z_R is:

$$Z_1 = Z_0\,\frac{Z_R \cosh P\,l + Z_0 \sinh P\,l}{Z_0 \cosh P\,l + Z_R \sinh P\,l}$$

$$= Z_0\,\frac{Z_R + Z_0 \tanh P\,l}{Z_0 + Z_R \tanh P\,l} \tag{13}$$

For a line short-circuited at the receiving end Equation (13) reduces to,

$$Z_1 = Z_0 \operatorname{Tanh} P\,l \tag{14}$$

For a line open at the receiving end

$$Z_1 = \frac{Z_0}{\operatorname{Tanh} P\,l} \tag{15}$$

Equation (15) is of particular interest since the power network quite often consists of spur lines tapped off the main line which is to be used for a communication channel. The effect of such spur lines is generally quite serious. Equation (15) may be further expanded to give both the resistance and reactance components.

$$Z_1 = Z_0\,\frac{\tanh \alpha\,l\,(1 + \tan^2 \beta\,l)}{\tanh^2 \alpha\,l + \tan^2 \beta\,l}$$

$$- j\,Z_0\,\frac{\tan \beta\,l\,(1 - \tanh^2 \alpha\,l)}{\tanh^2 \alpha\,l + \tan^2 \beta\,l} \tag{16}$$

By an inspection of Equation (16) it is seen that the reactive component goes through zero every 90 deg. and at the odd quadrants the resistance term becomes,

$$Z_1 = Z_0 \tanh \alpha\,l \tag{17}$$

At the even quadrants $Z_1 = \dfrac{Z_0}{\tanh \alpha\,l}$ (18)

Between the points of maximum and minimum resistance the reactive component predominates unless the value of $\alpha\,l$ is large. The value of the $\tanh \alpha\,l$ is always less than unity so that points of minimum impedance occur at the odd quadrants, that is whenever the value of $\beta\,l$ is such that the tangent of $\beta\,l$ is infinite. This occurs at all the odd multiples of 90 deg. or $\frac{\pi}{2}$ radians. The points of maximum impedance occur at all values which make the tangent of $\beta\,l$ equal to zero or at all the even multiples of 90 deg.

The interval between the maximum and minimum values may be determined by referring to Equation (10)

$$\beta l = \frac{2 \pi f l}{V} = \frac{\pi}{2} \text{ for first minimum value}$$

$$f = \frac{\pi}{2} \left(\frac{V}{2 \pi l} \right) = \frac{V}{4 l} = 4.65 \times 10^4 \times \frac{1}{l} \quad (19)$$

The presence of a transformer bank and other power apparatus terminating the spur line further complicates the problem of obtaining a satisfactory communication channel. The effect may be calculated from Equation (**13**), if the impedance of the apparatus is known.

Another method giving a better insight into the problem is to replace the terminal apparatus by a line of such length that its impedance is the same as the apparatus replaced. This equivalent line may be either open or short. If an open line is used it merely adds to the original line and the effects become more evident. The impedance of an open line is given by Equation (**15**). The length of the equivalent line must then be such that,

$$\tanh \theta_R = \frac{Z_0}{Z_R} \quad (20)$$

The sending end impedance of the line with its terminal apparatus is then the same as the original line plus the equivalent line or,

$$Z_1 = \frac{Z_0}{\tanh (\theta + \theta_R)} \quad (21)$$

Expanding Equation (**21**) into its real and imaginary components gives,

$$Z_1 = Z_0 \frac{\tanh (\alpha l + \alpha_R) [1 + \tan^2 (\beta l + \beta_R)]}{\tanh^2 (\alpha l + \alpha_R) + \tan^2 (\beta l + \beta_R)}$$

$$- j Z_0 \frac{\tan (\beta l + \beta_R) [1 - \tanh^2 (\alpha l + \alpha_R)]}{\tanh^2 (\alpha l + \alpha_R) + \tan^2 (\beta l + \beta_R)}$$

This equation is of the same form as Equation (**16**). It differs in that the real and imaginary components of the propagation constant applying to the line are increased by the components corresponding to θ_R of the transformer bank. By comparison with Equation (**16**) it will be seen that the maximum value of impedance is reduced and the minimum increased. Tests on transformers indicate that the impedance is principally a capacity reactance. If the impedance of the transformer bank remains substantially constant over the range considered, the number of cycles between peaks on the impedance curve of the spur line is the same as for the line without the transformer bank. The net effect of a transformer bank on this assumption is to shift the whole impedance curve such as Fig. 5 down in frequency without changing the distance between peaks. In practise it is found that the impedance of the transformer is not constant. The transformers therefore make the observed impedance curves differ from the simple case.

Application to a Typical Line

The application of the foregoing equations may be seen by considering a typical line such as shown in Fig. 1. The conductors are assumed to be 4/0 stranded copper with a spacing of 22 ft. between the outside conductors.

The d-c. resistance per loop mile is 0.518 ohms. The high-frequency resistance may be calculated by the method given in Circular 74 of the Bureau of Standards. The ratio of the high-frequency resistance to the d-c. resistance is obtained from a table giving values of a factor X_c. The value of X_c varies directly with the diameter and the square root of the frequency. The equivalent diameter of a stranded wire is assumed to be equal to the diameter of a solid wire having the

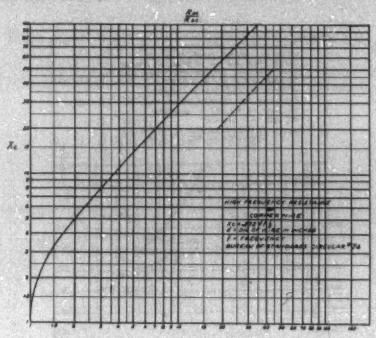

Fig. 8—High-Frequency Resistance of Copper Wire

same copper cross-section. The data in this table are plotted in the form of a curve in Fig. 8. The high-frequency resistance calculated by this method for 4/0 stranded copper at 50,000 cycles is 10.5 times the d-c. resistance or 5.44 ohms.

Measurements on a number of lines of this general type indicate that the actual line resistance (due to losses not understood at present) is approximately 250 per cent of the value calculated. The dotted curve shown on Fig. 8 will therefore be used. This gives a value of 13.6 ohms per loop mile at 50,000 cycles for the 4/0 stranded copper conductor.

It will be here assumed that over the range of frequencies being considered the high-frequency resistance varies as the square root of the frequency, and that the tabular values should be multiplied by a factor of 2.5. Values so calculated check closely with observed results.

The inductance per loop mile may be calculated from the formula,

$$L = 1.48 \times 10^{-3} \log_{10} \frac{2 D}{d} \text{ henrys} \quad (23)$$

where D is the distance between conductors, d is the

diameter of the conductor. The value for the typical case assumed above is,

$$L = 1.48 \times 10^{-3} \log_{10} \frac{2 \times 12 \times 22}{0.538} = (148 \times 10^{-3})$$

$\times$ (2.992) = 4.43 $\times$ 10^{-3} henrys per loop mile.

The capacity between wires per mile may be calculated from the formula,

$$C = \frac{0.0194}{\log \dfrac{2D}{d}} \times 10^{-6} \text{ farads}$$

$$= \frac{0.0194}{2.992} \times 10^{-6}$$

$$= 0.065 \times 10^{-6} \text{ farads} \qquad (24)$$

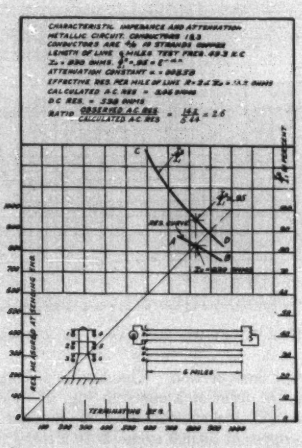

FIG. 9—CHARACTERISTIC IMPEDANCE AND ATTENUATION

The propagation constant obtained from Equation (4) assuming f = 50,000 cycles is

$$P = \sqrt{13.6 + j\,1390}\ \sqrt{0 + j\,2.04 \times 10^{-3}}$$

$$= 1.69 \angle 89^\circ 43.2'$$

$$= 0.00826 + 1.69 = \alpha + j\beta$$

The value of the attenuation constant calculated from Equation (8) is

$$\alpha = \frac{R}{2}\,\frac{1}{Z_0} = \frac{13.6}{2}\,\frac{1}{825} = 0.00825$$

Where Z_0 from Equation (9) is 825 ohms.

The value of the wavelength constant calculated from Equation (10) is

$$\beta = \frac{2\pi f}{v} = \frac{(2\pi)\,(50{,}000)}{186{,}000} = 1.69$$

Measurement of Characteristic Impedance

The characteristic impedance is known to be a resistance from Equation (6). As stated above the characteristic impedance is measured at the sending end of a finite line when the receiving end is terminated in the characteristic impedance. Trial values of resistance are inserted at the receiving end and the resistance measured at the sending end. These values are plotted in the form of a curve *A-B* as shown in Fig. 9. The measured value in this case is 830 ohms which agrees well with the calculated value of 825 ohms.

In taking the data for determining the characteristic impedance, ammeters are inserted at the sending and receiving ends as shown in Fig. 10. The ratio of the receiving end current to the sending end current is plotted in the form of a curve *C-D* for various values of terminating resistance. The attenuation ratio is then easily determined for the particular value of resistance that is found to be equal to the characteristic impedance.

Measurement of Line Resistance

The data shown in Fig. 9 were taken on a 6-mi.

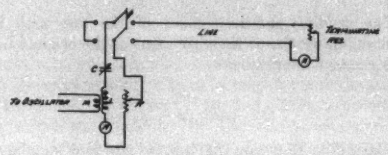

FIG. 10—APPARATUS AND CIRCUIT USED IN MAKING LINE MEASUREMENTS

line constructed as shown in Fig. 1. The attenuation ratio is 95 per cent. The attenuation constant is

$$E^{-\alpha l} = \frac{I_2}{I_1} = 0.95$$

$$\alpha l = 0.0515$$

$$\alpha = 0.00858$$

The effective resistance per mile of line obtained from Equation (8) is,

$$R = 2\,\alpha\,Z_0$$

$$= 14.2 \text{ ohms}$$

$$\text{Ratio } \frac{\text{Observed A-C. Res.}}{\text{Calculated A-C. Res.}} = \frac{14.2}{5.4} = 2.63$$

Values obtained by test on other lines vary between 2.0 and 3.0, the value 2.5 used in these calculations being a fair average.

Spur Lines

Fig. 5 shows the variation of impedance of a typical 4-mi. spur between 40 and 80 kilocycles. The reactance curve is seen to be a typical tangent curve except for the higher values. The reactance curve reaches a maximum and then passes through zero instead of following the tangent curve to infinity. This is also

evident from an inspection of Equation (**16**) since the values of $\tanh^2 \alpha l$ are small as compared to unity and small as compared to most values of $\tan^2 B l$.

The impedance presented by a 4-mi. spur varies over a wide range. The impedance at 46.5 kilocycles is 26,000 ohms and at 58.1 kilocycles 29.3 ohms. The impedance then rises again to a maximum of 21,200 ohms at 69.7 kilocycles. The interval between the points of maximum and minimum impedance as given by Equation (**19**) is

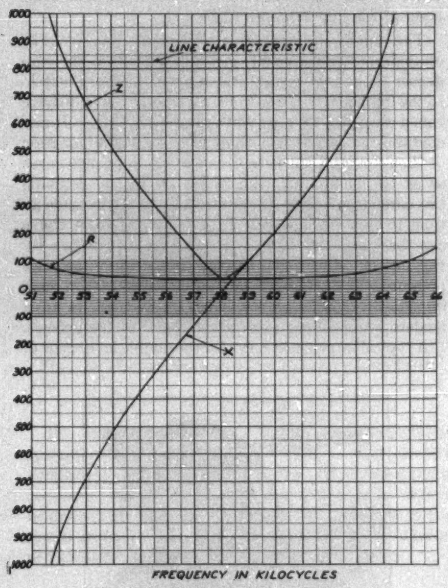

Fig. 11—Enlarged Section of Fig. 5

$$\frac{V}{4l} = \frac{186{,}000}{4 \times 4} = 11{,}600 \text{ cycles}$$

It should also be noted that the impedance between 52.5 kilocycles and 63.7 kilocycles is less than the characteristic impedance of the line and will in this band of frequencies absorb considerable energy in addition to the loss in the main line due to reflections. Fig. 11 shows an enlarged portion of the curve in Fig. 5. The presence of a single 4-mi. spur line tapped on the main line causes over 50 per cent of the available high-frequency channels to be subject to wide variations in signal level by switching conditions. If several such spur lines of varying lengths are present, it may be impossible to find a single frequency which will not be subject to excessive attenuation and variation with changes in the operating condition of the power system.

The presence of a transformer bank at the end of the spur line adds further complications. Switching the bank on and off changes the frequencies at which the points of maximum and minimum impedance occur.

For example, if the 4-mi. spur line were terminated in a large substation having a number of transformer banks connected in parallel, the points of maximum and minimum impedance might be lowered as much as 5.5 to 6 kilocycles. In general the shift is of the order of 10 to 15 per cent of the interval between the maximum and minimum values as calculated by Equation (**19**).

Most transformer banks act as a capacity reactance of a rather high order. Values of 4000 ohms reactance and 200 ohms of resistance are typical. From Equation (**20**) the value of the equivalent line angle θ_R is

$$\text{Tanh } \theta_R = \frac{Z_0}{Z_R} = \frac{825 \angle 0^\circ}{4000 \angle 87.2^\circ} = 0.206 \angle 87.2^\circ$$

$$\theta_R = 0.206 \angle 87.2^\circ$$
$$= 0.01007 + j\,0.206$$
$$\alpha_R = 0.01007$$
$$\beta_R = 0.206 \text{ radians} = 11.8^\circ$$

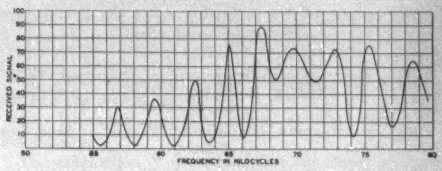

Fig. 12—Received Signal for 50- to 80-Kilocycle Band

Showing the effect of a spur line, approximately 34-mi. long at the receiving end

Substituting these values in Equation (**21**) lowers the point of maximum impedance from 26,000 ohms at 46.5 kilocycles to 20,000 ohms at 44.98 kilocycles. Similarly the point of minimum impedance is changed from 29.4 ohms at 58.1 kilocycles to 37.3 ohms at 56.58 kilocycles.

The energy absorbed by a spur line becomes more nearly constant as its length increases. Also as the length increases the interval in cycles between maximum and minimum values becomes less. The actual loss of energy in the case of a long spur line is not such a serious factor. The rapid variation of impedance does, however, cause considerable distortion of the modulated wave. In Fig. 12 are shown the results of a test made on a portion of a typical power system. The most noticeable feature is the rapid fluctuation of signal with frequency. This is due to the location of the receiver at a point approximately 34 mi. from the end of the line.

Increasing the attenuation either by raising the frequency or by increasing the length will tend to limit the variation of impedance. The impedance ap-

proaches the characteristic impedance as a limit as the length becomes great. The maximum and minimum values of impedance for varying values of the attenuation factor are plotted in the form of a curve in Fig. 13. It should be noted that a value of the attenuation factor of 0.55 or greater is required to prevent the minimum values of the spur line impedance from falling below 50 per cent of the characteristic impedance value. This would require the spur line to be at least 66.7 mi. long assuming the frequency to be 50 kilocycles and the

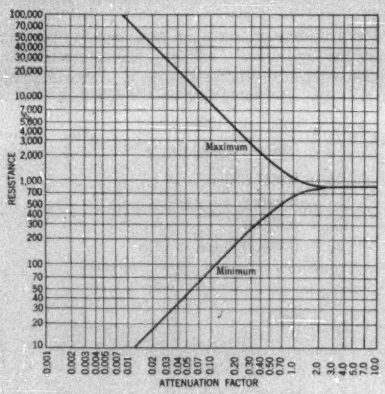

Fig. 13—Maximum and Minimum Values of Spur Line Impedance

type of construction as shown in Fig. 1. At 100 kilocycles the required length is only reduced to 47 mi.

From the foregoing it is apparent that the effect of spur lines is very detrimental. The short lines cause excessive transmission loss over wide bands of frequencies and the longer lines cause distortion due to the rapid variation of impedance with frequency. All types of telephone communication utilizing high-frequency currents require the transmission of a band of frequencies rather than a single frequency. The width of the band may be reduced to some extent by employing special types of modulation. The best that can be accomplished in this direction will not reduce the width of the band below 2000 cycles. This gives only a minor improvement without providing a general solution to the problem.

The detrimental effects of spur lines may be obviated by making certain changes in the transmission system. Any desired frequency can then be used, distortion is eliminated, wide fluctuations in signal level are prevented, and complications in the transmitting and receiving apparatus are avoided.

These results can be accomplished by inserting in the power conductors, equipment which will carry the power current without loss and at the same time introduce high impedances to the communication currents.

Resonant Line Chokes

Physically the resonant line chokes are dividing into two classes. The single layer type is used for introducing moderate amounts of resistance into the power line and the double layer type for introducing a high resistance to isolate an undesirable circuit. The single layer resonant line choke consists of an inductance coil similar to a standard lightning arrester choke and the necessary tuning equipment. The double layer type is similar except that the number of turns on the inductance coil has been doubled by the addition of another layer of winding as shown in Fig. 14.

The tuning equipment is shown in Fig. 15. The diagram of connections is given in Fig. 16. The condenser C_1 is used to tune the main inductance coil L_1 to the frequency it is desired to obstruct. An additional tuned circuit L_2, C_2 coupled to the main coil L_1. It also contains an adjustable resistance R_2. The function of the second circuit is to broaden the resonance curve so as not to injure modulation. By tightly coupling the second circuit to the first, the system is made to tune to two frequencies. In this way a single choke can be made to block two frequency bands. By varying the adjustment of this circuit the impedance presented by the choke can be changed from a high impedance over a narrow band of frequencies to a moderate impedance over a wide band of frequencies.

A typical impedance curve for the double layer type of

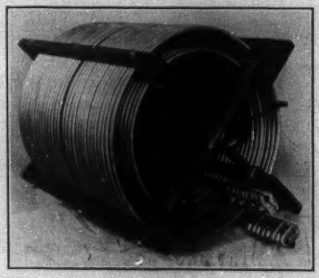

Fig. 14—Double Layer Resonant Line Choke

choke is shown in Fig. 17. Chokes are placed in each of the line conductors so that the total line-to-line impedance is of the order of 12,000 ohms as compared to the line characteristic impedance of 825 ohms. It is evident that a circuit isolated by this type of choke cannot divert any appreciable amount of the communication current.

In Fig. 18, curve No. 1 shows the resistance curve of a single layer choke adjusted to give a moderately high resistance at two frequencies. By the use of this type two channels can be cleared. A resistance of approximately 400 ohms per choke is sufficient to limit

the current in an undesirable circuit to a value that will not seriously affect the communication channel.

The adjustment may be changed so as to give a very flat impedance curve as shown in curve 2, Fig. 18. This is a very valuable characteristic, and is used where it is necessary to stabilize a power system and still transmit through the chokes as in the case shown in Fig. 6.

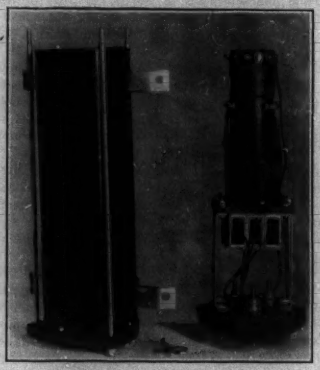

Fig. 15—Tuning Equipment for Resonant Line Choke

Conclusion

The ideal condition for a high-frequency telephone system is to have the line a simple circuit directly connecting the points between which communication is required. This ideal may be approximated as closely as desired by isolating the channel from the rest of the power system. This is made possible by the installation of resonant line chokes at appropriate points.

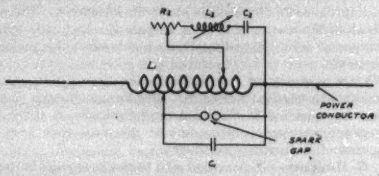

Fig. 16—Diagram of Connections for Resonant Line Chokes

Some of the advantages gained are:

1. Increased stability.
2. Improved quality of speech transmitted.
3. Reduction in the noise level due to the greater energy delivered to the receiver.
4. Freedom from variation in signal due to switching.
5. Simple transmitting and receiving equipment satisfactory on complex systems.
6. Increased reliability.
7. Ability to transmit through practically any type of system.
8. Ability to utilize all lines between communication points.

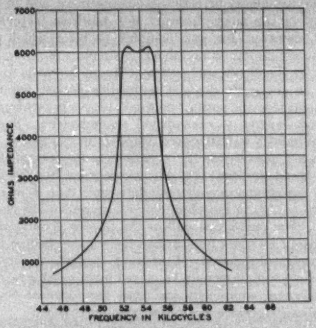

Fig. 17—Typical Impedance Curve for a Resonant Line Choke, Double Layer Type

9. Interference with equipment on adjacent power systems can be eliminated.
10. Reduction in field work at the time of installation.

The application of the established line theory to the analysis of power lines has materially advanced the art of telephone communication over power lines. It is now possible to predict with reasonable accuracy what frequencies can be successfully transmitted over a given power network. This materially reduces the extensive field work which has often been necessary

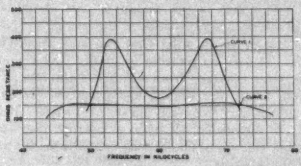

Fig. 18—Typical Resistance Curves for Resonant Line Chokes, Single Layer Type

when making an installation. If the analysis shows that there are conditions which prevent the use of the desired frequency bands, corrective measures may be taken by installing resonant line chokes at suitable points. System limitations are largely overcome by this procedure and the field of application of high-frequency communication materially increased.

Discussion

CARRIER-CURRENT AND SUPERVISORY CONTROL ON ALABAMA POWER COMPANY'S SYSTEM

(Robinson and Woodcock)

LINE CHARACTERISTICS FOR CARRIER COMMUNICATION

(Boddie and Curtis)

C. F. Boeck: The authors say that power-line carrier equipment should be as simple and reliable as possible. I agree that simplicity of terminal arrangement and in the configuration of the power network are desirable features. It is unusual, however, for any complex problem which involves contradictory factors to be solved by simple and direct methods alone or by the direct application of simple methods to any one factor. In the early days of power-line carrier telephony, it was thought that the power line would have a rather smooth and stable high-frequency transmission characteristic. Simple terminal equipment was therefore proposed with the idea of using ample power to over-ride the variations in the transmission line. Later it became evident that the transmission line did not have a stable high-frequency characteristic and that it was subject to variations due to switching operations and other changes in the line. The problem is still in the process of solution and it will probably be some time before the various factors involved are properly coordinated. I wish to suggest, therefore, that perhaps the emphasis should be placed upon reliable rather than simple terminal equipment and that the terminal equipment should be as simple as is consistent with reliability.

Power-line carrier telephone equipment initially was designed to use the transmission line without modification for a communication channel. The development, therefore, has been centered largely upon terminal apparatus and has resulted in the development by us of single-frequency duplex, single-sideband, carrier-suppressed equipment which was discussed at a recent Institute meeting.[1]

This equipment represents a distinct advance in the art and is giving reliable service over a complex power distribution network. It was installed after two years' experience with other terminal equipment had fully demonstrated its inability to meet the conditions imposed by this particular network.

Full advantage of improvements in terminal apparatus cannot be realized unless the transmission characteristics of the power line are favorable. The authors propose to improve the transmission characteristics by the use of high-frequency choke coils inserted in the power line at suitable places to isolate the detrimental effects of branch circuits. This method is a useful one and has a certain application. It has, however, some disadvantages. The first is the design of this choke coil. That involves in advance a knowledge of the carrier frequency which you are going to use in order that you may properly tune the choke coil. It may be possible to predict this frequency with reasonably close accuracy from a study of the line by using the information which Mr. Boddie has given us. If not, it may involve a preliminary frequency survey of the line. This method, moreover, is applicable to only a narrow frequency band. Furthermore, the use of choke coils complicates the addition of future carrier channels.

In Fig. 12 of the paper, the authors show how the transmission between two points in a power line is influenced by one small tap line somewhere between them. I should like to know if they have any data showing the changes in the transmission characteristics of that line resulting from isolating the spur line by choke coils.

Another method which may have some value in eliminating reflection effects is to terminate properly the transmission lines and the spur lines. This method has an additional advantage over the use of choke coils, if it is a feasible one, in that it will provide a wider frequency band and permit the use of several channels on the power line. An investigation showing what might be accomplished by this method I think would be desirable also.

Connected power machinery also influences the transmission characteristics of a power line and leads to reflection effects. It may be possible in the future design of power machinery to give some thought to it so that its effect upon carrier transmissions will be minimized.

An additional advantage of considerable importance to the power companies may accrue by improving the high-frequency transmission characteristics of the power lines in that these improvements may reduce the magnitude and effect of line surges and switching transients. However, until such time as we can determine definitely what methods shall be used to improve the carrier-frequency characteristics of a power line, we will have to use the lines more or less as they are and to depend upon terminal equipment to help us over the rough spots.

L. E. Anderson: I wish to make a few remarks as to the inconsistencies and disadvantages of the use of power-line carrier from an operating standpoint.

One of the first things I wish to mention is the interference which results between apparatus of different manufacturers. It will be necessary for the manufacturers to get together and make apparatus that will not interfere with other apparatus. We have on our system 10 low-power sets of one manufacturer and 2 high-power of another,

When we are using the high-power equipment, all the equipment of low power is inoperative due to relays being paralyzed. We have a condition where 4 of our stations on the low-power equipment are paralyzed from one of the high-powered sets about 400 mi. away. While people are talking in Alabama, we are paralyzed and cannot talk until such time as they are through. Thus, it will be necessary for the manufacturers to do something about getting the sets to operate together, or provide some means of getting rid of this interference.

We have found that in balancing a network, it is much harder to balance the hybrid coils for the high power than it is for the low power.

In operating through open switches on the double circuits we are able to get through with 3 phases of one line and 2 of another open, but then the level is so low that we cannot operate consistently.

Mr. Boddie mentioned the trouble experienced in spur lines, that is, the shunt impedance at 50 to 60 kilocycles. What we have been up against are the frequencies above 80 kilocycles. We have found that the 110-kv. current transformers offer a high impedance to frequencies above 80 kilocycles. The only method of eliminating this is to by-pass the high side with a resistance of such value that it does not affect the metering equipment.

We are planning to install within the near future, double resonant line chokes which Mr. Boddie mentioned. These chokes are expected to eliminate the frequencies at which the high-power sets are operating and at the same time pass the frequency of the lower-powered sets.

L. G. Huggins: The method used to charge storage batteries in the supervisory controlled sectionalizing stations was briefly mentioned in Messrs. Robinson and Woodcock's paper. I believe that a little further discussion on this will be of interest.

Many of the switching stations are very remotely located. No power is available at these points for charging storage batteries. To haul these batteries in a truck to a charging station at periodic intervals would entail considerable work and expense.

A very unique scheme of charging these storage batteries from the overhead ground wire has been devised. As mentioned in the paper, this ground wire is insulated for use as a carrier channel and is grounded by means of drainage coils. A current transformer is connected between the drainage coil and ground.

1. *Problems in Power Line Carrier Telephony*, W. V. Wolfe and J. D. Sarros, see p. 107.

The output of the secondary of this current transformer is rectified by a standard Rectox charger and is used to "trickle charge" the batteries. This arrangement has been in service for some time with encouraging results.

This same method of obtaining small amounts of power from high-voltage lines has been tried in Europe, I understand, to furnish power to farmers and other isolated customers at points where it is not practical to install transformers. The amount of power that can be obtained in such a way is rather small and the regulation for lighting service or any variable load is so poor that the method is very limited in its application.

R. G. McCurdy: In connection with the paper by Messrs. Boddie and Curtis it may be of interest to contrast the problems of transmission of high-frequency currents over power networks with those arising in transmission over telephone circuits, particularly with reference to the desirable frequency ranges to be employed.

In telephone lines, attenuations are in general larger than on power lines and these increase rapidly with increasing frequency. Because of the large number of circuits in close proximity on a telephone pole line on which it is desired to employ carrier-current transmission, cross-talk must be controlled. In general, the cross-talk difficulties and the expense of controlling them increase rapidly with increased frequency. In the present state of the art, these difficulties fix an upper limit on the frequency which it is economical to employ, at a figure somewhat below 50,000 cycles.

On the other hand, in the power-system line, attenuations are lower and crosstalk problems are much less acute. Since the upper limit is established by line attenuation rather than by crosstalk, it is practicable to employ much higher frequencies up to the neighborhood of 200,000 cycles.

While the problems of high line attenuations and crosstalk are much less difficult in the power networks than in telephone lines, other difficulties are present which are not present in telephone lines, such as branch lines, bridged loads, and changes due to switching. The telephone system essentially consists of a large number of point-to-point circuits as contrasted with the power system which is essentially a "broadcasting" system.

Messrs. Boddie and Curtis propose to overcome this broadcasting difficulty by connecting in the undesired paths resonant chokes, thus providing a direct path between the points between which it is desired to communicate. Since the impedances of a given choke increase with increasing frequency, it is practicable for a given loss to the power currents to obtain a more effective blocking action as the frequency of the carrier current is increased. It may also be noted that as the frequency is increased the choking becomes effective over a wider band.

The paper also shows that, aside from any actual transmission losses due to these branches and changes due to switching, considerable distortion may be introduced by them in a telephone channel due to changes in the loss with frequency over the band which must be transmitted. Increase in the frequency usually decreases the variation in these losses with a change in frequency over the required band. On the whole, it appears that the more desirable frequency range for such transmission over power circuits lies above rather than below 50,000 cycles.

This difference in desirable frequency range for transmission over the two classes of circuits has marked advantages from the standpoint of coordination between them, and as a telephone man it appears to me a fortunate circumstance.

It has been necessary in carrier systems on telephone lines, in order to avoid cross-talk, to avoid large power-level differences in the same frequency range on circuits on the same pole line and to employ elaborate transposition arrangements to control the crosstalk between circuits employing the same frequency and operating at about the same power levels. This, in general, has required that amplifiers on all carrier circuits on a given pole line be installed at the same point, and that all transmission in one direction employ the lower half of the frequency range and all transmission in the opposite direction employ the other half. Thus, differences in power level in the same frequency band, even of the order of the amplification ratio in the repeaters, are avoided.

Transposition arrangements in the telephone-line carrier systems, the balance of the lines and equipment, together with the power levels employed, are such that coordination is secured with neighboring power circuits as far as the incidental components in the telephone carrier frequency range present on the power circuits. As far as a power-line carrier system is concerned, these incidental carrier-frequency components establish a noise level above which the system must operate. Consequently, cases of proximity between power-line carrier and telephone-line carrier systems operating in the same frequency range would involve increasing the effect of the power circuit on the telephone system by the level of the power-line carrier system relative to the level of these incidental frequencies. Since it is not practicable to transpose the power circuits in a way which would be effective at these frequencies or to lay out the two systems in a way to avoid the large level differences as is done within the telephone carrier systems, frequency allocation constitutes the only practical method of coordination at present available. As mentioned above, it is considered that these two ranges are naturally divided about the frequency of 50,000 cycles, this figure being the upper limit for telephone-line systems and the lower for power-line systems.

In connection with the paper by Messrs. Woodcock and Robinson, I should like to refer briefly to the use of medium frequencies for supervisory control and its effect on inductive coordination. I think it will be of interest to contrast the problem when these frequencies are superposed directly in the power circuit with that introduced by the scheme described in this paper, whereby they are put on between a pair of insulated "ground" wires. When these voice frequencies are placed directly on the power lines, it is necessary that their level be comparatively high, above the level of the harmonics on the power system. This means that when power lines and telephone lines are in proximity and coordinated in such a way that they can live happily together, with the normal magnitudes of the harmonics on the system, increasing the level by the use of these supervisory control systems will bring in new troubles. When the frequencies are placed on the ground wires the noise levels over which they must work are very much lower. This should permit the use of power levels on the ground-wire pair about equal to the level of the harmonics normally present on the phase wires. Also, it would be more practical to transpose this ground-wire pair than would be the case with the phase wires. Thus the possibility of satisfactory coordination between supervisory control circuits and telephone lines looks very much more hopeful, when the ground wires are so employed.

W. I. Woodcock: We have encountered conditions similar to those mentioned by Mr. Boddie in connection with his Fig. 4, which shows a 4-mi. spur. In our case a tap line was run to Decatur from the line between Sheffield and Huntsville. Calculations show that this tap has about the same resistance as the line of Fig. 4. When this tap line was constructed communication on the main line was made impossible until the carrier frequency was changed.

C. E. Stewart: (communicated after adjournment) Messrs. Robinson and Woodcock refer to two systems of supervisory control. Attention is invited to a third method which consists of using high-frequency carrier-current impulses operating over the insulated overhead channel. This method eliminates the necessity for expensive coupling which is required if the impulses are transmitted over the power conductors. A supervisory equipment of this kind has been in successful operation since October

1926, on the Chicago, South Shore and South Bend Railroad System.[2]

C. A. Boddie: Mr. Boeck seems to think that simplicity and reliability do not go together. My experience, which applies to quite a variety of equipment, is that simplicity almost inevitably leads to reliability, and that complexity tends toward lack of reliability. That is the answer I would give to the system which he proposes, whereby a single frequency is used for communication. Any single-frequency system is inherently less stable than a corresponding double-frequency system. The particular system advocated suffers from the additional complications arising from the suppression of one sideband and the fundamental.

We must bear in mind that the communication which we are attempting to obtain is of vital importance. Stability and reliability are first and we should select for this service only those means which are unquestionable.

I would call attention to a feature which Mr. Woodcock and Mr. Robinson have stressed several times in their paper, namely the matter of maintenance. A commercial telephone system has a corps of men who are thoroughly conversant with all the details of their equipment. Under such conditions complicated equipment can be successfully operated, yet this same equipment will fail under the operating conditions met with on a power system. In view of the operating conditions on a power system I think that the equipment which Mr. Boeck advocates is fundamentally a move in the wrong direction.

I was much interested in Mr. Boeck's remarks on the operation of his new system on a line out West. The line he refers to is the Pacific Gas and Electric 220-kv. line. He made the statement that it was utterly impossible to meet the conditions on these lines with standard apparatus. I wish to oppose that view as I was on that particular project myself. I talked over that line with standard apparatus and wish to say that the quality of the talk delivered was of such a high order as to leave little to be desired. The equipment we used was of the same type as that which Mr. Woodcock has been using so successfully in Alabama, and which is in successful operation in many other parts of the country. It is of simple design in which doubtful elements have been carefully excluded. It provides true duplex operation by utilizing two frequencies. The known fluctuations in power-system conditions are met by providing 250 watts of communication energy.

It is true, as Mr. Boeck says, that the standard equipment furnished by his company demonstrated its "utter inability to meet the conditions imposed by this particular network." That statement is not general as applying to all equipment, but only to a particular design.

The power company in this case had already convinced itself that there was something about a 220-kv. line which made impossible its use as a communication circuit. They thought that the large corona losses were the chief obstacle. Imagining the line to be unusual they felt that the design of equipment which would work on this line must also be unusual, and very naturally accepted the design advocated by Mr. Boeck.

Mr. Anderson points out the difficulties he has had due to interference between two types of equipment operating on his system. These two types of equipment are in general similar except that one is designed to develop 250 watts of communication energy, whereas the other equipments develop only 1 watt normally. The interference arising between these two classes of equipment is due to some extent to the great difference in power level. The low-power equipment is particularly susceptible to interference due to the type of receiving circuit used. Discussions are going forward under the direction of the N. E. L. A. looking toward the elimination of interference between equipments of different manufacture. It might be said in general that most of the low-power equipments which have been manufactured to date are very susceptible to interference, as a result of the broad tuning properties of the receiving circuits which have been used. The circuits in the high-power equipment have been very greatly improved for the purpose of rendering these equipments less susceptible to interference, and to improve the quality of modulation. The manufacturers of the low-power equipment have already expressed a willingness to make improvements in their circuits when interference arises.

There are serious obstacles to go to such high frequencies as 200,000 as suggested by Mr. McCurdy. At these frequencies very short spurs of ¼ to ½ mi. offer serious obstacles. So many of these are encountered on any commercial power system as to render this band of frequencies practically unusable. Our experience is that the best operating region is in the vicinity of 50,000 cycles, and that the desirable range extends as low as 25,000 and as high as 100,000.

Mr. Boeck asked about Fig. 12 of our paper. The chokes for stabilizing the particular section of line have been installed, but no curves have been run to show just what improvements have been made. The installation of the chokes has resulted in a very noticeable improvement in the quality of modulation.

I neglected to point out the efforts we have made to avoid tuning to too narrow a band of frequencies. As shown in Figs. 17 and 18 we have a flat-top curve. This was obtained by the use of the coupled circuit shown in Fig. 16. Its sole purpose is to broaden the frequency band so as to prevent distortion of speech. Mr. Boeck feels that the installation of resonant choke coils will interfere with the free use of other channels on the line. These chokes, in general, act only at the particular frequencies for which they are designed, and have little or no effect on other frequencies. The installation of chokes will not appreciably interfere with the free use of a line for other channels. Here in the South there will soon be in operation four simultaneous two-frequency communication channels.

2. See *Carrier-Current Selector Supervisory Equipment*, Stewart and Whitney, A. I. E. E. Trans., Vol. XLVI, 1927, p. 431.

Impulse Flashover of Insulators

BY J. J. TOROK* and W. RAMBERG*
Associate, A. I. E. E. Non-member

Synopsis.—For the most part this paper is a discussion of the mechanism of insulator flashover. The breakdown of air between electrodes, with particular consideration of the effect of various factors upon the flashover characteristics of insulator strings equipped with arcing rings, is discussed. It is shown that critical dimensions and spacings of arcing rings exist for a given insulator string. If the arcing rings are designed to lie on the safe side of this critical point, cascading flashovers will not occur. If the arcing ring dimensions are below the critical values, cascading may or may not occur depending upon the nature of the applied voltage wave. Data from tests, which determined the limiting physical dimensions of arcing rings that will prevent cascading under all types of impressed voltages, are presented.

* * * * *

Introduction

TRANSIENT voltages on transmission lines commonly are attributed to three sources: switching, induction from cloud fields, and direct lightning strokes. The most troublesome surges are the last two classes.[1] These surge potentials often exceed the insulation value of the line and a flashover follows. The surge flashover forms an ionized path which is highly conducting, and a short circuit on the energized transmission line results. The subsequent interruption may last for only a few seconds. But, if the power short circuit current shatters an insulator, the line may be out of service for several hours, an occurrence which it is extremely desirable to avoid. Because the power current will naturally follow the path established by the surge, a thorough study of the nature of surge flashover is involved directly in the problem of preventing insulator shattering.

Wave Shapes and Specifications

First, however, some attention will be given to the specification and classification of impulse voltages. The waves produced by prevailing types of lightning or surge generators can be analyzed when divided into two parts, the front of the wave in which the voltage rises from zero to the crest value, and the back or tail which is the part beyond the crest.

A wave often is defined by merely giving the constants of the circuit and the time to reach crest value. This is improper for three reasons: 1, The calculated wave form may be incorrect because of the variable resistance of the sparks closing the circuit; 2, Leads and test apparatus may cause distortion; 3, The crest value is not stated. It is particularly essential that the crest value be given in the above form of specification, since two similar waves having different crest values will have fronts of different steepnesses although they may reach their respective crests in the same total time. Because of the lack of a standard specification, where flashover tests are involved the authors have been defining the wave front in terms of the average rate of voltage rise of the "active portion," in kilovolts per microsecond. By "active portion" is meant the part of the wave between the 60-cycle crest flashover voltage and the surge flashover voltage for the particular wave considered. This is illustrated in Fig. 1. Assume that an insulator has a 60-cycle crest flashover voltage corresponding to point A on the wave and that flashover occurs at point B when this particular wave is impressed. No surge with a crest value below A will cause a flashover. Also, the part beyond B has little

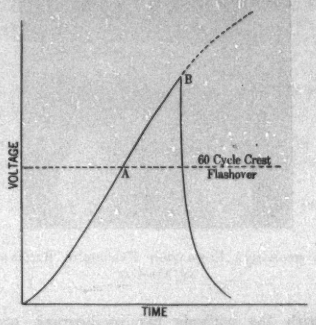

Fig. 1—Illustrating the Active Portion of an Impulse Flashover Voltage Wave

effect since the breakdown has already occurred. Therefore, the section AB has been designated the active portion. Symbolically the rate of rise is $(E_B - E_A)/(t_B - t_A)$ where E and t represent voltage and time respectively. The impulse ratio has been defined as the ratio E_B/E_A, but it should be remembered that this term is meaningless in itself. It must always be accompanied by the characteristics of the wave to which it corresponds. Only waves in which the active portion is on the front of the wave will be considered in this paper. When this is the case, a chopped wave is obtained and the effect of the tail of the wave is eliminated.

After trying and discarding most of the known systems for the determination of wave front, a new method was developed. The generator voltage is

*Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

1. For numbered references see Bibliography.

Presented at the Regional Meeting of the Southern District of the A. I. E. E., Atlanta, Ga., Oct. 29-31, 1928.

28-134

reduced by means of a resistance potentiometer, and the low-voltage wave is impressed upon a transmission line. Here, the wave is chopped with a sphere gap, resulting in a wave of the general form shown in Fig. 1. The chopped wave travels along the line and is impressed on a resistance capacitance circuit. From measurements of the chopped wave voltage and of the condenser voltage, the wave form can be calculated. This is a step by step method and necessarily long and tedious. Hundreds of voltage applications must be made for the determination of a given wave, and the method is valid only for the front of the wave. Check

Fig. 2—Streamer Formation Preceding Breakdown of an Air-Gap

tests with the cathode ray oscillograph prove its correctness.

Preliminary Tests

One of the first effects noticed was that the nature of the flashover of a string of insulators changed quite perceptibly with different types of applied voltage waves. When the active portion was at the crest of a wave or even on the front of a slowly rising wave the arc sometimes cleared all or nearly all the insulators. But, when breakdown occurred on the front of a steep wave, the arc formed between the metal parts of the insulators, cascading the string. Even with many forms of arcing rings and horns, cascading resulted when steep waves were applied. In designing arcing rings for an insulator string, the first consideration is to ensure that all arcs will take place between the rings. At the same time it is desirable to maintain or increase the flashover voltage of the string by improving the gradient along the string. However, the flashover voltage between rings under all conditions must be less than that of the string with improved gradient. Obviously, from the above, the most difficult condition to satisfy is the prevention of cascading with rapidly rising voltages, and this really includes all others.

We can now summarize the factors involved in a study of the surge flashover of insulators, as follows: the characteristics of the applied voltage, the breakdown of air at atmospheric pressure, the electrostatic field or voltage stress distribution, and hence, the nature of the electrodes.

Breakdown of Air

Laboratory experiments have furnished considerable information on this subject. The suppressed discharge,[2] in which the voltage is reduced suddenly to zero before breakdown is completed, shows very clearly the ionization processes. The air is left in an ionized condition, and the light emitted by recombination or by its secondary effects in the ionized sections can be seen or recorded photographically.

The photographs show the effect of extensive ionization especially at the most highly stressed points. Here, the stage of thermal ionization[1] is reached, and small white streamers are found growing from the electrodes as in Fig. 2. Since these streamers already have the properties of arcs,[3] they can be regarded as needle electrodes extending into the field. Therefore, the gradient at their tips will be very high, resulting in rapid growth.[3] As the streamers grow, the effective gap length is reduced and their development is accelerated. This process continues until the entire gap is spanned by the streamers. When this stage is reached, the gap is said to be broken down.

Thus, it is seen that the breakdown of air is a progressive process, and it requires a finite time or progresses at a finite speed. The speed of formation of the streamers depends upon the field through which they progress. Preliminary time lag tests with various types of fields have shown that the speed of formation is a function of the field distribution. The more homogeneous the field, the faster the streamers will develop, since, under these conditions, higher gradients at the tips of the streamers are possible. For instance, the streamers grow much faster in the uniform field of a sphere gap than in the non-homogeneous field of a needle gap. Since a definite time is required for the streamers to grow, in the breakdown of any gap, there is a definite time lag. A typical breakdown curve is shown in Fig. 3. The faster the applied voltage rises, the higher is the value that it will reach before breakdown occurs; and for the slower rates of rise, the breakdown voltage approaches the 60-cycle or static value. The nature of the time lag curve of an air gap depends upon the uniformity of the electrostatic field, and, therefore, upon the gap electrodes.

Breakdown of Insulators

The flashover of a string of insulators is not as simple a process as the breakdown of air between electrodes. A knowledge of the electrostatic field and of the stress

distribution along the string is necessary. Calculations based upon the potential theory advanced by Ollendorf[4] have been made and checked in the laboratory. Ollendorf replaces the insulator string by two spheres of suitable diameter, one representing the line insulator, and the other the ground insulator. He neglects the effect of the intermediate insulators upon the field by considering that their charges are bound. Thus, the effect of any charge is nearly neutralized by an equal and opposite charge displaced no more than the length of an insulator. Below breakdown values, voltage distribution curves calculated with these assumptions check closely with experimental curves. Fig. 4 shows the distribution on a plain string of seven suspension insulators raised to breakdown potential. Most of the stress is across the two units adjacent to the line. The seventh or line unit is stressed well above its breakdown value. However, there is a current between the porcelain sections of the string tending to equalize the stresses across the units. This is a conduction current through the air, and the maximum possible current density

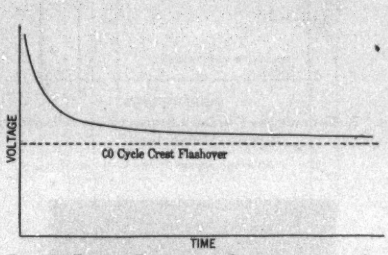

Fig. 3—Typical Breakdown Curve of an Air-Gap

without thermal ionization is low. Since time is required to transfer the charge, the average rate of voltage rise determines the equalization effect of this current. Therefore, with a rapidly applied voltage, one insulator will reach its breakdown value before the other units of the string. Streamers will start to form around this highly stressed unit. When it is shunted by streamers, its potential will be thrown across the remaining units causing a new voltage distribution. The process continues at an increasing rate until the breakdown is completed. A time lag or breakdown curve similar to that of an air gap exists for the flashover of an insulator string.

Arcing Rings

We may now consider a typical insulator string equipped with arcing rings. Data on a seven unit string equipped with 19-in. diameter rings of ¼-in. diameter material are available. The rings were located 3.8 in. from either end of the 39-in. string. Surges with a rate of rise greater than a certain critical value invariably cascaded the string. Surges with lower rates of rise struck between the rings. The comparative breakdown curves may be constructed as in Fig. 5. Since the rings have a lower 60-cycle breakdown value, the active portion of a given wave is reached at a lower voltage for the rings than for the insulators. The curves must cross at a point corre-

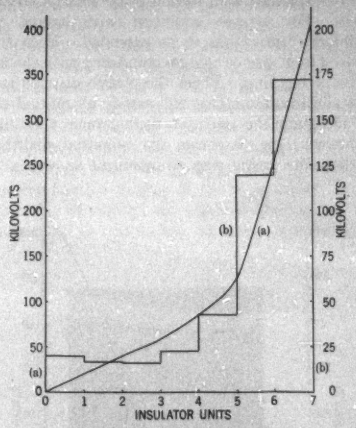

Fig. 4—Voltage Distribution (a) and Voltage per Unit (b) on a String of Seven Insulators

sponding to the critical rate of rise. It will be noticed that, for slowly rising waves, the active portion for the insulators may not be reached.

The following physical explanation may be made.

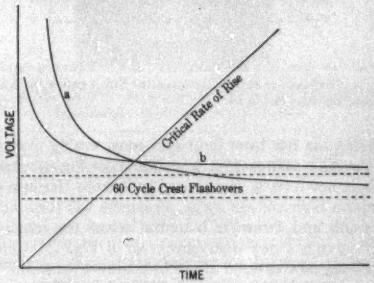

Fig. 5—Comparative Breakdown Curves for an Insulator String (b), and for an Air-Gap Between Arcing Rings (a)

For slow rates of voltage rise, the streamers forming from the rings span the entire gap before the active portion for the insulators has been reached. For moderate rates of voltage rise, the potential on the insulators reaches the active or streamer forming portion of the wave, but not until the ring streamers have almost completely spanned the gap. For rapidly

rising voltages, the ring streamer development is sufficiently slow to permit the insulator potential to rise above breakdown value. Hence, the insulator streamers develop into a cascading breakdown.

The rapidity of the insulator streamer development may be explained with the aid of potential distribution curves. For strings equipped with arcing rings, Ollendorf's theory has to be extended. Each ring is replaced by a ring of charges coinciding with the center of the arcing ring. Then, the charge distribution for rings and insulators that will satisfy all mutual effects and maintain the parts at their proper potentials is calculated. Fig. 6 shows the potential distribution for the same arcing ring arrangement as above. The

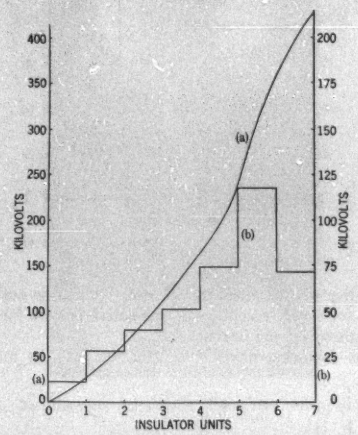

FIG. 6—VOLTAGE DISTRIBUTION (*a*) AND VOLTAGE PER UNIT (*b*) ON A STRING OF SEVEN INSULATORS EQUIPPED WITH ARCING RINGS, AT BREAKDOWN

distribution has been improved considerably over that of the plain string (Fig. 4). The most highly stressed unit is now No. 6, the second unit from the line end. Under a high voltage surge, streamers will form across this unit and throw its potential across the remaining units giving a new distribution as in Fig. 7. Units 5 and 7 are now stressed above breakdown value and will flash over simultaneously. Further analysis is impossible because of the changes in charge distribution, but it is probable that the remaining units will break down in pairs. Thus, the effective streamer development around the insulators is accelerated to such an extent that its growth is faster than that of the arcing ring streamers. Fig. 8 is a photograph of this type of breakdown on an eight unit string. The top insulator has already flashed over while units 2 and 4 are in the process of breaking down. The streamer formation from the rings is apparent.

Since it is possible to calculate the potential at any point around the rings, the gradients can be determined. Calculated gradients along the line between rings and along the units of the string are shown in Fig. 9. In this case the maximum gradients are at the surfaces of the arcing rings. Hence, the streamers will develop from the rings and meet in the middle of the field.

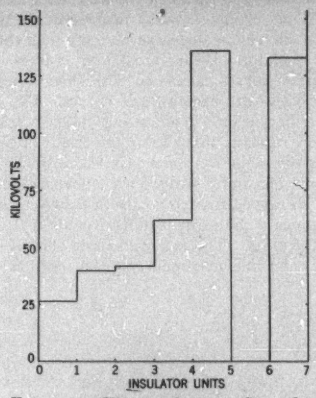

FIG. 7—VOLTAGE PER UNIT ON A STRING OF SEVEN INSULATORS WITH ARCING RINGS

After unit six has flashed over

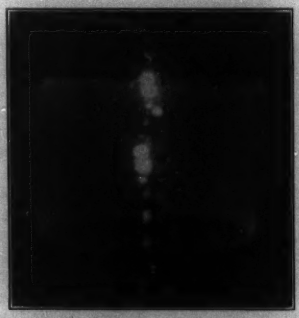

FIG. 8—SUPPRESSED DISCHARGE ILLUSTRATING THE SIMULTANEOUS BREAKDOWN OF TWO UNITS

Another important factor in the problem will now be considered. Assume that a string of insulators equipped with arcing rings as in Fig. 10 is subjected to an impulse voltage above the breakdown value. The metal parts of each insulator in the string will be raised to corresponding potentials, and a definite difference of potential or electric field will exist between each part of the string and each arcing ring and between the arcing rings. The gradient is highest at the ring

surface, and ionization will begin and the streamers will start to form at this point. There are two distinct components to the forces on the streamers, a ring-to-ring component and a ring-to-insulator-string component. These cause the streamers to branch out. The action is complicated by a progressive change in the field due to the conductivity of the streamers. However, the relative values of the components determine the rapidity with which the branches develop, and

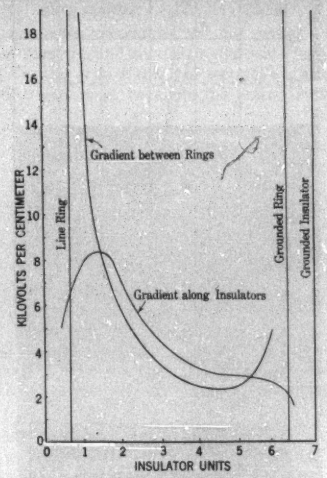

FIG. 9—VOLTAGE GRADIENTS AT BREAKDOWN ON A SEVEN UNIT INSULATOR STRING EQUIPPED WITH ARCING RINGS

therefore, whether the arc passes directly between the arcing rings or is drawn into the insulator string as shown.

Laboratory experiments indicate that the streamers upon formation are fairly high in resistivity, and probably only a part of the charge is transferred to their tips. For this reason, the streamers of opposing electrodes will have less effect upon each other than might be expected. However, as the streamer progresses, the insulator-streamer component increases due to the relatively higher potential of the insulators at this point. This effect is increased further under steep surges by the unbalanced rise of potential on the insulators. A slight bending of the streamer toward the insulators produces an increase in the streamer-insulator component which forces the streamer to develop still further in toward the insulators. Thus, a slight bending or branching of the streamers in the early stages of flashover will result in a cascading or partial cascading flashover. This point is particularly important for long strings such as 220-kv. transmission line insulation.

We may now determine the effect of various physical arrangements of the arcing rings upon their flashover characteristics. It is obvious that a homogeneous field between the rings is desired to speed up the streamer formation. The nearer the surface of the ring approaches that of a sphere, the more uniform the field will be. Therefore, the ring must be made of relatively large diameter material. To prevent the streamers from being drawn into the string, a ring of relatively large radius must be used. To aid in obtaining these results the spacing or distance between the rings can be reduced, and this is usually necessary. However, due to the improvement in voltage distribution along the string resulting from the grading effect, a flashover voltage comparable to that of the plain string can be maintained. Obviously, the grading effect is lessened by an increase in ring radius, and the most desirable

FIG. 10—SURGE FLASHOVER OF AN INSULATOR STRING WITH IMPROPERLY APPLIED ARCING RINGS

arrangement for a particular application will be a compromise.

ARCING HORNS

The arcing horn has characteristics similar to the needle gap. The stress distribution between opposing horns is very poor, since the section surrounding the horn tips has a relatively high gradient. Through a field of this nature the streamers will develop slowly, and the potential across individual units will have time to build up and cause cascading. To overcome this sluggishness of operation, the distance between the tips must be reduced considerably. This adjustment is doubly effective; the point at which breakdown begins or streamers start to form is lowered, and the breakdown distance is reduced. However, decreasing the spacing has the serious disadvantage that the

insulation of the line is lowered. When an arc exists, it must terminate at the horn tips and is quite likely to be blown into the insulators.

The characteristics of the arcing ring are inherently much better. The stress distribution is uniform permitting the streamers to grow rapidly. Consequently, spacings which do not lower the line insulation can be maintained. When an arc occurs between rings, it can travel around the rings. Hence, there is little chance of the arc being blown into the insulator string.

Tests

Test data on various arcing ring arrangements have been accumulated. Standard suspension insulator units 10 in. in diameter by 5⅝ in. spacing were used in the tests. Rings of ¼ in. diameter copper were first used to facilitate variation of the ring diameter and ring to ring spacing. The average rate of rise of the active portion of the generated wave was 7000 kv. per

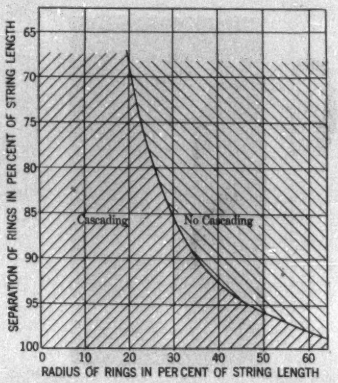

Fig. 11—Critical Arcing Ring Arrangements for Cascading and Non-Cascading Flashovers on a Seven Unit (39 in.) Insulator String

microsecond. This is somewhat steeper than lightning produces on transmission lines, as indicated by available data.[1] However, extensive measurements of voltage or wave form were not made.

First, the rings were set at a definite spacing, that is, a definite percentage of the string length. The radius of the rings was increased until cascading ceased entirely with repeated voltage application. Thus, the minimum ring radius for a given spacing was obtained. Similarly, the maximum spacing for a given ring radius was determined. Fig. 11 where critical ring spacings and ring radii are plotted in terms of the string length, shows graphically the results of such a test for a seven unit string. The curve divides the cascading and non-cascading regions. Thus, in order to keep the ring spacing or flashover distance above 90 per cent of the length of the plain string, for this particular case, the ring radius must be 40 per cent of the length of the string. Where the ring radius is limited by tower clearances, the curve gives at once the maximum ring spacing. Similar tests were made with ring material of 1½-in. and 2-in. diameter. The results for the seven unit string are shown in Fig. 12. For close spacings the larger diameter material is of no advantage. At greater separations the larger diameter material permits the use of rings of smaller radius.

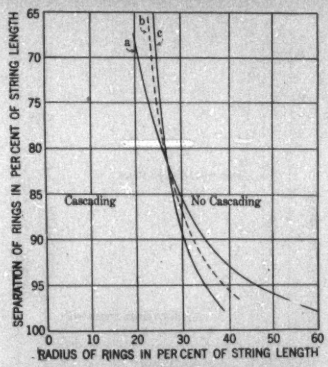

Fig. 12—Critical Arcing Ring Arrangements; (a) ¼-in. Diameter Material, (b) 1½-in. Diameter Material, (c) 2-in. Diameter Material; for Same String as Fig. 11

Fig. 13—Arcing Rings Designed for a 16-Unit Insulator String

With this information arcing rings for any particular set of conditions can be designed, as illustrated by the following example. Assume a line with seven suspension units 38.5 in. in length and with tower clearances which limit an arcing ring to a radius of 14.5 in. The ring radius is thus 38 per cent of the string length. For

two inch diameter material, Fig. 12 indicates that the maximum spacing for this ring radius is 96 per cent of the string length. To secure a slight margin of safety by avoiding the critical cascading point, a spacing of 90 per cent might be chosen. Thus, the most satisfactory arcing rings for these conditions have the following dimensions:

Material diameter	2 in.
Ring diameter	29 in.
Ring spacing	35 in.

In some cases, clearances will not permit a ring of suitable diameter, and an oval or elliptical ring must be used. For this type of ring, the sections at the ends of the major axis must be made the active or working

Fig. 14—Streamer Formation During Breakdown

parts. This is accomplished by making the spacing between the active parts less than between the sections around the minor axis. Fig. 13 shows oval rings designed for a 16 unit string. The arc striking sections are of large diameter pipe and furnish the minimum breakdown distance for the string. Care must be taken to avoid abrupt bends in changing from one section of the ring to another. Fig. 14 shows the streamer formation at the stage close to complete breakdown on the same rings.

Conclusions

1. The breakdown of air is a progressive process requiring a finite period of time for the streamers developing from one or both electrodes to span the distance between electrodes. The time required is a function of the average gradient of the field.

2. The flashover of a string of insulators is a similar though more complicated process.

3. The nature of the flashover depends upon the rate of rise of the applied voltage. Cascading of the units of an insulator string is more likely to occur the faster the rate of voltage rise.

4. Cascading flashovers occur on the application of rapidly rising voltages because of the limited values of the leakage and conduction currents which tend to equalize the stress distribution of the units of the insulator string.

5. The path formed by the surge flashover furnishes a low resistance path for the power follow current.

6. Arcing rings will prevent cascading flashovers provided the time lag characteristic curve for breakdown between the rings is below the corresponding curve for flashover of the insulator string. This condition may be satisfied by conforming to definite physical relations between the ring to ring spacing, the radius of the rings, and the diameter of the ring material, as functions of the length of the insulator string to be protected. Furthermore, the radius of the arcing rings must be great enough to prevent the streamers from being drawn into the insulator string after they have started to form at the active sections of the arcing rings.

7. Arcing horns are unsuitable because of their high time lag characteristic, the decrease in flashover voltage necessary, and the likelihood of the arc being blown into the insulator string.

The authors wish to express their appreciation of the assistance contributed by Messrs. J. Slepian, C. L. Fortescue, and P. H. McAuley.

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Discussion

For discussion of this paper see page 254.

A Study of High-Voltage Flashovers

BY JOSEPH T. LUSIGNAN, Jr.[1]

Associate, A. I. E. E.

Synopsis.—*A brief discussion is given of the various forms of electrical discharges as they are characterized by the physicists.*

Oscillographic records secured of voltages and currents, both before and during the flashovers, are reproduced and discussed. Polarity tests are described in which the electrode shape and spacing were found to determine the sign of the half-cycle at which flashovers would start.

Photographic results obtained with moving films are illustrated, showing the various stages of a flashover from the formation of the corona discharges, through the initial sparkover, to the development of the half-cycle arc channels. The flashovers studied ranged from several feet to 22 ft. in length.

Spectrographic analyses of the different stages of a flashover are described, wherein it is possible to gain some knowledge of several of the factors contributing to the conductivities of the various stages. Several of the spectra obtained are illustrated along with a comparison spectrum of lightning.

With the knowledge gained from the laboratory studies made, the probable history of a high-voltage a-c. flashover is summarized.

* * * * *

Introduction

THE object of the study described here was to secure a better understanding of the development of a high-voltage a-c. flashover, particularly with a view to investigating the successive stages of discharge involved, and the various factors contributing to their existence,—from the initial localized breakdown of air, or corona, to the final stage of a fully developed power arc. The entire experimental work was carried out in the Harris J. Ryan High-Voltage Laboratory at Stanford University. Due to the limited kilovolt-ampere capacity of the laboratory equipment and of the connected power line supplying it with energy, it was not possible to secure arcs of sufficiently high-current density and of adequate time duration to simulate a truly high-power arc as might develop on a transmission system. However, from the analyses that it was possible to make of the arcs formed, it is felt that a sufficient knowledge was gained of the various factors that would enter into the later stages of a power arc, to make possible the completion of the story, and in that way allow of the securing of the entire history of a high-voltage flashover.

Electrical Discharges in General

Our knowledge of the mechanism of electrical breakdowns is largely derived from the work of the physicists who, as a rule, have used short gaps with simple forms of electrodes and gases in order to bring in as few contributing factors at a time as possible. Although the particular problem here involved the study of 60-cycle discharges through comparatively long distances and under atmospheric conditions, it was nevertheless essential to turn to the findings of the physicists in order to secure better analyses of the results obtained.

Dr. K. T. Compton recently gave a valuable paper before the Institute[2] in which he described the nature of the electric arc and other related phenomena. He particularly shows the difficulty of always drawing sharp lines of differentiation between arcs, sparks, glow discharges, and coronas, due to the fact that they share so many characteristics. In counter-distinction to the spark, the arc is usually considered to involve the greater current density with a lower voltage drop, yet there are sparks obtained which carry more current with less voltage than some arcs. As a rule, physicists differentiate the two by means of the respective cathode voltage drops, the spark having considerably the greater of the two, while some add the condition that the arc must have a negative volt-ampere characteristic.

The engineer is used to defining the arc in various ways, but generally he associates it with types of conduction occurring when metal vapors from the electrodes furnish the conducting path, coupled with possible thermionic emission from the cathode, electrons being emitted in the same manner as from the hot filament of a vacuum tube. Physicists, though, have obtained arcs between rare earth oxides in air which are maintained solely by thermionic emission. There are still other continuous discharges drawing fairly heavy currents and with current characteristics of arcs, which exist in gases between non-volatile electrodes and without hot cathodes. These, therefore, involve neither metal vapors nor thermionic emission. The ordinary neon discharge tube has characteristics[3] which would probably allow it to be cited in this category.

The spectroscopist has characterized arcs and sparks in terms of the spectra produced. In general he would define an arc as any discharge leading to the excitation of lines or bands in the spectrum, indicating excited states of neutral atoms or molecules, or possibly the first stage of ionization. Sparks would be characterized as discharges leading mainly to spectral lines indicating higher stages of excitation and ionization. The difference depends mainly on the drops of potentials through which the colliding electrons, causing the radiations, have fallen.

1. General Electric Company, Pittsfield, Mass., formerly Fellow in Electrical Engineering, Stanford University.

2. See bibliography for references.

Presented at the Regional Meeting of the Southern District of the A. I. E. E., Atlanta, Ga., Oct. 29-31, 1928.

28-136

Oscillographic Study of Flashovers

Voltage and Current Waves. In this study, voltage records of flashovers were first secured by attaching a shielded water column resistance to one of the high-voltage electrodes and inserting an oscillograph element into the ground end of the column, so as to obtain current waves there, which would be indicative of the electrode voltages to ground. Two of the records obtained in this manner are shown in Figs. 1 and 2.

Fig. 1 shows the voltage wave of the initial part of a 20-ft. flashover, the formation point of the arc being on the half cycle where the sharp drop in voltage occurs. Each half cycle after that is seen to start with a brief rise in voltage which breaks down the gap again and allows the arc to re-form, whereupon the voltage drops for the remainder of the half-cycle. If the potential applied to the electrodes is allowed to decrease after the arc forms, then the latter may break of its own accord. Fig. 2 shows the corresponding voltage wave of the final stage of such an arc. The record represents two revolutions of the film, and starts at *a* on the irregular wave of the arc itself. Here it is again apparent how each half cycle starts with a brief rise in voltage which each time breaks down the gap, but as the record progresses and the current decreases due to the applied voltage being allowed to fall, the voltage has an increasingly harder task to maintain the half cycle discharges so that the initial half-cycle voltage needles continue to increase in magnitude. Finally at the point *b* the voltage is no longer able to re-form the arc for that half cycle and the regular open-circuit voltage wave starts, which continues around on the film a second time and drops to zero at *c* when the circuit breaker opens.

Fig. 1—Voltage Wave at Start of 60-Cycle Flashover

Arrow indicates point of breakdown

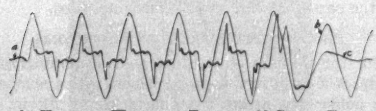

Fig. 2—Voltage Wave at End of 60-Cycle Flashover

Arc broke due to lowering of applied voltage

In order to secure simultaneous current and voltage records prior to flashover, a two-element oscillograph was mounted at the top of the water column. One element was connected into the water column circuit so as to obtain a wave indicative of the electrode voltage, while the other element was connected to an insulated lead running to the point of the electrode, thereby allowing a wave of the discharge current from the point only to be secured. One of the records obtained in this manner is shown in Fig. 3. The current wave is seen to be decidedly unsymmetrical with respect to the two halves of each cycle, indicating the rectifying effect that is always associated with a point in corona. On the positive current crests a series of discharge needles can be detected whereas on each of the negative crests usually but one appears. Current waves, taken as the voltage across the 20-ft. gap is slowly increased from pre-corona values, show that the positive current needles appear soon after corona starts and occur regularly on the positive half cycles. The negative half cycles of current increase in magnitude as the voltage rises, but the brief discharge needles do not appear until almost the point of flashover, when they usually occur but once on a crest. This feature will be seen later to check well with photographic observations taken on moving films.

Fig. 3—Corona Current Prior to Flashover

A. Corona current wave prior to flashover of 20-ft. point-gap
B. Corresponding voltage wave

Fig. 4—Flashover Current from Point Electrode of 20-Ft. Gap

Arrow indicates start of flashover of 14 ½ cycles duration

By using a calibrated shunt on the oscillograph element it was possible to secure a current wave of the flashover itself, the shunt, of course, diminishing the corona current values to negligible magnitudes. Fig. 4

shows an oscillogram taken in this manner. The approximate effective value of the first half cycle of flashover is 1.63 amperes.

Determination of Flashover Polarities. As an added step in the study of flashover characteristics, tests were made to investigate the polarities at which different gaps would arc-over to ground at 60 cycles, and in that way ascertain whether this phase of a flashover took place in any orderly manner. With direct current, the point-to-plane set-up has always been found to arc-over at a lower potential when the point was positive than when it was negative, except at small spacings of the order of a fraction of an inch;[4] therefore, this arrangement was tried first to see whether the same was true for alternating voltages. About 50 flashovers were taken with spacings ranging from 5 to 10 ft., and oscillographic records of the current in each case were

Fig. 5—Voltage Wave Just Prior to Flashover of 18-Ft. Point-Gap

Arrow indicates brief drop in voltage at wave crest due to transient sparkover, one cycle before start of flashover

secured in the ground end of the high-voltage winding. In every instance the flashover started on the half cycle when the point was positive.

A similar test was made on 50-cm. spheres with the bottom sphere grounded. With the spheres spaced 50 cm. apart, approximately 50 flashovers were taken, and found to start with the ungrounded sphere negative, which was just opposite from the results with the point-to-plane set-up. The sphere-gap was next opened up to a maximum spacing of 69 cm., and the arc-overs were found to start on both halves of the waves, that is, the spheres flashed over from the upper sphere on either the positive or negative crests. There were about twice as many initial positive flashovers as negative, but an insufficient number was taken (about 40 in all) with which to predict any fixed ratio.

From the above results it seemed that initial local breakdowns due to concentrated field intensities would cause a flashover to start positively from the ungrounded electrode, while, on the other hand, with a more or less uniform field, such as a sphere-gap at close spacing, the arc to ground would form on the negative crest of the wave. To test out this theory roughly, a short ⅜-in. barb was placed on the under surface of the ungrounded sphere, and the gap arced over at a 50-cm. spacing, where it had previously flashed over to ground on the negative crests. However, the barb caused the polarity to reverse and the flashovers started positively. With the same barb on the bottom sphere, the flashovers started on the negative crests, probably because of the lesser field concentration on the grounded sphere. The light from a quartz mercury arc lamp was also played on the spheres to ascertain its possible effect on the polarity of flashovers. It was found to produce no change, but the intensity of the ultra violet light reaching the spheres from the distance away that the lamp had to be mounted was afterwards determined to be very small.

Sixty-cycle polarity tests were also made on shielded and non-shielded suspension insulator strings of 10 and 14 units respectively, where it was found that practically all flashovers to ground started on the conductor positive crests. Occasionally one would take place on a negative crest, but the average was never more than about one out of every 25 discharges. These results would seem to fit in well with d-c. and impulse insulator flashover tests made by Peek[5,6] in which he found the arcs to form on lower positive potentials than negative.

The reasons for the behavior of the various forms of gaps with respect to the polarities of the flashovers can probably be traced to the difference in mobilities of the ions and electrons, and their paths as controlled by the particular electrostatic fields involved.

Photographic Studies of Flashovers

For the purpose of securing possible photographic analyses of flashovers, a special camera with a revolving film holder was made. A quartz lens of 2¼-in. diameter, loaned by Mr. Peek, was used in it, and the continuous film placed on the revolving cylinder which was arranged at the focal point of the lens. By driving the cylinder at different speeds it was possible to draw out the discharge figures on the films as much as desired; and by the use of shutters actuated by relays, the films could be exposed to any phase of a flashover.

Figs. 6 to 10 show some of the moving film records obtained in this manner. In each of the figures it is apparent how the path of the first breakdown, shown by the brief but heavy discharge at the start of the first half cycle, practically determines the path of the succeeding half cycles.

The manner in which the streaks of light are drawn out on the film after the last half cycle of arc proved of interest. They undoubtedly can be accounted for in the following manner: The arc itself is probably not more than a fraction of a millimeter in thickness

but at the points where it is folded or bent there are several millimeters or more of arc length in the line of vision. Therefore, at these folded points there will be much stronger light intensities which will have greater effects on the photographic film. The ionized gas remains partly incandescent even between half cycles when the current is passing through zero, which densitometer measurements verified, but it is only the more intense folds of the arc that appear to persist on the film between half cycle bands. Accordingly, after the last half cycle, it is these same folds that streak out as if by themselves. Undoubtedly the whole path across the gap remains incandescent, but only those parts that are bent to appreciable length in the line of vision seem to show up on the film, those of the greatest depth lasting longest. In some of the films obtained, these folds persisted as long as three cycles after the arc-overs had ceased, indicating that the gas in the gap remained incandescent that long.

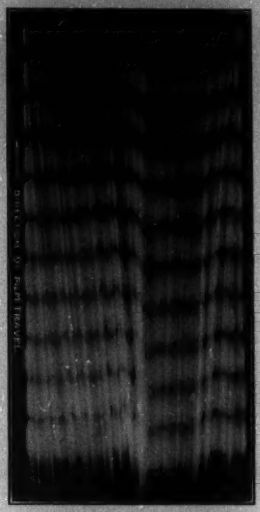

Fig. 6—Reproduction of Moving-Film Photograph of 10-Ft. Horizontal Flashover of 11 Half Cycles Duration Between Pointed Electrodes

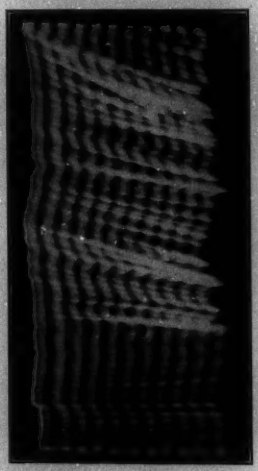

Fig. 7—Nine-Foot Vertical Flashover from Point to Floor

Moving-film photograph of flashover of 11 half cycles duration

Fig. 8—Start of Nine-Foot Vertical Flashover from Point to Floor

Arrow indicates single intense corona streamer on negative half cycle prior to flashover. Moving-film photograph with film speed approximately twice that of Fig. 7

Fig. 6 is an example of one of the faster moving films wherein the half cycle bands are drawn out to a width of about one inch each. The overload relays opened the circuit in this case in five and a half cycles so that eleven half cycle bands appear on the film. In one test, the generator field was quickly reduced at the start of arc-over so that the arc would break of its own accord after about twelve cycles. The moving film photograph taken in this case showed an appreciable decrease of incandescent gas particles after the last half cycle of arc-over, undoubtedly due to the very low final current density compared to the cases where the opening of the oil breaker terminated the arcs.

In the first work with the 10-ft. horizontal point-gap, both sides insulated, it was noticed that the brush discharges, from each point prior to arc-over, were only perceptible at the positive half cycles on the moving films. When the gap was next opened up to 22 ft., though, discharges appeared on every half cycle just before arc-over. On the films, the alternate half cycles of brush discharge were found to be strikingly similar,

Fig. 9

Fig. 10

Fig. 9—Moving-Film Photograph of 60-Cycle Flashover

11 half cycles duration, between 25-cm. spheres, 25 cm. apart

Fig. 10—Moving Film Photograph of 30-In. Discharge from Point to Plane

Air condenser in parallel with gap energized preliminary discharges on five positive half cycles preceding start of flashover

while the succeeding half cycles were different, indicating that the positive and negative discharges each had their own decided characteristics. As the point discharge first begins to appear on the moving film with the rising voltage, there can be seen on the negative half cycles, either no visible signs of discharge, or else one heavy stalk or jab with a corona head at the end of it (see Fig. 8), indicating a small voltage drop along its path, so that the field terminates largely at its end and produces a brush head effect there. As the voltage approaches arc-over these intense white negative jabs increase in length and brilliancy, and may occur more than once on an occasional half cycle. The positive discharges, however, seem to be more prolonged over their corresponding voltage crests, branching out continually and to greater lengths than the negative discharges, but with less luminous intensity than the latter. They come in earlier also, and consistently appear at each alternate half cycle. The fact that no long intense negative streamers appeared at the 10-ft. spacing, both electrodes insulated, apparently indicated that they required much higher voltage gradients.

In the work on space charge carried out at Stanford two years ago,[7] it was found that at the start of corona, the region between two conductors was built up to a slight negative potential above ground due to the corona rectification. The sign of the charge soon reversed with increase in voltage and near the flashover voltage the space had been charged up to an appreciable positive potential above ground. Since it is at this

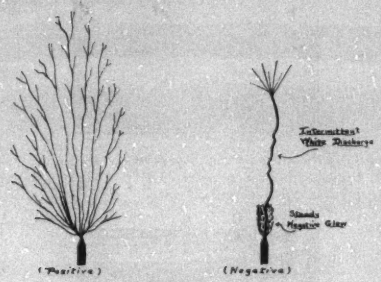

Fig. 11—Corona Point Discharges at One Million Volts

point that the long intense streamers appear from the negative electrode, it would seem that they merely serve to dart out at intervals to drain different portions of the field of the heavy positive space charges there.

By means of a stroboscope it was possible to check visually the electrode discharge figures obtained on the moving films. At the start of corona the discharges resembled very closely those found by Peek,[8] in that at the negative electrode there was a rather small even glow while the positive electrode exhibited a long dispersing discharge. Both increased in size as the voltage was raised until at a million volts above ground the positive discharge resembled a huge tree-like figure surrounded on all sides by a bluish haze, while at the negative electrode a rather short glow persisted from which intense white streamers, several feet in length, darted out at intervals of a second, more or less. Fig. 11 shows diagrammatically the approximate shape of the positive and negative discharges at a million volts above ground.

The physical appearances of the point brush discharges found above seem to display features which fit in rather well with the oscillographic records of corona current obtained. On these current waves, one of which is shown in Fig. 3, the many needle jabs on each positive half cycle probably serve to supply the tree-like channels continually branching out when the electrode is positive. On the negative current crests the smooth part of the wave is probably indicative of the current supply to the short negative glow, while the single sharp peak with it, if it occurs, would be associated with the ordinarily single heavy streamer that is seen on the negative half cycle on the moving film.

In order to determine the effect of additional condenser capacity in parallel with a discharging electrode, an 8 ft. by 10 ft. mesh plate was supported 4 ft. from the floor and connected to a point 30 in. above the floor. A moving film photograph of the discharge across the latter gap is shown in Fig. 10. The figure represents about one and a half revolutions of the film so that the record overlaps partially. The first visible breakdown occurs at *a* on a positive half cycle. Two discharges each appear on succeeding positive half cycles *b* and *c*. On the next one *d* three occur, and the next *e*, there are four. At the half cycle *f* after four intense discharges have occurred, the arc forms, and persists for the remainder of the record. The arc appears every half cycle and comes around on the film once more, finally ceasing at *g* after 11 half cycles of duration. At the early part of the arc stage, each half cycle is seen to be started by a rather intense breakdown flash. As the arc progresses and the conductivity increases this initial half-cycle flash is found to decrease in intensity.

Since the previous polarity tests showed positive discharges to be particularly favored on point-to-plane set-ups, apparently some of them temporarily bridged the gap above on several positive half cycles before arc-over so that the energy of the paralleling condenser would come in to intensify the brief breakdowns each time. Finally, though, when the voltage had been raised sufficiently, the condenser was able to pour enough energy into the breakdown to render the path sufficiently conducting for an arc to form. From this it would seem that added capacitance might tend to lower the flashover voltage of such a gap a trifle, since it would provide more energy for one of the earlier discharges. Extra resistance for damping or reactance directly behind the point, on the other hand, might have the opposite effect by checking the inflow of condenser energy.

The same test was repeated with 25-cm. spheres, spaced 25 cm. apart, as discharging electrodes, the top sphere being connected to the condenser and the bottom one grounded. There were no visible discharges across the gap on the moving film in this case prior to complete arc-over, as occurred with the point-to-plane arrangement, probably because no appreciable corona brushes appeared from the smooth spheres. The only apparent effect of the condensers was to intensify the initial heavy flash starting the first half cycle of arc-over.

In these last short gap set-ups the initial intense discharges between the electrodes preceding the first half cycle of arc-over appeared to be of highly oscillatory nature on the revolving films. With the longer electrode spacings, particularly those of approximately 20 ft. between points, and of 10 ft. from point to plane, these discharges were extremely brief so that no oscillations could be detected on the moving films. The current oscillograms though usually showed a high-frequency oscillation at the time of initial breakdown, but they were generally of not more than one cycle duration, probably due to the damping effect of the 400,000 ohms of protective resistance connected behind each electrode when attached to the 2,000,000-volt transformer set.

In photographs of laboratory flashovers of insulator strings, condenser-like discharges are sometimes visible along different paths than that over which the actual flashover finally develops. These are undoubtedly initial discharge breakdowns to ground at a time when the voltage was too low to provide sufficient energy ($1/2\ C\ E^2$) to render the path conducting enough for the arc to follow. Fig. 5 shows the oscillogram of a voltage wave several cycles before flashover of a 20-ft. point gap, where a brief drop in voltage at one crest can be seen, which probably occurred simultaneously with some such temporary pre-flashover condenser discharge.

Spectrographic Analysis of Flashovers

Spectrograms were taken next of the different stages of a flashover in an attempt to identify some of the factors contributing to its various conductivities. By arranging a shutter that would open for only the start of a flashover, it was possible to secure the spectrum of the initial intense discharge which was found to bridge the gap at the beginning of the first half cycle of arc-over on each of the moving film photographs, (of Figs. 6 to 10). Fig. 12A shows one of the spectra, obtained in this manner, of a 10-ft. discharge from a point to a plane. This was identified as the spark spectrum of air and is practically identical with that of lightning, as can be seen from a comparison with a lightning spectrum as obtained by Fox[9], which appears in Fig. 12B. Due to the different dispersions of the two spectrographs used, the separations of the spectral lines are of course not the same, but their order matches up perfectly. In every laboratory flashover so analyzed the initial discharge was found to give the spark spectrum of air. These spark lines were always found perceptibly stronger at the center of the gap than at the electrodes, probably due to the intense breakdown discharge at that point between the space charges

built up at the head of the advancing corona streamers from opposite electrodes.

In these spectrograms of the initial spark it was noticed that arc lines of metal vapor appeared at the electrodes. In the discharge of Fig. 12A, where an aluminum electrode was used, typical aluminum arc lines are visible at the left end. In order to ascertain how soon metal vapor appeared in a flashover, a moving film spectrogram of about one inch of the discharge at the electrode point was obtained, making use of a slit. This was carried out for aluminum, copper, and iron electrodes. In every case the presence of metal vapor was detected in the initial spark discharge starting the first half-cycle of arc-over, the aluminum showing up most intensely. Fig. 13 shows one of the moving film spectra obtained with an iron electrode. There were apparently two spark discharges at the start of the

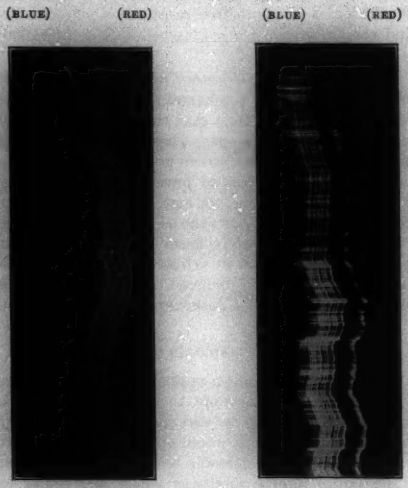

FIG. 12—COMPARISON OF LABORATORY SPARKOVER AND LIGHTNING SPECTRA

(a) Spectrum of initial spark bridging 10-ft. point-to-plane gap, which started first half cycle of complete 60-cycle flashover

(b) Spectrum of lightning stroke. (Fox-Yerkes Observ.)

first half-cycle of arc, as two spark spectra of air are visible on the film there. Faint arc lines of iron can be detected in these first spark discharges while the succeeding eleven half cycles of flashover show arc lines of iron vapor only. Since iron vaporizes at 2450 deg. cent. it would seem that temperatures sufficient for thermionic emission at the electrode are probably reached at the very first spark breakdown of the gap. Other factors than heat may possibly come in, though, to facilitate vaporization during the first air spark, since the initial high-voltage gradient at the electrode may provide sufficient bombardment by positive ions, or strong enough electromechanical forces to remove metal particles from the electrodes which would in turn be vaporized. It is interesting, nevertheless, to find metal vapor present within the first one thousandth of a second of flashover.

It was not possible to secure a recognizable spectrum of the arc itself in the center of the gap after the first sparkover, due to its continually changing its path over each half cycle. The light intensity of the arc part of the flashover was not great enough either, with the spectrograph used and the distances that had to be maintained from the gaps, to allow of the use of a slit or of a rapid shutter action. Therefore, one was unable to determine whether the arc lines of air only would be present in the center of the gap after the first breakdown spark. No spark lines were found other than those of the initial discharge. In some short spectrographic exposures made of the 10-ft. flashovers though, it was apparent from the metal arc lines visible that metal vapors extended out from the electrodes an appreciable distance within the first two cycles of flashover, at least 8 or 10 in. in the case of iron, and about 6 in. with aluminum.

Spectrographs were taken of the positive and negative corona discharges prior to flashover, using a stroboscope. Exposures of about 20 sec. each were necessary in this, so that only that portion of the discharge next to the point remained sufficiently stationary to give a recognizable spectrum. Here the band spectrum of molecular nitrogen was found for both the positive and negative discharges, with no evidence of any oxygen radiations or metallic excitation. This is somewhat in accordance with the results of others.[10,11] This band spectrum is typical of the arc form and it was hoped that the intense white negative streamers might be shown to involve spark radiation spectra, at least at their outer high-voltage ends. Each was too brief in duration, though, to allow of analysis by spectrographic means.

THE PROBABLE HISTORY OF A HIGH-VOLTAGE A-C. FLASHOVER

Factors Involved. Undoubtedly ionization by collision, as originally suggested by Townsend and later amended by others,[12] plays an important role throughout an entire high-voltage a-c. flashover. At the start it is the essential factor in breaking down the air column between electrodes. Later thermionic emission and metal vapors enter to enhance the conductivity of the channel, the former giving its pure electron supply from the electrodes, while the other provides a much more easily ionized medium for conduction. Metal vapor is particularly favorable to high-arc conductivity for the following reasons:

1. It has from one-half to one-third the ionizing potential that such gases as oxygen, hydrogen, or nitrogen have, thereby requiring lower voltage gradients for maintaining a given current through it.

2. It does not become the electron trap that such

atmospheric gases as oxygen do, for oxygen tends to remove electrons by forming ions, thereby hampering the discharge.

3. Metal vapors have more elastic impacts than gases like oxygen or nitrogen, in that way removing less energy from the ionizing electrons when they strike.

Process of Breakdown. When the potential across a gap or over an insulator string becomes great enough,

FIG. 13—MOVING-FILM PHOTOGRAPH OF SPECTRUM OF DISCHARGE AT IRON POINT ELECTRODE

Shows approximately 1 in. of 10 ft., 60 cycle, flashover to floor. Two brief discharges, each giving spark lines of air coupled with slight arc lines of iron, appear on the left at start of first half cycle of flashover. All eleven half cycles of flashover show arc spectrum of iron.

due either to the alternating voltage being increased, or an impulse wave superimposed, two corona streamers from opposite electrodes will meet and bridge the gap. (No experimental evidence yet has served to explain the exact details of this contact.) The condenser capacity of the electrodes and its attached parts will then discharge across, giving an intense initial spark. If, due to a too low applied voltage, the stored electrostatic energy ($1/2\,C\,E^2$) is not great enough to provide a sufficiently heavy discharge for starting adequate thermionic emission or metallic vaporization at the electrodes, the gap breakdown will cease, and the voltage wave will return to normal, similar to the one shown in Fig. 5. On the other hand, should the initial discharge be ample to create sufficient thermionic emission and metal vaporization at the electrodes, (and possibly a certain amount of sufficiently ionized and excited gaseous bodies in the gap), then the conductivity of the channel will be increased enough to allow the discharge to be maintained though the voltage wave continues to drop through the remainder of the half cycle, as seen in Fig. 1. On the next half cycle the discharge has to form again, but due to many of the particles in the gap being either already ionized or in an unstable state, it requires only a small voltage rise before the gap is again broken down, whereupon the voltage wave falls once more. As the arc-over current increases, which it surely must unless the effective applied voltage is dropped appreciably, less and less voltage is required to break down the gap each half cycle. Probably ionization by collision occurs at the beginning of every half cycle, although in a fully developed power arc, with copious thermionic emission and metal vaporization in existence, it would require but a small initial voltage for it to start the discharge each time. Thermal agitation and slight photoelectric effects probably enter to assist in the ionization process by "exciting" neutral bodies, but it is highly questionable whether they in themselves do any actual ionizing.

Fig. 14 is that of a photograph loaned by Mr. Harold Michener of the Southern California Edison Company. The main photograph shows a complete 50-cycle power arc. The insert in the corner is a fiftieth of a second exposure of the arc, taken after the latter had persisted for some time. This latter figure brings out well the tendency of each half-cycle discharge to develop its channel along the path of the rising ionized gases and vapors, however greatly distorted and lengthened this path may be. When the length of path becomes so great that its resistance drop exceeds the applied voltage, the arc will re-form along a new discharge channel at a lower point.

ACKNOWLEDGMENTS

Doctor Ryan suggested this study of flashovers and has been a valuable source of assistance and guidance throughout. Professor J. S. Carroll took part in all of the tests and is largely responsible for many of the results obtained. Mr. Bradley Cozzens of the laboratory staff also assisted in much of the work.

FIG. 14—TYPICAL POWER ARC ON TRANSMISSION SYSTEM

Photograph of complete 50-cycle arc on Southern Calif. Edison Co., with insert showing one-fiftieth second exposure of same

Doctor L. B. Loeb of the Physics Department of the University of California very generously helped the author to secure the facts regarding electrical breakdowns as known to physicists, and continually pointed out the meaning of many of the laboratory results obtained. Valuable assistance in the spectroscopic analysis of flashovers was secured from Professor F. S. Brackett of the University of California Physics De-

partment, and from Professor J. G. Brown and Dr. G. R. Harrison of the Physics Department at Stanford.

A subsequent paper involving the measurement of the flashover voltages themselves was presented before the Pacific Coast Convention of the Institute at Spokane, Washington, August 28-31, 1928 by Professor J. S. Carroll and Mr. Bradley Cozzens, page 1.

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Discussion

IMPULSE FLASHOVER OF INSULATORS

(Torok and Ramberg)

A STUDY OF HIGH-VOLTAGE FLASHOVERS

(Lusignan)

Atlanta, Ga., October 30, 1928

T. F. Ball: Whereas Dr. Lusignan's paper has brought out and helped to clarify many of the phenomena pertaining to high-voltage flashovers, still I feel that in order to get a more complete physical concept of the phenomena involved we should take into account the effect of having the electrodes and the spark-gap itself exposed to radiations of different frequencies, under the same conditions of pressure and voltage between the electrodes. This, of course, would entail an application of the well-known quantum theory. For example, let us consider the case in which we have two pointed electrodes of copper, each having the same radius of curvature, and separated a certain distance d apart in air under normal atmospheric pressure.

If a d-c. potential be applied to the electrodes so that one becomes positive and the other negative, it is obvious that as the voltage is raised the neutral gas atoms in the space between the electrodes would become more and more distorted until finally the outermost electrons in the atoms would be moving in such eccentric orbits as to allow them ample opportunity for escape from their parent atoms. This, of course, would mean that these electrons, which have just broken away from the neutral atoms, would then be free to move under the influence of the electric field, thereby causing an increase in the ionization already begun between the electrodes. This cumulative effect would naturally persist until a rupture or flashover occurred.

Moreover, those free electrons, which are contained within the negative electrode, will have a tendency to be pulled across the surface of this electrode, and then under the influence of the strong electric field they would be drawn toward the positive electrode with considerable velocity. Hence, we see that these fast moving electrons would aid greatly in causing additional ionization in the space between the electrodes. Such an effect would be a contributing factor in causing the flashover to occur. Now, if the electrodes were more pointed it is obvious that less work would have to be done by the electrons in coming across the surface of the negative electrode, and therefore those electrons which escape from the surface would possess considerably more energy than formally. These would tend, under the same conditions of pressure and voltage, to increase the ionization. Hence we see that a flashover would probably occur at a potential somewhat lower than when the electrodes had a larger radius of curvature. Such a supposition would be entirely in accord with a result obtained by Debije when he made use of the theory of images to prove that an electron does less work in escaping from a curved surface than from a flat surface.

Let us now return to Fig. 1 and suppose that the gap and both electrodes were subjected to electromagnetic radiation of a definite frequency, such as ultra-violet light or X-rays. Assuming the frequency of this incident radiation as ν vibrations per second, and the amount of work done by the electrons in passing across the surface of the electrodes as w, then we would have

$$V e = h \nu - w \tag{1}$$

or,

$$V e = h \nu - \phi e \tag{2}$$

where V is the retarding potential across the gap, e is the electronic charge, h is Planck's constant of action, and ϕ the electron affinity of the metal, of which the electrodes are composed.

We see from the above equations that for any given set of electrodes, separated a certain distance apart in air of a definite pressure, and having a definite difference of potential applied between them, we would expect the energy of the electrons emitted from the negative electrode to possess energy characteristic of the radiation falling upon the electrode itself. That is, the higher the frequency of the radiation the more energy the escaping electrons would possess. Moreover, the neutral gas atoms in the space between the electrodes would certainly be in a more excited state as a result of the absorption of quanta of larger energy content. Both of these effects would favor ionization so that the flashover would occur at a lower potential than when the gap was subjected to radiation of lower frequency.

I noted from the results of Dr. Lusignan's paper that under the conditions of his experiment he found that when applying an a-c. potential to a point-to-plane set-up he always obtained a flashover on the half cycle when the point was positive. He gave as an explanation of this phenomenon the fact that the electric field was more concentrated around the point than it was in the neighborhood of the plane. While I concur with him in the explanation of his experiment as carried out, still I believe that it would be possible to obtain a flashover on the half cycle when the point was negative instead of positive, if the gap and electrodes should be subjected to radiation of considerably higher frequency, such as ultra-violet light or X-radiations.

Whereas Dr. Lusignan has shown that ionization, thermionic emission, and metal vapors all play an important role in causing an a-c. flashover between two metallic electrodes, still his results do not take into account the effect of the photoelectrons emitted from the electrodes themselves as a result of the energy absorbed from the incident radiation falling on the spark-gap. I believe that some valuable information could be obtained by carrying out Dr. Lusignan's experiments under the conditions which I have cited, *i. e.*, subject the spark-gap to high-frequency radiations. I would also suggest that the radius of curvature of the electrodes be varied through a wide range in order to see what effect the shape of the electrodes would have upon the flashover, for any particular frequency of radiation. I would further suggest that the spark-gap be enclosed so that the pressure of the intervening dielectric could be varied.

Furthermore, I feel sure that such information might be of considerable importance in arriving at the most economical size of conductors and the best type of insulators to be used on transmission lines in high altitudes. This would be especially true in the case of very high-voltage transmission lines where there is always a considerable amount of energy wasted in the form of corona.

J. T. Lusignan: The need for properly shielding suspension insulator strings was recognized with the advent of the higher line voltages some time ago. The experimental study of this problem was probably first carried out in the West at Stanford University under Dr. Ryan in the interest of the Pacific Coast power companies, and in the East by Dr. Peek at Pittsfield, both of whose work was reported to the Institute about eight years ago (A. I. E. E. TRANS., 1920). Soon afterwards the necessity for shielding against impulse flashovers was recognized and the laboratory work in connection with the study of this was reported at various times (F. W. Peek, Jr., *General Elec. Rev.*, 1923, V. 26; A. I. E. E. TRANS. 1923, 1924; *Journal Franklin Inst.* 1923.)

The paper by Messrs. Torok and Ramberg describes a similar study of shielding under impulse conditions, and the manner in which they have represented with curves the arcing characteristics of the various shielding arrangements (Figs. 11 and 12) is very interesting.

In this paper which deals essentially with insulator flashovers, several references have been made to the mechanism of the breakdown of air itself that are a bit misleading. The authors state: "The more homogeneous the field, the faster the streamers will develop, since under these conditions, higher gradients at the tips of the streamers are possible. For instance, the streamers grow much faster in the uniform field of a sphere-gap than in the non-homogeneous field of a needle-gap." Of course, once the breakdown is started in a sphere-gap the rupture is complete in an extremely brief time, but if a needle gap of the same spacing is in parallel with it the needle-gap breakdown should start and be over long before the voltage has started appreciable action in the sphere-gap.

I believe that experimental data bear this out, for on one impulse sparkover curve that I have seen a 1000-kv. wave of a fraction of a microsecond front will break down a 90-cm. needle-gap, whereas the maximum spacing of meter spheres that this wave will rupture is approximately 45 cm. It is true that with an increase in wave-front steepness the needle-gap spacing must be decreased for sparkover at a given crest voltage, but even for the steepest waves encountered in practise it can still be sparked over at an appreciably greater spacing than a sphere-gap.

The above 1000-kv. impulse wave will also spark over 7 insulator units of 5¾-in. spacing each, representing a total distance of 102 cm. This then would indicate that the suspension string is faster than the needle-gap. Accordingly, this would seem to be the reason for the string cascading with the arrangement of simple arcing horns. With the large ring shields though, the fields at the ends of the insulator string are appreciably altered and the stress concentrations around the end units so much modified as to lessen the "sparkover speed" of the units and allow the discharge to occur between ring edges.

The authors state: "It is obvious that a homogeneous field between the rings is desired to speed up the streamer formation. The nearer the surface of the ring approaches that of a sphere, the more uniform the field will be." How close they have come to a homogeneous field in their design is questionable, for the flashover of Fig. 13, and the longest streamer of Fig. 14 are both from the sharp edges at the ends of the ring sectors.

I should like to ask the authors what the ring spacing of Fig. 13 was and what voltage was necessary for flashover, also what method they used for calculating the rather smooth field gradient curves of Fig. 9.

D. H. Rowland: The paper of Messrs. Torok and Ramberg confirms a great many of the laboratory facts arrived at some time ago by Dr. F. W. Peek and used in the development of the grading shield, the merits of which have been proved by a great deal of field experience.

Surge investigations are yielding more and more information regarding the exact nature of disturbances on transmission lines. Before many of these data were available, it was found by laboratory experiments that an insulator string equipped with an arcing device which would clear it properly of impulses of very steep wave front would also function properly when subjected to longer waves down to those approaching ordinary 60 cycles. Moreover, it was found that arcing horns would not give satisfactory results unless the spacing between the horns was cut down so far as to reduce materially the 60-cycle insulation.

Some of the earlier grading-shield installations consisted of galvanized iron pipe, bent into an oval shape and placed at the bottom of the insulator strings, in conjunction with a pair of arcing horns at the top. Later a ring was also placed at the top of the strings. In this way, it was possible to prevent insulator breakage under all types of surges and at the same time not cut down the 60-cycle flashover more than 12 per cent. It was also determined that such installations would raise the flashover voltage of steep waves 10 per cent or more.

A ring at the top and bottom allows power arcs following the initial surge to blow around the insulators rather than through them. To accomplish this properly, it is important for the rings to be continuous and unbroken. The device shown by Mr. Torok is lacking in this respect.

The latest laboratory information which is available indicates that rings of flat strap rather than pipe give better flashover values under steep impulses. This is no doubt due to the dissipation of energy in corona by the sharp edges of the rings. It is fortunate that the strap rings are also cheaper to manufacture. Very satisfactory results in the field have been obtained by this type of equipment, of which over 50,000 shields are now in actual service.

In the light of our present information concerning the exact nature of the disturbances taking place on transmission lines, it seems best to let the design of insulator protective devices be a compromise between what is indicated by laboratory data, field experience, and manufacturing costs.

J. H. Cox: The particular importance of the papers by Torok and Ramberg, and Lusignan is the relationship they bear to the flashovers experienced on transmission lines in service, due to the actual surges existing on those lines. If we knew more definitely the nature of the surges with which we have to contend we could more intelligently apply these principles of protection, and furthermore proceed more efficiently with our laboratory investigations.

The klydonograph of Mr. Peters has told us much during the past five years regarding transmission-line surges. It has told us that switching surges are unimportant; that potentials subsequent to flashovers are unimportant, with the exception of those on isolated neutral lines; and that lightning causes all those surges which give trouble on a well designed system. It has given data on the number of surges encountered during one storm, the polarity of such surges, and their magnitudes, as limited by line insulation. What we now need to know is the more detailed nature of these surges; the shape of the front, the total duration, the shape of the tail, and the magnitude to which lightning surges may go if unrelieved by line insulation limitation. Torok and Ramberg's paper demonstrates clearly the influence the wave shape has in one particular instance of vital importance.

Several groups are working on this problem of determining the detailed nature of lightning surges. It has been known for some years that the cathode-ray oscillograph would be an ideal instrument with which to obtain the picture of a lightning surge, if only it could be set to catch the picture when the lightning stroke occurred. At least two methods of doing this are being tried. One of these, the Kipp relay, was developed in Germany, and consists of a vacuum tube arrangement to start the oscillograph after a surge arrives. The worst feature of this is the delay of about 1 microsecond in getting into operation and that much of the surge is lost. The other method, Dr. Norinder's invention, provides a means of placing a cathode-ray oscillograph in operation for two hours or more at a time, able to catch any lightning surge during that time.

Two Norinder cathode-ray oscillographs have been installed on the 154-kv. transmission line of the Aluminum Company of America at Chilhowee, Tennessee. To make the test as comprehensive as the facilities now at hand permit, these oscillographs have been supplemented by klydonographs. Also a scheme has been devised to record the position of the stroke. This record merely consists of a vibrating oscillographic record of the time difference between the arrival of the electrical disturbance and the sound of thunder. These stations have been in operation during the latter part of the past season.

Since the installation of the oscillographs only one stroke in the immediate vicinity of the oscillographs has occurred. From this one record was obtained. This record, together with a detailed description of the oscillograph, and the other apparatus of this installation, will be in a paper to be presented before this Institute in the near future.

Klydonograph data indicated that very few surges per storm occur. Usually, there would be only one large surge and seldom more than two or three of any kind. This has been conclusively confirmed in this summer's tests. When I mentioned previously that only one stroke had taken place near the stations since the installation of the oscillographs, I did not mean in the general vicinity. Storms within sight and sound, and even near at hand, were rather frequent, and some caused fairly high surges at one of the outlying klydonographs. However, all but one caused no appreciable surge at the oscillograph stations. It seems to be a fact that in order to cause a surge of appreciable magnitude the lightning stroke must either strike the line directly, or within a few feet of the line, and then not far from the point of measurement. This means that to obtain the information needed we must work with very infrequent phenomena and that the work must be slow and expensive. Even with the klydonographs it has been an expensive proposition to place them as thickly as needed. Thus, the best efforts of the industry as a whole are necessary. Certainly the problem, while important to the manufacturer from the point of view of apparatus design, is even more important to the operating utility from the point of view of system design and operation. The problem is pressing and too large for any one firm or group to handle satisfactorily. Therefore, some form of coordinated effort by the entire electrical industry, including both operating utilities and manufacturers, is needed, and must be worked out. It is to be hoped that there will not be too long a delay for this cooperative effort to be effected.

L. E. Reukema: Doctor Lusignan's paper is a happy illustration of the engineer's increasing knowledge of the very extensive and valuable research of physicists, and of his willingness to make use of their results and, where applicable, to use their methods and the type of apparatus they have developed. At the same time it is a forceful illustration of the difference in magnitude of the quantities ordinarily investigated and measured by physicists and by engineers. Whereas physicists have been studying spark-gaps measured in inches or in fractions of an inch, Dr. Lusignan studies gaps of 10 to 22 ft., the voltages used increasing in a similar manner.

I have been particularly interested in seeking to explain some of the results reported. A physical picture, showing why the sparks occur as they do, should add to the value of the data obtained. For instance, Dr. Lusignan reports that an a-c. spark from a point to a grounded plane always occurs when the point is positive, yet a spark from one sphere to another grounded sphere always occurs when the isolated sphere is negative, provided the gap is not greater than the diameter of the spheres. At first thought, these results seem contradictory, yet a simple analysis enables one to predict exactly these results.

Flashovers, Point to Plane. Consider a positive point and a negative plane with the voltage between them gradually raised to the flashover point. Any electron or negative ion in the near vicinity will be attracted to the point, colliding with the air molecules and, as it approaches close enough to the point, attaining a speed sufficient to enable it to knock electrons (negative particles of electricity) out of these molecules, thus making the molecules positive ions. The electrons knocked out will also speed toward the point, knocking electrons out of the molecules they hit, the latter electrons ionizing in turn, so that for one electron originally attracted to the point many millions may hit it. The positive ions produced by the ionization are repelled from the positive point, and as they attain a high enough speed they may also knock electrons out of some of the molecules encountered. These electrons in turn ionize as they speed toward the point, this cumulative ionization resulting, in a small fraction of a second, in a flashover. The positive ions produced weigh almost 60,000 times as much as the electrons; therefore they cannot attain the speed in the electric field which the electrons attain. Since equal numbers of positive ions and electrons are produced, and electrons speed into the point faster than the positive ions can get out of the field, a positive space charge results.

This positive space charge so distorts the field that a potential between point and plane not quite sufficient to produce a flashover when the point is negative will cause a flashover when it is positive. Note from the accompanying Fig. 1 that the field at the positive point is such as to allow the field to build out as a pencil, whereas at the negative point the discharge is more fan-shape and extends out a much shorter distance. The reason for this is that between the positive point and a positive ion any space charge helps to speed the ion on its way away from the point, thus allowing it to ionize at a greater distance from the point than it could if the space charge were not there. A positive space charge between a negative point and a positive ion, however, can only

Corona from a positive point. Corona from a negative point.

Fig. 1

slow the ion down, as the repulsive effect of the positive space charge subtracts from the attraction of the negative point. Similarly, the positive space charge between a positive point and an electron, when that space charge becomes increasingly denser as the point is approached, speeds up the electron, and thus increases its ionizing power, whereas the same charge between a negative point and an electron traveling away from the point evidently must slow the electron down. This explains why the corona extends out farther from a positive point than from a negative point for the same voltage. Since flashover occurs when the corona from the point extends all of the way across the gap, evidently a spark will take place when the point is positive at a lower voltage than when it is negative.

The space charge, together with the difference in mass of the electron and the positive ion, explains the difference in shape of the discharge from positive and negative points. The positive ion, being of the same mass as the molecules with which it collides, gives up a large part of its kinetic energy to the molecule at each collision, regaining velocity from the field between collisions and moving on the average in as nearly a straight line as possible toward the negative point or away from the positive point. The electron, on the other hand, is about 60,000 times as light as the molecules with which it collides. Therefore, it rebounds from such collisions retaining almost all of its kinetic energy, such rebounds in all directions effectually preventing an electron from following anything even remotely resembling a straight line. Remember also that a positive space charge tends to pull an electron into the densest part of the charge and that the positive ions forming such a charge violently repel each other.

In view of these facts, consider the ionization near a negative

point. Electrons, ionizing as they move away from the point, are deflected in all directions, therefore the positive ions, produced by the ionizing of the electrons, approach the point from all such directions and the discharge is fanshape. At the positive point, positive ions tend to take as direct a path as possible away from the point and toward the plane. Therefore, the electrons resulting from the ionizing of molecules by these positive ions must start from some point near this straight line, and, in spite of the fact that collisions tend to deflect the electrons away from this line, the strong attraction of the positive ions concentrated along such a line tends to prevent the electrons from getting far away from it. Corona near a positive point, therefore, builds out as a pencil straight toward the plane.

Flashover between Spheres. Consider now two spheres, the lower one grounded, the space between the spheres being not greater than a sphere diameter. All of the lines of electrostatic flux leave the upper sphere, but only part of them end on the lower grounded sphere, the rest going to the ground. The potential gradient, being proportional to the electrostatic flux density, is therefore greater at the upper sphere than at the lower grounded one. If the gradient is just sufficient to allow ionization by positive ions at the upper sphere, it will, therefore, be insufficient to allow such ionization at the lower grounded one. Consider, now, what happens when an electron is either produced or strays into the field between the spheres. If the upper ungrounded sphere is negative, the electron is repelled from it and speeds into the lower sphere, ionizing as it goes, the resulting electrons continuing with it into the lower sphere and the positive ions produced moving up toward the upper sphere. The original electron, together with the electrons which it knocks out of the molecules with which it collides, may easily produce many millions of positive ions. Close to the upper sphere, where the gradient is greatest, some of these positive ions succeed in ionizing, thus starting a new stream of electrons toward the grounded sphere. These electrons produce more positive ions by collision and so on, this cumulative ionization resulting in a spark. Note that, though the gradient is sufficient for ionization by positive ions only when they are close to the upper sphere, electrons are able to ionize at the lower sphere also since their small mass and larger mean free path enable them to attain the ionizing speed at a lower potential than that necessary for positive ions.

Now consider the upper ungrounded sphere as positive and the voltage between spheres the same as before. An electron in the field will be attracted to the upper sphere, ionizing as it goes. Positive ions produced by such ionization will be repelled from the upper sphere. Those produced very close to the surface of this sphere may ionize while in this high field. However, the probability of ionization is small as the ions stay in this high field such a short time and are moving into a lesser gradient as they move away from the sphere. A few probably will ionize, but the electrons so produced are so close to the upper sphere that they have little chance to ionize more than once or twice before being pulled into the sphere, and the probability of the few positive ions produced by the latter ionization being successful in ionizing as they leave the sphere is negligibly small. At the potential gradient at which a spark takes place, the probability of ionization by electrons is many hundreds of times greater than that of ionization by positive ions. Remember also that the number of positive ions produced by an electron moving in a strong field is proportional to e (the base of the Napierian logarithm, 2.71828) raised to a power proportional to the distance which the electron travels. Therefore, if the electron in traveling a given distance can produce, with the help of the electrons which it knocks out of molecules, 100,000,000 positive ions (let us say), then in traveling half that distance it produces only the square root of 100,000,000 or 10,000 ions, and in traveling one-eighth of the distance it would produce only the eighth root of 100,000,000 or 10 positive ions. Evidently then an electron produced close to a positive sphere would have such a short path to follow that the number of positive ions it could produce would be too small to allow the cumulative ionization necessary to produce a spark. Therefore it is evident that a flashover will take place at a lower potential when the upper ungrounded sphere is negative than when it is positive.

All of this applies when the distance between spheres is not greater than the sphere diameter. However, when this distance is much exceeded, corona forms before the flashover. In other words, ionization takes place at less than flashover voltage and a space charge forms. When the space charge is present, the upper sphere acts as did the needle point, the lower grounded sphere and the ground together acting as the plane. There are then two opposing phenomena, the grounding of the lower sphere tending to cause the spark to take place when the upper sphere is negative, the space charge tending to cause it to take place when the upper sphere is positive, the result being that the flashover may take place for either polarity.

When Dr. Lusignan placed a ⅜-in. barb on the ungrounded sphere, the phenomenon was naturally that of point to plane and the spark always occurred with the ungrounded sphere (and the barb) positive.

The foregoing explanation also shows why flashover across an insulator string (of 10 to 14 units) always occurs on the half cycle when the conductor is positive, the phenomenon being similar to that of the point to plane. Corona forms before the flashover, forming a positive space charge and resulting distortion of field which speeds up the electrons and ions when the conductor is positive and slows them down when the conductor is negative.

Discharge Surges on Positive and Negative Half Cycles. Dr. Lusignan also calls attention to the fact that the brief discharge needles of corona at the ten-foot spacing between points, both points insulated, occur only on positive half cycles of current, and that they occur only when the voltage has almost reached the flashover point, and then occur several times each positive half cycle. At the 22-ft. spacing the same phenomenon occurs at positive half cycles, and in addition a very intense white discharge at intervals on a negative half cycle.

The phenomenon at the 10-ft. spacing may be explained easily by considering the relative mobilities of electrons and positive ions and the fact that chance has much to do with the amount of ionization occurring at any instant. Thus one electron together with all of the electrons which it produces as a result of ionizing collisions, may be responsible for (let us say) 22×10^{10} electrons arriving at the positive terminal in one surge. This would be equivalent to 40 ionizing collisions ($e^{40} = 22 \times 10^{10}$). Another electron in the same field might ionize fifty times, resulting in e^{50} or 47.5×10^{13} electrons surging into the positive terminal as a result of the ionizing activity of the particular electron considered and its progeny. Thus one electron may be responsible for 20,000 times as many electrons entering the electrode in one surge, as is another, and although 47.5×10^{13} electrons is small compared to 6.28×10^{18} (the number of electrons per coulomb) they enter the electrode in such a short period of time that, for an instant, they represent an appreciable current. Evidently in any positive half cycle several such extra large surges would be expected to occur, as the oscillograms show that they do.

That they would not be expected to occur on negative half cycles (for spacing of 10 ft. or under) is evident when one considers that the current at the negative terminal is due to the attraction of positive ions into that terminal. Since positive ions are so much heavier than electrons they move much more slowly. (At a gradient of 30,000 volts per cm., a-c., for instance, the positive ions would move 356.5 meters per second in the direction of the field, whereas electrons would move 121,600 meters or 340 times as fast.) The positive ions produced by one of the electrons ionizing fifty times, for example, as was considered above, would not be able to enter the negative terminal in one surge, as did the electrons at the positive terminal, but because of their much slower speed and the fact that their mutual

repulsion tends to prevent them from concentrating close to each other, they would come in gradually, thus ionizing out the irregularities in their production.

The logical explanation, it seems to me, for the intense white streamers noticed occasionally coming from the negative terminal at the 22-ft. spacing is that, at the high potential necessary to almost spark across such a gap, the potential gradient at the needle point is almost sufficient literally to tear electrons out of the negative point. A dense positive space charge close to the negative terminal would of course increase this gradient, and when the space charge for an instant happens to be greater than normal or happens to have a peculiarly advantageous configuration, the critical gradient necessary to tear electrons out of the point is momentarily exceeded and an intense surge of current at that terminal results. Such an intense charge, lasting such a short time, could not be due to the relatively slow moving positive ions entering the negative point, but must be due to electrons leaving it and leaving it in far too large numbers to be due to ionization by positive ions.

W. S. Kates and E. D. Tanzer: In the discussions of wave shapes and specifications, Messrs. Torok and Ramberg note the impropriety of defining an electrical wave by merely giving the constants of the circuit producing the wave and the time taken for it to reach crest value. Their discussion states three factors and two of these factors are very intimately related to the amount of energy available in the impulse testing set. Later in the paper the formation of streamers and the movements of charges in the electrostatic field are discussed. It is a readily apparent corollary of these facts that the entire region surrounding an insulator string has its local potential distribution progressively changed by the application of electrical stress, and such application implies that energy is supplied to the region. It seems necessary, therefore, that careful consideration should be given to the amount of energy available in the testing set producing the wave to insure that this testing set produces discharge phenomena at all parts of the region of stress which are identical with those produced by a transmission-line traveling wave wherein the energy may be very much larger than that available in the testing set.

It has been our privilege to witness several tests with impulse generators at the different high-voltage laboratories at voltages of one million or more. The energies available at these laboratories in the various testing sets are of the order of 10,000 to 15,000 watt-seconds, at pressures of 2,000,000 to 3,000,000 volts. With these amounts of energies, it has not always been possible consistently to reproduce desired tests. The writers distinctly remember a proposed demonstration flashover of a 20-unit insulator string. Previous to the demonstration, the string had been repeatedly flashed over and the apparatus was left unchanged, supposedly ready for the demonstration. Repeated subsequent attempts to flash over the same insulator string were unsuccessful in spite of any adjustments made to the testing set in an effort to do so.

In view of the relation of the two factors to the energy available in the testing set, and the limitations of these sets, we should like to ask the authors what precautions have been used and what tests have been made to insure that the energy capacity of their testing set was large in comparison to the amount of energy required to fully charge the dielectric of the insulator string and surrounding regions to potential causing flashover?

Under "Break-down of Air" the formation and progress of streamers during break-down are briefly discussed and they have been further discussed in a previous paper by one of the authors. It is notable that the mechanism of breakdown which they discuss is dependent on a continuous supply of energy to the repeatedly growing streamer and, even previous to this, to the dielectric medium surrounding the insulator string.

The discussion of the components of forces acting on the streamers is especially interesting in that it is a corroboration of the principles stated by F. G. Baum in his 1921 paper before the Institute. The previous statement of the principle was that the potential of the insulator string must be higher than that of the surrounding air at all points or the arc would tend to strike toward the insulators.

On the fifth page it is stated that there is a definite relation between the mean diameter of the ring and cascading. This principle is visualized by Fig. 11. We understand that when a ring of a small mean diameter is tested, the arcs will tend to strike toward the insulator string. As the mean diameter of the ring is increased, the tendency for cascading decreases and at some certain radius, all cascading stops. The radius at which cascading stops varies with the diameter of pipe of which the ring is made and an indication of the manner of this variation is shown in Fig. 4. Continuing this reasoning, we further deduce that if the mean diameter of the ring is further increased, a size will be reached at which the influence of a ring on the insulator string is small and cascading will again commence. In other words, the upper right-hand region of Fig. 11 noted as "no cascading" may not be a region. It would seem that a second line might be drawn, generally paralleling the curve of Fig. 11 and that only between this second line and the one shown is the non-cascading region. Accordingly, we should like to ask the authors whether they have made explorations of the territory shown as "no cascading" and, if so, would suggest that a discussion of this would be very valuable and pertinent.

S. Whitehead: (communicated after adjournment) I am very interested both by Dr. Lusignan's introductory and subsequent remarks as to the relation between spark and corona discharges and the arc. The work of Slepian and Compton has certainly made the distinction difficult and strengthened the view that the main differences is to be ascribed to the cathode fall of potential. I am, however, inclined to look at another aspect. Townsend's condition for discharge

$$\int \alpha \exp. \left(\int (\beta - \alpha)\, d\, x\right) d\, x = 1 \qquad (1)$$

(where α and β are the co-efficients of ionization by collision and the integral is taken over a line of force from electrode to electrode) enables one to predict the critical stress for corona when α and β are interpreted in a sufficiently general manner. This condition merely expresses the fact that ions must be generated as fast as they are removed from the field. In the arc, although such a condition must be obeyed, one is faced with the further difficulty of accounting for the high current density; whereas in corona, the magnitude of the currents can fairly well be provided by ionic and photo impact.

The spark, I am inclined to treat as follows: Equation (1) applies to corona in all its stages. At the beginning the field is not greatly modified by space charges, but as the voltage increases and the current and corona volume also increase, space charges come into play so altering the field and α and β that the condition (1) is satisfied. A point is reached, however, when the ionic density is too great for the condition to hold with the result that the left-hand side of (1), which is proportionate to the number of ions generated due to the passage of one ion from electrode to electrode, becomes greater than unity and the discharge changes into a self increasing type. Instability occurs and a large quantity of electricity passes until stability is again found under the new conditions of the arc and depending to some extent on the external circuit.

Thus spark may be looked upon either as exaggerated corona—if the condition prior to the sudden spark voltage drop is considered, or as an incipient arc—if the conditions after this voltage drop are examined.

The point at which condition (1) can no longer hold, however great the space charge due to the current, would correspond to the author's suggestion that the spark occurs when the corona streamers from the opposite electrodes meet.

The above views were suggested some years ago and later

mentioned in a monograph[1]. A further suggestion made then with regard to polarity of discharges may be of interest—namely, that positive corona such as from a positive wire to a negative cylinder, is continuous with low-pressure discharges, but that negative corona under the same conditions is not, and, if the wire is kept negative, the low-pressure discharge will form at a voltage continuous with the original spark potential of the arrangement. This view was founded merely on the cathode fall[2] of potential theory and was supported to some extent by Crooker's observations. It has since been confirmed in the course of experiments on another subject by the British Electrical and Allied Industries Research Association.

As Wolcott[3] showed several years ago, stray positive flux concentrations even on dielectric surfaces will cause discharges to take place at the voltage corresponding to positive corona even though the main sharp electrode may be negative. I have myself noticed this effect with dry insulating materials though not so frequently if moisture is present and a surface film might be thought to be of importance. Perhaps the author may have noticed similar effects.

I would also venture to question to some extent the author's statement that in a fully developed power arc the restriking voltage is very small owing to thermionic emission and metallic vapor. My own opinion, which is supported by experimental work with which I have been concerned, is that in the majority of cases the effect depends almost entirely on the temperature of the space between the electrodes and the resulting density of the gas or vapor. Again, thermal ionization does not usually become important for the initiation of sparkover until very exceptional conditions occur. As to the effect of metallic vapor, such experience as I have with large power arcs has made me discount to a large extent the part which it is alleged to play in the restarting of an arc although, admittedly, if present, the majority of metals considerably reduce the sparkover voltage.

In this latter connection, I should like to mention recent work on the electrodeless discharges. That minute impurities in a gas affect its sparkover potential has been very clearly shown, but it also appears that they can be removed by the discharge itself. Thus it is possible that small quantities of metallic vapor may occasionally be removed by a small subsidiary discharge without restarting the arc.

With reference to the author's use of ultra-violet light, I should like to say that I have myself found that it has no effect on sparkover between spheres at atmospheric pressure—at any rate to an accuracy of ½ per cent. Further I have found no definite difference with certain different metals such as might be expected if a photoelectric effect came into play. Further experiments are, however, being made.

J. J. Torok: The questions of sphere-gap and needle-gap breakdown and streamer development are quite important. It is therefore necessary that their characteristics be well known.

In the case of needle-gaps, streamers start at a much lower value than between sphere-gaps. Then under a slowly rising voltage, a low-voltage breakdown can be expected for a given spacing. These streamers, however, grow very slowly permitting, under a rapidly rising voltage, a maximum potential difference of many times that of steady-state breakdown. On the other hand, when sphere-gaps are considered, this is not the case. Breakdown, or streamer formation, does not start, regardless of the duration of the voltage previous to breakdown, until the critical or breakdown value has been reached, then the streamers develop at an exceedingly high rate, thereby permitting the voltage to rise only slightly above the steady-state breakdown value.

1. "Dielectric Phenomena: Electrical Discharges in Gases," (Benn, 1927).
2. Crooker, *Phys. Rev.*, 1906.
3. Wolcott, *Phys. Rev.*, 1918.

The voltage at which streamers start to form, and their respective rates of development must not be confused. With this distinction in mind the statements, "The more homogeneous the field, the faster the streamers will develop, since under these conditions, higher gradients at the tips of the streamers are possible. For instance, that the streamers grow much faster in the uniform field of a sphere-gap than in the non-homogeneous field of a needle-gap, can be taken literally.

A good illustration of the above points can be made from a test on needle-gaps and sphere-gaps. A pair of 75-cm. spheres spaced 56 cm. apart were connected in parallel with a needle-gap having the same spacing. Under slowly rising voltages the needle-gap broke down every time, but as the rate of rise of voltage was increased a point was reached where "suppressed discharges" appeared between the spheres. Further increase in the rate of rise caused the sphere-gap to break down, leaving the space between the needles in a partially broken down condition.

The flashover characteristics desired for a line insulator are precisely those of a sphere-gap. Breakdown must not take place unless the potential reaches a fixed value, regardless of variations of the voltage below that point. If the voltage rises above that value, breakdown must take place in order to protect the power apparatus. Should the breakdown characteristics be that of a needle-gap, *i. e.*, allow an exceedingly high over-voltage to pass on, its protecting ability is reduced. Briefly, it is desirable that the insulator and its equipment have a flashover voltage, under even the steepest surges, equal to the 60-cycle breakdown value.

The ring shown in Fig. 13 was the final compromise between laboratory results, field requirements, and cost. The reasons for the large end sections are presented in the paper. The sides were made straight and located closer to the insulators because of the tower clearances on the particular line for which it was designed. However, it is to be noted that the active surface is unbroken, *i. e.*, an arc formed between rings can be blown from one side of the ring to the other with ease, the straps furnishing the path on the sides. The abrupt change in section is the result of a compromise between laboratory results and cost. The original design embodied a gradual change in section, but the cost became so prohibitive that competition with less effective rings was out of the question.

The experimental determination of field stresses was made with the straw-and-neon-tube method, which is described fully in the Hermsdorf Schomburg pamphlets Heft 19-20-21- and 22. The calculated curves were arrived at by methods fully described by Ollendorff. The old capacity method had been abandoned because of its inaccuracy. The Ollendorff method of calculation, however, is rather tedious.

The question of surge-generator characteristics as brought up by Messrs. Kates and Tanzer is of considerable importance, as the voltage wave that is impressed upon the insulator exerts more influence upon the final result than any other factor. The characteristics of the various types of surge sets in use cannot be analyzed in this discussion. The characteristics of an ideal surge generator will be presented and the properties of the other sets can be compared.

The ideal surge generator would have no inductance and very high capacity. It should be possible to vary the wave front and the nature of the tail. It should also be possible to maintain the crest potential for a long period of time.

Some types of surge generators produce a high-voltage pulse for only a fraction of a microsecond and the potential then drops to a low value. In many cases breakdown will be started at the high values and completed after the voltage has dropped to low values. Under this type of wave, breakdown is an unstable factor. Moreover, very little can be learned of the test specimens' characteristics, as the voltage is too variable.

The ionization currents during the period of breakdown often mount up to thousands of amperes, but their period of develop-

ment is so short that the power taken is small compared to the capacities of large surge generators in use at present. The surge generator, then, must have an extremely low inductance to maintain the potential during streamer development. It is doubtful if such low inductance can be obtained with high-voltage surge generators.

The "no-cascading" area referred to by Messrs. Kates and Tanzer and shown in Fig. 11 was thoroughly explored by the authors. An outer limit has been found for the larger spacings, but the diameter of the rings necessary to reach that limit was so large that it was deemed unlikely that this region would be reached by any commercial ring.

J. T. Lusignan, Jr.: In reply to Professor Ball's question of the possible photoelectric effect on flashovers, we did not try any tests with light radiations of different frequencies except the short test on polarity alluded to in the paper. In view of the negligible effect of strong ultra-violet light sources on breakdown voltages in the laboratory as found by various experimenters (such as Dr. L. E. Reukema, A. I. E. E. Quarterly TRANS., Vol. 47, January 1928, p. 38) I seriously doubt whether any atmospheric short-wave radiations should cause differences in flashover characteristics at different altitudes, as Professor Ball suggested might occur. He also asked about tests at different pressures. These were not included in our work, although such research, probably using at first monatomic gases and simple electrodes, would serve to supplement the work of the physicists at short spacings, and enable us to have a better understanding of high voltage breakdowns of long gaps.

I am greatly indebted to Dr. Reukema for his very clear and thorough analysis of the breakdown process in unsymmetrical gaps. He has given some very convincing physical pictures of the factors fixing the polarity characteristics of the various gap arrangements.

Mr. Whitehead has introduced a mathematical interpretation of the discharge processes, using an equation apparently derived from Townsend's original theorem. Such simple relations for depicting the breakdown process at a given point in space are surely helpful in expressing the relative activity of the various factors and bodies involved at that point, but just how such an equation could be extended to involve the complete discharge across the entire gap without becoming hopelessly involved and unwieldy is not clear to me. For instance, the breakdown between two points starts first as corona extending out from each electrode. These localized brush discharges are supposedly of a low-voltage nature and give the molecular band spectra typical of arcs. When these two apparently arc-like corona brushes meet, though, the complete breakdown across the gap involves a spark radiation spectrum extending from point to point (see Fig. 12) which afterwards probably reverts to an arc. The interpretation of these various stages of a flashover of a gap in air is difficult enough when attempted from a physical standpoint, without seeking to develop reliable mathematical relations to do it.

Mr. Whitehead objects to attributing the low restriking voltage of the arc principally to the presence of thermionic emission and metal vapors. I perhaps did not stress sufficiently the effect of temperature in lowering this voltage also, but I believe that most physicists discount the ability of heat to be the primary factor in most low-conductivity discharges. It is surely helpful, though, to know that Mr. Whitehead's particular experiments apparently indicate temperature and air density to be deciding elements in the discharge process.

Electrification of the Textile Industry in the Southeast

BY EDWIN M. CLAPP[1] and A. G. STANFORD[2]
Non-member Member, A. I. E. E.

Synopsis.—This paper deals with the subject from the standpoint of the public utility and the application of the electrical equipment in the mills, outlining the past and present practise with forecasts of the practises and developments of the future. Brief discussions are included of the advantages and disadvantages of purchased power over generated power and of the individual drive over the group drive. Statistical data are given on the percentage of the total spindles now operated on electric drive in the Southeast as well as the range of horse power requirement of various textile machinery and the cost per spindle for electrifying mills on various cotton products.

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Part I

The Relation of the Public Utility to the Textile Mill

BY EDWIN M. CLAPP

THE pioneer textile mill electrification in the Southeast was made at Columbia, South Carolina, in 1893. This was a hydro development, using horizontal water-wheel generators. About a year later one of the mills at Pelzer, South Carolina, put in a hydro development. This Pelzer unit was unique at the time, in that power was transmitted 2¾ mi. and employed a 3300-volt, three-phase transmission line. About two years later the Anderson Power Company at Anderson, South Carolina, was organized to supply power to the mills at Anderson. In 1904, the Catawba Power Company at Rock Hill, South Carolina, started operation, serving several of the mills in the vicinity.

In North Carolina there were many small hydro sites which were early developed. This accounts to some extent for the impetus textiles gained in North Carolina during that period. Most of these original installations were mechanical drive, later changing to electric drive.

In the earlier years about the only advantages electric power was thought to have had was the elimination of the main line shafting in the mill; or an opportunity to develop a water-power site which heretofore had been unusable. In the case of central station power, an opportunity to cut fuel costs.

It was not long before the advantages of breaking the load up into smaller groups to get additional economies were established, and that the savings in power costs were not the major reasons for electrification.

There were many obstacles to overcome in the early period of development. The machinery manufacturer, the manufacturer of electrical apparatus, the central station, the consulting engineer and the mill itself all had their problems to solve. In those days each factor worked out his own salvation irrespective of the others. Today these four have joined hands, and a problem that concerns one has the interest of all.

The troubles of the central station were possibly the most serious. Each company was small, with no interconnection or tie lines. High line transmission was far from what it is today and regulation none too good. In many cases the systems were over-sold, and droughts in the late summer and early fall necessitated shutdowns.

The phenomenal growth of the textile industry in the Southeast during the last fifteen years is known to nearly everybody. A contributable factor to this development has been the coordination of the activities of the manufacturer of electrical apparatus, the machinery manufacturer, the central station, and the consulting engineer in working out problems of application, illumination, and continuity of service.

The fact that there are only a few mills in the Southeast that are not using some form of electric drive is prima facie evidence of the desirability and advantages of this form of drive.

There are approximately 17,000,000 spindles in the Southeast today and about 75 per cent of these spindles are driven electrically. Approximately 80 per cent of the electrically driven mills are using central station power. It is in most cases cheaper, power averaging from 1.75 per cent to 0.75 per cent per kw-hr., depending on the size of the load. It does not necessitate using capital funds for non-producing and non-dividend earning machinery. It is much more flexible and does not put the mill at the mercy of the labor problems and physical hazards of the mining and transportation industries.

After a few years, many of the original hydro developments found their basins filling up with silt, thereby cutting down their storage capacity and lessening their prime power. The timber was being fast cut down, which meant quick run-off during rainfall. Practically all of these same mills found it expedient to increase their capacity, which, in most cases, the solution of the

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8-140

problem was purchased power. A great many mills came into existence that were not adjacent to a hydro site. These mills installed what were considered at the time very modern steam plants. The majority of these mills found it profitable to double up after a few years' operation and simultaneously awoke to the fact that their power plants were not only obsolete but inadequate to carry the additional load. Again the central station solved the problem.

A question that is very often asked and very seldom answered is, "Why are the textile mills in the Southeast standardized on 550 volts?" The answer is that in the early days the underwriters limited open wiring to 750 volts; 550 volts was the nearest standard to this limit, and also meant a tremendous saving in installation costs over 440-volt or 220-volt motors.

The textile load to practically all the utilities in the Southeast is a very large part of their production and the major part of the industrial load. The desirability of a load of this type plus the unusual increase in demand have kept the utilities busy in coping with this situation.

Today there exists a network of transmission extending over the entire Southeast, all tied in together. Practically all trunk lines consist of dual conductors, and nearly all primary feeders and distribution have tie lines, thereby insuring continuous service.

Interchange of power helps to work out a balance between hydro plants located on different watersheds, compensating for diversity in rainfall over different sections, and tends to equalize the delivery of prime power over a period of time between a high-head storage station and a low-head, stream-flow station.

A large block of high-economy steam reserve is ready for service on call. These steam stations are located at strategic points on the system.

In contrast to the over-sold condition existing a few years ago the utilities have anticipated the increase in load for several years to come, have worked out a program to meet these requirements, and today there is a comfortable surplus of power. As this power is absorbed new developments are made. From now on the supply will be ahead of the demand.

Large synchronous condensers are being placed in operation at different points on the system to insure better regulation.

Transmission problems are being solved. The research laboratories and personnel of the manufacturer of electrical apparatus are contributing very helpful service in working out these difficulties. The public utilities in the Southeast are doing everything that can be done to insure reliable service and to deliver low-cost power to their customers.

The textile load has always enjoyed a high-load factor, but the introduction of the individual drive with a large number of small motors usually operated considerably under-loaded has created lower power-factor conditions. Some of the central stations in this district are working on a bonus proposition for power-factor correction that will make it worth while for the mill to install the necessary corrective equipment. The day of purchased power on a kv-a. basis is not far away.

In bleacheries, dye houses, finishing plants and rayon plants, which require a large amount of process steam, an ideal form of electrification is the installation of a non-condensing steam turbine, the exhaust being used for manufacturing purposes and the balance of the electric load being furnished by the central station.

The central station realizes that it cannot grow and develop unless the communities and industries it serves are growing and expanding. Recognizing the fact that the Southeast is a logical place for the development of the textile industry, it has a corps of men working continuously in other textile centers to bring new textile industries into this section. The combined efforts of this activity have resulted in several hundred thousand spindles being located in the Southeast. This, of course, has resulted in direct benefit to the utility, but the indirect benefits gained by the communities in which these new industries have been located are far greater.

The field of development for the use of electric power in the textile industry is unlimited. The tremendous research facilities of the electrical manufacturer and the machinery manufacturer, together with the consulting engineer in the field and the central station are working harmoniously together to keep continuously improving their responsibility to the industry.

With these great forces joined together the prospect for electrical development in the textile industry is exceedingly bright.

Part II

The Application of the Electric Drive in the Textile Mill

By A. G. Stanford

In the preceding section of this joint paper reference has been made to the sources of power that were utilized in the early days of textile mill electrification. At this point it will be of interest to review briefly the types of drives employed in the earlier installations.

The first complete mill electrification was made in the Columbia Duck Mill, Columbia, South Carolina, in 1893, and consisted of 200-hp. squirrel cage, 40-cycle motors.

In 1894 the Pelzer Manufacturing Company, located at Pelzer, South Carolina, installed a 250-hp., 3300-volt, 60-cycle synchronous motor with other large 220-volt induction motors.

Soon afterwards the Anderson Cotton Mill of Anderson, South Carolina, wishing to eliminate as much of the overhead shafting as possible, installed eight-hp., water cooled, double shaft extension motors, each to drive two spinning frames, the motor bracket being so arranged as to tie the end of the two frames together

and the motor shafts inserted directly into the cylinders. These motors were operated for some time in this manner but due to the difficulty experienced in maintaining the alinement of the frames, were later mounted on the ceiling and used similarly to the two-frame drive motors known today. This change was made after the first installation of four-frame drive spinning frame motors had been made in the Mayes Mill at Mayesville, North Carolina.

Limited by the then existing commercial designs of a-c. motors the large group drives were the logical consideration. These drives required the retention of a majority of the shafting and other mechanical idle load. The motors though sturdy and generous in design, were of low efficiency as compared with those of more modern design. The electrical losses added to the mechanical transmission losses resulted in an increase in the actual power required to operate the mills, together with an increase in some cases in the cost of power.

That the only principles applied in the first installations were fundamentally sound is witnessed by the fact that in comparatively recent years some of the installations made have followed largely the pattern of those installations which were made when the application of the electric drive to the textile industry was in its infancy.

With the view of reducing, as far as possible, the idle shafting load and to relieve the strain on the power plants occasioned by the use of the large size motors, the effort was early made to apply individual drives to a majority of the textile machines often through the medium of direct connection of the motor to the driven shaft. Many difficulties developed from this practise principally, in the case of the direct connected drives, to restricted range of speeds, and in general to inherent requirements of individual drive application to textile machinery which were not clearly foreseen at that time.

Like a pendulum the trend of textile mill electrification swung back strongly to the group drive but employing smaller units than in the earler installations. Special designs, such as the four-frame drive spinning motors were developed, which made it possible to eliminate heavy main line and countershafts and at the same time be free from the difficulties then encountered with the individual drives.

In later years, and even at the present time, the small group drive has retained a considerable amount of popularity, although, with the design of individual motors and controls perfected for practically every machine used in the textile industry, the application of individual drive motors is receiving more and more attention among owners and operators and is unquestionably gaining in popularity.

After the first few years of practical experimentation so much trouble was experienced with the control equipment, occasioning heavy maintenance expense, together with the problems of wiring and transmission of electrical energy, that for some time the progress of mill electrification was retarded until adequate improvements had been made in these items.

In 1906 the Profile Cotton Mill at Jacksonville, Alabama, decided on the use of a five-hp., 60-cycle, 550-volt motor to drive each spinning frame, the motor being direct connected to the cylinder. This application, though crude in many ways, as compared to the modern individual drive, was nevertheless a success and is still in operation using much of the original equipment.

In 1911 the Duncan Mills of Greenville, South Carolina, installed the most extensive application of individual drives then existing, this plant being individually driven throughout with the exception of the cards and roving. The spinning frame motors drove through pinions and gears to the cylinder.

In 1921 the Dixie Spinning Mill, now the Dixie Mercerizing Company, of Chattanooga, Tennessee, erected the first textile spinning mill in the United States to be completely equipped with individual drives. The success of this installation is evidenced by the fact that since the erection of the first unit three additions have subsequently been made and in each case the individual drive has been used throughout.

Ever since the first application was made the designing engineers of electrical equipment have been confronted with the problem of adapting their equipment for the operation of machinery which for many years had enjoyed a considerable degree of perfection in operation through means of mechanical transmission.

With the design of textile motors and controls highly perfected the manufacture of this equipment is largely a commercial problem. Much consideration is now being given to the various mechanical features in the equipment itself and in the physical application of this equipment to the textile machines. Great improvement has been made in the design of the motor bearings and many types of silent chains and other transmission equipment suitable for short center drives are available. The successful development of compact and inexpensive magnetic starters with push-button control stations has contributed towards the increased consideration given the individual drives.

Increased Production

That an increase in production per spindle-hour of mill operation can be realized in the majority of cases with the electric drive, as compared with the steam engine and mechanical drive, is well recognized by those who have carefully noted the results obtained in plants that have discarded the steam power equipment. In a 30,000 spindle spinning and weaving mill on narrow sheeting, which has been electrified within the past year, an actual increase in production of seven per cent has been realized, of which percentage the owners have attributed five per cent to the electric drive and two per cent to the fact that the entire mill had been operated slightly under speed on account of

the overloaded condition of the steam engine. Another similar mill of 13,400 spindles on wide sheeting has an annual increase in production of slightly over four per cent. Still another mill, which has come under the author's observation, is of 11,400 spindles on hosiery yarns, from which the records show an increase of approximately 3½ per cent since electrifying.

The foregoing examples indicate clearly the possibilities and these cases are of particular interest since no additional machinery was added and no changes whatever in the goods manufactured were made.

Satisfactory records are available from a large number of installations showing a range of from one per cent to as high as ten per cent increase in production. An average increase of three per cent may conservatively be expected between corresponding mills on steam and small group electric drive, and four to five per cent for mills on complete individual drive.

The reasons for the increase in production per spindle-hour of mill operation may be summed up as follows:

1. The greater speed at which the mill can be started up and stopped.
2. The greater flexibility in the operation of the several departments, enabling each to be run at its maximum production and to maintain a balanced output from the entire machinery organization.
3. A more constantly maintained speed at the value desired affecting not only the overall production but also the quality of the product.

In the textile industry, the speed at which the individual workers perform their duties does not have as great an effect on the output as is the case in numerous industrial activities where many parts must be handled before the manufactured article is complete. In the manufacture of textiles the production is largely controlled by the three factors mentioned above.

Group Drive Versus Individual Drive

From the standpoint of first cost only, either for a new mill or one to be changed to the electric drive, the group drive is considerably cheaper; however, this differential is considerably reduced in the case of a new mill where the cost of the shafting, belting, pulleys, etc., would have to be taken into consideration. No further explanation of the reason for the increased cost of the individual drive is necessary since it is evident that the cost per horsepower for motors, starters, and wiring increases as the rating of the motors decreases.

It has long been conceded that the individual drive is superior, and justifies itself from a first cost standpoint, for certain operations. For example, the picker room machinery, while performing only a preparatory operation, is readily adapted to the individual motor, while the elimination of the counter shaft and several large pulleys on the machine itself somewhat reduces the extra first cost of the individual drive.

In the author's opinion, the individual loom motor, is unquestionably a logical and superior application and practically no new weave mills have been equipped in the past few years and probably will not be in the future, with other than the individual motor drive on these machines. The few to be equipped with group drive have been controlled more from the standpoint of first cost than from lack of appreciation of the value of this application. The higher maintained speed and the similarity of operation of all of the looms, together with the freedom from variation due to unequal belt tensions, all contribute to a considerable increase in production and uniformity of finished product.

On account of the starting requirements the total connected motor horse power will be greater in the case of the individual drives, resulting in a lower plant load factor. However, any increase in the electrical losses occasioned by operating under these conditions is more than offset by the reduction in shafting and mechanical transmission losses. The newly developed double cage windings for squirrel-cage induction motors produce the high starting torque necessary for many textile machines without affecting the efficiency and power-factor characteristics after the motor has attained normal speed. This type of motor will permit of a reduction in the total motor horse power and reduce proportionately the cost of the individual drive.

In general the over-all power factor of the plant load is usually higher with the group drive than with the individual drive. In mills consisting mainly of small group drives power factors between 81 per cent and 86 per cent can usually be easily obtained, while for mills with individual drives the over-all plant power factor will usually vary between 75 per cent and 80 per cent, although the author has noted power factors as high as 85 per cent on this type of installation.

Cost of Electrical Installation

In Table No. 1 is given the actual cost per spindle to electrify textile plants, manufacturing cotton goods, together with the total connected motor horse power installed at the time each installation was made. This information has been compiled from actual records. The cost per spindle includes the cost of the electrical apparatus, power wiring, and lighting transformers installed complete, but does not include the cost of any substation or generative equipment, mill lighting system or low-tension feeders between the substation and the switchboard.

Variation in the cost per spindle of different plants on the same product and employing the same type of drives is occasioned by numerous factors such as the arrangement of the buildings and whether of one or more stories, all of which affect the cost of the wiring, while the number of spindles on the various machines being different in different plants affect to some extent the total connected horse power. Several of the items given in the table, cover additions to existing plants,

TABLE I

ACTUAL COST PER SPINDLE TO ELECTRIFY TEXTILE PLANTS MANUFACTURING COTTON GOODS

Y	Yarn mill	SW	Spinning and weaving mill
G	Group drive*	I	Individual drive throughout
IL	Individual loom drives	IS	Individual spinning drives
IT	Individual twister drives	Dr.	Drills
TC	Tire cord	Sh.	Sheeting

No. of ring spindles	Year installed	Goods mfg.	Type drive	Total motor hp.	Cost per spindle
12,000–Y	1923	40's–2 ply	I	1040	$4.13
8,000–Y	1926	28's–2 ply	I	650	3.00
19,000–Y	1927	60's–2 ply	I	1284	2.60
11,400–Y	1924	Hosiery yarn	IS–G	675	1.80
18,400–Y	1926	18's ply	G	1225	1.65
19,700–Y	1926	20's combed	G	1165	1.40
13,400–SW	1925	Wide sh.	G	1028	1.80
16,100–SW	1927	Narrow sh.	G	977	1.80
8,700–SW	1927	Dr–sh.	G	480	1.75
10,500–SW	1927	Dr–sh.	G	639	1.65
50,000–SW	1927	Narrow sh.	G	3139	1.63
30,000–SW	1927	Narrow sh.	G	1440	1.30
6,500–SW	1927	Dr–sh.	G	568	2.15
16,700–SW	1926	Towels and Merino yarn	G	1155	2.10
5,000–SW	1923	Narrow sh.	IL–G	335	3.65
13,600–SW	1923	Outing	IL–G	1027	3.56
14,400–SW	1923	Chambry	IL–G	900	3.05
8,400–SW	1926	Napped sh.	I	633	4.50
17,700–SW	1923	No. duck sh.	G	1437	2.05
12,900–SW	1926	No. duck	IT–G	1673	3.60
10,500–SW	1927	Duck	IS–IT IL–G	1062	3.90
14,200–SW	1925	Tire cord and Fabric	IT–IL G	1430	2.80
32,300–SW	1927	Tire cord and Fabric	IS–IT IL–G	2970	3.60
30,200–S	1927	Tire cord	IT–G	2864	4.37

*Items indicated as group drive include, in all cases, individual drive on the picker room machinery.

which cases usually are lower in installation cost than the original unit.

Horse Power Requirements

In Table No. II is given the average horsepower requirement of various textile machinery and the horsepower rating of motors used for individual drive. Where a range of horse power requirement or motor rating is given for any item, the indication is that the machine may be constructed of more than one size or number of spindles and the nearest standard motor rating should be applied for any specific installation.

The Electric Drive is Applicable to All Branches of Textile Industry

In dealing with the subject matter of this paper the author has necessarily given major consideration to those mills manufacturing cotton yarns and plain or coarse fabric, since the Southeast has long specialized in these products and has produced the major percentage of the total poundage of the United States. As far back as 1923 the Southeast produced 75 per cent of all of the duck (excepting tire duck and fabric), 77 per cent of the sheeting, 82.7 per cent of the denims, 91.5 per cent of the drills, and 93.2 per cent of the osnaburgs, together with various other percentages of all of the cotton textiles produced in the United States. In later years the percentages have increased and it seems but a short time ahead until all of the plain and coarse fabrics will be manufactured in this section.

What has been mentioned with reference to the value of the electric drive in textile plants manufacturing plain cotton goods, applies in an even greater extent to plants on woolen, silk, and rayon products, since increased production and uniformity of finished product is translated into increasingly greater profit as the value of the finished product increases. In the manufacture of knitted goods motor drives have been successfully applied to all machines.

TABLE II

Textile machine	Actual horsepower	Size motor for individual drive
Bale breaker	3/4–1 1/2	1–2
Vertical opener	5 1/2	7 1/2
Conveyor fan and condenser	4–12	5–15
Breaker—lapper—single beater	6	7 1/2
Intermediate lapper	4	5
Finisher lapper	4	5
Card—40 in. revolving flat cotton	1	1 1/4
Card—wool—full roller breaker	2	3
Card—wool—full roller finisher	3 1/2	5
Sliver lap	1/2	1
Ribber lap	1	2
Comber, with aspirator	1 1/8	2
Drawing frame	5 to 6 deliveries per hp.	
Slubber	45 to 55 spin. per hp.	
Intermediate	55 to 60 spin. per hp.	
Fine frame	60 to 80 spin. per hp.	
Jack frame	85 to 95 spin. per hp.	
Spinning frame—ring	30 to 100 spin. per hp.	
Spinning frame—mule	100 to 150 spin. per hp.	
Spooler, 100 sp	1/2	3/4
Warper	1/2	3/4
Twister	15 to 40 spin. per hp.	
Twisters—cabling	8–14	10–15
Slasher	3 1/2	5
Loom—light 40 in. automatic	1/3	1/2
Loom—heavy	1/2	3/4 and up
Loom—narrow on light silk	1/6	1/3–1/2
Winder—quiller	1/3–1/2	1/2
Winder—cap and butt—15 sp.	3 1/2	5
Winder—cone and tube 100 sp.	4	5
Brusher	3–8	5–10
Napper	4–8	5–10
Inspecting table	1/3–1/2	1/2–3/4
Quiller—silk, 40 sp.	1/3	1/2
Winder—silk, 100 sp.	1/4	1/3
Warp. Mill—silk	1/3–5/8	3/4

For bleaching, dyeing, and finishing plants, the electric drive has proved its worth and individual drives have satisfactorily been applied on practically all finishing plant machinery. Due to a lower operating load factor occasioned by the fact that many finishing machines do not operate as continuously as in the spinning and weaving mills, and because of the variable speed requirements the individual drive is especially desirable.

Future Developments

But little further improvement can be desired or expected in the motor and control equipment as now applied to textile machinery. There is, however, much that can be done, through cooperation between the textile machinery and electrical equipment manu-

facturers, with the view of making the motor equipment a more integral part of the complete machine. It is probable that many features in the design of the textile machinery, which are necessary from the standpoint of a belt drive, could be changed to adapt the individual motor equipment more readily and at the same time reduce the cost of the complete machine and drive. The future will undoubtedly see improvements along this line.

Other future developments will no doubt embrace many improvements along the line of automatic control and regulation of the electrical equipment which will govern, to a large extent, the operation of the machines and the quality of the goods produced.

Heat, electrically produced, for slashers and for mercerizing and finishing plants, as well as for the mill buildings, are now feasible and future developments will no doubt make these applications commercially practical realities.

Further improvements in power-factor corrective apparatus and motors having an inherently high power factor can be looked for in keeping with the increased interest manifested in load power factor.

Of particular interest to the textile and electrical industries, the public utilities, and the consulting engineers, is the ever increasing range of usages to which electrical energy is being applied throughout the textile industry. Aside from its importance as the motive power, such miscellaneous applications as the electric stop motions and signaling devices on individual machines, Klaxon and siren type horns for signaling the operators, electric calling and intercommunicating systems, motor driven drinking water systems, motor driven air compressor plants for cleaning, these being automatically controlled to maintain the required air pressure, and individual motor driven humidifier heads and air conditioning equipment may be mentioned, while good lighting is recognized as an income producing investment. In fact, it may safely be stated that electrical energy now serves a necessary and useful purpose, directly or indirectly, in some portion of all activities or operations in the textile industry.

Conclusion

The foregoing remarks should serve to stress the fact that the electric drive is the most logical type of motive power for all branches of the textile industry and that satisfactory motor and control equipment has been developed for individual motor drive on practically all machines. The degree of importance of the uniformity of the finished product as represented by the loss in value due to imperfect goods or seconds, the limiting factor of first cost and the age and type of machine should all be taken into consideration and carefully weighed in arriving at a decision to adopt the individual or the group type of drive for any specific machine.

With the constantly increasing interest in the Southeast as a logical location for plants manufacturing all types of fabrics and finished goods, electrification in the textile industry will ever be an interesting and fertile field for talent of the electrical engineers throughout this section.

Discussion

J. W. Fox: The cotton-mill man is primarily interested in continuous service, the maximum number of pounds from the spinner and the maximum product from a given investment at the least possible cost.

I want to impress the fact that this is the time to prepare to give him that continuous service during next summer's lightning storms. Whether it can be done by new protective devices or in other ways, steps should be taken now.

J. M. Oliver: Our transmission and distribution system covers a large area, and our system is interconnected with those of neighboring companies. We feel that we are doing everything within reason to keep apace with the development of the art to provide continuous service to our customers. The important load centers are provided with at least two, and in many cases more, sources of power supplied over different routes. The protective equipment is so arranged that if one line becomes faulty, it will be immediately isolated, leaving the other line or lines to continue supplying service. Line or equipment trouble causes only a momentary dip in voltage, lasting for only a fraction of a second. We have found no way to prevent these momentary voltage surges.

An analysis of the service to customers over different sections of the system indicates that at least 90 per cent of the interruptions which they suffered during the past lightning season were caused by these momentary voltage disturbances. The interruptions are the result of the operation of instantaneous undervoltage protective equipment on the customers' motors. Many of these undervoltage devices operate practically instantaneously when the voltage dips to from 50 to 80 per cent of normal. For several years some of the manufacturers have had on the market time-delay undervoltage devices. Most of our customers' motors are now protected with the instantaneous type undervoltage relays, and where the time-delay relay has been substituted, material improvement in service has been noted. The cost of substituting these time-delay devices for instantaneous relays is comparatively low, and the substitution certainly seems justified, in view of the improved service.

We have made a series of tests on textile-mill motors. These motors, operating under full-load conditions, were suddenly disconnected from the source of supply, and it was demonstrated that the voltage to the motor could be interrupted for a period as high as four seconds without any injury whatever to the motor or to the quality of the goods. I want to make a plea to manufacturers, to the salesman, and to the customer, to cooperate with us in providing this time-delay protection, which will eliminate these outages which now result from momentary voltage disturbances.

E. F. Pearce: It is needless to say that the power companies are intensely interested in giving the very best service to their customers that is humanly possible. We have been up against a very serious problem on this question of lightning service, and it was very forcibly impressed on us during July of this year. Under a normal year we disconnect from our lines about 6500 kv-a. of transformers that are damaged, either a coil is punctured, a bushing damaged, or the transformer may be completely destroyed.

From January 1 of this year until August 1, we disconnected 30,000 kv-a. of transformer capacity, and 22,000 of that was damaged during July. That gives a picture of what lightning does to the operating department and what their troubles are.

Aside from those difficulties, we have the momentary lightning

surges affecting the undervoltage release equipment of all our customers, particularly on the line that is affected.

It has been my experience that a cotton-mill man will probably stand one or two interruptions a week and not raise a very loud complaint about it, but when he begins to get five and six in an afternoon, or ten in a day, he complains bitterly. We experienced that during July. We saw that we had to do something, and with the cooperation of Mr. Oliver's department and his assistant, Mr. Logan, we went to various types of mills and made specific tests on equipment. We made a thorough investigation of all the switchboard control equipment, the wiring, the number of motors on each circuit, the control equipment on each motor, and got a complete picture of the electrical equipment in various mills.

As Mr. Oliver explained, we went so far as to determine what effect the slowing down of the motor would have on the product and also what effect the returning voltage would have on the motor. We had one particular case where the mill owner said, "Why, that proposition sounds all right, but my machinery won't stand it. I have certain twisters in my mill that will shut down in 1 to 1½ seconds."

We decided, with the customer's permission, to make various tests. We found that the very worst machine would stand 4 seconds without any trouble to the motor or to the goods being produced. We, of course, don't want any relay equipment that will handle a 4-second interruption. In this particular case we have called for 2½-second time-delay relays and we are satisfied that there will be no trouble from that equipment. I think that in the majority of cases 1 to 1½ seconds will be satisfactory.

The power company is perfectly willing to do everything possible to further the installation of time-delay relays. If there is any mill that is in trouble, we want to know about it and we are very glad to make a complete survey of that mill. I ask the cooperation of the manufacturers' salesmen and the engineers in helping us to put across this proposition because it is going to be our only salvation.

The degree to which this time-delay relay should be applied in certain mills warrants a great deal of study. There is a question whether we should put undervoltage relays on switchboard equipment and also on motor equipment. Certainly the cotton-mill operators object to leaving the undervoltage relay off each individual motor for the reason that when they do have an interruption for any great duration of time, if there is no undervoltage relay, it is necessary for an operator to go to each control and take the motor off the line. For that reason, they are inclined to insist that undervoltage devices be put on each individual starter, but if that is the case, then we can apply the time-delay relay to that particular control device.

In this connection, I should like to relate a concrete case of the application of a time-delay relay. About two years ago, we heard that a big development was going to be made in our territory, and up to that time this particular customer had always put in isolated plants, had never purchased power. We got busy with the directors of the company and interested them in purchased power. In the meantime, the equipment had been bought with the idea of operating on an isolated-plant system. As a result, there was certain relay equipment on some synchronous motor-generator sets and one relay was very delicate. It would drop out at about 80 per cent normal voltage, causing innumerable complaints about shut-downs. This customer was blaming us for the class of service we were rendering. We inspected the equipment and made certain recommendations, but they replied that their equipment was all right; that it was our fault and we should improve our service. They went so far as to install a turbine. They operated that turbine part of the time and part of the time they used it as a stand-by unit. When a storm would come up in an afternoon, they would put that turbine on the line and wouldn't wait for our lines to go out.

After several conferences with them, we put the proposition up to them to install certain relays, the same thing that we had been trying to do for the past six months. We went so far as to tell them, "We will put this equipment in for you. If it does what we say it will do, you pay us for it. If it doesn't do what we say it will, we will take it out and you haven't lost anything. You can go on with your turbine and disconnect from us entirely."

Finally the equipment was installed on that basis. It has been operating now for eight months and during July of this year that customer had only two shut-downs and if this relay equipment had not been installed I believe that he would have had over a hundred shut-downs during July.

As a result, he is entirely satisfied with our service. The service today actually is not better than it was before but the results were accomplished by the installation of those little relays that cost him, I think, $225.

George Wrigley: I should like to pledge our help, so far as we can, on this time-delay undervoltage protection. There are still some developments to be made, but it should take care of a great many interruptions. We ask that the power men also please do not neglect the other interruptions.

The mill man can be approached in all fairness and asked to pay for these things without their costing him anything at all. It is just a matter of where he puts his money. He can put it here or put it in losses of shut-downs because these interruptions are extremely expensive. I should like to carry the development forward on that basis.

Power Limit Tests
on Southeastern Power and Light Company's System

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Associate, A. I. E. E. Member, A. I. E. E.

Synopsis.—The general subject of stability and power limits has been discussed several times before the Institute. Most of the papers have been of a theoretical nature, presenting methods by which the power limit of a system could be calculated, or describing various schemes by which the power limit could be increased. Some papers have been written describing tests on miniature systems and a few, giving the results of tests on actual high-voltage systems on the Pacific Coast.

Calculations and tests on small models have indicated that power systems at or near the theoretical power limit could not be operated safely, since any slight disturbance would cause instability with the resultant separation of the power transmission system. Very often, tests on small models do not closely check the results obtained on the real system: therefore, as a result of certain troubles which occurred on the Southeastern Power and Light Company's System in 1925, apparently due to instability, it was decided to attempt to determine by actual tests the amounts of power which could be safely carried over certain transmission lines, and to compare the results with the calculated values. The General Electric Company offered to cooperate with the engineers of the Southeastern Power and Light Company in conducting such tests, and made them of further value by suggesting experimentation with certain schemes of high-speed excitation which could be applied to the generator excitation systems in an effort to increase the maximum power limit which could safely be carried over the transmission lines.

This paper presents the results of these tests and the conclusions to be drawn from them.

* * * * *

1. Introduction

THE Southeastern Power and Light Company serves a territory of considerable size and extent, covering practically two and one-half states. These states are Georgia, in which the Georgia Power Company operates; Alabama, which is served by the Alabama Power Company; and Mississippi, which is supplied by the Mississippi Power Company. The primary transmission system of these companies is operated at 110,000 volts. Previous to the consolidation of these individual companies into the Southeastern Power and Light group, 110,000-volt interconnections, for the exchange of energy, existed between them, the most important ties being between the Alabama Power Company and the Georgia Power Company. The two principal tie lines, and the ones on which these tests were conducted are shown on the map of the Southeastern Power and Light Company systems at the time of test, Fig. 1. During the fall of 1925, there existed a very serious water shortage in Georgia and the Carolinas, which, of course, limited the output of the hydroelectric plants in this territory, and would have necessitated curtailment of power to many industries had it not been for the system interconnections. The Alabama Power Company was more fortunate in having a normal amount of water for that season of the year, and sufficient steam reserve capacity to be able to deliver power over the two tie lines into the Georgia system, some of it being relayed into the Carolinas. The result of this situation was that the two tie lines on which the tests were conducted were loaded to capacity.

The Gadsden-Lindale tie line, which will be called the north line, was designed to carry economically a load of approximately 30,000 kilowatts, and the North Auburn-Newman, or South line, approximately the same amount. During this emergency condition, these lines each carried instantaneous amounts of power up to 50,000 kw., and during the 24-hr. period, an average of 40,000 kw. per line. The transmission losses and the voltage drop over the lines were high, but the power was needed and economics were temporarily forgotten. Unfortunately, the worse lightning occurs during the dry season, and with the lightning came more trouble than was expected. Several interruptions occurred on these tie lines for seemingly unaccountable reasons, and when one tie line would open, the other would immediately follow it.

The Martin Dam plant of the Alabama Power Company was not in active service during that year; hence, there were some 300 miles of transmission line between the Alabama Power Company's plants at Lock 12 and Mitchell Dam, or the government plants at Wilson Dam and Sheffield, and the Georgia Power Company's plants in the Tallulah Falls district in Northeast Georgia as will be seen from Fig. 1.

In order to secure the proper relay protection on these lines during light load conditions, it was necessary to set the relays to operate on values as low as 60,000 kv-a. This, of course, was close to the amounts of power which were being carried over each line during the times of peak load. Therefore after one or two interruptions in which both lines opened, it was at first supposed that the first line was tripped out due to lightning, and the second followed because of overload. However, patrol, and detailed inspection reports, failed to reveal any of the customary indications of insulator flashovers on either line, and a further check of the location of the storm at the time of the interruption

1. Southeastern Engineering Company, Birmingham, Ala.
2. General Electric Company, Schenectady, N. Y.

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served to indicate that the storm was not over either tie line.

Hence, it was decided to obtain more detailed and accurate information at the time of the next interruption, which was expected, and did occur shortly afterward. During peak load conditions, the relay settings on these lines were increased to approximately 100,000 kv-a. and patrolmen and operators were notified to obtain all the information possible on the storm location and any insulator flashovers.

The result was that at the time of the next case of trouble, the cause of the interruption on these two tie lines was definitely determined as being due to instability. A lightning storm occurred south of the plant at Mitchell Dam which flashed over an insulator string on the Mitchell Dam–Lock 18 line, and tripped this line; at the same time, the two tie lines came out, one after the other. Thus it was proved conclusively that these tie lines had opened, first one and then the other, owing to the fact that the limit of power which they could carry had been passed.

At the time of the flashover on the Mitchell Dam–Lock 18 line, there was no more power going over these lines than prior to the short circuit. However, owing to the short circuit, the voltage on the entire system was reduced, and the voltage on the Alabama end of the tie lines was so low that the amount of power which the South line was carrying exceeded the power limit of the line at that voltage. Thus, the Alabama and Georgia plants were pulled out of step and both lines had to open to separate the two systems.

Numerous conferences resulted between engineers of the power company and of several manufacturers relative to methods of calculating the safe power limit of the lines. The calculations involved many indeterminates and assumptions, but finally seemed to indicate that the maximum amounts of power which the lines could carry and still not be subjected to interruptions caused by flashovers on other parts of the system was in the neighborhood of 25,000 kw. per line. This was not even up to the amount of power which the lines had been designed to carry normally, and, consequently, study was given to methods of increasing the power which could be transmitted over them safely. These studies included synchronous condenser installations, faster relaying on the tie lines and other parts of the system by means of carrier-current pilot wire protection, and the speeding up of the excitation systems on the generators at the various plants. The last method,—that of speeding up the excitation systems on the generators,—seemed to be the most logical, and was by far the most inexpensive.

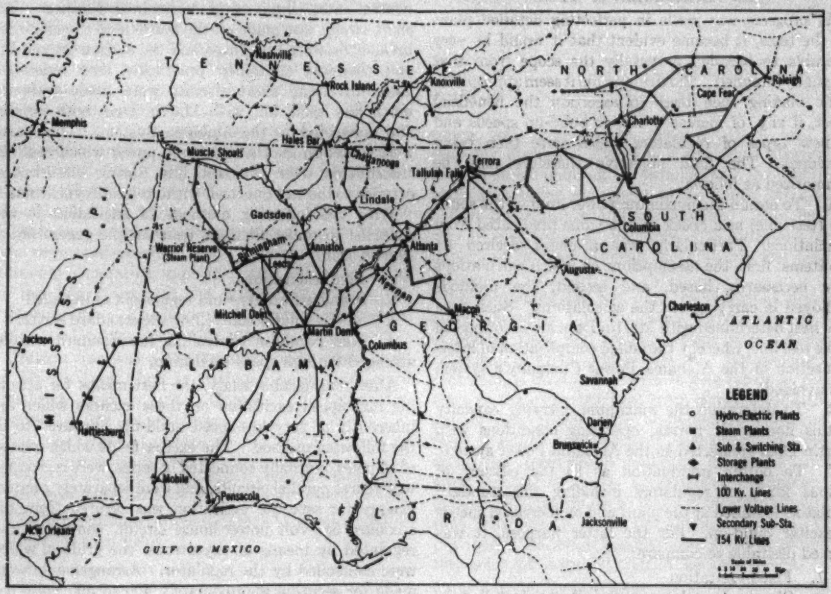

FIG. 1—MAP OF SOUTHEASTERN POWER & LIGHT COMPANY SYSTEM

After many discussions between the engineers of the General Electric Company and the Southeastern Power and Light Company, it was found desirable to make certain tests on these tie lines during the spring of 1927. The Alabama Power Company's plant at Martin Dam would then be completed and it was hoped that water conditions would be such that the Georgia Power Company would not require much power, thus making available for test the plant at Martin Dam and the tie lines. During the month of June, there was sufficient water at the Georgia plants to carry the Georgia load with practically no interchange over the tie lines, and as a result, the tests were made at that time.

II. Description of Tests

As progress was made in perfecting detailed plans for the tests, it became evident that it would be very desirable to broaden materially the scope from that originally intended. Particularly did it seem appropriate while making such tests to ascertain the beneficial effect, if any, of increased exciter build-up speeds and of new types of regulators which were then being developed. The final objective therefore may be summarized as follows:

A. To obtain the maximum power limit of the south line (test line) as a check on the value predicated from calculations. Particularly was a check desired on two items: first, the assumptions on which calculations were necessarily based, and second, the methods employed in carrying out the calculations. Such data were best determined with Martin Dam alone connected to the test line, whereby the added complication of interconnection of the Alabama Power Company's system was avoided.

B. To determine the maximum carrying capacity of this line under normal operating conditions with Martin Dam connected to the Alabama Power system.

C. To obtain information as to the efficacy of various kinds of regulators including the standard regulators installed at the plant and on increased speeds of exciter build-up. For the latter purpose, it was deemed desirable to compare:

1. Fixed excitation
2. Standard regulators and exciters as installed
3. New regulator in conjunction with
   a. Standard exciters
   b. Exciters arranged for increased speed of build-up.

The comparisons indicated under C were desired not only for steady-state operation, (*i. e.*, with no transients or disturbances on the system), but also for the condition of suddenly applied loads such as obtained when the north line was tripped and also during short circuits nearby on the system.

Three distinct kinds of test were therefore decided upon:

a. Steady-state pull-out between Martin Dam and the Georgia system both with Martin Dam alone connected to the test line and with Martin Dam and the test line connected to the Alabama system. For these tests when Martin Dam was connected to the Alabama system, the north line remained open.

b. Suddenly applied load by loading up the two tie lines in parallel and then tripping the north line to determine how much load could thereby be shifted to the test line without causing loss of synchronism. For this test it was expected to obtain points both above and below the value causing instability in order to determine as accurately as possible the maximum value of load which could be thus transferred.

c. Short circuit, obtained by closing a 110-kv. bus-tie oil circuit breaker at Martin Dam on a line-to-neutral short circuit of predetermined duration. The short circuit was made on the line side of the oil circuit breaker between one phase and the station ground bus and therefore contained practically zero resistance. Tests involving short circuits were made with the north line open, but with Martin Dam both isolated and connected to the Alabama System. These tests were to determine the maximum power which could be transmitted over the test line during disturbances exterior to the test line itself without loss of synchronism.

With the following methods of excitation it was expected to make all these tests on the generators at Martin Dam:

a. Fixed excitation
b. Standard exciters and regulators as installed
c. New regulator controlling the standard exciters
d. New regulator controlling the standard exciters arranged for high speed excitation.

After considerable study, the best means for attaining high-speed excitation on these exciters which are inherently of very low-speed build-up, appeared to be the following method: The exciter fields which consist of 10 poles, normally connected in series, were regrouped to give five parallel circuits, and were separately excited. Energy for separate excitation was secured from the auxiliary 550-volt power-house circuit, converted and regulated by means of thyratrons, the grids of which were controlled by the regulator. Arrangements were made for securing approximately 400 to 500 volts per second. Some tests were taken with the transformers on a half-voltage tap, giving about half that rate of rise.

III. Test Procedure

General Order of Tests. A total of approximately 50 tests was scheduled, but 31 were actually made, and these furnished practically all the data desired. Because most of the tests had to be made during the peak-load periods, and also because no testing could be done during lightning storms or other troubles on the system, no definite schedule could be adopted more than three or four hours in advance. In addition, it was felt that the results of one test might necessitate others not contemplated, or might eliminate some

proposed. As work progressed, several tests were eliminated, since data obtained indicated such tests to be unnecessary.

In order to obtain the data with different speeds of exciter build-up, it was necessary to make major changes in the generator excitation systems. These changes involved considerable time and in order to facilitate the work on the generators and reduce the time of testing to a minimum, the tests were grouped according to the types of excitation used.

The first group were those using the normal excitation from the direct-connected exciters furnished with the generators, but with different types of voltage regulators. Steady-state tests, dynamic tests, and those involving suddenly applied loads, and short circuits were made. The steady-state tests were made first with the generators at Martin Dam isolated from the Alabama system and connected only to the test line; and second, with the test line tied in with the Alabama system. After completing this program, the connections of the exciters were changed for the so-called high-speed excitation. First an exciter voltage build-up of approximately 250 volts per second was used; and then 400 volts per second.

Equipment Used. One, two, or all three of the 37,500-kv-a. waterwheel driven generators at Martin Dam were used on each test. Those tests, requiring additional generating capacity, were made by tying the test line directly to the system 110,000-volt bus at Martin Dam. The load was the normal commercial load of the Georgia Power Company.

The test line was a 110,000-volt single circuit from the Martin Dam plant in Alabama to the Boulevard Substation near Atlanta, Georgia. The characteristics of this line are as follows:

Total length, 139.8 mi. (224 km.)
Total resistance (R), 41.5 ohms per conductor
Total inductance (L), 273.3 millihenrys per phase to neutral
Total capacitance (C), 1.957 microfarads per phase to neutral

The line contained three different types of construction as described below:

A. *Martin Dam—North Auburn.*
Length, 26.92 mi. (43.3 km.)
Towers, H-frame wood poles
Spacing, 17 ft. (5.18 m.) horizontal
Ground wires, 2 1/0 A. C. S. R. Conductors
Insulation, Locke 5800, 10-in. disk, 7 units
Size conductor 397,500 cir. mils, A. C. S. R.
R—6.33 ohms
L—56.8 millihenrys
C—0.3715 microfarads.

B. *North Auburn to Georgia State Line.*
Length, 40.4 mi. (65 km.)
Towers, H-frame wood poles
Spacing, 14 ft. (4.27 m.) horizontal
Ground wires, 2 1/0 A.C.S.R. conductors
Insulation, Locke 5800
Size conductor, 397,500 cir. mils, A.C.S.R.
R—9.5 ohms
L—82.5 millihenrys
C—0.574 microfarads.

C. *Georgia State Line to Newnan.*
Length, 30.22 mi. (48.3 km.)
Towers, wood pole H-frame and Steel "H" Frame
Spacing, 13 ft. (4 m.) horizontal
Ground wires, none
Insulation, O. B. 25622
Size conductor, 397,500 cir. mils, A.C.S.R.
R—7.052 ohms
L—61.3 millihenrys
C—0.4304 microfarads.

D. *Newnan—Boulevard.*
Length, 42.29 mi. (68 km.)
Towers, double-circuit steel
Spacing, 9 ft. (2.74 m.) vertical, 15 ft. (14.57 m.) horizontal
Ground wire, none
Insulation, O. B. 25622
Size conductor, 2/0 hard drawn copper, 7-strand.
R—18.62 ohms
L—92.7 millihenrys
C—0.582 microfarads.

The map (Fig. 1), shows geographically the location of the test line, the plant at Martin Dam, and the Boulevard Substation, also the Gadsden-Lindale line which was used on the tests in which load was suddenly dropped. Fig. 2 is a single-line diagram of the test circuit showing the switches with their numbers and the metering equipment on the south line used to obtain the readings and oscillograms. This line, as may be seen from the diagrams, constitutes one of the important tie lines between the Alabama and Georgia systems.

For obtaining the test data, oscillographs, and portable laboratory type indicating meters were located at Martin Dam, and graphic meters were installed at Gadsden, Newnan, and Boulevard. Six oscillographs containing a total of 21 elements were set up at Martin Dam in the cable terminal room directly beneath the switchboard room. The oscillographs consisted of four standard three-element machines, one special wattmeter oscillograph and one six-element instrument containing three single-phase wattmeter elements. The reason for so many oscillographs was that they were set up to obtain duplicate results, since, previous to the test, it was unknown just how accurately the point of instability could be determined. Consequently, on the first tests, one group of oscillographs were started as the pull-out point was approached, and if these did not record the actual pull-out, the second group was started. Experience during the first two or three tests indicated that the approach of the pull-out point was readily determinable and on all further tests, one master switch was used to start all the oscillographs at one time and duplicate films were obtained. The indicating wattmeters, voltmeters, ammeters, and power-factor meters, were arranged on a table in the terminal room, between groups of oscillographs.

The graphic instruments installed at other points on the system merely as a check, were as follows:

Newnan—Voltmeter, wattmeter, ammeter, and power-factor meter.

Boulevard—Voltmeter.

Gadsden—Voltmeter, wattmeter, and power-factor meter.

Procedure of Making Tests. Due to the uncertainties in the testing schedule previously mentioned and the relatively large personnel actively engaged in conducting the tests, conferences were held two or three times each day to discuss the previous results and definitely schedule the tests to follow. This proved very satisfactory and as the work progressed, eliminated many tests which were found unnecessary thus saving considerable time and duplication of effort. These conferences also resulted in a minimum disturbance to the system, which of course was most desirable.

During all tests, telephone communication was maintained between one of the men reading the portable instruments in the test room, who also had control of the starting switch for the oscillographs, and the supervisor of the tests who was located in the plant switchboard room. Continuous telephone communication was also held between the plant superintendent at Martin Dam and the chief load dispatcher of the Alabama and Georgia systems.

One of the plant operators was stationed at the switchboard, and was directed by the supervisor of tests as to when to clear the test line after pull-out. Relays were used on the clearing switch, but they were given a relatively long time setting in order to insure complete data being obtained on the oscillograms. Had the relays been used to clear the line, in most instances the period in which the equipment was out of step would have been longer than necessary to obtain complete data, and therefore would have prolonged the system disturbances. The net result of this arrangement was that practically no complaints were received from customers of either company, and the only effect that the tests had on the operating system was that in one or two instances a rotary converter in Atlanta lost its synchronism and fell out of step. During all of the tests, there was no load lost.

In making the steady-state tests, the procedure was as follows: When all the oscillographs and system connections were ready, the chief oscillograph operator notified the supervisor of the tests, and he in turn notified the plant superintendent to request the Georgia dispatchers to drop a certain amount of generation. This load was picked up either entirely by the generators at Martin Dam, or by these machines and the Alabama system, depending on whether the tests were being made with the system tied in or not. The load on the test line was gradually built up to a point within 5 or 10 per cent of the calculated pull-out point. The voltage was then adjusted to as near 110,000 volts as possible and the power factor as close to unity as practical. Finally, with everything in readiness, the Georgia dispatcher was notified to drop load rapidly, and this in turn was picked up by the Martin Dam machines until the unstable point was reached. The circuit was cleared at the direction of the supervisor of tests after he was notified that the oscillographs had obtained the record of the pull-out phenomena.

On those tests in which load was suddenly dropped, the test line and the north tie line were loaded to a total load equal to that calculated to make the test line unstable when the north tie line was tripped. This necessitated another link in the telephone facilities and during these tests, the plant superintendent at Martin Dam gave orders over the phone to three parties; the Alabama and the Georgia dispatchers and

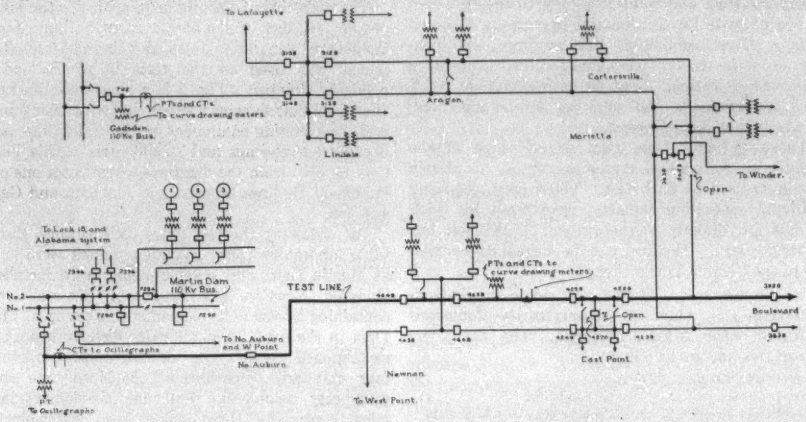

Fig. 2—System Connections for Power Limit Tests

the man stationed at Gadsden to read and report on the power over the north tie line. As the proper load was reached, the oscillographs were started and the man at Gadsden directed to trip the Lindale line

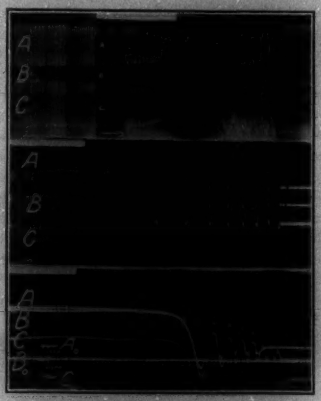

Fig. 3—Test No. 10. Steady-State Pull-Out of Test Line

Film No. 1—Line-to neutral volts on Martin Dam 110-kv. bus
Film No. 2—Current in test line
Film No. 4—Curve *A* Power, generator No. 2
Curve *B* Power in test line
Curve *C* Power, generator No. 3

The short-circuit tests were relatively simple and consisted of grounding one phase of the Martin Dam 110,000-volt bus by closing one of the bus tie oil circuit breakers on a previously made short circuit. The short-circuiting switch was closed by the plant operator on orders from the supervisor of tests immediately

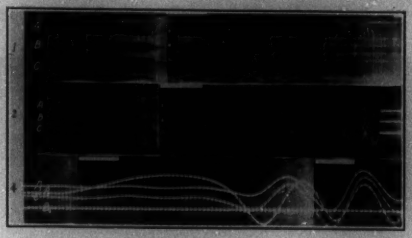

Fig. 5—Test No. 5B. Suddenly Applied Load by Tripping North Line; Synchronism Lost

Film No. 1—Line-to-neutral volts on Martin Dam 110-kv. bus
Film No. 2—Current in test line
Film No. 4—Curve A Power, generator No. 2
Curve *B* Power in test line
Curve *C* Power, generator No. 3

after the load on the test line had been adjusted and the chief oscillograph operator had notified him that the oscillographs had been started.

Dark room facilities were arranged in the plant, and practically all of the films were developed immediately

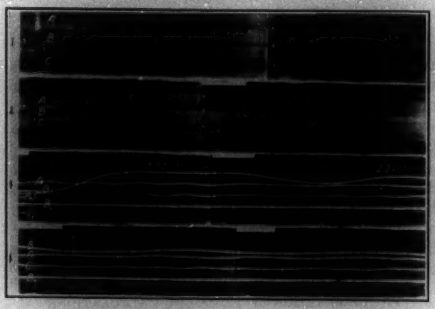

Fig. 4—Test No. 7B. Suddenly Applied Load by Tripping North Line; Synchronism Maintained

Film No. 1—Line-to-neutral volts on Martin Dam 110-kv. bus
Film No. 2—Current in test line
Film No. 4—Curve *A* Power, generator No. 2
Curve *B* Power in test line
Curve *C* Power, generator No. 3

immediately. If the test line failed to become unstable or appeared to reach instability too rapidly the tests were repeated with a slightly larger or smaller load as the individual case dictated.

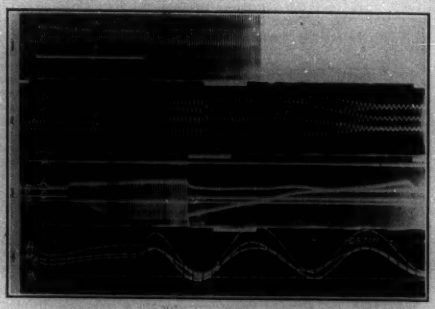

Fig. 6—Test No. 25A. Line-to-Ground Short-Circuit Test with Alabama System not Connected; Synchronism Maintained

Film No. 1—Line-to-neutral volts on Martin Dam 110-kv bus
Film No. 2—Current in test line
Film No. 3—Curve *A* Generator, slip-ring volts
Curve *B* Generator field current
Curve *C* Short-circuit current
Film No. 4—Curve *A* Power in test line
Curve *B* Power, generator No. 2
Curve *C* Power, generator No. 3

following each test. Thus, if for any reason the films were unsatisfactory, the test could easily be repeated.

IV—Curves and Tables of Results

The results of the tests are presented in the form of curves and tables. In addition, there are included reproductions of some of the oscillographic records, Figs. 3 to 9. From these oscillograms, it is possible to

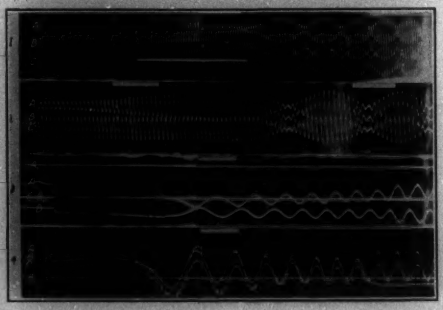

Fig. 7—Test No. 15B. Line-to-Ground Short Circuit Test with Alabama System Disconnected; Synchronism Lost

Film No. 1—Line-to-neutral volts on Martin Dam 110-kv. bus
Film No. 2—Current in test line
Film No. 3—Curve *A* Generator, slip-ring volts
Curve *B* Generator field current
Curve *C* Short circuit current

Film No. 4—Curve *A* Power in test line
Curve *B* Power, generator No. 2
Curve *C* Power, generator No. 3

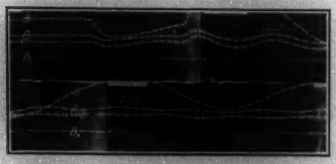

Fig. 8—Test No. 30B. Line-to-Neutral Short Circuit Test with Alabama System Connected; Synchronism Maintained

Curve *A* Power, generator No. 2
Curve *B* Power in test line
Curve *C* Power, generator No. 3

obtain a very comprehensive qualitative idea of the phenomena accompanying the disturbances set up by the various tests. The oscillograms of power were obtained on a three-unit instrument, each unit of which consisted of a three-phase assembly of wattmeter elements. One of the elements recorded power input to the test line; the other two, the power output of two of the Martin Dam generators. The moving part of these elements contained considerable inertia. The records of the tests for steady state pull-out and for the increase in power when tripping the north line are probably fairly accurate, since power changes in the test circuit were not very rapid. Owing to the inertia of the elements, however, the oscillographic record of power changes during the short-circuit tests are not entirely satisfactory. In analyzing the test data, therefore, it was necessary to make suitable correction of the values shown by the oscillograms to allow for the effects of inertia and damping. A check on the power input to the test line was obtained from the records of the single-phase wattmeter oscillograph.

Table I presents the values of power obtained in steady-state pull-out with Martin Dam alone supplying the test line. For the sake of comparison, there are also included values of power calculated for the test line, plus the terminal apparatus with fixed excitation on the generators and at the values of terminal voltage observed in each test. In Table II are shown the

Fig. 9—Test No. 23B. Suddenly Applied Load by Tripping North Line; Synchronism Maintained

Curve *A* Power, generator No. 2
Curve *B* Power in test line
Curve *C* Power, generator No. 1

corresponding values of power at steady-state pull-out with the Alabama System connected. As a means of comparison, there are also included in this table, values which it is estimated from the test data, the power would have attained at 110 kv.

Table III gives the results of the tests in which a line-to-ground short circuit was thrown on the 110-kv. bus at Martin Dam, and for comparison, an estimate of the same values corrected to 110 kv. Table IV includes the results of tests in which the north line was tripped and the corrected values for 110 kv.

V. Comparison of Tests and Calculations

One of the purposes of making the tests was to obtain a check on some of the basic assumptions necessary in carrying out mathematical computations of power limits. It is therefore interesting to compare the values obtained from the tests with values calculated for the same conditions. Such a comparison for the steady-state pull-out with only Martin Dam connected to the test line is given in Table I. In the test with fixed excitation on the Martin Dam generators, a value of power equal to 98 per cent of that calculated for similar conditions was attained. This agreement is

TABLE I

STEADY-STATE POWER LIMIT WITH MARTIN DAM ALONE CONNECTED TO TEST LINE

Test No.	Excitation	Kilovolts		Megawatts		Ratio test to calc.	Test megawatts as percentage of maximum for line alone
		Martin Dam	Atlanta	Test	Calc. for same E_g and E_r		
8	Form Y—Std. exc.	109.98	95.26	93.6	82.3	1.137	73.6
9	Form K—24 exc.	107.90	95.26	90.6	80.6	1.124	73.7
10	Hand control	106.86	99.76	78.6	80.2	0.980	62.5
18	Form W—250 V/sec.[1]	112.75	95.60	104.5	90.9	1.150	79.8
21-A	Form Y—500 V/sec.[1]	103.92	99.24	97.4	75.4	1.292	80.5
21-B	Form W—500 V/sec.[1]	109.98	93.87	93.0	83.6	1.112	74.5

1. See text page 9 regarding thyratron grid control

TABLE II

STEADY-STATE POWER LIMIT WITH ALABAMA POWER SYSTEM CONNECTED TO MARTIN DAM AND TEST LINE

Test No.	Kilovolts		Megawatts		Excitation	No. gen. used at Martin Dam
	Martin Dam	Atlanta	From test	Corrected to 110 kv.		
1	107	96.2	90	106	Hand control	2
2	111	101.	104	112.3	Form Y std. exc.	2
3	110	102.	102	110	Form W std. exc.	2
4	110	102.	97.2	104.9	Form K—24 std. exc.	2
19	112	96.	104.5	117.6	Form Y 250 V/sec.[1]	3
22	108	101.	102.7	113.8	Form Y 500 V/sec.[1]	3

1. See text page 9 regarding thyratron grid control.

TABLE III

MAXIMUM POWER PRIOR TO LINE TO GROUND SHORT CIRCUIT THAT COULD BE CARRIED WITHOUT LOSS OF SYNCHRONISM

Test No.	No. Martin Dam machines	Line kv.		Estimated maximum power in kw.		Duration of short circuit in seconds	Volt-sec. rise during 1st half swing of gen.[1]	Excitation
		Martin Dam	Atlanta	From tests	Corrected to 110 line kv.			
Alabama System not connected to test line								
11	2	102.5	105.0	28,000	31,450	0.857	−35.6	Hand control
12	2	103.0	105.0	28,000	31,300	0.882	−34.9	Form K—24 std. exc.
14	2	103.0	105.0	29,000	32,400	0.832	−20.8	Form Y—std. exc.
15	2	102.4	105.0	32,000	35,900	0.815	+12.7	Form Y 250 V/sec.[2]
16	2	102.0	105.0	32,000	36,100	0.807	− 2.0	Form W 250 V/sec.[2]
17	2	101.0	104.5	32,000	36,600	0.807	− 0.7	Form Y 250 V/sec.[2]
25	2	114.0	104.0	39,500	40,300	0.825	+49.3	Form Y 500 V/sec.[2]
26	2	115.0	102.8	48,000	49,000	0.608	+31.2	Form Y 500 B/sec.[2]
27	2	114.0	103.0	55,000	56,000	0.433	+43.9	Form Y 500 V/sec.[2]
29	1	114.5	103.0	22,500	23,100	0.915	−35.0	Form K—24 std. exc.
31	1	114.0	102.7	23,000	23,700	0.907	−35.0	Form K—24 std. exc.
Alabama System connected to test line								
24	2	112.0	101.5	55,000	58,500	0.682	+12.2	Form Y 500 V/sec.[2]
30	2	113.0	101.0	52,500	55,700	0.682	−16.7	Form K—24 std. exc.

1. See Appendix I. 2. See text page 9 regarding thyratron grid control.

TABLE IV

MAXIMUM POWER THAT COULD BE CARRIED OVER TEST LINE PRIOR TO TRIPPING NORTH LINE, WITHOUT LOSS OF SYNCHRONISM

Test no.	Line kv.		Estimated maximum power in kv.		Excitation
	Martin Dam	Atlanta	From test	Corrected to 110 line kv.	
5	114.0	102.0	36,000	37,500	Form Y—Standard Exciter
6	113.5	102.0	36,000	37,600	Form K—24 Exciter
7	114.0	104.0	36,000	36,700	Hand control
20	112.0	105.0	36,000	37,000	Form Y—250 V/sec.[1]
23	110.0	102.0	35,000	37,600	Form Y—500 V/sec.[1]

1. See text page 9 regarding thyratron grid control.

closer than the probable error in the test readings. Fig. 10 shows a comparison of calculated and test results during a line-to-ground short circuit. Figs. 11 and 12 present a comparison of calculated and test results when power over the test line is suddenly increased by tripping the north line.

VI. Notes on Test Results

It should be borne in mind that the results herein presented were obtained on commercial power systems under actual conditions of operation, and mostly during times of peak load. This very fact precluded the attainment of results of an accuracy approaching laboratory precision. Every test was accompanied by a severe disturbance on the system, and it was therefore

deemed desirable not to try to check any points unless for some reason the test had been a failure. There was no checking merely for the sake of checking. In some of the tests, the tripping of certain switches or the manipulation of load was necessarily in the hands of persons remote from the scene of test and carried out on telephone instructions. This condition did not increase the probability of satisfactory results.

The test was carried out and the power measurements

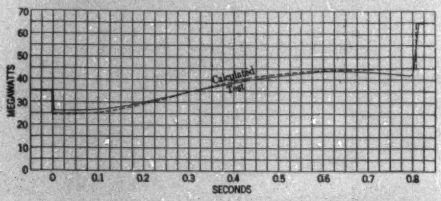

Fig. 10—Test No. 25A. Comparison of Calculated and Test Values of Instantaneous Power during Line-to-Neutral Short Circuit on Martin Dam High-Tension Bus

made at the input end of the test line. Practically all calculations made heretofore on the power limit of a line have considered the maximum amount of power obtainable at the receiver end. The relationship between these two is presented in Fig. 13. Curve *C* in this figure indicates the maximum power which could be obtained if the reactance of all terminal

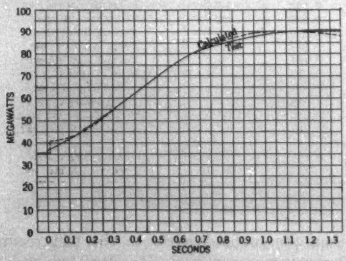

Fig. 11—Test No. 7B. Comparison of Test and Calculated Values of Instantaneous Power during Sudden Increase in Load by Tripping North Line

apparatus at both generator and load could be reduced to zero. It will be noted that the maximum power at the generator end occurs slightly after the systems have started to fall out of step and is somewhat greater than when the receiver is at a maximum. The absence of second harmonic in these curves is owing to the method of calculation, and does not detract from their value for comparative purposes.

In certain of the tables, reference is made to control of the Martin Dam generators by types *W* and *Y* regulators. The type *W* regulator is that described by Messrs. R. E. Doherty,[3] C. A. Nickle, and R. M. Carothers[4] in papers presented at the St. Louis regional meeting of the Institute in March 1928. The type *Y* is the commercial form thereof also referred to in those papers.

Some of the tests were made with the exciter fields

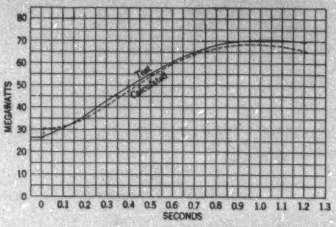

Fig. 12—Test No. 7A. Comparison of Test and Calculated Values of Instantaneous Power During Sudden Increase in Load by Tripping North Line

supplied from a thyratron. The grid control for the thyratron was from d-c. batteries. Such control does not produce an exciter voltage decay at the same rate as the voltage rise. From the standpoint of aiding the new type of regulator to provide positive damping for the system to smooth out oscillations as rapidly as possible, it appears that the rate of decrease of exciter voltage should be approximately equal to the rate of increase. For this reason, the new regulators probably could not show their maximum efficiencies.

Before starting the tests, the maximum power which could be transmitted over the test line to the Georgia

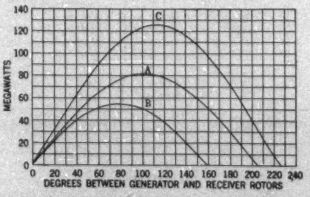

Fig. 13—Calculated Power Angle Curves for Test Line and Systems under Conditions of Test No. 10

Curve *A* Power at generator end of line under test conditions. The maximum of this curve is the value observed on all these tests

Curve *B* Power at load end of line under test conditions

Curve *C* Power at generator end of line if the power were limited only y the impedance of the test line

system under steady conditions was calculated. The calculation was based necessarily upon assumptions with regard to the reactance of the Georgia System and the values of voltage at each end of the test line

3. July Quarterly Trans., p. 944.
4. July Quarterly Trans., p. 957.

which could be used under the test conditions of abnormal loading of the south line. Data from which to make such assumptions were meager, and the preliminary estimates of maximum power were too low. In the tests, it was possible to carry with fixed excitation amounts of power nearer the maximum theoretical limit than had been anticipated. The margin left for improvement owing to the different excitation systems proved to be considerably less than expected. Considering this, and the unfavorable condition as regards thyratron grid control, as well as the possible

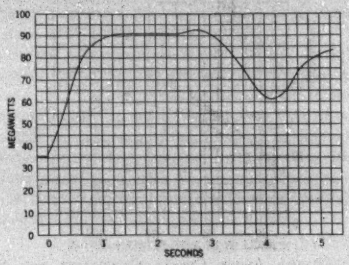

FIG. 14—FLAT TOPPED POWER WAVE ON FIRST SWING
Plotted from data obtained on test 5A

errors in the test results owing to the conditions under which they were made, no conclusive data were obtained on the efficiency of the new regulator, or of high-speed excitation.

An examination of the wattmeter oscillograms of tests in which the two systems were set to swinging with respect to each other, but not pulled out of synchronism, shows that the Alabama system as a whole, with respect to the Georgia system, swings at a periodicity of approximately two seconds, (Fig. 4). When the two systems are set into oscillation as the result of tripping the north line, there appears to be a fair degree of damping; but when the oscillation results from a short circuit, the damping between the two systems is negligible. When Martin Dam alone is connected to the test line, it swings, with respect to the Georgia system, with a periodicity of about ¾ sec. and there is very little damping, (Fig. 6). Fig. 8 shows that the Martin Dam swings with respect to the Alabama system with a periodicity of about ⅔ sec. with practically no damping. From the foregoing, it is evident that when, by sudden increase in load, the Alabama and Georgia systems were made to oscillate with respect to each other, there was an appreciable amount of damping, but very little of it is located in the Martin Dam station.

The exact reasons for the marked difference in the degree of damping out of oscillations between the two systems, when caused by short circuits, and when caused by sudden increases in load, form an interesting and profitable field for future study.

There was a certain amount of variation in the power over the test line, even at relatively light loads. This was owing to the tendency of the two systems to drift in speed with respect to each other. This variation, however, increased the difficulty of obtaining the desired degree of precision when making some of the tests. In the short-circuit tests and those when the north line was tripped, sometimes a test would be obtained which was considered to be very near the top limit. Then it would be attempted to try the next test with the power increased by approximately 2000 or 3000 kw. However, with the power over the test line varying by 5000 kw., in a somewhat irregular manner, the next test might turn out to be the same as the last, or it might show an increase of 5000 to 7000 kw. Owing to the undesirability of subjecting the systems to a large number of severe disturbances, it was not deemed desirable to try for too close a check of these points.

In looking at the oscillograms of some of the short-circuit tests and of those when the north line was tripped, it will be noted that on the first swing the power rose to a maximum value, the duration of which was considerable compared to the periodicity of the subsequent swings. The significance of these flat-topped waves on the first swing may best be explained by a mechanical analogy. A well balanced bicycle wheel, when raised from the ground, will come to rest with the air valve at the bottom. When the air valve is moved a certain amount, the wheel will oscillate

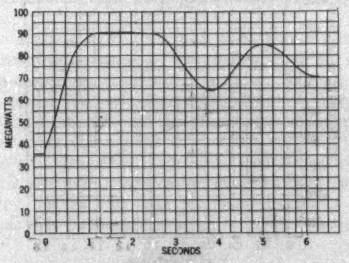

FIG. 15—FLAT TOPPED POWER WAVE ON FIRST SWING
Plotted from data obtained on test 7B

like a pendulum with a periodicity which is practically independent of the amplitude. However, if the wheel is given enough of an impulse to carry the air valve almost, but not quite, past center at the top, it will hesitate in this position for a considerable period. This hesitation at the maximum of the first swing is analogous to the flat-topped power wave observed on these tests. The existence of the flat-topped wave indicates that the systems were near to losing synchronism. The greater the duration of this first swing maximum the nearer they were to going out of step. This phenomenon provides a method for estimating the true

value of maximum power under certain test conditions when the two values actually obtained were so far separated as to leave some uncertainty as to what the maximum value actually was. The results presented in Tables III and IV are the maximum values of power estimated from the test results by examining the characteristics of the power waves and also by examining

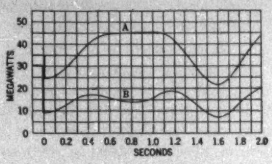

Fig. 16—Flat-Topped Power Wave on First Swing Occasioned by Line-to-Neutral Short Circuit Calculated for Conditions Observed on Test 25A

Curve *A* Generator power
Curve *B* Receiver power

the nature of the decrement in the voltage waves. Figs. 14 and 15 have been derived directly from the test data to show more clearly this flat-topped wave on the first swing.

To check the theory as to the cause of these flat-topped waves, calculations were made to see if they could be produced mathematically. A condition was assumed in which the swing was started by a short

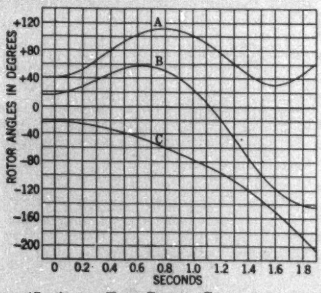

Fig. 17—Angle Time Curves Calculated for Same Conditions as Fig. 16

Curve *A* Total angle between generator and receiver rotors
Curve *B* Absolute angle of generator rotor
Curve *C* Absolute angle of receiver rotor

circuit; but the phenomenon should be the same whatever the cause. Fig. 16 is the result. In studying Fig. 16, Figs. 17 and 18 should also be noted. They show the relation between angle time and angle power, calculated for the same phenomenon. It is interesting to note from these curves that it is possible for the angle between load and generator to exceed the critical steady-state angle by a considerable amount without losing synchronism.

While making these tests, it was felt desirable to obtain some check on the expectation that a reduced duration of short circuit would be beneficial to stability. Tests were made, therefore, with the short circuit main-

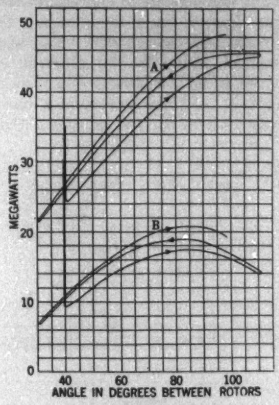

Fig. 18—Power Angle Curves Calculated for Same Conditions as Fig. 16

Curve *A* Generator power
Curve *B* Receiver power

Note that the Martin Dam generators and the load swing apart by over 110 deg. without losing synchronism

tained for differing times, and the results are shown graphically in Fig. 19. It should be remembered that these data show the result with a metallic short circuit from one phase-to-ground directly on the 110-kv. bus

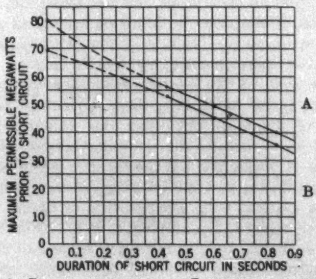

Fig. 19—Relation between Duration of Line-to-Neutral Short Circuit and Maximum Power That Can be Carried Prior to Short Circuit Without Losing Synchronism

Curve *A*—High speed excitation
Curve *B*—Constant slip ring volts

at Martin Dam. No tests were taken with any other form of short circuit. It is expected that similar results would be obtained if it had been possible to try shorts with varying amounts of resistance and reactance simulating conditions when a fault occurs at some

distance from the station, but they would not necessarily be comparable in magnitude.

Owing to the high total resistance of the test circuit, it is to be noted that when operating at values near the pull-out a very substantial part of the input to the test line was consumed in line losses. During the tests in which the north line was tripped throwing the total load on the test line, the total line losses were increased by some 15,000 kw. This is shown in Fig. 13. Thus additional load was created which tended to slow down both systems. Owing to the difference in the power-frequency and load-frequency characteristics of the two systems, this increased load undoubtedly had some effect on the test line loading.

In the tests for maximum power under no-disturbance conditions, after passing the maximum value at the receiver end, the generator load continues to increase owing to greatly increased line losses. At this time, apparatus at both ends of the line tend to slow down in absolute speed, thus keeping in "near" synchronism for quite a period. Thus, the process of pulling out of step is one requiring considerable time, probably of 15 to 30 sec. duration, possibly even longer.

VII. Conclusions and Summary

The safe loading of the test line for operating conditions, considering that the north line may be tripped at any moment as a result of a disturbance on it, appears to be about 35,000 kw. From the standpoint of maintaining synchronism during short circuits on other circuits, the safe loading of the test line depends

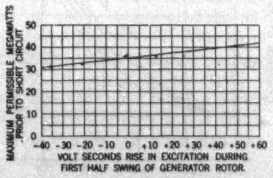

Fig. 20—Relation between Excitation and Maximum Power that can be Carried Prior to Line-to-Neutral Short Circuit of 0.83 Sec. Duration without Losing Synchronism

on the duration of the short circuit, and for a one-second duration, appears to be about 35,000 kw.

The gain in power which could be carried with the increased speeds of excitation used in these tests appears to be not great. Further consideration is given to the effect of excitation speeds in Appendix I, contributed by F. R. Longley, who has made most of the mathematical computations in connection with working up the short-circuit tests and those when the north line was tripped.

The use of a voltage regulator showed some gain in steady-state power limit over the value with fixed excitation. For reasons already noted in the text, the tests were not conclusive with reference to the efficacy of the new regulators.

While the Alabama and the Georgia systems showed a fair degree of damping when made to oscillate with respect to each other by load increases, there was very little damping evidenced when the oscillation was between Martin Dam and either of the two systems. This would indicate the desirability of providing waterwheel-driven generators with amortisseur windings.

A reduction in the duration of short circuits appears to have a very beneficial effect upon the amount of power which can be carried during a short circuit.

Information was secured concerning the equivalent

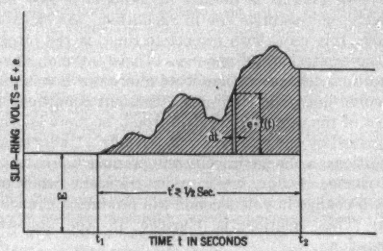

Fig. 21

reactance,[5] damping, and inertia of the systems on which the tests were made, which permit very close mathematical checks on the results attained.

The authors desire to express their appreciation of the assistance rendered by the executives of the companies concerned, in granting permission to make the tests; to the operators, load despatchers, maintenance crews, and engineers of the operating companies for wholehearted cooperation in carrying out the tests; and to members of the Engineering Department of the General Electric Company for valuable assistance in working up the data, and for many pertinent suggestions as to methods of analysis and interpretation.

Appendix I

The Effect of Excitation Upon the Maximum Power that can be Carried Through a Transient Disturbance

By F. R. Longley[6]
Associate, A. I. E. E.

An excitation system may be defined as an arrangement to control flux linkages of synchronous apparatus in some predetermined manner. The efficiency of the system depends upon its ability to produce the proper

5. By "equivalent reactance" is meant that value of reactance which, for purposes of calculation, is used for the actual reactance of a large composite system.

6. Electrical Engineer, General Electric Co., Schenectady, N. Y.

change in flux linkages the moment the change is required.

In most of the previous papers on high-speed excitation, consideration has been given only to the rate of rise of exciter voltage; that is, volts per second, or, in some cases, the maximum, or ceiling voltage, as a measure of the efficacy of the system.

To effect a change in flux linkages requires time. Neither volts per second nor ceiling volts alone will produce the change. They must have time in which to operate in order to overcome the self inductance of the field circuit and produce the required change in flux.

It will be noted that the excitation versus power curve in Fig. 20 is plotted in terms of a new unit; namely, volt-seconds rise in excitation. As explained below, this unit, with respect to time, is the integral of the excitation voltage-rise. Under normal, steady conditions, the excitation of an alternator is measured in volts, but under the usual transient conditions the effect of the excitation system would be more logically expressed by the new unit volt-seconds. Under steady conditions, a change in volts will produce a proportionate change in flux, while under transient conditions, only a change in volt-seconds will produce a change in flux. This relation is clarified in the following discussion:

Mr. R. H. Park has called attention to the following simple relationship existing between excitation volts, time, and the flux linkages of the field circuit of large alternators when the time considered is equal to or less than half a second: The *change* in flux linkages will be approximately equal to the *change* in volt-seconds; that is, to the *increase* in the area under the volt—time curve. The mathematical demonstration of this statement is given below.

The increase in volt-seconds for the time $t' \leqq \frac{1}{2}$ second is the shaded area illustrated in Fig. 21. During the time t' the rise in flux linkages will be approximately proportional to this area.

The rise e in slip-ring volts will have the relation

$$e = L\frac{di}{dt} + iR$$

where

i = rise in field current produced by e.
t = time.
L = inductance of field circuit.
R = resistance of field circuit.

For the short time considered, iR is small when compared to $L\frac{di}{dt}$ and may be neglected. Then

$$e \simeq L\frac{di}{dt}$$

$$i \simeq \frac{1}{L}\int_{t_1}^{t_2} e\,dt$$

The increase in field current i will be proportional to the increase in field flux ϕ or

$$i = k\phi$$

where k is a constant, and

$$\phi \simeq \frac{1}{kL}\int_{t_1}^{t_2} e\,dt$$

The integral expression $\int_{t_1}^{t_2} e\,dt$ is the shaded area in Fig. 21 and the rise in field flux ϕ is, therefore, approximately

Load-Time Curves of Synchronous Condenser rated ATI-12-30,000 M-600 S-13,800 Volts when connected to an infinite bus and the following excitations applied when the machine is carrying 10,000 leading kv-a. (0.05 seconds allowed for regulator to operate.)

Curve Number	Rate of Voltage Rise	Ceiling Voltage
1	∞	784
2	6400	900
3	5000	953
4	4000	1041
5	3200	∞

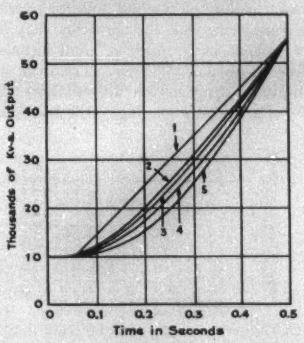

Fig. 22

proportional to the rise in volt-seconds. The change in volt-seconds is absolutely independent of the shape of the slip-ring volts versus time curve, and at the end of the time, t', the change in flux linkages of the alternator field circuit depends only upon the magnitude of the integrated area, shown shaded in Fig. 21 of change in excitation volts versus time.

As a proof that the increased iR drop produced in the field circuit of large alternators, by increasing the slip-ring voltage, is negligible under the conditions described, Fig. 22 is presented.[7] For each of the five

7. Fig. 22 appeared as Fig. 3 in *Super-excitation of Synchronous Condensers*, by D. M. Jones, presented at the Regional Meeting of the A. I. E. E., District No. 2, Baltimore, Md., April 17-20, 1928.

curves the applied increase in volt seconds was the same at 0.5 seconds although the rates of excitation voltage rise, and the maximum or ceiling excitation voltages, varied from 3200 volts per second to infinite volts per second, and from 784 volts ceiling to infinite volts ceiling. It will be seen that all the curves have the same ordinate at the instant when the volt seconds are the same, namely, 0.5 seconds. The condenser was assumed connected to an infinite bus and its rise in kv-a. would be proportional to its rise in flux. The curves in Fig. 22 were calculated, but curves of this type can be, and have been, checked by test with an exceedingly small error.

After the excitation voltage reaches its ceiling, it becomes constant and the rise in flux linkages of the alternator field circuit will thereafter be an ordinary exponential curve.

During transient disturbances, it is desirable to limit the maximum swing of the alternator rotor. This may be done by increasing the air-gap flux during the first half swing of the rotor, and therefore, by increasing the slip-ring volt seconds. If a certain number of volt-seconds is available during the first half swing it is best to apply them as quickly as possible.

From the above discussion, it is clear that in designing an excitation system of such characteristics as to be most effective in limiting the first swing by increasing flux linkages, two factors are paramount in importance. First, the maximum voltage applied to the slip-rings must be as high as permissible from other considerations; second, that maximum voltage must be attained as promptly as possible, commensurate with other design limitations. This in order to get to that maximum voltage as early as possible during the first swing, calls for an exciter capable of applying a relatively high voltage to the slip-rings, and also capable of an extremely high rate of voltage rise; that is, an exciter capable of producing the maximum change in volt-seconds.

In each of the tests described in this paper, the rise in volt-seconds was applied in a similar manner. The slip-ring volts started from a minimum and rose uniformly to a maximum. The change in volt-seconds should therefore provide a means of comparing the efficiency of the excitation systems in limiting the maximum rotor swings.

The duration of the short circuit for seven of the Martin Dam tests was so nearly the same, that an average value of duration could be used for each test with very little error.

The rate of excitation rise for each of the seven tests was different, and therefore permitted a direct comparison between the rise in volt-seconds and the maximum permissible power that could be carried prior to the short circuit without loss of synchronism. This relation is shown in Fig. 20. The negative volt-seconds are a result of the drop in slip-ring volts when the alternator field current increased during the short circuit.

The foregoing touches only a small part of a subject which has many other aspects. A more comprehensive treatment will be made in a paper to be presented before the Institute at a futuer date.

Discussion

A. Dovjikov: The studies which Messrs. Jones and Treat have been making on the system of the Southeastern Power and Light Company are extremely interesting as they provide data on power limits of actual systems under operating conditions rather than miniature laboratory conditions.

We note from the discussion under "Curves and Tables of Results" that the authors found that oscillographic wattmeters having any appreciable inertia were not accurate when the power was varying rapidly and that instantaneous type instruments were used for checking. The tests point out the care with which all metering equipment must be selected, having in mind its appropriateness for the conditions under which the measurements are made. Our experience was similar and led to the development of single- and two-phase instantaneous and three-phase average oscillographic wattmeters which are now available and are free from defects mentioned in the paper.

Referring to the conclusions reached by the authors, I believe that the statement that "the gain in power which could be carried with increased speed of excitation used in these tests appears to be not great," is at least too pessimistic. If my understanding of Fig. 20 is correct it appears as though a gain of 28 per cent in power limit was obtained by speeding up the excitation. This amount, of course, would be quite worth while.

Their conclusion relative to the recommendation of providing water-wheel-driven generators with amortisseur windings appears to be based on insufficient evidence. Several considerations, into which I have not sufficient time to go, make us believe that from the point of view of stability it would be better to omit damper windings, especially of the low-resistance type that have been previously recommended, from water-wheel-driven generators. In general, the studies which my associates have made relative to this subject indicate the undesirability of damper windings for water-wheel-driven generators except when their use is required by other considerations, as for example in connection with automatic operation.

F. R. Longley: Comparison of the three-phase and single-phase wattmeter oscillographs: The accompanying curves (Fig. 1, herewith) of the transient power during a single-phase line-to-ground short circuit show how closely the three-phase

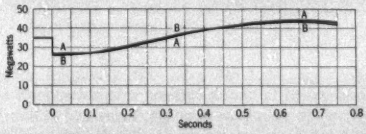

Fig. 1—Test Values of Instantaneous Power during Line-to-Neutral Short Circuit on Martin Dam High-Tension Bus

Curve *A*—Power from three-phase single-element oscillograph
Curve *B*—Power from single-phase three-element oscillograph

wattmeter oscillograph element and the three single-phase wattmeter oscillograph elements check, when the curve of the three-phase element has been corrected for the effect of its inertia.

Curve *A* was obtained from the film of the three-phase wattmeter after a few hours work. It required a week to obtain curve *B* from the terribly distorted film curves of the single-phase elements.

The effect of excitation upon the maximum power that can be transmitted through a transient disturbance: It should not be assumed that the total rise in power shown in Fig. 20 can be obtained by ordinary high-speed excitation.

Reference to Table III will show that the maximum gain in volt-seconds during the first half swing of the generator was 49.3 obtained during test No. 25, with a rise in exciter potential of 500 volts per second.

From Fig. 20, the maximum power with zero volt-seconds (constant slip-ring volts) is 35,000 kw. With 49.3 volt-seconds the maximum power becomes 41,000 kw. The gain is 6000 kw. or 17 per cent. All other tests show a smaller percentage gain.

The negative volt-seconds in Fig. 20 were produced by leaving a part of the resistance of the alternator field rheostat in the alternator field circuit between the exciter and the slip rings. The rise in field current during the short circuit produced an increase in the resistance drop between the exciter and the slip rings with a consequent reduction in the slip-ring volts. Any negative volt-seconds reduced the maximum power in the same manner that positive volt-seconds increased it. It must be remembered, however, that 100 per cent maximum power is that obtained with zero volt-seconds.

Robert Treat: The paper states that not entirely satisfactory results were secured in measuring power changes in the current when such changes were sudden. I think, however, that they are as reliable as many other parts of the test, for the reason that entire reliance in drawing conclusions was not placed in one oscillographic method of recording power. Checks were also made with indicating and recording instruments at both ends of the line. In addition to the one type of oscillograph which contained some inertia and which therefore, as stated in the paper, did not faithfully follow sudden changes in power, check oscillograms were also made with the type which records single-phase power. In finally working up the tests, fairly substantial agreement was obtained from all the methods.

When we came to work up the results, it was found that the single-phase wattmeter oscillograph, although it was substantially instantaneous and therefore free from the difficulties of the other instrument, recorded so faithfully the power in the circuit that it became difficult to interpret even those results because the power itself was no longer sinusoidal. On short circuits it contained harmonics and the correct value of power could not be obtained directly and readily from these oscillograms. It was necessary to magnify the graphs and integrate the areas to find the true power. In the end, however, a fairly substantial agreement was obtained.

It should be noted that a portion of the gain in power mentioned by Mr. Dovjikov was obtained simply in going from hand control to regulator control, and not all of it by increasing the speed of the exciter. As near as could be estimated from the test results, half of the total gain was attributable to control of the exciter voltage by regulator; that is, the control of the voltage of the exciter as installed. It is a particularly low-speed, sluggish exciter. Mr. Dovjikov questions our conclusion that amortisseur windings would be helpful to stability. I agree with him that it is not a simple phenomenon, but it would have been interesting if he had presented some substantiation of his opinion. Of course we were unable to make tests on the Martin Dam generators with and without amortisseur windings, so our conclusion of their value is inferred rather than proved. Nevertheless, when the test machines were set into violent oscillation with respect to the system, there was very little tendency toward damping, and the conclusion seems entirely reasonable that properly designed amortisseur windings would have been beneficial. This conclusion is supported by analytical work done both in connection with these tests, and for other studies.

So far as I am aware, this is the first time that tests of exactly this nature have been made on a power system. A high devotion to anything tending to advance the art, to permit them to improve their service, must have motivated the executives of the power companies concerned in embarking upon tests of this sort. It is a rather high tribute to the effectiveness of the operating personnel that service was not impaired in the least.

Some Photoelectric and Glow-Discharge Devices and their Applications to Industry

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Synopsis.—*During the past few years, intensive research work has been carried out on some photoelectric and glow-discharge devices. More recently, certain applications of these devices to the industry have been undertaken. This paper presents some of the results attained.*

* * * * *

Introduction

UNTIL very recently, applications of vacuum and gas filled devices in industry have been comparatively limited except for the familiar electric lamp, the X-ray tube, and the vacuum tube for radio receiving and broadcasting and for communication circuits. Thus far, industry has made very little use of photoelectric and glow-discharge devices, this delay in making widespread use of such devices being due no doubt to the fact that until recently they had not been sufficiently developed for applications where reliability, long life, and constancy are required. Conditions are changing rapidly, however, and as development work progresses, the applications of these devices will become more and more widespread.

Photoelectric devices have been used in laboratories for some time, and are now an essential feature in television, telephotography, radio-photography, and talking motion pictures. The photoelectric cell can solve many industrial problems, for this light-sensitive device, unlike the human eye, is unhampered by the tricks of human psychology and fatigue. Some glow-discharge devices operate on as little as a few millionths of a watt, and have an amplification factor of several millions. The possibilities of such sensitive apparatus must be apparent.

A number of new developments on photoelectric and glow-discharge devices have been pursued during the last two or three years and some of these have been completed, while others are still in the development stage.

It is the purpose of this article not only to describe some of these recently developed or recently improved devices,—such as photoelectric cells, selenium cells, photo-glow tubes, and grid-glow tubes,—but also to point out their fields of application and to describe several of the specific applications that have been undertaken, or are under way at present; as, for example, smoke recorders, fire and flash detectors, apparatus for the sorting of materials, paper-machine control, oilburner safety devices, etc.

1. Supply Engg. Dept., Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Regional Meeting of the Southern District of the A. I. E. E., Atlanta, Ga., Oct. 29-31, 1928.

Theory and Description of Devices

General. While differing in construction and in application, the devices discussed in this paper in reality have much in common. The extent to which these devices are related can be seen from the following general considerations.

1. Conductivity in High Vacuum. When sealed in a glass tube which is highly evacuated, two electrodes, such as *A* and *B* in Fig. 1a, would be perfectly insulated from each other for normal voltages, assuming no leakage took place over the surface of the glass.

If, however, one of the electrodes as *A* were to be heated to incandescence or illuminated with light of

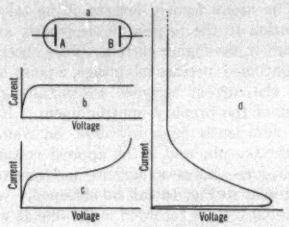

Fig. 1—(a) Elementary Vacuum or Gas-Filled Tube
(b) Current—Voltage Curve for Vacuum Device with One Active Electrode
(c) Current—Voltage Curve for Gas Filled Device with or without an Active Electrode
(d) Continuation of Curve (c)

suitable frequency the inter-electrode space would, upon application of a positive potential to electrode *B*, be found to be a relatively good conductor of electricity. In either case, if current be plotted as a function of applied voltage, a curve similar to that shown in Fig. 1B would be obtained. This curve is a fundamental one applying to the case of the conductivity of a high-vacuum space between two electrodes, one of which is source of electrons.

As will be shown later, the high vacuum photoelectric cell displays the general characteristic presented in Fig. 1b.

2. Conductivity of a Gassy Space. If the space

28-146

between the two electrodes in Fig. 1a be gas filled, upon applying a voltage it will be found that the electrodes are no longer as nearly perfectly insulated from each other as they were when the space was highly evacuated, even though no purposely applied ionizing agent be allowed to act on the gas. The space will also be found to conduct equally well in either direction.

The conductivity is attributed to spontaneous ionization; that is, a form of ionization not due to ordinary ionizing agents but rather to the so-called Ultra Gamma Rays or Cosmic X-rays, also known as "penetrating raditions." By these means, the neutral gas molecules are ionized or broken up into positive and negative ions, which under the influence of the applied voltage migrate respectively towards the negative and positive electrodes. This migration of ions constitutes a flow of current so small in magnitude as to require the most sensitive instruments to detect its passage. By plotting current as a function of voltage, a curve as shown in Fig. 1e can be obtained, differing from that shown in Fig. 1b in that a rising characteristic is obtained for the higher voltages. This increase in the current over and above what is accounted for by the ionizing agents previously mentioned occurs for voltages which are sufficient to give to the migrating ions sufficient velocity to ionize the neutral gas molecules upon colliding with them. The newly formed ions resulting take part in the migration to the respective electrodes, and by so doing result in the higher current values observed.

By a continued increase in voltage, a point is reached at which the current becomes self-sustaining and independent of the primary ionizing agent. If suitable resistors, sufficiently high in value, be connected in series with the tube and if the applied voltage be increased, a curve between current and inter-electrode voltage similar to Fig. 1d can be obtained,[2] where the most extreme voltage recorded is the one at which the current becomes self-sustaining. It will be recognized that the initial portion of curve *d* is the same as curve *c* drawn on a different scale. Over a considerable portion of the curve *d* in the region immediately above and following the sharp knee in the curve, the currents as obtained in the above mentioned observations are oscillatory in nature ranging in frequency from only a few pulsations per second at low voltages to frequencies above the audible range for the higher total circuit voltages. These same currents have further significance, in that they represent the "threshold" values of current. For each particular voltage there is a definite "threshold" or minimum value of current which must be supplied to the discharge before it is capable of progressing to that form of discharge characterized by the constant voltage drop portion of the curve. The manner in which currents and voltages are related is depicted by that portion of the curve *d* under discussion. It is the phenomenon as depicted in and applying to Fig. 1d which accounts for the action of the grid-glow tube.

If the space in Fig. 1a be gas filled as before, and if one of the electrodes were to serve as a source of electrons in itself, being either heated to incandescence or illuminated by light of suitable frequency, the device would have the same characteristics as depicted in curves *c* and *d*. In this case the effect of the penetrating radiations can be neglected, as the one active electron emitting electrode will normally serve as a source of a much greater number of ions than the penetrating radiations could account for. Neglecting the effect of the penetrating radiations, the device will exhibit a unidirectional conductivity, permitting the flow of current only when the inactive electrode is positive with respect to the electron emitting electrode. The gas-filled photoelectric cell having a single photoelectrically active electrode operates over a characteristic essentially that as shown in curve *c*.

Keeping in mind the significance of the threshold currents as brought out in a previous paragraph, and also the action of a device having a single electron emitting electrode as described above, it can be seen that this electron emitting electrode can be made to function as a controllable source of supply of threshold currents. Accordingly, with proper design, the device mentioned in the last paragraph above can be made to operate over the extended part of its characteristic as presented in curve *d*. The photo glow tube is a device which has this characteristic.

If a pulsating glow-discharge be established with an alternating voltage impressed across two initially inactive electrodes in a device similar to that shown in Fig. 1a, the currents which flow in opposite directions will be the same provided the two electrodes are equal in area. If, however, one of the electrodes is reduced in area, rectification will be obtained. The degree of rectification obtained will depend upon the relative areas of the two electrodes. The current which passes in any one direction is roughly proportional to the area of the electrode which, at the instant, is functioning as a cathode.

The application of glow-discharge devices to the general problem of rectification has not been made the subject of any part of this paper.

The following table will serve to show the relation between the various photoelectric and glow-discharge devices in agreement with the general discussion above.

TABLE I

	High vacuum	Gas space
Electrodes Inactive....		Two electrode glow tube Grid-glow tube
One active Electrode..	High vacuum Photo cell	Photo-glow tube Gas-filled photo cell

2. "On the Sparking Potentials of Glow-Discharge Tubes," James Taylor, *Phil. Mag.*, 269.3, Ser. 7, Feb. 1927.

Light-Sensitive Devices

1. Photoelectric Cell. The first observation of the photoelectric phenomenon was made in 1887 by Hertz, who noticed that a spark would pass across a spark-gap more readily when the electrodes were illuminated than when they were in darkness. A year later Hallwachs noted (a) that a negatively charged body readily loses its charge and (b) that an initially uncharged body

Fig. 2—Photoelectric Cells Showing Type of Base Mounting. The Two Sizes are Respectively (a) 2½ In. and (b) 1¼ In. in Diameter

acquires a positive charge when illuminated by ultra-violet light. In 1889 Elster and Geitel showed that the more electro-positive metals, such as rubidium, potassium, sodium, etc., would display this same effect when exposed to ordinary light of the visible spectrum. The effects are due to the emission of negative electrons by the photoelectrically active material, which emission takes place when that material is illuminated by light of suitable frequency.

The modern photoelectric cell modeled somewhat after the originals by Elster and Geitel takes advantage of this action and reduces the phenomenon to one having extremely practical uses.

Two such cells as developed by Dr. Zworykin of the Westinghouse Research Laboratories are shown in Fig. 2. The spherical portions of the large and the small size cell in parts *a* and *b* of Fig. 2 are respectively 2½ in. and 1¼ in. in diameter, both sizes being mounted in standard tube bases for ease in mounting and in making connections.

Both sizes are made in the high-vacuum and gas-filled types. The particular characteristics of each type will be pointed out in a later paragraph.

The larger cell is shown in greater detail in Fig. 3. The photoelectrically active cathode consists of a layer of photo active alloy deposited on the inner surface of the spherical portion of the cell. A cleared circular aperture serves as a window for admitting light to the active surface. A short wire lead makes connection

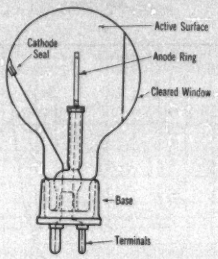

Fig. 3—Details of the 2½ In. Diameter Photoelectric Cell Showing Interior Structure

through the glass press between the active layer and one prong on the base.

The anode structure consists of a ring of metal ribbon supported in the center of the cell by a section of wire which passes through the press and out to a second prong on the base. Means are provided to insure freedom from internal stray leakage currents.

A typical curve of cell response plotted as current in microamperes per lumen of light flux at various voltages for a particular high-vacuum cell is shown in Fig. 4a. Curve 1 of Fig. 4b shows the relation between the same quantities for a particular gas-filled cell. The two types of cells display the typical curves which were presented in parts *b* and *c* of Fig. 1 as applying to high-vacuum and gas-filled devices having an electron emitting electrode.

The high-vacuum cell is seen to yield its saturation value of current at a comparatively low voltage. The gas filled cell while displaying a similar behavior at the lower voltages shows a rising characteristic on the higher voltage due to the additional conductivity result-

ing from the ionization of the gas. Curve 2 of Fig. 4b is the illumination limit curve applying to the same gas filled cell. Its significance lies in the fact that at any particular voltage, as for example V, at which time the cell passes i microamperes per lumen, the maximum amount of light flux which can be supplied to the cell without causing the cell to glow is represented by L lumens. This illumination limit curve is essentially the "threshold" current portion of the curve previously discussed and presented in part d of Fig. 1. If for any particular application it is considered desirable to take advantage of the extreme sensitivity which the cell has

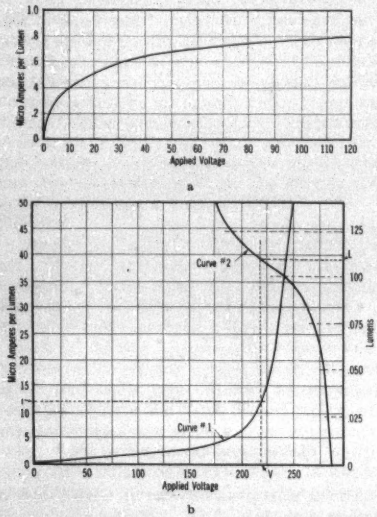

FIG. 4—CURVES SHOWING PHOTO-CELL CURRENT IN MICROAMPERES PER LUMEN OF LIGHT FLUX AS A FUNCTION OF APPLIED VOLTAGE FOR (a) THE HIGH VACUUM AND (b,1) THE GAS FILLED CELL

(Plotted in the above manner the characteristics of the 2½ in. and the 1¾ in. diameter cells are identical). In (b, 2) is shown the illumination limit curve

at the higher voltages, it can be seen from this curve that the luminous flux must be reduced. For the particular cell whose characteristics are presented in Fig. 4B, it is accordingly evident that if for example it is desired to operate at a sensitivity of 30 microamperes per lumen the maximum luminous light flux supplied to the cell must be held close to, but not above 0.102 lumens. It is to be pointed out that Curve 2 represents the limiting conditions, which in actual operation are purposely avoided by a safe working margin. Obviously the high-vacuum photo cell, by reason of its tendency towards saturation, is not subject to these limitations.

The photo cells described above are standard. For very special applications cells of special size and shape have been employed. In one instance, that of a sorting unit (described elsewhere), the cells are very small, being cylindrical in shape and 0.5 in. in diameter by 1.75 in. long.

2. *Photo-Glow Tube.* The photo glow tube developed by D. D. Knowles of the Westinghouse Research Laboratories is a device in principle essentially the same as a gas-filled photoelectric cell with the exception that it is designed to operate at a voltage above what was called the glow point voltage in the case of the photo cell previously described.

The photo glow tube is presented in more detail under the general heading of glow-discharge devices.

3. *Selenium Cells.* The selenium cell is a form of light sensitive device which functions by reason of the variation in conductivity of the metal selenium with illumination. While the change in conductivity of selenium is considered to be due to the release of photo electrons,[3] the normal construction and operation of such cells is such that they do not fit well into the summary in Table I. To date the selenium cell has not been reduced to the same degree of practicability as have the devices discussed above. Some cells have appeared on the market but they have been very low-voltage cells capable of delivering only small amounts of energy in the output circuit, which energy is insufficient to control sturdy relays. Successive cells of any one type display wide variations in response.

The current output of a selenium cell at constant voltage varies with the square root of the illumination. The speed of response of a selenium cell is considerably slower than that of a photoelectric cell but the current which passes is much greater. However, unless more constant cells are developed, which will make it possible to control sturdy relays without necessitating amplification, their usefulness will be very limited, since with these shortcomings they are inferior to photoelectric cells and amplifiers.

New cells of 110-volt rating able to deliver several watts in the output circuit are under development but as yet they are still a laboratory device.

GLOW-DISCHARGE DEVICES

1. *The Two Electrode Glow Tube—Glow Meter Bulb.* An example of a glow-discharge tube in its simplest form is the glow meter bulb shown in Fig. 5A. The bulb consists essentially of the two small coiled electrodes sealed into the glass bulb proper, which is filled with neon gas at low pressure. The finished bulb is small, being ½ in. in diameter and about 3½ in. long. Under an impressed voltage of sufficient magnitude the bulb becomes filled with the orange-red neon glow and passes current.

3. "Photoelectric Conduction in Selenium," R. J. Piersol, *Phys. Rev.*, 30, November 1927, p. 664.

z. Grid-Glow Tube. The grid controlled glow tube as developed by Knowles[4] consists essentially of a cylindrical aluminum cathode, an anode, and a grid, both of the latter being made of heavy nickel wire. The three electrodes are inclosed in a glass tube containing neon gas at a low pressure. A photograph of the tube is shown in Fig. 6a and a sketch showing the construction is shown in Fig. 6b.

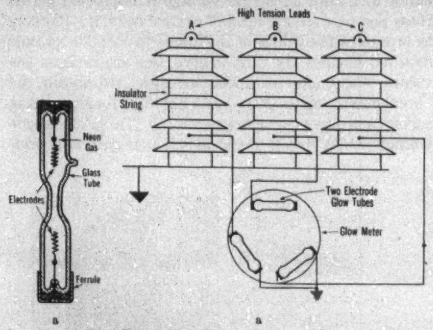

FIG. 5—(a) TWO ELECTRODE GLOW TUBE FOR ELECTRIC STATIC GLOW METER
(b) CONNECTIONS FOR ELECTRIC STATIC GLOW METER WHEN USED AS GROUND DETECTOR

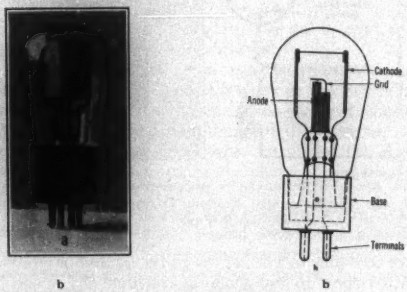

FIG. 6—(a) PHOTOGRAPH OF THE GRID-GLOW TUBE
(b) SKETCH SHOWING DETAILS OF CONSTRUCTION OF THE GRID-GLOW TUBE

The grid-glow tube can be used on both direct and alternating current but, as will be pointed out, its characteristics are somewhat different on the two sources.

The impedance offered to the flow of current from the aluminum cylinder, the cathode, to the central wire, the anode, is practically infinite under all conditions. The impedance offered to the flow of current in the reverse direction is quite low when the tube is glowing, amounting to about 20,000 ohms at 8 milliamperes. The glow resistance is not constant but varies inversely with the current. As a result, the drop over the glow is constant at about 160 volts at all times. To prevent the discharge from passing from a glow to a destructive arc a series resistor must be employed in series with the tube, which should limit the current to about 10 milliamperes (effective).

A unique feature of the grid glow tube is the fact that the voltage required to start the glow is subject to control within wide limits by means of the grid electrode.

A simple circuit for the grid-glow tube is shown in Fig. 7a. If the grid be free (thoroughly insulated) about 600 to 900 volts, d-c., depending on the particular tube in use, will be required to start the glow and operate the relay. This is due to the fact that under free grid conditions the grid will assume a high negative charge, which, if not allowed to leak off, will rise sufficiently to block the tube. The fact that the grid does assume this high negative charge gives rise to one form of control of the breakdown voltage.

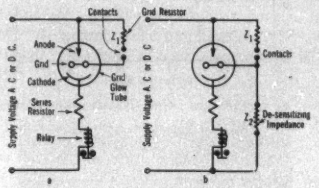

FIG. 7—SCHEMATIC CIRCUIT DIAGRAMS SHOWING BASIC FEATURES INVOLVED IN CONTROL OF THE GRID-GLOW TUBE

In (a) the use of the Grid Resistor functioning as an anode to grid leakage path is shown and in (b) the use of a de-sensitizing impedance is shown

Any conducting path placed in the grid to anode circuit will permit the grid charge to leak off and permit the tube to operate providing the leakage takes place at a sufficiently high rate as demanded by the "threshold" current portion of curve *d* of Fig. 1. The leakage path may be in the form of a condenser, a resistor, or its equivalent; as, for example, a photo cell or a flame. The controlling effect of a grid resistance for alternating current and direct current is shown in Fig. 8.

The amount of energy which must be handled by a pair of control contacts placed in series with the leakage resistor in Fig. 7a, is a very minute quantity. Accordingly it is important that dirt, dust, and moisture be kept out of the tube socket. A good quality socket should be used to avoid leakage in the socket itself. The ratio between the energy controlled in the output of the glow tube and the energy which the above mentioned contacts are called upon to handle is subject to control, and on occasion may be made very high. For instance, if the grid resistance in Fig. 7a is about

4. "The Grid-Glow Tube Relay, D. D. Knowles, *The Electric Journal*, April, 1928.

100 megohms, it will be found that a grid current of the order of one microampere will be sufficient to permit the main discharge to start for a certain safe normal anode to cathode voltage. Assuming the relay contacts to be of one ampere capacity, it is seen that the

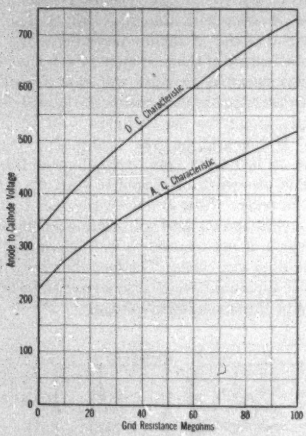

FIG. 8—GRID-GLOW GRID-RESISTANCE CONTROL CHARACTERISTICS FOR A-C. AND D-C. CIRCUITS

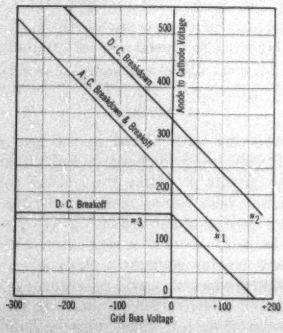

FIG. 9—GRID-GLOW TUBE CURVES SHOWING A-C. AND D-C. ANODE TO CATHODE BREAKDOWN AND BREAK OFF VOLTAGES AS A FUNCTION OF GRID BIAS VOLTAGE.

(Data taken with Zero Grid Resistor in use)

current ratio for this case is of the order of one million to one.

If the sensitivity of such a circuit be thought too great, a means of varying the sensitivity as shown in Fig. 7b can be used. For a given value of voltage between anode and cathode, the ratio between Z_1 and Z_2 will determine whether or not the tube is to break down. The sum of $Z_1 + Z_2$ governs the sensitivity; that is, the energy which is lost in the control circuit. By reducing the sensitivity in this manner the effects of extraneous leakage paths such as socket insulation resistance will be lessened.

If instead of connecting a leakage resistor between anode and grid, an actual voltage is impressed across these points, a second form of control results. Plotting as ordinates the values of the anode to cathode voltage required to cause the tube to glow, and as abscissas the values of the grid to anode voltages, we obtain the "breakdown" curves Nos. 1 and 2 of Fig. 9 for alternating current and direct current respectively. The grid to anode voltage or bias voltage is considered as negative or posi-

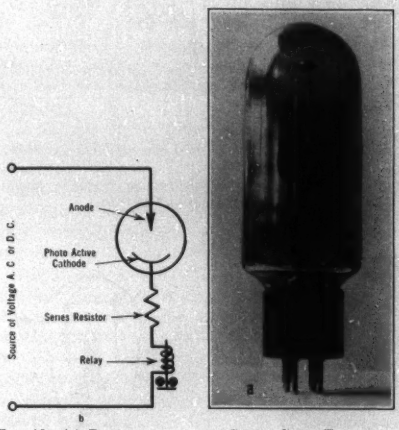

FIG. 10—(a) PHOTOGRAPH OF THE PHOTO-GLOW TUBE
(b) SCHEMATIC CIRCUIT DIAGRAM INVOLVING THE PHOTO-GLOW TUBE

tive according to whether the grid is negative or positive with respect to the anode at the time the anode is positive with respect to the cathode.

As was mentioned before, the voltage drop over the glow amounts to 160 volts. Accordingly the anode to cathode voltage must be at least 160 volts if the glow is to persist. Curve No. 3 of Fig. 9 shows the "breakoff" curve for d-c. operation. Since on a-c. the voltage wave falls below the 160-volt value in each cycle, the "breakdown" and "breakoff" curves for a-c. operation are seen to coincide.

On direct current, therefore, once the tube has been set in operation by a certain grid influence, it will continue to operate even after the influence has been removed. The glow can be stopped only by reducing the applied anode-cathode voltage below the 160-volt value. On

alternating current however, since the breakdown and breakoff curves coincide, the tube will start and stop glowing accordingly as the grid influence is applied or removed.

Advantage may be taken of such characteristics. In the circuit shown in Fig. 16, a glow tube is used to leave a permanent record of the occurrence of a transient in the light supplied to the photo cells. It is functioning in that circuit as an "instantaneous pickup and seal-in" type of relay without contacts.

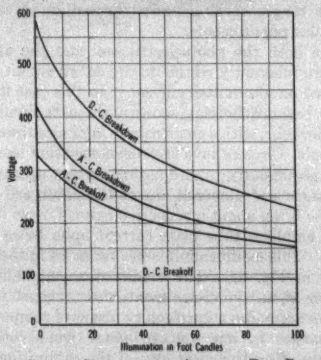

Fig. 11—Curves Showing the A-c. and D-c. Breakdown and Break Off Voltages as a Function of Illumination for the Photo-Glow Tube

Further uses of the grid-glow tube are brought out under the heading of "Applications."

3. Photo-Glow Tube. As mentioned before, the photo glow tube developed by Knowles is a device in principle essentially the same as a gas-filled photoelectric cell, with the exception that it is designed to operate at a voltage above what was called the glow point voltage in the case of the photo cell previously described.

One form of photo glow tube is shown in Fig. 10A. The anode is a centrally located wire having a small area exposed. The cathode, the film of metal deposited upon the inner surface of the glass, serves two functions. The cathode is coated with the photo-active substance and as such serves as a source of electrons when illuminated. Under an impressed voltage of sufficient magnitude, accumulative ionization of the gas with which the tube is filled sets in when the tube is illuminated, and a current will pass, the cathode forming one of the electrodes for the passage of this glow current as well.

Typical performance curves for the photo glow tube are shown in Fig. 11. These curves serve to bring out the fact that the glow, once started, can be made to stop upon a reduction in illumination when the tube is run on alternating current. On direct current however, the glow is self-sustaining and will persist either until the circuit is opened or the applied voltage is reduced below a value corresponding to the value for the constant drop over the glow discharge.

A resistor is normally connected in series with the photo glow tube to limit the glow current to a safe value of 8 milliamperes. A typical circuit diagram for the use of this tube is shown in Fig. 10b.

The response of the photo glow tube is practically instantaneous, and the current which the tube passes is of sufficient magnitude to operate small relays directly, no amplification being necessary.

As developed at the present time, the life of the photo glow tube on continuous duty is not very long. Consequently it should be applied only where the tube is normally not illuminated, and where only an abnormal condition causes breakdown.

Applications

Light Sensitive Devices.

1. Photo Cell Amplifier Unit. The normal current output of the photoelectric cell being of the order of micro-amperes is not readily put to use directly except through the use of very sensitive instruments; it requires that a degree of amplification be given it in order that less sensitive associated apparatus may be employed. A vacuum-tube amplifier may be used for this purpose. One form of such an amplifier is shown in Fig. 12.

Fig. 12—Illustration Photo-Cell Amplifier Unit Showing a 2½-In. Diameter Photo Cell and a Standard 201-A Radio Tube in Position on the Panel

The internal wiring of the amplifier unit is shown in Fig. 13a. The photo cell and the grid resistor are in series between terminals *A* and *X*. Their common point is run direct to the grid of the amplifier tube. The filament and plate leads to the amplifier tube are run out as shown. Except in special cases, terminals *A* and *P* are connected together, a connecting link being provided for this purpose. The performance of the unit as a whole is evident from an inspection of the typical vacuum tube characteristics as shown in Fig. 13c

and the scheme of connection shown in Fig. 13b. Assuming the photo cell is dark, it can be seen that the grid will be at a negative potential with respect to the filament as determined almost solely by the value of the C battery. If the photo cell be illuminated, a current will flow under the influence of the B and C batteries combined, from the positive terminal of the B battery

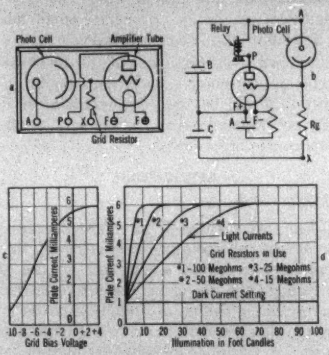

Fig. 13—(a) Internal Wiring Diagram of the Photo-Cell Amplifier Unit
(b) Schematic Diagram Showing Simple Circuit for Use with Photo-Cell Amplifier Unit
(c) Typical Dynamic Plate Current—Grid Bias Characteristic for an Amplifier Tube
(d) Typical Curves Showing the Amplifier Plate Current as a Function of Illumination on the Photo Cell for Various Values of the Grid Resistor

through the photo cell, and will then return to the negative terminal of the C battery after passing through the grid resistor, across which, as a result, there will exist a difference of potential. The polarity of this voltage drop over the grid resistor will be such as to tend to cause the grid to become positive with respect to the filament: in other words, the effective negative grid bias will have been reduced. From Fig. 13c, it can be seen that this reduction in bias voltage will result in an increased flow of plate current, and as a result the relay in the plate circuit will be energized.

Assuming an initial bias voltage of 9 volts, and assuming a 50-megohm grid resistor in use, it is seen from Fig. 13c, that upon illuminating the photo cell slightly, if 0.06 micro-ampere should flow, the voltage drop over the grid resistor would amount to 6 volts and the plate current would rise from one milliampere to three milliamperes. From the typical curve in Fig. 13d, for a 50-megohm grid resistor, it is seen that an illumination intensity of about 5 foot-candles should produce this result.

The unit responds very quickly to changes in illumination. The time lag is generally of the order of 0.001 to 0.0001 second. This figure is for the plate current in the vacuum tube. When a relay is used an additional time lag enters, depending on the speed with which the moving armature picks up and closes its contacts.

Typical curves showing milliamperes plate current as a function of cell illumination for various grid resistors in use are shown in Fig. 13d. By the proper choice of grid resistor, amplifier tube, and circuit voltage, the region of the curve wherein plate current is proportional to the illumination can be made to cover either wide or narrow ranges of illumination intensities to suit any particular requirements.

Since both the photoelectric cell and the amplifier tube are inherently rectifiers, the entire circuit can be operated on alternating current. In this case it is customary to shunt the relay, shown in the plate circuit of the amplifier tube, with a condenser to minimize the effect of the ripple. Circuits have been developed that lend themselves particularly to alternating current operation.

By means of a special circuit the photo cell can be made to bring about a reduction instead of an increase in the amplifier tube plate current upon being illuminated. This modification is incorporated in the circuit shown in Fig. 21. The modification in addition to bringing about the above mentioned reversed amplifying effect provides a satisfactory means of compensation for changes in line voltage which would otherwise seriously affect the constancy of the calibration of the amplifier unit.

The grid resistor used as a coupling resistor between the photoelectric cell and the amplifier tube is of the "thin-film" type shown in Fig. 14. The resistance proper is made up of a small helix of glass coated with a thin film of metal evaporated in the process of construction, from the concentrically located molybdenum filament. The filament is used only in the construction of the thin resistance film.

Other types of grid resistors are used for special requirements.

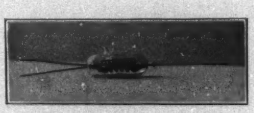

Fig. 14—The Thin Metal Film Type of Grid Resistor Used in The Photo-Cell Amplifier Units

2. Multi-Cell Circuit. An extremely useful modification of the simple photoelectric cell amplifier circuit of Fig. 13b, is the multi-cell arrangement shown in Fig. 15.

For a given voltage impressed across the series circuit composed of the cells and the grid resistor, the voltage drop over the resistor and the voltage per cell depends upon the number of cells, the illumination, and the current-voltage curve for each cell. For uniform

illumination and for identical cells the voltage per cell will be the same.

The cells generally used in such series arrangements are of the high vacuum type. By reason of the pronounced saturation effect of such cells the response of the series of cells to a reduction in the illumination on

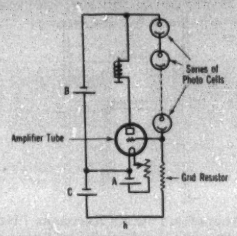

Fig. 15—Single Stage Multi-Cell Circuit for Amplification of Photo-Cell Currents Derived from a Group of Photo Cells Connected in Series

one cell only, as is manifested by a reduction in voltage drop over the grid resistor and a consequent reduction in amplifier tube plate current, is almost as pronounced as it is for a reduction of like amount in the illumination supplied to all of the cells of the series. In other words, the partial darkening of one cell of the series is very nearly equivalent to a partial darkening (to the same extent) of all of the cells of the series.

The response afforded by the single-stage circuit can of course be increased through the use of additional stages of vacuum tube amplification. A circuit showing such an arrangement is presented in Fig. 16. The details of this circuit are further taken up under the topic "Sorting of Materials."

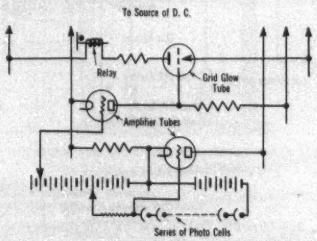

Fig. 16—One Form of Two Stage D-c. Amplification of Photo-Cell Currents. The Grid-Glow Tube is Shown in this Circuit Functioning as an Instantaneous "Pick-up and Seal-In" Type of Relay without Contacts

3. Sorting of Materials. Fig. 17 shows in schematic form the essential elements involved in the photoelectric examination of surfaces of materials. Light is supplied by the concentrated filament lamp and is brought to a focus at the surface under examination. The amount of light which the photo cell receives by reflection is dependent upon the reflection coefficient for the particular portion of the surface illuminated.

Any discontinuity or irregularity in the surface under examination which results in a variation from normal in the light flux supplied to the photo cell is made evi-

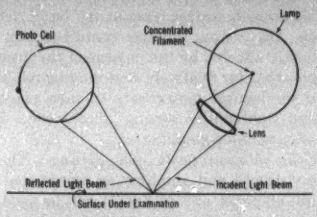

Fig. 17—Sketch Showing the Essential Elements Involved in the Application of a Photo Cell to the Examination and Sorting of Opaque Materials

dent by the amplifier circuits which are connected to the photo cell.

Obviously the actual area under examination is quite limited when a single photoelectric cell is used. The multi-cell arrangement discussed previously lends itself very well to the examination of more extended surfaces where it is desired to pick out irregularities extending over relatively small parts of the surface.

A photograph of a multi-cell unit employing special cells of only ½-in. diameter is shown in Fig. 18. This particular unit incorporates all of the essentials pointed

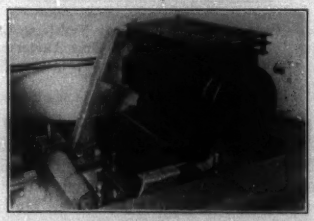

Fig. 18—Photograph Showing a Housing Built to Accommodate a Special Straight Line Filament Lamp, a Cylindrical Condensing Lens, and a Series Connected Group of Special One-Half Inch Diameter Photo Cells. The Unit is an Experimental Model Used in a Photoelectric Sorting Application

out in Fig. 17. The light is supplied by a special straight-line filament lamp and is brought to a straight line focus on the surface under examination by means of a cylindrical lens. The material under examination can be made to travel automatically in a direction at right angles to the line of light focused on the surface. Associated apparatus can be employed so that surface

irregularities of a certain magnitude will cause automatic separation of perfect from imperfect material. One of the amplifying circuits associated with this unit is the one mentioned before and presented in its elementary form in Fig. 16. In this particular circuit, advantage is taken of the d-c. seal-in characteristics of the grid-glow tube, with the result that even if the above mentioned surface irregularity be of very short duration, the output of the control circuit is left permanently affected by the passage of the irregularity. This gives time for relatively slow moving relays to act, and for mechanical movements to take place in the associated automatic sorting machinery. Fig. 19 shows an assembled amplifier unit.

4. Color Measurements and Sorting. The single photoelectric cell and amplifier as represented by the photo cell amplifier unit of Fig. 12 can be used in conjunction with color filters to aid in matching the colors of materials.

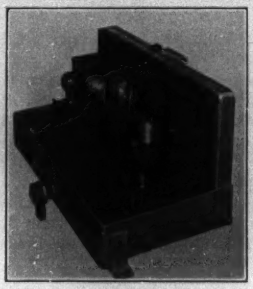

Fig. 19—Photograph Showing the Amplifier Unit Used with the Unit Shown in Fig. 18. This Amplifiee Makes Use of the Grid-Glow Tube in the Manner Shown in Fig. 16

By substituting a second photoelectric cell for the grid resistor $R\,g$ in Fig. 13B and by illuminating the two photo cells by light reflected from the surface or surfaces under examination, a difference in the reflection coefficients of the materials can within limits be detected regardless of the absolute value of those coefficients.

5. Photoelectric Smoke Recorder. A photoelectric smoke recorder has been developed for power plants for the purpose of recording the smoke density in the smoke stack.

While the plant operator has had available instruments such as the CO_2 meter and the boiler meter, which instruments assist in operating the boilers efficiently, there has been no device available which would give him a true indication of smoke. For this reason, methods like the Ringelman's Chart had to be resorted to. Evidently, any visual method that depends on watching the smoke issuing from the stack, is very approximate, as it depends upon the eye of the operator, and is entirely useless at night. Since smoke abatement is a subject that receives more and more attention, as is evidenced by smoke ordinances in all large cities, the photoelectric smoke detector should find a valuable field.

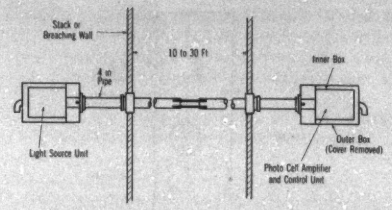

Fig. 21—Photoelectric Smoke Recorder Installation for Measuring Smoke Density in Smoke Stack

Fig. 20 shows a sketch of an outdoor installation and Fig. 21 a diagram of the circuit. A pipe extends through the breaching or stack on one end of which is mounted a light source and lens. On the other end is located the photoelectric cell and amplifier unit, similar to that shown in Fig. 12 plus other control equipment. In the boiler room is located the indicating or recording meter, calibrated in degrees of smoke, as well as an alarm which rings whenever the smoke density reaches a predetermined amount.

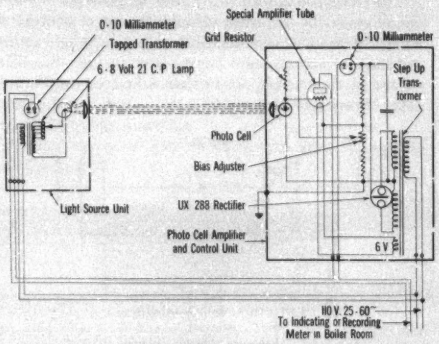

Fig. 21—Circuit Diagram of Photoelectric Smoke Recorder

The device operates entirely from the available a-c. supply, the light source and vacuum tube being operated at a rating that insures long life and the device being compensated for voltage variations of the supply source. The light source unit and the receiving unit are of weatherproof construction. They are so designed that no smoke or dirt can settle on the lenses, photo cell, or light source.

Among other applications of this device, an important one might be in the mining industry in connection with high-voltage ore dust precipitators, for indicating or recording the efficiency and operativeness of the precipitators.

6. Photoelectric Smoke Detector. This device is similar to the smoke recorder except that it is simplified mechanically. It consists of two units, the light

A B

FIG. 22—PHOTOELECTRIC SMOKE DETECTOR

a. Light source unit
b. Photo-cell amplifier and control unit

source and the photo cell amplifier unit, photographs of which are shown respectively in parts *a* and *b* of Fig. 22. The circuit diagram is similar to that of the smoke recorder shown in Fig. 21. Both the light source and the photo cell amplifier unit are operated directly from alternating current.

Whenever smoke interrupts the beam of light or cuts off part of the light, a relay in the amplifier circuit operates. An important application of this device is

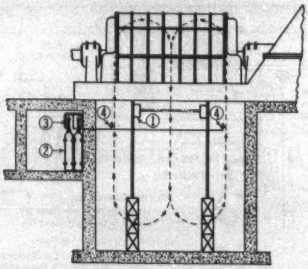

FIG. 23—APPLICATION OF PHOTOELECTRIC SMOKE DETECTOR TO GENERATOR PROTECTION

1. Smoke detector
2. Carbon dioxide cylinders
3. Electric-automatic-manual head
4. Discharge

in connection with fire extinguishing apparatus such as CO_2 equipments. It can be used for protection of electrical machinery such as generators, as shown in Fig. 23, and in many industrial fields, particularly where a fire is apt to spread quickly and cause damage in a short time, as in dipping tanks, lacquer spray booths, etc. So far thermostats have been used for automatically releasing the extinguishing equipment, but these thermostats are inherently slow.

Many other uses suggest themselves for this device, such as for counting and timing operations. An interesting application is that of a photoelectric flag switch in steel mills, suitable for starting shears when rolling light flat material. Various types of mechanical switches have been used to start the shears which cut up the material into definite lengths while the material is still in motion, but they do not stand up under severe service conditions. A light sensitive device obviates these difficulties.

7. Photo-Glow Tube Unit. A photograph of a photo glow tube unit is shown in Fig. 24. The circuit is similar to that shown in Fig. 10b, the source of voltage being 110 volts a-c. stepped up through a transformer having secondary taps for adjustments. As mentioned before, photo glow tubes have not been developed to a stage where they have a long life when the glow discharge passes continuously. However, if they are normally deenergized and have to function at more or less infrequent intervals, their life is very satisfactory. On account of the simplicity of such a unit it is to be preferred over the photoelectric cell unit described previously.

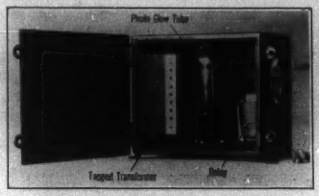

FIG. 24—PHOTO GLOW-TUBE UNIT FOR USE AS A SMOKE DETECTOR, FLASH DETECTOR, ETC.

One important application of this device is as a flash detector, in connection with the protection of oil circuit breakers, bus bar structures, etc. In this manner, fire extinguishing means can be released instantly upon the occurrence of a severe flash in a circuit breaker in order to prevent the spreading of an oil fire.

Another application is shown in Fig. 25. The device shown is an experimental unit and not the commercial device. It is used as a paper break recorder on a paper machine, and is installed both on the dry end as well as on the wet end of the machine. Whenever a break occurs, an alarm is sounded so that no time is lost in starting the paper going again. Also, a record is made on a recording meter to indicate the location of the break, the time of the occurrence, and its duration.

GLOW DEVICES

1. Electrostatic Glow Meter. One of the simplest applications of a glow discharge device is the Electrostatic Glow Meter. The three-phase glow meter consists of three small two electrode glow discharge

tubes of the type shown in Fig. 5a, suitably mounted and connected as indicated in Fig. 5b. By the use of a set of insulators functioning as a condenser-potentiometer, a voltage is impressed on each tube, which is proportional to the voltage between line conductor and ground. Normally, the three tubes give out a

Fig. 25—Experimental Installation of a Paper Break Recorder on a Paper Machine

bright orange-red glow which is characteristic of the neon gas with which they are filled. A grounded conductor is indicated by a darkened tube.

The electrostatic glow meter is especially applicable at high tension switching stations because of the fact that potential transformers are not needed. The insulator string potentiometer provides an adequate energy supply for the device.

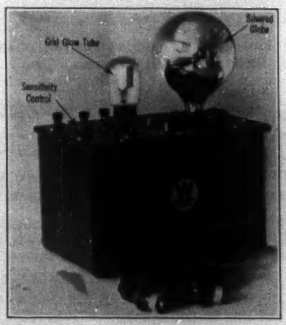

Fig. 26—Grid-Glow Tube Demonstration Set

2. Grid-Glow Tube Used with Contact Making Instrument. A means for controlling the action of a grid glow tube by means of a contact making instrument is afforded by the arrangement in Fig. 7b. As mentioned previously, the circuit constants can be so arranged that the active contacts are called upon to handle currents of very small magnitude. In the instance previously given, a one-ampere load was controlled with a contact load of only one microampere. When operated on alternating current the contacts can be placed in series with Z_2 instead of in series with Z_1 if it is desired that the glow tube become deenergized upon closing the contacts.

3. Grid-Glow Tube Demonstration Set. The grid-glow tube demonstration set pictured in Fig. 26 is a device which takes advantage of the condenser potentiometer form of control of the grid-glow tube. The

Fig. 27—Domestic Oil Burner Installation, Using Grid-Glow Tube Control Unit and Flame Terminal

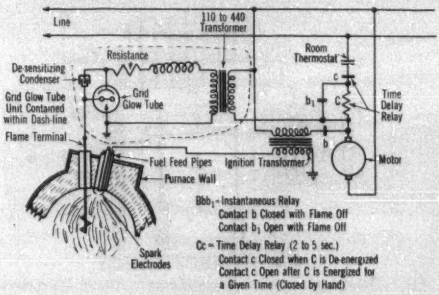

Fig. 28—Circuit Diagram of Domestic Oil Burner Control, Using Grid-Glow Tube Unit as a Combustion Safety Device

impedance Z_2 in Fig. 7b takes the form of a variable condenser, which provides a control on the sensitivity. The impedance Z_1 is accounted for in the capacity between the operator's hand and the silvered globe.

The device serves as a novel means for controlling moving displays in shop windows. When the demonstration set is properly installed and adjusted, it is only necessary for the onlooker to bring his open hand up to within an inch from the surface of the silvered globe, the glow tube being caused to glow and thereby energiz-

ing the relay which forms a part of the set. Through the circuits set up by the relay contacts, the various displays are thus brought into motion.

Means are provided so that a small metal foil disk attached to the inside surface of a window may be connected to the set so as to function in place of the silvered globe. A passerby need only bring the palm of his hand up to the outside surface of the window near the metal disk to start the set and actuate the display.

4. *Grid-Glow Tube Oil Burner Control.* A domestic oil burner installation, using a grid-glow tube combustion safety device, is shown in Fig. 27, and a diagram of the complete automatic control is shown in Fig. 28.

Until now, thermostatic devices, operated by the heat of the flame and located in the flue or in proximity to the fire brick of the furnace, have been used as a safety device, to prevent puff backs or more serious explosions in case of (a) belated electric ignition in starting the burner, (b) flame failure while the burner is operating. These puff backs occur if belated ignition takes place after a certain amount of atomized oil has been introduced into the furnace, about 5 sec. being sufficient to cause such trouble. Since these thermostatic devices take from 5 to 90 sec. to operate, depending on operating conditions, unsafe conditions result.

The grid-glow tube does not operate on the heat of the flame, but on its conductivity, the conductivity being zero with no flame. This conductivity is measured between a flame terminal and ground, if the flame or the conducting gases reach the grounded metallic furnace structure (see Fig. 28), or between two flame terminals. The description of the resistance control of the grid glow tube as given in the paragraphs relating to Fig. 8 under Part 2 of Glow Discharge Devices will serve as an explanation of the theory of operation.

The action of the grid-glow tube is instantaneous, and the relay *C*, Fig. 28, introduces a time delay of 2 to 3 seconds before shutting down the motor in case of flame failure. This time delay is necessary so as to allow sufficient time under normal condition for the fuel to reach the furnace after the starting of the motor.

Brief reference has been made in this paper to the initiative of our Research Department and interested research engineers in the development of the devices described in this paper, which is really a basis on which the art rests. The authors also wish to acknowledge the important work rendered by Dr. V. Zworykin, Mr. D. D. Knowles, and Mr. L. Sutherlin, as well as the Research Department as a whole, in the development of the applications and complete apparatus described in this paper.

Conclusions

On account of the wide scope of the subject discussed in this paper, it was not possible to cover the theory nor the applications of these devices in as much detail as might be desirable. However, it is intended to discuss several of the topics in this paper more specifically in other publications.

It is hoped that this article will act as a stimulus toward a broader recognition of the possibilities of tube devices in the industry, and that it will bring forth valuable suggestions which will result in other important applications.

Anomalous Conduction as a Cause of Dielectric Absorption*

BY J. B. WHITEHEAD† and R. H. MARVIN†

Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*The paper describes a series of experiments on the fundamental electrical properties of a number of waxes and oils, such as are used in composite high-voltage insulation. Particular attention has been directed to dielectric absorption as the origin of dielectric loss and the probable cause of deterioration and short life. The relation of the properties of the constituent parts to the properties of mixtures has been examined, with special reference to purification methods and the influence of small amounts of impurities. The studies have also afforded opportunity to test the validity of certain existing theories of absorption and loss. The method of experiment is the recording of short time absorption curves by means of the string galvanometer. Charge and discharge curves have been traced from a small fraction of a second following the beginning of charge or discharge, these curves having been brought much nearer to their beginnings than any heretofore recorded. It is this region near the beginning of these phenomena, which is most important in its bearing on dielectric loss.*

The principal results are summarized at the end of the paper, among the most striking being: (a) The extensive evidence that dielectric absorption is purely a conduction phenomenon; (b) The conductivity of dielectrics is highly anomalous in character, sometimes increasing, sometimes decreasing with increases in voltage and temperature. Absorption always follows the conductivity in these changes. (c) No existing theory, including that of Maxwell, can account for the variations in the absorption of mixtures of dielectrics here studied, and (d) Temperature elevation increases conduction and is a most serious factor in causing deterioration. The control of the processes involved in changes of anomalous conduction offers the best opportunitity for improvement of composite insulation.

* * * * *

I. Introduction

THE two most important properties of an insulating material are high dielectric strength and long life. The best insulators from this point of view are the crystalline, rigid, and refractory materials, such as porcelain, mica, quartz, and the like. These materials do not lend themselves to the insulation of wires and the various forms of conductor necessary in electric machinery. The composite and fibrous materials necessary in these cases have in general lower dielectric strength and shorter life. The short life is associated with the relatively high dielectric loss, always inherent in this class of materials. Under the combined influence of electric stress and temperature, progressive chemical and physical changes occur in the structure of the material. These changes are almost invariably in the direction of a deterioration of the important insulating properties: phase difference, resistivity, and electric strength.

The nature of these changes is not understood. The close correlation with changes in dielectric loss indicates clearly that an understanding and control of the loss will point the way to longer life. The evidence of recent years is that the greater part of dielectric loss is due to the phenomenon known as dielectric absorption, the nature of the origin of which is itself very obscure. The close relationship between dielectric absorption and dielectric loss has been shown experimentally,[37] and in fact the familiar decaying charging current curve under continuous voltage, typical of absorption, has been shown by Tank, Bouasse[37] and others to be sufficient to account for the essential characteristics of dielectric loss.

Unfortunately this does not help us very much, since in spite of the early recognition of the phenomenon of absorption (1843) and the constant study given it over many years by both physicists and engineers, it remains almost as obscure in its origin as during the early active period of its study. Only a single rational theory, that of Maxwell, has been proposed. Other suggestions of purely hypothetical character, and broadly qualitative, in relation to recent theories of atomic structure and ionic movement, have been proposed from time to time. None of these in the light of present knowledge is subject to experimental test. In fact, the same may be said of the proposal of Maxwell. Maxwell's theory, in short, has remained alive in spite of the absence of experimental proof, largely because of its simple rational character, because of its obvious sufficiency, if its premises be granted, and principally because no better theory has yet been proposed. As an indication of this condition, K. W. Wagner[40] has recently extended Maxwell's analysis to the alternating case and shown that many of the most striking experimental observations may thereby be satisfactorily explained.

The probable explanation of this vague position of Maxwell's proposal lies in the vast amount of experimental data accumulated in the general field of dielectric research, which contains many suggestions of other causes which might account for the phenomenon of dielectric absorption. Recently there has been a particularly striking increase in interest in the problem of dielectric conductivity and polarization. All this work brings prominently forward the evidence of the motion of ions through the body of the dielectric,

*Report to Engineering Foundation of an experimental research conducted in the Laboratory of Electrical Engineering of the Johns Hopkins University, Baltimore, Md.

†Both of the Johns Hopkins University, Baltimore, Md.

37. For numbered references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-31, 1929.

whether liquid or solid, inequalities of potential gradient, and various suggestions of an accumulation of space charge. We comment in further detail on this work later on. In general, it may be said that almost any single one of these various types of conductivity may be invoked with reasonable success as a qualitative explanation of the more conspicuous manifestations of dielectric absorption.

II. Purpose of the Work

It therefore appeared desirable to investigate more closely some of the more common dielectrics utilized in

Fig. 1—General View of Measuring Equipment

Left: string galvanometer and auxiliaries. Center: condenser, cover removed. Right: charge and discharge switch.

the manufacture of commercial insulation, with special reference to (1) A study of the charge and discharge absorption current curves at shorter intervals than heretofore observed, following the application and removal of voltage. This is the region of greatest influence on the value of dielectric loss. (2) To examine how closely these materials may be made by ordinary methods to approach the simple character postulated by Maxwell. (3) To study the dielectric properties of several substances, singly and in combination. (4) To study the mutual relationship of absorption and conduction as related to variations in temperature, electric gradient, and admixed impurities.

As materials for study we have selected some of the better known waxes and oils. These are, for the most part, good dielectrics, are used commonly in the manufacture of insulation, and lend themselves readily to convenient and reliable assembly for test. The several samples have been measured in a parallel plate condenser. The quantities measured are the charge and discharge current curves, the dielectric constant, the final conductivity, over a range of values of temperature and of electric gradient.

III. Apparatus and Methods

The condenser consists essentially of two circular brass plates. The lower of these is 60.96 cm. in diameter, and serves also as the bottom of a pan to contain the dielectric. This pan is mounted on insulators and is connected to the high side of the potential. The upper plate is 52.7 cm. in diameter and is surrounded by a guard ring 2.54 cm. wide, with air gap 0.32 between. See Fig. 2.

The back of the condenser plate is screened by metal cover, forming a part of the guard system. All metal surfaces of the electrodes, guard rings, and pan were nickel plated and polished. The computed capacity of the condenser at 0.249 cm., separation, is 0.000,784 μ f. (air).

The condenser as described is placed in an outer electrically heated oil bath, which in its turn rests in a deep bed of sand, the whole being contained in a stout wooden box, reinforced with steel bars and covered with galvanized iron with carefully soldered joints to render it air tight. This box is provided with a removable top,

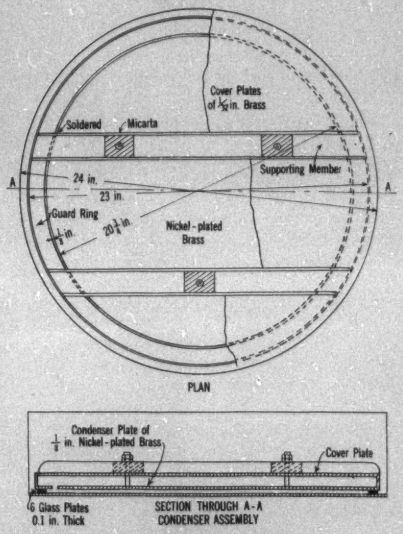

Fig. 2—Condenser Used in Tests

the joint between consisting of polished brass bars, sealed with wax. The whole provides means for temperature control and evacuation of the air during the preparation of the specimens.

After considering various methods and some preliminary experiments for measuring the very small absorption current of good insulators, we adopted the Einthoven[7] string galvanometer, for which see also the series of articles by Williams.[38]

The galvanometer itself consists of a powerful electro-

magnet with a narrow air gap, across which is stretched a coated quartz fiber. By a system of a condensing lens and microscope, light from an arc is passed through the galvanometer and the shadow of a portion of the string is focused on a revolving photographic film. The essential elements will be seen in Fig. 1. The film revolves at a known uniform speed and the shadow of the string gives a trace of the variation of the current passing through the string. Simple methods permit also the projection on the film of uniform vertical and horizontal scales. As the speed is slow, one revolution in 2 to 3 sec., bromide paper is sufficiently rapid for taking the exposure. It has the advantage of giving immediate and legible records without printing, and at great reduction of cost. The complete record is about 10 in. in length.

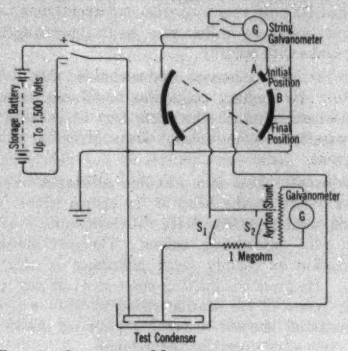

FIG. 3—CIRCUIT FOR MEASURING ABSORPTION AND CONDUCTIVITY

S_1 is closed during absorption test
S_2 is to stop swing
Connections are shown for discharge. For charge reverse connections to A and B

The sensitivity of the galvanometer can be varied over a wide range by altering the tension of the string. In our work the sensitivity usually ranged from 10^{-9} to 10^{-7} amperes per mm. The instrument is extremely sensitive to induced electrical disturbances, as well as mechanical vibration. Progressive elimination of all such disturbances, not always easy to find, is essential for satisfactory records.

An automatic switch operated by gravity was used for connecting the galvanometer into the condenser circuit. This switch was arranged to close the condenser circuit on charge and insert the galvanometer into the circuit after a small fraction of a second. Similarly, for discharge the condenser was short circuited for a brief interval, the galvanometer then being cut in. In the case of discharge the period of open circuit following charge was 0.005 sec., the period of short circuit 0.0022 sec., the galvanometer then being cut in. Similar intervals obtained for the period of charge, these figures being obtained by oscillograms. An approximate computation indicated that the initial transient of the geometric charge was negligible before the galvanometer was cut in the circuit. See Fig. 3.

The conductivity was measured on a D'Arsonval galvanometer of sensitivity 4×10^{-10} amperes per cm. The measurement was taken following the taking of absorption current on charge. The readings were taken shortly after the absorption record and at intervals during an hour thereafter, this being ample time to give a practically constant value.

The values of dielectric constant on the materials studied lie within a very narrow range as compared with the variations in conductivity, and it was not felt that a high degree of accuracy in its measurement was necessary. We therefore adopted the rapid and convenient method of measuring the current under alternating voltage as indicated in Fig. 4. The dielectric constant so taken is the ratio of the currents in the condenser, when filled with the dielectric in question, and when empty. The introduction of two auxiliary fixed condensers and of a Grondahl rectifier reduced the time required in these measurements to a short interval.

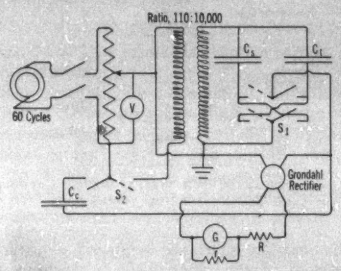

FIG. 4—CIRCUIT FOR MEASURING DIELECTRIC CONSTANT

C_t test condenser
C_2 and C_c, standard condensers. Value not necessarily known

The absorption and conduction of dielectrics are extremely sensitive to temperature variation so that the accurate determination of the temperature received special attention. Two methods were used: A resistance thermometer was placed in the oil below the condenser. This was used in melting the material and as a means of holding the temperature at a definite value. A second method was by the use of copper-Advance thermocouples, three placed on the central plate of the condenser and one in the oil. The couples were mounted in brass blocks placed immediately on the upper side of the condenser plate, and one below the condenser on top of the heater. When the condenser was filled all of these blocks were covered by the dielectric. The cold junctions were kept at 0 deg. cent.

The true temperature was assumed to be the mean of the values of all four couples. The resistance thermometer and the thermocouples were calibrated by reference to an accurate mercury thermometer, graduated to 0.2 deg.

For calibrating the string galvanometer and several of the instruments, and for measuring temperatures, the potentiometer as indicated in Fig. 5 was used.

As a source of potential a lead storage battery of 768 cells was available. Values of voltage used were 1500, 1000, and 500 volts.

IV. Description of Materials

Paraffin. One of the best of the better known insulators. Ours was purchased as "Refined Wax," melting point 128–130 deg. fahr. (53.4–54.5 deg. cent.). It was a hard white paraffin, being the purest and most refined grade obtainable from one of the large manufacturers.

Thorpe[33] gives the following definition of paraffin:

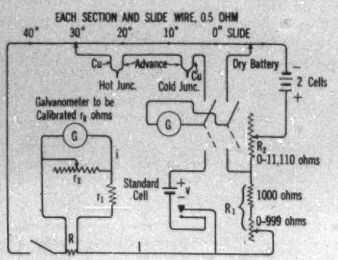

Fig. 5—Potentiometer for Galvanometer Calibration and Temperature Measurements

"A solid wax-like substance composed of saturated hydrocarbons of the series C_nH_{2n+2}. Melting point 38–57 deg. cent." Values of n from 12 to 60. The variety of constituents explains why commercial paraffin, although presumably it has a crystalline structure, softens over a wide range of temperatures. The paraffin used by us softened slightly at 35 deg. cent. and at 45 deg. cent. was plastic like putty.

Paraffin when melted and allowed to cool in air, absorbs a considerable quantity of air which is set free on solidification, causing the usual opaque appearance. Melting and cooling in even a moderate vacuum reduces this and makes it fairly transparent; Möller.[17]

Commercial paraffin may also contain noticeable quantities of water. Sometimes on first melting globules of water would collect in the bottom of the vessel. In one instance about half a teaspoonful was obtained from five pounds of paraffin, but this was exceptional. This excess of water is presumably mechanically entrapped in manufacture, since it at once falls down on melting. As in the case of transformer oils one would expect a certain amount of water to be actually dissolved. Since at 65 deg. cent., the temperature generally used for melting, the vapor pressure of water is 18.8 cm., which is much above the vacuum of around 5 cm., the water should be largely eliminated. Whether, however, sufficient water is taken up to affect the properties we were measuring is open to question, since with perhaps one or two exceptions, paraffin merely melted and cooled in air had as low absorption and conductivity as when vacuum treated.

Ceresine. This is a natural or fossil paraffin. The composition and structure are probably much the same, although some oxidation may have occurred. It is obtained by purifying ozokerite or mineral wax, which is found principally in Poland. As an insulating material it ranks with paraffin.

The particular material used resembled a hard white paraffin. Its melting point was 52-53 deg. cent. It absorbed air in much the same way as paraffin, and like paraffin became clear when given a vacuum treatment.

Spermaceti. This is a wax-like substance obtained from a cavity in the head of the sperm whale. The material used by us was white, rather opaque, and had a slight odor resembling tallow. Thorpe[33] gives the composition as mainly cetyl palmitate ($C_{16}H_{33}$) $C_{16}H_{31}O_2$. He gives the melting point as 41-49 deg. cent., but by repeated purification rising to 53.5 deg. cent. Our material showed a melting point of 45-48 deg. cent., depending upon the method used.

Unlike paraffin, a vacuum treatment produces no visible change. The structure is in all cases rather coarse and crystalline, and does not suggest good dielectric strength, which was also indicated by its failing during one of the tests. It did not show characteristics justifying extensive study.

Carnauba Wax. This is a vegetable wax. Thorpe[33] describes it as follows: "Obtained from the leaves of the carnauba palm, Brazil. Color: dirty yellow or greenish, melting point 83-86 deg. cent. specific gravity 0.990-0.999 at 15 deg. cent. The principal constituent is a myricyl ester of ceratic acid, $C_{27}H_{53}O_2$. $C_{30}H_{61}$, together with some minor constituents." The melting point was 85 deg. cent. specific gravity at 20 deg. cent. being 0.979. While hard, it is very brittle, has little adhesion to metal, and contracts greatly on cooling. As a consequence it cracks badly, and is very unsatisfactory as a dielectric.

Stearic Acid. This is obtained from animal fats. The composition is $C_{18}H_{36}O_2$. The melting point when pure is 69.3 deg. cent. It is crystalline in structure. Palmitic acid $C_{16}H_{32}O_2$, another member of the same series, is usually associated with it, and is very difficult to eliminate. Pure stearic acid is odorless. The grade used by us was that purified by alcohol, and presumably of good quality. The melting point was only 55 deg. cent., indicating the presence of palmitic acid, and possibly lower members of the same series. The

odor was quite strong, resembling that of tallow. It had a light cream color.

Stearic acid has high absorption and conductivity, making it an interesting material for our work. While the adhesion to metal is poor, which would tend to promote cracking, no trouble was ever experienced with the dielectric strength.

Refined Lubricating Oil. This is a high grade oil of excellent insulating qualities and was selected in order to study the effect of mixing it with paraffin. It was supplied by the Standard Oil Company of New Jersey, as their purest and highest grade insulating oil. It is clear oil with a bright yellow color, somewhat viscous, but flows quite freely at room temperature. The specific gravity at 26 deg. cent. is 0.877.

Black Oil. Furnished by request as a petroleum oil of poor insulating qualities for use in the study in mixtures. It is evidently in an intermediate stage of refining. The color is a dense black, probably indicating the presence of an asphalt compound. At 20 deg. cent. it flows with difficulty, and approaches the consistency of a grease. Even at 65 deg. cent. it was so viscous that it was difficult to remove from the condenser plates.

V. The Experiments

The chief observations were the charge and discharge absorption current curves as taken with the string galvanometer. These afforded data as to absorption and conduction. In each case they were taken over a range of temperature and of voltage, and the dielectric constant measured as well. The influence of combined temperature and reduced air pressure on the electric properties was also studied. Paraffin showing itself practically free of absorption, was used as a basic material for the study of its combination with other materials, and with "impurities."

A large number of observations has been taken. Complete presentation is unnecessary. We give, therefore, in Figs. 6 to 11, a series of typical charge and discharge current curves as observed on various specimens. In all we have taken over 200 such curves. They constitute the principal record on which our conclusions are based. The numerical values presented in Tables I to IV are measured directly from the photographic records. These tables are a condensed compilation from the more extended individual tables pertaining to the respective specimens. The materials and details of treatment corresponding to the individual specimens as numbered are given in Table V. Three standard temperatures were used, 25, 35, and 45 deg. cent., and three standard voltages, 500, 1000, 1500 volts; nine sets of tests on each sample. Occasionally higher temperatures were included, but as these usually approach the melting point of the solids, we have selected the three low temperatures mentioned.

From the charging current records the matter of chief interest is the excess of the current over the final conduction current, measured after one hour. The comparison of this excess of the current with the values of the discharge current indicates the possible presence of irreversible absorption current. In the tables are recorded the values of absorption current at 0.2, 0.4, and 0.8 sec., and also these values less the final conduction current denoted by i''.

From the records of discharge currents we have found it best to record the values taken at time intervals in geometric progression 0.1, 0.2, 0.4, 0.8, 1.6, 3.2 sec. The rate at which the current dies out is of significance, and is indicated by the ratio of the current at 0.2 and 0.4 sec., and also at 0.4 and 0.8 sec.

The maximum deflection is clearly indicated on many records. It is, however, reported in only a few instances in which the deflection was so small that the maximum was the only readable feature. We have not used it in our deductions. We have analyzed elsewhere the possible error in the maximum deflection and in general have shown that throughout our work the deflection of the string follows closely the variation of the current, except in a few instances of the higher sensitivity, at the beginning of the record. This conclusion has been proved by photographic records.

VI. Characteristics of Absorption Curves

Figs. 6 and 7 are records of charging current, Figs. 8 and 9 of discharge.

Many attempts have been made to express the absorption current curve by a simple algebraic relation. Our accurate curves beginning at brief intervals following short circuit furnish us with excellent material to test the validity of some of these expressions. Prominent among them are those involving the time to a constant negative power and that based on a negative exponential function of the time.

In Fig. 12 is shown a comparison between an actual curve and that given by the formula $i = I\,t^{-n}$, where I and n are constants. The constants have been so chosen that the curves intersect at 0.1 sec. and 2 sec. The agreement is seen to be fair. Such a curve is a straight line when plotted to logarithmic coordinates. A large number of our curves was so plotted and almost invariably gave a line slightly concave towards the time axis.

The negative exponential $i = I\,E^{-at}$ has also been plotted on Fig. 12, in the same manner. Its simplicity and definite value at zero time make it attractive, but the deviation from the actual curve is so great that it is scarcely even a rough approximation. This curve has particular interest from the fact that it is that indicated by Maxwell's analysis. This departure of the results of experiment from the indication of this simple theory has often been noted and is one reason for the discredit now commonly attached to the validity of Maxwell's proposal.

In view of the foregoing discrepancies we have preferred to describe the current curves directly in terms

of the ordinates at the intervals already mentioned, and to compare the variation among different samples in terms of the changes in the ratio of these values.

VII. Influence of Electrical Gradient

Early theoretical analysis of dielectric absorption, including Maxwell's, attributes to it a linear increase with electric gradient. Curie,[3] from experiments on a number of crystals, announces this as the first of his three well known empirical laws. We have found noticeable variations from this simple relation.

As regards the relation of conduction to electric gradient much confusion exists. Curtis[4] using samples about 1 cm. thick and measuring the conduction at 50 and 500 volts, found no difference with fair insulators, and only a slight increase at the higher voltage for poor and porous insulators. He states that with the best insulators, as paraffin and ceresine, the effect, if any, was masked by absorption. On the other hand Poole[19, 20, 21] investigating mica, glass, and celluloid at high potential gradients found a noticeable effect which could be approximately expressed by the formula $\lambda = A + B X$, where λ is the conductivity, X the potential gradient, and A and B constants. This has been confirmed by Schiller[23] also working on glass, and using values of X up to 500 kv./cm. Mündel[18] likewise experimented with glass plates at all voltages up to breakdown, and concluded that while at low gradients the current is proportional to the gradient, that is constant conduction, at high values the current is nearly proportional to the square of the gradient. Tadeschi[22] studied various paper products at high potential gradients, and found a marked increase in conduction with increasing gradient, although no definite relation

Fig. 6—Specimen No. 24—Carnauba Wax

Dec. 21, 1927—Charge—1500 volts
Temperature 46.1 deg. cent.
Sensitivity 6.28 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

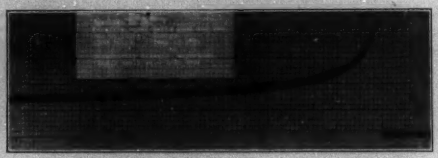

Fig. 7—Specimen No. 28—Paraffin, Equal Parts Good and Bad

Jan. 24, 1928—Charge—1500 volts
Temperature 45.2 deg. cent.
Sensitivity 6.24 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

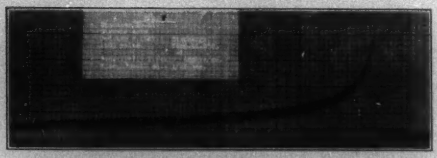

Fig. 8—Specimen No. 11—Stearic Acid

Oct. 6, 1927—Discharge—1500 volts
Temperature 44.8 deg. cent
Sensitivity 1.84 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

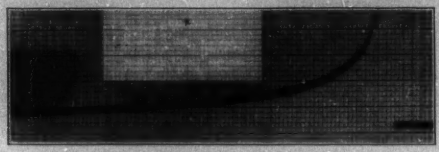

Fig. 9—Specimen No. 24—Carnauba Wax

Dec. 21, 1927—Discharge—1500 volts
Temperature 46.8 deg. cent.
Sensitivity 5.22 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

Fig. 10—Specimen No. 34—"Black Oil"

March 2, 1928—Discharge—1500 volts
Temperature 45 deg. cent.
Sensitivity 3.92 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

Fig. 11—Specimen No. 38—Paraffin 99 Per Cent, "Black Oil" 1 Per Cent

March 28, 1928—Discharge—500 volts
Temperature 25.1 deg. cent.
Sensitivity 2.09 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

was observed. Further instances could be cited, but these are sufficient to indicate that at gradients which are fairly high the conduction increases with the gradient. Our own results in general indicate the same relation.

The influence of electric gradient in our work may be

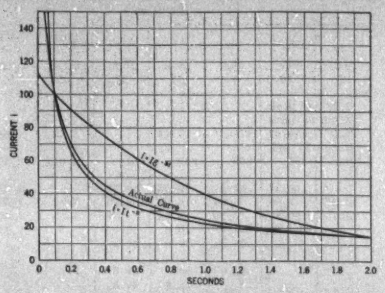

FIG. 12—COMPARISON BETWEEN ACTUAL ABSORPTION CURVE ON DISCHARGE AND EMPIRICAL EXPRESSIONS

Actual curve, paraffin, specimen No. 28, at 45.3 deg. cent. and 1500 volts. Plotted to arbitrary scale of 100 at 0.1 sec.

Empirical curves drawn to intersect actual at 0.1 and 2 sec.

studied from Tables I to IV. The influence on absorption is shown in the ratio of the current at 1500 volts to that at 500 volts, for two time intervals, 0.4 and 1.6 secs. after discharge. The influence on conduction is shown in the ratio of the final currents at 1500 volts and 500 volts. If the absorption were proportional to the electric gradient and the conduction constant, these ratios would all be "3."

In the case of absorption, we notice that there is a general tendency to a value less than "3" for these ratios and this effect is most noticeable at the higher temperatures. Perhaps the most noticeable feature is that the ratio of the currents is almost invariably less at 1.6 sec. than at 0.4 sec. The meaning of this is that the current is falling off more rapidly at the higher voltage. If this is correct then the ratio of all consecutive ordinates should be greater at 1500 volts than at 500 volts. We have examined the results on a number of our specimens from this point of view, and out of 23 cases in which the comparison is possible we have found the ratio is greater at 1500 volts, in 13 cases, in 10 others approximately equal to that at 500, and in no case definitely lower. The evidence, therefore, is clear that the absorbed charge falls off more rapidly the higher the voltage, thus showing a noticeable departure from Curie's empirical law. As regards the conduction a study of the table plainly shows large variations of the ratio of conduction current at 1500 volts to that at 500 volts from the value "3" in both directions. At 25 deg. there are two values out of 15, at 45 deg. there are seven out of 25 which are less than "3." When the ratios for the two temperatures are compared, in nearly every case it is greater at the lower temperature. Since, as will be seen, the conduction usually increases with the temperature, it thus appears that as the conduction increases the relative influence of increasing electric gradient becomes less.

In addition to the data here given for paraffin a large number of check tests was made on the values of conduction. All these agree in showing a marked increase with voltage. However, attempts relating them to a definite law such as that of Poole, for example, were not successful.

Instances in which the conduction has decreased with increase of voltage are of special interest. In the case of No. 30 refined oil and No. 33, equal parts of this

TABLE I

RESULTS OF TESTS AT 500 VOLTS AND 25 DEG. CENT.

UNIT OF CURRENT 10^{-8} AMPERE

		Charge						Discharge						
Specimen No. 1	Conduction current 2	i (0.2) 3	i (0.8) 4	i (3.2) 5	i'' (0.2) 6	i'' (0.8) 7	i'' (3.2) 8	i (0.2) 9	i (0.4) 10	i (0.8) 11	i (1.6) 12	i (3.2) 13	$i(0.2)/i(0.4)$ 14	$i(0.4)/i(0.8)$ 15
6	0.154	..	..	..	..	..	..	Trace at peak				..		
8	0.242	..	..	..	..	..	..	Trace at peak						
9	0.236	..	..	..	..	..	..	7.53	5.94	4.63	3.24	..	1.27	1.28
10	0.0176	..	..	..	..	..	..	0.54	at peak					
11	18.4	..	..	..	..	..	..	207.	129.	79.	43.	27.	1.68	1.64
24	0.007	15.0	7.55	3.17	15.0	7.54	3.16	..	10.9	7.85	5.34	3.44	..	1.39
25	0.065	46.6	22.8	9.45	46.5	22.7	9.39	..	31.1	21.4	14.5	9.45	..	1.45
28	10.96	..	42.8	26.9	..	33.8	15.9	58.2	41.4	28.6	19.2	12.3	1.40	1.45
29	1.116	..	33.6	11.9	..	32.5	10.8	..	55.4	30.3	16.3	9.2	..	1.83
32	0.091	1.69	1.54	1.43	0.78	0.63	0.52	1.5	at peak					
34	216.	..	..	..	..	..	..	..	..	..	18.6	11.8	..	..
35	194.	..	..	..	..	..	..	35.4	31.5	26.7	21.9	16.4	1.12	1.18
36	19.8	57.4	37.1	25.3	37.6	17.3	5.5	43.7	31.3	21.8	14.7	9.9	1.39	1.44
37	6.66	..	21.5	13.6	..	14.8	6.9	..	..	15.6	11.6	10.4	..	..
38	0.409	10.14	6.09	3.57	9.73	5.68	3.16	8.44	6.74	4.92	3.87	2.89	1.25	1.37
39	0.276	18.1	11.5	7.2	17.8	11.2	6.9	16.1	13.1	10.4	8.2	6.1	1.23	1.26

oil and paraffin, it might reasonably be explained as due to the approach to a saturation value of the current, as has often been noted in liquid dielectrics. The remaining instances are all paraffin. In the direction of possible explanation we report the following examples of a kind of slow moving polarization found with

TABLE II
RESULTS OF TESTS AT 1500 VOLTS AND 25 DEG. CENT.
UNIT OF CURRENT 10^{-6} AMPERE

		Charge						Discharge						
Specimen No. 1	Conduction current 2	i (0.2) 3	i (0.8) 4	i (3.2) 5	i'' (0.2) 6	i'' (0.8) 7	i'' (3.2) 8	i (0.2) 9	i (0.4) 10	i (0.8) 11	i (1.6) 12	i (3.2) 13	$i(0.2)/i(0.4)$ 14	$i(0.4)/i(0.8)$ 15
6	0.425	..	..	..	..	..	..	0.76 at peak						
8	1.92	..	..	..	..	..	..	1.40 at peak						
9	1.30	..	..	..	..	..	..	25.9	18.6	11.8	7.24	..	1.39	1.58
10	0.100	..	..	..	..	..	..	1.62 at peak						
11	138.	..	..	..	..	..	..	798.	486.	260.	159.	91.	1.64	1.87
21	0.885	..	4.60	2.88	..	3.71	1.99	..	18.5	13.4	9.56	6.53	..	1.37
22*	0.282	5.40	3.72	2.40	5.12	3.44	2.12	10.3	7.86	5.69	4.07	2.76	1.32	1.38
23†	0.294	10.6	7.56	4.47	10.3	7.27	4.18	14.7	10.8	7.98	5.48	3.74	1.36	1.36
24	0.031	..	23.0	15.2	..	23.0	15.2	..	33.5	23.7	16.0	10.3	..	1.41
25	0.343	..	96.4	30.9	..	96.1	30.6	..	..	69.5	47.0	30.3	..	..
28	57.2	..	..	98.5	..	..	41.3	..	..	90.6	60.3	37.0	..	..
29	14.6	..	..	61.3	..	..	46.7	..	..	..	74.6	41.3	..	..
32	0.023	0.432	0.308	0.308	0.409	0.285	0.285	0.6	0.4	..	..	..	..	..
34	845.	..	..	..	..	..	..	85.4	72.7	57.0	40.3	23.7	1.17	1.27
35	660.	..	..	..	..	..	..	..	..	54.0	40.9	29.3	..	..
36	64.5	191.	129.	91.9	127.	64.5	27.4	129.	96.5	67.4	46.4	30.2	1.34	1.43
37	20.4	..	72.4	47.6	..	52.0	27.2	..	60.2	44.1	31.1	21.4	..	1.37
38	1.56	23.9	14.3	9.58	22.3	12.7	8.02	21.8	16.7	12.7	9.7	7.5	1.30	1.31
39	0.690	..	22.9	12.7	..	22.2	12.0	37.2	27.1	19.4	13.4	9.4	1.51	1.40

*No. 22 is at 35 deg. cent.
†No. 23 is at 35 deg. cent.

TABLE III
RESULTS OF TESTS AT 500 VOLTS AND 45 DEG. CENT
UNIT OF CURRENT 10^{-6} AMPERE

		Charge						Discharge						
Specimen No. 1	Conduction current 2	i (0.2) 3	i (0.8) 4	i (3.2) 5	i'' (0.2) 6	i'' (0.8) 7	i'' (3.2) 8	i (0.2) 9	i (0.4) 10	i (0.8) 11	i (1.6) 12	i (3.2) 13	$i(0.2)/i(0.4)$ 14	$i(0.4)/i(0.8)$ 15
6	0.125	..	..	..	..	..	..	2.43	1.90	1.35	0.86	..	1.29	1.40
8	0.137	..	..	..	..	..	..	1.62	at peak					
9	1.32	..	..	..	..	..	..	7.69	6.37	4.80	3.40	..	1.20	1.33
10	0.048	..	..	..	..	..	..	1.78	1.30	0.98	0.70	..	1.38	1.33
11	88.1	..	..	..	..	..	..	186.	135.	100.	67.	41.	1.38	1.34
13	0.013	..	..	..	..	..	..	4.84	3.57	2.51	1.83	1.25	1.35	1.42
14	0.037	..	..	..	..	..	..	5.84	4.32	3.19	2.26	1.56	1.35	1.34
15	0.004	..	..	..	..	..	..	2.86	1.68	1.35	0.94	0.54	1.70	1.25
16	3.94	..	..	..	..	..	..	22.6	16.3	10.8	7.5	4.8	1.38	1.52
17	6.84	..	..	..	..	..	..	31.9	20.3	13.2	8.2	5.1	1.56	1.54
18	13.5	..	..	..	..	..	..	49.3	33.9	23.6	14.4	8.7	1.45	1.44
19	16.9	..	..	..	..	..	..	47.2	30.8	20.0	12.6	7.4	1.53	1.54
20	272.	..	..	..	..	..	..	98.8	56.7	30.6	14.7	6.5	1.74	1.85
21*	0.472	15.6	10.5	6.54	15.1	10.0	6.07	8.78	6.58	4.88	3.48	2.25	1.34	1.35
28	36.8	145.	86.2	56.6	108.	49.4	19.8	95.0	62.8	41.8	26.6	17.1	1.51	1.50
29†	10.4	..	70.2	33.9	..	59.7	23.4	..	100.	55.8	31.2	17.9	..	1.80
32	0.065	0.315	0.197	0.197	0.250	0.132	0.132	1.76	1.45	1.19	0.94	0.54	1.22	1.28
34	1500.	..	..	..	..	..	..	10.8	6.60	4.60	3.40	2.60	1.64	1.44
35	1405.	..	..	..	..	..	..	15.7	11.7	7.85	4.80	3.33	1.34	1.49
36	957.	1455.	1332.	1260.	498.	375.	303.	..	78.1	44.8	24.6	11.0	..	1.74
37	334.	864.	734.	660.	530.	400.	326.	186.	121.	76.1	47.2	25.4	1.54	1.59
38	46.3	..	153.	126.	..	107.	80.	82.5	54.1	35.6	22.0	12.8	1.53	1.52
39	1.47	49.8	32.1	20.5	48.3	30.6	19.0	71.8	48.4	32.4	20.5	12.4	1.48	1.49

*No. 21 at 35 deg. cent.
†No. 29 at 35 deg. cent.

TABLE IV

RESULTS OF TESTS AT 1500 VOLTS AND 45 DEG. CENT.

UNIT OF CURRENT 10^{-4} AMPERE

		Charge						Discharge						
Specimen No. 1	Conduction current 2	i (0.2) 3	i (0.8) 4	i (3.2) 5	i'' (0.2) 6	i'' (0.8) 7	i'' (3.2) 8	i (0.2) 9	i (0.4) 10	i (0.8) 11	i (1.6) 12	i (3.2) 13	$\frac{i(0.2)}{i(0.4)}$ 14	$\frac{i(0.4)}{i(0.8)}$ 15
6	0.488	..	..	..	..	..	..	8.40	5.93	3.40	1.94	..	1.42	1.74
8	0.350	..	..	..	..	..	..	3.56	1.36	1.40	..	..	1.64	1.63
9	2.54	..	..	..	..	..	..	26.0	18.6	11.9	9.70	..	1.39	1.57
10	0.200	..	..	..	..	..	..	4.38	3.14	2.33	1.62	..	1.40	1.35
11	350.	..	..	..	..	..	..	645.	432.	287.	180.	105.	1.49	1.50
13	0.119	..	..	..	..	..	..	11.2	8.32	5.98	4.08	2.67	1.34	1.39
14	0.189	..	..	..	..	..	..	19.0	14.5	10.3	6.78	4.52	1.31	1.40
15	0.032	..	..	..	..	..	..	..	..	..	..	..	..	..
16	11.8	..	..	..	..	..	..	51.8	38.3	25.9	16.5	10.0	1.35	1.48
17	73.3	..	..	..	..	..	..	117.	74.8	45.4	26.7	14.7	1.57	1.65
18	61.4	..	..	..	..	..	..	130.	88.5	57.8	35.8	21.4	1.47	1.53
19	114.	..	..	..	..	..	..	131.	83.0	50.8	29.5	14.7	1.58	1.63
20	1068.	..	..	..	..	..	..	245.	130.	64.1	31.3	12.5	1.90	2.03
21*	3.35	21.9	14.2	8.66	18.5	10.8	5.31	29.6	20.5	13.4	8.61	5.61	1.44	1.53
22	0.075	3.36	2.18	1.36	3.28	2.10	1.28	3.39	2.67	1.96	1.58	1.21	1.27	1.37
23	0.133	12.0	8.82	5.10	11.87	8.68	4.97	14.0	11.3	8.07	6.58	4.47	1.24	1.26
24	0.583	269.	100.	31.4	269.	99.4	30.8	326.	209.	133.	81.0	51.2	1.56	1.56
28	261.	465.	303.	218.	204.	42.	−43.	233.	145.	91.0	54.5	34.2	1.60	1.60
29†	73.6	..	..	169.	..	..	95.	..	..	198.	111.	62.7	..	..
32	0.225	2.84	1.55	0.64	2.61	1.32	0.41	4.66	3.52	2.52	1.62	0.97	1.32	1.40
34	6180.	..	..	..	..	..	..	16.4	10.0	6.88	4.72	3.72	1.65	1.46
35	4730.	..	..	..	..	..	..	24.5	17.1	11.2	7.14	4.28	1.43	1.53
36	3070.	4690.	4380.	4180.	1620.	1310.	1110.	..	277.	148.	72.0	30.5	..	1.87
37	1114.	2170.	1876.	1747.	1056.	762.	633.	353.	221.	128.	71.2	34.8	1.60	1.72
38	132.	..	320.	260.	..	188.	128.	175.	113.	70.9	42.2	23.6	1.55	1.59
39	2.00	92.0	47.6	16.0	90.0	45.6	14.0	71.8	48.4	32.4	20.5	12.4	1.48	1.49

*No. 21 at 35 deg. cent.
†No. 29 at 35 deg. cent

several of the better grade of specimens. Our tests have usually been made in the order 500, 1000, 1500 volts, the condenser being short circuited between tests. In a number of instances it was found that at 500 volts the current at 10 min. is large compared to that at 60 min. In applying the next higher voltage the current at 10 min. would differ but little from that at 60 min. A series of check measurements confirmed these observations. It will be noted that there is a suggestion of a sweeping out or removal of some conducting element, which on reversal of polarity is brought back and must again be swept out. The known magnitude of the absorption current makes it certain that this is a much greater effect than that due to a possible residual absorption from a previous charge.

VIII. Influence of Temperature

We find convenient reviews presenting knowledge of the nature of conduction in solid dielectrics by Curtis[5] and by Kraus.[12] The tendency is to attribute the conduction of insulating solids to ions of widely varying character. Some of these are of electrolytic type, others of greater mass, and occasionally the electron is invoked. Much uncertainty surrounds the question of the nature of these carriers.

There is a common tendency to consider an increase in conductivity with temperature as a requisite of electrolytic conducting, but as pointed out by Kraus, this is not necessarily so, since conductivity is the result of both mobility and dissociation and the temperature affects these in opposite ways, so that it is quite possible for conductivity to fall with rising temperature.

The majority of the investigations on the influence of temperature on dielectric conduction have been made on glass, various crystals, and impregnated paper. For materials such as we are now considering, Curtis[4] gives the variation in resistivity between 20 and 30 deg. cent. Paraffin (Parawax) is stated to have double the resistance, or half the conduction, at 20 deg. that it has at 30 deg. cent.

Considering the influence of temperature on absorption we find a number of investigations. Von Schweidler[29] gives results on glass at 18 to 47 deg. cent. He found that they were well expressed by the relation $t = I\,t^{-n}$. He found that n was approximately constant, while I increased with rising temperature. From this he concluded that the effect of temperature was to increase the magnitude of the absorption current, while not altering the form of the curve. This would be equivalent to an increase in the total absorbed charge with temperature.

Zeleny[39] prepared condensers of paper impregnated with paraffin, and also with ozokerite, in such a

manner as to contain various amounts of moisture. With temperatures ranging from − 70 to + 30 deg. cent., the throw on the ballistic galvanometer of 5 sec. period was measured, the condenser having previously been short-circuited for 0.3 sec. In all cases a marked increase in throw was observed as the temperature increased. These results are open to criticism because of the short period of the galvanometer. The observed values could be explained either as due to a greater absorbed charge, or else that this charge came out more readily at the higher temperature.

Wagner,[34] working on gutta-percha, balata, rubber, hard rubber, paraffin, and ceresine, over a considerable range of temperatures, using an a-c. method, indicates an increasing magnitude in the absorption current as the temperature increases, followed finally by a decrease as the melting point is approached. He also shows that the effect found with alternating stress would be equivalent with constant stress to a change in the rate at which the absorbed charge fell off, but to only a slight variation in the total absorbed charge, differing in this respect from von Schweidler.

Our principal results as regards temperature are contained in Tables I to IV. The influence on conduction is found in the ratio of the final current at approximately 35 deg. and 45 deg. to that at 25 deg. cent. at 500 volts. The voltage being the same at both temperatures, this is also the ratio of the conductivities.

The influence on absorption is brought out in the ratios of the currents on discharge at 35 deg. and 45 deg. to that at 25 deg. cent. After 0.4 sec., and 1.6 sec., and in a few cases for the peak values, all at 1500 volts.

In the majority of cases the conduction increases with the temperature. The large range of variation is noticeable. It is suggestive that the mixtures of distinctly different substances, Nos. 29, 33, 36, 37, and 38, all show large values. This is probably due to some dissociation among the constituents.

The number of instances in which conduction has fallen with a temperature increase is noteworthy. Examining each specimen we see that this occurs with the better quality insulators; paraffin in fair to good condition, ceresine, and refined oil. This is significant, and leads to the inference that it is related to the abnormalities found with these high insulators which were noted in considering the influence of voltage.

As to absorption, we find a general tendency to an increase with rise in temperature,—quite large in some cases. But just as with conduction we notice exceptions to this rule. That of ceresine, Nos. 22 and 23, is conspicuous. A duplicate test was made on No. 22 and is included to show the same tendencies in both tests. The interesting feature is that ceresine was also exceptional in its conduction, so that here is a substance which contrary to most materials has both decreasing conduction and absorption current with rising temperature. The other exception is "Black Oil," Nos. 34 and 35. This being a liquid at all temperatures, the decrease is in line with that always noted in passing from the solid to the liquid state.

Comparing the results we note all through a definite tendency for the conduction and absorption current to vary more or less proportionately. That is, the greater the increase in conduction, the greater the absorption current.

In comparing the magnitude of the absorption currents at a single time interval we must keep in mind that the curves from which they are taken may be falling at different rates. If we compare the ratios at different time intervals, we have an indication as to whether the total absorbed charge has changed, or merely that the rate of discharge has altered. We find 11 cases where we can compare the ratio of the currents at 0.4 and 1.6 sec. with the change in conductivity. In 6 instances the ratio of the absorption currents is less at 1.6 than at 0.4 sec., while the conduction has increased with the temperature. This agreement is striking, and in line with Wagner's conclusion that the chief influence of temperature is on the rate of discharge, and only slightly on the total absorbed charge. In the other 5 the ratio of the absorption currents is greater at 1.6 than at 0.4 sec. In 3 of these, all ceresine, the conduction decreases with the temperature. Thus these also fall in line. The other two, paraffin No. 9 and stearic acid No. 11, show an increase in conduction, and the relation is not so definite.

As is well known the effect of melting a dielectric is in most cases practically to remove the absorption. This was found to be true of paraffin. The mixture of paraffin and stearic acid No. 29, is interesting in this connection. Although both constituents have melting points above 50 deg. cent., yet the mixture even at 45 deg. cent. had the consistency of soft butter. Under this condition only a trace of absorption on discharge was found. At 54.4 deg. cent. the mixture was a clear liquid, and here also only a trace of absorption could be detected.

The "Black Oil," Nos. 34 and 35, is an instance of a liquid with a large reversible absorption. It is commonly assumed that liquids do not show this phenomenon. The high viscosity of the oil suggests itself in explanation. Even at about 70 deg. cent. it was difficult to wipe off from the condenser plates, whereas paraffin at this temperature would wipe off as readily as water.

Reviewing briefly, we note that in the case of solid dielectrics, in general both conduction and absorption current increase with rising temperature. As the melting point is reached the absorption falls off, usually almost disappearing on complete fusion. In some instances a decrease has been found in both conduction and absorption with temperature rise. In the case of both increase and decrease the proportionate changes in absorption and conduction are almost the same. There are indications that the rate at which the absorption current dies out is greater the greater the conduction.

TABLE V
DESCRIPTION OF SPECIMENS

Specimen No.	Material	Treatment
6	Paraffin	100 deg. cent. for 9 hr. in vacuum
8	Paraffin	No. 6 heater to 110 deg. cent. for 6 hr. in vacuum
9	Paraffin	No. 8 heater to 110 deg. cent. for 3 hr. in vacuum
10	Paraffin	65 deg. cent. for 6 hr. in vacuum
11	Stearic acid	Melted in air at 72 deg. cent.
13	Paraffin	Melted in air at 69 deg. cent.
14	Paraffin	No. 13 heated to 70 deg. cent. for 9 hr. in vacuum
15	Paraffin	No. 14 heated to 70 deg. cent. for 8 hr. in air
16	Paraffin	No. 15 heated to 85 deg. cent. for 8 hr. in vacuum
17	Paraffin	No. 16 heated to 85 deg. cent. for 8 hr. in air
18	Paraffin	No. 17 heated to 100 deg. cent. for 8 hr. in vacuum
19	Paraffin	No. 18 heated to 100 deg. cent. for 8 hr. in air
20	Paraffin	No. 19 heated to 145 deg. cent. for 8 hr. in air
21	Ceresine	Melted in air at 64 deg. cent.
22	Ceresine	No. 21 heated to 66 deg. cent. for 3 hr. in vacuum
23	Ceresine	No. 22 heated to 70 deg. cent. for 5 hr. in vacuum
24	Carnauba wax	Heated to 96 deg. cent. for 5 hr. in vacuum
25	Spermaceti	Melted in air at 64 deg. cent.
28	Paraffin. Equal parts of new paraffin and No. 20	Heated to 67 deg. cent. for 30 min. in vacuum
29	Equal parts by volume of paraffin and stearic acid	Heated to 67 deg. cent. for 2 hr. in vacuum
32	Paraffin	Heated to 65 deg. cent. for 4½ hr. in vacuum
34	"Black oil"	Oil poured in with condenser at 45 deg. cent.
35	"Black oil"	No. 34 heated to 69 deg. cent. for 3 hr. in vacuum
36	Paraffin, 91.7 %: "Black Oil," 8.3 %	Heated to 68 deg. cent. for 1½ hr. in vacuum
37	Paraffin, 96 %: "Black Oil," 4 %	Heated to 71 deg. cent. for 2 hr. in vacuum
38	Paraffin, 99 %: "Black Oil," 1 %	Heated to 68 deg. cent. for 3 hr. in vacuum
39	Paraffin. No. 20 filtered through Fuller's earth	Heated to 71 deg. cent. for hr in vacuum

We have noticed a number of cases of reversible absorption in liquids.

IX. Influence of Treatment

As far back as 1883 Hertz[10] showed that the conduction and absorption of benzine could be largely reduced by careful distillation. The injurious effect of even a trace of moisture in transformer oil has been long recognized.

We have heated all our materials at reduced air pressure for the removal of air and water, but in only a few cases was there definite evidence of improvement. One such instance is ceresine, where No. 22, vacuum treated, shows distinctly lower conduction and absorption current than No. 21, merely melted in air. On the other hand "Black Oil," although a poor insulator, was substantially unaltered by heating in a vacuum.

Paraffin when heated for a long time, shows increased conduction and absorption current. The same sample of material was heated for 8-hr. periods both in air and under vacuum, and with a gradually increasing temperature. All tests were made at 45 deg. cent. As a basis of comparison we take the conductivity at 1000 volts, and the absorption current after 0.2 sec. at the same voltage, as these are available for all specimens. These values are given in Table VI. The unit of current is 10^{-8} ampere. We note that so long as the temperature does not exceed 70 deg. cent. the injury to the paraffin is slight, if any. Above this temperature deterioration is more and more rapid. There is evidently a rough parallelism between the increase in conduction, and that in absorption current.

The general nature of the deterioration was indicated by the appearance of a greenish color, evidently due to solution of nickel from the condenser plates by some acid in the paraffin. This action is well known, although the exact composition of the acids had not been determined.

This deterioration of paraffin has been already noted by Mikola.[15] He found that after heating to 200 deg. cent. its conduction was much increased, and its electrical properties modified in other ways.

TABLE VI
INFLUENCE OF PROLONGED HEATING

Specimen	Treatment	Temperature	Conduction current at 1000 volts 45 deg. cent.	Absorption current at 1000 volts, 0.2 sec. after discharge, 45 deg. cent.
13	Air	69	0.062	7.92
14	Vac.	70	0.142	13.4
15	Air	70	0.051	6.24
16	Vac.	85	8.64	39.8
17	Air	85	25.6	68.4
18	Vac.	100	40.0	101.
19	Air	100	65.3	97.5
20	Air	145	706.	164.

A similar effect occurs in petroleum oils, such as transformer oil, and for these the subject has been much studied; see the series of papers by Haslam, Frölich, and Mead.[9,15]

We filtered the deteriorated paraffin through Fuller's earth, a clay having high absorbing properties, and commonly used in the refining of paraffin. The paraffin after filtration had largely lost its green color, and its electrical properties were greatly improved. (Specimen No. 39.) This study was not carried further, but it appears evident that the temperature deterioration of paraffin consists largely in the formation of other compounds sufficiently definite and different in properties as to permit separation by filtration.

X. Conduction and Absorption in Mixtures

We have already mentioned the apparent impossibility of finding a solid insulating substance meeting

Maxwell's requirement of conduction without absorption. Rowland and Nichols[26] found no absorption in calcite (Iceland spar). But later investigations on calcite show absorption, (Richardson[22] and Joffe[11]). In this connection we may call attention to the extensive tests by Lee and Lowry[13] on materials of the class we are considering. They give data on 31 different waxes, resins, and bitumens. They determined the d-c. resistance and the a-c. conduction at 1000 cycles. Comparing the reciprocal of the conduction with the d-c. resistance, it is in all cases lower, showing the influence of absorption. Also, in general, the lower the d-c. resistance the more pronounced was the effect of absorption.

Since no materials could be found exactly meeting Maxwell's requirements, it was necessary to study mixtures of materials of which one at least had noticeable absorption. Two methods suggest themselves. The first is to derive a mathematical relation, following the general method laid down by Maxwell, for a two layer dielectric, either or both of the layers having absorption. From such an expression the value of the absorption current could be calculated, and compared with the measured values. However, the absorption currents in the constituents are functions of the time, and the introduction of these functions into the relations leads to apparently insuperable mathematical difficulties. The other, and more feasible method is to analyze the measured values, and notice any possible relations between them which would be suggested by a combination of the characteristics of the ingredients.

To form an idea of the absorption which would occur if the mixture followed Maxwell's theory we may treat it as a two-layer dielectric, each layer having the capacity and conductance determined by the tests for that material. Maxwell has shown that the order in which the layers are arranged is immaterial. A mixture could be considered as an infinite number of layers of each constituent, and if all of those of the same kind were arranged together it would reduce to two layers. Evidently in arranging the materials in layers we have taken the case most favorable to absorption, since if the substances were arranged in parallel columns between the condenser plates there would be merely two dielectrics in parallel, and consequently no absorption. Of necessity we must leave to one side the question of the solution of one substance in the other,[31] and our discussion is thus only broadly qualitative as a test of Maxwell's theory for good insulating materials in their best available condition.

Considering Maxwell's expression for the current in the two-layer case we find that when the dielectric constants are nearly the same, and one constituent has much higher conduction than the other, the low conduction determines the magnitude of the conduction current, and the high conduction determines the magnitude of the absorption current.

Among others, mixture No. 28 was composed on equal parts by volume of new paraffin and the deteriorated paraffin, No. 20. This mixture was studied at 500 and at 1500 volts, and its properties compared with those of the constituents and with those computed from Maxwell's two-layer expression. The general results are as follows:

While the conduction of the mixture is lower than that of the poor paraffin, No. 20, its absorption currents are greater in all cases. The computed absorption currents at 0.4 sec. are fairly near the measured values, while at 1.6 sec. they are decidedly lower. If we assume that the computed value should be added to that of the constituents, we may do this approximately by adding the computed value to half that of No. 20, since the absorption of the fresh paraffin is low. Making this comparison, we see that it would much exceed the measured value at 0.4 sec., and would be somewhat low at 1.6 sec. While there is a hint of Maxwell's effect, yet it is not conspicuous, and the results are better accounted for by a change in the distribution or freedom of the ions present in the deteriorated dielectric.

Following this a mixture, No. 29, of equal parts by volume of paraffin and stearic acid was studied. The conduction of the mixture was only about one-fifth that of the stearic acid. The absorption current of the mixture was on the average 0.45 of that of the stearic acid, this being fairly constant. That is, as regards absorption, the effect is nearly the same as if, instead of being mixed, one-half of the condenser had been filled with paraffin, the other half with stearic acid. There is here no indication of any increase in absorption through mixture. The conduction of the mixture is much greater than if the materials were in layers, but less than if they were side by side. As regards the computed values, since the mixture had less absorption than if the stearic acid it contained had been put by itself in the condenser, that is less than half the value for the acid alone, it is evident that no increase in absorption was produced by mixing, and the Maxwell effect did not exist.

The next specimen was No. 33, a mixture of equal parts by volume of paraffin and refined lubricating oil. The oil had a low conduction, only a little higher than the paraffin and only a trace of reversible absorption. The mixture closely resembled the constituents, having a conductivity of the same magnitude, and only a trace of absorption. Here again, there is no evidence of any increase in absorption due to the mixture.

We finally tried what may be considered the addition of a conducting impurity to paraffin by mixing with it varying proportions of the highly conducting "Black Oil." The properties of the paraffin, of the oil, and of 8.33, 0.4, and 1 per cent mixtures respectively of the oil with the paraffin were studied separately. The values of conduction are best compared by expressing them as percentages of the value for "Black Oil," see Table VII.

TABLE VII
PER CENT CONDUCTION OF MIXTURES OF PARAFFIN AND "BLACK OIL"

	No. 35	No. 36	No. 37	No. 38
Per cent "Black Oil"	100	8.33	4.0	1.0
25 deg. cent. 500 volts	100	10.2	3.46	0.21
25 deg. cent. 1500 volts	100	0.8	3.10	0.24
45 deg. cent. 500 volts	100	68.1	23.8	3.30
45 deg. cent. 1500 volts	100	65.0	23.6	2.79

We notice that at 25 deg. cent., the conductivity is nearly proportional to the amount of oil, that is, the oil behaves as if the paraffin were not present. At 45 deg. cent., however, the conduction at each percentage is from 3 to 8 times greater than for the oil by such a proportional relation, indicating that at the higher temperature there is more reaction between the oil and the paraffin. The computed current for the two-layer case is omitted since, as already explained, it would be nearly the same as for paraffin alone, that is far less than any of the measured values of the mixtures.

Examining the absorption current we find that it is greater for all the mixtures than for the oil alone. There is evidently a maximum in the absorption in the vicinity of Nos. 36 and 37. Thus there is an approximate accord with Maxwell's theory. But the oil is a liquid and its absorption can scarcely be expected to follow the same relations as for a solid. As an instance, we notice that with the oil the absorption current falls off in going from 25 deg. cent. to 45 deg. cent., while with the mixture it increases. Examining the computed values for the two-layer case we find two noticeable peculiarities. The first is the small magnitude of the absorption currents, particularly at 45 deg. cent. The other is the rapid rate at which even these small values fall off as compared to the actual decrease. These figures definitely show the inability of Maxwell's theory to explain the absorption.

From the experiments described it may be concluded that a mixture of two solid dielectrics acts much the same as if the two were separate. In some cases there may be a slight increase in absorption, due probably to a change in the number and distribution of free ions. There is little or no evidence to show that Maxwell's theory applies in any way. The addition of a conducting liquid to a solid dielectric with which it is miscible causes a large increase in the absorption. The absorption current is much greater and falls off more slowly than can possibly be explained by Maxwell's theory.

XI. Irreversible Absorption

The presence of an irreversible absorption in certain cases has long been recognized. It occurs commonly in composite dielectrics made up of one or more originally liquid ingredients, as for example, impregnated paper. It usually appears when the final conduction has high value.

We have examined all our records for the presence of irreversible absorption, by studying the difference between the discharge current and the charging current reduced by the amount of the final steady conduction current. For the most part we have found evidence of only traces of irreversible absorption. There is, however, one notable group in which it occurs, *viz.*, mixtures of paraffin and relatively small amounts of "Black Oil" at 45 deg. cent. (Nos. 36, 37, and 38). In these cases the irreversible component of the charging current was from 2 to 7 times the corresponding value of the discharge current, increasing with the amount of admixed oil. This irreversible current undoubtedly contributes to the loss in the alternating case. That it can assume such large values is striking, and indicates the importance of its further study.

XII. Results and Discussion

The main conclusions of the work are as follows:

1. The shape of the current-time curve of the absorption current on discharge is essentially the same with every substance studied. Its striking peculiarity is a very rapid decrease at first, followed by a very gradual dying out. No simple mathematical formula has been found to express this relation satisfactorily. There is a fair approximation to the relation $i = A\,t^{-n}$. The variation from the negative exponential relation, as proposed by Maxwell, is very wide.

2. All the solid dielectrics showed appreciable absorption. Heating at reduced air pressure in no case reduced the absorption. The absorption was approximately proportional to the final conduction in all cases. There was no indication that the simple dielectric having conductivity but no absorption, as postulated by Maxwell, can exist.

3. Noticeable departures were found from Curie's law of proportionality between the ordinates of the current time curve on discharge and the charging voltage.

4. The absorption current on discharge falls off more rapidly the higher the charging voltage.

5. The conduction in general increases with the voltage. In a few cases there is evidence of the approach to a saturation current.

6. In general both conduction and absorption currents in solids increase with rise in temperature. As the melting point is approached the absorption current falls off, and usually almost disappears on complete fusion, as noted by other observers.

7. In some instances a decrease was found in both conduction and absorption as the temperature rose. While an exception to 6 they agree with it in showing a proportionate change in both conduction and absorption currents.

8. Reversible absorption in liquids, while not as conspicuous as in solids, is very noticeable in some cases. This shows that whatever the nature of absorption it is not fundamentally related to the solid state.

9. Any treatment of a dielectric, such as prolonged heating, which increases or diminishes the conduction, increases or diminishes the absorption current in much the same proportion.

10. There is evidence that a mixture of two solid dielectrics acts much the same as though the two were separate.

11. The addition to a solid dielectric of a conducting liquid which can mix with it causes a large increase in the absorption. The absorption current is much greater, and falls off far more slowly than can be explained by Maxwell's theory.

12. The solids tested gave no definite evidence of irreversible absorption current. However, mixtures of paraffin with a small percentage of conducting oil, showed at 45 deg. cent. a noticeable irreversible absorption current.

The most striking feature throughout these studies is the intimate connection between the absorption and the conduction. Increase or decrease in either, under the influence of other variables, is accompanied always by a similar change in the other. Other observers have noted a similar correlation. We have shown that it extends over such a wide range of materials and conditions as to make it certain that the ultimate seat of the phenomenon of absorption is to be found in the laws of ionic conduction.

Throughout the course of this work we have used Maxwell's theory as a sort of criterion for our results. We have done this partly because of the distinction of its originator and of the place which it apparently still holds in the minds of many physicists, but principally because it is the only available rational theory offering reasonable opportunity for experimental check. We have emphasized the failure of the theory to account for the observations in either magnitude or general qualitative relation. We therefore dismiss the theory as, if not positively disproved, at least useless for application to commonly available dielectrics.

The anomalous conduction found in dielectrics both liquid and solid offers a far more promising prospect for the explanation of dielectric absorption. We use the word "anomalous" since conduction in dielectrics rarely follows Ohm's law. If electrons are involved in this conduction, their contribution is in general overshadowed by that of much larger, heavier, and slower ions. This is true for both liquids and solids. All the work of recent years points to this type of conductivity. It may be seen at once how readily the conception lends itself to an explanation of absorption. The ions being large and slow, move towards the opposite electrodes; in doing so they upset the potential gradient; or, if by any cause their motion is interrupted, they may accumulate and by the resulting space charge cause the counter or polarization e. m. fs. often reported. There are, however, obvious difficulties in this simple picture, some of which we discuss below.

Curie[5] describes experiments on porous porcelain showing pronounced absorption, which decreased as the porcelain was dried. From this he suggests polarization as the cause of absorption, but makes no attempt to develop a complete theory.

Hartshorn[8] suggests that absorption is due to a non-conducting air film between the dielectric and the terminals, so that the actual contact exists at only a few points. The motion of the ions through the dielectric is stopped by the air layer, resulting in a counter electromotive force. The conduction current is determined by the occasional points of actual contact. As one of the proofs of this theory, he points to the absence of absorption in liquids, but from what we have already shown this is not a valid objection. It is also evident that this is nothing but an application of Maxwell's theory, the dielectric being one layer, the air film the other.

Richardson[22,23,24] gives the results of extensive tests on conduction and absorption. The materials studied were: quartz, Iceland spar, rock salt, glass, ebonite, and paraffin. He takes the view that absorption is a manifestation of polarization, and states as his conclusion[24] that the charging current in a dielectric with constant impressed voltage is expressed by the sum of two terms, one the rate of change of the charge in the dielectric, and the second the difference between the impressed e. m. f. and a polarization e. m. f. divided by a constant resistance. The second term gives the current at infinite time. The polarization is assumed as the cause of absorption, but no explanation of the polarization is offered.

Somerville and Buckley[30] show that in porcelain at high temperatures, as much as 1100 deg. cent., the current I due to an electromotive force E can be represented as $I = (E - e)/R$, where e is a counter electromotive force dependent on applied voltage, time of charge, and temperature, and R is the resistance. This is introduced as showing the possibility of a counter electromotive force of polarization where water is very definitely absent.

Joffe[11] has extensively investigated the absorption in crystals, and considers it due to the motion of both ions and electrons, resulting in a polarization in the crystal. He finds two distinct types of polarization; one represented by Iceland spar, the other by quartz. With Iceland spar, if the crystal is charged, and a thin layer, as little as 0.01 mm., removed from the side next to the cathode, the residual charge disappears. If the layer is removed from the side adjoining the anode, no change is noticed. With quartz the removal of layers from either side has only a small effect. The potential changes gradually and symmetrically through the crystal.

The fundamental difficulty in a proposal of polarization, due to space charge, as an explanation of dielectric absorption is the inconsistency of the idea of a free flow of charge through the mass of the dielectric, this charge being more or less abruptly halted at the surface of the

dielectric, where the latter is in contact with a metallic electrode. There is ample evidence of the motion of the ions, even in a solid dielectric. It is important to note, however, that Joffe reports only one case of a sharply defined surface layer of charge, with consequent high potential gradient to the electrode, and that even in so good an insulator as quartz, it was apparent that the internal accumulation of charge was spread over a much greater volume and the variation of potential gradient, although not uniform, was gradual rather than abrupt.

From the point of view of free motion of ions, and internal accumulation of charge with consequent reaction on the electric gradient, the work of Smekal[44] and of Schmidt,[45] as discussed by Rogowski,[46] is much more suggestive and acceptable. Smekal has tried to explain conduction in solids as due to the motion of free ions in sub-microscopic cleavages or surfaces of crystal separation. The ions are liberated from the crystal more readily in these spaces and follow easily the direction of the electrical intensity, probably in accordance with the laws of motion of gaseous ions. Schmidt and Rogowski have extended this suggestion to studies of the influence of voltage and temperature and show that there is marked agreement in the behavior of the conduction, so predicted, with the observations of experiment. Rogowski has also used these sub-microscopic spaces in the development of an interesting theory of dielectric breakdown.

The presence of such open spaces or cleavages in crystals even when prepared under most careful conditions, has been shown beyond question for many crystals. They offer a more satisfactory explanation of dielectric absorption than the proposal of an elastic and retarded polarization, such as suggested by Joffe, for we have the picture of a motion of free ions over limited distances, presumably in a gaseous atmosphere. The analysis of Rogowski explains the proportionality of current and potential gradient for low values of the latter, and also an increasing conduction with increasing gradient as proposed by Poole.[21] Looking further we may see that it leads without too violent assumptions to an explanation of dielectric absorption. It is easy to picture the accumulation of charges at the ends of the sub-microscopic spaces. Moreover, beyond this, simple mathematical analysis shows that there results a steadily decreasing current-time relation, in fact, the negative exponential relation, characteristic of absorption as suggested by many experiments. This interesting suggestion obviously depends on the presence of the sub-microscopic cleavages as described. The presence of these has been recognized only in certain crystals. A somewhat similar suggestion for noncrystalline solids, involving minute canals with electrolytic conduction, has recently been made by Böning.[47] We are, however, far from the proposal of a general theory covering all solid dielectrics.

We have already called attention to the variation of density of distribution of the ions in a conducting insulating liquid, when subject to electric stress. In many cases this results in a marked variation in potential gradient near the electrodes, as long ago shown by Mie,[41] Warburg,[35] and others. This leads to the question of the influence of this accumulated space charge on the external field as a possible explanation of the typical current-time curves, often noticed in viscous liquids and similar to those observed in solids. We have reported above one especially conspicuous case. The difficulty here, as in the case of polarization in solids, is to explain the failure of this accumulated charge to pass to the electrodes. We venture to suggest that this difficulty in the case of the liquids may be explained by assuming that an ion, owing to its intense electric field, surrounds itself with an atmosphere of neutral molecules which move with it, and which when an obstacle, such as an electrode, is encountered, prevents the charge ion from passing to it. Extending this conception we can easily conceive the accumulation of layers of space charge in the proximity of the electrodes. Simple mathematical analysis of the influence of this space charge on the applied electric field of a parallel plate condenser, leads to the negative exponential current-time relation approximating the charging current time curves, as often observed in experiment.

The increasing evidence that absorption and conduction always go hand in hand, and that the latter in solids frequently partakes of the character of that of gaseous and liquid ions, emphasizes clearly that further progress towards a better understanding of the phenomena in dielectrics can best be obtained through a study of this type of ion. Moreover, as it is the slow moving ions which are of the greatest interest and importance, we propose, as a continuation of this work, further studies of ionic mobilities and conductivities as found in liquid insulators of increasing viscosity, merging into the type of solid compounds such as studied in this paper and those commonly used for electrical insulation.

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Discussion

E. R. Thomas: Clerk Maxwell demonstrated that if two ideal condensers having leakage resistances such that the product of their respective capacitances and resistances are not equal, are connected in series, they produce the external phenomenon of causing voltage to build up at their terminals after an initial discharge, which is now called dielectric absorption, dielectric hysteresis, residual charge, etc.

This was demonstrated recently by oscillograph tests on a 1 - μ f laboratory standard condenser. The two 0.5 - μ f sections were connected in series and leakage resistances of approximately 1 and 2 megohms were connected across the respective sections. Such an arrangement gives an unequal product of r and c for the respective condenser sections.

This equivalent circuit after being charged at 50 volts and momentarily short-circuited showed a building up of a residual voltage to a maximum value of about 3.2 volts in 0.6 sec.

If this equivalent circuit produces the same phenomenon as is met with in commercial insulation then it would seem advantageous to use it as a unit of measure on commercial insulation to evaluate the degree to which it deviates from ideal insulation.

Suppose we pass a plane parallel and midway through a sheet of insulation between two plate electrodes, the degree of uniformity in conductivity of the two halves of the insulation will have a ratio to each other which is indicated by the ratio of the maximum residual voltage occurring after discharge to the original voltage charge. Thus on the assumption of the Maxwell equivalent circuit the degree of uniformity of the insulation resistance is a function of the percentage peak residual voltage compared to the original charge.

The time at which the peak residual voltage occurs after the initial discharge is a function of the insulation resistance and capacitance as well as the ratio of resistivity.

The discharge characteristic of the residual voltage is by this reasoning the difference of two negative exponentials with respect to time that is $e_t = A\,(\epsilon^{-xt} - \epsilon^{-yt})$ where A is a function of the original voltage charge and the degree of uniformity of insulation resistance and x and y are functions of the insulation resistance, the degree of uniformity, and the capacitance.

Mr. Marbury pointed out in his paper on dielectric absorption in 1925 similar data which he obtained in a point-by-point method.

I regret that Messrs. Whitehead and Marvin conclude from the test data presented that the Maxwell equivalent circuit fails in application to the commonly available dielectrics. When the magnitudes of the physical constants of the dielectrics investigated and described in their paper are considered, the period of time between discharge and the starting of the graphic record, even though it is only 0.005 sec., may be a relatively long time period as far as recording the true picture of the phenomenon. A method of investigation which starts from zero time would tend to be more conclusive.

G. M. J. Mackay: Dr. Whitehead has developed a very effective method for studying d-c. conductivity and absorption in a dielectric but he has investigated materials which are very complex from both the chemical and physical standpoints.

It should help very greatly in the interpretation of the relationship between d-c. and a-c. effects if these characteristics could be studied with relatively simple geometric assemblies or with an exaggeration of the variations which occur in commercial materials. For instance, in commercial insulation we have materials which consist of conducting portions in series with very good dielectric material; we have emulsions of water in oils, and conducting and non-conducting particles suspended in liquids. By making artificial systems of this nature and determining the characteristics, both according to Dr. Whitehead's method and by a-c. measurements, a good deal of light might be thrown upon the mechanism of dielectric losses. As an example, suppose two quartz plates with liquid between. On direct current there would be practically no conductivity because of the high insulating value of quartz. I am not sure what the absorption effects would show, but on alternating current the losses would be relatively high and dependent on the resistance of the liquid. Again, with an emulsion of water and an oil, the d-c. measurements would show a different relationship to

the alternating characteristics. If Dr. Whitehead would show us typical characteristics representative of different arrangements of this nature, it should be very helpful in the interpretation of both d-c. and a-c. phenomena.

Herman Halperin: During the past year and a half in Chicago, we have had in experimental operation six 1000-ft. lengths of cable operating at 132 kv. Five of the cables have "solid" insulation while the other one is of the oil-filled type. Periodically with the high voltage removed, measurements have been made of direct current versus time, using a 200-volt battery. Generally, the curve of charging current has been higher than the curve of the discharging current. The curves as plotted on logarithmic paper are not straight lines, but curves upward and downward and change from time to time indicating a disagreement with the Maxwell theory. To date we have not been able to learn much from the curves.

The impression that I get from the first paragraph of the article, is that, under the combination of influence under electric stress and temperature, progressive physical and chemical changes occur in the structure of the material with the result that radical changes in the insulating properties are bound to occur. This sweeping statement does not seem to be justified. Laboratory and operating data, which are not nearly as complete as they should be, indicate for most cable insulations, however, that there is a critical stress for a given operating condition below which the given insulation should operate without any material change in characteristics.

R. W. Atkinson: There has, in the past, been a great deal of theoretical discussion as to the validity of Maxwell's theory of absorption but very little quantitative data bearing on the matter. We have at last been presented with a practical amount of test data on the subject.

I have been very much interested to look over Dr. Whitehead's work showing the actual physical characteristics of dielectric absorption and showing that Maxwell's theory does not apply to the physical conditions that we get there. I think there have been many partial data showing things of that sort, but it is worth while to see the systematic work and know that we need to use another physical theory to account for the particular phenomenon.

Up until the time when dielectric losses became of large importance in cable insulation the particular factor of dielectric loss that was important undoubtedly was conduction; the dielectric losses were important at the higher temperatures of 60 deg. cent and above, that is, where the insulation consisted of paper saturated with a fluid oil having a considerable conductivity. Also it was possible to determine a pretty direct relation between the conductivity of the oil itself and the dielectric power factor of the cable saturated with that oil. In view of these facts, since the cable engineer was at that time primarily interested in power factor within that range, he was principally concerned with the conductivity of the oil which could be measured by simple means, as with direct current. Having developed facilities devoted to that particular phase of the subject, it has been possible to reduce dielectric losses within that range until it is now possible to bring those losses commercially to values that were considered impossible even in the laboratory a few years ago. With the reduction of losses of that type and with the possibility of largely increased stresses brought about by reduction in dielectric losses and by other means, we now find that the loss which is left over, which was formerly practically negligible, may become in the future a matter of larger relative importance. The matter of absorption, which we have been neglecting for a few years, may become vastly more important commercially than the conductivity. Thus only from the standpoint of interest in physics and of interest in helping us to understand the laws of the dielectric, this question of absorption in dielectrics is something that is very likely to become of increasing commercial importance.

A. Nyman: I should first like to ask Dr. Whitehead whether the absorption really determines or affects in any definite way the life of the dielectric.

H. H. Race: I agree with Dr. Whitehead that in 60-cycle phenomena the short-time point of view is certainly of interest. At the same time, in getting at the ultimate nature of absorption, if we can, the long-time absorption curves will have to be considered. A paper on some of this work was published in the A. I. E. E. Quarterly Trans., October 1928, p. 1044.

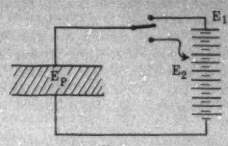

Fig. 1

I should like to present some data obtained this fall on hard rubber, which is as near a heterogeneous solid as we could possibly get. The experiment deals with the so called polarization potential which is built up within the body of the dielectric during a charge at constant potential. Suppose a potential E_1 is applied at the instant t_0 of dielectric and maintained constant until the charging current approaches a constant value.

In our previous work we found that this takes several days. There is evidence that during this time an electric potential difference called the polarization potential (E_p), has been built up within the body of the dielectric. The current which has been flowing during this time will be considered positive and is shown by the curve between the ordinates t_0 and t_1.

Now suppose the switch is thrown at the instant t_1 so that a constant potential E_2 less than E_1 is applied to the sample. If E_2 is less than E_p, a current proportional to $(E_p - E_2)$ will flow in a direction opposite to that in which it had previously been flowing. The current will continue to flow in this direction until E_p has dropped to a value equal to E_2 at which time the

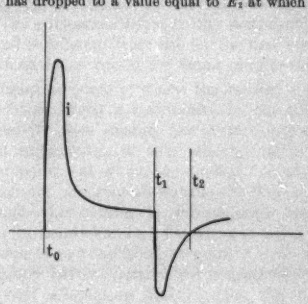

Fig. 2

current will be zero as shown by the point t_3. After this time the current will be positive and will approach the conduction current due to the positive potential E_2.

If several runs are made with exactly similar conditions of charge but with increasing values of E_2, the negative portions of the current curve will get smaller and the several points t_3 will come nearer to t_1 since the value of $(E_p - E_2)$ is getting smaller. If, as E_2 is increased, the first value could be found at which no reversal of current occurred, this would be the value of E_p for the particular charging conditions being studied. The data for five runs all having a charging potential of 750 volts are shown in the following table.

E_1 Volts	E_2 Volts	$t_2 - t_1$ Minutes
750	703	59
750	712	36
750	725	16
750	736	11
750	748.5	1

The values of $t_3 - t$, depend upon the final conduction currents of the specimen which are not definite because the apparatus errors are of the same order of magnitude.

The last run shows that E_p was greater than 99.8 per cent of E_1 and the first run shows that when E_2 was 94 per cent of E_1, E_p remained greater than E_2 for an hour. Therefore the important points are that after a long charge at a potential E_1, E_p is very nearly equal to E_1 and the relaxation of the material is relatively slow.

C. F. Hill: (communicated after adjournment) In this paper the authors have pointed out several conclusions of importance. Of these, the conclusions that Maxwell's theory of dielectrics does not fit experimental evidence and that a reversible absorption current has been measured for liquids, are the most important. It has been evident for some time that a model built after Maxwell's theory is difficult to make conform to our ideas of ionization, electric conduction, atomic and molecular structure and the like which we associate with electrical phenomena in matter. It would be very important and certainly worth some effort to try to tie up the action of polar molecules of these liquids, for which Professor Whitehead and Mr. Marvin have found reversible absorption, with the dielectric absorption. The application of Debye's theory to dielectrics to explain the anomalous properties is very promising.

One or two questions concerning the conclusions of this paper have arisen, the first of which concerns the reversible absorption of liquids. Do the results distinguish in any way if the reversible absorption is a space charge effect or if it might be a polarization on the electrodes? Also, it is not evident what the irreversible absorption current might be, and how detected or measured. Is there a plausible explanation of this lost component?

The extension of the current-time curve towards the zero time axis is much needed data and the authors have developed a method which has carried the curves to a small fraction of a second which is a worth-while step as it aids in a study of what is happening over the time when losses take place under a-c. voltage. Apparently to get much nearer the zero time axis we must resort to devices for measurement with time lags comparable with the periods of the electric-charge carriers themselves, which suggests that cathode-ray devices must be used before we get farther with this problem.

R. H. Marvin: Mr. Thomas' discussion of the possibility of applying Maxwell's theory and his illustration of tests on circuits made up with actual condensers and leakages develop very interesting relations and certainly give a qualitative idea of the phenomena, but, as we have pointed out in our paper, there are many ways in which the actual dielectric differs from any such simple representation. Perhaps the most striking is the great difference between the discharge curve as shown from actual samples and the exponential curve which would follow from any simple representation in accordance with Maxwell's theory.

Replying to Dr. Mackay on the connection between a-c. absorption and d-c. losses, that has not been attempted in this paper but we have other investigations under way at the University which are attempting to correlate these phenomena.

In connection with Mr. Halperin's discussion of the curves, we have pointed out ourselves that the absorption curves do not agree exactly with the relation $i = I\,t^{-n}$ but show a slight curve, and although in general this curve is in one direction, we also found cases where it curved in the other direction, so that it is quite certain that this curve is only an approximation, but it does seem to be a very fair approximation.

Replying to Mr. Nyman's inquiry as to the relation between absorption losses, all we referred to there, was that the absorption increased as the heating did, and the heating in turn with the deterioration.

Corona Ellipses

BY VLADIMIR KARAPETOFF[1]

Fellow, A. I. E. E.

Synopsis.—*The purpose of this investigation is to give a mathematical theory of the cyclograms of corona obtained by a cathode-ray oscillograph. In the case investigated a long wire of small diameter is connected to one terminal of an a-c. source; the other terminal of the source is connected to a concentric cylinder of considerable diameter or to a metal plate at some distance from the wire. A cathode-ray oscillograph with two pairs of deflecting plates, at right angles to each other, is so connected that one pair of plates causes deflections of the cathode beam proportional to the values of instantaneous voltage of the source, and the other pair of plates causes deflections proportional to the instantaneous values of the charging and loss current flowing into the wire.*

As long as the sinusoidal amplitude of the applied voltage is below the visual corona point, the charging current is also sinusoidal, in time quadrature with the voltage. The oscillograph record is therefore an ellipse, with the amplitudes of the voltage and the current as the principal semi-axes. When, however, the minimum ionization voltage is exceeded during a part of each alternation, the cyclogram ceases to be an ellipse, but consists of four portions per cycle, two of which correspond to the intervals of time during which the corona is extinct, and the other two when corona is present, with quite short transients in between.

F. W. Peek (A. I. E. E. TRANS., Vol. XLVI, 1927, p. 1009) published a number of such oscillograph records, with voltage amplitudes both below and above the visual critical point. In order to explain the mechanism of corona formation and the influence of the space charge upon the instantaneous critical voltage, he also produced "artificial corona," by using two condensers in series, one of which was shunted by a sphere gap.

The purpose of the present investigation is to give a mathematical theory of the observed cyclograms, on the basis of two condensers in series, with the space charge as a fictitious dividing line. The condenser nearest to the wire is assumed to be shunted by a conductance, and to have a resistance in series, to account for the actual motion of ions and the power loss. Approximate equations are derived for the current and the voltage as functions of time.

For the artificial corona it is shown that the composite curve consists of arcs of two ellipses, with their principal axes along those of the cyclograms. Assuming the visual critical voltage to be known, an expression is derived for the instant of the cycle at which the corona is re-established.

For the actual corona, it is shown that the cyclogram also consists of portions of two ellipses, only their principal axes are at some angles with the principal axes of the cyclogram. The theory is applied to one of Mr. Peek's records, and it is shown that both the shape of the experimental curves and the instants at which the corona is re-established check fairly well with the theory.

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I. Introduction

NUMEROUS laboratory tests and theoretical researches on corona formation about long cylindrical conductors of comparatively small cross-section have been made by various investigators; many noteworthy results have been recorded in Institute papers for over twenty years. The modern cathode-ray oscillograph, which is a further development of the original Braun tube, has made it possible to study the a-c. corona in a much greater detail quantitatively, from instant to instant, and thus has brought us nearer a rational explanation of the ionic mechanism involved.

A number of a-c. corona records (cyclograms), obtained by means of a cathode-ray oscillograph, was published by Lloyd and Starr[2] and by Peek.[3] Peek's Fig. 9A is reproduced in Fig. 1 below, using a somewhat different ratio of the scales for abscissas and ordinates. The beam was slightly diffuse (not focused), and rather than to draw an arbitrary average line, the boundaries of the actual trace on the photograph are shown by the cross-hatched strip. The theoretical ellipses drawn in the same sketch do not concern us yet.

The horizontal distances from the vertical axis are proportional to instantaneous values of the sinusoidal applied voltage between the wire and a metal plate. The vertical ordinates represent the corresponding instantaneous values of the current flowing into the wire. The cyclogram refers to the established conditions, and is different from one for the first few cycles. The cathode beam traced the figure counter-clockwise.

Beginning at point p, where the applied voltage is zero and the corona is maintained by the previously accumulated space charge (see Peek's paper for a physical explanation of this seeming paradox), we arrive at point b at which the applied voltage reaches its maximum. Shortly afterwards the flow of current stops and the corona is extinguished somewhere between the points b and s. On the decreasing voltage the corona is re-established at point h and continues until slightly beyond point d; $d\,k$ is again that portion of the next alternation during which the corona is extinct. It is re-established at point k. The two parts of the actual cyclogram, $p\,n\,b$ and $r\,q\,d$, are not quite alike, because of the difference in the mass and mobility of positive and negative ions.

The purpose of the paper is to outline a mathematical theory which explains the general shape of the observed corona cyclograms. The narrower of the two theoretical ellipses shown in Fig. 1 represents the current-voltage relations during those portions of the cycle when the corona is extinct. The broader ellipse gives a similar

1. Professor of Electrical Engineering, Cornell University, Ithaca, N. Y.

2. W. L. Lloyd, Jr., *Methods Used in Investigating Corona Loss by Means of the Cathode-Ray Oscillograph*, A. I. E. E. TRANS., 1927, Vol. 46, p. 997.

3. F. W. Peek, *The Law of Corona and the Dielectric Strength*; *ibid.*, p. 1009.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

relationship with the corona present. The theoretical transition points are indicated by the small circles at h and k. It will be seen that the agreement with the observed cyclogram is quite satisfactory considering the complicated nature of the phenomenon.

II. Two Condensers in Series, One of Which is Shunted by a Spark Gap

A. *The Circuit*

We shall first consider the circuit (Fig. 2) which, according to Peek, roughly imitates the conditions in

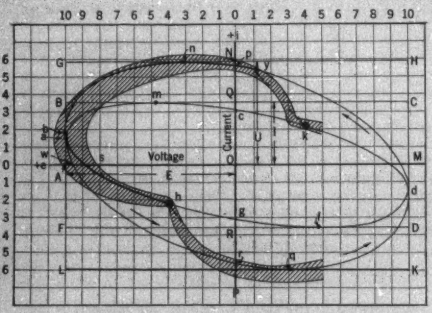


Fig. 1—An Actual Oscillogram of A-C. Corona (Peek) with the Corresponding Ellipses

an actual a-c. corona. This circuit consists of two condensers, C_1 and C_2, in series, the first one being shunted by a "glow gap" G_1. In this ideal glow gap,

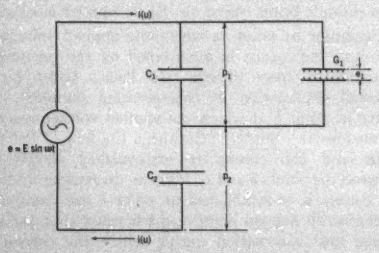


Fig. 2—Two Condensers in Series One of Which is Shunted by a Glow-Gap

as distinct from an actual "spark gap," a copious corona promptly takes place between the "hair brush" electrodes when the voltage between them reaches a value e_1. When the applied voltage is raised further, the discharge remains corona-like at the same voltage e_1 and does not change to streamers or spark-over. The excess voltage is supposed to be consumed in the condenser C_2. The corona stops instantly when the voltage p_1 drops below e_1. We shall not discuss here the question as to whether such a gap is realizable in practise or not, because the circuit shown in Fig. 2 is intended merely as an analog, to simplify the mathematical treatment of an actual a-c. corona around a conductor.

We shall disregard the non-periodic transient conditions of the first few cycles, and assume the phenomenon to be periodic, as shown in Fig. 3. In this cyclogram, the cathode beam undergoes two harmonic motions simultaneously. One is along the axis of abscissas, and the deflections are proportional to the total applied voltage $e = E \sin \omega t$. (See notation at the end of the paper.) The other motion is along the axis of ordinates, the deflections being proportional to the instantaneous values, i, of the current through the condenser C_2. The cyclogram is described by the cathode beam counter-clockwise. The points of discontinuity correspond to the instants at which the current suddenly increases when the corona is re-established. The corona is extinguished at the extreme right and left points where the voltage reaches its maxima, because at these instants the current becomes zero and is reversed.

Because of these discontinuities, the cathode beam

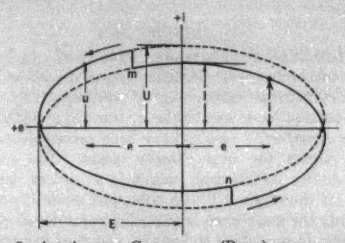


Fig. 3—An Actual Cyclogram (Peek) taken on the Arrangement shown in Fig. 2, with the Corresponding Ellipses

alternately follows two distinct curves, and it is necessary to consider the equation of each separately. It is shown below that both curves are arcs of ellipses.

B. *Corona Discharge is Out*

Let an instantaneous voltage across C_1 (Fig. 2) be p_1 and that across C_2 be p_2. Since by assumption we are considering that portion of an alternation when no current is flowing through the gap G_1, the charging currents of the two condensers are equal, and we have:

$$i = C_1 \, d\, p_1/d\, t = C_2 \, d\, p_2/d\, t \qquad (1)$$

The sum of p_1 and p_2 is equal to the instantaneous value of the applied voltage, so that

$$p_1 + p_2 = E \sin \omega t \qquad (2)$$

These are simultaneous differential equations for p_1 and p_2. Integrating Equation (1), gives

$$C_1 \, p_1 = C_2 \, p_2 + \Gamma \qquad (3)$$

where Γ is a constant of integration. Experiment

shows that the corona goes out, approximately, at the instant when the current is reversed, that is, at $\omega t = \pi/2$; at this instant $p_1 = e_1$, and $p_2 = E - e_1$. Substituting these values in Equation (3), gives

$$C_1 e_1 = C_2 (E - e_1) + \Gamma \tag{4}$$

Eliminating Γ from Equations (3) and (4) and solving the resulting equation with Equation (2) for p_1 and p_2, we obtain:

$$p_1 = (e_1 - P_1) + P_1 \sin \omega t \tag{5}$$

$$p_2 = -(e_1 - P_1) + P_2 \sin \omega t \tag{6}$$

where

$$P_1 = E\, C_2/(C_1 + C_2) \tag{7}$$

$$P_2 = E\, C_1/(C_1 + C_2)\,. \tag{8}$$

P_1 and P_2 are the voltages across the condensers C_1 and C_2 respectively when the total applied voltage reaches its maximum value, E. They are inversely as the corresponding capacitances and their sum is equal to E.

From Equations (1) and (6) we have:

$$i = \omega\, C_2\, P_2 \cos \omega t = I \cos \omega t \tag{9}$$

where the amplitude of the current is

$$I = \omega\, C_2\, P_2 \tag{10}$$

Let the instantaneous value of the applied voltage be e, so that

$$e = E \sin \omega t \tag{11}$$

From Equations (9) and (11) we obtain

$$(i/I)^2 + (e/E)^2 = 1 \tag{12}$$

This is the equation of an ellipse (Fig. 3) whose horizontal and vertical semi-axes are E and I respectively. Only the full-drawn portions of this curve appear on an actual experimental cyclogram.

C. Corona Discharge is Present

By assumption, while the corona is going, the glow gap, G_1, maintains a constant voltage, e_1, across the condenser C_1. Hence, in place of Equation (1), we now write

$$u = C_2\, d\, p_2/d\, t\,, \tag{13}$$

where u is the charging current of condenser C_2 while the glow gap is operating. By using two distinct letters, i and u, for the current, confusion is avoided as to which part of the cycle is meant. Equation (2) becomes

$$e_1 + p_2 = E \sin \omega t \tag{14}$$

Substituting the value of p_2 from Equation (14) in Equation (13), we get

$$u = U \cos \omega t \tag{15}$$

where the amplitude of the current is

$$U = \omega\, C_2\, E \tag{16}$$

Comparing this amplitude with that given by Equation (10), we see that U is greater than I in the same ratio in which E is greater than P_2. In other words, with reference to Equation (8),

$$U/I = E/P_2 = (C_1 + C_2)/C_1 \tag{17}$$

From Equations (11) and (15), we get

$$(u/U)^2 + (e/E)^2 = 1 \tag{18}$$

This result means that when a glow discharge is taking place, the relationship between the current and the voltage is again an ellipse. Its horizontal semi-axis, E, is the same as before, but the vertical semi-axis is equal to U, and is larger than I, as indicated by Equation (17). The arcs of this ellipse shown in Fig. 3 by solid lines are portions of this ellipse that would appear on an actual cyclogram.

D. Points of Discontinuity

To determine the instants of time at which the corona starts (m and n in Fig. 3), we put in Equation (5)

$$p_1 = -e_1 \tag{19}$$

The minus sign is necessary for the following reason: The constant of integration, Γ, in Equation (3) has been determined from the condition that $p_1 = +e_1$, at the points where the ellipse for i crosses the axis of abscissas. At these points the glow discharge goes out, because the total applied voltage begins to diminish. So when the discharge is re-established, it is due to a voltage, e_1, in the opposite direction. Putting $p_1 = -e_1$ at the instant $t = t_1$, and solving Equation (5) for $\sin \omega t_1$, we get:

$$\sin \omega t_1 = 1 - 2\,(e_1/P_1) \tag{20}$$

In this expression, the angle ωt_1 at which the corona is reestablished, can have only values greater than $\pi/2$, because the corona goes out at $\omega t = \pi/2$. A negative value of $\sin \omega t_1$ means that ωt_1 lies between 180 deg. and 270 deg. (point n in Fig. 3 and also Peek's Fig. 18B). A positive value of $\sin \omega t_1$ means that ωt_1 lies between 90 deg. and 180 deg. (Peek's Fig. 18C).

Equation (20) determines the points m and n for the simplified corona diagram shown in Fig. 2, when the values of e_1 and P_1 are known. However, in application to an actual corona around a conductor, the voltage e_1 is not known. Instead, it is the visual critical voltage, e_v, that can be readily measured. By definition, e_v is that value of the applied voltage (direct voltage or crest of alternating voltage) at which a corona barely appears or just goes out. Therefore, in Equation (20) it is convenient to express e_1 through e_v, in order to make the formula applicable to an actual corona.

Let both condensers be originally uncharged and let an increasing d-c. voltage be slowly applied to the combination of C_1 and C_2, in place of an a-c. voltage. At the instant just before a glow appears at G_1, we have for the total charge or electric displacement in each of the condensers:

$$e_1\, C_1 = p_2\, C_2 \tag{21}$$

Moreover,

$$e_1 + p_2 = e_v \tag{22}$$

Eliminating p_2 and solving for e_1, we get

$$e_1 = e_v C_2/(C_1 + C_2) \tag{23}$$

Substituting this value of e_1 in Equation (20) and eliminating P_1 by means of Equation (7), we finally obtain

$$\sin \omega t_1 = 1 - 2\,(e_v/E) \tag{24}$$

This formula may also be written in the form

$$e_1 = E - 2\,e_v \tag{25}$$

where $e_1 = E \sin \omega t_1$ is the instantaneous critical voltage at which the corona starts.

When $E = e_v$, $\sin \omega t_1 = -1$; $\omega t_1 = 270$ deg., and we have an ionized state for an instant only (Peek's Fig. 18A). When $E = 2\,e_v$, $\sin \omega t_1 = 0$; $\omega t_1 = 180$ deg., the discharge starts each time when the applied voltage passes through zero, and the ionized state continues for one-half of each alternation. When E has a value between e_v and $2\,e_v$, ωt_1 lies between 180 deg. and 270 deg., and the discharge continues for less than one-half of each alternation, as shown in Fig. 3. When E is greater than $2\,e_v$, $\sin \omega t_1$ is positive, ωt_1 lies between 90 deg. and 180 deg., and the corona glow lasts for more than one-half of each alternation (Peek's Fig. 18C).

E. A Mechanical Analog

The electrical conditions shown in Figs. 2 and 3 may be made clearer by the following mechanical analog. Fig. 4a shows a combination of two spiral springs, C_1 and C_2, which correspond in their behavior to the two condensers in the circuit in Fig. 2. The tops of the springs are fastened to the plate q. The bottom of the spring C_2 is fastened to a solid foundation gg. The bottom of the spring C_1 is attached to the plunger s which can slide through an immovable sleeve G_1. So long as the vertical force on C_1 in either direction does not exceed a certain value, e_1, the plunger remains stationary because of friction in G_1. When the force exceeds e_1, the plunger begins to slide, so that a force greater than $\pm\, e_1$ can at no time be applied to C_1.

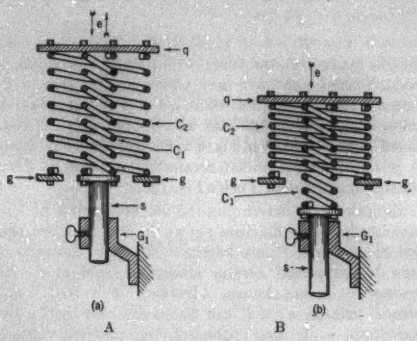


FIG. 4—A MECHANICAL ANALOG OF A-C. CORONA

Let a mechanical force, e, which varies harmonically with time, be applied to the upper plate q, causing the springs alternately to expand and to contract. When there is no slippage at G_1, this force is divided between the springs C_1 and C_2 in proportion to their elastances (force per unit deformation), just as in Fig. 2, without the corona, the applied voltage e is divided between C_1 and C_2 in proportion to their electrical elastances. When the plunger begins to slide, the spring C_2 takes more than its share of the force and becomes highly compressed (Fig. 4b), whereas C_2 is only compressed by the force e_1.

On the upward stroke, the compression in C_1 may be changed to tension long before C_1 has reached its normal state, and the plunger G_1 may begin to move upward before the total force e has been reversed. The parts of the mechanical cycle during which the plunger is moving correspond to the parts of the electrical cycle when a glow discharge takes place. By following this analog in greater detail, the reader may make clear to himself the cyclogram, the points of discontinuity, charge, and energy storage in C_2, the effect of the value of E as compared to e_v, etc. The applied mechanical force takes the place of the applied voltage; mechanical displacements are analogous to electric charges; and the velocity of the motion of the plate q represents the current.

III. ELLIPSES OF ACTUAL CORONA

The proper scientific way to treat mathematically a d-c. glow discharge about a round wire is by determining the distribution of electric space-charge density and the voltage gradient as functions of the distance from the geometric axis of the conductor. This can be done by using the divergence theorem and considering the mobilities of ions. While the mathematics of such a deduction is quite complicated and some uncertainties and simplifications are unavoidable, Otto Mayr[4] has succeeded in obtaining approximate expressions for d-c. corona in finite form. He also has shown that the relationship between the critical voltage gradient at the surface of the wire, and the values of mobilities of ions determined by separate experiments of a different kind, is in fair agreement with his theory.

The theory of the a-c. corona is much more involved, since the space charge and the velocities of ions are then functions not only of the distance from the axis of the conductor, but of time as well. Moreover, the phenomenon becomes dependent on the frequency of the supply, since an ion which at 60 cycles may travel during one alternation from the inner wire almost to the outer cylinder, at 5000 cycles may be caught and reversed near the inner conductor. Although Mayr shows by rough computations (*ibid.*, p. 276) that the conditions at 60 cycles are qualitatively the same as

4. Otto Mayr, "Raumladungsprobleme der Hochspannungstechnik", *Arch. f. Elek.*, 1927, Vol. 18, p. 270.

with direct current, yet so far no rational theory covering this case has been worked out.

Our problem here is much more narrow, namely to represent mathematically the general features of the appearance of corona cyclograms, such as shown in Fig. 1, and we shall attempt to do this in a semi-empirical way on the basis of two equivalent diagrams (Figs. 5 and 6) which are a generalization of that shown in Fig. 2.

A. *An Expression for the Current when Corona is Extinct*

Strictly speaking, the actual conditions in an a-c. corona cannot be adequately represented by an equivalent diagram of condensers, fixed glow gaps, resistances, etc., such as shown in Fig. 2, nor by any extension thereof. There is in reality no definite stationary cylindrical surface, concentric with the wire, which could serve as a boundary between some fictitious condensers, such as C_1 and C_2; nor is there anything in the mechanism of the corona to correspond to a "set" glow gap G_1. However, the diagram shown in Fig. 2 leads to a cyclogram in Fig. 3 which checks with the experiment. This suggests the possibility

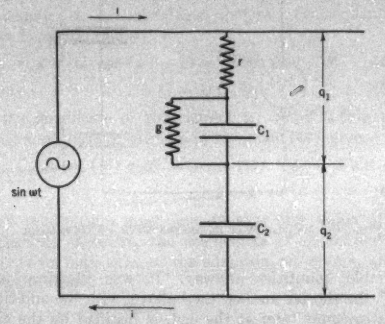


FIG. 5—AN IMPERFECT AND A PERFECT CONDENSER IN SERIES

that a more elaborate series combination (Figs. 5 and 6) of an imperfect condenser, C_1, representing the ionized portion of the gas adjacent to the wire, and a perfect condenser, C_2, representing the remote portions of the gas in which no ionization by collisions takes place, may adequately (although semi-empirically) reproduce the cyclogram in Fig. 1.

In Fig. 5, the imperfections in the condenser C_1 are represented by a fictitious series resistance r and a shunted conductance g. We shall first assume that there is no corona and no glow gap, that the combination of the two condensers is connected to a source of voltage, $E \sin \omega t$, and that permanent conditions have been established. The equivalent impedances of the condensers C_1 and C_2 are

$$Z_1 = r + (j\,\omega\,C_1 + g)^{-1} \tag{26}$$

$$Z_2 = (j\,\omega\,C_2)^{-1} \tag{27}$$

Hence, the partial voltage across C_2 is

$$q_2' = E\,\epsilon^{j\omega t}\,\frac{(j\,\omega\,C_2)^{-1}}{r + (j\,\omega\,C_1 + g)^{-1} + (j\,\omega\,C_2)^{-1}} \tag{28}$$

In this expression, $E\,\epsilon^{j\omega t}$ is to be thought of as a rotating vector whose length is equal to the amplitude of the applied voltage. This vector is multiplied by a complex fraction which changes its length and turns it by a constant angle. Hence, q_2' is also a vector in the same sense as $E\,\epsilon^{j\omega t}$. The voltage q_2 is marked with a prime sign to indicate that only the sinusoidal component of the complex exponential voltage is to be used later.

Multiplying the numerator and the denominator of the righthand side of Equation (28) by $j\,\omega\,C_2$, we get

$$q_2' = E\,\epsilon^{j\omega t}\,[1 + r\,j\,\omega\,C_2 + (j\,\omega\,C_2)/(j\,\omega\,C_1 + g)]^{-1} \tag{29}$$

Expanding the last fraction by long division, this equation is transformed into

$$q_2' = E\,\epsilon^{j\omega t}\,[1 + (C_2/C_1) + j\,r\,\omega\,C_2 + j\,g\,C_2/(\omega\,C_1^2) + \text{etc.}]^{-1} \tag{30}$$

Since r and g are small quantities (imperfections), the terms with the higher powers of g may be neglected. Combining the terms with j into one, we may finally write

$$q_2' = E\,\epsilon^{j\omega t}\,[1 + (C_2/C_1) + j\,n]^{-1} \tag{31}$$

where n is an empirical correction term. Thus, we may generalize the meaning of expression (31) by saying that *it represents the a-c. voltage across the perfect condenser C_2, no matter what the nature of imperfections in C_1 is, provided that they are sufficiently small.* In other words, from now on it is not necessary to think of these imperfections as being due to a particular constant resistance or conductance; the loss may be caused by collisions of residual ions and other factors in gaseous conduction. Equation (31) may also be written in the form[5]

$$q_2' = E\,\epsilon^{j(\omega t - \gamma)}\,C_1 \cos \gamma/(C_1 + C_2) \tag{32}$$

where γ may be called the imperfection angle of condenser C_1. For a perfect condenser, $\gamma = 0$, and Equation (32) agrees with Equation (8). The angle γ is determined by Equation (34).

For the sake of brevity we shall introduce

$$Q = E\,C_1 \cos \gamma/(C_1 + C_2) \tag{36}$$

so that Equation (32) becomes

5. To transform expression (31) into Equation (32), rewrite the factor by which $E\,\epsilon^{j\omega t}$ is multiplied in the form

$$K = C_1/(C_1 + C_2 + j\,n\,C_1) \tag{33}$$

and put

$$n\,C_1 = (C_1 + C_2)\,\text{tan}\,\gamma \tag{34}$$

This will give

$$K = C_1 \cos \gamma/[(C_1 + C_2)\,(\cos \gamma + j \sin \gamma)] \tag{35}$$

It remains to replace $(\cos \gamma + j \sin \gamma)$ by $\epsilon^{j\gamma}$, to obtain Equation (32).

$$q_2' = Q\, e^{j(\omega t - \gamma)} \tag{37}$$

The sine component of q_2', corresponding to $E \sin \omega t$, is

$$q_2' \text{ (sine)} = Q \sin (\omega t - \gamma) \tag{38}$$

So far we have assumed our circuit to be operating continuously, under established conditions, and with the initial charges equal to zero. In reality, the arrangement shown in Fig. 5, if it applies to a corona at all, can hold true only during those portions of each alternation during which ionization by collision is not taking place, but only motion and recombination of residual ions; in other words, when the corona is "out." Just before the corona goes out, the distribution of electric stress in the layers of the gas adjacent to the wire is quite different from that when the corona is extinct, more voltage being thrown upon the fictitious condenser C_2. Thus, during the portion of an alternation under consideration, C_2 behaves as if it were "pre-charged" by a d-c. source.[6] This extra charge adds a d-c. component to the voltage q_2' and causes a similar component to be subtracted from q_1'. Thus, Equation (38) should be generalized to

$$q_2 = Q \sin (\omega t - \gamma) + D \tag{39}$$

where D is the d-c. component of the voltage and corresponds to Γ in part II of the paper.

Experiment shows that the corona goes out when the applied voltage has reached its maximum value, E, and just begins to decrease. We shall assume that at such instants the voltage across the strongly ionized portion of the dielectric is e_1, as in Fig. 2. At the same instants, the voltage across the condenser C_2 is $q_2 = E - e_1$. Substituting this value for q_2 in Equation (39), at $\omega t = \pi/2$, gives

$$D = E - e_1 - Q \cos \gamma \tag{40}$$

so that Equation (39) becomes

$$q_2 = Q\,[\sin (\omega t - \gamma) - \cos \gamma] + (E - e_1) \tag{41}$$

The charging current through C_2 is still determined by Equation (1), because a constant initial charge has no effect on it. Thus,

$$i = C_2\, d\, q_2/dt = \omega\, C_2\, Q \cos\ \omega t - \gamma) \tag{42}$$

and the amplitude of the current is

$$I = \omega\, C_2\, Q \tag{43}$$

For the case in Fig. 2, $\gamma = 0$, and Equations (42) and (43) become identical with Equations (9) and (10). Thus, for the part of an alternation when no new ions are produced, the cyclogram curve is represented by the two equations

$$e = E \sin \omega t \tag{44}$$

$$i = I \cos (\omega t - \gamma) \tag{45}$$

It is shown in the Appendix that eliminating t from these equations gives an ellipse between e and i, but the principal axes of this ellipse are not equal to E and I and are not in the direction of those of the cyclogram. The ellipse $a\, m\, c\, d\, f\, g\, h$ in Fig. 1 corresponds to the Equations (44) and (45). The values of $I = 3.56$ divisions and $\gamma = 28$ deg. have been chosen by trials to fit the portions $a\, h$ and $d\, k$ of the cyclogram as closely as possible.

Equation (39) upon which Equation (45) is based, may be called semi-empirical for the following reason: The phenomenon under consideration is a "periodic transient" (using Dr. Steinmetz's expression), so that, strictly speaking, it is not legitimate to begin with Equations (26) to (28) which apply to a steady state only. The starting point should be equations similar to Equations (1) and (2), applied to Fig. 5. These would give a sinusoidal term and a transient exponential term. The result comes out quite involved for application to cyclograms, because of the exponential term containing t. Moreover, such an accurate procedure is hardly justifiable in view of the uncertainties in the quantities C_1, C_2, r, and g, which are fictitious and variable quantities anyway. It was therefore felt that the shorter method used above, with the addition of a constant term at the end, is justified by the circumstances of the case and by the specific purpose in view.

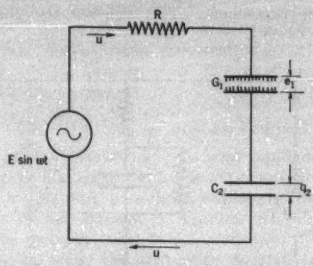


Fig. 6—A Glow Gap in Series with a Condenser

B. An Expression for the Current when Corona is Present

The assumptions made for this portion of the cyclogram are shown in Fig. 6. A comparison with Fig. 2 will show that a series resistance, R, has been added, to make the case more general and to account for the loss of power in the corona. The condenser C_1 has been omitted since it is shunted by a glow gap of constant voltage e_1, so that the charge on C_1 remains constant and does not enter in the equations. Or else, C_1 may be thought of as having been converted into the glow gap G_1; this mode of representation agrees with the physical facts more closely.

Equation (13) applies in this case, and Equation (14) is generalized by the addition of the ohmic drop, $R\, u$. Thus, the equations corresponding to Fig. 6 are:

6. V. Karapetoff, *Precharged Condensers in Series and in Parallel*, A. I. E. E. Proc., 1918, Vol. 37, p. 509.

$$u = C_2\, d\, q_2/dt \tag{46}$$

$$R\, u + e_1 + q_2 = E \sin \omega\, t \tag{47}$$

The correct procedure in the solution of Equations (46) and (47) is to substitute the value of u from Equation (46) in Equation (47) and to solve the resulting equation for q_2. However, this would bring in an exponential transient term of doubtful accuracy, since the whole equivalent diagram shown in Fig. 6 is only a crude approximation to an actual corona. We therefore reason as follows:

If the constant voltage, e_1, did not exist, both the current u and the voltage q_2, under steady conditions, would be sinusoidal. The presence of e_1 causes the condenser to become "pre-charged," that is, its voltage does not alternate between say Q_2' and $-Q_2'$ but between $+Q_2' - e_1$ and $-Q_2' - e_1$. On the other hand, the current remains sinusoidal because, according to Equation (46), it depends only on the rate of change of q_2 with the time. Thus, disregarding the transient conditions, we may put

$$q_2 = Q_2 \sin(\omega\, t - \beta) - e_1 \tag{48}$$

where β is a phase displacement or imperfection angle analogous to γ introduced before. Consequently, from Equation (46),

$$u = \omega\, C_2\, Q_2 \cos(\omega\, t - \beta) = U \cos(\omega\, t - \beta) \tag{49}$$

where

$$U = \omega\, C_2\, Q_2 \tag{50}$$

is the amplitude of the current u. Substituting the foregoing values of q_2 and u in Equation (47), gives

$$R\, \omega\, C_2\, Q_2 \cos(\omega\, t - \beta) + Q_2 \sin(\omega\, t - \beta) = E \sin \omega\, t \tag{51}$$

This relationship must hold true at any value of t. Equating, therefore, the coefficients of $\sin \omega\, t$ and of $\cos \omega\, t$ on both sides of the equation, we obtain, after simplification:

$$\tan \beta = \omega\, C_2\, R \tag{52}$$

$$U = \omega\, C_2\, E \cos \beta \tag{53}$$

When $R = 0$, $\beta = 0$, and Equation (53) agrees with Equation (16), Equation (49) becomes identical with Equation (15).

The equations

$$e = E \sin \omega\, t \tag{54}$$

$$u = U \cos(\omega\, t - \beta) \tag{55}$$

determine an ellipse, just as Equations (44) and (45) do; see Appendix. This is the ellipse $b\, n\, p\, d\, q\, r$ in Fig. 1. The values of $U = 5.9$ divisions and $\beta = 18$ deg. have been chosen by trial to fit the portion $y\, p\, n\, b$ of the cyclogram as closely as possible. Positive corona differs somewhat from negative corona so that the two halves of the cyclogram are not quite symmetrical. A somewhat different ellipse would fit the lower corona region better.

C. *Points of Discontinuity*

As explained in Sec. II, under D, we shall assume that the corona starts when Equation (19) is satisfied. Since in Fig. 5

$$q_1 + q_2 = E \sin \omega\, t \tag{56}$$

the condition of $q_1 = -e_1$ is equivalent to

$$q_2 = E \sin \omega\, t_1 + e_1 \tag{57}$$

where t_1 is an instant of time when the corona starts. Substituting this value of q_2 in Equation (41), we obtain

$$E \sin \omega\, t_1 - Q \sin(\omega\, t_1 - \gamma) = E - Q \cos \gamma - 2\, e_1 \tag{58}$$

To solve this equation for t_1, we introduce an auxiliary voltage S and an auxiliary phase angle θ such that

$$E \sin \omega\, t_1 - Q \sin(\omega\, t_1 - \gamma) = S \sin(\omega\, t_1 + \theta) \tag{59}$$

Equating the coefficients of $\sin \omega\, t_1$ and $\cos \omega\, t_1$ on both sides of Equation (59), we get

$$E - Q \cos \gamma = S \cos \theta \tag{60}$$

$$Q \sin \gamma = S \sin \theta \tag{61}$$

so that

$$\tan \theta = Q \sin \gamma/(E - Q \cos \gamma) \tag{62}$$

$$S = Q \sin \gamma/\sin \theta = (E - Q \cos \gamma)/\cos \theta \tag{63}$$

Therefore, S and θ may be considered as known quantities. Equations (58), (59), and (60) then give

$$\sin(\omega\, t_1 + \theta) = \cos \theta - 2\,(e_1/S) \tag{64}$$

When $\gamma = 0$, $\theta = 0$, and Equation (64) becomes identical with Equation (20). The statements made there regarding the limits of values of $\omega\, t_1$ apply here to $(\omega\, t_1 + \theta)$. Having calculated the values of θ and S from Equations (62) and (63), the value of $\sin(\omega\, t_1 + \theta)$ may be computed from Equation (64) and then the angle $\omega\, t_1$ determined.

The relationship between e_1 and e_v may be assumed to be represented by Equation (23), because at the visual critical voltage the imperfections represented by the resistances may be neglected.

From Equation (36)

$$C_2/(C_1 + C_2) = 1 - [Q/(E \cos \gamma)] \tag{65}$$

so that

$$e_1 = e_v \{ 1 - [Q/(E \cos \gamma)] \} \tag{66}$$

Dividing Equation (43) by Equation (53), we obtain

$$I/U = Q/(E \cos \beta) \tag{67}$$

Therefore, Equation (66) may also be written in the form

$$e_1 = e_v \{ 1 - (I/U)(\cos \beta/\cos \gamma) \} \tag{68}$$

By means of either Equation (66) or (68) e_1 may be computed if e_v has been determined experimentally and the ellipses drawn.

In Fig. 1, the points k and h have been determined as follows: (a) By means of Equation (68) the value of e_1 was computed using the value of $e_v = 18.1$ kv., given by Peek for this particular cyclogram; (b) Q was computed from Equation (67) for the given voltage

$E = 65.6$ kv. (c) The angle θ was determined from Equation (**62**) and S from Equation (**63**); (d) sin $(\omega t_1 + \theta)$ was found from Equation (**64**); (e) Having found the angle ωt_1, the abscissas $\pm E \cos \omega t_1$ gave the positions of the points k and h. It will be seen that these points agree quite well with the experimental discontinuities in the cyclogram.

IV. Conclusion

It has been shown that the general shape of corona cyclograms, such as in Fig. 1, may be adequately represented by arcs of two ellipses, whose equations can be derived from an equivalent diagram of the corona. It has also been shown that cyclograms of a circuit consisting of two condensers and a glow gap (Fig. 2) also consist of arcs of two ellipses. In the latter case the ellipses are "straight," whereas with the actual corona they are "oblique."

While experimental and theoretical work on the detailed mechanism of corona is going on, based on a statistical consideration of actual moving ions in the glow region, the computations and the point of view presented in this paper may be of some value in that certain fictitious "bulk quantities" (capacitances and resistances) are introduced which may be calculated from experimental data. Variations in these quantities with the conditions of the experiment may give an indication as to the corresponding changes in the state of the gas, changes which otherwise it may be difficult to measure or to express by numbers.

The particular equivalent diagrams used in this paper are perhaps the simplest and the crudest possible, so that the field is open for further refinements. For example, it is natural to assume that the space charge does not retain its position during an alternation, but contracts and expands, so that the values of the two capacitances, C_1 and C_2, vary harmonically with the time. However, such refinements will be of value only when suitable experimental material is available to check theoretical conclusions and when the cathode ray in an oscillograph can be better focused, so as to produce a much sharper record.

Appendix

Some properties of an oblique ellipse. By an oblique ellipse (Figs. 1 and 7) is meant an ellipse boxed in a rectangle whose bisectors ($A\,M$ and $Q\,R$) do not coincide with the principal axes of the curve. Let the ordinates of the ellipse be denoted by i and the abscissas by e. We shall prove that the curve may be represented by two simultaneous equations of the form

$$e = E \sin \alpha \tag{69}$$

$$i = I \cos (\alpha - \gamma) \tag{70}$$

where $E = O\,A$, $I = O\,Q$, $\alpha = \omega\,t$ is an auxiliary variable angle, and γ a constant angle which characterizes the ellipse. Equations (**44**) and (**45**) and Equations (**54**) and (**55**) are of this form.

A mathematical point may be thought of as performing a harmonic motion, $E \sin \omega\,t$, along the axis of abscissas and at the same time being subjected to another harmonic motion, $I \cos (\omega\,t - \gamma)$, along the axis of ordinates. It is well known in physics that a superposition of two such motions gives an elliptical path (for example, elliptically polarized light).

To prove that Equations (**69**) and (**70**) represent an ellipse, eliminate α between them. The result is

$$I^2 e^2 + E^2 i^2 - 2\,i\,e\,I\,E \sin \gamma - E^2 I^2 \cos^2 \gamma = 0 \tag{71}$$

This equation is of the form

$$a\,x^2 + b\,y^2 + 2\,h\,x\,y + c = 0 \tag{72}$$

which is the equation of a conic section referred to the origin at its center. The curve is an ellipse when

$$a\,b - h^2 > 0 \tag{73}$$

In our case

$$a\,b - h^2 = I^2 E^2 - I^2 E^2 \sin^2 \gamma = I^2 E^2 \cos^2 \gamma \tag{74}$$

and is intrinsically positive, so that the curve is an ellipse. The angle ϕ which its major axis forms with the axis of abscissas is determined by the equation

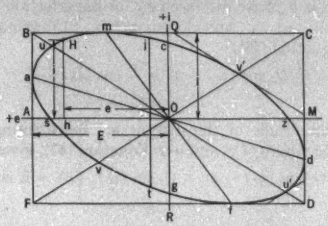


Fig. 7—Conjugate Diameters of an Ellipse

$$\tan 2\,\phi = 2\,h/(a - b) \tag{75}$$

In our case

$$\tan 2\,\phi = 2\,I\,E \sin \gamma/(E^2 - I^2) \tag{76}$$

On first sight the last equation may seem to be physically inhomogeneous, in that (amperes)^2 is subtracted from (volts)^2. However, in this equation both E and I are supposed to be expressed in some units of length, such as divisions of cross-section paper; otherwise ϕ has no meaning.

The lengths of the principal axes are determined from the "discriminating quadratic"

$$k^2 - (E^2 + I^2)\,k + E^2 I^2 \cos^2 \gamma = 0 \tag{77}$$

One value of $2\,k$ gives the length of the major axis, the other that of the minor axis.

Conjugate Diameters. Two diameters of an ellipse are said to be conjugate when any chord parallel to one is bisected by the other. For example, in Fig. 7, $a\,d$ and $c\,g$ are conjugate diameters. All vertical chords, such as $j\,t$, are bisected by $a\,d$. The tangent to the ellipse at the extremity of one of the conjugate

diameters is parallel to the other diameter. Therefore, a is the point of tangency with $B\,F$. For the same reason, the diameter $m\,f$, conjugate to $s\,z$, passes through the tangency points, m and f, of the horizontal sides of the circumscribed rectangle.

The diagonals $B\,D$ and $F\,C$ also determine a pair of conjugate diameters of the ellipse; that is, the tangents to the ellipse at points u and u' are parallel to $F\,C$, while the tangents at v and v' are parallel to $B\,D$. This may be proved as follows: Equations (69) and (70) give

$$d\,e/d\,\alpha = E\cos\alpha;\ d\,i/d\,\alpha = -\,I\sin(\alpha-\gamma) \quad (78)$$

Consequently, the slope of the curve, at a point determined by the coordinates e and i, is

$$d\,i/d\,e = -\,(I/E)\sin(\alpha-\gamma)/\cos\alpha \quad (79)$$

For the point u, from the geometry of the figure

$$(i/e)_u = I/E \quad (80)$$

On the other hand, from Equations (69) and (70)

$$(i/e)_u = (I/E)\cos(\alpha_u-\gamma)/\sin\alpha_u \quad (81)$$

Hence, for point u

$$\cos(\alpha_u-\gamma) = \sin\alpha_u \quad (82)$$

which means that

$$\alpha_u = 45^\circ + 0.5\,\gamma \quad (83)$$

Substituting this value in Equation (79), gives

$$(d\,i/d\,e)_u = -\,I/E \quad (84)$$

This proves that the tangent at u is parallel to $F\,C$, the slope of the latter line being $-\,I/E$. For the point v, the slope is equal to $+\,(I/E)$, so that Equation (79) is satisfied when

$$\alpha_v = 135^\circ + 0.5\,\gamma \quad (85)$$

Substituting the value of α_u from Equation (83) in Equations (69) and (70), we get

$$(O\,u)^2 = e_u^2 + i_u^2 = (E^2 + I^2)\sin^2(45^\circ + 0.5\,\gamma) \quad (86)$$

Similarly

$$(O\,v)^2 = (E^2 + I^2)\cos^2(45^\circ + 0.5\,\gamma) \quad (87)$$

Consequently,

$$(O\,u)^2 + (O\,v)^2 = E^2 + I^2 = (O\,B)^2 \quad (88)$$

Both E and I are of course supposed to be expressed in some units of length. One of the fundamental properties of the ellipse is that the sum of the squares of any pair of its conjugate diameters is constant. Hence, in our ellipse the sum of the squares of the principal axes, or of any other pair of conjugate diameters, is equal to the square of the diagonal of the circumscribed rectangle.

The angle δ between any two conjugate diameters is determined by the equation

$$\tan\delta = 2\,I\,E\cos\gamma/[E^2\sin 2\,\alpha - I^2\sin(2\,\alpha - 2\gamma)] \quad (89)$$

For the sake of convenience in computations and in fitting an experimental curve to an ellipse, some values of e, i, and $d\,i/d\,e$ are given in the table below, determined from Equations (69), (70), and (79). The ellipses shown in Fig. 1 were drawn with the aid of this table.

α	e	i	$d\,i/d\,e$
0	0	$I\cos\gamma$	$(I/E)\sin\gamma$
γ	$E\sin\gamma$	I	0
$45^\circ+0.5\,\gamma$	$\left\{\begin{array}{l}E\sin(45^\circ+0.5\,\gamma)=\\ E\cos(45^\circ-0.5\,\gamma)\end{array}\right.$	$\left.\begin{array}{l}I\sin(45^\circ+0.5\,\gamma)=\\ I\cos(45^\circ-0.5\,\gamma)\end{array}\right\}$	$-(I/E)$
90°	E	$I\sin\gamma$	infinity
$90^\circ+\gamma$	$E\cos\gamma$	0	$(I/E)/\sin\gamma$
$135^\circ+0.5\,\gamma$	$\left\{\begin{array}{l}E\sin(45^\circ-0.5\,\gamma)=\\ E\cos(45^\circ+0.5\,\gamma)\end{array}\right.$	$\left.\begin{array}{l}-I\sin(45^\circ-0.5\,\gamma)=\\ -I\cos(45^\circ+0.5\,\gamma)\end{array}\right\}$	(I/E)
180°	0	$-\,I\cos\gamma$	$(I/E)\sin\gamma$
$180^\circ+\gamma$	$-\,E\sin\gamma$	$-I$	0

Notation

a	parameter in Equation (72)
b	parameter in Equation (72)
C_1, C_2,	capacitances
c	parameter in Equation (72)
D	constant of integration in Equation (39)
E	amplitude of applied a-c. voltage e
e	instantaneous a-c. voltage
e_1	critical voltage of a glow gap
e_i	instantaneous critical voltage
e_v	visual critical voltage
g	shunted conductance
h	parameter in Equation (72)
I	amplitude of current i
i	instantaneous current
j	$\sqrt{-1}$
K	complex quantity defined by Equation (33)
k	either of the principal axes of an ellipse; Equation (77)
n	correction term in Equation (31)
P_1, P_2	voltage amplitudes defined by Equations (7) and (8)
p_1, p_2	instantaneous component voltages
Q	amplitude of voltage q_2', defined by Equation (36)
Q_2	voltage amplitude defined by Equation (48)
q_1, q_2	instantaneous voltages across condensers C_1 and C_2
q_2'	sinusoidal component of voltage q_2
R, r	resistances
S	auxiliary voltage defined by Equation (59)
t	time
t_1	instant at which corona is re-established
U	amplitude of current u
u	instantaneous current
u, v	subscripts referring to points u and v in Fig. 7
x, y	coordinates in Equation (72)
Z_1, Z_2	impedances defined by Equations (26) and (27)
$\alpha = \omega\,t$	variable time angle
β	imperfection angle, Equation (48)
Γ	constant of integration in Equation (3)
γ	imperfection angle of condenser C_1

δ angle between conjugate diameters of an ellipse

ϵ base of natural logarithms

θ auxiliary phase angle defined by Equation (59)

ϕ angle between the major axis of an ellipse and the axis of abscissas

$\omega = 2\pi f$ angular frequency

Discussion

E. B. Payne: In a cooperative research with the Utilities Research Commission we have conducted corona experiments at the University of Illinois. By means of tube amplifiers we have obtained oscillograms which record discontinuities in current-time curves. When such discharges occur in lead-covered cables we have found them to be associated with high-frequency disturbances which travel along the cable and may by proper apparatus be picked up at the cable ends.

J. B. Whitehead: It is of very great interest to see an effort to represent so highly complicated a phenomenon as the alternating-voltage corona by means, first, of a mechanical model and then of a simple electrical analogy.

I confess to some surprise that Professor Katapetoff has been able to advance so far in this type of analysis. He has himself called attention to the complexity of this phenomenon, and I hope that his remarks indicate that he is going to offer us something further in the way of a deeper and more fundamental analysis. Perhap that is the reason why he has not mentioned the recent work of one or two others on the fundamental character of corona and on the underlying laws of the travel of space charge in its reaction on the corona curves. I have particularly in mind the work of Holm, who has analyzed the travel of the space charge in a state of ionization, and under the influence of the sustained voltage corresponding to one-half of the alternating cycle. Holm has computed the limit of the travel of space charge, and used it to explain the shapes of the alternating-voltage curves as we see them.

The changes in the position of the critical voltage, on the alternating wave in the direction of the zero value of voltage, as described by Mr. Peek, are to be explained by the return of the space charge, which has gone out during one-half wave, under the influence of the succeeding wave of opposite polarity. This results in a very much higher potential gradient around the wire than would be due to the voltage itself, and so break down at an *apparently* lower value of voltage. It is this out-going travel and return of space charge that has engaged the attention of Holm.

In our own laboratory at Johns Hopkins, Dr. Willis two years ago made some very interesting extensions of Holm's theory, and since then one of our men, Dr. Waldorf, has been making some further studies in the hope of checking up the theories of Holm.

Flux Linkages and Electromagnetic Induction in Closed Circuits

BY L. V. BEWLEY[1]
Associate, A. I. E. E.

Synopsis.—It is shown that the flux linkages of a circuit may be changed in two very different ways—either the flux may be varied causing a voltage to be induced according to Faraday's Law of Electromagnetic Induction, or the turns may be varied by a substitution of circuit without inducing a voltage. In the Appendix it is mathematically shown that the flux may be changed either by transformer or cutting action, but that the presence of one or the other of these actions is dependent on the choice of reference axes. Thus any argument to the effect that one of them in particular is a necessary part of all induction phenomena is futile. It is possible to identify in every d-c. machine the building up of flux linkages so as to generate a voltage, and the reduction of flux linkages by a substitution of circuit so as not to generate a voltage. The alternate working of these two methods for changing the flux linkages of a circuit is an essential and necessary feature of every d-c. dynamo-electric machine. General criteria are introduced for ascertaining in any given case the nature of the changes in interlinkages which occur, and whether voltages are induced thereby. By way of application, a new restriction on the use of coefficients of inductance is pointed out, the sliding contact and homopolar machine are discussed, and finally a table has been prepared illustrating the various types of flux linkages found in familiar apparatus.

INTERLINKAGES

THE purpose of this paper is to examine the various changes in flux linkages that occur in electrical circuits, and to classify them with respect to the type of voltage generated. With this end in view a general equation is derived for calculating the voltage induced in a circuit of any shape moving or changing its configuration in a variable field of magnetic flux. Therefrom criteria are developed, and their application demonstrated, for determining in any given case the nature of the changes in interlinkages that take place and whether voltages are induced thereby.

There are many examples of changing flux linkages which are exceedingly difficult to analyze, and uncertainty may exist about the way in which the voltage is generated. Nor is it always easy to transpose the change in interlinkages that seem to occur to an equivalent switching operation and substitution of circuits, or to a rate of change of flux or turns. Oftentimes these phenomena are masked by each other, or seem to admit of a double interpretation. It is therefore convenient, if not essential, to classify the different methods for bringing about a change in flux linkages, in such a way and subject to such interpretation, that this uncertainty is reduced to a minimum.

In order that no ambiguity shall exist as to the meaning of the term *circuit*, as used in this paper, the following definition shall apply—

Any closed contour in space, whether in conducting media or not, and regardless if parts thereof are common to any other selected contours, constitutes a closed circuit.

On the basis of this definition, a network of linear conductors having two or more branches in parallel will consist of just as many separate and distinct circuits as it is possible to trace closed paths in that network, even though some of the conductors are common to two or more of those circuits. It is thus possible to construct a network of $n(n-1)/2$ complete circuits with n flexible linear conductors if only two junctions are used. Each circuit of a network will in general link a different amount of flux, and if the network is not constrained to move as a rigid body, will have different velocities. Consequently the voltages induced will be different, and if currents are permitted to flow they will automatically adjust themselves so that the resistance drop in each circuit will completely balance out the voltage induced in that circuit. In the case of conductors of large cross-section, all filaments of which are not linked with the same amount of flux, it becomes necessary to treat the conductor as a bundle of filaments in parallel, and to compute the voltage induced in each separate filament circuit. In such a case it may happen that a certain group of filaments forming a continuous surface are all linked with the same flux, and may therefore be treated as a group. For example, in calculating the skin effect of a tubular conductor where the return is too far removed to affect the current distribution, it is customary to treat the conductor as a nest of concentric hollow cylindrical elements.

In most cases of engineering practise, what constitutes a *turn* is usually so intuitively self evident as to require no explanation. But when tubes of induction are interwound with a circuit there may be some chance for confusion. The following arbitrary definition will therefore apply from the point of view of this paper.

If it is possible by means of imaginary lines to subdivide a circuit into a network of N cells such that each cell encloses the same flux ϕ and in the same direction, then the circuit is said to have N turns with respect to ϕ.

On the basis of the above definition, the actual physical loops may be made by either the circuit or by the tubes of induction, and the specification of the number of turns present is entirely arbitrary. In any particular case the induced voltage may be computed on the basis of N circuits in series each linked with a

1. General Transformer Engineering Dept., General Electric Co., Pittsfield, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

flux ϕ, or of a single turn circuit linked with a flux $N\phi$. To prove the equivalence it is only necessary to note that the voltage round any circuit C, which has been so subdivided into N cells 1, 2, 3 . . . N, is equal to the sum of the voltages round each cell, all taken in the same direction, thus

$$E_c = E_1 + E_2 + \ldots + E_N$$

If a circuit consists of n concentrated turns linked with a flux ϕ, then the interlinkages are

$$\Omega = n\,\phi \tag{1}$$

and their rate of change is

$$\frac{d\,\Omega}{d\,t} = n\,\frac{d\,\phi}{d\,t} + \phi\,\frac{d\,n}{d\,t} \tag{2}$$

The term $n\,d\,\phi/d\,t$ of Equation (2) accounts for those changes in interlinkages which are caused by varying the flux through the circuit. It expresses the Law of Electromagnetic Induction deduced experimentally by Faraday and stated as follows:

"Whenever the total flux through a circuit varies there is an electromotive force induced whose magnitude is proportional to the rate of diminution of the total number of tubes of induction threading the circuit."

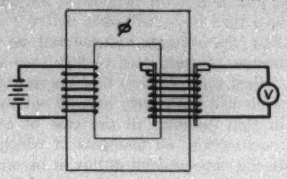


FIG. 1—CHANGING THE FLUX LINKAGES BY UNWINDING TURNS AND WITHOUT INDUCING A VOLTAGE

The law is universally true and applies when either or both the magnetic system and circuit are moving. While $n\,d\,\phi/d\,t$ is the general and most concise expression of the Law of Induction, it is nevertheless convenient to expand it into a more useful form in order to facilitate an understanding of its meaning and application. It is perhaps intuitively evident that the total flux threading a circuit may be changed either by varying the density of those tubes of induction already linked with the circuit, or by moving the circuit through the field. However, it is shown mathematically in Appendix I, by means of the Calculus of Variations and Vector Analysis, that $n\,d\,\phi/d\,t$ is composed of these two natural components. Two less general proofs are also included which are not dependent on the processes of mathematical physics, but whose solutions exhibit the same form as that of the general case, and from which the latter may be easily inferred. The expanded form of the Law of Induction developed in Appendix I is

$$e = -N\frac{d\,\phi}{d\,t} = -\sum N\left(\frac{\partial\,\phi}{\partial\,t} + \int_c \mathbf{B}\,.\,\mathbf{V}\times d\,\mathbf{s}\right)$$

$$= -\sum N\left\{\frac{\partial\,\phi}{\partial\,t} + \int_c \begin{vmatrix} \alpha & \beta & \gamma \\ u & v & w \\ dx & dy & dz \end{vmatrix}\right\}$$

$$= -\sum N\left(\frac{\partial\,\phi}{\partial\,t} + \int_c B_n\,V\sin\theta\,ds\right) \tag{16}$$

where the summation is to range over all of the circuits of concentrated turns which are connected in series.

The definitions of the symbols are given in the Appendix and in the attached list of symbols.

The first term of $d\,\phi/d\,t$ depends on the position and configuration of the circuit relative to the reference axes, and on the rate of change of the magnetic field. It is independent of the rate of motion or change in configuration of the circuit and is therefore called the "variational component," or referred to as "transformer" action.

The second or "motional" term of these expressions depends on the velocity of the elements of the circuit and on the instantaneous value of the components of flux density at the elements and normal to their planes of motion. It represents the "cutting" action of the moving circuit.

The two terms of the general e. m. f. equation thus have a real physical significance, but they are not invariant to a change of coordinate axes. In some apparatus, as for example in the case of the transformer, there is only one possible choice of axes. But in other instances, either or both terms may be present depending on the choice of reference axes. Thus, in the polyphase induction motor the voltage induced in the rotor is of the type $(\partial\,\phi/\partial\,t + B\,l\,v)$ or $\partial\,\phi/\partial\,t$ or $B\,l\,v$; corresponding respectively to axes taken on the frame, on the rotor, or rotating with the stator m. m. f.

The term $\phi\,d\,n/d\,t$ of Equation (2) accounts for those changes in interlinkages which are caused by varying the number of turns linked with a given rigid distribution of flux. It may appear that there are two possible interpretations to the meaning of $\phi\,d\,n/d\,t$. First, the turns may be changed without cutting the flux, as by winding them around it as indicated in Fig. 1, where the turns are wound onto a drum which revolves about a magnetic core. The electric circuit is completed through a slip-ring and brush. In such an arrangement the turns n are constant until the connection to the slip ring passes under the brush, when the number of turns changes abruptly to $(n \pm 1)$. Thus $d\,n/d\,t$ is infinite at that instant, but otherwise is zero. *No voltage is induced by this process.* A unique d-c. generator based on this process is described in the paper. A variation of the same scheme is in fact employed in every d-c. generator, and will be hereafter referred to as a *substitution of circuit.*

The second interpretation of $\phi\,d\,n/d\,t$ might seem to be the piling up of turns by cutting through the flux, as illustrated in Fig. 2. For such a point of view it is necessary to define a full turn as one linking all

of the specified amount of flux, and a partial or fractional turn as one linking only a part thereof. But such a definition is contrary to the idea of a turn as something necessarily integer, and leads to endless confusion and uncertainty in practise. In fact, if Equation (1) is defined on the basis of concentrated integer turns, then $\phi\, d\, n/d\, t$ cannot possibly admit of this second interpretation; for the several turns which make up the circuit of Fig. 2 are not concentrated, that is, they do not all link the same flux at the same instant. Under the conditions of such a definition each of the several turns must be regarded as separate circuits connected in series, and the changing flux linkages due to the motion of the coil are then fully accounted for by $n\, d\, \phi/d\, t$. It is therefore evident that any attempt to account for the change in interlinkages due to the motion of the circuit through the flux, by any term other than $n\, d\, \phi/d\, t$ would be superfluous and lead to a duplication of result, or else place an unnatural restriction on the definition of $n\, d\, \phi/d\, t$.

Thus it is seen that the interlinkages of a circuit

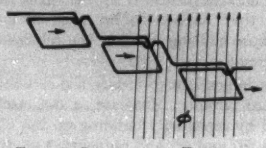


Fig. 2—Piling up of Turns

may be changed in two very different ways:

(1) The flux threading a circuit may be changed either by "transformer" or "cutting" action causing a voltage to be induced according to Faraday's Law of Induction.

(2) The turns linking the flux may be changed in such a way as not to cut through the flux, as by winding on turns or substitution of circuits, thus effecting a change of interlinkages *without* inducing a voltage.

The first of these two methods makes possible the generation of a voltage by electromagnetic induction. The second offers the only possible way for obtaining an average or d-c. component therefrom by means of a dynamo-electric machine.

The Average or D-c. Component of Voltage

The average voltage induced in a non-interrupted circuit over the time interval $(t_2 - t_1)$ is

$$e_{av} = -\frac{n}{t_2 - t_1}\int_{t_1}^{t_2}\frac{d\,\phi}{d\,t}\,d\,t = -\frac{n}{t_2 - t_1}\,(\phi_2 - \phi_1) \qquad (3)$$

It is evident that $e_{av} \to 0$ if averaged over a sufficiently long period of time, for the flux ϕ_2 cannot perpetually increase.

Suppose, however, that at time t_2 when the flux included by the circuit is ϕ_2, the interlinkages be reduced to some lower value ϕ_1 by effecting a substitution of circuit, $\phi\, d\, n/d\, t$; and then that the flux linkages of the new or substituted circuit be increased by increasing the flux. Any number of such cycles may be passed through in succession and the average voltage induced over all of the cycles is

$$e_{av} = -\frac{\sum n\,(\phi_2 - \phi_1)}{\sum\,(t_2 - t_1)} \qquad (4)$$

If every substituted circuit and cycle is alike this reduces to

$$e_{av} = -\frac{\phi_2 - \phi_1}{t_2 - t_1}\,n \qquad (5)$$

Thus a d-c. component of voltage may be obtained over any period of time, merely by providing some arrangement whereby new circuits may be continually substituted as the limit in flux linkages is reached for each. And therefore in any d-c. generator the voltage must be induced by a change of interlinkages $n\, d\, \phi/d\, t$; but the interlinkages must be held within finite bounds by a periodic reduction $\phi\, d\, n/d\, t$.

The most familiar arrangement of this kind is the ordinary d-c. generator, wherein a commutator functions as an automatic switch connecting the armature coils to the external circuit. At regular intervals each armature coil is disconnected from the external circuit on being short circuited by the brushes, and is then substituted back into the circuit, but with reversed connections.

As a second illustration of this method for obtaining a d-c. component of voltage, Fig. 3 shows a generator consisting of a magnetic core *c*, exciting winding *f*, and reversible revolving windings *a* and *b*. The flux cycle in the magnetic core, furnished by the exciting winding *f*, is a trapezoidal wave as shown in the figure. The generating cycle is divided into four periods, shown on the flux and voltage diagrams as

Period No. 1 — N turns on a, none on b.
Flux varied linearly from $-\phi$ to $+\phi$.
Constant voltage, since $d\,\phi/d\,t =$ constant.

Period No. 2 — Turns transferred from a to b. $(d\,n/d\,t.)$
Flux constant.
Voltage zero, since $d\,\phi/d\,t = 0$.

Period No. 3 — N turns on b, none on a.
Flux varied linearly from $+\phi$ to $-\phi$.
Constant voltage, since $d\,\phi/d\,t =$ constant.

Period No. 4 — Turns transferred back to a. $(d\,n/d\,t.)$
Flux constant.
Voltage zero, since $d\,\phi/d\,t = 0$.

Of course, whether the voltage generated during period No. 3 is of the same or different sign to that generated during period No. 1 depends on the direction of the windings, but as this is arbitrary it presents no problem.

The General Criteria

In the light of the foregoing developments, the following criteria are introduced as a means towards systematically determining the nature of the interlinkages which occur in electrical apparatus, and whether voltages are induced thereby. It is not supposed that these criteria will reduce the analysis to a simple mechanical process, but they are a step in that direction. The proposed rules are:

(a) Choose a set of coordinate axes as convenient and refer all changes in flux, or in position and configuration of the electric circuit, to these axes. (Transformation of coordinates, if properly done, is always permissible.)

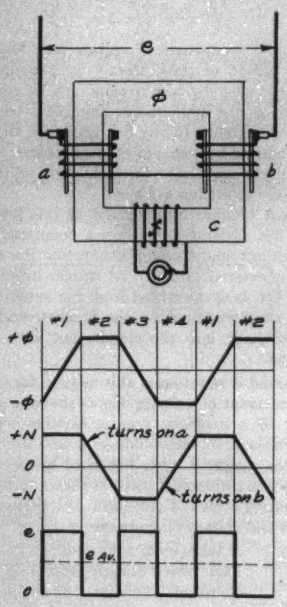


Fig. 3—D-c. Generator Making Use of the Principle in Fig. 1

(b) If that flux linked with the electric circuit at any instant, *i. e.*, with the circuit fixed, is a function of time with respect to the coordinate axes, it will induce in the circuit a voltage

$$e_1 = -\sum n \partial \phi / \partial t$$

where the summation is to include all those groups of concentrated turns of which the total circuit is composed, and ϕ is the flux linked with any group of n concentrated turns. By "concentrated turns" is understood all those turns connected in series which link exactly the same flux, although they may be physically widely distributed and at different places in space.

(c) If any of the elements ds of an electric circuit are moving with respect to the coordinate axes so as to "cut" the flux of the magnetic circuit, they will induce a voltage

$$e = -\sum n \int_c B_n V \sin \theta \, ds$$

where

V = velocity of the element ds
θ = angle between the direction of v and ds
B_n = component of flux density normal to the plane of V and ds
$\int_c$ = the integral for all the elements ds taken completely round the circuit.

(d) Any change of interlinkages which cannot be classified under either (b) or (c) is due to a substitution of circuit, $d\,n/d\,t$, and will necessarily involve some switching operation, sliding contact, or transfer of turns. No voltage will be induced thereby, except in so far as the flux itself is changed; either because the exciting m. m. f. is furnished by the turns themselves, or because they happen to be made of a magnetic material whose shifting changes the reluctance of the magnetic circuit.

(e) It is impossible to induce a d-c. voltage in an uninterrupted circuit. However, a d-c. voltage may be obtained by repeatedly building up the flux linkage by methods (b) and (c) and reducing them by method (d). In this way unidirectional voltages are induced during the increase of flux linkages, but no voltages are induced during their reduction.

Applications

A few examples are discussed under this section of the paper to fix in mind the principles involved, and the method of applying the general criteria. It is pointed out that there is a distinct restriction on the use of coefficients of inductance quite apart from saturation effects. The sliding contact and homopolar generator are briefly described and analyzed. Finally, a table has been compiled indicating the nature of the interlinkages which occur in some of the more familiar types of electric apparatus.

Coefficients of Inductance

It is sometimes convenient to express the total flux linkages of a circuit in terms of its coefficients of self and mutual inductance as defined by the equation:

$$N_k \phi_k = N_k (\phi_k' + \phi_k'') = \frac{N_k^2 i_k}{R_k} + \sum_s \frac{N_k N_s i_s}{R_{ks}}$$
$$= L_k i_k + \Sigma_s M_{sk} i_s \qquad (6)$$

where

N_k = total turns of circuit k
N_s = total turns of circuit s
ϕ_k' = flux linked with N_k due to $N_k i_k$
ϕ_k'' = flux linked with N_k due to $\Sigma N_s i_s$

TABLE I
TYPES OF INTERLINKAGE PHENOMENA

Apparatus	Reference axes	Circuit considered	Change of interlinkages		Voltage generated
			Increase	Decrease	
Transformer	On core	Either	$n\frac{\partial \phi}{\partial t}$	$n\frac{\partial \phi}{\partial t}$	a-c.
Syn generator	On field	Armature	$n\,Blv$	$n\,Blv$	a-c.
	On armature	Armature	$n\frac{\partial \phi}{\partial t}$	$n\frac{\partial \phi}{\partial t}$	a-c.
Polyphase ind. motor	On stator	Rotor	$n\left(\frac{\partial \phi}{\partial t}+Blv\right)$	$n\left(\frac{\partial \phi}{\partial t}+Blv\right)$	a-c.
	On rotor	Rotor	$n\frac{\partial \phi}{\partial t}$	$n\frac{\partial \phi}{\partial t}$	a-c.
	Rotating synchronously	Rotor	$n\,Blv$	$n\,Blv$	a-c.
D-c. generator	On field	At brushes	$n\,Blv$	$\phi\frac{dn}{dt}$	d-c.
	On armature	At brushes	$n\frac{\partial \phi}{\partial t}$	$\phi\frac{dn}{dt}$	d-c.
Unipolar generator	On core	At brushes	$n\,Blv$	$\phi\frac{dn}{dt}$	d-c.
Generator in Fig. 3	On core	At brushes	$n\frac{\partial \phi}{\partial t}$	$\phi\frac{dn}{dt}$	d-c.
Sliding contact	On core	At contacts	$\phi\frac{dn}{dt}$	$\phi\frac{dn}{dt}$	None

$\phi_k = \phi_k' + \phi_k''$
R_k = reluctance to ϕ_k'
R_{sk} = reluctance to ϕ_k''
i_k = current in circuit k
i_s = current in circuit s

Since the identity is to be true for all values of i_k and i_s, then

$$L_k = \frac{N_k^2}{R_k} \text{ and, } M_{ks} = \frac{N_k\, N_s\, i_s}{R_{ks}} \tag{7}$$

Now suppose that the currents and reluctances remain constant, but that N_k is varied (as by the winding on of turns in the experiment shown in Fig. 4).

The flux will then vary directly with the turns, $\phi_k' = N_k\, i_k/R_k$, and there will be induced in the circuit a voltage

$$e = -N_k \frac{d\,\phi_k}{d\,t} = -N_k \frac{d}{d\,t}\left(\frac{N_k\, i_k}{R_k} + \sum{}^s \frac{N_s\, i_s}{R_s}\right)$$

$$= -\frac{N_k\, i_k}{R_k}\,\frac{d\,N_k}{d\,t} = -\phi_k'\,\frac{d\,N_k}{d\,t} \tag{8}$$

This result may appear at first sight to violate the conclusion previously reached, *i. e.*, that the Law of Induction is $-N\,d\,\phi/d\,t$ and not $-d\,(N\,\phi)/d\,t$ or $-\phi\,d\,N/d\,t$. However, in Equation (3) the final result follows directly from $-N\,d\,\phi/d\,t$ and has the form $\phi\,d\,N/d\,t$ merely as a matter of coincidence. Had we proceded from the expression $-d\,(N\,\phi)/d\,t$ our result would have been twice too large.

In Equation (8) $d\,N_k/d\,t$ may be regarded as quite uniform and continuous, as is evident from a study of Fig. 4A. C is the magnetic circuit linked with a coil in which a current I is flowing. At the connection to the slip-ring R, the current divides as i_1 and $i_2 = I - i_1$ and flows in opposite directions to the brush b. This may be considered as a superposition of the full turn currents shown in Fig. 4B. The equivalent turns at full current then are $2\,I + i_1 = (2 + \theta/2\,\pi)\,I$. Thus the effect is exactly equivalent to the hypothetical existence of a uniformly changing fractional turn. Of course if the slip-ring were of non-uniform resistivity the division of current would not be proportional to the angle θ. If it were slit axially the change in N_k would occur abruptly.

The coefficients of inductance as defined by (7) are not sufficient to express the voltage due to a change in flux linkages of the type described above. For under such conditions there results:

$$\frac{d}{d\,t}\,(L_k\, i_k + \Sigma^s\, M_{sk}\, i_s) = i_k\,\frac{d\,L_k}{d\,t} + \sum{}^s\, i_s\,\frac{d\,M_{sk}}{d\,t}$$

$$= i_k\,\frac{d\,L_k}{d\,N_k}\cdot\frac{d\,N_k}{d\,t} + \sum{}^s\, i_s\,\frac{d\,M_{sk}}{d\,N_k}\cdot\frac{d\,N_k}{d\,t}$$

$$= i_k \frac{dN_k}{dt} \cdot \frac{d}{dN_k}\left(\frac{N_k^2}{R_k}\right)$$

$$+ \sum{}' i_s \frac{dN_k}{dt} \cdot \frac{d}{dN_k}\left(\frac{N_k N_s}{R_{sk}}\right)$$

$$= 2\frac{i_k N_k}{R_k} \cdot \frac{dN_k}{dt} + \sum{}' \frac{i_s N_s}{R_{sk}} \frac{dN_k}{dt}$$

$$= 2\phi_k' \frac{dN_k}{dt} + \frac{\phi_k''}{N_k} \frac{dN_k}{dt} \qquad (9)$$

In this result the term $2\phi_k' d N_k/dt$ is twice too large and the term $\frac{\phi_k}{N_k}\frac{dN_k}{dt}$ should not appear. This discrepancy arises from the fact that the coefficients of

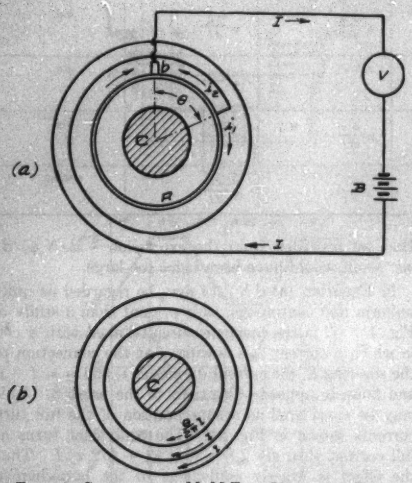


FIG. 4—CHANGING THE M. M F. BY UNWINDING TURNS

inductance L_k and M_{sk} contain by definition the number of turns N_k; but this factor is not subject to differentiation when these turns vary in such a way as to change the flux linkages without "cutting" the flux.

On the other hand, if they do vary in such a way as to "cut" the flux it is more reliable and more in accord with the physical facts to attribute the change in interlinkages as due to the changes in reluctances; rather than to introduce the motion of partial turns as previously discussed.

It is therefore evident that the coefficients of inductance are not directly applicable to cases of the type described above.

THE SLIDING CONTACT

The "Sliding Contact," Fig. 5A consists, in its most simple form, of a circuit completed through, and making a sliding contact with a magnetic core. As the contacts are moved across the core, the total flux linked with the circuit appears to change but no voltage is induced.

This is usually explained as a case of continuous and successive substitution of circuit. The iron core is considered as consisting of an infinite number of conducting filaments in parallel, as illustrated in Fig. 5B, and the process of moving the contacts is equivalent to progressively closing and opening switches at infinitesimal increments.

But in the light of the interlinkage criteria, the fact that no voltage is induced is evident at a glance, for:

a. Choose axes on the core.

b. There is no change of flux, hence $n\,\partial\phi/\partial t = 0$.

c. No moving element of the circuit "cuts" flux, hence $n\,B\,l\,v = 0$.

d. The change of interlinkages is due to a substitution of circuit. Therefore no voltage is induced.

THE HOMOPOLAR MACHINE

Fig. 6 shows the schematic form of the homopolar or unipolar generator. It consists of a bar magnet $N\,S$ around which revolves a conductor $c\,d$ on rings R. The circuit is completed through the upper and lower parts of the slip rings and the brushes b, thus forming two circuits in parallel having a common return through the brushes and external circuit. As the conductor revolves the flux in the upper circuit increases and that in the lower circuit correspondingly decreases, but as the flux is in opposite directions relative to the two circuits, the voltage generated is in the same direction. Although the flux through the upper circuit is apparently continually increasing, yet when the conductor reaches the brushes the flux included is the same as it was a revolution previously.

The paradox may be explained by reference to Fig. 6C. Let the conductor $c\,d$ be moved from A to B across the magnet face, generating a voltage. If this conductor could by some means be transferred back to A, each time that it reaches B, and in such a manner that the flux is not cut in a reversed direction, or the circuit interrupted, then conditions would be ideal for generating a d-c. voltage. The unipolar machine is just such an automatic arrangement for instantaneously effecting a change of interlinkages by a substitution of circuit, and in this respect Fig. 6C may be considered as its "development."

In effect, the unipolar machine is equivalent to making the distance $A\,B$ infinite. But this equivalence is not offered as an explanation of the phenomena of induction in the unipolar generator, for it fails to take into account the actual substitution of circuit which occurs, *an essential feature of any dynamo-electric machine generating direct current.*

Applying the interlinkage criteria:

(a) Choose axes as the frame.

(b) There is no change of flux, hence no voltage $n\,\partial\phi/\partial t$.

(c) The moving element cuts flux and always in the same direction, thus inducing a unidirectional voltage $B\,l\,v$.

(d) There is an automatic substitution of circuit each time that the conductor passes under the brush.

(e) The interlinkages are increased at a regular rate by cutting the flux, and induce a unidirectional voltage; but the interlinkages are reduced to zero by a substitution of circuit each time that the conductor passes under the brush. Thus a d-c. voltage is continuously generated.

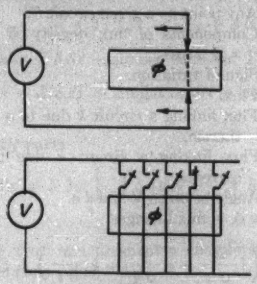


FIG. 5—SLIDING CONTACT AND EQUIVALENT SWITCHING OPERATIONS

CONCLUSIONS

The phenomena of changing flux linkages and electromagnetic induction in electrical circuits may be classified according to the following scheme:

Thus the flux linkages of a circuit may be changed by changing either the flux or the turns. The flux may be varied either by transformer action or by cutting action. (These are also known respectively as the *variational* and *motional* components of e. m. f.) But whichever action is involved is dependent on the arbitrary choice of the reference axes. Regardless of the way in which the flux through a non-interrupted circuit is changed, it will induce a voltage according to Faraday's Law. And if the period of time is taken sufficiently long, this e. m. f. must be alternating, or zero, for the flux cannot perpetually increase.

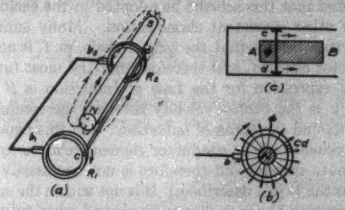
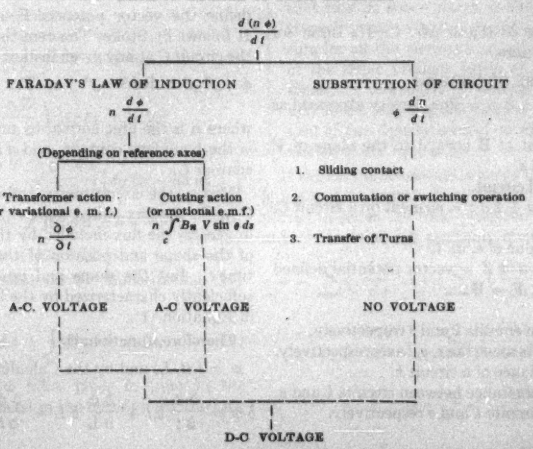


FIG. 6—HOMOPOLAR INDUCTION

But the flux linkages may also be changed by varying the number of turns linked. If this is done in such a way as not to change the flux itself, it is classified as a *substitution of circuit*, and includes the sliding contact, transfer of turns, and commutation or switching operations. *No voltage can be induced by a substitution of circuit.*

The generation of an average or d-c. component of voltage depends on the alternate use of $n\,d\,\phi/d\,t$ and $\phi\,d\,n/d\,t$. That is, the flux linkages must be increased



by increasing the flux and causing a voltage to be induced according to Farraday's Law; but these linkages must be held within finite limits by a periodic reduction with a substitution of circuit.

It is not intended to imply that it is always advantageous in analyzing a given case to follow the above scheme. It may often be more simple to deal directly with $n \frac{d\phi}{dt}$, than with its components. Nor is it suggested that this scheme be adopted to the exclusion of any other consistent classification. Many eminent authorities believe that the seat of the e. m. f. is at the conductor element, and therefore that the most fundamental expression for the Law of Induction is $B\,L\,\overline{V}$ where $\overline{V}$ is the *relative* velocity between the conductor elements and the tubes of induction. (In this paper V is the velocity of the conductor element relative to the coordinate system, and therefore is not necessarily the same as the $\overline{V}$ just described.) It is not within the scope of the present paper to discuss in general when tubes of induction have a velocity. Nevertheless, it is often convenient to assign velocities to them—especially when calculating the e. m. f. induced in a finite linear conductor element. (It may be parenthetically remarked that the process of calculating the voltage induced in an isolated element is also equivalent to assuming that the return circuit is at infinity.) Dr. H. B. Dwight recently discussed this $B\,L\,\overline{V}$ interpretation of the law, with certain necessary restrictions, in the April 1928 issue of the *Electric Journal*.

Finally, the author wishes to point out that the notions given in this paper have been presented from the engineering point of view, and with no attempt other than that given in Appendix II at correlation with the general mathematical theory of electricity and magnetism.

The author wishes to thank Mr. C. H. Biron for preparing the illustrations.

List of Symbols

$\mathbf{B}$ = $i\,\alpha + j\,\beta + k\,\gamma$ = flux density expressed as a vector.
B_n = Component of $\mathbf{B}$ normal to the plane of V and $d\,s$.
C = Any closed circuit.
$d\,\mathbf{s}$ = $i\,dx + j\,dy + k\,dz$ = element on a circuit C.
e = e. m. f.
e_{av} = average value of e. m. f.
$\mathbf{F}$ = $i\,X + j\,Y + k\,Z$ = vector potential defined by $\nabla \times \mathbf{F} = \mathbf{B}$.
I = Current.
i_k, i_s = Currents in circuits k and s respectively.
i, j, k = Unit vectors along the x, y, z axes respectively.
L_k = Self-inductance of a circuit k.
M_{ks} = Mutual inductance between circuits k and s.
N_k, N_s = Turns of circuits k and s respectively.
n, N = Turns.
$\mathbf{n}$ = Unit vector normal to a surface.
R_k, R_{ks} = Reluctances to fluxes due to i_k and i_s respectively.
t = Time.
u, v, w = Components of the velocity V along the x, y, z axes.
$\mathbf{V}$ = $i\,u + j\,v + k\,w$ = velocity of an element $d\,s$ expressed as a vector.
V = Absolute value of $\mathbf{V}$.
X, Y, Z = Components of the vector potential $\mathbf{F}$.
x, y, z = Any point on a given circuit C.
α, β, γ = Components of flux density B along the x, y, z axes.
δ = Sign of variation.
ϕ = Flux.
ϕ_k' = Flux linking a circuit k due to a current i_k therein.
ϕ_k'' = Flux linking a circuit k due to external currents.
θ = Angle between $\mathbf{V}$ and $d\,\mathbf{s}$.
Ω = $n\,\phi$ = flux linkages.
∇ = $\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)$ = Grad
Σ = Sign of summation, effective as implied.
$\int_c$ = Contour integral taken round any circuit C.
$\times$ = Vector or cross product.
$\cdot$ = Scalar or dot product.

Appendix I

General Equation of the Law of Induction

(Using Vector Analysis and the Calculus of Variations)

Consider an electrical circuit C situated in a field of variable magnetic flux density $\mathbf{B} = i\,\alpha + j\,\beta + k\,\gamma$. Then since $\mathbf{B}$ is a solenoidal vector, $\nabla \cdot \mathbf{B} = 0$, we may define the vector potential $\mathbf{F}$ such that $\nabla \times \mathbf{F} = \mathbf{B}$. It follows by Stokes Theorem that the flux linked with the circuit C at any given instant is

$$\phi = \int\int \mathbf{n}\cdot\mathbf{B}\,dS = \int\int \mathbf{n}\cdot\nabla\times\mathbf{F}\,dS = \int_c \mathbf{F}\cdot d\,\mathbf{s} = L \tag{1}$$

where $\mathbf{n}$ is the unit normal to any surface $\mathbf{S}$ of which C is the boundary contour, and $d\,\mathbf{s}$ is an element of the contour C.

Now as the circuit moves and changes its shape from C to C', the flux linked therewith also changes; so that in general the flux included by the circuit is a function of the shape and position of the circuit as well as of time t. But the shape and position of the circuit is sufficiently characterized by the line integral L as given in Equation (1).

Therefore, functionally

$$\phi = f\,(t, L) \text{ and by the Calculus of Variations} \tag{2}$$

$$\delta\,\phi = \frac{\partial\,\phi}{\partial\,t}\,\delta\,t + \frac{\partial\,\phi}{\partial\,L}\,\delta\,L = \frac{\partial\,\phi}{\partial\,t}\,\delta\,t + \delta\,L \tag{3}$$

since by Equation (1) $\phi = L$ at any given instant, and therefore $\frac{\partial \phi}{\partial L} = 1$.

Now if

$$\mathbf{F} = i\,X + j\,Y + k\,Z \tag{4}$$

and

$$d\,\mathbf{s} = i\,dx + j\,dy + k\,dz \tag{5}$$

we have

$$\delta \int_c \mathbf{F}\,.\,ds = \delta \int_c (Xdx + Ydy + Zdz)$$

$$= \int_c \delta\,(Xdx + Ydy + Zdz)$$

$$= \int_c (\delta\,X\,dx + X\,\delta\,dx + \text{similar terms in } Y \text{ and } Z).$$

$$= \int_c (\delta\,X\,dx + X\,\delta\,dx + \text{similar terms in } Y \text{ and } Z). \tag{6}$$

Integrating by parts

$$\int_c X\,d\,\delta\,x = X\,\delta\,x\Big]_c - \int_c \delta\,x\,d\,X = -\int_c \delta x\,d\,X \tag{7}$$

since at the start and finish of a completed contour $X\,\delta\,x$ has the same value. Also

$$\delta\,X = \frac{\partial X}{\partial x}\,\delta\,x + \frac{\partial X}{\partial y}\,\delta\,y + \frac{\partial X}{\partial z}\,\delta\,z \tag{8}$$

and $$d\,X = \frac{\partial X}{\partial x}\,dx + \frac{\partial X}{\partial y}\,dy + \frac{\partial X}{\partial z}\,d\,z$$

$$\therefore \delta \int_c \mathbf{F}\,.\,d\,s = \int_c \Big(\frac{\partial X}{\partial x}\,\delta\,x\,dx + \frac{\partial X}{\partial y}\,\delta\,y\,dx$$

$$+ \frac{\partial X}{\partial z}\,\delta\,z\,dx - \frac{\partial X}{\partial x}\,\delta\,x\,dx - \frac{\partial X}{\partial y}\,\delta\,x\,dy - \frac{\partial X}{\partial z}\,\delta\,x\,dz$$

$$+ \text{similar terms in } Y \text{ and } Z). \tag{9}$$

Collecting terms and rearranging

$$\delta\,L = \int_c \Big[\Big(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z} \Big)(dz\,\delta y - dy\,\delta z)$$

$$+ \Big(\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x} \Big)(dx\,\delta z - dz\,\delta x)$$

$$+ \Big(\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} \Big)\Big(dy\,\delta x - dx\,\delta y\Big) \Big]$$

$$= \int_c \nabla \times \mathbf{F}\,.\,\delta\,\mathbf{s} \times d\,\mathbf{s} = \int_c \mathbf{B}\,.\,\delta\,\mathbf{s} \times d\,\mathbf{s} \text{ where} \tag{10}$$

$$\delta\,\mathbf{s} = i\,\delta x + j\,\delta y + k\,\delta z \tag{11}$$

is the variation of a point on C. Substituting (10) in (3)

$$\delta\,\phi = \frac{\partial \phi}{\partial t}\,\delta\,t + \int_c \mathbf{B}\,.\,\delta\,\mathbf{s} \times d\,\mathbf{s} \tag{12}$$

If this variation in ϕ takes place in time $\delta\,t$ the electromotive force induced in the circuit is, by the Law of Induction

$$e = -\frac{\delta\,\phi}{\delta\,t} = -\Big(\frac{\partial \phi}{\partial t} + \int_c \mathbf{B}\,.\,\frac{\delta\,\mathbf{s}}{\delta\,t} \times d\,\mathbf{s} \Big) \tag{13}$$

But

$$\frac{\delta\,\mathbf{s}}{\delta\,t} = i\,\frac{\delta\,x}{\delta\,t} + j\,\frac{\delta\,y}{\delta\,t} + k\,\frac{\delta\,z}{\delta\,z} = iu + jv + kw = \mathbf{V} \tag{14}$$

is the vector velocity of a point on C. Therefore,

$$e = -\Big(\frac{\partial \phi}{\partial t} + \int_c \mathbf{B}\,.\,\mathbf{V} \times d\,\mathbf{s} \Big)$$

$$= -\Big(\frac{\partial \phi}{\partial t} + \int_c B_n\,V \sin\theta\,ds \Big) \tag{15}$$

where θ is the angle between $\mathbf{V}$ and $d\,\mathbf{s}$, and B_n is the component of $\mathbf{B}$ normal to the plane of $\mathbf{V}$ and $d\,\mathbf{s}$.

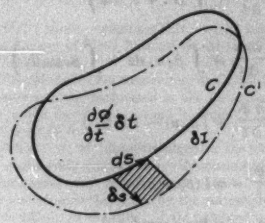


Fig. 7—Closed Circuit Moving and Changing Shape in a Variable Field of Flux

This general equation of the Law of Induction states that the electromotive force induced in a closed circuit C is equal to

(The time rate of change in magnitude of that flux in space which is linked at the given instant with the circuit C) + (The sum for all the elements round the circuit of the triple product *length of the moving element* × *component of velocity perpendicular to the length of the element* × *component of flux density normal to the plane of motion of the element.*) (16)

If the circuit consists of n turns the potential induced is n times as great. If several such circuits are connected in series the total potential induced is given by any of the following mutually convertible expressions:

$$e = -\sum n\,\frac{d\,\phi}{d\,t} = -\sum n \Big(\frac{\partial \phi}{\partial t} + \int_c \mathbf{B}\,.\,\mathbf{V} \times d\,\mathbf{s} \Big)$$

$$= -\sum n \Big(\frac{\partial \phi}{\partial t} + \int_c B_n\,V \sin\theta\,ds \Big)$$

$$= -\sum n \left\{ \frac{\partial \phi}{\partial t} + \int_c \begin{vmatrix} \alpha & \beta & \gamma \\ u & v & w \\ dx & dy & dz \end{vmatrix} \right\}$$

$$= - \sum n \left\{ \frac{\partial \phi}{\partial t} + \int_c [(\alpha v - \beta u)\, dz + (\beta w - \gamma v)\, dx + (\gamma u - \alpha w)\, dy] \right\} \quad (17)$$

The case of chief practical interest in electrical machinery is that of a coil with parallel coil sides moving through a field which is uniform in a direction along the coil sides perpendicular to the motion, and is zero along the ends. Taking the motion along the X axis, and an effective length of coil l along the Z axis, we have

$$\begin{cases} \mathbf{B} = i\,0 + j\,\beta + 0 \\ \mathbf{V} = i\,u + 0 + 0 \\ d\,\mathbf{s} = 0 + 0 + j\,dz \end{cases} \quad (18)$$

Then substituting in (17)

$$e = - n \left(\frac{\partial \phi}{\partial t} + \int_c \mathbf{B} \,.\, \mathbf{V} \times d\,\mathbf{s} \right)$$

$$= - n \left(\frac{\partial \phi}{\partial t} + \int_0^l \beta_2 u_2\, dz + \int_l^0 \beta_1 u_1\, dz \right)$$

$$= - n \left(\frac{\partial \phi}{\partial t} + l \beta_2 u_2 - l \beta_1 u_1 \right)$$

$$= - n \frac{\partial \phi}{\partial t} - n\, l\, (\beta_2 - \beta_1)\, u \quad (19)$$

since $u_2 = u_1 = u$ for a rigid coil.

This equation may be derived without the use of the Calculus of Variations as follows, and from it the more general Equation (17) may be inferred.

The flux linked with a coil in the xz plane having parallel coil sides along the z axis is

$$\phi = \int_0^l \int_{x_1}^{x_2} \beta\, dx\, dz = l \int_{x_1}^{x_2} \beta\, dx \quad (20)$$

But if the coil is moving, the limits x_1 and x_2 are functions of time t. Therefore the derivative of (20) must be taken according to the rule for differentiating a definite integral whose limits are functions of the parameter with respect to which differentiation is performed. Thus

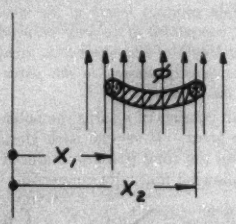


Fig. 8—Rigid Coil Moving through Variable Flux

$$e = - n \frac{d \phi}{d t} = - n \frac{d}{d t}\, l \int_{x_1}^{x_2} \beta\, d x$$

$$= - n\, l \left(\int_{x_1}^{x_2} \frac{\partial \beta}{\partial t}\, dx + \beta_2 \frac{d x_2}{d t} - \beta_1 \frac{d x_1}{d t} \right)$$

$$= - n \left(\frac{\partial \phi}{\partial t} + l \beta_2 u_2 - l \beta_1 u_1 \right)$$

$$= - n \frac{\partial \phi}{\partial t} - n\, l\, u \,.\, (\beta_2 - \beta_1) \quad (21)$$

This same procedure may easily be extended to the case of motion in two orthogonal directions.

Still another way of proving this result is to note from Fig. 8 that

$$\phi = f\,(t, x_2, x_1) \quad (22)$$

Then

$$- n \frac{d \phi}{d t} = - n \left(\frac{\partial \phi}{\partial t} \frac{d t}{d t} + \frac{\partial \phi}{\partial x_2} \frac{d x_2}{d t} + \frac{\partial \phi}{\partial x_1} \frac{d x_1}{d t} \right)$$

$$= - n \left(\frac{\partial \phi}{\partial t} + l \beta_2 u_2 - l \beta_1 u_1 \right)$$

$$= - n \frac{\partial \phi}{\partial t} - n\, l\, u\, (\beta_2 - \beta_1) \quad (23)$$

and this method also may be extended to the case of motion in two orthogonal directions.

Appendix II

Note on the Induced Voltage in an Element of Circuit

The term $\partial \phi / \partial t$ of Equation (15) accounts for the voltage induced in the circuit due to the time rate of change of that flux in space which at any given instant is linked with the circuit. It is accordingly computed as though the circuit were fixed in position. From Equation (1)

$$e_{variational} = - \frac{\partial \phi}{\partial t} = - \frac{\partial}{\partial t} \int_c \mathbf{F} \,.\, d\,\mathbf{s} = - \int_c \frac{\partial \mathbf{F}}{\partial t} \,.\, d\,\mathbf{s} \quad (24)$$

since C is fixed. The total induced voltage for the circuit may then be written in the form

$$e = e_{variational} + e_{motional}$$

$$= - \int_c \frac{\partial \mathbf{F}}{\partial t} \,.\, d\,\mathbf{s} - \int_c \mathbf{B} \,.\, \mathbf{V} \times d\,\mathbf{s}$$

$$= - \int_c \left(\frac{\partial \mathbf{F}}{\partial t} + \mathbf{B} \times \mathbf{V} \right) .\, d\,\mathbf{s} \quad (25)$$

since the dot and cross products of the latter expression are interchangeable.

Equation (25) was derived on the assumption that

the circuit was closed. Since, however, it assigns to each element ds of the circuit a definite portion of the total voltage induced in the complete circuit, it follows that the voltage between any two points a and b of the circuit may be taken as equal to

$$e_{ab} = -\int_a^b \left(\frac{\partial \mathbf{F}}{\partial t} + \mathbf{B} \times \mathbf{V} \right) . d\mathbf{s} = \int_a^b \mathbf{E} . d\mathbf{s} \qquad (26)$$

where $\mathbf{E}$ is the electric force at the circuit. To the integrand of (25) may be added any acyclic function ∇U, since the line integral of ∇U round any closed circuit is zero. Such a function exists when no changes are taking place in the magnetic field or circuit, and is therefore identified physically with the electrostatic potential. For this reason it has been omitted in (26). If the circuit is fixed, then $\mathbf{V} = 0$ and (26) reduces to the familiar rate of change of vector potential expression for the electric force.

$$\mathbf{E} = -\frac{\partial \mathbf{F}}{\partial t} \text{ for } C \text{ fixed} \qquad (27)$$

The advantage of Equation (26) is that the voltage induced in a *part of a circuit* may be calculated quite independently of other parts of the circuit, if the vector potential F is known. The three simultaneous differential equations of $\nabla \times \mathbf{F} = \mathbf{B}$ are not sufficient to uniquely determine $\mathbf{F}$. For if $\mathbf{F}$ is one solution, $(\mathbf{F} + \nabla \psi)$ where $\nabla \psi$ is any acyclic function, is also a solution; since $\nabla \times (\mathbf{F} + \nabla \psi) = \nabla \times \mathbf{F}$. The additional condition which Maxwell arbitrarily imposed was $\nabla . \mathbf{F} = 0$, but this is not entirely satisfactory.

The "second circuital equation" of Heaviside is obtained by taking the curl of Equation (27)

$$\triangle \times \mathbf{E} = -\frac{\partial}{\partial t} \triangle \times \mathbf{F} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (28)$$

However, it is not within the scope of the present paper to discuss the induction of voltage in parts of a circuit. This Appendix has been added merely by way of correlation with the methods found in such texts on Electricity and Magnetism as those by Maxwell, Livens, and Jeans.

Discussion

E. E. Johnson: Mr. Bewley has very well coordinated many of the ideas relating to the subject of electromagnetic induction, and he has also placed the much discussed sliding contact or substitution of circuit on what appears to be a rational basis. Furthermore, the rules which he has set down for determining whether a unidirectional continuously generated voltage may be generated are sometimes very useful, especially when some inventor presents an ingenious device for generating such voltages.

In Appendix I Mr. Bewley discusses the flux linkage of a circuit as well as the voltage generated in that circuit, and he mentions that if this particular circuit contains n turns, the voltage will be n times as great. This brings up the question of what is a turn. Although the concept of a turn is indispensable to electrical engineering, nevertheless in the application of Equation (1) in Appendix I, it is not necessary to think of a turn because if the integration is properly performed, turns do not enter into the expression.

Mr. Bewley, I think, recognizes this, and he brought out the point of the concept of turns, because we use turns so frequently in engineering practise.

It should be pointed out in connection with Equation (16) that the integration is to be performed in the direction that a right hand screw would have to be turned in order to advance in the direction of positive flux.

L. V. Bewley: As Mr. Johnson suggests, the idea of a "turn" is only an engineering convenience. Mathematically, the concept is unnecessary and rather inconvenient. Some attempt was made to emphasize this point by the statement in the paper following the definition of a turn. I am indebted to Mr. Johnson for pointing out the direction of the positive normal which applies to the equations and to Fig. 7. The origin of the rule for the direction of contour integration is to be found in the basic derivation of Stokes' Theorem.

For uniform isotropic media at rest, the complete specification of the electromagnet field is given by the pair of symmetrical first and second circuital equations of Heaviside. The latter follows directly from Equation (27) of this paper, by taking its curl as follows:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{F}) = -\frac{\partial \mathbf{B}}{\partial t}$$

Progress in High-Tension Underground Cable
Research and Development

BY G. B. SHANKLIN* and G. M. J. MACKAY*
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper deals primarily with the "solid" type paper insulated high-tension cable, the development field still ahead being extensive. A review is given of the progress made during the past four years. The research and development work which made this progress possible is outlined, only the more important features being considered.

Particular attention is given to void formation both in the factory and in the field, and methods of arriving at some understanding of this are demonstrated. The conclusion is drawn that void formation under service conditions is inherent in paper insulated "solid" cables. Methods of reducing the size and duration of voids, and increasing their pressure, thereby reducing the possibility of ionization deterioration, are carefully considered. Oil reservoir feed at the joints is of great value in this respect. This is dealt with in detail.

Research data on ionization and other characteristics of finished cable and cable materials are presented.

* * * * *

Introduction

PRONOUNCED progress has been made in high-tension underground cable practise during the past three or four years, resulting not only in cable of better quality and reliability but also in greatly improved operating conditions.

The main principles upon which this progress has been based are simple and sound. They are not entirely new and, at least in part, have been recognized by cable engineers for some years past. Knowledge of their relative importance, putting these principles effectively into practise, relegation of less important factors, and confirmation by tests and operating experience are comparatively new acquisitions.

The purpose of this paper is to present as briefly as possible a general outline of the research and development work conducted by one of the manufacturers during the past four years and the observations and conclusions drawn therefrom, as well as from operating sources. High-tension cable is not a self sustaining product. Proper methods of installation and operation are of as vital importance to successful service as are proper design and manufacture. It might be said that we have entered a new era in which there is general recognition of the fact that a high-tension cable is not an inert mass but is physically, electrically, and chemically "alive" and must be so regarded throughout its life. Therefore, no general observations and conclusions bearing upon improved cable service and life can be complete without careful consideration of operating conditions and experience.

It is realized that this is an ambitious program for a single paper of reasonable length and conciseness. In the first place, the authors have at their disposal a mass of research and test data accumulated by a number of workers in their company during the past four or five years. As is usually the case a good part of this is merely mute evidence of a groping for the truth, of experiments tried and discarded. There remains however, a very considerable mass of pertinent data bearing upon the conclusions we wish to set forth.

It would be undesirable, even if possible, to present all of these data herein, for much would be merely repetition of recently published work. Other cable engineers have not been idle during this period and a good part of our work necessarily parallels and in many cases confirms theirs. Where such is the case we will refer to their work and present our confirming conclusions without further proof.

Of the remaining data we will attempt to present only those of particular significance as they relate to the general trend of the paper.

Fundamentals of Cable Practise

The present trend of high-tension paper-insulated cable practise is towards the principles so clearly brought out by the liquid oil filled cable.[1] These principles, as we see them today, are almost a complete answer to the nearly insurmountable difficulties experienced with the older types of "solid" cable.

Briefly stated:

(a) Removal from the cable of all impurities such as moisture and gas (both free and in solution).

(b) Elimination of voids and prevention of void formation in service.

(c) Maintenance of positive pressure inside the cable at all times.

These three fundamental principles or rules have been more or less recognized for years and are axiomatic. They are given above in the order of their importance.

The whole trouble is that the older types of "solid" cable fall far short of these rules. Few thought so some years ago, but today it is a recognized fact.

The characteristics of "solid" cable, present or past, are inherently such that these three rules cannot be closely followed. Even the latest and most modern types of "solid" cable, such as we will describe in this

*Both of the General Electric Co., Schenectady, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

1. For references see Bibliography.

paper, represent a compromise in which are incorporated only the more vital features of these fundamentals. The success of the "solid" cable depends upon our ability to distinguish and meet these vital features.

Bearing this in mind we will attempt to rewrite rules (a), (b), and (c) in the form of compromise rules that will meet the inherently complicated characteristics of "solid" cable. No compromise rule can be axiomatic; therefore, there will always be a reasonable and just difference of opinion regarding the best cable practise so long as "solid" type cable is used. The compromise rules, as we see them, are:

(d) Remove all moisture and similar impurities from the cable.

(e) Remove, as far as possible, all gas (both free and in solution).

(f) Eliminate voids initially and, as far as possible, prevent or reduce void formation thereafter.

(g) Prevent ionization deterioration in the voids that do form.

(h) Maintain, as far as possible, positive pressure inside the cable.

(i) Accepting the fact that low pressure will exist in certain sections of the cable, maintain absolute tightness and integrity of outer covering at these points so that air and moisture cannot enter from outside.

(j) Do not expose the filling oil or compound directly to gas in the reservoirs, joints, or cable, attempting at all times to keep this oil as gas-free as possible. Particularly avoid exposure to air or oxygen.

As the paper progresses the reasons for these rules, many of which are obvious, will be brought out.

"Solid" vs. Oil Filled Cable

Internal ionization of voids* or gas spaces is now recognized as a vital and dangerous factor. Most of the progress made recently has resulted from a better understanding of void formation during manufacture, installation, and operation, and the causes and effects of ionization when conditions allow it to exist. More will be said of this later.

Our experience with the 132-kv. liquid oil filled cable has been of inestimable value in clearly bringing out these points. The amount of gas in the cable, the internal pressure, and other factors are under control and subject to exact measurement and calculation.

The so-called "solid" type cable is not adaptable to exact engineering study. The high viscocity of the compound and high, variable resistance to flow, longitudinally and radially, offer serious obstacles. We must confess that even with that learned from the liquid oil filled cable a story of just what goes on inside a "solid" cable during void formation and ionization is not yet completely worked out and probably never will be. It is felt, however, that the right line of attack is being followed and that in time the "solid" cable will be under closer control.

This paper will deal primarily with the "solid" cable, not only because it is the type used for all voltages up to 75 kv., but because the development field ahead of us is still extensive. It is a singular fact that this cable, the only type used for thirty years or more, is still in the developmental stage, whereas the liquid oil filled cable, a very recent product, has yielded almost completely to scientific attack, the remaining work being more in the nature of economy, simplification of fixtures, methods of installation, etc. We do not mean to say that the theory and principles are entirely established, but the gaps yet to be filled appear within reach.

No one can say at present what the ultimate development of "solid" cable will be, nor how far its field will be encroached upon by oil filled cable. No doubt there will evolve a legitimate, economic field for each type, but where the dividing line in voltage and current rating will be, time only can tell.

In this respect it might be pointed out that the safe working temperature range of oil filled cable is considerably more than that of "solid" cable. There are so many factors involved that the limits of both types are yet to be scientifically set, but the trend of present cable knowledge indicates that "solid" cable of the higher voltage ratings should, if anything, have its present temperature limits decreased rather than increased.

Since we are making a distinction between the liquid oil filled and "solid" types of cable perhaps a brief description is in order.

Liquid Oil Filled Cable

The liquid oil filled cable, as its name implies, is filled with a relatively thin oil that remains liquid at all temperatures met with. It is supplied with definite longitudinal channels through which this oil can flow more or less freely. A positive pressure is maintained in these channels at all times after oil filling of the cable is accomplished. During this vacuum and filling treatment all gas is removed and no voids or gas spaces initially exist.

Since pressure is maintained in the channels even during periods of maximum cooling, voids cannot exist for any appreciable length of time in the paper or elsewhere.

The final gas removal and oil filling treatment can be done in the factory on individual reels, or in the field on the line as a whole after splicing work is finished. In either case it is important that positive pressure be maintained at all times thereafter by expansible reservoir feed. In those cases where final filling treatment is given to the factory reels the problem obviously

*The true definition of the word "void" is, empty space free of all matter. In cable engineering the word is used to represent a gas pocket, since a true void cannot exist in cable insulation, due to the nature of the components. Throughout this paper the word void means a gas pocket in which the pressure might be anything from zero to the maximum met with.

remains of installing and splicing these reels without loss of oil pressure, entrance of air, or void formation.

Another very important reason for maintaining positive pressure is that it prevents sucking in of air or moisture at leaky lead pipes or other points of leakage, the cause of many failures in the past in cable of the "solid" type.

"Solid" Filled Cable

There are all varieties and kinds of "solid" filled, paper insulated cable, the characteristics of the filling compound and the degree of impregnation having much to do with their classification. Some are filled with a compound that is a solid at temperatures below 50 deg. cent., others with a compound that is so thick and viscous at the lower temperatures that for all practical purposes it should be classified as a solid, still others having filling compounds remaining a fair liquid even at the lower temperatures. They all, however, have certain points in common.

The compound must have sufficient viscosity and the cable cross section sufficient compactness to prevent "bleeding" or displacement of compound by air during manufacturing process and when the ends are exposed for sweating connectors, jointing work, etc. Also, migration of compound and piling up of pressure at the bottom of slopes must be avoided.

These requirements are the cause of all our difficulties and troubles. They place serious limitations on the cable and are the main points of distinction between the "solid" and oil filled types.

The compound is so viscous and the resistance to flow radially so great that it is impossible to maintain positive pressure at all points of the cross section. It is also impossible to prevent voids. Void formation in some form or other must be accepted as inherent in paper insulated "solid" cable and it should be designed, handled, and operated with this limitation in mind.

It can be seen from the above definitions that fundamentally the difference between the liquid oil filled cable and the "solid" cable is one of degree. It is simply a physical question of the viscosity of the filling compound and the resistance offered to flow both longitudinally and radially.

The fact that void formation and, frequently, pressures below atmospheric exist in "solid" cable explains many disconcerting failures in the past with cables carefully made of materials having highly desirable electrical characteristics such as low dielectric loss, high dielectric strength, high insulation resistivity, etc. Unfortunately, the physical characteristics and behavior were not so well understood and the problem we have today with the "solid" cable is almost entirely a physical one.

Failures of the type just referred to can be placed in two classes. First, those due to excessive void formation and internal ionization, without external help of any sort. We shall deal with these later in more detail.

The other class of failures is primarily due to entrance of air or moisture through accidental leakage. These leaks might be caused by pin holes and flaws in the lead sheath, formed during manufacture, or by punctures, gouges, or abrasions from foreign matter and sharp projections in the ducts. Electrolysis, chemical corrosion, and porous lead wipes have also furnished a source of leakage.

Whatever the cause of leakage, the results are usually the same. During periods of cooling, partial vacuum is formed in the cable. Air, and sometimes moisture, is forced in driving the compound back on each side of the leak, causing ionization to start, oxidizing the compound, forming additional water and acids, and increasing the dielectric loss. Ionization, in turn, attacks the filling compound and paper, generating gas and additional water and leaving a wax deposit, further enlarging and concentrating gas pockets at this spot.* This cumulative effect continues until eventual failure occurs. In the meantime, as the compound is pushed back, disintegration has spread several feet on each side of the point of leakage, depending upon how long this action has gone on before final failure. When the cable is opened up the part affected has a very dry appearance even though initially it might have been well impregnated. Wax deposit is usually prominent throughout.

In final appearance the two classes of failures can be identical under certain conditions, and since the short circuit arc usually destroys any direct evidence of leakage it is very difficult to distinguish clearly between them. In fact, the mechanism of deterioration and final failure is, doubtless, the same in both cases, the only difference being that the entrance of air (and moisture) initiates and accelerates the action in the second case.

Now, these two classes of failures have been gone into for the purpose of pointing out that it is possible greatly to reduce failures of the first class. The remaining part of this paper will deal with this problem.

Failures of the second class, however, can only be reduced, as far as we can see, in one of two ways,—either by eliminating all possible sources of leakage or by maintaining positive pressure inside the cable, so that air and moisture cannot enter and the leaks can be discovered and repaired before failure occurs. Maintenance of positive pressure at all points of a "solid" cable is apparently a physical impossibility. Perhaps someone will find a practical way of doing this some day. In the meantime absolute tightness of the outer cover is the only remedy.

In this respect, we wish to point out that with the modern, non-solidifying filling compounds tightness of the outer covering is of even more importance. Pressures below atmospheric tend to form when the

*This action will be referred to in detail later.

cable is cool and the liquid filling compound offers no obstacle to the entrance of air and moisture at leaks unless there happens to be positive pressure at these points.

Petrolatum Filled Cables

Previous to 1919 practically all American manufacturers used a mineral oil or compound containing various percentages of rosin or rosin oil. These mixtures were quite "tacky" and adhesive, staying in place well and more or less preventing migration and "bleeding." It was not appreciated at the time but the most valuable result of this property was that the compound clung to the walls of the cavities in the form of a good thick film, materially reducing the size of the larger voids where ionization trouble always begins and causing a more even distribution of smaller voids throughout the volume.

Most of these mixtures would be considered too stiff and viscous for use today, especially at the lower temperatures, but in other respects had all of the desired physical characteristics. Electrically, they left much to be desired. The dielectric strength was low and the dielectric loss so high and of such variable quality that frequently the cable would not meet the severe American conditions of duct operation and heavy loading, failures due to accumulative heating occurring.

Pressure was brought to bear upon the manufacturers to reduce dielectric loss. This was undertaken without due regard to void formation and ionization, factors then but little understood and considered of no great importance.

The result was that petrolatum was resorted to as a filling medium, either in pure form or with small amounts of rosin or rosin oil. Dielectric loss was thereby effectively reduced but ionization deterioration was accentuated. It was only after several years of field experience and confirmatory laboratory studies that a clear picture of the situation was formed. During this time the evidence was conflicting. The operating record of some systems was quite satisfactory; others showed an increase in failures, depending upon operating voltage, loading, etc. No yardstick was available for measuring results.

About three years ago sufficient evidence had accumulated upon which reliable conclusions could be based. It was then quite clear that high-voltage petrolatum filled cable was inherently more susceptible to ionization trouble than cables filled with rosin mixtures.

This, in itself, is not an alarming statement even though a large amount of petrolatum cable is in service today. Liquid oil reservoir feed at the joints has in part overcome ionization trouble. On some of the higher voltage systems very satisfactory improvements have been obtained in this way. The operating records are comparable with, and in some cases better than older records, and would not indicate in any way a general widespread collapse of petrolatum cable.

At the time the above conclusions were reached it was felt that the situation did not warrant going back to the use of high loss rosin mixture with the limitation in loading and maximum voltage rating that it sets. Experience had shown that both types of compound had undesirable features and progress demanded that better compounds be developed. These new compounds will be described later, but first we wish to discuss the characteristics of petrolatum.

Chemically and electrically petrolatum is all that could be asked for. It is quite stable, has low dielectric loss, etc., but physically it leaves much to be desired. In the first place, viscosity increases rapidly as the compound cools, the solidifying point being reached between 40 deg. and 50 deg. cent. The viscosity is so high below 50 deg. cent. that the resistance to flow both longitudinally and radially in a cable rises enormously and it is almost impossible to fill up voids that form below this temperature during factory impregnation.

The transition from liquid to solid state is, unfortunately, within the operating temperature range and the compound is constantly subjected to this transition as the temperature varies.

Petrolatum is a typical grease with practically no "tackiness" or film adhesiveness. It will not cling to the walls of the larger cavities during contraction. The result is that during the cooling period these larger cavities are drained dry by capillary attraction and remain in this state, surrounded by a rigid wall of stiff compound and fiber, until the temperature again increases.

It is during this period of large, low pressure voids that ionization trouble often begins in service. The mechanism of deterioration where the conditions are such as to allow sustained ionization to exist is more or less the same regardless of the type of compound used. Apparently, petrolatum does not deteriorate more rapidly than other compounds because of any peculiar chemical or electrical property but simply because its physical characteristics allow relatively large low pressure voids to exist for long periods of time, giving wax formation and gas evolution a good chance to get started and spread.

Improved Type of Compound

An ideal filling compound for "solid" cable would have the following characteristics, as far as our knowledge and experience go:

(a) Very low coefficient of expansion.

(b) Low viscosity at impregnating temperature, increasing to a value just sufficiently high at normal temperatures to prevent "bleeding" during the leading process and during installation in the field, and to prevent migration and building up of excessive pressure at points of low elevation. Solidifying point below the minimum temperature met with in service.

(c) Sticky and adhesive, forming a good strong film.

(d) Low dielectric loss, high insulation resistance, and low corresponding temperature coefficients.

(e) High dielectric strength.

(f) Chemically stable and free of all absorbed gases and other impurities.

In regard to property (a) all liquid compounds have coefficients of expansion within a relatively narrow range and, unfortunately, these are high compared with the coefficients of cellulose fiber, lead, and copper. As far as we know at present little can be gained in this direction.

An all-mineral oil compound, designated as No. 219, was developed about three years ago, having the desirable properties outlined above, with the exception of (a).

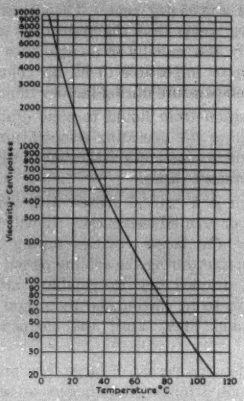


Fig. 1—Viscosity of No. 219 Compounds

The viscosity-temperature curve is shown in Fig. 1. It will be noted that the compound remains a liquid even at the lower temperatures. The property of adhesiveness or ability to form a thick film cannot be

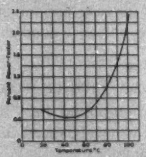


Fig. 2—Dielectric Power-Factor Temperature of Typical Cable at Normal Voltage at 60 Cycles

shown clearly with the type of test data we have available. Comparative tests by dipping strips of treated paper in the different compounds and then allowing them to drain at various temperatures show No. 219 and rosin mixtures to have about equal film adhesiveness, while petrolatum in liquid form leaves no film at all.

Fig. 2 shows a dielectric power factor—temperature curve of a typical cable insulated with wood pulp paper and impregnated with No. 219 compound. The dielectric loss and temperature coefficient are both quite low. A typical resistivity-temperature curve of this compound is shown in Fig. 3. In regard to dielectric strength No. 219 compound has the usual high values, 30 to 35 kv. in the standard 0.1 test gap, that all liquid oils free from impurities show. This test is useful

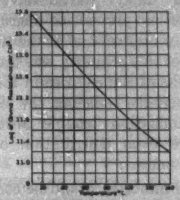


Fig. 3—Resistivity—Temperature

mainly in checking freedom from suspended impurities but is not otherwise a very useful guide.

In Table I are some typical dielectric strength results on unused cable insulated with wood pulp paper

TABLE I
DIELECTRIC STRENGTH

Typical Cable

Single conductor, 750,000 cir. mils, 0.750 in. paper.

Test (a), 25 deg. cent., 60 cycles:

303 kv. for 5 min., 348 kv. for 5 min., failed at 402 kv. in average time of 2 min.

Test (b), 25 deg. cent., 60 cycles:

214 kv. for 5 min., 235 kv., 259 kv., and 285 kv. each for 1 min., failed at 300 kv. in average time of 1 hr., 20 min.

Standard Short Time Test on Cold Bent Samples

25 deg. cent., 60 cycles

Type and size	In. thick	Average kv. break
3 cond. shielded 350,000 cir. mils round.....	0.312 in.	164.
1 cond., 300,000 cir. mils...	0.190 in.	114
1 cond., 250,000 cir. mils...	0.220 in.	134.
1 cond., 200,000 cir. mils...	0.520 in.	252.

and impregnated with No. 219 compound. The increased dielectric strength over older types of cable is not due to the compound alone. Wood pulp paper uniformly and compactly applied is also a contributing factor. The advantages of wood pulp paper are described elsewhere.

Impregnation

Compound and paper of the right properties and quality are the primary requisites for a high-voltage cable of high initial dielectric strength. There are other factors, however, of equal importance as regards quality of finished cable and durable service which we wish to discuss briefly in their proper order. Of these, impregnation should come first.

A great deal of improvement in impregnation has been accomplished in recent years. In addition to proper compound and paper, the carefully worked out process has added much. Exact control of temperature, more thorough removal of residual gas and moisture from the cable and compound by special process, time and method of soaking, prevention of bleeding during the leading process, etc., have all contributed towards more perfect and uniform impregnation.

To give an idea of this improvement, power factor—voltage curves for a typical unused cable are shown in Fig. 4. There is only slight indication of ionization over a wide range of voltage stress.

It can be safely stated that it is now possible to produce cable which, for all practical purposes, can be regarded as having perfect impregnation, initially, as shipped from the factory. Actually, there is a small amount of gas and void formation present but compared with the amount formed later during installation and service and also compared with the degree of impregnation obtained a few years ago with petrolatum cable, present impregnation can be considered almost perfect, provided, of course, that no incidental mistakes are made in routine manufacture.

Present manufacturing process and methods have greatly decreased such mistakes. This, together with

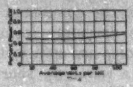


Fig. 4—Ionization Curve of Typical Cable at Room Temperature—60 Cycles

careful inspection and acceptance tests, has reduced the probability of trouble from this source to a very small amount.

It is impossible, by any method we know of, to determine directly this small initial total of voids or occluded gas* volume in "solid" cable of the present type, as shipped from the factory. By indirect methods, as described by Emanueli[1,2] and from theoretical considerations we estimate the total initial void volume at atmospheric pressure, 20 deg. cent. as between 0.10 and 0.20 per cent of the volume occupied by filling compound and fiber.

It is our opinion that this small total percentage of void volume consists of two components, one being the gas normally sealed in the cellular structure of the fiber and the other representing the residual gas from impregnation and leading process. The last mentioned void volume is a variable quantity, depending upon the temperature range and handling to which the cable is subjected before shipment. It can be more definitely controlled and reduced by attaching an expansible oil reservoir to each reel of cable after leading and leaving this attached during shipment, as is done with the oil filled cable. We are not convinced that sufficient benefit is derived thereby to warrant the extra expense, in view of the greater void formation during installation and service.

This question is worthy of further study; also, whether reservoir feed during installation and jointing is warranted, this being a practise learned from oil filled cable. If the cable cannot be held within a comparatively narrow temperature range during shipment and installation, reservoirs would undoubtedly be of real service.

The question of reservoir feed during operation is, to our minds, already answered by wide experience. It is unquestionably of very material benefit, although some doubts regarding its use are not yet entirely cleared up. We wish to deal with reservoir feed in more detail later.

It has been suggested frequently by engineers that perhaps perfect impregnation is not desirable—that it may be better purposely to leave a certain amount of gas in the cable to act as a cushion and assist in maintaining more uniform pressure. This reasoning is right in principle up to a certain point. The gas must, first of all, be kept out of the field of stress and be confined to electrostatically shielded points such as the space between conductor strands and the filler spaces of shielded three-conductor cable.

The real difficulty lies in initially confining the gas to these spaces. We have learned from past experience that entrapped gas is not under control. It cannot be confined to desired points and is just as liable to concentrate at random in sections of the cable length, causing the troublesome so-called "dry spots." We doubt that any manufacturer would court trouble by deliberately leaving gas in the cable. Perfect impregnation must always be the goal aimed at.

Another objection to leaving free gas in the cable is that it will not be permanently maintained in this state unless the exposed compound is gas saturated. Now, gas saturated compound makes it difficult to close up additional voids as they are formed in service. It is much better to have the compound originally gas free so that voids which form will be more quickly reduced by reservoir feed from the joints before ionization damage has a chance to get started.

The principle of a more elastic "solid" cable maintained under pressure is correct but unless practical methods can be devised for definitely segregating the elastic medium (gas) from the compound,* or developing a more elastic sheath,* we should rely upon other ways of accomplishing these results.

*The word "void" as used in this paper represents volumetric dimension and "occluded gas" represents the amount of free gas by weight that fills the void. A high pressure void would contain much more gas than a low pressure one of the same dimension.

*Study is being given to this and several designs have been considered. Economy has so far proved to be one of the chief obstacles.

Formation of Voids in Service

To assist in obtaining some idea of the formation of voids in service, we will take a specific example. The cable considered (Cable No. 1) will be a 66-kv. single-conductor cable of the following specifications:

Cable No. 1

Cond.—750,000 cir. mils stranded (no core).
Insulation—24/32 in. impregnated paper.
Sheath—9/64 in. lead.
Total area of cond.—0.785 sq. in.
Area of copper—0.588 sq. in.
Area of space between strands—0.197 sq. in.
Total area insulation—4.125 sq. in.
Area of paper fiber—1.650 sq. in.
Area of space between fibers—2.475 sq. in.
Area of lead sheath—1.160 sq. in.

The conditions of operation will be:

(a) Six cables (No. 1) in duct bank, installed at temperature of 20 deg. cent.

(b) Max. load, copper temp. 65 deg. cent., sheath 50 deg. cent. with ambient earth temp. 20 deg. cent.

(c) Load dropped from all cables, which eventually cool down to ambient earth temperature.

As a first step and to simplify the problem we will assume that:

First Step Assumptions

(a) The cable as installed, before load is applied, is perfectly impregnated with no voids present.

(b) Lead sheath has no elasticity whatever.

With ambient earth temperature still at 20 deg. cent., full load will be applied and the amount of expansion calculated. The volumetric expansion coefficients of the various components are taken as:

Lead	—0.000085
Copper	—0.000050
Compound	—0.0007
Fiber	—0.00009

The volumetric expansion, considering only the cross section is:

Copper	0.00132 sq. in.
Oil in strand space	0.00620 sq. in.
Paper fiber	0.00535 sq. in.
Oil in paper	0.06230 sq. in.
Total	0.07517 sq. in.
Exp. of sheath	0.01250 sq. in.
Stretching of sheath	0.06267 sq. in.

If load is dropped and the cable allowed to cool back to 20 deg. cent. a total void formation of 0.0627 sq. in. will exist, since it is assumed that the sheath has no elasticity. This is equivalent to 1.5 per cent of the space normally occupied by compound and fiber.

When load is applied the first time the pressure, of course, increases rapidly until the sheath begins to stretch. It is difficult to measure this pressure accurately for it is a variable function of time and temperature. The maximum pressure reached is in the order of 50 to 60 lb. per sq. in. directly under the lead sheath. It is very sensitive to even slight temperature fluctuations but if the full load temperature could be held steady for a sufficient length of time the pressure would drop gradually and become constant at about 15 to 20 lb. which is, as far as we can determine, the "set point" for the sheath under the conditions as given.

When load is dropped the first time pressure is lost rapidly and a vacuum forms. Assuming no leaks in the system and an absolutely gas-free, non-volatile compound, the voids would be at zero pressure, absolute, and remain so. This is never the case, however, gas being set free and, in consequence, the pressure builds up. The final recovery of pressure would depend upon the degree of gas saturation, stability of compound, and other factors. We regret that no qualitative test data are available with different degrees of gas saturation. It is an example of the large amount of uncompleted work relating to "solid" cable.

Tests show that the recovered pressure would generally fall short of atmospheric at all points and would be very low unless, as we suspect, the following action occurs.

When voids are at very low pressures and the stress above a critical value, ionization occurs, generating or freeing gas from the compound and increasing the pressure until the discharge is automatically extinguished. If the voids are sufficiently small ionization is extinguished in a relatively short while, the action being quite feeble. This will be dealt with in more detail later. We feel that in view of the very low vapor tension of the compound and from tests and other considerations that some such automatic partial recovery of pressure in the voids must take place. If this theory is correct a "solid" cable cannot remain entirely gas free for a very long while in service but probably never becomes entirely gas saturated unless "sustained" ionization of an entirely different and more severe nature takes place. This type of ionization causes the deterioration so frequently found. For this to take place the voids must be sufficiently large and the stress sufficiently high to sustain gas discharge even at relatively high pressure. This also will be dealt with later.

The ability to regain or retain gas pressure involves desirable and undesirable features. To illustrate this let us assume that Cable No. 1, which has now been through one heat cycle and has a total void volume of 0.0627 sq. in. at 20 deg. cent. is filled with a gas saturated, unstable compound. We will assume further that this allows the pressure to return to atmospheric, the cable still being at 20 deg. cent.

If full load is then suddenly applied the cable will heat up more rapidly than the gas can be re-absorbed. Assuming for the moment no re-absorption and a "short-time" elastic limit of 50 lb. per sq. in. for the

sheath the voids would be compressed to 23 per cent of their original volume by the time 50 lb. pressure is reached. The remaining expansion of compound would have to be taken care of by an additional permanent stretching of sheath equivalent to 23 per cent of the original amount, giving a total void volume for two complete heat cycles of 0.077 sq. in.

As additional heat cycles are gone through it is easy to show that progressively less stretching of sheath occurs until a final balance is reached at about the sixth cycle with a total void volume of 0.081 sq. in., equivalent to 1.9 per cent of the space normally occupied by compound and fiber.

This calculation ignores two important factors, the rate of re-absorption of gas by compound and the rate of "viscous flow" of lead sheath between the "short time" elastic limit of 50 lb. and the "set point" of 15 to 20 lb. We have no final data that would allow the inclusion of these two factors but since they must surely cancel each other to some extent the calculation is probably not too greatly in error. Possibly, with a freely "gassing" compound the sheath would be stretched more than this before the final balance is reached.

Assuming no re-absorption at all the sheath would be stretched an additional amount (1.6 per cent) while the pressure is dropping from 50 to 20 lb., giving now a total void volume of 3.5 per cent. This is the absolute maximum that could be reached. We do not believe it would be reached, for some re-absorption of gas must occur during the long period of "viscous flow" of the sheath.

Referring back to the calculation for the first heat cycle the sheath was stretched an amount equal to 1.5 per cent of the total insulation volume. This is equivalent to a radial sheath movement of only 7.0 mils and it was assumed that none of this was recovered after cooling. In the first place the cable is not a perfect cylinder and when the pressure inside drops below atmospheric there is a certain amount of response from the sheath tending to flatten out the distorted areas. This causes some closing of voids. The action is a complicated function of time, temperature, pressure, and distortion of sheath. Our work merely indicates roughly that it exists. We cannot yet assign numerical values.

Considering re-absorption and possible sheath elasticity, we feel justified in taking an average between the limits of 1.9 per cent and 3.5 per cent, giving a maximum possible void volume of 3.0 per cent for a gas saturated cable operating under the conditions given without reservoir feed.

Likewise, we feel justified in reducing the void volume for a gas-free cable from 1.5 per cent to 1.3 per cent to compensate slightly for sheath elasticity.

These observations and calculations are very illuminating when viewed in perspective. They point to several interesting conclusions which will be given in order.

(a) If a cable without reservoir feed, as initially installed, has a degree of impregnation equal to or better than 1.3 per cent total void volume (assuming these voids uniformly distributed along the cable length) its final void volume after "settling down" in service will be between 1.3 per cent and 3 per cent at 20 deg. cent., depending upon the degree of gas saturation. This is based on the assumption that the copper temperature never exceeds 65 deg. cent.

(b) Factory impregnation would have to be of very high order if 1.3 per cent is not to be exceeded in installed cable. Initial impregnation as near perfect as possible is, therefore, very important.

(c) The pressure in the voids at 20 deg. cent. after the cable has settled down might, theoretically, be anything from zero (absolute) to atmospheric, depending upon the degree of gas saturation and the action of temporary ionization.

(d) All other things being equal, any benefit derived from higher gas pressure is counteracted more or less by the voids being larger.

In addition to (d), another and more practical consideration rules out the saturated compound. It is impossible, by any methods we know, to obtain reliably and uniformly the high degree of initial impregnation required.

Ionization in Service

We have shown in an elementary and approximate way what goes on inside a cable during void formation in service. We have also given a measure of the total amount of void formation to be expected. The question naturally arises as to what relation this might have to the possibility of ionization damage in service.

In approaching this subject we wish to point out that our assumptions are based on a representative, compact cable cross section and a "stringy," adhesive compound that will cause a fairly uniform distribution of voids. If voids concentrate in the cable causing "dry spots," or if due to loose structure relatively large voids form at points of high electrical stress these assumptions do not hold.

The best method of determining the relation between total void volume and ionization can be found in Emanueli's published work.[1] We take the privilege of extrapolating and tabulating this in the form given below:

TABLE II

Impregnated paper insulated cable.

P. F. DIFF. = Difference in power factor from 20 volts per mil to 100 volts per mil, 60 cycles.

% VOID = Total void volume in per cent of insulation volume.

Atmospheric pressure, 20 deg. cent.			
P. F. diff.	% void	P. F. diff.	% void
0.25	0.4	2.0	3.2
0.50	0.8	2.5	4.0
0.75	1.2	3.0	4.8
1.0	1.6	4.0	6.4
1.5	2.4	5.0	8.0

Emanueli's tests were made on a single-conductor oil filled cable with small void volume and of the same general cross section as Cable No. 1, except that it had a hollow core. Presumably, the distribution of voids would be practically the same in Cable No. 1 and his results should hold fairly well in spite of rather broad extrapolation. At any rate, theoretical calculations check Table II surprisingly well. Voids have a tendency to collect (radially) in the largest available spaces. In this case, these spaces are the strand space and that directly under the lead sheath. Doubtless a good proportion of the void volume would be found here and since the strand space is electrostatically shielded and the space directly under the lead sheath is at a point of relatively low stress they do not contribute very materially to the measured ionization loss.

Table No. II would not hold so closely for ordinary three-conductor cable nor for shielded type three-conductor cable because of the different distribution of voids and electrical stress. Similar data are needed for these two types. A fair part of the total void volume of a three-conductor cable seems to concentrate in the filler spaces, forming relatively large voids. In an ordinary three-conductor cable these points are subjected to relatively high electrical stress. Therefore, we feel that test results would show a smaller total void volume than that in Table II for the same ionization loss. In other words, the loose structure is less favorable from an ionization standpoint. The shielded type three-conductor cable, on the other hand, should, if anything, show a larger total void volume than Table II because the filler space is electrostatically shielded. Shielded cable is a great improvement over ordinary three-conductor cable in this respect.

Following this same idea a shield could be placed directly under the sheath of single-conductor cable. It has been tried but its superiority has not yet been demonstrated. We believe it has possibilities in those cases where the sheath is stretched in service.

Standard specifications for single-conductor cable call for a maximum difference in power factor of 0.75 on factory test samples, which from Table II is equivalent to 1.2 per cent void volume. A great deal more experience and study are needed before actual safe limits of power factor difference can be definitely determined. There is no reason to believe, judging from operating experience, that the above rule is not conservative for properly designed cable. At any rate, it is interesting to note that the void volume of 1.6 per cent which this rule allows in the factory falls below the range of the limits, 1.3 per cent to 3.0 per cent, calculated for installed cable without reservoir feed. The data in Table II are based on short samples with voids at atmospheric pressure, whereas the installed 20 deg. cent. void pressure is probably higher or lower, depending upon conditions.

On the foregoing basis we might assume that the cable should operate safely without reservoir feed provided migration does not occur and the temperature cycle never exceeds a range from 20 deg. cent. to 65 deg. cent. Lower temperatures must be met in the winter, unfortunately.

Low Temperature Operation

Carrying the foregoing calculations a little further by assuming an extreme minimum no load, winter temperature of 0 deg. cent. it is easy to show that the void volume increases to 2.1 per cent for the unsaturated cable and about 4.0 per cent for the saturated cable. Furthermore, the pressure in these voids drops very materially. It is during this period of low temperature operation that accumulative deterioration from ionization gets a good start. Whether ionization damage would occur in either of the two cables considered we cannot say. One thing is certain, however,—if ionization damage occurs at all it will get its start during this period of low temperature, low pressure operation at no load. It is the danger period for high tension cable and every precaution should be taken to guard against it.

Possible safeguards are listed below:

(a) Take the line out of service during periods of no load in winter.

(b) Maintain a higher temperature in the conduit run by always having some cables loaded.

(c) Reduce the maximum allowable copper temperature range both for summer and winter loading.

(d) Use a compound that remains a fair liquid even at the lowest temperature met with.

Reservoir Feed for "Solid" Cable

Reservoir feed on high-tension cable lines is growing rapidly in this country although certain objections have been raised from time to time. Aside from cost the chief objections have been:

(a) The reservoir oil contaminates the cable filling compound increasing dielectric loss and lowering dielectric strength.

(b) Slow percolation of oil into the cable might cause progressive stretching of sheath and ultimate rupture due to rapid expansion when load is applied.

As far as (a) is concerned we feel sure that, provided the proper grade of oil is used, the contamination that is sometimes found is due to the occlusion or entrance of moisture, oxygen, or both into the reservoir, joint, or cable. A thin, high grade, low freezing point mineral oil should be used. The viscosity of a typical oil of this kind (Oil No. 2) is given in Fig. 6. Wide experience has been had with this oil in intimate contact with many types of insulating materials, including rosin, petrolatum, and the modern cylinder oil cable compounds. In no case has deterioration been found under proper conditions of use.

The harmful results of moisture are apparent. Equivalent results from the presence of oxygen are usually ignored. The cable ends during jointing are exposed to air for a long time and no attempt is made to remove

the absorbed oxygen afterwards. Often, also, the reservoir oil is maintained in direct contact with air, acting as an agent for transmitting an unlimited supply of oxygen into the cable. And yet oxidation of oil under heat with resulting formation of water and acids is widely recognized. It cannot be said how serious this action might eventually be in service but it surely increases dielectric loss to some extent. Riley's paper[3] and the discussion which followed deals with this subject in a most interesting way.

In regard to (b) reservoir feed has now been generally used for several years by some of the operating companies. During this time there has been no noticeable indication of a general swelling of cable sheaths. On the contrary, only a few isolated cases of ruptured sheath have occurred, to our knowledge, and in every one of these cases it was clearly accounted for by excessive pressure at the bottom of vertical risers, or by defective or badly damaged sheath.

Theoretically, it is possible under certain conditions for progressive swelling of sheath to occur within certain limits. Accepting, for the moment, that it does actually occur, no greater proof of the value of reservoir feed is needed for it would prove that the oil surmounted all obstacles and completely penetrated the cable, increasing the degree of impregnation to a high order. It is more than doubtful that such perfect impregnation can be obtained in this way unless the cable is initially well impregnated.

It has already been shown that impregnation must be better than 1.3 per cent to 3.0 per cent (depending upon the occluded gas pressure) before stretching of sheath occurs when load is applied. This is based on the assumption of no reservoir feed. A better degree of impregnation is allowable if reservoir feed is used. This will be demonstrated.

Assume, for purposes of discussion, that oil reservoir feed so thoroughly saturates cable that at some time in the future bursting of cable sheaths becomes pronounced. The remedy in this case is simple and consists of shutting off the reservoirs from the joints while the cable is partly loaded. The reservoirs can be permanently removed and used elsewhere or they can be left in place and used during certain periods for maintaining good impregnation. The line should surely be in better condition than if reservoir feed had not been used at all.

When it is considered that many cable lines in service today have an impregnation in the order of 5.0 to 8.0 per cent and doubtless there are others of even poorer impregnation, it is apparent that reservoir feed has a long way to go on these lines before any question of bursting sheaths will arise.

This statement is based on observation, records of oil absorption where reservoir feed is used, and examination and tests of removed samples. Responsibility for this condition should be shared by both the manufacturers and operators.

Poor factory impregnation and wrong materials have contributed part, rough handling during installation, excessive loading, and short circuit heating have contributed the rest. We know of cases (much rarer today) where short circuit temperatures as high as 150 deg. cent. have occurred. Even a single short circuit of this character would by stretching the sheath radially cause a void volume of 4 per cent or more.

It is not, however, radial stretching of sheath that causes the worst cases of void formation. Longitudinal stretching, forming the characteristic annular, wavy ridges so often found on the sheaths of old cable, is more to blame. This is caused by rough handling, excessive loading cycles, and short circuit heating.

To illustrate this, suppose both the copper and lead of a single conductor cable were at 20 deg. cent., no load. Suppose, further, that the copper heated suddenly to 100 deg. cent. during short circuit while the lead remained practically at 20 deg. cent. Ignoring the effect of stranding the copper would expand 16 in. per 1000 ft. stretching the sheath this amount. Upon cooling back to 20 deg. cent. this 16 in. of excess sheath would form itself into characteristic annular ridges, increasing the total void volume very materially. Migration and re-arrangement of voids would result in the "dry" cable so often found.

Returning to reservoir feed, there is every reason to believe that bursting sheaths in general will never be encountered. It is more probable that a final balance will be reached after the cable line "settles down" to a certain average degree of impregnation after it has absorbed all the oil that it can and that, thereafter, seasonal cycles will be gone through (upon which daily cycles will be more or less imposed) in which the cable will return to the reservoirs the same amount of oil as received. Records kept by the operating companies show a decided trend in this direction and theoretical considerations point the same way.

Since solid cable offers an extremely high resistance to oil flow, reservoirs should be connected along the cable length as frequently as possible; that is, at every joint and the shorter the distance between joints the better from this standpoint.

In the past it has been customary to use very small expansible reservoirs, *re-filling these as the cable absorbed oil.* The modern cable is initially so well impregnated with liquid compound that this practise would be questionable. This type of cable extrudes compound under load in a comparatively short time. If the amount extruded exceeded the available reservoir capacity an excessive pressure would result, bursting the reservoir or expanding the sheath. If this "pumping in" of oil continued long enough the sheath would eventually burst. *Reservoirs of proper capacity and adjustment are most important.*

Reservoir feed is of very material help in preventing so-called migration or formation of "dry spots" in the cable length.

Aside from the benefit to the cable proper liquid oil filling has proved the real solution of the high-tension cable joint problem. Such joints are more uniform in quality and much more reliable than joints filled with solid or viscous compound, particularly when a small positive oil pressure is maintained. The majority of leaks experienced in underground service occurs at or adjacent to the joint, due to porous lead wipes, screw fittings, etc. Positive oil pressure furnishes an opportunity to discover and repair these leaks before damage is done.

Additional Conclusions Regarding Reservoir Feed

Summed up briefly, some of the additional conclusions we have reached regarding reservoir feed after several years study and observations are:

The pressure at different points in the cable cross section varies from 0 to 125 lb. (absolute) or more during temperature cycles. The duration of these extremes of pressure can be controlled and reduced, but there is no known way of materially reducing the range throughout the paper thickness of solid cables. The pressure range is smallest at those points where the largest voids tend to collect. If the cable has an excess of gas, these larger voids can maintain some pressure, the range depending upon the size of the individual voids and the amount of gas present. The more gas there is in the cable the larger the voids in these positions tend to be, the type of compound used also being a governing factor. Ionization deterioration (when it occurs) is always found in these large voids; therefore reduction in their size and incidentally increase in their pressure, are highly desirable. The larger voids form in those positions that later, after the line settles down, are the main channels or arteries for reservoir feed. If oil penetrates these channels, the voids are reduced in size and ionization is less likely to occur. Excessive stretching of sheath seems improbable simply because a balance is finally reached after some sheath stretching of well impregnated, liquid compound cable and because reservoir feed cannot impart or maintain a sufficiently high degree of impregnation in cables filled with solidifying compound. In those cables having solidifying filling compound, gradual percolation of thin oil from the reservoirs "softens up" the compound, making it possible to reduce voids more easily, although never to the extent reached with liquid compounds.

Reservoir Feed as a Function of Time

A question that naturally arises regarding reservoir feed is concerned with the penetration of the thin oil into the cable and the transmission of oil pressure as related to distance and time. A final answer to this complicated problem with substantiating data is not available but progress has been made and some light thrown on the subject.

The behavior of "solid" cable in this respect varies widely and depends upon the initial degree of impregnation, the physical characteristics of the filling compound, the compactness of the physical structure, and the character of the temperature fluctuations.

As the cable cools transmission of oil and pressure from the reservoirs are accomplished by a combination of two actions. First, an infiltration or percolation of thin oil through the channels that are formed in the larger spaces. The main channels or arteries are in a general longitudinal direction in the strand space, directly under the sheath and in the filler spaces of three-conductor cable. The second action is a physical pushing back of the heavier compound, mostly in a radial direction, tending to compress the voids in the paper insulation and enlarge the main arteries. A maze of small capillary radial branches to the main arteries are sometimes formed in the paper, particularly in cable filled with petrolatum or other solidifying compound.

This action continues to penetrate the cable length as temperature cycles are gone through, searching out its own path and establishing and enlarging main channels of oil flow as it proceeds. Voids are reduced in this way partly by absorption of gas as the pressure builds up and partly by compression.

Finally a balance is reached after which there is a cyclic exchange of oil between reservoir and cable, depending upon the final degree of impregnation. The length of time required to reach this balance depends among other things upon the type of compound used, degree of initial impregnation, distance between reservoirs, cable contour, rate of sheath stretching, the difference in reservoir pressure and void pressure, rate of gas absorption and, finally, rate of gas generation and other effects of ionization. There are so many factors involved that no one can predict when this balance will be reached. Furthermore, the word "balance" is used in a relative sense. Change in any one of the many factors might upset this. Probably an exact 24-hr. cyclic exchange of oil will never actually exist but only an approximation of this.

Cable lines having reservoir feed are operating on long slopes of as much as 2 per cent grade without any apparent building up of head pressure, or appreciable transfer of oil from one reservoir to another. Where this might occur it can be easily taken care of by balanced pressure reservoir feed. Stop joints could be used to break up the head pressure, although this has so far proved unnecessary except in one or two cases where vertical risers existed. The balanced reservoir feed system will be described in a separate paper soon to be published.

The above theory regarding the penetration of oil into a cable is based on tests and examination of cable lengths removed from service. The additional theory of final balance and cyclic exchange of oil is not fully proved. The calculation of reservoir feed is very complicated. We will attempt to show the headway that has so far been made.

Calculation of Reservoir Feed

As in the preceding calculations, a start will be made by assuming an initially perfectly impregnated 66-kv. single conductor cable (Cable No. 1). The conditions of operation will be the same as previously, 20 deg. cent. earth, six loaded cables, etc., except that the steady load per circuit of three cables will be 45,000 kv-a., giving a copper temperature of 52.5 deg. cent.

When load is dropped from all six cables they eventually cool down to earth temperature (20 deg. cent.). During this cooling voids are formed. The rate of void formation without reservoir feed in a constant temperature medium can be calculated theoretically with good accuracy. The mathematics involve Bessel functions with the final solution involving Heaviside formulas. Mr. K. M. Miller demonstrated the method of attack in discussing the paper on 132,000-volt cable.[1] Messrs. F. H. Buller and R. H. Park independently developed the same method and carried it to a final solution. This work will soon be published. In view of this and the limitations of space the mathematics will not be dealt with, only the calculated results being given.

The calculations are based on a representative conduit cooling curve, the cables reaching earth temperature practically 72 hr. after load is dropped. The results are given in Fig. 5, and as curve *a* was carried only to a time of 110 min. where cooling is far from complete, the behavior during the first hour of time is shown clearly. Beyond this time the cooling curve approaches the zero line in the form of a simple exponential, the complicated harmonics having disappeared. By the end of two days the rate of void formation is negligible. The total area bounded by this curve represents the total volume of voids (0.577 cu. in. per ft. or 1.1 per cent). This can also be calculated by the simpler method already described.

Curve *a* is practically independent of the type of filling compound used, provided the cable is initially well filled. It represents fairly well the rate of void formation under the given conditions for all types of impregnating compound.

If perfect oil feed is obtained curve *a* also represents the rate of oil feed but due to the flow resistance offered the volume of oil fed into the paper usually lags behind the volume of voids, the amount of lag depending upon the viscosity of the oil, the compactness of wrappings, and the pressure in the feed channels.

We are developing similar methods for theoretically calculating oil feed curves. The problem is very complicated and a solution must be based on approximately assumed laws of oil flow in capillary passages and rate of gas absorption. The degree of accuracy reached in curve *a* can never be expected for calculated oil feed curves, but such calculations will help us to interpret measured results more accurately.

An actual measured oil fed curve furnishes the best method of getting at this problem but here, again, we run into difficulties. It is not feasible to measure an oil feed curve on a cable containing a compound as viscous as No. 219. The rate of feed is too slow for accurate measurement and there are other inherent objections. Such curves can, however, be taken on short cable samples filled with less viscous oil. Curve *b* in Fig. 5 represents a measured curve of oil feed for a cable similar to No. 1 but filled with a thinner oil, designated as Oil No. 1, the viscosity of which is shown in Fig. 6. The curve is an allowable correction of measured results in still air to correspond with the same typical conduit cooling conditions selected for curve *a*. Atmospheric pressure was maintained on the oil in the conductor channels throughout the test by use of a hollow core. The sheath was slightly loosened during the treating process but it is improbable that atmospheric pressure was maintained at this point. Curve *b* represents the radial flow of oil into the insulation thickness at constant pressure. We will first consider radial feed and show later the effects of longitudinal feed. It should be understood that no impossible attempt

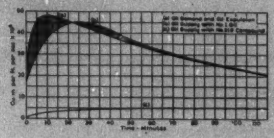


Fig. 5—Cable No. 1, Two Circuits at 45,000 Kv-a. Per Circuit

will be made to calculate oil feed for a cable filled with solidifying compound but only for cable filled with liquid compound of known viscosity.

Returning to Fig. 5, curve *a* will be called the oil demand curve and curve *b* the oil supply curve. Their close agreement even where such small quantities are involved, is not an accident,—it was done purposely. As a matter of fact, the first theoretical calculation of the demand curve might fall above or below the measured supply curve within narrow limits, as determined by the relative values of thermal resistivity, thermal capacity, and coefficients of expansion selected from the known range of such measured values for this type of impregnated paper. There is only one possible combination, however, that will "fit" the calculated demand curve to the measured supply curve and still take care of the known physical conditions, these conditions being that any voids which form during cooling must eventually be closed up, since no voltage stress that might generate gas was applied during the test and the gas saturation point was far lower than atmospheric pressure at 20 deg. cent.

Further work might account for the differences of curves *a* and *b* in some other way but we believe it is due to the supply lagging behind the demand during the first 22 min., thus forming voids. The wall of

compact paper insulation is so thick (24/32 in.) that the oil has a hard time penetrating it with such low pressure in the channels and such short periods of time. Supply curves measured at different pressures in the feed channels agree with this assumption. We are continuing this phase of the study and will publish the details later when more results are obtained with oils of different viscosities. In the meantime the tentative conclusion is offered that this difference is due to void formation.

The area between curves *a* and *b* up to 22 minutes of time represents the maximum volume of voids that are formed. From then on the supply is greater than the demand and the voids are gradually closed up. The area between the curves after they cross must equal the area before they cross.

From the behavior of curves *a* and *b* it appears very probable that at the point and moment each void begins to form there is an extremely low pressure. Gas is then drawn from the compound, filling the void and increasing the pressure, depending upon the amount of gas in solution or the vapor tension of the materials. When the voids begin to reduce in size the pressure increases and the gas is gradually re-absorbed. This would account for the relatively long and slowly decreasing "tail" of the area between the two curves. If this theory is correct the pressure in the voids should be practically atmospheric during the latter part of the "tail" period.

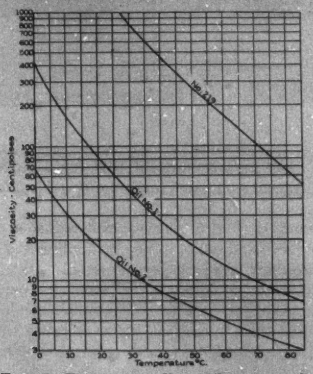


Fig. 6—Viscosity of Three Different Oils

This interpretation of curves *a* and *b* gives us an empirical measure of the rate of gas re-absorption under the conditions that actually exist in the cable.

A study of Fig. 5 is quite interesting. It shows that with a sufficiently thin oil and a compact cross section the voids which form are relatively microscopic in size and are quickly closed up. With a still thinner oil or a sufficiently high pressure practically no voids would form at all, the supply exactly following the demand.

Curves *a* and *b* in Fig. 5 can be used for estimating an oil supply curve for No. 219 oil, provided the relative viscosities and flow resistivities are known. The relation between flow resistivity and viscosity for the type of oil and paper we are considering is shown in Fig. 7. The measurements were made on stacks of carefully impregnated wood pulp paper sheets having parallel slits and arranged to simulate butt tape wrappings. Two thicknesses, 0.05 and 0.3 in., were used. One side of the stack was in oil at atmospheric pressure and vacuum drawn on the other side. From Fig. 7 it will be seen that flow resistivity is approximately proportional to viscosity. Space does not allow presentation of other data bearing on this subject.

From Figs. 5, 6, and 7 oil supply curve *c* in Fig. 8 was estimated for No. 219 oil. The mathematics of oil supply are not yet developed to a point where the accuracy of curve *c* can be vouched for. We believe, however, that it is reasonably close for the purpose of this paper. The oil demand curve *a* is replotted in Fig. 8 for comparison. The radical difference in total void formation and time lag between supply and demand for a heavy oil as compared with a moderately light oil is apparent from Figs. 5 and 8, one plotted in minutes and the other in hours. A short section of curve *c* is re-plotted in Fig. 5 to show this great difference.

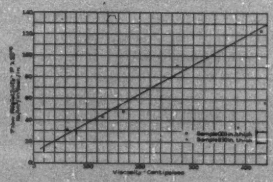


Fig. 7—Flow Resistivity—Viscosity of 0.005 in. Wood Pulp Paper Impregnated with and in Bath of No. 219 Oil

With No. 219 oil (ignoring possible mixture in service of thin oil from the reservoirs) maximum void formation (0.47 per cent) is not reached until 6 hr. have elapsed and voids are not reduced to a negligible value until 52 hr. have passed. Quite obviously with a solidifying compound, voids that form during cooling are never reduced to a negligible value except, possibly, after thin oil from the reservoirs has mixed with the compound and softened it. This should require an excessively long time for the two materials do not mix well below 60 deg. cent.

The demand and supply data in Fig. 8 were re-calculated to represent volumetric variation with time, as shown in Fig. 9, curves *a* and *c*. A similar volumetric supply curve *b* is given in Fig. 9 for Oil No. 1. There is such close agreement between curves *a* and *b* that the difference can hardly be noted. Fig. 9 ex-

plains at a glance the great superiority of oil filled cable over solid cable. Curve *b* might be called the "worst" condition for oil filled cable because it is not difficult to maintain a pressure higher than atmospheric in the feed channels if desired. Curve *c*, on the other hand, represents the "best" condition for solid cable. It is impracticable to maintain atmospheric pressure

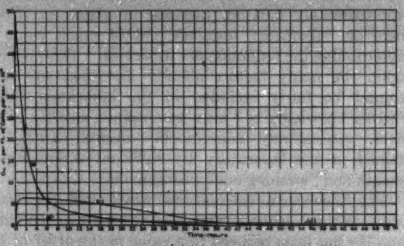


Fig. 8—Cable No. 1, Two Circuits 45,000 Kv-a. per Circuit

(a) Oil demand and oil expulsion
(c) Oil supply with No. 219 oil, 20 deg. cent. earth
(d) Oil supply with No. 219 oil, 0 deg. cent. earth
Atmospheric pressure in channels

in the feed channels throughout the whole length of cable during cooling. At those points where the pressure drops below atmospheric there will be greater void formation and more time lag. Also No. 219 oil is about the thinnest that can be used in solid cable.

Other Conditions of Service

With atmospheric pressure assumed as maintained in the feed channels by oil reservoirs, we now have a fair picture of what goes on in a solid cable of perfect initial impregnation when steady summer load is dropped and the cable allowed to cool down to earth temperature. During cooling voids gradually increase to a maximum value and then decrease, so that after about 52 hr. time the cable is again brought back to perfect impregnation, assuming no gas generated by ionization in the meantime. This same cable will now be briefly taken through other limiting conditions of service, one step at a time, as listed below. Figs. 6, 7, 8, and 9 will be used as a foundation for this study.

(a) Correction for longitudinal feed.
(b) Dropping full load in summer.
(c) Dropping full load in winter.
(d) Applying full load in summer.
(e) Applying full load in winter.
(f) Effect of daily load cycles.
(g) Effect of seasonal temperature cycles.
(h) Effect of initially poor impregnation.

(*a*) *Correction for Longitudinal Feed.* Longitudinal flow resistance is usually conceived as being, inherently, the determining factor of void formation in service, even in those cables filled with liquid compound. As a matter of fact, radial flow resistance is more of a determining factor. Longitudinal resistance is automatically lowered during operation of initially well impregnated solid cable by stretching of sheath and entrance of thin reservoir oil. Radial resistance, on the other hand, is almost a fixed quantity.

If we assume feeding reservoirs as being 400 ft. apart (it is important to avoid having reservoirs too far apart) each reservoir will feed 200 ft. of cable on each side of its position. From Fig. 8 maximum oil supply is 6.2×10^{-6} cu. in. per sec. per ft. of cable. There are approximately 520 drops of oil in a cu. in. Therefore, the maximum amount of oil required from the reservoir to feed 200 ft. of cable amounts to slightly more than one drop every two seconds. Furthermore, this does not flow through the entire 200 ft. but is more or less uniformly absorbed by the paper along the length. Even a very high resistance will take care of this small flow. If longitudinal resistance is higher than that required to maintain the above maximum rate of flow the rate will be lowered to fit this condition and the lag between demand and supply correspondingly greater.

It is difficult to obtain reliable measurements of longitudinal flow representing conditions of reservoir feed in service, where the oil must work down through the joint and cable wrappings to reach the conductor strand space. Resistance along the strand space is small compared with that through the end wrappings. Longitudinal resistance in single conductor cable can be considered as composed of two paths in parallel, one consisting of the joint and cable end wrappings leading to the strand space and the other of the space directly under the lead sheath. Of these two, that directly under the sheath is eventually the most effective and important, although after the thin reservoir oil has had an opportunity to "wash out" channels through the end wrappings, this path is far from being of negligible help.

Unfortunately, from this aspect, more direct access

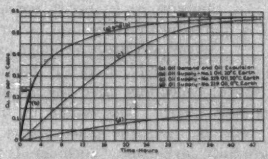


Fig. 9—Volumetric Change with Time-Cable No. 1

to the strand space cannot be obtained with the present standard design of joints. When viewed from another angle, however, this might be fortunate. It is possible that direct access of thin oil to the relatively large strand spaces might cause cumulative piling up of head pressure at the bottom of even slight grades.

From this same standpoint, corrugations or grooves of any appreciable size directly under the lead sheath would, after the thin oil worked in, convert the cable

into an oil filled cable with its attendant problems but without its inherent benefits.

There appears to be at least some promise in compromising by having very small corrugations of the right size under the sheath. This would merely serve to hasten final settling down of the line and reduce but not eliminate the final stretching of sheath. This possibility is being studied with caution. The difficulty of obtaining good initial impregnation is one objection.

The present cable has a tightly applied sheath and before heating the flow resistance directly under the sheath is large as determined by test. Loosely applied sheath is not desirable for there would be too much danger of trapping air during manufacture.

A relatively low longitudinal flow resistance is wanted immediately after installation to prevent excessive time lag between oil supply and oil demand. We believe the most practical way of obtaining this would be initially to "loosen up" cable of this type with circulating current about 25 per cent higher than maximum load current. The line should then be allowed to stand idle for a specified time to give the reservoir oil (under increased pressure) plenty of time to soak in thoroughly. Two or three, or even more, applications of this cycle before operating voltage is applied should be quite helpful.

The formula for calculating longitudinal pressure drop for uniformly distributed oil supply is:

$$P = \frac{s\,r\,l^2}{2\,A} \qquad (1)$$

where; r = average longitudinal flow resistance, at 20 deg. cent. in terms of lb. pressure drop per ft. of cable per cu. in. of oil flow per second.

P = total lb. pressure drop along cable length

l = length of cable in ft.

s = average radial oil supply or oil expulsion, cu. in. per ft. per sec.

A = temperature correction factor, assumed as proportional to change in viscosity.

In solid cable the supply, s, at any moment varies along the length, depending upon the pressure in the channels at each point. This cannot be included in the above formula until the mathematics of radial feed are more fully developed. Formula (1) is approximately applicable to solid cable only when the pressure drop in the channels is held within a reasonable average range above and below atmospheric. Also, with our present limited knowledge the factor A should be applied with caution.

Summing up the rest that is known of longitudinal flow resistance r, it might be stated that, initially, before the cable is heated r is so high that when load is first applied almost the same amount of sheath stretching along the length occurs with and without reservoirs. This much, at least, can be determined by laboratory test. With a tight sheath and before the cable has been heated the value of r is initially about 20,000 for 200 ft. of Cable No. 1 and this represents a flow entirely through the end wrappings and strand space. After the thin oil has washed out a path through the end wrappings r drops to about 1000. The first application of current equivalent to 45,000 kv-a. causes an almost uniform sheath stretching of approximately 0.01 in. increase in diameter. From here on, laboratory tests are not of much value in determining the reduction of r in service. This depends upon the many factors already described and these cannot be fairly included in the laboratory. Any further knowledge of r must be obtained directly in the field. Almost any reduction in r desired can be obtained in the laboratory by sufficient stretching of sheath and "washing out" with thin oil.

The theoretical calculations which will be shown indicate that ultimate stretching will be reached when and if r has been reduced to a value in the order of 100 to 200 under normal conditions of service. Whether this will cause bursting of sheath at the further points from the reservoirs we cannot say. It might be a coincidence, but the few meager field records available seem to show a final value of r lower than 100 after a year or more of operation under head pressure or reservoir pressure much higher than normal. The maximum stretching of sheath in these cases appeared to have been in the order of 0.020 to 0.040 in. increase in diameter. The records, however, were so incomplete that a great deal more study of r is needed.

Under average normal service conditions with 24-hr. daily load cycles the chief trouble would appear to be, not eventual bursting of sheath, but that the sheath will not be stretched sufficiently and r thereby sufficiently lowered to maintain the best possible impregnation.

(*b*) *Dropping Load in Summer.* Returning to the condition of dropping full load with 20 deg. ambient earth the effects of longitudinal flow will be briefly shown by selecting arbitrary limits of r for comparison.

From Fig. 8 the maximum radial oil supply for Cable No. 1 is 6.2×10^{-4} cu. in. per ft. per sec. at an average cable temperature of 37 deg. cent. (The average temperature at any time is directly proportional to the total volume of oil contraction as determined from Fig. 9.) With limiting values of r, 2000 and 200, the pressure drop along the channels in 200 ft. of cable necessary to maintain this radial feed can be calculated from Formula (1). These pressure drops are 93 lb. and 9.3 lb. respectively. The first is, of course, entirely out of proportion. It means that with a reservoir pressure of 5 lb. there would be a very appreciable time lag in oil supply over that shown in Fig. 8, and also the maximum rate of supply would be decreased to fit this condition. There are no means at present for estimating this but it would surely require several days at no load to fill up all the voids. This brings out clearly

the desirability of "opening up" the cable initially, before voltage is applied.

The second value, $r = 200$, gives a pressure drop well within the limits of good operation. With a reservoir giving a constant pressure of 5 lb. the pressure in the channels 200 ft. away would be only 4.3 lb. below atmospheric at the time of maximum supply and something better than this at other times. Curve *c* in Fig. 8 should, consequently, represent the average over-all effect fairly well. The void volume and time lag would be a little greater at the far end than shown in Fig. 8 but less at the near end.

(*c*) *Dropping Load in Winter.* With the cable gas free and perfectly impregnated (the same theoretical assumption used previously) the oil demand curve *a* in Fig. 8 still holds if the same load (45,000 kv-a.) is dropped in the winter with an ambient temperature of 0 deg. cent. and copper temperature of 32.5 deg. cent. The estimated rate of supply is shown in Fig. 8 as curve *d* and the volumetric supply as curve *d* in Fig. 9. These curves are based on the assumption that flow resistivity (Fig. 7) remains proportional to viscosity at these lower temperatures. The difference from summer conditions, caused by higher viscosity of oil and slower gas absorption, is apparent. The maximum void formation and time lag (0.8 per cent at 24 hr.) are both greater. It would require about 16 to 20 days to fill up entirely all voids with the channels maintained at atmospheric pressure.

Maximum oil supply from curve *d* is 1.25×10^{-6} cu. in. per sec. at an average cable temperature of 18.5 deg. cent. The pressure drop in the channels needed to maintain this supply with a resistance, $r = 200$, would be only 5.5 lb. for 200 ft. of cable. Maximum pressure drop in this case, however, would occur after about 30 hr. when the cable is at an average temperature of 2 deg. cent. and the supply is 0.6×10^{-6} cu. in. per sec., giving a drop of 13.4 lb. for 200 ft. of cable. This means that curve *d* would not hold closely for the far end of the cable, and it would require even a longer time than 16 to 20 days to fill up the voids entirely.

A cable having a resistance, $r = 2000$, would require such a long time to fill up voids that reservoir feed would be of little use at 0 deg. cent.

From the calculations it can be seen that winter operation, from an ionization standpoint, is more severe than summer operation. The practise of increasing winter loads above summer loads makes it even more severe.

(*d*) *Applying Full Load in Summer.* Curves *a* in Figs. 8 and 9 also represent oil expulsion curves when load is applied, assuming the cable perfectly impregnated at that time. Fig. 8 shows that the maximum rate of expulsion occurs in 10 min. and is 48.0×10^{-6} cu. in. per sec. If the sheath and cable were absolutely rigid the expelled oil would have to return to the reservoirs and the pressure in the channels would reach a maximum value at that time, after which it would fall off rapidly. The pressure drop for 200 ft. of cable, calculated from Formula (1), is shown as a function of time in Fig. 10. The following conditions were assumed:

(a) Perfect impregnation.
(b) Absolute rigidity of sheath.
(c) A value, $r = 100$.

The pressure drop for any other assumed value of r can be determined from the curve by direct proportion.

Fig. 10 shows a brief maximum pressure of 88 lb. (above the reservoir pressure) which drops off to 36 lb. in 50 min., and to 18.5 lb. in 2 hr.

The bursting pressure for pure lead sheath of these dimensions at 20 deg. cent. and with about this speed of pressure application is 125 lb. per sq. in. with considerable swelling first, (¼ in. or more). If the theoretical conditions in Fig. 10 actually occurred the sheath would be stretched very slightly to relieve pressure each time load was applied. The degree of stretching of lead sheath under conditions of transient pressure at

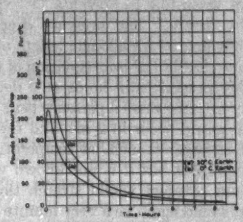


FIG. 10—PRESSURE AT FAR END OF 200 FT. CABLE NO. 1 WHEN LOAD IS APPLIED $r = 100$

various temperatures is not well understood. As a rough guess, the sheath would be stretched about 0.0005 to 0.001 in. in diameter during the first pressure applications shown in Fig. 10. Each succeeding application would cause progressively less stretching as r decreased below its initial value of 100. Whether the sheath would eventually burst is not known.

Standard antimony and tin alloy sheaths have a bursting strength of 200 to 300 lb. per sq. in. under the above conditions but with much less stretching before rupture. This increased strength would appear to come well within the requirements but pure lead sheath is preferred because it will stretch more easily and to a greater degree, automatically lowering longitudinal resistance to the required value and reducing the possibility of rupture.

The theoretical conditions in Fig. 10 do not hold in actual practise. If we assume an initial impregnation of 0.5 per cent at 20 deg. cent., atmospheric pressure (a better impregnation could hardly be expected after installation) it would act as a cushion and smooth off

the pressure peak in Fig. 10. Assuming that compression of the void volume to ⅛ its original size represents a pressure in the channels of 25 lb., which seems fair in view of the high pressure existing in the insulation thickness, we have left a volume of 0.33 per cent to take care of expansion. From Fig. 9, curve *a* this amount of expansion would be reached in 1¼ hr., practically in agreement with the corresponding pressure and time in Fig. 10.

This means that with $r = 100$ and 0.5 per cent initial impregnation the pressure would start from atmospheric and increase gradually to a maximum of 25 lb. in 1¼ hr., after which it would drop off. It cannot be said definitely whether this would stretch the sheath or not. Each application would cause a minute amount of stretching, if any, and a slight decrease in r would stop this altogether.

(*e*) *Applying Load in Winter.* Curve *a*, Figs. 8 and 9 represent the oil expulsion at 0 deg. cent. ambient earth provided the same load (45,000 kv-a.) is applied. The pressure curve corresponding to that at 20 deg. cent. is also plotted in Fig. 10. The maximum theoretical pressure reached in 10 min. is 436 lb. for 200 ft. of cable. Pressure drop, according to Formula (1), varies as the square of the length. The pressure drop would be correspondingly greater for a longer length of cable.

If we assume that initial impregnation of 0.5 per cent at atmospheric pressure still holds at 0 deg. cent. (an optimistically extreme case) a pressure peak of approximately 65 lb. would occur in 2 hr., and then decrease. Lead is much stronger at such low temperatures than at normal temperatures but it is not known whether the above transient pressure would stretch the sheath or not. If so, it would again be a very minute change and many such applications would be required to cause a noticeable effect.

There is very little likelihood of repeated applications of this nature at 0 deg. cent. in service. It has already been shown that a cable line would have to remain idle at 0 deg. cent. for 16 to 20 days before all of the voids caused by dropping load are filled up. In other words, if load is again applied before this time has expired there would be void volume additional to the initial 0.5 per cent.

Cold weather conditions probably determine the ultimate stretching of sheath. We have no way at present of determining this ultimate stretch but bursting of sheath appears improbable. A summary of the work, as far as we have carried it, has been presented, however, hoping that it will assist others in reaching a solution of this difficult problem.

A study is being made of the transient physical characteristics of lead sheath and we hope at a future time to present more definite conclusions.

(*f*) *Effect of Daily Load Cycles.* The lag effect between oil demand and oil supply during 24-hr. load cycles is apparent from previous considerations. From Figs. 8 and 9 this might even be approximately estimated. It means, of course, that in solid cable oil supply can never exactly equal oil demand and during periods of decreasing load a variable void volume will be added to the initial void volume, the amount of lag and nature of the load cycle determining the average total impregnation. This average will be worse for large loads than for small loads.

Maximum void volume will occur near or during periods of no load or low load. Likewise, during periods of high load the voids will be practically closed up. Obviously, high load factor is very desirable from this standpoint. With perfect initial impregnation of Cable No. 1 and assuming a 12-hr. duration of rapidly reached 20 deg. cent. no-load (low load factor) the maximum void volume would be about 0.5 per cent for $r = 100$. With a 4-hr. period of low load, and a gradual decrease in load (high load factor) the maximum void volume would be 0.25 per cent or better.

If accurate hourly records were taken of reservoir feed and pressure, as well as of cable temperature, it would be possible to determine approximately the degree of impregnation and many other interesting things related to the cable.

(*g*) *Effect of Seasonal Temperature Cycles.* There would be no time lag between oil demand and oil supply for slow seasonal temperature cycles (liquid compound filled cables). The important effect of seasonal changes in temperature appears in the time lag of daily load cycles, due to change in viscosity of oil.

(*h*) *Effect of Initially Poor Impregnation.* If the void volume initially in the cable appreciably exceeds the volume of expansion when load is applied there will be little if any extrusion of oil during daily load cycles, depending upon the value of initial impregnation, difference in void pressure and reservoir presence, and the value of longitudinal resistance. Nor will there be any large increase in pressure or stretching of sheath. The cable will continue to absorb oil until a balance is reached. If there is a relatively large quantity of gas still present after balance the extrusion will still be small in comparison with the total expansion.

Obviously, under these conditions the amount of oil absorbed depends upon the relative void and reservoir pressures. With voids and reservoirs both at atmospheric pressure and the cable on level ground no oil would be absorbed at all unless the reservoir pressure is increased.

With a poorly impregnated cable r would remain quite high even after balance is reached, since the sheath would be stretched little if at all.

Poor impregnation materially increases the thermal resistance of the cable; therefore, curve *a* in Figs. 8 and 9, which was calculated for a well impregnated cable, does not hold very closely. For this and other reasons exact determination of the effects of poor impregnation is very difficult.

Oxidation of Compound

In cable made of proper materials operating within

allowable temperature limits, and assuming that moisture from outside is excluded, the only form of deterioration so far noted, other than that caused by ionization, also results in an increase of dielectric loss.

Riley[3] concludes that this is nothing more than ordinary oxidation of compound, with its accompanying formation of water and acids. He claims that sufficient oxygen to produce this result is normally absorbed by cable during manufacture and installation.

Our work confirms his in effect but perhaps not in degree. We have found cases of as much as a 100 per cent increase in dielectric loss at 60 deg. cent. after two years of operation. Judging from laboratory tests it is hard to see how initially absorbed oxygen could fully account for such an increase, even though the original loss was extremely low, unless the amount initially absorbed was much more than would be expected. Further work might fully confirm Riley's conclusions but it is more reasonable to assume that other actions, such as reaction between paper and compound, or entrance of additional oxygen or water occurred during operation.

We have never found proof of chemical reaction between pure paper and pure compound at operating temperatures. On the other hand, there is abundant proof of oxidation of compound if sufficient oxygen is present.

The only conclusion that can be drawn with our present knowledge is that it is always wise to exclude oxygen as far as it is possible to do so and in this way limit the increase in loss to a safe value. The following precautions are worth while:

(a) Have the cable initially free of oxygen.

(b) Keep the outer covering absolutely tight in service and maintain, as far as possible, a positive pressure in the system.

(c) Do not open cable ends or joints in air when below atmospheric pressure.

(d) Remove, as far as possible, by vacuum treatment the oxygen absorbed during the exposure of jointing work.

(e) Fill the joints and reservoirs with an oxygen-free, unexposed oil.

(f) Avoid exposure of the reservoir oil to air or oxygen during service.

The seriousness of increased dielectric loss in service depends upon the ultimate value of this loss in comparison with copper loss. A loss which, initially, was only 0.5 per cent of the copper loss could increase 500 per cent without being regarded as especially serious. On the other hand an initial dielectric loss of 5 per cent, if increased 500 per cent would seriously affect "hot spot" temperature and bring back the dangers experienced years ago of accumulative heating failures. Obviously, the percentage ultimate increase of dielectric loss is of more importance the higher the voltage rating of the cable.

All oils and compounds deteriorate at high temperature in the presence of oxygen, the rate of deterioration increasing rapidly with temperature. We have exposed No. 219 compound to a temperature of 150 deg. cent. for one week under vacuum without increase in electrical conductivity or dielectric loss. This same exposure in the presence of oxygen increases the dielectric loss to a prohibitive amount for cable work.

A 155-ft. length of 25-kv. three-conductor, shielded type cable, insulated with wood pulp paper and impregnated with No. 219 compound, was put through 230 heating cycles from 20 deg. cent. to 60 deg. cent. at normal voltage by heating the conductor with current. The ends were exposed for some time while the terminals were being applied and no vacuum treatment was used afterwards. No reservoir feed was applied and, of course, the sample dropped below atmospheric pressure while cooling. The dielectric loss, 60 deg. cent. normal voltage, 60 cycles, at the start was 0.094 watts per ft. It increased gradually and appeared to become constant towards the end of the run, the final loss being 0.148 watts per ft., representing an increase of 57.5 per cent. No measurable or noticeable ionization occurred and this increase in loss might be accepted as that due to normal oxygen absorption (it was discovered during test that the terminals leaked slightly). The results are not conclusive, however, because of the relatively short time of test (40 days) and the lack of assurance that moisture might not have entered during test.

It might be claimed that regardless of the fact that no visible ionization damage was present in this test that part or all of the increased loss was caused by the weak, intermittent discharge previously described. We do not think so. Perhaps future work will prove us entirely wrong but it is hard to see how such a faint and relatively brief discharge could cause this effect. According to Hirshfeld, Meyer, and Connell[16] the discharge must also attack the paper fibers before water is formed. This would seem improbable with such a small degree of ionization while, on the other hand, the effects of oxidation are well recognized. We are planning an additional series of tests which will lead to more definite conclusions.

Ionization

This paper, so far, has dealt with methods for the prevention of ionization because there can be no doubt of its injurious action, and every effort should be made to eliminate it by proper design and operation. That it does exist in service there is abundant evidence. For this reason knowledge of its behavior is important.

A great deal of attention has been given to the subject of ionization in cables since it was first identified in 1917, resulting in a large amount of published matter. Some of the outstanding papers on this subject are given in the Bibliography. From these a fairly good knowledge is now available of the voltage stress at which ionization begins as related to size of voids and pressure. Very little is known, however, regarding

the laws of ionization loss in cables as related to size of voids, pressure, and voltage stress. Such knowledge would be of great value in determining the quality of cable, particularly in estimating more accurately the degree of impregnation. It should also throw additional light upon the rate of deterioration.

The effects produced by ionization in a void are, excessive energy loss, production of non-uniform stress distribution in the insulation, chemical changes in compound and paper, and perhaps a migration of the compound from the paper as demonstrated experimentally at very high stresses by Emanueli[2] and Dunsheath.[11]

Hitherto, studies of the mechanism of breakdown of insulation have been confined chiefly to the consideration of phenomena involved in solid or liquid materials. However, in most cases, the failure of high-tension insulation such as that of cable is initiated by electrical discharge through a gas in proximity to the insulation proper. We shall therefore briefly describe some of the characteristics of gaseous conduction which appear to be applicable to the phenomena appearing in cable.

There are always a few ions being produced by penetrating radiation in a gas under ordinary conditions and if a potential difference exists between the boundaries of the space, current will be carried which increases with the applied voltage until it becomes constant, this stage being the condition where the ions are discharged at the opposite electrodes at the same rate as they are being formed in the space. However, the voltage cannot be increased indefinitely without a further change in conditions. This occurs when the ions are speeded up to such an extent that they begin to ionize neutral atoms by collision. At this point the current begins to increase rapidly and ionization and recombination occur so plentifully that radiation may be visible as in corona discharge.

The whole space is now in a comparatively high conducting state and, depending upon the geometry of the enclosure, the nature and pressure of the gas, at a certain current density the whole system becomes unstable and breaks down into an arc-like discharge where the current increases, producing less and less resistance, so that the voltage across the gap falls.

During the first stage before the so-called saturation current is reached, Ohm's law is obeyed, but the current passing is extremely small, being of the order of 10^{-15} amperes per sq. cm., so that the energy loss is relatively very small.

A qualitative idea of the change in condition of the gas as the voltage is raised is given by the diagram[17] of Fig. 11 which shows the generalized characteristic of the electrical discharge between metal electrodes. The ordinates and abscissas of this diagram are logarithmic as the current when the arc stage is reached is many hundreds of times the current at the point O', and the voltage at O' is hundreds of times greater than that of the arc.

The source of potential E produces a discharge between electrodes with a voltage drop V, a resistance R being in the external circuit, and an ammeter to measure the current i. The external characteristic is

$$E = V - Ri \tag{1}$$

and the internal characteristic of the discharge itself is

$$V = f(i) \tag{2}$$

If the series resistance be kept constant and the voltage gradually increased the current is at first small and the volt-ampere characteristic is positive, as shown by the curve OO'. Beyond O' the current increases and the voltage decreases, the characteristic of the glow discharge, and at $G'A$ occurs the transition from glow to arc where very large currents may be carried with a very low voltage drop.

Depending upon the value of E and the amount of series resistance in the circuit, the changes from one stage of the discharge to another may be continuous or discontinuous. For instance, in the case of Equa-

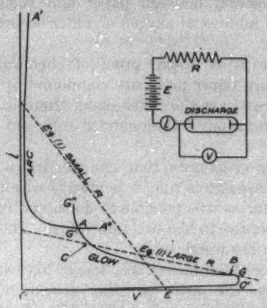


Fig. 11—Volt-Ampere Characteristic of Gas Discharge

tion (1), small R, the entire glow discharge may be skipped over. In cable containing gas spaces the dielectric acts as a current limiting device so that the characteristic may correspond to that of a discharge with a high series resistance.

It is obvious that in a cable, when a gas pocket is present, as the voltage increases the power factor will also increase so long as the discharge has a positive volt ampere characteristic. When, however, the glow stage is reached, a slight increase of current is accompanied by a very large drop in the voltage. Accordingly the energy dissipated in the gas space decreases because it becomes a relatively good conductor. The damage to the insulation, on the other hand, may be greater, and eventually a spark or destructive arc-like discharge may occur.

The ease with which a discharge may be started in a gas varies very greatly with the pressure, size of enclosure, and the nature of the gas. The variation with pressure of the sparking potential for air is shown in Fig. 12 for two different gaps, 1 mm. and 3 mm.

For any given distance between electrodes there is

some pressure at which discharge occurs at the minimum sparking potential of the gas. Thus with a gap of 3 mm. in air, sparkover occurs at a pressure of 1.5 mm. at about 340 volts. For a gap of 1 mm., the discharge occurs at a pressure of 5 mm. at the same voltage. At pressures above and below these minima, or critical pressures, the voltage necessary to produce breakdown of the gas increases both with increasing and decreasing pressure. Paschen's law summarizes these results by stating that the sparking potential of a gas is a function

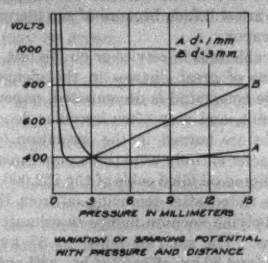


FIG. 12—DIELECTRIC STRENGTH OF AIR AT DIFFERENT PRESSURES AND TWO DIFFERENT GAP SPACINGS

only of the product of the sparking distance by the pressure of the gas, or in other words, of the mass of the gas, between the electrodes. If p, the pressure, and S, the gap length, are varied in such a way that the product $p\,S$ remains constant, the sparking potential will remain unaltered, and the product $p\,S$ is a constant for any gas. The following table[18] gives data for some of the ordinary gases.

	Minimum spark potential	p (mm) $\times$ S (mm)
Air	341	5.7
Nitrogen	251	6.7
Hydrogen	278	14.4
Oxygen	455	...
Carbon dioxide	419	5.1
Helium	261	27
Acetylene	468	....

In order to determine the characteristics of a cable containing gas spaces, measurements were made by E. S. Lee[19] on a length of single conductor cable 2/0 *A W G* (1.06 cm.) with 9/32-in. (0.714 cm.) treated paper insulation which after removal of its sheath was inserted in a brass tube and held centrally by copper spacing rings so as to leave an annular space of 0.120 in. (0.305 cm.) between the paper and the tube. The length under test was 10 ft. and the variation of power factor with voltage was measured with hydrogen, carbon dioxide, and air at different pressures in the annular space. The results are shown in Figs. 13 and 14.

The normal voltage rating of this cable would be about 12 kv. and curve 1 shows the power factor voltage characteristic in its normal state with lead sheath. Curve 2 shows the characteristic with sheath removed and after insertion in the brass pipe to provide an air gap between sheath and insulation. Here the typical break in the characteristic, the so-called ionization point, occurs at about 12 kv. after which the power

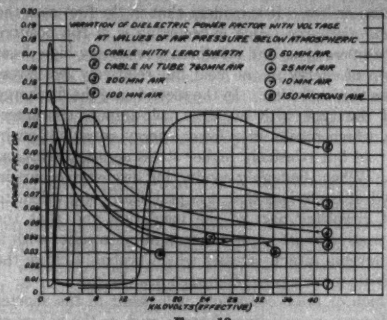


FIG. 13

factor increases rapidly to about 13 per cent and then decreases as the voltage is raised further. At this stage the discharge through the gas is of the glow type and is beginning to result simply in an extension of the conducting sheath inwards towards the insulation.

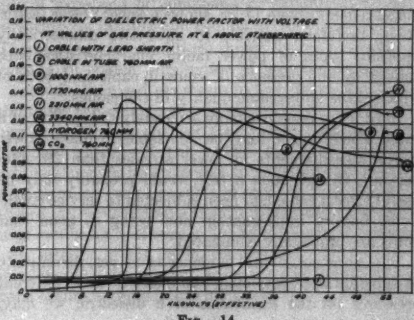


FIG. 14

On reducing the pressure of air the ionization point is lowered until corona eventually occurs at about 500 volts at a pressure of 0.15 mm. as shown in curve 8. In this case after a rapid rise to a power factor of about 18 per cent a decrease to about 3 per cent is found on raising the voltage to 17 kv. Increasing the gas pressure raises the ionization point as shown in curves 9, 10, and 11. Hydrogen (curve 13) shows greater ease of corona formation than air, and carbon dioxide (curve 14) requires a higher voltage than air for initiation of the discharge.

Further consideration of the curves of Fig. 13 shows the possibility of two definite and distinct types of characteristic which may produce failure in the neighborhood of the operating voltage. For instance, consider two cables operated at 14 kv., in one case having a void containing gas at atmospheric pressure and the other with a void having a gas pressure of 25 mm. or less. It is evident that in the first case the corona discharge which is occurring is of the positive volt-ampere type, and results in a relatively high dissipation of energy in the void but relatively less stress across the insulation. In the second case the discharge is of the type with negative volt-ampere characteristic, which results in a lower energy consumption in the void but relatively higher stress across the insulation. When the highly conducting discharge occurs in a void, it is obvious that it is practically equivalent to short circuiting part of the insulation and that the higher stresses set up on the insulation in series with it will develop lateral stresses which probably cause the dendritic markings so often seen near failures.

Which of these conditions will result in the shorter life depends upon several factors, such as the nature of the gas present, the effect of corona on the walls of the void, and the nature of the insulation. With cable materials, such as we are using at present, the low pressure voids are the most dangerous. But it is interesting to see that in the first case the measurement of power factor at 14 kv. and a higher voltage shows an increasing power factor; in the second case measurement above 14 kv. shows a lower power factor. It is, therefore, possible to have two diametrically opposite types of power factor characteristics both caused by the presence of voids in cable and both of which show undesirable quality in the cables.

In order to diagnose more accurately the quality of a cable, therefore, it may be advisable to make power factor measurements at lower stresses than 20 volts per mil, though such a test would probably only be useful after a cable had been subjected to service conditions, factory impregnation now being so good that voids are practically eliminated.

Though experimental evidence is, as yet, incomplete to substantiate completely the effect of these different pressures on the endurance of a dielectric we may, however, tentatively conclude that:

(1) Internal corona in insulation occurs less readily as the gas pressure in voids increases above the critical discharge potential of the contained gas.

(2) The voltage required to produce internal corona varies with the nature of the gas.

(3) The power-factor voltage characteristic shows two distinct parts, one corresponding to the positive volt-ampere characteristic of gaseous discharge, the other to the negative volt-ampere characteristic.

From the general characteristics of insulation, it is very surprising that when ionization weakness develops it does not always accelerate rapidly to ultimate destruction. In the case of cables, for instance, a hotspot may develop and get slowly worse over a period of days before finally breaking down. In some cases the hot spot may actually disappear. We suggest that such a result is caused by the gradual increase of pressure produced in a void by the action of the initial corona in producing an increasing quantity of gas from the compound or paper which tends to stabilize and retard the development of the corona discharge. Such an action may be seen, for instance, in an enclosed mercury arc in quartz, where if the current be increased above normal the arc will fade because of the increased pressure of the vapor.

When a cable fails in service or on life test, there are usually signs of great distress in the vicinity of the break. The compound is decomposed, paper charred, wax is present, and very often irregular dendritic discharge paths are burned in the insulation. Of a different character entirely were three punctures made on samples of liquid oil filled cable of the 132,000-volt type which were tested after removing oil from the hollow core and applying vacuum pumps to exhaust air while under stress. The samples failed at 57 kv., 80 kv. after 3 min., and 80 kv. after 22¼ hr., without development of hot spots and with clean, small punctures. The normal value for breakdown voltage on such cable completely filled with oil is about 450 kv. In the case of the failure under vacuum conditions, pressure could not develop so when weakness developed deterioration proceeded very rapidly.

Similar experience has been obtained with small capacitors during an investigation of characteristics of materials which were assembled in a glass bulb for subjection to refined vacuum treatment and endurance tests. With the bulb sealed off with high vacuum inside, it was found that such capacitors could not be operated above 1200 volts without immediate breakdown. However, upon admitting air at atmospheric pressure they operated at 3000 volts for 1000 hr. and more, a stress of 1500 volts per mil.

Another experiment has been performed, as follows: A glass bulb, 4 in. in diameter, with two electrodes and a mercury manometer attached, containing 50 cu. cm. of standard cable compound, was exhausted, while hot, to good vacuum. A glow discharge was then produced in the bulb by application of 2000 volts from a transformer across the electrodes. Gas was evolved at the rate of approximately 1.2 cu. cm. per hr. and after 28 hr. the discharge stopped and the voltage had to be raised to 2300 in order to start the discharge again. The actual pressure in the bulb rose in 75 hr. to 15 cm. but the voltage necessary to maintain the discharge did not increase correspondingly because of the gradual building up of a conducting deposit which shortened the gap.

We do not wish it to be inferred, however, that we think it advisable to generate gas in a cable in such a way as to displace oil or compound from the insulation

so that the gas space will be enlarged and perhaps ultimately extend from core to sheath. And, of course, we realize that the ideal method is to eliminate voids entirely, and to prevent their formation, but in some cases this may be impracticable. Nevertheless, we do feel that if voids exist it is preferable to have as high a gas pressure in them as possible, and if the whole cable is sealed under definite pressure these spaces need not necessarily grow larger, and may perhaps be constrained to move their boundaries only in a longitudinal direction.

As a result of these observations, we feel that these conclusions may be drawn:

(1) If voids exist in any given insulation it is desirable to have present as high a gas pressure as possible.

(2) Since it may be impracticable to use materials which are capable of maintaining pressures below the critical discharge pressure in insulation, it may be desirable to use those which readily develop as high pressures as possible, though not high enough to cause enlargement of voids which exist because of expansion and contraction phenomena, or to produce gas spaces in otherwise continuous media.

As a matter of fact, if a cable sheath could be constructed and installed with sufficient strength to withstand a pressure of a few hundred pounds per square inch, it should have as high a dielectric strength when filled with gas at that pressure as our present compound treated material. A cable embodying this principle has been described by Messrs. Fisher and Atkinson.[20]

It is also evident, in view of the preceding discussion, that no discharge can occur at any pressure if the voltage drop across a gas space is below the critical discharge potential. For air, this is 341 volts or about 240 volts r. m. s. Therefore, if voids in cable can be restricted to thicknesses across which the drop is less than this no internal corona can occur. Assuming for ordinary cable, an operating stress of 50 volts per mil and a dielectric constant for the insulation of 3.5, this would mean that voids less than 1.4 mils in thickness should cause no trouble. And if such voids could be filled with gas to atmospheric pressure they could be many times this thickness.

Formation of "X" Wax

The work of Hirshfeld, Meyer, and Connell,[5] on the chemistry of wax formation and changes in insulating materials is outstanding and when completed will greatly assist in clearing up this question. Their work indicates that ionic bombardment forms wax and hydrogen from the compound. It also causes chemical and possibly physical changes in the paper fibers, if sufficiently severe and sustained sufficiently long, yielding water and carbon dioxide among other things. The hydrogen and carbon dioxide react to form more water. It is this water and possibly other factors which cause gradual deterioration of the electrical properties, according to their tentative theory.

The formation of wax appears to be the result of a polymerization or condensation of hydrocarbon molecules with the evolution of more or less hydrogen or hydrocarbon produced by glow discharge, bombardment by electrons, or alpha particles. The theory of this reaction is discussed by S. C. Lind.[21]

This wax is one of the most stable organic compounds yet found and if it could be prepared in large quantity would probably be one of the best of insulating materials.

We have prepared this wax by the action of glow discharge on a wide variety of compounds such as paraffin base oils, asphalt base oils, rosin oil, rosin and petroleum base mixtures, water white oils, such as Nujol and kerosene, and a somewhat similar material from acetylene gas.

Conclusive evidence that wax is not formed by the action of stress alone on the oil is offered by capacitors which have operated for years at a stress of 400 volts per mil without the formation of wax though the oil with which they are filled develops wax by the E. T. L. test or by subjection to corona discharge. Furthermore, wax is never found in cable except at those points where voids are formed.

The formation of wax, therefore, seems to be simply a symptom of "corona disease" and as such is a useful criterion of the quality of a cable and an aid in diagnosing the source of trouble. Existence of wax in a cable after operation shows either that voids have existed as a result of poor initial impregnation or as a result of expulsion of compound or expansion of the lead sheath by heating and cooling cycles of operation.

The real cure for conditions shown by the formation of wax is, of course, elimination of gaseous ionization by methods discussed elsewhere in this paper, and the stability of the compounds to corona is of secondary importance, though we feel that rather than have a compound which will not evolve gas under corona conditions, it may be preferable to have the compound evolve as much gas as possible in order to raise the gas pressure in the void above that at which electrical discharge can occur. However, if oxygen can gain access to the interior of the cable, for instance through the joints, there is danger that water may be produced if corona occurs in the mixture of oxygen and hydrogen.

Laboratory Cable Treatment

In the latter part of 1923 extensive investigation was started to determine the best methods and materials for cable manufacture. A small treating plant was built in the research laboratory, consisting of a steam jacketed pipe, 5 in. inside diameter and 20 ft. long, attached to which was a compound tank. The outfit was provided with rotary oil vacuum pumps, condensing equipment, liquid air traps, and McLeod gage for measuring the degree of vacuum. With this apparatus, valves and joints being oil sealed, a vacuum of 0.010 mm. could be obtained, and liquid air could be

used to condense any residual water vapor in the apparatus.

Lengths of cable 19 ft. long were treated, two sizes being used: (1) 500,000-cm. conductor with 30/32 in. paper insulation and (2) 2/0 B & S conductor with 9/32-in. paper insulation. Five runs only were made with the larger cable after which the smaller cable was adopted as the standard size for investigation because of its greater convenience for testing and because three lengths could be treated at the same time.

About 50 separate runs were made with this apparatus to determine the effect of variations in (1) temperature of vacuum treatment, (2) time of vacuum treatment, (3) pressures during vacuum treatment, (4) temperature of impregnation, (5) pressure during impregnation, (6) preliminary air dry before treatment, (7) application of high pressure after impregnation, and (8) materials, both paper and compounds.

In order to determine the quality of the cable, with the 9/32-in. insulation, after leading, power factor was measured on one length at 25 deg., 40 deg., 60 deg., 80 deg., and 100 deg. cent. at voltages from 5.6 to 28 kv., and the two other lengths were placed on endurance test at 44 kv., one being operated at room temperature, the other at 85 deg. cent.

Relatively few tests were made with the larger cable but the following illustrates the effect on power factor of using different grades of petrolatum, first the standard factory compound and second, a clear water white material having an initial resistivity about six-fold that of the former, and a third containing a mixture of 78 per cent petrolatum and 22 per cent rosin. The paper used was a standard manila rope paper.

	Per cent power factor				
	25°	40°	60°	80°	100° C.
Standard petrolatum	0.29	0.34	0.63	1.17	1.93
Water white petrolatum	0.27	0.26	0.25	0.27	0.47
Standard petrolatum—22% rosin	0.48	..	..	..	6.60

The very low temperature coefficient of the pure petrolatum and the very high coefficient of the rosin mixture seem to be characteristic of such materials.

In the treatment of the 9/32-in. insulated cable the pressure during the vacuum period was varied from 0.01 mm. with liquid air condenser to 25 mm. without liquid air, the time from 77 hr. to 242 hr. Materials from manila paper to expensive linen paper, such as used in capacitors, and compounds from highly refined water white petrolatum to factory petrolatum used 62 times for impregnation, and mixtures of different oils with rosin were used. But no definite relationship to endurance could be established as a result of these variations. The life at 44 kv. of the different samples fell between 5 and 162 hr. The endurance at 85 deg. cent., however, was always much longer than at 25 deg. cent., in some cases reaching one or two thousand hours. Obviously, some essential factor, common to all the experiments, was being overlooked. The longer life at 85 deg. cent. when the expansion of the compound produced a more complete filling of the sheath, the successful operation of capacitors filled with oil at stresses of 400 volts per mil, and the work of Clark and Shanklin[4] and Shanklin and Matson[5] on ionization phenomena suggested that the limiting factor was the presence of voids in the insulation and that these had to be eliminated before any radical improvement could be made. Mr. E. G. Gilson,[23] of the research laboratory, also clearly demonstrated by experiments with petrolatum and cable in both glass tubes and lead sheath the effect of expansion and contraction upon the formation of voids. Temperature drops of 15 deg. cent. and less on tubes 6 ft. long showed separation of the compound with the formation of voids ¼ to 1 in. in size.

Experiments were then made by impregnating cable in the usual manner with transformer oil, but upon removal from the treating tank instead of being leaded they were wound with lead foil and placed in a loose lead sheath which was filled with the same oil. Upon testing samples of this kind submerged in oil, instead of failing in one or two hundred hours at 44 kv. they operated for 1000 hr. and more at 91½ kv., an average stress of 326 volts per mil with no sign of deterioration. Practically no change of power factor with voltage was found up to 100 kv., the power factor of about 0.5 per cent remaining constant. The short time breakdown voltage of this oil submerged cable, however, was only about 15 per cent higher than that of the average petrolatum filled insulation, though the results for a number of samples showed much greater uniformity. Had we taken the so-called dielectric strength as a criterion of quality instead of the endurance test at lower voltages, the advantages of oil filling would not have been so apparent. The difference in the relationship between endurance and dielectric strength shows conclusively that a rapidly increased voltage test to destruction is not a good criterion of the very wide differences which may exist in the quality of cable insulation. It is also very striking that while the petrolatum filled cables operated only a few hours at about 38 per cent of the breakdown voltage the oil submerged cables operated at 66 per cent of their breakdown voltage indefinitely.

There was then no doubt but that a great advance was possible in the design of cable for high voltage operation. However, before we had designed engineering methods for installation, we learned that the Pirelli Company of Milan had also arrived at the same conclusion and had already gone far towards the ultimate solution of the problems involved in engineering application. Accordingly, we combined forces and as a result Chicago and New York now have commercial lines of oil filled cable operating at 132 kv. with only 23/32-in. insulation when the highest voltage in commercial operation with ordinary insulation is 75 kv. with 24/32 in. of insulation.

Characteristics of Papers and Compounds

In order to ensure the uniformity of the finished cable characteristics and to develop materials of better quality it was necessary to establish definite and significant tests for paper and compound.

The usual mechanical tests and tests for conducting particles for papers were supplemented by:

(1) Determination of power factor of paper alone as a function of temperature.

(2) Determination of dielectric strength of impregnated paper.

(3) Determination of rate of percolation and retention of compounds in paper.

(4) Endurance tests on paper operated as sheets of dielectric in small condensers at a stress of 730 volts per mil, the paper being saturated and submerged in standard compound.

POWER FACTOR OF PAPER

In order to eliminate the uncertainty produced by other factors, such as quality of impregnating material and methods of treatment, the electrical characteristics

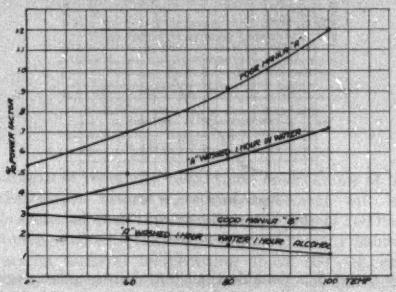


Fig. 15—Effect Upon Power Factor of Washing Paper with Solvents

of the paper itself were determined by making power factor measurements on a small condenser, using three sheets of dried paper as the dielectric between nickel plated iron electrodes. This condenser was arranged to be heated and was inserted in one arm of a Wien capacity bridge manufactured by the General Radio Company, the whole set-up being enclosed in a large box with one or two incandescent lamps burning in it to maintain lower humidity conditions than the open air.

This test as developed by Mr. C. V. Ferguson proved very satisfactory and as a result we were able to eliminate papers which produced a low insulation resistance in the finished cable and to assure uniformity of the product.

The most significant part of the characteristic is not so much the value of the power factor at room temperature as the rate of increase of power factor with temperature. In fact, with the best papers the power factor decreases with temperature up to 100 deg. cent. and this negative slope is more pronounced with wood pulp paper than with manila. The increase of power factor appears to be caused by the presence of conducting or hygroscopic materials and in many cases a poor paper may be improved by washing in water or in organic solvents, the most effective of which appear to be alcohol or chloroform. Fig. 15 shows characteristic curves for a poor manila paper, the same washed with water, with water and alcohol, and a good manila paper.

A positive power factor—temperature characteristic indicates direct leakage of current through the paper, possibly because of superficial films on the outside of the fibers. A negative slope probably means discontinuous conducting media, possibly within the fiber, when because of the current limiting capacity in series with it, the I^2R loss becomes less as the temperature is increased. The advantage of using an a-c. bridge method for measuring the losses in paper lies in being able to differentiate between these two types of loss, though it is also very much more convenient and easy to apply than a d-c. measurement. Loss because of so called "dielectric hysteresis" in the sense of molecular orientation is considered to be negligible, the more useful point of view being that of Dunsheath,[11] and Joffé,[23] in which the measurable losses correspond to those in a resistance parallel to, or in series with capacity.

DIELECTRIC STRENGTH OF IMPREGNATED PAPER

This test is made by impregnating single sheets of paper for 30 min. in standard compound heated to 150 deg. cent. allowing to stand for one hour and then applying an increasing voltage to the impregnated paper between two disks 2 in. in diameter until puncture occurs. Ten separate punctures are made on each sheet.

The dielectric strength of impregnated paper is proportional to its density and this test is a check on the uniformity of the paper in this respect and upon the retentivity of the paper for compound.

PERCOLATION TEST

For the study of the characteristics of paper and compounds with respect to impregnation, an apparatus, Fig. 16, was devised by Mr. C. V. Ferguson consisting of a funnel shaped vessel over which could be clamped any desired number of sheets of paper, and to which was attached a graduated glass cylinder with a side tube for vacuum connection.

The percolator and paper are placed in a thermostatic oven at the impregnating temperature and air is drawn through the paper by application of vacuum for two hours. A definite quantity of compound is then placed on top of the paper and the time noted when the first drops appear in the glass tube and after that the rate at which it comes through. The difference between the total amount collected and that placed on top of the paper is a measure of the retentivity of the paper for compound.

Typical results are shown in Fig. 17.

ENDURANCE TESTS

As a check on the quality of both paper and compound, small condensers are made up, using three sheets of paper about 4½ in. square, impregnated in the compound at 150 deg. cent. for 30 min., and tested at 11 kv. (733 volts per mil for paper 5 mils thick) between brass disks 2 in. in diameter, both paper and

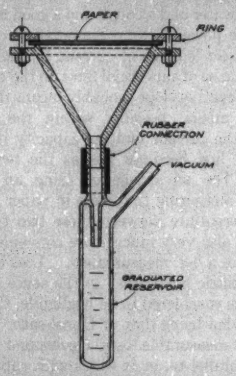


FIG. 16—APPARATUS FOR TESTING RATE OF IMPREGNATION OF PAPER

electrodes being completely submerged in the compound. Usually ten separate cells with similar material are tested at once. The temperature is maintained at 100 deg. cent. so that differences between solid and

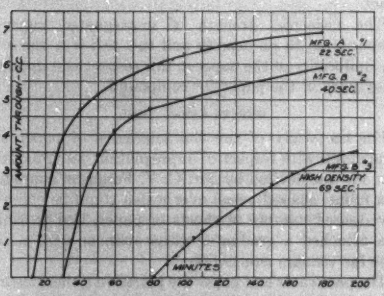


FIG. 17—FLOW OF COMPOUND THROUGH PAPERS OF DIFFERENT POROSITIES

liquid compounds, and the effect of voids are eliminated. The results, expressed as hours of endurance to failure, are plotted in order of their failure, as shown in Fig. 18, which gives an idea as to the uniformity of the materials which are being compared.

This endurance test was used to determine the effect on the dielectric properties of chemical additions to the paper or compound and the effect of mechanical changes in the nature of the paper, such as pinholes, open and closed butt joints, conducting particles, tears in the paper, etc.

In three sheets of paper one sheet might have small holes, open or closed butt joints, or tears, and very little effect would be produced on the endurance.

This, taken with a wide experience of similar nature on actual cable, indicates that there is little to fear with respect to the electrical qualities in having very many more tears present in cable than allowed by the present bending test. Indeed, it is hard to see why tears are

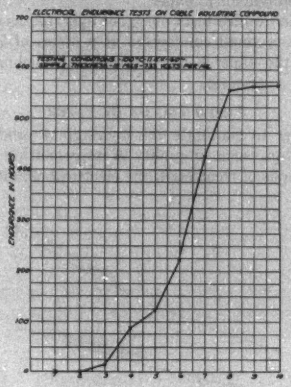


FIG. 18—ELECTRICAL ENDURANCE TESTS ON CABLE INSULATING COMPOUNDS

any more objectionable than butt joints, unless registration of the tears should also occur, or a relatively large void be formed.

COMPOUNDS

The most significant of the electrical tests for compound are:

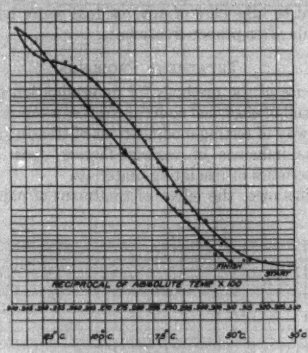
(1) Determination of d-c. resistivity as a function of temperature, and

(2) Determination of rate of decrease of resistivity under exposure to air at 100 deg. cent.

It was found that resistivity measurements made at temperatures below 100 deg. cent. had very little meaning because of the variable water content of an oil. This water is not necessarily driven off at 100 deg. cent. whether the oil is under vacuum or not. In fact, water may remain liquid without ebullition under oil up to many degrees above its boiling point. If water and oil in contact could be prepared free from gas it should be possible to heat water surrounded by oil up to the critical point of water, 365 deg. cent. without conversion to vapor, because there would be no way for the initial development of a vapor phase to

occur. However, in the ordinary oils we found that evolution of water vapor usually occurred at temperatures in the vicinity of 120-130 deg. cent. A plot of the change of resistivity of an oil with temperature is shown in Fig. 19, where a characteristic discontinuity is shown at these temperatures. Similar characteristics have been described by Shrader.[24]

We therefore found it necessary to determine this characteristic up to 150 deg. cent and to make comparisons of resistivity based upon observations at this temperature before we found agreement with the characteristics of the finished cable which had been impregnated with compound heated for many hours in vacuum tanks at temperatures above 100 deg. cent. It will be noticed that the curve of Fig. 19, determined with falling temperatures, shows a lower resistivity than with rising temperature above the critical point and higher below it. This difference is produced by oxidation at 150 deg. cent., and because of this, measurements should be made very quickly in order to eliminate this effect as much as possible. In order to do this a cell was designed by Mr. C. Dantsizen as shown in Fig. 20. When a small volume of oil is heated in the glass cell by a heating coil energized from an insulated X-ray tube filament transformer, about 20,000 volts are impressed on the electrodes from another transformer through a rectifying kenotron and the current read by a microammeter. Such a cell is used for factory control work where speed is a requisite, and complete measurements can be obtained in about 10 min. For more refined work a cell with guard ring, designed by Dr. C. W. Hewlett, to operate in an oil bath at a few hundred volts and for current measurements with galvanometer is used.



CONDUCTIVITY OF PETROLATUM LIQUIDUM AS A FUNCTION OF TEMPERATURE

Fig. 19

The rate of deterioration is determined by simply exposing a definite quantity of oil in a vessel at 100 deg. cent. to air and removing samples from time to time, measuring their resistivity. This is an important characteristic from the standpoint of permanence during factory operations. When the resistivity of the material in the factory falls below a certain value because of oxidation during the repeated exposures of the treating processes, it is purified by agitation with finely divided Fuller's earth, followed by filtration. This brings the resistivity back to, or better than that of the original material.

Too high a resistivity, obtained by excessive purification, does not appear to yield correspondingly better cable. In fact, endurance tests on some water white oils and petrolatums have indicated inferior dielectric properties.

The unreliability of a dielectric strength test as a measure of oil quality is shown by the results of measurement on a petrolatum after 1160 hr. heating in air at 100 deg. cent. when the dielectric strength fell only to 38.2 kv. from an initial value of 40.2 kv., the sludge formed, however, being allowed to settle. During the same period the resistivity fell from 500×10^8 to 1.5×10^8.

Measurement of Oxidation

An interesting method for the investigation of the changes produced in an oil by oxidation is that described by I. Langmuir[25] in his investigation of "Oil Films on Water."

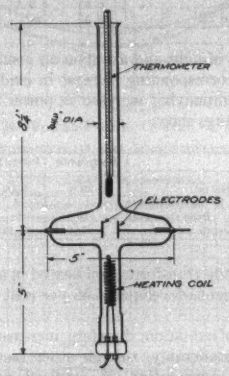


Fig. 20—Apparatus for Measuring Insulation Resistance of Compounds

A pure saturated hydrocarbon oil has no affinity for water. If a drop of such oil be placed on a clean water surface it remains as a lens without spreading. However, if an acid oil such as oleic acid be used, it will spread out into a film one molecule deep, because the end of the molecule having the OH group dissolves in the water, the other end with only hydrogen atoms exposed being soluble only in the oil. Similarly when an oil is oxidized, the OH group formed will cause the

oxidized portion to spread over the surface of the water. Accordingly by measuring the area of the film formed when a known quantity of oil is dropped on a clean water surface the actual quantity of oxidation may be measured if the size of the molecule is known. This is uncertain in many cases but for approximation purposes these molecules have been assumed to be of the size of a stearic acid molecule.

In order to compare different oils, one gram of oil was dissolved in 10, 50, or more cu. cm. of carefully purified benzol so that two or three-tenths of a cubic centimeter when dropped on water would give a film of about 1000 sq. cm. The benzol rapidly evaporates leaving the oil only on the water. Assuming the cross section of a stearic acid molecule to be 20×10^{-16} cm., the amount of oxidized material in the original oil in terms of stearic acid may then be calculated.

The per centage of hydrophils expressed as stearic acid in petrolatum refined to different degrees is given in the following table as well as the resistivities determined at 150 deg. cent.

	Hydrophils	Resistivity 150°
Red	1.82%	1.82×10^{10}
Amber	0.89	9.5×10^{10}
White	0.35	28.5×10^{10}

The effect of light and air upon an insulating oil also produces a corresponding increase in oxidized material and an accompanying increase in power factor as the following figures show:

	Hydrophils	Power factor
Original oil	0.33%	0.047%
6 hr. in open dish 40 cm. from quartz mercury arc	2.87%	0.135%

A high grade transformer oil heated in air for 120 hr. at 100 deg. cent. developed 4.55 per cent of hydrophil content.

Products of oxidation, therefore, increase the losses in oils very considerably, though other factors, such as sludging or aggregation of oxidized products into discrete particles may greatly modify the relationship between the actual quantity of material oxidized and the resistivity.

The presence of OH groups in an oil is responsible for the formation of emulsions of water and oil. If water is present in insulation containing only pure saturated hydrocarbons, it will occur undistributed in drops which may initiate local breakdown. In the presence of organic acids, however, the water may "chemically" combine so that it will be uniformly distributed throughout the oil and so be in a less dangerous condition. In order to determine whether or not rosin additions acted beneficially in this way, power factor measurements were made upon sheets of undried paper impregnated with oil alone, and with a rosin and oil mixture. The power factor—temperature characteristic is shown in Fig. 21, which shows no essential difference in the characteristics, although it might be expected that if the water were attached to the rosin the power factor would be less than in the case of the oil alone, or that the maximum point on the rosin curve would occur at a higher temperature because of its greater affinity for water. However, it is probable that the oils used for impregnation contain enough oxidized material to take care of any residual water left in the materials after proper treatment.

Treatment Problems

The use of the methods for determination of power factor and of electrical endurance gives a comparatively simple way of testing modifications of treating methods and the influence of various factors upon life and characteristics of insulating materials.

For instance, the effect of treatment of paper in oil

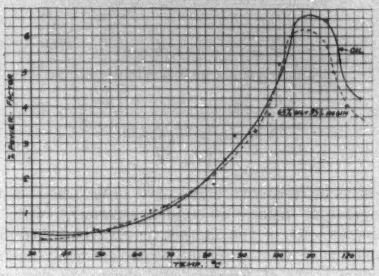


Fig. 21—Dielectric Loss Measurements on Samples of Undried Impregnated Paper

at different temperatures was studied, as shown in Figs. 22 to 24, the chief difference presumably being the amount of moisture left in the paper.

Fig. 22 shows the variation of the power factor of wood pulp paper, as a function of the drying time in air at 110 deg., and Fig. 23, the variation of impregnation time in 118 oil at 80 deg., compared with the characteristics of pre-dried paper impregnated in the same oil for 15 min. Fig. 24 shows the power factor characteristics of similar paper treated in the same oil at 100, 120, and 150 deg. cent. As the temperature is increased the power factor characteristic becomes flatter, and the time necessary for the elimination of water less, so that 15 min. impregnation at 150 deg. gives a better characteristic than 2 hr. at 100 deg. cent.

Fig. 25 shows the relationship between temperature of treatment and endurance of impregnated paper tested according to the condenser method described above. Three sheets of undried manila paper were dipped in petrolatum at the temperatures indicated

for ½ hr. and then placed under stress of 733 volts per mil at 100 deg. cent. until breakdown occurred.

It will be seen that the paper treated at 85 and 115 deg. cent. broke down immediately. In order to equal the life record of the 150 deg. treatment, samples had to be heated for 50 hrs. at 110 deg.

The effect of adding rosin to the oil is shown in Fig. 26 when a steeply rising characteristic is obtained similar to that of moist paper. This indicates that the losses

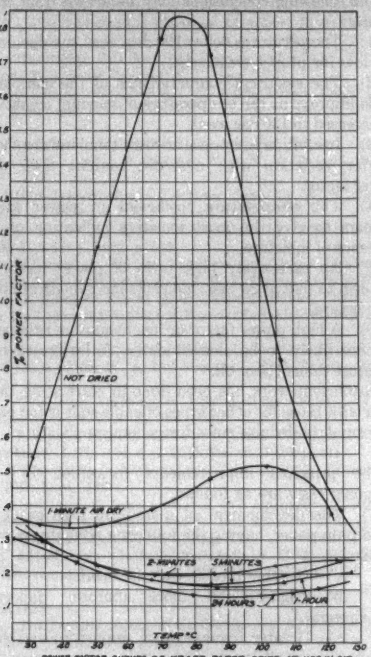


Fig. 22

in a rosin mixture are caused by direct leakage through the dielectric.

The effect produced by eliminating this conduction current is shown in Fig. 27 where a thin quartz plate with negligible losses is inserted in series with a piece of poor quality impregnated paper. The resultant characteristic has lost the high temperature coefficient of loss, as might be expected.

The results of the above experiments show that a high temperature coefficient of power factor, when caused by the presence of moisture, is followed by a short life of the insulation under stress. High power factors produced by other agencies do not show such definite results.

It is also apparent that high temperatures of treatment accomplish the elimination of water much more rapidly than low temperatures and may be more effective than vacuum treatment at lower temperatures.

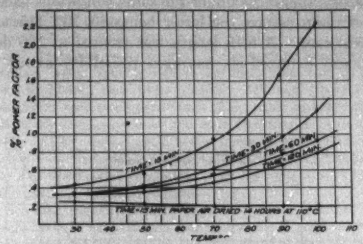


Fig. 23—Power-Factor Curves of Kraft

Paper treated in 118 oil at 80 deg. cent. for different lengths of time and under various conditions

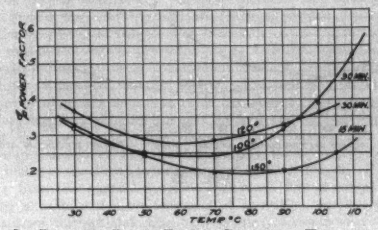


Fig. 24—Power-Factor Curves of Kraft

Paper treated in 118 oil at different temperatures

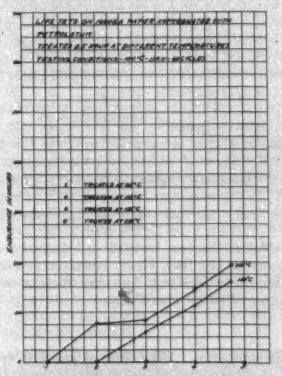


Fig. 25

Specimens in order of failure

However, when using high temperatures, the time of exposure must not be so long as to injure the paper mechanically, nor to oxidize the compound unduly. Compromise between the factors involved is therefore necessary.

In order to determine the characteristics of actual cable exhaust at different temperatures four pieces of lead-sheathed hollow-core cable, 23/32-in. Kraft paper insulation, were vacuum treated and impregnated in the laboratory apparatus under the same conditions except that of temperature, the latter being 80, 100, 115, and 130 deg. cent. A fifth length was treated at 130 deg. with badly oxidized oil, having a resistivity about 1/8 of the regular material. The time of vacuum treatment was 120 hr., and the pressure less than 1/10 mm.

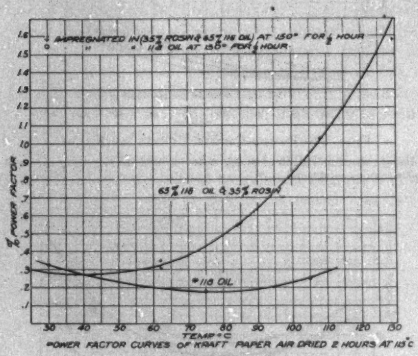


Fig. 26

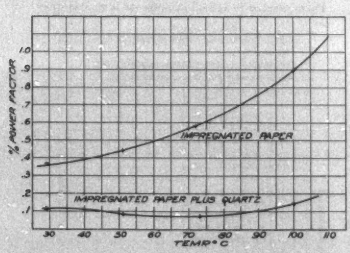


Fig. 27—Power-Factor Curves of Manila Paper Air Dried 2 Hours at 110 Deg. Cent. Impregnated in No. 118 Oil at 150 Deg. for ½ Hour, Showing Result of Putting Quartz Plate in Series with Paper

After treatment power factor was measured by the Schering bridge method at temperatures from 25 to 100 deg. cent. at 20 to 76.4 kv. and insulation resistance at 1000 volts d-c.

The following characteristics were obtained:

Temp. of vacuum treatment	Insulation resistance in megs. per mile		Per cent Power Factor		
			20 volts/mil		100 v/m
	25° C	100° C	25° C	100° C	25° C
80° C	195	8.1	0.80	6.6	0.90
100° C	509	24.3	0.43	1.3	0.41
115° C	823	50.2	0.43	1.1	0.40
130° C a	1722	125	0.41	0.89	0.37
Length treated with oil exceptionally low in resistivity					
130° C b	387	17.5	0.47	1.6	0.45

The following conclusions may be drawn from these results:

(1) Temperatures below 100 deg. cent. are decidedly too low for proper treatment.

(2) The electrical characteristics of the completed cable improve as the temperature of vacuum treatment is increased to 130 deg. cent.

(3) Insulation resistance at room temperature shows greater variation with moisture content than does power factor.

(4) The temperature coefficient of power factor appears to give the most sensitive indication of the presence of moisture.

(5) An excessively high ratio of power factor at 100 deg. to that at 25 deg. indicates incomplete removal of moisture.

(6) A high power factor or low insulation resistance is more likely to be characteristic of the presence of moisture than of poor oil.

Insufficient work has been done in correlating these data with those for other types of cable so it is too early to formulate definite rules for general application.

The principal improvements forwarded by the foregoing tests are as follows:

(1) The development of a uniform high grade of wood pulp paper, having the following advantages over manila:

(a) Greater mechanical uniformity.

(b) Higher and more uniform dielectric strength.

(c) Greater freedom from impurities and conducting particles.

(d) Lower and more uniform power-factor characteristics.

(e) Greater ease of drying.

(f) Better characteristics with respect to impregnating compound.

(2) The securing of insulating oils, uniform in quality, and the maintenance of their uniformity in characteristics during the factory process.

(3) The determination of the magnitude of effects produced by definite additions of impurities or mechanical changes in the insulating materials used.

(4) The determination of optimum conditions for treatment and impregnation.

In conclusion the writers wish to emphasize the fact that they have in this paper correlated the efforts of

many individuals. In addition to those mentioned in the text especial contribution of thought and effort has been made by Messrs. W. S. Clark, W. C. Hayman, C. A. Piercy, E. S. Lee, E. D. Eby, and V. A. Sheals.

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Discussion

M. T. Crawford: Operating experiences discussed by the Underground Systems Committee, N. E. L. A., last week in Pittsburgh, seemed to indicate that the mechanical performance of cables and splices under temperature changes, as referred to in this paper, make it imperative that modern highly impregnated cable be tightly packed and assembled, or else much thicker lead be used on splicing sleeves than has heretofore been the practise. Instances were reported of the swelling and bursting of ordinary ⅛-in. splicing sleeves on first application of load current to 11-kv., and 13-kv. cables. Although these cases represented a very small percentage of the cable recently made, a case in my own experience was so severe that a 1-mi. cable run of five 11-kv., cables could not be operated over half normal loading without excessive bursting of splices, and it was necessary to tap and bleed cable compound from the splices in order to put the cables in full-load operation.

In this case, the only unusual features of the installation were the facts that the cable sheath possessed extra strength due to a content of tin, and observations had been noted at time of factory inspection and tests of loose filler, and of slight looseness of sheath at ends of certain cable lengths.

Assuming, for instance, a cable sheath diameter one-half that of the splicing sleeves, and the same thickness and quality of lead in each case, (this approximating current practises) the wall of the sleeve will be subjected to twice as great a unit stress as the wall of the sheath, providing the cable is constructed loose enough to permit ready longitudinal flow of compound and transfer of pressure into the splices. The fact that the great majority of recent solid-type cable installations has not burst splices must be due to the fact that the cables are built so tightly that this ready longitudinal flow is not permitted, and the pressures developed are therefore largely taken up by slight stretching of the sheath in increments of the cable length. However, from this paper I conclude that ready flow of compound along a cable and some looseness of assembly, together with maximum impregnation, may be desirable features in solid type cables, for electrical reasons.

I should like to ask the authors:

1. If the above conclusions are in their opinion correct, and if fully impregnated cables *are* better with slightly loose construction, should not present splicing standard practise be changed, to include sleeves of proportionate strength to that of the sheath, or else be provided with reservoirs or expansion chambers filled with oil, even in the moderate voltage class solid-type cables.

2. If looseness of construction in solid-type cables is not a desirable feature, should not specifications cover this point more specifically in cables which are to be fully impregnated, in order that standard splices will not be damaged by an occasional variation in tightness which heretofore has not been considered of importance.

F. A. Brownell: A comparison of tests made on cable 5 years ago with the present might be of interest, to show the tremendous advancement that has been made in the art in the past few years.

The maximum time obtained on a test of 5 samples, 5 years ago, was 23 hr. at 192 volts per mil and 5 hr. at 224 volts per mil; present tests give 226 hr. at 202 volts per mil (without failure) and 95 hr. at 224 volts per mil.

Analysis of cable after tests made 5 years ago showed heavy wax formation, the entire test section was filled with dendritic designs and heavily carbonized spots, evidence of extreme heating and a rank odor which indicates heavy overstressing were found. The tests that we are making at present show the cable to be practically free from wax and relatively free from dendritic designs, except in the region of the failure. We do not get heating due to ionization that we did in the past. We have had cables on test for 25 hr. at 260 volts per mil with practically no rise in temperature.

One great advancement has been in the general use of shielded-conductor cable for voltages over 25 kv. In this type of cable the stresses are at right angles to the conductor at all times, thus eliminating the tangential stresses and the ionization and heating that occurred in the center filler space of the belted cable.

The soaking in of the compound with time, is no doubt a most important detail in manufacture. If the paper is not thoroughly saturated the free oil is drawn from between the layers into the fibers of the paper, after the cable has been in service for some time. This leaves voids in the cable which in time are ionized. We find plenty of evidence of this in cable that has been taken from service.

We find in many cases that in the region of failure on laboratory tests the insulation is water stained, which checks the findings of Hirshfeld, Meyer, and Connell.

Extreme care should be used in handling oil for joint filling. We have tested oil that has been shipped in sealed containers and it has shown a dielectric strength of 15 kv. Consequently all of our oil is tested before being used.

I should like to ask the authors what they consider the maximum safe voltage for a "solid" single-conductor cable.

I believe most of us are in agreement with the authors that the short time breakdown test is of little value and consider it a waste of time to make them.

I want to bring up the subject of wrinkled cable. The best tests that we have ever obtained were on cable that was rejected due to wrinkles. Analysis of failures on 12 samples tested showed that none of the failures occurred in or were due to wrinkles, although some of the cable was extremely wrinkled. I trust that manufacturers will not use this as an argument for selling us wrinkled cable. My object for bringing up this subject is to ask the authors if they can visualize what will happen in these wrinkles after the cable has been in service for a few years.

E. W. Davis and W. N. Eddy: The data tabulated on page 366 are of considerable interest in indicating the relative difficulty of removing moisture from the oil filled hollow-core type of cable with the sheath in place. In the course of a laboratory investigation the following impregnating procedure was applied to a piece of single conductor solid core cable insulated with 24/32 in. of wood pulp paper, without the lead sheath. The cable was subjected to 50 hr. atmospheric drying at 105 deg. cent., 12 hr. vacuum drying at 80 deg. cent., 3-mm. pressure, 10 hr. saturation at 80 deg. cent. with a semi-solid compound, and then cooled to room temperature. While this procedure appears far less rigorous than the authors' 120 hr. at 0.1-mm. pressure, the cable gave a power factor of 1.8 per cent at 100 volts per mil, 100 deg. cent. For a solid cable such a power factor is not considered sufficiently high to indicate the presence of an excessive amount of moisture. It is far lower than the hollow-core cable after 120 hr. at 80 deg. cent. and 0.1-mm. pressure.

Fig. 26 suggests that the addition of rosin to the compound might be expected to increase greatly the influence of temperature on the power factor. That this may not always be the case is indicated by the following experiments with single conductor No. 2/0 cable insulated with 9/32-in. wood pulp paper. Pieces of this cable impregnated in the laboratory with a cylinder oil (very similar in viscosity to the authors' No. 219 compound) gave a power factor of 1.1 per cent at 100 volts per mil, 80 deg. cent. while the power factor of the compound itself was 1.6 per cent at 80 deg. cent. Using the same impregnating procedure in each case, similar cable was impregnated with other compounds. A mixture of the same cylinder oil after some oxidation (compound power factor of 2.2 per cent with 15 per cent of a wood rosin) resulted in a cable power factor of 5.5 per cent and a compound power factor of 6.8 per cent. A mixture of the same cylinder oil unoxidized (1.57 per cent power factor) with 15 per cent of a gum rosin resulted in a cable power factor of 1.7 per cent and a compound power factor of 1.01 per cent. (All power factors given are at 80 deg. cent.) Tests on the two rosins separately showed no wide difference in electrical characteristics. The only difference in the mixing procedure was that the wood rosin was added to the cylinder oil with both materials at 120 deg. cent. and stirred at that temperature for 30 min., while the gum rosin in a wire basket was submerged in the cylinder oil with the latter at 120 deg. cent. until complete solution and for 30 min. thereafter.

The authors suggest that the electrical quality of cable insulation may not be so sensitive to the presence of torn tapes as is sometimes assumed. In the laboratory, high-voltage cables are continually being tested to destruction. During the last three years not one of these failures has been found due to torn paper. Only one was due to wrinkled paper. As a laboratory failure is not burned so badly as a service failure any torn paper at the fault would usually be easily seen.

Herman Halperin: Two leading European cable engineers recently made statements to me which are in agreement with the authors, that is, the development field ahead for the "solid" type of high-tension cable extensive is still quite excessive. These engineers cited considerable work being done in this direction in Europe.

On the basis of theoretical and field investigations in Chicago in the past four years in connection with oil-filled joints, I cannot be as hopeful as the authors are for the maintenance of impregnation of entire lengths of cable having solid insulation by means of reservoir feed at the joints.

In connection with a recent examination of over 100 samples of three different makes of 66-kv. single-conductor cable, which had been in service for 20 months, it was found that the oil from joints fed from gravity reservoirs traveled only 5 or 10 ft. into two of the makes, while it was present 20 or 30 ft. away from the the joints in the third make. The presence of the oil was determined by those present, including the manufacturers' representatives, by the appearance of the insulation, and perhaps there was some oil still farther in the cable, but this amount would be so small that it could do very little good in connection with the filling of void spaces during decrease in load.

On the other hand, there was one length of cable installed in a tunnel which is usually almost completely filled with water, but in connection with some special work the water was removed. Oil from the joint at the top migrated down the vertical length of cable (a distance of 85 ft.) and thereby caused a hydrostatic pressure of 35 lb. per sq. in. inside the cable. The sheath cracked at a defect. The distance the oil traveled was a few hundred feet, but the pressure of 35 lb. per sq. in. is considered very excessive for single-leaded sheaths.

If the pressures on the oil in the joint are to be limited to some value of a few pounds up to 10 to 15 lb. per sq. in., it seems to me that the oil will travel entirely too slowly to affect void formation materially during temperature drop in the middle of insulation 200 or 300 ft. away from a joint. There is the great resistance to longitudinal flow and then, even were there fairly free channels along the conductor and sheath, there is large radial resistance to flow of oil.

Our experience confirms some of the authors' theories in that we have found that with lengths of the same vintage of cables installed on two different lines and operating through different temperature ranges, the migration of oil from the joints into the cable was proportional to the temperature ranges. Furthermore, we have found with both single and three-conductor cables that while the total amount of oil taken is increasing, the rate of absorption is decreasing slightly each year. It is hoped that the authors' theories that the sheath will not rupture eventually are true.

Our experience with some three-conductor, 33-kv. cable and the 66-kv. cable, which was furnished us in the past three to

eight years giving considerable trouble in service, was that neither the operating temperature nor temperature range had any primary influence on the rate of failures. When such cable, however, is greatly improved and the product is uniform, it appears from theory that the temperature range will have an effect; but at present I do not agree with the authors that the present temperature limits should be decreased.

Another point is on the third page, where the authors say that when holes occur in the sheaths the water is allowed to come into the insulation and usually wax and ionization is found. Our experience does not agree with this generalization.

T. F. Peterson: It is quite gratifying to me to see that some of the conclusions reached by the authors are in line with contentions that have been held by myself and others in the Brooklyn Edison Company and other utilities using reservoirs on solid types of insulation.

It would seem to me, however, that the time has come for a really rational consideration of impregnation and reservoir use, covering cable throughout the entire range of operating voltage. We in the electrical industry are not unlike those in other spheres of life; we have a tendency to move in cycles; the pendulum motion seems to be quite characteristic of our motion. We get into a certain line of throught, run away with it, and finally slow down and retrace our steps. An illustration of this was had a few years ago when power factor of cable was predominant in our minds. All efforts were exerted to reduce the power factor of insulation. Finally compounds were introduced which, though they corrected power factor, ultimately gave rise to wax formation, and so had to be modified or abandoned.

It seems to me now that the question of ionization, after having been so fully studied, is going to lead us into a similar condition. Though we should bear all pressure on the higher voltage cables and endeavor to eliminate voids and air spaces as much as possible in this range, it does not necessarily follow that the same principles should be applied to the lower voltage cables. I have in mind a range from about 600 volts to 13,000 to 15,000. We have had illustrations of cable sheath and joint failure due to expansion of compound. Others have experienced difficulties with compound emerging from oil impregnated low-voltage cable into air-filled potheads, and the like. What is the solution to this? Obviously, reservoirs on such cable systems are impractical, since, for the most part, hard compound is used for filling joints.

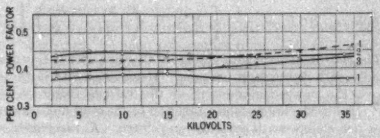
My contention is that in such cable it is perfectly permissible to allow a certain small percentage of air within the cable to provide for the expansion and contraction of compound without the development of extreme pressure or very low vacua. The cable can be designed so that these void spaces will not have detrimental effects at operating voltage, and will also serve to eliminate most of the difficulties that have been experienced. Therefore, in the case of low voltage cable, say to 13,000 volts, it would seem that 2 per cent of air running up probably to 6 and 7 might be allowable.

Carrying this thought a step further, it might be argued that we ought to have no impregnation but air, throughout the cable length. However, the factor entering and making that impossible is, of course, thermal conductivity of the insulation. With all air, the carrying capacity of the cable would be materially reduced. Nevertheless, it is perfectly possible to strike some happy balance of air content with which we will not run into the difficulties that have been cited this morning, but which will also yield desirable thermal characteristics of the insulation.

The range from perhaps 20,000 to 60,000 volts will probably be taken care of with a semi-solid insulation with reservoirs at joints and above that, of course, the regular oil-filled cable would serve.

I am stressing this point because I think it is necessary in this connecton to view also the tests that are applied to cables in determining the quality rating. Obviously if we think it permissible to have a certain percentage of air in lower voltage cables we shouldn't judge their serviceability by ionization characteristics, as in the case of high voltage cable, nor should we judge their serviceability by accelerated life tests that are obtained at gradients probably three or four times those used in operation.

C. L. Dawes and P. H. Humphries: At the Harvard Engineering School we are studying the progressive changes in cable characteristics as they undergo accelerated life tests. Five 10-ft. samples of 300,000-cir. mil, 3/16-in. wall, single-conductor cables were put on test at an average gradient of 225 volts per mil. These samples were submitted by four different manufacturers, one manufacturer submitting two samples. Four have already failed. An examination shows that in nearly every case the tapes adjacent to the copper were perforated by ionic bombardment. Moreover, the portions of the paper which lay over the interstrand spaces showed either deposits of wax or charred compound. This would seem to bear out the statement that voids have a tendency to collect in the strand spaces. This would also indicate that the interstrand spaces were not entirely shielded electrostatically. Otherwise, the ionic perforations and disintegrated compounds would not have occurred as we found them.



POWER FACTOR *vs.* VOLTAGE CURVES OF CABLE

Frequency — 60 cycles. Temperature — 22 deg. cent.

Curve	Life in hours
1	0
2	83¾
3	170½
4	470½

We are able to confirm the statement that for all practical purposes a cable may be perfectly impregnated as shipped from the factory. Among the cables under test, the initial power-factor curve of one is shown as 1 in Fig. 1 herewith. Curve 2 gives the power-factor curve after 83 ¾ hr. test; Curve 3 after 170½ hr. test, and Curve 4 after 470½ hr. test. The cable has been on test to date 532½ hr. and although its characteristics have been obtained daily, no appreciable change in the power-factor from that given by Curve 4 has been found. The relation of power and capacitance to voltage has also assumed constant values. The cable to date after 532½ hr. test has shown no indications of failure, whereas the last cable of the other four failed at the end of 124½ hr. Hence, it appears that the perfectly impregnated cable has long life.

On page 346 it is stated that a shield under the sheath of single-conductor cable may be tried. We have made tests on three such cables, submitted by three different manufacturers. The same manufacturers submitted simultaneously, cables nearly similar but without shielding tapes. The complete report of our tests was made to the N. E. L. A. Underground Systems Committee Meeting at Louisville, March 22, 1928. Even after severe bending tests, the cables with the shielding tapes showed no marked superiority over those having no shielding tapes.

Referring to Fig. 11, the authors show a decrease in voltage across a gas discharge after the current has reached a critical value. This is undoubtedly true where the discharge occurs in a comparatively long tube as indicated. However, we have investigated the discharge characteristics of a number of air films

as shown in Figs. 8 and 9 of our paper. The voltage increases to a maximum and constant value. Under the highest ionic current densities which we have been able to produce (4 microamperes per sq. cm.) there has never been any indication of a drop in voltage with increasing current. Hence it does not appear probable that the reversed type of curve occurs in cable insulation. Perhaps this is due to the more rapid recombination which occurs in restricted gas films. We are in complete accord with the statement that very little is known regarding the laws of ionization loss in cables. We have been attempting for some time to determine the laws of ionization of gas films and some of our results are given in our paper. When a few more data are obtained, we hope to be able to state definitely the effect of pressure, gap length, frequency, etc., on the loss.

We are particularly interested in Figs. 13 and 14 which give power-factor characteristics of a cable with artificial gas films at different pressures. Reference is made to the relation of negative slope in the power-factor curve to negative volt-ampere characteristics of the gas. This is contrary to our experience. We have investigated the properties of such gas films down to 10-in. pressure, and have never obtained a negative volt-ampere characteristic. The variation of the power factor curves is readily explained by Equation (12) in our paper, which may be rewritten as follows:

$$\text{P. F.} = \frac{1}{C\,\omega}\left[K + K_1\left(\frac{1}{E} - \frac{E_0}{E}\right)\right]$$

The rate of increase with voltage of the capacitance C increases rapidly with decrease in pressure. Hence, decrease in pressure lowers the power-factor curve as the voltage increases. The first term in the brackets is a function of the solid dielectric, and does not change with pressure or voltage. K_1 decreases with decrease in pressure. For example, at atmospheric pressure, $K_1 = 39.3$; at 10-in. pressure, $K_1 = 10$. E_0 obviously decreases with pressure. This can be readily seen in Figs. 13 and 14. Hence E_0/E is smaller for lower pressures than for higher pressures. Thus at the lower values of E, the parenthetical term is larger for the lower pressures than for the higher pressures. As E increases E_0/E becomes smaller in comparison with unity and the parenthetical term becomes more nearly constant. The lower value of K_1 for the lower pressures now causes the power-factor curve to decrease at a more rapid rate. Hence, the curves at different pressures should cross one another which they actually do in Figs. 13 and 14, the curves for lower pressures ultimately having the lower values of power factor.

R. W. Atkinson: I find myself in very good agreement with most of what is given in the paper by Messrs. Shanklin and Mackay.

As to specific points in the paper, I will comment on one only. Mention is made of the relation between the volume of void space, as defined, and the amount of ionization. We have considered that a given amount of ionization represents a considerably greater void space than is indicated by the authors. I think that the difference is largely in the method of measurement of the void space. As the authors point out, this is not a simple matter to determine, and I think that probably both of us should study that question more and find out whether the void space is actually that given by the authors or whether it may be an amount perhaps several times as large as that.

I should like to say a word in summary of the trend of development in the future, an analysis which is right along the line that the authors have given. In fact, I have taken part of it from there. I think the future development is going to be along two or three or four distinct lines, each one being carried forward and ultimately perhaps resulting in things rather different, or perhaps the trend may bring the different lines together. The oil-filled cable development is one of those. That development, as the authors point out, is going to be largely in the perfection of manufacturing and installation, and in the reduction in cost. In addition, there will be the development of stop joints that are much less expensive and can be placed at closer intervals.

Then another trend of development will be in the direction of an approximation of the oil-filled cable, an attempt to retain the conveniences of the solid type and the electrical advantages of the oil-filled type. There is no doubt there will be important developments in that direction.

An important direction of development of the solid type of cable will be in the mechanical characteristics of the compound so as to minimize void formation. As pointed out by different authors, an attempt to find a compound which is immune to the effect of the ionic bombardment has not proved at all promising.

The development in both the oil-filled and the solid type of insulation seems to be toward minimizing that bombardment by minimizing the voids. Of course we will have improvement in the characteristics of the paper and in the general method of application, and of course we will have improvements in the general matter of increasing uniformity and in the obtaining on all cables of results substantially equal to those obtained on the best of them.

There is one other phase of development that has been remarkable in its rate of activity recently and which undoubtedly will continue in the same direction. I refer to the shielded type of cable. Some may wonder why the shielded type of cable, since it has been known as many years as it has, did not take large commercial importance for some time, and then having taken a hold, has gone with such extraordinary rapidity. I think three reasons have contributed to that: first, of course, the very successful results of the earlier experience with the shielded type of cable; then, the recognition of its value; and finally, the fact upon which I commented at the N. E. L. A. Convention in 1923, that the better we make our insulation, the more advantage we obtain by the shielded construction. As we develop the quality of our insulation, the field for the shielded-type cable becomes greater and greater, and this is going to continue and bring its use down into the lower voltages as well as increase the predominance it already has in the higher voltage range.

E. S. Lee: There is an achievement recorded in this paper which is most inspiring.

In a most intensive effort to obtain better cable insulation many samples of different materials made up with different treatments were tried by subjecting them to 44 kv. at room temperature until they failed. The life of the many different samples fell between 5 hr. and 162 hr., which was certainly not encouraging.

However, similar samples maintained at a temperature of 85 deg. cent. continued to live under the applied voltage for many hours, reaching in some cases 1000 or 2000 hr. All that one had to do to get the sample to fail was to open the heater switch and let the ambient temperature come down to that of the room, after which the sample was sure to fail within a few hours.

What was the reason for this remarkable difference and long life at the higher temperature? Investigation revealed that it was simple due to the treating compound being liquid at the higher temperature and solid at room temperature. The suggestion of using a liquid oil filler was immediate.

Subsequent tests proved that by using liquid oil as a treating material the voltage rating could be doubled. This was a most remarkable achievement in insulation design and one not very frequently met. However, the difficulties of using liquid oil prevented the immediate application of so signal an advance.

Opportunity was later afforded by the New York Edison Co. and the Commonwealth Edison Co. for the installation of oil-filled cable for 132-kv. circuit. The cable installed was for double the voltage of the 66-kv. cable then in operation but had only 77 per cent of the insulation thickness. The electrical performance of that cable has been perfect.

The reason for the achievement is in the uniform and perfect filling of the insulation throughout the operating temperature range. The result came from intensive application in the

laboratory by observing carefully all the related phenomena. As it turned out, the only equipment necessary was a high-voltage transformer, a voltmeter, a clock, means for connecting the transformer to the cable sample, and means for holding the cable sample at 85 deg. cent. I point this out as encouragement to those who may not have extensive equipment available but who still may be in position to contribute worthily.

N. E. Buck: (communicated after adjournment) Oil reservoirs connected to the joints are probably of benefit to the cable adjacent to the joints in the following ways:

1. Gas bubbles can be expelled from the cable ends, and find their way into the reservoir. On the contrary, there can be no transfer of free gas from the reservoir to the cable.

2. The principal oil-flow in the cable will take place in the filler spaces, between sheath and insulation, or between conductor strands. Gas bubbles which may be present in these channels will probably be carried along with the oil flow. Thus when the heat cycle causes oil to be returned to the joint, the free gas which flows with it will become trapped in the reservoir. The converse effect cannot take place, as pointed out in (1).

3. The pressure, and therefore the amount of dissolved gas, will be lower when oil is being drawn into the cable than when it is being forced out. The net result will be a removal of dissolved gas.

From the above it is clear that there will be a marked tendency for the amount of free and dissolved gas to diminish, in the cable adjacent to joints equipped with reservoirs.

The flow of oil through the body of the insulation is probably small, and the gas entrapped in the capillaries is highly resistant to motion. The only way in which oil reservoirs at the joints could have a beneficial effect on the cable insulation remote from the ends would be to maintain a high enough pressure to keep the gas dissolved.

W. A. Del Mar: (communicated after adjournment) The authors reiterate the statement that a cable should be gas free, but neither they nor anyone else has offered any evidence to support this theory.

The statement that reservoir oil contaminates the cable compound and lowers dielectric strength is contrary to theory and experience. Indeed, operating companies report that the cable near the reservoirs is greatly improved in quality.

There is a notable absence of reference to the properties of cable compound with respect to gas absorption, although statements are made about the effects of gas dissolved in oil. This is a matter of great importance as it is known that oil dissolves gases in proportion to the pressure applied and that some oils, unlike water, hold less gas at low than at high temperatures. The published data on the subject are conflicting and experimental work is beset with difficulties.

D. W. Kitchin: (communicated after adjournment) Some work on the subject of gas and "x" wax formation done in our laboratory about three years ago is in agreement with the conclusion of the authors and may be of interest. It was possible to produce gas and "x" in all the materials mentioned by the authors except acetylene, which was not tested, when the arrangement was such that corona was *certain* to occur. A rigorous demonstration that stress alone could in no case form "x" or gas was more difficult. Tested in a glass apparatus in which corona could not occur in the evacuated space over the materials, none of them showed any effect in the liquid or molten state, but some greases, notably petrolatum, produced both gas and "x" fairly long periods of stress although initially no voids could be detected visually.

Petrolatum is plastic and requires a certain minimum pressure to make it flow. When allowed to cool in a highly evacuated vessel, the weight of the grease was not sufficient to make it contract absolutely in one piece and voids, in some cases microscopic ones, were produced.

After altering the apparatus so that the previously evacuated grease cooled under a mercury head of an inch or so, sufficient to overcome the stiffness of the compound, *all* the materials could be stressed indefinitely without change. These results serve to confirm the conclusions of Shanklin & Mackay on "x" formation but they also suggest some definite ideas on the value of the Electrical Testing Laboratories' plate test as a criterion of compound stability, which it seems appropriate to express at this time. Many cable engineers are familiar with the conflicting results obtained from this result. Successive tests on the same sample of compound often show contradictory results and often compounds form "x" on the test but not in service, and vice versa. Why does the plate test fail to furnish an infallible criterion of compound stability? Because the action of corona or ionized gas is necessary to cause the "x" and gas production, and corona can take place only when voids or gaseous spaces are present. The methods involved in performing the plate tests do not insure either the *presence* or *absence* of voids, so that whether a material fails or not in that test is largely a matter of chance. It is true that plastic materials like petrolatum are more likely to contain voids when placed between the plates than a viscous oil, and if a large number of plate tests are made on different compounds and the results treated by statistical methods a certain trend may be shown which might have been predicted by merely noting the physical properties of the materials.

The things we want to know and which the plate tests indicate only vaguely are, (a) what compounds form voids most readily, and (b) given the presence of corona, what compounds are most stable to it? To answer (a), measure directly the coefficient of expansion and the plasticity (not the viscosity), or most simply let the compound cool in a long glass tube and note the formation of voids in the body of the material as described in the papers of Shanklin and Mackay, and of Davis and Eddy.

To answer (b) subject the several compounds deliberately to corona under *comparable* conditions and measure the production of gas during a definite time interval.

Since deterioration due to corona is a *surface* action, the apparatus should insure the exposure of a *definite constant* area and the same stress should be applied in each case. A device of the type used in the Detroit-Edison investigation, but arranged to subject the material to corona instead of X-rays is proposed. Data secured in this way should give us a real measure of merit of the various compounds instead of the accidental results of a hit-or-miss method. We need more rational and scientific criteria of compound stability if we are to make progress in the development of better cables of the non-oil filled type. Also, since the plate test gives variable results under its uncontrollable conditions, the proposal to include it in specifications is fundamentally unsound. More scientific tests are suggested above to substitute for the plate test.

G. M. J. Mackay: In answer to Mr. Crawford's questions we wish to say that we feel that low-voltage cable should be free from looseness of assembly in order to assure a good mechanical structure and should be as completely impregnated as possible. Installation of reservoirs or expansion chambers at the joints would, of course, eliminate the troubles caused by expansion but the expense may not be justified. Reinforced sleeves or alloy metal at the joints should be sufficient to overcome the difficulty. Enlargement of the cable sheath will then take care of the expansion of the compound.

Specifications covering the type and strength of joint in the case of these cables should be useful to the operator in safeguarding his installation.

Mr. Brownell's testimony on the improvement of cable during the last few years is very gratifying. We feel, however, that it is too soon to make a prediction about the maximum safe voltage for a "solid" single-conductor cable. We rather expect, however, to see the principle of the oil-filled cable extended to lower

voltages than the raising of the present limit for the "solid" type.

Wrinkles, we are afraid, if not the cause of breakdown on test, will eventually be the spots where voids will occur during operation with the eventual formation of destructive ionization.

The table on page 366, mentioned by Messrs. Davis and Eddy, was inserted to emphasize the effect of temperature in drying paper insulation rather than the difficulty of eliminating water. The time involved was arbitrarily chosen and is longer than that necessary to attain constant conditions at 100 deg. cent. The high vacuum may have been disadvantageous, as far as rate of water evolution is concerned, because a current of air caused by leakage, indicated by a higher pressure, may help in sweeping the water vapor out through the condenser and pump. Their tests on rosin and oil combinations are very interesting, apparently showing the betterment of characteristics when oxidation by exposure is lessened by elimination of stirring while the materials are at high temperatures.

While we agree with Mr. Halperin that oil feed is not a perfect solution of the difficulties attendant upon the use of solid cable, we feel that it certainly very materially improves its endurance. His evidence that oil was found 20 or 30 ft. away from the joint indicates that voids would have existed in this region if oil were not supplied, and in this case were eliminated by the oil. The longer the cable operates the freer the oil flow becomes, whereas without oil, conditions would not improve.

Mr. Peterson is quite right, we think, in saying that there is no necessity for carrying all of the high-voltage technique to the construction and operation of low-voltage cables. Gas, if at sufficiently high pressure, should cause no danger from ionization in low-voltage cables but free admission of air would be dangerous on account of oxidation of the compound with resultant increase of power factor and formation of water.

Messrs. Dawes and Humphries have made some important and instructive criticism of the ionization characteristics in gas spaces. We agree with them that at the current densities involved the discharge itself does not have a negative volt-ampere characteristic and that it is unnecessary to assume one in order to obtain the characteristics shown in Fig. 13. Their equation will give the same form of curve.

We do feel, however, that at any given pressure of a gas once ionization has appeared it is not possible to increase the voltage above a certain limit. The lower the pressure the lower this voltage becomes and in the curves of 6, 7, and 8, Fig. 13, it is only a few hundred volts. The total voltage cross the cable and tube is only 500 when ionization exists. Because, however, the cable insulation is intended for 13,000 volts, this means that if the full voltage were suddenly put across such insulation the gas space would act practically as a good conductor, extending the sheath inwards, when almost the entire voltage would be impressed upon the impregnated paper insulation. While then the discharge itself does not have a negative volt-ampere characteristic, if the full voltage of 13 kv. be impressed across the impregnated paper and series air gap, in a sense the resulting characteristic may be said to have a virtual negative volt-ampere characteristic.

Mr. Del Mar has apparently misunderstood us when he quotes us as stating that reservoir oil contaminates the cable compound and lowers the dielectric strength. Only when the reservoir oil is in contact with air so that it deteriorates by oxidation is such an effect possible. Otherwise, of course, it is of the greatest advantage to use oil feed.

We are in complete agreement with Mr. Kitchin's criticism of the inclusion of so-called stability tests for compounds in present specifications.

Some Problems in High-Voltage Cable Development

BY E. W. DAVIS* and W. N. EDDY*

Member, A. I. E. E. Associate, A. I. E. E.

1. Introduction

A LARGE amount of research work on high-voltage cable is being carried on throughout the country in various laboratories by both the manufacturer and operator. It is generally recognized that a considerable amount of duplicate effort can be avoided and general progress increased by comparison and common discussion of the independent investigations.

In order to encourage such co-operation it was thought desirable to present briefly a few of the problems along this line that are receiving attention in the laboratory of a manufacturer.

2. Measurement of Dielectric Loss and Power Factor

In order to obtain greater sensitivity, the high-voltage bridge introduced by Dawes and Hoover[1] was installed for use in conjunction with the dynamometer method already available. This involved the design and construction of a high-voltage air condenser. As some features of this condenser may be of interest it is described in Appendix I.

Inherently the bridge methods are single-phase and therefore not so well adapted to three-phase measurements. However, the connection shown in Fig. 2 which has been developed for this purpose, has been found satisfactory, about its only disadvantage being that a setting without the cable connected must be taken for each test voltage.

3. Determination of Insulation Quality

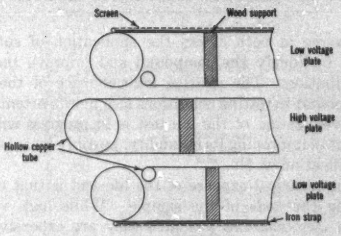
At present the determination of the insulation quality of high-voltage cable is an important problem for both the cable manufacturer and operator that is not yet satisfactorily solved. Everyone concerned with such cable is familiar with cases where cable withstood satisfactorily every test imposed by the manufacturer and operator but failed to give satisfactory service after installation.

Considerable experimental work is in progress with the object of improving the quality of cable insulation. It is obvious that the reliability of the experimental indications is entirely limited by the reliability of the criterion of quality that is used.

At one time a "short-time breakdown test" (increasing voltage rapidly enough to cause insulation failure within 30 min.) was considered a reliable indication of the insulation quality. More recent experience combined with extensive laboratory tests has shown the limited value of this test as compared with the "overvoltage life test." The test results in Table I are given as examples of the greater sensitiveness of the life test to insulation quality.

However, a considerable experience with the overvoltage life test suggests that even this test may not always give a true indication of relative quality. It is becoming accepted that the cause of most premature failures in service is fundamentally the presence in the insulation of voids that are formed by the shrinkage of the compound when it cools. While the service conditions (extreme temperature changes with the in-

A



B

Fig. 1—No Loss High-Voltage Air Condenser

* Simplex Wire & Cable Co., Boston, Mass.

1. A. I. E. E. Trans. 1926, Vol. XLV, *Ionization Studies in Paper Insulated Cables—I.*

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

sulation approaching a constant enclosing volume condition) encourage this void formation, the life test conditions (constant temperature with the insulation at the ends of the sample exposed to constant pressure) tend to discourage the void formation. Due to the different contraction and viscosity characteristics of different compounds, this tendency may vary with different compounds.

Table II shows the results of life tests on cable saturated with three different compounds at each of several testing conditions. It is seen that the life is in general increased by conditions unfavorable to void formation and decreased by conditions favorable to void formation, the tendency varying with the compounds. In connection with the influence of condition No. 2 on the cylinder oil, it has been generally found that when the life test is interrupted for several hours the life will be increased, the cause being apparently

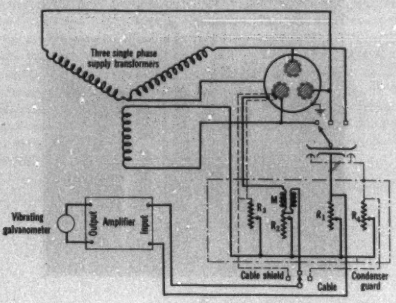


FIG. 2—A THREE-PHASE CONNECTION OF THE DAWES BRIDGE

the same in both cases—the application of sufficient heat to liquefy the compound and improve the void distribution. The relative independence of the rosin compound to testing conditions also merits attention.

Investigation of the life test is in progress with the object of improving its reliability, possibly by some such modification as No. 4.

The principal expense of the life test is that of preparing the ends of the sample. While high voltage cable terminals are available they are expensive and almost impossible to handle without overhead crane facilities. Fig. 3 shows a laboratory terminal that was developed to overcome these disadvantages in connection with a life test at 180 kv. on single conductor cable samples—750,000 cir. mils, 24/32-in. wall.

The ionization test or variation of power factor with voltage at room temperature has received considerable attention as an indication of insulation quality. Fig. 4 shows a comparison between the life test on several cables and an ionization test made just before the life test. While differing in type, dimensions, impregnating procedure, and treatment previous to the life test, the cables were all impregnated with the same compound. The ionization tests were made at 24 deg. cent. and the life test at 20-26 deg. cent.

It is seen that 17 of the 19 samples in Fig. 4 indicate a relation between the ionization test and the life test. A relatively flat power factor—voltage curve seems to correspond in general with a relatively long life. However, the two re-saturated samples give an unusually flat power-factor curve but poor life. They contradict

TABLE I

COMPARISON BETWEEN SHORT TIME BREAKDOWN AND LIFE TESTS

All short time breakdown tests made in accordance with 1926 Specifications for Impregnated Paper Insulated, Lead Covered Cable by Association of Edison Illuminating Companies, Section 20.

All 3-conductor cable tested with 3-phase voltage.

a—3-Conductor 350,000 cir. mils, 19/64 in. Wall, 9/64 in. Jacket.

	Construction No. 1	Construction No. 2
Short time breakdown at room temp., 10-ft. sample	201 kv. for 20 sec.	201 kv. for 15 sec.
Life test at 96 kv. room temp., 10-ft. sample	3.5 hr.	12.5 hr.
Life test at 78 kv. room temp., 250-ft. sample	22 hr.	99.5 hr.
b—Single cond. 750,000 cir. mils, 24/32 in. Wall—all samples 10 ft.		
	Impregnation No. 1	Impregnation No. 2
Short time breakdown at room temp	290 kv.	310 kv.
Life test at 180 kv	2.5 hr.	30 + hr.

*Test discontinued—no failure.

TABLE II

OVERVOLTAGE LIFE TESTS ON 10-FT. SAMPLES OF 1 × 2/0, 9/32-in. WALL IN HR. LIFE AT 65,000 VOLTS

Compound	Condition No. 1	Condition No. 2	Condition No. 3	Condition No. 4
70% Petrolatum B, 30% Transformer Oil C	30	24	215	1.0
Cylinder Oil	50	700	..	1.6
85% Cylinder Oil, 15% Rosin	9	..	9.5	2.2

Each figure the average of 3 tests.

Condition No. 1—Tested at room temp. 25-30° C. with thin oil in ends.

Condition No. 2—Tested after 300 hr. at 25-30° C., otherwise same as No. 1.

Condition No. 3—Tested at 40° C. ambient, otherwise same as No. 1.

Condition No. 4—Sample ends sealed at room temp., with pothead compound, then sample plunged in ice and voltage applied. Final sheath temp. 3-6° C.

Although not typical of factory practise, the saturation and handling of each sample was identical. All samples were cut from the same length of cable.

the relation suggested by the 17 samples. That is, while the ionization test may be a fairly reliable indication of insulation quality on certain cables, it is equally unreliable on other cables. Re-saturation may result in an excellent ionization curve but a deficient life.

Fig. 5 shows the power factor of the same 19 samples at 50 volts per mil. Apparently such a value is even less reliable than the ionization test as an indication of insulation quality.

4. Air Resistance of Paper

One of the most important properties of the paper is its compactness or denseness. This is usually measured by timing the passage of a given volume of air through a definite area of the paper under a known pressure

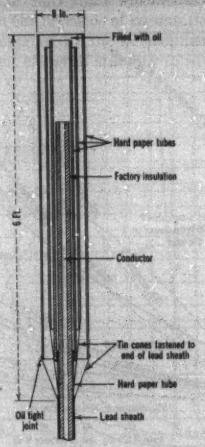


Fig. 3—180-Kv. Life Test Terminal

and is known as the air resistance of the paper. The most familiar units in this country are the seconds necessary for 100 cu. cm. of air to pass through one

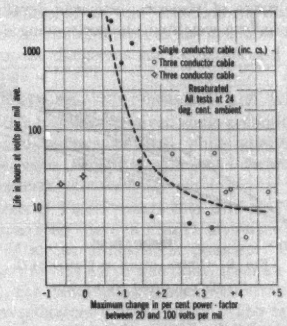


Fig. 4—Comparison between Life and Ionization Test

square inch of paper under 1.25 lb. per square inch pressure.

Fig. 6 shows that the air resistance has a pronounced effect on the dielectric strength of the saturated paper in sheet form and that the effect is independent of the kind of fiber, wood pulp or manila. The papers shown in Fig. 6 represent 12 different manufacturers and therefore should be representative.

In Table III are given the results of life tests on several different cables, both cables of each pair being alike in all respects except the air resistance of the paper. The results show that the paper air resistance

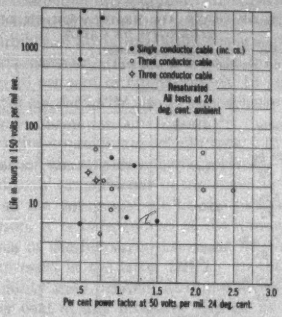


Fig. 5—Comparison between Life and Power-Factor Tests

has a pronounced influence on the dielectric strength of the cable.

Fig. 7 shows that the air resistance also has a marked influence on the penetration of the compound into the paper, the higher the former the slower the latter. The penetration rate of the compound is important to the manufacturer in its influence on the time necessary for saturation of the insulation.

5. Selection of Compound

As the power factor of the compound has a controlling

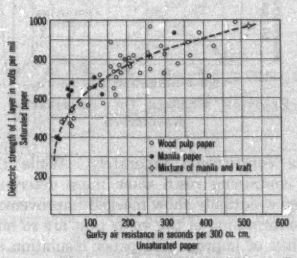


Fig. 6—Influence of Air Resistance on Dielectric Strength of Paper

influence on the power factor of the cable, the deterioration of the compound power factor with heating is of importance.

Without the presence of air, it is insignificant in comparison with the deterioration in the presence of air.

Fig. 8 shows the power factor deterioration of some representative compounds in the presence of air.

Each sample of compound was in the same shape container so that the area exposed to the air was always the same. All containers were immersed in an oil bath. Free natural circulation of air through the oven was provided. The curves show that there is material difference between compounds and that sometimes a blend of compound will show more deterioration than its constituents separately. While the compounds containing rosin show more deterioration at first, they eventually show marked improvement.

TABLE III
INFLUENCE OF PAPER AIR RESISTANCE ON DIELECTRIC STRENGTH OF CABLE

All life tests at room temperature, ambient.
A. R. = air resistance of unsaturated paper in Gurley secs. per 300 cu. cm.

Size of cable	Life test voltage	Cable No. 1		Cable No. 2	
		A. R.	Hr. life	A. R.	Hr. life
1/750,000 $\frac{24 \text{ in.}}{32}$	180 kv.	300	30 +	35–300†	1.0
1/ 2/0 $\frac{9 \text{ in.}}{32}$	52 kv.	138	116	30	49
3/350,000 $\frac{19 \times 9 \text{ in.}}{64}$	96 kv. 3 phase	140	12.5	40	3.5
*3/350,000 $\frac{19 \times 9 \text{ in.}}{64}$	78 kv. 3 phase	140	99.5	40	22

*Sample 250 ft., all others 10 ft.

†Inside $\frac{12 \text{ in.}}{32}$ of wall A. R. = 300

Outside $\frac{12 \text{ in.}}{32}$ of wall A. R. = 35

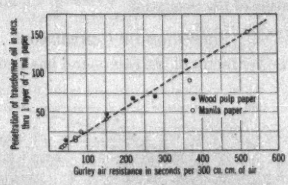


FIG. 7—RELATION BETWEEN OIL PENETRATION AND AIR RESISTANCE

As the formation of the voids that are so harmful to the quality of impregnated paper insulation is caused primarily by the contraction of the compound on cooling, the volume change of the compound with temperature is important. Fig. 9 shows this characteristic for a few typical compounds. It is seen that all the compounds are alike when in a liquid state, but that when they solidify the volume change approximately doubles. Therefore, on cooling from 50 to 25 deg. cent., a petrolatum should form more voids than a cylinder oil.

The viscosity of the compound below 50 deg. cent. is of importance because of its great influence on the formation and distribution of the shrinkage voids (2) and on the drainage of the compound. Fig. 10 shows this property on some typical compounds. It is seen that the movement of a petrolatum through the insulation should decrease sooner and more abruptly with decreasing temperature than that of a cylinder oil.

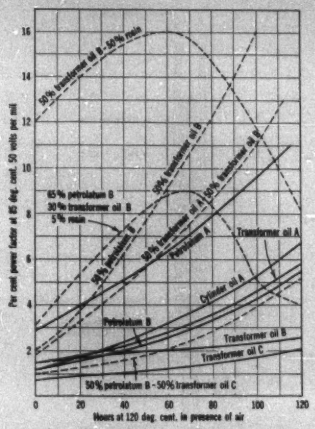


FIG. 8—ELECTRICAL DETERIORATION OF COMPOUNDS

Therefore, the petrolatum should begin to form voids at a higher temperature than the cylinder oil.

As no available viscosimeter was found that would give this range of viscosity, one was developed. It is substantially a weighted plunger moving down into a cylinder slightly larger than the plunger. The cylinder is full of compound which is displaced by the plunger and forced up through the annular opening between the plunger and cylinder.

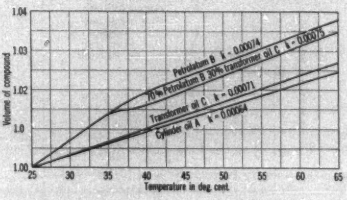


FIG. 9—CONTRACTION OF COMPOUNDS

K = Volume change per deg. cent.

Because of greater simplicity in preparation and testing, miniature samples of insulation (thickness of 50 mils and less) are in considerable use for laboratory comparison of compounds. When tested immersed, these samples represent almost ideal conditions for void elimination and when tested unimmersed, their

failure is usually caused by expulsion of compound. Because of the extreme contrast between each of these conditions and service conditions, it is believed that results of this test are of doubtful value in comparing compounds.

It is now recognized that the formation of gas and X by compounds in the presence of corona (ionized voids) can only be eliminated by the development of a compound of sufficient chemical stability to withstand such

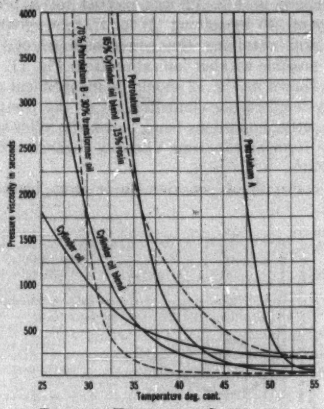


FIG. 10—VISCOSITY OF COMPOUNDS

conditions without forming gas. As the immediate development of such a compound is not promising, improved control of the void formation and distribution merits careful consideration as an available means of minimizing the dangerous gas formation and harmless X formation.

6. MECHANISM OF VOID FORMATION AND IONIZATION

Under this heading are given some preliminary conceptions and test results that, while admittedly not conclusive, were thought of sufficient interest for subjection to general discussion.

A—Influence of Void Pressure and Size on Ionization.

The upper solid curve in Fig. 11 shows the dielectric strength of air at atmospheric pressure. While the greater part of this curve was taken from an Institute paper (3), the relations have been well established by several investigators independently. The application of Paschen's law (breakdown of air constant as long as the product of the gap length and absolute pressure is constant) to this curve gives the dielectric strength at other pressures as shown.

The dotted lines represent the voltage across each void in insulation 300 mils thick at 100 volts per mil total stress. When this voltage exceeds the ionization voltage (solid curves), the void will be ionized. The curves show that increase of void pressure and more complete distribution of voids (many small instead of few large voids) both tend to decrease the possibility of ionization.

B—Void Pressure versus Air Content.

Air can exist in the insulation in two forms, in solution in the compound or separated. It is harmful only when it is separated as it is then subject to ionization.

The solubility of air in insulating oils is such that at atmospheric pressure, the oil will take into solution from 10 to 15 per cent or approximately 13 per cent of its volume of air, the latter also at atmospheric pressure. The influence of temperature is so slight that it can be disregarded for approximate calculations. In terms of the volume of air at atmospheric pressure, the solubility is directly proportional to the absolute pressure; in terms of the volume of air at the pressure under consideration, it is independent of the pressure. For convenience in the following discussion, the solubility will be expressed in the latter terms. In these terms, if the solubility is 13 per cent at atmospheric pressure, it is 13 per cent at any pressure. All air in the compound in excess of 13 per cent will be in the separated form (void).

An air content of 3 per cent at atmospheric pressure would mean that with decreasing pressure, no air would separate out above 17.5 cm. absolute $\left(\frac{3 \times 76}{13}\right)$. Hereafter, the pressure at which the air begins to separate

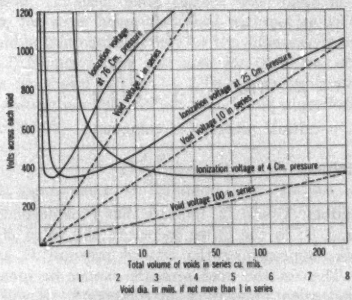


FIG. 11—INFLUENCE OF VOID SIZE PRESSURE AND ARRANGEMENT ON IONIZATION

Thickness of insulation—300 mils
Total voltage—30,000 volts
All voids the same size

out will be called the critical pressure of the compound. It is entirely dependent on the air content of the compound. There should be no separated air at pressures greater than the critical pressure.

If the air content exceeds 13 per cent at atmospheric pressure, the critical pressure will be greater than atmospheric and there will be separated air at atmospheric pressure and, therefore, voids even before the

cable is cooled. Thus, an air content in excess of 13 per cent at atmospheric pressure is definitely undesirable. Hereafter, only air contents less than this will be considered.

Let V_c = the volume of compound immediately surrounding a void, the compound that is subjected to a decreased pressure resulting from the formation of the void.

V_v = the volume of the void.

P_v = pressure of the void in cm. absolute.

a = air content of the compound in fraction of compound volume, the air at atmospheric pressure.

maximum solubility of the air in the compound = 13 per cent.

The total air content of V_c at any pressure $P_v = \dfrac{76\,a\,V_c}{P_v}$

from Boyles law.

The maximum dissolved air content of V_c at any pressure $P_v = 0.13\ V_c$. The separated air content of V_c at any pressure $P_v = \dfrac{76\,a\,V_c}{P_v} - 0.13\,V_c$. As the latter must fill the void,

$$V_v = \frac{76\,a\,V_c}{P_v} - 0.13\,V = \left(\frac{76\,a}{P_v} - .13\right) V_c$$

Rearranging

$$P_v = \frac{76\,a\,V_c}{V_v + 0.13\,V_c}$$

Let $\dfrac{V_c}{V_v} = R$ Then $P_v = \dfrac{76\,a\,R}{1 + 0.13\,R}$ (1)

Let P_p = critical pressure of compound

By definition $P_p = \dfrac{76\,a}{0.13}$ (2)

Equations (1) and (2) indicate that P_v is dependent on a and R; as the latter increases, the value of P_v increases, approaching the critical pressure P_p as a limit. That is, the void pressure cannot be greater than the critical pressure of the compound. It will be less because of the incomplete distribution of pressure through a stiffening compound—the same characteristic that was instrumental in forming the void in the first place. The stiffer the compound, the more incomplete will be the pressure distribution outwards from the void, the less the quantity of air available in the compound, and the lower the void pressure.

If V = viscosity of the compound and K the proportionality factor

$$R = \frac{K}{V} \qquad (3)$$

When $R = 1000$ pressure distribution can be taken as reasonably complete as from (1) $P_v = 99$ per cent of P_p.

If a pressure of 20 sec.* (thin transformer oil) is taken as representing complete pressure distribution

$K = 1000 \times 20$ and (3) becomes $R = \dfrac{20000}{V}$

allowing the approximate determination of R for any viscosity.

The curve in Fig. 12 was plotted for a pressure viscosity of 145 by solving (3) for R, then solving (1) for P_v and obtaining the corresponding total void volume possible without ionization from Fig. 11. The relation shown in Fig. 12 should be independent of the viscosity-temperature characteristic of the compound because, other conditions being equal, the voids should begin to form at the same viscosity regardless of the temperature.

This curve is of considerable interest because it

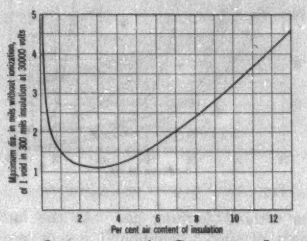


FIG. 12—INFLUENCE OF AIR CONTENT ON IONIZATION

indicates that in general, an air content in the neighborhood of 10 per cent is more desirable than one around 2 per cent.

C—Void Size versus Moisture.

For the same total void volume, the size of each void is dependent on the distribution of the voids, many small or few large. It is evident that the more complete distribution (smaller voids) is desirable because it decreases the tendency to ionization.

Table IV shows the results of some tests made with a glass tube approximately 9 in. long, ¼ in. diameter, a stop cock at each end. Each test was made by completely filling the tube between the stop cocks at 60 deg. cent., then closing the stop cocks and allowing the compound to cool to 25 deg. cent., when the resulting voids were counted. It will be noted that the dry compounds gave definitely less complete distribution (larger voids) than the compounds containing moisture.

This indication suggests an interesting advantage of compounds containing rosin. Because of the well-known tendency of rosin to evolve moisture when heated, such a compound is more liable to contain

*—pressure viscosimeter described above.

moisture in the cable and, therefore, to have smaller voids than a no-rosin compound.

When an enclosed volume of compound shrinks, the molecules of the compound are torn apart at some point (their cohesion is overcome), and a void is formed there. As the shrinkage continues, this void increases in size. As the surface of the void enlarges, some expenditure of energy is necessary to rearrange the molecules at the boundary of the void and compound. As the void becomes larger, this expenditure of energy increases. When the energy necessary to enlarge the void becomes greater than the cohesion of the compound, molecules somewhere else in the compound will be torn apart and a new void formed. This would indicate that a decrease in cohesion should increase the number of voids and, therefore, decrease the size of each void. As the surface tension of water is materially less than that of the compounds, the presence of moisture in the compound may appreciably decrease its cohesion. Further investigation of these relations is in progress.

TABLE IV
VOID DISTRIBUTION IN COMPOUNDS
Container 9 in. long by ¼ in. diameter. Cooling range 60 to 25 deg. cent.

Treatment previous to test	Total number of voids in container	Relative average dia. of each void
Cylinder oil No. 1 + 15% rosin		
a—After 1 hr. at 120 deg. cent.	45	0.68 d_1
b—After the original mixing	116	0.50 d_1
c—"b" after drying	14	d_1
d—"c" after addition of moisture	3600	0.16 d_1
Cylinder oil No. 2		
a—Fresh from original container	320	0.4 d_2
b—"a" after drying	20	d_2
Petrolatum No. 1		
a—After drying	12	d_3
b—"a" after addition of moisture	3000	0.16 d_3
c—"b" after drying	14	0.96 d_3
Petrolatum No. 2 + 15% rosin		
a—After drying	6	d_4
b—After 15 hr. at 200 deg. cent.*	75	0.43 d_4

*Rosin content reduced to 8 per cent by heating.

Conclusions

1. Reliability of insulation quality determination is essential to satisfactory progress in high-voltage cable development. The most reliable test available at present for this purpose is the overvoltage life test, but there is some evidence that even this test may not always give a true indication of insulation quality. Efforts are being made to increase its reliability by causing failure on test by exaggeration of the causes of failure in service, rather than by increase of the operating voltage alone.

2. While the ionization test is usually in fair agreement with the overvoltage life test as an indication of insulation quality, there are some instances, as that of a resaturated cable, where an indication of quality by the ionization test is not confirmed by the life test.

3. The air resistance of the paper used has a material influence on the insulation quality of the completed cable, regardless of whether the paper is wood pulp or manila.

4. While voltage tests on miniature samples of insulation are of considerable value along some lines, they are not believed to be a reliable basis for comparison of compounds.

5. The interdependence of the contraction-temperature and viscosity temperature characteristics of the compound in their controlling influence on the formation of voids should receive more general consideration.

6. Tentative indications are found that a decrease in air content of the insulation might increase the tendency to ionization because of the resulting decrease in void pressure.

7. Data are shown suggesting that ionization might be decreased by the presence of moisture in the insulation because of the resulting more complete void distribution and consequent smaller size of each void.

Appendix I—High-Voltage Air Condenser

A photograph and some details of the condenser are shown in Fig. 1. The high voltage plate is 58 in. by 80 in., the two low voltage plates each 50 in. by 72 in. At a spacing of 3 inches the total capacitance is 572 $\times 10^{-6}$ farads, the calculated and tested values checking very closely. No appreciable loss has been found in the condenser.

In order to minimize the expense and time of construction, (1) each plate surface was made in two pieces, the seam being smoothed with solder so that it could be distinguished only by the difference in color, and (2) copper tubing was used for the edges of the plates as shown in Fig. 1. This greatly simplified the construction and has not been found objectionable in any way.

Discussion

C. L. Dawes and P. H. Humphries: We are particularly interested in the experiences in removal of moisture and air. The equations preceding (1) in this paper show that in order to reduce the volume of absorbed air, the pressure must be very low. As stated in our paper,[1] we submitted comparatively small volumes of compounds to 2 mm. vacuum and 105 deg. cent., for 8 to 9 hr. before forming the sample. For considerable time after beginning, the foaming is so violent that we cannot apply full vacuum at once as the oil will be drawn over into the pump. The foaming and bubbling may continue 4 or 5 hr. Recently, on advice from a manufacturer, we attempted to prepare a sample of oil in two hours. The electrical tests showed it to contain both moisture and air in such comparatively large quantities that it was worthless for our purpose. Messrs. Davis and Eddy state that when miniature samples of compound are tested immersed the conditions for void elimination are almost ideal. We have tried several times to prepare samples in the open air, by immersing the electrodes very carefully. We have never been able under such conditions to prepare a sample which did not have such rapid increases in power factor that we considered the sample worthless with regard to its being representa-

1. *Ionization Studies in Paper-Insulated Cables-II*, by C. L. Dawes, H. H. Reichard, and P. H. Humphries, see p. 382.

tive of the compound itself. We were driven to devising the impregnating apparatus which we describe. We now go still further in the matter of vacuum and are able to obtain vacuum so close to absolute that we cannot read the differences of the meniscuses of the mercury. I do not believe that all manufacturers appreciate the large capacity of impregnating compounds for absorbing air and moisture.

J. B. Whitehead: Messrs. Davis and Eddy have called attention to the influence of air resistance on the dielectric strength. Most of us are agreed that the principal function of impregnated paper insulation is not only to hold the oil but also to serve as a barrier. It is, therefore, easily understood that if a paper has a lower air resistance, it has bigger openings in it, and therefore there is a greater cross-section of free liquid passages, and greater opportunity for secondary ionization to start in the oil itself. This secondary ionization in the oil results in an increasing conductivity, and so can result itself in an increasing power factor. It is, therefore, a factor in the deterioration of cable. This may explain the fact that beyond the question of the degree of impregnation, the character of the compound may have a bearing on the life of cables merely in its relation to voltage. If we have the process of secondary ionization going on and it is facilitated in some way, it means not only the presence of temporary loss, but it must also mean a progressive change in the character of the compound.

Wm. A. Del Mar: (communicated after adjournment) This paper puts on record a number of interesting facts of the kind that laboratory workers too often keep to themselves, largely because each one is too unimportant to be made the subject of a paper.

The relation between air resistance and dielectric strength of paper is checked very closely by our own tests, but that between air resistance and time of oil penetration shown in Fig. 7, does not always hold. The air resistance of paper is a function of the length of fibers, the hydration of the cellulose, and the amount of surface calendering. If the different air resistances are obtained by altering these three variables and not one, as was apparently done with the papers used by the authors, a curve is sometimes obtained showing a consistent decrease of time of oil penetration with increasing air resistance. In other words, it is quite possible to obtain a highly air-resistant paper which impregnates rapidly and conversely a very porous paper which impregnates slowly.

The authors make an excellent case for cylinder-oil compounds by showing both the lower thermal contraction and the lower and more uniform viscosity over the working range of temperatures (Figs. 9 and 10). These characteristics with the superior stability when used with rosin, are strong arguments for cable made with such compounds. Their argument in favor of moisture has the merit of originality and further developments will be awaited with interest.

F. M. Clark: (communicated after adjournment) Field service tests for determining insulation quality are not only slow in producing results, but in many cases are expensive and may lead to erroneous conclusions due to lack of definite knowledge of service conditions. The question is ultimately one for the chemist, a determination of the behavior of material under varying conditions, especially under electric stress. This phase of chemical behavior has been very largely neglected. The complexity of the oil and cellulose has served to concentrate chemical attention to the identification of a molecule, or molecules, rather than the behavior under conditions of interest to the "dielectrician."

The authors lean toward the overvoltage life test as a means of determining service quality, although frankly recognizing its defects. It does not appear possible, with our present state of knowledge, to eliminate any of the numerous tests in use without sacrificing something in the knowledge desired. Air-resistance tests on the paper alone merely give information concerning the "barrier action" of the solid insulation. Such tests throw no light on the stability of the treated product under low-voltage application. Initial power factor and power factor—temperature tests give an indication of the effectiveness of the drying process. Power factor—voltage tests throw some light on ionization tendency. The oxidation of the treating compound in air gives some information on its chemical and electrical stability, but such tests should be carried out at temperatures lower than 100 deg. rather than at 120 deg. cent. as done by the authors. The rate of oxidation is important, but just as important is the knowledge of the amount of water produced by the oxidation reaction. Oils and petroleum compounds vary according to their individual chemical characteristics in the products formed by oxidation. High voltage—high temperature tests on small treated samples give an idea of the chemical and electrical stability of the compound—paper combination, but a set-up to throw light on the behavior under ionization is difficult to obtain. Thermal expansion of the compound, air content of the compound, and other properties are tests which cannot be eliminated in the proper selection of cable material. Each property is of value and must be given proper consideration.

With regard to Fig. 8 of the article, it appears that the injurious effect of rosin is apparent only in the early stages of an oxidation test. The mixture of rosin and compound is eventually tending to approach the characteristics of the other constituents present. I should like to ask the authors if such behavior is shown in tests run at 100 deg. cent. or lower. The tendency of rosin to eliminate water when heated in air may give considerably different results with tests run at lower temperatures. I would also request the authors to describe the physical and chemical characteristics of the oils and petrolatum used in the experiments. This is necessary for a proper understanding of the results illustrated in Fig. 8.

The solubility of air in insulating oils is given as 10-15 per cent. Subsequent discussion is based on these figures. I should appreciate information from the authors concerning the test on which this amount of air solubility is based. Work which we have done in Pittsfield, and which I hope soon to present to the Institute, indicates that the air content of oils and compounds varies considerably from one type of compound to another. It is only in the lighter oils such as those approaching the transformer-oil type that the air solubility reaches values such as those given by the authors.

The authors state that the size of the void formed is a function of the water content of the oil or petrolatum and suggest an explanation based on the relatively high surface tension of oil as compared to water. Our experience has indicated that the surface tension of oil against air, at 25 deg. cent. is considerably less than the same value for water. Surface tension of oils under such conditions appears to be approximately 30 dynes per sq. cm., varying slightly with the state of oxidation, but never approaching the much higher value possessed by water. I would suggest that the smaller voids produced in the presence of water are caused by the precipitation of dissolved water from the oil as the temperature falls, such precipitation assisting in the formation of gas bubbles and securing a wider distribution of voids than is obtained with the dry oils. Such moisture separation, of course, would be equally, if not more dangerous to cable insulation than would be the void formation itself.

E. W. Davis: We are interested to learn that Mr. Humphries found two hours vacuum treatment of the compound to be insufficient for his miniature samples of sheet insulation. In the past, using paper of low air resistance, we have made many miniature samples with no more than two hours previous vacuum treatment of the compound. Our electrical tests on these samples indicated almost complete elimination of voids. For instance, some of them withstood 750 volts per mil for 18 months

without failure.[2] In fact similar results were obtained without any previous vacuum treatment of the compound.

Mr. Del Mar suggests that our different air resistances were obtained by variation of only one of the several factors influencing air resistance. This was not the case. All air resistance shown in the paper was obtained by variation of all the factors. While we have done some work on the separation of the factors it was considered too incomplete for publication.

We are glad to see Mr. Clark's recognition of the fundamental importance of the development of a reliable criterion of insulation quality. We believe this to be the greatest need of the industry today.

Mr. Clark suggests that the oxidation tests on the compounds (Fig. 8) should be made at 100 deg. cent. instead of 120 deg. cent. If the only purpose of the tests were to study the oxidation of the compound in service we would agree with him. However, in order to study the deterioration of the compound in the factory tanks we believe such tests should be made at temperatures greater than 100 deg. cent. It may be found necessary to make tests at several temperatures. What tests we have made at 100 deg. cent. show the recovery of the rosin compounds to be much less than at 120 deg. cent. and in some cases entirely absent.

Mr. Clark asks for a more detailed description of the compounds shown in Fig. 8. It probably should have been noted in the paper that the compound designations are the same in Figs. 8, 9, and 10. Thus Figs. 9 and 10 give considerable information as to the physical characteristics of the compounds in Fig. 8. Only wood rosin was used. The Saybolt Universal viscosity at 85 deg. cent. is 126 sec. for petrolatum *B*, 245 sec. for cylinder oil *A*, and 38 sec. for transformer oil *B*. While we cannot give a complete chemical description of all the compounds we can say that the petrolatums, cylinder oils, and transformer oils of Fig. 8 are all representative of the better grade paraffin-base insulating compounds that are being supplied the paper cable industry. The principal purpose of Fig. 8 was to show the wide difference in oxidation represented by such a group of typical compounds.

The air solubility figure of 10-15 per cent was based on tests with a known volume of air above and in contact with a known volume of carefully prepared oil which in turn was supported by mercury. By means of the mercury the air and oil were maintained at constant pressure until the air volume showed no further decrease. While our tests gave some indication of the influence of compound type they were not continued far enough to be conclusive.

The assumed value was purposely taken for "insulating oils" and not all types of compound. In this discussion disregard of the unknown but suspected decrease of solubility with increasing viscosity is not believed to influence the ultimate conclusion in any way (No. 6 on page 379). In fact it seems that the inclusion of this decrease in the discussion would tend to accentuate the relations considered. We are indeed glad to hear that Mr. Clark is soon to publish data on the influence of compound type on the solubility of air as available information on the subject is far from complete.

We agree with Mr. Clark that our suggested explanation of the moisture effect may not appear entirely sound. However, we do not feel able to accept his explanation without further investigation.

2. Further test results on these samples given in A. I. E. E. Trans., 1927, p. 215.

Ionization Studies in Paper-Insulated Cables—II

BY C. L. DAWES,* Member, A. I. E. E.
H. H. REICHARD,† Associate, A. I. E. E.
and
P. H. HUMPHRIES‡ Associate, A. I. E. E.

Synopsis.—*This is the second report to be presented before the Institute of the research investigation of ionization phenomena in paper-insulated, high-voltage cables, which is being conducted at the Harvard Engineering School under the auspices of the Impregnated Paper-Insulated Cable Research Committee. This paper is a report of progress and will be followed by further reports as the work progresses.*

The power dissipated as ionization loss in cables is much more harmful than the power dissipated in the solid dielectric. The paper presents methods of separating this ionization loss from the total dielectric loss occurring in high-voltage impregnated paper cables, the methods being verified experimentally. It consists essentially of seven parts as follows:

(1) A description of the apparatus and the method of impregnating cable paper whereby test specimens having practically no occluded gases can be produced. It is necessary to prepare and test samples in this manner in order to determine the law of their dielectric properties over a wide range of voltage gradient. It is only by knowing this law that it becomes possible to separate the ionization loss in the cable from the total dielectric loss. These experiments show that with three typical but widely different types of impregnating compounds, the power factor and capacitance are constant up to gradients as high as from 250 to 300 volts per mil; the power loss is proportional to the voltage squared under these conditions. These results are illustrated with curves.

(2) In order to verify the law of ionization loss in cables, as determined from the analysis of their dielectric-loss curves, it is necessary to know the character of the ionization-loss curve for thin air films under conditions of "restricted ionization." Such ionization-loss curves have been determined experimentally and the characteristics of two air films, of different thickness, are given in the paper. Above the ionization voltage, the ionization loss is a linear function of the voltage.

(3) By employing the fact that in a cable the power loss in the solid dielectric varies as the square of the voltage, even above the ionization voltage, it is a simple matter to separate this loss from the total dielectric loss in a cable, the remainder being the ionization power loss. The ionization power-loss curve concaves upwards directly above the ionization voltage. After all the gas films have become ionized, the curve becomes linear. This is confirmed by a synthetic curve determined by the data obtained in (b). This analysis is given for four different cables having widely varying degrees of ionization. As a result of this analysis, the power-loss curves of cables may now be expressed by a very simple equation. It is suggested that the constants of these equations be used as a basis for the quality of cables.

(4) Cable models, of glass and intervening air spaces, were made and their dielectric-loss curves determined as with the actual cables. The analysis of the curves gives results identical with those obtained with actual cables. This seems to verify the results obtained with the cables themselves.

(5) It is possible to express the power-factor curve by a simple equation, from which its maximum, etc., may be determined. In reality the power-factor curve consists of three simple curves.

(6) The energy current of a cable may be analyzed into three simple components, one of which is determined by the loss in the solid dielectric and the other two by the ionization characteristics of the cable.

(7) The character of the capacitance curve, which varies in different cables, is determined by the positions and thicknesses of the gas films. Further experimental data and study are necessary before an accurate interpretation of these curves can be made.

Introduction

THIS paper presents further results of research work being conducted at the Harvard Engineering School in the study of Ionization in High-Voltage Impregnated Paper Cables. The first paper was presented at the Midwinter Convention in 1926. The work is conducted under the auspices of the Cable Research Committee which is a subcommittee of the National Electric Light Association, the American Institute of Electrical Engineers, and the Association of Edison Illuminating Companies. The National Electric Light Association provides the Research Fellow for the work. The work is under the immediate supervision of a Cable Research Committee made up of members of the faculty of the Harvard Engineering School and one of the authors is chairman of this committee.

In this paper are presented further laws of ionization phenomena which occur in high-voltage paper-insulated cables. These laws are substantiated by experimental data taken under widely different conditions.

Procedure

The dielectric-loss measurements which are given in this paper were made on a bridge, the principle of which was given in the first part of the paper.[1] Since that time, however, many improvements have been made in the bridge which have increased its accuracy and ease of manipulation. Detail of this bridge and its accessories will be described in a paper§ which will shortly come before the Institute, hence will not be repeated here.

As is well known, the dielectric medium of paper-insulated cables consists of impregnated paper, the impregnating material, and thin gas films which may be air or gases from the impregnating material, all in series. In this paper, the insulating paper and the impregnating material will be referred to as the solid dielectric and the air and vapors as gases.

The variation of the dielectric characteristics of the solid material with voltage is quite different from the

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1. For numbered references see Bibliography.

§*Some Problems in Dielectric Loss Measurements* by C. L. Dawes, P. L. Hoover, and H. H. Reichard.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

variation of the dielectric characteristics of the gaseous films with voltage. This fact provides a means of separating each of these two losses from the total loss in a cable. This is important, for, as is well known, the energy which is dissipated in the gas films is very insidious and destructive in its effects on the dielectric and hence is much more harmful to the life of the cable than the energy dissipated in the solid material. Thus the energy which is responsible for tree designs and ultimate failure of the cable may be determined quantitatively and used as a criterion of cable quality. However, before it is possible to make this separation of losses, it is necessary to determine experimentally the individual dielectric characteristics of both the solid material and the gas films as functions of voltage gradient.

Dielectric Characteristics of the Solid Material

The separation of the losses within the cable dielectric can be accomplished only when the dielectric characteristics of cable paper, impregnated under conditions that remove essentially all occluded gases, are known.

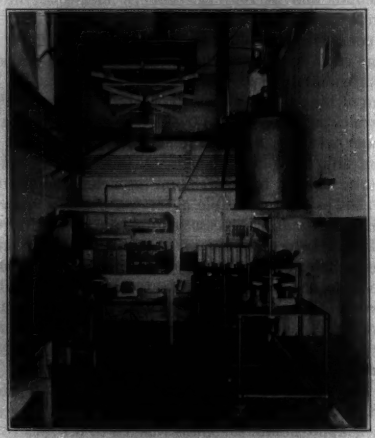

Fig. 1—Photograph of Paper-Compound Impregnating Apparatus

Thus a solid dielectric having no perceptible ionization is obtained.

Although the dielectric characteristics of paper and impregnating compounds have been determined experimentally a number of times and such results have been published occasionally, the authors know of only two instances[2,3] where the experimental work was conducted under such carefully controlled conditions that the results might be used to confirm the theories which are developed in this paper. In order to substantiate these theories beyond question, it seemed necessary to conduct a series of experiments on cable papers impregnated with different types of compounds under carefully controlled conditions. To conduct these experiments properly, it became necessary to design and construct special apparatus for impregnating and testing. Fig. 1 shows a photograph of the bridge and the impregnating apparatus with the dome unbolted from the base of the apparatus and raised. A diagram is shown in Fig. 2A.

The sample is made and tested in a circular brass vessel, Figs. 1 and 2A, the vessel itself forming the lower electrode. As this lower electrode is connected to the

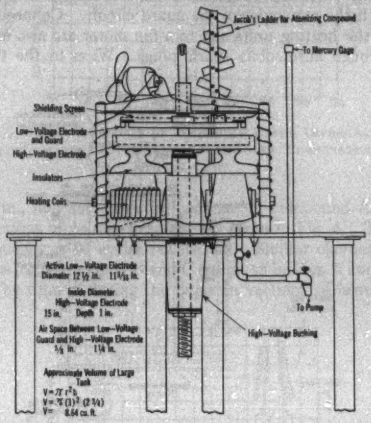

Fig. 2A—Diagram of Paper-Compound Impregnating Apparatus

high-voltage side of the transformer secondary, it is set on three high-voltage, pin-type insulators. The inside diameter of the electrode is 15 in. (38.1 cm.) and the depth is 1 in. (2.54 cm.).

The low-voltage electrode is a circular brass disk ½ in. (1.27 cm.) thick and originally 12.50 in. (31.8 cm.) diameter, but later reduced to 11.187 in. (28.4 cm.) to permit the obtaining of voltage gradients up to 300 volts per mil. This electrode is surrounded by a guard ring 1-in. (2.54 cm.) radial thickness separated from it by 0.1 in. (2.54 mm.) and made a unit with the electrode by means of strips of bakelite which hold the two together. The low-voltage electrode is also shielded above with copper screening attached to the guard rings (Fig. 2A). This low-voltage electrode unit is fastened rigidly through a block of horn fibre insulation to a 1-in. (2.54 cm.) brass rod which runs in a rigid bearing above. This keeps the two plates in exact alinement at all times. The upper electrode can be raised and lowered by means of a ⅜-in. (0.95 cm.) brass rod attached to this larger brass rod and extending outside the top of the dome through an air-tight gland, Fig. 2B. The details of the

method of making this joint tight are also shown in Fig. 2B.

To permit high-voltage tests to be made while the prepared sample is within the impregnating apparatus, a 35,000-volt bushing is set in the base plate of the apparatus to carry the connections from the high-voltage electrode to the high-voltage side of the power supply, Figs. 1 and 2A.

The low-voltage lead is carried through the base plate by means of an automobile spark plug. This lead is shielded throughout its entire length. The iron-work of the apparatus itself is a portion of the shielding and is balanced to the potential of the low-voltage electrode with the remainder of the guard circuit. Connections to the heating units and to a fan motor are also made through automobile spark plugs. Wires to the thermocouples are carried into the apparatus through bushings sealed with special compounds.

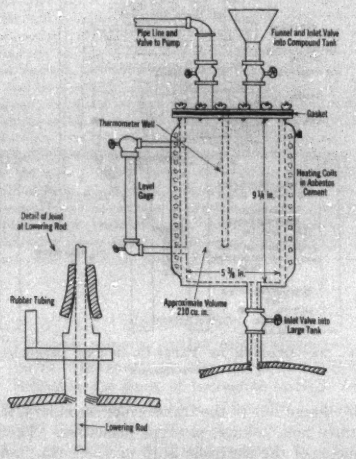

FIG. 2B—DETAIL OF SMALL COMPOUND TANK AND METHODS OF PRESSURE SEALING

The entire apparatus is covered with a boiler-iron dome, which bolts to the base plate. This dome, when in use, is heat lagged. A special rubber gasket is used to seal the joint between the dome and base plate. Great difficulty was experienced in making the apparatus sufficiently tight to obtain the necessary high vacuum. It was particularly difficult to make the high-voltage bushing joint air tight. This difficulty was finally overcome by designing a type of seal similar in principle to that used for the lowering rod (Fig. 2B).

A small, totally enclosed tank, having a capacity of approximately 0.91 gal. (3.44 liters), is fastened to the top of the dome, (Fig. 2B). This tank is provided with a thermometer well and a glass gage for determining the amount of compound at any time. The tank is surrounded with resistor wire to permit heating of the compound, and is heat lagged. A connection to the vacuum pump is made so that the compound in the tank may be subjected to vacuum as well as temperature, before it runs down to the sample. The compound is allowed to run down into the dome to the testing vessel by means of a valve. A "Jacob's ladder" (Figs. 1 and 2A) is provided so that the compound trickles slowly down into the test vessel, thus exposing a very thin layer to the vacuum for a considerable time before it reaches the sample.

The procedure is to place 12 circular pieces of wood-pulp cable paper in the brass test vessel (Figs. 1 and 2A) and subject them to a temperature of 105 deg. cent. and a high vacuum for several hours to remove the last traces of moisture.[3] The upper test electrode must be raised above the paper during this process to permit the air and moisture to escape readily. While the paper is being subjected to this drying and evacuating process, the compound in the small totally enclosed chamber on top of the dome is maintained at 2 to 3 mm. vacuum and at 105 deg. cent. by means of the current in the resistor wire which surrounds it. The paper alone was subjected to the temperature of 105 deg. cent. and 2 to 3 mm. vacuum for a period of approximately 9 hr. The valve in the pipe at the bottom of the small tank was then opened slightly. This permits the compound to trickle slowly down to the test vessel over the "Jacob's ladder," the vacuum of 2 to 3 mm. still being maintained. A thin layer of the oil is thus exposed to the vacuum for a considerable time, so that as far as possible all occluded gas is removed. This impregnating process requires something like one-half hour before the test plate can be lowered down to the specimen by means of the rod which extends through the top of the dome. After the test plate had been thus lowered, the specimen was allowed to remain at a temperature of 105 deg. cent. and a vacuum of 2 to 3 mm. for a period of not less than 9 hr. before it was ready for test. The samples tested consisted of 12 circular sheets of wood-pulp paper, each having a diameter of 15 in. (38.1 cm.), so that they just fitted within the test vessel. Their approximate thickness was 6.5 to 6.7 mils (0.165 to 0.170 mm.) per sheet.

Three different compounds were chosen for impregnating the paper and producing test samples. These compounds were supplied by three different cable manufacturers. Their viscosity curves are shown in Fig. 3. The first compound is a light oil, specially prepared, so that amorphous constituents are removed. It has a flow point of approximately − 20 deg. cent. This is designated as Compound A; the second is a paraffin-base cylinder-oil compound of medium viscosity and is designated as Compound B; the third is of a petrolatum base, containing a small percentage of rosin and is designated as Compound C. Although Fig. 3 shows that for temperatures down to 60 deg. cent.

and even less, the viscosity of Compound C is considerably less than that of Compound B, yet at room temperature the viscosity of Compound C is very much greater than that of B, being very plastic.

These three compounds, therefore, not only include the ranges of viscosity of compounds generally used for paper-cable impregnation, but are fairly representative of different types of compounds used in practise. Hence, any general deductions from results obtained with these three compounds will be more likely to apply to compounds in general.

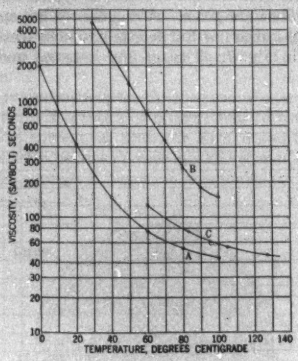

FIG. 3—VISCOSITY CURVES FOR COMPOUNDS A, B, AND C

The method of impregnation has already been described. Tests were made immediately after impregnation and at atmospheric pressure, since it is practically impossible to conduct high-voltage tests in a vacuum

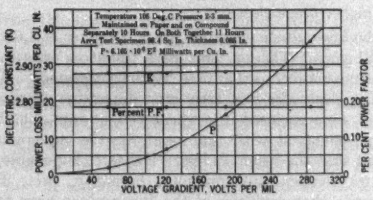

FIG. 4—CHARACTERISTICS OF WOODPULP PAPER IMPREGNATED WITH COMPOUND A, FREQUENCY = 60 CYCLES, TEMPERATURE 19.5 DEG. CENT.

because of flashovers within the apparatus. After a sample has been properly prepared in a vacuum, test results are not affected by conducting the tests at atmospheric pressure. This has been confirmed by Emanueli.[2] Tests must, however, be conducted within three or four days after removing the vacuum, for if the sample stands too long, moisture commences to permeate it. Tests were made over a considerable range of frequencies and temperatures. For the purpose of this paper, however, data taken at room temperature and at a frequency of 60 cycles per second only are necessary. A paper giving the complete results of all these tests will be published later.

The power factor, the dielectric loss in milliwatts per

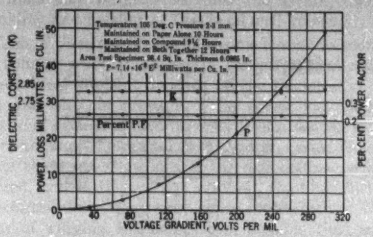

FIG. 5—CHARACTERISTICS OF WOODPULP PAPER IMPREGNATED WITH COMPOUND B, FREQUENCY = 60 CYCLE, TEMPERATURE 22 DEG CENT.

cu. in., and the dielectric constant of compound A as functions of voltage gradient are given in Fig. 4. The frequency is 60 cycles per second. Similar characteristics for compounds B and C are given in Figs. 5 and 6.

The time of drying the paper in vacuum, of heating

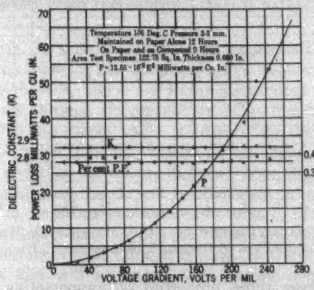

FIG. 6—CHARACTERISTICS OF WOODPULP PAPER IMPREGNATED WITH COMPOUND C, FREQUENCY = 60 CYCLE, TEMPERATURE 24 DEG. CENT.

the compound in the small tank, the temperature, etc., are given with each figure.

In none of the figures is there any indication of an ionization voltage. The power factor and dielectric constant and hence capacitance remain essentially constant at all voltages. At the higher voltage gradients, a very slight increase in power factor and capacitance was observed during the time required to balance the bridge. We have shown that this increase is due to the slight increase in temperature caused by dielectric loss in the sample. The following three conclusions may be drawn from these curves:

With the most careful impregnation, which removes nearly all traces of occluded air, at gradients up to 250 to 300 volts per mil, and at room* temperature, the power factor is essentially constant, the capacitance is essentially constant, the power loss varies as the voltage squared. This last conclusion follows from the other

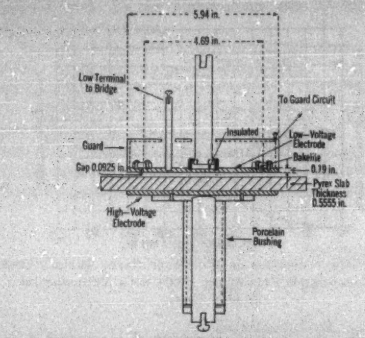

FIG. 7—DIAGRAM OF ELECTRODES USED IN TESTING THIN LAYERS OF AIR

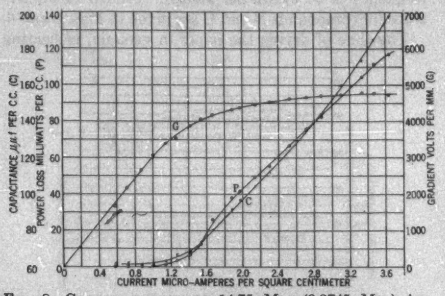

FIG. 8—CHARACTERISTICS OF 14.75 MIL (0.3745 MM.) AIR FILM, FREQUENCY = 60 CYCLES, TEMPERATURE 19 DEG. CENT.

two. These three properties of impregnating compounds are very important, for they form the basis of the methods of analysis which are given later in this paper.

DIELECTRIC CHARACTERISTICS OF GAS FILMS

Gas films form a portion of the cable dielectric. It is important to know the character of their dielectric-loss curves as a function of voltage gradient, for this gives additional means of verifying the ionization-loss curves that result from analyzing the dielectric-loss curves of cables.

In the first paper (I)[1], Figs. 29 and 30 showed some of the dielectric properties of a 2-mm. air film under conditions of "restricted ionization." Since that time we have continued further the investigation of the electrical properties of air films independently of the Cable Research. The apparatus has been improved considerably and the measurements are now made with much greater precision than formerly. This work is a research in itself and will require a separate paper for its presentation.

Since gas films, as well as solid materials, constitute the body of the impregnated cable dielectric and their electrical characteristics form one basis for the method of analysis that follows, two sets of typical curves for such air films are given. A diagram of the apparatus used for the experimental determination of these curves is given in Fig. 7. The air film is tested under con-

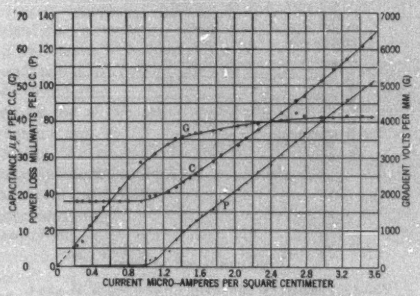

FIG. 9—CHARACTERISTICS OF 27.65 MIL (0.703 MM.) AIR FILM, FREQUENCY = 60 CYCLE, TEMPERATURE 21.5 DEG. CENT.

ditions of "restricted ionization," the current flow being limited by a slab of Pyrex glass 0.555 in. (1.411 cm.) thick. The lower surface, which is silvered, rests on the high-voltage electrode.

The low-voltage electrode has a diameter of 4.69 in. (11.91 cm.). The air films were tested in practically the same manner as that used for the compounds. The testing electrodes and associated apparatus shown in Fig. 7 were enclosed in a vacuum-tight iron chamber almost identical with that used for the compounds, except that the height is less. Hence, it is possible to control temperature, pressure, humidity, etc. The dielectric characteristics of the Pyrex alone were first measured. The dielectric characteristics of the air film and the Pyrex in series were next measured. The electrical characteristics of the air film are then determined by deducting at any given current the characteristics of the Pyrex alone from the characteristics of the air film and Pyrex in series.

In Fig. 8 are given the following characteristics of a 0.3745-mm. (14.75-mil) air film as a function of current: the voltage gradient, the power loss per cu. cm., and the capacitance per cu. cm. Similar characteristics for a 0.703-mm. (27.65-mil) air film are given in Fig. 9.

*At temperatures above from 35 deg. to 50 deg. cent. this is not true. Further discussion will appear in a paper devoted to the characteristics of cable compounds.

These curves are given as typical of the characteristics of air films under the conditions of "restricted ionization."

The voltage gradient below ionization is proportional to the current. After ionization has begun, the rate of increase of voltage gradient becomes less and less until the voltage gradient ultimately becomes substantially constant. This characteristic of gas films has already been described in Part I.[1] It is also in accordance with the investigations of Townsend,[6] although his investigations were made with direct current and at much lower pressure. The fact that the voltage gradient across the gas films becomes constant is important in that it causes the power factor to reach a maximum and then to decrease with further increase of voltage on the cable.

After ionization begins, the increase of power loss in the air films is practically proportional to the further increase in current, as shown in Figs. 8 and 9. That is, the relation between power loss and current is linear. In a large number of measurements under various conditions, this has invariably been found to be the case. A very slight curvature occurs in the lower portion of the curve, but as a whole the characteristic is practically a straight line.

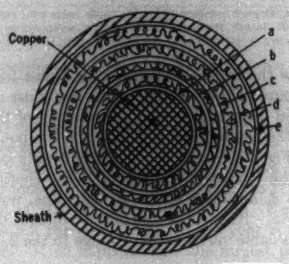

Fig. 10—Successive Gas Films within Cable Dielectric

Hence, the power loss in the solid dielectric varies as the square of the voltage; above the ionization voltage the relation of the power loss in the gas films to the current is linear. These two relationships give a method of separating the two losses in the cable dielectric.

The relation between capacitance and current in ionized air films is almost identical with the relation of power to current. That is, after ionization has begun, the increase of capacitance with current is also linear. The rate of increase of capacitance depends, however, on the thickness of the air film, the rate of increase of capacitance becoming less with decrease in the thickness of air film. These capacitance relationships are also important in that they may give a basis for determining the character of the gas films within the dielectric of impregnated paper cables.

Power Characteristics of Gas Films Distributed in Cable Dielectric

Within the cable dielectric, the gas films are distributed indiscriminately throughout the insulation. There may be a greater number in some places than in others, due to voids occurring during impregnation; gas films may be produced near the sheath, due to wrinkling and bending. This is all well known.

Each of these gas films will become ionized as soon as its voltage gradient exceeds a certain critical value. This critical gradient depends on the thickness of the film and also on pressure. The power loss characteristic of each film then becomes linear, similar to the characteristics shown in Figs. 8 and 9. With films of equal thickness, those nearer the copper first become ionized, and as the voltage is raised, the films further and further from the copper ionize successively. With films of varying thickness, it is possible that some films further from the center will ionize sooner than some nearer the center. Probably this effect occurs only occasionally, but in any event it does not in any way affect the general relationships which we are attempting to prove.

The character of the total power-loss curve is, however, determined by the radial positions of the gas films. As a whole, the gas films near the copper ionize first and the others throughout the dielectric ionize successively from the copper outwards as the voltage is raised.

In Fig. 10 a cross-section of an impregnated paper cable is shown. For simplicity, five concentric gas films *a*, *b*, *c*, *d*, and *e* are shown distributed radially outwards through the insulation. Gas film *a* is adjacent to the copper and *e* is adjacent to the sheath. Now consider that the voltage on the cable is being raised and that the change in capacitance of the cable is so small that the current is practically proportional to voltage. Below the ionization voltage, the loss in the gas films is zero. When the voltage gradient across *a* reaches its critical value at E_1, Fig. 11, the power loss in this film will be linear with further increase in voltage, as is shown by the straight line *a*. As the voltage is further raised, film *b* becomes ionized at voltage E_2 and the relation of its power loss to voltage is given by the straight line *b*. In a similar manner, films, *c*, *d*, and *e* ionize at voltages E_3, E_4, and E_5 and their power characteristics are given by lines *c*, *d*, and *e*, Fig. 11.

At any voltage, the total power loss in the gas films is obviously the sum of the ordinates of these power curves at that voltage. Thus, in Fig. 11, between voltages E_1 and E_2 the power loss is given by that portion of curve *a* alone. Between E_2 and E_3 a portion of curve *b* is added to *a*, etc. Up to E_5 the power curve *P* is a broken line. In the actual cable, however, the gas films relatively are much smaller in magnitude and very much more numerous than shown in Fig. 10.

Hence, that portion of the curve up to voltage E_5 will be a smooth curve.

After the last gas film has become ionized, the power curve P becomes a straight line. This is true, irrespective of the slopes of the curves a to e. That is, the point f on the curve is the criterion of complete ionization of all the gas films within the cable. Not long ago, it was thought that this condition was reached when the

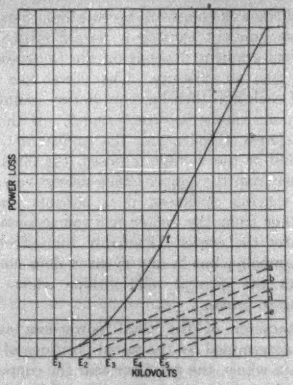

FIG. 11—POWER LOSS IN CABLE DIELECTRIC DUE TO SUCCESSIVE GAS FILMS

power-factor curve tended to become horizontal. That this is erroneous has already been shown in (I).[1]

ANALYSIS OF CABLE POWER CURVE

Fig. 12 shows the three well-known characteristic curves of a 10-ft. length of 500,000-cir. mil, 3/16-in.

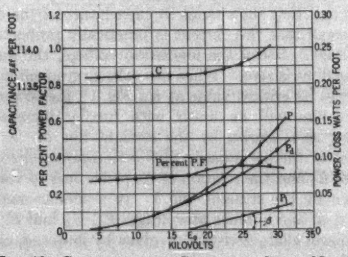

FIG. 12—CHARACTERISTIC CURVES OF CABLE NO. 14

500,000-cm., single-conductor 3/16-in. paper cable with metallized shielding tape 10-ft. length, frequency 62.1 cycle, temperature = 20 deg. cent.

(4.76 mm.) paper, 8/64-in. (3.18 mm.) lead, single-conductor cable with a metallized shielding tape, which is designated as Cable 14. The power curve P, the power-factor curve, and the capacitance are plotted against kilovolts. An examination of these curves shows that this cable has relatively little ionization.

In Fig. 14 this same power curve P ($a\ b\ d$) is plotted on logarithmic paper. Up to 17.5 kv., the ionization voltage (see Fig. 12), this logarithmic power characteristic is a straight line having a slope of 2.0. This shows

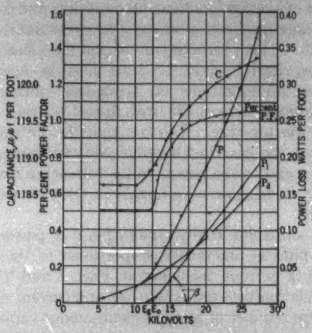

FIG. 13—CHARACTERISTIC CURVES OF CABLE NO. 12

500,000-cm., single-conductor 16/64-in. paper cable with copper shielding tape 10-ft. length, frequency 60.6 = cycle, temperature = 20 deg. cent.

that below the ionization voltage the power loss varies as the square of the voltage. This loss must all occur in the solid dielectric, so that this portion of the curve confirms the results obtained with the compounds and given in Figs. 4, 5, and 6. A discontinuity occurs at point b, the ionization voltage, and the characteristic from b to d, although still a straight line, now has a slope of 2.3. Above the ionization voltage, the total loss in this cable varies as the voltage to the 2.3 power.

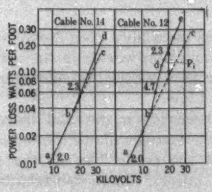

FIG. 14—LOGARITHMIC PLOTS FOR FIGS. 12 AND 13

If it be assumed that above the ionization voltage the power loss in the solid dielectric still continues to vary as the square of the voltage, the loss in the solid dielectric will be given by the line $b\ c$ where $b\ c$ is a continuation of line $a\ b$. This assumption is justified by the results obtained with the three compounds which are given in Figs. 4, 5, and 6.

A small error is introduced in considering $a\ b\ c$ a straight line. After ionization begins, the voltage across the gas films increases slowly and soon becomes constant and the rate of increase of the voltage across

the solid dielectric must become greater. (See Fig. 20 in Bibliography 1.) If power loss were plotted as a function of current, the line $a\,b\,c$ would be straight. It follows that if the total voltage were proportional to the current, the line $a\,b\,c$ would be straight. The departure of the relation of current to voltage from the linear relation is accounted for by the change in capacitance of the cable. If the capacitance remained constant, the voltage and current would be proportional to each other and the line $a\,b\,c$ would be straight. A study of Fig. 12 shows that the maximum change of capacitance per foot with cable 14 is from 113.6 to 113.9 $\mu\,\mu$ f., a change of 0.3 per cent. In cable A, Fig. 16, the change is 5.2 per cent. The 0.3 per cent is obviously negligible. It so happens that the largest percentage changes in capacitance occur when the ionization losses are greatest, so that under most conditions the error introduced into the ionization losses by the foregoing assumption is small. If conditions warrant it, obviously it is not difficult to make this correction for values of power above the ionization voltage. Let it be desired to find the true power dissipated in the solid dielectric at any voltage E_1. Multiply E_1 by K_c, where K_c is the ratio of the capacitance at voltage E_1 to the initial value of capacitance. The true value of power for voltage E_1 is the value corresponding to the voltage $K_c\,E_1$. In this paper the authors do not consider it necessary to make this correction.

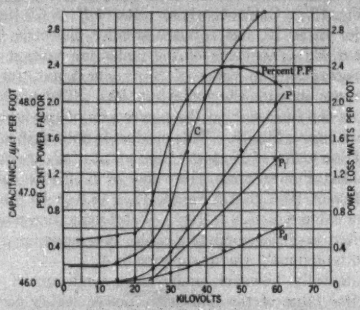

FIG. 15—CHARACTERISTIC CURVES OF CABLE B

No. 00 single-conductor, 9/32-in. paper cable with 10-ft. length, frequency = 60 cycle, temperature, 20.4 deg. cent.

It follows that in Fig. 14 the line $a\,b\,c$ gives the power loss in the solid dielectric and the line $a\,b\,d$ gives the total loss. The difference between the two must be the power loss in the ionized gas films.

This difference is transferred and plotted in Fig. 12 as the curve P_i. It is a linear curve and its equation is of the form

$$P_i = K_1\,(E - E_0) \qquad (1)$$

where K_1 is the slope of the line ($= \tan\beta$, Fig. 12), E the voltage, and E_0 the intercept of the line with the voltage or X-axis. $K_1\,E_0$ is the intercept of the line, when extended, with the power or Y-axis. These relationships are shown in Fig. 18. The equation of the curve, P_i, Fig. 12, is

$$P_i = 2.38 \times 10^{-6}\,(E - 17{,}500) \text{ watts per ft.}$$

The loss in the solid dielectric for this cable is given by the curve P_d, Fig. 12, and the equation of the curve is

$$P_d = 1.21 \times 10^{-10}\,E^2 \text{ watts per ft.} \qquad (2)$$

The equation for the total power-loss curve P is the sum of (1) and (2) and is given by

$$P = K\,E^2 + K_1\,(E - E_0) \text{ watts per ft.} \qquad (3)$$

Fig. 13 gives the power, power-factor, and capacitance curves of a 500,000-cir. mil, 16/64-in. (6.35-mm.) paper, 8/64-in. (3.18-mm.) lead, single-conductor cable with a copper shielding tape. This cable is designated as 12. A comparison of these curves with those of Fig. 12 shows that the cable has more ionization than cable 14. Fig. 14 also shows the power curve of the cable, Fig. 13, plotted logarithmically. As before, the line $a\,b$ gives the loss in the solid dielectric before ionization commences, and the broken line $a\,b\,d\,e$ gives the total loss. Under the assumptions already made, the straight line $a\,b\,c$ gives the loss in the solid dielectric, and the loss in the ionized gas films is the difference of the curves $a\,b\,d\,e$ and $a\,b\,c$. This difference is plotted in Fig. 13, giving the curve of ionization power P_i. The dielectric power curve P_d is also plotted in Fig. 13. The equation of these two curves is

$$P_i = 1.28 \times 10^{-6}\,(E - 12{,}500) \text{ watts per ft.} \qquad (4)$$

$$P_d = 2.24 \times 10^{-10}\,E^2 \text{ watts per ft.} \qquad (5)$$

Fig. 15 gives the power, power-factor, and capacitance curves of a 00, single-conductor, 9/32-in. (7.14 mm.) paper cable, which is designated as Cable B. A study of these curves shows that the cable has a much higher degree of ionization than the other two.

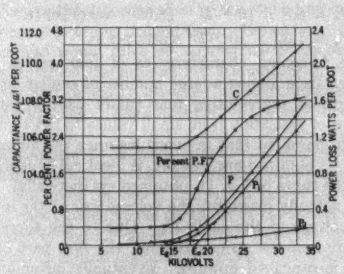

FIG. 16—CHARACTERISTIC CURVES OF CABLE A

500,000-cm. single-conductor ¼-in. paper cable, 10 ft. length, frequency = 58.3 cycle, temperature = 20 deg. cent.

The power curve for this cable is plotted logarithmically in Fig. 17. As before, the broken line $a\,b\,d\,e$ gives the total power loss and the straight line $a\,b\,c$ gives the dielectric power loss. The difference of the

two, which gives the ionization power loss, is plotted in Fig. 15 as curve P_i. Likewise, the dielectric power curve P_d is plotted in Fig. 15. The equations of these two curves are

$$P_i = 3.81 \times 10^{-8} (E - 24{,}000) \text{ watts per ft.} \quad (6)$$

$$P_d = 1.67 \times 10^{-10} E^2 \text{ watts per ft.} \quad (7)$$

Fig. 16 gives the power, power factor, and capacitance curves of Cable A, a 500,000-cir. mil, 8/32-in. (6.35-mm.) paper, 8/64-in. (3.18-mm.) lead, single-conductor cable.

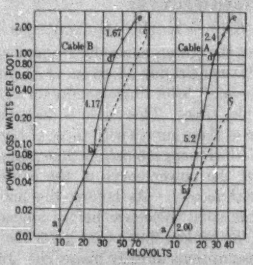

Fig. 17—Logarithmic Plots for Figs. 15 and 16

This cable has a very much higher degree of ionization than the other three. The power curve for this cable is plotted logarithmically in Fig. 17. Again, the total power is given by the broken line $a\ b\ d\ e$ and the dielectric power by $a\ b\ c$. The difference of the two, which gives the ionization power, is plotted in Fig. 16 as curve P_i. Likewise, the dielectric power curve P_d is plotted in Fig. 16. The equations of these two curves are

$$P_i = 8.8 \times 10^{-8} (E - 18{,}000) \text{ watts per ft.} \quad (8)$$

$$P_d = 1.62 \times 10^{-10} E^2 \text{ watts per ft.} \quad (9)$$

The foregoing characteristics of four cables were selected from among a large number of similar characteristics, as representing four cables having varying degrees of ionization from small values to large values. It is to be noted that in every instance the ionization curve P_i, obtained analytically, is identical in character with the curve of Fig. 11, obtained synthetically. The curve actually intercepts the voltage axis at the ionization voltage of the cable E_g. Due to the successive ionization of the series gas films, the first part of the curve is concaved upward. Eventually, after ionization is complete, it becomes a straight line which when extended intersects the voltage axis at E_0.

It is to be noted in cables 14 and 12 (Figs. 12 and 13) that the difference between E_g and E_0 is small. In fact, so far as it is possible to tell, E_g and E_0 coincide. This would seem to indicate that the gas films are not distributed throughout the dielectric, but rather concentrated near one radius, so that complete ionization occurs almost instantaneously.

For comparison, the characteristic constants of these four cables are tabulated below.

Cables	14	12	B	A
K	1.21×10^{-10}	2.24×10^{-10}	1.67×10^{-10}	1.62×10^{-10}
K_1	2.38×10^{-8}	1.28×10^{-8}	3.81×10^{-8}	8.8×10^{-8}
E_0	17,500	12,500	24,000	18,000
E_g	17,500	11,500	24,000	14,000
$K_1 E_0$	4.05×10^{-2}	16.0×10^{-2}	0.953	15.8×10^{-2}

This tabulation shows that the cables with highest ionization loss have the largest values of the ionization constant K_1, which is the slope of the ionization curve. (See Fig. 18.) Cable A, whose value of K_1 is 6.9 times that for cable 12, has a much smaller dielectric loss constant K. The value of the ionization voltage E_g is important. The value of E_0 is significant, for it determines in part the ionization loss. The product $K_1 E_0$ is the intercept on the power axis of the ionization-loss curve, Fig. 18. This product is not significant, since a low value of K_1 and a high value of E_0 are both desirable. For example, the value of $K_1 E_0$ for cable 14 is 4.05×10^{-2}, that for cable 12 is 16.0×10^{-2}, and that for cable A is 15.8×10^{-2}. Yet if ionization power is a criterion of the worth of a cable, 14 is superior to 12 and both obviously are far superior to A.

Incidentally, however, under some conditions $K_1 E_0$ gives the value of the ionization power which corresponds to maximum power factor as will be shown later.

The authors feel, however, that the following constants are criteria for cable quality:

K is important, for it determines the loss per foot in the solid dielectric.

K_1 is very important, for it defines in a large measure the ionization loss which is progressively harmful to the cable.

E_g is important, for it defines the voltage at which

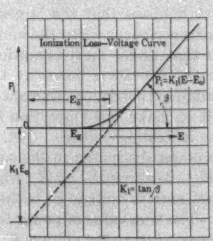

Fig. 18—Graphic Analysis of Power Loss

ionization begins, and its value is closely related to E_0.

Another advantage of the foregoing analysis is that it gives a simple method for the determination of the power dissipated in cables for voltages far in excess of values determined experimentally. It is merely necessary to extrapolate two straight lines, the line $a\ b\ c$, Figs. 14, 17, to obtain the power loss in the solid dielectric, and the straight line P_i, Figs. 12, 13, 15, 16, to obtain the power dissipated in ionization.

The power loss P_d in the solid dielectric is only harmful

because of its heating effects. It limits the kv-a. rating of the cable and, if excessive, may cause thermal disintegration of the dielectric. It is a function of the dielectric properties of the impregnating compound and paper in combination. The constant K is a criterion of this loss and is readily determined for any cable by the foregoing method of analysis.

The power loss P_i in the gas films is not only harmful because of its heating effects, but also because it is accompanied by ionic bombardment of the solid dielectric. This was shown in Part I[1]. This loss is accompanied by perforations of the paper, carbonization, chemical change in the compound, and tangential stresses. These effects are all cumulative and frequently result in tree designs and localized burn-outs. The constant K_1 is a criterion of this loss and is also readily determined by the foregoing method of analysis.

Hence, it may become desirable to rate cables, not only on the power loss per foot, but also on the ratio of ionization to dielectric loss, or similar factors.

When comparing the ionization power loss per foot

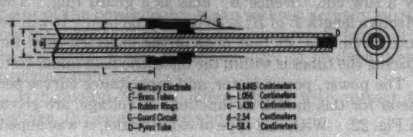

Fig. 19—Glass-Air Cable Model, Single Air Space

in different cables, their cross-sectional size must also be taken into consideration. For example, Cable A (Fig. 16) shows much higher ionization power loss than Cable B (Fig. 15). Yet the fact that Cable A has 500,000-cir. mil copper and Cable B has only 133,000-cir. mil copper, and the volume of insulation in Cable B is considerably less than in Cable A must be taken into consideration. A more complete discussion of such factors will be presented when more data are available.

Cable Models

The authors have analyzed the characteristics of a large number of cables and in every instance the analysis has given results which are in accordance with those just described. It was deemed advisable, however, to apply these same methods to a cable model composed of Pyrex glass tubing and intervening air spaces. This was done in Part I of the paper to substantiate the theory of restricted ionization. The advantage of such models is that they are composed of a definite solid dielectric whose characteristics are very stable. The gas films are definite and the geometry of both the solid dielectric and the air films is known. Moreover, it is possible to determine the dielectric properties of the glass alone, which leaves only one unknown quantity, the dielectric properties of the gas film. Thus, data taken with such a cable model and analyzed in the same manner as that used with the foregoing cable data, should give results which are similar. This is found to be the case.

In Fig. 19 is shown a simple cable model. In all cable models Pyrex tubing was used, since the variation of power and power factor with temperature is known to be small.[4] The cable model, Fig. 19, is composed of a Pyrex tube having inside and outside diameters of 0.6465 cm. (0.254 in.) and 1.056 cm. (0.416 in.). This is surrounded by brass tubing whose inner diameter is 1.430 cm. (0.563 in.). This brass tubing is in three sections, a center section whose length is 58.4 cm. (23.0 in.), which corresponds to the active sheath of a cable, and the end sections G which are guards. The two guard sections are separated from the active section by saw cuts and are held in alinement by outside bakelite cylinders, so that there is no solid dielectric in the dielectric field under measurement. This brass sheath is completely surrounded by a second brass tube whose inside diameter d is 2.54 cm. (1.0 in.). This outer tube is connected to the guard sections, so that the entire active section of brass tubing is completely shielded. The glass tubing is filled with mercury, which constitutes the center electrode of the model. Provision is made for filling the space between the glass tube and the brass sheath with water. This short-circuits the air space and permits the dielectric characteristics of the Pyrex tube alone to be determined.

Fig. 20A gives the electrical characteristics per meter of the Pyrex tube alone, the space between the inner brass tube and the Pyrex tube being short-circuited with water. The capacitance and power factor are substantially constant. A few air bubbles in the water did cause a very slight rise in power factor. However, this effect in the determination of the ionization loss in the air space is almost insignificant. Hence, since these few bubbles would have no appreciable effect on the principles which the authors are attempting to prove, the time and difficulty involved in their removal did not seem justified. The current curve i is linear and the power increases as the square of the voltage. These relations appear to be true generally of dielectrics at room temperature, provided all occluded gas is removed.[4]

Fig. 20B gives the power factor, the power curve P, and the capacitance curves per meter of the model without water in the air space. These curves are typical of dielectrics containing occluded gases. The curve P_G gives the power loss in the glass taken from Fig. 20A. For any given voltage, Fig. 20B, the value of P_G is obviously not equal to the value of power given by this same voltage, Fig. 20A, for in Fig. 20B the total voltage is equal to the vector sum of the voltages across the glass and air film in series. Correction was made by finding the current $E\,C\,\omega$ at any voltage E, Fig. 20B, and then taking from the power curve P_G, Fig. 20A, the loss corresponding to this same value of current. This method disregards the loss in the glass due to the harmonics which are produced in the current wave by ionization. Since, however, the loss in the glass is

small compared with the ionization loss, little error is introduced into the curve P_A, Fig. 20B.

The curve P_A, Fig. 20B, gives the power loss in the air film, P_A being obtained by deducting the power loss in the glass, given by curve P_G, from the total power-loss curve P. The resulting ionization power-loss curve P_A is identical with those obtained for cables. From the ionization voltage E_g, the curve begins to be concave upwards in accordance with the synthetic curve, Fig. 11, based on the fundamental characteristics of the solid dielectric and gas films in series. That

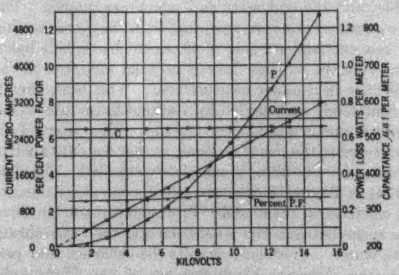

FIG. 20A—CABLE MODEL, GLASS DIELECTRIC SINGLE AIR SPACE—CURVES

Frequency = 60 cycle, temperature = 20 deg. cent.
$P = P_A$ = Power loss in glass = $56 \times 10^{-10}\ E^2$ watts per meter

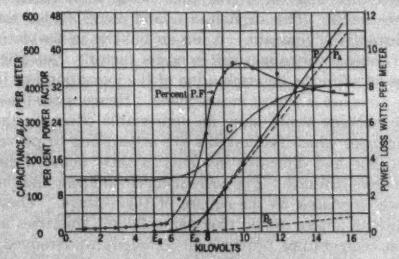

FIG. 20B—CABLE MODEL, GLASS-AIR DIELECTRIC SINGLE AIR SPACE—CURVES

Frequency = 60 cycle, temperature = 20 deg. cent.
P = Total power loss, P_A = Power loss in air (ionization loss) = $12.24 \times 10^{-4}\ (E - 7100)$ watts per meter, P_G = Power loss in glass

is, it appears to be almost certain that the relation of ionization power loss in a cable to voltage is a linear relationship after all the gas films have become ionized.

In order to simulate more nearly the conditions existing in a cable, in which the gas films are distributed throughout the insulation, a second cable model of Pyrex tubing was constructed having three concentric gas films in series. This produces very nearly the type of cable illustrated in Fig. 10. A diagram of this model is shown in Fig. 21. B is a polished brass rod having a diameter of 4.78 mm. (0.188 in.). It is surrounded by a Pyrex tube G_1 having a 1.0 mm. (0.0394 in.) wall, giving an air space A_1 of 1.6 mm. (0.063 in.). G_2 is a second Pyrex tube surrounding G_1, giving an intervening air space of 1.75 mm. (0.0689 in.). G_2 is surrounded by a brass tube B_1 having an inside diameter of 1.9 cm.

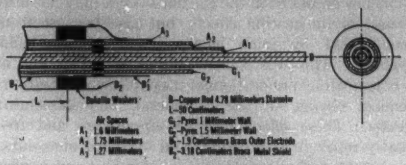

FIG. 21—TRIPLE AIR SPACE CABLE MODEL

(0.748 in.), which gives an air space A_3 of 1.27 mm. (0.05 in.) between G_2 and B_1. B_2 is another brass tube surrounding B_1 and acting as a shield. B_2 is connected to B_1', which is identical to B_1 but separated from it by a saw cut. Hence B_1' acts as a guard ring. The effective length L of B_1 is 50 cm. (19.68 in.). It will be noted from Fig. 21 that none of the dielectric used for spacing the tubes is within the dielectric field under test.

The power, power-factor, and capacitance curves per meter for this model, as functions of voltage, are given in Fig. 22. With this type of cable model, it was hoped that the discontinuities in the power curve, which occur as the successive air spaces become ionized, would be

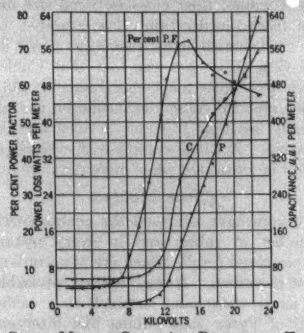

FIG. 22—CABLE MODEL, GLASS-AIR DIELECTRIC TRIPLE AIR SPACE CHARACTERISTICS

Frequency = 60 cycle, temperature = 19 deg. cent., P = Total power loss per meter

apparent in the power curve. Such a theoretical curve has already been shown in Fig. 11. A study of Fig. 22, however, shows that the loss in the glass alone, as compared with the ionization loss, is not even noticeable in the power curve. Below the ionization voltage, for example, the power curve as far as one can tell coincides with the voltage axis. Hence, it would not be expected

that the discontinuities which occur as successive air spaces are ionized would be shown on this power curve. It is to be noted, however, that except for the usual curved portion at its beginning, the power curve is linear. This is still further confirmation of the authors' theory of the relation of ionization loss to voltage in cables.

The capacitance curve, Fig. 22, does show, however, discontinuities, which undoubtedly are due to the successive ionization of the air spaces.

Power-factor Curve. It follows from Equation (3) that the power factor

$$\text{P. F.} = \frac{K E^2 + K_1 (E - E_0)}{E^2 C \omega} \tag{10}$$

where ω is 2π times the frequency.

If the variation of capacitance C with voltage is known, Equation (10) gives a means of extrapolating the power-factor curve. With most cables, the capacitance C varies only a few per cent with voltage, so that the law may be very closely approximated. It thus becomes possible to determine graphically the maximum point of the power-factor curve without actually carrying the cable to the high stresses that would frequently be necessary, if this maximum were to be determined experimentally.

If Equation (10) be differentiated with respect to voltage, the equation becomes

$$\frac{d\,(\text{P. F.})}{d\,E} = \frac{E^2 C\,[2\,K\,E + K_1] - [K E^2 + K_1 (E - E_0)]\left[2\,C\,E + E^2 \dfrac{d\,C}{d\,E}\right]}{E^4 C^2 \omega} \tag{11}$$

It is interesting to note that if the term $E \dfrac{d\,C}{d\,E}$ is small compared with $2\,C$, the point of maximum power factor occurs when $E = 2\,E_0$. This condition is closely approximated with Cables 12 and B, Figs. 13 and 15. (See also Fig. 23.)

If this value, $E = 2\,E_0$, be substituted in Equation (1), the ionization power under these conditions of maximum power factor becomes $K_1 E_0$, which is the intercept of the ionization power curve with the power axis. (See Fig. 18.)

Equation (10) may be divided into three terms.

$$\text{P. F.} = \frac{K}{C\,\omega} + \frac{K_1}{E\,C\,\omega} - \frac{K_1 E_0}{E^2 C\,\omega} \tag{12}$$

The first term is nearly constant and is equal approximately to the power factor of the cable below the ionization voltage E_0, since the second and third terms vanish below the ionization voltage. The second term, whose denominator is equal to the charging current of the cable, is approximately a rectangular hyperbola lying in the first quadrant. The third term, whose denominator is equal to the volt-amperes of the cable, is a hyperbolic type of power function of the form $y = x^n$, where $n = -2$. This therefore represents an "inverse square" curve which lies in the fourth quadrant and approaches zero with increasing values of voltage.

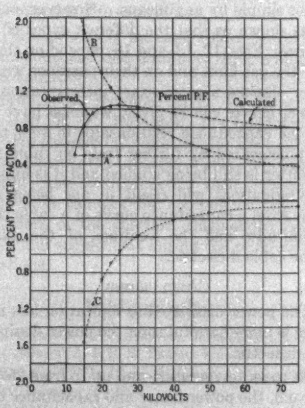

FIG. 23—ANALYSIS OF PER CENT POWER FACTOR CABLE NO. 12

$$\text{Per cent power factor} = A + B - C = 100\left(\frac{K}{C\,\omega} + \frac{K_1}{E\,C\,\omega} - \frac{K_1 E_0}{E^2\,C\,\omega}\right)$$

Each of these three terms, Equation (12), is plotted in Fig. 23, using the data of Cable 12, Fig. 13. The magnitude of each term is given. It is obvious that the second term is responsible for the descending portion of the power-factor curve, which occurs when the voltage is carried to sufficiently high values. The computed power-factor curve is carried well beyond the experimental curve.

It thus becomes possible to extrapolate the power-factor curve of a cable to values of voltage which it may be inconvenient or even impossible to obtain experimentally.

Current Curve. The variation of the total current in a cable with voltage requires but little comment. The current is

$$I = E\,C\,\omega \tag{13}$$

since in any ordinary cable the effect of the energy current on the absolute value of the total current is negligible.

It is, however, interesting to study the component currents in a cable.

The energy current,

$$I_e = \frac{P}{E} \tag{14}$$

substituting the value for P, Eq. (3) in Eq. (14)

$$I_e = \frac{K\,E}{C} + K_1 - K_1\,\frac{E_0}{E} \qquad (15)$$

Neglecting change in capacitance, the first term is proportional to the voltage and is the solid-dielectric energy current. The second and third terms are the two components of the ionization energy current. That is,

$$I_i = K_1 - K_1\,\frac{E_0}{E} \qquad (16)$$

The first term is constant, as shown in Fig. 24. The second term is a rectangular hyperbola which lies in the fourth quadrant, as shown by the curve I_i', Fig. 24. As the voltage E increases I_i' decreases. Hence the ionization energy current itself is a rectangular hyperbola having the current axis as one asymptote and a horizontal line K_1 units from the voltage axis and parallel to it as the other axis. The ionization energy current thus approaches K_1 as a limit, as shown in Fig. 24. Therefore, the energy current of a cable may be analyzed into three simple components, one practically proportional to the voltage, a second which is constant, and hence independent of the voltage, and a third which is inversely proportional to the voltage. Fig. 25 shows the ionization energy currents and the total energy currents for cables 14, 12, and A. The relative degree of ionization of these three cables can be readily determined by comparing these curves.

Capacitance Curves. The curve showing the increase

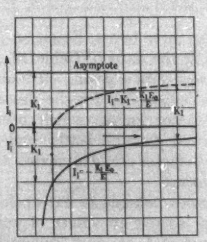

FIG. 24—GRAPHICAL ANALYSIS OF IONIZATION ENERGY CURRENT

$$I_i = K_1 + I_i = K_1 - \frac{K_1\,E_0}{E} = \frac{P_i}{E} = K_1\left(1 - \frac{E_0}{E}\right)$$

$$I_e = \frac{P}{E} = K\,E + K_1 - \frac{K_1\,E_0}{E} = \frac{K\,E^2 + K_1\,E - K_1\,E_0}{E}$$

in capacitance in cables after ionization begins may take three different forms. It may be practically a straight line, Fig. 16; it may be concave, upwards, Fig. 12; or it may be concave downwards, Fig. 13.

In Figs. 8 and 9, which give the electrical characteristics of flat air films, the increase of capacitance with voltage, after ionization begins, is substantially linear. The capacitance curve is, therefore, similar in character to the power curve. Also the rate of increase of capacitance with current increases as the thickness of the air film decreases. This may be seen from a comparison of Figs. 8 and 9. The rate of increase of capacitance for the 0.3745-mm. (14.75-mil) air film, Fig. 8, is greater than for the 0.703-mm. (27.65-mil) air film, Fig. 9.

In the cable, the gas films are in series with the solid dielectric and in series with one another. The resulting capacitance of the cable is determined, therefore, from a

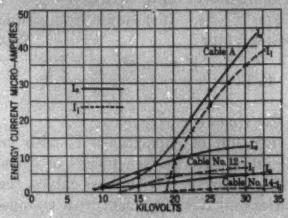

FIG. 25—I_e AND I_i FOR CABLES 14, 12, AND A

reciprocal relationship of individual capacitances in series. Hence, after ionization begins, the resultant capacitance of the cable is determined by the reciprocal relationships of the fixed capacitance of the solid dielectric, and the variable capacitances of the gas films which in themselves ionize successively, and after ionization each has a linear increase in its capacitance.

So far, the authors have not been able to express the resultant capacitance in a mathematical relationship which would show clearly the effects of various factors on the resultant capacitance of the cable. The relationship is not simple by any means. However, our study so far has shown us that the different types of capacitance curves, Figs. 12, 13, and 16, depend on the thicknesses of the gas films and on their positions within the dielectric. Before a more exact interpretation of the capacitance curves can be given, we must have further data on the capacitance relationships in very thin gas films, supplemented by further tests on cable models.

The matter is still being studied and it is hoped that these investigations will result in our being able to determine from a capacitance curve of a cable the relative magnitudes and the locations of the gas films within the cable dielectric.

CONCLUSIONS

1. The power dissipated as ionization loss in cable insulation is more harmful than the power dissipated in the solid dielectric.

2. At room temperature and up to from 250 to 300 volts per mil, the power factor and capacitance of cable insulating paper impregnated with typical compounds are essentially constant with variation in voltage and the power loss varies as the voltage squared, provided the impregnation is conducted in such a manner as to remove practically all occluded gases.

3. The relation of power loss to voltage in thin gas films is substantially linear for voltages above the ionization voltage.

4. From the relationship in (3), it may be shown that in single-conductor cable insulation above the ionization voltage the relationship of ionization power to voltage is a curve which is concaved upwards until all the gas films are ionized. With further increase in voltage the relationship becomes linear.

5. By applying (2) to the power curve of a cable, it becomes possible to analyze it into two components, one giving the loss in the solid dielectric and the other giving the loss due to ionization. The ionization power curve so determined becomes linear and similar in character to the curves determined by (4) and (3).

6. It thus becomes possible to express the power curve by a simple equation and to extrapolate it, if necessary.

7. The constants of the power equations may be criteria of cable quality.

8. The analysis of power loss into two components, one giving the power dissipated in the solid dielectric and the other giving the power dissipated in ionization, is confirmed by tests made with glass cable models.

9. It becomes possible to express the power-factor curve by a simple equation and to analyze it into three simple components. Also the power-factor curve may be extrapolated and the point of maximum power factor can be determined.

10. The energy current of a cable can be analyzed into three simple components, a term nearly proportional to the voltage, a constant term, and a term inversely proportional to the voltage.

11. The character of the capacitance curves depends on the thickness and distribution of the gas films within the cable insulation.

The authors are indebted to D. W. Roper, F. M. Farmer, and W. F. Davidson, the members of the Paper-Cable Research Committee, for their suggestions and counsel during the progress of this investigation; to the several cable companies for their cooperation in supplying cable samples and compounds; and to Prof. H. E. Clifford of the Harvard Engineering School for his advice and suggestions during the progress of these investigations.

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Discussion

E. W. Davis and W. N. Eddy: Because of the well known need for a criterion of insulation quality that is more reliable than the power factor—voltage relation and as convenient to determine, it is encouraging to see that the problem is receiving the attention of capable investigators like Professor Dawes and his assistants.

Preliminary inspection of the paper gives the impression that the authors' ionization loss P_i might be superior to the power factor—voltage relation as a quality criterion. However, the calculation of P_i from a considerable amount of available dielectric loss data indicated that P_i is no more reliable or sensitive than the power factor.

The authors base their method of separating P_i from the total loss on the assumption that without ionization the losses increase with the square of the voltage (the power factor showing no change with increasing voltage). This assumption appears reasonable for the authors' purposes because their discussion is limited to room-temperature data. At higher temperatures several different investigators have found in some cases the power factor *decreasing* with increasing voltage, indicating that the losses are not increasing as rapidly as the square of the voltage. This is probably due to the effect of the temperature and the liquidation of the compound on the anomalous properties of the insulation. While the rate of increase of loss with increasing voltage has been found to increase with temperature in the case of insulation impregnated with some compounds, it has been found to decrease in the case of similar insulation saturated with other compounds. Such data would indicate that the common interpretation of a flat power factor—voltage curve as meaning no ionization, may be rather limited in application.

It is suggested that the influence of temperature and character of the insulation on the rate of loss increase with voltage might well be considered for future investigation by Professor Dawes.

W. B. Kouwenhoven: At the Johns Hopkins University we are conducting an investigation to determine the correlation, if any, between the absorption under continuous potentials and the losses under alternating voltages of the same magnitude. In our experiments we are planning to study the continuous- and alternating-current characteristics of the impregnating compound alone, and of the paper alone. Then we will impregnate the paper with the compound and obtain the resulting properties of the impregnated paper.

We considered very carefully the use of flat samples for this work, and in fact designed apparatus for testing flat samples. After further study, however, we decided to use wound samples with the paper wound under tension, as in a cable, and we have redesigned our equipment to use this type of specimen.

Although Mr. Dawes' curves of power factor against voltage, Figs. 4, 5, and 6, are flat and show apparently that there are no voids present in his specimens, I still think that in a flat sample there is more danger of voids than in a tightly wound specimen.

In this type of apparatus it is always difficult to obtain a vacuum-tight seal. In the apparatus used by Drs. Whitehead and Marvin in making their study of *Anomalous Conduction as a Cause of Dielectric Absorption*, we were confronted by the problem of sealing. We experimented with rubber and a number of other materials with more or less unsatisfactory results. The best thing that we found for a vacuum-tight seal was a wax consisting of three parts of rosin mixed with one part of beeswax. This wax is made by melting the rosin, adding the beeswax, and mixing thoroughly. Before hardening, it should be poured into molds. It may be removed by slightly warming the bottom of the mold. In applying this wax to a joint two methods may be used. It may either be cut into strips and laid on the joint and melted in place by the use of a bunsen flame, or it may be melted in a small dish and applied by means of a brush. The surface should be warm enough to melt the wax, but care should be taken

not to burn it. Both surfaces of the joint are usually coated with the wax and the surfaces placed in contact. Then the joint is warmed by means of a bunsen flame sufficiently to soften the wax after which it is allowed to harden. The joint is then ready for use. When it is desired to break a joint the wax is easily softened by heating.

T. F. Peterson: The authors have done a very valuable work in making these tests on various samples of cable, compiling the data, comparing them with experimental models, and finally plotting the data according to logarithmic coordinates and then determining equations. These equations, however, are empirical and must be used as such. All too often experimenters, and especially those using the results of experimental work, are prone to take these empirical equations and use them as if they actually represented the functional relations of the phenomena involved.

I want to point out that in the case at hand these phenomena are very complex and are not necessarily represented as simply as Professor Dawes has done. About a year ago we had two papers on subjects similar to these being presented this morning, and at that time I derived some theoretical equations for the power factor—voltage curves of insulation containing air. These, of course, were based on uniform distribution of voids and uniform size. Although the expressions derived were rather complicated, they served very well in plotting curves similar to those which have been attained by Professor Dawes in this case.

I am citing this because it leads to a criticism of one of the figures in the present paper. To my way of thinking, the dotted curves of Fig. 11 should not be straight lines throughout their entire length, nor should they be parallel. What is actually represented in each of these curves is the loss in certain air films. As the voltage increases, the power factor or the equivalent phase angle between the current through air film and the voltage across it varies and, therefore, we do not have a straight line or linear relation between the power loss in the air space and voltage until a voltage is reached somewhat in excess of the initial ionization voltage for that void. That would mean that at the outset these curves would be sloping upward until finally a point is reached where the voltage is considerably in excess of the initial ionization and then we can say that the power factor in the void space is unity and, of course, the curve would be a straight line from that point on.

Inasmuch as this represents the loss in the void space at a certain radius from the center of the conductor, it is obvious that, as the ionization progresses outward, there will be more air (based on uniform distribution), *i. e.*, more void space in ionization, and hence the constant coefficient which would represent the slope of each of these curves will be variable and will be greater the greater the distance out from the center of the conductor. These curves ought, therefore, to have increasing slopes as we progress from E_1, E_2 to E_5.

The other point that I am not quite clear on is immediately below that particular curve. The statement is made that it is not generally conceded now that the power factor curve does not have a tendency to become flat when ionization is complete.

I might say that the work I have done seems to indicate that the curve does flatten out when ionization is complete. In fact, I should say that ionization is complete just a little in advance of the flattening out of the power factor curve. This can be shown pretty definitely by means of the mathematical equations.

J. B. Whitehead: It is more or less tacitly assumed in all three of the papers presented in this session that the invariable cause of the rise in the power factor—voltage curve is the presence of air. Certainly in these days no one will controvert the fact that the presence of air in the cable will give a rise in the power factor curve and probably in the vast majority of cases it is the principal cause.

However, we have shown that in laboratory samples both air and moisture can cause a rise in power factor curves, in insulation, which on dismantling appears to be extremely well impregnated.

Further we have shown that a decreasing power factor curve, a curve in which the power factor falls with voltage, usually to be found at the higher temperatures, is in very well impregnated insulation.

The flat curves shown by Professor Dawes, and which we have obtained many times, are shown by him as at relatively low temperature. If these samples were carried up in temperature, say to 60 or even to 80 deg., I venture to predict that he would find a decreasing power factor curve. In one of our papers, we have offered a speculative suggestion as to an explanation of this.

In contrast with the foregoing, I call attention to the variation which may occur from an entirely different cause, namely, the quality of the paper. In experimenting with some different qualities of paper, we have found that under the same conditions of evacuation and drying it is not possible to obtain the same final value of conductivity of the paper. In the various samples we have taken this final conductivity as a convenient indication of the amount of moisture remaining in the paper.

In those papers in which the conductivity cannot be reduced to the standard which we have adopted as pertaining to excellent paper, we have found that this decrease in the power factor curve at the upper temperatures is not found. We, therefore, emphasize the fact that the shape of the power factor curve may be varied considerably, due to other causes than that of large air voids. It is worth speculation as to whether the ionization test, as we have it today, might not be extended and modified to embrace tests at higher temperatures as an indication not only as to the original state of the paper as regards air in large voids, but also as regards the presence of moisture which would cause an increasing power factor—voltage curve. I take it that everyone will agree that a decreasing power factor—voltage curve is eminently desirable.

Professor Dawes has made an excellent case for a derivation of his very simple law of loss due to gaseous ionization. In view of all the factors that must enter in the gaseous ionization, I think it is extremely remarkable that this result should come out.

It has been shown long since that in the ionization of gas films at the moment when ionization begins there is a considerable increase in the pressure of the confined gas states. This pressure is not due to the temperature rise but due to the fact that a greater number of ions arises immediately.

Temperature will have a bearing on the loss due to ionization. Also above a certain voltage, ionization in an air film and accompanying oxidation undoubtedly reach saturation or constant values.

We have reported experiments in which the character of the gaseous discharge in a gas film changes very definitely after a short time, and we have attributed this to the fact that whatever oxygen is present in the gas film is consumed or rather goes into combination with the adjacent insulation and as a consequence the energy loss following such a process, *i. e.*, during the subsequent constant condition, must be less than it was during the time that it was arising.

It would appear that these various matters must play some part in the process of ionization and in the behavior of cable insulation.

The decrease in power factor shown by Messrs. Shanklin and Mackay for artificial gas films is quite different in character from that I referred to earlier, *i. e.*, the decreasing power factor curve of good impregnated paper. In the latter case, the values of power factor involved are of very much lower order of magnitude than those pertaining to the gas films, and I should like to ask a comment from one of the authors as to their explanation of the decreasing power factor of these gas films. It seems to me that it is a result of the stable condition of ionization which I have referred to as a saturated condition of ionization, in which practically all molecules are being ionized and where there is no progressive change thereafter.

W. V. King: (communicated after adjournment) As a com-

ment on the paper of C. L. Dawes, H. H. Reichard, and P. H. Humphries on *Ionization Studies in Paper Insulated Cables* may I submit the results of analyzing the dielectric loss curves on two separate samples of type H-3 conductor cable as tested by the Research Bureau of the Brooklyn Edison Company?

The solid dielectric losses vary approximately as the square of the voltage. The equations expressing these losses as a function of the voltage are

$$P_{d_1} = 12.5 \times 10^{-4}\, E^{2.08}$$

and

$$P_{d_2} = 10.5 \times 10^{-4}\, E^{1.99}$$

where P_d = Solid dielectric loss in watts per foot

and E = Potential in kv.

The ionization voltages for the two samples are

$$E\, o_1 = 21.3 \text{ kv.}$$

$$E\, o_2 = 17.0 \text{ kv.}$$

The voltages at which ionization will commence will depend upon the pressure in the void, the size of the enclosure, and the different distribution of the voids from the copper conductor to the lead sheath. Many investigators have shown that an increase in the void pressure and a distribution of many small voids instead of several large ones both tend to raise the ionization point.

The total dielectric losses above ionization voltage vary as the 2.31 power of the voltage which compares very favorably with the results of Dawes, Reichard, and Humphries on a single-conductor cable with metallized shielding tape and designated as cable 14 in their paper. The equations expressing the total dielectric losses above ionization voltage are

$$P_1 = 6.18 \times 10^{-4}\, E^{2.31}$$

and

$$P_2 = 4.27 \times 10^{-4}\, E^{2.31}$$

The ionization losses, after ionization is complete, do not vary exactly as a linear function of the voltage. However, a very small error will result if the points are averaged and a straight line is drawn to represent the variation of ionization loss with voltage after ionization is complete.

C. L. Dawes, P. H. Humphries, and H. H. Reichard: Referring first to the discussion of Messrs. Davis and Eddy, the authors did not intend to convey the impression that P_i should be the sole criterion of cable quality. They are of the opinion, however, that it may be *one* of the factors which determine cable quality. Experimental work to determine the relation of cable constants to cable life is now being conducted and when sufficient data are obtained, it may be possible to state more definitely the relation of P_i to cable life.

In our paper we found that the loss in the solid dielectric does vary as the square of the voltage, but the method of analysis is not at all dependent on this fact. A careful study of the method shows that it is necessary merely to assume that solid-dielectric power loss follows the same law above the ionization voltage as it does below the ionization voltage. We realized that at the higher temperatures the power factor did not necessarily remain constant with increase in voltage and a footnote to this effect is given at the bottom of the fifth page. As stated in the first paragraph of the Synopsis, this is a report of progress and the authors hope to present a paper somewhat later which will deal more specifically with phenomena at higher temperatures and some of the other matters mentioned by Messrs. Davis and Eddy.

We hope that Dr. Kouwenhoven and his associates will use cylindrical samples rather than flat samples, since this will give an excellent opportunity to compare the results obtained with the two types of samples. If their results differ from ours, there will be opportunity to reconcile the differences. On the other hand, if the results are in accord, it will show that either flat or cylindrical samples may be used for such tests. Dr. Kouwenhoven feels that there is greater opportunity for voids with flat samples than with cylindrical samples. As a matter of fact, we chose flat samples because we believed they gave less opportunity for voids. With a flat sample it is possible to expose the paper and compound to a high vacuum and to apply the compound to the paper without the upper electrode being in contact with the paper. This electrode is then lowered while still in the high vacuum. This process prevents the occluding of any considerable amount of air within the sample. Since these experiments were performed, improvements in sealing the apparatus together with a pump of larger capacity have enabled us to obtain vacuums as close to zero as we can read the mercury meniscus, so that the amount of occluded air is now almost zero. It is difficult to seal this apparatus with wax and resin as Dr. Kouwenhoven suggests, on account of its construction. We now find that porthole rubber makes very satisfactory gaskets and is very convenient to use.

I agree with Mr. Peterson that our results are to some extent empirical, but to date even the fundamental work on dielectrics has, for the most part, been empirical. We are submitting the results as we find them and as is frequently the case with pioneer research, later investigators may be able, with improved methods, to add refinements to the relationships which we now offer. We do believe, however, that the characteristics of both solid and gaseous dielectrics are more fundamental than Mr. Peterson realizes. For example, the fact that the voltage across an ionized gas ultimately becomes constant with increase in current is one of the fundamental discoveries of Townsend, and for the most part is very likely due to the effect of space charge. Mr. Peterson's criticism that in Fig. 11 the power lines are not necessarily parallel is justified although the figure is not incorrect; the slope of the different power curves should have been shown as being different in order to generalize the discussion. As a matter of fact, the curves were not parallel in the original sketch, but somehow the figure finally appeared in the paper with the lines drawn parallel. The general theory concerning the curved and straight portions of the resultant curve, however, still holds, as is stated in the paper, "This is true irrespective of the slopes of curves *a* to *c*." The individual power curves should be substantially straight, however, in spite of Mr. Peterson's reasoning to the contrary. He appears to rest his agreement on the origin being at E_o. As a matter of fact, in the region under discussion, the rate of increase of voltage E decreases until it becomes zero (that is, E becomes constant) and at the same time the power factor increases until it becomes nearly constant. These changes are such that in the equation for power, $P = E\, I \cos \theta$, the term $E \cos \theta$ is nearly constant in this region for any given gas. The coefficients which represent the slope of each of the curves in Fig. 11 depend on the thickness of the gas film and on the pressure. The coefficient does not depend on the geometrical position of the void in the cable insulation as stated by Mr. Peterson. The slope of the resultant curve obviously increases as the voltage is raised until all the voids become ionized as stated in the paper.

We do not recall having stated in a previous paper that the power-factor curve does not have a tendency to become flat when ionization is complete. We have shown, however, that the power-factor does reach a maximum and then actually decreases and that the reasoning as to the character of the power-factor curve previous to our work had been fallacious. Mr. Peterson, however, seems to be in accord with our conclusion that ionization is complete at point *f*, Fig. 11, which is in advance of the "flattening out," or better, the maximum value of the power-factor curve.

Like Dr. Whitehead, we find with the different cable compounds both increasing and decreasing power-factor curves at the higher temperatures and we intend to analyze our results under these conditions as the work progresses. The committee, as well as ourselves, believes it wiser to confine our efforts for the time being to work at room temperature, until some of the many problems that have already arisen have been solved. We shall then consider the matter of higher temperatures. We agree with Dr. Whitehead that it does appear to be remarkable that the

power curve for ionized gases, at least its lower portion, is so simple. Some of the reasons for this have been brought out earlier in this discussion. At the higher current densities, however, the voltage and power factor tend to become essentially constant, which would make the power proportional to the current after this condition is reached.

Referring to Dr. Whitehead's query as to the cause of the decreasing power factor of gas films, we intend to analyze the electrical characteristics of gas films in a later paper. We do not feel that the current density reached in occluded gas films is sufficiently high to produce the condition of high saturation as Dr. Whitehead suggests. At this time the increase in the power factor of the gas films themselves appears to be due to the increased power loss and the enormous increase in capacitance of the films themselves. The increase in capacitance is due to increased space charge. If the air film be considered as a series circuit having resistance R and capacitance C in series, $\tan \theta = 1/R\,C\,\omega$, where θ is the power-factor angle and $\omega = 2\pi$ times the frequency. With increase in current, R will first increase rapidly to a maximum and then decrease slightly to a nearly constant value. C, however, will always increase rapidly with voltage. Hence θ ultimately decreases and the power factor increases with voltage. With the current densities which we have been able to attain, the power factor reaches a maximum value of 0.75 or 0.80 and then becomes nearly constant. It is the combination of the increasing power factor of the gas films and the constant power factor of the solid dielectric which gives the increasing and then decreasing power factor of the two in series, such as is shown by the curves in Figs. 13 and 14 of the Shanklin and Mackay paper.

Mr. King's results are in part a confirmation of our own work. We have found in some instances recently that the ionization power loss P_i increases more rapidly with voltage than is indicated by the linear relationship. This was found to be due in part to heating of the cable while taking measurements. However, after this factor had been eliminated, the P_i curve of some cables that had been undergoing accelerated life tests still showed a slight upward curvature. We attribute this to the losses caused by the creepage currents which result from tangential electrical stresses. This loss probably does increase more rapidly with voltage than the linear relationship shows. Mr. King's results seem to bear this out, since it is well known that tangential stresses are large in unshielded 3-conductor cables. It would be interesting to learn of the experiences of others in this connection.

Reduction of Sheath Losses in Single-Conductor Cables

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Synopsis.—The use of single-conductor, lead-covered cable for high-voltage, three-phase transmission lines results in sheath losses ranging from 25 to 300 per cent of the conductor losses for cables installed in separate ducts, unless special methods for the reduction of the losses are used. Some of these methods, while practically eliminating sheath losses, cause a-c. sheath potentials which may be injurious. In this connection the authors have developed a new scheme of bonding and a new bonding device, which appear to have marked advantages.

This article consists of a general discussion relative to the reduction of sheath losses with special reference to laboratory tests and field work done on 132 miles of single-conductor cable of the Commonwealth Edison Company.

The economics of sheath losses and of the methods for their practical elimination are discussed. The theories of sheath losses and induced voltages are outlined and correlated, and new formulas and curves are developed. An analytical and graphical comparison of sheath bonding connections is presented. Investigations are reported on tests regarding the nature and extent of possible corrosion of sheaths caused by a-c. sheath voltages.

I. Introduction

WITHIN recent years the use of single-conductor lead-covered underground cable for three-phase voltages ranging from 11 to 132 kv. has been rapidly increasing. The principal reasons are:

(a) To transmit large quantities of power, for which three-conductor cable would be unwieldy;

(b) To obtain phase isolation;

(c) To gain advantage of the inherently higher unit dielectric strength of the insulation in single-conductor cable.

The growing importance of single-conductor transmission cable is shown by the fact that the percentage used as compared to the total cable used in the United States increased from 4 per cent in 1926 to 8 per cent in 1927.[1]

If the cables are installed in separate ducts and operated with solidly bonded sheaths, the sheath losses will range from 25 to 300 per cent of the conductor losses, thereby considerably decreasing the carrying capacity and increasing operating costs. If the sheaths are made discontinuous and bonded in special ways to prevent sheath currents, problems arise to provide satisfactory insulating joints, bonding apparatus and bonding connections, and to limit the sheath corrosion that may be caused by unneutralized sheath voltages.

In 1926 the Commonwealth Edison Company began the installation of single-conductor underground cable as three-phase, 60-cycle, 66-kv. lines and as 12-kv. leads to transformers feeding these lines. Since calculations showed that if the sheath losses were eliminated, the carrying capacity of these two sizes of cable would be increased approximately 20 and 70 per cent respectively, with a corresponding increase in the cost of the installed cable and conduit of 1 or 2 per cent, it was decided to make some trial installations with various kinds of insulating sleeves, bonding apparatus, and methods of bonding. Because the results of the preliminary installations were favorable, this practise was extended to all 66-kv. lines and 12-kv. leads in 1927.

The authors developed a new device, namely, a three-phase sheath bonding transformer, and also a new sheath bonding connection, which appear to have marked advantages over previous bonding devices and connections.

At the end of 1928 the various devices and the most promising schemes of connections were installed on 114 miles of 66-kv. cable and 18 miles of 12-kv. cable.

A general consideration of the entire problem of eliminating sheath losses and reducing sheath voltages on single-conductor a-c. cables will be attempted in this article with special reference to the work done by the Commonwealth Edison Company in the past two years.

II. Economics

Preliminary findings, which have been confirmed, show that the elimination of the sheath losses in three-phase, 60-cycle, 66-kv. lines consisting of single-conductor 750,000-cm. cables installed in conduits with other cables, increases the average yearly carrying capacity from approximately 50,000 to 60,000 kv-a., or about 20 per cent. Since the total investment cost of the installed cable is approximately $60,000 per mile of line or about $1.20 per kv-a. mile, the increased investment value resulting from the elimination of sheath losses is, therefore, approximately $12,000 per mile of line. An additional saving is made in the cost of reduced sheath losses which is about $1000 per mile of line per year, assuming 55,000-kv-a. average yearly rating and a 60 per cent load factor. Capitalized at 10

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1. See Bibliography.

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per cent per year, this represents an additional investment saving of $10,000 per mile of line.

Extra investment is required for (1) insulating joints, (2) insulated cable saddles and bond wire, (3) special fire-proofing to minimize corrosion due to a-c. voltages, and, perhaps, (4) bonding devices such as reactors or transformers. The additional investment cost required per manhole is about $65 for the first three items and $125 to $250 for all four items. Using even all four

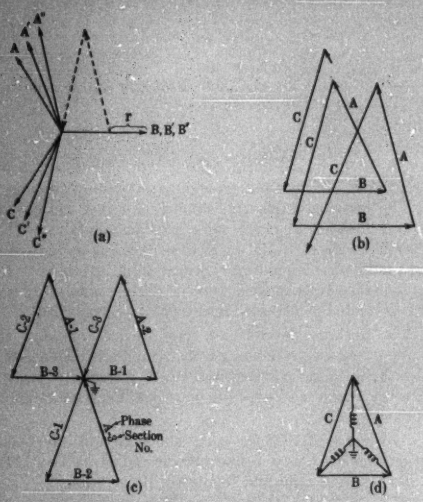

FIG. 1—SHEATH VOLTAGE VECTOR DIAGRAMS

(a) Induced voltages for equilateral rectangular and flat cable spacings, Note, $\sum E'' = r \neq 0$
(b) Continuous cross bonding, one circuit of Fig. 2c
(c) Cross-bonding, Fig. 2b
(d) Continuous cross-bonding, star-connected transformer, Fig. 2a

items and assuming twelve manholes per mile, the average additional cost of elimination of sheath losses is approximately $1500 per mile. This is only one-eighth of the increased value of a 66-kv. line due to the increased rating. The cost of the losses saved would pay for the additional investment cost in one or two years.

If 66 kv. cables only were installed in a given conduit, the elimination of sheath losses would result in an increase of carrying capacity of 30 per cent, while the extra cost would be the same.

On lower voltage, higher current cables, the gain is much larger. For the 12-kv. leads feeding the transformers of the 66-kv. lines, the standard practise in Chicago is to install three 1,750,000 cm. cables per phase. All nine cables are usually in one conduit. In this case the increase in carrying capacity obtained by the elimination of sheath losses is about 70 per cent (see cable 2 in Table I).

TABLE I

CALCULATED RATINGS, INDUCED SHEATH CURRENTS, LOSSES, AND VOLTAGES FOR TYPICAL CABLE INSTALLATIONS

Item No.	Location	Oper. volt kv.	Cable: Conductor: Size cir. mils	Conductor: Diam.-inches: Int.	Conductor: Diam.-inches: Ext.	Thickness 64th inch: Insul.	Thickness 64th inch: Lead	Diam. over lead inches	Frequency	Cable Arrangements: Spacing	Cable Arrangements: Inches between centers	Ratio for sheath to copper, %: Currents	Ratio for sheath to copper, %: Losses	Kv-a. rating, sheaths solidly bonded	Results of eliminating sheath losses: Increase in carrying capacity: Kv-a.	Increase in carrying capacity: %	Decrease in copper temp. °C	Induced voltage per 100 ft. of sheath*: Outer cables	Induced voltage per 100 ft. of sheath*: Middle cable
1	N. Y. City	11	600,000	0	0.89	12	7	1.5	25	O* O O	1.7	3	1.2	9,400	56	0.6	0.4	0.44	0.44
2	Chicago	12	1,750,000	1.0	1.88	20	9	2.8	60	O O O	6.0	37¶	220¶	12,300	3,700	70	40	4.50	3.52
3	N. Y. City	44	500,000	0	0.82	30	8	2.0	60	O* O O	2.3	14	12.5	34,000	1,400	4	2	0.95	0.95
4	N. Y. City	44	500,000	0	0.82	30	8	2.0	60	O O O	6.0	30¶	56¶	32,500	4,500	14	6	2.62	1.94
5	Paris	60	297,000	0	0.63	36	6.3	1.95	50	O† O O	2.1	6.1	4.0	31,000	420	1.4	1.4	0.47	0.47
6	Chicago	66	750,000	0	1.00	48	9	2.85	60	O O O	6.0	40¶	88¶	50,000	10,000	20	6	2.36	1.83
	Chicago	132	600,000	0.75	1.20	46	‡	3.15	60	O O O	6.5	57¶	100¶	95,000	25,000	26	10	2.36	1.83

*In one duct
†Lute covered and buried in contact
‡Double sheath, 100 mils plus 90 mils thick

¶Average for all three cables
*Based on kv-a. rating with *no* sheath losses

The maximum allowable length of conduit between manholes is another important economic consideration. The cost of a manhole suitable for 66-kv. cable and joints is about $500. The additional cost of three 66-kv. joints and oil reservoirs plus the joints on lower voltage cables, fireproofing, racking, etc., will average $600 per manhole. Each manhole eliminated along a typical 66-kv. line will represent, therefore, a saving of $1100, besides additional savings in the unit cost of pulling and handling longer cable lengths.

Induced sheath voltages are directly proportional to cable length. Because of a-c. electrolysis and other reasons, it may be impracticable or destructive to operate the cables with sheath voltages exceeding 12 volts to ground or 22 volts between cables. Without bonding devices on 66-kv. cables, for example, this limit is reached with about 500-ft. lengths, or under practical conditions with about 13 manholes per mile. If a series type of bonding device as shown in Fig. 2E, or the cross-bonded, star-ground type of connection as shown in Fig. 2G is used, the lengths may be increased 100 or 60 per cent, respectively, without increasing the voltages between sheaths and ground, and with only negligible sheath losses.

The Commonwealth Edison Company has found it desirable to increase the maximum conduit lengths up to the length of a city block, about 660 ft. This reduces the average number of manholes per mile from 13 to 9, thereby saving over $4000 per mile of line. The increased investment cost for the necessary bonding devices for connection Fig. 2G, that is, one in every other manhole, is approximately $600 per mile. In view of this large saving, the company is trying over 90 H & M sheath bonding transformers for installation in new conduit built in 1928, where the sections of conduit are 550 to 700 ft. long.

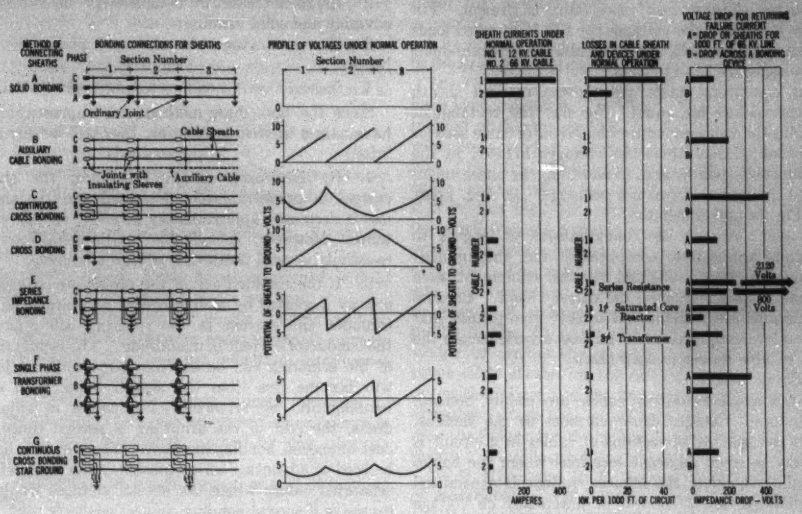

FIG. 2—COMPARISON OF CONNECTIONS AND RESULTING SHEATH VOLTAGES, CURRENT, AND LOSSES FOR 12-KV. AND 66-KV. THREE-PHASE 60-CYCLE SINGLE-CONDUCTOR CABLES

1. Lengths of sections 1, 2, and 3 are 330, 400, and 470 ft. for 66-kv. cables and 55 per cent as long for 12-kv. cables
2. Cable (1) 1,750,000 cir. mils, 12 kv.; flat arrangement with 6 in. between centers; 20,000-kv-a. load per circuit
3. Cable (2) 750,000 cir. mils, 66 kv.; flat arrangement with 6 in. between centers; 60,000-kv-a. load per circuit
4. Voltage profiles are for outer cables on connections *B*, *E*, and *F*, and profiles are for section 1-*C*, 2-*B*, and 3-*A* for connections *C*, *D*, and *G*.
5. Impedance drops are for fault current of 3000 amperes returning over sheaths with manholes every 400 ft.

III. SHEATH VOLTAGES AND LOSSES

A. *Sheath Voltages*

The induced sheath voltages in single-conductor lead-covered cables on a-c. circuits vary logarithmically with the ratio of distance between cable centers to the sheath radius, directly with the frequency and magnitude of the conductor currents, and directly with the length of section between insulating joints. When cables are closely spaced, the adjacent and remote portions of the sheaths are unequally affected, a condition commonly called the "proximity effect." For cable sheaths in contact the proximity effect becomes very pronounced, but for cables installed in separate ducts with six or seven inches separation between

centers, it is small. The proximity effect has little effect on sheath voltages and is usually neglected. Various formulas for sheath voltages and losses have been developed; these formulas, neglecting proximity effect, have been collected and extended in the Appendix.

For purposes of illustration, the calculated induced voltages for typical kinds of cables are shown in Table I, and for the 66- and 12-kv. cables of the Commonwealth Edison Company in Fig. 2, the calculations being made on the basis that no other influence, such as other single-conductor circuits, iron pipes, etc., is present. For cables in contact the voltages per 100 ft. at rated loads are less than one volt, while for cables in separate ducts the voltages are 2 to 4½ volts.

For equilateral cable spacing the induced sheath voltage vectors are equal. For flat and rectangular spacings the voltages induced in the outer cable sheaths are usually larger than the voltages induced in the middle cable sheath, and, in addition, the phase angles of the induced voltages are no longer 120 deg. apart (see Fig. 1A and Appendix).

At line terminals the configurations of lengths to potheads are usually complicated and the induced voltages are greatly affected by other equipment, such as the busses which close the current loops. Because calculations are difficult and usually give results that are too low, estimates based on past experience are the most reliable guides in such cases.

B. Sheath Losses.

If the single-conductor cables are solidly bonded, the induced voltages cause currents in the sheaths. When the spacing of sections of cables in a circuit is uniform between adjacent locations where the cables are solidly bonded, the induced voltage is neutralized as generated and there will be practically no voltage between the sheaths.

In addition to the effect of the conductor currents, the sheath currents are mutually inductive between themselves, so that in general the sheath current cannot be obtained by dividing the induced sheath voltage (open circuit) by the vector sum of the sheath resistance and sheath reactance. The sheath currents must be solved for simultaneously; and, as shown in the Appendix, the currents and losses in outer cables in flat or rectangular spacing are generally unequal.

In order to give an idea of the magnitude of sheath currents and losses, the ratios of currents and losses in the sheath as compared to the conductor are shown in Table I. In general if the sheath losses are eliminated and the cables are in separate ducts, the carrying capacity is increased 15 to 70 per cent for usual cable sizes, or for a given load the copper temperature is decreased from 5 to 45 deg. cent. For cables in contact, the sheath losses, including the proximity effect, are 1 to 15 per cent of the conductor losses. (Precise calculations for cables in contact are difficult because of proximity effect.)

The sheath currents and losses are graphically shown in Fig. 2 for the 66- and 12-kv. cables of the Commonwealth Edison Company.

IV. Methods of Reducing Sheath Losses

When each phase of a given three-phase line installation is to be carried in a separate metal-covered cable, one of the following methods might be used for materially reducing the sheath losses:

1. Large increase in sheath resistance and solid bonding;
2. Large increase in conductivity of protective covering and solid bonding;
3. Two-conductor cable for each phase and solid bonding;
4. Special sheath bonding connections and devices.

Since the first three methods are impracticable or have only a limited application, they will be discussed briefly.

1. By increasing sheath resistance, the sheath current is proportionately decreased and the sheath loss is decreased approximately inversely as the resistance. However, no suitable plastic metal having a resistivity several times that of lead is known.

2. If the conductivity of the protective covering is greatly increased by adding copper wire armoring, for example, the current in the covering will approach the conductor current in magnitude. The conductivity of the armoring can be made so large that the losses will become less than the sheath losses with solid bonding and no armoring. For cables in adjacent ducts, the cost of the armoring is several times the cost of special bonding methods. The armoring might be used to advantage on submarine cables or on widely separated cables where the sheath voltages would be excessive if special bonding connections were used.

3. If a two-conductor "D" or concentric cable is used for each phase, with the two conductors connected to opposite ends of transformer phase winding, the two currents will have equal and opposite effects on the sheath and cause no induced voltage. This method requires special transformers (6-phase), but appears to have some merits for use at 10 or 15 kv. This scheme, however, also has the advantage of allowing continuation of operation with open delta connection when one cable fails. For 22 kv. and higher voltages, the tendency is for the cable size to become impracticable and for the cost per kv-a. of capacity to become considerably more than for single-conductor cable with special bonding.

V. Sheath Bonding Methods

A. General Considerations. All special methods of sheath bonding have as their principal purpose the elimination of sheath losses. Some methods are designed also to result in smaller voltages between sheaths and to ground. Many bonding methods are possible, although only a limited number of connections is

practicable. Various connections (see Fig. 2) will be discussed later in detail.

For a given cable arrangement, conductor current, and system frequency, the sheath voltage *induced* per cable length is a fixed quantity. The voltage from end to end of a cable sheath cannot be reduced except by allowing sheath currents to flow and to neutralize partially or totally the voltages in the sheath where they are induced. It is possible, however, by the bonding devices and connections later described to *shift* the relative position of the induced voltage vectors of the several cable sheaths in such a manner that the voltages between sheaths and to ground are reduced.

The major considerations in the selection of connections, methods, and devices for special bonding are as follows:

1. Elimination of sheath losses and increase of cable current-carrying capacity;
2. Reduction of normal induced voltages between sheaths and to ground to keep corrosion due to a-c. voltages at a minimum;
3. Limitation of abnormal sheath voltages during failure to the lowest possible values.

The above objects must be accomplished *without* causing the following objectionable features:

1. Excessive losses in the sheath bonding devices;
2. Introduction of triple or other harmonic currents into the sheath circuit causing inductive interference with telephone circuits;
3. Interference with proper current drainage to prevent d-c. electrolysis; also adverse effect on operation of the a-c. sheath bonding method by flow of stray d-c. currents;
4. Excessive size, weight, space, or cost of bonding devices.

B. Solid Bonding. If the sheaths are joined with ordinary joints and solidly bonded (Fig. 2A), the installation is simple and avoids the introduction of new methods and apparatus. The sheath losses, however, will be of such size, especially if the cables are in separate ducts or farther apart, that the reduction in carrying capacity and the increase in sheath losses and operating costs are excessive, as previously indicated and as shown by the graphs in Fig. 2.

C. Bonding One End Only to Auxiliary Cable. Sheath losses may be eliminated by connecting only one end of every length of sheath to some auxiliary cable (Fig. 2B) or to the sheaths of other cables in the same conduits. This connection requires either the use of an extra duct and an auxiliary cable at considerable expense, or placing dependence on other cables which may not be permanent. One bond is more apt to become open by accident or mistake than bonds at each end, and the cable sheath may be left "floating" with the possibility of acquiring potentials dangerous to cable maintenance or to personal safety.

This method, however, has been used by the company for special lengths at line terminals consisting of sections of cable going to potheads, transformer neutrals, delta ties, etc. In these cases the induced voltages are irregular in magnitude and phase, and the cable sections do not lend themselves readily to other schemes of bonding. The usual practise of the Commonwealth Edison Company is to place one or more insulating sleeves in the sheaths of these special lengths. One end of each insulated section is connected to the station ground bus, and in case the section has a pothead, the pothead end is the end that is usually grounded. The end of the line in regular conduit is solidly bonded in the last manhole and also connected to the same ground bus.

D. Cross Bonding. Two methods of cross bonding have been suggested. In 1914 L. Emanueli[2] devised a method of sheath bonding in which the cable sheaths were cross-bonded continuously along the complete line, (Fig. 2C). With such a method and irregular cable lengths (which are of necessity always present), it is evident that the voltage vector diagram will not repeatedly retrace a triangle; instead, it will form three erratic triangular helices, one for each sheath circuit (Fig. 1B). The voltages to ground are uncontrolled and may become excessive, depending on the chance succession of unequal cable lengths. (See voltage profile in Fig. 2.) Also, during failures it is not desirable to confine the returning failure current to a single cable sheath for the full length of the line between the failure and the terminal, on account of the excessive voltage that would exist on the cable sheath near the failure due to the high impedance of one cable sheath. (Note last column of Fig. 2.)

W. E. Kirke and H. R. Searing[3] devised a method of cross-bonding, (Fig. 2D), in which the cables are solidly bonded in every third manhole and transposed in the two intermediate manholes. With this method the sheath voltages of the three sheath circuits between solid bonded points each trace the three sides of the triangle, starting at ground potential and returning to the same potential. If the cable spacing is not equilateral or if the cable lengths are not equal, the induced voltage triangles will not close and a residual voltage remains which is neutralized by circulating current over the three cable lengths (see "r" Fig. 1A and Equation 4 of Appendix). Since the impedance of three cable sheaths in series is presented to a relatively small differential voltage, the resulting sheath currents caused by unequal cable lengths are in general quite small; and the sheath losses (see Fig. 2) are usually not more than 1 or 2 per cent as great as for solidly bonded sheaths.

The ground potential is definitely fixed in every third manhole at the corner of the voltage vector triangle. As shown in Fig. 1C, the complete vector diagram for all three sheath circuits consists of three triangles with one common point (ground). The maximum voltage between sheath and ground is equal,

therefore, to the induced voltage of a length of cable as shown in the voltage profile (Fig. 2D).

This method has the advantages of simplicity and low cost, since it requires insulating joints in only two-thirds of the manholes and no additional bonding devices. Where conduit lengths are very unequal and where lines are frequently cut and interchanged, as on station properties, this method is often inconvenient.

Cross-bonding has been used in over 90 per cent of the work of the Commonwealth Edison Company. However, the company has installed insulating joints in all manholes in order that other bonding schemes could be used later if the sheath voltages with cross-bonding proved too high.

E. Reactance Bonding. In 1914 L. Emanueli[3] and in 1920 P. Capdeville[4] set forth the general principle of making a 50 per cent reduction in potential between sheaths and to ground by using impedances connected in series with the cable sheaths, the coils being interconnected and grounded at the mid-points (see Fig. 2E). The impedance of the devices is made considerably higher than the impedance of the sheaths, with the result that very little current flows and the voltage drop is almost entirely in the device. Emanueli used single-phase transformers for the purpose, while Capdeville describes both resistance and reactance (Fig. 2E), preferring a single-phase iron cored reactance designed for enclosure inside the cable joint sleeve.

A method of bonding cable sheaths through saturated iron core reactance coils has been devised by R. W. Atkinson.[5] The coils are connected in series with the cable sheaths, and their midpoints may also be interconnected and grounded (Fig. 2E). A second purpose of these coils is to limit the value of abnormal voltage which can exist on the cable sheaths due to excessive current in one or more of the conductors during a failure or other cause. For this purpose the coils are designed with closed iron cores proportioned to operate normally at or near the saturation point. The coils normally draw only a small exciting current. If, however, the voltage which is induced in the sheath and applied to the coil tends to rise to an excessive value due to abnormal conductor current, the iron core will saturate and the effective impedance of the reactor will greatly decrease. This will allow a very large current to flow which will neutralize the excessive induced voltage in the sheath. Abnormal voltages from this cause are limited to a small amount over the saturation voltage of the coil, perhaps 200 to 300 per cent of normal.

Although reactance coils accomplish their intended functions they introduce several objectionable features, among which the following may be named:

1. Since they are single-phase and operate near the saturation point, triple harmonic exciting currents are introduced into the sheath circuits, and may cause inductive interference on parallel telephone or signal circuits. (Triple harmonic currents equal to 15 per cent of the exciting current were found in test on such devices.)

2. It frequently happens that cable sheaths carry stray direct currents which may easily be of the same order of magnitude as the normal exciting current of the reactors (10 to 20 amperes). The iron core may become saturated with d-c. flux, increasing losses in the sheath and reactor and causing even harmonics in the exciting current, which may result in telephone interference.

3. During return flow of failure current along the sheath circuit the voltages or impedance drops across the coils are limited only by flux saturation in the iron cores. These voltages are added in series with the sheath voltage drop along the entire length of an isolated section of a line, and the total voltage drop, including the single-phase coils, may become excessive (see Table II and Figs. 2 and 6).

4. The three separate devices increase the investment cost and in addition require considerable manhole space.

TABLE II

COMPARATIVE VOLTAGES AND LOSSES FOR VARIOUS SHEATH BONDING CONNECTIONS

Connection	Fig. 2	Calculated* losses in per cent of losses with solid bonding			Calculated sheath voltage drops along 66-kv. line during failure†-volts	
		Cable sheaths	Device	Total	Across device	Per 1000 ft. of line
Solid bond	A	100	..	100	..	125
Aux ground	B	0	0	0	..	200‡
Continuous cross bond	C	0.2	..	0.2	..	415
Cross bond	D	3	..	3	..	135
Series resistance	E	0.5	5	5.5	800	2120
Sat. core reactor	E	1	2	3	65	250
1-φ transformer	F	0	2	2	100	320
3-φ transformer	E	1.5	0.9	2.4	14	160
Cross bond—star	G	1	0.5	1.5	..	135

*Average maximum limit.

†Isolated circuit with 3000 amperes returning on its sheaths with man holes assumed 400 ft. apart.

‡Assumed other cables in three adjacent ducts.

F. Resistance Bonding. Resistances may be connected as shown in Fig. 2E. They have several times the losses of equivalent reactance devices and must have large thermal storage to withstand line failure currents of several thousand amperes. Metal grids would probably be required and manhole water conditions would require enclosing them in a case. All these factors necessitate excessive size, and hence relatively large cost and inconvenience in installation.

Resistance bonding does not have the desirable voltage limiting characteristic for minimum voltage drop during the return flow of line failure currents. The summation of the $I\ R$ drops for a section of isolated line during the flow of failure currents might be several thousand volts per mile of line (see Table II and Figs. 2 and 6). Finally, resistance bonding would greatly

complicate protection against d-c. electrolysis, making effective "sheath drainage" practically impossible.

G. Partial Current Flow Bonding. Either series resistances or reactances may be chosen of such values that any desired percentage of the sheath current with sheaths solidly bonded will be allowed to flow. With such a scheme there is partial neutralization of the induced sheath voltages and thus a compromise is made between sheath losses and sheath voltages. This reduction may be made in addition to the 50 per cent obtained by grounding the coil mid-taps as in Fig. 2E. In no case can a reduction of more than 50 per cent in

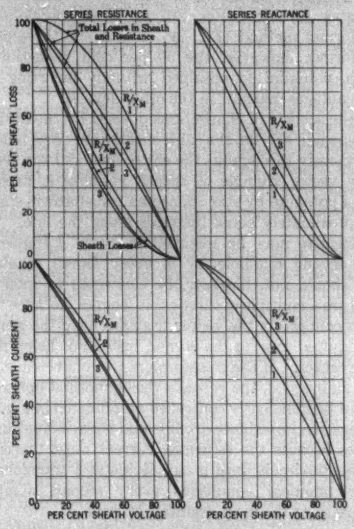

FIG. 3—SHEATH CURRENT, SHEATH LOSS, AND TOTAL LOSS *vs.* SHEATH VOLTAGE

Using series resistors or reactors in single conductor cable sheath
The ordinates give values as compared to the values which obtain for solid bonding, while the abscissas give voltages as compared to those which obtain when no sheath current flows

sheath voltage be obtained without introducing sheath losses.

Fig. 3 shows in percentages the possibilities of this method for ordinary conditions where the ratio of sheath resistance to reactance varies from 1 to 3—the values usually encountered with 60-cycle cables.

If resistance bonding is used and numerical values are considered it is quickly discovered that the heat losses in the resistor itself are very large. For example, if sheath potentials are limited 50 per cent by this method, then, as shown in Fig. 3, the sheath currents are reduced to 50 or 60 per cent of the values which they would attain with solid-bonding, and the total losses in both the sheath and the resistor are from 50 to 70 per cent of the value for solid-bonding. Of these losses about half occur in the resistors and for typical cables are easily of the order of 200 to 300 watts per resistor.

If reactance bonding is used and sheath potentials are limited to 50 per cent of the induced voltages by this method, the currents are 60 to 70 per cent of the values with solid bonding, and sheath losses are 45 to 60 per cent of values for solid-bonding. The losses in the reactors will be small. The reactors usually must have an air gap in the core to prevent saturation of the iron.

With either resistance or reactance a serious practical disadvantage of this method of sheath voltage reduction is the necessity for separate individual adjustment of every device to values appropriate for the various sheath lengths if uniform conditions are desired along the line. In all other respects the connections, advan-

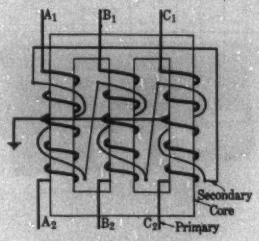

FIG. 4—WIRING DIAGRAM OF THREE-PHASE H & M SHEATH-BONDING TRANSFORMER

Series primary-wound secondary type for use in connection *E* of Fig. 2

tages, and disadvantages are the same as already discussed for reactance coils and resistors.

H. Single-Phase Transformers. Both foreign[3, 4] and American engineers have suggested single-phase transformers for sheath bonding (Fig. 2F). The iron core is placed either inside or outside the joint sleeve and around the copper conductor, which acts as a one-turn primary. The secondary winding is connected across the insulating sleeve so that its induced voltage exactly opposes the induced sheath voltage and prevents the flow of sheath currents. Individual adjustment of every coil is required. The method has serious mechanical disadvantages and introduces also practically all the disadvantages described for single-phase iron cored reactors.

I. H & M Sheath Bonding Transformers. In order to retain the advantages of 50 per cent reduction in the voltages between cable sheaths and to ground by using connection 2-*E*, or a 40 per cent reduction by using the newly devised connection 2-*G*, and at the same time not to incur the disadvantages listed for reactance or resistance bonding, the H & M three-phase sheath bonding

transformer has been devised by the authors. This transformer consists essentially of three primary coils wound on the legs of a three-phase iron core. The secondary winding may consist of an individual coil on each of the three core legs, the coils being delta-connected as shown in Fig. 4. This type of transformer is called the wound secondary type. A cheaper method of obtaining the secondary is to place a bar of copper around each end of the entire core, as shown in Fig. 5. This design is called the bar-secondary type. It is less effective electrically during cable failures.

The normal three-phase reactance of the primary coils is high, only a small exciting current flows, and losses are negligible. (The losses in the transformers themselves are only a fraction of one per cent of the losses which would obtain with solid bonding. See Table II.) The secondary coils function normally only as a tertiary winding preventing the flow of triple harmonic currents in the sheaths. When there is a line failure the single-phase failure current divides in

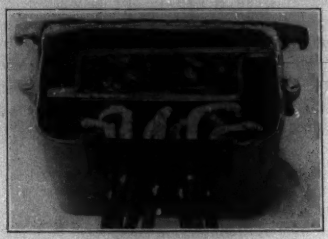

FIG. 5—H & M THREE-PHASE SHEATH-BONDING TRANSFORMER

Bar secondary type—(cover removed)
Case is normally filled with a hard compound

the first transformer or two adjacent to the fault. The current returns nearly equally divided on the three cable sheaths and therefore in parallel through the three primary coils of almost all of the transformers along the line. Because of the secondary winding the device then behaves exactly as a short-circuited transformer, and the series reactance of the transformer is due to leakage flux only (not saturation flux) with the result that the reactance drop is minimum in value. Consequently, in the case of cable failure current returning along the sheaths, the transformers cause only a very small increase in the total impedance drop, as indicated in Figs. 2 and 6.

Stray direct currents flow according to the resistance of the sheath circuit, and divide about equally in the three coils. The d-c. flux is in parallel in the three legs of the iron core and must return through an air path, and thus these stray direct currents flowing through the transformers will have no undesirable effect on the a-c. characteristics. Also, the transformer coils present very small resistance to the flow of direct current, and introduce no complications into d-c. electrolysis mitigation. The three-phase transformer allows large economies in size, weight, and convenience of installation.

To summarize, the three-phase transformers have the following characteristics: high three-phase impedance, minimum single-phase impedance, very low losses, practically no current harmonics, no interference with stray d-c. flow or electrolysis mitigation, and economy of installation.

The primary coils of the three-phase transformer may be connected in series with the sheaths and the mid-taps grounded as shown in the series reactance or resistance methods (Fig. 2E) or in the single-phase transformer method (Fig. 2F). With the three-phase transformer, however, the 50 per cent reduction in sheath potential is obtained without incurring the disadvantages inherent in the other devices.

In the scheme of connections devised by the authors and shown in Fig. 2G, the cable sheaths are cross-bonded continuously throughout the entire line, and the center of the vector triangle of sheath voltages is fixed at ground potential by star connected transformers spaced at approximately equal intervals along the line. The transformers may be installed in every manhole, in every second manhole, or in every fourth manhole. They should not, however, be connected at intervals which are a multiple of three cable lengths, because with such a connection the vector triangles have the ground point shifted over to the corner, instead of being fixed at the center. If the transformers are connected at too great an interval, the voltages during normal operation will not be sufficiently controlled as indicated in Fig. 1B; furthermore, impedance drops would be excessive during failures because the returning current must travel on one sheath circuit until it can divide at a transformer. Under practical conditions every second manhole appears to be the most desirable interval.

This connection permits a considerable saving in investment cost for the following three reasons: only half as many coils are required as for other connections; the coils are wound for about 60 per cent as much voltage as for the series connections; and only three coil leads are required for the sheath bonds.

Under practical conditions the maximum sheath voltages to ground are reduced to about 60 or 65 per cent of the induced voltages instead of to 50 per cent as with the series connections (see Table II and Fig. 2E and G).

J. General Comparison of Voltages and Losses. In order to make a general comparison of the various sheath bonding connections, Fig. 2 and Table II have been prepared. The profiles apply only to the normal voltage on the cable sheath. The maximum voltages occur in all cases in the manholes where the insulating sleeves are installed.

Table II and Fig. 2 give also a comparison of the abnormal voltage rises along the sheaths of a three-

phase 66-kv. line. It is assumed that a typical fault current (3000 amperes) is confined entirely to the three sheaths (in parallel) of the given line for all cases except for continuous cross-bonding where the fault current is confined to the one sheath circuit, and for auxiliary bonding where current returns on sheaths of three adjacent cables. In the practise of the Commonwealth Edison Company, the maximum distance to which the fault current will be confined entirely to a given line is one-half mile and occurs in conduit built in 1928. The company's practise for single-conductor cables as well as other cables is to bond them to other cable sheaths wherever possible, thereby reducing the impedance of the return path for the fault current. For the same reason the sheath bonding transformer neutral points or mid-taps are bonded solidly to other cables in the manhole wherever possible. On the 12-kv. leads to transformers the maximum length has been 2000 ft., the usual length, however, being only 500 or 600 ft.

In most of the sheath bonding methods under practical conditions a small amount of sheath current and losses are present. Some losses are present also in sheath bonding devices. The sum of these losses for the various connections under average field conditions is given in Table II and Fig. 2.

K. Field Tests and Experiments on Connections. Voltage tests have been made on all 66-kv. and 12-kv. single conductor cables bonded for elimination of sheath losses and on several short sections of line specially bonded for tests. In all cases the measured sheath voltages check those calculated by the ordinary logarithmic formulas (given in the Appendix) within about 10 per cent. The degree of accuracy of the voltage formulas is apparently greater than is required due to the uncertainties and complications of actual installations.

Tests have been made on connections *A*, *B*, *D*, *E*, and *G* of Fig. 2 and on some other less promising connections not shown. The results of all these tests verified the accuracy of the theories and formulas.

No measurable difference in sheath voltages was obtained with any of the connections with the cable dry or submerged in water. This does not indicate that leakage currents did not flow between the sheaths, but merely that the total amount is small compared to solid bonding currents and that because of symmetry of connections very little leakage current flows through the bonding devices.

No resistance devices were tested, but there is apparently no good reason to expect any departures from theory. Tests were made using single-phase Atkinson saturated core reactors and H & M three-phase sheath bonding transformers, both devices being rated at 22 volts per phase. In one case the latter device has withstood return failure current from a line service failure about one-half mile distant with no apparent effects.

Fig. 6 shows the much lower voltage drop for single-phase current flowing equally in all three coils of a three-phase transformer as compared to three single-phase saturated core reactors or resistors in parallel. The higher current values are similar to those which would occur in the field for return failure currents.

The vectorial relations shown in Fig. 1A or the summation voltage Equation 4 of the appendix for flat spacing was checked by disconnecting cable sheaths at one end of a group of cross-bonded 66-kv. cables installed in three nearly equal lengths of conduit. With one set of bonds open at the end of a group of three lengths, the voltages between cables were nearly zero, while the voltage between each sheath end and ground was about 35 per cent of the average voltage induced per sheath length. When the bonds were closed, sheath currents measured about 10 per cent of the solid-bonded values in spite of practical equality of the cross-bonded lengths.

A confirmation was made of the interesting fact that

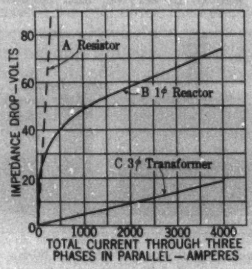

FIG. 6—IMPEDANCE DROP DURING SINGLE PHASE LINE FAILURE FOR SHEATH BONDING DEVICES

A = Resistor or open core reactor
B = Three 1-phase saturated core reactors in parallel
C = 3-phase H & M sheath bonding transformer (wound secondary type)
(Normal rated phase voltage, 22 volts for all devices)

for solidly bonded cables in either flat or rectangular spacing the voltage summation is not zero. The residual voltage which is cumulative along the entire line causes current to circulate between the three sheaths in parallel and the earth or surroundings, entering the cable sheaths at one end of the line and leaving at the other. This circulating current was measured for a 66-kv. line and totaled about 1/30th of the conductor current. The actual numerical value is of little significance since the impedance of the earth return is not known.

VI. INSULATING JOINTS

In any method of eliminating sheath losses by special bonding, insulating joints are required for interrupting the electrical continuity of the sheath circuit. An insulating joint should be mechanically rugged, impervious to moisture, fluid tight under all operating conditions of internal pressure and vacuum, and should have an insulating ring or surface of sufficient length to

prevent excessive current leakage across the insulator. Also, it must operate satisfactorily when totally submerged in manhole water for years at a time.

A number of insulating sleeves has been devised. The leading types are as follows:

(1) An assembly of parts mechanically held together;

(2) A lead sleeve soldered directly onto a metallic coating on the ends of a porcelain insulator tube;

(3) A material such as Bakelite molded on brass.

The latter method, as incorporated in the H & T insulating sleeve, has been used for almost all the work of the Commonwealth Edison Company. In their practise the brass end rings of the sleeve are soldered to lead tubing which forms a part of the containing sleeve for the joint or slip-over sleeve for installation on the cable sheath. A picture of a longitudinal cross-section of the device as part of one-half of a 66-kv. joint sleeve is shown in Fig. 7. No leaks have occurred in these insulating sleeves in service, and several which had

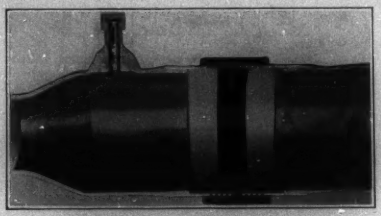

Fig. 7—Cross-Section of One-Half of 66-Kv. Joint Sleeve with H & T Insulating Sleeve

operated for over 2 years under water with voltages of 15 to 35 volts across the Bakelite were inspected and all found to be in good condition.

When a common oil reservoir is used for the three joints, it is necessary to put insulators in each of the three oil pipe lines. Insulators suitable for this service are available. In one design of the H & T sleeve, the nipple for the oil pipe is molded directly into the Bakelite ring. This avoids the necessity for an additional insulator in the oil pipe line and at the same time it may be used as one of the filling nipples for the joint sleeve.

VII. A-C. Electrolysis

Any method of eliminating sheath losses results in the presence of a-c. potentials between sheaths and to ground. The principal problem introduced by these voltages is the production or increase of sheath corrosion.

For lines in regular conduits, the Commonwealth Edison Company has operated single-conductor cables from a few months to two years with a-c. sheath potentials at full load of 5 to 12 volts to ground and 8 to 20 volts between sheaths. For lengths to potheads the voltages have been about the same, excepting for one temporary installation, where they were 100 per cent larger. In addition, a large number of laboratory tests has been made and is still being conducted for the purpose of determining the chief factors in corrosion produced by a-c. voltages and the practical maximum value that these sheath voltages should be allowed to attain. These tests and field observations have demonstrated that under field conditions corrosion due to a-c. voltages may occur. The principal factors appear to be as follows:

(a) Current density, which is dependent on the voltages between cable sheaths and to ground and on the resistivity of the surrounding manhole water, conduit, and fireproofing.

(b) The chemical nature of the water, ducts, and fireproofing.

(c) Superimposed d-c. potentials.

(d) Temperature.

The field studies have been made on cables installed in the company's ordinary conduits which are made by pouring concrete around precast concrete ducts to form a monolithic mass. The usual distance between duct centers is 6 or 6½ in.

As the company's standard fire-proofing has been rope and cement, the first tests were made with that covering. In the laboratory and field it was found that severe pitting was produced under the rope leaving distinct chainlike rope marks. Pitting occurred also under bubbles of air entrapped next to the sheath with cement directly applied with cheese-cloth binding. Since it was apparent that such pockets would be very difficult to eliminate entirely,—one air bubble or crack might cause concentration and a pit,—the opposite course was pursued by providing a uniform cushion of unimpregnated asbestos tape wrapped next to the sheath, over which was applied standard rope and cement fireproofing. This distributed the area of attack, laboratory tests showing that the severity of the pitting for given a-c. potentials was reduced at least 50 per cent, as compared to what was obtained with only rope and cement. It was discovered in the laboratory that certain asbestos preparations, cements and dips, had a very pronounced action in promoting corrosion, chiefly, it appeared, as a result of forming a good electrolyte and increasing the current density for a given voltage.

Tests were made in the field to determine the relative resistivities of the several elements for the purpose of obtaining an approximate idea of the important factors affecting the current densities. The range of resistances found was as follows:

RESISTANCE PER FT. OF CABLE LENGTH*—OHMS

	Max.	Min.	Ave.
Bare cable sheath in manhole water	35	1.5	10
Wet fireproofing on cable	100	10	50
Cable sheath in submerged ducts	1500	250	400

*690 sq. cm. of surface per foot of cable (cable overall diameter—2.8-in.)

Laboratory tests have been made on ⅛-in. lead wires in test tubes with the wires submerged in waters taken from nine manholes and with the voltages such that the average current densities were maintained at 8.6, 12, and 20 milliamperes per sq. cm. The test tubes were kept at temperatures of 18 to 25 deg. cent., which are typical temperatures of manhole waters in the summer. The tests, which were run with 150 cells for periods of 300 hours, showed that corrosion occurs with a large variation in severity. It seems that the only general conclusion that can be safely drawn in these tests is that waters which are practically neutral chemically and of high specific resistivity will cause the smallest amount of corrosion with a-c. voltages.

Increased corrosion at higher temperature was determined by running duplicate test cells at room temperature and at 45 deg. cent. with current densities of 10 milliamperes per sq. cm. It was found that corrosion progressed two to four times as rapidly at the higher temperature. In all tests in small cells it was found that the active chemicals in the water were used up more or less rapidly and that with a larger volume of water, or with renewed water, the corrosion was greatly increased. In the field, of course, available water supply is often very large.

In some laboratory tests, conditions at the duct mouth were simulated. The cables were protruding from the dummy conduit with 6 in. between centers, the protruding portions being fireproofed with rope and cement. It was found that excessive corrosion could take place in the protruding portions with a potential of 10 volts between the sheath and ground, or 20 volts between cables. It was found also in these tests, and confirmed by a special field installation, that the rate of corrosion increased with the temperature. Other tests showed that the resistivity of manhole waters and cement ducts or fire-proofings decreased about 15 per cent, if the temperature is increased from 15 to 25 deg. cent. and 50 per cent for a change from 10 to 40 deg. cent. The increase of corrosion with temperature appears to be due to both the stimulation of chemical activity and to an increase in current density.

Cables that operated a year with voltages of 5 to 12 volts between sheath and ground and in conduits which were normally dry have been found upon removal to have only a very thin layer of corrosive products, probably only one or two mils thick, which could be easily rubbed off with a rag. Other cables which had operated a year and a half under water with maximum potentials of about 12 and 24 volts to ground showed pitting. The maximum depths of the pits were found in the manholes under rope and cement fireproofing and were approximately 4 and 15 mils deep, respectively. The cable that was in the ducts showed only surface discoloration of 1 or 2 mils thickness except in one case where the cable had operated with a sheath potential of 18 volts to ground and corroded to depth of over 20 mils at two isolated spots. It appears that the cable at these spots was intermittently wet.

Hayden[6] found that a-c. electrolysis could be greatly reduced, even entirely removed or reversed, by superimposing a d-c. voltage about 1.5 per cent of the a-c. voltage with the sheath negative. From tests made in the company's laboratory on cable samples and on lead wire in the test tubes it was found that the converse may be equally true and that a positive d-c. potential of 0.5 volt combined with a-c. potentials of 10 or 20 volts may be destructive out of all proportion to the sum of the effects to be expected from either potential alone. Apparently, when single-conductor cables are operating with induced a-c. potentials on the sheaths, special precautions will be necessary to keep the sheaths at zero or negative d-c. potentials relative to the surroundings.

As a result of their studies, Hayden and the Bureau of Standards[7] and other investigators have concluded that only under very unusual circumstances will a-c. corrosion exceed 1 per cent of the theoretical effect of an equal direct current. Most of the results are based on weighing the samples to detect the total amount of removed metal. But with a-c. electrolysis just as with d-c. electrolysis, the chief damage has been found to be pitting of the sheath surface. These pits may penetrate entirely through the lead sheath long before the loss of weight uniformly distributed would become serious. Pitting may be caused by impurities in, or adhering to, the sheath, by differential aeration at the bottom of the pit due to corrosion products left behind, or by breaks in the coating of corrosion products which protect and reduce corrosion over most of the surface.

It appears that so-called self-corrosion, which may be caused by chemicals from the surrounding soil or conduit structure, by differential galvanic action, or by non-uniformity or impurities in the lead, may be increased by a-c. potentials. This applies also to the increased action at the air-water line.

From two years of laboratory and field experience it appears that for cables submerged in water, an a-c. potential of 12 volts between the cable sheath and ground is a practical safe limit for Chicago. It is realized, however, that conditions vary in different cities or parts of one city and that conclusions based on experience gained over such a short interval of time may have to be altered as a result of future data.

Similar corrosion problems may be encountered when lead-covered cables are buried in the earth as has been done occasionally in Europe and the United States. The effects of soil corrosion itself have been studied and a good summary appears in Technologic Paper No. 368, Bureau of Standards, 1928. If the sheath is covered by a fibrous covering, then there is still the possibility of sheath corrosion on account of the covering not remaining impervious to moisture. The various methods for reducing sheath losses and possible corrosion described in this article for cables in ducts are equally applicable to buried cables.

VIII. Conclusions

1. For cables separated as in ducts the elimination of sheath losses results in increased load ratings of 15 to 80 per cent, and decreased total cable losses of 20 to 75 per cent.

2. Experiments have verified the calculated sheath voltages for the various bonding connections.

3. If sheath voltages on regular sections of cable must be reduced because of corrosion due to a-c. electrolysis, it appears very desirable to use the bonding connections shown in Fig. 2E or 2G for which the three-phase sheath bonding transformer seems most suitable.

4. Satisfactory insulating joints are essential for the practical elimination of sheath losses and successful designs are now available.

5. A-c. electrolysis is a complicated phenomenon and few definite conclusions can be drawn. Field tests and experience in Chicago indicate that 12 volts to ground is a practical safe limit.

Acknowledgment

The authors wish to express their appreciation of the very helpful assistance given by Messrs. D. W. Roper; Karl Horine, and their assistants in obtaining the field and laboratory data.

Appendix

Sheath Voltages, Currents, and Losses

by K. W. Miller

1. Symbols and Assumptions. The formulas for sheath voltages, currents, and losses given in Table III and the numerical values plotted in Figures 8, 9, 10, and 11 are based on the following assumptions:

1. No proximity effect.

2. Mathematically perfect cables and spacing, and equal sheath resistances.

3. Conductor currents accurately 3-phase and equal.

4. Open-circuited sheaths not connected at more than one point and solidly bonded sheaths not grounded at more than one point.

5. No disturbing magnetic or conducting bodies within the field of influence.

B-phase conductor current is taken as the reference vector in all cases. Phase rotation is *A*, *B*, *C*. The following abbreviations are used.

E_1 and I_1 = sheath voltage and sheath current in cable A

E_2 and I_2 = sheath voltage and sheath current in cable B

TABLE III

FORMULAS FOR SHEATH VOLTAGES, CURRENTS, AND LOSSES FOR SINGLE-CONDUCTOR CABLES OPERATED 3-PHASE*

EQUATION NUMBER	CABLE ARRANGEMENT NUMBER AND DIAGRAM	I	II EQUILATERAL	III RECTANGULAR	IV FLAT	V 2-CIRCUIT	VI 2-CIRCUIT
	SHEATHS OPEN CIRCUITED: $I_1 = I_2 = I_3 = 0$						
1	$E_1/j\,I_B =$	$-x$	$\frac{x}{2}(-1+j\sqrt{3})$	$\frac{1}{2}[(-x+\frac{a}{2})+j\sqrt{3}y]$	$\frac{1}{2}[(-x+a)+j\sqrt{3}y]$	$\frac{1}{2}[(-x+\frac{b}{2})+j\sqrt{3}y]$	$\frac{1}{2}[(-x+\frac{b}{2})+j\sqrt{3}y]$
2	$E_2/j\,I_B =$	$+x$	x	x	x	$(x+\frac{a}{2})$	$(x+\frac{a}{2})$
3	$E_3/j\,I_B =$		$\frac{x}{2}(-1-j\sqrt{3})$	$\frac{1}{2}[(-x+\frac{a}{2})-j\sqrt{3}y]$	$\frac{1}{2}[(-x+a)-j\sqrt{3}y]$	$\frac{1}{2}[(-x+\frac{b}{2})-j\sqrt{3}y]$	$\frac{1}{2}[(-x+\frac{b}{2})-j\sqrt{3}y]$
4	$\frac{(E_1+E_2+E_3)}{3j\,I_B} =$	0	0	$a/6$	$a/3$	$\frac{1}{6}(a+b)$	$\frac{1}{6}(a+b)$
	SHEATHS SOLIDLY BONDED: $I_1+I_2+I_3=0$						
5	$E_r/I_B =$	0	0	$+\frac{j\,Ra}{6(R+jZ)}$	$+\frac{j\,Ra}{3(R+jZ)}$	$+\frac{jR(a+b)}{6(R+jZ)}$	$+\frac{jR(a+b)}{6(R+jZ)}$
6	$\frac{I_1}{I_B} =$	$+\frac{jx}{R+jx}$	$-\frac{jx(-1+j\sqrt{3})}{2(R+jx)}$	$\frac{-(1-\sqrt{3}N)+j(M+\sqrt{3}}{2(M+j)(N+j)}$			
7	$\frac{I_2}{I_B} =$	$-\frac{jx}{R+jx}$	$-\frac{jx}{(R+jx)}$	$\frac{-j}{(N+j)}$			
8	$\frac{I_3}{I_B} =$		$-\frac{jx(-1-j\sqrt{3})}{2(R+jx)}$	$\frac{-(1+\sqrt{3}N)+j(M-\sqrt{3})}{2(M+j)(N+j)}$			
9	$\frac{W_s}{I_B^2R} =$	$\frac{2x^2}{R^2+x^2}$	$\frac{3x^2}{R^2+x^2}$	$\frac{3(M^2+N^2+2)}{2(M^2+1)(N^2+1)}$			
10	WHERE:		$y = x$ $Z = x$	$y = (x+a/2)$ $Z = (x-a/6)$	$y = (x+a)$ $Z = (x-a/3)$	$y = (x+a+b/2)$ $Z = (x+a/3-b/6)$	$y = (x+a-b/2)$ $Z = (x+a/3-b/6)$

*Single-phase operation, Case No. 1, is given for comparison.

E_3 and I_3 = sheath voltage and sheath current in cable C

$\Sigma E/3$ and E_r = residual voltages per unit length of circuit for connections (*e. g.*, see Fig. 1A)

W_s = Total sheath losses in cables of one circuit

R = Resistance of sheath per unit length of cable

s = Spacing between adjacent cable centers

r = Mean radius of cable sheaths

$jx = 2j\,\omega \log_e s/r$

$ja = 2j\,\omega \log_e 2$

$jb = 2j\,\omega \log_e 5$

$M = R/y$ and $N = R/z$

y and z as specified in Equations 10.

If the foot is chosen as the unit of length then at 60 cycles $a = 1.594$, $b = 3.705$, and $x = 5.30 \log_{10} s/r$, all times 10^{-6} ohms per foot to neutral. The values of a, b, x, y, and z, and the voltages given in Fig. 8 for other frequencies and lengths may be found by direct proportion. The value of x may be found from Fig. 8 by multiplying the values of curve (II A, B, C) by 10^{-4}.

2. Discussion. The expression for the voltage drop per unit length of a conductor of resistance R in a magnetic field, due to alternating currents flowing in itself and in parallel conductors is of the general form

$$E = I\,R + \sum^{n} 2\,j\,\omega\,I_n \log c/d \qquad \text{(A)}$$

where the symbols and limiting assumptions need not here be defined. If "proximity effect" (or the unequal induction for points around the circumference of closely spaced cables) is neglected, it is possible to set up n simultaneous equations similar to Equation (A) for the n cable sheaths. These equations can be expressed simply in terms of the spacings between cable centers and the mean sheath radii. They may be solved simultaneously to obtain the unknown sheath potentials and currents in terms of the conductor currents providing the assumed conditions are such that the current summation for the group is zero. (This is always true for the assumed three-phase conductor currents. In order to insure zero current summation in the sheaths the ideal case is usually considered with sheaths isolated or not grounded at more than one point.)

With similar or identical assumptions this method of solution has often been used by various authors. Sheath voltages with open circuited sheaths, even with a large number of mutually inductive cables, can easily be computed by this method. The sheath voltage formulas given in the upper portion of the table can be found in various works on this general subject.

For convenience in computing and for easy comparison, the magnitudes of these induced voltages are plotted in Fig. 8. The curves are labeled with Roman numerals corresponding to the cable arrangements shown in Table III and with letters to identify the phases to which the curve applies. Thus, for example, III-A, C denotes voltages for A and C phases of rectangular spacing. Some curves are identical for different arrangements of cables.

The error in induced voltages due to neglect of the proximity effect is estimated to be less than 10 per cent for cables in contact and the error rapidly decreases with wider spacings.

The algebra involved in solving the n simultaneous equations for sheath currents with solidly bonded sheaths and any cable arrangement, other than equi-

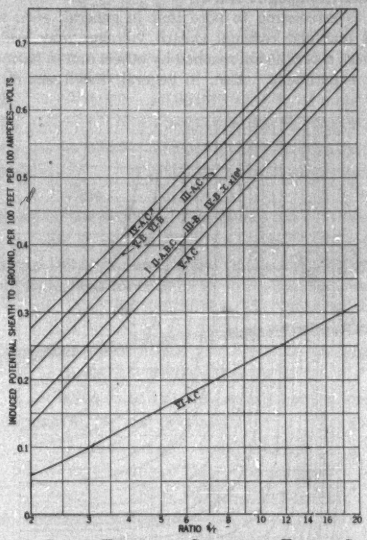

FIG. 8—SHEATH VOLTAGE TO GROUND FOR VARIOUS SPACINGS

lateral, is quite laborious. It is gratifying to find that for most ordinary cable arrangements it is possible to derive general type formulas (given in the lower part of Table III) for sheath currents and losses covering all the three-phase cable arrangements shown in the table and, in fact, several other arrangements of one or two circuits having geometrical phase symmetry about a line or a point.

The current formulas tabulated were derived separately and also from more general arrangements which, for the particular cases, degenerated into the expressions given. In particular, the general expressions for two parallel circuits became identical to that for one circuit when the two individual circuits were greatly separated. Also those for one circuit with equilateral spacing reduced to expressions identical to

the well known relations derived in other ways by several investigators and given for arrangement II in the third column of the table. The limiting conditions for zero and infinite sheath resistance are properly satisfied.

Therefore, subject to the approximations discussed, it becomes possible to express sheath voltages, currents, or losses for a large variety of typical cable arrangements in the form of master equations or curves and to solve for the numerical values after finding a few easily obtained ratios. Obviously the equations are applicable to similar arrangements with phases rearranged in cyclical order or to reversed phase rotation by interchanging A and C.

It is interesting to note that in general, even with solid bonding, residual sheath voltages may still be present, a conclusion reached by others and in harmony with experiment. Also, in general, the losses in the

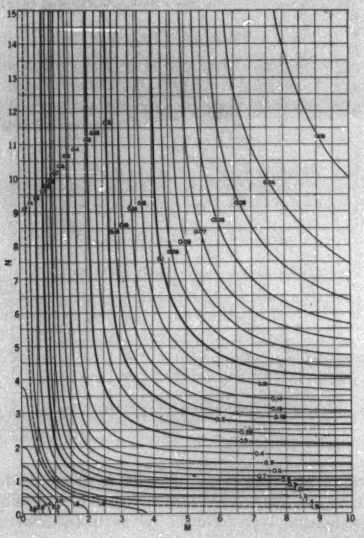

FIG. 9—RATIO $\dfrac{W_s}{I_B^2 R}$

outer cables of flat or rectangular spacings are unequal. Since resistance R cannot be factored out of the expressions for sheath losses there is no value of "effective spacing," as far as losses are concerned, which is independent of sheath resistance.

For convenience in computation Equation (9) has been plotted in Fig. 9. It is easily shown that $W_s/I_B^2 R$ is equal to the magnitude of $(I_1^2 + I_2^2 + I_3^2)/I_B^2$. This enables us to find the total sheath losses for one circuit by computing the losses $I_B^2 R$ in one sheath if full conductor current flows in it and then merely applying the ratio to this $I_B^2 R$ value. Similarly the losses in sheaths A and C can be found by applying (to $I_B^2 R$) the ratios of I_1^2/I_B^2 and I_3^2/I_B^2 plotted in Figs. 10 and 11. To obtain the current squared ratios it is only necessary to form the simple ratios M and N for the particular cable arrangement as expressed in Equations (10) of the table. Entering Figs. 9, 10, and 11 with M and N as abscissas and ordinates, the current squared ratios may be read off

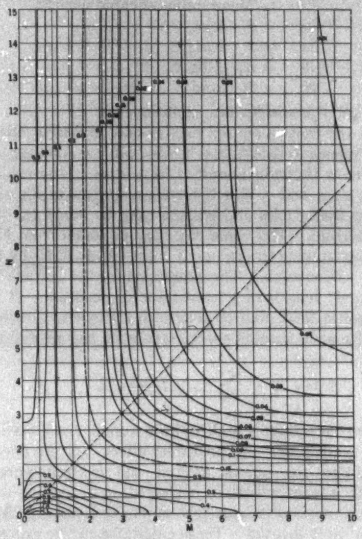

FIG. 10—RATIO $\left|\dfrac{I_1}{I_B}\right|^2$ FOR A-PHASE

at once by interpolating between the curves. To find the ratio of currents I_2^2/I_B^2 for the middle cable, phase B, either Fig. 10 or 11 may be entered with the value of N *only* and the ratio read at the point of intersection of the ordinate N with the dotted 45 deg. line. A check may be obtained by comparing the sum of ratios for the three phases with the total from Fig. 9.

It will be found that for cables in contact the losses inclusive of proximity effect will ordinarily be less than 25 per cent in excess of the values computed by the approximate formulas or curves while for cables in separate ducts the error is much less. Also in practical field installations the sheaths are grounded at more than one point so that the residual sheath voltage causes a current to flow over the line and return through earth or surrounding cables. In general it is impossible to predict the effective impedance of this earth return

but the sheath losses will be increased a few per cent by the additional current required to consume this "residual" voltage.

The values of the "effective" conductor resistance and reactance with solidly bonded cables are equal to the real and j components respectively of the following equations for the effective conductor impedances Z:

$$Z_A = (R_c + j X_c) - (I_1 R - E_r)/I_A \quad (11)$$

$$Z_B = (R_c + j X_c) - (I_2 R - E_r)/I_B \quad (12)$$

$$Z_C = (R_c + j X_c) - (I_3 R - E_r)/I_C \quad (13)$$

where R_c and X_c are respectively the conductor resistance and the conductor reactance from the cable center to the mean sheath radius. The other vectorial sheath voltages and currents and sheath resistance R are already defined. A study of these equations indicates the existence of unequal transformer action between phases (except for the equilateral case where the equations reduce to the familiar simple form). Complete discussion of these equations lies outside the subject matter of the present article.

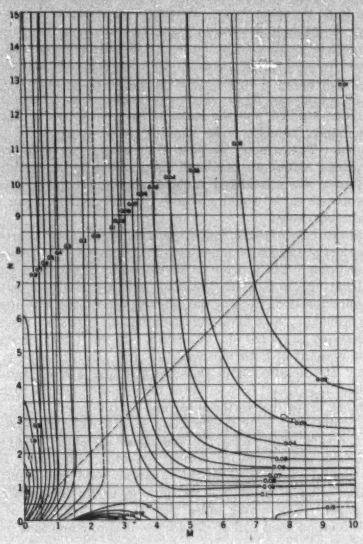

FIG. 11—RATIO $\left|\frac{I_3}{I_B}\right|^2$ FOR C-PHASE

3. Examples. As an illustration of the use of the equations and curves an example will be given for 66-kv. 3-phase, 60-cycle, 750,000 cir. mils single conductor cable, one circuit, flat spacing, operating at 60,000 kv-a. or 525 amperes per phase. The necessary dimensions are as follows: Spacing—6 in., overall diameter—2.8 in., sheath thickness 9/64 in., from which the following constants are easily obtained:

Mean sheath radius $r = 1.33$ in.

$x = 5.30 \times 10^{-5} \log_{10} 6/1.33$

or from Fig. 8 curve (II A, B, C) $= 3.46 \times 10^{-5}$ Ω/ft.

Constant $a = 1.59 \times 10^{-5}$ Ω/ft.

Sheath resistance $R = 9.3 \times 10^{-5}$ Ω/ft.

Therefore from Equation IV-10, Table III:

$$y = (x + a) = 5.05 \times 10^{-5}$$

$$z = (x - a/3) = 2.93 \times 10^{-5}$$

$$M = R/y = 1.84$$

$$N = R/z = 3.17$$

$I_B^2 R = (525)^2 \times (9.3 \times 10^{-5}) = 25.65$ watts/ft. cable

Making these numerical substitutions into the formulas (1, (2), (3), and (4):

$E_1 = (-0.023 - j\,0.00491)$ volts/ft. of cable

$E_2 = (\quad 0 + j\,0.01816)$ volts/ft. of cable

$E_3 = (+0.023 - j\,0.00491)$ volts/ft. of cable

$\Sigma E/3 = (\quad + j\,0.00278)$ volts/ft. of cross bonded line.

from which the numerical magnitudes are

$E_1 = 2.35$, $E_2 = 1.82$, $E_3 = 2.35$ and $\Sigma E/3 = 0.28$ volts per 100 ft. of line. These values of E_1, E_2, and E_3 could have been found directly from Fig. 8 curves IV-A, C and IV-B by multiplying the values of E (at $s/r = 6/1.33 = 4.5$) by the ratio of currents 525/100.

Numerical substitution into Equation IV-5 gives:

$E_r = (0.00076 + j\,0.00253)$ volts/ft.

$= 0.264$ volts/100 ft. of solidly bonded line

Equations (6), (7), and (8) yield:

$$I_1 = 215 - j\,28 = 217 \text{ amperes}$$

$$I_2 = -48 - j\,151 = 158 \text{ amperes}$$

$$I_3 = -167 + j\,179 = 245 \text{ amperes}$$

from which sheath losses are:

$$W_1 = 4.37 \text{ watts/ft.}$$

$$W_2 = 2.32 \text{ watts/ft.}$$

$$W_3 = 5.58 \text{ watts/ft.}$$

which check with

$W_s = 12.26$ = total sheath losses by Equation (9).

If, as is usually the case, only the sheath losses are required they may be found directly by entering Figs. 10, 11, and 9 with $M = 1.84$ and $N = 3.17$. The sheath to conductor current ratios squared for cables A, B, C, and total are found to be 0.17, 0.091, 0.22, and 0.48 respectively. Multiplying $I_B^2 R = 25.65$ watts by these ratios we find practically the same sheath losses $A = 4.35$, $B = 2.30$, $C = 5.65$, and total $= 12.30$ watts per foot as before.

It is of interest to compare sheath voltages and losses for this same cable with various cable arrangements, conditions being otherwise identical. The results are

easily obtained from the curves and are as follows:

Cable spacing	Induced sheath voltage per 100 ft. line		Sheath losses watts per ft.			
	A & C phases	B phase	A phase	B phase	C phase	Total per circuit
II Equil.	1.82	1.82	3.08	3.08	3.08	9.24
III Rect.	2.06	1.82	3.59	2.69	4.36	10.64
IV Flat.	2.35	1.82	4.35	2.30	5.65	12.30
V	1.69	2.30	6.28	2.82	8.86	17.95
VI	0.76	2.30	2.80	2.82	2.80	8.47

The average error in using the graphical solution is estimated to be within ± 5 per cent. Uncertainties of cable spacing, sheath thicknesses, and temperatures, etc., make further accuracy unnecessary.

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Discussion

H. R. Searing: The United Electric Light & Power Company has used the cross-bonding method (*D* in the paper) for several years with entire success, and can subscribe to the limit of 20 volts between sheaths set forth by the authors. We adopted this value as a limit in order to be certain that sheaths would not attain dangerous potentials during short-circuit conditions, but careful inspection has not disclosed any deterioration of sheaths due to a-c. electrolysis.

We feel that care should be taken in drawing conclusions regarding a-c. electrolysis in that the sheath deterioration observed, especially under fireproofing, in manholes, may be caused at least in part by other agencies than induced a-c. currents.

We wish to correct a statement in Table I, item 4, regarding cable arrangement. These cables are arranged rectangularly as shown by the authors' cable arrangement III.

For several years we have felt that more satisfactory operation could be obtained by the use of single-conductor cables. Recent cable development has indicated that triplex cable of the oil-filled type may be supplied for operation at 66 kv. and below. In view of this development, it is our feeling that the use of single-conductor cable would, in general, be limited to circuits operating above this voltage.

Triplex cables are to be preferred from a regulation standpoint. The regulation of circuits using single-conductor cables, especially where they are installed in separate ducts, is much poorer than that with triplex cables of equal rating. This difference in circuit regulation tends to complicate operation of a system having both types of cable.

E. R. Thomas: Mr. Riley, (in a paper on sheath losses in single-core cable which was printed in the November 1927 issue in the *Journal* of the Institution of Electrical Engineers, London) gave results of measurements on flat spacing of cables for various ratios of spacing to sheath radius. He found, contrary to the equations of Messrs. Halperin and Miller, that the losses in each of the two outer sheaths were the same. Several years ago I developed a procedure for calculating the sheath current and losses for the general case of unequal spacing and by applying these formulas to the cable constants used by Mr. Riley I obtained very good agreement.

I am in agreement with the paper regarding the magnitude of the voltages induced per loop unit of length between pairs of sheaths, but I am not in agreement with the computed voltage to ground for each sheath, which for the middle cable sheath for the three cases of equilateral, rectangular, and flat spacings are shown to be the same value. Accurate experimental data would be very desirable to determine whether sheath losses are unequal in the two outer sheaths in the case of flat spacing and what the values of induced voltages to ground are for various arrangements.

P. S. Creager: I have made some preliminary design calculations on a proposed 26-kv. single-conductor cable. The scheme of bonding studied was the same as the one shown in Fig. 2*c* of the paper, namely, cross bonding with every third joint grounded. The primary purpose of the study was to determine the voltage across an insulated joint under fault conditions. This factor was considered to be of paramount importance as the proposed cable would probably be installed in a run with several other cables.

As in this type of cable, faults would be single-phase to ground, the method of symmetrical components appeared to be necessary for a solution. The scheme as developed by Mackerras, Lewis, and Bekku in the *General Elec. Rev.*, with certain modifications and approximations to meet the different conditions, was applied to the problem. As a first approximation, all of the fault current was assumed to return on the sheath. It was found necessary to take account of both the resistance and the reactance of the sheath. It was also found that the resultant voltage across a joint had to be found by combining the induced voltages with the impedance drops due to the returning fault current. The final result, while admittedly approximate, gave a value of about 300 volts across a joint under the conditions specified. While no calculations were made for normal operating conditions, some rough mental calculations on the equations used gave results of the same order of magnitude as those given by the authors in their paper. The writer takes this as a confirmation of the general method used and of the admitted approximations which he made.

H. C. Lewis: I should like to add something on the question of regulation previously referred to by Mr. Searing. The regulation is of considerable importance in the case of parallel cables. Inasmuch as it is not always practical to select ducts, and to get equal duct spacing, unequal impedances may result which will cause great difficulties in paralleling cables. In some cases this will make it impractical to parallel cables due to improper division of load. Information is desired from the authors as to how they actually took care of this difficulty.

P. D. Morgan and S. Whitehead: (communicated after adjournment) In discussing the economic aspects the authors have made several important assumptions the validity of which is not proved in the paper. These are as follows:

a. That the life of the cable is the same and the liability of failure no greater due to corrosion and to the insertion of insulating sleeves with their complicated oil attachments, etc., than when solid bonding with continuous lead is employed.

b. That the labor cost of installation and upkeep is no greater than with solid bonding.

c. That the danger of electrostatic sparking and consequent ignition of gases accumulated in manholes is negligible.

If these assumptions are only partly true the net economic result may be quite different from that stated in the paper.

We have recently carried out an investigation into methods of laying single-conductor cable for 60 kv. and above in connection with the high-voltage "grid" system now under construction in Great Britain, and an account is shortly to be published in the *I. E. E. Jl.* (England).

The British Electrical and Allied Industries Research Association, as a result, decided to recommend wherever possible the laying of cables in triangular spacing with regular transposition, as close together as possible and with solid bonding and earthing at every joint. As an alternative where special conditions render it desirable, flat spacing with regular transposition at each joint and solid bonding and earthing of the sheaths at every third transposition (*i. e.*, at every complete turn of the lay) may be employed. Where two or more three-phase circuits have to be laid in close proximity, mutual inductive effects between them will be eliminated by coordination of the frequencies of transposition. For example, where two circuits are employed the frequencies of transposition will be in the ratio of 3 to 1. These considerations apply both to cables laid in ducts and direct in the ground, the latter method being the usual practise in this country.

A comparison of the above with the authors' method reveals the following features:

Advantages of authors' method:

a. Increased permissible loading under certain conditions.

b. Reduced sheath losses.

Disadvantages of authors' method:

c. A-c. electrolysis of sheath of unknown amount.

d. Increased initial and maintenance costs due to insulating sleeves, transformers, etc.

e. Increased interference with nearby communication circuits due to the conductors not being transposed. (This is chiefly of importance where the power and communication cables are in the same trench, such as when supplying private telephone lines for the power-supply authority.)

f. Possible danger due to electrostatic sparking and to personnel, caused by the voltage between sheaths and to earth.

For cables laid direct in the ground, the authors' method would be difficult to apply in practise as in order to obtain the increased permissible loading it would be necessary to separate the cables. For cables in ducts, as an alternative to the authors' method we are investigating the possibilities of obtaining increased loading by a reduction in the thermal resistivity between the cable and duct wall. Advantage (b) above is not considered of much importance economically as on a high voltage system the copper losses (of which the sheath losses are only a fraction) are generally much less than the losses in the transformers, etc.

Regarding the disadvantages mentioned, it is considered that (f) above may be a real menace as extensive explosions have sometimes occurred due to gases accumulated in cable tunnels.

On the theoretical side of the paper the authors' remarks as to the inequality of sheath losses in the outer cables in flat and rectangular spacing, and in the existence of a "residual" voltage to earth with solid bonding, are in agreement with recent theoretical and experimental work in this country. Our theoretical formulas differ slightly from those of the authors, but it is believed that the differences are not material.

The authors mention "proximity effect" losses. We incline to Cramp's nomenclature "sheath eddy losses," *i. e.*, losses occurring due to eddy currents whose vector sum is zero over the cross section of the sheath, and find it more convenient to estimate these separately although their magnitude is affected by the presence of circulating sheath currents. We agree that they may nearly always be neglected with high-voltage cables.

With regard to the impedance of the earth return for residual currents, the present writers' experience with overhead lines inclines them to the view that this is not greatly affected by the presence of other currents, the "balanced" components, and may therefore be calculated by the methods developed for overhead lines (*e. g.*, Rudenberg, Pollaczek, or Carson). Further attention is being given to this matter.

With regard to the effect of copper-wire armoring, it is more usual in this country, in the few cases where non-ferrous armoring is used, to use copper tape. The present writers have incorporated the effect of such armoring into their theory of sheath losses, but this section will not be published in the account referred to. Our conclusion is that with tape armoring, reduction of losses below those for plain lead sheaths by reducing the resistance of the tape is not practicable. As the resistance of the armoring decreases, the losses rise to a maximum and then fall. At small spacings of the cables it is impracticable to attain even the maximum loss by decreasing the resistance of the tape. At very large spacings, the maximum is shifted towards higher resistances so that the fall of loss with resistance can be realized but it is only at impracticably large spacings that a reduction below the unarmored case can be obtained with tape of practicable dimensions. With wire armoring, it is, of course, easier to obtain a low resistance armoring. Nevertheless, our opinion is that, in order to obtain a reasonable reduction of losses even with wire armoring, it would be necessary to introduce rather heavy armoring, particularly if bronze wires are used as is more usual in this country.

It is to be observed, with pleasure, that at the present time, agreement is being reached between different countries as to the method of evaluating sheath losses.

Herman Halperin and K. W. Miller: Mr. Searing has pointed out that his company has selected 20 volts between cable sheaths as a safe operating limit because this results in safe values of sheath potentials during failures. It appears that in general emphasis should be given to the a-c. electrolysis factor since the possibility of corrosion from this cause is present continuously while the cables are loaded.

Regarding the spacing of the cables shown in Table I, nine-tenths of our own single-conductor cables are installed with rectangular spacing but conditions were shown for flat spacing since that is the worst condition from the standpoint of both sheath losses and voltages.

We have not yet encountered the problem of poor regulation or poor load division between single-conductor and three-conductor lines. Our 66-kv. lines are comparatively short (1½ to 10 mi.) and, with transformers included, the line impedance is usually a small fraction of the total. Load division between lines or reversal of power flow is accomplished by tap changers on the transformers working under load. We have not yet attempted to parallel single-conductor and three-conductor cables at the same voltage. Where two or more single-conductor lines are paralleled it is important to keep the lengths nearly equal for cables installed in separate routes or to properly transpose the cables to minimize mutual effects between circuits when they are in the same conduits.

Replying to Mr. Thomas, equations and experimental data have been presented by several investigators, recently by Dr. Arnold in the January issue of the *Journal* of *I. E. E.*, and we have also our own field data all showing unquestionably the inequality of sheath currents and losses in the outer cables with flat or rectangular spacings. The existence of residual voltage induced along a solidly bonded line with flat or rectangular spacings has been theoretically and experimentally confirmed. A residual voltage is also induced along a line with sheaths cross bonded (copper conductors not transposed). We have measured this voltage in the field and found it to check with theory.

It is interesting to know that Mr. Creager has checked our work for the cross bonding connection in Fig. 2b and found that the voltage drops along the sheath during a single-phase failure is of the same magnitude as computed by us. The insulating sleeves are never called upon to withstand more and usually

less than the voltages given in the last column of Table II during a typical fault. Excepting for resistance bonding they can easily and safely be subjected to abnormal voltages of this order of magnitude even when submerged in water.

Mr. Lewis' questions on line impedance and parallel operation have already been answered.

Messrs. Morgan and Whitehead have contributed some very interesting information on English practise and raised some very practical questions. If sheath potentials are limited to reasonable values (we have found 12 volts to ground is practical for our duct conditions) then sheath corrosion due to a-c. electrolysis will be little or nothing. All cable sheaths are exposed to the hazards of chemical corrosion, d-c. electrolysis, and mechanical damage which, like possible a-c. corrosion, if present, are usually random and localized in occurrence. All factors considered, placing, of course, a reasonable limit on a-c. sheath potentials, it does not appear that cable life is appreciably reduced below the age of obsolescence or a natural insulation life by operating with small a-c. sheath potentials. Our wide field experience is confirming this.

The additional installation cost for eliminating sheath losses was included in the economic study and is only 1 or 2 per cent of the total line cost. The economic savings both in increased line carrying capacity and decreased losses are so large and real as to demand serious attention.

In this country on typical installations of single-conductor cables the sheath losses would be from 50 to 250 per cent of copper losses if the sheaths were solidly bonded. Decreasing the thermal resistance exterior to the sheath does not avoid the extra cost of the sheath losses. Also, for a given permissible maximum operating copper temperature and within practical limits of reduction of thermal resistance, the percentage increase in rating made possible by eliminating sheath losses is still very attractive.

It appears that there are three factors in English practise which tend to minimize sheath losses with solid bonding:

a. The cables are generally comparatively small and have high sheath resistance.

b. The usual frequency of 50 cycles is lower than the standard 60 cycles in the United States.

c. It is feasible to lay buried cables at close spacings or in contact.

All these factors decrease the necessity or advantages of special sheath bonding.

Oil-filled joints with their attachments are an item entirely independent of methods of sheath bonding. Maintenance costs are not affected by special sheath bonding. Occasional inspections are required by all cables for miscellaneous reasons and special bonding of single-conductor cables has not increased inspection costs. We have at present over 3000 insulating sleeves in service and there has not been one leak or defect developed in the 2½ years of service with the type of sleeve generally used by us.

Telephone cables are seldom run in the same conduits with power cables. Even if they were there is nothing inherent in any of the special sheath-bonding schemes which prevents transposing the copper conductors at any desired intervals—in fact such transpositions are standard on all of our lines.

The danger from sparks igniting gas is rather remote. The sheath voltage is far too low to cause "static" discharges. The cables are insulated from each other by fireproofing in the manholes and cannot come in contact in separate ducts.

When the copper conductors are transposed, "residual" sheath voltages for solid bonding or any sheath-bonding connection will also be transposed out. Then, unless the sheaths are very substantially grounded at intermediate points, no "residual" current will flow. Field measurements for cables in ducts have shown that the leakage resistance to ground, even with cables submerged in water, is so high compared to sheath resistances as to have little effect on sheath potentials with any of the sheath connections. When this is borne in mind, it is seen that transposing copper conductors along a long line will allow the residual sheath potentials to trace a vector triangle instead of being consumed by current flow as would occur with an untransposed line. Unless the length of line between transpositions is short, appreciable "residual" a-c. sheath voltages to ground can result even with solid bonding.

The authors are in entire agreement with the remarks on copper armoring. The method was included for completeness in listing all possible methods available for reduction of sheath losses.

Losses in Armored Single-Conductor Lead-Covered A-C. Cables

BY O. R. SCHURIG,* Member, A. I. E. E. H. P. KUEHNI,* Non-Member and F. H. BULLER* Associate, A. I. E. E.

Synopsis.—*The losses occurring in single-conductor armored cables whose sheath and armor are bonded and grounded at more than one point are—besides the copper loss and the dielectric loss—the circulating-current losses in the sheath and in the armor, and the additional iron losses in an armor of magnetic material. The circulating-current losses are due to the currents induced in the sheath and armor circuits by the fluxes linking these circuits when these circuits are closed by the bonds or grounding connections. Since the losses obtained in such cables are widely variable, depending on the cable design, both designers and operating engineers desire to know the loss magnitudes obtained in different cases and to have an understanding of the factors tending to give low losses.*

In this paper the sheath and armor losses have been analyzed for a steel-wire armored cable, a copper-wire armored cable, a steel-tape armored cable, and a cable enclosed in an iron pipe, and have been compared with the losses occurring in a plain lead-covered cable without armor. The data used include test data and calculated values.

It is shown that a steel-wire armored 350,000-circular mil single-conductor copper cable with sheath and armor short circuited by low-impedance bonds had a total loss (exclusive of dielectric losses), of 2.8 times the conductor loss, by test in a single-phase 60-cycle circuit at 4 ft. cable spacing, at 260 amperes. The corresponding loss in a similar cable without armor, with lead sheath short circuited, was 2.4 times the conductor loss. Thus the steel-wire armored cable had an additional loss due to the armor of about 20%, this relatively small loss increase being due to (a) an armor of individual wires wound with a large lay, so that the path of circumferential magnetic flux at the armor is broken up by many non-magnetic spaces, and the component of magnetizing force along the armor wires is a small part of the resultant magnetizing force; (b) low-resistance circulating-current circuits. A steel-tape armor or an iron pipe surrounding a single-conductor cable usually introduces higher losses.

A cable similar to the steel-wire armored one but having an armor of copper wires has a calculated total loss (exclusive of dielectric loss) of only 1.3 times the conductor loss, in a single-phase circuit at 4 ft. spacing, at 60 cycles, the armor current itself being 92 per cent of the conductor current.

The reduction of losses due to diminished cable spacing in a single-phase circuit is shown. When the cable spacing is reduced from 10 ft. to 1 ft. the overall losses (exclusive of dielectric losses) for a steel-wire armored 350,000-circular mil cable are reduced about 8%, while those for a similar cable with copper-wire armor drop only about 3%.

A loss-calculating procedure for determining the losses in single-conductor armored cables in a single-phase circuit, or in a balanced three-phase circuit with equilateral spacing, is presented. The procedure involves (a) the calculation of the circulating currents in sheath and in armor by formulas derived, which takes into account the effect of armor iron on reactance values, (b) the computation of the circulating-current losses as $R\,I^2$ values, (c) the determination of the extra iron losses in the armor with the aid of suitable test data, or by calculations when sufficiently reliable. Test data on effective armor resistance and on extra iron losses are given for a steel-wire armored cable. These data are applicable to loss calculations for certain steel-wire armored cables. Corresponding test data for cables with steel-tape armor or with other types of magnetic armor have not been obtained. In the absence of these test data the calculations are less reliable.

The relative total annual costs of a steel-wire armored cable and of a similar copper-armored cable, both cables having a conductor cross section of 350,000-circular mils, were compared. The lower operating cost of the copper-armored cable was found to have a lower total annual cost than that of the steel-wire armored cable at load factors above 60%, the steel-wire armor having the advantage economically at lower load factors.

* * * * *

INTRODUCTION

It is the object of this paper to present the results of test data and of calculations on losses in certain armored single-conductor lead-covered cables, and with the aid of the data to point out the effects of the chief design variables on the total losses, and on the principal loss components. The relative merits of steel-wire armor and copper-wire armor are discussed both in respect to losses and total annual costs at different load factors. A calculating procedure is given for the determination of the circulating currents in sheath and armor circuits for a pair of cables in a single-phase circuit or for three cables in a symmetrical three-phase circuit (triangular spacing) with balanced currents, the formulas derived taking into account the increase in inductance values due to a magnetic armor. Test data on effective armor resistance and on extra iron losses for a steel-wire armored cable are given such that with their aid, and with the aid of the circulating-current formulas, the losses in certain steel-wire armored cables may be computed. It is not intended in this paper to give data or formulas broadly applicable to the calculation of losses in cables with steel tape armor or with other types of magnetic armor differing widely from the steel-wire armor tested, although the circulating-current formulas are applicable to a variety of cables with magnetic armor.

It is usually considered desirable in practise to bond together and ground by means of low-impedance connections the sheaths and armors of single-conductor armored cables at more than one point so as to keep the armor and sheath at substantially ground potential. Therefore the cases considered throughout most of the paper will be those for cables whose sheath and armor are bonded with low-impedance bonds from cable to cable (or grounded) at several points. The bonding

*All of the General Electric Company, Schenectady, N. Y.

For numbered references see Bibliography.

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and grounding, of course, introduces circulating currents due to the fluxes passing between cables or between cables and ground.

Under present practise the case of *armored* cables not bonded or grounded (*i. e.*, having no circulating currents) is of more academic than practical interest. Nevertheless data and discussions are included for cables having the sheath or the armor open circuited, or both sheath and armor open circuited.

Throughout the paper, unless otherwise mentioned, cables in a single-phase circuit, equivalent (for the purpose of this paper) to cables in a balanced three-phase circuit in a symmetrical triangular arrangement at the same spacing are dealt with. Unsymmetrical multi-conductor layouts of steel-armored cables have not been covered. In such layouts the circulating currents and losses will differ for the different cables of the group. However, the average loss per cable will often not differ much from the loss calculated for each cable of a pair of going and return cables at the proper spacing. It seems that more thorough-going researches might profitably be made into the subject of losses in steel-armored cables in unsymmetrical multi-conductor layouts.

In analyzing the circulating currents in single-phase circuits in this paper the sheaths and armors of the going and return cables are considered as bonded together at two or more points by short bonds of negligibly small impedance. Any circulating currents flowing through the earth by reason of the particular system of grounding employed are not taken into account here. Since the effect of such earth currents upon the total circulating-current losses will usually be sufficiently small to be negligible (provided highly conducting short bonds with low-resistance joints are used between the cables) and since the need for low-impedance bonding connections is generally recognized, the loss data based on tests and calculations made without earth currents are considered as representative of actual cases in good practise where ground connections are used.

The component losses occurring in single-conductor a-c. cables with lead sheath and iron armor, when both sheath and armor are bonded and grounded, so as to give circulating currents in sheath and armor, are

(a) conductor loss (copper loss)
(b) lead sheath circulating-current loss
(c) armor circulating-current loss
(d) eddy-current loss in lead
(e) extra eddy-current and hysteresis losses in armor
(f) dielectric losses

This paper deals primarily with the lead sheath and armor losses, *i. e.*, with items b, c, and e, which have not been so well understood and have formed the subject of recent investigations. The other component losses, which in the main have been better understood for design purposes, will not be analyzed here and will be only briefly referred to at the end of the paper.

Circulating-Current Losses in Lead Sheath and in Armor

The circulating-current loss in the lead sheath (item b) is due to the circulating current induced in the sheaths of adjacent cables by the flux passing between the cables when the sheaths are bonded together or grounded at two or more points. In the case of a pair of going-and-return cables (without armor) whose lead sheaths are bonded at both ends, the sheath circulating current follows the path indicated in Fig. 1 by the arrows *a b c d e f g*. The flux inducing this current is indicated by ϕ_1 and ϕ_2. The sheath circulating-current loss is then equal to the sheath resistance times the square of the sheath circulating current.

The armor circulating-current loss is produced in a similar manner by the circulating current induced in the armor circuit by the flux passing between adjacent cables when the armors of the cables are bonded together (or grounded) at more than one point. In addition there will often be eddy-current and hysteresis

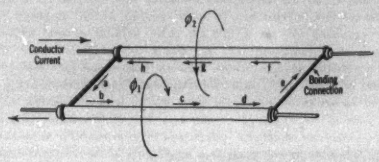

Fig. 1—Circulating Current in Sheaths of a Pair of Lead-Covered Single-Conductor Cables (not Armored) in a Going-and-Return Circuit

Circulating current is indicated by arrows *a b c d e f g h*. The current is induced by fluxes ϕ_1 and ϕ_2 linking the loop indicated

losses in the armor to be considered separately.

For a circulating-current path of very low resistance the circulating current will approach the conductor current and will then be practically opposite to it in phase; *i. e.*, the so-called phase angle will be nearly zero. Increasing the resistance of the circulating-current path will diminish the circulating current and materially increase the phase angle. When there are two circulating-current paths a change in the resistance of one of the circulating-current paths will change the currents and phase angles for both circulating currents. The above matters are, of course, not new, having been covered in earlier literature (see bibliography).

The behavior of armored single-conductor cables in regard to losses due to circulating currents in lead sheath and in armor may be analyzed by referring to the known characteristics of unarmored lead-covered single-conductor cables carrying circulating current in the lead.

Unarmored Lead-Covered Single-Conductor Cables. When the sheaths of cables are bonded at the ends, sheath circulating-current losses will be obtained

increasing with spacing between going and return conductors and varying with sheath resistance.

Let us examine more closely the effect of sheath resistance on circulating current and losses for a particular cable as indicated by curves of current and losses in the sheath in Fig. 2. The values are calculated for cable spacings of 1 ft. and of 4 ft. for a single-phase circuit, and cover a range of sheath resistances of from 0 to 0.4 ohm per 1000 ft. per cable. At the maximum sheath current, equal to the full conductor current, (*i. e.*, for the hypothetical case of zero sheath resistance) the circulating-current loss is obviously zero. While

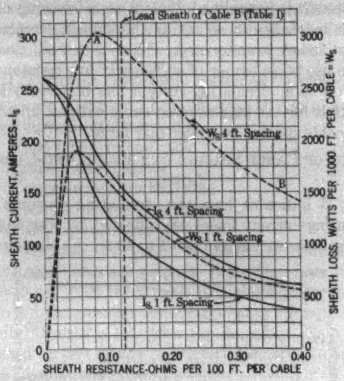

FIG. 2—SHEATH CURRENT AND SHEATH LOSS *vs.* SHEATH RESISTANCE

Single-phase circuit with 1-ft. and 4-ft. spacing. No armor
Cable B, dimensions given in Table I, except that sheath resistance is considered variable
Sheath bonded as in Fig. 1
Conductor current 260 amperes at 60 cycles per second
All values calculated

the sheath current falls with increasing sheath resistance, the sheath loss first rises to a maximum, and then falls, again approaching zero at infinite sheath resistance. Attention is called to the fact that a particular sheath design for each cable and for each spacing gives a maximum-sheath circulating-current loss. Values of sheath resistance higher or lower than this critical value (at point A for 4 ft. spacing in Fig. 2) will give lower circulating-current losses than those for the critical sheath resistance. For the types of cable usually encountered in practise, and at moderate spacings, the sheath resistance is above the critical value. Attention is also called to the fact, indicated in Fig. 2, that the critical sheath resistance for a given cable is diminished when the spacing between going and return cables is reduced.

A particular 350,000-circular mil unarmored lead-sheath cable B, as specified in Table I, when used in a single-phase going and return circuit at 4 ft. center spacing between cables and when carrying 260 amperes conductor-current at 60 cycles, will according to Fig. 2 have a lead-sheath circulating-current of 150 amperes or about 58 per cent of the conductor-current when the sheaths are bonded at more than one point. The corresponding loss due to lead-sheath circulating-current is 2880 watts per 1000 ft. or 1.4 times the conductor loss. Thus it is seen that the sheath circulating-current for this cable without armor introduces a very considerable loss equal to 140 per cent of the conductor loss, at 4 ft. spacing. Any increase in sheath resistance will reduce the loss as seen in Fig. 2. If the sheath resistance is lowered, a reduction of sheath circulating-current loss will not be obtained until the sheath resistance is made less than half in this particular case.

The simple case of cables with a single non-magnetic circulating-current path has been reviewed in so far as the characteristics of circulating-current magnitudes and losses are concerned, because these characteristics will now be used in a discussion of armored cables to indicate the effect of armor design on total circulating-current losses.

ARMORED LEAD-COVERED CABLES

Total Circulating-Current Losses. When an armor is added to an ordinary lead-covered cable, and when in a cable circuit both armor and lead sheath are bonded at 2 or more points, so that the bond at each point connects the sheath to the armor of the same cable and also connects from cable to cable, the armor is in multiple with the lead sheath. Thus the addition of the armor is at least approximately equivalent to a lowering of the sheath resistance in so far as the sum total of circulating-current losses is concerned, neglecting the difference between sheath and armor inductances. Hence, if the lead-sheath resistance itself were not higher than the critical value (as *A* in Fig. 2) the addition of the armor would actually tend to reduce the total circulating-current losses. These considerations, of course, do not include the extra eddy-current and hysteresis losses in a magnetic armor (to be discussed later).

The effect of the addition of an armor on the total of circulating-current losses will be illustrated by an example: Cable B in a single-phase 2-wire circuit will again be considered. Its sheath resistance is 0.128 ohm per 1000 ft. per cable. The sheath circulating current and sheath loss at 260-ampere conductor-current at 60 cycles and 4 ft. spacing for this unarmored cable are 150 amperes and 2880 watts per 1000 ft. per cable, respectively, from Fig. 2, as already mentioned. If an armor of 0.12 ohm effective resistance* per 1000 ft. is added to this cable and bonded to the sheath, we may consider, as a rough approximation for estimating the total circulating-current losses, that the armor and sheath are in multiple having a combined resistance of 0.062 ohm. From the upper loss curve in Fig. 2, the

*The armor in question is the steel-wire armor specified for cable A in Table I.

combined circulating-current losses in sheath and armor would be 2840 watts per 1000 ft. for a resistance of 0.062 ohm. By actual test of a 50-ft. length of the cable in question the sum of armor and sheath circulating-current losses is 3100 watts per 1000 ft. per cable in a single-phase circuit at 4 ft. spacing. (See item 4 Table II.) The difference between the two values for total loss is seen to be less than 10 per cent, the higher value by test being caused by the additional inductance due to the magnetic armor, this factor (which is appreciable) not having been allowed for in the rough calculation.

The figures indicate that the use of an armor of only moderately low resistance at the 4-ft. spacing between

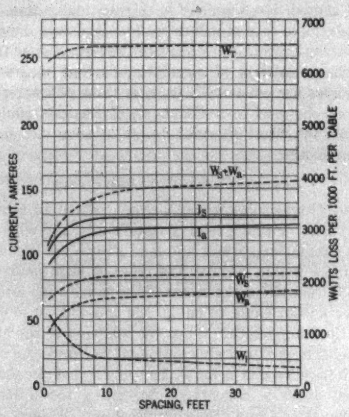

FIG. 3—CALCULATED SHEATH AND ARMOR LOSSES *vs.* SPACING

Single-phase circuit for cable A (see Table I)
Conductor current 260 amperes at 60 cycles per second
I_s = Sheath current
I_a = Armor current
W_s = Sheath circulating-current loss
W_a = Armor circulating-current loss
W_i = Eddy-current and hysteresis loss in armor iron
W_T = Total loss including copper loss and dielectric loss. Copper loss = 2080 watts. Dielectric loss = 150 watts

cables, need not increase the total of the circulating-current losses. However, at a lower cable spacing, a considerably lower armor resistance would have to be used in order that the sum of the circulating-current losses in the armored cable may be less than those in the unarmored cable.

The approximate calculation just given is not recommended for estimating losses in armored cables, because it does not take into account such factors as the difference between the inductances of armor and sheath circuits, and the effect of a magnetic armor on inductance values. Also this procedure would not give the values of the circulating currents in sheath and armor, which currents must be used in determining the extra eddy-current and hysteresis losses in magnetic armors.

To summarize: To lead-sheath single-conductor cables (without armor) whose sheaths are bonded or grounded at 2 or more points, a lowering of the sheath resistance at constant cable spacing and constant conductor current, results in a greater circulating current, but will give a lower circulating-current loss if the lead sheath resistances considered are not higher than a certain critical value (see Fig. 2). If, however, the sheath resistance exceeds this critical value, a further increase in sheath resistance will give a lower circulating loss. For the type of cables usually encountered in practise (both armored and unarmored) the sheath resistance is above the critical value.

The addition of an armor bonded at both ends to the lead sheath is roughly equivalent to a lowering of the sheath resistance in so far as the sum of circulating-current losses is concerned. Thus it is even possible by the use of an armor of sufficiently low resistance to obtain a total circulating-current loss materially lower than that for the lead-covered cable without an armor, if the cable spacing be sufficiently large.

EFFECT OF CABLE SPACING ON CIRCULATING-CURRENT LOSSES

Fig. 3 indicates how the circulating currents and circulating-current losses are affected by varying the spacing between cables from 1 to 40 ft. in a single-phase

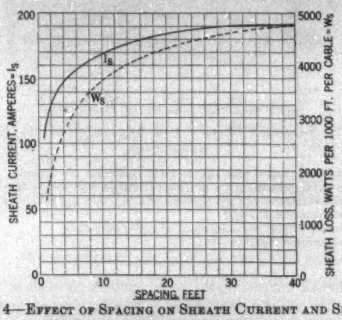

FIG. 4—EFFECT OF SPACING ON SHEATH CURRENT AND SHEATH LOSS FOR UNARMORED CABLE

Single-phase circuit
Calculated values for cable B (see Table I)
Conductor current = 260 amperes at 60 cycles per second

circuit, for the steel wire armored cable A of Table I. An increase of spacing from 1 to 4 ft. increases the total circulating-current losses about 20 per cent, while an increase of spacing from 1 to 40 ft. increases the total circulating-current losses about 45 per cent. For the copper-armored cable C the effect of spacing is very much less, the increase in total circulating-current losses being about 3 per cent when the cable spacing is increased from 1 to 40 ft.

In the case of the unarmored cable B of Table I, the increase of circulating-current loss with spacing is larger than in either of the above cases, as indicated in Fig. 4.

The increase in circulating-current losses with increased spacing is due to the increase in the circulating currents themselves, since the inductances of sheath and armor circuits are increased. In the case of the copper armor the small increase of circulating currents with spacing is due to the almost complete demagnetization produced by the circulating currents.

The effect of spacing on total losses is much less than that on the circulating-current losses alone, because the eddy-current and hysteresis losses decrease with increased spacing. Thus for cable A, the increase in total loss, with an increase of spacing from 1 to 40 ft. is only 5 per cent as seen from the upper curve for Fig. 3.

The effects of various cable designs and cable operating conditions on losses will now be considered with the aid of results based on test data and on calculations.

Losses for Different Kinds of Cables

The kinds of cable to be compared in regard to circulating currents and losses are:

A. Steel-wire armored cable
B. Cable without armor
C. Copper-wire armored cable
D. Cable with iron-pipe armor
E. Steel-tape armored cable

Their constants are given in Table I. Circulating currents and losses are given in Table II. Loss values are given per 1000 ft. of cable in a single-phase circuit of two similar parallel conductors carrying equal and opposite currents at 60 cycles per second. The values are based on test results (see description of tests below) and on calculations as well.

The ordinary lead-covered cable without armor, (cable B) is seen in Table II to have a lead-sheath circulating-current loss 1.4 times the conductor loss at 4 ft. spacing between cables, making the sum of the losses (exclusive of dielectric loss) 2.4 times the conductor loss, the sheath circulating current being 150 amperes or 58 per cent of the conductor current. If the cable were operated without bonding the lead sheaths or without grounding at more than one point, the entire circulating-current loss would of course be eliminated and the total loss would be substantially equal to the conductor loss. Hence, operating the plain lead-covered cable with the sheath circuit open clearly gives a saving of more than 50 per cent in total losses, in comparison with the losses occurring when the sheaths are short circuited through bonds of negligible impedance.

If now the same cable is equipped with a particular *steel-wire armor*, (*cable A*), its operation with sheath and armor open circuited is much less attractive, in respect to losses, than the plain unarmored cable with sheath open circuited, because of the armor eddy-current and hysteresis losses, which bring the total loss to 2.3 times the conductor loss (omitting dielectric losses).

The highest total loss for the steel-wire armored cable is obtained when the sheath alone is short circuited, for which case the total loss is 3.6 times the conductor loss at 4 ft. spacing. The armor eddy-current and hysteresis losses have dropped to less than half the value obtained in the previous case (with sheath and armor open) because of the demagnetizing effect, upon the armor, of the sheath current; but a large sheath circulating-current loss, of more than double the conductor loss, is caused by the sheath-circulating current of 185 amperes (71 per cent of the conductor current).

If the armor of this cable is shorted, with the sheath left open, (item 3, Table II) the demagnetizing action of the circulating current (in the armor) is less than in the

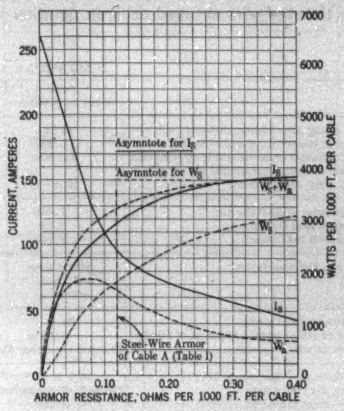

Fig. 5—Calculated Values of Sheath and Armor Currents and Losses *vs.* Armor Resistance

Single-phase circuit with 4-ft. spacing

For cable A (Table I) except that armor resistance is considered variable from 0 to 0.40 ohm per 1000 ft. per cable

Conductor current 260 amperes at 60 cycles per second

Average circumferential permeability at armor wires assumed constant and equal to 10 throughout the entire range of armor resistance

I_a = armor circulating current
I_s = sheath circulating current
W_a = armor circulating current loss
W_s = sheath circulating current loss

preceding case (item 2, Table II), the circulating current being smaller. Hence the iron armor eddy-current and hysteresis losses are higher than in the preceding case. The armor circulating-current loss is about equal to the conductor loss or about half the value of the sheath circulating-current loss of the case of item 2. Thus the overall losses are 2.8 times the conductor loss when the armor alone is shorted, as against 3.6 times the conductor loss when the sheath alone is shorted, for a cable spacing of 4 ft.

Finally if both sheath and armor of cable A are shorted by bonds of low impedance, as is usually the case in practise, the sheath and the armor carry lower currents and have smaller circulating-current losses than those occurring in the cases of items 2 and 3

respectively. Moreover the total demagnetizing effect upon the armor is sufficiently increased, so that the armor eddy-current and hysteresis losses are lower than in any of the preceding cases, thus making the total losses about 5800 watts per 1000 ft. per cable (or 5.8 watts per ft.) at 260 amperes conductor current, the armor eddy-current and hysteresis losses being only about 10 per cent of the total losses. The ratio of total loss (exclusive of dielectric loss) to conductor loss is 2.8 for this steel-wire armored cable at 4 ft. spacing with both sheath and armor bonded, while the corresponding ratio is 2.4 for the unarmored cable whose sheath is bonded or grounded at 2 or more points. Hence the steel-wire armored cable in question has a total loss less than 20 per cent in excess of the total loss of a similar but unarmored cable, when both cables are operated with low-impedance bonds.

Since various armor designs permit of a wide range of the value of armor resistance, it is of interest to show the effect of armor resistance on currents and circulating-current losses. Fig. 5 has been prepared for this purpose. The range of armor resistance covered is from 0.40 ohm per 1000 ft. per cable down to very low values. The circulating-current losses and current values in sheath and armor for the particular steel-wire armored cable A (see Table I) are shown at the ordinate drawn for 0.12 ohm. As the armor resistance is diminished below this value (at constant average circumferential permeability at the armor wires in Fig. 5) the combined circulating-current losses decrease more and more rapidly although the armor circulating current increases sharply.

For the copper-wire armored cable C the combined sheath and armor losses (see Table II) are only about 30 per cent of the conductor loss, on account of the low armor resistance (9 per cent of the steel-wire effective armor resistance at 100 amperes and 60 cycles). The armor current itself is 92 per cent of the conductor current at the 4 ft. spacing. Hence the overall losses for this cable, with sheath and armor bonded and grounded at several points are only 1.3 times the conductor loss or only half as much as the overall losses for a similar cable with the steel-wire armor described.

TABLE I

DIMENSIONS AND CONSTANTS OF CABLES

	Cable A steel wire armored	Cable B unarmored	Cable C copper armored	Cable D in iron pipe	Cable E steel tape armored
Voltage rating (line-to-line kilovolts)	44 kv.	44 kv.	44 kv.	....	....
Diameter of copper conductor—inches	0.682	0.682	0.682	0.682	0.528
Copper conductor cross-section in circular mils	350,000	350,000	350,000	350,000	211,600
Resistance of copper conductor per 1000 ft. at 25 deg. cent. in milliohms	30.8	30.8	30.8	30.8	50.9
Inside diameter of lead sheath	1.888	1.888	1.888	no lead sheath*	0.688
Thickness of lead sheath	0.13	0.13	0.13	no lead sheath*	0.086
Diameter over lead sheath	2.148	2.148	2.148		0.86
Armor type and description	single layer steel wire armor	no armor	single layer copper wire armor	2-in. iron pipe	double layer steel tape armor†
No. of armor wires (bands)	32	..	32	....	2 bands each 1 in. wide
Diameter of armor wires (thickness of bands)	0.18 in.	..	0.18 in.	....	0.035 in. thick
Lay of armor wires (bands)	10 times the armor dia.	..	10 times the armor dia.	....	1 in.
$\sqrt{\text{d-c. ohms/cm.}^2}$ for armor	15.4×10^{-6}	..	1.76×10^{-6}	13.8×10^{-6}	....
μ max. by d-c. for armor	770 at B = 7500 gausses	..	1.0	1260 at B = 6300 gausses	....
Diameter under armor, in.	2.398	..	2.39	2.0	1.03
Diameter over armor, in.	2.758	..	2.758	2.375	1.17
Lead sheath resistance per 1000 ft. at 25° C. in milliohms	128	128	128	68.5*	505
Effective armor resistance per 1000 ft. in milliohms	120 at 100 amp. induced armor current	..	10.72	....	....
Calculated armor d-c resistance per 1000 ft. 25 deg. cent. in milliohms	94	..	10.72	50.5	....

*Copper secondary conductor representing the lead sheath consisting of two rectangular conductors in multiple, each 0.485 × 0.125 in., placed along side the primary cable within the iron pipe

†The two tape layers wound in opposite directions.

The cable enclosed in an iron pipe (cable C), when tested with both the copper-wire secondary circuit (representing the sheath) and the pipe armor short circuited, had a total loss of about 4 times* the conductor loss. While the total loss in this case is higher than the corresponding values for the other cables considered so far—on account of the closed magnetic circuit surrounding the conductor—attention is called

*The losses for a similar cable *with a lead sheath* enclosed in the iron pipe will be materially larger than 4 times the conductor loss, because the usual lead sheath will have a considerably higher resistance than that of the copper wire secondary used in place of the sheath in cable D. The low-resistance copper secondary circuit (resistance about half of that of sheath for cable A) was used to show the effect of a higher than normal amount of demagnetization on the losses in the pipe armor.

to the very considerable reduction in iron pipe losses brought about by the demagnetizing action of the circulating currents, as shown in Table IV: When both circulating-current paths were open, the heavy eddy-current and hystersis losses in the iron pipe raised the total loss to 25 times the conductor loss, while the demagnetizing action of the circulating currents in the sheath circuit and pipe armor reduced the eddy-

TABLE II

Circulating currents and loss data per 1000 ft. of cable in single-phase circuit, for various kinds of cable operated both with and without bonding (or grounding) of sheath and armor. Conductor current = 260 amperes (except cable E which had 225 ampere) at 60 cycles per sec. Dielectric loss is not included. Bonds are considered to be of negligible impedance. Cable constants given in Table I.

Item	Cable	Test condition	Circulating currents amps. Lead sheath	Circulating currents amps. Armor	Conductor loss watts	Sheath circul. current loss watts	Armor circul. current loss watts	Armor eddy current & hyst. loss watts	Sum of losses watts	Ratio Total loss* / Conductor loss	Cable spacing ft.
1*	A steel wire armor	Sheath & armor open no circulating currents	0	0	2080	0	0	2740	4820	2.3	4
2*	A	Lead sheath shorted armor open	185	0	2080	4380	0	1040	7500	3.6	4
3*	A	Lead sheath open armor shorted	0	136	2080	0	2220	1460	5760	2.8	4
4*	A	Both lead sheath and armor shorted	117	106	2080	1750	1350	600	5780	2.8	4
4a†	A	Both lead sheath and armor shorted	123	110	2080	1930	1460	550	6020	2.9	4
5†	B Cable without armor	Lead sheath shorted	150	0	2080	2880	0	0	4960	2.4	4
6†	C Copper wire armor	Lead sheath and armor shorted	23	240	2080	68	620	0	2768	1.3	4
7*	D iron pipe armor	Copper wire secondary circuit & pipe circuit shorted	195‡	42	2100	2680‡	3460		8240	3.9	4.5
		Conductor current 225 amperes at 60 cycles per sec. From tests by W. L. Middleton and E. W. Davis. *Elec. Wld.* Nov. 18, 1916, p. 1003									
8	E Steel tape armor	Lead sheath and armor shorted	168	22	2580	14,250	15,000		31,820	12.3	4
8a†	E	Lead sheath and armor shorted	185	21	2580	16,450	1220	7680	27,930	10.8	4

*Values based on test data adjusted for 25 deg. cent. temperature and corrected for resistance and reactance of bonds. The results for the cases having circulating currents thus apply to cables having bonds of negligible impedance.

†Values for items 4a, 5, 6, and 8a were calculated.

‡In copper wire secondary representing the sheath.

*Not including dielectric loss.

TABLE III

Circulating currents and losses by test for steel-wire armored 350,000-cir. mil cable
For cable specifications see Table I cable A.
Single-phase circuit, 23 ft. loop with 50 ft. length of cable, 4 ft. spacing.
Conductor current 260 ampere, at 60 cycles. Tests at room temp. (Aver. 20° C.)

	Test condition	Circulating currents, amp. Sheath	Circulating currents, amp. Armor	Conductor loss watts	Sheath circul. current loss watts	Armor circul. current loss watts	Armor eddy current & hyst. loss watts	Sum of losses watts
a	Sheath and armor open	0	0	101	0	0	137	238
b	Sheath shorted armor open	175	0	101	200	0	52	353
c	Sheath open armor shorted	0	128	101	0	111	73	285
d	Both sheath and armor shorted	110	100	101	79	65	30	275

Resistance values by test for 50 ft. length of cable:

Conductor resistance	= 1.5 milliohms for 50 ft.
Lead sheath resistance	= 6.25 milliohms for 50 ft.
Resistance of leaf sheath plus bond	= 6.51 milliohms for 50 ft.
Effective armor resistance at 100 amp.	= 6.0 milliohms for 50 ft.
Effective armor resistance at 128 amp.	= 6.25 milliohms for 50 ft.
Resistance of bond in armor circuit	= 0.5 milliohms.

TABLE IV

Circulating current and losses from test data for 350,000-cir. mil copper cable enclosed in iron pipe with secondary copper conductor* to represent lead sheath.‡ Values in table are for a 50-ft. cable enclosed in 50 ft. of iron pipe, and are based on best results obtained for the following: Single phase circuit; 21 ft. loop with 46 ft. cable, 4.5 ft. spacing; Iron pipe armor covered only 5.2 ft. of cable on each side of loop about half way along the loop. Conductor current 260 amp. 60 cycle, test at room temperature.

Test condition	Circulating currents amps.		Conductor loss watts	Secondary copper conductor circul. curr. loss watts	Pipe circul. curr. loss watts	Pipe eddy curr. & hyst. loss watts	Sum of losses watts	Ratio of total loss conductor loss
	Secondary copper conductor	Iron pipe						
a No circulating currents	0	0	105	0	0	2500	2600	25.
b Copper secondary conductor shorted pipe circuit open	211	0	105	157	0	153	415	4.
c Pipe circuit shorted copper secondary conductor open	0	184	105	0	←1580†→		1685	16.
d Both copper secondary circuit and pipe circuit shorted	195	42	105	134	←173†→		412	3.9

*Secondary copper conductor placed along side of primary cable conductor inside of pipe: the secondary conductor representing the lead sheath in so far as circulating currents are concerned.

†Total pipe loss including circulating current loss.

‡For cable dimensions see Cable D in Table I.

current and hysteresis losses in the iron pipe sufficiently to bring the total loss down to 4 times the conductor loss, in spite of the addition of the circulating-current losses. These results again bring out the great effect of the circulating currents in sheath and armor towards reducing the eddy-current and hysteresis losses in a magnetic armor.

Cables with steel-tape armor have been tested by Middleton and Davis and by others (see Bibliography). The results of tests by Middleton and Davis[21] on a 211,600-circular mil steel-tape armored cable (cable E Table I) are given in Table II item 8. One feature of the results is the high armor loss of 5.8 times the conductor loss. The other interesting feature is the high sheath loss, due to the high sheath current passing through the sheath circuit of relatively large resistance. Both sheath and armor losses are of a materially larger order than in the steel-wire armored cable type, similar to cable A, because the tape armor offers a magnetic path of relatively low reluctance around the cable, thus tending to increase the induced voltages in sheath and armor, and to produce high eddy-current and hysteresis losses in the iron.

Eddy-Current and Hysteresis Losses. The preceding data and discussions indicate that the overall losses in steel-armored cables with sheath and armor circuits bonded and grounded at more than one point can be kept within reasonable limits—*i. e.*, within values that are not far in excess of those for unarmored cables likewise bonded and grounded—provided the armor eddy-current and hysteresis losses are kept low. The principal means to this end are indicated by the data of Tables II and IV, and are the following: (1) an armor design of individual wires wound with a large lay so that the magnetic path around the circumference is broken up by many non-magnetic spaces, and the component of magnetizing force along the armor wires is a small fraction of the resultant circumferential magnetizing force. Thus the magnetizing force at the armor due to the several currents is rendered relatively ineffective in producing eddy-current and hysteresis losses; (2) low-resistance circulating-current circuits, so as to keep the resultant magnetizing force itself (at the armor) low by a large demagnetizing effect due to circulating currents.

If both of the above features are incorporated in the cable design, as has been aimed at in the case of the steel-wire armored cable A, item 4 in Table II, the extra armor eddy-current and hysteresis losses will represent only a small fraction of the overall losses, (say 10 per cent at the 4 ft. spacing for this type of cable), and the overall losses themselves will not be very considerably in excess of those for a similar cable without armor, (cable A, item 4 having overall losses about 20 per cent higher than those of cable B item 5 in Table II).

A very material reduction in the permeability of the armor will also tend, as a rule, to reduce the armor eddy-current and hysteresis losses for given dimensions and otherwise fixed constants. However, a general rule for the permeability and resistivity requirements of armor material—with a view to keeping the overall losses low—cannot be given, because changes in the values of armor permeability and resistivity have many-fold effects (*i. e.*, on armor resistance, sheath and armor inductance values, circulating currents in sheath and armor circuits and the resulting losses, and armor eddy-current and hysteresis losses). The choice of the best permeability and resistivity values in any particular case must be determined by analysis of the individual losses for the cable spacing to be used.

Particular Values of Combined Eddy-Current and Hysteresis Losses for the Steel-Wire Armor of Cable A. In the discussion of the eddy-current and hysteresis losses in a steel-wire armor, as in the case of cable A, reference must be made to the armor resistance, because the effective armor resistance offered to the flow of the armor circulating current includes certain iron losses—namely those due to the flux component passing

at right angles through the armor wires, as discussed below under "armor resistance." If, then, the armor circulating-current loss is obtained as the product of this armor resistance times the square of the armor-circulating current, the extra eddy-current and hysteresis losses remaining after the armor circulating-current loss is allowed for will be the iron losses due to the flux passing lengthwise through the armor wires.

In the tests of the steel-wire armored cable A, the extra iron losses in the armor were obtained (1) with sheath and armor short circuited, by subtracting from the total measured loss (exclusive of dielectric loss) the conductor loss and the circulating-current losses

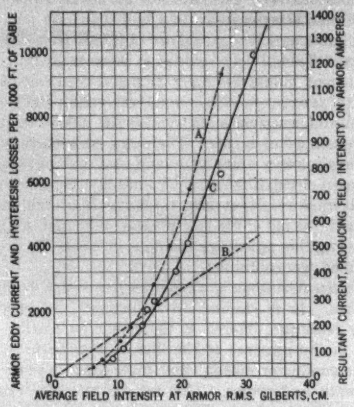

FIG. 6—DATA ON EDDY-CURRENT AND HYSTERESIS LOSSES IN IRON-WIRE ARMOR

A. Total open-circuit eddy-current and hysteresis losses combined in armor for 1000 ft. of cable A in Table I against circumferential magnetizing force H at the armor; values based on tests with armor open circuited

B. H values r. m. s. gilberts per cm. against resultant current acting on the armor. H is the resultant field intensity acting circumferentially at the center of the wires, assuming no magnetic armor present. The curves are based on actual test values

C. Extra eddy-current and hysteresis losses in steel-wire armor for cable A, obtained in the presence of circulating currents in the armor

Frequency 60 cycles per second

in the sheath and armor, and (2) with armor only short circuited (the sheath being open), by subtracting from the total measured loss the conductor loss and the armor circulating-current loss. These tests were made for a variety of induced circulating currents in the armor and gave curve C in Fig. 6, in which the extra eddy-current and hysteresis losses (combined) are plotted against H, the resultant r. m. s. field intensity acting circumferentially at the center of the armor wires assuming a non-magnetic armor. The field intensity H was computed for the vector sum of the conductor current, sheath current, and half the armor current. The lower part of the curve rises somewhat more rapidly than the square law, due to the increase in permeability. It should be mentioned that the actual magnetic intensity in the iron for any point on the curve is less than the average H value in Fig. 6, because of the air gaps in the circumferential flux path and because of the lay of the armor wires. The data in Fig. 6 apply of course only to the steel-wire armored cable specified under A in Table I, at 60 cycles per sec.

A curve of open-circuit armor loss (*i. e.*, when the steel-wire armor is open circuited) is also shown in Fig. 6, curve A. These losses average about 25 per cent higher than those of curve C, for reasons already explained. The open-circuit armor loss (curve A) is not used in the analysis of the losses of cables operating with sheath and armor bonded.

TESTS

Tests were made for measuring the circulating currents and the various loss components (1) for a steel-wire armored 350,000-circular mil lead-covered cable (cable A in Table I) and (2) for a plain copper 350,000-circular mil cable in an iron pipe (cable D in Table I). Attention is also called to the data in Table II on a 211,600-circular mil single-conductor cable with steel-tape armor, reported by Middleton and Davis, *Electrical World*, November 18, 1916, p. 1003.

The tests reported in the present paper are of the following kinds:

Steel-Armored Cable A in Single-Phase Circuit.

(a) Loss measurements with both sheath and armor open (no circulating currents), giving data on total armor eddy-current and hysteresis losses at different conductor currents.

(b) Loss and circulating-current data with lead sheath short circuited and armor open.

(c) Loss and circulating-current data with lead sheath open and armor short circuited.

(d) Loss and circulating-current data with both lead sheath and armor short circuited.

(e) Measurements of steel-wire armor effective resistance at different currents, both in the presence and absence of currents flowing in the conductor within.

(f) Resistivity and permeability data on samples of steel wire from the armor (given in Table I).

(g) Impedance measurements both in the presence and in the absence of circulating currents (reported below under a separate heading).

Cable D enclosed in iron pipe with secondary copper conductor to represent lead sheath:

(a) Loss measurements without circulating currents.

(b) Loss and circulating-current data with secondary conductor short circuited and pipe armor open.

(c) Loss and circulating-current data with secondary conductor open and pipe armor short circuited.

(d) Loss and circulating-current data with both secondary conductor and pipe armor short circuited.

The test method and equipment are indicated in Fig. 7. The test cable itself was 50 ft. long and was set

up in a loop of parallel sides at 4 ft. center spacing. The bonds were carefully soldered.

Circulating currents were measured by means of a Rogowski coil* with a thermocouple-type milliammeter, the complete unit being capable of ready calibration when the coil was placed around a conductor carrying a known current. The Rogowski coil was used in order that the extra impedance due to the current-measuring device be negligible in circulating-current measurements.

Losses were measured by an astatic reflecting dynamometer wattmeter. All tests were made at 60 cycles.

Test Results for Steel-Wire Armored Cable. The results of the tests in respect to loss values and circulating currents for the steel-wire armored cable are

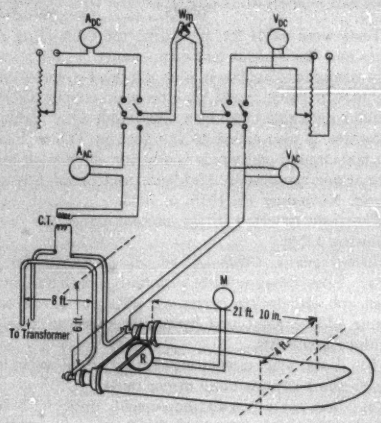

FIG. 7—SCHEMATIC DIAGRAM OF TEST SET-UP AND TEST CIRCUITS

A_{DC} = D-c. ammeter
V_{DC} = D-c. voltmeter
A_{AC} = A-c. ammeter
V_{AC} = A-c. voltmeter
W_m = Wattmeter
M = Thermocouple type milliammeter
R = Rogowski coil
$C.T.$ = Current transformer

given in Table III. At the bottom of the table, the resistance values for the various circuits are given, including the bond resistances. In the table, the values for "sum of losses" were obtained from wattmeter readings, while the values of conductor loss, sheath loss, and armor circulating-current loss were obtained by multiplying the respective resistances by the square of the corresponding test values of current. The difference between the measured total loss and the combined conductor loss and circulating-current losses gave the extra armor eddy-current and hysteresis loss values as tabulated. The resistances for the conductor and for the sheath were measured directly, while the effective armor resistance was based on loss measurements described below.

Attention is called to the fact that the circulating currents in this test were somewhat affected by the resistance and reactance of the end bonds because of the short loop length of 23 ft. In a similar cable circuit 1000 ft. long, the effect of the bonds would be negligible, and the circulating currents obtained would therefore be somewhat higher than the test values given in Table III. The amount of this increase in the circulating currents has been calculated to be about 6 per cent. Thus circulating-current data and loss data per 1000 ft. of cable at 4 ft. spacing are obtained from the test values in Table III by increasing the measured circulating-current values by 6 per cent and then calculating the $I^2 R$ product for the new current values and for the sheath and armor resistance values per 1000 ft. omitting the resistance of bonds. The extra eddy-current and hysteresis losses were also adjusted to the slightly increased circulating-current values in accordance with the average resultant field intensity H at the armor, and with the aid of the law of losses indicated in Fig. 6. Following this procedure the data in Table II, already discussed, were obtained.

Effective Resistance of Steel-Wire Armor to Circulating Currents. It is desired to determine the effective armor resistance offered to the armor circulating currents in the presence of the fluxes (in the armor) due to the conductor current and the circulating currents in sheath and armor. This resistance under exactly the conditions stated was not measured in these tests. Instead, the following approximate value of effective armor resistance was measured: Current at 60 cycles per second was passed through the armor and was returned through the copper conductor within, in an opposite direction and the total input into the complete series circuit was measured. From this total was deducted the conductor loss. The extra losses in the iron, *i. e.*, those due to the flux component passing lengthwise through the armor wires, (which extra losses were a relatively small quantity in these measurements) were also determined according to curve C, Fig. 6, and deducted. The resulting value divided by the square of the current gave the effective armor resistance as plotted in curve A, Fig. 8. Hence this resistance takes into account the losses due to all currents (including eddy currents) flowing lengthwise in the armor wires, but is not exactly the value desired, because in the test from which it was derived, the relative magnitudes of armor and conductor currents and their phase relationship were only roughly representative of those in a cable operated with sheath and armor short circuited. However, the resistance so obtained is considered to be a closer approach to the desired value than the resistance (plotted in curve B Fig. 8) based on the

*An air-core current transformer looped around the particular current to be measured. See Rogowski, W. S., Steinhaus, W., "Measuring the Magnetomotive Force," *Arch. für Elek.*, 1912-1913, pp. 141 and 511.

armor loss measured across the armor terminals when the armor only carries current, because the resistance of curve *B* is obtained for a current distribution in the armor in the absence of the flux due to the currents in conductor and lead sheath.

The resistance of curve *A*, Fig. 8, was used in the equations for sheath and armor circulating-current calculations (see Appendix for formulas) and gave current values within 5 per cent of the test values for the steel-wire armored cable.

The resistance of curve *A* is seen to range from 6 to 7.5 milliohms for 50 ft. per cable for cable A (at 60-cycle armor currents from 100 to 200 amperes), these resistance values being from 1.3 to 1.6 times the measured d-c. resistance of the 32 armor wires in multiple. The calculated effective 60-cycle resistance of a single armor wire from cable A, Table I, was 1.15 times the measured d-c. resistance at 100 amperes.

The circulating-current loss in the armor calculated as the product of effective armor resistance (curve *A*, Fig. 8) times the square of the armor-circulating current will of course not include the iron losses due

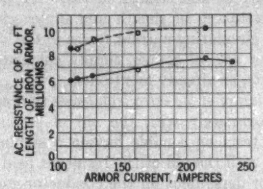

FIG. 8—MEASURED A-C. RESISTANCE OF 50-FT. LENGTH OF STEEL-WIRE ARMOR FOR CABLE A (SEE TABLE I) AT 60 CYCLES PER SECOND

A. Effective armor resistance to armor circulating current; derived from total loss measurement with armor and conductor in series carrying opposite currents, after subtracting copper loss. The small amount of extra iron loss has also been deducted.

B. Armor resistance derived from a-c. loss measurement with current carried by armor only (conductor and sheath open)

D-c. armor resistance by test = 4.7 milliohms for 50 ft. of cable

to the flux component passing lengthwise along the armor wires, and these extra iron losses will have appreciable magnitudes in the steel-wire armored cable even though operating with sheath and armor short circuited, as indicated in Table II item 4 and in curve *C* of Fig. 6 already discussed.

Cable Enclosed in Iron Pipe. In order to obtain some further data on the effect of circulating currents on armor losses, an insulated 350,000-circular mil cable without a lead sheath but passing through iron pipe was tested, the results obtained being those in Table IV. To simulate lead-sheath circulating currents a secondary copper conductor was passed through the pipe and was provided with bonding connections. Measurements were made by the methods already indicated. The results are given in Table IV for a 50 ft. length of cable enclosed in iron pipe, while the test circuit had only 10.4 ft. of cable enclosed in iron pipe. Consequently the results obtained in the presence of circulating currents are only approximately applicable to large lengths of armored cable, because of the resistance and reactance of end connections. However, the sensitiveness of circulating currents to changes of resistance in sheath or armor circuits is less for the cable with iron pipe armor than for the one with steel wire armor, because of the greater inductance due to the magnetic pipe. The results of the tests on the pipe-armored cable have not been corrected for the effects of resistance and reactance of bonding connections, since the purpose of the tests was to show merely the approximate order of magnitude of the losses in the pipe armor.

CALCULATION OF LOSSES

The calculation of the losses in sheath and armor for steel-armored single-conductor cables may be subdivided into the following three problems:

(1) Establishing formulas for the circulating currents in sheath and armor, these currents to be expressed in terms of certain reactances and resistances pertaining to the cables.

(2) The evaluation of the constants required in (1) for numerical calculation of the circulating currents.

(3) The computation of the circulating-current losses in sheath and in armor and of the extra armor eddy-current and hysteresis losses with the aid of known values of the circulating currents.

These three problems will be discussed separately.

Method Used in Derived Formulas for the Circulating Currents in Sheath and in Armor for Single-Conductor Armored Cables in a Single-Phase Circuit. The formulas for the circulating currents are derived in the appendix. The method used may be briefly outlined as follows:

First the fundamental Kirchhoff equations for voltage drops around sheath circuit and around the armor circuit were written. These equations were developed for a long single-phase circuit carrying equal and opposite currents in the two cables. The sheath-circuit equation, for instance, expresses that the sum of the voltages induced in the sheath due to the three currents plus the $R\,I$ drop in the sheath circuit is zero. The armor circuit equation is similar. The induced voltages themselves were expressed in terms of flux linkages.

In working out the final formulas for sheath and armor currents the terms involving flux linkages were converted into corresponding terms involving reactances, such that each reactance term in the equations for current is defined by definite flux linkage terms, partial flux linkages being taken into account by integration. The other values contained in the equations are the resistances, for sheath and armor respectively, and the conductor current.

In computing flux linkage terms, skin effect and proximity effect were neglected—but skin effect was taken into account in the evaluation of the resistance

constants. Due allowance was made for the increase in flux linkages on account of the presence of the magnetic armor. The shielding action of the armor in linkage calculations was also analyzed. For the steel-wire armored cable considered in this paper (cable A in Table I) this shielding action was found to be negligible in so far as mutual-inductance flux linkages are concerned.

For cables in which the extra armor-iron losses are a material factor, exact circulating-current calculations would require taking into account the effect (on flux linkages) of the component of the conductor current consumed in supplying the extra iron losses; *i. e.*, the losses not included in the $R I^2$ losses due to the armor circulating current. The factor in question was omitted from the equations derived in the appendix, and therefore the results based on them will be approximate for a cable whose armor has the extra iron losses, the errors increasing with the magnitude of the extra iron losses. The current supplying the extra iron losses could be considered mathematically as a third circulating current flowing in a circuit having a definite resistance and inducing definite voltages in the various circuits. An alternative to carrying out this procedure would be to make the first solution of sheath and armor currents with the equations given, to obtain an approximate value for the extra losses in the iron with these currents, and then to recalculate the circulating currents in terms of a new conductor current adjusted for the current consumed by the extra iron losses. Either of these procedures would materially complicate the calculations. In the calculations for the circulating currents for the steel-wire armored cable A, the currents were not more than 5 per cent higher than those based on tests, although the current consumed by the extra iron losses was neglected in the calculations. For the steel-tape armored cable E one of the circulating currents, similarly calculated, was 10 per cent too high. In other cases, the errors may be still larger, according to the magnitude of the extra iron losses.

Evaluation of Constants. The reactance constants required in the calculation of circulating currents are defined by flux-linkage terms tabulated in the appendix. In calculating flux linkages due to a magnetic armor the average circumferential permeability at the armor is required. Data for several armor designs have been published by Harvey and Busby.[2] The three resistance constants are the effective conductor resistance, the effective lead sheath resistance, and the effective armor resistance. The conductor resistance is equal to the d-c. value for the smaller sizes, but for the larger sizes, skin effect must often be considered, by methods well established. The effective lead sheath resistance offered to circulating currents may be taken as equal to the d-c. value without appreciable error.

The effective armor resistance to circulating currents, in the case of magnetic armors, must be calculated with due consideration of permeability and resistivity values or obtained by test. No general rule applicable to resistance calculations for different kinds of armors can be given because the methods will differ according to the shape and bulk of armor material, the arrangement of the armor conductors, and the effective current penetration. Exact calculations would be rather difficult. However, rough calculations can often be made with a fair degree of accuracy using such approximations as fit the particular case without resorting to new researches. If a sample of the cable is available, the armor resistance may be measured approximately, as indicated in the test described.

For the steel-wire armored cable tested, (cable A in Table I) the effective armor resistance at 60 cycles per second was found to be from 30 to 60 per cent higher than the d-c. resistance, depending on the current (see Fig. 8).

For steel-tape armored cables the authors have no test data giving the armor resistance.

Calculation of Sheath and Armor Losses When the Circulating Currents and Circuit Constants are Known. When the armor and sheath currents are determined and the resistances of sheath and armor circuits are known, the circulating-current losses in the sheath and in the armor are, of course, simply calculated as the $R I^2$ products, as already mentioned.

The calculation of the extra iron losses in armor can be made approximately with the aid of empirical data derived by tests on a cable having a similar type of armor, and with the aid of the resultant field intensity at the armor dependent on the conductor current and on the circulating currents (see data in Fig. 6 derived from tests on a steel-wire armored cable). Corresponding data for other types of iron armor have not been obtained.

In the absence of empirical iron-loss data, the extra iron losses in the armor can often be roughly determined—with less reliability—with the aid of suitable approximations carefully chosen for particular cases. Results so obtained will frequently be sufficiently good to predict at least the relative merits of different designs. Further tests would have to be made to establish for general use such methods of calculation for single-conductor cables with magnetic armor.

Results of calculations of losses for steel-armored cables are given in Table II, items 4a and 8a, for the steel-wire armored cable A and for the steel-tape armored cable E. The circulating currents were obtained by the formulas in the Appendix, using calculated reactance values. For cable A, the steel-wire armored type, the values of armor resistance and of extra iron losses used were those based on the test curves of Figs. 8 and 6 respectively, while for cable E, all values were calculated.

For cables A and E the calculated current values are seen to be within 5 per cent and 10 per cent, re-

spectively, of the test values, and the agreement between calculated and test values of overall losses is within 5 per cent and 15 per cent respectively. For the steel-tape armored cable the predicted values of armor eddy-current and hysteresis losses are considerably in error, indicating that in cases where the iron losses are a larger part of the overall losses, the calculations may be rather uncertain.

Other Losses. The conductor loss (copper loss) will not be discussed because its calculation is well understood, except for the proximity effect (due to neighboring conductors), which effect will be somewhat reduced by the shielding action of the sheath and armor around the conductor. The combined effects due to proximity and shielding will generally be small and of little significance in so far as the conductor loss is concerned.

Eddy-current losses in the lead sheath may be of two kinds: (a) those set up in the lead by the concentric fluxes, due to the current in the copper conductor within and the sheath current itself, and (b) those due to the fluxes from neighboring cables cutting through the sheath. The losses of the first kind are negligible for lead sheaths of the customary dimensions, because of the high resistivity and because of the relatively thin wall. The losses of the second kind also will be negligible when the sheaths and armors of the cables are bonded by low impedance bonds, but may become appreciable in the absence of circulating currents, for large cables at close spacings.[20] Additional losses in the sheath due to the longitudinal flux set up by the armor current will generally be negligible, especially when the lay of the armor wires is large.

Dielectric losses will not be discussed here. The reader is referred to the extensive literature on the subject.

Reactance of Armored Cables

Test results on reactance of armored cables are given in Table V. The purpose of the table is to show values of cable reactance, effective resistance, and impedance, per 1000 ft. per cable, in single-phase circuits at 4 ft. spacing between cables, for

steel-wire armored cable A (see Table I for specifications)

bare copper cable of the same copper diameter as that of cable A, but without sheath or armor

steel-tape armored cable E.

In the steel-wire armored cable, the reactance is highest for the condition when armor and sheath are both open circuited, such that the demagnetizing effect of circulating currents is absent (item 1 in Table V). As this demagnetizing action is increased—by shorting the armor alone (item 3), then by shorting the sheath alone (item 2), and finally by shorting both sheath and armor (item 4)—the reactance per cable drops steadily, till in the last case with sheath and armor both carrying circulating currents, the reactance has dropped to nearly ⅓ of the value obtained with both sheath and armor open, at 4 ft. cable spacing. It is also of interest to point out that the reactance for the steel-wire armored cable with both sheath and armor short circuited through low-impedance connections, has a reactance of less than half of that for the plain copper conductor alone without sheath or armor, the spacing between conductors being 4 ft. in all cases.

The relative values of effective cable resistance, based on overall loss values, exclusive of dielectric losses, need no further mention in view of the discussion of losses already given.

Calculations of conductor reactance for single-conductor steel-armored cables carrying circulating currents may readily be made, with the aid of the expressions for flux linkages and reactances given in the Appendix and with the aid of additional flux linkages inside of the sheath. The effect of the magnetic armor on reactance values and on circulating currents must, of course, be considered as indicated in the appendix.

Economic Considerations

A submarine cable is usually furnished with an armor for mechanical protection; the purpose of the armor is not so much to prevent a small sharp object from damaging the lead sheath, as is the case in an armored cable intended for burying in the earth, but rather

TABLE V

Reactances and effective cable resistances per 1000 ft. per cable in a single-phase circuit for various kinds of cables operated both with and without bonding (or grounding) of sheath and armor. Conductor current = 260 amperes (except cable E which had 225 amperes) at 60 cycles per second. Values based on test data, except item 5

Item	Cable	Test condition	D-c. cable resistance milliohms	Eff. cable resistance milliohms	Cable impedance milliohms	Cable reactance milliohms	Center spacing between cables. Ft.
1	A	Sheath and armor open. No circulating currents	30.8	71.5	176	161	4
2	A	Lead sheath shorted armor open	30.8	111	148	98	4
3	A	Lead sheath open armor shorted	30.8	85	141	113	4
4	A	Both lead sheath and armor shorted	30.8	86	103	57	4
5	bare copper cable	Bare 350,000-cir. mil cable. No lead sheath no armor	30.8	30.8	127	123	4
6*	E steel tape armor	Lead sheath and armor shorted	50.9	628	674	245	4

*Results taken from test by Middleton and Davis, Bibliography reference No. 21.

to prevent an undue strain on the sheath while the cable is being laid and after it is placed in service. Such a strain might be caused by the tension which occurs in the process of laying, by the action of tides and currents in causing the cable to more down stream, or possibly by a boat with a dragging anchor. For these reasons the wire armor is used rather than a band steel armor (which has inherent high losses); and it is usually possible to obtain all the mechanical strength necessary with a copper armor.

If a portion of the armored cable rises above the water level or runs in ducts which are not at all times completely submerged, then any additional losses due to the armor may seriously affect the rating of the line as a whole. If, however, the armor is terminated at a point which is always below water level and the land section of the cable is completely unarmored, then the losses in the armored cable will not affect the rating of the line appreciably, because the armor throughout its length will be in contact with the water, which is capable of removing very rapidly the heat produced.

It is thus apparent that under practical conditions of operation of submarine single-conductor cables, neither the mechanical strength of the higher-loss steel armor nor the negligible gain in rating which would be obtained by the use of the low-loss copper armor is really an important factor in deciding which type of armor should be used. The choice rests primarily on the economics of the situation.

To illustrate this, Fig. 9 has been prepared. The total annual cost of an armored cable line has been plotted against load factor. This total annual cost is based on a single-phase or symmetrically-spaced three-phase circuit, with a 4-ft. spacing between conductors and is expressed as a percentage of the total annual cost at 100 per cent load factor of a similar unarmored circuit, carrying the same load and arranged in the same way. Since the cost of the copper-armored cable is considerably higher than that of the steel-armored cable, and since the losses of the copper-armored cable are considerably lower than the losses in the steel-armored cable, it is evident that at low load factors the steel-armored cable will tend to be the more economical, while at high load factors, the copper-armored cable tends to be more economical. This is clearly brought out in Fig. 9. The figure indicates that at 58 per cent load factor the economy of the two types of cable tends to be about the same. At higher load factors, the copper-armored cable has the advantage, whereas at lower load factors the steel-armored cable appears to be the more economical. Since the effect of cable spacing on overall losses is quite small for a wide range of spacings (above, say, 2 ft. for the cables in question) the economy curves given for 4 ft. spacing will also apply approximately over the wider range of spacings.

The values used were obtained by averaging economic data furnished by several operating companies. If the fixed charges were exceptionally high or the cost of losses were exceptionally low, the steel-armored cable would probably show up to better advantage, whereas, if the converse conditions held, the copper-armored cable would tend to be favored.

It should be particularly noted that the present economic calculation is based on the assumption that the same load may be carried by either the steel-armored or the copper-armored cable. This assump-

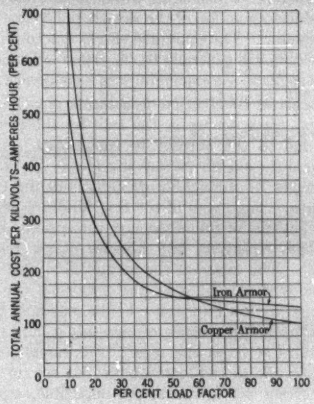

Fig. 9—Relative Annual Costs per Kv-a-Hour at Different Load Factors for Steel-Wire Armored Cable (Cable A in Table I) and for a Similar Cable with Copper-Wire Armor (Cable C in Table I)

tion is perfectly valid so long as the armored portion of the cable line is at all times completely immersed, since under these conditions the sheath and armor losses are very thoroughly dissipated by the surrounding water and produce only a very small temperature rise. If, however, a portion of the armored cable is above water level either continuously or for a portion of the time during which it is in operation, the losses in the lead sheath and in the armor of that part of the cable will tend to produce much greater temperature rises than they would under water and the current capacity of the cable will be thereby greatly reduced. Since the losses in the copper-armored cable are so much lower than those in the steel-armored cable, the current-carrying capacity of the copper-armored cable under these conditions will be considerably higher than that of steel-armored cable, and the economic study should probably be based on the assumption that the two cables are loaded to the maximum capacity that heating will permit. If this is done, the economic comparison will be more favorable to the copper-armored cable, especially at the higher load factors.

Appendix

Derivation of Formulas for Calculating the Induced Circulating Currents in Lead Sheath and Armor

Consider the case of a single-phase circuit, going-and-return cable running parallel to each other, the spacing between center to center of the two cables being D. Lead sheath and armor of both cables at the near end and at the far end of the cable circuit are assumed to terminate in heavy cross bonds of negligible impedance through which the induced armor and lead-sheath circulating currents will flow, these currents being induced by the flux passing between the two cables.

Assumptions on Which the Calculations are Based

(I) That there is no circumferential saturation in the armor, if the latter is magnetic. Values of circumferential permeability in the space occupied by the armor have been given by Harvey and Busby[2] for certain cable armors.

(II) That there is no decrease in the reluctance of

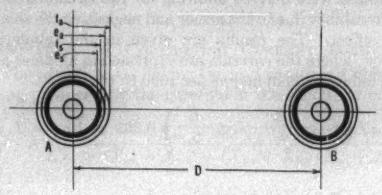

Fig. 10—Schematic Cross Section through a Single-Phase Cable Circuit

Subscripts s and a apply to the lead sheath and armor respectively

the path of a line of force due to cable A (Fig. 10) passing through the armor of cable B if that armor is magnetic. If the cables are close together, and the circumferential permeability is high, this assumption might lead to considerable error.

(III) That sheath and armor currents are uniformly distributed, in computing the linkages.

The above assumption is not made in determining the resistance of the armor. The effective a-c. resistance is used.

(IV) That the currents in cable B, its sheath, and its armor, are concentrated at the centers of the conductors when considering the effects of cable B on A. This assumption is justified, if proximity effect is neglected.

(V) When writing the voltage equations for sheath and armor, the induced voltages due to the currents in conductor, sheath, and armor are included together with the resistance drop. When extra iron losses are present in the armor in addition to the circulating-current losses, the extra iron losses may be considered as circulating-current losses in a third circulating-current circuit, separate from the armor circulating-current circuit. Naturally in a mathematical analysis the current in this third circulating-current circuit will have to be taken into account as contributing to the induced voltages in sheath and armor. In the derivations given below the third circulating current, representing the extra iron losses in armor, and its effect on induced voltages are neglected to simplify the calculations. When the extra iron losses are of a considerable order of magnitude, the approximation in question will, of course, introduce material errors, as already discussed.

List of Symbols

General.

e_s = inside radius of lead sheath, inches
f_s = outside radius of lead sheath, inches
e_a = inside radius of armor, inches
f_a = outside radius of armor, inches
f = frequency, cycles per second
r_s = resistance of lead sheath, ohms
r_a = resistance of armor, ohms
D = spacing between cables, center to center, inches
μ = circumferential permeability of armor, that is, permeability in the circumferential direction
$\omega = 2\pi f$
R = a large distance tending toward infinity, and compared to which D is negligible.

Currents.

I_c = current in cable conductor, taken as reference vector
I_{ca} = current in conductor of cable A
I_{cb} = current in conductor of cable B
I_s = current in lead sheath, vector value
I_{sa} = current in sheath of A
I_{sb} = current in sheath of B
I_a = current in armor, vector value
I_{aa} = current in armor of A
I_{ab} = current in armor of B

Magnetic Flux Linkages (Refer to Fig. 10)

λ_{cs} = partial linkages with sheath of A due to I_c integrated from e_s to f_s
λ_{ss} = partial linkages with sheath of A due to I_s integrated from e_s to f_s
λ_{cs}' = total linkages with sheath of A due to $(I_c + I_s)$ integrated from f_s to e_a
λ_{cs}'' = total linkages with sheath of A due to $(I_c + I_s)$ integrated from e_a to f_a
λ_{as} = total linkages with sheath of A due to I_a in armor of A integrated from e_a to f_a
λ_{As} = total linkages with sheath of A due to $(I_c + I_s + I_a)$ of A integrated from f_a to R
λ_{Bs}' = partial linkages with sheath of A due to $(I_c + I_s + I_a)$ in conductor, sheath, and armor of B, integrated from $D - f_a$ to $D + f_a$
λ_{Bs}'' = total linkages with sheath of A due to $(I_c + I_s + I_a)$ in conductor, sheath, and armor of B, integrated from $D + f_a$ to R

λ_{ca} = partial linkages with armor of A due to $(I_c + I_s)$ in conductor and sheath of A, integrated from e_a to f_a

λ_{aa} = partial linkages with armor of A due to (I_a) in armor of A integrated from e_a to f_a

λ_{Aa} = total linkages with armor of A due to $(I_c + I_s + I_a)$ of A integrated from f_a to R

$$\lambda_{A_a} = \lambda_{A_s}$$

λ_{Ba}' = partial linkages with armor of A due to $(I_c + I_s + I_a)$ in armor, sheath, and conductor of B, integrated from $D - f_a$ to $D + f_a$

λ_{Ba}'' = total linkages with armor of A due to $(I_c + I_s + I_a)$ in conductor, sheath, and armor of B, integrated from $D + f_a$ to R

λ_i = partial internal linkages within the iron wires with the current I_a, and linkages with that current of longitudinal flux produced by the spiraling of the armor wires

λ_{AS} = sum of all linkages with the lead sheath of A

$$\lambda_{AS} = \lambda_{cs} + \lambda_{ss} + \lambda_{cs}' + \lambda_{cs}'' + \lambda_{as} + \lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}''$$

λ_{AA} = sum of all linkages with the armor of A

$$\lambda_{AA} = \lambda_{ca} + \lambda_{aa} + \lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}'' + \lambda_i$$

Inductance Coefficients.

The expressions of the flux linkages, as shown below, consist in each case of the product of a term containing a current, or the vector sum of several currents, and a factor depending on the size and the configuration of the sheath and armor circuits. This latter term is the coefficient of inductance. In general

$$\lambda = I\,L$$

Reactance Coefficients.

The reactance coefficient is in each case equal to $2\,\pi\,f$ times the inductance coefficient. The reactance terms used in the final formulas correspond to the following flux linkages:

X_{cs} corresponding to λ_{cs}
X_{ss} corresponding to λ_{ss}
X_{cs}' corresponding to $\lambda_{cs}' + \lambda_{cs}''$
X_{ca} corresponding to $\begin{cases} \lambda_{ca} \\ \lambda_{as} \end{cases}$

because L_{as} and L_{ca} are identical

X_{As} corresponding to λ_{As}
X_{Bs}' corresponding to λ_{Bs}'
X_{Bs}'' corresponding to λ_{Bs}''
X_{aa} corresponding to λ_{aa}
X_{Ba}' corresponding to λ_{Ba}'
X_{Ba}'' corresponding to λ_{Ba}''
X_i corresponding to λ_i
X_m corresponding to $\begin{cases} \lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}'' \\ \lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}'' \end{cases}$

because $L_{As} + L_{Bs}' + L_{Bs}'' = L_{Aa} + L_{Ba}' + L_{Ba}''$

X_{AS} corresponding to λ_{AS}
X_{AA} corresponding to λ_{AA}

Computation of the Flux Linkages

The general procedure outlined by Alexander Russell[22] and later used in this connection by Clark and Shanklin[12,10] was adopted for the computation of flux linkages. This procedure obviously applies directly, except that it must be modified in the calculation of all linkages within the armor of, say, cable A (Fig. 10), due to currents in the conductor, sheath, and armor of that cable, by the introduction of a factor μ to take care of the circumferential permeability. Also it is necessary, in some cases, to consider the magnetic shielding effect of the armor in computing λ_{Bs}' and λ_{Bs}''.

It should be pointed out that the circumferential permeability of a steel-wire armor is low, probably varying from 5 to 15 for a single wire armor as shown by tests made by Harvey and Busby.[2] Under these conditions its shielding effect is small, probably not exceeding 25 per cent for $\mu = 10$, as shown by Professor DuBois.[23] Since these components of linkage are not large, it seemed to be a reasonably good approximation to neglect the shielding effect altogether. A calculation based on 100 per cent shielding does not differ materially from one based on zero shielding.

In accordance with the above, the flux-linkage formulas were derived allowing for the circumferential permeability due to the armor and neglecting the shielding effect. The results are given in the tabulation below, where the currents are expressed in amperes and the inductances in henrys per 1000 ft. of cable.

$$\lambda_{cs} = I_c\left(1 - \frac{2\,e_s^2}{F_s^2 - e_s^2}\log_e\frac{f_s}{e_s}\right) 0.305 \times 10^{-4} = I_c\,L_{cs} \tag{1}$$

$$\lambda_{ss} = I_s\left[2\left(\frac{e_s^2}{f_s^2 - e_s^2}\right)^2 \log_e\frac{f_s}{e_s} + \frac{1}{2}\,\frac{f_s^2 - 3\,e_s^2}{f_s^2 - e_s^2}\right] 0.305 \times 10^{-4} = I_s\,L_{ss} \tag{2}$$

$$(\lambda_{cs}' + \lambda_{cs}'') = (I_c + I_s)\,2\left[\log_e\frac{e_a}{f_s} + \mu\log_e\frac{f_a}{e_a}\right] 0.305 \times 10^{-4} = (I_c + I_s)\,L_{cs}' \tag{3}$$

$$\lambda_{as} = I_a\,\mu\left(1 - \frac{2\,e_a^2}{f_a^2 - e_a^2}\log_e\frac{f_a}{e_a}\right) 0.305 \times 10^{-4} = I_c\,L_{as} \tag{4}$$

$$\lambda_{As} = 2\,(I_c + I_s + I_a)\left[\log_e\frac{R}{f_a}\right] 0.305 \times 10^{-4} = (I_c + I_s + I_a)\,L_{As} \tag{5}$$

$$(\lambda_{Bs}' + \lambda_{Bs}'') = -2(I_c + I_s + I_a)\left[\log_e\frac{R}{D}\right] 0.305 \times 10^{-4} = -\,(I_c + I_s + I_a)\,(L_{Bs}' + L_{Bs}'') \tag{6}$$

$$\lambda_{aa} = (I_c + I_s)\,\mu\left[1 - \frac{2\,e_a^2}{f_a^2 - e_a^2}\log_e\frac{f_a}{e_a}\right] 0.305 \times 10^{-4} = (I_c + I_s)\,L_{ca} \tag{7}$$

$$\lambda_{aa} = I_a \mu \left[2 \left(\frac{e_a^2}{f_a^2 - e_a^2} \right)^2 \log_e \frac{f_a}{e_a} + \frac{1}{2} \frac{f_a^2 - 3 e_a^2}{f_a^2 - e_a^2} \right] 0.305 \times 10^{-4} = I_a L_{aa} \tag{8}$$

$$(\lambda_{Ba}' + \lambda_{Ba}'') = (\lambda_{Bs}' + \lambda_{Bs}') = -2 (I_c + I_s + I_a) \left[\log_e \frac{R}{D} \right] 0.305 \times 10^{-4} = -(I_c + I_s + I_a)(L_{Bs}' + L_{Bs}'') \tag{9}$$

$$(\lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}'') = (\lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}'') = (I_c + I_s + I_a) \left[\log_e \frac{D}{F_a} \right] 0.305 \times 10^{-4} = (I_c + I_s + I_a) L_m \tag{10}$$

λ_i has not been computed. Tests indicated that it is negligible for wire armors with large lay (Cable A in Table I).

$$\lambda_{AS} = \lambda_{cs} + \lambda_{ss} + (\lambda_{cs}' + \lambda_{cs}'') + \lambda_{as} + (\lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}'') \tag{11}$$

$$\lambda_{AA} = \lambda_{ca} + \lambda_{aa} + (\lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}'') + \lambda_i \tag{12}$$

Derivation of Circulating Current Formulas

The total flux λ_{As} linking the lead sheath of Cable A, being produced by the currents in the conductor of A, the sheath of A and the armor of A and also by the currents in the conductor, sheath, and armor of B, induces a voltage E_{AS} on the lead sheath of cable A. This voltage

$$E_{AS} = -\frac{d}{dt} \lambda_{AS} \tag{13}$$

Applying Kirchhoff's law to the lead sheath and armor of Cable A the following voltage expressions are obtained:

$$\left.\begin{aligned} I_{sa} r_s &= -\frac{d}{dt} \lambda_{AS} \\ I_{aa} r_a &= -\frac{d}{dt} \lambda_{AA} \end{aligned}\right\} \tag{14}$$

where r_s is the lead sheath resistance and r_a is the armor resistance of cable A.

From the flux linkage Equations (11) and (12) replacing λ_{AS} and λ_{AA} by their respective components and assuming the currents to be sinusoidal, the voltage Equations (14) become

$$I_{sa} r_s = -j\,\omega\,[I_c L_{cs} + I_s L_{ss} + (I_c + I_s) L_{cs}' + I_a L_{as} + (I_c + I_s + I_a) L_m] \tag{15}$$

$$I_{aa} r_a = -j\,\omega\,[(I_c + I_s) L_{ca} + I_a L_{aa} + (I_c + I_s + I_a) L_m + I_a L_i] \tag{16}$$

From the symmetry of the cable circuit it follows that

$$I_{sa} = -I_{sb} = I_s$$
$$I_{aa} = -I_{ab} = I_a$$

Replacing ωL by X in Equations (15) and (16)

$$I_s r_s = -j\,[I_c X_{cs} + I_s X_{ss} + (I_c + I_s) X_{cs}' + I_a X_{as} + (I_c + I_s + I_a) X_m] \tag{17}$$

$$I_a r_a = -j\,[(I_c + I_s) X_{ca} + I_a X_{aa} + (I_c + I_s + I_a) X_m + I_a X_i] \tag{18}$$

Dividing Equations (17) and (18) through by I_c and rearranging these equations

$$\frac{I_s}{I_c} = \frac{-j\,[(X_{cs} + X_{cs}' + X_m) + \frac{I_a}{I_c}(X_{ca} + X_m)]}{r_s + j\,(X_{ss} + X_{cs}' + X_m)} \tag{19}$$

$$\frac{I_a}{I_c} = \frac{-j\,[(X_{ca} + X_m) + \frac{I_s}{I_c}(X_{ca} + X_m)]}{r_a + j\,(X_{aa} + X_m + X_i)} \tag{20}$$

Solving the two Equations (19) and (20), assuming $X_i = 0$, the final equations for the circulating currents in lead sheath and armor are obtained.

$$\frac{I_s}{I_c} = \frac{-[(X_{ca} + X_m)^2 - (X_{cs} + X_{cs}' + X_m)(X_{aa} + X_m) + j\,r_a (X_{cs} + X_{cs}' + X_m)]}{r_s r_a + (X_{ca} + X_m)^2 - (X_{aa} + X_m)(X_{ss} + X_{cs}' + X_m) + j\,[r_s (X_{aa} + X_m) + r_a (X_{ss} + X_{cs}' + X_m)]} \tag{21}$$

$$\frac{I_a}{I_c} = \frac{-[(X_{ca} + X_m)(X_{cs} - X_{ss}) + j\,r_s (X_{ca} + X_m)]}{r_s r_a + (X_{ca} + X_m)^2 - (X_{aa} + X_m)(X_{ss} + X_{cs}' + X_m) + j\,[r_s (X_{aa} + X_m) + r_a (X_{ss} + X_{cs}' + X_m)]} \tag{22}$$

Non-Magnetic Armor

The same formulas (21) and (22) for the circulating currents may be used. When computing X_{cs}', X_{ca}, and X_{aa}, however, μ should be put equal to 1.

Three-Phase Circuit

The calculation of the three-phase circuit is rather long, and has been omitted here. It is not difficult, however, to show that in the case of a three-phase equilaterally-spaced circuit, the same formulas hold as for the single-phase circuit, if the fact that $\Sigma I_c = \Sigma I_s = \Sigma I_a = 0$ is used.

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Discussion

E. W. Davis and W. N. Eddy: The test results on the authors' steel-armor-wire cable (Cable *A* in the paper) give a sheath and armor loss at 260 amperes and 4-ft. spacing of 3940 watts per 1000 ft. of cable with both sheath and armor bonded. Tests made in our laboratory on a cable very similar to the authors' (4/0 instead of 350,000 cir. mil, No. 4 armor wire instead of No. 7, and an 18-in. armor wire lay instead of 22-in.) gave a sheath and armor loss of 4600 watts per 1000 ft. of cable under the same testing conditions. It is believed that the difference in loss between the two cables can be entirely accounted for by the slight differences given in their dimensions, all of which would tend to make the losses greater on the second cable.

Tests on the No. 4/0 cable submerged in sea water with both sheath and armor bonded showed the sheath and armor losses to be only 75 per cent of the same losses in air, because of the more complete bonding provided by the sea water. The losses in clean fresh water were found to be the same as those in air.

The authors mention the demagnetizing action of the armor and sheath circulating currents. This can be shown very clearly by actual measurement of the flux in the sheath and armor. For instance, such measurements on a 6-in. piece of the sheath and armor of the No. 4/0 cable mentioned before gave, at 250 amperes conductor current, 0.45 kiloline of flux with no bond, 0.40 kiloline with 62.5 amperes bond current, and 0.31 kiloline with 135 amperes bond current.

These measurements also bring out the greater flux density found in the low-voltage steel-tape-armored cables. The authors' steel-tape cable *E* (Middleton and Davis) gave at 250 amperes conductor current, 7.9 kilolines with no bond and a corresponding decrease with increasing bond current. That is, the flux on this cable was *eighteen times* that on the No. 4/0 high-voltage steel-wire-armored cable at the same current.

Fig. 1 is submitted herewith as an interesting comparison of some of the loss measurements available. It indicates a consistent influence of the iron-circuit circumferential reluctance on the losses. It also shows an interesting comparison between the bonded and not-bonded losses. At low flux densities, whether caused by low conductor current or high circumferential reluctance, the not-bonded losses are greatest while the reverse is true at higher flux densities. One reason why the new high-voltage armored cables show such relatively low losses is that because of the additional insulation, the cable diameter is relatively large and the circumferential reluctance of the magnetic circuit correspondingly smaller.

The authors' calculation of the circulating-current losses is based on the value of the sheath resistance used. As described in the paper it was measured by passing current through the conductor and the sheath in series. It seems doubtful that such a resistance is always equal to the true resistance of the sheath. The circulating current is induced in the steel armor wires by the flux from the conductor current. As this flux is directly proportional to the circumferential reluctance of the armor it must be maximum at the center line of the armor wires, decreasing to a very low value outside and inside of this line, because of the rapid increase of air gap and decrease of iron in the magnetic circuit (circumferential to the cable). Since the circulating current is induced by the flux it would seem that its density should be greatest where the flux is strongest and least where the flux is weakest. Any change in conductor current should change the flux distribution and result in at least some change in the true resistance of the armor.

On the tenth page of the paper the authors, using their measured sheath resistance, offer an explanation of the increase in circulating-current losses due to the addition of steel armor wire to a lead-covered cable that gives reasonable agreement with test values. Yet it would seem even more reasonable to expect that the true effective resistance of the armor might be considerably higher than their value (which would tend to give lower losses) but that the flux density in the iron was so much greater than in the lead that the induced current density was greater and the losses greater. The combination of these effects might easily account for the test results obtained.

O. R. Schurig: The data on armored cables in the discussion by Messrs. Davis and Eddy are a valuable contribution to this subject. Their test results on the effect of water on losses in submarine cables are particularly interesting. The data are consistent with those given in the paper and need no further comments.

In their discussion it is stated, with reference to the measured value of armor resistance used in the paper, that "it seems doubtful that such a resistance is always equal to the true resistance."

Attention is called to the fact that no such claim is made in the paper. On the contrary, a full paragraph on effective resistance of steel-wire armor to circulating currents is devoted to pointing out the nature of the approximations made, the complications involved in measuring the true armor resistance, and the reasons why the resistance measured "is not exactly the value desired" (to quote from the paper). Nevertheless the results of loss calculations with the values from Fig. 8, curve *A*, differ by not more than 10 per cent from the test data, and the calculated circulating-current values are within 5 per cent correct. These results amply justify the use of the test values of armor resistance for performance calculations.

In further discussing the armor resistance, Messrs. Davis and Eddy refer to the distribution of flux in the armor wires, to the effect that the flux "must be a maximum at the center line of the armor wires." There is no doubt that the air-gap flux between the armor wires is densest at the point where the gap is smallest. However, this condition does not by any means determine the distribution of flux in the armor wires themselves, the latter being conditioned upon skin effect and proximity effects. That is, the flux passing across the armor wires (and also the current flowing lengthwise along the armor wires) will tend to crowd toward the upper side of the armor wires or toward the under side of the armor wires, according to whether the armor alone carries current or whether the armor current is induced, as in normal operation bonded, by the resultant flux from conductor and sheath currents. In other words, Messrs. Davis and Eddy are not correct in stating that the flux and the circulating current in the armor are a maximum at the center line of the armor wires, and conclusions drawn from this reasoning cannot be expected to be reliable.

In the last paragraph Messrs. Davis and Eddy discuss the effect of steel-wire armor resistance upon circulating current losses and suggest that the effective armor resistance might be considerably higher than the values used in the paper. All the evidence in the paper indicates, however, that the armor resistance values used in the calculations are rather too high than too low. A higher armor resistance would give still higher losses and greater errors of calculation in the case of cable *A* (see calculated results in item 4a, Table II and loss curves with changing armor resistance in Fig. 5). Moreover, an analysis of the current distribution in the armor under operating conditions and under the conditions of the test for armor resistance (see Fig. 8, curve *A*) also suggests that the armor resistance values used may be slightly high rather than too low.

In regard to the explanation of the increase in circulating current losses due to the addition of steel armor wires to a lead-covered cable, we are in agreement with Messrs. Davis and Eddy on the point that a magnetic armor will tend to give greater circulating-current losses than a non-magnetic armor having the save effective resistance, because of the additional inductance caused by the magnetic armor.

Lightning

Progress in Lightning Research in the Field and in the Laboratory

BY F. W. PEEK, Jr.*

Fellow, A. I. E. E.

I. Introduction

RECENT progress in the mastery of lightning problems through combined research in the laboratory and field has been so rapid that it seems important at this time to make a review of the present status of the various phases of the subject. While there is still much to learn, lightning may be said to be now at least on an engineering basis since it is expressed numerically in volts and amperes. It has been removed from the realm of the "medicine man."

The following indicate how rapid the progress has been: The wave shape of lightning has been pictured by the cathode ray oscillograph; the time required for a cloud to discharge has been measured by the cathode ray oscillograph; the attenuation of lightning waves traveling on a transmission line has been determined; natural lightning waves have been reproduced in the laboratory where their effects on transmission lines, insulators, insulation, transformer and protective apparatus have been studied at will; a lightning generator producing over 3,600,000 volts has been constructed and waves from this generator have been sent over transmission lines to test full size tranformers and other apparatus to determine how to make them highly resistant to lightning; scientific work on the time lag of gaps and insulation has been extended, etc. The above list is not complete but will serve to indicate how much progress has been made. The important phases will now be discussed in detail.

II. Laboratory Research

The Lightning Generator. Up to the early part of 1927 the laboratory lightning work had progressed so far,[1] that it seemed important to double the 2,000,000 volts available at that time. This high voltage was desirable so that full size apparatus could be tested and results obtained without extrapolation. A 3,600,000-volt generator was built and is in satisfactory operation; and an extension is now available so that about 5,000,000 volts is obtainable. Double the directly generated voltages due to reflection have been measured at the ends of transmission lines.

A radically new method was devised by the author to obtain the very high voltages.[2] The effect is of adding two, three, four, or more of the original generators in series at the proper instant so that all of the respective impulse voltages add together. No rectifiers are used. The a-c. voltage is applied directly to each unit generator. At that instant on the crest of the wave when each unit is fully charged, gap sparkovers take place that connect the generators in series and the impulse occurs. The connections are shown in Fig. 1A. The condensers of the three generators C_1, C_2, C_3 are charged from the transformer to a crest voltage corresponding approximately to the G_1 gap setting. Sparkover occurs on G_1 followed immediately by sparks on G_2 and G_3. Resistance R_a, R_b, and R_c, R_1 and R_2 permit the small 60-cycle changing current to flow but are, in effect, infinite to the very high impulse current. The result is as shown in Fig. 1B. Only three gaps are in series on 3,600,000 kv. and four on 5,000,000 kv. One, two, three, four, or more steps can be used. The wave shape is determined by R, L, and C. Waves varying in

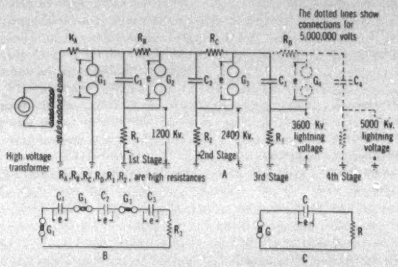

Fig. 1—Circuit Diagram of Lightning Generator

duration from a few microseconds to a thousand microseconds have been experimented with. A capacity of at least 0.0034 microfarads is generally used per unit. The maximum energy is about 14,000 watt-seconds. Fig. 1C shows the original single-unit generator.

The above generator is more satisfactory for very high voltages than where the a-c. voltage is rectified, and capacities are charged in multiple and then discharged in series. To obtain the above voltage with the Marx connection,[3] 35 to 50 units with 35 to 50 spark-gaps would be required. However, the Marx arrangement is satisfactory for low or moderate voltages.

Fig. 2 shows a 3,600,000-volt flash from the lightning generator. The maximum sparkover distance possible with such a voltage depends upon what wave shape of surge the lightning generator is adjusted to give. With a surge of a very short duration, a sparkover of

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(On January 25, 1929, after the original writing of this paper, the voltage of the lightning generator was increased to 5,000,000 volts, and laboratory lightning research started at that potential).

1. See bibliography for all references.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

only 9 ft. can be secured at 3,600,000 volts crest. Longer distances can be broken down with long waves, as will be explained later, as much as 20 ft. being possible with a 1000-microsecond front. The lightning generator condenser units are shown in Fig. 3.

(*a*) *Measurements at very high voltages*

Fig. 2—3,600,000-Volt Artificial Lightning Stroke

Fig. 3—Lightning Generator

Voltage. Considerable skill and experience is necsary to measure high lightning voltages of very short duration. During the pioneer work, it was necessary to check the voltage measurements in a number of ways. The voltages were first calculated from the circuit arrangement; a check by 100-cm. sphere-gaps up to 1500 kv. was made; readings were taken with a surge voltage recorder[1] or klydonograph;[5] as a final method measurements up to the maximum voltage were made with a capacity potentiometer. Surge-voltage recorder records of 5,000,000 volts from the lightning generator are shown in Fig. 4.

Wave Shape. In the first studies of transients, wave shapes could not be pictured directly; it was necessary to calculate them. The cathode ray oscillograph[6] now affords a means by which oscillograms can be taken readily. It is interesting that these oscillograms measuring time in microseconds check the early work.[2] Fig. 5 shows a typical oscillogram. This particular wave reaches its crest in a fraction of a microsecond and then decays to half value in five microseconds. Surges are frequently measured along a high-frequency timing wave as in Fig. 6.

As a matter of convenience, such waves will be

Fig. 4—Surge-Voltage Recorder Records of 5,000,000-Volt Artificial Lightning

reduced to rectangular coordinates as in Fig. 7. The Fig. 7 wave has been used extensively in sparkover tests and is believed to approximate the effects of actual lightning.

(*b*) *Sparkover of spheres, points, and insulators*

The full line curves in Figs. 8, 9, and 11 give the sparkover voltages for different gaps with the Pittsfield high voltage laboratory "standard wave" and other waves. In making these curves, the full effect of the wave was used; that is, the impulse for a given gap setting was increased until sparkover just did occur. In addition curves are given for sparkover on the crests of the waves. Breakdown takes place on the crests of the wave with applied crest voltages considerably higher than those just necessary to cause sparkover. Fig. 12 shows that the lightning sparkover depends upon the length of the string or the spacing of the units. Five and three-fourths in. seems to be approximately the right spacing for 10-in. diameter disks.

(*c*) *The effect of wave shape on the lightning sparkover. Polarity*

With the exception of gaps between electrodes

producing a uniform field the lightning or impulse sparkover voltage is always appreciably higher than the 60-cycle sparkover voltage. The steeper the wave or the shorter the duration of the transient, the higher the crest sparkover voltage. With an exceedingly steep or short wave there may even be a measurable increase for spheres. The lightning breakdown voltage will thus vary because lightning surges vary. The ratio of the lightning to the 60-cycle crest sparkover voltage is

Fig. 5—Typical Five-Microsecond Wave of Lightning Generator

Fig. 6—Wave Measured with Oscillatory Sweep on Cathode Ray Oscillograph

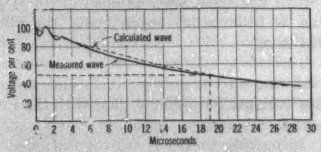

Fig. 7—Lightning Wave of 3,600,000-Volt Lightning Generator Calculated and Wave as Measured by the Cathode Ray Oscillograph

always greater than unity. Some years ago, this was termed the impulse ratio.[2] Under the usual severe lightning conditions in practise, insulator sparkover voltages give an impulse ratio of two. This has been well established by comparing the lightning sparkover voltages of insulators as measured in the field by the surge voltage recorder and the klydonograph with the 60-cycle sparkover voltage. The impulse ratio is thus an indication of the effective duration of the wave. The wave in Fig. 7 gives approximately an impulse ratio of two. Points measured on transmission lines are indicated on one curve of Fig. 11. The standard high-voltage laboratory wave was established before the field data were available. The laboratory lightning thus corresponds to the wave causing sparkover and damage in practise, for if the actual lightning wave were of effectively longer duration the impulse voltages could not be so high.

Whether the front or the tail of the wave is the

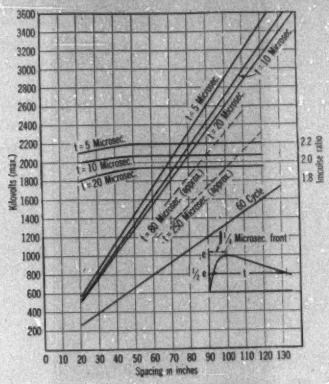

Fig. 8—Point-Gap Sparkover for Different Lightning Waves

See Fig. 5 for oscillogram of five-microsecond wave.

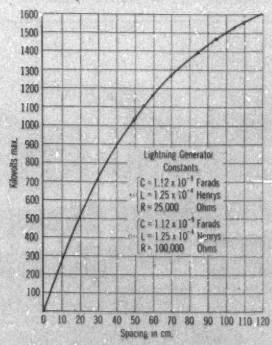

Fig. 9—Sphere-Gap Sparkover

100-cm. diameter spheres—one sphere grounded.

controlling factor in determining the lightning sparkover depends upon the voltage applied. This is well illustrated in the oscillograms of Fig. 13 representing actual test records on a 19.9-cm. point-gap. The same wave shape was used throughout these tests. In the first tests the voltage was increased until sparkover occurred at 50 per cent of the applied impulses. An impulse wave with a crest voltage of 175 kv. was required for

breakdown while the 60-cycle crest sparkover value for the same distance was 125 kv. The impulse ratio was accordingly 1.40. The actual sparking points on the wave are indicated by the crosses. An interesting

FIG. 10—LIGHTNING SPARKOVER OF 100-CM. SPHERES FOR WAVE OF VERY SHORT DURATION

fact was found here probably for the first time—namely, that sparkover actually took place after the tail of the wave had decreased below the 60-cycle

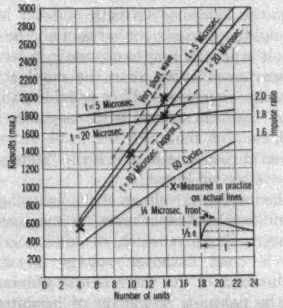

FIG. 11—LIGHTNING AND 60-CYCLE SPARKOVER CURVES OF SUSPENSION INSULATORS FOR DIFFERENT WAVES

See Fig. 5 for oscillogram of five-microsecond wave

value. Apparently the breakdown effect, once started by the overvoltage, continues, so that the spark actually forms after the wave has fallen below the minimum 60-cycle crest sparkover voltage. This is of great theoretical interest, but space does not permit further discussion here. A wave 57.5 per cent in excess of the minimum impulse sparkover voltage was next applied to the gap. As can be seen from Fig. 13, sparkover still took place on the tail but at a higher value. Breakdown of the gap occurred on every applied impulse. With 130 per cent excess voltage, sparkover took place practically on the crest of the wave, while at 178 per

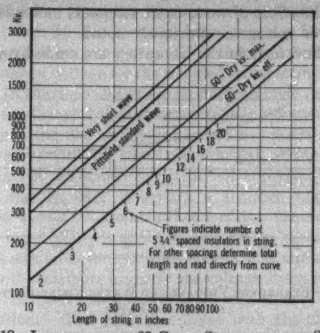

FIG. 12—LIGHTNING AND 60-CYCLE SPARKOVER FOR STANDARD 10-IN. DIAMETER SUSPENSION TYPE INSULATOR

Length of string and actual physical length between center lines of bolt holes; not actual arcing distance or creepage distance

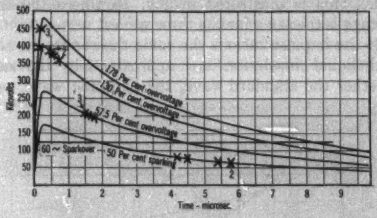

FIG. 13—SPARKOVER OF A POINT-GAP WITH A CONSTANT WAVE SHAPE AT VARIOUS APPLIED VOLTAGES

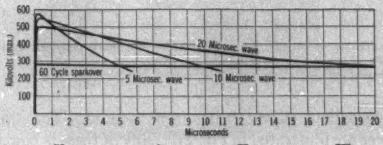

FIG. 14—VARIATION OF SPARKOVER VOLTAGE FOR WAVES OF 5, 10, AND 20 MICROSECONDS DURATION

50-cm. needle-gap—sparking 50 per cent of applied impulses

cent overvoltage, it occurred on the front of the wave. The corresponding impulse ratios for this short gap cover a range from 1.4 to 3.50. Fig. 14 shows the variation in sparkover voltage with waves of the same front but with 5, 10, and 20 microseconds durations above the 60-cycle sparkover voltage.

Tests were also made with waves rising more or less uniformly at various rates and with breakdowns always occurring on the fronts. The results are shown in Fig. 15. For the gap used in Fig. 15, the impulse sparkover voltage approximately equals the 60-cycle sparkover when the time from application of voltage to complete breakdown is approximately 1000 microseconds. This shows the effect of wave-front steepness increasing the sparkover voltage as previously noted.

Standard tests in the laboratory are made by gradu-

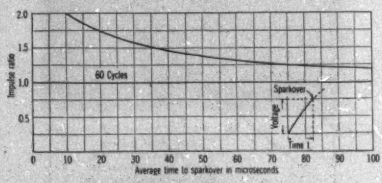

FIG. 15—SPARKOVER VOLTAGE FOR WAVES RISING UNIFORMLY AT VARIOUS RATES

ally increasing the impulse voltage until sparkover occurs on 50 per cent of the applications. The instantaneous breakdown voltage when the front is relatively short then depends largely upon the duration of the tail. This is illustrated in Fig. 16 where the crest voltage is the same for the three waves.

If the impulse ratio is the same the results on solid and liquid insulation are approximately the same whether the standard wave, the lightning wave, or a

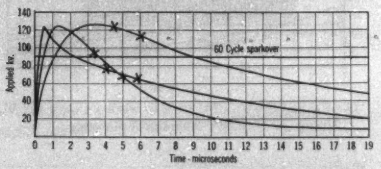

FIG. 16—EFFECT OF WAVE FRONT ON SPARKOVER

Sparkover of 13-cm. needle-gap with given impulse ratio and different waves

Crosses indicate average sparking range

wave with a slowly rising front is used. The three waves are illustrated in Fig. 17. Any of these is equally effective for testing purposes.

A gap between spheres has very little time lag if the spacing is not greater than the diameter of the spheres. In general, therefore, the impulse ratio is practically unity, and the sphere-gap indicates the voltage at the crest of the wave. However, because it is generally desirable to know the effective duration of the wave as well as its crest, in making tests a "time-gap" is necessary. The suspension insulator is a very good gap for this purpose. An example will best illustrate the use of such a gap. Assume that it is desired to compare the lightning sparkover voltage of two entirely different types of bushings, but that it is not possible to do this in the same laboratory with exactly the same waves. A sphere-gap measurement would give the crest of the wave, but equal sparkover voltages would not indicate equivalent bushings unless the shapes of the waves were known. However, a very good comparison can be obtained by the insulator "time-gap" even if the waves differ considerably. This can be done by placing an insulator string in parallel with the bushing, applying impulses, and adding or removing units from the string until 50 per cent of the sparks occurs on each. The equivalent breakdown strength of the bushing is thus obtained in terms of line insulators. Since the impulse ratio of bushings and insulators vary together up and down over a wide range with varying wave shapes, the effect of variations due to such differences is eliminated and a good comparison is obtained. The lightning sparkover voltage of the bushing for any particular wave can then be determined from the lightning sparkover curve of the insulator string. The insulator time-gap also offers a convenient method for

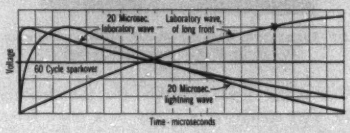

FIG. 17—SPARKOVER VOLTAGE FOR VARIOUS WAVES

comparing the lightning strength of solid insulation. Since the lightning sparkover varies with the length of the string, it is usually best to express it in terms of the 60-cycle sparkover rather than the number of units, whose spacing may vary. It is possible to use other "time-gaps" such as spheres with resistance in series, gaps in oil, etc., but the suspension insulator seems best for practical purposes because it is the "time-gap" that limits the voltage on lines.[1]

When the maximum voltage of the lightning impulse causing an insulator sparkover is measured by a sphere, surge-voltage recorder, or klydonograph and the 60-cycle crest flashover voltage is known, the effective duration of the wave is also obtained. For example, the lightning sparkover of insulator strings measured on the 220-kv. lines of the Pennsylvania Power & Light Company were found to average about 2000 kv. For these insulators the 60-cycle sparkover was about 1000 kv. This indicates an average impulse ratio of 2.0. The usual impulse ratios of natural lightning varies between 1.8 and 2. The crosses in Fig. 11 for four, ten, and fourteen unit insulator strings are flashover voltages due to natural lightning as measured by surge voltage recorders. In a few cases, impulse ratios as high as 2.7 were obtained. These

impulse ratios show that the effective duration varied from 1 to 20 microseconds, where the effective duration is the time that the voltage is above half voltage, or approximately the time above the 60-cycle sparkover. Such waves are illustrated in Figs. 5 and 7 and were actually measured by the cathode ray oscillo-

Fig. 18—Lightning Applied to Shielded and Non-Shielded Insulator Strings in Parallel. Sparkover on Non-Shielded String.—80-Microsecond Wave

graph. Thus a wave giving an impulse ratio between 1.8 and 2 on line insulation represents the average severe field conditions, and the standard laboratory wave, established long before measurements were available, is confirmed as simulating practical conditions. The lightning wave secured on the Pennsylvania Power & Light Company line this last summer had a duration above half voltage of about 20 microseconds.

In the early work[2] on transients it was found that where dissimilar electrodes were used, lower voltage sparkovers occurred with the smaller electrodes positive. This fact has been checked in these recent tests, and the greatest differences in voltage result between a point and a plane. With long insulator strings the polarity effect is not appreciable.

(*d*) *The grading shield*

An important development is the grading shield for insulators.[7] The grading shield bears about the same relation to the insulator string as the ground wire does to the line. An important function of the grading shield is to cause even distribution along the string. This strengthens considerably the path along the insulator surfaces to lightning and forces the arc to take place between the rings which may be set for a lightning sparkover voltage higher than that of the non-shielded string. Destructive cascading is thus prevented. In this way the gain in voltage may be as much as 10 per cent to 12 per cent, and can be checked by comparing the lightning sparkover of the non-shielded string with the needle-gap lightning sparkover of the distance between rings. For instance, the sparkover voltage of a 16-unit non-shielded string from Fig. 11 is 2050 kv. at 20 microseconds. For 85 in. (from Fig. 8) between rings, when the sparkover occurs on a shielded string, it is 2200 kv. For the 20-microsecond wave this is usually over 10 per cent for long strings. For very steep waves it may be more. That there is considerable advantage in voltage for the shielded string is illustrated in Fig. 18 where an impulse of 80 microseconds duration above the 60-cycle sparkover voltage is symmetrically applied to the two strings connected in parallel. The flashover occurs on the non-shielded string. The difference in sparkover voltage is not appreciable with longer waves. Fig. 19 shows a shielded and non-shielded string. Careful adjustment to prevent cascading must be made at the steeper waves because less time is available for allowing corona and surface discharges to distribute properly the voltage stress over the units. Sufficient time is available with slower voltage applications. The gain in voltage with the shield is incidental as its important function is to prevent cascading.

While the sparkover voltage to lightning waves may be increased by the shield, the 60-cycle sparkover voltage may be lowered. This is not a handicap because lightning surges having an impulse ratio of unity and thus corresponding to 60-cycle waves have never been observed in practise. The dry 60-cycle shielded spark-

Fig. 19—Typical Lightning Arcs on Shielded and Non-Shielded Insulator Strings

over voltage might be somewhat increased by using *very* large shield surfaces free from sharp ends or points. However, there can be no gain in practise in this way because the large surfaces would be reduced to equivalent "points" in 60-cycle voltages when wet by the first raindrop. Lightning sparkover voltage is not affected by rain.

The oscillogram in Fig. 20 shows the successive cascading of the units in a non-shielded string. That shields prevent deterioration of the units in a string through improved distribution of voltage stresses is forcibly illustrated in tests. After a few lightning sparkovers, insulator units fail in the non-shielded strings while there are no failures in the shielded strings.

In addition to the actual increase in lightning spark-

Fig. 20—Cathode Ray Oscillogram of Successive Breakdown of Units in Non-Shielded Insulator String of Three Units

(a) Line unit cascade
(b) Second unit cascade
(c) Complete string flashover

over voltage discussed above, there is also an apparent increase which is probably of more importance. When the energy of the lightning generator is limited, it is necessary to supply a higher voltage to a shielded string to cause sparkover. This apparent increase in sparkover voltage may be of a higher order than the actual increase. The extra voltage must be generated because of the energy dissipated by the "barrel" of corona between the edges of the rings. (See Fig. 21.) The gain has been observed when the energy available approximated that in an average span and should be an approximate measure of the effect in practise since there is one shield for each line per span. This energy dissipating effect by corona has been made use of by purposely designing grading rings of flat strap material in place of smooth surfaced pipes.

The results of lightning sparkover tests with the strings excited at normal 60-cycle voltages were not different from tests on non-excited strings.

From the above it can be seen that a successful shield must grade and increase the strength along the string so that sparkover is forced to occur between rings rather than over the surface of the insulators with the shield at the same time maintaining a high 60-cycle flashover voltage; that the design must be such as to dissipate the maximum energy by corona and thus have the effect of increasing the impulse sparkover voltage; that single sharp points or sudden surface changes are undesirable; that no practical gain results from large rounded surfaces.

From the standpoint of clearing the dynamic arc, complete round or oval rings are highly desirable as a track for the arc when blown by the wind. Anchor points at the ends of a sectionalized shield may cause it to wrap around the string.

Horns cannot prevent cascading without a serious reduction in voltage because they do not properly grade the string. They must be adjusted for a lightning sparkover voltage lower than that of the weak non-shielded string.

(*e*) *Wood poles*

The insulating value of a wood pole to lightning voltages has been measured up to 3,600,000 volts. The measurements show that the strength of wood poles of such varying degrees of wetness and dryness as might occur in practise, range from 100 to 300 kv./ft. A good average value is 180 kv./ft. Thus, a pole 35 ft. high, with a 5-ft. crossarm, would have a lightning sparkover voltage of $40 \times 180 = 7200$ kv. The insulator would add very little to a pole of this length. However, when the length of wood in series with the insulator is not over 10 ft., from 75 to 100 per cent of the insulator flashover voltage may be considered as added to that of the wood to comprise the

Fig. 21—Dissipation of Lightning Energy by Corona on Insulator Shields

total pole insulation. A practical example of this is a pole made conducting by a lightning rod to prevent splitting, with the insulation depending upon the insulator and crossarm. In a case of this kind, part of the insulation of the pole could be utilized and protection from splitting afforded at the same time by placing a gap in series with the lightning rod. To prevent splitting, the 6-3-1 ratio shown in Fig. 22 should be used.

Whether a pole is wet or dry makes very little

difference on the lightning voltage necessary to cause complete flashover. However, when a pole is quite wet, incipient sparks will take place over the insulator string at voltages approximately equivalent to the lightning sparkover voltage of the insulator string above.

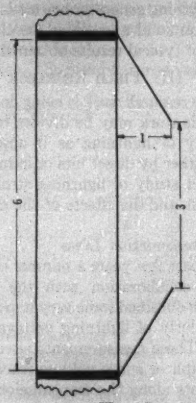

Fig. 22—Proportions for Wood Pole Protective Gap

While wood poles without rods or ground wires may have very high lightning sparkover voltages, there is always a danger of long delays due to split and burned poles that could not be tolerated on important lines. The porcelain insulator is more reliable. However, by use of gaps as illustrated above, part of the insulating value of the wood may be used to advantage on certain secondary lines. Fig. 23 shows a lightning flashover test being made on a 20-ft. pole.

(*f*) *Effect of bus structures on lightning voltages—tower structures*

Tests made on models in the laboratory show that the bus structures of outdoor stations should be of material assistance in reducing transient voltages. There are several effects that help. The grounded steel work acts as a very effective ground wire system which may reduce induced voltages very considerably. Tests on line models built to scale often show as low as one-third voltage when bus structures are added. Full effect of this is not obtained in practise due to the limited physical length of the structures compared to the cloud. A wave traveling to the bus structure would be reduced in voltage due to the reduction in surge impedance. The massed capacity effect of the bus would prevent high voltage reflection. The effects in practise should be quite effective for waves chopped short by insulator arc-overs. Several extra ground wires of a half mile or more in length extending out from a station, should, because of reduction in surge impedance, be very effective in reducing the voltage of incoming waves. On the other hand, tests show that the omission or reduction in the ground wires at the station causes a rise in voltage.

Tests on models have been very useful in determining the best arrangement of ground wires, the effect of high towers at river crossings, etc. Tests are also under way to determine the practicability of protecting towers from direct strokes by rods.

(*g*) *Transformers and Transformer Insulation.*

The new lightning generator has made possible invaluable studies on full size transformers and insulation arrangements. It has long been recognized that the internal insulation of a transformer should be stronger than the bushing while the bushing in turn should be stronger than the adjacent line insulation. Research on transformers has been made by applying lightning waves over a line insulated in the usual way. The general method is to apply gradually increasing impulses until the insulators spark over. Insulator units are then added until either the bushing sparks over or the internal insulation fails. If failures occur internally, the weak points are then

Fig. 23—Lightning Sparkover of 20-ft. Wood Pole

strengthened until the flashovers occur on the bushings. The insulator is ideal as a voltage limiting gap for such tests because it performs the same function in practise, limiting the surges in duration as well as magnitude.

Cathode ray oscillograph records are taken and the voltage distribution is measured throughout the winding. This last measurement is of extreme importance since it shows that in the usual transformer the voltage

distribution is not constant but varies with steepness and duration of the impulse or the frequency of the transient. High frequencies and steep impulses may cause excessive voltages at any part of the winding. The ideal transformer would be one in which the voltage distribution was the same for all frequencies and wave shapes. Fortunately it has been possible to accomplish these results by the shielded design, which is an entirely new type. It will not be necessary to go into details here as this transformer is described elsewhere.[8]

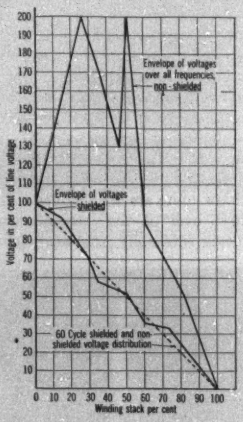

FIG. 24—VOLTAGE DISTRIBUTION AT ALL FREQUENCIES OF SHIELDED AND NON-SHIELDED TRANSFORMERS

Fig. 24 shows the results of tests on an actual transformer. In the shielded transformer the impulse and high-frequency distribution is shown to be practically the same as the 60-cycle distribution. The transient distribution curve, which is the envelope of voltages for all high-frequency transients and impulses, is quite startling. It shows that the shield reduces local transient voltages as much as 80 to 1 and that lightning failure in a non-shielded winding may occur anywhere in the winding depending upon the wave. In the shielded winding free from localized stresses under all waves, breakdown is as definite as an insulator flashover. The above lightning tests are, of course, design tests and not intended for commercial testing. This follows because dismantling every transformer in the routine factory tests to detect possible internal failures is impractical. However, the shield removes the necessity for lightning tests since the impulse distribution becomes the same as that at 60 cycles.

Briefly the reason for the varying distribution of voltage in a non-shielded transformer is as follows: The initial lightning distribution is determined by the distribution of the capacity in the windings and the 60-cycle or long duration voltage distribution by inductance. If the voltage distribution as determined by these factors is not the same an oscillation results until the distribution corresponds to that of the inductance. The shields make the capacity and inductance distribution correspond. The action of the capacity is instantaneous and there is no oscillation.

Insulation

The new lightning generator has made possible extensive research on solid and liquid insulation. Fig. 25 gives a curve of typical results obtained.

III. FIELD RESEARCH

Considerable research work is being done with natural lightning. This work may be divided into two classes:

(a) A study of lightning as it appears on transmission lines either by direct hits or induction.

(b) A direct study of lightning strokes, the clouds producing them, and the effects of the strokes on rods, etc.

Research on Transmission Lines

During the past few years a number of the operating companies in collaboration with the manufacturing companies have obtained some very important measurements, particularly of lightning voltages on transmission lines.[9,10] These measurements were obtained with the klydonograph or surge voltage recorder connected at various points along the transmission lines.

(*a*) *Voltage, polarity, wave shape, limitation of voltage.* The surge-voltage recorder measures the maximum of the wave and indicates the polarity. By comparing the insulator sparkover voltage resulting

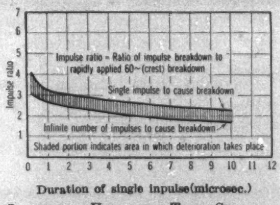

FIG. 25—LIGHTNING VOLTAGE. TIME STRENGTH OF SOLID INSULATION

Typical curve—actual values will vary for different kinds of insulations

from lightning with the 60-cycle sparkover voltage the impulse ratio is obtained. The impulse ratio gives a good indication of the effective duration of the wave. The lightning voltages causing sparkover on transmission lines in various sections of the country give an impulse ratio of the order of two. Waves giving such ratios are shown in Figs. 8, 11, 12. The maximum impulse ratio observed was approximately 2.8.

In the many measurements made, the very low voltages, necessarily induced, were mostly positive, indicating a negative cloud. Most of the excessively high voltages,—probably direct strokes,—were negative, also indicating negative clouds. A few high positive and

low negative voltage surges indicated that some clouds are probably positive. The general indications were that the waves were non-oscillatory.

Measurements on different lines in various parts of the country and on the 220-kv. lines of the Pennsylvania Power & Light Company with 14 and 4 unit insulation show that the maximum lightning voltage on transmission lines is limited to the lightning spark-over of the insulators. Measured points are shown on the laboratory curve, Fig. 11.

(*b*) *Attenuation.* Successive surge voltage readings

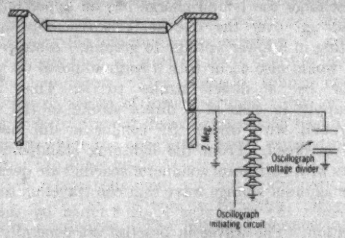

Fig. 26—Method Employed in Measuring Natural Lightning Voltages

of a given lightning wave traveling on transmission lines have shown how the waves attenuate or decrease in voltage. The attenuation or loss in voltage per mile was found to vary as the square of the crest voltage. Thus really high voltages cannot travel far.

Measurement of Lightning Waves and Time Required for a Cloud to Discharge with the Cathode Ray Oscillograph

Fig. 27—Oscillogram of Natural Lightning Made on Short Lines—Pittsfield 1928. Drawn Dotted Lines Indicate Approximate Fronts

A portable cathode ray oscillograph, of the Dufour type developed by Mr. Lee and his staff of the General Engineering Laboratory[6] of the General Electric Co., made possible during the past summer the measurement of wave shapes of actual lightning. In order to make proper use of the oscillograph, it was necessary to devise a means of establishing the cathode beam and the sweeping circuit and to have the complete set-up connected to the line as the lightning wave reached it. With the equivalent "switching circuit" developed for this, the complete operation was accomplished in about one microsecond—that is, one-millionth of a second. Part of the wave front would accordingly be lost unless special means were taken to prevent it. One way to accomplish this is to side track the wave around a loop about 1000 ft. long, requiring about one microsecond to travel it, and take it coming back. However, the results so far indicate that this refinement will usually not be necessary. The actuation of the oscillograph is done by means of spark "switches" controlled by the lightning surge. Connection to the line was made by an insulator potentiometer.

Measurements were made on short horizontal antennas and on actual transmission lines. The measurements of the antennas were made at Pittsfield while the transmission line measurements were made on the 220-kv. lines of the Pennsylvania Power & Light Company in cooperation with their engineers.

(*a*) *Pittsfield measurements.* In the Pittsfield measurements the antennas consisted of three parallel wires 120 ft. long and 40 ft. above ground. The wires were

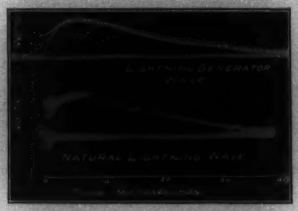

Fig. 28—Comparison of Natural Lightning Wave Measured on Transmission Lines with Cathode Ray Oscillograph with an Artificial Lightning Wave Measured in the Same Way

grounded through a 2,000,000-ohm resistance and connection to the oscillograph was made as in Fig. 26. With this arrangement the lines assume a potential opposite to that of the cloud when the lightning discharge takes place. Since the charge cannot move along this short line, but must be dissipated by leakage, the potential of the conductors rises at a rate and to a magnitude dependent upon the collapse of the cloud field. The time for this conductor voltage wave to reach maximum is thus a measure of the time required for the cloud to discharge. Fig. 27 shows two of the four antennas waves obtained. The wave fronts are of the order of one to two microseconds. The induced voltage crests on the antennas were from 50 to 75 kv. The storm clouds in each case were at least a mile away. Such wave measurements, indicative of actual cloud discharges, are very helpful in determining the shape of surges themselves on transmission lines. A mathematical analysis to correlate these field results with theory is now being undertaken.

(*b*) *Pennsylvania measurements.* A very good wave

of natural lightning obtained on the 220-kv. line is shown in Fig. 28. The front of this wave practically reaches its maximum in 5 microseconds, decreases to half value in 20 microseconds, and reaches zero in 40 microseconds. The oscillating ripple is apparently due to a local flashover[12] and is not really part of the original wave. A reproduction of this wave by the laboratory lightning generator is also shown in Fig. 28. The effects of the wave are very similar to the standard wave of Fig. 7 and the impulse ratio for insulator sparkover corresponds to those determined by the surge recorder or klydonograph readings. This oscillographic study of lightning surges on actual lines is to be continued on a greater scale next year.

Study of Direct Strokes

Fig. 29 is an illustration of the method used in studying the effects of direct strokes. Photographs showed certain office buildings struck. An examination was made to see if any of these hit fell within the protective cone of other buildings as established by former laboratory tests.[1] It was found that they were in agreement with tests on models.

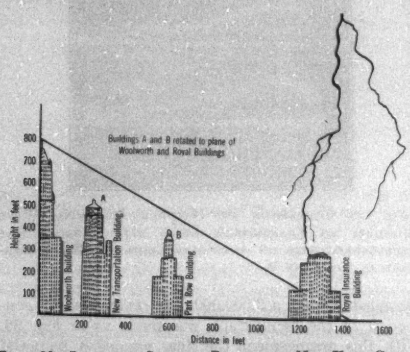

FIG. 29—LIGHTNING STRIKING BUILDING IN NEW YORK CITY—PROTECTIVE CONE IS NOT VIOLATED—FROM PHOTOGRAPH

Estimated cloud height, 2800 ft.

IV. LIGHTNING ON TRANSMISSION LINES AND APPARATUS

A consideration of the foregoing in conjunction with former work shows that a fairly clear picture of what takes place during a thunder-storm is now available. It is reassuring that the large advances in research made during the past year tend to confirm and clarify what had already been done. Because of this advance in research it seems desirable at this time to re-examine present transmission practise and see if changes are desirable.

Voltage

The available data still confirm the rule that the maximum induced voltage on a transmission line depends upon the height of the conductors above ground or

$$V = h\,g\,\alpha$$

Where V = volts, h = average height of conductor in feet, and α depends upon rate of cloud discharge and the initial distribution of bound charge on the line. Calculations indicate that under the usual assumed bound charge conditions, an induced voltage wave high enough to cause insulator flashover can result only from a rapid collapse of the electrostatic field of the cloud. Accordingly this must involve a traveling wave of steep front or of short effective duration. With a slow rate of discharge the bound charge has an opportunity to spread out over the line as it is released, thereby resulting in a lower voltage to ground. A steep wave front would also occur with a surge imposed on a conductor by a direct stroke to it. This would also apply in case of a direct stroke to the tower or ground wire where the conductor did not become involved. From the lightning standpoint low towers with horizontal conductor spacings are desirable. The maximum voltage wave that can travel on the line and reach the apparatus is determined by the line insulation. The waves in practise are generally non-oscillatory and have a wide variety of shapes. However, the waves usually causing insulator sparkover give an impulse ratio of the order of two (2.0), and indicate an effective duration of 1 to 20 microseconds above the 60-cycle flashover value.

Loss in Voltage as the Lightning Wave Travels on the Line—Attenuation

The distance that lightning can travel at high voltage on a transmission line is of extreme practical importance. The following table of data from the curve in Mr. Lewis' paper[10] bears this out.

Lightning voltage kv.	Distance of travel for reduction to half voltage (mi.)	Distance of travel for reduction to 0.80 voltage (mi.)
4000	1.5	0.4
3000	2.1	0.5
2000	3.0	0.7
1000	6.3	1.6

For example, 4000 kv. is reduced to half voltage or 2000 kv. in 1.5 miles. This indicates that the badly exposed section of a line normally insulated for 2000 kv. could be highly over-insulated without much danger of subjecting the normally insulated section to excessive voltages. It is only necessary to extend the highly insulated line several spans beyond the exposed section. It also indicates that moderate reduction of insulation at a substation, for instance, should not appreciably increase the outages. For example, 2000 kv. is reduced 20 per cent in traveling less than a mile and 50 per cent in three miles. Any outages within the reduced insulation section must be attributed to local storms and would not be due to line surges coming into the station from outside normally or abnormally insu-

lated sections. Further data on the attenuation factor on other lines are of course desirable.

The Ground Wire

Statistical data still confirm the value of the ground wire. These data indicate that outages due to insulator arc-overs are reduced from one-half to one-tenth or more after the installation of the ground wire. On one 220-kv. line the installation of the ground wire over two sections did not seem to have any decided effect on the lightning arc-over voltages. The reason for this seems to be that, because of the peculiar location of the line, the arc-overs are caused mostly by direct hits. Where the spans are long, where the ground wires are near the line conductors, or where the ground resistances are high, a direct stroke is likely to involve the line conductor. This suggests taking special precaution in design of towers, ground wires, or protecting rods to prevent direct strokes side flashing to conductors where the lines are in badly exposed places and subject to direct hits.

The laboratory tests point out that a very important function of the ground wire in reducing voltages is generally overlooked. The lightning corona loss is materially increased by ground wires which greatly limits voltage. The action is the same as on the grading shield in Fig. 21.

The surge impedance of the lines is reduced by the addition of ground wires. A reduction in the surge impedance reduces the voltage of traveling waves. It is thus important to bring all ground wires up to the steel bus structure or to the station. Extra ground wires extending about one-half mile from the station should be of great value for station protection. Voltages caused by local storms would be reduced and waves from distant storms would be lowered to harmless values by attenuation and reduction in surge impedance.

Lightning Proof Transmission Lines and Coordination of Transformer Insulation and Line Insulation

It is interesting to consider again if a lightning proof transmission line is possible or practicable. It has been shown that the lightning voltage is independent of the operating voltage and depends upon the height of the line; that the ground wire greatly reduces the lightning voltages; and that the lightning flashover voltage of insulation and the breakdown voltage of apparatus is known. A consideration of these factors shows that a line of moderate height, protected with ground wires and properly insulated, could usually be made lightning proof against induced voltages at a reasonable cost. This comparison is made by calculating the maximum induced voltage from the formulas given above, making proper allowance for the ground wire, and using insulator strength in excess of these values as determined by Figs. 11 and 12. In order to make a line immune from direct strokes the necessity of ground wires above the line is almost obvious because, no matter what the insulation, the limit will be the sparking distance from line to ground. With ground wires the stroke would usually take place to the wires and then along the wires to the tower, preventing insulation arc-over. However, where the line is badly exposed to direct strokes special precaution should be taken in the design of the tower so that side flashes are not likely to take place to the conductor. Rods might be used and special precautions taken as to length of span, ground resistance, distance from conductor to ground wire, etc. Extra ground wires in sections may be necessary to assure immunity against direct strokes.

The limit of the voltage in any line is the lightning flashover voltage of the insulator. It is very important, therefore, when designing a system, so to proportion the insulation that the transformer lightning breakdown voltage is higher than the lightning breakdown voltage of the line insulators in the vicinity of the station.[11] It is obviously not good engineering to make the transformer the weakest link in the insulation chain. The insulators on the rest of the line may be as strong as desired.

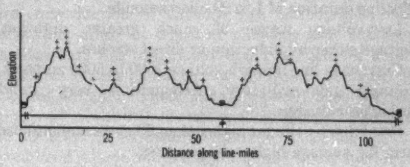

Fig. 30—Distribution of Flashovers along a Transmission Line

+ = Insulator flashovers

The grading shield is as important to the insulator string as the ground wire is to the line. It not only reduces the maximum stress but increases the arc-over voltage and causes the arc to clear the string. The horn and similar arcing devices cannot cause the arc to clear without a serious reduction of the flashover voltage.

In addition to the above factors, the location of the line is very important. See Fig. 30, which suggests extra insulators and ground wires on the exposed sections. The ideal line would thus be as low in height as practicable, be protected by one or more ground wires, and be well insulated with insulators protected by grading shields. The transformer insulation should be somewhat stronger than the bushing flashover voltage, which in turn should be higher than the flashover voltage of the insulators in the immediate vicinity of the line. By immediate vicinity is meant that the coordinated insulation should start within 75 ft. of the apparatus and preferably extend out at least one-half mile. As pointed out above, such arrangement of the insulation should not increase outages. As a precaution, extra ground wires may be added on the coordinated section to provide against local dis-

turbances. With extra ground wires the lightning voltages can be reduced in proportion to the insulator strength. This coordination of insulation has been in effect now for several years on one system with results as anticipated. It is not intended to take the place of the lightning arrester, however, for good lightning arresters are to be recommended as in the past.

V. Summary of Characteristics of Lightning

Voltage — order of 100,000,000
Current — order of 100,000 amperes
Energy — order of 4 kw-hr.
Power — order of thousand billion hp.
Time — order of a few microseconds
Gradient at breakdown 100 kv/ft.

Time for cloud to discharge from one to ten microseconds.

Discharge generally non-oscillatory. Total energy dissipated in world by lightning 1,200,000 kw. continuously. (Very approximate.)

Lightning on Lines

High-voltage waves of few microseconds front and effective duration of 1 to 20 microseconds.

Low-voltage waves of much greater duration. Voltage either by induction or direct strokes.

Current in line for high voltage 2000 to 5000 amperes. Voltages non-oscillatory. Attenuation very rapid for higher voltages.

Breakdown voltages higher than 60-cycle and impulse ratio for average high voltages, two.

Voltage increased directly as height of line—usual ground wires reduce to about one-half. Maximum voltage $V = g \alpha h$ but limited by lightning arc-over of insulator.

Transformer insulation should be stronger than adjacent line insulation.

Approximate lightning proof line seems feasible.

Laboratory—Voltages higher than ever observed on lines have been obtained and design tests made on full size apparatus.

Acknowledgments

The laboratory cathode ray oscillographic work described here was carried out by Messrs. H. L. Rorden and J. C. Dowell of the laboratory staff. The transmission line field work with the oscillograph was done by Messrs. C. M. Foust, E. A. Evans, and N. Rohats. To Mr. G. D. Heye is due credit for the assembling and plotting of the impulse flashover results obtained with the lightning generator.

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Discussion

For discussion of this paper see page 468.

Theoretical and Field Investigations of Lightning

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Fellow, A. I. E. E. Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper gives a review of some recent developments in the methods of studying lightning phenomena. The Norinder form of cathode ray oscillograph and its application in Tennessee are discussed, together with the information secured.

The second part gives the theory of traveling waves along transmission lines. Reflections at open and grounded ends are considered. A mirror scheme of an infinite series of waves on a double infinite line equivalent to actual waves along a finite line is developed.

The third part discusses the manner in which surges are actually produced on lines by lightning and the effect of ground resistance on the protection afforded by ground wires, both with respect to induced and direct strokes.

The Importance of Lightning Research. During the last few years the increasing importance of the solution of the lightning problem in electrical systems, particularly in long transmission circuits, has resulted in a tremendous increase in the amount of attention given the problem all over the world. The problem has changed from the earliest status of mere scientific interest and the status of perhaps ten years ago, that of convenience, to the present status of very definite necessity. Electrical systems are being reconstructed with the idea of generation in large plants at the most economic point and interconnection between such plants to take advantage of diversified load factors. This requires long distance transmission of large amounts of power between these plants and to the load centers, which, added to the natural growth, involves expenditures in the central station industry of nearly a billion dollars a year. Of this, possibly 150 million dollars a year is expended for transmission circuits. Under the present conditions which demand substantially complete reliability, lightning has been found to be the operating condition which limits the design of transmission circuits. The electrical industry must accept the fact that these enormous expenditures give full warrant for whatever research expenditure and effort may be necessary to provide the solution to the lightning problem.

I. Recent Developments in Field Investigations of Lightning

Status of the Work. The situation has been recognized by many workers in the industry and widespread efforts toward the solution of the problem have been made. In Germany there is a society set up for the purpose of studying the nature of these voltages in transmission circuits and this society has been active for the past few years in a broad program of apparatus development and field study. This work is supplemented by some individual effort on a smaller scale in other European countries. In this country the use of the klydonograph has been widespread and the interest has extended to practically the entire industry. Notwithstanding the great advances in the understanding and knowledge brought about by these studies it has been shown that even the very complete use of these instruments has not given all that is necessary and the indications are clear that continued work along the same lines will not solve the problem. The need has been recognized for new instruments and these have been developed and given preliminary trial in actual use. These new instruments and the work which has been done with them will be described.

Application of Cathode Ray Oscillograph. Since the first introduction of cathode ray oscillographs into laboratory study of transient phenomena it has been realized that this instrument would be ideal for determination of the character of lightning voltages in exposed circuits. There have, however, been barriers to this use of the cathode ray oscillograph which seemed at the start to be insurmountable. That the instrument itself is large and is not well suited to use outside of good laboratory conditions, and that it requires rather elaborate accessory equipment, including vacuum pumps of high quality, are disadvantages but these are not insurmountable. The major difficulty has been in providing means for having the oscillograph in operation on the arrival of the transient. If the electron jet is permitted to strike the photographic film or plate in one spot for more than one ten-thousandth of a second or so, a large part of the film becomes so badly fogged as to prevent any decipherable records. It is essential to the solution of the problem to secure records of the entire duration of the abnormal voltages and particularly of the beginning of the wave fronts. Thus, if means are devised to anticipate a transient which may occur at any time, it is necessary that the oscillograph be started by the transient itself, and with zero delay. This is a problem of no small magnitude.

The earliest approach was made some years ago in Germany[2] by use of a Kipp relay. With this arrangement the cathode tube is continuously energized but the jet is deflected to one side until the transient arrives. The transient voltage is led directly to the relay which by an ingenious arrangement brings the

1. Of the Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

2. D. Gabor, (*Archiv für Elektrotech.*, Band 18, Heft 1)—(Fortschritte im Oscillographieren von Wanderwellen.)

jet quickly to the edge of the film and then more slowly sweeps it across the film providing the time scale. The transient voltage is led to the voltage deflecting plates of the oscillograph through a delaying cable to compensate for the time required to bring the jet to the film.

A second approach was made by G. F. Harrington in July 1926. In this scheme the source of high unidirectional potential for the electron jet is blocked by means of a three-element vacuum tube, the initial

Fig. 1—Oscillogram Taken at Open End of Rankin-Wilmerding Line

grid potential of which is neutralized by the transient itself. Thus, the oscillograph is ready to operate but the electron jet is prevented from forming by the blocking effect of the tube. When the transient arrives and neutralizes the initial grid potential the electron jet is formed. The time required to form the jet may be compensated for by use of a delaying cable as with the Kipp relay.

These schemes appear promising but the results which have been made public thus far, as well as our experience, indicate that before records can be obtained of the whole wave front some further developments will be necessary. Our experience indicates that there is an irregularity in the time intervals to bring the jet to the film after the arrival of the transient, which is comparable to the time of building up of the wave fronts to be measured.

Early Field Trials. The early work in the laboratory with the Harrington scheme had indicated the need for further trials on a circuit of considerable length in order to evaluate this deficiency and attempt to find ways around it. In order to provide for such work and to carry on other investigations beyond the limitations of concentrated laboratory equipment, permission was obtained in 1926 from the Duquesne Light Company for two years use of a six-wire circuit extending from Rankin to Wilmerding, a distance of about five miles, close to East Pittsburgh, Pennsylvania. A surge generator of conventional type was set up at the Rankin end of the line and measuring equipment, including klydonographs and a Dufour type oscillograph, was provided at the Wilmerding end. Transients were introduced into the line by direct discharge of the surge generator at the Rankin end and also by the use of an artificial cloud arrangement by which a half mile of one or more conductors over the Rankin end of the circuit were charged and discharged inducing and releasing bound charges in the continuous conductors. The transients thus introduced were observed with the oscillograph at the Wilmerding end and by means of the klydonograph at an intermediate point.

Representative records from this test are shown in Figs. 1, 2, and 3, which illustrates that the major benefit of this kind of work is additional quantitative determination of the line constants affecting surge propagation. Observations were made illustrating the control of surge current by surge impedance of the line, reflec-

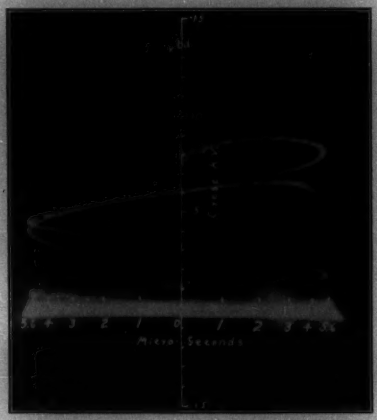

Fig. 2—Oscillogram Taken at End of Line with Lightning Arrester, Rankin Wilmerding Line

tions from open and grounded terminals, attenuation with travel on the line, partial reflection by choke coils, and reduction of voltage by various kinds of protective equipment.

In securing this information use was made of a control system between the two stations by which the surge was timed to precede the beginning of operation of the oscillograph by a definite amount. This scheme is illustrated in Fig. 4. Attempts to make the oscillograph wholly automatic by means of devices such as described above, gave no further benefit than to demon-

strate the need for some improvement to make them dependable in securing the first part of the transient. Various methods were tried to overcome these difficulties but none proved successful.

The Norinder Cathode Ray Oscillograph. During these tests it came to our attention that successful work had been done by Dr. Harald Norinder of the

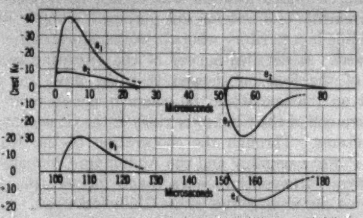

FIG. 3—WAVES RECORDED AT END OF RANKIN-WILMERDING LINE, REPRODUCED FROM OSCILLOGRAMS FIGS. 1 AND 2

e_1 Waves are successive arrivals of surge with line open
e_2 Waves are successive arrivals of same surge but with a lightning arrester at the end of the line

Swedish Royal Board of Waterfalls, Upsala, Sweden, in making cathode ray oscillograms of actual lightning voltages. This indicated the availability of a means for the solution of the problem and accordingly arrangements were made for Dr. Norinder to visit this country to discuss his work and the work being done here.[3] It proved that Dr. Norinder's results had been secured by means of an ingenious improvement over the previously known forms of cathode ray oscillographs by which

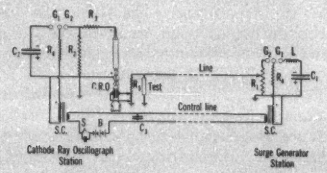

FIG. 4—CONTROL SCHEME USED IN RANKIN-WILMERDING LINE TEST

operation is made entirely automatic on the arrival of the transient. Fig. 5 illustrates the scheme invented by Dr. Norinder.

When it is desired to prepare to make a record of an anticipated disturbance with this instrument, the necessary potential is applied between the anode and cathode resulting in the formation of the electron jet, the useful portion of which passes through the opening in the anode and, since no deflecting potential is applied to the upper deflecting plates, the electrons strike the target and are stopped. Stray electrons are prevented from reaching and fogging the film by the lower diaphram, the orifice of which is wholly shielded by the target. When the transient arrives a proportionate deflecting potential from a potentiometer is applied to the upper deflecting plates and the beam is caused to move off the target. Below the target the beam is again bent back by the reversed potential. By proper proportioning of the length and spacing of the two pairs of deflecting plates the beam is caused to pass through the orifice in the diaphragm for all values of applied potential. The relay function is thus performed by the oscillograph itself and a complete record is made of any transient whenever it may occur. With this

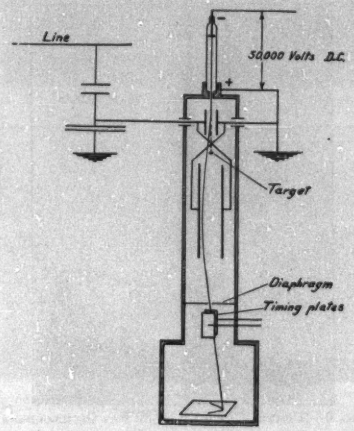

FIG. 5—DIAGRAMATIC SCHEME OF NORINDER OSCILLOGRAPH

scheme it is possible to have the oscillograph in operating condition for a period of two hours or so without any appreciable fogging of the film so that the operation, technique, and field studies reduce to the mere attention necessary to maintain the proper vacuum, etc. However, during the development and laboratory trials of the instrument it was found necessary to take precaution by minor design details against various difficulties, mostly due to the effect of normal line voltage on stray electrons, in order to make the full possibilities of the instrument available in studies on operating circuits. Fig. 6 shows the first oscillograph of this type made for use in this country.

Instruments and potentiometers for the purpose of recording transient voltages on operating lines are designed and used so that the normal line voltage does not deflect the beam beyond the limits of the target.

3. Dr. Harald Norinder, "Some Electrophysical Conditions Determining Lightning Surges," *Journal Franklin Institute*, June 1928.

Dr. Harald Norinder, *The Cathode-Ray Oscillograph as Used in the Study of Lightning and Other Surges on Transmission Lines*, A. I. E. E. Quarterly TRANS., Vol. 47, No. 2, p. 446.

The record of transient voltage thus begins at slightly over the crest value of normal line voltage and extends to the limit of the instrument. It is possible to make records of voltages less than line voltage by filtering schemes using capacitors and resistors. In general, however, the interest is limited to those voltages which exceed the line voltage by a considerable margin. For this reason such schemes have not been used here.

In order to provide the time scale the film may be mounted on a drum and rotated at a high speed. The mechanical limitations prevent speeds in excess of approximately 200 ft. per second. With such an arrangement 100 microseconds would appear as approximately one-quarter inch so that the records show only the magnitude and general shape of the transient. More detailed records can be secured by sweeping the beam electrically across the film or by oscillating the

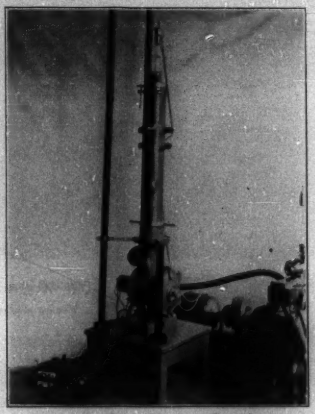

Fig. 6—Norinder Type Cathode Ray Oscillograph

beam at right angles to the direction of voltage deflection. With these schemes any desired time scale can be secured. The deflecting plates used for either the sweeping or oscillating scheme are placed below the target so that they are ready to operate at all times and become effective as soon as the beam is deflected around the target by the transient.

Use of Norinder Oscillographs in Field Studies. Laboratory trials substantiated the experience of Dr. Norinder in establishing the usefulness of the instrument. It was recognized, however, that the complete solution of the lightning problem will require a rather extended use of such devices and that the expense and effort necessary to install and operate the required number of instruments will be great. It was felt to be necessary, therefore, to get final assurance by a trial installation in the field that the instrument will give all the information which is needed. It was further felt to be worth while to secure such information as could be obtained, even with the small number of installations which could be made in the short time available, during the 1928 lightning season. Arrangements were accordingly made for an installation of two instruments in Tennessee and a second installation of a single instrument in Illinois.

The Tennessee location was chosen as it met the requirements of a test for this purpose—a highly insulated line in a territory where lightning storms are very frequent. A detailed survey of the operating record and of the location showed that the most suitable point on the line would be where it passes over the western end of the Chilhowee ridge of the Great Smoky Mountains. Through the cooperation of the Knoxville Power Company and the Aluminum Company of America, arrangements were made for locating two stations, one on each side of the Chilhowee ridge.

Each of the stations was designed to give as complete information as is possible. Each oscillograph, which gives the voltage record on a single conductor, was supported by a series of three klydonographs each for three-phase records, one located alongside the oscillograph and one approximately a mile away in each direction along the line. It was felt to be highly desirable, if not entirely necessary, to get a fairly accurate record of the location of the cloud discharge relative to the line and test station. Accordingly, provision was made to measure and record the difference in time of arrival of the electrical and audible disturbances by the use of a microphone, antenna, and Osiso. The microphones were located about a mile on each side of the test station and the electrical impulses due to the noise of the thunder were brought into the station over circuits provided for the purpose. The Osiso was arranged to start automatically upon occurrence of any disturbance and to record the impulses from the microphone and those originating in the antenna at the station. The difference in time between the record for the antenna and for the microphone provides a measure of the distance from each microphone location to the cloud discharge. By plotting circles with radii corresponding to this distance and with centers at the microphone locations the position of the cloud discharge is determined to be at one of two points where the circles intersect. These points are on opposite sides of the line but are equidistant from the line and from the point where the oscillograph is connected so that there is no advantage in distinguishing between them. The general arrangement of the test stations is indicated by the diagram of Fig. 7 and by the photographs of Figs. 8, 9, 10, and 11.

In order to supplement the electrical records with direct photographs of the lightning flashes without requiring too much attention of the operators, special cameras were provided which photograph the entire sky on a single plate. These cameras which make use of the principle of the "fish-eye camera," but which

secure the results by the use of lenses, were developed by Mr. R. Hill in England.[4] The lens scheme is shown in Fig. 12.

The oscillographs and accessories were designed and built during the period between January and about the first of June, 1928. Installations in the field were completed by the early part of July and from this time the stations were ready to record any lightning which occurred.

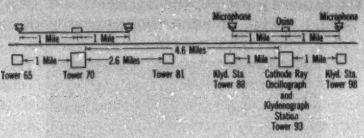

FIG. 7—LAYOUT OF TEST AT CHILHOWEE, TENN.

FIG. 8—TEST HOUSE AT TOWER 70, CHEOAH-ALCOA LINE

FIG. 9—OSCILLOGRAPHIC APPARATUS AT TOWER 93, CHEOAH-ALCOA LINE

The original plan was to provide a third station in Illinois through the cooperation of the Public Service Company of Northern Illinois, but the preliminary work of building the instruments and making the field installation was not completed in time to warrant beginning the test until the lightning season of 1929.

It was planned to make the first few records using the rotating film in order to get a general picture of the nature of the surges and then to take later records using the oscillator for the time scale deflections.

FIG. 10—THREE-PHASE KLYDONOGRAPHS POTENTIOMETERS

FIG. 11—LOOKING NORTH FROM CHILHOWEE MOUNTAIN, SHOWING TOWER 81

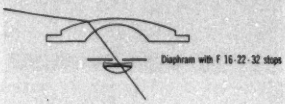

FIG. 12—LENS ARRANGEMENT OF HILL CAMERA

Unfortunately, however, only a single disturbance of appreciable magnitude appeared on the line during the period between the time when the apparatus was set up until November 4 when the test was discontinued for the year. This record is shown in Fig. 13. The corresponding klydonograms at the same station are shown in Figs. 14, 15, and 16. No voltage was recorded on either the klydonographs or the oscillographs at the other station, four and one-half miles away.

The microphone and Osiso arrangements for determining the location of the lightning disturbance were

4. R. Hill, "A Lens for Whole Sky Photography," *Proc. of the Optical Convention*, 1926, Part II.

not in service at the time of this record. The storm conditions were, however, observed by the operators and the following general ideas developed. There were many discharges between clouds apparently directly over or nearly over the line without causing any line disturbances of sufficient magnitude to be recorded by either klydonograph or oscillograph. Strokes to earth were observed in the near vicinity of

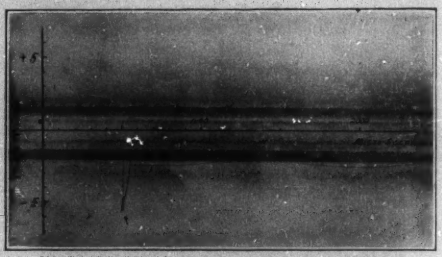

Fig. 13—Oscillogram of Lightning Surge on Transmission Line. 750 Kv.

the line without causing any voltage of sufficient magnitude to make either klydonograph or oscillograph records. The particular stroke which gave the records

Fig. 14—Klydonograph Record at Tower 88, Corresponding to Record of Fig. 13

shown in Figs. 13 to 16 was observed by one of the operators to be extremely close to the station but later examination did not show the exact place where it struck the earth.

Accomplishments of the First Year's Work. Perhaps the accomplishment of greatest importance resulting from the work during this summer is the demonstration that complete oscillograms of lightning voltages can be secured by the use of the Norinder form of cathode ray oscillograph. The klydonograph record shows that only one voltage of appreciable magnitude occurred on the line at an oscillograph station during the test. A complete oscillogram of this surge was secured. It shows the beginning, the entire wave front, the crest, and the entire duration of the transient.

A second finding of major importance is that any

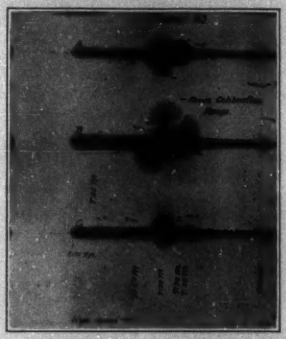

Fig. 15—Klydonograph Record at Tower 93, Corresponding to Record of Fig. 13

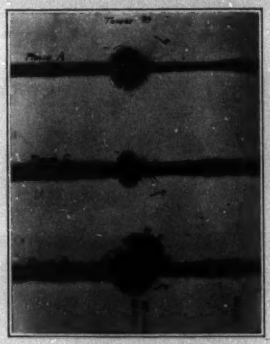

Fig. 16—Klydonograph Record at Tower 98, Corresponding to Record of Fig. 13

large number of serious overvoltages at any one point on a line during a single season is not to be anticipated. This fact has also been indicated by the previous extensive work with the klydonograph. Its significance in connection with lightning research is that it will be necessary to make a considerable number of installations and to operate them for several years in order to secure records covering the entire range of lightning voltages and operating conditions for various territories.

To be certain that any plans for control or protection are adequate it will be necessary to know the maximum rate of rise of voltage, the maximum crest value which might be reached, and the maximum duration. To determine the extent of protective measures which will be economically justified it will probably be necessary also to learn the relation between these maximum values and the average or ordinary values.

It is indicated by the klydonograph work of the past and also by observation of service results that all transmission lines, regardless of the insulation used, are liable to lightning flashover at times. We may conclude from this that the maximum value of lightning voltages exceeds any insulation which has been used thus far. It is possible, even probable, that the margin between the maximum lightning voltage and the present maximum insulation strength is considerable and before adequate control or protection can be provided this margin must be determined. Thus, in order to determine the crest value of lightning voltage which may be reached and to determine the average or ordinary value at the time of flashover it will be necessary to provide in some way for measurements on a line overinsulated beyond the present practicable values. This might be done by providing in some of the installations for possibly doubling the number of insulator units used in a section of the line a few miles long adjacent to the oscillograph. As an alternative it might be done, as has been proposed, by building a special line a few miles long securing a very high value of overinsulation by the use of some such material as wood or rope which, while not satisfactory for continuous service, would serve under transient voltage conditions.

Need for Coordination of Effort. It is the plan to continue the tests in Tennessee and in Illinois and probably several more instruments will be installed during the next season at other locations. With the proposed program we could in course of time obtain sufficient data to determine the characteristics of lightning waves so that their effect on transmission lines and connected apparatus could be predicted and improvements in design and construction would result. The time required, due to the necessarily limited scope of our work or that of any other similar group, to carry out the necessary program would, however, be more than the electrical power industry would care to countenance for an investigation so vital to the future of power development. Since in all probability the major advantages that will result from carrying out this program will be in transmission line design and construction, in which the manufacturers of electrical power apparatus do not participate, we feel that it would be to the advantage of the electrical power industry as a whole to speed up this investigation by taking an active part in it, so that many more observation stations could be installed and the necessary data obtained in a reasonable time. While the financial benefits that will accrue from the complete determination of the characteristics of lightning surges cannot be predicted with assurance, it is the experience of American industry in all lines that once a problem of major importance is accurately defined, a satisfactory solution is promptly secured. When we consider that the annual expenditure of the industry in transmission is of the order of 150 million dollars, it seems clear that there is ample justification for spending the million dollars or so necessary for adequate lightning research.

There is general recognition that a solution of the lightning problem is necessary if the full possibilities of the development of the electrical power industry are to be realized. The work reported here demonstrates that the means necessary for obtaining a complete solution of the first part of the lightning problem are available. Similar work being conducted by other groups along somewhat different lines will doubtless result in availability of equally satisfactory means. The problem should not be confined to the manufacturers nor to the operators but should be participated in by the whole industry. It cannot be solved by either the manufacturers or operators working alone nor properly by any small group. The solution requires the financial and technical support of the whole industry, and now that the equipment is available it is only necessary to work out the necessary organization of effort.

II. Theoretical Discussion of the Propagation of Lightning Surges

Waves on Wires. The fundamental equations for current and voltage are:

$$-\frac{\partial v}{\partial x} = R i + L \frac{d i}{d t} \tag{1}$$

$$-\frac{\partial i}{\partial x} = K v + C \frac{d v}{d t} \tag{2}$$

From (1) we have

$$-\frac{\partial^2 v}{\partial x^2} = \left(R + L \frac{d}{d t}\right) \frac{\partial i}{\partial x}$$

Substituting from (2) for $\frac{\partial i}{\partial x}$ we have

$$\frac{\partial^2 v}{\partial x^2} = \left(R + L \frac{d}{d t}\right)\left(K + C \frac{d}{d t}\right) v$$

$$\frac{\partial^2 v}{\partial x^2} = L C \frac{d^2 v}{d t^2} + (K L + R C) \frac{d v}{d t} + K R V$$

Dividing by $L C$ and denoting $\frac{1}{L C}$ by α^2

$$\frac{\alpha^2 \partial^2 v}{\partial x^2} = \frac{d^2 v}{d t^2} + \left(\frac{K}{C} + \frac{R}{L}\right) \frac{d v}{d t} + \frac{K R}{L C} \cdot V \tag{3}$$

Take as a tentative solution

$$V = A e^{M + \gamma x}$$

Substituting this in (3) we have

$$\lambda^2 + \lambda\left(\frac{K}{C} + \frac{R}{L}\right) + \frac{K\,R}{L\,C} = \alpha^2 \cdot \gamma^2$$

Solving for λ we obtain

$$\lambda = -\left(\frac{K}{2\,C} + \frac{R}{2\,L}\right) \pm \sqrt{\alpha^2\,\gamma^2 + \left(\frac{K}{2\,C} + \frac{R}{2\,L}\right)^2 - \frac{K\,R}{L\,C}}$$

Let $\gamma = \pm\, j\, p$

$$\lambda = -\left(\frac{K}{2\,C} + \frac{R}{2\,L}\right) \pm \sqrt{\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2 - \alpha^2 p^2}$$

$$= -\left(\frac{K}{2\,C} + \frac{R}{2\,L}\right) \pm j\,p\,\alpha\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}$$

The solution will therefore have the form

$$V = \sum e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t}\left[\left\{\frac{A + j\,B}{2}\,e^{jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} + \frac{A - j\,B}{2}\,e^{-jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)}\right\} + \left\{\frac{C + i\,D}{2}\,e^{jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} + \frac{C - j\,D}{2}\,e^{-jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)}\right\}\right] \quad (4)$$

In the above solution $e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t}$ is called the attenuation factor and

$$\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}$$

is called the distortion factor. It will be noticed that the latter is unity, denoting no distortion, when $\frac{K}{2\,C} = \frac{R}{2\,L}$. This simple principle is the theoretical basis for the Pupin coil which by increasing L reduces the ratio $\frac{R}{2\,L}$ until it is equal to $\frac{K}{2\,C}$. It will further be noted that there are two pairs of conjugate functions which combined form two real functions which are the real solution of the differential equation.

Let us now consider the value of the first wave represented by the terms in the first pair of brackets of Equation (4), at a point $x = x'$ when $t = 0$. For the sake of clearness we shall consider the attenuation and distortion factors to be both unity. The value is given by

$$|\,V\,|_{\substack{x = x' \\ t = 0}} = \sum\left(\frac{A + j\,B}{2}\,e^{jpx'} + \frac{A - j\,B}{2}\,e^{-jpx'}\right)$$

Now consider a point on the line at the end of time $t = t'$, let the value of x for the point be $x = \alpha\,t' + x'$. Then, substituting these values in Equation (4), making the attenuation factor and the distortion factor both unity, we shall have:

$$|\,V\,|_{\substack{x = \alpha t' + x' \\ t = t'}} = \sum\left(\frac{A + j\,B}{2}\,e^{jpx'} + \frac{A - j\,B}{2}\,e^{-jpx'}\right)$$

the same value as before for $x = x'$ and $t = 0$. Consequently, the first bracketed term of Equation (4) represents a positively traveling wave having velocity α. Similarly, the second bracketed term represents a negatively traveling wave having the same velocity.

It will be noted that the distortion factor apparently reduces the velocity of propagation and that the amount of reduction varies with the frequency, being higher the lower the frequency. It is important that this apparent velocity be not confused with the velocity of propagation which remains unchanged and is always α or $1/\sqrt{L\,C}$. It will be clear, after considering the matter, that the only way in which the original $\psi\,(x)$ can change during propagation is by a progressive change in the relative phase positions of its constituents, this is the effect produced by the distortion factor. The discontinuities on the wave will be propagated at the velocity α, as will be apparent due to the fact that the distortion factor approached unity for large values of p, and these determine the phases of the discontinuities.

It will be noted that Equation (4) is a perfectly general solution for any system of waves on an infinite line whose value can be expressed, when $t = 0$ by a function of x. It will not be necessary nor advisable to maintain this generality throughout this discussion. It will be assumed that for $t = 0$ the wave is given in the neighborhood of $x = 0$ by a charge distributed on the line between limits $+\,a$ and $-\,a$ and that the potential between these limits due to this charge is given by $2\,\varphi\,(x)$ at the time $t = 0$. In which case the charge will divide into two equal charges moving in opposite directions. So that in Equation (4) B must be equal to A and D must be equal to $-\,B$.

On this assumption Equation (4) may be expressed in the form

$$V = \sum e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t}\left[\left\{A\cos p\left(x - \alpha\,t\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}\right) - B\sin p\left(x - \alpha\,t\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}\right)\right\}\right.$$

$$+ \left\{ A \cos p \left(x + \alpha t \sqrt{1 - \frac{1}{\alpha^2 p^2} \left(\frac{K}{2C} - \frac{R}{2L} \right)^2} \right) \right.$$

$$\left. + B \sin p \left(x + \alpha t \sqrt{1 - \frac{1}{\alpha^2 p^2} \left(\frac{K}{2C} - \frac{R}{2L} \right)^2} \right) \right\} \Bigg] \tag{4a}$$

This form of solution is appropriate for some analytical work but is not well suited for expressing a non-periodic function of (x). To do this we must use Fourier integrals. If we have a function $\varphi(x')$ which represents a non-periodic function $\Psi(x)$ between the limits $-a$ and a, and if the value of the non-periodic function is zero for all values of the independent variable outside these limits, the non-periodic function may be expressed by the infinite definite integral.

$$\psi(x) = \frac{1}{\pi} \int_0^{\infty} d\,p \int_{-a}^{+a} \varphi(x') \cos p\,(x - x')\, d\,x' \tag{5}$$

where $(\varphi(x))_{-a}^{+a} + (0)_{+a}^{\infty} + (0)_{-a}^{-\infty} = \psi(x)$.
The usual statement of the integral is

$$\psi(x) = \frac{1}{\pi} \int_0^{\infty} d\,p \int_{-\infty}^{+\infty} \psi(x') \cos p\,(x - x')\, d\,x' \tag{6}$$

The statement given in (5) is appropriate for the problem as usually presented. Where $\psi(x')$ between limit $-a$ and $+a$ can be represented only by several functions of the form $\varphi(x)$ the integration between $-a$ and a must be divided to correspond with the limits imposed on the several functions. The solution appropriate for an infinite line in which a single charge of definite finite distribution is induced by lightning is, therefore

$$V = e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} \frac{1}{\pi} \int_0^{\infty} d\,p \int_{-\infty}^{+\infty} \varphi(x')$$

$$\left\{ \cos p \left(x - x' - \alpha t \sqrt{1 - \frac{1}{\alpha^2 p^2} \left(\frac{K}{2C} - \frac{R}{2L} \right)^2} \right) \right.$$

$$\left. + \cos p \left(x - x' + \alpha t \sqrt{1 - \frac{1}{\alpha^2 p^2} \left(\frac{K}{2C} - \frac{R}{2L} \right)^2} \right) \right\} d\,x' \tag{7}$$

Since t does not enter into the integration, the form of the integral is given when $t = 0$ and is

$$V_0 = e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} \frac{1}{\pi} \int_0^{\infty} dp \int_{-a}^{+a} \varphi(x') \{ \cos p\,(x - x')$$

$$+ \cos p\,(x - x') \}\, d\,x' \tag{8}$$

where it should be noted that the wave must be considered as consisting of two equal parts, one of which becomes the positive and the other the negative traveling wave.

The above solution is appropriate for lines of infinite length subject to a single lightning stroke and may be used for investigation of surges over long lines where only the attenuation of incoming surges are of interest. For short lines the end effects become of major importance. In looking into these problems the form of solution given by (4) which is analytically the equivalent of (7) will be found most convenient and we shall use it in the form

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + j\,B)$$

$$\left\{ e^{jp\left(x + \alpha t \sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right.$$

$$\left. + e^{-jp\left(x + \alpha t \sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \tag{9}$$

which is the appropriate form for the vector representation of a single non-periodic induced surge traveling over the line. The true solution will be the real part of the solution obtained by (9). It will be found later on that such a surge will become periodic as the result of the boundary conditions set up by the finite line. It will be noticed that in Equation (5) the coefficient of the sine term of the negative wave is of opposite sign to the coefficient of the sine term of the positive wave. This is the necessary condition on an infinite line for the positively traveling wave and the negatively traveling wave to be exact counterparts of each other. Similarly in vector representation in the negative traveling wave the exponential $e^{-p(x + \alpha t)}$ corresponds to the exponential $e^{p(x - \alpha t)}$ since time is always positive whereas x is positive for the positive wave and negative for the negative wave.

From (2) we obtain the symbolic expression for i, namely

$$i = -\frac{K + C\,D_t}{D_x}\, V \tag{10}$$

where $D_t = \dfrac{d}{d\,t}$ and $D_x = \dfrac{\partial}{\partial\,x}$. Applying this to Equation (9) we have

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + j\,B) \sqrt{\frac{C}{L}}$$

$$\left\{ \sqrt{1 - \frac{1}{\alpha^2 p^2} \left(\frac{K}{2C} - \frac{R}{2L} \right)^2} + \frac{j}{\alpha\,p} \left(\frac{K}{2C} - \frac{R}{2L} \right) \right\}$$

$$\left\{ e^{jp\left(x - \alpha t \sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right.$$

$$- e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} \Bigg\} \tag{11}$$

the real solution being the real part of the above equation as in the case of the potential. The quantity $\sqrt{\frac{C}{L}}$ is called the surge admittance, $\sqrt{\frac{L}{C}}$ being the surge impedance. It will be seen that if the distortion factor is close to unity the current is equal to the potential multiplied by the surge admittance. If distortion is present the current will be out of phase with the potential and will have a somewhat different form.

Equations (9) and (11) refer only to an infinite line over which a single surge caused by a released bound charge is traveling.

The type of line referred to above may be called a doubly infinite line. Let us now consider a singly infinite line open at one end and let the value of x at this end be l_1. Let us consider Equation (11). It will be obvious that in the form given it is not general enough for this particular problem as the initial condition imposed when $t = 0$ makes the negatively traveling wave identically zero for $x = > l_1$. To generalize this equation we shall retain the original coefficient of the negatively traveling wave but add another term A' whose value must be consistent with the conditions $V_0 = 2\,\varphi\,(x)$ when $t = 0$. The equation so altered is still a solution of the general equation and is given for $x = l_1$ by

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} \sqrt{\frac{C}{L}}$$

$$\left\{\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}+\frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\}$$

$$\left\{(A+jB)\,e^{jp\left(l_1-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$\left.- A'\,e^{-jp\left(l_1+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{12}$$

where, since for values of greater than α_1 the original negative traveling wave is zero it does not appear in the equation. Now the condition imposed by the open end is that I shall be identically zero. This condition will be satisfied only if we make

$$A' = (A + jB)\,e^{j2pl_1} \tag{13}$$

The solution for a singly infinite line open at $x = l_1$ is therefore

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A+jB)\sqrt{\frac{C}{L}}$$

$$\left\{\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}+\frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\}$$

$$\left\{e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$\left.-(1+e^{j2pl_1})\,e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{14}$$

An examination of this equation shows that when $t = 0$ the negatively traveling wave given by the term having coefficient $(A + jB)\,e^{2l_1}$ will be zero at every point within the singly infinite line. The solution for the potential is obtained by reversing the operation (10) but it may be obtained directly from (14) by inspection. It is given by

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A+jB)$$

$$\left\{e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$\left.+(1+e^{j2pl_1})\,e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{15}$$

This equation shows that at the end of the singly infinite line the potential reaches twice the value obtained by the wave traveling in any other part of the line. In a similar manner it may be shown for a singly infinite line open at $x = -l_2$, the solution for current and e. m. f. are:

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A+jB)\sqrt{\frac{C}{L}}$$

$$\left\{\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}+\frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\}$$

$$\left\{(1+e^{j2pl_2})\,e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$\left.-e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{16}$$

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A+jB)$$

$$\left\{(1+e^{j2pl_2})\,e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$+ e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} \Bigg\} \tag{17}$$

The waves having the factors e^{j2pl_1} and e^{j2pl_2} are reflected waves and it will be noticed that at the positive end of the line the positively traveling wave is reflected into a negative traveling wave of opposite sign in the case of the current, and of the same sign in the case of the potential. The multiplying factor e^{j2pl_1} is introduced in the case of reflection of a positive traveling wave into a negative traveling wave and the factor e^{j2pl_2} in case of reflection of a negative traveling wave into a positive traveling wave. We may now write down the solution of the finite line having ends at $x = l_1$ and $x = -l_2$, the total length of the line is $l = l_1 + l_2$. By taking into account the successive reflection we arrive at the factors for the positive and negative waves; they are:

Positive: $(1 + e^{j2pl_1})\,(1 + e^{j2pl} + e^{j4pl} + \ldots e^{j2plr})$

Negative: $(1 + e^{j2pl_1})\,(1 + e^{j2pl} + e^{j4pl} + \ldots e^{j2plr})$ (18)

In this manner, starting with the idea of a lightning wave induced in a doubly infinite line we arrive at a solution for a finite line. The solution is:

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)\sqrt{\frac{C}{L}}\left\{\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\}$$

$$\left\{(1 + e^{j2pl_1})\, e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} - (1 + e^{j2pl_1})\, e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{19}$$

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)$$

$$\left\{(1 + e^{j2pl_1})\, e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + (1 + e^{j2pl_1})\, e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{20}$$

These solutions have been obtained by considering the terminal conditions separately and they show that the result is an oscillatory solution, but as a result of (19) and (20) we can combine the three terminal conditions and obtain a Fourier expansion for $2\,\varphi\,(x)$, when $t = 0$, which will give us the solution direct in its simplest form. We shall deal with this later on after we have taken up the singly infinite line grounded at one and the finite line grounded at both ends.

In the singly infinite line grounded at the positive end $x = l_1$ the potential at the end must be identically zero; we may therefore, following the same principles as in the case of the singly infinite line, write down the solution. It is

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A + j\,B)$$

$$\left\{e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + (1 - e^{j2pl_1})\, e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{21}$$

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A + j\,B)\sqrt{\frac{C}{L}}$$

$$\left\{\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\}$$

$$\left\{e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} - (1 - e^{j2pl_1})\, e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{22}$$

with similar expression for a singly infinite line grounded at the negative side. In the case of the finite line grounded at both ends we have the following factors for the positive and negative waves:

Positive: $(1 - e^{j2pl_1})\,(1 + e^{j2pl} + e^{j4pl} + \ldots)$

Negative: $(1 - e^{j2pl_1})\,(1 + e^{j2pl} + e^{j4pl} + \ldots)$

The solution for this case is:

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)\sqrt{\frac{C}{L}}$$

$$\left\{\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\}$$

$$\left\{(1 - e^{j2pl_1})\, e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$- (1 - e^{j2pl_1})\, e^{-jp\left(x + \alpha t\sqrt{-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \Bigg\} \tag{23}$$

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + jB)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)$$

$$\Bigg\{ (1 - e^{j2pl_2})\, e^{jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)}$$

$$- (1 - e^{j2pl_1})\, e^{-jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \Bigg\} \tag{24}$$

Let us now examine Equation (**20**) at $t = 0$. A term of the Fourier expansion for the positive traveling wave is:

$$(A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)\,(1 + e^{j2pl_2})\, e^{jpx}$$

or $(A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)\,(e^{jpx} + e^{jp(x+2l_2)})$

Now it will be observed that in the case of the variable $(e^{jpx} + e^{jp(x+2l_2)})$ the second term will have the same value as the first term if we take the value of x for it is equal to $x - 2\,l_2$. The second term therefore might be expressed as arising from the reflection of the original wave at $t = 0$ in a mirror placed at the end $x = -\,l_2$ of the line. The factor $(1 + e^{j2pl} + e^{j4pl} + \ldots)$ shows that all the positive waves are derived by repeating the original wave and its reflection, so that from the beginning of one to the beginning of the next the distance is $2\,l$ or twice the length of the line. If we take the negative traveling wave we find that it arrives from a similar system of waves which may be ascribed to the reflected image of the original wave in a mirror at the positive end of the line, which wave and its reflected image are repeated every $2\,l$ length in the positive direction.

Let us now add to the succession of positive waves to the left of the line a succession of positive waves corresponding to the negative waves to the right of the line and vice versa. It will be evident that the positive waves to the right and the negative waves to the left will produce no effect on the line at any time but will remain entirely imaginary as far as the line is concerned. The result will, however, have the effect of making the system of waves harmonic from $-\,\infty$ to $+\,\infty$. If we imagine a mirror, placed at the two ends of the line and the original static wave reflected in the two mirrors the result will be a double infinite set of images in the two mirrors, which will be an exact representation of the system of waves required to give the solution of Equation (**20**); as the wave divides into positive and negative traveling waves these images will also divide into positive traveling waves, which will be the system of waves as given by the solution, Equation (**20**).

Since the solution at $t = 0$ is given by a doubly infinite periodic function of x it may be represented by a Fourier series of integral harmonics of the wave length. In this case, since the system of waves is unidirectional, the actual wave length will be $2\,l$ and in addition to even harmonics there will be a constant term in the expansion.

Examining Equation (**24**) we find that the original static wave may be replaced by a doubly infinite wave of wave length $2\,l$ and which may be obtained by considering a mirror which reflects an inverted image to be placed at each end of the line. See Fig. (17). Similarly, if we have to do with a finite line grounded at one end and open at the other, the system of images obtained by placing an ordinary mirror at the open end and a mirror reflecting an inverted image at the other end will give the appropriate expansion for $\varphi\,(x)$ when $t = 0$. In this case the wavelength will be $4\,l$. Fig. 17.

In obtaining the appropriate Fourier expansion in any case of a finite line with like ends we need consider only the position of the system between $-\,2\,l_2$ and $2\,l_1$ as the rest of the system simply repeats this portion periodically, the fundamental wave length being $2\,l$. Both Fourier expansions will in general have even as well as odd harmonics and in the case of the positive reflection there will be a non-harmonic term. Where the surge starts at the middle of the line, the wave length for the positive reflection becomes l and there are even as well as odd harmonics; in the case of the negative reflection the wavelength remains $2\,l$ but there are no even harmonics.

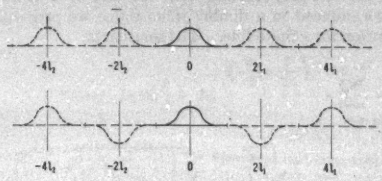

FIG. 17

If we denote the function of x when $t = 0$ obtained in this manner by $2\,\psi\,(x)$, so that $\psi\,(x)$ represents the static function from which the positive and negative waves are propagated, we shall have for the open finite line

$$\psi\,(x) = \text{The real part of } A_0 + \sum_{1}^{\infty} (A_n + j\,B_n)\, e^{jn\frac{\pi}{l}x} \tag{25}$$

And for the line grounded at both ends

$$\psi\,(x) = \text{The real part of } \sum_{1}^{\infty} (A_n + j\,B_n)\, e^{jn\frac{\pi}{l}x} \tag{26}$$

where

$$A_0 = \frac{1}{2l}\int_{-2l_2}^{+2l_1} \psi(x)\, d\, x \tag{27}$$

$$A_n = \frac{1}{l}\int_{-2l_2}^{+2l_1} \psi(x) \cos \frac{\pi n}{l} x\, d\, x \tag{28}$$

$$B_n = \frac{1}{l}\int_{-2l_2}^{+2l_1} \psi(x) \sin \frac{\pi n}{l} x\, d\, x \tag{29}$$

The solution for the finite line open at both ends may therefore be stated as follows:

$$V = \sum_{n=0}^{n=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} \left[A_0 + (A_n + j\, B_n) \left\{ e^{j\frac{n\pi}{l}\left(x-\alpha t\sqrt{1-\frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + e^{-j\frac{n\pi}{l}\left(x+\alpha t\sqrt{1-\frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{C}-\frac{R}{2L}\right)^2}\right)} \right\} \right] \tag{30}$$

The expression for the current is obtained by changing the sign of the second exponential term and multiplying the whole quantity within the summation sign by

$$\sqrt{\frac{C}{L}} \left\{ \sqrt{1-\frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2} + \frac{j\, l}{\alpha\, n\, \pi}\left(\frac{K}{2C}-\frac{R}{2L}\right) \right\} \tag{31}$$

which is the true surge admittance.

For the finite line grounded at both ends the potential will be

$$V = \sum_{n=0}^{n=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A_n + j\, B_n) \left\{ e^{j\frac{n\pi}{l}\left(x-\alpha t\sqrt{1-\frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + e^{-j\frac{n\pi}{l}\left(x+\alpha t\sqrt{1-\frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} \right\} \tag{32}$$

and the current may be obtained by the same procedure used in the case of the open finite line. In each case the actual solution is the real part of the summation.

In the case of a finite line open at one end and grounded at the other the wave length is $4\,l$ as can be ascertained by means of the mirror scheme which was described above and the expression for the potential will be:

$$V = \sum_{n=0}^{n=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A_n + j\, B_n) \left\{ e^{j\frac{n\pi}{2l}\left(x-\alpha t\sqrt{1-\frac{4l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + e^{-j\frac{n\pi}{2l}\left(x+\alpha t\sqrt{1-\frac{4l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} \right\} \tag{33}$$

and the current may be obtained by changing the sign of the second exponential term and multiplying the whole quantity within the summation sign by (31).

In the above solutions the value of $\frac{n\,\pi}{l}$ and $\frac{n\,\pi}{2\,l}$ is the same as the coefficient p in the theory of doubly infinite and singly infinite lines.

The principle of conservation of energy affords a general means for introducing into the general solution of the differential equation the effect of any physical discontinuity. This principle states that the total energy of the system together with that dissipated must be constant.

The principle shows us at once why the initial wave when $t = 0$ divides into two equal waves flowing in opposite directions for in order to satisfy this principle the total energy must remain constant and equal to the total energy in the system at time $t = 0$ less the dissipated energy. The kinetic and potential energy in the traveling wave at any point of the line are equal. Therefore, the original wave must divide into two equal waves, traveling in opposite directions and having one-half its crest value except for the effect of the energy dissipated which has the effect of reducing this value with time. Each wave has now one-half the energy of the original wave less the dissipated energy and in each wave one-half the energy is electrostatic energy, or potential, and the other half is electromagnetic energy, or current. Thus, the system satisfies the condition imposed by the principle of conservation of energy.

Again, considering the terminal condition at an open end, the electro-kinetic energy must be zero as all the energy at the terminal must become potential energy. This condition is satisfied when two equal potential waves traveling in opposite directions are coincident in value at the point under consideration. Thus the potential wave is reflected positively while the current wave is reflected negatively. Similarly for a closed end the potential energy must be zero and therefore the potential wave is reflected negatively while the current wave is reflected positively.

These specific conditions were of course imposed upon the equation when the terminal conditions $i = 0$ or $V = 0$ were introduced. Where the discontinuity is not complete as in those two cases, it becomes necessary

to introduce specifically the principle of energy into the boundary condition at the point of discontinuity.

In the case of the finite line we perceive that at the terminals there must be a periodic recurrence of the phenomena due to successive reflection as the energy must remain in the system until dissipated, and we perceive at once that this condition may be exactly represented by an initial distribution periodic with respect to x in an infinite line which resolves into two traveling waves of equal value traveling in opposite direction. Two of the nodal points of this system are at the terminals of the finite line, and as would be expected with like terminal conditions at either end the wave length of the initial distribution would be twice the length of the line and in the case of opposite condition at either terminal it would be four times the length of the line.

We shall now direct our attention to a singly infinite line connected at the point $x = l_1$ to a similar line having different transmission constants.

We shall from now on confine ourselves to the solution of the specific differential equation of the electric circuit. We shall for brevity designate

$$\frac{K}{2C} + \frac{R}{2L} \text{ by } u$$

$$\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} \text{ by } v$$

$$\sqrt{\frac{C}{L}}$$

$$\left\{\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C} - \frac{R}{2L}\right)\right\} \text{ by } Y$$

The reciprocal of this or the surge impedance

$$\sqrt{\frac{L}{C}}$$

$$\left\{\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} - \frac{j}{\alpha p}\left(\frac{K}{2C} - \frac{R}{2L}\right)\right\} \text{ by } Z$$

Then we have for the doubly infinite line

$$V = e^{-ut} \sum_{p=0}^{p=\infty} (A + jB) \left\{ e^{jp(x-vt)} + e^{-jp(x+vt)} \right\} \quad (32)$$

$$I = e^{-ut} \sum_{p=0}^{p=\infty} Y (A + jB) \left\{ e^{jp(x-vt)} - e^{-jp(x+vt)} \right\} \quad (33)$$

The actual values being the real part of these vector quantities. It should be noted that the surge impedance being equal to

$$\sqrt{\frac{L}{C}}$$

$$\left\{\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} - \frac{j}{\alpha p}\left(\frac{K}{2C} - \frac{R}{2L}\right)\right\}$$

the radical sign over $\frac{L}{C}$ implies the signs $\pm$. The positive sign applies to positive traveling waves and the negative sign applies to negative traveling waves. This accounts for the change in sign of the negative traveling current wave as compared with the negative traveling potential wave.

Let us now consider both lines as having the same attenuation and distortion factors.

At the junction point of two lines the conditions imposed are

(a) The current shall be continuous.

(b) The potentials at the junction point on either side shall be the same.

Let the junction point of the two lines be at the point $x = l_1$ of the first line. It will be obvious that if the original static potential distribution $2\,\varphi\,(x)$ from which the wave on the first line originated had the value zero at the point only the positive traveling wave will be involved at the junction but since the discontinuity at the junction may give rise to a reflected wave which will travel in a negative direction, we shall assume for V_1 the value

$$V_1 = V_{+1} + V_{-1} \quad (34)$$

There will be no negative traveling wave in the second line and therefore

$$V_2 = V_{+2} \quad (35)$$

We have, therefore

$$V_{+2} - V_{-1} = V_{+1} \quad (36)$$

$$Y_2 V_{+2} + Y_1 V_{-1} = Y_1 V_{+1} \quad (37)$$

We have, therefore

$$V_{+2} = \frac{2\,Y_1}{Y_1 + Y_2} V_{+1} = \frac{2\,Z_2}{Z_1 + Z_2} V_{+1} \quad (38)$$

$$V_{-1} = \frac{Y_1 - Y_2}{Y_1 + Y_2} V_{+1} = \frac{Z_2 - Z_1}{Z_1 + Z_2} V_{+1} \quad (39)$$

No particular advantage is obtained by considering this case on the basis of reflected images as the expansion cannot be effected in terms of a Fourier series. We shall therefore confine ourselves to the statement of the solution, as we did in the case of the singly infinite lines. The potential and current in the first line are given by

$$V_1 = e^{-ut} \sum_{p=0}^{p=\infty} (A_1 + jB_1)$$

$$\left\{ e^{jp(x-vt)} + \left(1 + \frac{Y_1 - Y_2}{Y_1 + Y_2} e^{j2pl_1}\right) e^{-jp(x+vt)} \right\} \quad (40)$$

$$I_1 = e^{-ut} \sum_{p=0}^{p=\infty} (A_1 + j\,B_1)\,Y_1$$

$$\left\{ e^{jp(x-vt)} - \left(1 + \frac{Y_1 - Y_2}{Y_1 + Y_2}\, e^{j2pl_1}\right) e^{-jp(x+vt)} \right\} \quad (41)$$

The potential and current in the second line are given by

$$V_2 = e^{-ut} \sum_{p=0}^{p=\infty} (A_1 + j\,B_1)\, \frac{2\,Y_1}{Y_1 + Y_2}\, e^{jp(x-vt)} \Bigg\} \quad (42)$$

$$I_2 = e^{-ut} \sum_{p=0}^{p=\infty} Y_2\,(A_1 + j\,B_1)\, \frac{2\,Y_1}{Y_1 + Y_2}\, e^{jp(x-vt)} \Bigg\} \quad (43)$$

For the finite line of length l connected on either end to infinite lines of the same constants having different surge impedances from the first line but the same attenuation and distortion we may write

$$\frac{Y_1 - Y_2}{Y_1 + Y_2} = a$$

and taking into account successive reflections, we have

$$V_1 = e^{-ut} \sum_{0}^{\infty} \Big[(A_1 + j\,B_1)\,(1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)$$

$$\left\{ (1 = a\,e^{j2pl_1})\, e^{jp(x-vt)} + (1 + a\,e^{j2pl_1})\, e^{-jp(x+vt)} \right\} \Big] \quad (44)$$

$$I_1 = e^{-ut} \sum_{p=0}^{p=\infty} \Big[(A_1 + j\,B_1)(1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,Y_1$$

$$\left\{ (1 + a\,e^{j2pl_1})\, e^{jp(x-vt)} - (1 + a\,e^{j2pl_1})\, e^{-jp(x+vt)} \right\} \Big] \quad (45)$$

The potential and current in the second line will be obviously

$$V_2 = e^{-ut} \sum_{p=0}^{p=\infty} \left\{ \frac{2\,Y_1}{Y_1 + Y_2}\,(A_1 + j\,B_1) \right.$$

$$\left. (1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_1})\, e^{jp(x-vt)} \right\} \quad (46)$$

$$I_2 = e^{-ut} \sum_{p=0}^{p=\infty} \frac{2\,Y_1\,Y_2}{Y_1 + Y_2}\,(A_1 + j\,B_1)$$

$$\left. (1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_1})\, e^{jp(x-vt)} \right\} \quad (47)$$

The potential and current in the third line, since $Y_3 = Y_2$ is

$$V_3 = e^{-ut} \sum_{p=0}^{p=\infty} \left\{ \frac{2\,Y_1}{Y_1 + Y_2}\,(A_1 + j\,B_1) \right.$$

$$\left. (1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_1})\, e^{-jp(x+vt)} \right\} \quad (48)$$

$$I_3 = e^{-ut} \sum_{p=0}^{p=\infty} \left\{ -\frac{2\,Y_1\,Y_2}{Y_1 + Y_2}\,(A_1 + j\,B_1) \right.$$

$$\left. (1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_1})\, e^{-jp(x+vt)} \right\} \quad (49)$$

The solutions given above are not of much value for analytic work but are useful when plotting the potentials graphically. The analytic solution would depend upon an infinite series of images of the initial wave of decreasing magnitude and can only be expressed as a Fourier integral but the physical picture is of benefit when making a graphical analysis.

An important case is that in which two transmission lines feed into a common line and surges are released on the two lines having different magnitudes. We shall consider the two in which the surges are propagated as No. 1 and No. 2, the other line being No. 3. The propagated surge in No. 1 and No. 2 will be V_{+1} and V_{+2} and the transmitted surge in No. 3 will be V_{+3}. We shall assume the value of the surge admittance of No. 1 and No. 2 to be both Y and of No. 3 to be Y_3. Then assuming all lines to have the same attenuation and distortion

$$V_3 = V_{+1} + V_{-1} \quad (50)$$

$$V_3 = V_{+2} + V_{-2} \quad (51)$$

$$Y_3\,V_3 = Y\,(V_{+1} + V_{+2}) - Y\,(V_{-1} + V_{-2}) \quad (52)$$

In these equations V_{+1} and V_{+2} are supposed to be known, V_3, V_{-1}, and V_{-2} are unknown.

We have by substituting from (50) in (52)

$$(Y + Y_3)\,V_{-1} + Y\,V_{-2} = (Y - Y_3)\,V_{+1} + Y\,V_{+2},$$

and by substituting from (51) in (52)

$$Y\,V_{-1} + (Y + Y_3)\,V_{-2} = Y\,V_{+1} + (Y - Y_3)\,V_{+2}$$

Solving for V_{-1} and V_{-2} we have

$$V_{-1} = \frac{2\,Y\,Y_3\,V_{+2} - Y_3^2\,V_{+1}}{2\,Y\,Y_3 + Y_3^2} \quad (53)$$

$$V_{-2} = \frac{2\,Y\,Y_3\,V_{+1} - Y_3^2\,V_{+2}}{2\,Y\,Y_3 + Y_3^2} \quad (54)$$

and

$$V_3 = \frac{2\,Y\,Y_3\,(V_{+1} + V_{+2})}{2\,Y\,Y_3 + Y_3^2} \quad (55)$$

Still another important case is where a line of surge

admittance Y_0 terminates in two lines of surge admittances Y_1 and Y_2. In this case, by similar methods

$$V_1 = \frac{2\,Y_0}{Y_0 + Y_1 + Y_2}\,V_0 \qquad (56)$$

Extending this still further to the case of a line of surge admittance Y_0 terminating in three lines of surge admittances Y_1, Y_2, and Y_3, the equation for the transmitted voltage is:

$$V_1 = \frac{2\,Y_0}{Y_0 + Y_1 + Y_2 + Y_3}\,V_0 \qquad (57)$$

A resistance between the junction and ground acts the same as an additional infinite line having the same surge admittance.

III. The Effect of Ground Resistance of Overhead Ground Wire Protection of Transmission Lines

Overhead Ground Wires as Protection from Induced Waves. Equations (40) and (41) are of great importance for estimating the actual protection afforded by overhead ground wires to transmission lines when the value of the ground resistance is taken into account. In this problem the ground resistance may be regarded as an infinite line having a surge impedance equal to the ground resistance. The two adjacent sections of the overhead ground wire both feed into the common ground and may be regarded as in multiple. Since what we desire is a quantitative relation between a perfect ground and grounds having various values of resistance, it will serve our purpose to consider the potential to extend over three spans and to be constant over this whole length. The spans being 1000 ft. in length this would mean that the line surge is a square topped wave of about three microseconds wave-length. Assuming a typical 220-kv. line with two ¾-in. ground wires spaced 30 ft. apart the surge impedance will be 290 ohms or the surge admittance 0.00344 for one pair of ground wires. For two pair feeding with a common ground it will be 0.00688. If the ground resistance is 80 ohms it will be equivalent to an infinite line having admittance 0.0125. We have then

$$a = \frac{Y_1 - Y_2}{Y_1 + Y_2} = \frac{-\,0.00562}{0.01938} = -\,0.290$$

Y_1 being the surge admittance of the ground wires in parallel and Y_2 the admittance of the ground resistance. Let the mean height of the transmission line be 50 ft. and of the ground wires 60 ft. Then if the gradient due to the thunder cloud is 50 kv. per foot, the induced potential will be 2500 kv. and on the ground wires 3000 kv. We may assume that the ground wires when at zero potential would reduce the potential on the transmission lines 50 per cent. When the potential of the ground wire is not zero a first approximation will be to add to the 50 per cent voltage on the transmission line the same percentage of 50 per cent as the ground wire potential at that point is of its full voltage, due regard being paid to signs.

Fig. 18 shows the potentials of ground wires and potentials of line on this basis for several reflections with a ground resistance of 80 ohms.

Fig. 19 shows the potentials of ground wires, line

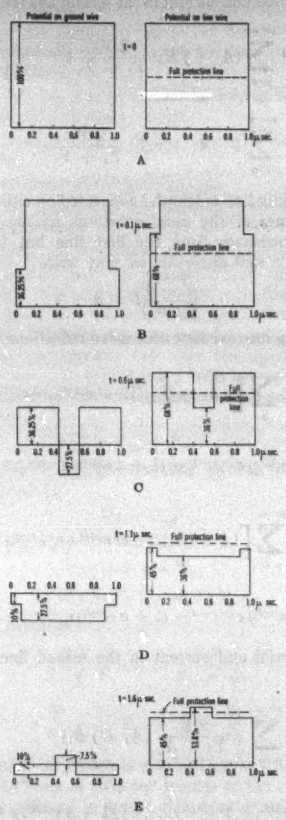

Fig. 18—Progress of Wave on Transmission Line and Ground Wire After the Release of a Bound Charge

Resistance of tower and ground = 80 ohms

wires, and difference of potential at the tower for ground resistance of 20 ohms.

Figs. 20 and 21 show the same voltages at the tower for resistances of 80 and 200 ohms respectively.

It will be noted that for an interval of two microseconds the lower resistance gives the lower voltage with respect to ground. The curves of difference of potential

between line conductor and ground wires, which is the same as the tower top, seems to favor the higher ground resistance. However, the tendency to arc over an insulator string depends not only on the potential to tower top but also on the potential to ground. Furthermore, the performance of ground wires during

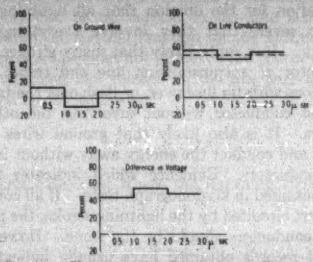

FIG. 19—INDUCED VOLTAGES ON LINES WITH GROUND WIRES

Voltages at the towers—based on equal induced voltages on ground and line wires ground resistance R = 20 ohms

direct strokes is more important and the following discussion will show that for direct strokes a low ground resistance is essential.

Overhead Ground-Wires as Protection against Direct Strokes. A second important practical application for these formulas is for the case of a direct stroke of lightning. We shall first take up the case where the lightning stroke hits the middle of the span.

The phenomenon of lightning discharge has been clarified to a large extent by the experiments of Torok[5]

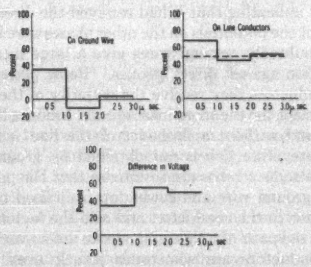

FIG. 20—INDUCED VOLTAGES ON LINES WITH GROUND WIRES

Voltages at the tower—based on equal induced voltages on ground and line wires ground resistance R = 80 ohms

in suppressed discharges between sphere gaps. This phenomenon of suppressed discharge is very frequently observed in thunderstorms and the laboratory duplication of it has helped a great deal in the understanding of lightning discharges.

[5]J. J. Torok, "Surge Impulse Breakdown of Air," TRANSACTIONS A. I. E. E., Vol. 47, No. 2, April 1928, p. 349.

As a result of the study of Torok's suppressed discharges we have formed the following picture of what takes place just before and during a lightning discharge. The thunder cloud has first built up sufficient potential to produce local ionization by collision; as a result, space charges are formed in the neighborhood of the cloud which still further increase local ionization. The ionization begins to take place along preferred paths and the space charges become most dense in the neighborhood of these paths. Finally the density of the ionization paths becomes so great that the temperature along these paths becomes sufficient to cause thermionic emission of electrons from the molecules. This results in an intense stream of electrons which progressively produce other electrons by collision followed by thermal ionization. The result is that the front of the charged space rapidly approaches the earth with accelerated motion until a complete conducting circuit is formed

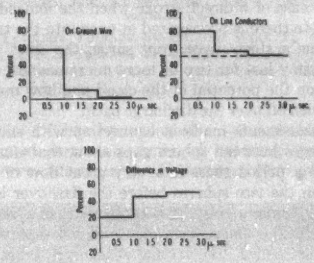

FIG. 21—INDUCED VOLTAGES ON LINES WITH GROUND WIRES

Voltages at the towers—based on equal induced voltages on ground and line wires ground resistance R = 200 ohms

by the ionized paths from cloud to earth, which culminates in a lightning stroke.

It should be noted that the front of the charged space is not at the same potential as the cloud at any time. The effect of the space charge is to produce a drop of potential from cloud to front as the front progresses; but the result of this progression of the space charge is the increase of the intensity of stress in the air path from its front to the earth, and further ionization of the air path, unless the cloud is not able to support the additional charge required of it and maintain potential sufficient to complete the flashover. In the latter case the flashover is suppressed.

It is obvious that the intensity of a lightning stroke varies widely depending upon the height and nature of the cloud, which determines the quantity of charge that can be dissipated in a single stroke. Many strokes never reach the earth as frequently the progress of the streamer is stopped in mid-air due to insufficient energy to support and urge it forward. Others barely reach the earth and do so with little energy. Still others, of course, are intense in varying degrees.

In the initial stage before the streamer or ionized paths have reached the earth, the front of the ionized and charged space may be propagated at an average velocity of from one-twentieth to one one-hundredth the velocity of light. The first figure is the approximate value of this velocity as obtained between spheres, the second is the order of this velocity as obtained along strings of insulators. In other words, the preliminary or setting up stage of the flashover from a cloud may take from 25 to 125 microseconds, and during this time low potential surges flow from the ends of the line to the point of influence. Obviously, during this stage the field stress near the earth directly under the streamer point increases and beyond a short distance away it decreases. Thus, if a stroke takes place at some distance from a line the bound charges can travel away in both directions faster than the decrease of the field gradient. This is why a stroke must be very close to a line in order to induce a surge of appreciable magnitude. In the case of a direct stroke when the ionized path is complete there is propagated at once into the transmission line a steep wave front surge, the crest value of which may last for two or more microseconds, depending upon the potential of the cloud and how much of it is drained off over the discharge path.

Measurements made in connection with suppressed discharges between sphere gaps show that during the initiation period there is a very great flow of current between the two spheres before the flashover is established. Consequently, it is probable that a very large portion of the charge from the cloud is dissipated in the streamer before the actual flashover takes place, amounting to a large proportion of the total charge that passes from the cloud. This helps to lower the crest value of the wave when the flash hits the transmission line.

When the flashover takes place the cloud is discharged over the ionized path in the form of a surge traveling like any other traveling wave over a conducting path and since the surge impedance of the path which it must follow decreases progressively as it approaches the earth, the crest of the propagated wave will decrease in potential progressively as it approaches earth so that although the potential of the cloud may be something like one hundred million volts, the propagated wave when it reaches a transmission line may not have a crest of more than four or five million volts. This crest of the wave of the lightning stroke when it hits the line is further reduced by reason of the fact that its surge impedance is very much less at the striking point than at the cloud so that the actual wave propagated from the cloud as it approaches the earth along the ionized path is reduced in crest value. It may be assumed that when it strikes the transmission line it has a surge impedance of the same order as that of three transmission line conductors in parallel and, since it divides into two waves at this point, the crest value of each of these waves depends upon the surge impedances of the conductors struck and the lightning path, and the voltage of the lightning as it enters the line. We may assume that the maximum crest from the stroke lasts two or more microseconds. This wave may enter the ground wires, the transmission lines, or both, and it will be reflected at the towers in both the earth and other conductors involved. There is no foundation for the opinion that all lightning strokes are so severe that they involve everything in the vicinity. It is quite likely that many strokes strike a conductor of a transmission line and travel in both directions with no flashover, or flash over the insulators on that conductor without involving the other conductors. It is also likely that ground wires may be struck and conduct the energy away without involving the conductors. Evidence which indicates this has been obtained in klydonograph tests. If all conductors are short circuited by the lightning stroke, the potential of all conductors should be the same. However, the highest records obtained have usually indicated that the surge on one conductor was considerably the highest indicating that it had received the direct stroke and that the potentials on the other phases were induced.

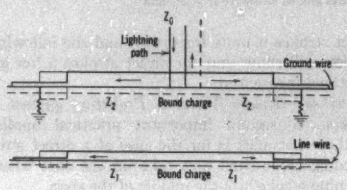

Fig. 22—Diagram of Direct Stroke to Ground Wire in Middle of Span Showing Voltage Waves

Undoubtedly, ground wires give a large measure of protection against direct strokes. Being placed above the conductors they receive the majority of the strokes and, except for the more intense ones, conduct the surge to ground without a flashover of the line insulators. Complete protection is not obtained by ground wires because some strokes are so intense that the potentials of the ground wire and tower top are raised to where they flash to the conductor, and also the factors determining the path of a lightning stroke are so varied that line conductors are sometimes struck even though there are ground wires above them. It is obvious that ground wires will be most effective if they are made of good conducting material and if the tower ground resistance is low. A steel ground wire is likely to be melted in two by the heat generated in carrying off the lightning surge and become involved in the line conductors. Therefore, it is important that a conductor having at least a surface of good conducting material be used as ground wires.

Figs. 22 and 23 show the nature of the resulting voltage waves when a stroke hits the ground wires in the

center of a span and at the tower respectively. The waves traveling in each direction from the point hit may be calculated by Equations (56) and (57). Assuming that the stroke strikes the ground wire only, there is a potential induced on the line conductors by the traveling wave but, since the line conductor is insulated and since the wave travels in both directions from a point there can be no current in the line conductor and therefore no mutual reaction on the ground wire. Thus, the line wire does not influence the waves on the ground wire and the voltage on the line wire may be calculated by the equation;[6]

$$V_1 = \frac{z}{Z_2} V_2 .$$

In the first case, that where the lightning strikes the

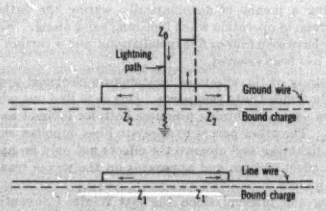

Fig. 23—Diagram of Direct Stroke to Tower with Ground Wires Showing Voltage Waves

middle of the span, there is a reflection when the wave reaches the tower. The voltages at this point are:[6]

$$V_{1t} = V_1 - \frac{z}{Z_2 + 2R} V_2$$

and

$$V_{2t} = \frac{2R}{Z_2 + 2R} V_2$$

where V_{1t} is the voltage on the line conductor at the tower, V_{2t} the voltage of the ground wire, and hence the tower top; V_1 and V_2 the oncoming voltages on the line wire and ground wire respectively; Z_2 the surge impedance of the ground wire, z the mutual surge impedance, and R the tower and ground resistance.

Assuming the same typical 200-kv. line chosen above, the surge impedance of the two ¾-in. ground wires in parallel Z_2, is 290 ohms, that of three one-in. line conductors in parallel, Z_1, is 205 ohms, and the mutual surge impedance z, is 96 ohms. The surge impedance of the lightning path at the line height will be assumed to be, $Z_0 = 200$ ohms. The surge admittances then are $Y_0 = 0.005$, $Y_1 = 0.00488$, $Y_2 = 0.00344$. The surge impedance of one line conductor is 470 ohms and the surge admittance 0.00222. With these constants the voltages at the tower were calculated in terms of the lightning voltage V_0 at the point of contact for various ground resistances and the results plotted in Fig. 24. In addition to curves for V_1 and V_2, a curve of $V_2 - V_1$, or the potential across

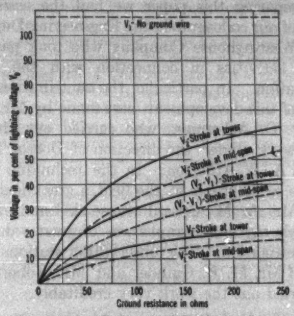

Fig. 24—Surge Voltage on Ground Wire V_2, on Line Wire V_1 and Difference of Voltage as a Function of Ground Resistance in Ohms in Terms of the Lightning Voltage at the Line V_0

the insulators, has been plotted. Although based on an assumed surge impedance of lightning path, V_0, these curves illustrate the importance of low ground

Fig. 25—Illustration Showing Lightning Stroke near Cheoh-Alcoa Line

resistance in obtaining the highest measure of protection from ground wires against direct strokes. The value given for V_1 with no ground wires was calculated on the basis that the lightning stroke hit only one

6. J. H. Cox and J. Slepian, "Effect of Ground Wires on Traveling Waves," *Elec. Wld.*, September 22, 1928.

conductor. This illustrates that a large measure of protection is obtained from ground wires even with a high ground resistance.

ACKNOWLEDGMENT

In preparing this paper we had the assistance of several of our colleagues in the Engineering Department of the Westinghouse Company who took part in the various activities in connection with the lightning investigation. In particular the contributions to the success of this development were made by Mr. E. Beck and Mr. O. Ackerman, who carried out the actual design work under the direction of Doctor Norinder and Mr. A. L. Atherton. In the technique of handling the oscillograph in the field, a great deal of credit is due Mr. A. M. Opsahl whose skill and experience in cathode ray oscillographs was of great value in the field investigation. The observation stations were in charge of Mr. M. E. Gainder and Mr. Robert Sparks who carried out the work very creditably.

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Discussion

THEORETICAL AND FIELD INVESTIGATION OF LIGHTNING

(FORTESCUE, ATHERTON, AND COX)

FIELD AND THE LABORATORY

(PEEK)

NEW YORK, N. Y., JANUARY 29, 1929.

K. B. McEachron: It is a matter of considerable interest to me that just a year ago at the Mid-Winter Convention in discussing Dr. Harald Norinder's lecture I stated that we had succeeded in finding a means of automatically setting the cathode ray oscillograph in operation when lightning came along. Since that time we have seen four or five installations, and a certain amount of data has been secured.

The study of the effects of lightning on electrical systems is naturally divided into two parts. One is to find out what kind of surges natural lightning produces both for indirect and direct strokes. The other part is to apply to lines impulses similar to natural lightning and observe the effects not only on apparatus but the effect of lines and apparatus on the waves traveling in the conductors.

During the interval since the last Winter Convention, the lightning-arrester research laboratory of the General Electric Company has begun a study of artificial lightning on the Turners Falls transmission line, which extends from Pittsfield to the Connecticut River, almost 40 mi. away.

A portable impulse generator which gives an open-circuit voltage of nearly half a million has been constructed. At the Pittsfield end a cathode ray oscillograph has been set up. By the use of this equipment it is possible to apply to the transmision line voltages of any desired wave shape and to record what happens to those waves when they strike apparatus in the station. This can be repeated as often as desired and thus the results may be checked accurately.

The problem of attenuation is easily solved by this method because we are able to apply to the transmission line not only full waves but chopped waves which correspond to what happens when lightning flashes over insulation, so that the effect of the various factors which influence attenuation may be determined.

This method of investigation is particularly valuable in studying the effects of protective devices. Considerable effort has been expended in the past two or three years on one system in particular to secure quantitative data on the protective value of lightning arresters when functioning under natural lightning discharges. Because of the fact that we are unable to have the arrester there and not have it there for the same lightning surge, the data have been inconclusive, but now we are able to go to the transmission line and apply a definite wave both with and without the arrester and find out what the degree of protection is.

This work has brought out an interesting side line. Until a year ago we were quite satisfied to make all our calculations on transmission lines in terms of perpendicular waves. Once we began to put on these lines waves which were not perpendicular, we were no longer satisfied with calculations of perpendicular waves, and it became necessary to develop means of calculating waves which are not perpendicular and which do not have perpendicular tails. This has been quite successfully worked out, and in a forthcoming article in the *General Electric Review* will be given a method which I believe is new for calculating the

effect of capacity and inductance, and combinations of capacity and inductance and resistance at the terminals of lines.

J. H. Cox: As in the past, Mr. Peek's paper has given us some new and valuable information on the subject of Lightning. However, I feel that the note of definiteness and finality in many of his statements and deductions is not warranted by the available facts, and coming from one prominent in the field of lightning research, is to be regretted, as it is likely to be misleading to those not so familiar with the actual situation. One would be inclined, after reading Mr. Peek's paper, to believe that the whole story about lightning is known, and to feel that any more work along this line is unnecessary. Such a feeling would, indeed, be unfortunate in view of the vital need of more lightning data, and of the fact that we are now only fairly started on the most difficult and expensive part of the lightning research program. As Messrs. Fortescue and Atherton and I attempted to bring out in our paper presented at this meeting, lightning still is the limiting factor, preventing the realization of the possible advantages of transmission systems. We do not have lightning-proof lines, we do not know enough about lightning, and we need more data.

True, the klydonograph has yielded much valuable information, but it has definite limitations. Data from it indicate rather definitely the magnitude of lightning surges as limited by line insulation, the number of surges occurring at one point, the attenuation along the line, and the polarity of surges. They indicate only approximately the steepness of the wave front, practically nothing of the duration and the magnitude to which lightning surges would go if not limited by line insulation, and nothing of the shape of the tail of the waves. Mr. Peek states that surges recorded by the klydonograph indicate that insulation has impulse ratios of from 1.8 to 2.8 to lightning surges. The surges indicating these values probably flashed over on the front of the wave and therefore indicate something of the wave front but nothing about the intended duration of the native lightning surge. The only klydonograph records that indicate anything about the duration of lightning surges are those of surges which did not flash over the line. In our rather wide experience with this instrument we have only recorded one surge indicating a high impulse ratio when there was not a simultaneous line flashover, and we have a great number of data which indicate that it is possible for a surge to flash over a line without a power follow and a trip out. Also, one such record is far too few on which to draw any conclusions, in that direction, particularly in view of the limitations of station logs. The conclusion indicated by the klydonograph records is that line insulation is not capable of withstanding any surges much above the 60-cycle crest flashover voltage for their entire duration, and that the impulse ratio for lightning surges is very little greater than one. From laboratory data this indicates that the usual duration of lightning surges is not less than 20 microseconds, but something over 20 microseconds, although this cannot be conclusive.

All of the above merely emphasizes that, although the klydonograph has greatly enriched our knowledge of lightning, there is crying need for further information. This can be obtained with the present forms of the cathode ray oscillograph. At the present time we have the sum total of two authentic oscillograms of lightning on transmission lines in the United States, one obtained by the Westinghouse Company and one by the General Electric Company. In addition to these there are those obtained on low-voltage lines in Europe, as discussed by Dr. Norinder before the Institute at the 1928 Winter Convention. These are far too few on which to base conclusions and none of them is all that could be desired. However, the experience in obtaining them was sufficient to assure that entirely satisfactory oscillograms can be secured. Undoubtedly, they will be secured, but it will take a great deal of effort and all possible support is needed. In view of this, it would be most unfortunate to delay the program of this work by misleading those from whom support is needed, to believe that the lightning surge problem is solved.

J. J. Torok: Mr. Peek has mentioned these phases of "Transient Phenomena" that are most interesting to the operating engineer. He has dealt with the characteristics of transients as they appear upon the transmission line including some of their effects upon apparatus. The conclusions drawn are in general taken from data obtained in the laboratory. The laboratory experiments are in turn justified, chiefly by the close agreement of the impulse ratio of insulators as determined by klydonograph studies in the field and by more exact methods in the laboratory.

In the study of insulator flashover, it happens that the boundary conditions are of cardinal importance, since they occur rather frequently. The waves corresponding to low impulse ratios are of low magnitude and long duration, thus simulating a 60-cycle wave. To reduce the number of flashovers incurred from such waves, it is only necessary to arrange the insulators and auxiliary equipment to give a high 60-cycle breakdown value. In general, the increase in 60-cycle breakdown is accomplished by creating a more homogeneous field with large-surface electrodes. Hence, grading rings and arcing rings should have large surfaces to prevent low voltage surges, that are not detrimental to the transformers, from flashing over the insulators and causing outages.

Large-surface electrodes possess very desirable characteristics under rapidly rising surges of high magnitude. They set up a more homogeneous field in which the breakdown process is far more rapid once it is started. As a result they have a low impulse ratio. The deteriorative effect of the surges upon solid insulation is a factor that increases with the voltage, thereby branding any protective device giving high impulse ratios as detrimental.

Thus it is seen that large-surface electrodes have the most desirable characteristics for all types of surges. Simple laboratory tests have shown that rain drops do not change the characteristics of large-surface rings to any appreciable extent.

Mr. Peek has tried to show the process of insulator flashover by means of a volt-time cathode ray oscillogram. Much can be learned from such records. However, a study of sphere-gap breakdown has shown that only a part of the story is told in volt-time oscillograms, since a kink in a wave may come from other causes than breakdowns. Dr. J. Slepian suggested that the streamers developing during the process of breakdown might draw an appreciable current. Accordingly various tests were carried out on sphere-gaps as well as insulators. In homogeneous fields, streamer currents of over 3000 amperes have been recorded. In a non-uniform field these currents are considerably lower. In the latter case the streamer currents become appreciable only after the ionization process has been carried to the final stages.

Recent work on insulator strings equipped with sharp-edged arcing rings has shown quite conclusively that the purely corona currents shown in Fig. 21 of Mr. Peek's paper are too small to detect with an oscillograph sensitive to a current of 5 amperes. The observations were made up until 8 microseconds after the potential had exceeded the 60-cycle flashover value. The wave had an abrupt front and a flat top maintained at 1.25 times the 60-cycle breakdown value. The current did not become appreciable until the streamers or actual breakdown had started, after which flashover is inevitable. With currents as small as those due to corona between rings only, it would take at least a 100-mi. line of such construction to reduce a 900-kv. surge to 700 kv., which was the 60-cycle flashover value of the arrangement tested. In view of these results, we would be interested to have Mr. Peek explain more fully how enough energy to prevent a flashover can be absorbed on one insulator string.

P. H. McAuley: In the Marx surge generator which Mr. Peek criticizes, the supply voltage is rectified and the condensers are charged in parallel and discharged in series. The group

voltage is limited by the a-c. voltage available and by the voltage rating of the rectifiers. Standard equipment is used throughout.

With Mr. Peek's system the charging period of the condensers necessarily is limited to 1/240 sec. Furthermore, unless the polarity of the generated surge is left to chance, a synchronous timing switch or a tripping device must be used. To obtain a small number of gaps in the discharge circuit, which is of little practical advantage as a matter of fact, it is necessary to use special high-voltage condensers of large dimensions and of limited capacity. An increase in voltage or capacity is complicated compared to the other connection.

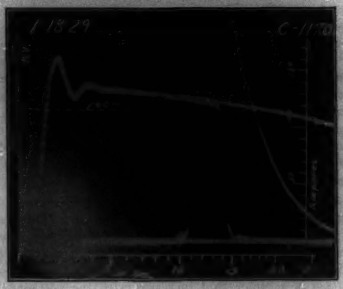


Fig. 1

Perhaps the main advantage of the Marx-connected generator is its flexibility and ease of handling. This is very well illustrated in connection with cathode ray oscillograph work. Fig. 1 herewith shows an oscillogram made with 3 successive applications of voltage to an insulator string.

1. Full wave without flashover
2. Flashover at 14 microseconds
3. Current on flashover

Note the perfect synchronization of the two voltage waves up to the time of flashover. The same perfection is obtained with a time scale many times as large. The completeness of detail compared to Figs. 5 and 6 of Mr. Peek's paper is very noticeable.

C. F. Harding: The papers at this session confirm the belief even more positively than the many previously presented upon this subject, that the cathode ray oscillograph, equipped for field use with adequate accessories and automatic control devices, is the only instrument suitable for the study of lightning transients. Although it may be supplemented by the klydonograph for determining approximately the attenuation of the transient, only the cathode ray oscillograph, automatically operated by the transient itself, will provide a permanent record of the magnitude, steepness of wave front, shape, and duration of the lightning surge.

The first step, therefore, looking toward the solution of the problem of adequate lightning protection, is to secure several such oscillographs, obtain simultaneously the maximum number of records upon transmission lines, upon exposed primary distribution systems at various voltages throughout the country and even upon secondary distribution systems. Secondly, such transients as have been found to exist should be reproduced by means of lightning generators in the laboratory and the cathode ray oscillograph used to check the type and magnitude of the transient used and its effect upon the equivalent line or lightning-protective equipment.

Such an oscillograph has been designed, constructed, and used by R. H. George, Research Associate of the Purdue University Engineering Experiment Station at LaFayette, Ind.[1]

Its advantages, particularly for field use, as demonstrated last summer upon the 140-kv. transmission lines of the Consumers Power Company of Jackson, Michigan are as follows:

(1) A sharply focused electron jet of high photographic intensity is available at all times, without fogging the film. Such a beam has been in readiness for hours at a time without film fogging.

(2) The transient itself initiates the exposure, whether it be positive or negative, and reproduces all of the surge after the first one-fourth or one-half microsecond. Advantage is taken of the voltage, proportional to the rate of change of leading current in the condensive circuit of an air-core, close-coupled transformer to affect the grid potential of one of two vacuum tubes, the plate voltage of which controls the electron-jet release of the oscillograph.

(3) The jet is readily focused to a sharp image upon the film by means of a new electrostatic focusing control operated from outside the vacuum. This process is not sensitive to gas pressure within the vacuum chamber.

(4) This oscillograph, which is of the hot-cathode type, provides many advantages, such as satisfactory operation with the wide range of gas pressure likely to exist within the oscillograph chamber in a field installation and with a variety of voltages ranging from 500 to 20,000 volts as an accelerating potential for the electron jet. Any gas pressure below 30 microns may be employed. The deflecting plates may be adjusted from outside the vacuum chamber.

This oscillograph is portable, entirely self-contained, and operated from any 110-volt, 60-cycle source with less than 1 kw. of input. The transient deflecting potential and the transient initiating voltages are supplied from simple aerials or capacitance potentiometers placed near the line upon which the surge is to be measured.

(5) A film holder, of the daylight-loading type, provides from 75 to 100 exposures, each 3 by 5 in. in area, with one loading. The vacuum is not broken during this period and the unexposed area of film may be moved into place for exposure in a predetermined sequence by manual or automatic operation.

(6) A timing wave from an oscillator, ranging from a 10,000- to a 500,000-cycle sine wave, is provided to spread out the transient which is easily recorded with a 10-kv. beam when traveling across the film at 125 mi. per sec.

It is hoped that many parallel investigations may be made during the next two or three lightning seasons in order that the interruptions to service may be reduced to a minimum.

Herman Halperin: From reading the papers, I get the impression there are various degrees of confidence in knowledge of lightning and its effects. It seems to me that considerable investigation is still necessary. For instance, there are only a few oscillograms on wild lightning.

It would be well to extend these studies to distribution circuits ranging from 115 volts up to, say, 15,000 volts, since the operation on such circuits affects the customers directly.

All of the oscillograms on wild lightning show single strokes although meteorologists say that frequently several strokes occur very close together in time and space.

Perhaps the effects of such multiple flashes would be different in line apparatus as compared to the effects of single flashes.

G. D. Floyd: The Hydro-Electric Power Commission placed in operation October 1, last year, a single circuit of 220-kv. line, extending from Ottawa to Toronto, a distance of about 200 mi.

In going over the literature that has been accumulated on lightning surges and correlated data, we found that apparently

1. This instrument is described in *A New Type Hot-Cathode Oscillograph*, by R. R. George, which will appear in the A. I. E. E. Quarterly Trans., July 1929.

the greatest number of data had been accumulated on the magnitude of the lightning surge on the transmission line itself and a relatively small number on the surge that might appear at the terminal apparatus.

I feel that there seems to be a difference of opinion among investigators on this subject, so I want to make a plea for those engineers who have made close and comprehensive study of the question of insulation strength and insulation surges, if it is possible, to get together. In the situation that exists, the buyer of apparatus doesn't know where he is.

I do think that Mr. Peek's paper was a little optimistic. There is a remarkable agreement between the actual lightning surge that he recorded and the one that he produced. I think one is very optimistic indeed if he would say from the result of ten records obtained in actual surges that the agreement would be maintained in all cases. I think that even a thousand records would be necessary before one could conclusively say that the actual surge was alike in all cases. It may be that if a hundred were taken, ten of them would appear like the record Mr. Peek has obtained and the other ninety would be altogether different.

C. E. Ambelang: Referring to Mr. Peek's paper, we have at present seven lines of over 200 mi. total of single circuit on double-circuit towers, with provision for two ground wires, one ground wire installed above each circuit, half way between the center of the tower and the conductor.

Last year we had three lightning interruptions in these seven lines, two of which were, we believe, direct strokes, one being over all three phases, and the second one involving 12- and 33-kv. lines on wood poles that were underneath the 132-kv. line.

Our insulator strings have a lightning flashover value, according to Mr. Peek, of 1350 kv. Using a 60 per cent reduction due to ground wire we have an induced lightning voltage of 3375 kv., which gives us a potential gradient for an average height of a conductor, 72 ft., of 46.8 kv. per ft.

After applying this to a wood-pole line, a 35-ft. pole would give us 1638 kv. induced voltage on our wood-pole lines. Mr. Peek in his paper gives an average insulating value of wood poles of 180 kv. per ft., which would make a 35-ft. pole with a 5-ft. crossarm have a lightning sparkover value of 7200 kv. That is a potential gradient of 206 kv., which is four times that on the tower lines in the same territory.

Citing from our records, there are three pole lines operating in the same vicinity. These lines are without ground wires.

Line *A*—12-kv. line, 17 mi. in length, had 33 cases of insulator flashover due to lightning last year. Eighteen were on guyed structures and 15 on unguyed structures.

Line *B*—33-kv., 17½ mi. long, 15 cases of flashover, 8 on guyed and 7 on unguyed poles.

Line *C*—33-kv., 19 mi. long, had 11 cases of flashover, 4 on guyed poles and 7 on unguyed poles.

It is these unguyed structures that we are worrying about because we ought to have some way of reducing the flashovers, especially if that insulating value of the wood pole is anywhere near correct. We should like to ask Mr. Peek what his explanation would be of the large number of cases of lightning flashovers on the wood-pole lines.

Mr. Peek states, "When a pole is quite wet, incipient sparks will take place over the insulator string at voltages approximately equivalent to the lightning sparkover voltage of the insulator string."

Could it be possible that such sparks could be responsible for the trouble which we have experienced, these sparks occurring within the limit of the voltage induced by lightning during ordinary storms?

E. S. Lee: An oscillogram of a lightning wave taken last summer is shown in the accompanying illustration Fig. 2. This was taken July 27, 1928 on the 220-kv. Wallenpaupack-Siegfried transmission line of the Pennsylvania Power and Light Company by means of a General Electric cathode ray oscillograph. The record shows that the surge voltage was positive in polarity and uni-directional, reaching its maximum in 5 microseconds, and decreasing to half value in 20 microseconds, and to zero in 40 microseconds. Superimposed on the surge at the peak was a highly damped oscillation of 2,000,000 cycles probably caused by induction from the insulator flashover which occurred on an adjacent conductor.

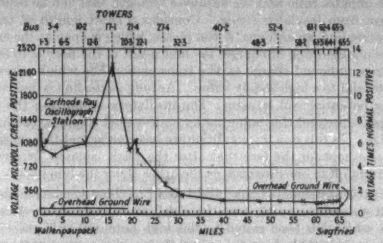


Fig. 2—Values of Voltage Recorded by Surge Voltage Recorders Along the Line

Surge voltage recorders connected to Y conductor Wallenpaupack-Siegfried 220-kv. line. Pennsylvania Power and Light Co.

As far as is known, this is the first cathode ray oscillogram of a surge voltage due to lightning obtained on a high-voltage power transmission line in the United States. It is remarkably similar in time characteristics to the waves which have been used in the laboratory. Fig. 28 in Mr. Peek's paper shows the almost exact similarity between the wave form of this surge and one used in his laboratory for experimental work. Also the cathode ray oscillogram shown by Messrs. Fortescue, Atherton, and Cox, in their Fig. 13, shows the wave form of a surge voltage with essentially these same time characteristics, if I interpret their oscillogram correctly.

When this oscillogram was made, simultaneous records of voltage were made by surge recorders connected to the same conductor (the West conductor) at various points along the line. These are shown graphically in Fig. 3 herewith. The maximum recorded voltage was at tower 17–1, the value being between

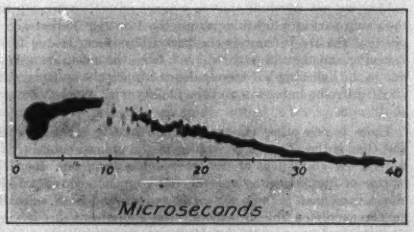


Fig. 3—First Cathode Ray Oscillogram of Lightning Surge on Transmission Line in America

2100 kv. and 2450 kv. At tower 1–3 near this cathode ray oscillograph station the voltage measured was 1000 kv. At the end of the line the voltage was 1260 kv., showing the characteristic increase at the line terminal. Towards the other end of the line the voltages measured decreased rapidly in value.

Line inspection following this surge showed that at tower 16–3 three insulators were broken on the middle conductor and practi-

cally all the insulators were flashed on the east conductor. A surge voltage recorder connected to this conductor at tower 1–3 recorded 1660 kv. The insulator string on the middle conductor at this tower flashed over. At this tower the spacing between the shielding rings had been reduced to provide protection for the terminal apparatus.

P. H. Thomas: I want to raise one point in connection with the impulse ratio and the flashover strength that can be counted upon from insulator string. Consider a short length of transmission line, an antenna, if you please, which is open at the ends. Upon a lightning discharge, the first result of the flash is to leave a free, unbound charge on the transmission line which raises its potential to a high steady value. As long as the line is insulated, this potential will remain. The insulators supporting the line must withstand this potential on its so-called 60-cycle insulating value. Is this not the condition we have on actual transmission lines except for the fact that after the flash, current or charge can run off in one direction or the other along the line itself?

If this statement is true, and I don't see how we can escape it, and we assume an exposure in which there is a large section of line which has been raised to this high potential, will there not almost inevitably be time enough before the central portion of the exposed section can be discharged by the melting away from the ends, to allow the strength of insulators to be reduced nearly to the 60-cycle value? The only assumption on which we can conclude that a high impulse ratio can be counted upon is that the section of line affected is very short. This apparently is usually the actual fact.

Instead of picturing actual lightning as a flash between a large flat level metal plate and the earth which is another plane surface, let us picture the charged cloud as a highly irregular upper plate with probably a funnel shaped portion pointing downward toward the earth from which the discharge actually occurs. Then it would seem very reasonable to assume that the only portion of a transmission line which has the maximum induced potential would be of very short length. Whether this would be less than one span or more than two or three spans, I don't know. In such a case it is easy to believe that a charge once accumulated and suddenly freed by the flash will actually begin to flow off even before the discharge is completed. In such a case we will get a wave form for the tail of the surge in the line, similar to the forms which we see on the oscillograph. It is only on this assumption of very short line exposures, as I see it, that we can explain such tails to the surge.

May we not conclude that in a surge on a transmission line, the rising part of a lightning surge, the 1 or 2 or 5 microseconds portion, the steep front is the part whose form is due to the lightning and the tail part, which is from the crest down, is not due to the lightning but merely depends upon the amount of line affected by the induction and the facility with which it flows off at the end.

There is one other matter I should like to mention briefly and that is the effect of resistance in the ground connection of towers. A great deal of importance has been put upon this matter of ohmic ground resistance. I am not convinced that ohmic resistance is important. If you ground both ends of a long transmission line and pass alternating current through it at 60 cycles and measure the total inductance and resistance of the loop, you will find that there is a very large and predominating amount of reactance in the ground return.

The net result is this: That we must consider even in lightning, I am sure, the effect of the reactance of the path of the current in the ground as well as its resistance.

In measurements of ground resistance and reactance at 60 cycles, the reactance is greater than the resistance. At lightning frequencies the reactance of that circuit is multiplied 100,000 times, while the resistance does not change except for skin effect and is much less than the increase in reactance. From this it would seem to follow that the resistance is relatively unimportant in lightning surges.

J. R. Eaton: The Consumers Power Company cooperating with Purdue University has been studying the effects of lightning on transmission lines in a manner very similar to that described in the paper by Messrs. Fortescue, Atherton, and Cox. The cathode ray oscillograph used in the study is of a type recently developed by R. H. George of Purdue University as mentioned in C. F. Harding's discussion.

Fig. 4 herewith is an oscillogram of a laboratory wave made with the instrument operating exactly as when set to record lightning transients. From this record it is seen that the instrument gets into operation in less than ¼ microsecond, and records the complete wave shape thereafter.

Fig. 4

The instrument was connected to a 140-kv. line through a capacitance potentiometer quite similar to the one used by the Westinghouse Company as was described by Mr. Cox.

During the period of operation no lightning discharges took place in the immediate neighborhood of the oscillograph station. Storms on distant sections of the system produced flashover which resulted in general system disturbances. Four cathode ray oscillograms were recorded as the result of such surges but the wave shapes obtained were probably the record of secondary disturbances rather than of the lightning itself.

The results of the past summer's work has demonstrated to our satisfaction that this oscillograph is well suited for recording the wave shape of lightning surge voltages.

In the early spring the investigations will be continued with the set in operation in the middle of a 100-mi. line. This point has been chosen with a view to eliminating wave distortion due to reflections at the end of the line and to lightning arrester operation.

Edward Beck: One of the most interesting aspects of Mr. Peek's paper is the confirmation of the high-speed cathode ray oscillograph as a tool for lightning research. This instrument now gives us knowledge where before there has been much conjecture. For example, it was formerly believed among certain investigators that lightning disturbances were exceedingly steep,—fronts of a few hundredths of a microsecond were frequently mentioned. Before the cathode ray oscillograph was available it was thought that waves of such steepness were easily reproduced in the laboratory by circuits whose calculated constants would lead to the expectation of waves with fronts of this order. The oscillograph has shown that factors not always considered previously, enter into the calculations of impulse generators, particularly where steep-front transients are concerned, and that exceedingly steep waves are not generally procured except by very special methods. On the other hand, it was found that the agreement between calculations and actual measurement for waves with longer fronts is excellent.

We are led to believe by oscillographic studies that the fronts of natural lightning transients are not as steep as formerly believed. Both of the oscillograms of transients on high-voltage transmission lines presented before this convention are apparently in agreement that the fronts are of the order of five microseconds. Of course the data so far collected are too meager to draw definite conclusions at this time, and no one would be rash enough to say that the problems have been solved. With the tools now at our command, we are on the threshold of complete knowledge, but the work has but little more than begun. One of the oscillograms shown today is complete, showing the entire front. There is still much information to be collected concerning the various transients apt to occur, such as their coordination with the location of strokes, rate of cloud discharge, and details regarding the transient fronts. But it has been established that these things can now be measured, and with oscillographs that make it possible to obtain complete records, the information will not be long forthcoming, particularly with concerted effort. The two oscillograms published, while they agree as to front, indicate already a variation in duration. Whereas the Pennsylvania transient indicates a duration of 20 microseconds above line voltage, the Tennessee record shows a duration of 50 microseconds above normal voltage. On this evidence I must disagree with Mr. Peek's conclusions that the duration of lightning transients may be 1 to 20 microseconds only. The similarity of the fronts lead us to believe that a wave with a quarter microsecond front is a very steep one, whereas formerly this was considered relatively slow and much more rapid ones were thought frequent.

I should like to ask Mr. Peek what is the crest voltage of the lightning transient shown in Fig. 28 of his paper, also whether the time lag of the oscillograph introduced by the external switching arrangement is sufficiently definite and constant so that the missing parts of the wave fronts can be interpolated with accuracy. Our laboratory investigations show that irregularities of several microseconds occur.

Philip Sporn: In connection with Mr. Floyd's discussion of Mr. Peek's paper, it might be of interest if I mention some experience we had in putting surge voltage recorders on equipment at terminal points. For quite a number of years we have made a practise of installing permanent recorders at all the large transformer installations and treating them as part of the equipment. We did that particularly on complicated transformer designs. Over a period of three years, we have obtained a considerable number of data that we believe has helped us in our design and operating problems.

H. L. Wallau: (communicated after adjournment) In Mr. Peek's paper reference is made to Mr. Lewis' work in connection with the attenuation of lightning surges along a transmission line.

The information usually desired is the distance along a transmission line which a surge will travel before the initial voltage $E\,o$ is reduced to some given value E.

The formula as given there by Mr. Lewis' is not in convenient form for this solution but may be converted to that below.

$$L = 6200\ Y/E\,o$$

in which

L = distance in miles

$6200 = 1/K = 1/0.00016$

$Y = (1 - x)/x$

where x is a decimal fraction such that $x\,E\,o = E$.

It will be seen that for a decrease to one-half of the original value of surge voltage Y becomes unity and $L = 6200/E\,o$.

For other values of E the value of L so obtained is increased or decreased by the corresponding value of Y as shown below.

Value of E in per cent of $E\,o$	Multiplying factor to be applied to $L = 6200/E\,o$
90	1/9 or 0.11
80	2/8 0.25
70	3/7 0.43
60	4/6 0.67
50	5/5 1.00
40	6/4 1.50
30	7/3 2.33
20	8/2 4.00
10	9/1 9.00

H. L. Melvin: (communicated after adjournment) Referring to the section of Mr. Peek's paper on the subject of wood poles, I am assuming that the data and information were taken from the tests made during the past year for several of the operating companies under the general supervision of the company by which I am employed. These tests were undertaken for the purpose of obtaining some fundamental data on the impulse insulation characteristics of wood-pole lines constructed for 66-kv. to 132-kv. operation. It is hoped that sufficient actual operating experience will be obtained by the close of the coming lightning season on the several transmission lines, on which the results of the tests are being tried in an experimental way utilizing the insulating value of wood in varying ways and degrees, to check the information obtained in the laboratory. It may also be possible to obtain some idea of the value of increased impulse insulation on the performance of transmission lines.

As Mr. Peek has stated the species of wood, amount of moisture or contamination seems to have little influence on the impulse sparkover values. Quite a variation was observed between the samples tested, but this is as would be expected since ordinary wood should not have uniform insulation characteristics. A value of 170 kv. per ft. plus or minus 20 per cent is being used for wood poles and 190 kv. per ft. plus or minus 20 per cent for wood crossarms and timbers of smaller cross sections suitable for long wood guy insulators. These values are for tail of wave sparkover and the specific test wave used. They also include practically all the test points.

It might be stated that the 6-to-3-to-1 ratio of wood to air gap to clearance of the gap from the wood for designing by-pass horn gaps to protect wood poles from damage would be critical for an average specimen and the particular test wave. To provide a factor of safety for the average pole and provide protection for poles as low as 20 per cent under the average a ratio of 10-to-3.6-to-1.75 is being used. These relations cannot be used for crossarm protection as the voltage appearing on the arm is a function of the relation between the type and amount of porcelain and the crossarm length. Whether by-pass air gaps employing these dimensions will protect poles under actual lightning conditions will require field experience.

To obtain sparkover values for combinations of porcelain insulation and wood it was necessary to make tests on the combinations by varying the amounts of porcelain, crossarms, and pole through the ranges which might be used in practise. The sparkover values of the component parts cannot be added directly, which is to be expected, and it is also impracticable as yet to give any rule which can be applied for combining the individual values to get the overall sparkover voltages.

If the results of these tests are confirmed by field experience and a considerable improvement in operating performance can be expected from lines with high impulse insulation strength, having available complete test data of this character should make it possible to improve the conventional wood-pole designs and develop modified designs which will utilize the wood to a much greater degree and more effectively. This may not be

practicable in all situations, however, on account of the possibility of crossarm and pole fires.

Possibly it would be well to caution against the application of these very incomplete data as there are many other factors to be taken into account and additional data required in a co-ordinated design. This discussion should simply be taken as a brief outline of the tests and their possible application.

Harald Norinder: (communicated after adjournment) The paper presented by Messrs. Fortescue, Atherton, and Cox gives a full and complete description of a field research station for investigation of lightning transients on transmission lines. This paper has been completed by a theoretical discussion of the propagation of lightning surges on transmission lines and of the effect of ground resistance on the protection afforded by overhead ground wires. The theoretical part of the paper completes in a very happy manner the existing literature, particularly as the authors' treatment is in such a form that it is easily applied to actual problems on transmission lines. The effect of ground-wire protection has been treated by the authors in an original manner which will have considerable importance in all experimental work on ground-wire protection.

It is with great satisfaction that we can state that several fully equipped stations for the investigation of lightning surges have been erected in locations in the United States where the expected frequency and violence of thunderstorms is high. There is thus a good opportunity for securing valuable records in a relatively short time. It is unfortunate that only one surge of large magnitude has occurred in these locations. However, this does not diminish the general and future value of the station. For a number of years, I have had the opportunity to observe the occurrence of lightning transients on lines in Sweden. Sometimes a whole group of thunderstorms will pass in the vicinity of the transmission lines without producing any lightning surges at all. On the other hand, the lines have carried whole sets of disturbances in other storms. There are many accidental circumstances which govern the production of surges on electrical systems. Hence I am not astonished that more records were not secured last summer.

In this connection it will be of interest to recapitulate the lightning researches which have been carried on in the Laboratory of the Swedish Royal Board of Waterfalls near Uppsala. Ten years ago the Board of Waterfalls decided to proceed with a systematic research of the general field conditions during thunderstorms. The motive for this work was a desire to obtain records of field changes produced by lightning discharges and thus to have data for studying the development of lightning surges on transmission lines under full load conditions. This extensive research in Sweden was suggested by the Chief Director of the Swedish Royal Board of Waterfalls, W. Borgquist. His opinion was that researches along these lines were a practical necessity. He believed that it was quite necessary to revise our ideas on the overvoltage problem and that this revision could be undertaken only with the aid of extensive experimental work. The same point of view was later taken also in other countries. At the present time the necessity for such work is clear to everyone who is concerned with the possibility of service of long transmission lines. At the beginning of the researches, much of the literature failed to disclose any information on the field forces or general field condition during thunderstorms. Opinions were divided and the conclusions arrived at were sometimes not in accord with existing physical facts and laws. Therefore it was necessary to begin at the beginning.

The first steps in the research were to study the field conditions by extensive experimental work. In the existing literature it was assumed that the field forces were of the order of 3 kv. per ft. Observations at three or four stations soon showed field forces at least ten times higher. These results have later on been fully confirmed by the extensive investigations carried on in the United States by means of klydonographs.

From these higher field forces we could derive conclusions regarding the distribution of charges that were dissipated by lightning strokes. With high field forces, the regions must be relatively small and their radii sometimes cannot have been more than approximately 1000 ft. Simultaneous records from three stations operating at considerable distances from each other fully confirm these conclusions. This relatively small extent of the discharged regions made it possible to explain why a transmission line often does not show the expected frequency of occurrence of lightning surges during the storm. The field changes were sometimes extended over such a limited region that only an inconsiderable part of the transmission line was influenced. Data thus obtained were of great value in estimating the order of magnitude of surges on transmission lines, but it was not possible, by this method of observation, to secure any records of the character of the wave front and duration of surges.

Our next problem was to analyze the variations with time of the gradient in free atmosphere during the lightning disturbance. Such observations should give valuable indication as to how to take up the most important problem, namely, direct records of transients on transmission lines operating under normal conditions.

For this purpose I constructed a cathode ray oscillograph consisting of a closed cathode tube, a photographic lens system, and a revolving drum. By this arrangement, it was possible to record variations of 0.0001 sec. The oscillograph was connected to a suitable antenna circuit so balanced that it followed the lightning variation without appreciable distortion. The first results were obtained during the summer of 1921. They showed that the lightning discharges were unidirectional or quasi-periodic. It was sometimes quite evident that it took considerable time to develop the discharging process. In 1923 the cathode ray tube was connected to a transmission line. Thus I was the first to have opportunity to observe visually lightning transients on a transmission line. These direct visual observations were of value for the later construction of a special cathode ray oscillograph for recording these transients.

The construction of such a special instrument was begun in 1922 and during the season of 1925 we had the good fortune to take the first records of transients on the transmission line. Since then the instrument has been improved in many respects. In its present form it is identical with the cathode ray oscillograph described by the authors of the paper. With this oscillograph I have recorded numerous lightning transients on a 20-kv. transmission line during normal operation. The results I presented during the 1928 Winter Convention of the A. I. E. E.[2]

The cathode ray oscillograph necessitates several accessory pieces of apparatus, the most important of which is the high-vacuum pump arrangement. We have put forth considerable effort in the development of an effective high-vacuum pump. Another important adjunct is an indicator for the transient. The transient may be observed visually on a separate cathode tube with a fluorescent screen. This tube is connected in parallel with the cathode ray oscillograph. Hence it is possible by visual observation of the spot on the tube to follow the records, their amplitude, and their polarity.

Comparison of my records obtained at the Uppsala Station where I had used a method of coupling to the line analogous to the one used in the Tennessee location shows full agreement in the general shape and structure of the surges. In the immediate future, we shall take up particular studies of the wave-front variations. For that purpose we have need of several cathode ray oscillographs working simultaneously. Although in general the records secured more or less agree, it is dangerous to conclude from this that we will be able to discover some one typical kind of lightning surge. One must not forget that the primary phenomena which occur during thunderstorms are too shifting to allow regular forms of transients on the circuits. This

2. A. I. E. E. Quarterly Trans., Vol. 47, April 1928, p. 446.

conclusion applies particularly to disturbances from direct strokes. The most recent studies of the character of lightning discharges show the most variable discharge forms. I will illustrate this by some typical records of lightning discharges at the Uppsala Station. These records which are reproduced in Fig. 5 herewith were secured during the thunderstorm season of 1928.

During these researches we connected the cathode ray oscillograph to an antenna circuit with a time constant of a few microseconds. The circuit would thus follow the field variations without distortion. The method of investigation has been described by me in the *Franklin Inst. Jl.*, Vol. 205, June 1928. Only one oscillograph could be devoted to these researches. Hence it was not possible to study the variations at more than one location. In order to save time it was necessary to record several lightning strokes on the same film. By means of the simultaneous indications of the visually observed cathode tube, it was easily possible to separate the records on the film.

In Fig. 5 there are reproduced 8 separate lightning strokes with high amplitude. Besides these there are some smaller records produced by more distant strokes, sometimes of a pronounced slow moving characteristic. The remoteness of the 8 strokes cannot be given with full certainty in all cases. The

Fig. 5—Lightning Discharges Recorded with Cathode Ray Oscillograph at the Uppsala Station, Sweden, During the Summer of 1928

strokes marked with K and I were at a distance of either 3 or 3.2 kilometers. B was 3.9 kilometers away. The stroke marked A was 1.4 kilometers away and C and F either 2.8 or 2.1 kilometers. In Fig. 5 we note only two, namely A and L, which showed pronounced positive charge in the antenna circuit. B was quasiperiodic with changing charge in the circuit. The remaining ones showed negative charge in the antenna circuit.

Comparing, for instance, B, C, and I, we note essential differences in their general form of variation with time. This is physically well explained by the very shifting field structures in a thunderstorm atmosphere. In only a few instances do uniform conditions exist. It is possible that the discharge of C, F, and E are produced one after another with no significant differences in the general structure of the field.

The time to reach a crest value is very variable. We need only to compare K with I to observe this. In one case the crest value is reached in a few microseconds, in the other case we estimate the time to reach crest value to be about 800 microseconds. Of particular interest are the superimposed high-frequency voltage variations with amplitudes reaching 40 per cent of the voltage variation of the main discharge. These variations are very pronounced in B and L. In the future we shall take another time scale to analyze more completely the superimposed high-frequency variations. Using only one record station I have not found it possible to get a physical explanation of these superimposed fluctuations. As I have already pointed out in the *Franklin Inst. Jl.*, they are of interest with regard to the severe atmospherics in radio receivers.

The phenomena of lightning discharges were discussed before the Franklin Institute. In that discussion I based my opinion on researches made before the season of 1928. The results of the past season are in full agreement with earlier ones. The discharge process of a lightning stroke must, as a conclusion from our records, be preceded by partial streamer discharges which penetrate the dielectric and thus produce an ionization over increasing regions. The phenomenon is thus similar to the suppressed partial discharges that have been observed by Torok between sphere-gaps and described by him before the Institute.[3] Messrs. Fortescue, Atherton, and Cox have in their paper given a picture of what takes place just before and during a lightning discharge in the atmosphere. A close examination of the records of Fig. 5 show that their conclusions are quite true.

We have observed in some of the records that such discharges with the above mentioned suppressed discharges are pronounced. Such a one is K in Fig. 5. The crest value was in this case reached within a few microseconds. Careful examination of the records, however, shows that the stroke was prepared by a set of streamers of short individual duration. The total number of the streamers in this case has been estimated to be about 50 during a time of 1000 microseconds. Thus it has taken quite a long time to build up an ionization process sufficient to permit the main stroke to occur.

Not in all cases have we been able to study the discharge process as clearly as in this case. The record obtained depends somewhat on the location of lightning discharge with respect to the measuring circuit. In the above mentioned case the orientation must have been a very favorable one.

A further problem of interest is the general variation in form of lightning strokes when the strokes constitute a connection between cloud and earth. At least one and probably two such strokes were recorded in Fig. 5. Either C or F constitutes the general form of a lightning stroke between cloud and earth. At the moment of one of these two discharges a lightning stroke was observed to hit a wet pole of a 3000-volt line. This pole was about 2 kilometers away from the station. The 3-kv. line was connected to the 20-kv. line through a transformer. This line went out of service through its protective apparatus. It is thus evident that one of these two records C or F was caused by a stroke between cloud and earth. I cannot definitely say which one of them it was. The other stroke caused a heavy surge on another 20-kv. transmission line at some hundred meters distance from the one mentioned above. In the case of this surge, however, we have no certain observation of a direct stroke on the line.

We can conclude that discharges C and F represent types which occur when the stroke occurs between cloud and earth. Neither the lines nor the transformers were injured by this direct stroke, a circumstance that may be considered a little astonishing at first thought. We must remember, however, that there is pronounced difference between different lightning strokes.

There is no doubt that we must calculate with lightning strokes having energy varying in amount from 50 or 100 coulombs down to a fraction of a coulomb. It is obvious that in general direct strokes can pass into the poles and transmission lines without any hazard at all.

It is not possible to draw general conclusions from some few records. It is dangerous to make generalizations. The problem of lightning surges on transmission lines can be cleared up only by extensive and intensive research. My results secured by field studies are proof that such work is possible.

In our work in the study of lightning surges, we have had at

3. *Surge Impulse Breakdown of Air*, by J. J. Torok, A. I. E. E. Quarterly Trans., Vol. 47, April 1928, p. 349.

our disposal only one oscillograph. This has brought about a deplorable limitation in our work. For instance it has not been possible to make simultaneous records of surges at different points on a transmission line at the same time. Our efforts for future work are directed to the equipment of our station with several oscillographs which may work simultaneously. Visitors in our laboratory have sometimes expressed the opinion that "A cathode ray oscillograph is too complicated an affair." This is true, but requires qualification. The real fact of the matter is that lightning or a lightning disturbance is a complicated phenomenon and it is rather remarkable that research on such complicated phenomena can be carried on with apparatus not more complicated than our cathode ray oscillograph.

Objections have been made that it will always be difficult to engage people skillful enough to carry out such work. This is not the case as has been borne out by experience with the Tennessee station. There it has been possible to build and put into operation two fully equipped stations within a short time. Moreover, it will always be possible to find young scientists and engineers who have interest enough in the problem to surmount all difficulties which may exist. It is especially fortunate that the results of such research on lightning are of the utmost practical importance, besides being of great scientific interest. Such a happy coincidence will always be stimulating. It should be pointed out that it is not only for protection of transmission lines that the research on lightning is important. The protection of buildings and oil tanks and the elimination of atmospherics at radio receiving stations are other phases of the problem which await exact solution.

When discussing the future of lightning studies among interested people, I have always predicted that the United States will probably constitute the most fitting region for extended lightning and thunderstorm studies. My reasons for stating this are three: relatively high frequency and the violence of lightning storms in America, the great development and importance of transmission lines and other structures requiring protection, and last but not least, the traditions from the days of the venerable Franklin, whose keen experiments are worthy of the highest admiration.

C. L. Fortescue: (communicated after adjournment) I agree with Mr. Peek's remarks to the effect that lightning investigation has reached a stage where it may be said to be on an engineering basis. However, I wish to emphasize the fact that as Mr. Peek remarks, there is a great deal more to learn about lightning before we can confidently say we know enough to predict how a given transmission line will behave under lightning conditions.

I was very much interested in Mr. Peek's description of his new lightning generator which is apparently a modification of the Marx scheme which we are using in our high-voltage laboratory. The difference is that in Peek's generator the system of condensers is charged direct from the a-c. system, which means that the charge must be built up in the condensers in one-quarter cycle, and since the charging must be effected through high resistances, this appears to me to be a serious limitation. The limitation of the Marx scheme, which uses direct current for charging, is not in the matter of charging the condensers but in the limit of the rectifier tubes which seems to be about 100,000 volts direct current. Consequently, with a d-c. scheme many more sections are required. On the other hand there is no limit to the time of charging. However, I do not agree with Mr. Peek that the Marx scheme using direct current is only applicable to low-voltage surges. In our Trafford Laboratory we have a 2,000,000-volt generator which has been extremely satisfactory and which could, if necessary, with equal reliability, be extended to much higher voltages.

The gratifying progress that has been made within the last few years in the study of lightning phenomena naturally leads one to be somewhat optimistic in regard to what is known about lightning surges at the present time. My impression on first reading Mr. Peek's paper was that he presents too rosy a view of what has been achieved up to the present time in regard to lightning investigation and the same impression might be conveyed to those unfamiliar with the difficulties of the investigation and the immense amount of work that has yet to be done before we can claim to have a real working knowledge of lightning and its effects on transmission lines. A further reading of the paper indicates to me that Mr. Peek's reaction was that of a man who has been working for years on a more or less speculative program and begins to see the path broadening out and as a result many of his speculations are being confirmed by actual observation and he naturally remarks "lightning has been removed from the realm of the 'medicine man' and brought to more or less of an engineering basis."

As regards the wave form of lightning, I am inclined to disagree with Mr. Peek in the impression he conveys in his paper that it has a very definite wave form which is closely approximated by his laboratory wave; for example, where impulse ratios of 2 and larger are obtained, with the wave form Mr. Peek uses, the same impulse ratios could be obtained on lightning waves building up at a much slower rate but causing flashover before the crest is reached. Thus for such a wave building up at the rate of 4000 or 5000 kv. per microsecond, an impulse ratio of two or more may be obtained on a 14-unit string. I gather that this is illustrated in Fig. 17 of Mr. Peek's paper, where he shows three waves. One of these is a 20-microsecond laboratory wave, the second an intermediate wave, and the third one having a very long front; all three of them give the same impulse ratio when flashing over an insulator string. Mr. Peek's laboratory wave has a certain advantage from a laboratory point of view and this is that with a very high crest value impressed, where the flashover takes place at the front, he has an extremely steep wave, much steeper than I believe is possible due to lightning.

It is difficult on purely theoretical grounds to deduce how steep a wave front may be obtained from a direct stroke of lightning to the line; possibly some day we may be able to approximate the value from calculations but at the present time we do not know enough about the behavior of lightning, the volume of the cloud that is discharged, etc., to be able to tell how much of the line is affected due to a direct stroke.

I am inclined to think that Mr. Peek is over-optimistic in his estimate of the rate of rise of the lightning wave which he shows. Our experience is that the time required to initiate the cathode stream may vary considerably, so that the steepness of the wave front of the natural lightning wave which he shows in Fig. 28 may be a great deal slower than he supposes. I should like Mr. Peek to give in his discussion the details of his initiating scheme and such tests as he has made to determine its speed of response. However, I take it that Mr. Peek thinks that any wave which will give an impulse factor on a string of insulators, as high as that obtained with lightning, is satisfactory and he does not insist that his laboratory wave is necessarily the wave representative of actual lightning. In fact, one might say that the effect on insulator strings of 10 or more units, of the most severe lightning waves as recorded on the klydonograph up to date is the same as the effect of a wave having a slope of 4000 to 5000 kv. per microsecond and having a crest value such that flashover takes place on the front or just at the crest.

Regarding Part IV, I agree with Mr. Peek except in the following statement which he makes: "However, the waves usually causing insulator sparkover give an impulse ratio of the order of two (2.0) and indicate an effective duration of one to 20 microseconds above the 60-cycle flashover value." As I have remarked in the last paragraph, this might just as well be caused by a long-front wave which, if the insulator did not flash over, might give a much higher value than when the flashover takes place on the front. I agree with Mr. Peek that lightning corona loss may very materially decrease the voltage due to lightning

when ground wires are employed. I do not agree that corona from grading rings will have an appreciable effect in preventing flashover of insulator strings.

In regard to the question of lightning-proof transmission lines and coordination of transformer insulation and line insulation, I agree in general with Mr. Peek's conclusions but it is necessary to be very cautious in regard to such phrases as "lightning-proof transmission lines." As I have stated in our paper, the direct stroke of lightning may vary all the way from a very light stroke to an extremely heavy stroke. We have no information yet as to the possible potential that may be built up in a transmission line due to a heavy stroke of lightning. We do not know therefore whether a system of ground wires could be designed which would adequately protect transmission lines against the most severe strokes. However, I myself am quite optimistic and believe that in the course of time we shall be able to achieve a transmission line which is practically lightning-proof.

Mr. Peek's conclusions from his Pittsfield tests as to the time for a cloud discharge do not satisfy me. The necessary ionization phenomena are much slower than might be inferred from his results. Our work with suppressed discharges indicates a much longer time than he gives; however further data will no doubt enable us to determine the actual time of discharge. It is my belief that the minimum time is considerably more than 10 microseconds and the lower limit given by Mr. Peek of one microsecond is absolutely impossible.

To sum up, I think Mr. Peek has presented in his paper a very interesting summary of our knowledge up to date in regard to lightning phenomena. In some respects it presents perhaps too rosy a picture, which may mislead those who are not familiar with the difficulties to be overcome in obtaining scientific data on these phenomena, but as usual he has presented a very worth while paper.

J. H. Cox: The work being carried on in connection with artificial surges on the Turner Falls transmission line described by Mr. McEachron cannot be too highly commended. There is a great deal still unknown regarding the performance of surges on actual lines. A great amount of theoretical work has been done on this subject but the values which have been used for the line characteristics may need modification. It is certain that these can be determined with the use of artificial surges in a small fraction of the time which will be required if use is made only of natural lightning surges.

A start was made on this problem in 1927, in the tests on the Rankin-Wilmerding line described in the authors' paper. These tests were on a line insulated for 33 kv. Mr. McEachron is extending this information with his tests which are on a line insulated for 66 kv. Plans are now being made for a similar test on a 220-kv. line to be performed cooperatively by the Public Service Electric & Gas Company and the Westinghouse Electric & Manufacturing Company. With the tests now in progress information on the performance of surges on transmission lines covering the entire range of lines now in service should soon be available.

It is true, as Mr. Floyd points out, that economics do not permit the study of the effect of surges on actual terminal apparatus to the point of destruction. However, even this study can be made economically feasible by the use of full-size models which present the same effect as actual apparatus but which cost only a fraction of the amount of money. We have done considerable work with apparatus models in the laboratory and the value of this work can be greatly increased by combining it with surge tests on actual lines. Furthermore, the effect of the apparatus on surges can be studied with actual apparatus without particular hazard to that apparatus.

The oscillograph described by Professor Harding has considerable advantage in certain types of work. Where the taking of an oscillogram of natural lightning depends upon getting the oscillograph into operation after the arrival of a surge, the time delay is cut down and made definite by having a source of ionization available, as with the use of the hot cathode. It has been our experience with the use of the cold cathode, where the electron jet depends upon ionization by collision, that the time required to bring the jet to the film is too long and too irregular to be satisfactory. If Professor Harding is able to obtain a good jet in one-quarter microsecond and operate with the oscillograph evacuated only to 30 microns, the instrument is certainly valuable.

Mr. Halperin has stressed the need for lightning information in connection with the lower-voltage circuits. This accentuates the need for wider spread application of cathode ray oscillographs. With the present number in use and the more urgent need in the higher voltage range there are no instruments available for distribution-line investigation. It is agreed that this information is necessary and in time a sufficient number of oscillograph applications must be made to obtain it.

Mr. Thomas has discussed the nature of a lightning surge. However, his discussion pertains only to the induced stroke. We have come to the conclusion that the induced stroke is relatively unimportant in connection with high-voltage transmission lines. It is true, as he says, that the surge can travel away on a continuous line during the discharge of the cloud. This lengthens the surge as he describes, and we are of the opinion that it lengthens it to a point where its voltage is not important. The relatively slow rate of discharge of the thundercloud is shown by some of Dr. Norinder's records on isolated antenna where the bound charge could not get away. The extremely short line presents the same effect as an antenna but this is not a usual case.

Although we would state the reasons somewhat differently, we agree with Mr. Thomas that the 60-cycle crest voltage is the criterion of the strength of the line insulation. Laboratory tests have shown that the lower limit of impulse flashover of line insulation is very little more than the 60-cycle crest flashover for surges 10 microseconds or more in duration. All evidence indicates that lightning surges, whether induced or direct, are at least this long.

Mr. Thomas is correct in saying that the ground resistance is not the only important factor in the impedance to surge current. Not only reactance but capacity also is involved. The resistance plus the surge impedance of tower footing and ground return must be considered. Probably the reason that more attention has been given to the ground resistance is because very little is known regarding the surge impedance of towers and grounds and there is not much that can be done regarding the surge impedance. This is one of the factors which will be studied in the investigations of surges on transmission lines.

Some comments have been made regarding the disagreement existing between various investigators in this field and the suggestion has been made that these investigators get together and present a united front to the industry. We do not think that this would be particularly desirable since it would be misleading in indicating that the problem was solved when that is actually not the case. The reason there is disagreement is because certain factors are not definitely known and therefore are speculative. Different individuals interpret the existing evidence differently and therefore disagree. So long as this is the case the public has the right to be informed and should not be led to believe that the questionable factors have been determined. However, we feel sure that although there is a certain amount of disagreement as to some of the features of the problem, there is complete accord in the desire for further knowledge on the subject and the conviction that a great amount of further research work is necessary.

F. W. Peek, Jr.: I shall confine my closing remarks as closely as possible to those phases of the discussion that have to do with engineering. It is my opinion that most of the differences, possibly because of the short time that the paper has been available, have been due to hasty reading. For instance, Mr. Cox states that after reading my paper "one would be

inclined to believe that the whole story of lightning is known." In the first paragraph I state, "While there is still much to learn, lightning may be said to be at least on an engineering basis." I purposely was brief in the introduction in order to allow space for the numerous data included in the paper. The data at least were carefully obtained and the reader may use them in reaching his own conclusions. The fact that the lightning voltage has been increased to 5,000,000 and that very extensive field and laboratory work is under way indicates that there was no intention on my part of discouraging further work. In fact, I am still doing everything possible to expedite this type of research just as I have done in the past.

I shall answer the discussions of Messrs. Cox, Torok, McAuley, Beck, and Fortescue in a group since their points over-lap to a considerable extent.

Data are given on the sparkover of sphere-gaps, points, insulators, etc., up to 3,600,000 volts and for waves varying from about 5000 microseconds (60 cycles) duration to less than 1 microsecond duration. The whole possible range is covered. Data were also obtained on the front of waves reaching the crest in 1 to 100 microseconds. This wide range was covered for a comparison of all possible results in practise. In making comparative tests several waves have been used as standard. This was necessary for obvious reasons. These standard waves are the ones that give the same lightning flashover voltages as obtained in practise. The crosses in Fig. 11, which show the sparkover voltages measured on 4-, 10-, and 14-unit strings for natural lightning, correspond to laboratory values for the 5 to 20 microsecond range. This time is given above the half-voltage value or while the wave is active in causing sparkover (for impulse ratio of 2). The total length of these waves is thus 10 to 40 microseconds. Impulse ratio for this range is of the order of 2. It is quite probable that lower impulse ratios will be observed in practise, particularly at the lower voltages.

I have not stated that all lightning waves are of the same shape as the one obtained on the Wallenpaupack line; on the contrary, lightning waves, because there are all sizes and conditions of clouds, must occur in a very wide variety of ways. A large range of waves must be expected. However, where the voltages are very high the waves must be steep and due to rapidly discharging clouds. What I did state was this: Measurements with the surge-voltage recorders give the maximum value of the voltage in case of an insulator sparkover by lightning. An approximation of the steepness of the wave or the effective duration is obtained by comparing the sparkover as measured by the surge voltage recorder or klydonograph with the crest 60-cycle sparkover. In the many measurements that have been obtained on high-voltage lines, the lightning, where the voltage was high enough to cause sparkover, approximated twice the 60-cycle value, or the impulse ratio was 2. This means waves of an effective duration or steepness of front as indicated in Figs. 5 and 11 or for effective durations (time above the 60-cycle arcover value) of 5 to 20 microseconds. Of course the actual length of the lightning waves may be very much longer than these values. For instance, a sparkover may occur on the front of the wave reaching crest in 10 microseconds.

It can be seen why the wave fronts are likely to be steep when voltages are high. There are two ways in which high voltages can be obtained—either by direct stroke, which must be applied very suddenly and thus be steep and give a high impulse ratio, or by an induced stroke from a rapidly discharging cloud. The induced voltage upon a line depends upon a charge on a given length. If the charge is spread over twice the length the voltage is reduced to one-half. The induced voltage occurs when a bound charge on a line is released by the discharge of the cloud. If the cloud discharges slowly the charge is released slowly and spreads over a considerable portion of the line since it travels at the rate of about 1000 feet in a microsecond. The induced voltage then cannot be high. On the other hand, if the cloud discharges rapidly the charge does not spread and the induced voltage is very high. For example, assume a 1000-ft. cloud or a 1000-ft. section of charged line and a gradient of 100 kv. per ft. ($V = g\ h\ \alpha$). If the cloud discharges instantly the induced voltage on a long transmission line 40 ft. high would be 4000 kv., for a cloud discharging in 5 microseconds, 1000 kv., and for a cloud discharging in 10 microseconds, 600 kv. These voltages, however, would obtain only with a discharge in the near vicinity of the line. All the above waves would be considered steep. The voltage values show that high induced voltages on transmission lines require a rapidly discharging cloud. Although there must be all kinds of waves during a thunder storm, the highest voltage ones must be fairly steep, and the lower voltage ones less steep. It is because of this that the laboratory tests have covered such a wide range of waves and data over this range have been given in the paper.

I dislike to enter any discussion other than an engineering one on the lightning generator. My original lightning generator operated without a rectifier and is described in my paper *Effect of Transient Voltages on Dielectrics*, A. I. E. E., September 1915, to which I refer you. In this early investigation it was found convenient to use the term "microsecond" as a unit of time. The term "impulse ratio" was also first used in this paper as a convenient method of giving the effect of the lightning voltage. In the wide range of tests in that paper it was necessary to calculate the wave shape and determine the time of the transients in that manner. However, some of these tests have since been repeated and actual measurements by the Dufour oscillograph have checked this early work. This lightning generator contains inductance, capacitance, and resistance. In fact it is similar to all other electrical apparatus in that respect. However, the important point is that it does develop 5,000,000 volts directly (probably approximately 10,000,000 volts by reflection) with considerable energy and produces waves of any shape desired. These waves are readily measured by the cathode ray oscillograph. In fact, I might state that this oscillograph is used in the laboratory to the same extent as a voltmeter or an ammeter.

The tests on the grading shield were made with top and bottom rings. The loss is caused by a multitude of incipient arcs distributed around and between the strap rings. These arcs are much more intense than the corona but not sufficiently developed to permit the dynamic voltage to follow. As much as 25 per cent of the voltage has been dissipated in this manner. The question as to the relative value of the strap ring and a ring of a very large section has risen. There is no particular gain in the use of a ring of large section, since when it becomes wet with the first rain drops its arcover voltage is reduced to the needle-gap value.

The natural lightning waves measured on transmission lines are very accurate and definite. The time for actuating the oscillograph was well established by means of a parallel oscillograph. The method used in making the oscillograph self-actuating employed the three-electrode gap.

I was very much interested in Mr. Thomas' remarks. My discussion on the cloud discharge above applies. The crest, shape, and length of lightning waves depends upon the initial distribution of charge on the line and the rate of discharge. The length of the wave approximately equals the length of the bound charge plus the distance of travel during the time the cloud is discharging. Of course a large part of the tail of the wave has no effect in causing insulator arcovers since it is at a very low voltage. The effective part of the wave is that portion above the 60-cycle arcover of the insulator. The maximum value of voltage is lower the longer it takes the cloud to discharge, and becomes less with decreasing length of the initial charge or with the size of the cloud. The front depends upon the size of the cloud, or rather on the length of the charge on the line and the rate of discharge.

I cannot give Mr. Ambelang a definite reply without further information. However, the incipient arcs over insulators when

the poles are wet, to which I refer in my paper, may cause phase-to-phase arcs at fairly low voltage. Low-voltage arcovers may also result due to the shortened path at the guys. In designing a wood-pole line, to take advantage of the insulating value of the wood, these various factors must be considered.

It has been our goal to devise methods for predetermining lightning voltages on transmission lines, to obtain the lightning strength of insulators, to determine the best methods of protection and of making apparatus highly resistant to lightning, to study the origin, travel, and decay of lightning waves on transmission lines, and to determine the effects of lightning waves on inductance coils and transformers. Of particular interest has been the cathode ray study of what happens inside of a transformer when a lightning impulse strikes it. External waves cause tremendous internal stresses in transformers. The result of this study has been the development of a non-resonating transformer. In this new transformer, localized stresses are reduced as much as 100:1 compared to the ordinary transformer. Our general method has been to make tests in the laboratory on models and full size apparatus, to send lightning waves over lines and determine their effects on insulators and apparatus, to make extensive tests in the field, and to coordinate the results by a mathematical study. I have given the above brief description of our methods in answer to Mr. Floyd's question regarding the effect of lightning on terminal apparatus.

It is quite true, as in the case of all pioneer work, that a certain amount of speculation was necessary in the early stages of the research. Progress is made in that way. However, the necessity for speculation in this work has now become very much reduced due to the definite knowledge that has already been gained.

As to a lightning-proof line, I am quite certain that it is coming. In this connection I might point out that principles developed in research applied to existing lines have reduced outages as much as 10:1. The main problem now is to determine economic methods of preventing outages due to direct strokes.

The operating data given by Mr. Sporn are extremely interesting. Statistical data from the operating companies, if carefully made, would be of great value to us in solving these problems. I might state here that we are already greatly indebted to the engineers of many operating companies, and advancements that have been made are greatly due to their help and cooperation.

1927 Lightning Experience on the 132-Kv. Transmission Lines of the American Gas and Electric Company

BY PHILIP SPORN[1]

Member, A. I. E. E.

Introduction

WITHIN the past two years a series of papers,[2,3,4] was presented before the Institute on the performance of a number of 132-kv. steel tower lines on the system of the American Gas and Electric Company during the years 1925 and 1926.

It is proposed in this paper to continue and give the 1927 history of the 132-kv. transmission system investigated and described in the previous three papers.

General

Fig. 1 shows the 132-kv. transmission network in question. As has been previously pointed out, it comprises approximately 948 mi. of line, which was in service for all or a part of 1927; the total circuit miles being approximately 1266 in service for the same period. Details with regard to the territory traversed, the nature of the country, and details with regard to the generating stations, and the points where they feed into the network have already been described.[3]

1927 Performance

A brief summary of the principal characteristics of the various lines, together with their 1926 and the 1927 lightning performance, is given in Table I. It should be noted in this connection that while no quantitative data are available, the opinion gathered from all parts of the various indicates that in general, 1927 was a year of less severe and less frequent lightning than 1926.

An examination of the table shows the following with regard to the various lines:

Glen Lyn-Roanoke. The number of outages, while still high, was considerably less in 1926 than in 1927. Further, the number of outages, as compared with the performance of the other lines in the same territory, does not show up very much out of line. The number of cases of damage to insulators, conductors, and hardware, in proportion to the number of outages, is unusually high; the line, however, had no arcing protection devices of any sort during that entire year. It is pertinent also that the cases of damage on this line were determined by an actual climbing inspection of every tower on the line. The damage to the insulators varied from light marking of the glaze to the shattering of two or three units in a string; and the damage to the conductor varied all the way from slight pitting and small blistering to the burning of three and four strands in the conductor.

Glen Lyn-Switchback. This line was not in service a sufficiently long period during 1927 to furnish any authoritative data.

Lima-Fostoria. The number of outages was about half of what it was in 1926. The decreased amount of lightning during 1927 would, however, furnish the reason for this.

It will be seen that the total number of cases of damage exceeds the number of outages, the probable reason being that the cases of damage cited include a

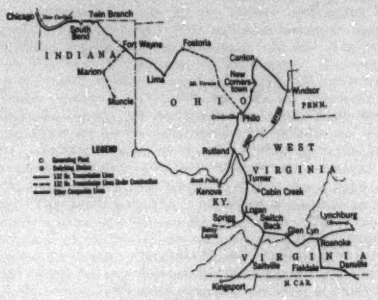


FIG. 1

number of cases accumulated from previous years that had not been detected until 1927.

The damage to the insulators in this case varied from mere burns on as many as three or four units in a string to broken insulators, while the conductor damage varied from pitting and light burning for a few feet from the point of contact, to burning of as many as four strands.

Lima-Twin Branch. The 1927 performance of the Lima-Twin Branch line when compared with its performance during 1926, bears about the same relationship as the 1927 performance of the Lima-Fostoria line bears to its 1926 performance. The cases of damage here are definitely known to represent accumulated damage from the previous year. The cases of damage disclosed in every instance but one, only a slight blistering of a considerable number of units

1. Electrical Engineer, Amer. Gas and Elec. Co., New York, N. Y.

2. *Lightning and Other Experience*, Sindeband and Sporn, A. I. E. E. TRANS., Vol. 45, p. 770.

3. *1926 Lightning Experience on 132-Kv. Transmission Lines*, Philip Sporn, A. I. E. E. TRANS., Vol. 47, p. 668.

4. *Surge Voltage Investigation on the 132-Kv. Transmission Lines of Amer. Gas and Elec. Co.*, Philip Sporn, A. I. E. E. Quarterly TRANS., Vol. 47, October 1928, p. 1132.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

TABLE I
CHARACTERISTICS AND PERFORMANCES OF 132-KV. LINES OF AMERICAN GAS AND ELECTRIC COMPANY 1926 AND 1927

Characteristics / Column No. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Line designation	Glen Lyn Roanoke	Glen Lyn Switch-back	Lima Fostoria	Lima Tw. Branch	Logan Sprigg	Philo Canton	Philo Crooks-ville	Philo Turner	Ros-noke Dan-ville	Ros-noke Reusens	Rutland So. Point	Saltville Kingsport	Switchback Logan	Switchback Saltville	Turner Logan	Tw. Branch So. Bend	Windsor Canton	So. Bend Mich City
Length of line (miles)	65.0	30.0	45.6	128.5	21.0	73	15.4	118.7	65.0	43.0	50.3	56.0	50.0	46.0	40.2	4.9	55.0	40.0
Number of circuits	2	2	1	1	1	2	1	1	1	1	1	1	2	1	2	2	2	1
Number of ground wires	1	1	1	1	None	1	1	1	1	1	1	1	1	1	None	1	2	1
Rings and horns	No	Yes	No	Yes	No	Yes	No	No	No	Yes	No	Yes	Yes	Yes	Yes	Yes	No	No
1926 Lightning Performance						1925 1926												
One circuit only out	16		9	23	3	12	3	11	0	13	3				16	0	9	..
Both circuits out	4		..	..	..	2	..	..	..	..	..				2	0	2	..
Total outages	20		9	23	3	88 15	3	11	0	13	3				18	0	11	..
Circuit outages for 100 mi. of line per yr. 1926	48*	0†	19.8	17.9	31*	120 20.5	19.5	10.6*	0*	36.6*	6.3*	0†	0†	0†	44.8	0	20	..
Circuit outages per 100 mi. of circuits per year 1926	24*	0†	19.8	17.9	31*	60 10.3	19.5	10.6*	0*	36.6*	0.3*	0†	0†	0†	22.4	0	10	..
1927 Lightning Performance																		
One circuit only out	21‡		4	8	7	3‡	1	7	16	16	7	..	..	..	20	1	2‡	4
Both circuits out	3		..	..	..	1	..	..	..	..	..	..	..	..	4	1	0	..
Total outages	24	0	4	8	7	4	1	7	16	16	7	..	..	..	24	2	2	4
Circuit outages per 100 mi. of line per yr. 1927	37	0*	8.8	6.2	33.3	5.5	6.5	5.9	24.6	37.2	13.9	0*	0*	0*	57.2	41	3.6	10.0
Circuit outages per 100 mi of circuit per yr. 1927	18.5	0*	8.8	6.2	33.3	2.8	6.5	5.9	24.6	37.2	13.9	0*	0*	0*	28.6	20.5	1.8	10.0
Damage to insulators	17	1	7"	28"	0¶	0	0	2	14	2	0	0	0	0	2	1	0	1
Damage to conductor and hardware	14	3	5	0	0¶	3	0	1	5	7	0	0	0	0	4	0	0	0
Total cases of damage	20	3	9	28	0¶	3	0	2	14	7	0	0	0	0	5	1	0	1

*Corrected for 1 calendar year (in service only part of year.)
†Not in service.
‡One circuit out of service at the time of F. O. in 1 case.
¶Superficial inspection—Towers not climbed.
"Probably partly accumulation of 2 previous years.

in a string, the number varying from three to all ten in the string. In none of these cases was any conductor damage noted, indicating that these cases of damage had resulted from a cascading arc that did not stay on long enough or did not do enough damage to be detected from a ground inspection. This would explain why they were not discovered, therefore, until a tower climbing inspection was made.

Logan-Sprigg. This line, which is a wood pole steel crossarm line unusually well insulated (even without giving any weight to the insulation of the wood itself) showed up during 1927 even worse, from the standpoint of outages, than it showed up in 1926, and except for the Roanoke-Reusens and the Turner-Logan line had the largest number of outages per hundred miles of circuit. The line has no ground wire.

It will be observed that no cases of damage were discovered, but on the other hand, the inspection that the line received was mainly a ground inspection.

Philo-Canton. The number of outages on this line was only 30 per cent of the number that occurred in 1926, which is accounted for partly by the mildness of the 1927 lightning previously referred to. It is interesting, however, to compare this with the number of outages in 1925. As will be seen, in 1925, when no ground wire was installed on the line and no protective devices of the type being employed now were utilized, the number of outages per hundred miles of line was more than twenty times as many as occurred in 1927. During 1927, there was no case where damage of any kind to the insulators occurred, although there were cases where the conductor was pitted. In some cases it was actually burned, but no strands burned through. In one case the horn and ring assembly were quite badly burned, and in the other case they were definitely pitted.

Philo-Crooksville. The 1927 experience with regard to outages is in line with the experience on the Philo-Canton line. No damage of any kind could be traced to the one outage that took place.

Philo-Turner. The number of outages in this case also was about 50 per cent of the number that took place in 1926. Two cases of damage occurred but these were very severe; in one case four insulators were shattered on one phase, and four insulators on another, and eight strands of wire burned through. No arcing protection of any kind was employed at that tower. In another case on a strain tower, where double strings were employed and a standard set of arcing horns (this particular circuit had been equipped with no arcing protective devices of any kind except at dead end points where standard arcing horns were used) one-half of a double string was completely opened by the burning through of one unit at the line end. The same arc chipped the other twelve units in the string and burned the tips of the horns but did not damage the conductor.

Roanoke-Danville. There are no data for the previous year with which to compare the 1927 performance of this line, but it will be observed that its 1927 performance is on about a par with the performance of the Glen Lyn-Roanoke line. The number of flashovers per hundred miles of line is moderately high. The number of cases in which damage occurred is here again relatively high as compared to the number of outages, but again the inspection was very thorough, so that it is certain that all cases of damage were noted. The damage in most cases confined itself to pitting, although in many cases this consisted of severe pitting of all or a portion of the insulators in a string, with blistering of wire for as much as five feet from the point of suspension. In some cases, however, insulators were actually broken and strands of wire were burned, although not through. This line, as will be noted, had no arcing protective devices. It is also well to note in connection with the damage that the possible fault current in the case of a flashover is a minimum for this line as compared with all the other 132-kv. lines on the system.

Roanoke-Reusens. The 1927 experience practically duplicated the 1926 experience, the number of flashovers being unusually high. At the present time (October 1928) ground resistance measurements are being taken on the line, which it is hoped will give some clue as to the reason for the abnormally high level of outages on the line. However, all cases of damage on this line were again determined by climbing of every tower and are unusually interesting in view of the large number of outages. Of the seven cases of damage, it will be seen that only two represent damage to insulators, and these were very light spots on the glaze. The conductor, too, was pitted only in two instances, whereas in every case burning and sometimes very severe burning was found on either the rings or the horns, and often on both.

Rutland-South Point. The number of outages is not unusually high but higher than in 1926. As was previously pointed out, however, such data for 1926 were obtained for a portion of the season only. So far as it is possible to find out, no damage occurred on the lines. It is definitely known that no serious damage occurred, but the line was subject only to ground inspection during this period.

Saltville-Kingsport, Switchback-Logan, Switchback-Saltville. These three lines went into service so late in 1927, that the data obtained cannot be used to form any conclusions in regard to their lightning performance during that year.

Turner-Logan. The number of outages on this line was even higher in 1927 than in 1926, and with the exception of the Roanoke-Reusens line and the Logan-Sprigg line, had the highest number of outages per hundred miles of circuit. Neither the Turner-Logan nor the Logan-Sprigg line has a ground wire. The Turner-Logan line is, however, a steel tower line. The number of cases of damage is unusually small in view of the large number of flashovers. This must be

qualified, however, by noting that this line again was not subjected to a thorough inspection for its entire length, but was so inspected for only half its distance. Only one case of really severe damage was found, and this was represented by a single broken insulator. All other cases consisted of very slight pitting of insulators or conductor, or both.

Twin Branch-South Bend. On a mileage basis, the number of flashovers this line had in 1927 was high, but it is to be noted that this line is only approximately five miles long. No flashovers were reported in 1926. On the basis of the two year average, therefore, the experience lines up very well with what would be expected. The damage that occurred consisted of very slight blistering to a number of insulators but it is possible here, as in the case of the Lima-Fostoria and Lima-Twin Branch lines, that this was a hangover from the previous year.

Windsor-Canton. The number of flashovers on this line was unusually low, being only 18 per cent of the number that occurred in 1926. It will be seen that this line had the least number of flashovers of any line on the entire system. The line has the shortest span and is the lowest above ground of any line on the system and is the only line that has two ground wires.

South Bend-Michigan City. The number of flashovers on this line is of the same order as those on the Lima-Fostoria, Lima-Twin Branch, and the Twin Branch-South Bend lines. The one case of damage consisted of slight blistering of all insulators on all three phases.

Discussion of 1927 Experience with Regard to Several Phases of Design

1. Effect of Ground Wire. On the two circuit lines with ground wire in service during all of 1927, there occurred 16.2 outages per one hundred miles of line. On the two circuit lines without ground wire in service all of 1927, there occurred 59.2 outages per one hundred miles per line, a ratio of 3.7. In the case of one of the single circuit lines—a single circuit wood pole line—although the insulation is considerably higher than the average, there occurred 33.3 outages per one hundred miles of line. The Windsor-Canton line with two ground wires had only 3.6 outages per one hundred miles per line per year. It is believed that this experience has further and effectively demonstrated the positive value of the ground wire in reducing the lightning voltages on the transmission line and even more effectively reducing the number of outages.

2. One vs. Two Ground Wires. The Windsor-Canton line with two ground wires showed the lowest number of outages per one hundred miles of line per year, namely, 3.6. The average for the entire system is 16.4, including single and double circuit lines and also those with and without ground wires. This average, it is to be noted, covers only those lines that were in service the entire year. On the other hand, it must be borne in mind that the Windsor-Canton line is a much shorter span line than all of the other steel tower lines (the Windsor-Canton average is 8.85 towers per mile). However, the employment of two ground wires has undoubtedly contributed to this performance.

3. Relative Shielding of the 3 Phases by Ground Wires. In Table II is shown the location of trouble on insulators and wire, separated into the top, middle, and bottom conductors. Disregarding the Turner-Logan line (because of the fact that it had no ground wire) and the Lima-Fostoria and Lima-Twin Branch lines (because of the fact that the trouble discovered in 1927 represented, without a doubt, trouble that had accumulated from previous years and that had not previously been noted) we find 21 cases of conductor damage on the top phase, 21 on the middle, and 29 on the bottom. This would indicate that for the case where one ground wire was employed immediately above and centrally with respect to the two top conductors, the shielding furnished by the ground wire to the top and middle conductors is such that the net average lightning voltage on the two, in spite of the difference in height between the two, is the same; but the bottom conductor, in spite of its still lower level and therefore lower induced lightning potentials, receives so much less shielding than the other two that the result is a higher net induced voltage on it than on the upper two conductors. It would seem, therefore, that another ground wire properly placed with respect to the bottom conductor, would more nearly equalize the net induced voltages and therefore the number of flashovers and number of damages on all three phases. This experience is extremely interesting since calculations on its shielding effect made before the installation of the ground wire, based on the work of Mr. Peek, indicated an expected lightning voltage on the top and middle wires of approximately the same value and an approximately 15 per cent higher value on the bottom wire.

TABLE II
LOCATION OF TROUBLE ON INSULATORS AND WIRE
1927

Line	Total	Top	Middle	Bottom
Glen Lyn-Roanoke	20	6	10	12
Glen Lyn-Switchback	3	1	2	2
Lima-Fostoria	9	8	0	1
Lima-Twin Branch	28	27	2	0
Philo-Canton	3	2	1	1
Philo-Turner	2	2	0	1
Roanoke-Danville	14	5	3	8
Roanoke-Reusens	7	3	4	4
Turner-Logan	5	3	4	0
Twin Branch-South Bend	1	1	0	0
South Bend-Michigan City	1	1	1	1
Totals	93	59	27	30
Totals excluding Lima-Fostoria and Lima-Twin Branch	56	24	25	29
*Totals excluding Lima-Fostoria Lima-Twin Branch & Turner-Logan	51	21	21	29

*Lima-Fostoria and Lime-Twin Branch excluded as damage appears to be accumulation of trouble since line was first built, and not for 1927 only Turner-Logan excluded, being a line without ground wire.

The Turner-Logan line, having no ground wire,

had three cases of trouble or damage on the top, four on the middle, and none on the bottom conductors.

The large number of cases of trouble on the top conductor on the Lima-Fostoria and Lima-Twin Branch line, it is confidently believed, is due to cases of trouble accumulated from the time when no ground wire at all was used on the line. It has already been pointed out that the first complete tower inspection by climbing was made in 1927.

4. *Use of Protective Devices.* Assuming that the cases of trouble on the Lima-Twin Branch line date back mostly to the period preceding the use of the ground wire and the arcing protective devices, we find on that line, on the Roanoke-Danville line, on the South Bend-Michigan City line, and on the Glen Lyn-Roanoke line, 67 per cent of the observed cases of trouble, although these lines represent only 31.5 per cent of the total line mileage. As already pointed out in the discussion of the individual lines, the damage to the lines where rings and horns were employed was in general confined to blistering of the wire and a marking or slight blistering of one or two insulators and only very rarely was a strand burned in two. On the other hand, many cases of burning were found on the rings and horns, although these were in no case serious enough to require replacement of the assembly. Where no rings or horns were employed, however, the damage was not only numerically more plentiful, but from a severity standpoint, was far heavier, and in one case actually burned one side of a double string right through.

5. *Ground Resistances.* Table III shows the tower ground resistance in cases where damage was found. It will be seen that in most cases the resistance was of the order of from two to five ohms, although in one case a resistance of 28 ohms was found, and in another, a resistance of 16 ohms.

TABLE III
TOWER GROUND RESISTANCE AT TOWERS WHERE FLASHOVER OCCURRED

Lima–Twin Branch Tower	Ohms Gr. resist.	Tower	(Ohms) Gr. resist.
7	1.2	176	2.4
111	2.5	186	2.2
112	2.5	189	2.2
138	2.8	221	Not measured beyond Tower 210
139	2.4	222	
141	2.8	240	
142	2.2	244	
143	2.3	252	
144	2.2	265	
152	2.0	281	
165	1.3	288	
169	2.0	291	
170	2.0	296	
175	2.0		
Philo–Canton			
82	28.0	160	6.9
83	2.4		
Philo–Turner			
3	1.5	55	Not measured
Turner–Logan			
86	6.25*	115	1.5*
94	Ground resistance too high for instrument to record	152	16 and 12*
95	1.0*		

*Includes effect of ground wire as well as tower

Table IV shows the maximum, minimum, and average tower ground resistance of the lines tested. The order of resistance encountered is of such low level that the data do not seem to warrant any definite conclusions with regard to the effect of ground resistance on the frequency of flashover at a particular point. The data on ground resistance which are being obtained at the present time on the Glen Lyn-Roanoke, Roanoke-Danville, and Roanoke-Reusens lines, may give some additional information on this point.

TABLE IV
TOWER GROUND RESISTANCES (Ohms)
Tower Only—Ground Wire Detached

	Maximum	Minimum	Average
Glen Lyn—Roanoke			
Glen Lyn—Switchback			
Lima—Fostoria	7.0	0.5	2.0
Lima—Twin Branch	11.4	0.8	2.5
Logan—Sprigg			
Philo—Canton	74.0	0.6	7.7
Philo—Crooksville	21.0	0.8	5.0
Philo—Turner	24.0*	0.7*	3.2*
Roanoke—Danville			
Rutland—South Point			
Saltville—Kingsport			
Switchback—Saltville			
Turner—Logan	100.0‡	1.0	11.8¶
Twin Branch—South Bend			
Windsor—Canton			
South Bend—Michigan City			
Turner—Cabin Creek	110.†	0.5	4.5

*Test on 20 towers.
†12 towers, readings not obtainable due to high resistance of ground
‡3 towers, readings not obtainable due to high resistance of ground.
¶74 towers out of 156.

6. *Single Circuit vs. Double Circuit Flashovers.* On the two circuit lines in operation throughout the entire period of 1927, with and without ground wires, 16 per cent of the outages tripped both lines, only one line tripping in the remaining 84 per cent of the cases. This would tend to confirm further the theory put forth previously that in the case of a double circuit line, the flashover reduces the energy in the surge sufficiently to lower the head of the wave to such an extent that the second circuit will not be subject to enough potential to flash it over after flashover on the first wire has once started. The fact that the percentage of flashovers on two circuit lines with ground wire, in which both circuits went out, is the same as the general percentage, would further confirm this.

1927 History in Light of Klydonograph Investigation

A brief description of the 1927 klydonograph investigation was given before the Institute last summer.[4] Reviewing the 1927 experience in the light of that investigation the following stand out:

1. The effectiveness of the ground wire which the

1927 experience has indicated was shown by the low voltage recorded on the ground wire at the time of surge. About 20 kv. maximum was recorded which is small in comparison with the line lightning voltage. Its effectiveness is further indicated by the relative voltages recorded on the three phases. These agreed fairly well with theory. Although the klydonograph investigation was not extensive enough to be definitely conclusive, the operating experience gathered in 1927 reinforced the klydonograph data in so far as they went on that point.

2. On some of the lines 2100 kv. was recorded in the klydonograph investigation with a resulting line outage. On the other hand, a recorded voltage of 1450 kv. resulted in no line outage. This tends to confirm laboratory tests made with artificial lightning applied to insulator strings, that is, with lightning voltages of the order of 1400 kv. no flashover would be expected, while with 2100 kv. flashover always resulted.

3. The 1927 experience showed, further, that the damage as a result of flashover was confined to a single tower. This checked the data on attenuation of lightning which were obtained in both the 1927 and 1928 klydonograph investigations. The klydonograph data show that the destructive value of even a very high surge is lost in from one to five miles.

Design Developments During 1928

In the paper of the 1926 experience, attention was called to some of the design changes which were incorporated as a result of that experience. During 1928, a further attempt was made to incorporate the previous year's experience in transmission design with the following results:

1. The careful attention that had been given in the previous years to the line entrance problem was continued in every case again where a new substation was built and the ground wire was brought into the station structure; in most cases the number of the ground wires so brought in was greater than the number of ground wires on the line itself. Many stations that had been built in previous years, and had not been so taken care of, were taken care of in this respect during the year.

2. The practise of using grading rings on insulator strings was continued, and extended to some lines already in service, where such protection was not used in 1926.

3. The practise that had been started in the previous year of grading the line by reducing the insulation for a mile or so on each side of a station was continued. The data on attenuation obtained during the year continued to confirm the soundness of this practise.

4. Whereas in previous years an assembly consisting of a ring at the bottom and a horn at the top was employed, a change was made during 1928 to an assembly consisting of a ring at the top and a ring at the bottom. Laboratory experience with this arrangement indicated a probability of cleaner arcing and less liability to cascading. The old assemblies already in place consisting of a ring and horn combination were, however, left undisturbed.

5. In the designing and the construction of new lines further stress was laid on reducing the average span, thus lowering the average height above ground where the contour of the country permitted this to be done without undue economic penalties. The use of a single ground wire was continued in general although two ground wires were employed in some special cases.

In every case where two ground wires were not employed provision was made to permit the installation of the second ground wire if further operating experience continued to check the beneficial effects of the second ground wire so far indicated.

Future Data and Investigation

The experience gathered in 1927 showed such great benefits resulting from the keeping of systematic data and from their proper interpretation, and the data gathered in 1928, in so far as they are available at the present time, have confirmed this so strongly, that it is planned to continue the keeping of these data and to tighten up those points where looseness has existed. Specifically it is planned to keep a systematic record of storms as they are experienced on the system and to co-relate these data with line flashover and damage. This was already done in 1928, and will be continued along the direction of arranging for patrol after severe thunderstorms. Again, the thorough inspection of towers at least once every season by actual climbing which was inaugurated in 1927, will be extended during the coming year.

The klydonograph investigation which was started in 1927, was continued in 1928, with particular stress in the latter year on the question of attenuation and the performance of lightning arresters. Some of the data obtained in this investigation have already been presented before the Institute and the balance it is hoped to summarize during the coming year. It is further planned to continue the klydonograph work in 1929, with the help of Dufour oscillographs.

Summation of Experience

Summarizing the 1927 experiences, it is believed that the following have been fairly definitely established or have been more definitely indicated:

1. The effectiveness of the ground wire was further established.

2. Some data were obtained that would indicate very definitely the effectiveness of two ground wires where properly employed.

3. It was shown that the ground wire equalizes the lightning voltages on all three wires of a vertically arranged line, besides reducing the lightning voltages. Where one wire was employed, if equalization did not result, it was in the direction of reducing the lightning

voltage on the top and middle conductors to a value below that on the bottom conductor where the ground wire was placed immediately above the top wire.

4. The use of properly designed arcing protective devices has in all probability resulted in a certain reduction in the number of flashovers, and has very definitely minimized cascading where flashovers did finally result. Where cascading does occur, the use of arcing protective devices results in the reduction of the damage to such an extent as to be of minor importance from an operating standpoint.

5. In cases where tower resistances are not particularly high the data showed nothing conclusive with regard to the effect of resistance on lightning flashover.

6. The two-circuit line having its circuits arranged vertically on the same tower has shown itself to be very reliable from the continuity of service point of view. In approximately only 15 per cent of the cases does outage result on both circuits, one circuit only going out in the remaining 85 per cent of the cases.

7. The localization of damage in case of flashover confirms very definitely the field data obtained by klydonograph and indicates a very rapid attenuation of surges. In fact, attenuation indicated would appear to be more rapid than would be expected from the relationship as given by any heretofore published formula.

The author acknowledges with thanks the cooperation and help furnished by the operating organizations of the Appalachian Electric Power Company, of the Indiana & Michigan Electric Company, and of the Ohio Power Company in gathering the field data, and the assistance of Mr. I. W. Gross in co-relating it and in the preparing of the paper.

Discussion

For discussion of this paper see page 492.

Power Transmission and Distribution

CONCLUDING REPORT OF THE 1927-1928 SUBCOMMITTEE

Synopsis.—This report gives the latest data of the subcommittee on the subject of the protection of transmission lines against lightning by the use of overhead ground wires, special construction, etc. Reports of power companies are included and recommendations of the committee are submitted.

* * * * *

Protection of Transmission Lines From Interruptions Due to Lightning*

As indicated in the 1928 Report of the Committee on Power Transmission and Distribution,[1] the subcommittee dealing with the general subject of the protection of overhead transmission lines from interruptions due to flashover, undertook to obtain more data on the performance of various remedial agencies with a view to presenting more authoritative inferences at a later date.

An attempt is made herein to present the most pertinent information obtained. Unfortunately, the replies to the questionnaire sent out have been comparatively few, and of the replies received many were lacking in definite data on one or more points because records were not available.

The subcommittee wishes to express its thanks to all who cooperated in supplying data of any character.

A. Performance of Lines Protected by Ground Wires

1. General.

The data received further confirm the value of ground wires in improving line reliability as formerly reported. Generally speaking, lines so protected show fewer outages per 100 mi. of circuit (and per 100 storms, where storm data were available) than unprotected lines. This is borne out by substantial reductions in outages on individual lines initially operated without, and later equipped with ground wires, on which comparative data for several seasons were submitted. Of course, the number of interruptions varies widely with the territory in which the lines are located, and the frequency and severity of storms normal to the territory.

Interruptions on steel lines equipped with ground wire were consistently less (of the order of 20 to 50 per cent) than on lines not so equipped.

The data submitted for wood pole lines were less consistent, but in those cases where several years were reported, a comparison of the average number of interruptions gave substantially similar results.

*CONCLUDING REPORT OF THE 1927-1928 SUBCOMMITTEE OF THE POWER TRANSMISSION AND DISTRIBUTION COMMITTEE OF THE A. I. E. E.

H. L. Wallau, Chairman
H. H. Dewey, A. E. Silver, E. C. Stone
C. L. Fortescue, P. Sporn, F. R. Weller.

1. A. I. E. E. Quarterly Trans., Vol. 47, October 1928, p. 1217.

Presented at the Winter Convention of the A. I. E. E., at New York, N. Y., Jan. 28-Feb. 1, 1929.

2. Effect of Ground Resistance.

Data submitted by the Pennsylvania Water and Power Company throw some light on this subject.

For three lines protected by ground wires, all operating at the same voltage, but of somewhat different heights and with different average tower earth resistances, the following comparison has been set down for the year 1927:

Performance Comparison	During 1927		
Line designation	12.	56.	1516.
Circuit mileage	80.	80.	31.
Storms reported	44.	44.	19.
Breaker openings due thereto	11.	7.	5.
Breaker openings per 100 mi.	13.8	8.75	16.1
Breaker openings per 100 storms	25.0	15.9	26.3
Number of ground wires	1.	2.	1.
Approximate average percentage of shielding due to same	25.	40.	25.
Height of top conductor (approximate)	54.	58.	66.
Max. kv. induced voltage in top wire based on height and ground wire shielding at a gradient of 100 kv. per ft.	4000.	3500.	5000.
Average tower earth resistance—ohms	65*	55†	500 Est
Variation in same	10–277	10–277	

*Towers built on steel grillage.
†Towers built on concrete footings.

The storm data were not available for all of the three years reported, the interruptions due to lightning per 100 mi. of circuit being as follows for these three lines:

Line Designation	1925	1926	1927
12	13.8	8.75	13.8
56	30.	20.	8.75
1516	48.4	48.4	16.1

This company also submitted a print showing:

a. The individual earth resistance per tower for approximately 150 consecutive towers, of lines 12 and 56.

b. The average profile of lines 12 and 56.

c. The accumulated flashovers at each tower for 17 years on line 12 and for 13 years on line 56.

The following indications were derived therefrom:

1. Flashovers were relatively more numerous at points of higher elevation.

2. Flashovers were relatively more numerous at towers having the higher earth resistances. Towers having earth resistances below 50 ohms were in general less subject to flashovers, the percentages being 39 for line 12 and 24 for line 56 for the periods reported.

3. Line 12 (steel grillage 65 ohms resistance) averaged 4.6 flashovers per year, while line 56 (concrete footings 55 ohms resistance) averaged but three flashovers per year for the section of lines plotted on the print submitted.

This reduction is in part due to the greater shielding effect of the two ground wires on line 56.

Two other lines of the same nominal voltage operated by the above company in the same general territory, but constructed without ground wires, showed 86.4 and 91.2 interruptions per 100 storms in 1927. Line 1112 with the better record, had tower resistances varying from 29 to 161 with a probable average of the order of 70 ohms, while line 1314 gave measured values of 297 and 410, and was probably quite comparable to line 1516 in its average earth resistance.

The interruptions per 100 miles of circuit (storm data not available for 1925 and 1926) were as follows:

Line designation	1925	1926	1927
1112	82.6	121.5	32.6
1314	51.7	90.	50.

The average interruptions per 100 mi. of circuit for the three lines protected by ground wires were 23.1 as against 71.4 for the two lines not so protected.

3. Comparison of protection afforded by two ground wires as compared with a single ground wire.

The following memorandum, submitted by the Pennsylvania Water and Power Company, indicates a betterment from the use of two ground wires, compared with one:

INSULATOR FLASHOVER RECORD

"At the 19th Convention of the Pennsylvania Electric Association a paper entitled 'A Fifteen-Year Insulator Flashover Record' was presented. The purpose of this paper is to bring this record up to date by adding the information collected during 1926 and 1927.

"The conclusions drawn from the data presented in 1925 are as follows:

1. During a lightning surge the top conductor of the circuit without ground wire is unquestionably raised to a higher potential than the lower one.

2. The maximum trouble from lightning on any one line can be expected at the highest elevation.

3. The more insulation on a line the freer it will be from lightning troubles.

4. A ground wire will reduce the induced surge voltage on the top conductor at least and to that extent will be beneficial to the operation of the line.

5. Arcing horns at the bottom of an insulator string will reduce the lightning flashover voltage of the string.

"The addition of the 1926 and 1927 record seems to add further more conclusive proofs to the conclusions already drawn. The complete record up to date is as follows:

TABLE NO. 1
WIRE POSITION

Circ. No	Line to ground			Line to line			Three phase	Remarks
	T	M	B	T M	T B	M B		
1	0.46	0.49	0.36	0.10	0.07	0.07	0.10	1 G. W.
2	0.49	0.50	0.51	0.27	0.08	0.11	0.16	1 G. W.
5	0.17	0.23	0.46	0.12	0.04	0.21	0.04	2 G. W.
6	0.16	0.31	0.31	0.13	0.12	0.19	0.18	2 G. W.
11	2.10	0.94	1.15	1.05	0.73	0.42	1.05	No G. W.
12	3.14	0.63	0.52	0.42	0.63	0.21	0.63	No G. W.
13	3.50	0.67	0.50	1.00	0.67	0.08	1.67	No G. W.
14	2.83	0.75	0.42	0.67	0.58	0.50	0.92	No G. W.
15	0.20	1.01	1.21	0.40	0.20	1.62	1.41	1 G. W.
16	1.21	0.81	0.61	0.61	....	0.40	0.20	1 G. W.

"The figures in the above table are expressed in percentages of the number of insulator strings exposed and years of service,

$$\frac{\text{Number of Flashovers}}{\text{Number of Insulator Strings} \times \text{Years of Service}}$$

"A record of circuits Nos. 1, 2, 5, 6, and 15 indicates that a ground wire has reduced potential on the top wire so that the number of flashovers on the top wire is always less than the ones on the middle or bottom wire. Circuit No. 16 seems to furnish an exception to the rule. This circuit, however, has an insulation of nine units per string giving a length of string of 4 ft.-10 in. and reducing the clearance from the bottom of the insulator string to the brace on the middle crossarm to 3 ft.-10 in. Apparently the insulation is past the balanced condition and the arc jumps from cable to the crossarm brace in preference to going across the insulator string. We have been unable to find any concrete evidence on the towers to this effect, but the amount of power transmitted over the line being limited and the relays set to trip almost instantaneously the burns from the power arcs are extremely slight and it is quite possible that although the original flash occurs to the crossarm brace, the power arc may rise immediately to the crossarm above the insulator. All evidence of power arcs so far has been on the insulator string itself or the crossarm above the insulator.

"In order to find out if ground elevation and ground resistance have any influence on the flashovers, a part of lines No. 12 and No. 56 was selected and the towers on this section measured for ground resistance. Curves were plotted on the same sheet showing accumulated flashovers on each tower, the profile of the line, and individual tower ground resistance. The general appearance of these curves is roughly the same and it is particularly noticeable that although high resistance does not always follow with high elevation, the number of flashovers is always a maximum on the section of line having high average ground resistance. This would also help to explain the apparently poor performance of line No. 1516. This line is built on limestone bed-rock and consequently can be expected to have an average high-resistance ground. The same thing holds true of line No. 1314, which also shows an unusual number of flashovers per mile.

TABLE NO. II

Line no.	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	Total	Remarks
12	23	10	2	11	11	16	5	10	15	8	111	1 G. W.
56	17	16	8	16	16	19	5	32	16	7	152	2 G W.

"Table No. II shows a comparison of the number of flashovers on lines No. 12 and No. 56, which are parallel and practically identical in construction with the exception that line No. 12 has a single ground wire and line No. 56 has two ground wires.

"On a direct comparison the line with the double ground wire seems to have a poorer operating record than the line with the single ground wire. There is, however, a good reason for this as will be explained later. If we make the same comparison between the 60 cycle lines and express the number of flashovers in percentages of insulator strings exposed and years of service, we get the figures of 6½ per cent for line No. 1112, 7.4 per cent for line No. 1314, and 3.7 per cent for line No. 1516, which indicate a decided advantage for the ground wire.

"During 1926 the arcing horns were removed from circuits No. 12 and No. 14 and below is given a tabulation of the flashovers on these circuits as compared with parallel circuits No. 11 and No. 13 before and after the arcing horns were removed. As seen from the table there was no decided difference in the number of flashovers on the parallel circuits before the removal of the horns. After the horns were removed, however, the circuits without horns showed a decided improvement. This improvement roughly amounts to 53 per cent. Going back to lines No. 12 and No. 56 we have two circuits on No. 12 line without horns and two circuits on No. 56 line with horns. If we allow the 53 per cent increase in flashovers due to the horns, No. 56 line should have 170 flashovers on its record as against 152 which actually took place. Or in other words, the record for No. 56 line, if the horns had been left off, would have been 99 flashovers as compared with 111 for line No. 12. This seems to indicate that the two ground wires actually furnish more protection than the single ground wire as is to be expected.

BEFORE REMOVAL OF HORNS

	Circ. No. 11				Circ. No. 12				Circ. No. 13				Circ. No. 14			
	1 Ph.	2 Ph.	3 Ph.	Total	1 Ph.	2 Ph.	3 Ph.	Total	1 Ph.	2 Ph.	3 Ph.	Total	1 Ph.	2 Ph.	3 Ph.	Total
1924	2	1	0	3	6	0	0	6	1		2	3	0	1	4	5
1925	11	6	2	19	15	6	4	25	13	3	3	19	12	4	4	20
1926	6	1	0	7	3	1	0	4	7	3	2	12	8	3	3	14
Total	19	8	2	29	24	7	4	35	21	6	7	34	20	8	11	39

AFTER REMOVAL OF HORNS

	With horn circ. No. 11				No horns circ. No. 12				With horns circ. No. 13				No horns circ. No. 14			
	1 Ph.	2 Ph.	3 Ph.	Total	1 Ph.	2 Ph.	3 Ph.	Total	1 Ph.	2 Ph.	3 Ph.	Total	1 Ph.	2 Ph.	3 Ph.	Total
1926	18	9	7	34	14	5	2	21	16	9	7	32	13	7	0	20
1927	10	8	3	21	9	3		12	18	5	5	28	14	8		22
Total	28	17	10	55	23	8	2	33	34	14	12	60	27	15	0	42

"In the above compilation, the various lines have been compared directly without reference to the number of storms in the various seasons or any comparison as to the actual number of storms for each individual line. All of the lines are, however, located in a comparatively small territory and any one storm passing over the territory is apt to affect all of the lines. The results do not reveal anything unexpected but they may be useful as an actual proof of the theories advanced of the ground wire and the results obtained in the laboratory."

KEY TO PENNSYLVANIA WATER AND POWER COMPANY LINES REPORTED ON ABOVE

Line number	Carries circuits	From Holtwood to
12	Nos. 1 and 2	Baltimore
56	Nos. 5 and 6	Baltimore
1516	Nos. 15 and 16	Lancaster
1112	Nos. 11 and 12	York
1314	Nos. 13 and 14	Coatesville

Data submitted by the Georgia Power Company further confirm theory in regard to the greater shielding effect of two ground wires versus one.

Of two circuits on the same tower line, each 107 mi. long, one of which was protected by a ground wire its entire length, while the other had a ground wire over certain sections, aggregating 15 mi. in length, the circuit entirely protected had four outages in 1927 and the other, eight. In each case three of these outages were accompanied by the burning in two of a line conductor. On the basis of the total interruptions, the first circuit was twice as reliable as the first. Omitting wires burned off (possibly as a result of direct strokes) the remaining interruptions were as one is to five.

Of nine wires burned off in 1927, three were on unprotected circuits, three on circuits with a ground wire

over the opposite circuit, and three on circuits with a ground wire above them, but having only one ground wire on the tower. None was burned off on lines where each of the two circuits was protected by an individual ground wire, although considerable mileage of this type was within the area in which the other lines were affected.

Considering all lines constructed with one or two ground wires as being protected this company's record for 1927 for practically equal mileages of 110-kv. lines shows interruptions of 93 per 100 storms for protected lines and 400 interruptions on the same basis for unprotected lines.

4. *Earth Resistances.*

One company (Dayton Power and Light Company) reporting stated in a general way that marked improvement had resulted from the installation of a ground wire on 66-kv. wood pole lines and gave the following data in regard to it:

"These lines (54 mi.) operated until August 1925 (20 months) when the installation of the static wire was completed. The installation covered a period of about six weeks.

"The static wire consisted of 1-5/16 in. strand conductor attached to the end of a 4-ft. bayonet above the pole top, connected electrically to the bayonet and grounded at the base of each pole with a No. 4 copper wire stapled to the pole soldered into the end of a ¾-in. pipe, 6½ ft. long driven at the base of each pole. Every tenth ground rod was left disconnected from the static wire for test purposes. Tests made March 9 and May 8 1928 show resistances as follows:

Pole—Ground pipe		Static wire	
March 9	May 8	March 9	May 8
92.5	70.	17.5	20.
147.5	110.	27.5	20.
43.	37.5	5.	7.5
24.5	22.5	5.5	7.5
30.5	25.	5.5	5.
35.	40.	10.	10.
23.	22.5	9.	7.5

"The above readings are in ohms obtained with a Leeds & Northrup vibrating a-c. ohmmeter. An auxiliary pipe was driven, and three readings taken, *viz.*: between the auxiliary pipe and the ground pipe at the pole; between the auxiliary pipe and the static wire; and between the static wire and the ground pipe driven at the pole, from which the above readings were obtained algebraically.

"The high readings shown on the pole ground pipe are due, in each case, to a thin layer of soil over gravel, the low readings being in deep soil. The soil is clay loam mostly over gravel. In some places limestone is encountered eight to ten ft. below the surface.

"Prior to the completion of the static wire 40 flashes were encountered, 99 insulator units being destroyed. After the completion of the static wire up to the present time (May 1928) 7 flashes were experienced with 21 units destroyed. Also prior to the completion of the static wire 5 poles and one crossarm were destroyed by lightning. None have been destroyed since."

5. *Shattering of Wood Crossarms.*

Dallas, Texas, reports as follows for 60-kv. wood lines equipped with ground wire and similar lines not so equipped:

	With ground wire		Without ground wire	
	1926	1927	1926	1927
Circuit mileage	318.3	318.3	844.4	895.9
Arms shattered	0.	1.	39.	11.

A reduction in crossarm shattering was obtained in 1927 by grounding the hardware at each structure with a ground rod (but without installing an overhead ground wire) on 696.6 mi. of line.

For the 60-kv. lines reported the average interruptions per 100 mi. of circuit, for the years ending with 1927, were as follows:

Steel Lines with one ground wire	3.5
Wood Lines with one ground wire	6.3
Wood Lines without ground wire	25.6

B. Special Construction with a View to Reducing Lightning Outages

One company (Public Service of N. J.) reports, "There is a study being made on the use of wood construction for 26-kv. lines so as to take advantage of the inherent insulation in the wood poles. This study was initiated because it was found that the majority of flashovers were occurring at corner poles which were heavily guyed, and therefore, most of the insulation provided by the wood was short circuited. In connection with this problem we are studying methods of insulating guy wires so that the insulation at the pole will be retained."

Another company (Cleveland) is at present building a short (5 mi.) 33-kv. wood line protected by insulated ground wire. Steel pins and crossarms are used and these are all bonded. The dry flashover of the ground wire insulators is (86 kv.) about half that of the line insulators (165 kv.). The ground rods (every fourth pole) are carried down the pole on insulators to a point about ten feet from the ground. Insulators having a flashover of 125 kv. are cut in all guys at the pole.

This line is a tap line from an unprotected line exposed to severe lightning storms and it is hoped that by comparing the flashovers occurring on this section with those on sections of the main lines on either side of the tap point, some estimate of the comparative value of this type of construction may be obtained.

The Tampa Electric Company reported definite experience on three types of construction shown in the accompanying illustration.

In 1926, 83 mi. of circuit of Type *A* construction

were operated at 66 kv. One hundred and twenty electrical storms were experienced resulting in 57 "kick-outs" or 47.4 per 100 storms.

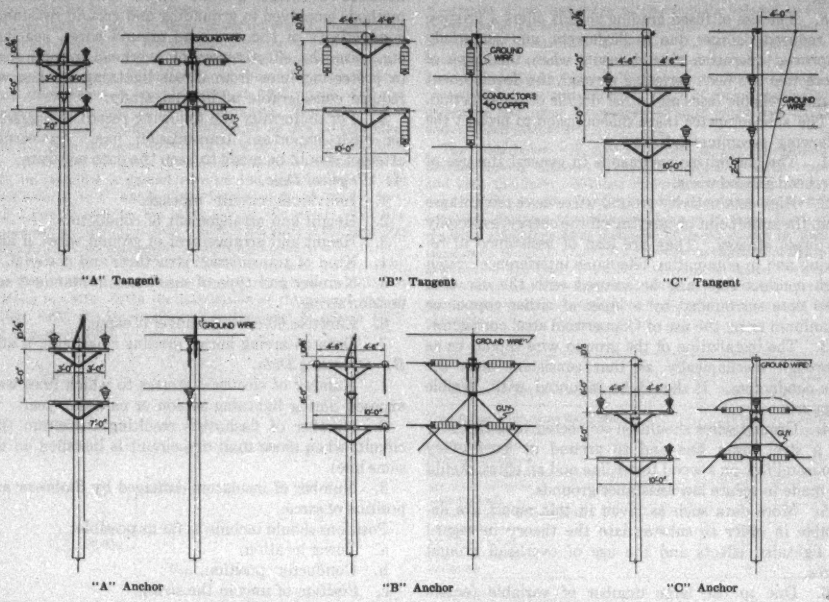
In 1927, 30 mi. of circuit of Type *A* construction were in operation, 30.3 mi. of Type *B* construction, and 29.7 mi. of Type *C* construction. One hundred and twenty-one electrical storms were experienced on each with the following operating record:

Construction	Kick-outs	Insulator replacements
Type A	34	10
Type B	2	6
Type C	8	0

Mileage and storms being equal for each line, results are directly comparable. Construction *B* with seven suspension insulators and therefore a very much higher flashover value than Construction *C* naturally gave the best results, but the improvement of construction *C* over *A* is very marked. Of course, a single year's experience cannot be taken as conclusive. It should be noted that these circuits were among the very few reported which were operated with ungrounded transformer neutrals.

C. Inferences

The following inferences may be drawn from the data submitted in connection with the preliminary report made to the committee, and those subsequently received and partially presented herein.



Three Types of Construction for 66-Kv. Wood Pole Line

These types are used by the Tampa Electric Company

1. Properly installed, an overhead ground wire materially increases line reliability by reducing flashovers due to lightning.

2. In general, points of higher elevation along transmission lines are more subject to flashovers than those of lower elevation.

3. Flashovers on lines equipped with ground wires are more likely to develop at structures where the value of the earth resistance is high, especially if materially above 50 ohms.

4. Earth resistances vary widely with the character of the soil, and, in the case of steel structures, with the nature of the foundation, making tests necessary if reasonable assurance as to their probable values is desired for any given installation.

5. In practise the installation of multiple ground wires on a transmission line appears to check theory, affording greater protection than a single wire.

6. Shattering of wood poles and crossarms is greatly

reduced by properly installed overhead ground wires and also by bonding and grounding all hardware at every pole.

7. Special construction may be resorted to with expectancy of reduced lightning outages on wood pole lines.

8. The use of fused grading shields offers a promise of reduced outages due to flashovers, and materially improved operation should result when this type of protection has been perfected through the development of more reliable fuses and other details of construction.

The subcommittee takes this occasion to present the following recommendations:

1. The committee commends in general the use of overhead ground wires.

2. High conductivity ground wires have advantages from the standpoint of carrying off the energy, especially of direct strokes. They are also of assistance in relaying and in eliminating telephone interference. Such high conductivity may be secured with the use of a steel core surrounded by a layer of either copper or aluminum or by the use of Copperweld steel conductor.

3. The installation of the ground wire should be as thorough mechanically as that consistent with the line conductors. It should be mounted with flexible supports.

4. Ground wires should be connected to each tower in a steel tower line and to ground at least every thousand feet on a wood tower line and an effort should be made to secure low resistance grounds.

5. More data such as given in this report are desirable in order to substantiate the theory in regard to lightning effects and the use of overhead ground wires.

6. Due to the large number of variable factors which enter into the performance of transmission lines and the functioning of overhead ground wires during lightning disturbances and the limited amount of operating data available which can be used in making comparisons, it is impossible to draw definite conclusions regarding the effectiveness of overhead ground wires for improving transmission line performance. It is therefore urged that the companies which have been keeping accurate records of transmission line performance continue this work, and other companies having transmission lines with and without overhead ground wires which are subject to comparable storm influence, and whose insulation strengths are uniform throughout their lengths and which are connected to the system by means of automatic switching equipment, prepare to keep records on these lines including storm data, insulator replacements, damage to structures, etc. Data on transmission line operation submitted to the Institute should include all influencing factors in the design, construction, and operation of the lines on their performance. Accumulation of data for a number of years is necessary in order to average out as many variables as possible.

7. From the rather limited amount of data obtained comparing performance of lines with and without overhead ground wires it would seem that a marked reduction in the number of flashovers due to lightning should be experienced on lines equipped with properly installed overhead ground wires. The influence of methods employed in grounding and ground resistance, conductivity of the overhead ground wires, spacings, etc., also the effectiveness of overhead ground wires in protecting lines from direct lightning strokes, will require considerable additional study.

8. For uniformity the following record is suggested for each important transmission line. An earnest attempt should be made to keep the data accurate.

A. Physical Data.

1. Individual circuit mileage.
2. Height and arrangement of conductors.
3. Height and arrangement of ground wires, if any.
4. Kind of transmission structures and material.
5. Number and type of insulators in standard suspension string.
6. Effective 60-cycle flashover of same.
7. Data on arcing horns, grading rings, etc., if any.

B. Operating Data.

1. Number of electrical storms to which lines were exposed during lightning season or calendar year.
2. Number of flashovers resulting therefrom (per circuit, when more than one circuit is installed on the same line).
3. Number of insulators damaged by flashover and position of same.

Positions should include as far as possible:

a. Tower location,
b. Conductor position,
c. Position of unit in the string.

4. Any pertinent facts which may be uncovered in the investigation of a case of lightning damage.

The accumulated data should be forwarded early in February of each year to the Chairman of the Power Transmission and Distribution Committee so that the material may be studied and reported upon in the Committee's Annual Report.

Discussion

1927 LIGHTNING EXPERIENCE ON 132-KV. TRANSMISSION LINES

(Sporn)

PROTECTION OF TRANSMISSION LINES FROM INTERRUPTION BY LIGHTNING (REPORT OF SUB-COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION

(Wallau)

New York, N. Y., January 29, 1929

J. H. Cox: In Mr. Sporn's paper a suggestion was made that a second ground wire should possibly be so placed as to be most influential on the lowest conductor, and thus equalize the induced potentials on the three wires. This would necessitate placing it some place below the top of the line. We believe that in so placing it, part of its value would be lost. A ground wire is undoubtedly effective in conducting many direct strokes

to ground without involving the line conductors in the stroke. In order to do this it is essential that the ground wires be placed above the line wires and in a position favorable to take the direct hits of lightning.

C. E. Ambelang: Referring to Mr. Sporn's paper, I should like to present a few of the facts which we have, showing protection afforded by ground wires.

Durng 1928 we had only three interruptions due to lightning on ouriines. In 1927 we had two flashovers on the lines equipped with g lound wires. Fig. 1 herewith shows three lines which I will usr to illustrate the protection of circuits by ground wires. These eines were put into operation in 1925. Line *A-B* was equippeld with ground wire at the beginning. During 1925 and 1926 it operated without any interruptions due to lightning. Line *A-C* is on the same towers for 10¾ mi. In 1925 and 1926 line *A-C* tripped out seven times each year due to lightning. In 1927 we installed a ground wire on the opposite side of the tower from *X* to *C*, which made the line equipped from *A* to *C* with a ground wire on the other side of the tower. The line tripped out six times that year. Before the start of the lightning season in 1928, we equipped the line with a ground wire directly over the circuit. There were no interruptions during 1928. Line *C-D*, without ground wire, tripped out 9 times in 1925 and 19 times in 1926. With the installation of the ground wire in March, 1927, the interruptions were reduced to one in 1927, and to none in 1928.

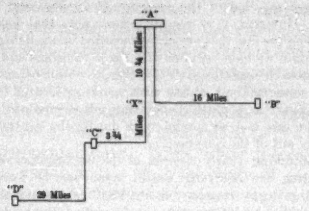


FIG. 1

We made a check of the lightning storms and found that in that territory there was practically the same number in each year.

James S. Mahan and M. G. Lloyd: In connection with the papers on field investigations of lightning, it may be of interest to present the statistical results of an investigation of the damage done by lightning to the properties of electric light and power companies undertaken from the insurance standpoint.

In 1924 the Western Actuarial Bureau instituted an investigation among electric light and power companies in the territory west of the Alleghany Mountains reaching to the Pacific Coast and extending from Canada to the Gulf of Mexico. Arrangements were made with 248 companies in this territory to supply data regarding the value of the property exposed and the nature and financial extent of the damages caused by direct strokes of lightning or by surges occurring on their lines during lightning storms. The data secured covered principally the calendar years 1925 and 1926. Data were received from 240 companies scattered over 30 states. Losses were reported by 165 of these companies, reports of nearly all of the companies covering a period of at least two years. The only state in the specified area from which losses were not reported was Utah.

The total value of the properties exposed to damage by lightning, of the companies reporting, amounted to about $819,756,000. This value includes power house and substation buildings, contents, and out-door apparatus. The total losses attributed to lightning over a 2-year period amounted to slightly more than $631,000, or about 0.04 per cent per annum.

Included in the reports were statements as to whether lightning arresters and choke coils were used to protect lines and equipment, and the type of arrester used. In presenting the results, nothing is said about types of arresters involved, since the total number of arresters of each type involved are not known, and consequently the number of cases of damage involving each type of arrester would not serve to discriminate between types of arrester as to the relative protective value of each. Neither has any attempt been made to indicate the value of arresters in general, since without knowledge of the amount of protection provided, conclusions cannot be drawn as to the extent to which protection was actually afforded. It is, of course, impossible to determine how much damage would have resulted if arresters had not been used at all.

The table below will give some idea of the distribution of damage among different classes of equipment within stations and upon the lines. Without counting interruptions to service due to circuit-breakers being tripped or fuses blown, and without counting instances of additional damage to fuses, there were reported altogether 4450 instances of damage, and in 4099 of these cases protection had been provided in the form of lightning arresters of one type or another. It may be stated that losses were heavier and more frequent on those systems whose operating conditions resulted in equipment being used at or near its rated capacity.

Number of pieces of apparatus damaged	Total	Having arrester protection
Transformers, including 296 bushings	1774	1610
Metering equipment	1280	1250
Lightning arresters	137	..
Rotating machines	117	110
Oil switches, including 30 bushings	146	133
Air-break switches	71	61
Switchboards	13	13
Bus structures or insulators	35	34
Line insulators	408	371
Line conductors	241	215
Pole or crossarm	215	155
Potheads	5	5

S. M. Jones: (communicated after adjournment) In Mr. Sporn's paper and in other papers and summaries of lightning studies presented from time to time, the analyses of interruptions have been presented in tables in each of which the final figure has been the number of interruptions per mile or per hundred miles of circuit. Our own company has done considerable investigating similar to the American Gas and Electric Company, has prepared tables quite similar to Mr. Sporn's, and has attempted to draw certain conclusion from them and comparisons between the operation of different lines. The criticism which follows, therefore, applies to the work of all of us who have been making lightning investigations.

First, I do not believe that any commonly accepted definition as to what constitutes an interruption to a circuit has ever been definitely established. Certainly this should be the first step so that tables prepared by different companies would all be on a common basis. Some operating departments classify a line as being interrupted when the circuit is dead. This includes prearranged interruptions, and also interruptions on a line which may be due to trouble on other circuits which interrupt the power supply to the line in question. Obviously, this is not a proper classification for lightning investigations. In our studies we have classified a circuit as being interrupted only when one or both ends of the line have been automatically disconnected from the station buses due to trouble or flashover on that particular circuit.

Another criticism is that we have not yet arrived at a common term of interruptions per unit or units of something tangible

which can actually be used as a basis of comparison for the performance of lines. In order to bring these summaries of interruptions to a common basis there are several factors which come to mind which must be considered, and there may be others. P. H. Thomas mentioned and elaborated on one of them, namely, that of ground resistance. As he said, it is probable that in so far as lightning voltages of steep wave front are concerned, we should study not the resistance, but the impedance of the ground return path. Personally, I believe we have spent rather too much time and money in trying to determine ground resistances and have not gone very far, and have neglected some other perhaps more important considerations which affect lightning flashovers.

The circuits to be strictly comparable must, of course, operate at the same voltage and have the same type of construction (towers, insulators, ground wires, and conductors). They must also pass through similar types of country since a difference in terrain has an appreciable effect on the ground resistance or reactance. This factor will be one of the hardest to determine and apply in some terms of interruptions per unit of impedance.

To my mind one of the most important factors to be considered and one which can be fairly readily taken care of to bring analysis of circuits to a common basis, is the number of storms or lightning flashes within a limited distance of the transmission line. Thus, instead of arriving at a figure of interruptions per mile, it would be interruptions per mile per storm or lightning flash.

In practically every section of the country most storms follow a rather definite path which should be determinable from weather reports obtained by operators or by storm intensity recorders. The reason that this factor becomes so important is that in some cases a storm path may be parallel to certain transmission lines, while in the case of other circuits the path may be at right angles to them. It would naturally be expected that where storms parallel a line there would be more interruptions than if they merely crossed over it. The figure of interruptions per mile does not consider this feature.

Still another factor of major importance affecting circuit outages is the possible short-circuit kv-a. on the line. Investigations on the Southeastern Power and Light Company system, particularly on some 110-kv. lines approximately 250 mi. long and having only one source of supply, have indicated that during the past two years all insulator failures have been within 50 mi. of the source end of the line. It was also found that many flashovers have occurred at points near the distant end of the line which did not cause outages, indicating that there was not sufficient power current to maintain the arc once it was established by the lightning flashover. This factor might very easily be considered and we would then arrive at a figure of interruptions to a line due to lightning, per mile, per lightning flash (within some specified distance of the line), per 100,000 kv-a. of possible short-circuit current.

Unquestionably, there are other factors which must be considered in any transmission system analysis, and many others will be brought forth as our investigations continue, but those previously mentioned must certainly be included to obtain any basis of comparison of the performance of different transmission circuits.

Philip Sporn: Mr. Jones' comments on the methods of recording and interpreting interruptions and keeping complete data on lightning storms are most vital to a logical analysis of lightning troubles. On our 132-kv. system we record as one interruption a circuit-breaker opening, whether at one or both ends of the circuit, when caused by apparent lightning trouble on the 132-kv. circuit itself. Our relay system is such that we rarely interrupt a 132-kv. circuit except when trouble occurs on that circuit. Thus, low-voltage circuit lightning interruptions do not cause 132-kv. interruptions, and the outages or interruptions reported in my paper include only cases of trouble on the 132-kv. circuit, known or apparently due to lightning.

In the matter of securing storm data, we are making a practise of recording lightning storms observed at points along the 132-kv. lines where operators are located. In this way we can obtain a general idea of the severity, frequency, and course of storms. However, it is readily conceivable that local storms occur along the line that are never recorded in our lightning data. In fact, data we have secured with surge recorders and klydonographs seem to bear this out. Further, to obtain complete storm data on some 1000 mi. of line with operators located from 30 to 120 mi. apart as a routine operating job is impossible; and to undertake this task as special work during the lightning season is equally out of the question.

By comparing yearly records of the performance of existing lines during the lightning season much can be learned even though the storm frequency is not identical from year to year. Also, considerable information can be gained by the comparison of similar lines with different soil conditions, under different exposure, and with different connected kv-a. generator capacity.

The measuring instruments for lightning comparison on transmission lines are at present crude and their calibration not standardized as Mr. Jones points out, but I feel we are making distinct progress in helping to solve the lightning problem.

Power Factor and Dielectric Constant in Viscous Dielectrics

BY DONALD W. KITCHIN*

Associate, A. I. E. E.

Synopsis.—This paper gives the results of a study of the peculiar variation with temperature and frequency of the dielectric constant and power factor of rosin, rosin oil, and castor oil. It includes data showing at several frequencies the relation of dielectric constant and power factor to the composition of vulcanized rubber.

Electrical double refraction in rosin at different frequencies and temperatures is discussed in relation to its behavior as a dielectric. It is shown that the viscosity is a decisive factor controlling both the electrical and optical behavior. The facts are important in themselves but it is possible to interpret them by a modern physical theory, the Debye[1] dipole theory, which it is believed has not hitherto been applied to commercial dielectrics. On this theory the anomalous change of dielectric constant and power factor with temperature and frequency is attributed not to impurities or heterogeneity of structure but to molecules containing electric doublets which try to orient themselves in an electric field. The rotation of the dipole molecules in a viscous medium gives rise to frictional heat loss expressed as power factor, and also to a contribution to the dielectric constant which vanishes when the dipoles are prevented from responding by too great viscosity or too high frequency.

For the sake of intelligibility, an outline of the dipole theory is first presented and then the experimental results are discussed on the basis of that theory.

* * * * *

I. Introduction

THE behavior of dielectrics in alternating fields has been the subject of considerable experimental and theoretical investigation. The majority of the theories advanced to explain the anomalous behavior have been based on models of various degrees of complexity, and are for the most part modifications of Maxwell's[2] theory. In some cases the models have been purely electrical—*e. g.*, combinations of capacities and resistances so chosen and arranged as to fit a given set of experimental data. While these theories are often very useful in the practical study of complex dielectrics, such as impregnated paper, etc., it is the belief of the writer, based on the experimental results of this paper, that the behavior of viscous, homogeneous liquid dielectrics, with respect to the change of power factor and dielectric constant with temperature, can best be interpreted on the basis of molecular orientation according to the Debye dipole theory, in particular his theory of anomalous dispersion and absorption.

If an oil like transil oil is tested at 60 cycles, it is generally found that the power factor increases with rising temperature because of the increasing leakage current, while the dielectric constant drops because of the decrease in density. On the other hand, certain viscous liquids, like rosin oil, show at high frequencies a rise in dielectric constant accompanied by a pronounced power factor maximum. If the testing frequency is raised, the maxima of both power factor and dielectric constant are shifted to higher temperatures. This type of behavior is the main concern of the present paper. An attempt is made to show that the observed facts can be accounted for by a molecular mechanism which also enables us to predict approximately how a certain material will behave under given conditions.

It should be stated at the outset that the work was originally undertaken for industrial purposes with no thought of any molecular theories, and for that reason the materials studied were not so well defined chemically as those a physicist might choose for a rigorous test of the Debye[1] theory. Nevertheless the results are in good qualitative agreement with the theory and it is hoped that they may serve the double purpose of interesting electrical engineers in the application of molecular theory to practical dielectric problems, and at the same time of showing that the study of power loss in commercial materials can lead to facts important to the physical theory.

Since the dipole theory of dielectrics may not be familiar to all who are interested in insulating materials, an outline is given here. The concept of dipole molecules and their probable behavior in alternating fields under certain conditions is discussed. For a full presentation of the theory, the treatise of Debye should be consulted.†

II. Molecular Behavior in Electric Fields

By convention the dielectric constant of vacuum is taken as 1. In matter subjected to the electric field, this contribution of empty space is supposed still to be present in addition to that of the material itself. Since it is uninfluenced by any changes in the material, it can be dismissed and our attention turned to the manner in which the molecules of the material contribute to the electrical properties.

1. Displacement Electrons. In the molecules of a dielectric the electrons are bound by quasi-elastic forces to positions of equilibrium with respect to the positive nuclei from which they can be displaced by an

*Research Laboratory, Simplex Wire & Cable Co., Boston, Mass.

1. See Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

†An English text "Polar Molecules" by Debye is announced by the Chemical Catalog Co.

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electric field only to a certain distance without disrupting the molecules. The displacement of bound electric charges is called polarization. In each volume element of a polarized dielectric, the centers of positive and negative charge no longer coincide. The relative motion of the charges in an alternating field gives rise to a charging current and contributes to the dielectric constant. The motion of the electrons in following the field may be considered to take place without lag and with no dissipation of energy.

2. *Concept of a Dipole Molecule.* There exist substances whose molecules are polarized even in the unstressed state on account of their atomic configuration. Each molecule forms a permanent electric doublet, called a dipole, with lines of force analogous to those of a permanent magnet. The product of the separation of centers of charge by the amount of charge of either sign gives the fixed dipole moment which is of the order of magnitude of 1×10^{-18} (cm. $\times$ electrostatic units). Dipole molecules are those which fail to fulfill certain conditions of symmetry. The leading substances whose molecules are dipoles, such as water, the alcohols, nitrobenzene, are characterized by very high dielectric constant. The contribution of the displacement electrons to the dielectric constant is small compared to that due to dipole orientation in these substances.

3. *Dipole Orientation.* Since the dipole molecule contains an electric doublet, it is evident that it will tend to line up in the direction of an electric field. We might think of the molecules of a gas or liquid behaving like a large number of compass needles in a magnetic field. The actual behavior is not so simple, however, because in a gas or liquid the molecules are in rapid motion. The effect of any ordinary field is only a relatively small one superimposed on the thermal motion. Calculation by Debye's[1] theory shows that to orient all the molecules of an ordinary dipole liquid at room temperature, a potential gradient of the order of 20 million volts per inch would be required. Nevertheless, at any instant with a given field there is a certain average component in the direction of the field of all the dipole moments which contributes to the polarization. The higher the temperature, the more vigorous the thermal motion and the less the tendency of the molecules to respond to the field.

It is of interest to consider the effect of increase of temperature on the dielectric constant, with respect to the three contributions. The contribution 1 of empty space is unaffected. The contribution of the displacement electrons decreases because the density decreases. If dipole molecules are present, the effect is twofold; the number of dipole molecules in unit volume is diminished and also the average response per molecule, so that the drop in dielectric constant is more marked. A striking example of such a drop in dielectric constant is shown by the right branch of the curve for ether in Fig. 1. Table I shows the same behavior in the case of the alcohols at temperatures above their freezing points. Thus, the normal behavior of a liquid dielectric containing dipole molecules is to exhibit a more rapid decrease of dielectric constant with rising temperature than can be accounted for by the density change alone.

4. *Influence of the Medium on Dipole Orientation.* Since the response of the displacement electrons to the field merely produces a state of strain in the molecules without tending to turn them, it is evident that the state of aggregation of the material will have but slight influence on their contribution. It is quite different

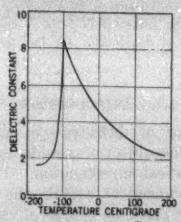

FIG. 1—CURVE SHOWING CHANGE OF DIELECTRIC CONSTANT OF ETHER WITH TEMPERATURE ISNARDI[4]

with the dipole molecules. For a dipole to respond to the electric field, it is necessary for a molecule, as a whole, to turn, and any factor which affects the ease of turning will affect the response.

a. *Effect of Arresting Dipole Motion Suddenly.* Suppose we could in some way fix the dipoles so that they could no longer follow an alternating field. Then their contribution to the dielectric constant would disappear. In this way we might recognize the dipole contribution. Freezing a dipole liquid produces this result. Some examples from a paper by Ratnowsky[3] are given in Table I. The curve dielectric constant *vs.* temperature for ether shows it clearly.

TABLE I

Substance	Temp. deg. cent.	Diel. const.
Amyl Alcohol	20	16.0
	−100	30.1
	frozen	2.4
Ethyl Alcohol	20	25.8
	−120	54.6
	frozen	2.7
Methyl Alcohol	20	31.2
	−100	50.0
	frozen	3.07
Water	18	81.1
Ice	. .	3.16

Thus, neglecting the change of density, we may say that the dipole contribution in amyl alcohol at − 100 is roughly 28, in ethyl alcohol at − 120 it is 52, in methyl alcohol 47, and in water 78.

In these cases, the dielectric constant increases with lowered temperature down to the freezing point.

b. Effect of Arresting Dipole Motion Gradually. If instead of suddenly arresting the dipole orientation we could apply a gradual braking action, the dielectric constant, instead of dropping abruptly, would fall gradually to the value corresponding to the contribution of displacement electrons and of free space. This gradual slowing down of the dipole motion occurs in liquids which on being cooled exhibit a great increase in viscosity instead of freezing to crystalline solids. Rosin oil, castor oil, and glycerine (see Bock[8]) are good examples of this type of liquid. The dipole orientation is opposed by the viscosity of the liquid and when it becomes very great, the molecules can respond only to a slowly changing field. They are not fixed, as in a frozen liquid, but have only become sluggish. In a one million cycle field, the response of the dipoles finally becomes negligible as the temperature is lowered, because the direction of the field changes too rapidly for them to follow. The dielectric constant for this frequency is then due only to the displacement electrons and to free space. The dipoles can still respond to a less rapidly changing field and the dielectric constant for 60 cycles is higher. A simple analogy would be a bottle half full of molasses. If you invert it, the molasses will take a certain time to flow down. If you turn it over and over rapidly, the response to gravity will be slight and it will behave like a solid. If the molasses is warmed or diluted to decrease the viscosity, it will respond to more rapid turning.

5. Losses Due to Dipole Friction. The motion of the dipole molecules turning in a viscous environment gives rise to frictional losses, so that some of the energy of the field is dissipated in the form of heat. The magnitude of the resulting power factor is determined by two factors, namely, the amount of motion, of which the dielectric constant is a function, and the viscosity opposing the motion. At temperatures where the viscosity is low, the dipole power factor (so called to distinguish it from that due to leakage) is small. On the other hand, at viscosities great enough practically to prevent orientation the power factor is again small. The maximum occurs in the intermediate region of temperature where the resultant of motion and high viscosity is greatest. This is the region where the dielectric constant decreases with falling temperature, as shown by the experimental results of the present work. As already pointed out, in viscous liquids, as distinguished from crystalline solids, the dipoles are not fixed and can therefore follow a field which reverses sufficiently slowly. It seems reasonable to suppose that with a given liquid in a definite state, the molecules can follow an alternating field with ease up to some range of frequencies where it becomes difficult for them to respond. We might then speak of a *characteristic frequency* for a given material. By this is roughly meant that the molecules can readily follow a field of lower frequency than the characteristic frequency, but that at higher frequencies the orientation falls off. A more precise conception of the characteristic frequency is given in the following sketch of the Debye[1] theory of anomalous dispersion.

III. The Debye Theory of Anomalous Dispersion and Absorption

1. A number of liquids which at ordinary frequencies are characterized by unusually high dielectric constants was found by Drude[6] and others to suffer a marked decrease in dielectric constant with increasing frequency in the very high frequency region of the electrical spectrum. This decrease was accompanied by a pronounced energy absorption. Water showed a drop from $\epsilon = 81$ to $\epsilon = 3$. The following table, quoted from Debye[1] shows the behavior of amyl alcohol and glycerine.

TABLE II

Amyl Alcohol 18 deg. cent.		Glycerine 15 deg. cent.	
Frequency	Diel. Const.	Frequency	Diel. Const.
Low	16	$25 \times 10^6 \sim$	56.2
$150 \times 10^6 \sim$	10.8	$150 \times 10^6 \sim$	39.1
$412 \times 10^6 \sim$	4.7	$400 \times 10^6 \sim$	25.4
		$111 \times 10^8 \sim$	2.53

Such decrease of dielectric constant with increasing frequency is called "anomalous dispersion," a term derived from Optics where "normal dispersion" means increase of refractive index n with increasing light frequency. In the vicinity of an absorption band the opposite change may occur, and is called "anomalous dispersion." The accompanying pronounced absorption of light energy is called "anomalous absorption." On the Maxwell electromagnetic theory $\epsilon = n^2$ and therefore the change of ϵ with frequency is also called "dispersion" and is "anomalous" when ϵ drops with increasing frequency. The energy loss in this region is likewise called "anomalous absorption." Anomalous dispersion and absorption are therefore defined as a decrease in dielectric constant with increasing applied frequency accompanied by a power factor maximum.

As Debye says, it is natural to attribute this anomalous dispersion to the fact that whereas the displacement electrons can respond to alternating fields of any frequency, dipole orientation becomes increasingly difficult with rising frequency. Therefore, anomalous dispersion and absorption should occur only in dipole liquids.

Consider such a liquid between the plates of a condenser. In the unstressed state, the thermal motion of the molecules gives rise to a perfectly random arrangement and the vector sum in any direction of the individual dipole moments is 0. There is no polarization. If an electrostatic field is now applied, there is a tendency for some of the slower moving molecules to line up in the field. The perfectly random arrangement of the molecules in the unstressed liquid changes to a somewhat more orderly one. If the field is removed, the molecules revert to the previous random state.

Whereas the displacement electrons spring back to their unstressed positions in the molecules instantaneously, the dipole molecules require a finite time to return to the original random state. The arrangement produced by the field falls off exponentially, and the time required to drop to the $1/e$th part is called the "*relaxation time*," τ ($e = 2.718$, the base of the natural logarithms). The relaxation time is thus a measure of the quickness of the response of the dipole molecules. The reciprocal of the relaxation time $\frac{1}{\tau}$ gives the "*characteristic frequency*" f_0. It depends in a given substance on the size of the molecule, the temperature, and the viscosity. If the assumptions are made that the molecules are spheres of radius r, and that the frictional resistance opposing their turning obeys the same law as with an ordinary sphere, we have, according to Debye:

$$f_0 = \frac{1}{\tau} = \frac{k\,T}{8\,\pi^2\,\eta\,r^3} = \frac{T}{\eta} \text{ times a constant} \qquad (1)$$

f_0 = characteristic frequency
r = radius of sphere equivalent to the actual molecule
η = coefficient of viscosity
k = Boltzmann constant, 1.37×10^{-16} erg per deg. cent.
T = absolute temperature

This relation is an ideal one which can be expected, when applied to viscous liquids like rosin and castor oil, to give only the correct order of magnitude. Knowledge of f_0 shows in what frequency range anomalous dispersion and absorption should occur. As long as the applied frequency f is low compared to f_0, the dipoles are able to respond to the field, and the dielectric constant is independent of the frequency. On the other hand, when f is much higher than f_0, the field changes faster than the dipoles can turn, so that the dipole contribution to ϵ disappears. Moreover, since the alternating field produces practically no turning, the frictional loss vanishes and the dipole power factor becomes negligible. The region of ϵ drop and maximum power factor occurs where f and f_0 are comparable.

By means of statistical reasoning, Debye obtains relations which permit the calculation of the power loss from the change $\epsilon_0 - \epsilon_\infty$ where ϵ_0 means the dielectric constant at frequencies much lower than f_0 and ϵ_∞ that at frequencies so high that the dipole response has disappeared. The energy loss is maximum when

$$\frac{f}{f_0} = \frac{\epsilon_\infty + 2}{\epsilon_0 + 2}\sqrt{\frac{\epsilon_0}{\epsilon_\infty}} \qquad (2)$$

For materials like rosin oil in which the change in ϵ is small, the power factor maximum comes almost exactly at $f = f_0$.

The general nature of the phenomenon of anomalous dispersion and absorption is more readily shown by reproducing the curves calculated by Debye for glycerine, than by giving the mathematical development of the theory. The results which are plotted against $\log \frac{f}{f_0}$ are the refractive index n and the index of absorption α. Since they are functions of the dielectric constant and energy loss respectively, their behavior with respect to frequency is similar and it is unnecessary to convert them to the usual electrical quantities.

In the curves showing the experimental results, the general resemblance to these theoretical curves may readily be noted. The direction of the experimental curves is opposite, because the ratio f/f_0 is decreasing with rising temperature.

2. *The Frequency Region of Anomalous Dispersion.* In the past it has been difficult to obtain conclusive data to support the theory of anomalous dispersion. The reason lies in the extremely high frequencies necessary to produce the effect. Consider the case of water. At 27 deg. cent. (300 absolute), the viscosity coefficient $\eta = 0.01$. If the radius of the molecule is taken firs

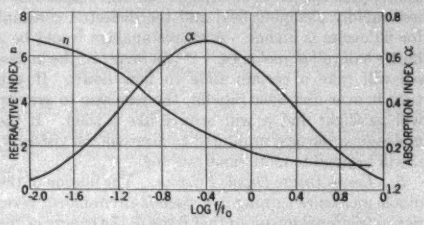

FIG. 2—THEORETICAL CURVES SHOWING ANOMALOUS DISPERSION AND ABSORPTION IN GLYCERINE [DEBYE[1]]

as 1×10^{-8} cm. for single molecules, and as twice that amount for associated molecules, we obtain by Equation (1) characteristic frequencies of 50 billion and 6 billion cycles. The phenomenon of anomalous dispersion and absorption has been studied in this frequency region by waves obtained from extremely minute Hertz oscillators. Optical methods were used to measure n and α. Such waves are difficult to produce and to control, and are highly damped and variable in intensity. The outstanding work of this sort has been done by Nichols and Tear.[7]

If the region of anomalous dispersion could be shifted into a more accessible frequency region, such as that of the radio waves, the measurements would be more reliable and easier to make. Quoting from Debye—"Particular emphasis, at least qualitatively, may be placed on the prediction that (a) substances with large molecules and great viscosity must show noticeable anomalous dispersion and absorption even at relatively long waves, and (b), that the dispersion and particularly the absorption will be very sensitive to temperature, since the coefficient of viscosity is very dependent on

temperature." He points to the behavior of amyl alcohol and glycerine (Table II) in support of the shift to lower frequencies. The experimental results of the present paper are in agreement with both predictions. It is shown that even at frequencies as low as 60 cycles it is possible to obtain anomalous dispersion and absorption. Thus, the range of the phenomena has been extended from the extreme frequency region of billions of cycles to that with which electrical engineers are concerned.

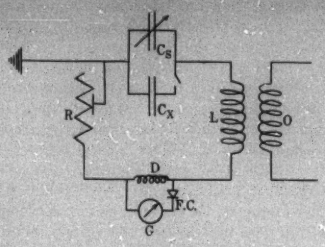

FIG. 3

C_s standard variable condenser
C_x test condenser
G micro-ammeter
R decade resistance
L inductance
D drop coil of a few turns
$F.C.$ crystal rectifier
O coupling to oscillator

IV. Experimental Method

The apparatus and method were very simple. The first series of experiments was made at one million cycles, the temperature being varied. A simple Hartley oscillator using a UX 201-A tube supplied the high-frequency current. Wavemeters were used to measure the frequency.

The measuring circuit is a simple resonant circuit with the experimental and the standard condenser in parallel. With C_x in and $R = O$, the circuit is tuned to resonance by varying C_s. The voltage drop across coil D causes a current, rectified by the crystal, to flow through G. The coupling to the oscillator is adjusted to give a suitable galvanometer deflection. Then C_x is removed, the circuit retuned, and R increased to give the same deflection. Then $C_x = C_{s2} - C_{s1}$, the difference in the two readings of C_s. The equivalent series resistance of C_x is obtained by the relation:

$$R_x = R\,(C_{s1}/C_x)^2$$

From the measured values, the power factor is obtained by the equation

$$P\,F = 2\,\pi\,n\,C_x\,R_x.$$

For the work at higher frequencies up to 11 million cycles, the same principles were followed. A suitable oscillator and measuring circuit were used. Instead of the decade resistance a set of resistances was made, consisting of short straight lengths of No. 40 cupron wire, soldered to heavy copper lead and sealed into glass tubes. Insertion of these resistors in the circuit caused very little detuning.

The test condensers consisted of coaxial metal cylinders. Tests showed the power factor and the capacity variation with temperature of the empty condensers to be negligible.

V. Discussion of Experimental Results

It was not until after a number of tests had been made that the molecular interpretation suggested itself. Tested at one million cycles, a sample of a cable compound containing 5 per cent of rosin showed higher power factor at room temperature than at 200 deg. fahr., whereas at 60 cycles the behavior was the reverse. The ingredients were then tested separately. Transil oil tested at one million cycles between 70 deg. and 160 deg. fahr. showed negligible power factor. Light amber petrolatum showed a gradual decrease in power factor from 0.16 per cent at 74 deg. to 0.03 per cent at 210 deg. The dielectric constant decreased with rising temperature as expected. The peculiar behavior of a commercial rosin oil (50 per cent rosin, 50 per cent mineral oil) shown in Fig. 4 gave the first hint of an explanation of the anomalous change in power factor and dielectric constant. Inspection of the viscosity curve shows that the power factor maximum and the drop in dielectric constant occurs in a temperature region where the viscosity increase is very rapid. This suggests the idea that the power factor and a part of the dielectric constant depends on some mode of motion which is greatly influenced by the viscosity of the medium. Dipole orientation discussed above readily accounts for this behavior.

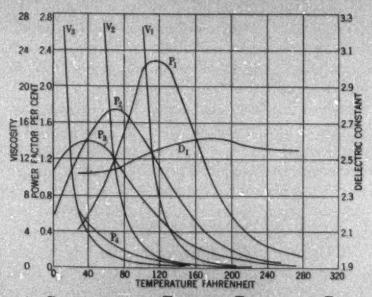

FIG. 4—CURVES SHOWING EFFECT OF DILUTION ON POSITION OF POWER-FACTOR MAXIMUM AT 1 MILLION CYCLES

D = dielectric constant
P = per cent power factor
V = viscosity Stormer—minutes per 100 turns

Subscripts refer to composition of mixtures:
1 100 per cent rosin oil (containing 50 per cent rosin)
2 80 per cent rosin oil 20 per cent Transil oil
3 50 per cent rosin oil 50 per cent Transil oil
4 20 per cent rosin oil 80 per cent Transil oil

1. *Effect of Dilution.* If the behavior was due to

some chemical change in the dissolved rosin rather than to the effect of viscosity, dilution with a neutral oil (*i. e.*, one of negligible power factor) like Transil oil, should merely decrease the power factor without shifting the position of the maximum. If, on the other hand, the dipole explanation is correct, dilution with Transil oil to decrease the viscosity should shift both the power factor and dielectric constant curves for constant frequency to lower temperatures. Obviously, since the concentration of dipoles is decreased by dilution, the dielectric constant drop and the maximum power factor are diminished. Fig. 4 shows that the curves for one million cycles shift in the predicted manner. For each concentration of rosin oil, the corresponding curves show the same relative positions. It is readily seen that an enormous increase of viscosity is necessary to cut down the orientation sufficiently to cause a large power factor decrease. This is to be expected, since the great increase in dipole friction with rising viscosity partly counteracts the effect of diminishing orientation. The same reasoning explains the fact that the dielectric constant always starts to fall at lower viscosity, *i. e.*, higher temperature, than where the power factor maximum is reached.

Substitution of the more viscous Nujol for Transil oil in a 50 per cent mix with rosin oil shifted the position of the power factor maximum from 40 deg. to 55 deg. fahr. This is additional proof of the orientation theory, since the mere substitution of one neutral oil for another should have no chemical effect on the rosin content. Replacing the Transil oil with 50 per cent of heavy cylinder oil caused a still greater shift to 75 deg.

An interesting fact was shown by the behavior of a mix of 50 per cent rosin oil in 140 deg. paraffin. At 122 deg. it was apparently solid, yet the power factor curve showed no break. The explanation lies in the fact that the apparent consistency is not a true indication of the actual environment opposing the motion of the dipole molecules. In the stiff, salve-like mixture there is a distributed liquid component in which the dipoles turn freely until this liquid component becomes very viscous. An analogy is electrolytic conduction in a dilute gelatine gel where the ions pass unhindered by the apparent solidity. Of course, the viscosity of the liquid component is affected by the change in composition as the paraffin separates out. This kind of behavior of apparent solids has led some investigators to attribute power factor maxima and dielectric constant drops to change in state of the material. In such cases a change of applied frequency should furnish a criterion. Lower frequency should shift the curves into the solid region, while much higher frequency should cause the whole change to occur in the liquid state.

2. Effect of Change of Frequency. Equation (2) shows that when $\epsilon_0 - \epsilon_\infty$ is small, the power factor peak and the drop in ϵ occur very nearly at $f = f_0$. According to Equation (1), f_0 is proportional to T/η, for a given material. The viscosity curves show the change of viscosity with temperature to be very great, so that a relatively small change in the temperature of rosin, for example, causes an enormous change in the characteristic frequency. It is therefore much more convenient to work at fixed frequency and variable temperature, *i. e.*, f constant and f_0 varying greatly, than to keep f_0 constant (constant temperature) and employ a correspondingly wide range of applied frequencies. Although the behavior observed when this method is used cannot strictly be called "anomalous dispersion" as defined in Section III, it is obviously due to the same cause, being determined by the ratio f/f_0. On this basis, at higher fixed applied frequencies the curves would shift to higher temperatures. Fig. 5 shows the results at 60 cycles, 1 million, and 10 million cycles, obtained with Hercules Wood Rosin *F*. The two higher frequency curves and the viscosity curve are plotted from data obtained by the writer; the data for the 60-cycle curve and the resistivity curve were kindly furnished by the Hercules Powder Co. These curves show some very interesting features with regard to the orientation theory.

a. Dielectric Constant of Rosin vs. Temperature and Frequency. The course of the dielectric constant curves is in good agreement with the theory. At 80 deg. fahr. ϵ is the same for all three frequencies. This value $\epsilon = 2.68$ is due to the displacement electrons alone, as shown by the fact that the power factor has dropped to negligible values. The viscosity of the rosin at this point is so enormous that even the slightest dipole motion would give rise to high frictional losses, so that we may be sure that the contribution of dipole motion to the dielectric constant is practically nil. Thus, when the dipoles are practically fixed—*e. g.*, down to 60 cycles, the dielectric constant is independent of frequency. On the other hand, the ϵ curves again converge on the high temperature side, because there the viscosity is very low, resulting in a characteristic frequency high compared to the three applied frequencies, and the dipole response is complete. At 180 deg. the dielectric constant for 60 cycles reaches a maximum, while it is still low for the two radio frequencies. At this point the dipoles respond fully to the 60-cycle field, but are too sluggish to follow the higher frequency fields. Above this temperature ϵ for 60 cycles drops because while the dipole response no longer increases, the density decreases. At 260 deg. and at about 310 deg. the dipole response to the 1 and 10 million cycle fields is complete.

b. The "V" Curve of Power Factor vs. Temperature. It may be noted that the 60-cycle power factor curve shows an upward bend above 210 deg. fahr., a behavior not shown by any of the radio frequency curves. Inspection of the resistivity curve gives the explanation. While at 210 deg. the dipole power factor has become very slight, the leakage is rapidly increasing. The leakage current becomes comparable with the 60-cycle charging current. At radio frequencies, on the other

hand, although the leakage current is even greater because of higher temperature, it is negligible compared with the charging current and gives no upward turn to the curves. Power factor minima resembling this part of the curve (commonly called V curves) have frequently been observed in tests on cable compounds. Various explanations have been given. Hochstädter[8] attributes the right branch to leakage increasing with temperature, and the left branch to "hysteresis." This explanation seems to be the correct one if "hysteresis" is taken to mean loss due to dipole friction. Dunsheath[9] disagrees with Hochstädter's explanation and attempts to account for the V curves on the basis of his $I^2 R$ theory. This theory is based on a model consisting of capacities and resistances. The increase with rising temperature is accounted for by assuming the capacities and resistances in parallel, so that decrease in resistivity causes rising power factor. On the low temperature side of the minimum the capacities are assumed to be in series, so that now increase in resistivity as the temperature falls again causes a rise in power factor. On this basis, the maximum could only be explained by assuming that the parallel arrangement was again restored at the proper point. This theory seems to the writer to be highly artificial, and incapable of explaining either the observed influence of viscosity in the rosin mixtures or anomalous dispersion and absorption in pure liquids containing only a single molecular species.

c. Relation of Spacing and Width of Power Factor Curves. The comparative widths of the curves of

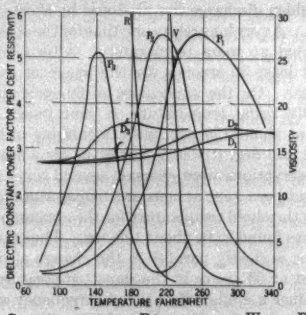

FIG. 5—CURVES SHOWING BEHAVIOR OF WOOD ROSIN AT DIFFERENT FREQUENCIES

V viscosity Stormer in minutes per 100 turns
R resistivity in ohms $\times 10^{12}$ per cu. cm.
P per cent power factor
D dielectric constant
Subscripts refer to frequency:
(1) 10 million cycles
(2) 1 million cycles
(3) 60 cycles

power factor and dielectric constant (Fig. 5) and the way they are spaced with respect to temperature are good qualitative evidence of the correctness of the orientation theory. By Equation (1), f_0 is proportional to T/η. Since the change of η is very great compared to that of T, it is simpler to neglect the direct effect on f_0 of change of T and consider only the effect of change of η. Then on the orientation theory the

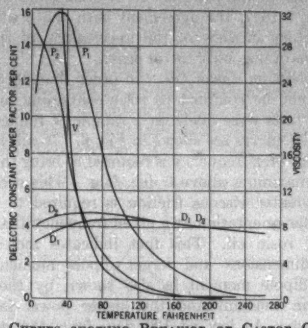

FIG. 6—CURVES SHOWING BEHAVIOR OF CASTOR OIL AT 2 FREQUENCIES

V viscosity Stormer in minutes per 100 turns
P per cent power factor
D dielectric constant
Subscripts refer to frequency:
1 10.9 million cycles
2 1.415 million cycles

anomalous dispersion and absorption curves for equal multiples of frequency would be of equal width and spaced at equal temperature intervals, provided the viscosity changed exponentially with temperature. That is, if the viscosity *vs.* temperature curve were exponential, the spacing of the power factor and dielectric constant curves for 100, 1000, 10,000, etc., cycles would be uniform. But inspection of the viscosity curve for rosin shows that the rate of increase of viscosity with falling temperature does not progress exponentially. Thus, starting at 300 deg. and taking the ratios of successive viscosities at 10 deg. intervals downward, we find 2 to 1, 2.5 to 1, 3.1 to 1, 5.5 to 1.

The result is that the 60-cycle curve, being in a region of viscosity increase which is much more rapid than that in the higher temperature radio frequency region, is narrower than the one million-cycle curve, which in turn is narrower than the ten million-cycle curve. Also, the one and ten million-cycle curves are farther apart relative to the frequency difference than the 60-cycle and one million-cycle curves. The viscosities corresponding to one and ten million-cycle power factor maxima are approximately in the ratio 10 to 1 as they should be.

To test further the spacing of the power factor peaks with respect to viscosity, tests were made at 1.415 $\times 10^6$ and at 10^7 cycles on two other samples, one of commercial rosin oil (50 per cent rosin), the other 50 per cent by weight wood rosin F in Transil oil. Since the curves are of the typical shape, they are omitted.

The peaks for the rosin oil occur at 120 deg. and 152 deg. fahr. The viscosities at these temperatures are in the ratio 6 to 1. The peaks for the diluted wood rosin are at 96 deg. and 136 deg. and the corresponding viscosities in the ratio 6.5 to 1. Since the frequency ratio is 1 to 7, the agreement with the prediction is satisfactory in view of the uncertainty of the exact positions of the power factor peaks.

d. Tests on Castor Oil. In order that all the tests might not be confined to rosin-containing materials, similar tests were made on a sample of refined castor oil. The curves are shown in Fig. 6. It will be noted that very high frequency is required to bring the power factor maximum above 0 deg. fahr. This shows that much greater viscous friction is required to prevent molecular orientation at a given frequency than in the case of rosin oil. This fact indicates molecules of small dimensions and larger dipole moment. The larger dipole moment is also shown by the higher dielectric constant and the higher power factor at maximum. As low as 130 deg. fahr. the dielectric constant is seen to be independent of frequency up to 10.9 million cycles. The tests were not carried to low enough temperature to make the dielectric constant curves for the two frequencies come together as the rosin curves did, but it is evident that they would. Castor oil thus shows behavior similar to rosin and rosin oil and the results with rosin are not due to any peculiarity other than its dipole nature and high viscosity.

e. Numerical Check of Equation (1). It is of interest to substitute in the equation $f_0 = k\,T/8\,\pi^2\,\eta\,r^3$ the values for rosin and castor oil to see if a reasonable figure for the molecular radius results in each case. Since the power factor maximum occurs practically at $f = f_0$, and the temperature and viscosity are known, Equation (1) can be solved for r. For rosin when $f = 10^7$, the peak is at $T = 396$ deg. absolute. At this temperature $\eta = 0.63$. Solving for r gives for rosin $r = 4.8 \times 10^{-8}$ cm.

Similar calculation gives for castor oil the value $r = 2 \times 10^{-8}$ cm. The fact that these results are comparable with molecular sizes obtained by entirely different methods (X-rays, spreading of oil films, etc.) gives a striking proof of the fundamental correctness of the orientation theory.

3. Optical Evidence of Orientation. Some results of optical tests on rosin are included because of the additional support they give to the orientation theory. The work was done with Professor Hans Müller of the Massachusetts Institute of Technology and is more fully treated in a joint paper.[11]

The phenomenon of electrical double refraction, called the Kerr[12] effect, is well known. Many materials become doubly refracting in a strong electrostatic field. The behavior in liquids is observed by placing the material in a cell between Nicol's prisms set at 45 deg. to the direction of the electric field and at right angles to each other and passing through a strong beam of light. In the unstressed condition the liquid has no influence on the light and the crossed nicols extinguish the beam. If under stress the liquid becomes doubly refracting, the plane polarized light entering the liquid through the first nicol becomes elliptically polarized and some light gets through the second nicol. An image of a slit placed between the first nicol and the source of light may then be observed with a suitable eyepiece.

Since in the unstressed liquid the dipole molecules are arranged in a perfectly random manner, the refractive index is the same in all directions. But in an electric field the dipoles tend to orient, so that the average number of molecules having their dipole axes in line with the field is greater than at right angles. The refractive indices parallel and at right angles to the field are different, giving double refraction. Non-dipole liquids show some Kerr effect but polar liquids show it much more strongly. Since this effect depends on dipole orientation, the time required for the effect to disappear after sudden discharge should equal the relaxation time already defined for electrical polarization. Then, by reasoning similar to that which applies to the change of dielectric constant, the effect should fail to appear in an alternating field of frequency too high for the dipoles to follow. This prediction was confirmed by the behavior of rosin. At room temperature, solid rosin showed a good Kerr effect with a static field of 10,000 volts, but none whatever with a 60-cycle field even though the stress was increased to 75,000 volts. The sudden discharge was produced by short circuiting the cell. A very long relaxation time for the optical effect was noted at room temperature. After discharge, it took at least 30 seconds for the image of the slit to disappear. On the application of 60-cycle stress and gradually rising temperature, a faint image was observed at about 120 deg. fahr., increasing in intensity until at 170 deg. it was as strong as for a static field. The observations were checked several times after the rosin had been kept at constant temperature for 24 hr. and the threshold temperature for the appearance of the effect was found to be about 100 deg. Thus the optical behavior is closely parallel to the change of dielectric constant at 60 cycles in the same temperature region. To produce a similar effect at radio frequency was more difficult. A high voltage field of 1.5 million cycles was produced by a Tesla coil driven by an oscillator using a UX-852 75-watt vacuum tube. The power loss in the liquid rosin caused peculiar convection effects which blurred the image of the slit. However, the disturbance due to convection required about 1 second to appear after the field was applied, so that during the first few instants an image could be observed. The difficulties of observation precluded an accurate determination of the threshold temperature. The Kerr effect could be observed unmistakeably above 240 deg. which is in approximately the correct region. Thus, the optical

behavior of rosin at different applied frequencies gives strong additional support to the orientation theory advanced to explain anomalous power factor and dielectric constant changes.

4. *Electrical Behavior of Rubber-Sulfur Compounds.* Electrical tests at various frequencies on samples of vulcanized rubber containing different amounts of combined sulphur (Figs. 7 and 8) show very interesting features which may be related to the dipole theory. Curtis, McPherson, and Scott[10] showed the interesting relations of power factor and dielectric constant at 1000 cycles to composition of vulcanized rubber. They found a power factor maximum of 8 per cent at 13 per cent sulphur, and a dielectric constant maximum of 3.75 per cent at 10.5 per cent sulphur. Both properties showed a minimum at 19 per cent. These workers pointed out the fact that the maxima and minima occurred at compositions corresponding to simple formulas and claimed that "compound formation is the fundamental basis for the interpretation of the observed changes." The power factor maximum would correspond to $(C_5H_8)_3S$, and the dielectric constant maximum to $(C_5H_8)_4S$. No explanation is given of the fact that the two maxima occur at different compositions.

Figs. 7 and 8 show that the positions of the maxima can be shifted over a wide range of composition by changing the test frequency and that, therefore, the fact that at 1000 cycles the maxima did coincide with compositions corresponding to simple chemical compounds is without significance.

The suggestion that certain rubber-sulphur chemical combinations are probably dipole molecules is due to Mr. C. R. Boggs. He pointed out the fact that while the untreated rubber molecule is electrically neutral, as is shown by electrical tests, the chemical addition of sulphur in such a manner as to produce an unsymmetrical molecule would produce a dipole. In other words, if more S groups became attached to the rubber molecule on one side of the original center of charge than on the other, a dipole molecule would result. If further S groups were added so that the arrangement became symmetrical, the molecule would again lose its dipole properties. Such a compensating effect is well known in other substances. Methane CH_4 is electrically neutral. If a strong dipole-producing group, like the nitro group, is added to form nitro-methane CH_3NO_2 by replacing one H, the molecule shows a high dipole moment, as indicated by the dielectric constant 56. The addition of more NO_2 groups, however, progressively reduces the dipole moment until with the symmetrical tetra-nitro-methane $C(NO_2)_4$ the dielectric constant falls to the neutral value of 2.1. Many other examples might be given, but the above shows well the progressive compensation.

In the low sulphur range the chances are greater that the compounds formed will be dipoles. As the percentage of sulphur is increased, the probability of compensation increases. Thus, the composition indicated by the per cent of combined sulphur at maximum power factor or dielectric constant would certainly not correspond to the formula for the molecule of maximum dipole moment. Starting with 0 per cent S and increasing the amount of combined sulphur, we should therefore expect to find at first a rise of power factor and dielectric constant to maximum as the number of dipoles increased, followed by a drop in both properties as the further addition of sulphur atoms eliminated the dipoles by compensation. The mechanism of the dipole power loss would here also be one of heat loss due to the mo-

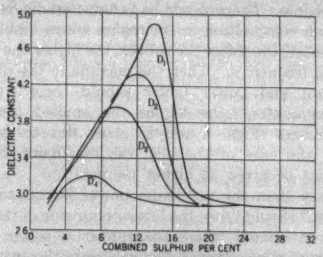

FIG. 7—CURVES SHOWING DIELECTRIC CONSTANT *vs.* PER CENT COMBINED SULPHUR AT 4 FREQUENCIES

Subscripts refer to frequency:
1 0 cycles, *i. e.*, static stress
2 60 cycles
3 1000 cycles
4 380,000 cycles

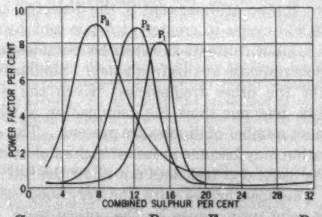

FIG. 8—CURVES SHOWING POWER FACTOR *vs.* PER CENT COMBINED SULPHUR AT 3 FREQUENCIES

Subscripts refer to frequency:
1 60 cycles
2 1000 cycles
3 380,000 cycles

tion. In this case, however, the molecules would probably be held by elastic forces, so that in responding to the changing field they would subject the surrounding medium to rapid stretch and release. Since rubber is an imperfectly elastic material, this internal motion would give rise to frictional losses measured electrically as power loss. The above discussion considers only the rise and decay of dipole effect due to change in number of dipoles. Another important factor is the increasing hardness of the medium. As the sulphur content

increases, the rubber becomes more and more rigid, so that even if the number of dipoles were constant, the amount of motion in a given field would change greatly with changing composition due to the change in opposing forces. Thus, the phenomenon in vulcanized rubber is more complicated than in the liquid materials previously studied, in that simultaneously the number of dipoles and the stiffness of the medium is changing.

However, it is possible to predict qualitatively some electrical features that should be observed if the application of the orientation theory to rubber is permissible. There should be a characteristic frequency for each composition in the region where dipole molecules exist; the softer the rubber, the higher the characteristic frequency. Then the maxima in the dielectric constant and power factor should occur at lower sulphur contents the higher the applied frequency. The curves (Figs. 7 and 8) show this to be true. As in the case of the oils, the maximum dielectric constant occurred at higher temperature—*i. e.*, lower viscosity than the maximum power factor, so in the rubber it should lie on the low sulphur side of the power factor maximum where the rubber is softer. Comparison of the corresponding curves in the two figures shows that this is so. The progressive drop in the values of the dielectric constant maxima for increasing frequency is readily explained. From 0 per cent S to 14 per cent, at least, the number of dipoles is increasing nearly linearly as the d-c. dielectric constant curve shows. But, although there are more dipoles at 10 per cent S than at 6 per cent S, the dielectric constant for 380,000 cycles is greater at 6 per cent because at higher sulphur contents the stiffness becomes too great for dipole response at this frequency. Similar relations hold for the other frequencies. Even the d-c. test probably does not show the composition at which the maximum number of dipoles are present. The speed of orientation may become so low at high sulphur contents that the usual d-c. test does not allow the full orientation. This idea agrees with the very long relaxation time in solid rosin shown by the optical tests.

A thorough study of the electrical properties of vulcanized rubber might throw considerable light on the nature of the vulcanization reactions. The application of the orientation hypothesis to rubber is only tentative and considerable critical experimental work would be needed to place it on as firm a basis as in the case of viscous liquids.

Conclusions

1. The peculiar temperature variation of dielectric constant and power factor at different frequencies of rosin, rosin oil, and castor oil, and the anomalous change in electric double refraction of rosin have been shown to be functions of the viscosity.

2. The influence of viscosity on both the electrical and optical properties has been explained on the Debye theory of dipole orientation.

3. The electrical behavior of vulcanized rubber samples of various compositions suggests the presence of dipole molecules.

4. Very accurate measurements on viscous, dipole materials of high purity would be of great value in checking and extending the theory.

The results given in this paper are believed to be of value both to electrical engineers and to physicists. It is hoped that they will help to interest practical students of dielectrics in a physical theory which hitherto has been tested mostly only at frequencies and on materials remote from practical purposes.

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Discussion

F. M. Clark: Mr. Kitchin is to be congratulated for presenting to a body of engineers DeBye's di-polar theory. With the proper understanding of the tenets of this theory, we have a means of explaining a large number of the difficulties which we now meet in our study of insulation.

The discussion of DeBye's theory has been voluminous in the past few years. The important contribution which Mr. Kitchin has made is the presentation of an explanation of power factor in the same terms as physicists are now explaining, or attempting to explain, the dielectric constant. Di-polar materials have been considered to possess an inherently high power factor. Mr. Kitchin shows that this is not necessarily so under all conditions of measurement. Given the right condition of test the power factor may be relatively small.

It must be borne in mind that di-polar materials possess fundamentally less resistance to direct conduction and a greater tendency to electrolytic type conduction. In going from a non-polar to a di-polar material, therefore, we are passing from a substance of high resistivity and a conduction which is chiefly electronic to a material of relatively low resistivity and a conduction which may be partly or entirely of the electrolytic type. The non-polar materials are of low power factor. The di-polar materials, according to Mr. Kitchin, may possess either high or low power factor depending upon the conditions of test.

Mr. Kitchin illustrates the effect of solidification by reference to test data on ether. Ether solidifies at −113 deg. cent. From

this fact it is rather difficult to understand the conclusions drawn by the author. However, reference to the original work of Isnardi shows that there is a slight error in the curve cited by Mr. Kitchin. The maximum reached by the original author was obtained at approximately −110 deg. cent and a marked drop in dielectric constant occurred between −110 and −115 deg. cent. With this correction in Mr. Kitchin's curve, the conclusions which he draws appear justified.

Reference is made to Bock's work with glycerine. Glycerine solidifies at a low temperature but when once solid melts on warming up at about +15 deg. cent. With dielectric-constant measurements under decreasing temperature, therefore, we are testing a viscous material. With dielectric-constant measurements with rising temperature after the glycerine has once solidified we are dealing with a solid. If the work of Bock were done on glycerine as the temperature was slowly lowered, the data obtained apply directly to the phenomenon with which Mr. Kitchin is dealing, orientation in a viscous liquid. If the work of Bock were done on a rising temperature, the phenomenon is that of orientation in solids under a slowly alternating field. Perhaps Mr. Kitchin can tell us the conditions under which Bock's work was carried out.

G. F. Kennedy: I might explain that there is a difference in dielectric characteristics between Hercules "FF" wood rosin and Hercules "FF" wood rosin No. 11. The rosin on which these electrical measurements were made is Hercules "FF" wood rosin No. 11, which is superior in dielectric characteristics to the Hercules "FF" wood rosin because the water-soluble material which is present in all rosins, both gum and wood, has been removed.

Rosins contain, for the most part, unoxidized and oxidized abietic acid or its isomers, the unoxidized acid having the formula $C_{20}H_{30}O_2$ and the oxidized acid the formula $C_{17}H_{26}O_5$.

Oxidized abietic acid is insoluble in 70 deg. Baumé petroleum ether at 25 deg. cent., so that the percentage of this material in rosins can readily be determined. Rosins contain, on an average, about 7 per cent of oxidized abietic acid and increase or decrease in viscosity at a given temperature in proportion to the amount of the oxidized acid present. Commercial abietic acid, which contains only ¼ per cent of oxidized abietic acid is of comparatively low viscosity.

There is a marked change in power factor as we pass from Hercules "FF" wood rosin No. 11 to commercial abietic acid. The peak is at 122 deg. fahr. and at that temperature, at 60 cycles frequency, is 4.6 per cent, diminishing to 0.2 per cent at 176 deg. fahr. This is due to the removal of the oxidized abietic acid.

I agree with Mr. Kitchin and Mr. Davis that rosins, both gum and wood, will progressively give off water on heating, for oxidized abietic acid has an oxyhydryl group which dehydrates completely by simple heating to 130 deg. cent., (266 deg. fahr). Water cannot be obtained from unoxidized abietic acid.

In regard to rosin oil, if you heat rosin or abietic acid to a relatively high temperature, you will change the characteristics of the abietic acid by breaking down the carboxyl group and you will form what is termed pyrogenated abietic acid. Rosin oil consists of this pyrogenated abietic acid entrained in the distillation dissolved in the liquid constituents of the oil which are many and of a complex nature. This pyrogenated abietic acid will in time crystallize from the oil. It will not oxidize, however, but for some reason that we have not yet determined, has poor dielectric characteristics.

A. Nyman: The dipole theory appears to be a very beautiful theory and apparently the tests of Mr. Kitchin are substantiated very fully. I am not quite in agreement with some of the conclusions of Mr. Kitchin. As I understand it, the reduction of the power factor at the higher temperature is explained by the reduction of viscosity and therefore a freer movement of the dipoles. However, the reduction of the power factor at the lower temperatures apparently is not explained except for Mr. Kitchin's suggestion that it may be due to the reduced conductivity; in other words, to a higher resistance and therefore a reduction of losses due to that.

It is my understanding that if you try to account for dielectric losses by conductivity alone, the results will be very inconsistent. In other words, the conductivity could account for only a very small proportion of absorption losses. On the other hand, the suggestion was made by Mr. Kitchin that it may be due to the fact that the dielectric constant is reduced; in other words, the picture I get is that when the substance cools down, the viscosity is increased enormously, but because the dipole can't orient itself as freely as before, the dielectric constant has been reduced.

Referring, for instance, to Fig. 5 of Mr. Kitchin's paper, I notice that the dielectric constant from the maximum position of about 3.4 has been reduced to about 2.7; that is about 20 per cent. At the same time, if you compare the maximum of the power factor, which is over 5 in all the three curves, it has been reduced to a figure which is probably around 0.3, that is a reduction of about 95 per cent. It seems to me there is a possible explanation of this which is not mentioned in the paper. On observing the figures, in particular Fig. 4, it seems to me that the maximum of power factor is in each case at a portion of the viscosity curve where there is the maximum change; in other words, the viscosity passes from a high value of the solid to a fairly low value of the liquid.

I have a mental picture of the change in the composition of the material with regard to dipoles during this passage from high viscosity to low viscosity. I have a picture that some of these dipoles change from sort of a solid state to a liquid state; other dipoles still remain solid, and you get a kind of a mixture of two kinds of dipoles, the solid dipoles and the liquid dipoles. As the temperature rises, more and more of these dipoles become liquid and therefore the whole substance approaches to a condition of a liquid, and the viscous substance that I have a mental picture of is a mixture of the two kinds of dipoles. If that picture is correct—and I am not at all sure that my physics of the thing is correct—then it seems to me that there is a possibility of applying again the Maxwell Law to this mixture; in other words, as long as you have a mixture of two kinds of dipoles, you have really two kinds of dielectrics, and the Maxwell theory was based on this unhomogeneous material which contained two types of dielectrics. So, although the statement of the author discards Maxwell's theory, it seems to me that it is possible to bring it back again on this supposition that there are two kinds of dipoles in a viscous substance.

J. B. Whitehead: Mr. Kitchin's work is the first attempt that I have seen to apply Debye's theory of dielectric constant to the phenomenon of energy losses in dielectrics. For some time past I have been not only making some study of Debye's theory but I have talked with a number of physicists, with the idea of developing whether or not in its present state it offers promise of explaining the losses that we find in dielectrics. I may say that my interest has been particularly directed to losses at relatively low frequencies, such as we have in commercial circuits. I have not been able to develop from the physicists any such suggestion; in fact, their opinion has been that a correlation could only be found at the very high frequencies which Mr. Kitchin has employed.

The suggestion has been that the amounts of energy that are involved in the oscillation or rotation of the dipole molecules at ordinary frequencies are quite small, and of quite different order of magnitude from those that we meet in the dielectrics of insulation. So I am particularly glad to see this work of Mr. Kitchin's, but I should like to point out that it is confined to this high range of frequencies, where it is to be expected that some such relationship would be found. The correlation that he has brought out, however, between power factor and viscosity, seems to be exceedingly important because it ties in a fundamental

physical theory with the ordinary physical constants of materials that we work with.

C. F. Hill: (communicated after adjournment) The paper by Mr. Kitchin is a very interesting one in that it is a first attempt by the practical scientist towards the use of Debye's theory of dipoles in the solution of our dielectric problems. Since the dipole theory has become pretty well established, it has been evident that it might form a rational explanation of many or most of our dielectric phenomena. The time lag of orientation of the polar molecule, for example, is similar to dielectric absorption as is also the time lag of the associated Kerr effect. Viscosity takes care of losses which we associate with absorption so that even for solids we need not disturb our ideas in accepting such an explanation. While it might be questioned if there are dipoles in solids, we do find the Kerr effect in glass for example with a very large time lag which suggests that it might be worth while to attempt to apply the theory there. The very long chains forming molecules in most of our solid dielectrics could even be imagined to have a temporary electric moment under voltage.

Mr. Kitchin's paper especially points out the effects of viscosity. We are reminded that a viscous displacement theory by Pellat forms one of our dielectric theories and gave us the equation for the absorption curve, $i = A\,t^{-n}$ which so far most accurately fits experimental results. The fact that many liquids which possess dipoles and show very large variation of the dielectric constant, apparently lose most of the dipole characteristics when they pass to the solid form may point to the fact that we are overlooking the dipole nature of molecules in many of our solid dielectrics. It seems probable that a study of the Kerr effect might throw some light on this as even in glass where it has been detected it may be masked by other effects.

An experience which was not understood at the time but which may have a bearing on the subject is worth repeating. Three or four years ago some condensers filled with transformer oil were used in a million-cycle broadcasting circuit. The oil apparently deteriorated very rapidly and became so hot it could not be used. A saturated oil such as Nujol was substituted and ran perfectly cool. Later both oils ran perfectly cool under ten million cycles. Mr. Kitchin's results show a resonance frequency for which power factor reaches a maximum. The voltage for broadcasting purposes was relatively high of course and might possibly magnify the effect but the results conform to Mr. Kitchin's results. This also raises the question if a decided increase in voltage used in power factor measurement might not be used to advantage in such a study. The difference between the saturated and unsaturated oil molecules (degree of symmetry) may have some influence on the tendency towards dipole characteristics.

Turning to Debye's theory, it would seem most logical to assume a rather stable dipole molecule considerably below the ionization voltage. Debye has assumed his dipoles to have a definite moment. The fact, if true, that substances with strong dipole characteristics have relatively high conductivity, would not necessarily come from Debye's theory, in my opinion. Debye's theory would also impose another characteristic on dielectric absorption if the two are the same, namely, that the space charge should be uniformly distributed. Some results by Joffe, for example, show a non-uniform distribution.

Some experimental work which has not been explained as yet is that of Eguchi on Elektrets. Since he used Canawba wax and resin it seems logical that his permanent polarization effects must be closely associated with the phenomena just reported by Mr. Kitchin. The conclusion that one should draw from Mr. Kitchin's paper, it seems to me, is that Debye's dipole theory may have considerable to do with our anomalous dielectric properties and can have without upsetting our ideas to any extent.

D. W. Kitchin: In order to avoid misconceptions regarding the scope of the orientation theory, the fact should be emphasized more strongly than was done in the paper, that it does not apply to every type of power loss. Its value lies in the explanation it gives of the anomalous changes of dielectric constant, the pronounced power-factor maxima in the corresponding temperature regions, and the influence of viscosity on both the electrical and optical properties.

An example may serve to make the above point clear. Mr. Clark, if I understand his discussion correctly, calls attention to the fact that although dipole materials have inherently high power factor, the magnitude depends on the conditions of measurement. In rosin we may have at least two types of power factor, which I have called "dipole power factor" and "leakage power factor." Curve P_3, Fig. 5, gives, I believe, a clear separation of the two effects. The 60-cycle power factor at 210 deg. fahr. is the sum of the dipole power factor and the leakage power factor. The leakage power factor becomes negligible below 210 deg. fahr., while above 210 deg. the 60-cycle dipole power factor is negligible. The orientation theory obviously does not apply to the rising 60-cycle power factor above 210 deg. But at 1,000,000 cycles, the dipole power factor is a maximum at 210 deg., and at 10,000,000 it is at about 250 deg. At some temperature, say at 270 deg., P_3 would cross P_2 and the power factor be equal for both 60 and 1,000,000 cycles and yet be due to entirely different mechanisms in the two cases.

I believe Mr. Clark is correct in his point that dipole materials in general have higher conductivity than non-dipole substances. Examples are water, nitrobenzene, and the alcohols, which have rather high conductivity. A dipole molecule may be considered as an intermediate stage between a neutral molecule and a dissociated molecule, and in a dipole liquid it is probable that some dissociation occurs. In such a case a high power factor at low frequency would be almost entirely due to leakage in spite of the presence of dipoles. But by increasing the testing frequency over a very wide range, we should reach a region where the dipole power factor was predominant. The curve power factor *vs.* frequency should therefore give a *V* curve, in some cases a very wide, shallow one.

Mr. Clark's remarks regarding Fig. 1 and the results of Bock with glycerine bring up two interesting points. The dielectric constant of ether instead of rising continuously as the temperature is lowered to the freezing point (−116 deg. cent.) reaches a sharp maximum at −108, or 8 deg. above the freezing point. Similar behavior is shown by other liquids in Isnardi's paper. One explanation would be that near the freezing point, the mutual influence of the molecular fields becomes so strong that response to the external field is negligible.

Bock measured the change of dielectric constant with temperature at 1.36 meters. The maximum was 36 at about 50 deg. cent., dropping to about 4 at 0 deg. He quotes the results of Graffunder who made similar measurements at 400, 670, and 1000 meters. At these lower frequencies the maxima reached about 56. The course of the dielectric constant curves is quite similar to those of rosin and castor oil, except that the effect is greatly accentuated by the much larger change of dielectric constant. Undoubtedly, both workers were dealing with glycerine in the supercooled state so that the behavior resembled that of rosin and castor oil.

It should be pointed out that rosin, glass, fused quartz, and similar materials, although popularly termed "solids," are in reality supercooled liquids of extremely high viscosity and hence media in which dipole molecules can turn if given sufficient time. Therefore, the dipole effect may occur at the proper frequencies, in some cases, conceivably 1 cycle per hour, day, or month, although it might be masked by leakage and other effects. As shown in Section *V*-3, the relaxation time of the dipoles in rosin at rosin temperature is of the order of 1 minute and there is no reason to suppose it may not be much longer in still more viscous media.

The permanent "elektrets" of Eguchi in which polarization exists for years may be an example. An alternative explanation of these elektrets would be that the rosin-carnauba mix has sufficient plasticity, or minimum shearing stress, to resist permanently the tendency of the molecules to return to a random distribution. Adams (*Franklin Inst. Jl.*, Vol. 204, p. 469, 1927) has shown that when the temperature is raised the rate of decay of polarization becomes measurable.

The suggestion of the physicists consulted by Professor Whitehead that the energy loss due to orientation at low frequency is negligible is correct, I believe, except in the case of liquids of extreme viscosity. In other words, there exists a sort of reciprocal relation, as pointed out in Section *V-2-c*, such that if you increase the viscosity 1000-fold by lowering the temperature, you get the power factor maximum at roughly $\frac{1}{1000}$ of the first frequency. I believe, therefore, that rosin and similar materials offer an exception to the point that the dipole power loss effect is confined to radio frequencies. The thing I have wanted to emphasize in my paper is the fact that the dipole effect is not confined to any particular range, but depending on the molecular size, the viscosity, and the temperature, may occur anywhere in the electrical spectrum from billions of cycles per second in the case of non-viscous liquids to 60 cycles and doubtless lower in very viscous supercooled liquids. Failure to detect a dipole power factor may therefore merely mean that the tests are not made in the range where it becomes predominant. Ordinarily, however, the materials used in cables, etc., would not be expected to exhibit the effect at ordinary frequencies and I believe Professor Whitehead is perfectly justified in not applying the dipole theory in his work.

Mr. Nyman's hypothesis of a mixture of solid and liquid dipoles to which Maxwell's theory would still apply is ingenious, but not in accordance with the facts. He speaks of a change from liquid to solid state in rosin in the temperature range where the slope of the viscosity curve changes rapidly. But no solidification takes place in this region. Moreover, by changing the testing frequency the power-factor peak may be shifted to correspond to quite different portions of the viscosity curve. Fig. 5 gives a clear example of this point.

Professor Karapetoff has asked about the connection between this paper and that by Kitchin and Müller in the December *Physical Review*. The two papers deal with practically the same data but from different points of view. Anomalous dispersion in dipole liquids has been familiar to physicists since the classic experiments of Drude. But in practically every case it was confined to the extremely high frequency region of the electrical spectrum. The fact that the phenomenon could by choice of suitably viscous materials having large molecules be found at frequencies as low as 60 cycles should be of great interest.

On the other hand, as Professor Whitehead has pointed out, it has been difficult to see how the dipole theory could be applied to practical insulating materials. The present paper was given in the hope of showing the conditions under which the dipole theory could be so applied.

A Graphical Theory of Traveling Electric Waves
Between Parallel Conductors

BY VLADIMIR KARAPETOFF[1]
Fellow, A. I. E. E.

Synopsis.—The analytical theory of simple traveling waves along parallel conductors is well known. Its disadvantage is that the relationships among the incoming, reflected, transmitted, and absorbed currents and voltages are expressed by a number of simultaneous equations difficult to grasp, from a physical point of view, in their entirety. For this reason, a graphical theory has been developed according to which all the quantities involved are represented in a so-called star diagram and the whole phenomenon conveyed to the eye quantitatively.

The star diagram is then applied to the following practical cases of a simple rectangular-front non-attenuated long traveling wave:

(a) Reflection from and absorption in a non-inductive terminal resistance;

(b) Reflection from an open-circuited and from a short-circuited end of a line;

(c) The case of critical resistance;

(d) Repeated reflections from the ends of a line;

(e) Discharge of a wave to the ground, directly or through a resistance;

(f) Passage of a wave through the junction point of two conductors having different values of surge impedance;

(g) Same as (f), only the junction provided with a series or shunted resistance,

(h) Effect of a lumped series inductance or shunted capacitance in reducing the steepness of a wave front.

In each case, the results agree with those deduced analytically, for example by R. Rüdenberg in his book "Elektrische Schaltvorgänge." Some reciprocal relationships are pointed out at the end and further problems suggested to which the star diagram may be applied.

Introduction

THE purpose of this paper is to explain graphically the fundamental properties of electric traveling waves between parallel conductors (such as transmission lines), especially where a wave strikes an obstacle, for example, a resistance, an inductance, an open end, a junction point between a cable and an overhead line, etc. While the general analytical theory of such waves is well known (see the references below), a graphical representation of the quantities involved is believed to be new, and it may give a clearer idea of the important relationships useful in the solution of some practical problems. The so-called *star diagram*, developed for this purpose, is the foundation of the graphical treatment given below.

1. General Properties of Electric Waves Traveling Along Parallel Conductors

Whenever some electric or magnetic conditions in a circuit are changed suddenly, a traveling electromagnetic wave starts from the point of disturbance and is propagated into the various conductors which constitute the circuit or the network. The wave is gradually attenuated because of the resistances of the circuit, is reflected at local obstacles, experiences changes in the values of its current, voltage, and shape upon entering conductors of different characteristics, etc. Ultimately, its energy is completely converted into heat.

Traveling hydraulic waves of similar character occur, for example, in a city water supply system. Should a pipe burst and the pressure be suddenly lowered locally, a wave of depression would start from the fault and spread over the whole system. Should a valve be suddenly closed in one of the branch pipes, the water hammer so created would also spread in the form of a pressure wave throughout the system, and after numerous reflections its power converted into vibrations and heat.

In this paper, only long rectangular-front waves (Fig. 1) are considered, first, because they are simpler to be treated theoretically, secondly, because other kinds

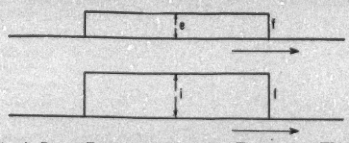

Fig. 1—A Long Rectangular-front Traveling Wave of Current and Voltage

of waves may be composed out of such simple waves, and thirdly because they are at least as harmful for electrical apparatus (if not more so) as any other waves known. In Fig. 1 the voltage wave, e, and the current wave, i, are shown separately, although in reality e and i are but two characteristics of the same traveling wave of electric energy. The direction of propagation is indicated by the arrowheads and the wave front is denoted by f.

The actual physical conditions corresponding to the symbolic representation in Fig. 1 are shown in Fig. 2. B is a battery, s is a switch, and a and b two parallel conductors. When the switch is closed, a wave starts to the right, charging the conductors. This means that an electrostatic field is established, shown by the vertical lines, and a magnetic field, indicated by the

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dots and the crosses. These two fields, together with a cross-section of the conductors, are also shown to the right.

Theory and experience show that such a wave, between two parallel conductors of comparatively small cross-section, is propagated nearly at the velocity of light (or of other electromagnetic disturbances) in the dielectric medium in which the conductors are immersed. Thus, for overhead lines the wave velocity is that of light in air, for a cable it is that of electromagnetic waves in impregnated paper, etc. This does

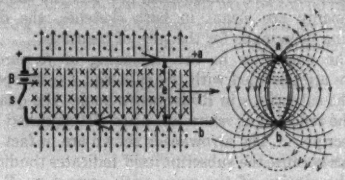

Fig. 2.—The Magnetic and Electrostatic Fluxes Accompanying a Traveling Wave Between Parallel Conductors

not mean that the actual carriers of electricity in the conductors (ions) move at such enormous velocities, nor is it necessary to assume this velocity of motion for the electrostatic and magnetic lines of force. All we know is that the electromagnetic state advances at this velocity. This difference between the velocity of propagation of a wave (or state) and that of the material particles in it, is well illustrated for water waves in W. S. Franklin's "Electric Waves," pp. 15 to 25. If the depth of a canal (Fig. 3) is D, the velocity of propagation of the wave front is

$$v = \sqrt{g D} \tag{1}$$

where g is the acceleration due to gravity. On the other hand, the average velocity, v, of the particles of water is much lower, being represented by the expression

$$v = V\,(h/D) \tag{2}$$

In an electric conductor, the velocity of motion of electrons which constitute the current is quite low, but as soon as the electrons in a cross-section begin to move, they disturb the equilibrium and cause the electrons in the next cross-section also to move. So the velocity of the wave itself simply characterizes the rate at which consecutive carriers of electricity are set in motion. Similarly, it is not necessary to assume the lines of force to be traveling at the velocity of light. New lines of force are built up as soon as a voltage and a current reach a point on the line, while the older lines of force remain at places where they have been created.

This difference between the two velocities may be also illustrated as follows: Think of a front of soldiers with their heads turned to the right, and let each be instructed to begin turning his head slowly to the left as soon as he sees the preceding man beginning the motion of his head. It will take but a fraction of a second for each man to begin his motion after his neighbor, so that the wave front of motion will be propagated along the row quite *rapidly*, whereas the actual motion of the heads may be quite *slow*.

There is nothing contradictory in the fact that the current in Fig. 2 seems to flow in an open circuit. Positive electricity may be thought of as flowing through the upper conductor to the right, creating a displacement current in the dielectric downward and thereby forcing a flow of positive electricity in the lower conductor to the left. Or else, the same process may be thought of in terms of negative electrons, with the polarity and direction of motion reversed. Similarly, there is no contradiction in the existence of a considerable voltage difference, e, between two adjacent points of the same conductor, just before and behind the plane f. New magnetic lines of force arising in that plane induce an e. m. f. which balances the voltage e. The relationship shown in Fig. 2 could not exist if f were a stationary plane, but as the front of the wave moves at the proper velocity, the transient conditions at new points of the conductors make stationary values of e and i possible behind.

We shall limit our discussion to conductors whose ohmic resistance and leakage conductance are negligible. In other words, the attenuation of a wave will be disregarded. Not only is the theory much simplified thereby, but in a study of destructive action of traveling waves it is not safe to count on an attenuation because in a most unfavorable case a wave may originate in the immediate vicinity of a device upon which it impinges, so that there is practically no attenuation.

With this assumption, and denoting the inductance of

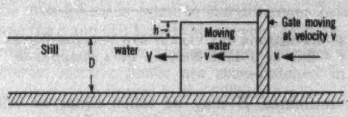

Fig. 3—A Water Wave in a Canal as an Analog to an Electric Traveling Wave

a line or cable by L and its capacitance by C, both per unit length, the following well known relationships hold true.[2]

(a) The velocity of propagation of a traveling wave is

$$V = 1/\sqrt{L C} \tag{3}$$

With proper units, for overhead conductors, V comes out equal to the velocity of light in air. In a power cable, the velocity of propagation is much lower, of the order of magnitude of one-half of that along overhead conductors.

(b) When a wave is propagated along an ideal

2. For a proof, see the references at the end of the paper.

conductor with uniformly distributed L and C, its shape remains undistorted and identical with that at the entrance into this particular conductor. A sudden change in line constants, or a concentrated (lumped) impedance, causes changes in the values of e and i, and in the shape of the wave front.

(c) For a given conductor, the ratio of a wave voltage to its current is constant; that is

$$e = Z i \tag{4}$$

where

$$Z = \sqrt{L/C} \tag{5}$$

Z is known as the *surge impedance* of the line. The reciprocal of Z,

$$Y = \sqrt{C/L} \tag{5a}$$

is called the *surge admittance* of the line. In preliminary estimates, Z may be taken equal to 500 ohm for overhead lines and to 50 ohm for power cables. Whereas L and C usually are understood to be per unit length, Z is expressed in ohms (not in ohms per unit length).

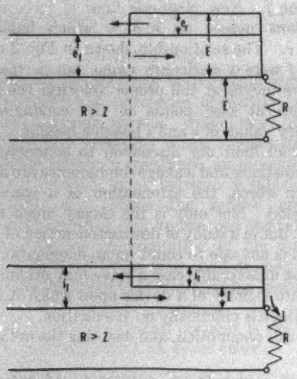

FIG. 4—REFLECTION OF CURRENT AND VOLTAGE WAVES AT THE END OF A LINE BRIDGED BY A RESISTANCE

(d) The electrostatic energy of a traveling wave, per unit length, is $0.5\,C\,e^2$; the corresponding electromagnetic energy is $0.5\,L\,i^2$. In a pure traveling wave, such as are considered here, the amounts of these two energies are equal, so that

$$C\,e^2 = L\,i^2 \tag{6}$$

Extracting a square root of both sides of this equation, gives Equation (4). Thus, the law of surge impedance follows from that of equipartition of energy, or vice versa. At a terminal, or at a joint of two conductors with different values of Z, some magnetic energy is converted into electrostatic, or vice versa.

2. THE STAR DIAGRAM OF A RESISTANCE TERMINAL LOAD

In Fig. 4, a single-phase line of surge impedance Z is shown bridged by a lumped resistance R at one end. A traveling electric wave which is propagated from left to right, is partly reflected and partly enters the resistance, to be absorbed there. Knowing the voltage and the current in the original wave, it is required to determine those in the reflected and absorbed parts of the wave. The voltage notation is shown in the upper sketch, the current notation in the lower sketch. Of course, the same transmission line and the same wave of energy are meant in both sketches, the division having been made for the sake of clearness.

The quantities in the impinging or *forward-going* wave are provided with the subscript f (corresponding to the subscript v in Rüdenberg's book); the quantities in the reflected or *return* wave are distinguished by the subscript r. The arrowheads shown are in fact superfluous because the subscript itself indicates the direction of the motion of the wave with respect to R. The quantities e_f and i_f are shown above the axes of abscissas to indicate that in this particular case they are positive. The positive sign of i_f means that the current is flowing to the right. If i_f were negative, the current (positive ions) would be flowing to the left, whereas the wave itself (the front of the electromagnetic state) would still be moving to the right. This happens in a wave of depression.

The wave e_r is shown on top of e_f to indicate that its voltage is also positive and is added to e_f. On the other hand, i_r is shown negative, that is, subtracted from i_f. The negative sign of i_r means that the current flow is to the left. In this case, this happens also to be the direction of the motion of the wave front. In other cases, however, for example if R were quite low, i_r may be positive, the actual motion of positive ions being to the right, and the motion of the wave front to the left.

In this paper, the fronts of the waves marked e_f and i_f are always assumed to move to the right, and those of e_r and i_r to the left. The actual current flow, whether in the wave i_f or i_r, is always to the right when these currents are shown positive, and vice versa.

The quantities i_r and e_r have no real existence. After the incoming wave, e_f, i_f, has reached the resistance R, the voltage changes from e_f to E and the current from i_f to I. The region in which this change has taken place spreads to the left at a velocity equal to that of the incoming wave. It is more convenient, however, to consider the actual voltage E as a result of superposition of the incoming voltage e_f and a fictitious reflected voltage, e_r, so that

$$E = e_f + e_r \tag{7}$$

Similarly, the actual current I, flowing into the resistance R, may be thought of as an algebraic resultant of the currents i_f and i_r; in other words,

$$I = i_f + i_r \tag{8}$$

In addition to these equations, we also have the following relationships, according to Equation (4):

$$e_f = Z\, i_f \tag{9}$$

$$e_r = -\, Z\, i_r \tag{10}$$

The minus sign in Equation (10) is necessary because when e_r is positive and moves to the left the actual current in the reflected wave also flows to the left, so that i_r is negative. For the resistance R itself, we have

$$E = I\, R \tag{11}$$

The foregoing five equations contain six quantities, e_f, e_r, i_f, i_r, E, and I. If one of these quantities, for example e_f, is given, the equations may be solved for the remaining five. The algebraic solution and an analysis of the results will be found in some of the references

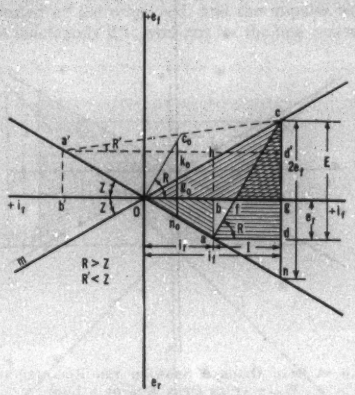

FIG. 5—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES SHOWN IN FIG. 4

given below. Here we propose to solve the problem graphically, using the *star diagram* shown in Fig. 5.

In Fig. 5, positive values of i_f are measured along the axis of abscissas, to the right of O; positive values of i_r to the left of O. Positive values of e_f are measured along the axis of ordinates upward, positive values of e_r downward. The two inclined axes, $m\,c$ and $a'\,n$, are drawn at an angle to the axis of abscissas corresponding to the surge impedance Z. This means that if, on the chosen scale of currents, $O\,g_0$ equals one ampere, then $g_0\,k_0$ and $g_0\,n_0$ are each numerically equal to Z volts on the chosen scale of voltages. Consequently, in accordance with Equation (9), any point on the line $m\,c$, such as c or h, gives values of e_f and i_f which satisfy Equation (9). In other words, the line $m\,c$ takes the place of Equation (9). For the same reason, the line $a'\,n$ takes the place of Equation (10). It will be seen that for points on this line to the right of O, e_r is positive and i_r is negative. The opposite is true for points to the left of O.

Let now $O\,g_0$ be numerically equal to one ampere, as before, and let $g_0\,c_0$ be numerically equal to R volts. Then any right-angle triangle similar to $O\,g_0\,c_0$ will give values of E and I which will satisfy Equation (11). It remains to choose the size and the position of a triangle in such a manner as to satisfy Equations (7) and (8) as well. It will be seen that the triangle $a\,c\,d$ satisfies these conditions. Its vertical side, E, is equal to the sum of the voltages e_f and e_r, and its horizontal side, I, is equal to the difference between i_f and i_r. However, in this particular case i_r is negative so that in reality I is equal to the algebraic sum of i_f and i_r.

Thus, in place of solving the foregoing five simultaneous equations, we simply mark on the star diagram the point c corresponding to the given value of e_f, draw $c\,a$ parallel to $c_0\,O$ and complete the triangle $a\,c\,d$. All the five unknown quantities can then be scaled off.

The product of the wave voltage by the corresponding current gives the power in the wave, that is, the electric energy which passes through a given point on the line, per unit time. Therefore, the area of the triangle $O\,c\,g$ is a measure for the energy flowing towards the resistance R, per unit time. Similarly, the area $O\,a\,b$ is a measure for the energy reflected and flowing into the line per unit time. The area $a\,c\,d$ represents the energy absorbed in the resistance per unit time. Accordingly we must have

$$\text{area } a\,c\,d = \text{area } O\,c\,g - \text{area } O\,a\,b \tag{12}$$

This relationship may be proved geometrically as follows: The areas $a\,h\,c$ and $a\,b\,g\,d$ are equal because the altitude $a\,d$ is the same, and the base, $a\,h$, of the triangle is twice the base, $a\,b$, of the rectangle. Subtracting the area $a\,b\,f$ from both and adding $f\,c\,g$, we find that

$$\text{area } a\,c\,d = \text{area } b\,h\,c\,g \tag{13}$$

But the area $b\,h\,c\,g$ is equal to the difference between the areas $O\,c\,g$ and $O\,h\,b$, and this proves Equation (12).

Throughout this paper, the important triangles which characterize electric power are cross-hatched in the same manner, as indicated in the following table:

TABLE I

Kind of energy	Direction of shading
Incoming wave.....	diagonally rising to the right
Reflected wave.....	diagonally rising to the left
Locally absorbed.....	horizontal
Transmitted through the junction.....	vertical

The latter shading is not used in Fig. 5, but will be found in some other star diagrams below, for example in Figs. 13 and 16.

Instead of scaling off the unknown currents, voltages, or impedances from the diagram, they may also be determined analytically from the geometry of the figure. For example, the triangles $a\,c\,n$ and $O\,c_0\,n_0$ being similar, we have

$$2\,e_f/E = (Z + R)/R \tag{14}$$

from which any of the four quantities may be computed if the remaining three are known.

In the particular case shown in Fig. 5, R is greater than Z, and, as a result, the reflected voltage is positive and the reflected current negative, as shown in Fig. 4. For another value, R', less than Z, the relationships become those indicated by the dotted triangle $a'\,c\,d'$. The reflected voltage, $a'\,b'$, is negative and the reflected current, $O\,b'$, is positive.

Critical Resistance. An important case arises when R equals Z. The triangle of reflected energy, $O\,b\,a$, collapses to point O and the triangle $a\,c\,d$ coincides with $O\,c\,g$. This means that $e_r = i_r = O$, and the whole energy of the incoming wave is absorbed in the resistance R. This fact is receiving an increasing attention, both in the design of protective apparatus and in impulse testing where reflected waves are liable to vitiate the results.

When $R > Z$, the wave is reflected without a reversal in the sign of the voltage, but with a current reversal (Fig. 4). Consequently, the total "piled up" voltage E is higher than that in the incoming wave. Such a combination of Z and R (or of Z and some other device) may be conveniently called a *step-up joint*, by analogy with a step-up transformer. Also, a resistance R, greater than Z, may be called a step-up load. On the other hand, when $R < Z$, the voltage E at the joint is lower than e_f, because e_r is negative. In such a case, I is greater than i_f because i_r is positive. Such a joint may be called a *step-down joint*. A resistance R, which is lower than Z, may be called a step-down load.

Whether a load is of step-up or step-down nature, some energy of an incoming wave is reflected and has to be dissipated elsewhere. Thus, from the point of view of energy absorption, a critical resistance, $R = Z$, deserves serious attention. It is possible that in the future lightning arresters and other over-voltage protective devices will be provided with such a series resistance (as is sometimes recommended in European literature). This resistance would absorb an incoming wave and thus prevent subsequent line oscillations due to repeated reflections, sometimes accompanied by high-voltage resonance oscillations in transformer windings. On the other hand, it must be remembered that such resistances would be large and expensive because the line current may follow the breakdown.

3. Wave Reflection at an Open End and at a Short-Circuited End of a Line

In the star diagram shown in Fig. 5, the terminal resistance R may have any value from zero to infinity. When R is infinitely great, $a\,c$ becomes a vertical line, point a moves to point n, and the triangle $a\,c\,d$ degenerates into the straight line $n\,c$. Thus, when a wave is reflected at an open end of a line, the conditions are as shown in Fig. 6. The reflected voltage is equal to the incoming voltage, so that the actual voltage at the open end rises to the value $E = 2\,e_f$. The reflected current is equal and opposite to the incoming current, to make the total current at the open end equal to zero. This is to be expected since there is no further conductor for the current to flow into. The cross-hatched triangles of incoming and reflected energy are of the same shape and area, the whole energy of the incoming wave being reflected back into the line. Thus, at an open end of a line a *total step-up reflection* takes place, the voltage of a wave being reflected without a reversal in sign, whereas the current is reversed in sign. The magnetic energy of the wave is converted into electrostatic. In Fig. 5, for any value of R between $R = Z$ and $R =$ infinity the reflection is of the same "step-up" kind, except that it is *partial* and not total,

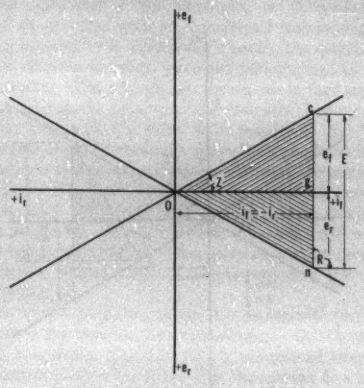

Fig. 6—A Star Diagram Showing the Reflection of a Wave at an Open End of a Line

the balance of the energy being absorbed in the resistance.

When the transmission line is short-circuited, that is, when $R = o$, the hypotenuse $a\,c$ in Fig. 5 becomes horizontal and the star diagram assumes the shape shown in Fig. 7. The energy is again completely reflected, but with a reversal in the voltage sign, so as to make $E = O$. On the other hand, the current is doubled, and the whole electrostatic energy of the wave is converted into electromagnetic. This is a case of *total step-down reflection*. A similar kind of step-down reflection takes place at any value of R between zero and Z (Fig. 5), except that the reflection is *partial*, and some energy is absorbed in the resistance.

Since the energy of a traveling wave is totally reflected whether the end of a line is open or short circuited, it is of interest to consider a few of the simplest cases of multiple reflection of waves at both ends of a line. This will give an idea of the most dangerous conditions which may be expected in practise.

(*a*) *Line Open at Both Ends.* Consider a fairly long line and a comparatively short thunder-cloud

over its middle portion. This cloud will induce a certain distributed electric charge in the nearest part of the line. When the cloud is discharged to the ground or to another cloud, the line charge is released and one-half of it starts in each direction in the form of a traveling wave. Let the voltage of each wave (or impulse) be e, the current i, and the length l. Having reached one of the ends of the line, the impulse is stopped, its current becomes zero, and its voltage rises to $2\,e$. But such a "chunk" of electricity of length $0.5\,l$ cannot remain in an equilibrium with the uncharged portion of the line, and so a return wave starts immediately. The voltage again drops to e and the current becomes $-\,i$, because at these values Equations (4) and (6) are satisfied. The same process is repeated at the other end, and the impulse would persist indefinitely if it were not for the line resistance

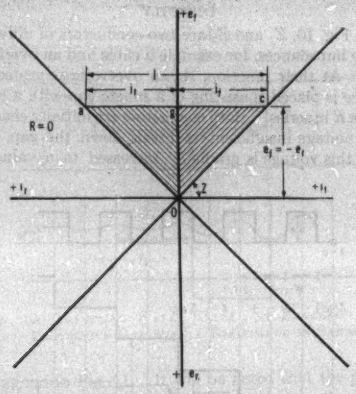

FIG. 7—A STAR DIAGRAM FOR THE REFLECTION OF A WAVE AT A SHORT-CIRCUITED END OF A LINE

and leakage, both of which exponentially reduce the impulse to zero as it travels along the line.

(*b*) *Line Grounded at Both Ends.* The phenomenon is similar to that described above, except that at each end the current is doubled and the voltage is instantly reduced to zero, to reappear in the reflected wave with a reversed sign. There is a general notion among engineers that grounding a line would automatically conduct dangerous impulses to the ground. It will be seen from the foregoing discussion that a traveling wave or impulse is simply reflected from a zero-resistance terminal and continues to surge back and forth until dissipated in the line itself. The only difference with an open end is that the current is doubled, instead of the voltage. The proper way to absorb a wave effectively is to provide a terminal resistance R. If $R = Z$, the whole impulse will be absorbed on the first trip; otherwise, it may take two or more reflections to reduce the energy to a negligible amount.

(*c*) *Line Open at One End and Short-Circuited at the Other End.* The reflections occur as described above, only the impulse approaches the open end of the line each time with its voltage polarity reversed, owing to a preceding reflection from the short-circuited end. It also approaches the latter end with its current polarity reversed, due to a reflection at the open end. Thus, after four trips back and forth, the impulse starts on its fifth trip with its current and voltage polarities the same as in the beginning.

(*d*) *Battery at One End, Line Open at the Other End.* We shall assume the battery to be of zero internal impedance and of voltage e. At the instant $t = O$ the battery switch is closed and remains closed. A wave of voltage e and current i, the latter determined by Equation (4), starts from the battery towards the opposite end of the line. Let T be the time required for the wave front to reach the open end, this interval of time being determined by the length of the line and the velocity of propagation. The subsequent history of the wave is shown in Table II.

TABLE II

Wave trip No.	Wave voltage	Wave current	Resultant voltage	Resultant current
1	$e_f\,(+)$	$i_f\,(+)$	e	$+\,i$
2	$e_r\,(+)$	$i_r\,(-)$	$2\,e$	0
3	$e_f\,(-)$	$i_f\,(-)$	e	$-\,i$
4	$e_r\,(-)$	$i_r\,(+)$	0	0
5	$e_f\,(+)$	$i_f\,(+)$	e	$+\,i$

The reflected voltage on the first return trip (trip 2) is positive, the reflected current negative. Hence, in the parts of the line over which the reflected wave has passed, the actual voltage is $2\,e$ and the current is zero, as indicated in the last two columns of the table. The battery, being of zero impedance, acts towards the returning wave as a short-circuited end of the line, the battery voltage being used up in maintaining the first outgoing wave. This explains the third line in the table, the values in the last two columns being obtained by adding $-\,e$ and $-\,i$ to the preceding values. After a second reflection from the open end, the reflected wave starts with a voltage $-\,e$ and a current $+\,i$, wiping out the previously existing resultant values $+\,e$ and $-\,i$, and clearing the line of all current and voltage. Thus, when a wave starts again on its fifth trip, the same conditions obtain as on the first trip, and were it not for the energy dissipation in the resistance of the line, the phenomenon would go on indefinitely.

Let a cathode ray oscillograph be connected at the open end of the line to measure the voltage there, and another similar oscillograph connected at the battery end to measure the line current. With an ideal line, the records would be of the shape shown in Fig. 8 by heavy lines, in accordance with Table II. The voltage at the receiver end is zero until the wave reaches it at the instant $t = T$. Then the voltage rises to e, and at

the same instant (because of the reflection) rises to $2\,e$. The oscillograph indicates this latter value until the reflected wave front reaches the battery and returns with the sign reversed, wiping out $2\,e$. For the next two trips the voltage at the receiver end is zero, then again rises to $2\,e$, etc. The current wave may be explained similarly.

Real oscillograms look more like the irregular curves indicated by lighter lines. Because of a steady absorption of the energy of the wave in the resistance of the conductors, the wave front does not remain abrupt,

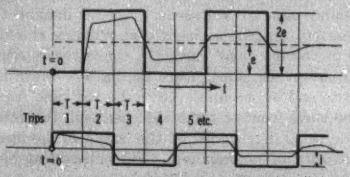

Fig. 8—General Shape of Theoretical and Actual Oscillograms of a Wave Voltage Reflected at the Open End, and of the Corresponding Current at the Battery End

and moreover there is a steady diminution in the amplitudes of both the voltage and the current. With the battery voltage equal to e, ultimately the receiver-end voltage acquires this value, and the flow of current stops.

(*e*). *Battery at One End, the Line Short-Circuited at the Other End.* This case differs from the preceding one only in the character of the reflection from the distant end, namely, the wave is reflected with a reversal in the sign of the voltage, whereas the current is doubled. This is shown symbolically in Table III.

It will be seen that in this case the resultant current increases indefinitely, as would be expected with an ideal short-circuited line. In reality the current tends to a finite limit determined by the resistance of the line.

Let an oscillograph be connected at the short-circuited end of the line. It will record zero current for an interval of time T (Fig. 9) after the closing of the switch, that is, until the wave reaches the short-circuited end. Then the current will suddenly rise to the value i in the wave, and immediately increase to $2\,i$, because of a step-down reflection. The latter value will persist until the reflected wave will have traveled back and forth once, at which instant the current will rise to $3\,i$ and immediately to $4\,i$; etc. There would be no object

TABLE III

Wave trip No.	Wave voltage	Wave current	Resultant voltage	Resultant current
1	$e_f\,(+)$	$i_f\,(+)$	e	$+\,i$
2	$e_r\,(-)$	$i_r\,(+)$	0	$+\,2\,i$
3	$e_f\,(+)$	$i_f\,(+)$	e	$+\,3\,i$
4	$e_r\,(-)$	$i_r\,(+)$	0	$+\,4\,i$
5	$e_f\,(+)$	$i_r\,(+)$	e	$+\,5\,i$

in connecting an oscillograph to measure the voltage at either end of the line. At the battery end the voltage is always equal to e and at the other end it is always equal to zero. By connecting an oscillograph across the line, at its middle point, a record would be obtained shown in Fig. 9. The voltage is zero during a time interval equal to $0.5\,T$, then it rises to the value e and remains at this value until the wave front has reached the short-circuited end and back, wiping out $+\,e$ with $-\,e$. The phenomenon is repeated with each new passage of the wave front over the mid-point of the line.

The resistance, R, of the line modifies these theoretical curves, as shown by the irregular lighter lines. The current finally reaches the value $i_0 = e/R$, and the voltage across the middle of the line tends to the ultimate value $0.5\,e$.

4. Discharge to Ground over a Resistance or Directly

In Fig. 10, Z_1 and Z_2 are two conductors of different surge impedances, for example a cable and an overhead line. At their junction, A, an overvoltage protective device is placed consisting of a sphere gap with a resistance R in series. Both conductors are initially charged to a voltage insufficient to break down the gap, and then this voltage is gradually increased to a value E_0

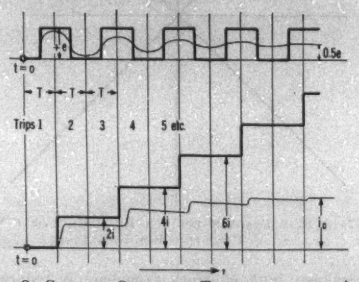

Fig. 9—General Shape of Theoretical and Actual Oscillograms of a Wave Current Reflected at the Short-Circuited End of a Line, and of the Corresponding Voltage in the Middle of the Line

at which the gap breaks down. The subsequent phenomenon is as follows:

(a) A current, I, flows to the ground.

(b) The voltage between A and the ground drops to a value $E = I\,R$, where E is less than E_0. The resistance of the arc is assumed to be negligible in comparison to R.

(c) Waves of equal voltage depression start in both directions; in conductor 1 this wave is denoted by e_{r1} (e – return – one) and in conductor 2 by e_{f2} (e – forward – two).

(d) Currents begin to flow into the ground connection from both conductors. The current wave in conductor 1 is denoted by i_{r1}; the current is positive

because it flows to the right, although the wave itself is propagated to the left. The current i_{f2} in conductor 2 is negative because it flows to the left, although the wave is propagated to the right. With equal values of voltages, the currents in the two waves are unequal; see Equation (4). The double arrowheads at the top of the figure indicate the directions of the currents themselves, the single arrowheads at the bottom show the direction of propagation of the waves.

The quantitative relationships are shown in the star

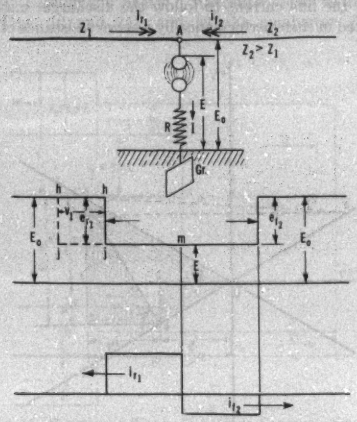

FIG. 10—TRAVELING WAVES CAUSED BY BREAKDOWN OF A PROTECTIVE GAP WITH A RESISTANCE IN SERIES

diagram in Fig. 11. It will be noted that the positive directions of i_{f2} and e_{f2} are the same as those of i_{r1} and e_{r1}. The triangle $a\,c\,d$ represents the current and the voltage in the ground resistance; the triangle rests on the rays $a'\,a''$ and $c'\,c''$ drawn at angles corresponding to the surge impedances Z_1 and Z_2 respectively; see the explanation to Fig. 5. The triangle $O\,a\,b$ gives the current and the voltage in the wave in conductor 1, the triangle $O\,d\,g$ those in conductor 2. These three triangles satisfy the following conditions:

$$E = R\,I;\ e_{r1} = -\,Z_1\,i_{r1};\ e_{f2} = Z_2\,i_{f2} \qquad (15)$$

$$I = i_{r1} - i_{f2};\ E = E_0 + e_{r1} = E_0 + e_{f2} \qquad (16)$$

Since there are no other conditions to be satisfied, Fig. 11 gives a correct representation of the phenomenon shown in Fig. 10. In checking Equations (16), it must be remembered that the quantities i_{f2}, e_{r1}, and e_{f2} are negative, so that an addition in Fig. 11 means a subtraction in Equation (16), and vice versa. We also have

$$\text{area } a\,d\,c\,s = \text{area } a\,p\,q\,s + \text{area } p\,q\,c\,d \qquad (17)$$

The area $a\,d\,c\,s$ is a measure of power input into the resistance R; the area $a\,p\,q\,s$ is a measure for the power input into R from conductor 1, and the area $p\,q\,c\,d$ for that from conductor 2.

The wave energy relations are also indicated by the shaded areas as follows: in Fig. 10, consider the wave front $h\,j$ at a certain instant of time and the same front one second later, at $h'\,j'$. The distance between the two is equal to the velocity v_1 of wave propagation in the medium surrounding conductor 1. During the second under consideration, part of the initial stored electrostatic energy on the length $j\,j'$ of the line is converted into the electromagnetic energy of the current i_{r1} on the same length of the line, and an amount of energy equal to $E\ i_{r1}$ is delivered to the resistance R. The amounts of electrostatic and electromagnetic energy on the part $j\,m$ of the line and in the region to the left of j' need not be considered since no energy changes take place in these regions during the second under consideration. Thus, the energy equation is

$$0.5\,E_0^2\,v_1\,C_1 = 0.5\,E^2\,v_1\,C_1 + 0.5\,i_{r1}^2\,v_1\,L_1 + E\,i_{r1} \qquad (18)$$

In this formula, the expression on the left-hand side represents the initially stored electrostatic energy, and the terms on the right-hand side represent the remaining electrostatic energy, the electromagnetic energy, and the input into R (from the left) respectively. Eliminating v_1 by means of Equation (3) and using the value of Z_1 from Equation (5), Equation (18) becomes

$$0.5\,E_0^2/Z_1 = 0.5\,E^2/Z_1 + 0.5\,i_{r1}^2\,Z_1 + E\,i_{r1} \qquad (19)$$

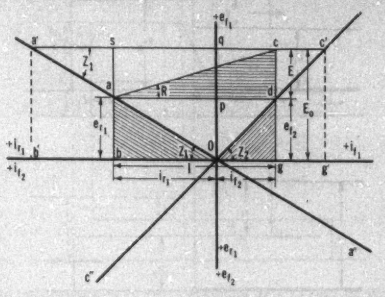

FIG. 11—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES SHOWN IN FIG. 10

In Fig. 11, this equation corresponds to the relationship

$$\text{area } a'\,q\,O = \text{area } a'\,s\,a + \text{area } a\,p\,O + \text{area } a\,s\,q\,p \qquad (20)$$

In the triangle $a'\,q\,O$ the vertical side is equal to E_0 and the horizontal side is E_0/Z_1; hence its area is equal to $0.5\,E_0\,(E_0/Z_1)$. Similarly, the area $a'\,s\,a$ is equal to $0.5\,E.\,(E/Z_1)$. The horizontal side of the triangle $a\,p\,O$ is i_{r1}; hence, its vertical side is $i_{r1}\,Z_1$, and the area equals $0.5\,i_{r1}\,.\,(i_{r1}\,Z_1)$.

With a given initial voltage E_0, the triangle $a'\,O\,c'$ is fixed. The power absorbed in R is represented by the rectangle $a\,s\,c\,d$ inscribed in this triangle, and the area

of the rectangle is a function of the resistance R. When R (and consequently E) decreases without limit, the rectangle approaches in the limit the straight line $a'c'$ and its area tends to zero. When R increases without limit, and consequently E approaches E_0, the rectangle approaches the straight line $O\,q$ and its area also becomes infinitesimal. Thus, there is a value of R for which this area, and consequently the power absorbed in the resistance, becomes a maximum.

By writing down an analytical expression for the area of the rectangle as a function of E, and equating its first derivative with respect to E to zero, the area may be shown to be a maximum when $O\,p = p\,q$, that is, when

$$E = 0.5\,E_0 = -\,e_{f2} = -\,e_{r1} \tag{21}$$

In this case, the area of the rectangle is equal to one-half of that of the triangle $a'\,O\,c'$. This means that with the most favorable value of resistance R one-half of the stored energy of the line is drained to the ground on the first trip of the waves of depression. On this trip, the initial voltage, E_0, is depressed to one-half of its value. If the distant ends of the two conductors are open, the waves will be reflected without change in the sign of the voltage and the remaining half of the stored energy will be drained on the return trip. With any other value of ground resistance, more reflections are necessary in order to discharge the line completely.

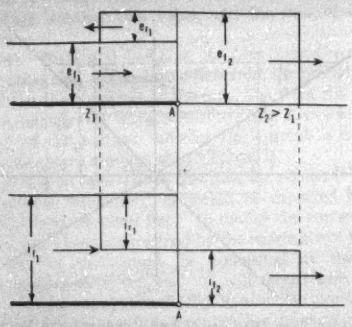

FIG. 12—A PARTIAL REFLECTION OF A TRAVELING WAVE AT THE JUNCTION POINT OF TWO CONDUCTORS WITH DIFFERENT SURGE IMPEDANCES

The most favorable value of R may be calculated as follows:

$$a\,d = E\,R^{-1} \tag{22}$$

$$a\,d = b\,O + O\,g = -\,e_{r1}\,Z_1^{-1} - e_{f2}\,Z_2^{-1} \tag{23}$$

When the condition (21) is fulfilled, Equations (22) and (23) give

$$R^{-1} = Z_1^{-1} + Z_2^{-1} \tag{24}$$

In other words, the optimum resistance is equal to the combined surge impedance of the two conductors in parallel.

When $R = O$, and consequently $E = O$, the initial line voltage is depressed to zero and the largest possible current (equal to $a'\,c'$) flows to the ground. However, practically no energy is conducted to the ground, and the potential energy of the charged line is merely converted into kinetic. At the distant open end, the voltage $e_{r1} = -\,E_0$ is reflected and the wave comes back again raising the line voltage to E_0, only with the sign reversed. Thus, protective devices (which do not allow the line current to follow the discharge) may be assisted in their performance by a series resistance of the

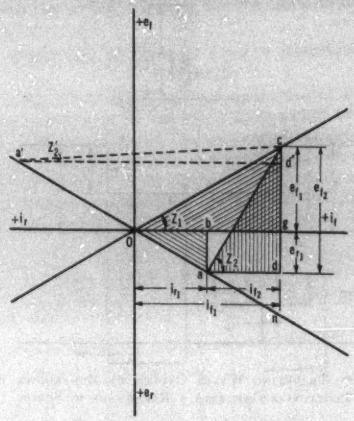

FIG. 13—A STAR DIAGRAM FOR THE CASE SHOWN IN FIG. 12

order of magnitude indicated by Equation (24). Not only is the energy of over-voltage converted into heat more rapidly, but the line disturbance is much less because the voltage drops to only one-half of its initial value.

5. PASSAGE OF A WAVE THROUGH A JUNCTION BETWEEN TWO CONDUCTORS

(*a*) *Plain Junction.* In Fig. 12, a junction of two conductors is shown at A. The surge impedance, Z_1, of the conductor 1 is assumed to be smaller than that, Z_2, of the conductor 2. A traveling wave, e_{f1}, i_{f1}, coming from the left, strikes the junction point A. Because of Z_2 being greater than Z_1, the junction point possesses step-up characteristics, so that the transmitted voltage, e_{f2}, is greater than e_{f1}, whereas i_{f2} is smaller than i_{f1}. The reflected wave is indicated by e_{r1}, i_{r1}.

The corresponding star diagram is shown in Fig. 13, and is practically identical with that in Fig. 5. The transmitted quantities, e_{f2} and i_{f2}, take the place of E and I, and the surge impedance Z_2 is substituted for R. The triangle $a\,c\,d$ is cross-hatched vertically, to indicate

that the corresponding power is transmitted through and not absorbed at the junction (see Table I). Equations (7) to (11) hold true in this case, except for changes in the notation. The subscripts f and r become $f1$ and $r1$, Z becomes Z_1, R is changed to Z_2, and E and I to e_{f2} and i_{f2} respectively. For this reason, the star diagrams in Figs. 5 and 13 are identical. When Z_2 is smaller than Z_1, the triangle of the transmitted wave assumes the shape $c\,a'\,d'$, and the joint acquires step-down characteristics.

(*b*) *Closing of a Switch.* Assume an open switch at A (Figs. 12 and 14), and let the conductor Z_1 be

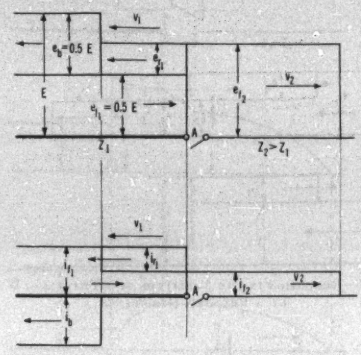

FIG. 14—TRAVELING WAVES CAUSED BY CLOSING A SWITCH

permanently connected to a source of d-c. voltage E, there being no traveling waves as long as the switch is open. When the switch is closed, a wave, $f2$, starts forward from the junction. The energy of this wave comes at first from that stored in conductor 1, until the source E has been reached; hence, a wave of depression, $r1$, is produced in conductor 1. The problem is to determine the intensities of these two waves.

In order to be able to use the star diagram and to avoid a discontinuity in the phenomenon, due to the closing of the switch, the initial static distribution of electricity of voltage E in conductor 1 may be thought of as a result of superposition of two equal waves traveling in the opposite directions. The forward going wave will have the voltage $e_{f1} = 0.5\,E$ and the corresponding current, i_{f1}, determined by Equation (4). The other wave, moving to the left and distinguished by the subscript b (balancing), is of the voltage $e_b = 0.5\,E$, and its current is $i_b = -\,i_{f1}$. The sum of these waves will give a static distribution of electricity of voltage E and current zero.

We shall assume the switch to be closed at the instant when the front of the wave $f1$ has just reached the switch, and the tail end of the b wave has just passed it. This assumption satisfies the following three conditions:

(a) The actual static distribution E extends to the switch previous to its being closed.

(b) The wave $f1$ meets a closed switch and proceeds into the second conductor as though through a plain junction (Figs. 12 and 13), that is, without a discontinuity in the circuit conditions.

(c) The closing of the switch has no effect upon the wave b because at that instant this wave already lies entirely within conductor 1 and moves away from the switch.

Thus, in so far as the component wave $f1$ is concerned, the star diagram in Fig. 13 applies, provided that e_{f1} is put equal to $0.5\,E$. To the left of the front of $r1$, the distribution is static, of voltage E. All the currents and voltages are indicated in Fig. 14.

(*c*) *Series Resistance between Conductors.* This case (Fig. 15) is similar to that shown in Figs. 12 and 13, except that a resistance R is interposed between the two conductors, to absorb part of the transmitted wave energy. The surge impedance Z_1 is assumed to be

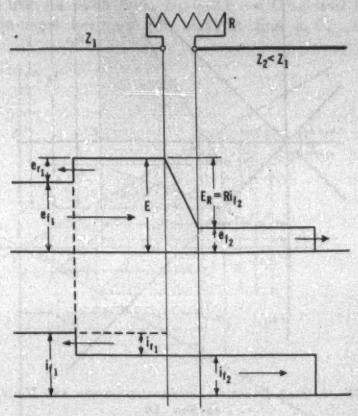

FIG. 15—REDUCTION IN THE AMPLITUDE OF A TRAVELING WAVE BY A SERIES RESISTANCE

greater than Z_2, and the resulting conditions are shown in the star diagram in Fig. 16. A similar diagram may be drawn for the case when Z_1 is smaller than Z_2.

The star diagram shows the following conditions fulfilled: Equation (4) for the waves $f1$, $r1$, and $f2$; Equation (11) for the resistor R; Equation (7) for both conductors; and Equation (8) for conductor 1. Since there are no other conditions to be fulfilled, the diagram gives a complete solution of the problem. The four triangles are properly cross-hatched to indicate the amounts of energy incoming, transmitted, reflected, and absorbed (see Table I). The positive directions of e_{f2} and i_{f2} are not indicated on the coordinate axes;

the physical sense of the problem is such that these quantities can have only positive values.

(*d*) *Shunted Resistance between Conductors.* The diagram of connections is shown in Fig. 17, where p is a resistance to the ground. The total current arriving at A is divided into two parts: a portion, I_p, flows to the ground; the remainder, i_{f2}, passes into the second conductor.

The corresponding star diagram is shown in Fig. 18. The familiar four triangles are shaded to indicate the amounts of energy incoming, transmitted, absorbed, and reflected. The relations obtaining between the various currents may be checked with reference to the waves themselves shown in Fig. 17; the same applies to the voltages.

It will be seen from Fig. 18 that $a\,d = E_p\,p^{-1}$,

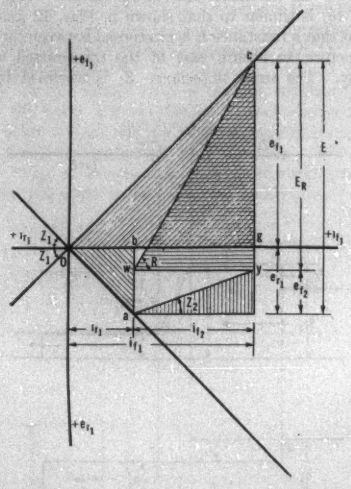

FIG. 16—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES IN FIG. 15

$d\,m = E_p\,Z_2^{-1}$, and $s\,m = E_p\,Z_1^{-1}$. But $a\,d$ plus $d\,m$ is greater than $s\,m$; hence

$$p^{-1} + Z_2^{-1} > Z_1^{-1} \tag{25}$$

This means that the total value of the combined impedances of p and Z_2 in parallel is less than Z_1; consequently, the joint may be expected to possess step-down characteristics. In fact, e_{f2} is smaller than e_{f1}, although Z_2 is greater than Z_1. Without p, the joint would have step-up characteristics (Figs. 12 and 13).

6. EFFECT OF A LUMPED SERIES INDUCTANCE IN REDUCING THE STEEPNESS OF A WAVE FRONT

In Fig. 19, a junction of two conductors is shown, with an inductance L between them. This inductance will be assumed to be devoid of both resistance and capacitance, and, electrically speaking, so short as to be practically concentrated at one point. When a vertical-front traveling wave, $f\,1$, first strikes the inductance, only an infinitesimal current can pass through, because of the counter—e. m. f. $E_L = 2\,e_{f1} = L\,d\,i_{f2}/d\,t$. In other words, at the first instant the wave front is reflected as if from an open end of a line (Fig. 6). This

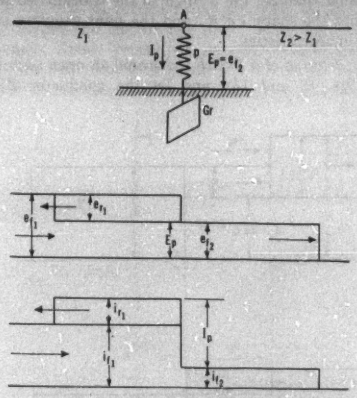

FIG. 17—REDUCTION IN THE AMPLITUDE OF A TRAVELING WAVE BY A SHUNTED RESISTANCE

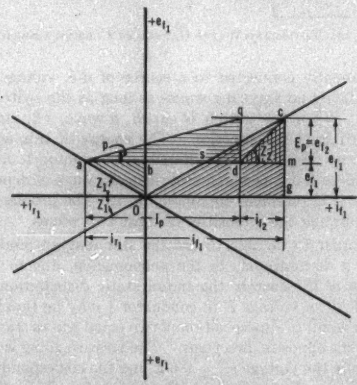

FIG. 18—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES IN FIG. 17

is indicated in the sketch of the wave by making $a\,b = b\,c$, and also by making $m\,n$ stand not only for i_{f1}, but also for $-\,i_{r1}$ at the first instant, so that the total current at m is equal to zero.

As more and more current trickles through L, the value of $d\,i_{f2}/d\,t$, and consequently of E_L, decreases; a voltage, e_{f2}, is built up across the line 2, and a trans-

mitted wave, f_2, begins to be propagated in the second conductor. In Fig. 19, this wave is shown *in space*, but an identical curve would indicate the current and the voltage at B *in time*, because the wave is propagated in the conductor 2 without distortion and any value found along the conductor at some time was produced at point B.

After an infinitely long time, the current flow through the inductance will become steady and the voltage E_L will approach zero. In reality, these conditions are

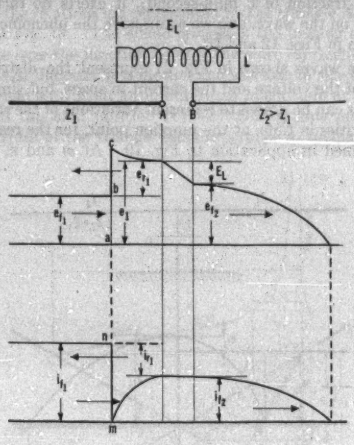

FIG. 19—ATTENUATION OF A WAVE FRONT BY A SERIES INDUCTANCE

reached within a small fraction of a milli-second. The junction then behaves as though L did not exist (Figs. 12 and 13). The purpose of the inductance is to make the wave front in conductor 2 more slanting and therefore the wave itself less harmful to the connected apparatus. It will be seen, however, that this advantage is gained at the cost of a steep reflected wave front in conductor 1.

The quantitative relationships are shown in Fig. 20, where the familiar four triangles may be distinguished by the usual cross-hatching. Since all the currents and voltages are now functions of time, the latter must be introduced into the star diagram explicitly. This is done by means of an exponential "timing" curve, $a_0\,u\,w$, whose equation is deduced below. This curve gives values of E_L for various instants of time. Knowing $C\,c = E_L$ at a given instant, and the slope of Z_2, the triangle $c\,a\,d$ for the wave $f\,2$ may be drawn; the triangle $a\,b\,O$ for the reflected wave, $r\,1$, becomes thereby determined. The problem is therefore solved for that particular instant.

At any instant the power in the incoming wave must be equal to the sum of the amounts of power in the reflected and transmitted waves plus the rate of increase in the energy stored in L. Graphically this means that

$$\text{area } O\,C\,g = \text{area } O\,b\,a + \text{area } a\,c\,d + \text{area } h\,c\,C \qquad (26)$$

The area $h\,c\,C$ is a measure for the rate of energy storage in L because the base, $c\,C$, of the triangle is equal to E_L and the altitude, $d\,a$, equals i_{f2}. To prove Equation (26) we write

$$\text{area } a\,h\,c = \text{area } a\,b\,g\,d \qquad (27)$$

because the triangle and the rectangle have the same altitude, $a\,d$, but the base of the triangle is twice that of the rectangle. Subtracting the area $a\,b\,f$ from both sides of Equation (27) and adding the area $f\,c\,g$, we obtain

$$\text{area } b\,h\,c\,g = \text{area } a\,c\,d \qquad (28)$$

We also have

$$\text{area } O\,b\,h + \text{area } h\,c\,C = \text{area } O\,b\,a + \text{area } h\,c\,C \qquad (29)$$

Adding Equations (28) and (29) term by term, gives Equation (26).

At the instant $t = O$, when the front of the wave $f\,1$ has just reached the inductance L, the junction behaves like an open end; $E_L = 2\,e_{f1} = C\,a_0$, and the triangle $a\,c\,d$ becomes the straight line $a_0\,C$. The

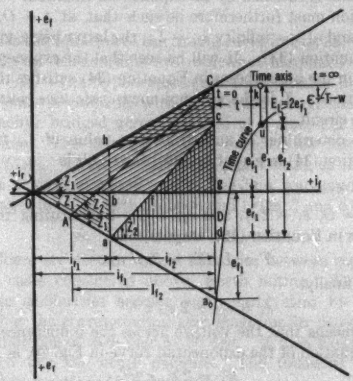

FIG. 20—A STAR DIAGRAM CORRESPONDING TO FIG. 19, WITH AN EXPONENTIAL TIMING CURVE

transmitted current, i_{f2}, is zero, and $i_{r1} = -\,i_{f1}$. After an infinitely long interval of time, $E_L = c\,C$ becomes equal to zero, and the triangle $a\,c\,d$ assumes the position and shape $A\,C\,D$. The diagram then becomes identical with that in Fig. 13.

Thus, the whole process of smoothing down the wave front by an inductance is represented in the star diagram by a gradual transformation of the straight line $a_0\,C$ into the triangle $A\,C\,D$. For any position of point c, the length $c\,C = k\,u$ gives the value of E_L, and the corresponding abscissa, $C\,k$, gives the instant of time at which this value of E_L occurs.

Equation of the Timing Curve. At any instant of time, t, the difference between the voltages e_1 and e_{f2} (Fig. 19) is equal to the voltage E_L across the inductance. Hence

$$e_1 = e_{f2} + L\, d\, i_{f2}/d\, t \tag{30}$$

To eliminate i_{f2} from this equation, we write (Fig. 20)

$$C\,D = I_{f2}\, Z_2; \quad D\, a_0 = I_{f2}\, Z_1 \tag{31}$$

where I_{f2} is the final value of i_{f2} at the time t = infinity. Adding the two Equations (31), gives

$$2\, e_{f1} = I_{f2}\,(Z_1 + Z_2) \tag{32}$$

so that

$$I_{f2} = 2\, e_{f1}/(Z_1 + Z_2) \tag{33}$$

Consequently, at any instant t,

$$i_{f2} = 2\, e_{f1}\,(1 - \epsilon^{-t/T})/(Z_1 + Z_2) \tag{34}$$

The reason for the added exponential factor in Equation (34) is as follows: In a straight-forward analytical solution of the problem (Rüdenberg, p. 397) one would supplement Equation (30) by a number of linear algebraic equations such as Equations (7) to (11). The result of elimination of all but one variable (i_{f2}) will be a linear differential equation of the first order, with constant coefficients. It is well known that such an equation is satisfied by an exponential function. This function must furthermore be such that at $t = O$, $i_{f2} = O$ and at t = infinity $i_{f2} = I_{f2}$, the latter being given by Equation (33). It will be seen that the exponential term in the parentheses in Equation (34) satisfies these conditions. T is the still unknown *wave time constant* of the circuit.

To determine T, substitute the value of i_{f2} from Equation (34) in Equation (30); the result is

$$e_1 = e_{f2} + 2\, e_{f1}\,(L/T)\, \epsilon^{-(t/T)}/(Z_1 + Z_2) \tag{35}$$

At $t = O$, $e_1 = 2\, e_{f1}$ and $e_{f2} = O$. Substituting these values in Equation (35), we get

$$T = L/(Z_1 + Z_2) \tag{36}$$

Thus finally

$$e_1 = e_{f2} + 2\, e_{f1}\, \epsilon^{-t/T} \tag{37}$$

This means that the voltage across the inductance, or the equation of the exponential curve in Fig. 20, is

$$E_L = 2\, e_{f1}\, \epsilon^{-t/T} \tag{38}$$

T being given by Equation (36).

The foregoing theory and the star diagram have been deduced for the general case of Z_1 different from Z_2. If $Z_1 = Z_2$, the equations and the diagram are somewhat simplified. Another interesting case is that of Z_2 equal to zero, that is, when the inductance L is bridged across the end of the line Z_1 (Rüdenberg, p. 405). In Fig. 20, $a\,c$ becomes a horizontal line which moves from a_0 to C. It will be readily seen that this represents a gradual transition from an open line to a short-circuited line (Figs. 6 and 7).

7. Effect of a Lumped Shunted Capacitance in Reducing the Steepness of a Wave Front

In Fig. 21, the vertical front of a traveling wave, $f\,1$, is assumed to arrive at the junction of the conductors Z_1 and Z_2 at the instant $t = O$. Some of the energy is reflected, some goes to charge the capacitor C, and the remainder passes into the conductor Z_2. Assuming the capacitor to be initially uncharged, it will act at the first instant like a short-circuit to the ground, so that the total voltage at the junction drops to zero and no current passes into the second conductor. After the capacitor has been fully charged (which theoretically takes an infinite time, but in reality occurs within a small fraction of a milli-second), it exerts no further effect on the wave, and we again have the phenomenon shown in Figs. 12 and 13.

The waves shown in Fig. 21 represent the distribution of the voltage and the current in space, but similar curves can be drawn to represent variations of the same quantities in time, at the junction point, for the reason explained in application to Fig. 19. At m and n, the

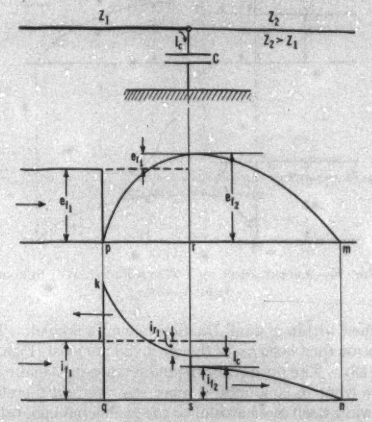

Fig. 21—Attenuation of a Wave Front by a Shunted Capacitor

transmitted wave is zero ($t = O$). At the same instant, at p and q, the reflected wave, $e_{r1} = -\, e_{f1}$, and $i_{r1} = +\, i_{f1}$, corresponding to a reflection at a short-circuited end of a line (Fig. 7). Then both e_{r1} and i_{r1} decrease and change their signs, so that the joint gradually acquires the step-up characteristics corresponding to $Z_2 > Z_1$. The discontinuity in the current at s is due to the charging current, I_c, of the capacitor, the instant s being an intermediate one, before the capacitor is fully charged. At t = infinity, this discontinuity is reduced to zero.

It will be seen that a capacitor smooths down the front of the transmitted wave, but causes a steep front of the reflected current wave and increases the total current in the first conductor to a double value. It is

instructive to compare the curves in Fig. 21 with those shown in Fig. 19.

The star diagram of the waves in Fig. 21 is shown in Fig. 22. The four cross-hatched triangles represent the amounts of instantaneous power, incoming, outgoing, reflected, and absorbed in the capacitance. The exponential curve $a_0\,u\,w$ gives values of the capacitor current, I_c, for different instants of time, and corresponds to the E_L curve in Fig. 20. At the instant $t = O$, $I_c = a_0\,C$, corresponding to a short-circuited end of the conductor 1 (Fig. 7). At $t =$ infinity, $I_c = O$, and the relations are represented by the triangle $A\,C\,D$, as in Fig. 13. The whole process of the passing of the wave over the junction point is therefore represented in the star diagram as a motion and increase in size of the triangle $a\,c\,d$ from an infinitesimal magnitude at point a_0 to the final position $A\,C\,D$. During this motion, the sides of the triangle remain parallel to themselves.

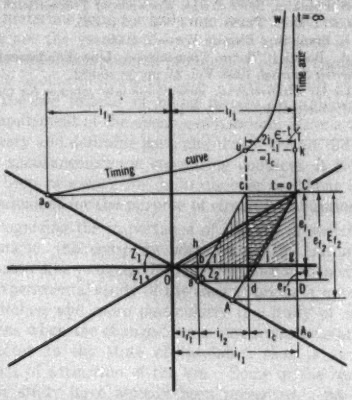

FIG. 22—A STAR DIAGRAM CORRESPONDING TO FIG. 21, WITH AN EXPONENTIAL TIMING CURVE

The law of conservation of energy requires that

$$\text{area } O\,c\,g = \text{area } O\,b\,a + \text{area } a\,c\,d + \text{area } c\,d\,C \quad (39)$$

The latter area is equal to the product of the capacitor current by the voltage at its terminals, and therefore is a measure for its power input. To prove Equation (39), we shall first compute the area of the trapezoid $h\,b\,g\,C$. The sum of its parallel sides is equal to $d\,c$ and its altitude is $b\,g$. Hence,

$$\text{area } h\,b\,g\,C = 0.5\,d\,c \times b\,g \quad (40)$$

Next we compute the area of the trapezoid $c\,a\,d\,C$. Its altitude is $d\,c$ and the sum of its parallel sides is equal to $b\,g$. Consequently

$$\text{area } c\,a\,d\,C = 0.5\,d\,c \times b\,g \quad (41)$$

Thus, the areas of the two trapezoids, $h\,b\,g\,C$ and $c\,a\,d\,C$, are equal. Adding the area $O\,h\,b$ to the first, and $O\,b\,a$ to the second, Equation (39) is proved.

Equation of the Timing Curve. When $t = O$, $e_{f2} = O$; when $t =$ infinity,

$$E_{f2} = 2\,e_{f1}\,Z_2/(Z_1 + Z_2) \quad (42)$$

see Equation (14). The voltage e_{f2} varies exponentially between these two limits, for the reason explained in the derivation of Equation (34). Thus, at any instant t,

$$e_{f2} = 2\,e_{f1}\,Z_2\,(1 - \epsilon^{-t/T})/(Z_1 + Z_2) \quad (43)$$

where T is the unknown wave time constant of the circuit. For the junction point we have

$$i_{f1} + i_{r1} = i_{f2} + C\,d\,e_{f2}/d\,t \quad (44)$$

Here

$$I_c = C\,d\,e_{f2}/d\,t \quad (45)$$

is the charging current flowing into the capacitor. Substituting the value of e_{f2} from Equation (43) in Equation (44), we get

$$i_{f1} + i_{r1} = i_{f2} + 2\,e_{f1}\,(C/T)\,Z_2\,\epsilon^{-(t/T)}/(Z_1 + Z_2) \quad (46)$$

At $t = O$, $i_{f2} = O$, $i_{r1} = i_{f1} = e_{f1}/Z_1$. Substituting these values in Equation (46), gives

$$T = C\,Z_1\,Z_2/(Z_1 + Z_2) = C/(Y_1 + Y_2) \quad (47)$$

where the surge admittances Y_1 and Y_2 are the reciprocals of Z_1 and Z_2; see Equation (5a). Equation (45) becomes

$$I_c = 2\,i_{f1}\,\epsilon^{-t/T} \quad (48)$$

This is the equation of the timing curve in Fig. 22. The foregoing relationships are somewhat simplified when $Z_1 = Z_2$. The hypotenuse $a\,c$ is then parallel to $O\,C$, and point A coincides with O. To deduce the case of a capacitor bridged across the end of a line, put $Z_2 =$ infinity. The triangle $A\,C\,D$ degenerates into the line $C\,A_0$. Thus, the reflection begins at a_0, as though with a short-circuited end of the line, and gradually changes to A_0 as with an open-end (Figs. 7 and 6).

8. RECIPROCAL RELATIONS

There are certain formal analogies between some of the cases considered above, worth noting. Some of these similarities become particularly clear by comparing the corresponding star diagrams in pairs, and turning one of each pair by 90 deg. It then looks very much like the other diagram. The following cases may be mentioned in particular:

Case	*vs.* Case	Figs.
Open end	Shorted end	6 and 7
Series resistance	Shunted resistance	16 and 18
Series inductance	Shunted capacitance	20 and 22
Current curve	Voltage curve	19 and 21
Voltage curve	Current curve	19 and 21

These formal graphical analogies follow from the equations underlying the phenomena. For example, where with a parallel connection currents are added, voltages are added with a series connection, etc. See A. Russell, "Alternating Currents," Vol. I, Chap. 21; the principle of duality and reciprocal theorems.

The subject is not followed here any further, but it

is quite possible that in some more complicated cases of protective apparatus, having obtained a solution for a certain combination, a solution for another arrangement of apparatus may be "guessed at" on the basis of the foregoing analogies and reciprocal relations.

9. Further Applications of the Star Diagram

The purpose of this paper being to explain the general theory and the use of the star diagram, only a few of the simplest applications have been considered. The next step will be to apply this diagram to more involved practical cases. The following problems on reflection and refraction of traveling waves may be suggested in particular:

(a) A transmission line which is divided at a point into two or more branches with different values of Z.

(b) Combinations of capacitance and inductance for protective purposes; for example, those shown in Rüdenberg's book on pp. 430 to 442.

(c) Same, with damping resistances; *ibid.*, pp. 442 to 446.

(d) Influence of short connecting conductors between two long conductors; *ibid.*, pp. 418 to 430.

(e) Graphical theory of short rectangular-front pulses, rather than long waves; *ibid.*, pp. 400, 404, 409, 443, 444, 445. Problems on short pulses may be solved by assuming an indefinitely long rectangular-front wave followed by an identical wave of opposite polarity which wipes out all but the short front portion of the original wave. Perhaps it is only necessary to draw star diagrams for both waves, with a proper time lag between them, and add the results algebraically.

(f) Mutual induction caused by traveling waves; *ibid.*, pp. 387 to 396.

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Theory of the Deion Circuit-Breaker

BY J. SLEPIAN*

Synopsis.—Three major features incorporated in the Deion circuit-breaker are discussed. They are: deionization at solid surfaces, the function of the static balancer, and cold electrode arcs.

THE switching of electric power circuits calls for elements which, subject to control, shall function sometimes as good electrical conductors and at other times as good insulators. When "closed" the element must pass hundreds or thousands of amperes with only at most a few volts drop; its resistance or impedance must be of the order of a fraction of an ohm. When "open" it must withstand hundreds or thousands of volts, with the passage of at most a few milliamperes; its resistance must be in the hundreds of thousands of ohms. Also it must be able to change from the one state to the other in a fraction of a second. So far, the only materials found which can meet these requirements are the gases, and arcs in air, and arcs in the vapors and decomposition products of oil are regularly serving to control power circuits. Careful study shows that the arc instead of being merely an unpleasant accompaniment of the opening of a switch, plays a very necessary and desirable part, and that if the arc did not occur spontaneously on separating contacts, it would have been necessary for us to discover or invent it or its equivalent for the purpose of circuit interruption.

Recognizing the importance of the arc in switching equipment, the company with which the author is associated five years ago began an extensive theoretical and experimental study of the electric arc as it appears in switches, and more particularly the study of what happens when the change-over occurs from the state of conductor to the state of insulator, that is at the mement of extinction of the arc. Some of the results of this study have already been presented.[1] As was perhaps to be expected, the study revealed some new and interesting possibilities in the application of arcs to circuit interruption and the Deion a-c. circuit-breaker in which these possibilities have been developed, promises to take its place with or perhaps replace the oil circuit breaker for certain classes of high power work.

While there are many details of the Deion circuit-breaker which are of great scientific interest and which required months and even years of intensive work for their mastery, there are three general principles which stand out above the others and which will be described here.

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1. *Extinction of an A-C. Arc*, J. Slepian, A. I. E. E. Quarterly TRANS., Vol. 47, October 1928, p. 1398.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

I. DEIONIZATION AT SOLID SURFACES

The subject of the rate of recovery of the dielectric strength of the space carrying a short a-c. arc immediately after arc extinction has been treated in some detail in a previous paper.[1] There it was shown that the ability to withstand the first few hundred volts was recovered almost instantly, but that later increments of dielectric strength were recovered at a very much slower rate. This is brought out in the curve of Fig. 1 which is derived from the data of Fig. 6 of the paper referred to.

Theory and experiment indicate that the first 250 volts are borne almost entirely by a thin layer of gas immediately adjacent to the cathode. Electrons readily leave this layer, but others to replace them cannot enter from the metal. The positive ions discharge into the cathode. Thus this cathode layer is deionized exceedingly rapidly. The subsequent slow further growth of dielectric strength is due to the further growth of the deionized cathode layer, and the disappearance of the ions in the other parts of the gas space by recombination.

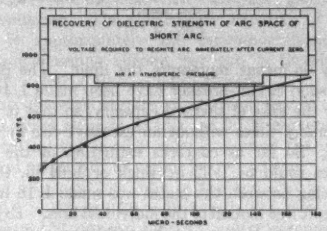

FIG. 1

It is evident then that it is the slow rate of recombination of the ions in the arc space away from the electrodes which limits the applicability of the arc in air for interrupting high voltages. A fairly obvious suggestion would be to reduce as far as possible the arc space remote from a cathode, and so far as possible cause all the arc to play in space close to a cathode; in other words to use a large number of short arcs in series. This is what is done in the Deion circuit-breaker.

As at present developed, the Deion breaker consists of a stack of copper plates 1/16 in. thick separated by 1/16 in. spacers. The arc which is drawn on contacts below this structure is blown by a magnetic field into

the stack. The arc is thus broken into short arcs in series, each of 1/16 in. length. There are thus in each inch of structure eight cathodes with their eight immediately adjacent rapidly deionizing gas layers. Immediately after the current passes through its zero value in its normal cycle each cathode layer is almost instantly deionized, and acquires the ability to withstand 250 volts much faster than any practical power circuit of corresponding voltage can supply the 250 volts. Thus the voltage necessary to reignite the arcs after the current zero is eight times 250 or 2000 volts per inch of structure. Hence the structure will interrupt circuits whose voltage is not over $2000/\sqrt{2}$ or 1414 volts r. m. s. per inch length.

This seems like a very high voltage for arcs in air,

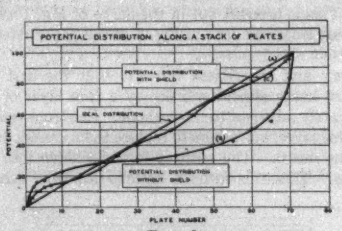

FIG. 2

but it by no means represents the limit. The plates in the Deion circuit-breaker have been made 1/16 in. thick for the sake of thermal capacity, so that it may operate many times without an excessive rise in temperature. So far as concerns extinction of the arc, the plates could be 1/32 in. thick or 1/64 in. thick. The spacing between the plates was made 1/16 in. so as to permit free motion of the individual arcs. Experiment shows that 1/32 in. spacing still leaves sufficiently free motion of the arc. Using these last figures there would be 21.3 plates per inch, and therefore the structure would interrupt 3760 volts r. m. s. per inch.

In the present Deion breaker the circuit volts per plate is kept very much less than the theoretical limit of 175 volts r. m. s.; in fact it is less than 130 volts. This is partly for the sake of having a factor of safety, and partly because when a voltage is impressed upon a long stack of plates insulated from one another, the potential does not divide among the plates in a uniform manner. This lack of uniform voltage distribution is compensated for in the Deion breaker by a static shield as described in the next section but the compensation cannot be made exact so that a sufficient margin between the theoretical limit and the working volts per plate is necessary.

II. The Function of the Static Shield

When voltage is applied to the ends of a long uniform stack of plates insulated from one another, the potential does not divide uniformly among the plates, but the potential differences between successive plates at the end of the stack may be many times the potential differences between successive plates in the middle of the stack. This is a consequence of the elementary principles of electrostatics, and need not be gone into in more detail here. The example of this phenomenon best known to the electrical engineer is probably the non-uniform distribution of potential among the units of a long string of suspension insulators.

The distribution of potential among the members of a stack of plates in a Deion breaker has been studied experimentally by Mr. B. P. Baker. A resistance-bridge method with telephone receiver as null instrument proved satisfactory. The stack studied consisted of 72 plates of 1/16-in. copper, spaced 1/16 in. apart, each plate having the shape of the letter P with round part 17 cm. diameter, and straight part 17 cm. long.

When unshielded, the potential distribution was found to be that shown by curve *b*, Fig. 2. The departure from uniform distribution given by curve *a* is very great. The point of inflection of curve *b* marks the potential of space remote from the stack. That it did

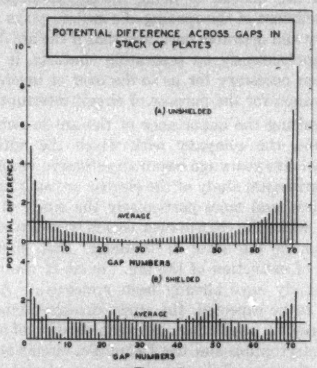

FIG. 3

not occur at the center of curve *b*, and that curve *b* was not symmetrical, are due to the influence of the frame supporting the stack.

Fig. 3A gives the relative potential differences across the gaps formed by the plates. It shows that the end gap received an impressed potential 9.1 times as great as the average voltage per gap. So large a ratio might be expected to diminish seriously the average volts per gap upon which the breaker could operate, and this had been found to be the case.

A static shield was then made up consisting of a micarta cylinder formed to fit over the stack, and with pieces of tin foil imbedded in it of such shape and size as to make uniform the distribution of potential among

the plates. The resulting curve as determined experimentally is given by *c*, Fig. 2. This curve follows the ideal distribution more closely, and as shown in Fig. 3B, the maximum gap potential is 2.6 times the average as compared with 9.1 for the unshielded stack. The curve of Fig. 2C is for the first static shield made. Later designs of static shield give still better results.

At first thought it would seem that the average volts per gap at which the unshielded stack would operate in a Deion breaker would be reduced in the ratio 9.1 so that if 176 volts r. m. s. is the theoretical limit for a single gap, 176/9.1 or 19.4 volts per gap would be the limit for the unshielded stack, and 1375 volts r. m. s. would be the limit for the whole stack. Similarly for the shielded stack, 176/2.6 or 67.8 volts r. m. s. would be the limiting average voltage per gap, and 4810 volts r. m. s. would be the limit for the whole stack.

Actually, however, the unshielded stack was found by test to be good for nearly 6000 volts r. m. s. and the shielded stack for more than 10,000 volts; how much more was not determined. The limit for the case of perfect uniformity of voltage distribution would be about 12,500 volts.

To explain these results it is necessary to examine in more detail the mechanism of reignition of an arc immediately following a current zero. In the previously quoted paper[1] it was pointed out that the almost instantly acquired ability to withstand 250 volts had a close relation to the voltage required to maintain a glow on copper electrodes. A glow is a discharge requiring several hundred volts at its cathode as compared with 10 or 20 volts required by an arc. It seems probable that when an arc space is broken down by the application of more than 250 volts immediately following a current zero, the current is first passed in some form of discharge like a glow, and only after a little time does the discharge break down into an arc. The time to change from the glow form to the arc will be very short if considerable current (that is several amperes) flows in the glow, and will probably be only a few hundredths of a microsecond. If, however, the current flowing in the glow is limited (that is a fraction of an ampere) the time for change from glow to arc may be many microseconds).

In the Deion breaker, immediately after the current zero, the voltage tending to reignite the arc in the stack of plates rises at a rate depending on the nature of the circuit in which the breaker is working. In a fast practical power circuit this reignition voltage may reach its maximum value in about one microsecond.[1] During this period of rising reignition voltage the overstressed gaps in the deion stack may be subjected to more than 250 volts, even though the average volts per gap impressed is much less than 250. These gaps then pass current, but the magnitude of this current is limited to that of the charging current of the remainder of the structure where the gaps are not broken down. If this charging current is small the discharge in the overstressed gaps will remain in the glow form, and will require in each gap a voltage of not less than 250 volts. The overstressed gaps then continue to hold more than the average voltage per gaps so that the remaining gaps are not overstressed and the arcs in the structure do not reignite.

If the charging current is too high, however, the discharge in an overstressed gap may break down from the glow form to an arc. It then takes only 20 volts which is much less than the average volts per gap, and thus causes more voltage to be thrown upon the remaining gaps. This may make the next overstressed gap break down into an arc with further raising of the voltage on the remaining gaps. This process may continue until all the remaining gaps are overstressed, and then reignition of all the arcs in the stack takes place.

As an example, consider the unshielded stack of 72 plates described above when operating in a circuit of 5000 volts r. m. s. or 7070 volts crest. This corresponds to 99.6 volts per gap average, and at this voltage the breaker was found to work. Assume that this voltage tending to reignite the arcs is applied in one microsecond at a uniformly increasing rate so that the average rate of application of voltage per gap is 99.6×10^6 volts/sec. The electrostatic capacity of a single gap is approximately 150 micro-microfarads. The average charging current per gap is therefore $99.6 \times 10^6 \times 150 \times 10^{-12} = 0.015$ ampere.

The charging current of the end gap by Fig. 3A is 9.1 times as great as this, or 0.136 ampere. After the critical voltage 250 volts is reached the current is carried principally as a glow in the end gap. The time taken to reach 250 volts in the end gap is $250/(9.1 \times 99.6 \times 10^6)$ or 0.28×10^{-6} sec. The glow therefore carries 0.136 amperes for 0.72×10^{-6} seconds. After this time the transient is over and on 60 cycles the glow current is correspondingly reduced. A copper cathode can carry 0.08 to 0.10 amperes as a glow indefinitely without changing into an arc.[2] Hence it would seem that 0.136 amperes would remain as a glow for 0.28×10^{-6} seconds.

At 7000 volts r. m. s. for which the unshielded stack fails, the charging current, and therefore also the current in the glow when the end gap discharges is 0.190 ampere, and the time that the glow must hold is increased to 0.61×10^{-6} seconds.

For the shielded stack at 10,000 volts r. m. s. at which it works in a breaker, the rate of rise of voltage in the end gap is by Fig. 3B $2.6 \times 10{,}000 \times \sqrt{2} \times 10^6/71$ or 518×10^6 volts/second. The charging current is then 0.078 amperes, and the glow should carry this current indefinitely.[2] Actually it needs to

2. W. Burstyn, *Elektrotech. Zeitsch.*, Vol. 41, p. 505, 1920. The current 0.32 amperes given by Burstyn is the current taken by the closed contacts in a 440-volt circuit. On allowing for the drop in the glow, 310 volts, the current on opening the contacts would drop to 0.095 ampere.

carry it only for $(1 - 250/518) \times 10^{-6}$ or 0.48×10^{-6} seconds. It seems then that with this size of plates, and with the shielding as obtained in Fig. 3B there is a large factor of safety in operating the 71 gaps at 10,000 volts or at 140 volts r. m. s. per gap average.

III. Cold Electrode Arcs

As explained in Section I, the efficacy of the Deion breaker resides in having all parts of the arc in close vicinity to deionizing surfaces. This is accomplished in the heavy current switch by blowing the arc into a stack of closely spaced plates so that no part of an arc is farther than 1/16 inch from a deionizing cathode. If the short arcs in this structure stood still for the duration of one-half cycle, 1/120 second, the structure would quickly be destroyed by the welding together of some plates and the burning of holes through others.

At the time this work was begun, it was believed that the cathode of an arc was necessarily at a very high temperature. In fact, the theory generally accepted then required thermionic emission of electrons from the cathode for the maintenance of the arc, and for most metals the temperature for so intense a thermionic emission is far above the boiling point. How well entrenched this theory was may be seen by referring to a paper by K. T. Compton, (*Phys. Rev.*, Vol. 21, 1923, p. 269), and Seeliger, "Physik der Gasentladungen" (Leipzig 1927, p. 360 *et. seq.*). On this account much work was done on speeding up the deionization of an arc by causing it to play through the openings of gauze sheets, and thus avoiding the development of arc terminals on the deionizing structure.

However, in the course of this work, it was forced upon the author's attention that sometimes arcs were obtained which did not have a hot cathode. Therefore, although it required considerable courage to take a stand opposite to that espoused by so many eminent authorities, the thermionic emission theory of the cathode of an arc was abandoned, and arcs with cold cathodes were accepted as possible. Experiment soon showed that by moving the terminals of the arc sufficiently rapidly over the electrode surfaces melting could be avoided even for very heavy currents. Arcs of more than 20,000 amperes have thus been carried on copper electrodes for more than 0.01 seconds with only slight oxidation of the electrodes.

A theory of the cathode of an arc was also developed based on the hypothesis that the metal itself is not necessarily at a temperature sufficient for thermionic emission but that a layer of gas or vapor immediately adjacent to the cathode is so intensely ionized, perhaps in virtue of very high temperature, that the arc current can be carried to the cathode by positive ions only.[3]

Shortly after this, papers by Stolt[4] described experiments with rapidly moving arcs of moderate current (up to 12 amperes) which seemed incompatible with the thermionic theory of the cathode of an arc, and this, with some theoretical work on heat balance at the cathode, is causing general abandonment of the thermionic theory.[5] Another theory of the cold cathode arc has been proposed by Langmuir,[6] who states that electrons are drawn from the cathode by intense electrostatic forces arising from space charges developed close to the cathode. However, a theory of the cold cathode arc is not essential for the understanding of the Deion breaker. Merely the possible existence of arcs with cold cathodes must be accepted.

In the Deion breaker the melting of the electrodes is prevented by causing the arc terminals to move very rapidly over the electrode surface by means of a magnetic field. The first experiments were carried out with a stack of long straight plates as the deionizing structure. It was found, however, that the velocity of the arc terminals necessary to prevent melting of the electrodes was so great that for 10,000 amperes, plates more than ten feet long would be necessary. These would be prohibitive from the standpoint of size for most applications, and would require too expensive a magnetic field structure.

This difficulty was surmounted by causing the arcs moving with high velocity to retrace over and over again an annular path. On adopting this expedient a very important new advantage was obtained. The deionizing structure became an almost completely closed structure. The arc, when once driven in, could not get out again, and had to stay in until its extinction at the end of the half cycle. Thus the danger of the arc getting across live parts outside the switch and causing short circuits was practically eliminated.

To calculate the speed necessary to prevent melting of the electrodes it is necessary to know the current density, and the voltage of the arc. Observation of the oxidized trail of arcs with currents about 10,000 amperes led to the estimate of 30,000 amperes/cm.^{2} as the current density in the cold cathode arc. The voltage of the 1/16-inch long cold cathode arcs was found to increase with current, (rising characteristic) and for more than 2000 amperes is given by

$$E = 30 + 0.79 \times 10^{-3}\, I \qquad (1)$$

where I is the current in amperes. The influence of the current then is a moderate one, and it is not until currents approaching 40,000 amperes are reached that the arc voltage is doubled.

The watts input per cm.^{2} of arc section is then

$$W = (30 + 0.79 \times 10^{-3}\, I) \times 30{,}000$$
$$= 9 \times 10^5 + 23.9\, I \qquad (2)$$

The diameter of the arc section, assuming it to be circular, is given by

$$1/4\, \pi\, d^2\, 30{,}000 = I \qquad (3)$$

3. J. Slepian, *Phys. Rev.* 27, p. 407, 1926.
J. Slepian, *Jour. Franklin Inst.* 201, p. 79, 1926.
4. H. Stolt, *Ann. d. Physik* 74, pp. 80-104, 1924.
H. Stolt, *Zeits. f. Physik* 262, pp. 95-101, 1924.
5. K. T. Compton, A. I. E. E. Trans., Vol. XLVI, 1927, p. 868.
6. I. Langmuir, *Zeits. f. Physik* 46, p. 282, 1927.

A point of the electrode surface will be exposed to the action of the arc for the time taken by the arc section to pass over the point. For a point over which the center of the arc section passes this time will be

$$t = \frac{d}{v} \tag{4}$$

where v is the velocity with which the arc moves.

Lastly, the temperature rise T in degrees C at the surface of a semi-infinite solid into whose surface W_1 watts per cm.^{2} is flowing is given by

$$T = \frac{2\,W_1}{4.18\,\sqrt{\pi\,k\,c\,\delta}}\,\sqrt{t} \tag{5}$$

where k is the heat conductivity of the solid in cal/cm.^{3} deg. cent. C is the heat capacity in cal/cm.^{3} deg. cent., and δ is the density in grams/cm.^{3} Substituting (3) and (4) in (5)

$$T = 2.19 \times 10^{-2}\,\frac{W_1\,I^{1/2}}{\sqrt{k\,c\,\delta}\,v^{1/2}} \tag{6}$$

$$v = 4.8 \times 10^{-4}\,\frac{W_1^2}{k\,c\,\delta}\,\frac{I^{1/2}}{T^2} \tag{7}$$

For copper we may take $k = 0.72$, $c = 0.096$, and $\delta = 8.0$. Let T be the temperature rise for melting, *i. e.*, 1000 deg. cent. Then (7) becomes

$$v = 8.69 \times 10^{-10}\,W_1^2\,I^{1/2} \tag{8}$$

Of the energy developed in the arc given by (2), it is uncertain how much goes into the cathode, and how much into the anode. A pessimistic estimate of the velocity needed to avoid melting will be obtained if all the heat W is assumed to go into one electrode, that is $W_1 = W$; an optimistic estimate will be obtained if the heat is supposed to divide equally between the electrodes, *i. e.*, $W_1 = 1/2\,W$. Substituting from (2) then, these two estimates become

$$v = 8.69 \times 10^{-10}\,(9 \times 10^5 + 23.9\,I)^2\,I^{1/2} \quad \text{(pessimistic)} \tag{9}$$

$$v = 2.17 \times 10^{-10}\,(9 \times 10^5 + 23.9)^2\,I^{1/2} \quad \text{(optimistic)} \tag{10}$$

The velocities given by (9) and (10) are quite large numerically for large currents as is seen in the following table. Experimentally the necessary velocities seem to be nearer to the optimistic figures than the pessimistic.

TABLE I

I Amperes	Velocity to prevent melting meters per second	
	Pessimistic	Optimistic
4,000	543	136
8,000	925	231
12,000	1350	338
16,000	1805	451

It will be observed that the speeds in the table are all greater than the velocity of sound. This occasioned no particular difficulty. Magnetic fields of about 0.3 I gauses were sufficient to give these velocities.

In the above calculation it was assumed that the arc section was coherent and circular. This was probably the case for small currents. Above 15,000 amperes there was some evidence from the character of the trails, and by indications of probe electrodes, that the arc was broken up into several separate arcs in parallel. It may be that at this high current in the correspondingly high field, the air in the 1/16-inch space was set into such violently turbulent motion that the arc was broken up. If the arc is divided into several parallel arcs, the velocity needed to prevent melting is lessened.

The above calculations give the temperature rise at a point in the electrode surface for a single passage of the arc over it. After the arc passes, the temperature of the point drops to nearly its original temperature before the arc reaches it again. With each rotation of the arc, the plate as a whole is raised in temperature, so that this rise in plate temperature must be added to the temperature of the point which the arc is passing over. Calculation readily shows, however, that this rise in temperature of the plates as a whole is too small to affect the above calculation seriously.

Discussion

For discussion of this paper see page 545.

The Structural Development of the Deion Circuit-Breaker up to 15,000 Volts

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Associate, A. I. E. E. Associate, A. I. E. E.

Introduction

UP to the present time, the interruption of an alternating-current circuit has been accomplished generally in one of two ways. The arc may be drawn between contacts located in some insulating liquid such as oil. In this case, the insulating value of the oil depreciates with each current interruption until it reaches a point where it must be renewed. On the other hand, the arc may be drawn in air with no means of extinguishing it other than lengthening it to such an extent that the generated voltage is no longer able to maintain it. For modern generating voltages, this requires arcs of great length and, for the upper range of transmission voltages, results in arc lengths which are impractical. As a result of this limitation, the oil circuit breaker has assumed a position of paramount importance on modern operating systems.

There has been a growing demand for some time, on the part of operators, for a circuit-interrupting medium which does not involve the use of oil. The chief reasons for this demand are removal of possible fire hazards and simplification of maintenance problems. The demand has been recognized by manufacturers and although research work toward this goal has been carried on for a number of years, no satisfactory, general-purpose apparatus of this kind has been placed on the market up to the present time.

At the Westinghouse Electric and Manufacturing Company, fundamental research in this field has been carried on for a long period and has served to give a deeper insight into the nature of arc conduction. It has suggested the use of means for deionizing the path of an arc drawn in air other than by merely extending it to a great length. Experimental circuit interrupters have been made utilizing such deionizing means in a variety of forms and the name "Deion circuit-breaker" has been applied to these devices.

The work on all of the methods of deionizing an arc stream has contributed much to the fundamental knowledge of arc phenomena and some of them may be developed further for practical application in the future. One of the most promising of these various methods was developed and applied to the Deion circuit breaker described in this paper.

In this circuit-breaker, an arc is drawn in air and forced into a deionizing chamber where it is broken up into a multiplicity of short arcs which are moved over metal plates at a velocity sufficient to prevent burning. This movement of the arc is maintained over an annular path until the current wave reaches zero, after which the arc stream between the metal plates is deionized, quickly changing from a good conductor to a good insulator. A further discussion of this theory of deionization is found in other papers presented before this and previous Institute meetings.[2]

Development of the "Deion" principles has been carried to the point of building and testing circuit-breaker structures up to 15-kv. ratings with rupturing capacities comparable to some present-day heavy-duty oil circuit-breakers in the power-house class. These breakers have been subjected to extensive laboratory and field tests, successfully interrupting three-phase grounded and ungrounded short circuits above 15,000 amperes at 12,000 volts consistently. The results of a recent series of field tests with one of these breakers is the subject of another paper presented before the Institute.[3]

General Construction

The 15-kv. Deion circuit-breaker shown in Fig. 1 is made up of three single-pole units, each consisting of a deionizing chamber, an arc-drawing mechanism with main contacts for carrying load current, and a controlling mechanism. The deionizing chamber consists essentially of a stack of thin copper plates spaced a short distance apart to form a series of gaps, the general arrangement being shown in Fig. 2. In these gaps are placed insulating spacers which enclose arc runways, each having a straight entering portion and a circular portion. One of these plates with its insulating spacers in position is shown in Fig. 3. Approximately 130 volts r. m. s. has been found to be a safe working voltage for each gap. On this basis, a large number of gaps is required for a 15-kv. circuit-breaker. These gaps are divided into groups and separated by coils connected in such a way that the magnetic fields of adjacent coils are in opposition, which causes flux to be diverted radially through the gaps, as shown in Fig. 4. The terminals of these radial field coils are so connected that they are automatically inserted in the circuit by the arc itself as it moves into the deionizing chamber. This is accomplished by short plates placed directly under the radial field coils for the purpose of developing

1. Electrical Engineer, The Westinghouse Electric & Manufacturing Co.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

2. *Extinction of an A-C. Arc*, J. Slepian, A. I. E. E. Quarterly Trans., Vol. 47, Oct. 1928, p. 1398.

3. "Field Tests on the Deion Circuit Breaker," B. G. Jamieson, see p. 535.

29-35

a high voltage on the section of arc subtended by the coils, as discussed later in this paper.

A blow-out magnet of conventional design, while adequate for small deionizing chambers working at low voltages, did not develop a suitable blow-in field for the larger chambers necessary for 15-kv. service. A blow-in magnet of entirely new design was, accordingly, constructed for this breaker. In this magnet, the coils are wound so as to cover the entire space in which the arc is drawn and extended. The cores on which these coils are wound are fabricated of iron and insulating material in such manner that the field in the air gap is concentrated at the necessary points for rapid movement of the arc into the deionizing chamber. The cores and coils are supported by a laminated return circuit passing over the top of the deionizing chamber. This return circuit acts also as a partial return path for the flux of the radial field coils previously referred to, and as a shield against stray magnetic fields. The general appearance of this magnet may be seen by referring to Fig. 1.

Fig. 1—Three-Pole Deion Circuit-Breaker with Continuous Rating of 2000—Amperes—15,000 Volts

Fig. 2—Deionizing Chamber with Blow-in Magnet and Electrostatic Shield Removed to Show General Arrangement of the Plates, Radial Field Coils, and Vents

Fig. 3—Plate from Deionizing Chamber with Insulating Spacers in Position, Showing the Trails Left by the Arc Terminals in Moving Over the Runway

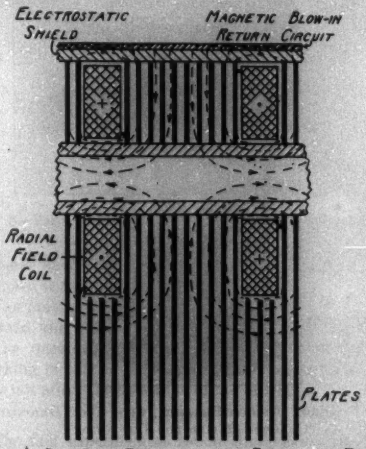

Fig. 4—Schematic Relationship of Plates and Radial Field Coils for the Deionizing Chamber, Showing the Manner in Which the Radial Field is Produced

Due to the high inductance of these blow-in field coils, it is necessary to use a special form of intermediate auxiliary contact when the continuous current-carrying capacity of the breaker is such as to make it inadvisable to connect the coils permanently in series with the main circuit. These contacts are of heavy section and are operated at high pressure. In order to obtain positive,

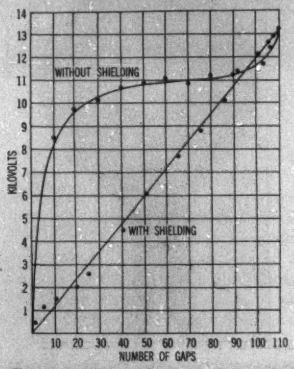

Fig. 5—Distribution of Potential Across the Gaps of the Deionizing Chamber with and without the Electrostatic Shield

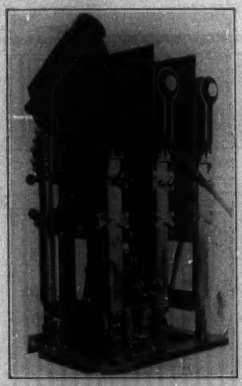

Fig. 6—Deion Circuit-Breaker with One Deionizing Chamber Raised for Inspection

quick transfer of the current to the blow-in coils, without undue flashing, this contact is equipped with an arc chute employing the Deion principle which was previously developed for use on standard industrial contactors.

A practically uniform distribution of recovery voltage over the gaps between metal plates in the deionizing chamber is an essential condition for satisfactory operation of the Deion circuit-breaker. In the longer deionizing chambers required for the higher voltages, it is necessary to use a shielding device to prevent a concentration of voltage across the end gaps, due to the electrostatic capacity of the metal plates to surrounding space. The electrostatic shield, which also serves as insulation between the deionizing chamber and the blow-in magnet, has layers of metal foil imbedded in it. These layers of metal foil are so shaped and located that each plate is forced to assume its proper electrostatic potential. Fig. 5 shows potential distribution across the deionizing chamber. The deionizing chamber, the blow-in magnet, and the electrostatic shield form a complete structural unit which is hinged at the rear support so that it may be rotated upward for inspection as shown in Fig. 6.

This unit is mounted at the top of insulating uprights which also support the main current-carrying parts and

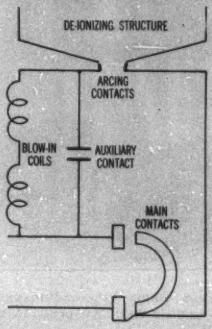

Fig. 7—Electrical Circuit of a Deion Circuit-Breaker in Which the Blow-in Field Coils are Normally in Parallel with the Main Contacts

the arc drawing mechanism. These uprights are securely anchored to a structural steel base as shown in Fig. 1. A movable parallel-path brush of conventional design, engaging stationary contact blocks, completes the main power circuit in the breaker. The moving arcing contact is carried on an individual arm, so linked to the main current-carrying contact as to secure proper sequence of opening and closing and arranged to provide a high speed of separation in drawing the arc. The arcing contacts are of familiar design, the moving element rolling on the stationary member and drawing the arc at the tips. Movement of the arcing contact operating arm also operates the intermediate transfer contacts which are isolated at the rear of structure. Operation of this transfer contact is such as to insure proper sequence of opening and closing with the main arcing contacts.

The electrical circuit of a Deion breaker is shown in Fig. 7. On opening the circuit the main contacts part

first, followed by the transfer contacts and then by the arcing contacts. As the transfer contacts part, sufficient voltage is developed across them to deflect the current to the blow-in coils. This insures the presence of a blow-in field when the arc is drawn.

The insulating uprights carrying the deionizing chamber and the arc drawing mechanism comprise a complete pole unit which may be operated by its own individual closing mechanism as a single pole breaker. Three of these pole units may be assembled on a structural steel base and operated by a single closing mechanism, for three-phase service. Barriers between pole units permit a spacing the equivalent of that in a modern oil-insulated breaker of comparable rating. This circuit-breaker is well adapted to use in isolated-phase service either with individual closing mechanisms or through remote control from a common mechanism.

Operation

The theory of the Deion circuit-breaker deals only with an arc after it has been drawn and is in no way connected with the method of securing a tripping impulse. Inasmuch as standard closing mechanisms are used to operate this breaker, tripping may be obtained in any conventional manner applicable to modern breakers of other types. The operation of the breaker up to the time of drawing the arc is similar to that of conventional circuit interrupters drawing an arc in air.

When the arc is drawn on the arcing contacts, the action of the blow-in field moves its terminals onto stationary arc horns very quickly, permitting the movable arcing contact to continue its opening stroke independent of the motion of the arc. The speed of arcing contact separation is sufficient to insure adequate break distance of the contacts at the time of interruption. To prevent possible retardation due to movement of the arc into a closed chamber, vents are provided at each end. As shown in Fig. 8, parallel copper plates spaced quite close together are placed in the vents to confine the arc and at the same time permit passage of air from the chamber. As the arc travels up the horns past the vents, it impinges on the metal plates of the deionizing chamber. As illustrated in Fig. 3, the lower end of these plates have a tapered slot. When a number of these plates are stacked together, these slots form a groove, roughly V-shaped, into which the arc is forced. The contour of the groove is such that as the arc moves upward, its cross section is decreased and the current density increased, with a corresponding increase in arc voltage. When a sufficiently high arc voltage is reached, the arc strikes to the plates, forming a series of short arcs which move into the circular portion of the plates under the influence of the blow-in field.

The portion of the groove below each radial field coil is made up of short plates introducing several gaps across the terminals of the coil. The increase in arc voltage resulting from the presence of several short series arcs in parallel with the coil insures sufficient voltage across its terminals to deflect the current into the more inductive path of the coil winding. Under the influence of the radial fields, the short arcs trace an annular path around the circular portion of the plates. This motion continues until a zero of current is reached, at which time deionization prevents further flow of current. Under these conditions, arcs have been found to move around this path more than 15 times in ½ cycle, with a velocity several times that of sound. This high speed results in what has been termed the "Cold Cathode Arc."[4]

Fig. 3 shows a plate taken from a Deion circuit-breaker on which approximately 300 rupturing tests were made, where arc terminals of currents up to 14,000

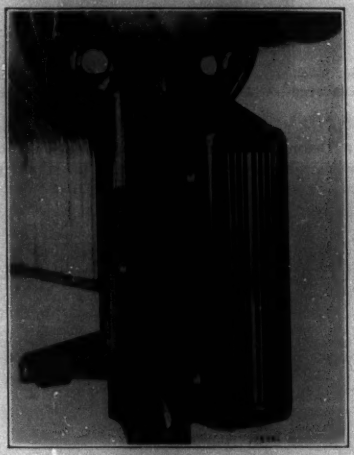

Fig. 8—Vent of Deionizing Chamber Showing Parallel Copper Plates

amperes have passed over the metal surface thousands of times without deterioration.

The mottled area extending from the end of the slot upward and around the circular portion of the plate is the retraced trail left by the arc. This mottled marking is only a very thin film of copper oxide and does not affect the operation of the breaker.

Tests

The first experimental Deion circuit-breakers built and tested were single-pole units. A large number of tests were made at 13,200 volts, 60 cycles, with two 20,000-kv-a. generators supplying the short-circuit current. The magnitude of short-circuit current was controlled by series air-core reactors. Currents interrupted varied from 1 ampere to 17,000 amperes r. m. s.

3. J. Slepian, *Jour. Franklin Inst.*, 201, p. 79, 1926; J. Slepian, *Phys. Review*, 27, p. 407, 1926.

at 13,200 volts and to 28,000 amperes r. m. s. at 6600 volts.

Figs. 9, 10, and 11 show typical oscillograms taken during these tests. The oscillogram shown in Fig. 9 was made on one of the earlier designs, while Figs. 10 and 11 were made on a later model. It may be noted that all of these oscillograms have the same general features, the most noticeable of which are the shape of the arc voltage wave and the short period of arcing.

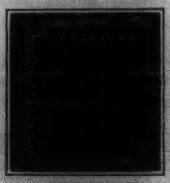

Fig. 9—Oscillogram of Single-Phase Short Circuit Interrupted by a Single-Pole Deion Circuit-Breaker

This is the characteristic form of arc voltage produced by a Deion circuit-breaker. The voltage is at first very low, while the arc is on the contacts and horns, and it may be regarded as simply an arc in open air. After it has been broken up into a series of short arcs and moved into the deionizing chamber, the arc voltage varies slightly with the current.

At currents above a few hundred amperes, the arc

Fig. 10—Oscillogram of Single-Phase Interruption on a Single-Pole Deion Circuit-Breaker. (Note that voltage across the blow-in coil was measured in this case)

voltage has a rising characteristic with increasing current, resulting in a convex arc voltage wave as shown in the oscillograms. At very low currents a concave arc voltage wave is obtained which is characteristic of the usual inverse arc voltage-current curve. The increasing arc voltage with decreasing current does not result in voltage surges as the current approaches zero. As the current wave passes through zero and is unable to increase in the opposite direction, the voltage reverses and rises almost instantaneously to a value depending on circuit conditions.

From the foregoing, it is apparent that a given deionizing structure will have a constant arc-voltage characteristic. That is, the arc voltage will depend only upon the current and the deionizing structure and will be independent of the open-circuit voltage. For large values of current, this arc voltage is high enough to reduce the current to less than its normal value. This is especially apparent when a given deionizing structure is tested at considerably less than its normal voltage.

In no case have tests shown that the Deion circuit-breaker has any undesirable effects on the system to which it is connected. In view of the fact that the arcing time is very short as compared to most conventional circuit-breakers, it is possible for the Deion cir-

Fig. 11—Oscillogram of Test on Single-Pole, 15-Kv. Deion Circuit-Breaker, Showing its Operation at Less Than ½ Normal Voltage

cuit breaker to remove short circuits from a system in a minimum of time with a minimum of system disturbance.

A Three-Pole 15,000-Volt Deion Circuit-Breaker

The three-pole, 15,000-volt Deion circuit-breaker described in this paper was given several series of interrupting tests, a part of which were laboratory tests and the remainder field tests. The most comprehensive laboratory series consisted of 250 rupturing tests at currents varying from 13,100 amperes to 586 amperes at 13,200, 7600, and 6600 volts, grounded and ungrounded, with both star and delta generator connections. These tests were made at an average of 12 tests per day, the highest number on a single day being 32. The total of 250 tests was made without a failure to clear the circuit and with very little maintenance. One hundred and fifty of these tests were made with no maintenance whatever. During the remainder, only minor adjustments were made and several of the arcing contacts were renewed. At the end of these tests, the breaker was in satisfactory operating condition.

Fig. 12 shows current and voltage characteristics of an ungrounded short circuit interrupted by this Deion circuit-breaker. The generator voltage in this test was

13,200 line-to-line and the average current in the three phases was 8170 amperes r. m. s. The oscillogram shown in Fig. 13 was made on the same test by means of instantaneous watt oscillograph elements and from it the total arc energy can be obtained. Fig. 14 is a side

Fig. 12—Oscillogram Showing Line Current and Line to Neutral Voltage in a Three-Phase Ungrounded Short Circuit, Interrupted by a Three-Pole Deion Circuit-Breaker

view of the breaker made simultaneously with these oscillograms, while the breaker was interrupting the circuit. Fig. 15 is similar to Fig. 12 except that the

Fig. 13—Instantaneous Power Oscillogram Made Simultaneously with Oscillogram Shown in Fig. 12

short-circuit is grounded, the average of the three currents in this case being 9630 amperes r. m. s. The photograph made with this oscillogram is shown in Fig. 16, which is a rear view of the breaker. Light from the arc as it moves up the horns may be seen through the vent of the right-hand pole. The light from the arcs in the other two poles is not visible because the camera was not directly in line with the vent plates. It is also possible to see the arc chutes of the transfer contacts, directly below the vents.

It is apparent that a large part of the arc energy liberated in interrupting a short-circuit will be absorbed by the metal plates of the deionizing structure. This arc energy has been measured in a large number of tests by means of the oscillograph, as mentioned before. Fig. 17 shows the results of some of these tests, the total arc energy per three-phase interruption being plotted

Fig. 14—View of Deion Circuit-Breaker Taken Simultaneously with Oscillogram Shown in Figs. 12 and 13

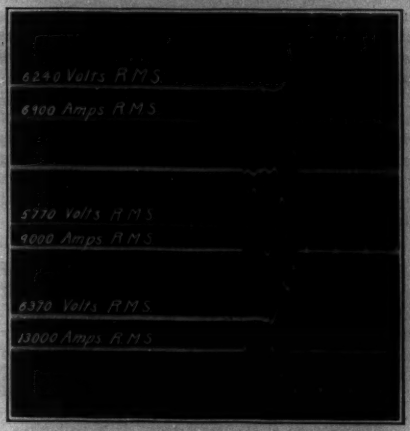

Fig. 15—Oscillogram of a Three-Phase Grounded Short Circuit Interrupted by a Three-Pole Deion Circuit-Breaker

against the average of the r. m. s. currents interrupted in the three phases. The average arc energy per interruption per pole is small in comparison to the large thermal capacity of the deionizing structure, so that the breaker is capable of withstanding much more severe operating duty than is encountered in modern applications. To investigate this, several series of laboratory tests were made on the three-pole breaker described in this paper. These series consisted of 12-CO interruptions at two minute intervals in a total time of 24

minutes, with the average value of the r. m. s. currents interrupted exceeding 8000 amperes. Currents interrupted in individual phases varied from approximately 5000 amperes to 12,000 amperes r. m. s. At the end of

Fig. 16—View Taken with Oscillogram Shown in Fig. 15

each one of these series of tests, the breaker was in satisfactory condition to perform further interrupting duty. In all series of tests in which the deionizing structure was heated to its limiting temperature, it was only necessary to allow the breaker to cool before further interruptions could be made.

It is obvious that a given deionizing structure will have a thermal limitation in regard to the number of interruptions it can make in rapid succession. It is also obvious that the thermal capacity of the deionizing structure can be controlled, and that by proper design a given structure can be made to have a factor of safety over any practical operating condition.

Application

From the manner in which the Deion circuit-breaker functions, it is apparent that zero points in the current wave play a most important part in its operation, which makes it most effective as an a-c. device. Strictly speaking, the type of deionizing chambers referred to in this paper is applicable to d-c. circuit interrupters, but there are some indications at present that the voltage at which a given structure will function on d-c. may be in the order of 0.2 of the limiting a-c. voltage, so that the advantages to be gained by the use of this form of structure on d-c. circuits are not at the present time outstanding. Due to extensive development over a long period, sufficient data and experience have been obtained to make possible the extension of the Deion principles to a large number of different classes of commercial a-c. switching apparatus. Industrial contactors operating on the Deion principles have been in service in considerable numbers for periods up to one year operating at as high as 440 volts. A limited number of three-phase Deion circuit-breakers has been operating under service conditions on 2300-volt circuits for more than one year. The performance of both the industrial contactors and the 2500-volt circuit breakers has been entirely satisfactory and represents a considerable advance over results obtained with conventional magnetic blowout devices applied to this class of service. Extensive tests have also been made on Deion circuit-breakers at 4500 and 7500 volts, the results of which warrant the belief that they can be placed in heavy duty service without encountering serious difficulties. This general development has led up to the building and testing of heavy-duty three-pole Deion circuit-breakers for operation in the 15,000-volt class. The results obtained in laboratory and field tests, as presented in this and other papers before the Institute, warrant the belief that Deion circuit-breakers as at present developed are applicable through the power-house class of breakers at modern generating voltages.

With reference to higher voltages, there appears to

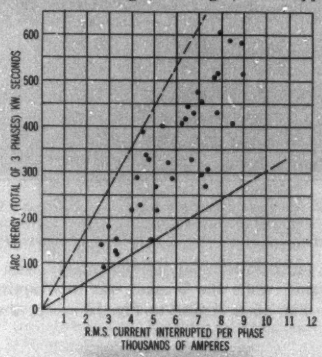

Fig. 17—Curve Showing Relationship Between Total Three-Phase Arc Energy and R. m. s. Current Interrupted on a Three-Pole 15,000-Volt Deion Circuit-Breaker

be no great difficulty in extending the Deion principles beyond 25 kv. However, there are certain detail problems involved which are not yet worked out and the discussion of developments along this line will be left for future papers.

Acknowledgment

This paper would be incomplete without acknowledgment to the Commonwealth Edison Company of Chicago for its courtesy in extending the use of its operating equipment for test purposes and to the various members of its engineering and testing organization for their efficient arrangements made for conducting the tests.

Acknowledgment is made also to Mr. H. M. Wilcox for his guidance in carrying on some phases of work on the Deion circuit-breaker, and for his very material assistance in the preparation of this paper.

Discussion

For discussion of this paper see page 545.

Field Tests of the Deion Circuit Breaker

BY B. G. JAMIESON[1]

Fellow, A. I. E. E.

Synopsis.—The operating principles of a new type of air circuit breaker for alternating currents are herein outlined together with results of field test of a 2000-ampere, 15,000-volt, three-phase unit. The special testing facilities of the Commonwealth Edison Company used for the test, the performances of the air circuit breakers under test, and the effects of the test currents on the power system are described.

* * * * *

THE power industry has been confronted by switching problems from its infancy. Development of the a-c. generator opened up vast possibilities for the deconcentration of power units in localities most favorable for generating purposes with transmission of power over long distances to consumers. With the advent of cheap power, consumption in the most favored industrial and mercantile districts increased, calling for larger and larger concentrations of generating equipment with more extended and complicated distribution systems. At each step in this advance, the question of adequate and dependable apparatus for control of the flow of energy has arisen for solution.

With the growth of the power industry, the oil circuit-breaker has assumed a commanding position as the best-adapted apparatus available for switching of power circuits in normal service and, through various combinations of control relays, for clearing these circuits under fault or abnormal operating conditions. This has come about if not with full agreement of the operating companies, at least with their acquiescence, since there was admittedly no known interrupting medium better adapted to the service.

Acquiescence on the part of the operators, however, was not entirely without misgivings and the hazard involved in the use of large quantities of oil in connection with the interruption of heavy short circuits has been ever present in the operators' minds and has been emphasized by their representative associations. Co-operation between operating companies and manufacturers has resulted in the development of types of construction which make this hazard somewhat more remote in the modern oil circuit-breaker, and as a further safeguard, some circuit-breakers have been installed out of doors although their logical location is inside the power station with the exception of breakers operating at the higher transmission voltages which admittedly must be located out of doors. This latter move has, however, increased the work and cost of maintenance, due to difficulties encountered from the presence of moisture in oil together with other troubles arising from varying weather conditions, so that the possibility of service interruption is still present in some measure.

In view of these facts, the attitude of the power industry has been one of open-mindedness toward the advent of any means of circuit interruption applicable to heavy-current circuits in the generating and distribution voltage classes, which should not involve the use of oil. If this interrupting means be applicable in the transmission voltage class as well, so much the better, but air insulation for electrical clearances runs into considerable distance for the upper range of transmission voltages and the conventional oil breaker in this class may carry advantages in the form of compactness, due to its oil insulation, that offset other disadvantages. By far the greater proportion of present-day switching is carried out on the low side of the transformers, and any practical interrupting medium which will remove the oil hazard from this class of switching service has a permanent field of usefulness.

It was with considerable interest, therefore, that the author of this paper was informed by a responsible manufacturer of switching apparatus that a new type of circuit-breaker, operating without oil, has been developed to the point of interrupting consistently the maximum short-circuit current available from a 40,000 kv-a. test circuit operating at 13,200 volts. Further discussion disclosed that the interrupting performance of this device, called the Deion circuit-breaker, is based on a means for deionizing the arc stream at the zero point of the current wave to such extent that the impressed voltage is not sufficient to re-establish the circuit and permit current to pass on the succeeding alternation. By this means, an arc is extinguished in air without resorting to the expedient of extending it to great length as is characteristic of the present-day conception of the air-break circuit-breaker. A discussion of the theory of this device and its operation is contained in other papers[2] to be presented before this meeting of the Institute and need not be presented in further detail here.

Upon the statement of the manufacturers that the development of this circuit-breaker had reached a point where they were desirous of obtaining data as to its

1. Vice-President, Engineer of Inside Plant, Commonwealth Edison Company, Chicago, Ill.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

2. *Theory of the Deion Circuit-Breaker*, by Dr. J. Slepian.

Structural Development of the Deion Circuit-Breaker up to 15,000 Volts, by R. C. Dickinson and B. P. Baker.

29-37

performance under interrupting duty on an operating system, arrangements were made for a series of tests at the Crawford Avenue Station of the Commonwealth Edison Company, and it is the purpose of this paper to discuss these tests together with the results obtained and the impressions formed from performance of the breaker on test.

The Deion Circuit-Breaker

The breaker supplied by the manufacturer for these tests was a three-pole, 2000-ampere, 15,000-volt,

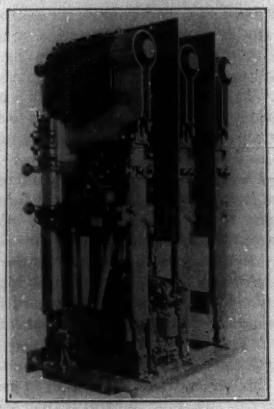

Fig. 1—Three-Pole, 2000-Ampere, 15-Kv., Deion Circuit-Breaker

Used at Crawford Avenue test circuit for the group-phase tests

electrically-operated Deion circuit-breaker of the multiple-single-pole form of construction as shown in Fig. 1. The three-pole units were mounted upright on a common structural steel base at 16-inch centers, and were operated through a single shaft by means of a conventional solenoid mechanism. The base or common mounting frame is at ground potential but the contact operating linkage for each pole unit is alive at line potential and is mounted on upright insulating posts supported by the grounded base. Operation of this linkage is obtained through insulating pull rods. The deionizing chamber which is the real interrupting medium, is mounted at the top of the structure in such relationship to the contacts that an arc drawn from the arcing members is moved into the chamber by a magnetic blow-in field of new design especially developed for this device. This breaker being of 2000-ampere normal current-carrying capacity, the blow-in coils are of necessity in shunt relationship to the main power circuit and are switched into series relationship with this circuit by a pair of auxiliary transfer contacts operating in proper sequence with the main current-carrying and the arcing contacts.

The manufacturer stated that this breaker was the first model of a commercial form of the Deion circuit-breaker which had gradually been evolved from development work covering an extensive laboratory testing experience with a number of experimental forms of construction. It was pointed out, however, that as a broader experience was obtained with the operation of this breaker under service conditions, and its application was extended to installations of varying requirements, conditions would undoubtedly be encountered pointing to the desirability of modification in some details. The general form of construction he believes well adapted to the application of this principle in service.

At the end of the initial series of tests with the breaker in the form supplied by the manufacturer, the

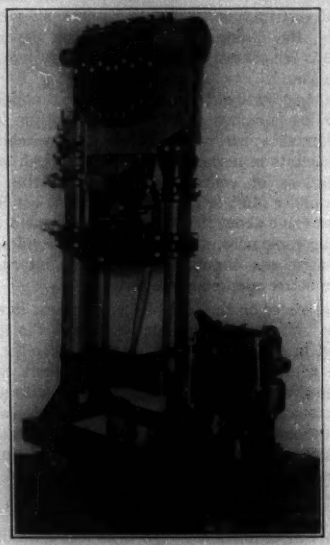

Fig. 2—One of the Deion Circuit-Breaker Pole Units Used for the Isolated-Phase Tests

author of this paper suggested that a further series of tests be made on the basis of an isolated phase application. The three-pole units were accordingly mounted on individual structural steel bases, each with its individual-solenoid closing mechanism, and connected to the testing circuit as individual units with no mechanical tie between poles other than the short-circuiting bar which was connected across the terminals of the three units. Fig. 2 shows the general arrangement of one of

the pole units used for this test. It was at first proposed to use the identical units for this test which had been tested on the previous series but the time allotted for completion was short and it developed that the necessary alterations could be completed more quickly by diverting three additional pole units then nearing completion in the factory and this course was finally adopted.

Test Equipment and Methods

A testing circuit with permanently installed recording equipment had been set up at Crawford Avenue Station over two years ago in connection with the general testing program of the Commonwealth Edison Company. Considerable pains were taken to make this installation conform in all respects to the rules for standard test procedure adopted by the Institute. The purpose of this installation was not solely to conduct interrupting tests on circuit-breakers but to make a study as well of the effects of a fault and its interruption on the remainder of the system linked to the fault. It has been apparent for some time that inability of large operating systems to withstand shocks is a serious limitation to their usefulness as sources of power for commercial and industrial purposes. The question assumes increasing importance as the trend continues toward larger and larger concentrations of generating equipment on the one hand and the linking of heavy systems through interconnection on the other. A study of these limitations is essential to the successful operation of large power systems and this phase of the work was, accordingly, given a prominent place in the program of tests. Such a study does include, however, consideration of various means of removing a fault and the time required for the clearing agent to perform this function.

The test circuit proper is connected to the 12,000-volt distribution system through suitable switches in Crawford Avenue Station. It consists essentially of approximately 500 ft. of 22-kv. underground cable and roughly the same length of three-phase overhead line mounted on wooden poles and insulated for 66-kv. Sheet metal houses of portable construction are located at intervals directly under the overhead line with tapped connections bringing the three phases into a rack in each house. These houses are used as testing cells and as enclosures for the permanent circuit equipment, some of which is indoor apparatus. Hand-operated disconnects are located in the overhead line at the pothead connection to the cables, permitting complete isolation of all test apparatus without recourse to switches in the station. Two backing-up oil circuit-breakers are permanently connected on the test circuit side of the disconnects. One of these breakers is used for closing purposes in the event of tests being made on the "CO" basis and its controls are arranged to retain the contacts in the closed position until tripped manually in order that its opening operation may not cloud the results to be obtained from the test breaker. The second breaker is arranged to open through relay control at a predetermined time interval after the short circuit has been applied, to clear the circuit in event of failure of the test breaker. Fig. 3 shows a schematic diagram of the test circuit connections and its relation to the distribution system.

A larger house of the same general construction as the test cells is used as a control room with all circuit controls and recording instruments permanently installed. A 125-volt battery is installed for solenoid operation, and 110- and 220-volt a-c. circuits are also available for control purposes when desired. The control room is also used as an observation post when tests are in progress. Telephone connections to the load dispatcher's desk and to other stations are available for the coordination of test operations with general operation of the system.

The main control table has been especially developed

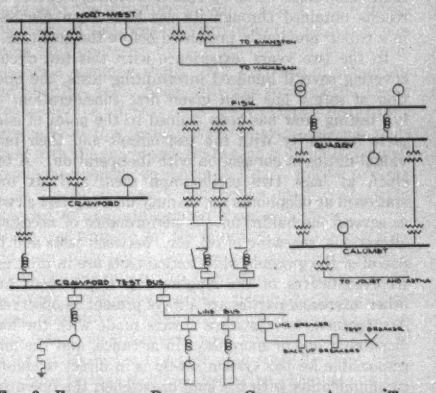

Fig. 3—Elementary Diagram of Crawford Avenue Test Circuit and System Connections

for use with this circuit. All test operations are controlled through a motor-driven controller, consisting essentially of a number of adjustable cams rotating on a common shaft and so arranged as to close or open the desired control circuits in proper sequence and at predetermined time intervals. Two years experience with this controller indicates that it is far preferable to control through relay operation since all operations are related to a common source, the revolving shaft, and variables are thus reduced to a minimum. Strictly speaking, test operations conducted with this device are electrically non-automatic but it is felt that the ability to increase or decrease the duration of short circuit accurately in very short steps is a valuable feature in conducting tests of this nature.

Two six-element Westinghouse oscillographs are mounted on the control table and permanently con-

nected to the motor-driven controller. The use of permanent connections throughout permits standardization of calibration for oscillograph circuits and test operators soon become sufficiently familiar with the circuit to make proper adjustment of resistances for any given oscillograph element and for any system set-up to obtain satisfactory films without preliminary trial tests. Adequate test records are thus obtainable with a minimum of test operations. Dark-room facilities are provided in the control room, permitting the development of films as each test is made in cases where it is desirable to study the results of one test before proceeding with the next.

As a part of the study of the system under fault conditions, Hall recorders are installed at points in the system where experience indicates the greatest voltage disturbance will occur. In this manner, valuable information is obtained as to the transient effects at different points due to various system set-ups under fault conditions. The operation of this device and the results obtained through its use have been discussed in a paper[3] previously presented before the Institute.

In the two years' experience with this test circuit, covering several hundred interrupting tests, the question of safety has been given first consideration. A full testing crew has been trained to the point of complete familiarity with the test circuit and their individual duties in connection with its operation. A test chief, at least two oscillograph men, and six men stationed at telephones are on duty at every test as well as several mechanics for the performance of necessary alterations, renewing of oil, etc., between tests and for patrol of the grounds while actual tests are in progress. Representatives of the Engineering Department and other interested parties are always present as observers. Arrangement for tests are always made with the load dispatcher one or more days in advance, and the man responsible for the system set-up is in direct telephone communication with the load dispatcher, the operating gallery of Crawford Avenue Station, and with the operators of any neighboring stations at which disturbance is anticipated, during the progress of all tests. To obtain the benefit of high generating capacity on the system, all tests are made during regular working hours of the business day, rather than at night or on Sundays.

In order to secure protection for the generators, the usual practise is followed of so arranging the system set-up that no power bus is left without the protection of a reactor. In addition to the backing-up breaker in the test circuit, two more protective breakers are involved in the test bus at Crawford Avenue Station as protection for the generators, although control relays are so set that these circuit-breakers are called upon for service only in event of failure for any reason of the first backing-up breakers to function, or in the event of a fault between this breaker and the test bus.

3. *The Hall High-Speed Recorder*, E. M. Tingley, A. I. E. E. Quarterly TRANS., January 1928, p. 252.

All of this testing experience was brought to bear on the two series of tests made with the Deion circuit-breaker. As shown in Fig. 3, one 66-kv. cable line to Northwest Station and three 22-kv. cables to Fisk Street Station were connected to the 12,000-volt test bus at Crawford Avenue Station, and thus to the test circuit for these tests. Various transformers and reactors were shorted out of these circuits as the tests proceeded, in order to secure the desired steps in the range of short circuit current values. In keeping with the usual test practise on this circuit, no temporary reactors were used in the test circuit nor were any temporary cross connections made in station circuits except in the case of one test for which it was desired to secure a very low value of short circuit current (approximately 1100 amperes) and which it was not feasible to obtain using the available system connections.

With the reactance of these circuits reduced to the minimum value consistent with the safety provisions outlined in a previous paragraph, the maximum short circuit current at the test breaker was approximately 10,000 amperes, average current interrupted for the three phases. Under these conditions, and for all short circuit currents up to this value, the currents in the three phases were approximately equal, the current decrement is comparatively small, the degree of assymmetry is not very pronounced, and the delta recovery voltage is approximately equal to the initial voltage.

When tests with the Deion circuit-breaker had reached this point of maximum current obtainable with the portion of the system connected, the breaker's performance in interrupting the circuit with a very short duration of arcing had been sufficiently consistent to warrant the belief that it would function satisfactorily over the next succeeding steps in the current range and a generator was, accordingly, connected direct to the test bus in Crawford Avenue Station in order to increase the current value at the point of short circuit. This means of securing additional power had never been attempted for in previous circuit-breaker tests, due to the fear of severe system disturbance following a prolonged duration of fault with the current values involved. The results obtained were, therefore, regarded with considerable interest aside from performance of the test breaker.

Three different generating units, varying in capacity from 60,000 to 75,000 kv-a., were used in this manner during the series of tests with the Deion circuit-breaker. These generators are driven by steam turbines and it was considered advisable to have them carrying some load rather than to run light when the short circuit was applied, in order to minimize the possibility of tripping out the steam end. They were, accordingly, operated to feed power into the system during these tests in amounts varying from 10,000 to 38,000 kw. The circuits were so arranged that closing in on the short in the test circuit had the effect of short circuiting

the generator, and clearing the test circuit automatically returned the generator to the system again. Permanent generator reactors to the value of one-eighth of an ohm were connected between the generator and the test bus. The generator neutral was grounded through approximately four ohms resistance with no neutral ground other than this on the various buses involved in the circuit. Tests made with the system alone were entirely ungrounded. No extensive cable system was directly connected to the test bus except that two idle three-phase cables were allowed to remain connected at times to note the effect on arc rupture.

The addition of steam driven generating capacity proved very satisfactory on the Deion circuit-breaker tests, and added quite materially to the current values obtainable on the test circuit. For tests made on the "OCO" basis, the load on the generator was varied from 10,000 to 28,000 kv. With this maximum load, the first-cycle short-circuit current was approximately 19,000 amperes r. m. s., and the current interrupted approximately 14,000 amperes, average for the three phases. For "CO" tests, the generator load was increased to a maximum of 38,000 kw. for tests where the motor-driven controller was set to give a duration of short circuit of from two to three cycles. This maximum load gave first-cycle short-circuit currents of approximately 30,000 amperes r. m. s., and the current interrupted was approximately 22,000 amperes, average for the three phases.

A total of 38 tests was made with a generator feeding varying amounts of load into the system and in all cases, the generator returned automatically to feeding the system as soon as the short was cleared, without system disturbance other than four volt dips in lamp voltage in a few cases and without injury to the generator. The maximum duration of short circuit was 16 cycles, varying from this time down to four cycles for maximum currents, except for two tests in which difficulty was encountered in operation of the test breaker and in which the circuit was cleared by the backing-up breaker after a period of 49 cycles (as determined by the motor-driven controller). On none of these tests did the generator drop out of step nor was any effect noted on low voltage releases on the system and but very little effect on lamps in the station. No so-called transients appeared during the course of these tests.

Connection of the generator to the test bus produced a noticeable effect on short circuit characteristics as compared with tests which involved use of the system alone, and this effect became more marked as the loading of the generator was increased. Considerable variation appears between the values of short circuit current for the individual phases, the degree of assymmetry is more pronounced, the current decrement becomes larger, and at the maximum currents tested, several cycles were required for the delta recovery voltage to approximate the initial voltage.

In general, the test procedure as outlined here has proved very satisfactory. It is believed that the short circuit current value of 30,000 amperes r. m. s. on the first cycle, obtained on the Deion circuit-breaker tests, represents about the maximum to which the test circuit as at present equipped should be subjected (if safety of operation is to be maintained). Future developments in connection with the testing program of the Commonwealth Edison Company contemplate further tieing the 66-kv. transmission system to the 12,000-volt bus through three 20,000 kv-a., 66,000/12,000-volt transformers. It is estimated that when backed by the entire system, short circuit currents of 50,000 amperes may be obtained at 12,000 volts with almost instant 100 per cent recovery voltage. With suitable modifications in the test circuit, interrupting tests at these values are contemplated without fear of the results.

Description of Group-Phase Tests

The first series of Deion circuit-breaker tests was made with the structure shown in Fig. 1. The three-pole units of this breaker were mounted on a common frame and operated by a single-solenoid mechanism through a common shaft. For purposes of simplicity and identity, tests made with this form of the Deion breaker will be referred to as "group-phase" tests in the remainder of this paper. For the same reasons, tests made with the single pole units each with an individual closing mechanism, as shown in Fig. 2, will be referred to as "isolated-phase" tests since these tests were intended to simulate isolated phase conditions.

The test breaker for the series of group-phase tests had been subjected to previous interrupting duty under laboratory conditions, in connection with the manufacturer's development work. According to his statement, this duty covered the range of current values from 500 to 9000 amperes with voltages from 6600 to 13,200 at 60 cycles, together with some 25-cycle tests. He states further that the deionizing chambers on this breaker had never been dis-assembled since the start of these laboratory tests, and that no other maintenance had been given the breaker during this time, except that it had been equipped with a new set of arcing contacts and auxiliary transfer contacts just previous to its delivery at Crawford Avenue, the purpose being to note the deterioration of these contacts on interrupting duty under system conditions.

In laying down a program for these tests, the question of duty cycle arose. It was desired that these tests should conform in all respects to the rules for standard test procedure adopted by the Institute and these rules stipulate that the test duty shall consist of two "OCO" operations with a two-minute interval. This rule, quite obviously, was adopted with the characteristics of the oil circuit-breaker in mind. It was believed that this duty cycle should be regarded as a minimum requirement but that the characteristics of the Deion circuit-breaker would permit more latitude in the

selection of a duty cycle than was practicable with the oil breaker, and it was felt that data should be obtained from these tests to determine the effect of a more severe duty cycle. The manufacturer was of an open mind on the subject and it was finally decided to retain the time interval of two minutes between operating cycles but to increase the number of operations to three. This number was selected rather as a matter of convenience than with any reference to breaker characteristics. The oscillograph equipment in use with the test circuit allows three exposures at short intervals after which a sufficient interval of time must elapse for insertion of a new film. The 3-OCO duty cycle, therefore, became a convenient one and was used throughout these tests. After some tests had been made, the time interval was reduced to one minute to observe the effect on the performance of the breaker, and as there was no apparent difference in performance, the remainder of the tests were made on the basis of three "OCO" operations at one-minute intervals.

In accordance with the usual test procedure, a 25-kv. potential test to ground was applied across the three phases of the complete testing circuit, including the test breaker, for one minute. This test was satisfactory and the interrupting tests proceeded as soon afterward as the system set-up could be made.

The current value for the first interrupting test was approximately 1100 amperes and required the special system connection previously referred to. This current value was the lowest of the series and the test is of interest as showing the interrupting performance of the Deion breaker at currents comparable to those involved in normal switching operations. Of the nine phase interruptions on this test, seven phases were interrupted with two half-cycles of arcing while two phases showed arcing in parts of three half-cycles. Analysis of these results indicates that the arc is not necessarily extinguished at the first zero occurring after it originates. Apparently an appreciable time required for the blow-in field to move the arc into the deionizing chamber and in the event of a zero occurring during this transmission period, the arc is not extinguished but persists until it has entered the chamber. The results of subsequent tests indicate that arcing may appear in parts of more than one half-cycle on any interruption, with the possible exception of very heavy currents having relatively large inherent blow-in effect.

The six succeeding tests were all made within a period of slightly less than two hours and used the system only as a source of power. Short-circuit current values ranged from 2000 to 10,000 amperes in the six steps. For the 18 three-phase interruptions in this group, in no case did any phase show arcing for more than two half-cycles and in many cases, the arcing was confined to a single half-cycle.

Fig. 4—Oscillogram of Short-Circuit Test at 4200 Amperes

Together with Hall record, taken during group-phase tests
(For test data, see Table I, Test No. 11-7)

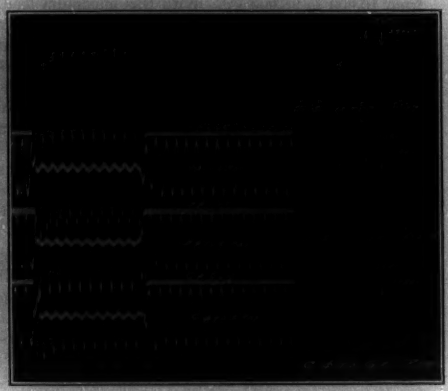

Fig. 5—Oscillogram of Short-Circuit Test at 10,000 Amperes

Together with Hall record, taken during group-phase tests
(For test data, see Table I, Test No. 11-19)

Fig. 4 shows an oscillogram of a three-phase interruption on one of these tests in which the current interrupted was approximately 4200 amperes. Together with this oscillogram is shown the record obtained during this test from the Hall recorder at Fisk Street Station, five miles away, which was the nearest generating station supplying the energy. In Table I, Test No. 11-7, are given the data for this test which is representative of the breaker performance in this portion of the current range. Fig. 5 shows another

TABLE I

TABULATED DATA TAKEN FROM OSCILLOGRAMS SHOWN IN FIGS. 4, 5, 6, AND 7

Test No.	R. m. s. current						R. m. s. volts		Arcing period (Cycles)		
	Initial			Interrupted			Before test	Re-established			
	A	*B*	*C*	*A*	*B*	*C*			*A*	*B*	*C*
11- 7	5,820	5,100	7,300	4,440	4,120	4,230	11,600	10,800	0.500	0.500	0.445
11-19	13,000	11,000	11,700	10,150	10,000	9,740	12,000	11,600	0.500	0.334	0.445
11-27	15,700	19,700	20,300	11,500	12,400	11,700	12,400	10,200	0.445	0.445	0.400
11-31	21,700	25,100	29,600	15,400	14,100	16,200	12,700	11,300	0.450	0.330	0.330

representative oscillogram with the corresponding record from the Hall Recorder, made during one of these tests at 10,000 amperes. Data for this test are given in Table I, Test No. 11-19. This was the highest current value obtained, using the system as a sole source of power.

The performance of the test breaker having been consistent throughout the tests up to this point, it

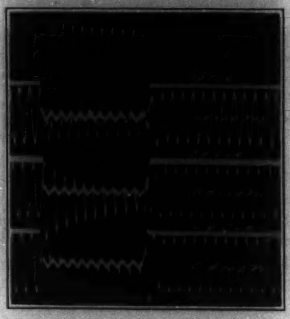

FIG. 6—OSCILLOGRAM OF GROUP-PHASE SHORT-CIRCUIT TEST AT 12,000 AMPERES

(For test data, see Table I, Test No. 11-27)

was decided to connect No. 2 Generator at Crawford Avenue to the test bus in order to increase the value of short circuit current. Four tests were made under this condition with the generator load varying from 10,000 to 28,000 kw. and the short circuit current value from 10,000 to 15,000 amperes. One test was made approximately one-half hour after the series of six tests just described and the remaining three the next morning in a period of slightly over one hour.

Of the twelve three-phase interruptions included in these four tests, eleven were satisfactory as to performance of the test breaker. Fig. 6 shows a representative oscillogram of a test at approximately 12,000 amperes, and Fig. 7, one at 15,000 amperes. No records were made with the Hall recorder during this series. The data for these two tests are given in Table I, Tests No. 11-27 and 11-31.

On the twelfth and last three-phase interruption, heavy flashing was observed on the breaker which persisted until the circuit was cleared by the backing-up breaker. The reason for this flashing was not entirely clear but upon examination of the breaker, it was found that two pieces of arc-resisting material had become loosened and dropped out of the breaker at some time during the test. It was concluded that this material had fallen between breaker parts of opposite potential, thus establishing an arc outside the control of the magnetic blow-in field and this arc had persisted until the circuit was cleared by other apparatus. Developments on subsequent tests, to be discussed later in this paper, throw additional light on the performance of the breaker during this test.

As nearly as could be determined, flashing on this

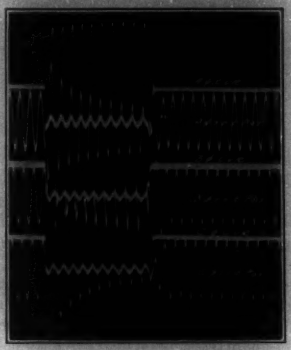

FIG. 7—OSCILLOGRAM OF GROUP-PHASE SHORT-CIRCUIT TEST AT 15,000 AMPERES

(For test data, see Table I, Test No. 11-31)

test had started in the contact operating levers of Phase *C* pole unit and moved to a position across the terminal studs of that phase. A short-circuiting bar had been placed across the lower studs of the three pole units for the group-phase tests and through this bar, an arc finally became established across the terminal studs of each pole unit. The breaker was still in good condition, aside from some scorching of insulating surfaces, and a light deposit of carbon.

A review of the test results obtained up to this point showed that eleven interrupting tests had been made

with the breaker at short circuit currents of from 1100 to 15,000 amperes r. m. s., average for the three phases. Of the 33 three-phase interruptions involved, breaker performance on 32 had been satisfactory and consistent while one was unsatisfactory and this at a current value slightly lower than had previously been interrupted by the breaker. This demonstration of breaker performance was believed to be sufficiently satisfactory to warrant further testing work and the author of this paper suggested a series of tests, using the three pole

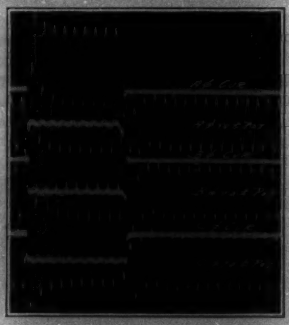

FIG. 8—OSCILLOGRAM OF SHORT-CIRCUIT TEST AT 4000 AMPERES TAKEN DURING ISOLATED-PHASE TESTS

(For test data, see Table II, Test No. 14-5)

units operating independently without a mechanical tie, which should simulate isolated-phase conditions.

DESCRIPTION OF ISOLATED-PHASE TESTS

Three single-pole Deion circuit-breaker units, as shown in Fig. 2 and previously described in this paper, were supplied by the manufacturer for the series of isolated-phase tests. These units were new, having had no previous interrupting duty, and were assembled in this form especially for these tests so that a period of about 30 days elapsed between the group-phase and the isolated-phase tests.

The three-pole units were mounted in the test cell on approximately 30-inch centers which was about the maximum spacing the cell would allow leaving the desired clearances and working space. It was believed that this spacing would permit interruption of short circuits without appreciable magnetic influence due to inter-phase action. Barriers of ½-inch thick asbestos were erected between the poles in such manner as to preclude the possibility of flashing between phases. The short-circuiting connection was placed across the lower terminal studs of the three units as in the group-phase tests, but there was no other mechanical tie between the units.

The first five tests were made, using the system only as a source of power and with short circuit currents of from 2500 to 10,000 amperes. All of these tests were made on the basis of a 3-OCO duty cycle at one-minute intervals as in the group-phase tests. Of the 15 three-phase interruptions involved, in no case did arcing appear on any phase for more than two half-cycles and in a great many cases, it appeared in one half-cycle

FIG. 9—OSCILLOGRAM OF SHORT-CIRCUIT TEST AT 7500 AMPERES TAKEN DURING ISOLATED-PHASE RESTS

(For test data, see Table II, Test No. 14-11)

only. Fig. 8 shows a representative oscillogram of the breaker interrupting approximately 4000 amperes and Fig. 9, a test at approximately 7500 amperes. The data for these tests are given in Table II, Test No. 14-5 and 14-11.

One single-phase short circuit test was made with the maximum current available on this set-up. For this test, the pull rod was disconnected on Phase *A* with the contacts in the open position, and the remaining two poles operated as usual on the 3-OCO duty cycle. No unusual features were observed in connection with this test. It may be noted that an effect on the test breaker similar to that of a single-phase short circuit

TABLE II

TABULATED DATA TAKEN FROM OSCILLOGRAMS SHOWN IN FIGS. 8, 9, 10, AND 11

Test No.	R. m. s. current						R. m. s. volts		Arcing period (Cycles)		
	Initial			Interrupted			Before test	Re-established			
	A	*B*	*C*	*A*	*B*	*C*			*A*	*B*	*C*
14- 5	5,120	6,520	6,240	4,030	4,030	4,350	11,500	11,000	0.445	0.445	None
14-11	8,740	9,340	8,620	7,760	7,300	7,780	11,700	12,200	0.334	0.665	0.334
14-31	22,000	14,400	20,600	11,600	11,000	11,400	12,800	11,300	0.443	0.521	0.626
14-43	22,000	25,400	25,600	14,200	15,900	15,600	12,200	11,800	0.500	0.500	0.700

was observed in several cases of the preceding three phase interruptions, in which one phase cleared slightly before the other two and one of the remaining pole units apparently did all the work of interruption since little or no arcing appeared in the third phase. In all of these cases, arcing in the second phase was confined to two half-cycles or less.

Performance of the test breaker having been satisfactory up to the point of maximum current available with the system set-up, No. 1 generator at Crawford Avenue was connected to the test bus and a 3-OCO test was made at a current value of approximately 13,000 amperes. The test breaker interrupted the circuit each time but some flashing was observed on Phase *A* at the third interruption. Inspection of the test breaker indicated that a small arc had been established across the terminals of the blow-in coil, which in this case were quite close together. The oscillogram showed that Phase *B* had functioned normally in interrupting its circuit but that Phase *A* had started to open about three cycles before the other two and that the arc had persisted in this phase until Phase *C* opened and cleared the circuit.

It was concluded that breakdown across the coil terminals had deprived Phase *A* of its blow-in field, resulting in a very slow movement of the arc toward the deionizing chamber and this arc apparently had not completely entered the chamber when Phase *C* cleared the circuit. It was believed advisable to inspect the deionizing chamber on Phase *A* before proceeding with further tests and it was completely dismantled. No signs of injury were found other than slight burning at the entrance of the chamber, evidently caused by slow movement of the arc. After a thorough cleaning the chamber was re-assembled and mounted on the pole unit.

The next test consisted of three "OCO" operations at 3500 amperes, using the system only, in order to check the performance of the breaker before proceeding with heavier currents. A single "OCO" test was then made, using the generator, with a short circuit current of approximately 14,000 amperes. The test breaker interrupted this circuit with arcing in parts of two half cycles on Phase *A* and with a single half cycle of arcing on the two remaining phases. The oscillogram showed, however, that Phase *A* was cleared two cycles before Phases *B* and *C*. The reason for this difference in operation was not entirely clear and the test was repeated with approximately the same result. A cycle-counter test on the three-pole units with no load showed that Phase *A* was opening consistently from 2 to 3 cycles ahead of the other two. Closing power for the isolated phase tests was applied to the three closing coils in parallel. Further investigation of the pole unit for Phase *A* showed that its contacts were not reaching the full-closed position before the closing current was cut off by the auxiliary switches on the other two pole units, which resulted in this unit beginning to open earlier than the rest. In view of these facts, it was decided that certain alterations should be made in this breaker before undertaking further "OCO" tests at heavy current values.

This difference in operation, however, did not affect the performance of the breaker when tested on the "CO" operating cycle and it was decided to proceed with testing on this basis in order to obtain data as to the breaker's interrupting performance at still heavier currents. All succeeding tests with this breaker were, accordingly, made with operating cycle of three "CO" interruptions at one minute intervals except the first which was a single "CO" interruption at approximately 7000 amperes, using the system only, as a check on the timing of the control circuit. The next test consisted of three "CO" interruptions with a current value of approximately 7500 amperes, using the system only, as a further check. This test was satisfactory and No. 3 generator at Crawford Avenue was connected to the test bus for all succeeding tests.

Fig. 10—Oscillogram of Isolated-Phase Short-Circuit Test at 11,000 Amperes

(For test data, see Table II, Test No. 14-31)

A series of tests was then made with 5 steps in current value, the first of which was 11,000 amperes. The generator load was then increased for the next two tests, in two steps, to a maximum of 38,000 kw., giving a short circuit current value of approximately 13,000 amperes. The duration of short circuit for the first three tests has been from seven to eight cycles and, for the next two tests, this time was reduced in two steps to a minimum of 2½ cycles in order to secure the benefit of increase in current value due to displacement. The maximum value of current interrupted on these tests was 22,400 amperes r. m. s., average for the three phases, and the maximum current interrupted in any single phase was 25,800 amperes r. m. s. Figs. 10 and 11 show oscillograms made during these tests, the corresponding test data being shown in Table II, Tests No. 14-31 and 14-43.

Performance of the test breaker had been satisfactory throughout these five steps in current value of from 11,000 to 22,400 amperes. The 15 three-phase interruptions covered by the tests had all been made within a period of 4 hr. 45 min., and the last twelve interruptions were made within a period of 1 hr. 40 min. The deionizing chambers had become very warm due to the rapid interruption of heavy currents, and it was decided to make an additional test with the same set-up in order to observe the breaker's performance under extreme temperature conditions within the deionizing chamber.

Of the three interruptions on this test, the first was normal and the second not greatly abnormal but, on the third interruption, very heavy flashing was observed on all phases and this flashing presisted until the circuit was cleared by other apparatus. Inspection of the test breaker showed that an arc had been established between opposite potentials in the contact operating levers of all pole units and had apparently persisted there until the circuit was cleared. Insulating surfaces were badly scorched and there was a considerable deposit of carbon, but otherwise the breaker was not seriously damaged.

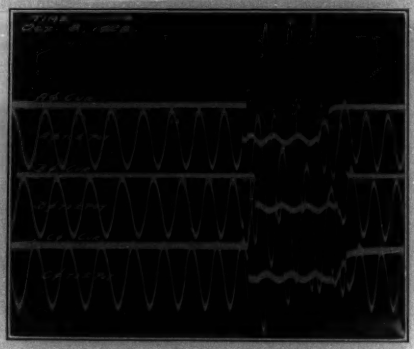

Fig. 11—Oscillogram of Isolated-Phase Short-Circuit Test at 15,200 Amperes

(For test data, see Table II, Test No. 14-43)

The oscillogram for the third interruption showed that the short-circuit current value at the time the breaker started to open was slightly less than 15,000 amperes r. m. s., average for the three phases, or about 67 per cent of the current value successfully interrupted on a test shortly before this. In view of this fact, and of the results of the preceding tests, it was concluded that this last test had demonstrated a limitation of the Deion circuit-breaker, not in maximum current rupturing ability, but in the severity of the operating duty to which it may be subjected.

Summary of Observations and Deductions

Severity of Test Duty Imposed. It was anticipated in arranging for these tests that the Deion circuit-breaker would yield a performance on the same test duty superior to that of an oil breaker because of the absence of oil, higher speed, and definiteness of arc suppression. Accordingly, the number of cycles of operating duty was increased from two to three and the time interval between cycles reduced from two minutes to one minute. Also, while no special effort was made to increase the number of tests to a maximum in a repetitive sense as in all system tests certain time allowances must be made for necessary supervisory functioning and for ascertainment of effects on the system, it was realized that the factors which were assumed would enable the more severe operating duty would also allow a considerable increase in the number of tests possible within a given period. Also it was appreciated that the service requirements for this type of breaker would not necessitate the extremes required of breakers in industrial service from the standpoint of frequent operation.

It was expected that the choice of this more severe operating duty would expedite the whole schedule of tests toward the end that the limitations of the breaker would become apparent more quickly and in a sense procedure on this basis might be regarded as an approach towards a destruction test rather than the establishing of safe operating interrupting capacity rating.

Speed of Test Program. On the first day of the group phase tests a total of 21 three-phase interruptions with currents varying from 2000 to 10,000 amperes was made in a period of 2 hr. 23 min. without evidence of limitation on the part of the breaker. On the succeeding day nine three-phase operations with currents varying from 11,000 to 15,000 amperes were made in a period of 1 hr. 9 min. This series of tests was ended by a disability of the breaker which, upon investigation, appeared to be partially due to heating of the deionizing chamber.

On the first day of the isolated phase tests, 12 three-phase interruptions with currents varying from 7000 to 13,000 amperes were made in the period of 1 hr. 42 min. The test breaker interrupted all of these short circuits when difficulty of a mechanical nature ended this series.

On the final day of the isolated phase unit tests, a total of 15 three-phase operations with currents varying from 12,600 to 22,400 amperes was made in the period of 2 hr. 40 min. These tests were made in the afternoon with breaker starting at a temperature somewhat above the ambient due to seven three-phase interruptions at varying currents made during the morning. This series of tests was ended by the temperature of the deionizing chamber becoming excessively high at the beginning of the final group of tests.

Performance of Test Breaker. In the foregoing text mention was made of the fact that the group-phase

breaker had undergone a protracted series of tests in the factory, and it should be stated that the isolated-phase breaker was assembled on short notice especially for the field tests and perhaps, therefore, not a commercial standard to the same extent as the group-phase breaker Considerable difficulty was experienced in the mechanical adjustments on the isolated-phase breaker, but all of the original contacts with the exception of one remained intact throughout the tests.

Very few renewals were required for the group-phase breaker. Throughout its test there was no renewal of insulation except that following the final interruption which was a failure due to the excessive temperature, the renewal of insulated parts in the vicinity of the arc contacts would have been required for further testing. During the tests there were no renewals of arcing of auxiliary contacts and despite the previous factory tests the main current carrying contacts were apparently in a condition to carry normal rated current without excessive heating at the end of the Chicago tests.

Summarizing the performance, the two breakers jointly interrupted 82 short circuits. Of the 82 operations, two were unsuccessful in clearing the circuit. These two failures are especially interesting from the point of view comparable with oil breaker failures. In neither of these cases was there any damage to the test house or barriers or other equipment in the test circuit. There was no resultant fire and due to the absence of oil-throwing the work of preparing the test cell for succeeding tests was very much expedited. The fact that practically instantaneous determination of the extent of the damage was possible should be a feature of paramount interest to all operating companies.

Another feature of interest in these tests was the extent to which it was possible to visualize the performance. In contrast to the oil breaker, witnesses at a distance of one hundred yards could judge from the flashes any irregularities in the closure or displacement of time with regard to the sequential phase operations or inequalities in amount of gas or smoke emitted. Moving pictures which were taken disclosed the presence of gas and sparks to an extent not possible with the naked eye, and it might be well to re-emphasize the importance of such records. At the same time it is realized that only experience in judging the performance by the movie camera films or actual presence at the same tests justifies conclusions from either of these visual indications of performance.

Limiting Factors in Interrupting Capacity. In this series of tests certain factors recognized as determinants of interrupting capacity were carefully measured or observed. They are: maximum current values, resultant temperatures, mechanical strength, arcing time, and also restoration voltage. Of these the current values reached 25,800 amperes without definite signs of distress on the part of the breaker. On account of the rapidity of successions of the test cycles the temperatures reached a value which appeared to limit the safe interrupting capacity of the breaker. Need of reinforcement of the mechanical strength in the construction of blow-out coils was indicated. One factor, however, which is regarded as a limiting influence in the operation of oil breakers, namely, the production of restoration voltage of relatively high value, as far as this factor may be attributed to the action of the breaker, was absent, and since according to the theory of the Deion breaker the extinguishing of the current at the zero value is a fundamental characteristic of its operation, the Deion breaker should be given full credit for the absence of this limiting feature of performance.

Indeterminate Factors of Interrupting Capacity. While these tests indicated a superiority of performance of the Deion breaker from relatively low ranges of current values, its performance at extremely low values such as that due to magnetizing current or charging current of cables was not disclosed, and this test must be made before the performance of the breaker can be compared to that of an oil breaker. Neither did the tests permit observation of the action of the breaker on closure under the maximum current duty since these values were employed only in the "CO" tests.

Attention is again invited to the fact that while the performance of the breaker must be ultimately judged from service on an operating system, the expected advantages of the Deion breaker were established by the test performance. Also it appears that the limitations of the breaker are more capable of definite determination than those of oil breakers, and finally it should be remembered that the tests as conducted were but tests of a unit the first of a type and that as such they may be fairly construed to indicate ultimate results from the deionizing type of circuit breaker not possible with types having a less scientific principle of arc disposal.

Discussion

THEORY OF THE DEION CIRCUIT BREAKER

(Slepian)

STRUCTURAL DEVELOPMENT OF THE DEION CIRCUIT BREAKER

(Dickinson and Baker)

FIELD TESTS ON THE DEION CIRCUIT BREAKER

(Jamieson)

New York, N. Y., January 30, 1929

P. H. Chase: These papers very forcibly illustrate the effectiveness of a determined attack upon a major problem in the electrical field; the solution undoubtedly resulted from research of a broad basic character and also, which is very important, from the willingness to consider and accept new theories.

Undoubtedly the oil circuit breaker of the present familiar type is the most unsatisfactory piece of apparatus on the power system. This is probably not due primarily to defects in workmanship, design, or application, but is due largely to the fact that the problem of arc extinction is an extremely difficult one, and the recognized methods of accomplishing it leave much to be desired, particularly when the requirements are severe. While the problem is an old one, it is acute at this time when

the tremendous size of power systems and the tendency to link them together through heavy ties has resulted in the existence of short-circuit conditions that are very difficult to handle.

The successful development of the deion breaker is of particular interest to the central-station engineer. It opens a vista of many possible revolutionary changes in design, construction, and central-station practise and promises many long-desired operating advantages. I want to refer to a few of them.

Of major importance is the promise of elimination of oil from all circuit breakers, with several desirable consequences: a pronounced decrease in life hazard; a decrease in the liability of property damage, with resultant lower fire insurance rates; elimination of oil-handling equipment; less complicated fire-fighting equipment; possibly less space required for isolation of equipment, and decreased inspection and maintenance costs.

Then, there is another series of possibilities that grow out of the apparent increased reliability and speed of operation. Among the foremost are the facilitation of relaying and the improvement of the selective clearing of faulty equipment in lines, which has an immediate effect in better service. Again, the higher speed of operation should have a decided improvement on the system operating stability.

In connection with high-voltage transmission lines, we should have more hope of clearing insulator flashovers on one phase, before there is even time for the arc to travel in a high wind and involve adjacent phases or adjacent circuits. There should be decreased damage at the point of fault. All of these point toward better operating reliability and better service afforded by the central stations.

S. M. Dean: There is one thought that occurs to me which is aside somewhat from the engineering aspect of the question. Those of us who have to deal with budget approval as well as system plans usually find the least definite part of our presentation to our executives to be the question of oil circuit-breaker performance. Inevitably they ask the question, "When we spend all this money are the infernal things going to blow up in ten years or in five years?" That question is very difficult to answer. We are going to be in a much better position to show them that in case the problem is attacked basically.

Operating Speed. On cable systems, the C–O operation of this breaker is of decided importance, and its high speed on such a cycle is very gratifying. Requirements as to stability are very uncertain. One may desire to connect a cable system in one manner today only to find it must be connected in another tomorrow, and to be able to have a margin in the speed of the circuit breaker is very useful.

Interrupting Capacity. While the tests to date on this breaker have been a repeated series of C–O or O–C–O tests, on cable systems particularly the single C–O shot is the thing of importance. The large interrogation point in our minds is, What will it do on the single shot? Will it open a short circuit of 1,500,000 kv-a. or is there some inherent limitation to a lower value? What immediately interests us on our particular system is higher interrupting tests on a single C–O basis.

Maintenance of the Breaker. In spite of our best efforts to deal with condensation and dampness we have had trouble. We don't want to be pessimists, but if insulations less subject to moisture troubles can be employed it will better satisfy us.

Safety revolves immediately around the question of no oil. To companies who look forward to large concentration in built up city areas with indoor switching, the elimination of oil, and with it the fire and explosion hazard, is a very real advantage.

S. M. Viele: From the point of view of a railroad man very much interested in heavy electric traction, the development of a circuit breaker which does not embody the use of oil, is a subject of great interest to railroads contemplating any major electrification.

Our past experience with oil, which is probably not materially different from that of the central station people, as well as the expense of its use, warrants the statement that oil should be reduced to a minimum. Oil elimination from circuit breakers offers the greatest opportunity in reduction of attendant hazard. This is particularly true in unattended substations. The development of air circuit breakers of the deion or equivalent type, for potentials to 25 kv., is one of major importance.

There is one thing, however, that must not be overlooked in this development, that is, the time required for operation must be reduced from that now utilized in the usual breaker equipment. It is essential that development should embody a time of operation for the selection of the circuit, relay operation, and functioning of the breaker not exceeding the remainder of the half cycle of current wave during which the short circuit occurs.

In other words, interruption of the circuit should take place approximately at the first zero of the current after the inception of the short circuit.

Naturally, faster performance than the above should not be carried to a point where such advantage is offset by the resultant increased potential of the circuit interrupted.

Such time performance as previously recommended would increase the dependability of the breaker equipment, increase the stability of systems, decrease the territory effected by short circuits, and eliminate many of the dilatorious effects which now result from short circuit.

We hope for the complete success of the development of an oil-less circuit breaker.

R. C. Mason: The interest of the Deion circuit breaker to electrical engineers centers chiefly in the high voltage which may be satisfactorily opened with only moderate dimensions. Besides this feature, there are involved other theoretical considerations, concerning the nature of the arc, which are of interest to physicists.

Dr. Slepian and his associates have brought to the attention of physicists most forcibly the fact that cold-cathode arcs can exist. It has long been known that discharges in gases could be obtained with cold electrodes, but always the discharges were glows, with very small current densities, of the order of milliamperes, and with large cathode falls, of the order of hundreds of volts. Here, however, has been described a heavy-current arc, with a current density of 30,000 amperes per sq. cm., and with a cathode fall certainly not over 30 volts, maintained between electrodes which were relatively cold. For some three or four years, the old idea that thermionic emission of electrons from the cathode was the essential and complete mechanism of the arc has been failing to explain completely some types of arcs. In some arcs, only part of the current could be given in that way. Now, we are offered evidence of an arc in which it is hard to imagine that any appreciable thermionic emission takes place.

To account for that part of the current not carried by electrons, two theories have been proposed: one, by Slepian, suggests that positive ions, arising from intense thermal ionization close to the cathode, carry the current to the cathode; the other, by Langmuir, proposes that a very high field pulls electrons out of a relatively cold electrode. The latter theory is possible, even though the total cathode fall is only a few volts, if it is concentrated in such a small space that the gradient is of the order of millions of volts per cm.

The volt-ampere characteristic of an ordinary arc is negative; that is, the voltage decreases with increasing current, and may be represented by an equation of the type,

$$V = a + b/I^n$$

From Equation (1) we see that the arc voltage of the cold-cathode arc in the Deion circuit breaker increases with increasing current, as given by,

$$V = a + b\,I^n$$

Arcs of this type, then, should operate stably in parallel, because of their positive volt-ampere characteristic. It is interesting to note that evidence is found of the existence of parallel arcs.

Whatever may be the final theory of the cold-cathode arc, the work of Dr. Slepian should offer a further stimulus to the study of a new type of arc phenomenon.

J. W. McNairy: Until recently the oil circuit breaker was practically unchallenged for applications above 2300 volts; however, successful commercial operation on 12-kv. single-phase circuits of air breakers of somewhat different design from that described in the papers presented is now an accomplished fact, the rupturing capacity being over 35,000 amperes.

We have followed with particular interest the series of papers presented by Dr. Slepian covering laboratory investigations of the phenomena associated with the electric arc in air as related to the performance of air circuit breakers. These papers bring up many interesting points, particularly with reference to the effect of metallic plates and screens introduced into the arc stream at right angles.

In following Dr. Slepian's theory as outlined in connection with the Deion breaker, it appears that the basic principle involved is the utilization of the rapid increase of dielectric strength of the ionized gas because of the quick accumulation of space charges immediately adjacent to numerous cathodes in series at the *instant the current wave* passes through its normal zero.

However, on examining the oscillograms contained in the papers by Mr. Jamieson, and Messrs. Dickinson and Baker, it was noted that in the majority of cases interruption occurred in advance of the time of normal zero current as indicated by the preceding half cycle and that the current just prior to the interruption was, in many cases, in excess of 50 per cent of the preceding current peak. Figs. 8 and 10 of Mr. Jamieson's paper and Fig. 12 of the paper by Messrs. Dickinson and Baker are cited as examples.

It appears that the normal current rise has been stopped and the current decreased to zero considerably in advance of the time of normal zero by some action in the circuit breaker other than the deionization of the gas adjacent to the cathodes which is assumed to take place only at the time the current would normally be zero or very near zero. In view of the above, it appears that there must have been some phenomena which resulted in a very considerable increase in the resistance of the arc while carrying relatively high currents, the resistance being sufficient to reduce the short-circuit current at a rate much faster than the normal decrement. The currents involved are too great to have been carried as a glow discharge at the cathode and following the preceding current zero.

Therefore the characteristics of this breaker under discussion, from the standpoint of performance in the circuit as indicated by the oscillograms, appear not to differ greatly from existing types of air and oil breakers for similar service designed to reduce ionization of the gas when carrying instantaneous short-circuit currents of considerable magnitude.

This theory is further substantiated by Figs. 13 and 14 of Messrs. Dickinson and Baker's paper. The average power dissipation in one unit of the breaker over a period of one-half cycle is of the order of 20,000 kw., the maximum being very much greater in an operation involving 8500 amperes and 6000 volts for the unit, the product of which is 51,000 kw. Since the existence of voltage across the breaker simultaneously with current through it results in power liberated in the arc, it would appear that the phenomena associated with the arc in the breaker while carrying heavy short-circuit currents are of as great importance in studying its performance as the phenomena at the normal current zero. If the recovery of dielectric strength occurred principally at zero no appreciable energy would be dissipated in the arc during the interruption. The standpoint is further borne out by the marked increase in temperature of the metallic disks noted during the short-circuit tests.

I cite this characteristic in particular as I shall presently show a slide of an oscillogram taken during tests on an air circuit breaker designed to bring about deionization of the main body of the arc while the short-circuit current is flowing.

In the opening paragraph of the papers by Messrs. Dickinson and Baker a statement is made to the effect that, with the exception of the oil breaker, the only other method generally used for extinguishing an arc in air has been by lengthening to such an extent that the circuit voltage would no longer maintain it. In view of this statement it seems desirable to call attention to the type of air circuit breaker previously referred to and now in commercial operation, the successful operation of which is dependent largely upon deionization of an arc in air. A photograph was taken of such a circuit breaker when interrupting a single-phase short circuit on a single element at 13,000 volts, 25,000 amperes r. m. s., the entire voltage after interruption appearing across this single element. This photograph demonstrates visually that the length of arc obtained was only a fraction of that which would be necessary in open air to insure interruption. The successful operation is, therefore, attributed to rapid deionization of the main body of the arc.

In this connection it has been repeatedly demonstrated that it is entirely possible to open circuits of from 1 to 5 amperes at very high voltages by unconfined arcs in open air. Disconnecting switches capable of opening exciting currents for large transformers at 135 kv. with only a few feet of arc have been in successful operation for some time. Since there is only one cathode involved, the successful interruption of the circuit appears to be due to the relatively rapid deionization of the main body of the arc by recombination resulting from rapid cooling, and not by a space charge accumulating at the cathode. As the current is increased, however, the energy dissipation is not sufficient to prevent continuous thermal ionization unless auxiliary means for energy dissipation or extremely long arcs are provided.

The method involved in the breaker consists primarily in taking advantage of the possibilities of energy dissipation in greatly accelerating the deionization of the main body of the arc so that the resistance can be made to increase during the time when the current through the arc is comparatively heavy, as well as at the normal zero of the current wave.

In a breaker of this type the arc is subdivided into a number of very narrow parallel streams, the columns of ionized gas being flattened in such a way as to bring all portions into intimate contact with a cooling medium, in this case made of insulating materials. The ionized gas is moved under the action of blowout coils so that fresh cooling surfaces are continually encountered by the arc stream.

It is usually assumed that ionization in the main body of the arc is maintained thermally since the voltage gradient is not sufficient for ionization by collision. The temperature of the main body of the arc is greatly lowered and re-combination of ions accelerated by the above arrangement, particularly during periods near zero of the current wave where the energy supplied is relatively small.

By virtue of the cooling agents introduced the energy required for maintaining sufficient ionization to insure a stable arc is greatly increased and the resultant voltage which can be developed even with relatively heavy currents through the arc is correspondingly increased. Oscillograms taken during the test of this circuit breaker show that deionization takes place both with current through the breaker and at the normal zero point of the current wave.

This arrangement is particularly effective for high-voltage direct current where no zero point is available.

Photographs and comparative records taken in open air have shown conclusively that the arc length necessary in a device of this type is only a fraction of that required in open air.

Another method utilized on breakers of this type is the introduction of arcing horns of high-resistance material so that mechanical resistance is introduced as the arc travels along the

horns absorbing considerable energy. This is a further important factor in the successful performance.

One of the problems usually encountered in the design of circuit breakers, particularly the air type, which depend for their successful operation upon the action of magnetic coils in series with the circuit, is the loss of effectiveness at relatively low currents.

For a-c. applications the most difficult practical case, as cited by Mr. Jamieson, is usually that involving the interruption of low-power-factor exciting currents for one or any number of transformers at normal voltage. Inasmuch as the breaker, subject of the papers presented, depends for its success upon the rapid circular movement of the numerous arcs around the deionizing plates, it seems reasonable to assume that at relatively small currents the action of the blowout coils is weakened and if the current becomes sufficiently low, this action might cease altogether. While currents of 1 or 2 amperes, as pointed out previously, may be broken on the main contacts of an air break without the necessity of deionization by auxiliary means, currents of the order 25 to 100 amperes particularly at low power factor usually require the operation of the arc-extinguishing agents. A statement of the observed effect of the reduced speed of rotation of the arcs at currents of this order on the temperature of the cathode spots and the effect on burning and the amount of metallic vapor liberated would be of considerable interest.

Dr. Slepian's theory of the cold cathode arouses considerable interest. The effectiveness of rapidly moving the arc over arc horns in limiting burning is unquestioned. This reduction in burning, however, is usually attributed to the wide distribution over a considerable area of the total burning which might otherwise be concentrated on a single cathode spot, a slight amount of burning always being present. The importance of preventing oxidation or burning of the metallic plates cannot be over-estimated because of the danger of strike-back through this metallic vapor. I believe this phenomenon has been observed by Dr. Slepian and commented on in one of his previous papers on this subject.

The development of several arcs in parallel, noted by Dr. Slepian, is interesting under conditions of heavy currents. The question naturally arises as to whether or not with very heavy currents the number of arcs might not increase to such an extent that the burning would be practically continuous on the surfaces of the deionizing plates. It would be interesting to know whether or not Dr. Slepian considers that the maximum current which the breaker is capable of rupturing at normal voltage is limited by the increase in the multiplicity of the arcs. It seems reasonable to assume that as the current is increased the volume of ionized gas necessary to carry this current until interruption occurs must necessarily increase.

The effect of temperature on the interrupting ability of the Deion breaker, as noted in Mr. Jamieson's paper, (p. 544), prompts the question as to what the temperature of the ionizing plates was at the time failure to interrupt occurred. It would appear that in order to impair seriously deionization of the region adjacent to the cathode, the temperature would have to be sufficiently high to maintain a considerable thermal ionization either of the air in contact with the plates or the thermal emission from the plates themselves.

R. M. Spurck: Mr. Jamieson in his paper brings out that the performance of the breaker in interrupting charging currents and exciting currents of transformers had not been disclosed. It seems necessary to complete the test record of the breaker to include such tests. They are particularly important because in the interruption of such currents, the voltage tending to re-establish the arc as the current passes through zero is a maximum under those conditions. Also, at the low currents, the blow-in effect is probably very small.

Another feature on which I should like to hear discussion is that of the insulation of the Deion chamber. When the breaker is open, this chamber is connected across the open contacts of the breaker. What insulation test across the contacts at 60 cycles will the breaker in the open position withstand? Also, how is this insulation affected by arcing in the Deion chamber?

As I understand Dr. Slepian's theory, the Deion chamber described has as a limit the ability to extinguish an arc in a circuit having a re-establishing voltage of 12,500 volts. Will the breaker operate satisfactorily in interrupting a circuit between two systems that may fall out of synchronism? In this case, on a 15,000-volt system, the *normal* re-establishing voltage would be about 17,000 volts.

J. D. Hilliard: Mr. Slepian's clear statement of his conception of the theory of operation, his determination of the limiting value of factors affecting satisfactory operation, and his mathematical treatment of the subject will greatly aid in the development of this interrupting device, which is one of the earliest methods of interrupting an electric circuit.

The claim is made that the recovery voltage of the Deion breaker is less than with an oil circuit breaker doing the same duty, but no statement is made as to the magnitude of the recovery voltage in the two cases. Personally, I believe the difference, if there is any, will be found to be small. In testing for recovery voltage on various breakers it is necessary that all breakers be tested on the same circuit, under the same conditions, because the circuit itself largely determines this voltage. They should also be given a considerable number of shots as the record of a single shot is of only small value. Tests as above indicated in which alternate shots would be made with the various breakers so that the characteristics of all could be definitely determined would be of the greatest interest.

The claim is also made that the Deion breaker interrupts at the zero value of the current wave; the same statement is true for the ordinary oil circuit breaker and also for a properly constructed magnetic-blowout oil circuit breaker.

Mr. Jamieson stresses the "absence of oil or oil fire." It is not necessary that either oil throw or an oil fire at the breaker accompany the interruption of an electric circuit by an oil circuit breaker. As a matter of fact, the modern oil circuit breaker does not throw oil or emit a flame during the interruption. The breaker may have an internal short circuit but no oil or fire will appear at the breaker as it is all conducted away by the waste piping connected to the breaker and without ignition or spilling of oil. Personally I believe the oil circuit breaker would stand a good chance of winning in a competitive test as its thick steel jacket prevents any ionized gas escaping to cause a short circuit on adjacent conductors. I am certain that the oil circuit breaker would stand an internal short circuit of longer duration and then be placed in operation with less repairs than would be required by any high-voltage air breaker, because usually a small quantity of oil to replace that blown out through the safety blow-off pipe and perhaps the renewal of burned arcing contacts is allt hat is required. Breakers with pressure-limiting devices have been given many severe tests and an oil breaker so equipped should be no more liable to explosion than the ordinary steam boiler with a proper safety valve attached and provision is now made for attaching the pressure limiting devices to all General Electric Company's modern breakers.

I anticipate that for low-voltage currents, air breakers of various kinds will prove satisfactory but I am skeptical concerning their success at higher voltages, as the ionized gas liable to be ejected at times will be a great hazard. The starting of an arc by the ejection of ionized gas has been observed by means of the motion-picture camera and this demonstration has proved the absolute necessity of preventing its escape if one is to safeguard against short circuits.

F. C. Hanker: Those of us who have known of Dr. Slepian's research have been greatly interested in the very early studies which he has made of the extinction of the arc. He has described those in some of his previous papers. The simplicity

with which he described those years of research makes it difficult to evaluate the importance of the results, but the effect is recognized by those who have followed circuit-breaker development.

Mr. Chase has called attention to the possible effects of this development on the design of switchhouses. That is one of the important features that will come out of a construction differing from existing devices and which eliminates the oil tank around the contacts.

In the early days, the first way known of interrupting a circuit was by separating the contacts in air. These varied from the crude plug-type switches to the more fully developed stick-type circuit breakers and were sufficient for the limited generating capacity.

The next development was the oil breaker. Some twenty-five years ago, we considered the results of the development as sufficient. We soon found, however, that with concentration of power there were limitations. While I am hopeful that Mr. Hilliard's prophecy will come true, unfortunately we have had failures on oil circuit breakers.

Mr. Jamieson called attention to the acquiescence of the operators to the development of the oil circuit breaker. All operators as well as manufacturers will agree there has been a very considerable advance in the design of oil circuit breakers to meet the conditions of today. Those who have been in contact with operating men have heard of their objection to oil breakers quite frequently. They haven't been at all hesitant in bringing the necessity of the elimination of the oil hazard to our attention. There have been important improvements in the mechanical structure of breakers but nevertheless all of those developments are in the direction of greater mechanical strength to resist higher pressures and interrupt the circuit by what may be termed "brute force." It is refreshing to have a development such as Dr. Slepian's which gives us what we might call a scientific method of interrupting a circuit.

It is hoped that the development of the Deion breaker which has been tried out in service on lower voltages, and is now being tried out on higher voltages, will follow through on the advantages that have been shown in the past. All the operators will welcome the elimination or minimizing of the present hazard of very high-power circuit interruption.

J. B. MacNeill: The present status of this development is about as follows. First, 2500-volt service: we have a small number of these breakers in actual service on our shop circuits. Second, 15,000-volt service: We are now building 6 single-pole units for 12,000-volt isolated-phase service on Mr. Jamieson's system in Chicago. These breakers are to be installed, I believe, in Washington Park Substation sometime in the near future to obtain actual field experience.

The extension of the development to other fields seems quite easy. For instance, the problem that Mr. Viele raised of handling 11,000-volt contact-line circuits on railway electrifications is being worked out and shows great promise. The extension to 25,000-volt service is actually being worked on and bears considerable promise. It is probable that within a year we shall have field data for 25,000 volts comparable to those which have been secured in Chicago at 12,000 volts.

With the deionizing chamber developed to the point of positive knowledge that it will handle 130 volts per gap up to say 25,000 or 30,000 amperes, there is a definite unit to work with which can be built up in series as necessary, or possibly, as has been suggested, in parallel. The use of these chambers in series makes the application of the device to 25,000-volt service a problem somewhat secondary to its development for 12,000 volts.

We expect to be able to present something on these other developments before the Institue in due time. At present the device is not offered for general sale. The art of switching has grown up empirically one step at a time, and it is not well to launch a radically new type of device on the market without the most complete operating experience.

Mr. McNairy and Mr. Spurck raised certain questions regarding the action of this device on small current values. You may be quite sure that the Chicago people did not overlook that point; although their system was not arranged so that the device could be demonstrated at low currents, they requested quite a comprehensive set of tests, which have since been made.

C. A. Corney: It is certainly gratifying to central-station engineers to learn of a development such as has been described in this paper. Any development which points the way to the possible elimination of oil as a medium in which to extinguish arcs will be carefully scrutinized and no doubt rapidly adopted as soon as it may have proved itself adequate and dependable in service.

There are several inherent features of the circuit breaker here described which appeal to me as particularly advantageous.

1. Elimination of oil hazard—as already mentioned.
2. Extremely rapid breaking of the arc once established resulting in:
   a. Reduced system disturbance at time of fault
   b. Lessened tendency to shake off synchronous apparatus
   c. Better selectivity in relaying, as a consistently quick break should permit closer timing among associated relays in any group, as it eliminates one of the variables now encountered in the overall timing of any system.
3. The apparent ability of the breaker to handle repeated short circuits without having to be derated should be particularly advantageous where the breaker is to be applied to automatic reclosing duty.

There would appear to be no reason why this breaker should not be built in the three-pole class using the truck type of construction with its attendant advantages. It should also be lighter in weight and more accessible for repairs than the equivalent oil circuit breaker.

K. B. McEachron: The Deion circuit breaker is of very great interest to me because it employs principles which have been used for quite a good many years in lightning arresters.

Early in the arrester art it was found that if an arc was subdivided into a large number of small arcs, it was possible to extinguish quite large currents. The theory at that time was not so well known as Dr. Slepian has shown us today, but this principle was employed in actual lightning arresters of that time.

It is quite well known that a single gap connected to a circuit whose voltage does not exceed 220 volts will suppress quite large follow currents. It has been common practise in designing lightning arresters for such service to use only single gaps when the voltage did not exceed 220 volts. This voltage limit is not far from the 250 volts mentioned by Dr. Slepian in his paper.

A single gap on high voltage, however, is quite another proposition. There a very small current will maintain an arc. It seems to be necessary to divide that arc into a large number of small arcs in order to stop the flow of current by the use of gaps.

The use of the static shield is also a principle which has been used for a long time in connection with the control of voltages across gaps. It was called an antenna in connection with a lightning arrester and its purpose was to distribute the impulse voltage across the gaps so that they would break down in cascade.

I should like to raise one or two questions which occur to me because of lightning-arrester practise. It is a race, as Dr. Slepian says, between the rate of deionization and the rate at which the voltage across the gaps builds up. I am wondering if we do have circuits in practise where the rate at which the voltage builds up will exceed the rate of deionization.

In connection with that same question, it would be interesting to know whether or not in the reactive circuit without the capacity in parallel the voltage is still 250 volts for recovery.

I should like to know whether or not there has been any attempt to use any other metals than copper. There has been a use in the past of metals containing zinc and other low-melting materials in an effort to produce the so-called non-arcing metal.

What does Dr. Slepian feel is the limitation in voltage of such a circuit breaker? Is it limited in ability to hold the disks in line by the static shield or is it limited in some other manner in voltage?

L. B. Chubbuck: (communicated after adjournment) To any engineer who has had experience with oil circuit breakers for many years, the Deion oil-less circuit breaker is of great interest. While oil circuit breakers have recently been greatly improved there is always present the hazard of oil fire due to poor maintenance, the use of too small a breaker, etc. Short-circuit tests on the Deion circuit breakers are remarkable in their absence of noise or blaze as experienced with conventional air breakers. The Deion breaker is fast, and evidently requires little maintenance as evidenced by the large number of consecutive interruptions at the high test currents. The breaker is very accessible for inspection, and I assume in the final design provision will be made for readily replacing the deionizing chamber or plugging in a complete pole unit.

R. H. Park: (communicated after adjournment) I understand that the abscissas of the curve of Fig. 1 were calculated, not tested, and that the calculation did not consider the effect of the capacity between turns of the coil used. I believe that this fact has resulted in an error in the abscissa scale. The effect of this capacitance will be to increase the values of time shown on the present scale. Thus 0 might be replaced by 40, 40 by 55, etc. If this is the case, as appears probable, the theory propounded by the author of almost instantaneous recovery to a strength of 250 volts, would have to be discarded.

A. H. Kehoe: (communicated after adjournment) In a symposium on an outstanding development such as the Deion circuit breaker, it is easy to overlook the valuable contribution made by the Commonwealth Edison Company in permitting its system to be used for high-capacity tests required to establish ratings of the new switch. In fact, Mr. Jamieson's paper records, by inference only, what appears to me to be one of his principal accomplishments; this was to obtain consent for such tests of those in responsible charge of his company's operations.

The effects of the tests on this system should act as a stimulus to other operating companies in supplying the necessary capacity for similar tests when proper results can be obtained in no other way than to utilize the operating system as the laboratory.

Being one of the operators mentioned by Mr. Jamieson who has frequently stated his misgivings concerning existing oil circuit breakers, it seems but just to the manufacturer to state that to me the development of the Deion circuit breaker appears to mark the turning of the ways, and the time may come when the circuit breaker can be treated like other electrical equipment rather than as an explosive hazard of large dimensions. It is hoped that this development will act as an incentive to all manufacturers to make material improvements in their present lines of switches.

P. H. Thomas: (communicated after adjournment) The physical or mathematical explanation as to the exact manner in which the electrons constituting the current in an electric arc escape from the body of the cathode into the arc space is given differently by different observers. I should like to suggest a conception or picture of this operation which I have held for over 20 years and which has seemed to me to fit the observed results very well. This conception was worked out well to explain the action of the cathode in mercury vapor rectifier and allied apparatus.

As stated by Dr. Slepian, there seems to be no serious resistance to electrode flow of current from a cold cathode; this is true not only of liquids such as mercury but of metals. The most extreme example of which I have knowledge is the use of a cold mercury vapor vacuum device with mercury electrodes as a breakdown gap for operating high-frequency Tesla coils. In such operation there must be enormous amperages passing with the current flow starting and stopping, corresponding to exceedingly high alternating frequencies. The operation of this device as far as I know it suggests no limitations as to the suddenness with which current may be produced nor its amount and the low temperature of electrodes appears to have no retarding effect; not only this, but these very high abrupt currents are at very low voltages once the flow is started. Thus, not only is a cold cathode no restraint on very heavy flow of current but apparently, at least in vacuum devices, the greater the current the easier the flow, as far as the cathode is concerned. I should perhaps say that this statement is not based on direct scientific measurements but on the results and observation of a large amount of laboratory work with various types of mercury vapor apparatus.

Aside from the *energy* required for passing an electron from a cathode to the vacuum space, a certain strong electric field or electric *force* is required to separate the electron from the solid matter of the electrode. This may be looked upon as the supplying of the necessary force to overcome the attraction of the electron lying outside of a solid surface for this solid body; such an electron has no difficulty moving within the body of the electrode as ordinary current, since attraction of the solid particles on one side of an imaginary plane through the electron is balanced by the attraction of the solid particles on the other side. Suppose now at the point of exit of the electron from a cathode there is a very violent vaporization of the solid material of the electrode concentrated at a small area on this surface and that the concentration of evaporation is so great that the density of the material changes by gradual stages from the solid condition behind the surface outward until at some relatively large distance from the position of the surface the density becomes reduced until the normal gaseous pressure appears. That is to say, the concentration of evaporation at the critical location on the surface is so great and diminishes so gradually that there is no line of demarcation between the solid material and the vacuum space. In such a case the electron could presumably pass from the cathode into the vacuum space by way of this path through a gradually reducing density without the necessary existence of any overwhelming electrical force to produce the motion. No such condition could reduce the amount of *energy* required for separating the electron from the electrode.

It would appear that this condition of high vaporization is exactly the condition that would be expected to exist at the cathode spot of an arc. It could exist only as long as there is sufficient current to supply enough energy for the necessary amount of evaporation. The necessary densities may easily exist at small currents on account of the possibility of the phenomena involving very restricted areas.

This theory is very strongly supported by calculations made, as I understand it, in connection with mercury vapor rectifiers of commercial proportions showing that the gas pressure at the immediate surface of the cathode spot exceeds one atmosphere during operation.

If this view can be co-ordinated with the fundamental theory of electronic phenomena it very beautifully explains the behavior of the cathode carrying current in a vacuum in many different applications. In this view the force resisting starting of current flow in an unexcited cathode is somewhat analogous to surface tension and when the cathode is excited the surface tension may be said to be punctured with a blast of vaporized material and electrons passing out through the hole.

Cannot this analogy be extended so that we may imagine the inside of a solid or liquid metallic conductor to be the equivalent of a vacuum free space for the passage of electrons and carrying space charges during current flow, with collisions and losses of energy as in the vacuum space and operating at a low voltage and that the electrode starting resistances and energy losses are related to the passing of the electrons by the boundary surfaces between the two spaces.

Joseph Slepian: The curve of Fig. 1 of my paper, which I

consider of rather fundamental importance, was derived by computation from tests which consisted of determining the limiting value of shunting resistance which would just cause an arc to go out in a particular circuit. It was found that when the circuit voltage had a peak value of less than 250 volts, the arc would go out without any shunt. This surely means that the arc space recovered the ability to withstand 250 volts faster than the circuit could develop 250 volts, immediately after the current zero. I described this fact by saying that the arc space recovers the ability to withstand 250 volts, "almost instantly" as compared with its ability to withstand larger voltages. Of course, strictly, the recovery cannot be instantaneous.

The curve of Fig. 1 was calculated by an approximate method and I have since calculated it more carefully and obtained a considerable departure from that given in the paper. The "almost instantaneous" recovery of 250 volts is, of course, still retained, but the rate of recovery of higher reignition voltages is much slower. This is shown in the accompanying figure. The curve is the envelope of the exponential curves which show the recovery-voltage transient following arc extinction. I have left in the figure portions of these exponential curves to show their relation to the resultant curve.

Mr. Park has raised the question as to the influence of distributed capacity in the circuit upon these calculations. I have already discussed this point in my former paper (see reference 1 of the present paper.) In the particular series of tests from which the curve was derived the current was limited by a small reactor whose natural period was of the order of 100,000 cycles. A quarter period of this frequency would be 2½ microseconds and I conclude, therefore, that the recovery of the first 250 volts of dielectric strength of the arc space is "almost instantaneous" at least to this extent. There are, however, good reasons drawn from the physical theories of the arc to believe that this recovery of dielectric strength is much faster. It is generally believed that a cathode of an arc requires either a very large gradient at the cathode surface or a high temperature in the gas space next to the electrode. The high gradient is maintained by space charges and to maintain these space charges requires a current density of several thousand amperes per sq. cm. This puts a limit to the density of ionization which is required for the striking of the arc at low voltage. It may readily be shown that the density of ionization will fall far below this figure due to recombination in a fraction of a microsecond. On the basis of high temperature in the gas next to the electrode also, we are lead to a similar result. The layer of gas involved is only 0.0001 cm. thick and this will cool in a very small part of a microsecond. Hence, on either basis after a small fraction of a microsecond, the arc must be restruck by break-down of the gas in a manner similar to that in which ordinary spark-gaps break down and such a break-down requires a minimum of several hundred volts.

Mr. McNairy has pointed out that in the Deion circuit breaker the zero point of the current wave is advanced to a considerable extent at the moment of arc extinction and that the current is considerably reduced while the arc is in the deionizing chamber, and asks whether some aid in arc extinction does not arise thereby. To a certain extent, there is some advantage resulting from the advancing of the zero of the current wave and the reduction in the maximum value of the arc current. As a result of the former, the voltage tending to reignite the arc does not rise to the peak of the generated voltage but to a smaller value, and the latter causes the rate of application of this voltage to the arc terminals immediately after the current zero to be slower than it otherwise would be. However, no advantage of this is taken in the Deion circuit breaker. Enough plates are used so that the arc will be extinguished even if the voltage tending to reignite rises to the full peak value of that generated in the circuit, and the plates recover the ability to withstand this voltage faster than it can be supplied by any practical circuit. Hence, they do not need the slowing down of this rate of rise resulting from the reduction in current.

This advance of the current zero and reduction in the maximum current in the arc is a consequence of the voltage required to maintain the arc in the deionizing chamber. This voltage is not negligible in comparison with the circuit voltage but in the case of the 15,000-volt Deion breaker amounts to 3000 volts for smaller currents and rises to 6000 volts for very large currents. This arc voltage is something that we would have much preferred to do without. If it were possible, we would bring the deionizing means into play only at the current zero, and leave the arc absolutely unimpeded until the current zero. Then there would be no energy liberated as heat in the switch and the ideal circuit breaker would have been obtained. However, the deionizing means while waiting for the current zero does affect the arc and abstract energy from it, although this energy is small compared to that abstracted by most other forms of circuit breakers. It is conceivable that we might take advantage of the effect of the arc voltage upon the current by reducing the number of plates for a given voltage but in that case, we would require a certain minimum time of residence in the chamber by the arc prior to extinction. We prefer, however, to have the chamber ready for extinction at the first current zero and to act whether the arc enters the chamber just shortly before the zero or near the beginning of a half cycle. When the arc enters the chamber near the end of the half cycle, tests show that the breaker operates entirely satisfactorily and with correspondingly very small heating.

The only type of switch I know in which the arc energy is quite negligible, is the vacuum switch. In this switch the arc voltage is only some 20 or 30 volts and there is practically no influence upon the course of the current in high-voltage circuits. The arc is extinguished at the current zero, however, by the "almost instantaneous" disappearance of the arc-conducting vapors.

Mr. McNairy makes the point that moderate currents can be interrupted on high-voltage circuits by a single break and believes this to be at variance with the theory given in this paper which states that an unshunted single break will handle only 250 volts. I have tried always to qualify this statement by saying that it holds only for a short arc. By a short arc, I mean one which is short compared to the linear dimensions of its cross-section. For long arcs this limit of 250 volts does not hold. I brought this out in my former paper and have curves showing how circuits of considerable voltage may be interrupted in moderate space by confining the arcs so that their section will be small in comparison to their length. The curves of that paper bring out very definitely the point that Mr. McNairy makes, that arcs confined to slots will be extinguished in a circuit of given voltage at much shorter length than arcs in the open air.

Concerning the phenomenon of the existence of arcs in parallel in the deionizing chamber for the larger values of current, Mr. McNairy asks whether this produces any limitation in the current-interrupting capacity of the circuit breaker. So far as I can see it does not. As a matter of fact it would seem to me that it makes the instantaneous temperature rise of the points of the metal plate surfaces less than it would be if the arc remained a single continuous arc. Of course, the heating up of the metal plates as a whole will be larger in proportion with the larger currents. Since we have found the current density in these arcs to be 30,000 amperes per sq. cm., it is quite evident that the arc cannot have sufficient extent to cover the whole of any one plate at one time with less than millions of amperes.

The failure of the circuit breaker when the deionizing chamber was permitted to reach too high a temperature has been noted. This limitation is tied up entirely with the properties of the fibrous insulation used for separating the copper plates. Without this limitation the deionizing chamber might be expected to be effective up to the melting point of copper. By using special

high-temperature insulation for the insulating spacers between the copper plates we have operated Deion circuit breakers successfully and repeatedly when the deionizing chamber temperature was over 300 deg. cent.

Mr. Spurck has raised the question as to how a Deion circuit breaker would act in separating two 15,000-volt systems which had fallen out of step. Regarding this problem, the Deion circuit breaker is in no way different from any other circuit breaker. When two systems are 180 deg. out of phase, they are really in series with each other so far as the circuit breaker is concerned and the circuit breaker is called upon to open a 30,000-volt circuit. In general, a 30,000-volt circuit breaker would be necessary for this service. However, systems fall out of step under conditions in which the generating voltage is usually very greatly reduced and so the usual 15,000-volt circuit breakers are able to meet this exigency. There is no reason why the Deion circuit breaker should be different from others in this respect.

Mr. McEachron's points concerning the relation of the Deion development to old lightning-arrester art are very good. As a matter of fact, in my former paper I mentioned this and took some of my data from investigations made on lightning arresters some eight or ten years ago. However, although the principles seem clear and obvious now, so that it would seem almost that I should apologize for myself and my colleagues for having taken so many years to apply such well known principles, nevertheless, there was much hard work, both mental and physical, before it was realized that the reignition of an arc is a phenomenon quite similar to the break-down of spark gaps and that it can be affected in the same way by static fields.

We have tried metals other than copper for the plates in the Deion circuit breaker and with success. Iron has advantages in certain types of switches. We have used brass and even aluminum. Because of its high electrical and thermal conductivity, copper is advantageous and does not require so high a speed of the arc to prevent the melting at the surface of the electrode.

I find the discussion of Mr. Thomas very interesting and thought-provoking. There is no doubt that the obstacle to the free egress of electrons from the surface of the metal is tied up with the discontinuity in material which exists at the surface. If it were possible to have a continuous transition from the dense metal to the attenuated gas or vapor, then the work function for electrons would disappear and arcs could be struck between separated electrodes with very low voltages. I find it difficult to see how this continuous transition from metal to vapor can take place without a tremendous amount of vaporization, but I shall be open-minded on this point for we may yet discover definite proof of the loss-of-work function under certain circumstances.

B. P. Baker: The question of the ability of the Deion circuit breaker to interrupt low currents has been raised by several gentlemen and I believe has been suggested in the paper by Mr. Jamieson.

The Deion circuit breaker has not been tested as extensively at low currents (less than 1000 amperes) as it has been at high currents. However, several hundred tests have been made which show that it is capable of successfully interrupting currents from a fraction of an ampere to many thousands of amperes.

Operating characteristics are essentially the same at low currents as they are at high currents. The arc is usually extinguished at the end of the first half-cycle after it has passed into the deionizing plates. The time required for the arc to get off the contacts and move up the arcing horns to the deionizing structure varies inversely with the current. At 3000 amperes and above, the movement of the arc is very fast and it usually gets off the contacts and into the deionizing chamber in time to be extinguished at the end of the first half-cycle. At 2000 amperes, one or two half-cycles are required. At 1000 amperes three half-cycles may be necessary. The time in general increases as the current decreases, until at about one ampere under the most unfavorable conditions one second may be consumed. This time may be shortened by using a stronger magnetic field to move the arc; however, as long as there is no injury resulting from this slow movement there seems to be no necessity for making any changes. Below 20 amperes the operating characteristics and time required for operation varies considerably with the current conditions and applied voltage. The most highly inductive circuit where the current is limited by air-core reactors, gives the longest arcing time.

When a bank of condensers is used as the load, to simulate the charging current of a cable system, the operation is quite different from that obtained on an inductive load. At 7.8 amperes, 13,800 volts, single-phase on a single-pole unit, the arc is always extinguished upon separation of the contacts. At 17.4 amperes, 13,800 volts, single-phase, the arc is extinguished as soon as the contacts separate; however, re-ignition frequently occurs one half-cycle later, when the generated voltage reverses and conspires with the condenser voltage to give double voltage across the breaker. This re-ignition may continue until the arc moves into the deionizing structure. In this case the time duration of the arc is very nearly the same as with an inductive circuit for a similar current.

Mr. Dean raised another question with reference to the ability of the Deion circuit breakers to withstand high voltages immediately after interrupting short-circuit current and suggested the advisability of using some sort of an air disconnect switch in series with the Deion circuit breakers to take voltage off the breaker when the circuit has been opened and the breaker is standing idle in the line. In response to this question I will say that in one of our earlier designs, the 2300-volt breaker, we did make use of an air disconnect switch which was mechanically operated by the operating mechanism of the circuit breaker itself. This switch was designed so that several half-cycles after the arc had been extinguished in the deionizing chamber, the switch was automatically opened, thus removing potential from the circuit breaker. After examining this type of construction for its ability to withstand voltage we found that the gaps between the grids which were required to operate at 140 volts r. m. s., when interrupting current, were capable of withstanding a 1000-volt potential test after deionization is complete. This immediately removed the necessity for using the disconnect switch.

Insulating the remainder of the breaker seems to be only a question of following conventional practise in that there is no inherent limitation involved. We, therefore, believe that it would be cheaper and easier to insulate the low-voltage breakers a little better and eliminate the complication arising from using the disconnect switches. There are, however, some values of voltage at which it might be easier to provide a disconnect switch mechanically operated by the circuit breaker mechanism than it would be to provide sufficient insulation to give the same factor of safety and with that in mind, each breaker design is gone over thoroughly from this point of view and a decision between the disconnect switches and straight insulation is made on an economic basis.

Mr. Spurck also raised a similar question concerning the ability of the circuit breaker to withstand voltage immediately after short circuits. I believe he had in mind conditions where there might be some residual ionization left from the arc, rather than the ability of the circuit breaker to withstand voltage at some considerable time after the circuit had been opened. We have tried to make tests to answer this question, but I am not certain that the tests which have been made will quite furnish the desired information. It is not feasible to apply a test voltage of several times operating voltage immediately after a short circuit has been cleared. However, in making tests in

accordance with the A. I. E. E. standards, where voltage is left on the line for several minutes after the short circuit is interrupted, we have found no signs of distress in the Deion circuit breaker. In addition to that we have, immediately after a series of short-circuit tests, disconnected the Deion circuit breaker from the power terminals and applied test voltage without finding the least indication of trouble. From theoretical considerations we have every reason to believe that the Deion circuit breaker is just as capable of withstanding voltage a few cycles after the short circuit has been interrupted, as it is several hours after the interruption. In other words, the deionizing process is certainly complete in much less than one second.

In recognition of the fact that insulation in this circuit breaker is dependent on creepage surfaces rather than puncture through oil as in the case of the oil circuit breaker, we did not feel content with making this breaker stand 2¼ times the rated volts plus 2000 volts, as specified by the A. I. E. E. standards, but have designed the 15-kv. Deion breaker for a potential test of 50,000 volts. We feel that with this additional insulation the Deion circuit breaker will be just as capable of withstanding voltage surges as any other piece of apparatus on the line.

From Figs. 13 and 14, Mr. McNairy has calculated that the average power dissipated in one unit of the breaker over a period of one half cycle, to be of the order of 20,000 kw. He has used this figure to substantiate the theory that there must have been some phenomena which resulted in a very considerable increase in the resistance of the arc while carrying relatively high current. This phenomenon we have chosen to call "arc voltage," which for the 15,000-volt Deion circuit breaker under discussion may be represented roughly as $E = 36{,}000 + 0.0951$ for currents above 2000 amperes, where I is the instantaneous current in the arc. Fig. 17 might have been used as a more striking proof of the existence of an arc voltage. In this case the highest point, 600 kw-sec. for the 3 phases, is the equivalent of an average of 24,000 kw. per phase during the half-cycle of arcing. In contrasting the theory of operation of the Deion circuit breaker with that of other circuit breakers we have been concerned but little with what happens in the arc other than at the current zero, so long as the natural or forced zero, as the case may be, does not produce any surges on the system, or impair the circuit for future use.

A further exception has been taken to a statement to the effect that, with the exception of oil breakers, the only other method generally used for extinguishing arc in air has been by lengthening. We are still of the opinion that even though it would be difficult to prevent deionization (recombination) in the main body of the arc, it is necessary to use much more lateral restriction than has hitherto been used to obtain the same amount of deionization per inch of arc as has been obtained in using short series of arcs.

Automatic Reclosing High-Speed Circuit Breaker

Feeder Equipment for D-C. Railway Service

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Synopsis.—*High-speed circuit breakers have been used in both feeder and machine circuits of railway substations. It is the purpose of this paper to point out some of the advantages gained by placing this type of breaker in the feeder circuit and also to describe some of the more common types of feeder equipment using a well known form of high-speed circuit breaker. A brief summary of the breaker characteristics is given, together with the results of short circuit tests made on 600-volt reclosing feeders employing this type of breaker.*

The reclosing action is ordinarily obtained by means of a relay which, in conjunction with a load indicating resistor, measures the resistance of the external circuit. The action of this relay is purposely delayed, after the opening of the breaker, in order to permit feeder conditions to reach a stable value and particularly to enable all transient and counter e. m. f. effects to disappear before the reclosing devices are given control. The latter effects usually disappear within a few seconds of opening the breaker on the usual type of railway circuits. In a majority of cases a time delay of 10 to 30 seconds (after the opening of the breaker) elapses before the reclosing devices are given the control of the breaker. After this time delay has expired and load conditions have reached the desired value the circuit breaker is reclosed.

In some cases, especially for sectionalizing purposes, it has been found satisfactory to take the reclosing indication and control from the load side of the breaker. This feature requires that some other breaker undertake the re-establishment of voltage on the load circuit. By means of supervisory control the range of such equipment can be extended so as to pick up a dead section during abnormal operating conditions.

A majority of 3000-volt installations has been made in connection with main line electrification where the desired reclosing operation is a combination of load indicating and load limiting functions. Voltage increments on the load of too large a value may increase the tractive effort to such values as to cause wheel slippage of the locomotive or snapping of draw-bars. Consequently the load voltage is raised in graduated steps by progressively short circuiting portions of load limiting resistors placed in the feeder circuit.

* * * * *

Characteristics of the High-Speed Air Circuit Breaker

THE type of automatic reclosing equipment described in this paper makes use of a well known magnetically held high-speed air-circuit breaker. A brief explanation of this type of circuit breaker, as illustrated in Fig. 1, may aid.

The breaker is a self-contained device. It is closed by means of a closing or reset coil which seats an armature against the pole faces of the holding magnet. During this operation the auxiliary contacts move between the positions corresponding to the open and closed positions of the breaker, but the main contacts do not close until the closing or resetting mechanism is returning to the open position. Due to this function, which is obtained by a suitable system of levers, etc., the breaker is free to trip during the closing operation.

A set of springs is attached to the moving contact. These springs exert a certain amount of opening force when the breaker is held in the closed position. The above mentioned armature is also attached to the moving contacts and holds the breaker in the closed position (once the armature is seated against the holding magnet pole faces) so long as sufficient holding flux passes through the armature.

The breaker may be opened in two ways; *viz.*, by decreasing the holding coil current or by shunting flux from the holding armature. By decreasing the holding coil current, a point is reached where the opening springs over-power the holding effect and the contacts consequently open. The breaker is tripped on overcurrent by means of a trip coil or bucking bar which is so placed in relation to the magnetic circuit as to shunt or transfer enough flux from the armature, thereby decreasing the holding effect and allowing the contacts to be opened by the opening springs. When the feeder current is gradually increased a point will be reached where the breaker is tripped in the above manner. This particular trip point is sometimes designated as the "steady" current trip point, as contrasted with a trip point where the line current is rapidly increasing.

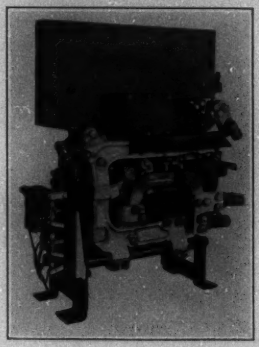

Fig. 1—600-Volt D-c. 2000-Ampere Magnetically-Held High-Speed Air Circuit Breaker

1. Switchgear Engg. Dept., General Electric Co., Philadelphia, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

By using a circuit in parallel with the trip coil or bucking bar, and proportioning the resistances and inductances of these circuits, it is possible to divide the currents by means of the resistances, when the line current is gradually increasing; and by having a relatively larger amount of inductance in the paralleling circuit or inductive shunt, it is possible to force a greater portion of the line current into the tripping circuit during rapid rates of current rise such as obtained on short circuits. This characteristic of lowering the trip point (in terms of line current) on rapid rise of current is sometimes referred to as "discrimination," since by virtue of certain load and circuit characteristics the breaker is able to trip on one value of line current when the load is gradually increasing and at another (lower) current value when a fault occurs. This feature results in decreasing the trip point of the breaker when it is most needed.

The "steady" trip point of the breaker is usually varied by changing the reluctance of the holding magnetic circuit. The amount of "discrimination" is varied by means of laminations placed on the external shunt.

The holding coil is connected in shunt with the source, which fixes the polarity of the holding flux. In order to shunt the flux, set up in the armature by the holding coil, the current in the tripping circuit must flow in a certain direction. If it flows in an opposite direction it merely tends to increase the flux in the armature as already set up by the holding magnet. Due to this relation of the magnetic circuits the tripping characteristic of the breaker may be said to be polarized and

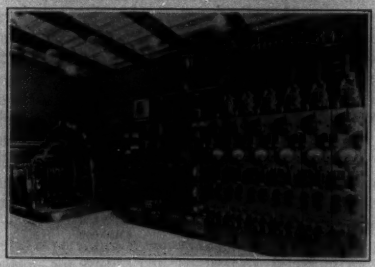

Fig. 2—Automatic Reclosing High-Speed Circuit Breaker Feeder Equipments as Installed in a 600-Volt Automatic Railway Substation

current must flow in a given direction in the line, in order to trip the breaker. Such a characteristic assists in obtaining selective tripping action when using certain system interconnections. A particular application using this characteristic is pointed out under a following heading.

The arc obtained when opening the breaker under load or faulty line conditions is extinguished by the well known combination of magnetic blowout and arc chute. The quick acting qualities of this type of breaker are obtained mechanically by using parts of small inertia combined with considerable force in the opening springs and electrically, by using a laminated structure around the tripping circuit, together with a suitable design of magnetic blow-out and arc chute.

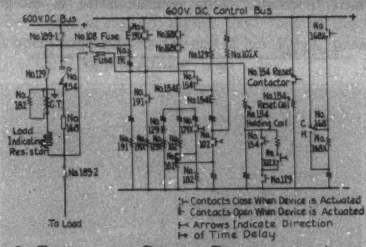

Fig. 3—Elementary Circuit Diagram of Automatic Reclosing Feeder

For 600-volt service, using a periodic load indicating scheme, and suitable for stub-multiple feed in either direction

The mechanism which operates the auxiliary switches is arranged in such a way as to allow the breaker to open quickly without interference from the auxiliary switches.

Advantages of Placing the High-Speed Circuit Breaker in the Feeder

Fig. 2 shows a typical installation of high-speed circuit breakers as part of reclosing feeder equipments in an automatic substation. By placing these breakers in the feeder circuit it is possible to remove any suddenly applied overloads or faults without any serious effect on the remaining feeders supplied from the station. The feeder handles its own circuit conditions without reflecting them back to the machine, which might cause the insertion of steps of machine load limiting resistors, reduce the voltage on the remaining feeders, and consequently decrease the car speed.

If conditions warrant the use of a high-speed circuit breaker in the machine circuit, together with corresponding breakers in the feeder circuits, it is possible to select the trip points and discrimination so as to permit the feeder breaker to trip first. This particular combination is not used frequently since it is felt that the high-speed breakers in the feeder circuits provide suitable protection to the machine under short circuit conditions, while the customary load limiting resistors afford sufficient protection on gradually increasing overloads.

There are conditions when the machines in an automatic substation are shut down, for example during light load periods, but the feeder breakers are closed so that the station bus becomes a tie point between the feeders, some of which may be feeding power into

the bus and the balance feeding outwardly. All feeder breakers in this case are connected so as to trip only on outgoing current. This inherent polarized tripping characteristic of the breaker has been described above. When a fault occurs on one of the feeders, only that particular feeder is tripped, since the remaining feeders are either carrying current below their trip points or else the current is flowing through their trip circuits in a reversed direction. This characteristic is of great

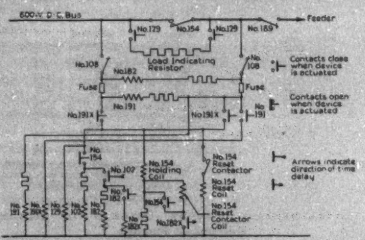

FIG. 3—ELEMENTARY CIRCUIT DIAGRAM OF AUTOMATIC RECLOSING FEEDER

For 600-volt service using a continuous load indicating scheme and suitable for stub-multiple feed in either direction

importance during conditions of through or back feed, when the machines in the station are shut down, or when these breakers are applied to cross-tie and sectionalizing points. When breakers having non-polarized tripping characteristics are applied to such points it will be found that it is usually necessary to rely on an excess current flowing through the breaker on the faulty circuit in order to obtain the desired tripping operation. If this condition is not fulfilled more than one breaker is apt to trip.

Other operating conditions may sometimes find two or more breakers in series (at different substations) with the fault current flowing through them in a direction to produce tripping. Due to the decreased voltage across the holding coil of the breaker nearest the fault, it will be found that its trip point is reduced below that of the breaker nearer the source. This results in tripping the breaker nearest the fault and isolates only that section of the system having the fault on it.

The use of a high-speed circuit breaker as contrasted with slower forms of circuit interrupters, results in a material reduction of the peak current of the faulty feeder. A comparison of the interrupting characteristics of common forms of air circuit breakers is given more fully in the comments pertaining to Fig. 8.

The trip-free characteristic of the high-speed breaker permits it to be closed on heavy overloads or faults where the reclosing relays are in the open position. The breaker may be closed in such cases (contrary to the indication of the reclosing devices) by means of supervisory or manual control. Where the breaker is not trip free it is desirable to have two breakers in series, one being closed before the other, thereby giving a trip-free combination.

The characteristics of this type of high-speed circuit breaker make it desirable for heavy city service and in addition for 1500- and 3000-volt applications. In some applications it is necessary to energize certain devices from a storage battery. When a storage battery is used the polarized tripping characteristic is still maintained, but the advantage of decreased holding coil voltage of the breaker nearest to the fault is lost. The number of cases where the storage battery has to be used, with a resulting sacrifice of the latter characteristic, is small and the advantages gained by a steady holding current and simpler devices, as in the case of 3000-volt d-c. circuits, appear to justify the use of a battery.

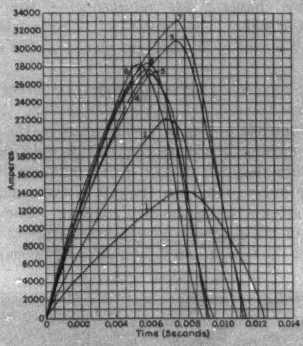

FIG. 4—HIGH-SPEED AIR CIRCUIT BREAKER

Opening a short circuit at 600-volts with different combinations of machines supplying power. Resistance of external circuit 0.002 ohm

Curve No.	Kw. connected to station bus	Maximum rate of rise, amperes per second
1	2,000	3,050,000
2	4,000	4,250,000
3	6,000	6,650,000
4	7,000	6,050,000
5	8,000	7,400,000
6	10,000	7,600,000
7	11,000	7,000,000
8	12,000	7,000,000

When long feeders of small cross-section are used, which appears to be confined principally to interurban systems as a class, the regulation of the feeder system may be so great as to cause unnecessary tripping of breakers nearest the load. Such regulation usually occurs when the machine in the substation is shut down, and an excessive drop is obtained over the positive and negative circuits under normal load conditions. However, such a system affects more devices than the mag-

netically held high-speed circuit breaker. In general, it will be found that whenever the regulation of the d-c. system is not excessive the above mentioned characteristics of the high-speed circuit breaker can be used to good advantage.

The value of circuit resistance between stations together with the size and type of load is useful in determining the application of this type of equipment.

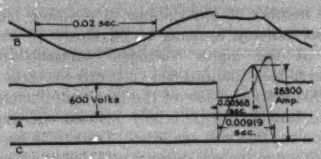

Fig. 6—Oscillogram of Short-Circuit Current on 600-Volt D-c. Reclosing Feeder

Using a high-speed circuit breaker as shown in Fig. 1, 12,000 kw. of conversion apparatus connected to bus. External circuit resistance 0.002 ohm. Curve *A*, line volts; Curve *B*, incoming station volts; Curve *C*, d-c. feeder amperes

Description of 600-Volt Feeder Equipment

High-speed air-circuit breakers were first applied to d-c. reclosing service over five years ago. The construction of this type of equipment is illustrated in Fig. 2. The elementary circuit connections agree substantially with those shown in Fig. 3, the design in the latter illustration being a slight simplification of the original.

This type of feeder can operate on either stub or multiple feed and obtain its control current from either the bus or the feeder. The breaker, No. 154, is tripped on steady current, and has the discriminating feature as pointed out above. In addition, the feeder is provided with a thermostat, No. 168, which disconnects the outgoing circuit when the feeder cable has overheated. The thermostat resets after the circuit has been disconnected and sufficient time has elapsed for the feeder cable to cool.

The reclosing devices operate in accordance with the external circuit characteristics in somewhat the same manner as the circuit breaker, No. 154. A load indicating resistor of comparatively low ohmic value (usually 1 ohm) is periodically connected in shunt with the open breaker, No. 154. This connection is made by the isolating contactor, No. 129, and suitable timing relays so that a limited amount of current is supplied to the feeder circuit for load indicating purposes. A current transformer is connected in series with the load indicating resistor, and in the secondary circuit of the transformer there is connected the coil of the reclosing relay No. 182. If a short circuit exists on the feeder the rate of current rise in the load indicating circuit will be sufficient to operate the reclosing relay, No. 182, so that the circuit breaker, No. 154, will not close. After a certain time delay has elapsed, the above cycle is repeated, and kept up in this manner until suspended by manual or supervisory control, or the fault has disappeared. In case the fault disappears and the load is not excessive, the resulting rise of current is not sufficient to operate the reclosing relay, No. 182, and consequently the feeder breaker recloses.

Control power from either side is obtained by the voltage directional relay, No. 191, which is usually connected so as to close its contacts when the voltage of the feeder is slightly higher than that of the bus. This operation energizes the coil of No. 191-X, which in

Fig. 7—High-Speed Circuit Breaker Opening a Short Circuit at 600-Volts

Power being supplied by four 1000-kw. and four 2000-kw. 25-cycle converters. Photograph taken simultaneously with oscillogram as given in Fig. 6. (Maximum current 28,300 amperes)

turn connects the control bus to the feeder side of the breaker. Under the normal operating condition the bus voltage is usually equal to or higher than that of the feeder, in which case the contacts of No. 191 are open and the coil of No. 191-X is deenergized. In this case, control power is taken from the bus side of the circuit breaker, No. 154.

Another design of reclosing feeder is illustrated in elementary form by Fig. 4. This feeder uses a continuous load-indicating scheme. In other words, the

load-indicating resistor is capable of having full operating voltage impressed continuously across its terminals. This condition is obtained during short circuit conditions on the feeder.

The reclosing relay No. 182, used in this class of feeder is of the so-called "balanced" type. The restraining (or opening) coil is connected across the load-indicating resistor. The actuating (or closing) coil is connected across the source of voltage. With a short circuit on the feeder, the restraining coil has full operating voltage impressed on it. This action causes the contacts of No. 182 to open and remain open. Sometime later, as determined by the setting of the time delay relay No. 102, the actuating coil is connected across the operating source. Both the actuating and restraining coils now have full operating voltage impressed across their respective circuits. The relay

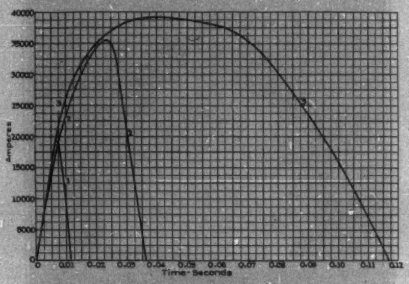

FIG. 8—COMPARATIVE TIME-CURRENT TRIPPING CHARACTERISTICS

Of correspondingly rated magnetically-held high-speed circuit breaker (Curve 1), mechanically latched circuit breaker with fast moving parts and magnetic blowout on arcing contacts (Curve 2), and mechanically-latched circuit breaker with carbon arcing contacts (Curve 3). Tests were made with the same external circuit conditions and with the same amount of connected power

(No. 182) is so adjusted that under this condition the restraining coil overpowers the actuating coil and the relay contacts remain open. As the load resistance increases, the voltage drop across the load indicating resistor decreases. This decrease also takes place across the restraining coil. Consequently, a predetermined load value is reached where the actuating coil overpowers the restraining coil and the relay contacts close. The latter operation energizes an auxiliary relay, No. 182-X, which in turn closes the circuit breaker, No. 154. The reclosing relay, No. 182, is so adjusted that it operates at a fixed voltage ratio over a wide range of operating voltage. By this characteristic the feeder recloses on a fixed value of load resistance, regardless of the value of the source voltage. In case another station is supplying the load (multiple feed) the feeder recloses when the same voltage ratio is obtained.

Control power is obtained from either side by means of the voltage directional relay, No. 191, and its auxiliary, No. 191-X. An isolating contactor No. 129, is used to disconnect the load indicating resistor when the feeder is taken out of service, by opening the control power switch No. 108. The maximum amount of current taken by the resistor is usually greater than that which can safely be interrupted by No. 108.

RESULTS OF CURRENT INTERRUPTING TESTS ON 600-VOLT RECLOSING FEEDER

The number of applications of reclosing feeders to 600-volt railway systems is obviously in excess of those at the higher voltages. Consequently, the number of tests made at 600-volts exceeds those at 1500 and 3000 volts. A few of the results of current interrupting tests might be briefly pointed out, especially those on the 600-volt systems. Tests have been made at the other voltages in order to establish the correctness and suitability of the design, but these tests were not carried to the extent of those made at 600 volts.

An extensive set of tests was made on a street railway property using the type of feeder as illustrated in Fig. 2. One feeder was installed in a station where the connected synchronous converter capacity (25 cycles) could be varied, in certain steps, from 2000 to 12,000 kw. One set of tests was made in which the short circuit was applied immediately outside the station, giving an external circuit resistance of 0.002 ohm. The results of these tests as given in Fig. 5 show that the peak current increased as the connected machine capacity increased from 2000 to 6000 kw. From 6000 to 12,000 kw. there was no corresponding increase, in fact most of the peak values were slightly under the peak value corresponding to the 6000 kw. condition. These tests show that as the generating or conversion capacity of the station increased from 2000 to 6000 kw., the peak current was largely determined by certain characteristics of the machines. Beyond the 6000 kw. condition, the characteristics of the external circuit predominated, so that the amount of connected machine capacity, beyond this point, no longer served as an indication of the maximum current to be expected when interrupting the short circuit with a high speed circuit breaker. With a short circuit more remote from the station it was found that the peak value was obtained at 4000 kw.

Fig. 6 is reproduced from an oscillogram taken with 12,000 kw. connected to the bus. The feeder current curve, *C*, corresponds to curve No. 8 in Fig. 5. The maximum rate of rise of current in these tests was approximately 7,000,000 amperes per second. Fig. 7 gives an indication of the amount of arc obtained when interrupting the current as given in Fig. 5, curve No. 8, and Fig. 6. This photograph was taken during the same time that the current was being recorded by the oscillograph. The above series of tests was made by having the breaker in the closed position and then applying the short circuit by another device. In other

words, the breaker operated on a *C O* cycle, which represents the normal interrupting cycle for reclosing service. This same type of breaker, as illustrated in Fig. 1, has successfully interrupted currents as high as 61,500 amperes at 600 volts, on a *C O* cycle. In this particular set of tests the total external circuit resistance was 0.001 ohm and the connected machine capacity consisted of four 4000-kw. 25-cycle synchronous converters. The maximum rate of rise in the latter set of tests was approximately 14,000,000 amperes per second.

The comparative time-current tripping characteristics of the high speed and other more common forms of circuit breakers may be obtained from Fig. 8. These tests were made at 600 volts, with the same external circuit conditions and with the same amount of connected conversion apparatus. Curve 1 shows the time-current tripping characteristic of the high-speed circuit breaker as illustrated in Fig. 1. Curve 2 is the corresponding characteristic of a relatively fast panel-mounted mechanically latched circuit breaker having a magnetic blowout associated with the arcing contacts. Curve 3 illustrates the corresponding characteristic of the usual panel-mounted air circuit breaker (mechanically latched) having carbon arcing tips.

The ratio of the overall times of these breakers (from 0 to 0 current) is approximately in the order of 1:4:12. Curve 2 shows how the overall time of the usual panel-mounted device can be decreased by making it faster mechanically, so that its tripping mechanism is quickly released and the moving parts start sooner, together with using a magnetic blowout to extinguish the arc in a shorter time. The type of breaker, whose characteristic is covered by curve 2, is fast enough so that its tripping mechanism is released during the opening operation of the high-speed circuit breaker. The latter time, however, is not sufficient for the slower device to operate and extinguish the arc, but does permit it to unlatch and drop open shortly after the faster device has interrupted the circuit.

Description of 1500-Volt Feeder Equipment

High-speed circuit breakers have been applied to 1500-volt reclosing service. Such an equipment, as illustrated in Fig. 9, makes use of the continuous load indicating scheme as outlined in connection with Fig. 4. As installed, the high-speed circuit breaker is mounted in back of the panel on suitable supports, so that ample clearance is obtained in the vicinity of the arc chute. The design and operation of the 1500-volt high-speed breaker agrees substantially with that of the 600-volt breaker, as may be seen by comparing Figs. 1 and 9.

The control of the equipment illustrated in Fig. 9 is so arranged that if a disconnecting switch is accidently opened while the feeder is carrying current, the corresponding high-speed breaker will open. This operating sequence is obtained by means of suitable auxiliary switches on the disconnects, which are opened when the disconnecting switch is being opened, thereby de-energizing the holding coil of the high-speed circuit breaker. This particular feeder is arranged so that it can be opened by supervisory control and have the reclosing function restored by the same means.

The load indicating resistor is shown to the left of the panel, while the resistors for the various 1500-volt coil circuits are shown on a frame swung out to the right of the panel.

A number of these reclosing feeders are in service and are giving successful results.

Another application of this type of equipment has been made to cross-tie and sectionalizing points. Fig. 10 illustrates a typical system consisting of three generating stations, designated as Nos. 1, 2, and 3,

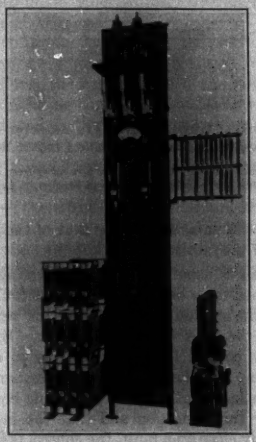

Fig. 9—1500-Volt, 2000-Ampere D-C. Reclosing Feeder

Using a high-speed circuit breaker and a continuous load-indicating scheme. Circuit breaker and load-indicating resistor shown respectively, to the right and left of the panel

feeding four track circuits. At convenient points along the system there are installed certain cross-tie and sectionalizing busses. This arrangement results in confining the fault to a particular section without a loss of service on other sections and, in addition, permits the feeder and trolley copper to be used to advantage during normal operating conditions.

If a short circuit should occur on Track No. 1 in the section fed from Station No. 2, not only will the corresponding feeder breaker at Station No. 2 be tripped, but also the two breakers which feed each end of the faulty section from the adjacent cross-tie and sectionalizing busses will be tripped. The selective operation of the breakers at these points is obtained by having them trip on outgoing current only, which is readily available due to the polarized tripping characteristics of the type of high-speed breaker covered by this

paper. Another characteristic used advantageously in such an installation is the reduced holding coil voltage of the breaker nearest the fault. This characteristic together with current flowing in the proper direction gives the desired selectivity. Breakers on a remote bus section distribute the current between themselves and even though the current flowing through them is in the proper direction for tripping, it is usually found that this current is considerably less than that through the breaker feeding the fault. In addition, the voltage at the remote bus is much higher than that of the busses adjacent to the fault so that the trip point is not

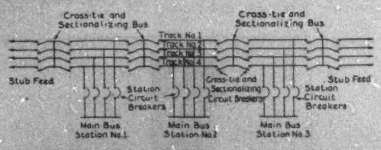

FIG. 7—TYPICAL ONE-LINE DIAGRAM SHOWING APPLICATION OF CROSS-TIE AND SECTIONALIZING CIRCUIT BREAKERS

To a four-track system. Current must flow away from cross-tie and sectionalizing bus in order to trip circuit breaker on faulty section

lowered by the same amount as that of the breakers at each end of the faulty section.

The breakers usually reclose when voltage has been re-established for a definite time on the faulty section, either by the station breaker or by the sectionalizing breaker at the other end of the section. By means of supervisory control it is also possible to reclose the sectionalizing breaker from its corresponding bus. Due to the fact that no load indicating equipment is used under this condition and that the breaker may consequently close on a fault, the control is so arranged that the breaker can close only once when supervisory control calls for closing from the bus. If this provision were not made, it would be possible for the breaker to "pump" due to a fault, while the supervisory devices were holding their "closing" indication. If the breaker does trip, it is necessary for the load dispatcher to turn the supervisory control to the "trip" position and then call for closing again. This method of control has been used for a number of years in connection with oil circuit breakers operated in conjunction with supervisory systems.

The stub feeders at the end of the system obviously obtain their control from the bus at all times. Those feeding in multiple with other feeders can reclose from the bus (as mentioned above) or in response to voltage restoration, on the track circuit. The normal reclosing operation is on voltage restoration. Provisions are made in the control so that if a breaker is reclosed from the bus, thereby energizing its line, the connection for reclosing on voltage restoration can be made without dropping out the breaker. This permits a transfer in the reclosing indication without an interruption to the load.

Fig. 11 illustrates a truck-mounted unit for 1500-volt cross-tie and sectionalizing service. The breaker is located in the upper portion of the truck where suitable clearances and barriers are provided. The covers of the 1500-volt relays, which are mounted on the panel at the front of the truck, are arranged so that they cannot be removed until the truck is withdrawn from the housing. Accidental contact with the 1500-volt circuit is averted by means of the latter arrangement and, in addition, by placing the balance of the 1500-volt devices in back of the steel panel.

DESCRIPTION OF 3000-VOLT FEEDER EQUIPMENT

The application of reclosing feeders to 3000-volt d-c. railway circuits includes a number of problems not encountered at the lower voltages. Principal among these is the fact that this voltage has been applied to main line electrification with long trains and also with longer distances between stations. Tests have been

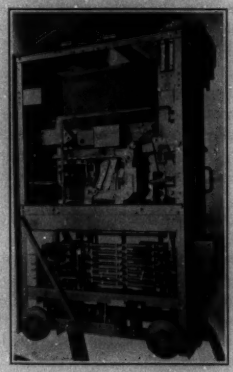

FIG. 11—SIDE VIEW OF TRUCK-MOUNTED 1500-VOLT 2000-AMPERE EQUIPMENT

With high-speed circuit breaker, for cross-tie and sectionalizing purposes (Barriers removed)

made in the field, showing that if too large a voltage increment is placed on the locomotive there is danger of suddenly increased tractive effort which will cause the wheels to slip or the draw-bars to snap.

The investigation of this problem has led to the development of a combined load-indicating and load-limiting feeder. The load-indicating resistor determines whether a short circuit exists on the feeder and if load conditions warrant reclosure. If so, the load-indicating resistor is short circuited, leaving the lower ohmic-value load-limiting resistor in circuit. The purpose of the latter resistor is gradually to raise the feeder voltage by suitable increments.

This sequence is accomplished by using the connections and equipment given in Fig. 12. Two high-speed breakers, Nos. 172 and 173, are used in the feeder circuit. At this voltage a high-speed breaker has about the same cost as that of any other interrupter that may be used as a resistor shunting device, No. 173. Upon the occurrence of overload either No. 172 or No. 173 opens, followed by the opening of the other breaker. After a time delay the load indicating devices come into

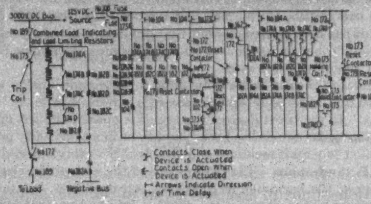

FIG. 12—ELEMENTARY CIRCUIT DIAGRAM OF 3000-VOLT COMBINED LOAD-INDICATING AND LOAD-LIMITING RECLOSING FEEDER

With resistors arranged to apply voltage in graduated steps

action and, if conditions warrant, the load indicating resistor is short circuited by device No. 174-A, which in turn is controlled by the reclosing relay No. 182-A. Four steps of load limiting resistance now remain in service. Three of these steps are progressively short circuited by suitable contactors controlled by the reclosing relays Nos. 182-B, 182-C, and 182-D. These particular relays are so interlocked that one cannot operate until the previous one has functioned at its particular value of drop out voltage. In other words, the relays close the contacts progressively when the difference of voltage across the open breaker terminals is less than a certain amount. These amounts become smaller as the last steps are being short circuited. The last relay in the sequence, No. 182, recloses the high-speed circuit breaker, No. 173.

In this way the load limiting portion of this type of feeder serves as an accelerating resistor. The most severe condition is one where the locomotive is in full parallel outside the station and is being fed from the remote station. The difference in voltage across the open terminals of the reclosing feeder breaker, is equal to the drop in the positive and return from the remote station. Such differences in voltage may be as high as 1200 volts. Under these conditions the reclosing feeder has to decrease this difference in voltage, by suitable steps, to zero or, in other words, raise the locomotive voltage from 1800 to 3000 volts.

In order to proportion suitably the resistors used in the load limiting portion of the feeder it is necessary to know not only the size and type of the load, together with other operating conditions, but also the resistance between stations, in the positive and return circuits.

Extensive tests have been made on this type of feeder. A typical train load of suitable tonnage was first supplied through a feeder having sufficient resistance to give the effect of the feeder and return of the remote station. The reclosing feeder was permitted to operate when the train was immediately outside the station, thereby giving the greatest difference in voltage across the reclosing feeder terminals. Fig. 13 is typical of the results obtained from these tests. Curve *A* is the current supplied from the remote station. Initially no current is being supplied by the nearby station. As soon as the combined load-indicating and load-limiting resistors are connected in the line, a slight current is obtained from station B, and a corresponding decrease at A. The next step in the curves is due to short circuiting the load indicating resistor. This action is followed by the progressive short circuiting of the steps

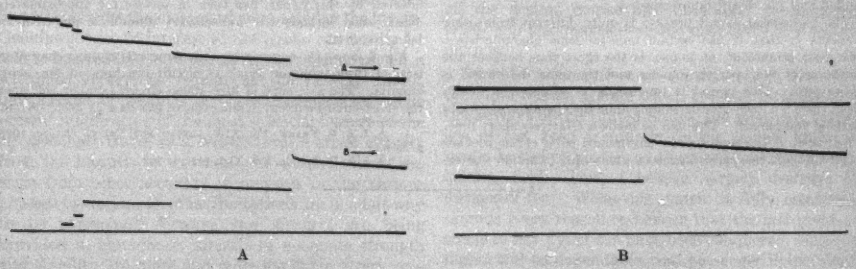

FIG. 13—OSCILLOGRAM SHOWING THE MANNER IN WHICH A 3000-VOLT LOAD-INDICATING AND LOAD-LIMITING RECLOSING FEEDER

Goes into service and takes over the load initially supplied from another source. Curve *A*, current supplied to load from remote source. Curve *B*, current supplied to load from nearby source. Time progress from left to right

of load limiting resistor, attended by increases in current from station B and corresponding decreases from station A. When the last step of resistance is short circuited, curve A decreases to zero and station B takes the entire load, since the load is practically outside the station in which the reclosing feeder is located. In this set of tests the controller was in full parallel. No jolts took place in the acceleration of the locomotive although it was apparent on the locomotive voltmeter that the voltage was being raised.

The resistors are protected by thermostats which disconnect the feeder and suspend further reclosing operation until the resistors have cooled and the thermostats have reset.

In the foregoing text the reclosing characteristic of the feeder is given more consideration than that of the high-speed circuit breaker. It is felt, however, that the characteristics and advantages of the high-speed circuit breaker as pointed out under other headings do not require repetition at this point. The protection afforded to the 3000-volt machines makes it a valuable part of the reclosing equipment.

Discussion

Chester Lichtenberg: This paper indicates that the high-speed breaker is particularly suited for railway service on d-c. circuits of 1500 volts and above and for a-c. service up to 15,000 volts.

The high-speed circuit breakers described have been in use for about fifteen years. They consist essentially of a pair of contacts which in opening project an electric arc into a chute. Under the influence of a magnetic field that electric arc is distended and the circuit is interrupted.

The high-speed circuit breaker is quite different from other breakers in that it has certain unusual time characteristics. The most prominent, of course, is the short time between the initiating of the opening impulse and the time the circuit is interrupted. This period is interesting to examine in detail. First, there is a fixed time depending upon the operation of certain relaying equipment. This can be made a relatively short time. Then, there is the time for the mechanical parts of the breaker to go through the performance of unlatching, whether the retention be mechanical or electrical. This is followed by the time for the contacts to part, the time for the arc to be sprung, and the time for the arc to be distended until the circuit is interrupted. All of these times in the high-speed type of breaker have been reduced. It responds and opens in about one cycle of a 25-cycle current.

Mr. Anderson has given in great detail some very interesting control circuits for these high-speed circuit breakers. They may appear to be rather simple, but as he points out in connection with the tripping and reclosing of a circuit which feeds a resistor load like lamps or heaters, sometimes a little change in the circuit makes a big change in the operation.

The earlier high-speed circuit breakers were used for tripping loads irrespective of the reclosing features. In the latter designs, it has been made apparent that not only must the breaker trip rapidly but it also must reclose as soon as the circuit condition makes reclosure safe.

In some of the earlier reclosing designs, the circuit breaker was tripped and reclosed repeatedly through a resistance to try the circuit. In the latest designs the circuit is opened and left open until the feeder circuit has been found to be in a sufficiently good condition to permit reclosure and the breaker to stay reclosed.

A. E. Anderson: The description of the high-speed circuit breaker at the beginning of the paper is rather brief, but those who would like to obtain more detailed information concerning the design of the circuit breaker may find certain papers, by Messrs. J. W. McNairy[1] and J. F. Tritle,[2] which have been presented at meetings of the Institute in the past, of interest.

A question has been raised on the widespread use of high-speed breakers in the feeder circuits. The principal advantages of the high-speed circuit breaker in feeder circuits may be covered by 1500- and 3000-volt applications. On such systems, it is practically necessary to have the high-speed circuit interrupter.

The amount of maintenance required for this type of high-speed circuit breaker is not necessarily excessive. Some of the earlier forms have been in service for ten years without any considerable amount of maintenance, and the present design, as covered by this paper, has been in service for approximately eight years without any unnecessary amount of maintenance being required.

Fig. 8 shows the comparative time-current tripping characteristic of three common types of circuit breakers on d-c. short circuits. The a-c. type of high-speed circuit breaker will open the short-circuit current in one cycle or less on a 25-cycle system.

1. A. I. E. E. Trans., Vol. XLV, 1926, p. 962; Vol. 47, October 1928, p. 1276.
2. A. I. E. E. Trans., Vol. XLI, 1922, p. 262.

Uses of Radio as an Aid to Air Navigation

BY J. H. DELLINGER[1]

Non-member

Synopsis.—Use of radio for guiding airplanes along fixed airings during fog or other conditions of low visibility is the principal topic of this paper. A directive radio beacon system is described together with a receiving system which gives simple and direct visual indications of the location of the airplane. Methods of air navigation on other than established courses are also discussed as well as simple radio communication between plane and ground.

* * * * *

THE possibilities of radio as an aid to flight are being actively developed. This development includes the following lines:

A. Communication
B. Course navigation
C. Field localizing
D. General

Under D are included miscellaneous developments, for example, the use of radio methods in connection with altimeter devices. This paper is largely devoted to item B, presenting a successful system of guiding airplanes along fixed airways during fog or low visibility. I shall speak particularly of the work of the Bureau of Standards because I am most familiar with that. It is, however, only one of many organizations pursuing active work and making contributions in this field. These organizations include transport companies, communication corporations, and research organizations, as well as government departments.

At the present time the transportation of passengers by air is far from the ideal service expected in the future. Genuine service of interest to the public can hardly be said to be available until the air traveler can count on a scheduled service as regular as the railway trains, independent of weather or other contingencies. The present nullification of the most essential feature of the air passenger travel as a serious service arises entirely from the hazards of weather. As we shall doubtless learn from other accounts of progress in the various phases of aviation, all other limitations are in a fair way to be overcome. Airways and airports are being provided in abundance, aircraft of adequate strength and stability are more and more available, every provision of comfort and convenience is offered the air traveler, and yet air traffic can still be halted when low visibility prevents the pilot from seeing his landmarks or lights on the ground.

It is impossible to exaggerate the solitude and helplessness of an airplane flying in dense fog. Deprived of all landmarks, under incessant strain at the controls to maintain equilibrium and direction, the aviator must frankly abandon dependence upon his senses and navigate according to the information conveyed by his instruments. It is contrary to all human instinct to throw overboard the testimony of the senses and stake life itself on a mute instrument dial. Not every pilot can do it, and unquestionably the oceans hide the sad remains of more than one hero whose only mistake was failure to learn "instrument flying" before he essayed the great adventure.

By means of the familiar instruments such as the altimeter, turn indicator, and compass, a pilot can continue flying in fog, but it is only by radio means that he can be certain to keep on a given course and find his landing field when the ground is invisible. Accurate as a compass may be, it cannot tell the pilot how much he is drifting sidewise due to cross winds, nor what actual progress he is making forward because of the unknown effects of head or tail winds. Unless radio aids are used, fog always brings the hazard of getting off the desired course into unfamiliar or dangerous areas, and also makes even the possibility of a landing unknown.

By radio means, however, particularly by the use of the radio beacon system which is being established on the airways of the United States, the pilot can, regardless of fog, keep accurately on his course, know the points he is flying over, and proceed unerringly to the landing field. This, I believe, largely destroys the menace of fog. When this system is fully established there is every reason to believe that the last great obstacle to safe flying will have been conquered, scheduled flights will be dependable, and passenger flying can be considered established as a serious service.

Directional Radio Off the Airways. Before describing the radio beacon system I should like to indicate briefly the possibilities of navigation by radio on other than the established airways. The beacon system will mark out the airway routes but will give no aid to the flyer on an independent course. There are several ways in which radio can be adapted to this navigational need.

One is the system used in Europe: radio direction-finding stations are maintained by the governments at

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various airports, and each airplane carries both a transmitting and a receiving set. Upon request by radio from an airplane, two or more of the direction-finding stations determine the direction of travel of radio waves from the airplane; combining their determinations they calculate the airplane's position and send this information by radio to the airplane.

A second means of radio navigation for the independent flyer is the use of a radio direction finder on the airplane. By steering a course in the indicated direction of a radio station on the ground, the airplane can be certain of reaching that point, the accuracy of the indicated direction increasing as the objective is approached. Direction finders are used extensively as a navigational instrument on marine vessels and on lighter-than-air craft. On airplanes their use is difficult. It is considerably more difficult to protect them from error and disturbances caused by the engine ignition and other sources aboard an airplane. They have been used successfully to some extent, and their use will doubtless increase; they do, however, require expert handling. This method of navigation has the inherent limitation that it does not prevent wind-drift from shifting the air-plane off its course; the method does eventually bring the airplane to its destination, although by a circuitous route if there is a side wind.

Fig. 1—The First Radio Beacon Tower on the Civil Airways
Experimental station of the Bureau of Standards at College Park, Md.

A third method of furnishing navigational aid to the independent flyer is the rotating radio beacon. This is a radio transmitting station, located at an airport, which has a rotating directive antenna. This causes a sort of beam of radio waves to sweep constantly around. A special signal indicates when the beam sweeps through the north. A pilot listening for this beacon's signal with his receiving set can determine his direction by the time elapsing between the north signal and the instant when the beam is heard with maximum (or minimum) intensity. The elapsed time is determined by means of a stop-watch, which can be calibrated to read direction.

The Airway Radio Beacon. The radio beacon system for the United States airways has been designed to operate with the minimum of apparatus and attention on the airplane. The objective in its development was to place a simple visual indicator on the airplane instrument board to tell the pilot whether he is on the course or how far off, which should operate without any effort or attention by the pilot. This has been successfully accomplished, and navigation over the official air routes thus has the advantage of a superior means of radio navigation not available to the independent flyer off those routes. The three methods of radio navigation for the independent flyer described in the foregoing require the pilot to listen with headphones through the roar of noise on the airplane. Also, each of them requires other apparatus besides a radio receiving set; thus in the first method the airplane must carry a radio transmitting set, in the second a direction finder, and in the third a special type of stop-watch.

While the radio beacon system for the airways attained practical development only this year, its origin goes back to 1920. At the request of the War Department, the Bureau of Standards undertook to develop a directive radio system for airplane navigation. A method was devised in which radio waves were transmitted alternately from two directive antennas placed at an angle with each other. Equality of signal intensity from the two antennas along a certain line or zone determined a course which an airplane could follow. The system was tried out successfully in Washington and in Dayton, Ohio. In succeeding years the Army engineers at Dayton developed the system further.

Fig. 2—Goniometer used in Radio Beacon Station
To orient the course marked out by the beacon in any desired direction.

When the Aeronautics Branch was formed in the Department of Commerce in 1926, it determined that radio aids would be necessary on the civil airways, and

assigned their development to the Bureau of Standards. As part of this work, the Bureau undertook to perfect the radio beacon, particularly by developing a visual indicator so that a pilot would have a direct indication, on his instrument board, of his location.

FIG. 3—AIRPLANE SHOWING 10-FOOT VERTICAL ROD ANTENNA

The required radio equipment on the airplanes is reduced to a short pole antenna and a simple receiving set weighing a few pounds, plus the indicator on the instrument board which tells the pilot whether he is on the course or how far off. All of the expensive and

FIG. 4—INSTRUMENT BOARD WITH BEACON INDICATOR MOUNTED ABOVE THE OTHER INSTRUMENTS

powerful apparatus necessary for the system is on the ground, maintained by the Government.

The radio beacons operate in the frequency band 285 to 315 kilocycles. Airway radio telephone stations are to communicate with airplanes in flight, in the band 315 to 350 kilocycles. These are allocated to air service by the 1927 International Radio Convention. For the present the beacons are adjusted to the frequency of 290 kilocycles, and the telephone stations to 333 kilocycles.

The directive radio beacon is a special kind of radio station, usually located at an airport, just off the landing field. Instead of having a single antenna like an ordinary radio station, it has two loop antennas at an angle with each other. Each of these emits a set of waves which is directive, *i. e.*, it is stronger in one direction than others. When an airplane flies along the line exactly equidistant from the two beams of radio waves, it receives signals of equal intensity from the two. If the airplane gets off this line it receives a stronger signal from one than the other.

FIG. 5—VISUAL BEACON INDICATOR

Comprising the pair of reeds to show deviations to either side of course and single reed (at right) which indicates when airplane is passing over a marker beacon

The current in the two antennas is of exactly the same frequency, but is modulated at a different low frequency in each, *i. e.*, the current in one antenna has a tone of 65 cycles impressed on it, and the current in the other antenna has a tone of 85 cycles impressed on it.

The indicator on the instrument board of the airplane shows when the signals from the two beams are received with equal intensity, by means of two small vibrating reeds. When the beacon signal is received the two reeds vibrate. The tips of these reeds are white in a dark background so that when vibrating they appear as a vertical white line. The reed on the pilot's right is tuned to a frequency of 65 cycles and the one on the left to 85 cycles. It is only necessary for the pilot to watch the two white lines produced by the vibrating reeds. If they are equal in length, he is on his correct

course. If the one on his right becomes longer than the other, the airplane has drifted off the course to the right. If he drifts off the course to the left, the white line on the left becomes longer. Thus if the pilot leaves the regular course either accidentally or to avoid a stormy area, the radio beacon will show him the way back.

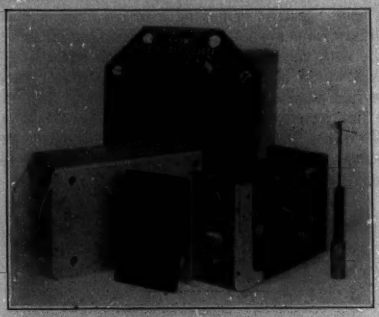

FIG. 6—INTERIOR OF BEACON INDICATOR

Showing electromagnets, pair of reeds in place, lamp, and detail of detached reed

The whole receiving system comprises a small indicator unit on the instrument board weighing one pound, a receiving set weighing less than 10 pounds, and a 10-pound battery. The same receiving set can be used to receive radiotelephone messages, by plugging in a pair of headphones. The receiving system is very

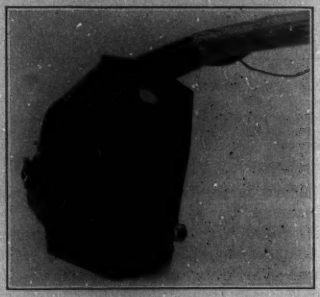

FIG. 7—COMPLETELY SHIELDED MAGNETO

Illustrating how all parts of the electrical circuits on the airplane have to be enclosed in a a grounded metal sheath

little affected by interference, including static, other radio stations, and airplane ignition interference, which has hitherto been the bar to satisfactory use of radio on airplanes.

The beacon stations will probably be placed at airports in general averaging about 200 miles apart. The Airways Division of the Department of Commerce Aeronautics Branch has begun a program of installing them on the various airways. The directive beacons, with a straight airway between them, will be supplemented by small marker beacons at intervals (perhaps 20 miles) along the route. These are simply very low-power radio transmitting stations serving as mile-posts. A characteristic signal from a marker beacon will show on the visual indicator aboard the airplanes what point is being flown over.

Thus the radio beacon system guides the airplane along the airway regardless of fog, informs the pilot of the distance passed over, and brings him to the landing field. There are two other services which directional radio can eventually perform to complete the conquest over fog, the providing of a field localizer and a landing altimeter, *i. e.*, to mark out clearly the landing area and to indicate distances above ground in the act of landing. While it is not yet certain whether radio or

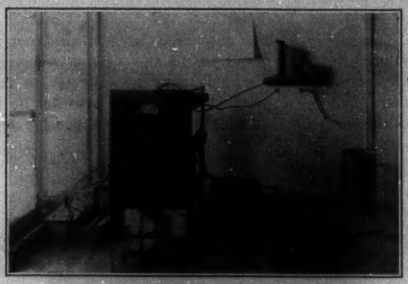

FIG. 8—TRANSMITTER FOR MARKER BEACON

A 10-watt transmitter located at intermediate points between airports to show distance flown along the airway

other methods will be the best means to provide these two services, the need of a field localizer is already partially met. When the pilot arrives at the radio beacon station and flies over it, there is a sudden deflection of his indicator which enables him to ascertain the location of the radio beacon station within 100 feet. This is accomplished by virtue of the peculiar properties of the vertical pole antenna on the airplane, and is of material assistance when landing during poor visibility.

The practicability of this system, both for course navigation and field localizing, may be illustrated by a recent trial flight. On a day of low visibility, a pilot unfamiliar with the route took the air in Philadelphia for Washington with no maps or instructions as to landmarks; he was told to proceed to Washington (a distance of 120 miles) and land at College Park field solely in accordance with the guidance given by the beacon indicator on his instrument board. He not only flew in a straight line to Washington, but when over College Park field, which he had never seen before,

the special deflection of the indicator told him he was at his journey's end, whereupon he landed.

Valuable as directional radio is, it is perhaps not as fundamental a service to the aviator as simple radio communication between airplane and ground. On the United States civil airways, radio telephone stations are being installed to inform the pilots of weather and landing conditions. This instantaneous service is a powerful addition to flight safety. The radio telephone messages may be received by the same simple receiving set used for the radio beacon signals.

As time goes on there will be more and more demand for two-way telephony between airplane and ground.

Fig. 9—Radio Telephone Transmitting Station at Bellefonte, Pa.

The first of the airways stations established to give weather and other information to airplanes in flight

A number of demonstrations has shown that such communication can readily be provided with quality sufficiently good to justify connection to the regular telephone exchange. In some of the demonstrations, officials sitting at their desks in Washington had two-way conversations on their regular desk telephones with other persons in an airplane. On one of these occasions Assistant Secretary MacCracken at his desk in Washington demonstrated the applicability of this service by warning the occupants of the airplane of the rising of a severe storm near the landing field.

The possibilities of radio in flying have been illustrated in some of the spectacular transoceanic flights. The Southern Cross, on its remarkable trip from California to Australia in the summer of 1928, was in touch with the world throughout the trip, by means of high-frequency (short wave) radio communication. The successful flight of Goebel and Davis from California to Hawaii in 1927 was made by the aid of the radio beacon of the aural type. A long flight over sea, terminating in a relatively small objective like the Hawaiian Islands, is extremely hazardous if undertaken without radio aid. Navigation by compass is subject to the indeterminate effects of wind drift, and the airplane's path may easily be shifted entirely away from the objective.

Any practical scheme for transoceanic air service would seem to require directional radio aid. It would be imperative for a system such as that involving a number of seadromes anchored at intervals across the ocean. Navigation on such a system without directional radio could not be considered; there is no other known means of being sure to arrive at the next airdrome.

Exploration by air is another instance where radio must be used. An exploring party takes unnecessary risks if it neglects directional radio aids to reach its objective or to find the way back to its base. This is recognized by Commander Byrd who is taking direction finding equipment along on the airplanes which he will use in exploring the Antarctic Continent.

The principal use of radio, however, will doubtless come on the regular commercial airways. The radio beacon system developed for airways is now being subjected to the test of routine operation.

As the radio aids have been slow in coming, compared with the advances in airplane design, engine reliability, and airway development, there has been a constantly increasing percentage of aviation accidents due to the hazards of weather. Radio seems the answer to those hazards, and there is ground for hope that not only this percentage of accidents but the whole number of accidents will become vanishingly small when the present possibilities of radio are realized in practise. Commercial reliability of air travel seems to depend directly upon the use of radio.

Discussion

H. W. Drake: I should like to ask what is the reaction of the aviators themselves to devices of this character? I am led to ask that question because of some familiarity with the feelings of locomotive enginemen in regard to the numerous improvements that are brought about in railroad practise from time to time.

Harry Diamond: The psychology is apparently the same in aviation as in railroading. The average pilot is not sufficiently trained in flying by instruments. The instruments, being mechanical, are subject to failure and therefore, even though the percentage of failure is very, very small, the pilot prefers to use his instincts.

The owners of aircraft are strongly in favor of the beacon system since they think it will incline their pilots to rely more upon instruments and thereby to fly by instruments. Such pilots as have used it on the lighted airways, on the transcontinental route, are very strongly in favor of it.

The Predominating Influence of Moisture
and Electrolytic Material Upon Textiles as Insulators

BY R. R. WILLIAMS[1] and E. J. MURPHY[1]
Non-member Non-member

Synopsis.—*Attention is called to the practical importance of water in all insulators and especially to the extreme electrical sensitivity to moisture of textiles as a class.*

Significant amounts of electrolytic impurities occur in many insulators.

In textiles in the presence of moisture such impurities are responsible for very conspicuous features of electrical behavior. Data are given showing their effect on the insulation resistance of cotton and silk.

The insulation resistance of textile fibers in moist air rises greatly with duration of d-c. voltage, accompanied by many evidences of electrolysis of aqueous solutions of impurities in the textile.

The instantaneous insulation resistance of fibers decreases with increase of the measuring voltage as previously shown by Evershed. However, this fact does not necessarily support his idea of kinetic redistribution of water in textiles, as this behavior is also compatible with the nature of electrolytic conduction.

Electrolytic impurities may be washed out of textiles and substantial practical improvements effected thereby. The increase of resistance is of the order of 50 times.

Fibers are classified according to their electrical behavior in a manner which is also in harmony with their chemistry as follows;

(1) Animal fibers.

These are of protein nature and are characterized by high moisture content at ordinary humidities and by great electrical sensitivity to further increments of moisture, yet possess excellent insulating properties under usual atmospheric conditions.

(2) Vegetable fibers.

These are of cellulosic nature and are characterized by lesser moisture absorption and lesser electrical sensitivity to further increments of moisture, yet possess relatively poor insulating properties over the range of prevalent atmospheric conditions.

(2a) Cellulose acetate.

This absorbs little water and accordingly has excellent insulating properties under like conditions.

The differences in electrical behavior of the two main classes of textiles are believed to be due to differences in the space patterns according to which water is distributed within the individual fibers. The patterns are probably determined directly or indirectly by the chemical composition of the fibers and associated with the colloidal structure.

* * * * *

A GREAT diversity of materials is used for insulating purposes. No simple descriptive term includes them all as the term "metals" includes commercial conducting materials. Yet in spite of this diversity it is to a great extent the quantity and mode of distribution of water in all insulators that determines their relative excellence. Were it not for the accumulation of moisture in it or on it the cheapest and mechanically most convenient material could, with rare exceptions, be used for the most exacting service.

At first glance it might seem possible to select insulating materials very simply according to moisture content, but a few illustrations will serve to show that wide contrasts exist in the response of insulations to a given amount of moisture. At one extreme is gutta percha, the classical insulation of submarine cables. If dry at the outset, it very gradually absorbs one or two per cent of moisture from the sea, but undergoes only a slight change in electrical characteristics in the process. Thereafter its water content and electrical properties are extremely stable in use. Rubber insulations used in air partake of these properties to some degree. Fluctuations in their electrical behavior never are large or sudden so long as they are mechanically intact. At the opposite extreme are the textile insulations which, especially if unimpregnated, are subject to every whim of the weather. Their water contents rise suddenly with corresponding changes in the relative humidity of the atmosphere, and the dielectric qualities faithfully reflect the moisture supplied from the air. A one or two per cent increment of moisture affects gutta percha scarcely at all but an equal amount has a most profound effect on the textiles.

The phenolized fibers, the impregnated papers, the cellulose esters, insulating varnishes and enamels, as well as glass and porcelain are intermediate between the "waterproof" insulations and the textiles in their sensitivity to atmospheric moisture. We refer to these insulations, of course, in the forms in which they are ordinarily used, for brevity neglecting distinctions which might properly be made as to relative importance of surface and volume characteristics in the several cases.

Diverse as are the insulators in use, they have another common property of importance. They often contain, or have deposited on their surfaces, electrolytic material which dissolves in the absorbed water to form conducting solutions which are injurious to the insulating qualities of the material. This fact seems to be second in importance only to the prevalence of water in insulating materials. These electrolytic substances may be present as part of the natural constituents of the insulating material or as accidental contaminants; they may consist of the saline or organic constituents of the vegetable tissues which furnished the raw material, of by-products of the processes of manufacture, of degradation products of the insulating substances resulting from atmospheric oxidation or hydrolysis, or

1. Both of the Bell Telephone Laboratories, Inc., New York, N. Y.

Presented at the Winter Convention of the A. I. E. E., Jan. 28-Feb. 1, 1929.

of atmospheric dust. Illustrative of the diversity of electrolytic material in commercial insulating materials are the natural ash constituents of textiles, pulp woods and other materials of vegetable origin; saline diluents of dyes used in fibrous materials; the quebrachitol of the latex of the rubber tree; acid resins produced by the atmospheric oxidation of rubber and gutta percha; and the free phenol present in phenol condensation products.

While the importance of moisture and of electrolytic contaminants in practical insulations has long had some recognition by electrical engineering opinion, especially in the telephone field, the foregoing general philosophy has been emphasized in the minds of the authors and their associates by the results of extended experimental studies of submarine insulation[2] and of textiles.[3]

Important contributions to the knowledge of the quantitative relations between the electrical properties of insulating materials and the moisture which they take up from the air have been made by Evershed,[4] Curtis,[5] Kujirai and Akahari,[6] Setoh and collaborators,[7] and other investigators. But in no published work, so far as we are aware, have data been given showing the quantitative relationships between the electrical properties of textile insulations and the electrolytic material which they contain. Data of this kind were obtained in the investigation of textiles mentioned above. Part of these data have been reported elsewhere,[3] but the investigation is being continued and a further report will be made when it is completed. It will require much further work to establish in detail the importance of contamination with aqueous solutions of electrolytes for every commercial insulating material. However, the presentation of the main thesis as a general one is abundantly justified by our constantly growing experience with cases in which such contamination of a variety of insulating materials has actually been found responsible for poor insulating qualities and for corrosion of metallic conducting or supporting elements in contact with them in electrical systems. This paper is intended to emphasize the importance of moisture and electrolytic material on the behavior of textiles as insulators and to discuss briefly the relation of electrical characteristics to physical structure and chemical constitution, so far as possible with the available facts.

General Characteristics of Textiles

It is obvious that the rapidity of response of textiles to atmospheric moisture is due first of all to their fibrousness which permits ready access to the interior of the mass through the large surfaces exposed. By contrast the relative stability of rubber insulations for example is clearly due in part to the smaller ratio of surface to volume.

Since textiles are composed of fibers, it might seem that the resistance of a thread or the serving on a wire should depend largely on the resistances of the contacts between fibers. Further, the fibers themselves have superficial irregularities which would suggest that their resistance might vary widely from fiber to fiber of the same material. Table I-A shows that single fibers of cotton and silk have a resistance[8] which, considering the nature of the material, is surprisingly uniform for different fibers taken from the same material. Similarly, Table I-B shows that threads[9] of cotton and silk also have a uniform resistance; this suggests that the interfiber contacts do not have a large effect on the resistance of the thread as a whole. Table I-C shows the resistance of the servings on wires; the resistances are for short twisted pairs (2 in. long). This shows also that even where the voltage is applied transversely to the long axis of the fibers,—which would tend to make contact resistances more important than when the voltage is applied parallel to the long axis—the resistance of different samples of the same material is fairly uniform. These facts suggest that interfiber contact resistances are only secondary or negligible in determining the resistance of a thread or other mass of fibers. Further evidence of this is given by the data in Table II, which show that, even when the length of a thread considerably exceeds the length of a single cotton fiber, the resistance is approximately proportional to the length; if interfiber resistances were large, the resistance per unit length would increase considerably with the length of the thread measured.

The above results also suggest that electrical conduction takes place primarily through moisture in the interior of the fibers rather than through moisture condensed on their surfaces. Other evidences that this is the case may be found in the relationships of conductivity to humidity, moisture content, and electrolyte content, as well as the absence of any obvious relationship between the physical dimensions of dif-

2. Williams, R. R. and Kemp, A. R., *Jour. Frank. Inst.*, 35 (1927). Lowry, H. H. and Kohman, G. T., *Jour. Phys. Chem.* **31**, 23 (1927).

3. a. Murphy, E. J. and Walker, A. C., "Electrical Conduction in Textiles. I. Dependence of the Resistivity of Cotton, Silk, and Wool upon Relative Humidity and Moisture Content," *Jour. Phys. Chem.* **32**, 1761, (1928).

b. Murphy, E. J., "Electrical Conduction in Textiles. II. Alternating Current Conduction in Cotton and Silk," *Jour. Phys. Chem.* **33**, (1929), p. 197.

c. Murphy, E. J., "Electrical Conduction in Textiles. III. Anomalous Properties of Conduction in Textiles," *Jour. Phys. Chem.* **33**, (1929).

4. Evershed, *Inst. of Elec. Eng. Jl.* (London) **52**, pp. 51-83, 1914.

5. Curtis, Bur. of Standards, *Sci. Paper No.* **234** (1915).

6. Kujirai and Akahari, *Sci. Papers, Inst. Phys. & Chem. Res.* (Tokyo), **1**, pp. 94-124, 1923.

7. Setoh and Toriyama, *Sci. Papers Inst. Phys. & Chem. Res.* (Tokyo), **3**, pp. 285-323, 1926.

8. The experimental procedure is described elsewhere.[3a]

9. Because of their uniformity, small samples of thread (½ in. lengths) have been used in this laboratory as a convenient means of comparing the insulating quality of cottons and other textiles.

ferent classes of fibers and their electrical behavior.

While the form of the sample is not of predominating importance with reference to the insulation resistance of either cotton or silk, the marked contrast, except at very high humidity, between cotton and silk, in all forms of samples should be noted. Both these facts and other available data justify the inference that the dielectric properties of textiles are determined primarily by the composition or internal structure of the fibers, not by the twist of threads or the lay of servings.

TABLE I

A. RESISTANCE OF COTTON AND SILK FIBERS

R is the resistance in megohms of a single fiber ½ in. long. Time allowed for equilibrium, 20 hr. or more. Room temperature.

Humidity, 99 per cent (about)			Humidity, 77 per cent (about)		
	R			*R**	
Fiber No.	Cotton	Silk (Tussah)	Group No. *	Cotton	Silk†
1	2600	3300	1	563,000	
2	4700	4500	2	736,000	
3	3800	5140	3	680,000	
4	5600	4740	4	822,000	
5	3000	6650	5	625,000	
6	4600		6	577,000	
7	6000		7	822,000	
8	4500		8	733,000	
9	5500		9	736,000	
10	6000		10	830,000	
11	4000		11	653,000	
12	4000		12	824,000	
			13	760,000	
			14	867,000	
			15	725,000	
			16	938,000	
			17	820,000	
			18	682,000	

*Each group consisted of 60 single fibers attached to the electrodes so that they were in parallel. *R** is the average resistance per single fiber calculated from that of 60 in parallel. The experimental technique is described elsewhere (3a).

†The values of *R** for silk at this humidity were found to be of the order of several million megohms, *i. e.*, beyond the limit of accurate measurement with equipment available at the time the study of single fibers was under way.

B. RESISTANCE OF COTTON AND SILK THREADS

Length ½ in. Temperature 25 deg. cent.

Sample	Resistance Megohms	
	Cotton Humidity 77%	Silk (Spun) Humidity 90%
1	4160	21,100
2	4220	22,700
3	4100	28,000
4	3730	14,500
5	4020	30,600
6	3820	
7	3900	
8	3715	
9	4050	
10	4100	
	Aver. 3982	23,380

The moisture content of each sort of textile depends directly on the humidity of the atmosphere. Fig. 1 shows the best data available for the moisture content of silk, wool, cotton, and cellulose acetate in equilibrium with air over considerable ranges of relative humidity. The data for silk and wool were taken from a paper by

C. RESISTANCE BETWEEN TWISTED PAIRS OF COTTON AND SILK INSULATED WIRES

Humidity 77 per cent Temperature 25 deg. cent.

Sample	Resistance Megohms	
	Cotton	Silk (Tussah)
1	5.45	2200
2	3.96	1890
3	5.20	1685
4	3.96	1685
5	3.50	2250
6	4.05	1970
7	5.60	
8	5.15	
9	4.37	
10	4.76	
11	4.00	
12	4.37	
	Aver. 4.53	1942

TABLE II

RESISTANCE OF DIFFERENT LENGTHS OF COTTON THREAD

Humidity about 77 per cent room temperature

Length Inches	Resistance Megohms	
	Total	Per Inch
0.5	21,700	43,400
1.	41,750	41,750
3.	153,000	51,000

Schloessing;[10] those for cotton are due to Urquhart and Williams;[11] while those for cellulose acetate represent the figures of Wilson and Fuwa,[12] who also give corresponding data for several textiles and many other substances. It is sufficient for our present purpose to emphasize the orderly dependence of moisture content upon the relative humidity of the atmosphere without

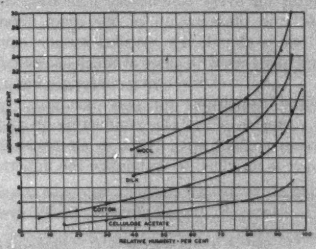

FIG. 1—DEPENDENCE OF MOISTURE CONTENT OF TEXTILES UPON RELATIVE HUMIDITY OF ATMOSPHERE WITH WHICH THEY ARE EQUILIBRATED

discussing secondary phenomena or the full significance of the curves.

The relation of electrical behavior of each textile to relative humidity is also very close. Fig. 2 shows the

10. Schloessing, Th., *Bul. Soc. Encour. Indust. Nat.* 8, 717 (1893); C. R. 116, 808, 1893. Text. World Record, Boston, Nov., 1908, p. 219.

11. Urquhart and Williams, *J. Textile Inst.* 15, 143, (1924).

12. Wilson, R. E. and Fuwa, Tyler, *Ind. & Eng. Chem.* 14, 913 (1922).

insulation resistance of each of the above fibres plotted against relative humidity over the upper part of the range of atmospheric humidities. It is not practicable to plot the resistance over the entire range of humidity directly in this way, on account of the wide range of insulation resistance values which are obtained. In order to depict the fact that there is a consistent relationship throughout the range, we have plotted in Fig. 3 Log Insulation Resistance *vs.* Relative Humidity as far as values are at present available. When considered together, these three charts show that the insulation resistance of a textile depends on its moisture

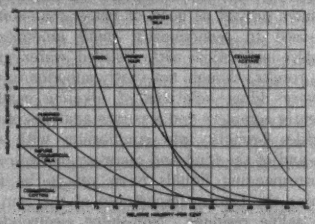

FIG. 2—INSULATION RESISTANCE OF ½-IN. LENGTHS OF TEXTILE THREADS AS AFFECTED BY RELATIVE HUMIDITY OF ATMOSPHERE

The purified cotton and purified silk had been submitted to a washing procedure to remove electrolytes. The impure silk was a commercial specimen representing somewhat more than usual contamination with electrolytes, while the commercial cotton is representative of its class

content, which in turn is a function of relative humidity. In the series of papers previously mentioned,[3] the electrical behavior of textiles in relation to relative humidity and moisture content is discussed more fully.

Aside from the common property of dependence of electrical characteristics of textiles upon their moisture contents, several other phenomena of electrical behavior have been encountered which are common to all. If any textile within our experience, including cellulose acetate (or even glass), be brought in contact with two electrodes of opposite polarity in the presence of atmospheric moisture, the electrical properties of the material undergo a change with a rapidity dependent on the current and in turn upon the voltage, the length of path, and the humidity. Such a change in the properties of insulating materials with continued application of voltage has been discussed recently by Granier who advanced the explanation[13] that it is due

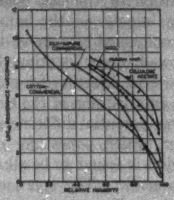

FIG. 3—LOG INSULATION RESISTANCE OF ½-IN. LENGTHS OF TEXTILE THREADS *vs.* RELATIVE HUMIDITY OF THE ATMOSPHERE

For description of samples see Fig. 2. For tabulated data see reference 3a

to the presence of electrolytic impurities in the materials. However, the great magnitude of this change which may occur in textiles when freely exposed to ordinary atmospheres seems to have been very little appreciated.

In our experiments the insulation resistance rises to a value perhaps 10 to 100 times the original value, depending on the nature and condition of the fiber. A few typical cases are given in Table III. This phenomenon will be referred to as polarization. This rise in resistance appears to be largely due to substantial denudation of some intermediate portion of the fiber of electrolytic impurities which in general tend to accumulate in the vicinity of the electrodes. The phenomenon involves the possibility of chemical reactions between the products of the electrolysis and the material of the electrodes, as well as the evolution of gases and the conversion of soluble salts into insoluble products. As regards the mineral constituents of cotton, it can be observed by ashing the polarized fiber that the ash

TABLE III

RATE OF CHANGE OF RESISTANCE WITH TIME OF APPLICATION OF VOLTAGE FOR SOME FIBROUS MATERIALS

Separation of Electrodes ½ in. Humidity 97 per cent (approx.) Voltage 275

Cotton		Silk		Cellulose Acetate Silk		Paper	
Time Secs.	Resist. Megohms	Time Secs.	Resist. Megohms	Time Secs.	Resist. Megohms	Time Secs.	Resist. Megohms
70	19.8	30	8.1	80	9.3×10^3	55	4.54
150	33.6	90	12.8	1250	1.01×10^4	100	8.30
200	40.5	120	19.8	2300	1.06×10^4	200	11.2
300	45.1	153	26.7	3200	1.11×10^4	260	19.8
400	50.0	220	40.5			520	32.2
500	57.5	300	47.8			615	38.5
900	68.2	640	54.8			1000	42.7
1900	82.0	1250	58.4				
3000	91.0	2100	79.5				
4000	99.0	2320	107.0				
		3000	153.0				

13. Granier, J., *Soc. Fran. de Elec. Bull.*, 3, 480 (1923).

lies largely in those portions which were adjacent to the electrodes and especially to the cathode. The electrical resistance is very unequally distributed along such a polarized fiber or thread, the positions of maximum resistance depending upon the conditions of polarization.[8c] Interruption of the current after polarization leads to a gradual restoration of the original electrical properties and a redistribution of the ash constituents with a speed depending largely on the humidity. Reversal of polarity is accompanied by a rapid drop in insulation resistance, followed upon continued application of voltage in the reverse direction by polarization in the opposite sense. Interruption of the circuit after polarization leaves large potential differences on the opposite ends of the fiber, which persist for several minutes. The cathode region is found to be alkaline in reaction, the anode region acidic. The polarized fiber is therefore a concentration cell.

The electrolysis of cotton may be carried out experimentally in another way. If cotton yarn is immersed in water in each of a series of cells separated from one another by a parchment paper membrane and a direct current is passed through the cells for some hours, the impurities tend to accumulate near the electrodes, with the development of acidity at the anode and alkalinity at the cathode, as is usual in the electrolysis of a saline solution. If the samples of cotton yarn be now removed and brought into equilibrium with an atmosphere of standard humidity, the insulation resistance is found to vary fairly regularly with the original position of the sample in the series of cells, being greatest in some intermediate cell and diminishing toward either electrode. The precise position of the maximum varies with the nature of impurities present in the system. The highest insulation resistance may be many times that of the original cotton.

Perhaps the most significant evidence of the importance of electrolytic impurities in silk, wool, cotton, and to some extent other textiles, is the fact that their electrical characteristics can be greatly improved by thorough washing with water though without altering qualitatively the general nature of the electro-conducting phenomena which characterize them. Chart II illustrates the result of washing upon the insulation resistance of cotton and silk threads. The improvement in insulation resistance of cotton and silk upon washing ranges commonly from fifty to one hundred fold, under any of the commonly prevailing conditions of atmospheric temperature and humidity. This improvement is accompanied by diminution of the ash content, in the case of cotton from about 1.0 per cent to 0.15 or 0.25 per cent. It produces only a slight reduction in the equilibrium moisture content of the cotton over the ordinary ranges of atmospheric humidity. The sensitivity of the washed cotton to continued application of voltage is much less than that of the original, but polarization still occurs. Commercial silks are similarly affected by washing.

If the mineral contents of cottons which have undergone washing are compared quantitatively with the original contents a decrease is observable, particularly as to potash, but the calcium and magnesium contents are much less altered. Fairly complete removal of potash is apparently essential to good electrical characteristics, but improvement electrically has been attended in some cases by an actual increase in content of alkaline earths. This suggests that interchange of electrolytic impurities between the textile and the water is involved as well as actual removal of electrolytes by the water. Thus in general hard natural waters, *i. e.*, those containing calcium and magnesium salts, have proved as good or better than soft waters when used in economically small amounts. Very exhaustive extraction with distilled water gives excellent results, though not vastly superior to washing with very dilute solutions of alkaline earth salts. Sufficiently complete and accurate analyses of samples of textiles brought into equilibrium with washing liquids and of the kind and quantity of electrolytes in the corresponding liquids have yet to be made to determine the precise importance of the composition of the saline residues. Non-saline electrolytes have also to be considered. This matter requires extended study and the experimental data are reserved for future publication.

The commercial value of such treatments of insulating yarns have proved to be very substantial. The utilization of the products forms the subject matter of another paper from the Bell Laboratories.

Another common property of textiles is known as the Evershed effect. Evershed[14] found in various insulating materials, including textiles, that insulation resistance does not obey Ohm's law but is less if a larger measuring voltage is used. Evershed's finding as to cotton has been verified by us. This result is easily obtainable if conditions are maintained so that little polarization occurs. But if extensive polarization is allowed to take place the reverse effect is observed and the ultimate resistance is higher in proportion to the voltage used. The conditions which favor polarization are, of course, considerable voltages, prolonged application, high relative humidities, and short paths through the insulation. Evershed's work apparently did not involve any special attention to the time of application of voltage.

Evershed's explanation of decreased insulation resistance with increased voltage involves the assumption that much of the water contained in insulations is originally in the form of isolated pools and therefore of no conductive effect at the instant of application of voltage. In support of his theory of "dormant" water he lays great stress upon his observation that the volume of water present in the insulating materials is far in excess of that which would be required to furnish

14. Evershed, S., *Inst. Elec. Eng. Jl.* **52**, 51 (1914).

the observed conductivity if the water were in the form of continuous filaments of uniform cross section. This argument seems impressive and conforms to our own ideas of the distribution of water in textiles. However, according to Evershed, this pool water is electrokinetically spread out into conducting films under electric stress, thus accounting for decreased resistance with increased voltage. A tendency to such movement of water cannot be denied. But it is difficult to harmonize Evershed's conception of electro-endosmotic movement of water as the predominating phenomenon with all the facts regarding textiles with which we have had experience; for example, with the fact that the Evershed effect is greatest when the electrolyte content of the textile is high. Electroendosmose in systems designed for its ready detection usually diminishes with increasing electrolyte content except when the concentrations are very small.[15] Further a decrease of resistance with increase of voltage has been noted in other systems in which electrolysis is unquestionably involved and in which electro-kinetic redistribution of water seems improbable.

Though sufficient support for an alternative cannot be furnished at present, electrokinesis does not constitute the sole possible explanation of the Evershed effect. The analogy between the moist fiber and an electrolytic cell conforms to a number of other corollary facts about the Evershed effect which are discussed in a more specialized paper by one of the authors.[30] The various properties which have been discussed are in agreement with the view that the cardinal principle of conduction in textiles is the electrolysis of aqueous solutions.

Distinctive Characteristics of Each Fiber Species

The several kinds of fibers exhibit a number of curious contrasts in the relation of electrical behavior to hygroscopic properties, some of which at first glance appear contradictory. For convenience in discussion let us classify the commercial fibers into two main groups: (1) the animal fibers, and (2) the vegetable fibers, and a sub-group (2a), the cellulose ester fibers of which the so-called cellulose acetate silk is the sole representative of commercial importance at present. It will be seen by reference to Fig. 1 that over the entire range of relative humidity the animal fibers, silk and wool, absorb more water than the natural vegetable fibers. This is true whether we deal with fibers in their natural impure state or after a washing process which has been shown to improve greatly the electrical characteristics of both types of natural fibers. Cellulose acetate absorbs less water at any given humidity than either class of natural materials.

We have seen that for any given kind of fiber there is an orderly dependence of electrical properties upon the moisture content of the fiber and in turn upon the relative humidity of the atmosphere. The more water present in any given fiber the poorer are the electrical properties. If the amount of water in fibers were the sole determinant of electrical characteristics we would expect the animal fibers to be, at a given humidity, the poorer electrically of the classes enumerated above. But this is emphatically not the case. With respect to electrical properties, we find that the vegetable fibers are inferior to the more hygroscopic animal fibers and are also inferior to the least hygroscopic variety of commercial fiber, *viz.*, cellulose acetate. To make the existing contrast clear we have plotted in Fig. 4 for the several textiles, Log Insulation Resistance *vs.* Log Moisture Content. When so plotted the values for each kind of fiber fall approximately on straight lines throughout the range of actual measurement. The relative position of the curves for animal fibers to the right and above that for cotton[16] means that the animal fibers have the better insulating qualities in spite of higher hygroscopicity.

The slopes of these lines have an even greater significance for they indicate the relative sensitivity of the fibers to an increment of moisture. Since the slope for the animal fibers is greater, it is evident that the animal fibers are more sensitive electrically to moisture than cotton. Under a given set of conditions they are not only wetter than cotton, but are more sensitive to the effects of further increments of moisture and yet they have a higher insulation resistance.

In one respect alone can we say that cotton is preferable as an insulating material. It has the merit of being more nearly uniform in behavior under a variety of weather conditions. When the amount of moisture taken up from the surrounding atmosphere by silk or wool is doubled, the electrical leakage increases by a factor of 50,000 to 100,000, while that for cotton rises by a factor of only 600.

The position and slope of the values for cellulose acetate are of great interest as the curve coincides with that for cotton, indicating that moisture affects these two fibers in a very similar manner. The essential electrical difference between these two appears to be satisfactorily accounted for by the fact that cellulose acetate absorbs less water than cotton at any given relative humidity of the atmosphere. The conversion of cotton which is essentially cellulose into cellulose acetate by the process of acetylation, has, as could be predicted on chemical grounds, reduced its hygroscopicity but apparently has not modified its structure greatly. Cellulose acetate is therefore put in a subgroup rather than in an independent classification.

The reader will have been led to ask several questions. Why do the several kinds of fibers differ so much in absorption of water and particularly why does not a

15. Powis, Frank, *Zeit. Physik. Chem.* 89, 91, 1914 and Burton, E. F., *Coll. Symp.*, Monograph IV, 132, 1926.

16. The threads referred to are approximately, but not precisely, of the same size. This variable is by no means sufficient to account for the higher position of the animal fibers as compared with cotton.

given amount of water affect them all alike electrically? He will want to know if the mere fact of animal, vegetable, or artificial origin is associated with a particular type of electrical or hygroscopic behavior. He will wonder whether there is any fundamental resemblance between the silk which is rapidly extruded by a worm and the hair which grows so slowly on a sheep's back. He will inquire whether there is sufficient justification for classifying other vegetable fibers with cotton. To these questions only partial and to some extent speculative answers can be given.

We are justified on chemical grounds in classifying the fibers in the same way which we have found to be convenient for discussion of their hygroscopic and electrical properties. How much importance should be attached to this correspondence between the chemical and electrical classifications cannot be determined at present. However, the correspondence seems suggestive and deserving of a brief discussion. The first class, that of animal fibers, has a common chemical nature in that they consist largely of proteins. Proteins are molecular aggregates of colloidal size composed in turn of simpler substances known as amino acids. Each of the constituent amino acids contains both an acidic and a basic group, so that in acid solutions they behave as bases and in alkaline solution they behave as acids. This so-called amphoteric property is due to the presence of an acidic oxygen nucleus and a basic nitrogen atom, which are almost invariably adjacent to one another. Amphoteric properties persist in the proteins which are formed by union of many amino acids in a single molecule. This is illustrated by the fact that either amino acids or proteins in a solution which is subjected to a d-c. voltage tend to migrate to positions intermediate between the electrodes where the acidity is such that they are equally ionized as bases and as acids. The combination of adjacent acidic and basic groups within a single molecule gives them a salt-like property which may be significant. It is reasonable to associate the hygroscopic quality of the proteins with these groups as the molecules are usually without groups of polar character other than the paired groups mentioned.

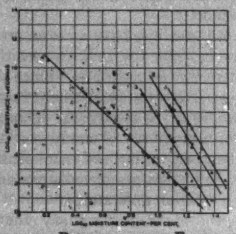

Fig. 4—Insulation Resistance as a Function of Moisture Content of Textiles

1. Wool yarn
2. Silk threads purified (○ sample 1 ● sample 2)
3. Silk threads impure
4. ○ Cotton threads, + cellulose acetate threads

That their common protein character is responsible in some way for the properties of principal interest from the insulating standpoint is rendered the more probable by the close resemblance of silk and wool, as shown by the approximate parallelism of their curves in Fig. 4. This resemblance is shared in considerable measure by other hairs than wool.

The second class of fibers, coming from the vegetable world, are alike in being composed of cellulose, a substance like the protein in having a high molecular weight but unlike it in that its polar groups are hydroxyls which have a faintly acidic rather than amphoteric nature. These are the groups in cellulose with which water is likely to associate itself. Such data as are available concerning vegetable fibers other than cotton, notably linen, ramie, manila hemp, and wood pulp, indicate a strong resemblance not only chemically but hygroscopically and electrically.

The sub-class embracing only cellulose acetate as a commercial fiber is chemically more neutral and non-polar in type than other cellulose fibers, with which fact it is reasonable to associate its lower hygroscopicity and consequent better electrical characteristics. It is probable that cellulose nitrate and cellulose ethers will be found to fall in this class but artificial silks other than cellulose acetate absorb more water and appear on chemical grounds to be better classified with the cellulose fibers of natural vegetable origin.

It cannot be decided from available information whether the similarities and differences in electrical properties among the textiles are traceable directly to chemical similarities and differences or indirectly to physical (colloidal) structures which in turn are determined by chemical composition. In either case it is possible to account for the high sensitivity of all the fibers to moisture and the variation in sensitivity from species to species by assumptions as to the distribution of water in them. Water which collects in any isolated form in the material will have little electrical effect compared with that which forms continuous filaments. The distinction we are making is essentially the same as that of Evershed when he referred to part of the water as "dormant," though we do not attach the same importance as he does to electrokinetic redistribution of water under electric stress. Each increment of water may be considered as undergoing partition into two portions, one causing a large increase of conductivity and one having a negligible effect, in a ratio determined in some fashion by the structure or nature of the material and the humidity of the atmosphere. The ratio of the two portions which are in equilibrium via the surrounding atmosphere will be subject to constant readjustment under changing conditions.

The fact that the electrical characteristics of the two classes of fibers as affected by moisture appear to be specific properties of the substances involved suggests

some highly regular distribution pattern of conducting water paths determined by the chemical or physical (colloidal) structure of the material. Such a regular pattern may involve only water condensed upon the surfaces of the elements of structure in such a way that the thickness of the film varies regularly from point to point through the material. Accumulation of water at thick points would have little electrical effect, while that in thin portions would be very significant. An alternative regular mode of distribution would involve water in part dispersed in solution or chemical combination within the units of structure of the material and in part in fairly uniform thin films on their surfaces, in which case the latter would have the major electrical consequence. While such a regular form of distribution seems preferable, it is perhaps not the only way of accounting for the electrical properties observed.

The curves as shown in Chart IV for both cotton and silk are straight lines within the experimental error but the assumption is not justified that they can be projected as straight lines to zero humidity. At the lower ranges of humidity the resistance of silk is so high as to exceed the limits of our present technique of measurement. Over some range below 40 per cent relative humidity it may well be that the sensitivity of the animal fibers to increments of moisture is less than that of the cellulose fibers. If so, the break in the curve would have great interest in connection with determining the mode of water distribution and in turn the colloidal structure of the materials in question. It is hoped that an extension of the study in this direction will be possible in the future.

The authors wish to acknowledge their indebtedness to their colleagues, whose names appear as authors of kindred papers, for their advice and assistance. Also we wish especially to thank Dr. Homer H. Lowry, whose discernment has stimulated the development of evidence necessary to several of the more important deductions.

Discussion

For discussion of this paper see page 581.

Purified Textile Insulation for Telephone Central Office Wiring

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and

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Associate, A. I. E. E.

Synopsis.—This paper outlines methods by which silk and cotton insulation can be purified and improved. It gives the results of tests on the insulation characteristics of these materials before and after purification and explains the testing procedures. One of the findings is that the purified cotton may be substituted for ordinary commercial silk.

IN another paper, *The Predominating Influence of Moisture and Electrolytes upon Textiles as Insulators,* (see p. 568) Messrs. Williams and Murphy have shown that the electrical properties of textiles are closely associated with their moisture content and impurities in the textiles. In particular, water-soluble salts become ionically conducting in the presence of moisture and the ions migrate along the paths of initially low resistance to the electrodes with which they react chemically, causing serious corrosion. The resulting corrosion products, themselves electrolytes, accelerate the process of current transfer and may easily lead to a complete failure of the insulating textile at the point of greatest concentration. Conversely, if the impurities are removed, the insulating properties of the textile are improved initially and, furthermore, are not subject to cumulative deterioriation due to concentration of conducting salts and electrolytic corrosion products at the weaker points. It is the purpose of this paper to show how these principles are borne out by field observations and laboratory tests, and to show in a general way the extent to which the insulating properties of silk and cotton can be improved commercially with particular application to telephone central office wiring.

Since the early days of telephone development work, silk and cotton have been the standard insulating materials for wire insulation in telephone central office apparatus, supplemented in later years by enamel insulation. Relatively low voltages have always been used in the telephone plant, 24 to 48 volts being the usual voltages which are carried continuously in cables, while intermittent a-c. and d-c. potentials generally do not exceed 100 to 150 volts. Therefore it has been generally accepted that telephone cables, once installed and properly protected from accidental high voltages, could be depended upon to have a substantially indefinite life. In general the insulation of these cables has been satisfactory, but breakdowns have occurred which could not be attributed to faulty operating conditions or to manufacturing defects. A study of this subject showed that it was possible under certain conditions to get discolored or faded spots in the insulation and corresponding corroded or pitted spots in the tinned copper conductors. It was also observed that the textile insulation at such spots showed a strong concentration of water soluble salts. Also, cables in which such conditions occurred measured relatively low in insulation resistance with the current leakage concentrated at these points. These observations led to the conclusion that silk and cotton would be decidedly improved as insulating materials if they were made less susceptible to deterioration under telephone service conditions.

Aside from the consideration of improving silk and cotton to assure greater insulation stability, considerable thought has been given to the possibility of improving the insulating characteristics of cotton to such a degree that it could be substituted for the more expensive silk. The importance of this work with respect to its bearing on the cost of telephone service can be better appreciated from the fact that about 2000 pounds of silk are required daily to provide for the growth of the country's telephone requirements, which if replaced with cotton would reduce raw material costs by a very substantial sum.

The desirability of reducing the quantity of silk required in the telephone plant does not arise entirely from this phase of the economic question. The problem of supply and demand has at times entered into the matter. For example, shortly after the close of the world war the supply of insulating silk was limited and the price prohibitively high. Substantially the same condition arose a few years later, which leads to the conclusion that silk is inherently much more subject to violent fluctuations in available supply and cost than cotton. Therefore, with demands for telephone equipment rapidly increasing, we have decidedly greater assurance of an adequate supply of insulating material at reasonable cost if cotton instead of silk is used.

Purification Process

With the foregoing as an introduction to indicate the economic advantages to be gained by improving the electrical characteristics of cotton and silk, the following is intended to show what has been accomplished by the commercial application to silk and cotton thread of

*Bell Telephone Laboratories, Inc., New York, N. Y.

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29-32

the processes referred to by Messrs. Williams and Murphy for removal of objectionable impurities.

Since such impurities are soluble in water, it will be inferred that the purifying process consists in a thorough washing with water. In effect, this is the case. The process, however, for both silk and cotton, being based on substantially complete removal of the ionically conducting salts, especially those of sodium and potassium, prescribes the use of water of low saline content. It also means that the washing is best accomplished by a continual flow which after passing through the textile

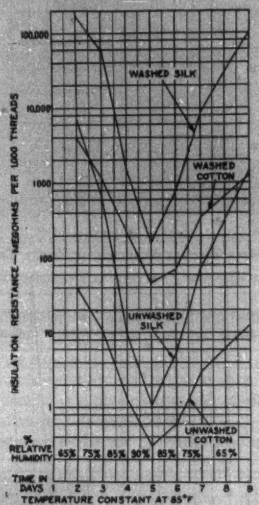

FIG. 1—TYPICAL D-C. RESISTANCE CHARACTERISTICS OF WASHED AND UNWASHED SILK AND COTTON THREADS OF EQUAL SIZE

is considered to be contaminated and is not used again.

Where cotton is to be dyed and washed, the washing consists in an additional operation applied to the cotton immediately following the dyeing operation without the necessity of drying between processes.

CHARACTERISTICS OF PURIFIED INSULATIONS

Obviously, the first consideration in the insulation of electrical conductors is to provide an insulating medium of sufficient dielectric strength to withstand the working potentials to which they are subjected. Also, the d-c. insulation resistance must be high enough to prevent undue d-c. energy loss. A comparison of the electrical resistance of the cotton and silk at relative humidities ranging upward from 65 per cent to 90 per cent and down again to 65 per cent, before and after washing, is shown by the graphs in Fig. 1 as determined by samples prepared and tested by the method and testing apparatus shown in Figs. 6, 7, and 8 and described later. The same comparison is shown in Fig. 2 except that these graphs show the insulation resistance of wire insulated with the washed and unwashed textiles. In addition to the insulation resistance requirement, it is required that the energy losses at talking and carrier current frequencies must be maintained at the minimum point consistent with the space limitations permitted for the conductors. The effect of purification of the textiles on this characteristic expressed in capacitance and conductance, measured at 1000 cycles per sec. between the wires of twisted pairs is shown in Fig. 3 and Fig. 4. The data represented by these graphs converted into transmission loss units are illustrated in Fig. 5. As the same thickness of insulation was used in all cases, the graphs are on a comparative basis. It should be noted that the graphs are illustrative of the effects of purification on the electrical properties of cotton and silk as insulation and should not be considered as applying quantitatively to telephone circuits.

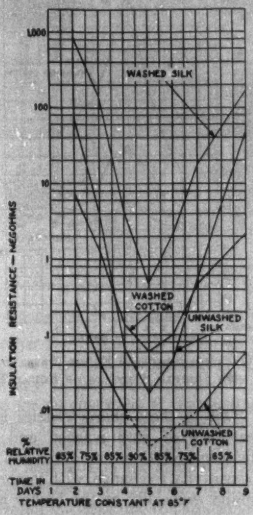

FIG. 2—D-C. INSULATION RESISTANCE OF 50 FT. OF TWISTED PAIR WIRE INSULATED WITH DOUBLE SERVINGS OF EQUAL THICKNESS

From a telephone transmission point of view, perhaps the most significant fact to be observed is the large reduction in capacitance and conductance at relative humidities of 75 per cent and higher. These characteristics which largely determine transmission efficiency are relatively low for both silk and cotton at 65 per cent and below, but in commercial textiles in general use for insulating purposes they increase very rapidly as the relative humidity increases. The characteristics of

purified textiles are not as markedly different from those of unpurified textiles at 65 per cent relative humidity as at higher humidities, but their rate of increase as the humidity increases is greatly reduced. This fact is of particular importance in the maintenance of a standard level of voice transmission through toll offices where suitable repeater gains and balance must be maintained. Losses, if fixed in value and not excessively large, can be compensated for, but if they change with every change in atmospheric moisture content the compensation problem becomes serious.

Method of Testing

Two fundamental characteristics of silk and cotton made it necessary to do a large amount of experimental work before a practicable shop test method could be established to determine whether or not the textiles were washed to the point of meeting the requirements

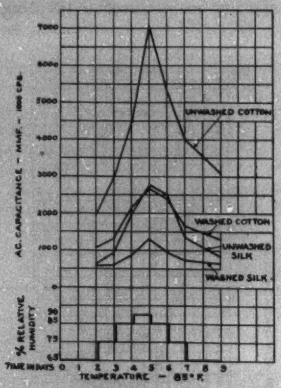

Fig. 3—A-c. Capacitance of 50 Ft. of Twisted Pair Wire Insulated with Double Servings of Equal Thickness

established. One of these characteristics is the high electrical resistance of both washed and unwashed textiles at the lower relative humidities and the other the extreme sensitivity to change, with minor change in relative humidity especially at the higher humidities. The first mentioned characteristic precludes the use of any but measuring instruments of the highest degree of sensitivity and makes desirable the use of comparatively high humidities, and the second characteristic means that the specimen must be tested under exceedingly well-controlled relative humidity conditions. Furthermore, the problem is complicated by the polarization effect discussed in the paper by Williams and Murphy and the fact that this effect varies in magnitude with humidity and with the degree of purity of the textiles. The problem was finally solved by the development of the test equipment shown in Figs. 6, 7, and 8.

Figs. 6 and 7 show a heat insulated glass tank of about one cubic foot capacity fitted with an insulating cover in which holes normally closed with stoppers are used to introduce the test samples. The humidity is

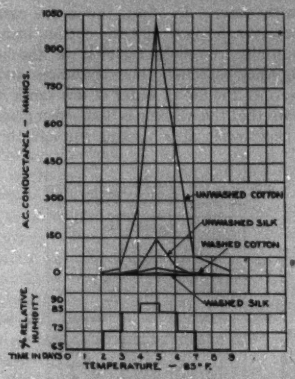

Fig. 4—A-c. Conductance of 50 Ft. of Twisted Pair Wire Insulated with Double Servings of Equal Thickness

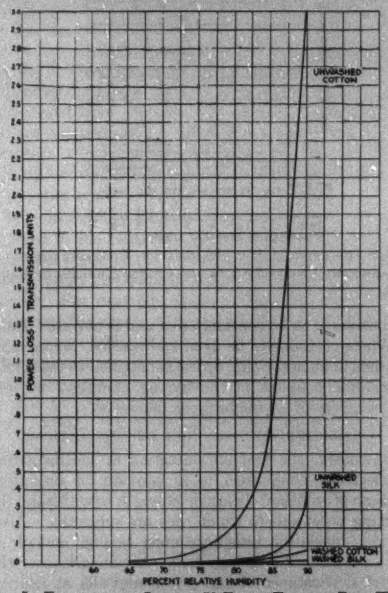

Fig. 5—Transmission Loss in 50 Ft. of Twisted Pair Wire Insulated with Double Servings of Equal Thickness

maintained by means of sulphuric acid or a saturated salt solution in the bottom of the tank and constant temperature within very narrow limits is maintained in

the tank by placing the entire assembly inside a cabinet or oven automatically controlled to ± 0.5 deg. fahr. Due to the heat insulation it has been found that temperature variations within the tank are reduced to the vanishing point for all practical purposes.

This is very important as it has been found that fluctuations in the temperature of the test chamber

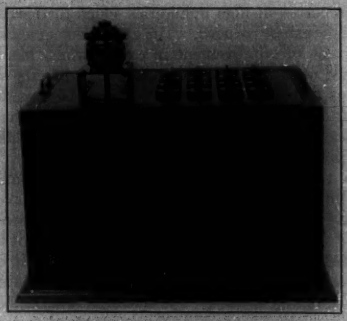

Fig. 6—Humidity Cabinet for Conditioning Samples

introduce large errors in the insulation resistance of the samples. The errors, however, are attributed not to temperature effects on the samples but to variations in relative humidity produced by the temperature changes and the considerable time required for equilibrium to be restored after such changes occur.

Another source of error in textile testing is found in

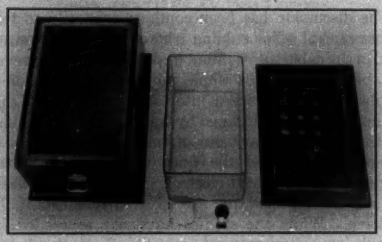

Fig. 7—Humidity Cabinet Disassembled

the fact that the values of insulation resistance are affected by the humidity condition to which the sample has been exposed prior to the test. To avoid error from this source, all test samples are conditioned by drying in a desiccator at the approximate temperature of the test tank before being placed in the tank.

The samples are prepared by winding a number of turns of the textile around the electrodes inserted in the stoppers as shown in Fig. 8. Care is taken not to handle the textile itself during the winding process as perspiration from the hands is likely to contaminate the thread. Samples are left in the tank over night as there is considerable evidence to show that several hours are required for them to come to complete equilibrium. A temperature of 100 deg. fahr. and relative humidity of 75 per cent has been found suitable for cotton testing and 100 deg. and 87 per cent relative humidity for silk.

The number of turns of yarn or thread wound around the electrodes will vary with the size of the thread since the winding space is fixed and a single layer of thread is applied. For No. 30/2 cotton approximately 90 turns, 180 parallel threads, have been found to give satisfactory readings. This same space accommodates about 256 turns, 512 parallel threads, of No. 62/1 spun silk.

The distance between the electrodes is not particularly critical. That is, it is not important, for ex-

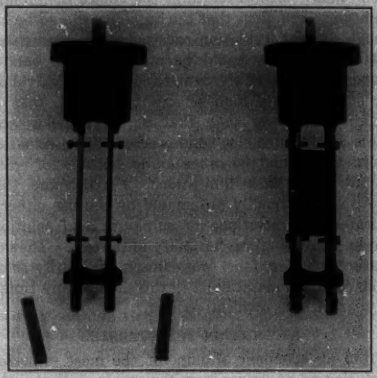

Fig. 8—Electrodes on which Samples are Wound for Test

ample, whether the distance is ⅝ in. or ¾ in. It is important, however, that having decided upon a certain separation, say ¾ in., this separation be accurately maintained for all electrodes if the readings are to be comparative. Of course if the separation is too great, an unreasonable number of turns of textile is required to bring the resistance of the sample within the range of the galvanometer. On the other hand, if the separation is too small the error due to variation in separation for different sets of electrodes increases in magnitude.

In actual practise, ¾ in. separation with a winding space of 2 in. accommodating, as mentioned above, about 90 turns of No. 30/2 cotton has been found to be fairly satisfactory. This arrangement gives galvanometer readings of the order of 2000 megohms for washed silk and 1000 megohms for washed cotton as compared with 12 meghoms and 5 megohms for unwashed silk and cotton respectively. It is obviously necessary to maintain a high degree of insulation resistance between the electrodes. This is accomplished by using hard rubber for the stoppers in which they are mounted and preventing surface leakage by coating the end of the stoppers with ozokerite wax. The electrodes them-

selves are gold or platinum plated to prevent oxidation or corrosion. Observing these precautions, it is possible to obtain readings sufficiently consistent to distinguish not only between washed and unwashed textiles but to determine differences in degree of purification in various lots of washed textiles.

The question may be asked as to why 75 per cent relative humidity and 100 deg. fahr. was selected for cotton and 87 per cent relative humidity and 100 deg. for silk. These values were, within reasonable limitations, more or less arbitrarily selected and further experience may show that some other values are preferable. However, for the following reasons, 75 per cent and 87 per cent at 100 deg. fahr. were selected as offering definite promise of giving consistent results.

The main considerations in the choice of humidity conditions were, first, that the humidity should be high enough so that insulation resistance measurements of sufficient accuracy could be made using a band of threads as described above and a commercial galvanometer of reasonably high sensitivity; second, that the humidity be lower than that at which polarization effects would introduce serious error. The humidities chosen are within the range found suitable for cotton and silk under these limitations. Furthermore, these conditions are readily obtainable by the use either of saturated salt solutions or sulphuric acid solutions, thereby increasing the flexibility of the test. The temperature 100 deg. fahr. was chosen arbitrarily as one which could be maintained in the shop at any time of the year without artificial cooling.

Application to Apparatus

From an economic standpoint the most important conclusion to be drawn from the graphs is that cotton can be improved by washing to such an extent that it becomes a better insulator than the ordinary commercial insulating silk in general use. Since the cost of washing silk and cotton is nominal, usually less than 5 per cent of the cost of the material, the engineer given purified textiles may either take advantage of marked improvement in quality of electrical characteristics by using washed silk, or may substitute washed cotton for silk and realize substantial economies without degrading the product. As an example of how this applies to Bell System apparatus, central office distributing frame wire with annual requirements of more than 400 million conductor feet is now insulated with two coverings of silk where three were formerly required. The resultant wire is superior electrically to the old wire and the annual saving in silk amounts to about 70,000 pounds.

As another example, telephone cords of various types have been reduced substantially in cost with no impairment in quality by substituting two washed cotton braids for the cotton and silk braids formerly used. Altogether, various types of textile insulated wire aggregating annual requirements in excess of two billion conductor feet have either been changed to employ washed textile insulation or are scheduled for change as soon as possible because of corresponding economies in manufacturing cost or improvement in electrical properties.

The foregoing is intended to show what has been accomplished on a commercial scale at reasonable cost in the way of improving the insulating properties of silk and cotton. There still exists a rather wide margin in insulating properties between washed silk and washed cotton at high humidities which further study may show can be reduced. The graphs do not show the magnitude of improvement in cotton which has been obtained occasionally in laboratory experiments which leads us to hope that presently it may be possible to process cotton in a way that will result in its having electrical properties equal to those of washed silk for many practical purposes.

The question naturally arises as to the permanence of the improvement effected by the purification process. We have attempted to answer this question by periodic tests of washed silk and cotton insulated wire over an extended time, the test samples being exposed to ordinary room conditions where they could accumulate the normal quantity of dust. The results show no tendency for the insulation to revert to the constants of unwashed insulation. This appears logical since there is no particular reason to expect contamination by accumulation of such impurities as sodium or potassium salts from ordinary exposure to the air. Furthermore, in service, telephone office wiring is protected from the effects of dust by braided textile coverings or by the application of waxes or varnishes where the individual wires are exposed.

Conclusion

The discussion has been confined primarily to telephone central office cabling where silk and cotton are used in the cable core without impregnation. However, it is believed that the whole subject of purification of textiles becomes of general interest when it is stated that the improvements obtained by washing are not nullified by the supplementary use of impregnating waxes or varnishes. That is, the improvement in dielectric properties and reduced electrolysis obtained by washing and by impregnating are apparently substantially additive. While the studies have not proceeded far enough to cover comprehensively all of the better known impregnating waxes, asphalts, varnishes, etc., they have proceeded to the point where we can say that this is the case for the beeswax-paraffin waxes and certain asphaltic compounds. These findings are in line with the generally known fact that impregnation of textiles with wax compounds does not prevent, though it does retard, the absorption of moisture which in the presence of soluble salts causes conducting paths to be established, probably through the embedded textile fibers. Consequently, such materials as fabric base insulating tapes, varnished linens and cambrics, electro-magnet coil winding insulation, all being sensitive electrically to moisture, should be benefited to a

substantial degree by purification of the fibrous components.

Therefore, while there is still much to be learned about the behavior of silk and cotton with respect to their electrical characteristics under various treatments and conditions, the study has progressed to the point where the following statements can be made.

1. The removal of water soluble salts which are present in both silk and cotton not only results in a very decided improvement in their insulating properties, but reduces the sensitivity to change of the a-c. characteristics with changes in atmospheric moisture conditions.

2. The improvement which can be realized is great enough to permit the substitution of washed cotton for silk where ordinary commercial silk has been found to give satisfactory results.

3. The use of purified textiles in cables carrying continuous d-c. potential will reduce electrolysis and consequently prolong the useful life of such cables about in proportion to the extent to which the purification process is carried.

In presenting the foregoing discussion, the authors wish to acknowledge their indebtedness to engineers of the Western Electric Company whose work in cooperation with silk suppliers has been largely responsible for the development of commercial methods of purifying insulating silk. Acknowledgment must also be made of the importance of the fundamental and research work which underlies the engineering result briefly described by this paper.

Discussion

INFLUENCE OF MOISTURE AND ELECTROLYTES UPON TEXTILES AS INSULATORS

(Williams and Murphy)

PURIFIED TEXTILE INSULATION FOR TELEPHONE CENTRAL-OFFICE WIRING

(Glenn and Wood)

New York, N. Y., January 31, 1929

William Fondiller: The problem of improving textile insulation has been attacked by various methods in the past, usually by treating the textile with some moisture-resisting compound such as wax or mineral soap in an effort to make the thread moisture-repellent. These methods have effected very slight improvement. It wasn't until the fundamental work done by Mr. Williams and his associates on the nature of the conduction in textiles that real improvement was effected.

The value of these 600,000 pounds of silk, a large part of which the authors tell us may eventually be superseded by cotton, at around three dollars a pound for silk, evidently means an economy in the neighborhood of a million and a half dollars a year.

Bela Gati: If the Bell concern or other manufacturers have built common-battery central offices in tropical countries, I should like to hear something about their experience in this matter.

S. J. Rosch: I should like to ask Mr. Williams just how the moisture content was measured. In the case of a wood-pulp paper which I know is being used in the telephone cable industry, I find that samples of the same paper given to different investigators and under the same conditions of humidity will yield different percentages of moisture.

I am also interested to know, after the cotton has been washed and used in service, just how the entrance of moisture into the fibers once more is retarded? A paper which is thoroughly dry and is brought into atmospheric conditions immediately absorbs moisture again. Even fibers which have been impregnated in waxes are not immune from absorbing moisture.

I should also like to ask Mr. Williams whether he has investigated jute in the same way as he has the cotton and silk fibers.

E. J. Murphy: In reply to Mr. Rosch, we didn't actually measure the moisture content on our own samples. We took the data from the literature on the moisture content of textiles and applied it to our own case. The error caused by not measuring the moisture content of the samples actually used is not large enough to have any essential effect on the conclusions to be drawn from the results we have obtained.

There is what might be called a hysteresis effect in the absorption of moisture by textiles. When the humidity is decreasing, the amount of moisture in the textiles is greater than when it is increasing. The difference is quite considerable. It amounts to as large an amount as the difference between the moisture content for humidities that are different from each other by about ten per cent, under some circumstances. Because of this "hysteresis" effect it is necessary to take into account the previous history of the samples as regards their exposure to humid air if consistent results are to be obtained by different investigators making moisture absorption measurements on the same samples under the same humidity conditions.

Washing the water-soluble material out of a textile does not prevent the entrance of moisture nor cause an appreciable reduction in the amount of moisture which it absorbs. The essential effect of washing is to increase the general level of resistance; that is, for any given moisture content, the resistance of a sample of washed cotton is from 50 to 100 times greater than that of the original cotton. This is shown by the fact that the logarithm of resistance *vs.* logarithm of moisture content curves for washed cotton samples are parallel to those for the original cotton.

In further reply to Mr. Rosch, we have done no experiments on jute so I don't know whether the resistance-moisture content relationship is similar to that of cotton or not, but it seems probable that it is since jute is a cellulosic material.

E. B. Wood: In view of the point which was brought up regarding trouble in tropical countries, we may say that the curves we have shown apply mainly to the problems of the telephone engineer in temperate climates. It is only those parts of the curves above perhaps 90 or 85 per cent relative humidity that might apply to the more severe conditions of the tropics and you will note that there particularly we get a great benefit from the washing of the textiles. At the lower humidities, particularly under the dry conditions which we have in the winter time, there is very little difference between the washed and unwashed materials. In other words, under dry conditions all textiles are good insulators. It is only when they get moisture in them that we get the low insulation resistance shown on the curves. Therefore, the purification of textiles, it would seem, would be of special interest to those who are interested in the question of insulation for the tropics.

Another point is the corrosion effects. The remark is made in one of the papers that we have had corrosion effects even in our temperate climates and we know from experience that the corrosion troubles are far more serious in the tropics. Since such troubles are caused largely by the presence of impurities in the textile insulation, apparatus for the tropics seems a rather important application for the use of washed textiles.

There is one more point I wish to bring out, namely, that it is not our intention to convey the impression that we are likely in the near future to abandon the use of silk. It is evident from the curves that washed silk is still appreciably superior to washed cotton, and there are many circuits in which the advantage to be gained by the use of washed silk will make it desirable to continue its use. Furthermore, to offset the tendency toward reduction of silk due to the use of washed materials, we have the ever increasing demand for textile insulation to take care of the natural growth of the telephone system.

Vector Presentation of Broad-Band Wave Filters

BY R. F. MALLINA* and O. KNACKMUSS*

Member, A. I. E. E. Non-member

Synopsis.—The function of a broad-band wave filter of the iterative ladder type in the attenuation band, and outside the attenuation band can be explained very simply when expressed in terms of two characteristic vectors Z_a and Z_b. Drawing the diagram of these vectors, it becomes obvious that the angle between them is the phase shift of the filter and that the natural logarithm of the ratio of their magnitudes is the attenuation.

The diagram also shows very plainly the relationship between a mid-series and a mid-shunt structure, and the equations for such filters can be derived in a very simple manner from the geometry of one vector triangle.

It is hoped that this simplified presentation of types of filters which are so extensively used in radio, acoustical, and in mechanical engineering will be helpful in understanding their physical meaning.

1.0 Broad Band Filters of the Ladder Type in General

SUPPOSE we measure the impedance of a network as illustrated in Fig. 1 at the point 1 and obtain at the frequency f, the value $\bar{Z}_i = \bar{Z}_k$. The impedance $\bar{Z}_i$ we call the input impedance, $\bar{Z}_k$ the iterative impedance.[1]

Then we cut off section a, measure the impedance at point 2, and obtain again $\bar{Z}_i = \bar{Z}_k$. Cutting off sections b and c and always obtaining the measurement $\bar{Z}_i = \bar{Z}_k$ shows that Fig. 1 is a network whose input impedance is equal to the terminating or iterative impedance. A structure of this type is called an iterative structure. The broad-band wave filter of the ladder type is a special case of an iterative structure, and it is this type of filter with which this paper deals.

As will be seen later, it is necessary that with a change of frequency the terminating impedance $\bar{Z}_k$ must be varied in a certain manner so that at all times the input impedance $\bar{Z}_i$ is equal to $\bar{Z}_k$.[2]

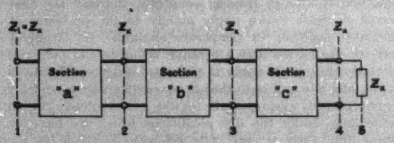

Fig. 1—General Arrangement of an Iterative Structure

A broad band wave filter structure will allow current to pass in a certain frequency band without attenuation, whereas outside this band the current is attenuated considerably.

This fact accounts for the name filter.

1.1 Two Characteristic Types of Filter Sections. The following two types of filter sections will make $\bar{Z}_i$ equal to $\bar{Z}_k$. The one is called a "mid-series section" (Fig. 2), and the other a "mid-shunt section" (Fig. 3). The mid-series section Fig. 2 is also called a T section and the mid-shunt section Fig. 3 a π section. There are other special types of filter sections which, however, will not be considered in this paper.

2.0 The Mid-Series Filter Section

2.1 Input Impedance. That the input impedance

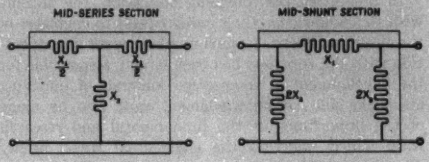

Fig. 2 Fig. 3

Figs. 2 & 3—Two Characteristic Types of Filter Sections

$\bar{Z}_i$ of a mid-series filter section (Fig. 4) can be made equal to the terminating impedance $\bar{Z}_k$ is shown in the impedance diagram, Fig. 5.

For the purpose of our first illustration, let us choose conditions so that $\bar{Z}_k$ is a pure resistance.

Adding to the terminating impedance $\bar{Z}_k$ the series impedance $X_1/2$ the vector Z_a (Figs. 4 and 5) is obtained. The sum of the reciprocal of Z_a and the vector $1/X_2$ gives us the vector $1/Z_b$ whose reciprocal is then Z_b. Adding $X_1/2$ to Z_b we find that the resulting vector $\bar{Z}_i$ is equal to $\bar{Z}_k$. In other words, having given $\bar{Z}_k$ and X_1 we choose X_2 to have a value such that $\bar{Z}_i = \bar{Z}_k$. The sequence of these operations is indicated in Fig. 5A.

Expressed in vector mathematics we have:[3]

$$\bar{Z}_k + \frac{X_1}{2} = Z_a$$

$$\frac{1}{Z_a} = \frac{|1|\,\epsilon^{j0}}{|Z_a|\,\epsilon^{j\beta/2}} = \frac{|1|}{|Z_a|}\,\epsilon^{j(-\beta/2)} = \frac{|1|}{|Z_a|}\sqrt{\beta/2}$$

*Victor Talking Machine Co., Camden, N. J.

1. As far as possible the same symbols will be employed in this paper as are used in K. S. Johnson's "Transmission Circuits for Telephonic Communication." (A complete list of symbols appears in Appendix A.)

2. In practise, of course, there is no such terminating impedance having the correct value at every frequency. However, it is possible to change certain elements of the network and obtain a close approximation to filter conditions.

3. The symbol $|Z|$ indicates magnitude of Z. The symbol Z without the bars represents a vector having magnitude and direction.

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29-44

In other words, if Z_a is a vector with angle $\left(+\frac{\beta}{2}\right)$ the reciprocal $1/Z_a$ is a vector with angle $\left(-\frac{\beta}{2}\right)$

$$\frac{1}{Z_a} + 2\frac{1}{2X_2} = \frac{1}{Z_b}$$

$$Z_b + \frac{X_1}{2} = \mathfrak{Z}_i$$

$$\therefore \quad \boxed{\mathfrak{Z}_i = \mathfrak{Z}_k} \tag{1}$$

These vector operations may be repeated for every filter section (Fig. 1) and the diagram (Fig. 5) will

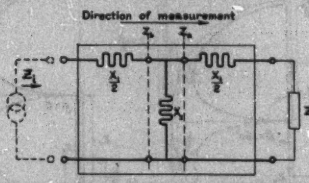
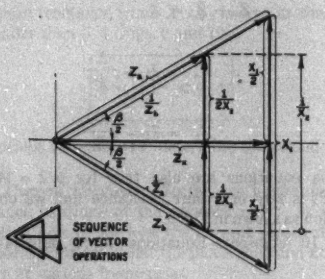


FIG. 4—CIRCUIT OF MID-SERIES FILTER SECTION

always remain the same. Therefore, we may say in general terms, that the input impedance $\mathfrak{Z}_i$ is equal to the iterative impedance $\mathfrak{Z}_k$ and this in turn is equal to the terminating impedance $\mathfrak{Z}_k$ (Fig. 1 at points 1, 2, 3, and 4).

2.2 Iterative Impedance $\mathfrak{Z}_k$. It is clear from the



5A

FIG. 5—VECTOR DIAGRAM OF A MID-SERIES FILTER SECTION

diagram in Fig. 5 that the vector $1/2\,X_2$ must be of such a value that the vector Z_a is in line with the vector $1/Z_b$. If this is not the case, the input impedance is not equal to the terminating impedance and the structure is not the iterative structure described in paragraph 1.1.

From the vector geometry of Fig. 5 we obtain:

$$\frac{X_1}{2} + \frac{1}{2X_2} = Z_a + \frac{1}{Z_b}$$

$$\therefore X_1 X_2 = Z_a Z_b \tag{2}$$

$$\boxed{Z_a = \mathfrak{Z}_k + \frac{X_1}{2}} \tag{3}$$

$$\boxed{Z_b = \mathfrak{Z}_k - \frac{X_1}{2}} \tag{4}$$

Substituting (3) and (4) in (2)

$$X_1 X_2 = \left[\mathfrak{Z}_k + \frac{X_1}{2}\right]\left[\mathfrak{Z}_k - \frac{X_1}{2}\right] = (\mathfrak{Z}_k)^2 - \left(\frac{X_1}{2}\right)^2$$

$$\mathfrak{Z}_k = \sqrt{X_1 X_2 + \left(\frac{X_1}{2}\right)^2}$$

$$\boxed{\mathfrak{Z}_k = \sqrt{X_1 X_2}\sqrt{1 + \frac{1}{4}\frac{X_1}{X_2}}} \tag{5}$$

In Fig. 5, vector X_1 and vector $1/X_2$ have the $+j$ direction; such a structure is called a low pass filter. However, Equation (5) may also be obtained when X_1 and $1/X_2$ have the $-j$ direction. This structure is a high pass filter.

In Equation (5) the reactances X_1 and X_2 are functions of frequency.

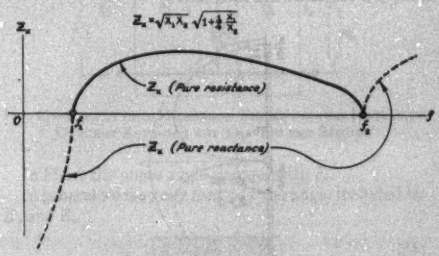


FIG. 6—ITERATIVE MID-SERIES IMPEDANCE OF ONE TYPE OF BAND-PASS FILTER

Taking the special case of a band-pass filter (Fig. 38a) we may plot Equation (5) and obtain a curve as illustrated in Fig. 6.

It appears from the figure that $\mathfrak{Z}_k$ takes the value zero at two points on the frequency scale, one at f_1 and one at f_2. They are called the cut off frequencies. As will be seen later, between these cut-off points the current passes through the structure without attenuation, whereas it is considerably attenuated outside the frequency cut-off points.

2.3 Iterative Impedance Z_k in Vector Diagram. How the impedance Z_k is changed from a resistance into a reactance at the points f_1 and f_2 may be illustrated by using the vector diagram of Fig. 5 and varying the value of the impedance vectors X_1 and $1/X_2$ from zero to infinity.

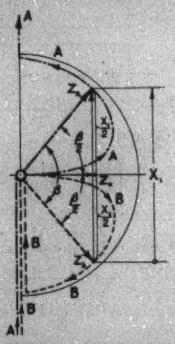


FIG. 7—POSITION OF THE CHARACTERISTIC VECTORS Z_a AND Z_b IN THE FREQUENCY BAND

Let us start with a frequency at which the angle β is the angle enclosed by Z_a and Z_b in the band-pass filter diagram of Fig. 7.

By varying the frequency up and down (Equations (3) and (4)), the vector Z_a describes the dot-and-dash line, the vector Z_b the dash-line.

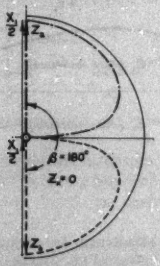


FIG. 8—POSITION OF THE CHARACTERISTIC VECTORS Z_a AND Z_b AT THE UPPER CUT-OFF FREQUENCY f_2

The arrows A and B indicate the directions in which the vectors Z_a and Z_b move when the frequency is increased.

If we increase the frequency, we get one limit for Z_k as pure resistance when $\beta = 180$ deg. (Fig. 8).

Then

$$Z_a = -Z_b = \frac{X_1}{2} \tag{6}$$

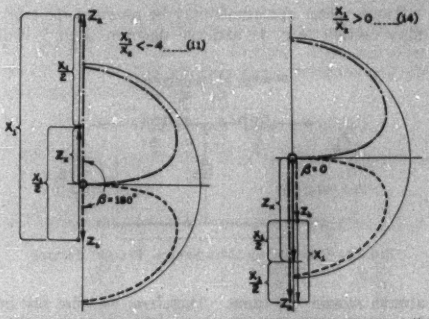
or in other words, Z_a and Z_b are equal in magnitude but have opposite directions. $Z_k = 0$ in this case, as is obvious from Fig. 8.

If we decrease the frequency we get the other limit for Z_k as pure resistance.

Then

$$Z_a = Z_b = \frac{X_1}{2} = 0 \tag{7}$$

2.4 Resistance and Reactance Limits of Z_k. The question is now, what happens to Z_k when we have increased or decreased the frequency beyond these limits.



FIGS. 9 & 10—POSITION OF THE CHARACTERSITIC VECTORS Z_a AND Z_b OUTSIDE THE FREQUENCY BAND

From Fig. 7 we can see that X_1 is the difference between the vectors Z_a and Z_b or

$$Z_a - Z_b = X_1 \tag{8}$$

also

$$Z_a = Z_k + \frac{X_1}{2} \tag{3}$$

$$Z_b = Z_k - \frac{X_1}{2} \tag{4}$$

These equations are also true for $\beta/2 = 90$ deg. Then Z_k is zero or a pure reactance and we obtain a condition as shown in Fig. 9.

Fig. 10 represents Equations (8), (3), and (4) when $\beta = 0$.

If we let $X_1/2$ and $1/2\,X_2$ in Fig. 5 increase until $\beta = 180$ deg. and $Z_k = 0$, we obtain a vector diagram as shown in Fig. 11. From Fig. 5 it is obvious that assuming the angle β to be 180 deg.:

$$\frac{X_1}{2} = Z_a \tag{9}$$

$$\frac{1}{2\,X_2} = -\frac{1}{Z_a} \tag{10}$$

$$\therefore \frac{X_1}{X_2} = -4 \tag{11}$$

If we let X_1 in Figs. 5 and 7 decrease until $\beta = 0$, it is clear that

$$\frac{X_1}{2} = 0 \tag{12}$$

$$\frac{1}{2X_2} = 0 \tag{13}$$

$$\therefore \frac{X_1}{X_2} = 0 \tag{14}$$

In this way, we may say that assuming the vectors

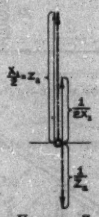


FIG. 11—POSITION OF THE VECTORS Z_a AND $1/Z_a$ AT THE UPPER CUT-OFF FREQUENCY f_2

X_1 and X_2 the vector Z_b must be a pure resistance when X_1/X_2 is smaller than zero and greater than -4 (Fig. 7)

$$\boxed{0 > \frac{X_1}{X_2} > -4} \tag{15}$$

and a pure reactance when X_1/X_2 is greater than zero and smaller than -4 (Figs. 9 and 10)

$$\boxed{0 < \frac{X_1}{X_2} < -4} \tag{16}$$

Equations (11) and (14) determine the cut off points f_1 and f_2.

2.5 Current Relations. So far, we have considered impedance relations of the network only. It will be interesting now to see how the current passes through a mid-series filter and what the phase shift and attenuation are at various frequencies.

From Fig. 12 we see that the voltage across Z_b is equal to the voltage across Z_a

$$I_1 Z_b = I_2 Z_a$$

$$\frac{I_1}{I_2} = \frac{Z_a}{Z_b} \tag{17}$$

$$\therefore \frac{|I_1|}{|I_2|} = \frac{|Z_a|}{|Z_b|} \tag{18}$$

This equation may be expressed in the exponential form and we obtain

$$\frac{|I_1|}{|I_2|} \equiv \epsilon^A = \frac{|Z_a|}{|Z_b|} \tag{19}$$

$$\therefore \boxed{A = ln \frac{|Z_a|}{|Z_b|}} \tag{20}$$

The index A is called the attenuation constant and is the natural logarithm of the ratio of the current magnitude entering the section to the current magnitude leaving it.

2.6 Phase Shift β. Since in the circuit Fig. 12, the current phase shift is equal to the angle between the current I_1 entering the section and the current I_2 leaving it, and since by (17)

$$\frac{I_1}{I_2} = \frac{Z_a}{Z_b} \tag{17}$$

it is obvious that the angle β in Fig. 7 is the phase angle of the filter (Appendix B).

In Fig. 10, Z_a and Z_b have the same direction, $\beta = 0$, and $X_1/X_2 > 0$.

In Fig. 9, Z_a and Z_b have opposite directions $\beta = 180$ deg. and $X_1/X_2 < -4$.

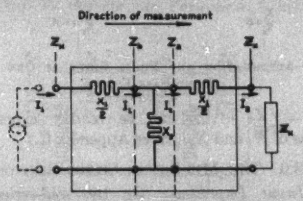


FIG. 12—CIRCUIT OF A MID-SERIES FILTER SECTION INDICATING CURRENT ENTERING AND LEAVING THE SECTION

In Fig. 7 the phase angle changes with Z_b.

In general we may say that β is the angle included by Z_a and Z_b.

$$\boxed{\beta = \angle Z_a + \angle Z_b} \tag{21}$$

Fig. 13 shows that the phase shift changes very abruptly near the cut off points.

2.7 Attenuation A. The attenuation in circuit Fig. 12 is zero when the magnitude of the current vector I_1 is equal to the magnitude of the current vector I_2. This is the case in Figs. 7 and 8.

By Equation (19)

$$\frac{|I_1|}{|I_2|} \equiv \epsilon^A = \frac{|Z_a|}{|Z_b|} \tag{19}$$

Letting $|Z_a| = |Z_b|$ (Figs. 8 and 7).

then $A = 0$

In Figs. 9 and 10, Z_a and Z_b are different in magnitude and there will be attenuation of the current.

$$\frac{|I_1|}{|I_2|} = \frac{|Z_a|}{|Z_b|} = \epsilon^A \tag{19}$$

$$\therefore A = ln \frac{|Z_a|}{|Z_b|} \tag{20}$$

Assuming the special case of a band-pass filter (Fig. 38a) and plotting Equation (20) in terms of frequency, we obtain a curve which changes very abruptly near the cut off point and which indicates that there is no attenuation in the frequency band between the cut off points f_1 and f_2 or between $X_1/X_2 = 0$, and $X_1/X_2 = -4$ (Fig. 13). Here is presented the basis of the statement made in paragraph 1.0 for definition of a broad-band iterative filter.

The attenuation $A = ln\,|Z_a/Z_b|$ and the phase

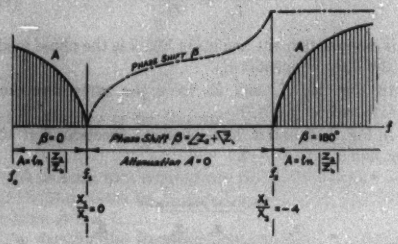


FIG. 13—ATTENUATION AND PHASE SHIFT OF ONE TYPE OF BAND-PASS FILTER

shift $\beta = \angle Z_a + \angle Z_b$ can be readily expressed in functions of X_1 and X_2. (See Appendix C.)

3.0 THE MID-SHUNT FILTER SECTION

3.1 Input Impedance. On the mid-series filter, Figs. 4 and 5, we have shown that the input impedance Z_i must be equal to the terminating impedance Z_k. The same must be true with a mid-shunt filter. As will be seen from Fig. 15, the vector diagram is identical for both types and the input impedance Z_i' is also equal to the terminating impedance Z_k'.

Starting with Z_k' and $2X_2$ (Figs. 14 and 15), we obtain:

$$\frac{1}{Z_k'} + \frac{1}{2X_2} = \frac{1}{Z_b}$$

$$Z_b + X_1 = Z_a$$

$$\frac{1}{Z_a} + \frac{1}{2X_2} = \frac{1}{Z_i'} = \frac{1}{Z_k'}$$

$$\therefore \boxed{Z_i' = Z_k'}$$

or the input impedance Z_i' is equal to the terminating or iterative impedance Z_k'.

3.2 Iterative Impedance Z_k'. The terminating impedance Z_k' may be obtained in a similar manner as for the mid-series filter.

$$X_1 X_2 = Z_a Z_b \tag{2}$$

$$\boxed{\frac{1}{Z_b} = \frac{1}{Z_k'} + \frac{1}{2X_2}} \tag{22}$$

$$\boxed{\frac{1}{Z_a} = \frac{1}{Z_k'} - \frac{1}{2X_2}} \tag{23}$$

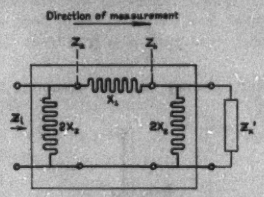


FIG. 14—CIRCUIT OF A MID-SHUNT FILTER SECTION

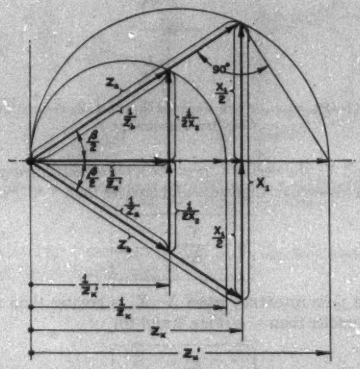


FIG. 15—VECTOR DIAGRAM OF A MID-SHUNT FILTER SECTION

FIG. 15A—SEQUENCE OF VECTOR OPERATIONS

Solving for Z_k' in terms of X_1 and X_2 we get:

$$\boxed{Z_k' = \sqrt{X_1 X_2}\,\frac{1}{\sqrt{1 + \frac{1}{4}\frac{X_1}{X_2}}}} \tag{24}$$

In Equation (24) X_1 and X_2 are functions of frequency. Plotting Z_k' (24) in terms of frequency for a band-pass filter (Fig. 38a) we see from Fig. 16

that Z_k' has a value of zero at the cut-off f_1, and a value of infinite resistance and infinite capacitive reactance at the cut-off f_2.

3.21 Iterative Impedance Z_k' in Vector Diagram. The variation of Z_k' when the frequency is changed may also be shown vectorially.

If we decrease or increase the frequency, we vary Z_k and Z_k' as was shown in Figs. 7, 8, 9, 10, and 15. The variation of Z_a and Z_b with the frequency will be exactly the same for the mid-series as well as the mid-shunt filter. Z_k and Z_k', of course, vary differently. From Fig. 15 we see that $1/X_2$ is now the difference between the vectors $1/Z_b$ and $1/Z_a$ and we have:

$$\frac{1}{Z_b} - \frac{1}{Z_a} = \frac{1}{X_2} \tag{25}$$

$$\frac{1}{Z_a} = \frac{1}{Z_k'} - \frac{1}{2X_2} \tag{23}$$

$$\frac{1}{Z_b} = \frac{1}{Z_k'} + \frac{1}{2X_2} \tag{22}$$

Figs. 17 and 18 illustrate the vector positions outside the band.

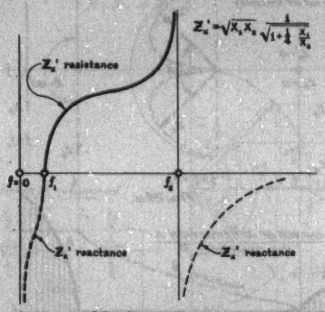


Fig. 16—Iterative Mid-Shunt Impedance of One Type of Band Pass Filter

3.3 Resistance and Reactance Limits of Z_k'. Since the vector diagram (Fig. 5) of a mid-series filter and the vector diagram (Fig. 15) for a mid-shunt filter are identical, the frequency limits between which Z_k and Z_k' are pure resistances are also the same. That is to say, the cut-off points f_1 and f_2 are common for both.

Therefore, Z_k' will be a pure resistance when

$$\boxed{0 > \frac{X_1}{X_2} > -4} \tag{15}$$

and a pure reactance when

$$\boxed{0 < \frac{X_1}{X_2} < -4} \tag{16}$$

Another interesting relationship may be shown at this point.

From Fig. 15 we see that

$$X_1 + \frac{1}{X_2} = Z_a + \frac{1}{Z_b} = Z_k + \frac{1}{Z_k'}$$

$$\therefore \boxed{X_1 X_2 = Z_a Z_b = Z_k Z_k'} \tag{26}$$

This equation is very convenient for checking from mid-series to mid-shunt filters.

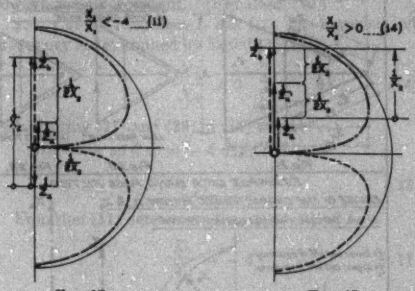


Fig. 17 Fig. 18

Figs. 17 & 18—Position of the Characteristic Vectors Z_a and Z_b outside the Frequency Band

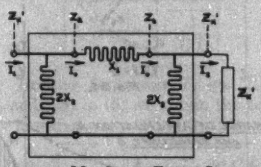


Fig. 19—Circuit of a Mid-Shunt Filter Section Indicating Current Entering and Leaving the Section

3.4 Attenuation A and Phase Shift β. It will be shown now that Equation (17) holds true for the mid-shunt as well as for the mid-series filter. (Fig. 19)

$$I_1 Z_k' = I_0 Z_a$$

$$I_2 Z_k' = I_0 Z_b$$

$$\therefore \frac{I_1}{I_2} = \frac{Z_a}{Z_b} \tag{17}$$

The phase shift and the attenuation are therefore identical for a mid-series and for a mid-shunt filter section of the same ladder type structure:

$$\boxed{\beta = \angle Z_a + \angle Z_b} \tag{21}$$

$$\boxed{A = ln \frac{|Z_a|}{|Z_b|}} \tag{20}$$

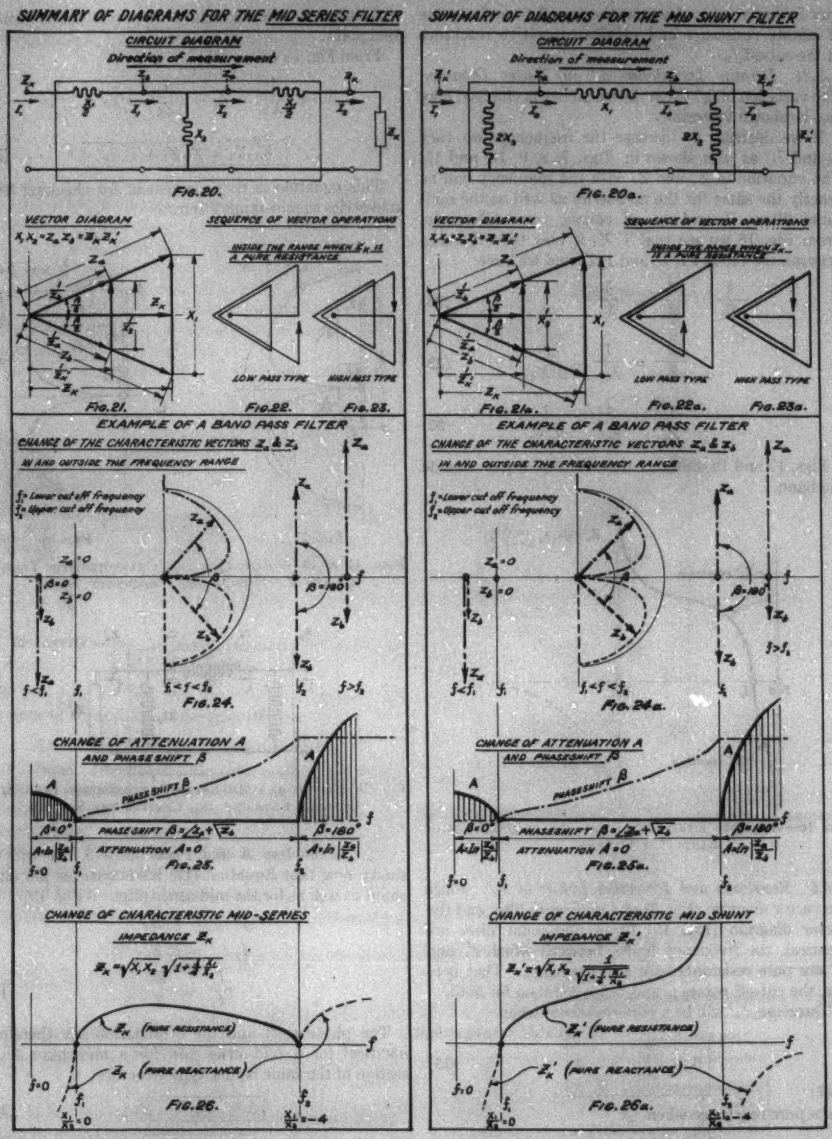


Fig. 26—Comparison of Characteristics of Mid-Series and Mid-Shunt Types of Filters

Summarizing the various steps in the treatment of the mid-series and mid-shunt filters, we can make a compact picture from which the action of a broad band iterative wave filter in and outside the frequency band is quite obvious. The chart refers to band pass filters of the low pass type. The difference between a filter of the low pass type and the high pass type is illustrated in Figs. 22 and 23.

4.0 The Low Pass Filter

In a low pass filter of the two element type (Fig. 27) the series reactance X_1 is a pure inductive reactance, and the shunt reactance X_2 is a pure capacitive reactance.

$$X_1 = j\,\omega\,L \tag{27}$$

$$X_2 = \frac{1}{j\,\omega\,C} \tag{28}$$

$$X_1\,X_2 = \frac{\omega\,L}{\omega C} \tag{29}$$

With Z_k properly chosen, we know from Equation (2) that

$$X_1\,X_2 = Z_a\,Z_b \tag{2}$$

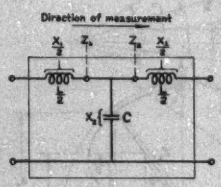


Fig. 27

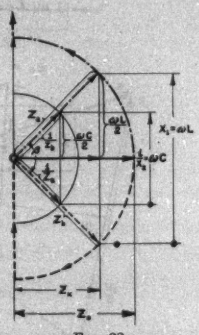


Fig. 28

Figs. 27 & 28—Circuit and Vector Diagrams of a Low Pass Filter Section of the Ladder Type

Substituting (29) in (2) we get:

$$Z_a\,Z_b = \frac{\omega\,L}{\omega\,C} = \frac{L}{C} \tag{30}$$

Within the band $|Z_a| = |Z_b|$

$$\boxed{|Z_a| = |Z_b| = \sqrt{\frac{L}{C}} = Z_0} \tag{31}$$

Z_0 is called the "nominal iterative impedance" or the iterative impedance at the frequency when the term X_1/X_2 in Equation (5) is equal to zero. In the case of the low pass filter this frequency is zero.

Since by Equation (31) the magnitudes of Z_a and Z_b are equal and independent of frequency, it is clear that if we vary ω the impedance vector Z_a describes a section of a circle in the first quadrant, and Z_b one in the fourth quadrant, (Fig. 28). When the angle β has reached 180 deg. or when $\omega\,L/2 = Z_a$, then Z_a and Z_b are pure reactances. A further increase of ω causes an increase of Z_a and a decrease of Z_b in the direction of the j axis. When ω increases, Z_a therefore increases, while Z_b decreases. The product, however, must always remain constant. (Equation (30).

4.1 Cut-Off Frequency f_1 and f_2. The lower cut-off frequency is determined by Equation (14)

$$\frac{X_1}{X_2} = 0 \tag{14}$$

Substituting (27) and (28) in (14) we get

$$-\,\omega^2\,C\,L = -\,4\,\pi^2 f_1^2\,C\,L = 0$$

$$\therefore\ \boxed{f_1 = 0} \tag{32}$$

Equation (11) determines the upper cut-off f_2

$$\frac{X_1}{X_2} = -\,4 \tag{11}$$

or in the case of the low pass filter

$$\frac{X_1}{X_2} = -\,\omega^2\,C\,L = -\,4\,\pi^2 f_2^2\,C\,L = -\,4$$

$$\therefore\ \boxed{f_2 = \frac{1}{\pi}\sqrt{\frac{1}{C\,L}}} \tag{33}$$

4.2 The Iterative Impedance Z_k and Z_k'. Substituting (27) and (28) in (5) we obtain the equation for the iterative impedance of a mid-series filter.

$$Z_k = \sqrt{\frac{L}{C}}\sqrt{1 - \frac{1}{4}\,\omega^2\,C\,L} \tag{34}$$

Substituting (31) and (33) in (34) we get

$$\boxed{Z_k = Z_0\sqrt{1 - \left(\frac{f}{f_2}\right)^2}} \tag{35}$$

Similarly we may obtain the equation for the iterative impedance of a mid-shunt filter.

$$\boxed{Z_k' = \frac{Z_0}{\sqrt{1 - \left(\frac{f}{f_2}\right)^2}}} \tag{36}$$

4.3 Phase Shift β. By Equation (21)

$$\boxed{\beta = \angle Z_a + \angle Z_b} \tag{21}$$

or in terms of L and C, as shown in Fig. 28.

$$\sin\frac{\beta}{2} = \frac{\frac{\omega\,L}{2}}{Z_0} = \frac{\omega\,L}{2}\sqrt{\frac{C}{L}} = \frac{\omega}{2}\sqrt{L\,C} \tag{37}$$

4.4 Attenuation A. By Equation (20)

$$\boxed{A = ln \frac{|Z_a|}{|Z_b|}} \tag{20}$$

or in terms of L and C (Appendix C)

$$\text{Cosh } \frac{A}{2} = \frac{\frac{\omega L}{2}}{Z_0} = \frac{\omega L}{2} \sqrt{\frac{C}{L}} = \frac{\omega}{2} \sqrt{LC} \tag{38}$$

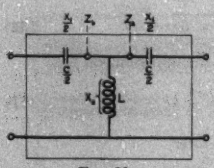


Fig. 30

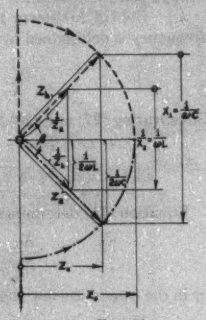


Fig. 31

Fig. 30 & 31—Circuit and Vector Diagram of a High Pass Filter of the Ladder Type

5.0 The High Pass Filter

In a high pass filter of the two-element type (Fig. 30) the series reactance X_1 is a pure capacitive reactance and the shunt reactance X_2 is a pure inductive reactance

$$X_1 = \frac{1}{j\,\omega\,C} \tag{39}$$

$$X_2 = j\,\omega\,L \tag{40}$$

The following formulas for the high pass filter may be readily derived by the same method as used for the low pass filter:

$$\boxed{f_1 = \frac{1}{4\,\pi} \sqrt{\frac{1}{LC}}} \tag{42}$$

$$\boxed{f_2 = \infty} \tag{41}$$

$$\boxed{Z_k = Z_0 \sqrt{1 - \left(\frac{f_1}{f}\right)^2}} \tag{43}$$

$$\boxed{Z_k' = \frac{Z_0}{\sqrt{1 - \left(\frac{f_1}{f}\right)^2}}} \tag{44}$$

6.0 The Band Pass Filter

6.1 Band Pass Filter of the Low Pass Type. One of the most common filters, especially in mechanical structures, is a filter which has an inductance and a relatively small capacitance in series, and a capacitance in shunt.

$$X_1 = j\,\omega\,L_1 + \frac{1}{j\,\omega\,C_1} \tag{45}$$

$$X_2 = \frac{1}{j\,\omega\,C_2} \tag{46}$$

The circuit diagram and the impedance diagram of such a structure are shown in Figs. 32 and 33.

The cut off points are determined by Equations (11) and (14)

$$\frac{X_1}{X_2} = 0 \tag{14}$$

Substituting (45) and (46) in (14)

$$\frac{j\,\omega_1\,L_1 + \frac{1}{j\,\omega_1\,C_1}}{\frac{1}{j\,\omega_1\,C_2}} = 0$$

$$j\,\omega_1\,L_1 + \frac{1}{j\,\omega_1\,C_1} = 0$$

$$\omega_1 = \sqrt{\frac{1}{L_1\,C_1}}$$

$$\boxed{f_1 = \frac{1}{2\,\pi} \sqrt{\frac{1}{L_1\,C_1}}} \tag{47}$$

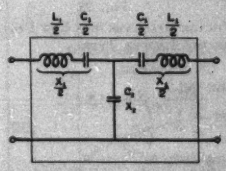


Fig. 32

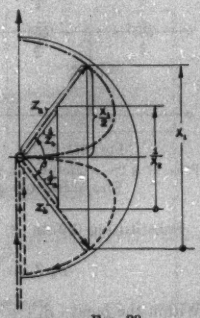


Fig. 33

Figs. 32 & 33—Circuit and Vector Diagram of a Band Pass Filter of the Ladder Type

Substituting the values for X_1 and X_2 in Equation (11) the other cut-off point is obtained

$$\frac{X_1}{X_2} = -4 \tag{11}$$

$$\frac{j\,\omega_2 L_1 + \dfrac{1}{j\,\omega_2 C_1}}{\dfrac{1}{j\,\omega_2 C_2}} = -4$$

$$\omega_2 = \sqrt{\frac{4 + \dfrac{C_2}{C_1}}{L_1 C_2}} \text{ and}$$

$$\boxed{f_2 = \frac{1}{2\pi}\sqrt{\frac{4 + \dfrac{C_2}{C_1}}{L_1 C_2}}} \tag{48}$$

7.0 Filters with Resistance

7.1 Vector Diagram. If the sections of a filter consist of reactances and resistances (Fig. 34) the vector

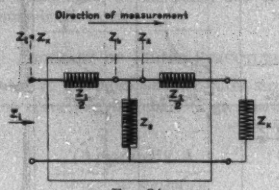


Fig. 34

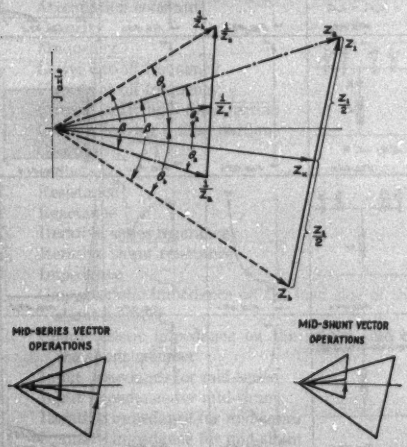


Fig. 35—Circuit and Vector Diagram of a Filter Section Having Small Resistance in the Shunt and Series Elements

diagram assumes the character as shown in Fig. 35. The vectors Z_a and $1/Z_b$ as well as the vectors Z_b and $1/Z_a$ are no longer in line.

The vectors Z_1 and Z_2 are not pure reactances and therefore they are not parallel to the j axis. Nevertheless, the formulas (20) and (21) derived for filters with pure reactances hold true.

7.2 Iterative Impedance Z_k. It will be shown now vectorially that also in this type of filter the input impedance can be made equal to the iterative impedance and equal to the terminating impedance. This will be the case when the vector diagram (Fig. 35) is the same for every added filter section.

Starting with vector Z_k and adding $Z_1/2$ the vector Z_a is obtained. The angle of this vector is $\angle\,\theta_a$ and the angle of $1/Z_a$ is $\angle\,\theta_a$. In other words, the two angles are equal, but opposite in sign. From circuit diagram (Fig. 34) we see that $Z_i - Z_1/2 = Z_b$ and since $Z_i = Z_k$ we may write

$$Z_k - \frac{Z_1}{2} = Z_b$$

This must also be the case in the vector diagram (Fig. 35). Having thus obtained vector Z_b with angle $\angle\,\theta_b$ we get $1/Z_b$ with angle $\angle\,\theta_b$.

In order to obtain a filter structure the vector Z_2 must be such that

$$\frac{1}{Z_2} = \frac{1}{Z_b} - \frac{1}{Z_a}$$

Since it is clear that

$$\frac{Z_a}{Z_b} = \frac{\dfrac{1}{Z_b}}{\dfrac{1}{Z_a}}$$

and since the angle β of the triangle enclosed by Z_a and Z_b is equal to the angle β of the triangle enclosed by $1/Z_a$ and $1/Z_b$ these triangles are similar

$$\frac{1}{Z_b} \div Z_a = \frac{1}{Z_2} \div Z_1$$

$$\therefore Z_a Z_b = Z_1 Z_2 \tag{49}$$

It is also clear from Fig. 35 that

$$\boxed{Z_a = Z_k + \frac{Z_1}{2}} \tag{50}$$

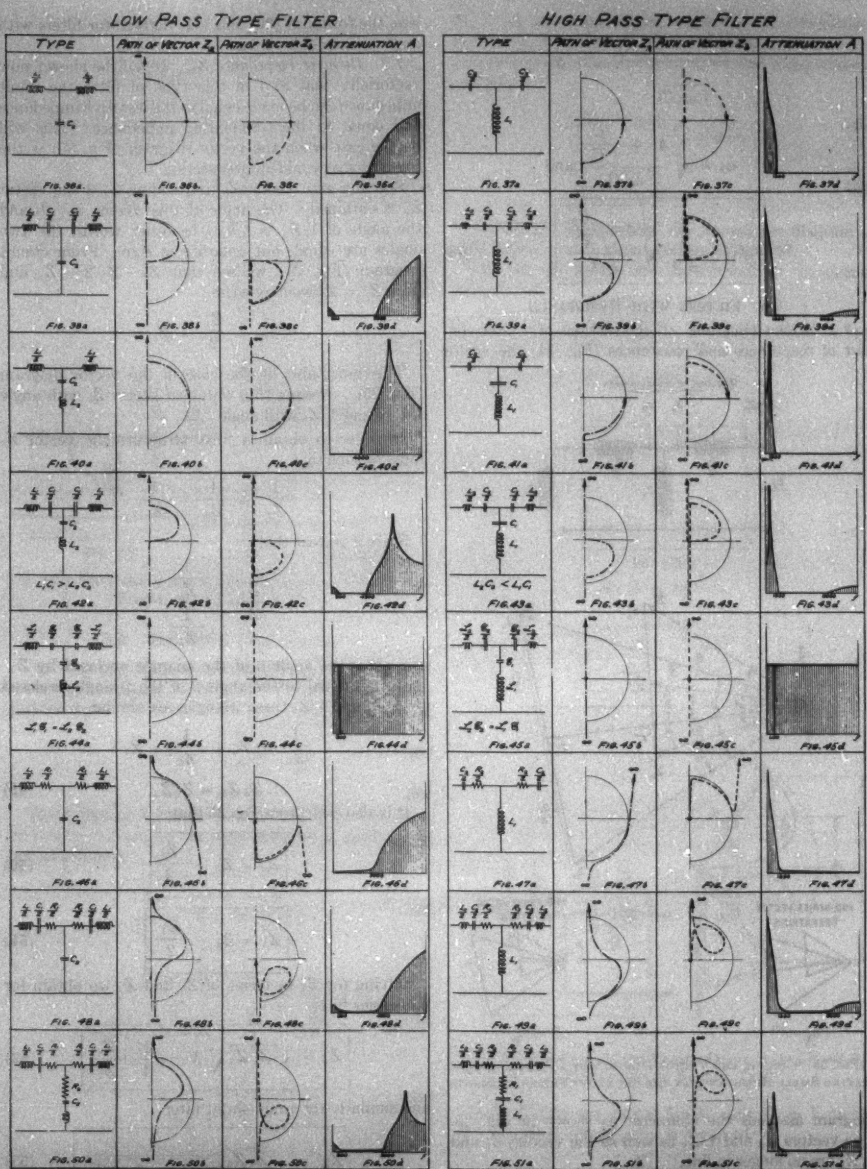
$$\boxed{Z_b = Z_k - \frac{Z_1}{2}} \tag{51}$$

Solving for Z_k in terms of Z_1 and Z_2 we obtain for mid-series filter

$$\boxed{Z_k = \sqrt{Z_1 Z_2}\sqrt{1 + \frac{1}{4}\frac{Z_1}{Z_2}}} \tag{52}$$

and similarly for a mid-shunt filter

$$\boxed{Z_k' = \sqrt{Z_1 Z_2}\,\frac{1}{\sqrt{1 + \dfrac{1}{4}\dfrac{Z_1}{Z_2}}}} \tag{53}$$



FIGS. 36 TO 51—SUMMARY OF THE MOST COMMON TYPES OF BROAD BAND FILTERS OF THE ITERATIVE LADDER TYPE SHOWING PATHS OF THE CHARACTERISTIC VECTORS Z_a AND Z_b, ALSO THE ATTENUATION

which two equations correspond with the pure reactance Equations (5) and (24)

7.3 Phase Shift β and Attenuation A. The same relations are true for phase shift and attenuation as with the reactance filter, therefore

$$\boxed{\beta = \angle Z_a + \sphericalangle Z_b = \angle \frac{1}{Z_b} + \sphericalangle \frac{1}{Z_a}} \tag{21}$$

$$\boxed{A = l\,n \frac{|Z_a|}{|Z_b|}} \tag{20}$$

The difference between a filter with resistance and one without resistance is the fact that the frequency cut off points are not sharp and there will always be a certain amount of attenuation in the band, and phase shift variation outside the band.

8.0 Summary of Filter Structures

A summary of the most common filter structures is given in the accompanying chart (Figs. 36a-51d).

Appendix A

List of Symbols

A	Attenuation constant
C	Capacity
f	Frequency
f_1	Lower cut-off frequency
f_2	Upper cut-off frequency
I_1	Current entering a filter section
I_2	Current leaving a filter section
j	Operator $\sqrt{-1}$
L	Inductance
R	Resistance
X	Reactance
X_1	Iterative series reactance
X_2	Iterative shunt reactance
Z	Impedance
Z_a	Characteristic impedance on the load side of the shunt member
Z_b	Characteristic impedance on the source side of the shunt member
Z_i	Input impedance for mid-series
Z_i'	Input impedance for mid-shunt
Z_k	Iterative impedance for mid-series
Z_k'	Iterative impedance for mid-shunt
Z_0	Nominal iterative impedance
Z_1	Series impedance
Z_2	Shunt impedance
β	Phase shift or phase constant
ϵ	Natural base 2.718
θ_a	Angle of vector Z_a
θ_b	Angle of vector Z_b
ω	$2\pi f$
$\lvert\ \rvert$	Absolute or scalar value of the complex quantity enclosed
$\lvert Z_a \rvert$	Magnitude of vector Z_a
$\equiv$	Equal by definition
$l\,n$	log to base ϵ
$\angle$	Sign of a positive angle
$\sphericalangle$	Sign of a negative angle
$<$	Is less than
$>$	Is greater than
	R Resistance
	X Reactance
	L Inductance
	C Capacitance
	Z Pure reactance or pure resistance
	Z Any kind of impedance
	G Generator or source

Appendix B

By Equation (17)

$$\frac{I_1}{I_2} = \frac{Z_a}{Z_b}$$

If α_1 is the phase angle of the current vector I_1, and α_2 the phase angle of the current vector I_2 the vectors I_1 and I_2 may be expressed in the following way:

$$I_1 = |I_1|\,\epsilon^{j\alpha_1}$$
$$I_2 = |I_2|\,\epsilon^{j\alpha_2}$$

$$\therefore \frac{I_1}{I_2} = \left|\frac{I_1}{I_2}\right| \frac{\epsilon^{j\alpha_1}}{\epsilon^{j\alpha_2}} = \left|\frac{I_1}{I_2}\right| \epsilon^{j(\alpha_1-\alpha_2)} = \left|\frac{I_1}{I_2}\right| \epsilon^{j\beta}$$

where β is the phase angle between I_1 and I_2

Similarly

$$\frac{Z_a}{Z_b} = \frac{|Z_a|}{|Z_b|}\,\frac{\epsilon^{j\angle Z_a}}{\epsilon^{j\angle Z_b}} = \frac{|Z_a|}{|Z_b|}\,\epsilon^{j(\angle Z_a - \angle Z_b)}$$

Equation (17) may therefore be written

$$\frac{|I_1|}{|I_2|}\,\epsilon^{j\beta} = \frac{|Z_a|}{|Z_b|}\,\epsilon^{j(\angle Z_a - \angle Z_b)}$$

This formula is only true, if

$$\frac{|I_1|}{|I_2|} = \frac{|Z_a|}{|Z_b|} \tag{18}$$

and $\beta = \angle Z_a - \angle Z_b$

or $\beta = \angle Z_a + \sphericalangle Z_b$ (21)

Appendix C

The attenuation constant A in terms of the characteristic vectors Z_a and Z_b is given by Equation (20).

A may be readily expressed as a function of X_1 and X_2.

By Equation (19)

$$\epsilon^A = \frac{|Z_a|}{|Z_b|} \tag{19}$$

and by definition of a hyperbolic cosine

$$\text{Cosh}\,\frac{A}{2} = \frac{1}{2}\left(\epsilon^{\frac{A}{2}} + \epsilon^{-\frac{A}{2}}\right)$$

$$\therefore \text{Cosh}\,\frac{A}{2} = \frac{1}{2}\left(\sqrt{\frac{|Z_a|}{|Z_b|}} + \sqrt{\frac{|Z_b|}{|Z_a|}}\right)$$

$$= \frac{1}{2}\,\frac{|Z_a| + |Z_b|}{\sqrt{|Z_a|\,|Z_b|}}$$

By Equation (2)

$$|Z_a|\,|Z_b| = |X_1|\,|X_2| \tag{2}$$

$$\therefore \text{Cosh}\,\frac{A}{2} = \frac{1}{2}\,\frac{|Z_a| + |Z_b|}{\sqrt{|X_1|\,|X_2|}}$$

Similarly

$$\text{Sinh}\,\frac{A}{2} = \frac{1}{2}\left(\epsilon^{\frac{A}{2}} - \epsilon^{-\frac{A}{2}}\right) = \frac{1}{2}\,\frac{|Z_a| - |Z_b|}{\sqrt{|X_1|\,|X_2|}}$$

Outside the band when $X_1/X_2 > 0$ (Fig. 10) the vectors Z_a and Z_b are pure reactances and have the same direction. From Fig. 10 it is obvious that

$$|X_1| = |Z_a| - |Z_b|$$

$$\therefore \text{Sinh}\,\frac{A}{2} = \frac{1}{2}\,\frac{|X_1|}{\sqrt{|X_1|\,|X_2|}} = \frac{1}{2}\sqrt{\frac{|X_1|}{|X_2|}}$$

The vectors X_1 and X_2 having the same direction, we may substitute

$$\frac{X_1}{X_2} \text{ for } \frac{|X_1|}{|X_2|}$$

$$\therefore \text{Sinh}\,\frac{A}{2} = \frac{1}{2}\sqrt{\frac{X_1}{X_2}}$$

$$\boxed{A = 2\,\text{Sinh}^{-1}\left(\frac{1}{2}\sqrt{\frac{X_1}{X_2}}\right)} \tag{53}$$

Outside the band when $X_1/X_2 < -4$ (Fig. 9) the vectors Z_a and Z_b have opposite directions. Fig. 9 shows that

$$|X_1| = |Z_a| + |Z_b|$$

$$\therefore \text{Cosh}\,\frac{A}{2} = \frac{1}{2}\,\frac{|X_1|}{\sqrt{|X_1|\,|X_2|}} = \frac{1}{2}\sqrt{\frac{|X_1|}{|X_2|}}$$

The vectors X_1 and X_2 having opposite directions we may substitute $-X_1/X_2$ for $|X_1/X_2|$

$$\text{Cosh}\,\frac{A}{2} = \frac{1}{2}\sqrt{-\frac{X_1}{X_2}}$$

$$A = 2\,\text{Cosh}^{-1}\left(\frac{1}{2}\sqrt{-\frac{X_1}{X_2}}\right) \tag{54}$$

The relation between the phase angle β and X_1 and X_2 may be read from Fig. 5.

$$\sin\frac{\beta}{2} = \frac{\dfrac{|X_1|}{|2|}}{|Z_a|}$$

$$|Z_a| \div \frac{1}{|Z_a|} = |X_1| \div \frac{|1|}{|X_2|}$$

$$|Z_a| = \sqrt{|X_1|\,|X_2|}$$

$$\therefore \sin\frac{\beta}{2} = \frac{1}{2}\,\frac{|X_1|}{\sqrt{|X_1|\,|X_2|}} = \frac{1}{2}\sqrt{\frac{|X_1|}{|X_2|}}$$

Since in the band the vectors X_1 and X_2 have opposite directions we may write.

$$-\frac{X_1}{X_2} \text{ instead of } \frac{|X_1|}{|X_2|}$$

$$\sin\frac{\beta}{2} = \frac{1}{2}\sqrt{-\frac{X_1}{X_2}} \text{ and}$$

$$\boxed{\beta = 2\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{X_1}{X_2}}\right)} \tag{55}$$

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Discussion

C. H. Dagnall: An interesting vector diagram results when the output current is expressed in terms of a constant input current. The locus of the output current vector is a circle in

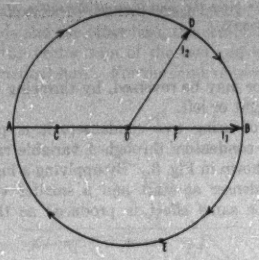

Fig. 1

the transmission bands and a diameter of the circle in the attenuation bands. A dissipationless structure terminated by its iterative impedance is assumed. In Fig. 1 herewith, let I be held constant. The vector I_2 will terminate on the line A B in the attenuation bands and on the circle in the transmission bands. For the structure of Fig. 2 herewith the locus will start at O, continue to A, then *clockwise*, around the circle past D, B, and E to A, and finally along A O to O. For the structure of Fig. 36 of the paper the locus would be B-E-A-O, the starting point B corresponding to zero frequency.

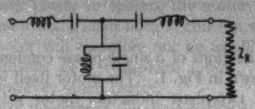

Fig. 2

When the filter possesses an infinite attenuation peak, such as that of Fig. 40, the terminus of I_2 goes to the origin at the peak and then moves out along O A or O B. For the structure of Fig. 43 the locus would be F-O-A-D-E-A-C.

It will be noted that the diagram represents both the attenuation and phase characteristics. The phase constant B is the angle by which I_2 *lags* I_1. In multisection filters the terminus of I_2 may go around the circle several times in one transmission band, since the phase constants of the several sections must be added.

The Condenser Motor

BY BENJAMIN F. BAILEY[1]

Fellow, A. I. E. E.

Synopsis.—*After a brief description of the construction and connections of the condenser motor the necessity of varying the capacitance is discussed, and the performance at start and under load is considered.*

Locus diagrams illustrating the operating performance in detail are given, followed by a more detailed discussion of starting torque. The Appendix gives the mathematical derivation of many of the formulas discussed.

THE connections of a single-phase condenser motor are shown in Fig. 1. The motor itself is identical with a two-phase induction motor with the exception of the fact that the two windings are not necessarily alike. Winding 2 may have more or less turns than winding 1. The total weight of copper in the two, however, is approximately the same. The rotor is identical with that of any polyphase motor. It usually is of the squirrel-cage type although a wound rotor may of course be used.

To obtain the best results, the capacitance should be

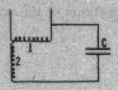

FIG. 1—SIMPLE CONDENSER MOTOR

large when the motor is being started and should be gradually reduced as the speed is increased. If it were practicable to adjust the capacitance to exactly the proper value corresponding to each value of load and speed, the motor itself would operate practically as a two-phase motor, the combination of the condenser and line operating as a phase changer to convert the single-phase supply into a two-phase supply. In prac-

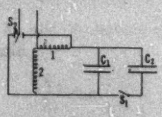

FIG. 2—CONDENSER MOTOR, REVERSIBLE AND WITH VARIABLE CAPACITANCE

tise a fixed value of capacitance may be satisfactory, providing a starting torque of about 50 per cent of full load running torque is sufficient. If more starting torque is necessary the motor may be connected as shown in Fig. 2. The switch S_1 is closed when the motor is at rest and is opened (usually automatically) when the speed is sufficiently high. Fig. 2 also illustrates a method by which the direction of rotation of the motor may be reversed, by throwing the switch S_2 to the right or left.

Instead of using two condensers it is possible to supply the condensers through a variable ratio transformer as shown in Fig. 3. By applying a high voltage to the condenser at start and a smaller voltage for running, the same effect is produced as though the

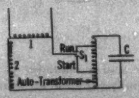

FIG. 3—CONDENSER MOTOR WITH VARIABLE TRANSFORMER

capacitance were changed. With this scheme the efficiency will necessarily be a little lower due to the losses in the transformer.

The vector diagram of a condenser motor is shown in Fig. 4. This was plotted from an actual test of a small motor under full load. In this case the motor was a standard two-phase motor.

The current I_1 in phase 1 lags, as usual, by a con-

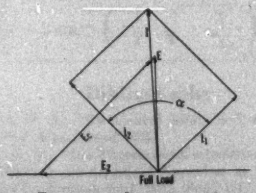

FIG. 4—VECTOR DIAGRAM OF CONDENSER MOTOR. (FULL LOAD)

siderable angle behind the line voltage E. Due to the introduction of the condenser, the current I_2 in phase 2 may be made to lead the line voltage. When the proper capacitance is used the two currents are approximately at right angles to one another and, if the two windings are alike, are nearly equal. Under these conditions the motor will operate just as though it were a two-phase motor and of course with the same efficiency.

From the above it will be apparent that we can build a single-phase motor having at full load practically the same efficiency as a two-phase motor and operating

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at or near 100 per cent power factor. It is self evident that its characteristics will be much better than those of a single-phase motor of the usual construction which necessarily operates at a lower efficiency and power factor than a two-phase motor.

In starting performance, the condenser motor is somewhat superior to the two-phase motor. Since one current leads and the other lags, the combined starting current is the vector sum of the two and is less than their arithmetical sum. For the same reason the power

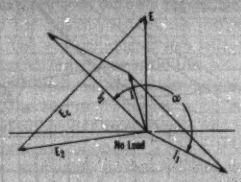

Fig. 5—Vector Diagram of Condenser Motor. No Load

factor is excellent and is usually close to 100 per cent. The motor will develop even more torque than a two-phase motor and the current required is substantially less; in fact, the torque per ampere is nearly double that of a two-phase motor.

When operating at or near synchronous speed the condenser motor like the usual single-phase motor has a nearly uniform rotating field, and consequently there is a nearly uniform induced voltage in the auxiliary winding, and the applied voltage across winding 2 will also be nearly constant. The induced voltage will be dependent upon the ratio of the number of turns in the two windings. When the windings are alike, the voltage across winding 2 will be nearly equal to the line voltage, the ratio being of course not exact, due to the resistance and reactance drops in the two windings. This voltage across the winding 2 will moreover be nearly 90 deg. out of phase with the voltage in winding 1. The voltage across the condenser, being the vector sum of the line voltage and the voltage of winding 2, will be approximately 1.41 times the line voltage with a one to one ratio. With any other ratio the voltage of winding 2 and the condenser voltage will be correspondingly changed.

The action of the motor under varying loads is shown in the vector diagrams of Figs. 4, 5, and 6. These diagrams have been plotted from tests upon a small two-phase motor connected as shown in Fig. 1. As previously explained, E_2, the voltage across phase 2, remains approximately constant both in magnitude and direction. The voltage across the condenser is marked E_c and this likewise remains approximately constant. Since the current in the condenser (which is also the current in winding 2) must of course lead E_c by 90 deg. and be proportional to it, it also varies but little in magnitude and direction.

When the motor is operating under no load (see Fig. 5) the power in phase 2 will obviously be more than necessary to operate the motor. It follows that the current in phase 1 must lag more than 90 deg. behind the line voltage so that it is returning power to the line. The vector sum of I_1 and I_2 is I, the line current. This must be of such a magnitude and phase as to give the necessary power to operate the motor. It will be obvious that at light loads both I_1 and I_2 are quite large and the motor will therefore not be so efficient under these conditions as a two-phase motor. The power factor is however much better.

The vector diagram of the same motor at 50 per cent over-load is shown in Fig. 6. The power in phase 2 remains nearly the same as at no load or full load and the current and power in phase 1 must therefore increase. The line current I is now somewhat lagging although the power factor is still very high. Again it will be seen that since I_1 and I_2 are not equal the efficiency must be somewhat lower than that of the same motor operated two-phase. The power factor, however, is excellent and the efficiency better than that of a plain single-phase motor.

Locus Diagrams

These relations can perhaps be more readily visualized from the locus diagram of Fig. 7. The curve

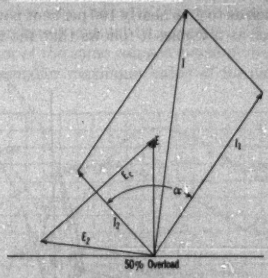

Fig. 6—Vector Diagram of Condenser Motor 50 per Cent Overload

marked I_1 represents the locus of the vector I_1 as the load is changed. The small circles represent the positions of the end of the vectors corresponding to no-load, full load, and 50 per cent over load and are taken from the same tests as those used in drawing Figs. 4, 5, and 6. Similarly the short curves marked E_2 and I_2 represent the loci of the vectors representing the voltage across phase 2 and the current in phase 2. The curve marked I is the locus of the line current. In this particular case the current was leading at light load, in phase with the voltage at a little over full load, and slightly lagging for 50 per cent over-load. The power factor throughout this range of load was very close to 100 per cent.

Fig. 8 shows a locus diagram for the same motor but connected so that phase 2 has twice as many turns as phase 1. The voltage across the phase 2 is of course nearly twice as great as before. The condenser voltage obtained by drawing a line from any point on the curve E_2 to the end of the vector E is more nearly at right angles to E than before. Since the current I_2 must be at right angles to the condenser voltage it is brought

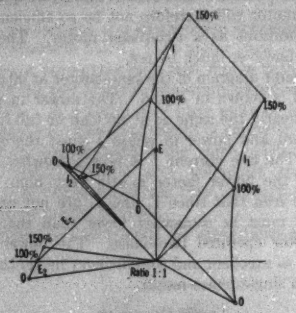

Fig. 7—Locus Diagram of Condenser Motor. Winding Ratio 1 to 1

more nearly into phase with E. Since the capacitance used was such as to give nearly 100 per cent power factor to the motor as a whole, it follows that the current I_1

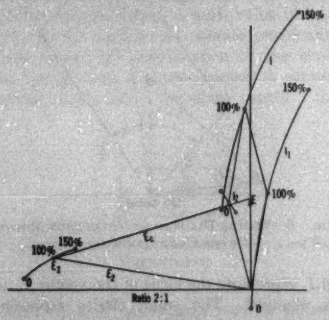

Fig. 8—Locus Diagram of Condenser Motor. Winding Ratio 2 to 1

was forced to be more nearly in phase with the line voltage. The vector sum of the two is represented by the current I (the line current) and this is at nearly 100 per cent power factor throughout the range of the motor.

The advantage of having more turns in winding 2 than in winding 1 is that a higher voltage is applied to the condenser and consequently the capacitance can be greatly reduced; in fact the condenser used in making the tests represented in Fig. 8 was approximately one-half as large as that used in the tests of Fig. 7. It will of course be apparent that since the two currents I_1 and I_2 are no longer at right angles the motor is not operating so efficiently. In fact the conditions approach those of the ordinary single-phase motor,

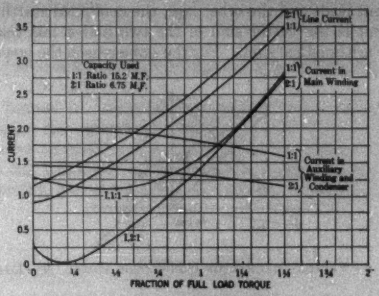

Fig. 9—Torques and Currents in the Different Windings of Condenser Motor

since the current I_1 and I_2 do not differ very greatly in phase. The power factor is still excellent but the efficiency of the motor is somewhat reduced.

Fig. 9 has been plotted from the same data used in Figs. 4 to 8, and shows the variation of the various currents with the torque. Similarly, Fig. 10 shows the variation of the $I^2 R$ losses in the different windings.

Comparison of Watts

In Fig. 11, the total watts input to the motor and also the watts in each of the windings have been plotted.

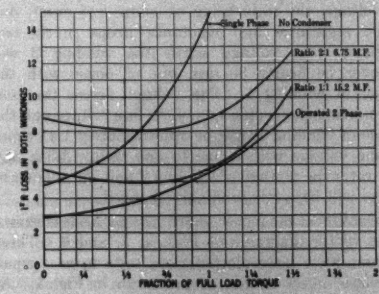

Fig. 10—Torque and $I^2 R$ Losses in Condenser Motor

The watts in the auxiliary winding are nearly the same with either connection and decrease only slightly as the load increases. The watts in the main winding are in both cases negative with light loads and of course increase as the load increases. The total power required with the two to one connection is greater than

with the one to one connection on account of the reduced efficiency.

Effect of Changing Capacitance

It will be evident from the preceding discussion that the characteristics of a motor will be radically modified by any change in capacitance. The results of a test upon the same motor, previously referred to, have been embodied in Fig. 12, which shows the currents plotted in their proper phase relation, the output being held constant while the capacitance was varied.

It will be noted in both of these curves that the power component of the current, that is, the projection of the current upon the voltage axis, is less with the current somewhat lagging than it is with 100 per cent power factor. At full load the minimum power input and consequently the highest efficiency is obtained with a capacitance of about five microfarads and at half load with about three microfarads. The lowered efficiency, as previously explained, is due to the fact that the current does not divide between the two windings in

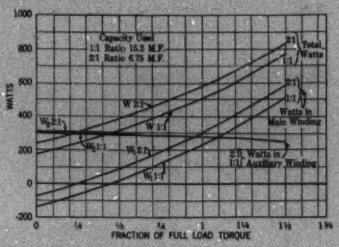

Fig. 11—Torques and Watts in Condenser Motor

the best ratio. In general, therefore, the capacitance which gives the best power factor will not necessarily be that which gives the highest efficiency. Particularly in the case of motors with a large number of turns in winding 2 compared with winding 1, it will be necessary to use a capacitance giving a somewhat lagging current if the best efficiency is to be obtained. With a one to one ratio, the points of best power factor and best efficiency will usually come more nearly together.

Starting Torque

As shown in the appendix the starting torque is given by the equation

$$T = Q N_1 N_2 I_1 I_2 \sin \alpha.$$

In other words, it is proportional to the ampere turns in the winding 1, the ampere turns in the winding 2, to the sine of the angle between the two currents, and to a constant which depends upon the construction of the motor.

The maximum current will exist in circuit 2 when the circuit is in resonance but under these circumstances the angle between the two currents will be unfavorable and consequently the greatest possible torque will not be developed.

It is shown in the appendix that the maximum starting torque is obtained when

$$X_2 = \frac{1}{C\ \omega} = -K^2 Z_1$$

in which Z_1 is the impedance of circuit 1, C is the capacitance of the condenser in farads, and $\omega = 2\pi f$. K is the ratio of turns in winding 2 to the turns in winding 1.

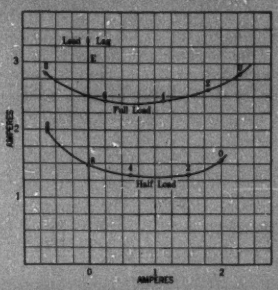

Fig. 12—Locus Diagram of Condenser Motor. Variable Capacitance and Constant Output. Winding Ratio 2 to 1

The negative sign indicates that the reactance X_2 is due to a condenser and not to a reactor. If we should use a reactor of the same value it would give us another but much smaller maximum value of torque. In the

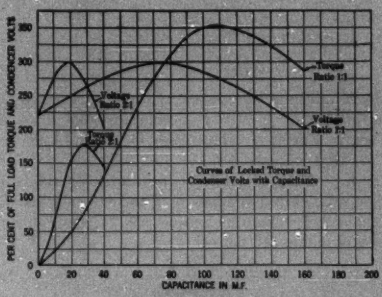

Fig. 13—Variation of Torque and Condenser Voltage with Capacitance

following the negative sign will be ignored when we are dealing with capacitance only. If the two windings are formed with coils of the same dimensions and if they contain the same weight of wire, the resistances, inductances, and impedances will all be in the ratio of K^2, K being the ratio of the turns in winding 2 to those in winding 1.

In the above formula, then, $K^2 Z_1$ is the impedance of

winding 2 and for maximum torque this impedance should be equal to the reactance of the condenser. The condition for resonance on the other hand would be that the reactance of winding 2 should equal the reactance of the condenser.

It is shown in the appendix that the locked torque is given by the following equation:

$$T = \frac{K N_1^2 E^2}{Z_1^2} \cdot \frac{R_1 C \omega}{K^4 Z_1^2 C^2 \omega^2 + 2 K^2 X_1 C \omega + 1}$$

in which the symbols have the same significance as before.

In Fig. 13 I have plotted the graph of this equation as computed for a particular motor with $K = 1$ and $K = 2$. The values of Z_1, X_1, and R_1 were obtained from readings with the rotor locked. Tables I and II show some comparisons between observed and computed torques.

TABLE I
For $K = 1$

C in microfarads	0	40	60	80	108	160
T calculated	0	133	231	311	353	286
T observed	0	130	210	285	355	....

TABLE II
For $K = 2$

C in microfarads	0	10	15	20	27	40
T calculated	0	66.5	115.5	155.5	176.5	143
T observed	0	66.2	114	155	178	147

Using the capacitance to give maximum torque, namely $C = \frac{1}{K^2 Z_1 \omega}$ we obtain the following for the maximum torque with any given ratio of turns:

$$T_m = \frac{N_1^2 E^2}{2 K Z_1^2} \cdot \sqrt{\frac{Z_1 + X_1}{Z_1 - X_1}}$$

Inserting the known values of the constants we find that for $K = 1$, $C = 108$ microfarads and $T_m = 353$ per cent. The test values were 115 microfarads and 355 per cent.

For $K = 2$ the calculated capacitance for maximum torque was 27 microfarads and the test showed 28, the calculated maximum torque was 177 per cent and the tested value 178 per cent.

Relation of Torque and Volt Amperes in the Condenser

As shown in the appendix the equation for torque may be written as follows:

$$T = \frac{K N_1^2 R_1}{Z_1^2} E_c I_c = \frac{K N_1^2 R_1}{Z_1^2} \cdot E_c I_2$$

In other words, the torque is directly proportional to the product of the volts across the condenser and the current flowing in it. This is important since the cost of the condenser (at least with voltages of 440 and above) is nearly directly proportional to the volt amperes in the condenser, rather than to its capacitance. That is, the cost of the condenser increases with the voltage it must withstand as well as with its capacitance.

If we insert the values of the constants of this particular motor we obtain the equation

$$T = 0.116\, K E_c I_2$$

In Fig. 14 I have drawn the curves corresponding to the above equation for $K = 1$ and $K = 2$. I have also plotted the test resulted on this motor and reasonably good correspondence is shown between the calculated and the test results.

The variation of the voltage across the condenser as we change the capacitance is also shown in the curves of Fig. 13. The voltage, for any ratio between the turns, is the same as the line voltage for $C = 0$, rises rapidly as C is increased up to the point of resonance and then decreases. The maximum torque, as we should expect from the theory, is found with a value of C somewhat greater than that which gives resonance.

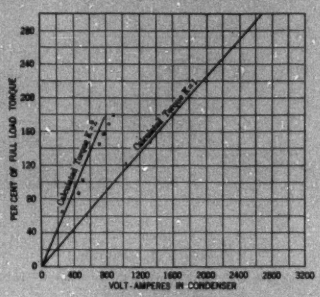

Fig. 14—Variation of Torque with Volt Amperes in Condenser

The condenser voltage at maximum torque is, however, usually decidedly above line voltage.

According to the theory as developed in the appendix the condenser voltage at the point of maximum torque is given by the following formula:

$$E_c = -E \sqrt{\frac{Z_1}{2 (Z_1 - X_1)}}$$

It will be seen that this value is independent of the ratio K and in the case of this particular motor the computed value is 274 volts. The actual values agreed fairly well with this, being 260 where $K = 1$ and 280 volts where $K = 2$.

Summary

From the preceding it will be obvious that as we increase the number of turns in winding 2, the maximum locked torque decreases. However, the capacitance needed decreases even faster so that if locked torque only is to be considered the ratio K of the turns

should be as great as possible and still permit the necessary torque to be developed.

It will also be apparent from the equation for maximum torque that this may be made as great as desired provided we make K small enough. Excessive torques, however, are obtained at the expense of large currents and excessive cost of condensers.

Starting Efficiency

The starting efficiency of any induction motor may be obtained by the following equation:

$$\eta = 0.142 \frac{\text{Lb. ft.} \times \text{Sync. Speed}}{\text{Volts} \times \text{Amperes}}$$

The voltage is that applied to the motor and the current is that in the supply line. For a motor of a given synchronous speed and voltage, it will be seen that this ratio is really the torque per ampere multiplied by a constant.

In Fig. 15 I have plotted the starting efficiency of this motor for the two ratios used. It will be seen that the efficiency increases with the torque and reaches

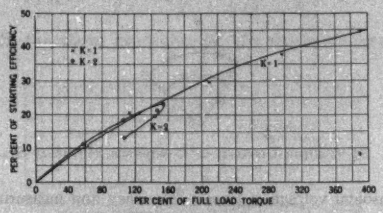

Fig. 15—Relation between Starting Torque and Starting Efficiency

higher values with the smaller values of the ratio K. The maximum efficiency obtained was 45 per cent. In comparison it may be mentioned that in split-phase motors the starting efficiency rarely exceeds 10 per cent and in polyphase squirrel-cage motors it is usually about 20 per cent. This particular motor was also tested as a two-phase motor and its starting efficiency was found to be 26.3 per cent. If the capacitance of the condenser used is increased indefinitely these curves of starting efficiency of course turn back and finally reach zero.

Complete Torque Curves

In the actual application of the motor we need to know not only the locked starting torque but the torque throughout the complete range of operation. In Fig. 16 I have plotted speed-torque curves for another motor with a ratio of turns of 1 to 1 and with various values of capacitance. It will be seen that the torque with the larger values of capacitance is substantially maintained until the motor has reached normal operating speed. As the capacitance becomes smaller there is more of a tendency toward development of points of low torque at certain speeds. This will be particularly apparent in the curve for 20 microfarads. The point of lowest torque occurs at about $^1/_7$ of synchronous speed and would seem to indicate the presence of a backwardly rotating 7th harmonic.

I have also plotted the curve of current taken when 40 microfarad condenser was used. It will be seen

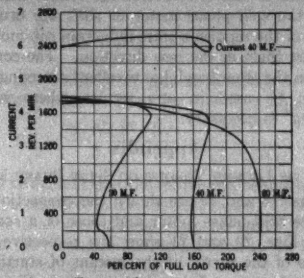

Fig. 16—Speed Torque Curves of Condenser Motor

that the current remains nearly constant throughout the entire starting period.

The torque developed when an attempt is made to reverse the motor at full speed is sometimes of im-

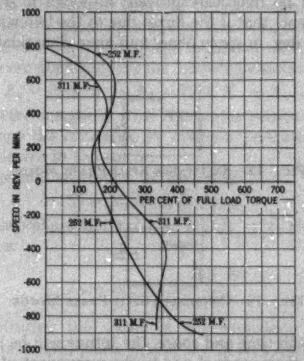

Fig. 17—Speed Torque Curves of Condenser Motor showing Reversing Torque

portance. The action under these conditions is shown in Fig. 17. These curves were taken on a different and much larger motor. Points below the line indicate the torque developed when the motor was running in opposition to the direction of the rotating magnetic field. In other words, they show what would happen if the motor were reversed at full speed. It will be seen that the torque developed at negative speeds is very great.

While this reverse torque is not of importance in the case of most motors, it is decidedly important in the case of motors used to operate elevators, hoists, or other similar devices. The ordinary split-phase motor with automatic switch for cutting out the starting winding and the repulsion start-induction run single-phase motors have no reserving torque. It is necessary to bring them to rest and reverse the connections in the case of the split-phase motor or shift the brushes in the case of the repulsion start-induction run motor and then start them in the new direction. The condenser motor, on the other hand, has excellent reversing torque and can be used in any service where so-called "plugging" is necessary.

Appendix

Theory of Locked Torque of Split Phase Motors

In Figs. 18 and 19 are shown the connections of a resistance split-phase motor and those of a condenser motor. Figs. 20 and 21 show the respective vector diagrams of the motor at the instant of starting. I_1 lags in both cases. I_2 lags less than I_1 in Fig. 20 and leads in Fig. 21. There is therefore an angle α between the two currents and this angle is much greater in the condenser motor. In place of a condenser or a resistor we might use a reactor in series with phase 2. This would give only a small angle between the currents and hence would be ineffective.

It can be rigorously proved (although it is nearly self-evident) that if the fluxes due to the two windings, differ 90 deg. in space phase and have sinusoidal variation in both time and space, the torque will be proportional to their product and to the component of one at right angles to the others. This can be expressed in the equation $T = Q_1 B_1 B_2 \sin \alpha$.

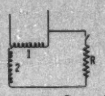

Fig. 18—Resistance Split-Phase Motor

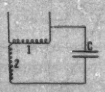

Fig. 19—Condenser Split-Phase Motor

In which B_1 and B_2 are the fluxes, α is the time angle between B_1 and B_2, and Q_1 is a constant.

If we neglect saturation, B_1 and B_2 are proportional to the respective ampere turns $N_1 I_1$ and $N_2 I_2$ and

$$T = Q_2 N_1 I_1 N_2 I_2 \sin \alpha.$$

Here the purpose is to treat these three motors together. All that we do in these various connections is to change the resistance or reactance of phase 2. We shall assume that the constants of winding 1 are fixed. We can increase the resistance of circuit 2 as much as we please but we cannot reduce it below a certain value. Likewise, we can increase the reactance of circuit 2 as much as we wish. We can also reduce it if we wish, however, or even make it negative by using a condenser.

Let the locked impedance, resistance, and reactance of winding 1 be respectively Z_1, R_1, and X_1 and those of winding 2, be Z_2, R_2, and X_2. The constants of the *entire* circuit of which winding 2 is a part including any resistance, reactance, or condensive reactance used are designated by Z, R, and X. These constants are to be determined in an actual motor by applying a

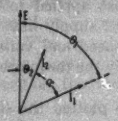

Fig. 20—Vector Diagram of Resistance Split-Phase Motor at Start

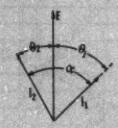

Fig. 21—Vector Diagram of Condenser Motor at Start

sinusoidal voltage of known frequency and measuring the volts, watts, and amperes. Then

$$Z_1 = \frac{E}{I_1}, R_1 = \frac{W_1}{I_1^2} \text{ and } X_1 = \sqrt{Z_1^2 - R_1^2}.$$

Similar equations hold for Z_2, R_2, and X_2 and for Z, R, and X. The latter three are of course taken with the resistors, reactors, or condensers used connected in the circuit.

It is important to note that the constants of circuit 2 are not affected by current in circuit 1 and vice versa. This is due to the fact that the windings are 90 electrical degrees from one another, and therefore current in circuit 1 does not induce voltage in circuit 2.

Adopting proper units for torque we may omit the constant and write

$$T = N_1 N_2 I_1 I_2 \sin \alpha.$$

$$\alpha = \theta_1 - \theta_2$$

$$\text{Sin } \alpha = \frac{X_1 R - R_1 X}{Z_1 Z} \text{ and}$$

$$T = \frac{N_1 N_2 E^2}{Z_1^2} \cdot \frac{X_1 R - R_1 X}{R^2 + X^2}$$

In a given motor, E_1, N_1, N_2, X_1, R_1, and Z_1 are fixed. If the motor is to be of the resistance split-phase type,

$X = X_2$ and the only variable is R. On the other hand, in a condenser motor, $R = R_2$ and X is the variable. A graph of this equation is shown in Fig. 22.

The condition for maximum torque is readily found. Assuming a resistance split-phase motor in which R is the variable, we have for maximum torque,

$$\frac{d\,T}{d\,R} = 0 = \frac{X_1\,(R^2 + X^2) - 2\,R\,(X_1\,R - R_1\,X)}{(R^2 + X^2)^2}$$

and solving for R

$$R = \frac{X}{X_1}\,(R_1 \pm Z_1)$$

Since Z_1 is always greater than R_1 it is evident that one of the above values is negative. We have negative reactors, *i. e.*, condensers, but unfortunately we have no negative resistors. Hence only the positive value has any immediate importance for us.

Substituting the value of R which will give us the maximum torque and calling the maximum torque as a resistance split-phase motor T_{MR} we get

$$T_{MR} = \frac{N_1\,N_2\,E^2}{Z_1^2}\,\frac{X_1^2}{X_2}\cdot\frac{1}{2\,(Z_1 + R_1)}$$

In the above expression for the maximum torque

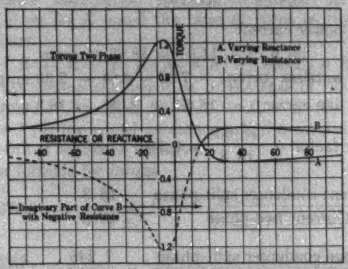

FIG. 22—CURVES OF TORQUE AND TOTAL RESISTANCE OR REACTANCE

the constants Z_1, R_1, X_1, and N_1 of the circuit 1 are rather rigidly fixed by the design of the motor. Since the circuit 2 is used only during the starting period, the number of turns and consequently its resistance and reactance can be varied within wide limits. Considering everything except N_2 and X_2 as constants and making use of the fact that X_2 is proportional to N_2^2 we may write

$$T_{MR} = \frac{\text{Constant}}{N_2}$$

This indicates that we can increase the torque of a resistance split-phase motor indefinitely by decreasing the number of turns in the starting phase and adjusting the resistance to correspond.

THE CONDENSER MOTOR

We shall now consider the effect of changing the reactance of circuit 2, the resistance being kept constant. We proceed as before except that we now differentiate with respect to X and have

$$\frac{d\,T}{d\,X} = 0 = \frac{-R_1\,(R^2 + X^2) - 2\,X\,(X_1\,R - R_1\,X)}{(R^2 + X^2)^2}$$

$$X = \frac{R}{R_1}\,(X_1 \pm Z_1)$$

Using this value of X and subsituting in the expression for torque we have for the maximum torque

$$T_{MC} = \frac{N_1\,N_2\,E^2}{Z_1^2}\,\frac{R_1^2}{R_2}\,\frac{\pm 1}{2\,(Z_1 \pm X_1)}$$

It will be evident that the torque will be much greater when X_1 is negative in the above equation. This means of course that a condenser must be used. If we let X_c equal the reactance used in addition to the inherent reactance X_2 of circuit 2, then

$$X = X_2 + X_c \text{ or } X_c = X - X_2 = \frac{10^6}{2\,\pi\,f\,C}$$

where C is expressed in microfarads and f is the frequency. From this equation we can readily calculate the capacitance necessary to give maximum torque.

It will be of interest to calculate some of the characteristics of an actual motor of typical design.

The motor chosen was a ½-hp., four-pole, two-phase, 60-cycle, 220-volt squirrel-cage induction motor. The phases were alike and had the following constants

$$Z_1 = Z_2 = 22.8,\ R_1 = R_2 = 15.9,\ X_1 = X_2 = 16.6$$

These were measured with the motor rotating slowly to average the constants for different positions of the rotor relative to the stator.

Our general expression for torque is

$$T = N_1\,N_2\,I_1\,I_2 \sin\alpha.$$

In a two-phase motor $N_1 = N_2$, $I_1 = I_2$ and $\sin\alpha = 1$, or

$$T_2 = N_1^2\,I_1^2 = \frac{N_1^2\,E^2}{Z_1^2}$$

Substituting this value in the expression for the torque of a split-phase motor, we have, since here the phases are alike

$$T = \frac{N_1\,N_2\,E^2}{Z_1^2}\,\frac{(X_1\,R - R_1\,X)}{R^2 + X^2} = T_2\,\frac{X_1\,R - R_1\,X}{R^2 + X^2}$$

If we consider our motor as a condenser motor, R becomes constant and equals 15.9. For simplicity we may take $T_2 = 1$ and by varying X, compute the value of T. The result is shown in curve A of Fig. 22. The reactance plotted is the total reactance of circuit 2 and is here equal to the external reactance plus the reactance of the winding 2.

With a reactance of 16.6 ohms, *i. e.*, no external

resistance or reactance, the torque is of course zero since the phases are exactly alike and the angle α is zero.

The reactance to give the greatest torque is given by the equation

$$X = \frac{R}{R_1}(X_1 \pm Z_1) = \frac{15.9}{15.9} \cdot (16.6 \pm 22.8)$$

$$= 39.4 \text{ or } -6.2 \text{ ohms.}$$

The first value would of course be obtained by using a reactor of 22.8 ohms and the second by using a condenser of the same reactance.

Substituting these values in the equation for the torque we find $T_m = -0.201$ or $T_m = 1.245$.

It will be noted that these torques are in opposite directions and that the torque using a condenser is approximately six times that using a reactor.

Curve B was obtained in a similar manner by assuming X constant at 16.6 ohms and varying R. Calculation shows that the maximum torque will be obtained when the total resistance of phase 2 is 39.7 ohms. Using this value in the equation for torque we find $T = 0.213$. (Curve A and Curve B happen to be nearly alike since the resistance and reactance are nearly equal.)

The portion of Curve B to the left of the point 15.9 cannot be used in practise since it corresponds to negative values of resistance.

Comparison With Two-Phase Motor

In the expression used above we assumed that the factor T_2 or the torque as a two-phase motor was equal to unity. It will be evident, then, that the results are factors by which we may multiply the torque developed as a two-phase motor to find the torque as a split-phase motor. This particular motor with the proper resistance added to one phase develops single phase approximately one-fifth as much torque as when operated two-phase. With the proper value of capacitance added to one phase it develops single phase nearly 125 per cent of its two-phase torque.

Locked Currents

As a Two-Phase Motor. Here we have two equal currents and to make a fair comparison we must of course consider the starting current as double that of one phase or

$$I_s \text{ (two-phase} = \frac{2 \times 220}{22.8} = 19.3 \text{ amperes.}$$

As a Condenser Split-Phase. The current in phase 1 is

$$I_1 = \frac{220}{22.8} = 9.65 \text{ amperes}$$

$$\text{Power component of } I_1 = 9.65 \times \frac{R}{Z}$$

$$= 9.65 \times \frac{15.9}{22.8} = 6.72$$

$$\text{Wattless component of } I_1 = 9.65 \times \frac{X}{Z}$$

$$= 9.65 \times \frac{16.6}{22.8} = 7.02$$

In the circuit 2 the resistance is the same as in circuit 1. Using a condenser of 116 μ f. having a reactance of $-22.8\ \omega$ the total reactance of circuit 2 is $16.6 - 22.8 = -6.2$ ohms.

The impedance is

$$Z = \sqrt{15.9^2 + 6.2^2 = 17.16} \text{ and}$$

$$I^2 = \frac{220}{17.16} = 12.82 \text{ amperes.}$$

Power component of

$$I_2 = 12.82 \times \frac{15.9}{17.16} = 11.91 \text{ amperes}$$

Wattless component of

$$I_2 = 12.82 \times \frac{(-6.2)}{17.16} = -4.64 \text{ amperes}$$

Total power current

$$= 6.72 + 11.91 = 18.63 \text{ amperes}$$

Total wattless current $= 7.02 - 4.64 = 2.38$ amperes

Total line current $= \sqrt{18.63^2 + 2.38^2} = 18.8$ amperes

$$\text{Power factor} = \frac{18.63}{18.8} = 0.99$$

$$\frac{\text{Current condenser split-phase}}{\text{Current two-phase}} = \frac{18.8}{19.3} = 0.973$$

$$\frac{\text{Torque condenser split-phase}}{\text{Torque two-phase}} = \frac{1.245}{1} = 1.245$$

The condenser motor not only develops more torque but it does it with less current. The power factor is also better being 0.99 compared with 0.696 for the two phase. It is frequently assumed that no split-phase motor can compare with a polyphase motor in starting efficiency. Actually the condenser split phase is much better. If we had selected a condenser to give the greatest starting efficiency (instead of the greatest starting torque) the contrast with the two-phase motor would have been still greater.

In a similar manner we can compute the locked current as a resistance split-phase motor, and the power input with the two connections.

Comparison of Various Connections

The results of the computations are embodied in Table III. This table applies of course to the par-

TABLE III

	Locked torque	Locked current	Torque / Current	Power factor	Power	Torque / Power
Two phase		1	1	0.696	1	1
Condenser split phase	1.245	0.973	1.28	0.990	1.39	0.894
Resistance split phase	0.213	0.760	0.280	0.795	0.855	0.249

ticular motor considered or to any motor having R_1 and X_1 in the same ratio and having the two windings alike. In general R_1 and X_1 are likely to be nearly equal so the table is fairly representative of average conditions.

It should of course be noted that if a motor were being designed as a resistance split-phase motor, the number of turns in the winding 2 would be made less than those in winding 1. This would increase the starting torque, but at the expense of a larger current.

Likewise in a condenser motor some economy could in general be effected by using more turns in winding 2 than in winding 1. This would reduce the starting torque, but since this is greater than in the polyphase motor, it could still be made great enough.

Further Conclusions

As previously pointed out, condenser motors are frequently wound with the same weight of wire and the same distribution in both windings. In this case, if we let K equal the ratio of turns in the two windings

$$N_2 = K N_1, R_2 = K^2 R_1, X_2 = K^2 X_1, \text{and } Z_2 = K^2 Z_1$$

If we confine our attention to the case of the condenser motor $R = R_2$ and $X = X_2 + X_3 = K^2 X_1 + X_3$ where X_3 is the added reactance and if a condenser is used will of course be negative.

Our formula for torque

$$T = \frac{N_1 N_2 E^2}{Z_1^2} \cdot \frac{X_1 R - R_1 X}{R^2 + X^2}$$

becomes

$$T = -\frac{K N_1^2 E^2}{X_1^2} \cdot \frac{R_1 X_3}{K^4 Z_1^2 + 2 K^2 X_1 X_3 + X_3^2}$$

It is frequently convenient to express the torque in terms of the capacitance used. This is readily done by substituting

$$X_3 = -\frac{1}{C\ \omega}$$

Then

$$T = \frac{K N_1^2 E^2}{Z_1^2} \cdot \frac{R_1 C\ \omega}{K^4 Z_1^2 C^2 \omega^2 - 2 K^2 X_1 C\ \omega + 1}$$

The condition for maximum locked torque becomes

$$X = K^2 X_1 + X_3 = \frac{R}{R_1} (X_1 \pm Z_1) = K^2 (X_1 \pm Z_1)$$

and

$$X_3 = \pm K^2 Z_1 \text{ or } C = \frac{1}{K^2 Z_1\ \omega}$$

If we use the positive sign it means that X_3 is a reactor, if the negative sign it is a condenser. Since the latter gives the greater torque it alone will be considered.

Substituting, we have

$$T_m = \frac{N_1^2 E^2}{2 K Z_1^2} \sqrt{\frac{Z_1 + X_1}{Z_1 - X_1}}$$

From this equation it is evident that the maximum torque is inversely proportional to K and may be made as large as desired by making K small, *i. e.*, using a small number of turns in winding 2.

Comparison with Two-Phase Motor. In a two-phase motor we have

$$T_m = \frac{N_1^2 E^2}{Z_1^2}.$$

$$\text{Then } \frac{\text{Torque condenser split-phase}}{\text{Torque two-phase}} = \frac{R_1}{2 K (Z_1 - X_1)}$$

This shows that the torque of the condenser motor may be more or less than that of the two-phase motor, depending upon the value of K and the relative value of Z_1, R_1, and X_1. If the motor has average characteristics, R_1 and X_1 will be nearly equal. If we assume that the phases are alike so that $K = 1$ and $R_1 = X_1$ then $Z_1 = \sqrt{2 X_1}$ and the ratio becomes 1.21.

The Condenser Voltage. It is important that we know the voltage across the condenser since its cost is dependent upon the voltage it must stand as well as upon its capacitance. The condenser current is obviously the same as the current in circuit 2 and

$$I_c = I_2 = \frac{E}{Z} = \frac{E}{\sqrt{K^4 Z_1^2 + 2 K^2 X_1 X_3 + X_3^2}}$$

and the voltage across the condenser

$$E_c = I_c X_3 = \frac{E X_3}{\sqrt{K^4 Z_1^2 + 2 K^2 X_1 X_3 + X_3^2}}$$

If we use the value of X_3 which gives maximum torque *i. e.*, $X_3 = -K^2 Z_1$. We have

$$E_c = \frac{-E K^2 Z_1}{\sqrt{K^4 Z_1^2 - 2 K^4 X_1 Z_1 + K^4 Z_1^2}}$$

$$= -E \sqrt{\frac{Z_1}{2 (Z_1 - X_1)}}$$

This result is *independent of the ratio K.* It may be quite large if X_1 is nearly equal to Z_1. Assuming as before that $R_1 = X_1$ and $Z_1 = \sqrt{2}\, X_1$ we have

$$E_c = -1.31\, E$$

or in the average motor we may expect at start a condenser voltage about 30 per cent above the line voltage,

if maximum torque is developed. Since for maximum torque $X_3 = -Z_2$, it is obvious that the voltage across winding 2 will be equal to the condenser voltage.

Relation of Starting Torque to Volt Amperes in Condenser. The volt amperes in the condenser is given by

$$E_c I_c = \frac{E^2 X_3}{K^4 Z_1^2 + 2 K^2 X_1 X_3 + X_3^2},$$

$$\text{also } T = -\frac{K N_1^2 E^2}{Z_1^2} \cdot \frac{R_1 X_3}{(K^4 Z_1^2 + 2 K^2 X_1 X_3 + X^2_3}$$

$$\text{therefore } T = -\frac{K N_1^2 R_1}{Z_1^2} \cdot E_c I_c$$

Hence we arrive at the very important result that the torque is proportional to the volt amperes in the condenser and hence roughly proportional to the cost of the condenser.

$$\text{Also, torque per volt ampere} = \frac{T}{E_c I_c} = \frac{-K N_1^2 R_1}{Z_1^2}$$

From this it is obvious that if we wish the torque per dollar to be high we should use a larger value of the ratio K. On the other hand as we increase K the maximum torque we can obtain becomes less. We should therefore make the number of turns in winding 2 as great as possible and still obtain the necessary starting torque.

Discussion

For discussion of this paper see page 629.

The Fundamental Theory of the Capacitor Motor

BY H. C. SPECHT*

Member, A. I. E. E.

Synopsis.—*A fundamental theory of the motor and capacitor is given partly by the algebraic method and partly by graphical method. The variables in the design of a complete capacitor motor unit for any desired performance are discussed. A few examples of unbalanced phases and performance are given. The suitability for various classes of service is discussed briefly.*

Introduction

An ordinary two-phase motor may be used as a capacitor motor, one phase being connected directly, and the other in series with a condenser, to a single-phase circuit. The performance, however, may not be all that is desired. The hp. rating, especially, may have to be reduced from its normal two-phase value, in order to obtain sufficient relative pull-out torque.[1] By varying the capacitor continuously as the load changes, operating characteristics approximating those of the two-phase motor could be obtained. This, however, is not practical, and only one or two taps from the capacitor are permissible, generally one for the starting and one for the running load. Since the capacitor phase, with a constant capacity, will give almost constant torque, the torque for overloads has to be furnished by the main phase. For example, if the torque at full load is equally divided between the two phases and if the main phase is able to develop only twice its full load torque, then the combined pull-out torque will be only two and one-half times the full-load torque. On applications requiring motors with starting and pull-out torques of high values, this handicap will necessitate the designing of a motor having the phases unbalanced in order to use a smaller capacitor and thus keep the cost of the unit within reason.

The capacitor motor could be designed with a power factor of nearly 100 per cent and an efficiency nearly equal to a similarly rated two-phase motor. However, in order to obtain a smaller capacitor, a reasonable sacrifice in power factor and efficiency may be accepted.

In order to have a clear understanding of the various characteristics of the capacitor motor it may be well to deal first with the general theory of a capacitor motor, assuming the stator to be wound two phase and the windings spaced 90 electrical degrees apart.

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1. See I. Biermanns, *Archiv. für Elektrotechnik*, Vol. XVII, p. 519; Franklin Punga, *Archiv. für Elektrotechnik*, Vol. XVIII, p. 267; B. F. Bailey, *Elec. Wld.* 1928, pp. 597, 647.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

General Theory

(*A*) *Motor at Stand Still.*

The fundamental equation of starting torque for any kind of an electrical motor is as follows:

Starting torque = Rotor ampere turns × flux × cosine of the angle between their vectors × a constant.

Φ_1 = Flux of main phase.
Φ_c = Flux of condenser phase.
i_1 = Stator amperes in main phase.
i_{1c} = Stator amperes in condenser phase.
i_2' = Current in the rotor of main phase and reduced to stator winding turns.
i_{2c}' = Current in rotor of condenser phase and reduced to stator winding turns.
$e_1 = e_2'$ = Rotor voltage induced by Φ_1, and reduced to stator winding turns.
$e_{1c} = e_{2c}'$ = Rotor voltage induced by Φ_c and reduced to stator winding turns.
E = Line voltage.
E_c = Terminal voltage of the capacitor phase.
e_c = Capacitor voltage.
t_1 = Stator winding turns in main phase.
t_{1c} = Stator winding turns in capacitor phase.
k_1 & k_2 = Main phase winding distribution factors.
k_{1c} & k_{2c} = Capacitor phase winding distribution factors.
f = Line frequency.
s = Slip.
p = Number of poles.
C = Capacity microfarads.
r_1 = Resistance in ohms of stator main phase.
r_{1c} = Resistance in ohms of stator winding in capacitor phase.
r_2' = Ohmic resistance of rotor reduced to main phase winding turns.
r_{2c}' = Resistance in ohms of rotor reduced to stator condenser phase winding turns.
x_1, x_{1c}, x_2' and x_{2c}' = The corresponding leakage reactances.
$x_c = \dfrac{10^6}{2\pi f C}$ Capacitance in ohms.
ψ_1 = Angle between i_2' and Φ_c.
ψ_2 = Angle between i_{2c}' and Φ_1.
α = Angle between Φ_1 and Φ_c.

T_1 = Torque in $k\,g\,m$ developed by main phase.

T_2 = Torque in $k\,g\,m$ developed by condenser phase.

$$T_1 = 2.3 \times p \times \Phi_0 \times i_2' \times t_1 \times \frac{k_1}{k_2} \times \cos \psi_1 \times 10^{-10}\, k\,g\,m \quad (1)$$

$$T_2 = 2.3 \times p \times \Phi_1 \times i_{2c}' \times t_{1c} \times \frac{k_{1c}}{k_{2c}} \times \cos \psi_2 \times 10^{-10}\, k\,g\,m \quad (2)$$

$T_1 + T_2$ = Total starting torque.

If the windings of the two phases have equal amounts of copper the formula for torque may be written as follows:

$$^{*}T = \frac{e_2' \times e_{2c}'}{2\,\pi\,f} \times p \times \frac{r_2'}{(r_2')^2 + (X_2')^2} \times \sin \alpha \times 10^{-9} \times \text{Const.} \quad (3)$$

From this formula it follows that the maximum starting torque for different rotor resistances occurs when $r_2' = x_2'$ providing all other values remain the same. The induced voltage in the rotor depends on the stator impedance drop and in the capacitor phase also on the capacity. As the induced voltage varies the torque changes proportionately. Further, the torque depends on the angle α and this, for maximum starting torque, should be close to 90 deg. When figuring the torque it is convenient to use the graphical method as this gives a clear picture and helps in making changes necessary to obtain the best results. First of all the currents and their power factors are figured.

$$i_1 = \frac{E}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} \quad (4)$$

$$\cos \varphi_1 = \frac{r_1 + r_2'}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} \quad (5)$$

$$i_{1c} = \frac{E}{\sqrt{(r_{1c} + r_{2c}')^2 + (x_{1c} + x_{2c}' - x_c)^2}} \quad (6)$$

$$\cos \varphi_{1c} = \frac{r_{1c} + r_{2c}'}{\sqrt{(r_{1c} + r_{2c}')^2 + (x_{1c} + x_{2c}' - x_c)^2}} \quad (7)$$

The other values are obtained from the graphical method. (See Fig. 1.)

The locus of the vector i_{1c} is a circle with the diameter of $\frac{E}{r_{1c} + r_{2c}'}$ and its center on the vertical line of $O\,E$. Therefore the locus of the voltage vectors is also a circle, the center M_{c1} of which is determined by the intersection of the perpendicular erected at the center of $O\,E$ and $A\,E$.

Also the locus of the vector of the induced voltage in the rotor is a circle and its center M_{c2} is determined by the intersection of the perpendiculars through the center of $O\,B$ and $O\,C$. $O\,B$ represents the rotor voltage of the capacitor phase at a given capacity and $O\,C$ represents the rotor voltage at resonance. After the circles are determined it is an easy matter to pick from the diagram the voltages for any current because the angularity in regard to the individual vectors must be the same. The maximum starting torque for the various capacities occurs when $\sin \alpha \times e_{2c}'$ = maximum, which is at the point where the tangent at this rotor voltage circle of e_{2c}' is parallel to vector e_1. This point is marked in Fig. 1 with T_m, the corresponding point is also indicated on the circle for the condenser voltage e_c and the current i_{1c}. The point T_m on the current circle is the tangent point to i_1 and T_m on the voltage circle E_c and e_c is the tangent point to $O\,E$ or line voltage.

It will be noted from the diagram, that with an appreciably smaller value of current and capacity near the point T_m, the torque is not much smaller. Therefore, in order to keep the starting current as low as possible, it will be advisable to stay below the maximum point.

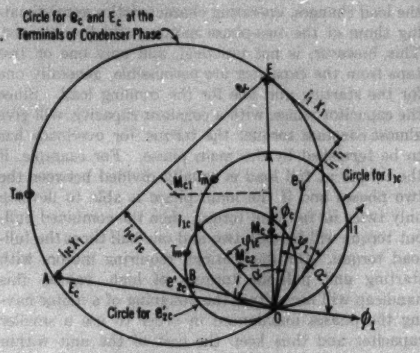

Fig. 1—Vector Diagram of Motor at Standstill

Since the capacity required to give a starting torque equal to or more than full load torque is so great, the capacitor may cost more than the motor, a series transformer should be used in connection with the condenser. The connection diagram most commonly used is shown in Fig. 2.

The advantage of this scheme is indicated very well by the fact that the capacity required decreases inversely with the square of the transformer voltage ratio and that at a certain voltage the cost of the condenser for the same volt-amperes is the lowest. The transformer also makes possible, by means of a transfer switch, the use of different effective capacities for both the starting and running conditions without breaking the condenser circuit. This is highly desir-

*See E. Arnold, *Wechselstrom Technik*, Volume 5.

able. This transfer switch may be of the centrifugal type or the magnetically operated type. The magnet coil of the latter type is connected, preferably, in the main winding circuit since the current of this winding varies through a wider range.

In designing the transformer it should be observed that the magnetizing reactance reduces the effective capacity, and that the watts loss also reduces the overall motor efficiency. Therefore, the transformer must be of ample size.

Considering starting torque only, it will be the cheapest proposition to work the main phase of the motor heavy and the capacitor phase light. How the value of the starting torque changes with an unbalanced winding system, is illustrated as follows.

Example. The amount of copper in both phases may be assumed as equal. The capacitor phase may, however, have twice as many turns as the main phase and therefore only half the cross section. The ohms resistance and the leakage reactance will be four times, and for simplicity the capacity may be only ¼. Therefore,

i_{1c} = ¼ with its power factor remaining the same.

According to formulas (1) and (2) we find;

T_1 = ½ the value of the motor with balanced windings because the flux is half as great.

T_2 = ½ the value of the motor with balanced windings as the current is only ¼ as great and winding turns twice as many.

The resultant torque is therefore only decreased ½ while the condenser capacity has been reduced to ¼. The starting current in the line is decreased to 73 per cent of value with balanced windings, providing both current vectors remain 90 deg. apart.

Generally it can be stated that the starting torque decreases approximately in the inverse ratio of the winding turns in the capacitor phase providing the

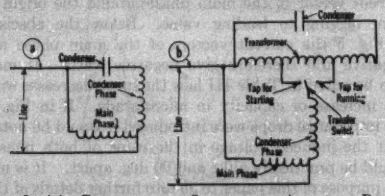

FIG. 2—(A) CONDENSER IN SERIES WITH AUXILIARY PHASE
(B) CONDENSER IN PARALLEL WITH TRANSFORMER

amount of copper in both phases is kept the same and the capacity decreased in the inverse ratio of winding turns squared. If the ratio of the amount of copper in the two phases is changed, the results are certainly different because the induced voltage in the rotor or the corresponding flux depends on the impedance drop in the stator winding. For commercial reasons, however, it is in some cases permissible to reduce the total copper section of the capacitor phase and still meet the required torques. However, in doing this the change in the performance of the motor under running load must also be given consideration.

(*B*) *Motor under Running Load.*

When the motor is running each phase will, in addition to its main flux, produce due to rotation a flux at

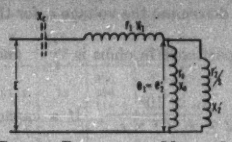

FIG. 3—CIRCUIT DIAGRAM OF MOTOR IMPEDANCE

right angles to the main flux. This field is a little smaller than the main field due to the rotor impedance drop. Therefore, for satisfactory operation it is necessary that the flux produced by the main and capacitor phase are at least approximately equal and displaced 90 deg. in time phase. If this is not the case, the rotational voltage produced by the main phase will not be equal to the transformer voltage of the capacitor phase and vice versa. This unbalanced voltage will cause a circulating current of such magnitude as to establish equilibrium. Such circulating currents result in a motor of lower performance and therefore in practise the capacitor motor will be designed with balanced flux condition without circulating current at normal operating load.

Although the method of calculation given in the following is limited in application to the balanced flux condition, it is a short and simple method for getting quick results which are sufficiently accurate for practical use. The impedance of an induction motor may be represented by the well known circuit diagram shown in Fig. 3.

In determining the complete vector diagram it is simplest to start out with the induced voltage in the rotor winding. According to the size of the motor the induced rotor voltage is generally from 4 to 8 per cent less than the line voltage. If at the end of the calculation this assumed voltage be found incorrect, the corrections can easily be made.

Since the magnetizing circuit is in parallel with the rotor circuit the corresponding conductance and susceptance must be used in our calculations.

$$i_1 = e_1 \sqrt{(g_0 + g_2)^2 + (b_0 + b_2)^2}$$

$$\cos \alpha_1 = \frac{g_0 + g_2}{\sqrt{(g_0 + g_2)^2 + (b_0 + b_2)^2}}$$

$$g_0 = \frac{r_0}{r_0^2 + x_0^2} \qquad g_2 = \frac{r_2'/s}{(r_2'/s)^2 + x_2'^2}$$

$$b_0 = \frac{x_0}{r_0^2 + x_0^2} \qquad b_2 = \frac{x_2'}{(r_2'/s)^2 + x_2'^2}$$

Having thus determined the value i_1, for both windings, lay off (Fig. 4) first the vectors i_{1c} and e_{1c} for the capacitor phase, then add the impedance drop of the stator winding. The vector $O\,A$ then represents the terminal voltage at the capacitor phase winding.

A line from A at right angles to i_{1c} and an arc with a radius equal to the line voltage around the point O as a center will determine the voltage e_c for the capacitor. Then the capacitance in ohms is $\frac{e_c}{i_{1c}}$ and in microfarads it is $\frac{i_{1c} \times 10^6}{2 \times \pi \times f \times e_c}$. If a capacitor with a transformer is used, the impedance drop of the transformer which is in series with the stator winding is also to be added.

The vector e_1 must be nearly equal to and at right angles to e_{1c}. By the angle α_1 the vector i_1 is determined. The stator impedance drop added to e_1 should end again in the point B. If the two windings of the

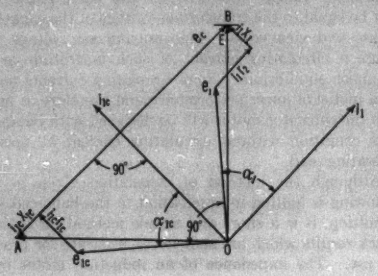

FIG. 4—VECTOR DIAGRAM OF MOTOR UNDER RUNNING LOAD

stator are not alike the induced voltage e_{1c} will be $= e_1 \times \frac{t_{1c} \times f_{1c}}{t_1 \times f_1}$ where t_1 and t_{1c} represent the corresponding winding turns and f_1 and f_{1c} their winding distribution factors. If the capacity obtained should not give the desired pull-out torque, the windings or the value of microfarads of the capacitor may be changed. It will then be found that the induced voltage e_{1c} in the rotor will change very little as long as the change stays within reasonable limits, because the voltage e_{1c} is governed somewhat by the induced voltage e_1 of the main phase. Certainly the current in the capacitor phase will change almost in ratio with the change of capacity. If the capacity is made considerably greater than a balanced condition would require, the voltage in the capacitor phase will go up, and by its transformer action, will reduce the amperes and watts in the main phase considerably. The watts may even become negative. On the other hand, the capacitor phase will take more load in both current and watts. Naturally the line amperes and watts input will go up, resulting in lower efficiency and correspondingly increased heating of the motor. As an example, the following full load data were obtained by test on a ¼-hp. 110-volt, 60-cycle, 4-pole motor, having equal windings in the phases, and is given below in Table I.

It will be noted that at 33 μf. the phases are fairly well balanced and that the watts input from the line is at lowest value and the power factor is practically unity. With approximately twice the value of μf. the current and watts input are enormously increased and also the terminal voltage on the capacitor phase as well as the capacitor voltage is much higher. At light loads and particularly at no load, this increase in voltage on the capacitor and increase of watts input always exists on capacitor motors having approximately balanced phases at full load. This is another reason for designing the capacitor motor with a strong main phase and a weak capacitor phase.

As an example, Table II gives some test results of such a capacitor motor and for comparison the results of an ordinary split phase motor having the same starting torque. It will be noted that the capacitor motor has better performance. For the same starting torque the starting amperes are only half as great. The power factor and efficiency at full load are appreciably higher. The voltage on the capacitor phase is higher than the line voltage because the stator winding of the capacitor phase has approximately twice as many turns as the main winding.

As a matter of interest, in Table III and Figs. 5 and 6 are shown the great unbalance of the current and voltage vectors for various capacities at no load for a motor similar to the motor discussed in the preceding paragraph. It is interesting to see how by transformer action the current of the condenser phase drags the current vector of the main phase around the origin O from lagging to leading value. Below the abscissa in Fig. 5 the current vectors of the main phase are negative and therefore give negative watts. It may also be noted in Table III how the speed decreases with the increase of capacity in microfarads. If in Fig. 6 the impedance drops were introduced it would be noted that the induced voltage in the rotor of both phases would be practically alike and 90 deg. apart. It is not the purpose of this paper to go into further details of the theory of unbalanced voltage conditions in split phase or polyphase motors. The theory given by Fortescue*, Slepian†, Biermanns‡, etc. may by some modifications, also be applied for unbalanced capacitor motors. However, these methods are rather complicated for daily use in design work and a more simple practical method would be highly welcomed by designing engineers.

Such extreme unbalance as shown in Figs. 5 and 6

*Fortescue, TRANSACTIONS A. I. E. E., Vol. 37, p. 1027.
†Slepian, *Elec. World*, 1920, p. 313.
‡Biermanns, *Archiv für Elektrotechnik*, Vol. XVII, p. 526.

TABLE I

Capacity in μ f.	Line		Main phase		Capacitor phase			Cap. volts	Pull-out torque in oz. ft.
	Amps.	Watts	Amps.	Watts	Volts	Amps.	Watts		
22	2.6	269	2.37	179	98	1.25	90	150	24
33	2.45	265	1.73	110	115	1.94	155	158	26
44	2.7	283	1.22	53	126	2.85	230	172	29.5
64	4.2	398	0.55	−25	154	4.55	423	180	34

TABLE II

(1) As cap. motor	Main phase			Aux. phase			Line			% Eff.	% PF	Cap.			oz. ft. T	
	V.	A.	W.	V.	A.	W.	V.	A.	W.			V.	Mfd.	FL	P O	Stg.
No load.....	110	2.8	12	247	0.8	86	110	2.65	98	..	..	276	7.5	0	..	..
Full load.....	110	3.4	219	231	0.7	72	110	3.63	291	64.2	73	250	7.5	12	41	..
Locked.....	110	19.	1640	92	1.92	148	110	19.4	1715	..	..	118.5	45	..	..	26
(2) As ord. split ph. motor																
No load.....	110	3.71	96	..	..	..	110	3.71	96	..	..	..	..	..	..	..
Full load.....	110	4.61	311	..	..	..	110	4.61	311	60	61.2	..	..	12	39	..
Locked.....	110	22.0	1805	110	20.2	1980	110	41.6	3785	..	..	..	..	..	..	26

TABLE III

	Main phase			Capacitor phase			Line			Capac. volts	R. P. M.	Mfd.
	V.	A.	Watts	V.	A.	Watts	V.	A.	Watts			
1	110	1.6	0	102	0.65	50	110	1.15	47	163	1798	10.6
2	110	1.475	−27	112	1.05	81	110	0.775	52	171	1798	16.3
3	110	1.31	−137.6	159	3.32	294	110	2.13	156.4	196	1795	45.3
4	110	1.7	−168	179	5.02	493	110	4.12	318	204	1790	64.8
5	110	2.25	−147	202.5	6.83	722	110	6.38	578	202	1775	88.5
6	110	3.28	− 25	214.5	8.64	990	110	9.63	990	185	1760	126
7	110	5.1	+390.5	201	9.26	950	110	12.15	1320	132	1730	188
8	110	6.0	+571	163	9.0	840.5	110	12.6	1365	100	1690	239
9	110	6.35	+681	160	8.8	645	110	12.4	1295	65	1610	315

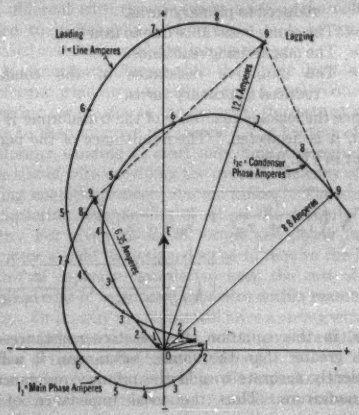

FIG. 5—CURRENT VECTORS FOR VARIOUS CAPACITIES AT NO LOAD

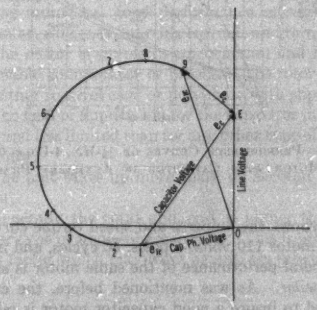

FIG. 6—VOLTAGE VECTORS FOR VARIOUS CAPACITIES AT NO LOAD

will not occur in practise. Generally it will be attempted to have the two phases at full load more nearly balanced and for such a case the performance can easily be determined. Due to the rather high effective ohms of the capacitor, the watts, as well as the current in the capacitor phase remain almost constant and all the variation in load is taken by the main phase. With the increase in load the watts and current in the capacitor phase will slightly decrease due to the higher impedance drop in the main phase and lower frequency of rotation. After the motor has pulled out, the current in the capacitor phase assumes the lowest value and stays practically constant to standstill. However, if the capacitor is much greater than required for balanced

condition at full load, *i. e.*, in the case of having the capacitor phase connected to the starting tap of the capacitor, then the current in the capacitor phase naturally will increase with the load because the capacitor phase is capable of carrying more load. After the motor has pulled out, the current stays practically unchanged since the total impedance in the capacitor phase has become almost constant. For example,

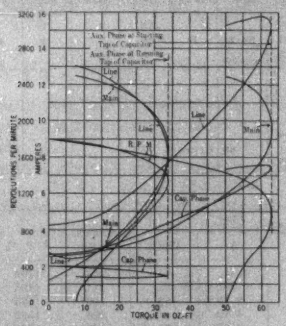

FIG. 7—SPEED-TORQUE-AMPERE CURVES OF A ¼-HP., 4-POLE, 60-CYCLE MOTOR

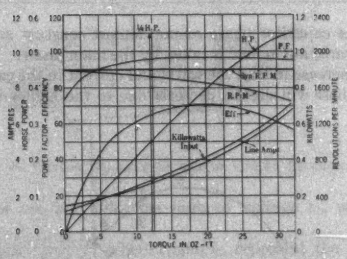

FIG. 8—PERFORMANCE CURVES OF ¼-HP., 4-POLE, 60-CYCLE MOTOR WITH CAPACITOR IN AUXILIARY PHASE

Fig. 7 shows the amperes and speed torque curves of a ¼-hp. motor 110 volts, 4 poles, 60 cycles, and in Fig. 8 the general performance of the same motor is shown.

Capacitor. As was mentioned before, the capacity required to insure a good capacitor motor is relatively high. The fact that the effective capacity varies with the square of the voltage, makes possible and desirable the use of a series auto transformer, as shown in Fig. 2. How far the voltage on the capacitor may be raised economically depends on the cost of the capacitor per kilovolt-ampere capacity and on the cost of the transformer. The cost of the transformer is quite an item. In cases where the line voltage is relatively high or where the starting torque required is low, the capacitor unit using a condenser only without a transformer may be more economical. By adding a transformer some of the capacitor effect is sacrificed, due to the magnetizing volt-amperes in the transformer.

The impedance diagram of the capacitor shown in Fig. 2B may be represented as shown in Fig. 9.

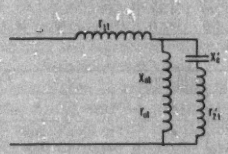

FIG. 9—IMPEDANCE DIAGRAM OF CAPACITOR

r_{1t} = The ohmic resistance of the primary winding of the transformer.

r_{2t}' = The ohmic resistance of the secondary winding reduced to primary turns.

r_{0t} = The ohmic resistance due to iron loss.

x_{0t} = The magnetizing reactance.

x_c' = The inductive resistance of the condenser reduced to primary turns.

Since the leakage reactance of the transformer is very small, it is neglected. The admittance of the parallel circuit in Fig. 9 is:

$$Z = \sqrt{\left(\frac{r_{0t}}{r_{0t}^2 + x_{0t}^2} + \frac{r_{2t}'}{r_{2t}'^2 + x_c'^2}\right)^2 + \left(\frac{x_0}{r_{0t}^2 + x_{0t}^2} - \frac{x_c'}{r_{2t}'^2 + x_c'^2}\right)^2}$$

Since, in this equation, the reactive resistances are much greater than the ohmic resistances, it will be sufficiently accurate to add the primary resistance r_{1t} in quadrature. Thus the total impedance of the capacitor is:

$$Y_t = \sqrt{r_{1t}^2 + \frac{1}{\sqrt{\left(\frac{r_{0t}}{r_{0t}^2 + x_{0t}^2} + \frac{r_{2t}'}{r_{2t}'^2 + x_c'^2}\right)^2 + \left(\frac{x_{0t}}{r_{0t}^2 + x_{0t}^2} - \frac{x_c'}{r'^2_{2t} + x_c'^2}\right)^2}}}$$

For low watt losses in the capacitor resulting in not much reduction in over-all efficiency of the capacitor motor, the resistances r_{0t} and r_{2t}' will be comparatively so small that they may be neglected and the above equation may be simplified and condensed as follows:

$$Y_t = \sqrt{r_{1t}^2 + \frac{1}{\left(\frac{1}{x_{0t}} - \frac{1}{x_c'}\right)^2}}$$

The equation shows very clearly how much damage the magnetizing reactance X_{0t} may do. As the saturation in the transformer iron increases, the reactance X_{0t} decreases and the impedance Y_t will increase. For a given voltage this results in a smaller current through the capacitor phase and hence less torque. The general rule that the starting torque of an induction motor increases as the square of the voltage holds true on a capacitor motor only as long as the iron in the transformer is not saturated. When the iron becomes highly saturated the torque may even decrease with increasing line voltage.

The saturation of the transformer iron exists only at the starting connection because at the running connection the primary winding turns are much greater. Generally the designing engineer is inclined to increase the flux density at starting to the highest possible limit; however in this case it is not wise to do so because the magnetizing volt-amperes directly reduce the capacity and with it the starting torque. It is somewhat different with the condenser itself as the voltage applied to it for such a short time may be greatly increased over its normal continuous rating.

It is somewhat different with the ohmic drop in the transformer winding as this gives only a small reduction in starting torque due to the fact that this acts at right angles to the inductive resistance. The heating of the transformer winding at start may limit the reduction in the size of wire providing the overall efficiency of the running conditions meets the guarantee. The size of the capacitor depends first of all on the design of the motor. For given values of torque and power factor the motor should be designed so as to require the least amount of capacity in order to keep the cost of the combined unit of motor and capacitor within reasonable limits. Ordinarily the motor will have a strong winding in the main phase and a weaker winding with a different number of turns in the capacitor phase.

The ratio of winding turns in the capacitor depends on the cost of the condenser. At a certain voltage the cost of the condenser per kv-a. will be least, which makes this the desirable voltage to figure on using, provided the winding cost of the transformer remains the same for the different ratio in turns. The lowest priced condenser may require a very high number of turns having a small wire section in the secondary winding, thus actually increasing the cost of the unit. In that case a larger condenser requiring a lower voltage will give a more economical design. The above statements show that the art of the design of a capacitor motor consists of properly balancing the design so as to obtain a satisfactory unit at a reasonable cost. This is far more difficult than in the design of an ordinary induction motor due to the large number of variables to be considered and especially so due to the additional cost of the capacitor.

Application of Capacitor Motors

As far as the electrical performance is concerned, the capacitor motor may be used for almost any class of service where practically constant speed is desired. It has great flexibility of design. On applications where high power factor, high starting torque, and low starting current are required, the capacitor motor finds a good field. The possibility of obtaining various speeds by changing the capacity or, when wound slip-ring polyphase rotors are used, of changing the resistance in the rotor circuit, insures the motor a field for such applications as blowers and similar devices. Further, where reversing service or braking is required, the capacitor motor may be used. The possibilities of application of the capacitor motor are so numerous that many pages covering its field could be written.

Summary

A summary of the above article on the capacitor motor is as follows:

1. The motor has good performance in respect to power factor, efficiency, starting current, and torques.
2. The motor is simple in construction and has no objectionable commutator or brushes. The rotor can be wound either squirrel cage or polyphase with slip rings.
3. The field of application for the capacitor motor is broader and less limited than for any other type of single phase motor.
4. The unit takes up more space on account of the capacitor.
5. Generally the total unit cost is higher than for any other type of single-phase motor. This is probably the only factor which may react against its present day use.

Discussion

For discussion of this paper see page 629.

The Revolving Field Theory of the Capacitor Motor

BY WAYNE J. MORRILL*

Associate, A. I. E. E.

Synopsis.—The paper presents an accurate theory of the split-phase motor, both as regards starting and running performance. The general equations for an unbalanced two-phase motor are first derived, and the results are then applied to the special case of the capacitor motor. Evidence of the validity of the theory is given in the form of curves, comparing test results with calculations.

The principal factors affecting practical capacitor motor design are discussed, and finally the performance characteristics of the motor are compared with those of repulsion-start induction motors. It is concluded that the capacitor motor has important advantages, which will justify its extensive use.

The complete derivation of the theory is given in an appendix.

Introduction

With the rapid increase which has recently taken place in the number of motors applied to such semi-continuously operated domestic loads as household refrigerators and oil burners, there has risen a demand for fractional horsepower motors of very high quality. Such motors operating on house lighting circuits as they do and running at all times of the night and day, must be quiet, have high operating characteristics, low starting current to prevent flicker of lights, and no radio interference. Of all the types of single-phase fractional horsepower motors available, the capacitor motor seems best suited for this sort of service. It not only has all of the characteristics necessary, but in addition is probably the simplest and most reliable of all high quality single-phase motors.

A number of manufacturers has appreciated the advantages of the capacitor motor and at the present time there are several different makes available. The renewed interest which has been shown in capacitor motors, by the actions of the motor manufacturers as well as by the attitudes of the power companies and the device manufacturers, makes it appropriate to discuss their theory and characteristics.

The capacitor motor as a type is not new. Some thirty years ago Doctor Steinmetz and his associates developed such a motor and secured a number of basic patents on it. Some of these motors were actually built and placed in service, but their manufacture was soon abandoned largely because of the difficulty, at that time, of securing capacitors of sufficiently reliable construction.

After this experience, there was a long period in which little interest in them was shown except for an occasional review which always brought the same answer; that is, until a cheap and reliable capacitor could be found, the capacitor motor would be commercially impractical.

In recent years great strides have been made in the manufacture of capacitors. The experience gained in their manufacture for the radio industry as well as for power-factor correction has been largely responsible for the improvements made. On the basis of this experience, there has been a definite effort made to develop a capacitor which would be satisfactory for use with motors, and the result is that at the present time the cheap and reliable capacitors for which the motor manufacturers have been waiting are available.

The purpose of this paper is to present the revolving field theory of the capacitor motor, and to show how, by the use of this theory, it is possible to explain and calculate the phenomena which appear in the operation of the motor. In addition to presenting the theory, an effort will be made to show that the possession of a sound theory is of great assistance in the design of a line of motors.

Of interest are the single-phase motor equations which can be obtained as the special case of a capacitor motor in which the auxiliary phase carries no current. It is believed that this method of treatment of a single-phase induction motor represents an advance in the art. By means of this treatment, the complete torque equation, including the alternating single-phase torque, is obtained.

A capacitor motor is really an unbalanced two-phase motor, in which both of the stator phases are connected directly across the same line. For this reason the equivalent circuit and general equations for an unbalanced two-phase motor will be first obtained, and after a brief discussion of the possible capacitor motor connections, these equations will be applied to the calculation of capacitor motor performance.

The General Equations of an Unbalanced Two-Phase Motor

So far as the writer is aware, the general equations for the unbalanced two-phase motor were first derived

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29-38

by Mr. P. L. Alger, early in 1924, as an extension of the A. I. E. E. article[1] he published at that time. The derivation of the general theory given here is largely founded on that work. In carrying on the further studies described in this paper, the writer has received encouragement and many helpful suggestions from Messrs. A. F. Welch, A. R. Stevenson, Jr., P. L. Alger, and C. J. Koch, to whom he wishes to express his obligation.

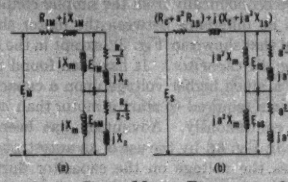

FIG. 1—SINGLE-PHASE MOTOR EQUIVALENT CIRCUITS

In the revolving field theory of single-phase motors the pulsating sinusoidal flux produced by the stator winding is resolved into two equal sinusoidal waves of flux gliding in opposite directions around the periphery of the air-gap at synchronous speed. The effect upon the stator produced by each of these revolving fluxes and the induced forward and backward revolving rotor fluxes can be represented by a parallel circuit of two branches. One branch is the magnetizing reactance and the other is composed of the secondary leakage reactance in series with the secondary resistance divided by the slip.

The equivalent circuit for a single-phase motor having a primary impedance of $R_{1M} + j\,X_{1M}$ and the primary winding of which is called the M phase is represented in Fig. 1A.

In this equivalent circuit the impressed voltage equivalent to the voltage induced in phase M by the forward revolving field of flux is $\bar{E}_{fM}$ and the impressed voltage equivalent to the voltage generated by the backward revolving field of flux is $\bar{E}_{bM}$.

If instead of the M phase there is an S phase displaced backward in position by 90 deg., electrical, from the position of the M phase and having a times as many turns as the M phase, the single-phase equivalent circuit for the S phase is shown in Fig. 1B. In the S equivalent circuit the primary impedance is $a^2\,R_{1S} + j\,a^2\,X_{1S}$ and the external series impedance is $R_c + j\,X_c$.

If both the M and the S phases exist on the motor and are excited simultaneously, their fluxes superimpose without distortion and the equivalent circuits are the same as before except that in addition to the forward and backward voltages self induced in each phase there is a forward and backward voltage due to the fluxes of the other phase. Under this condition the equivalent circuit becomes as shown in Fig. 2 wherein the divided circuits have been replaced by series impedances of equal value.

Since the M phase is displaced forward by 90 electrical degrees from the S phase, the voltage generated in the M phase by the S forward flux must lag by 90 time degrees the voltage which the same flux produces in the S phase and since the turns on the M phase are to those on the S phase as 1 is to a, the magnitude of the M voltage must be $\dfrac{1}{a}$ times that of the S phase.

From these considerations the equation for the impressed voltage equivalent to the voltage produced in the M phase by the S forward flux is:

$$\underset{Sf}{\bar{E}_M} = -\,j\,\frac{1}{a}\,\bar{E}_{fS} \tag{1}$$

By a similar line of reasoning the impressed voltages equivalent to the other mutually generated voltages can be shown to be the values given on the equivalent circuit.

The equations for the voltages impressed on each of the primary phases are, from Fig. 2:

$$\bar{E}_M = \bar{I}_M\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,\bar{I}_S\,a\,[(R_f - R_b) + j\,(X_f - X_b)] \tag{2}$$

$$\bar{E}_S = \bar{I}_S[R_c + a^2(R_{1S} + R_f + R_b) + j\,X_c + j\,a^2(X_{1S} + X_f + X_b)] + j\,\bar{I}_M\,a\,[(R_f - R_b) + j\,(X_f - X_b)] \tag{3}$$

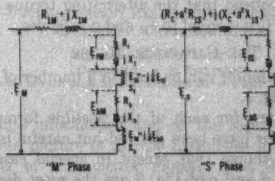

FIG. 2—UNBALANCED TWO-PHASE MOTOR EQUIVALENT CIRCUIT

CURRENT EQUATION FOR AN UNBALANCED TWO-PHASE MOTOR

The simultaneous solution of (2) and (3) gives for the currents of an unbalanced two-phase motor:

$$I_M = \frac{\bar{E}_M\,\{[R_c + a^2(R_{1S} + R_f + R_b)] + j\,[X_c + a^2\,(X_{1S} + X_f + X_b)]\} + j\,\bar{E}_S\,a\,[(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2\,(R_{1S} + R_f + R_b)] + j\,[X_c + a^2\,(X_{1S} + X_f + X_b)]\} \times \{[R_{1M} + R_f + R_b] + j\,[X_{1M} + X_f + X_b]\} - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \tag{4}$$

$$I_S = \frac{\bar{E}_S\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,\bar{E}_M\,a\,(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2\,(R_{1S} + R_f + R_b)] + j\,[X_c + a^2\,(X_{1S} + X_f + X_b)]\} \times \{[R_{1M} + R_f + R_b] + j\,[X_{1M} + X_f + X_b]\} - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \tag{5}$$

1. See Bibliography.

Equation for Average Torque of an Unbalanced Two-Phase Motor

Since only current and flux waves traveling at the same speed can act together to produce average torque, the average torque in synchronous watts produced by an unbalanced two-phase motor is equal to the sum of the watts consumed by the forward field in each phase minus the sum of the watts consumed by the backward field.

If the current in the M phase be:

$$\bar{I}_M = A + j\,B \tag{6}$$

and that in the S phase be:

$$\bar{I}_S = \mathbf{g} + j\,\mathbf{h} \tag{7}$$

the equation for the average torque can be shown to be:

$$T_{avg} = [I_M^2 + a^2 I_S^2][R_f - R_b] + 2\,a[A\,\mathbf{h} - B\,\mathbf{g}][R_f + R_b] \tag{8}$$

Since:

$$A\,\mathbf{h} - B\,\mathbf{g} = I_M\,I_S \sin\phi \tag{9}$$

Equation (8) may be reduced to (110) of Appendix I.

In addition to the average torque of Equation (8) there is an alternating torque produced through the action of the forward flux and backward current and vice-versa.

Equation for Alternating Torque of an Unbalanced Two-Phase Motor

In Appendix I, Equation (111), it is shown that the maximum value of the alternating torque is:

$$T_{\underset{Max}{\sim}} \doteq \sqrt{[I_M^4 + a^4 I_S^4 + 2\,I_M^2 I_S^2\,a^2 \cos 2\,\phi][(R_f - R_b)^2 + (X_f - X_b)^2]} \tag{10}$$

It will be noticed that the alternating torque ceases to exist when $I_S\,a$ is equal to I_M and ϕ is 90 deg.

The Capacitor Motor

A capacitor motor can be built in a number of forms as shown in Fig. 3.

The equations for each of the possible forms of a capacitor motor have been derived but careful investigations have shown that except in special cases the two forms in d and c of Fig. 3 are the most desirable from considerations of simplicity and economy.

The "shaded pole" and "permanent split" types are simpler than either d or e but their field of application is limited because their standstill torques are so low as to preclude their use on any loads which do not have a fan characteristic. Due to the unstable nature of the starting or capacitor phase, a peculiarity of capacitor motors which will be touched upon later, the starting phase takes an increasing amount of current as the motor speed approaches synchronism and as a result the normal and light load efficiencies are considerably reduced. A high-resistance rotor, due to the speed reduction which it produces, and the lesser amount of initial S phase current needed to secure the starting torque, can be used in a "shaded pole" or "permanent split" motor without suffering loss in efficiency, and since a reduction in the size of the capacitor is obtained at the same time, high-resistance rotors are usually employed in such motors. The poor speed regulation obtained with a high resistance rotor permits of variable speed operation by variation of the impressed voltage.

Normally, there is nothing to be obtained from the "series" connection shown in Fig. 3c which cannot be got more economically from the simpler connections.

There is no difference between the motor shown in Fig. 3d and that shown in Fig. 3e except in the method of applying the capacitor. It has been found possible to impress a much higher voltage upon a capacitor for the short time required to start a motor than could be impressed continuously. Advantage has been taken of this fact in Fig. 3e by the use of an auto transformer which raises the voltage on the capacitor during the starting period and then by a change in taps reduces the voltage to a value safe for continuous operation. In addition to the change in voltage, the transformer

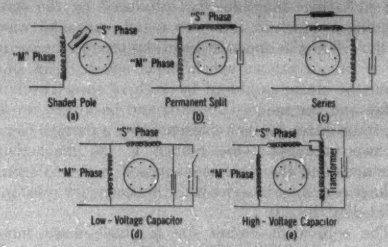

Fig. 3—Capacitor Motor Diagrams

makes it possible to impress upon the capacitor the exact voltage which is desired, regardless of the motor design.

The Capacitor Motor Current and Torque Equations

For purposes of analysis it is possible to represent either the capacitor of Fig. 3d or the capacitor transformer unit of Fig. 3e by a resistance in series with a condensive reactance. If these values be substituted for the external impedance $R_c + j\,X_c$ in the unbalanced two-phase equations and the voltage $\bar{E}_M$ be substituted for $\bar{E}_S$ the unbalanced two-phase equations apply directly to a capacitor motor.

The unbalanced two-phase torque equations are not affected by the above substitutions and apply without change to a capacitor motor.

Capacitor Motor Output

As shown in Equation (112) the equation for net output is:

$$W.\,O. = \{\,[I_M^2 + a^2 I_S^2]\,[R_f - R_b] + 2\,I_M\,I_S\,a\,[R_f + R_b] \sin\phi\,\}\,(1 - s) - W_f \tag{11}$$

wherein W_f is the watts friction loss.

Total or Line Current of a Capacitor Motor

The total current $\bar{I}$ of a capacitor motor is equal to the sum of $\bar{I}_M$ and $\bar{I}_S$ and if it be represented as follows:

$$\bar{I} = C + jF \tag{12}$$

the equation for the input may be written:

$$W.I. = C\,E_M + W_i \tag{13}$$

in which W_i is the iron loss.

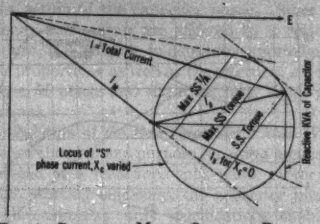

Fig. 4—Capacitor Motor Starting Diagram

Standstill Torque Equation

At standstill the torque Equation (8) becomes:

$$T_{SS} = 2\,a\,[A\,\mathbf{h} - \mathbf{g}\,B]\,[R_f + R_b] \tag{14}$$

If at standstill the total impedance of the S phase plus its external impedance be defined:

$$a^2 R_S + R_c = a^2 (R_{1S} + R_f + R_b) + R_c \tag{15}$$

$$a^2 X_S + X_c = a^2 (X_{1S} + X_f + X_b) + X_c$$

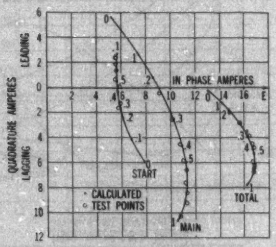

Fig. 5—Vector Current Relations for Starting Condition of a ¼-Hp. Capacitor Motor

Slip varied from "1" to "0"
. Calculated ⊙ Test points

CONSTANTS

R_{1M}	= 2.02	X_{1M}	= 2.79	X_M	= 33.4
R_S	= 2.06	X_S	= 1.06	a	= 1.18
R_{1S}	= 5.12	X_{1S}	= 2.31	Wf	= 13
R_c	= 3	X_c	= − 14.5	Wi	= 24

Equation (14) may be written in the following form:

$$T_{SS} = 2\,a\,E_M \left[\frac{-A(a^2 X_S + X_c) - B(a^2 R_S + R_c)}{(a^2 X_S + X_c)^2 + (a^2 R_S + R_c)^2}\right](R_f + R_b) \tag{16}$$

Equation for Capacitor Reactance Required to Produce a Given Standstill Torque

If Equation (16) be solved for X_c the equation for the capacitor reactance required to produce a given torque is obtained:

$$X_c = \frac{a\,E_M\,(R_f + R_b)}{T_{SS}} \left\{ -A \pm \sqrt{A^2 - \frac{4\,T_{SS}}{2\,a E_M (R_f + R_b)}\left[(a^2 R_S + R_c)\left\{B + \frac{T_{SS}\,(a^2 R_S + R_c)}{2\,a\,E_M\,(R_f + R_b)}\right\}\right]} \right\} - a^2 X_S \tag{17}$$

Starting Diagram for Capacitor Motors

Equations (14) and (17) are useful but a starting diagram has been devised which furnishes graphically

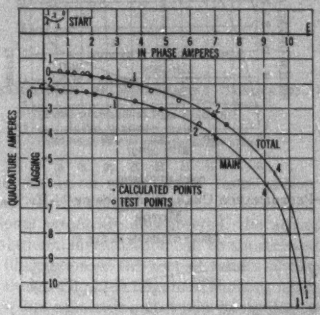

Fig. 6—Vector Current Relations for Running Condition of a ¼-Hp. Capacitor Motor

Slip varied from "1" to "0"
. Calculated ⊙ Test points
Constants are the same as Fig. 5 except R_c = 9 and X_c = − 172

the same information and at the same time gives a complete picture of all the quantities involved in the starting of a capacitor motor. This diagram is so simple and convenient to use that it is desirable to present it at this time.

If the locus of the total current $\bar{I}$ as X_c is varied be plotted, the result is the circle shown in Fig. 4. From this circle the various starting quantities can be read in the manner indicated on the diagram.

It will be noticed that the maximum torque per ampere occurs when the total current is tangent to the circle and the maximum torque occurs at the point where a line parallel to $\bar{I}_M$ is tangent to the circle.

Capacitor Motor Characteristics

The calculated current loci for a capacitor motor as its speed is varied are shown in Figs. 5 and 6. Loci

for both the starting and running connections are given and the torque and performance corresponding are given in Fig. 7.

On the preceding figures the circled points are test results. It will be noted that the calculations check the tests remarkably well.

On both "current loci" and "speed torque" curves

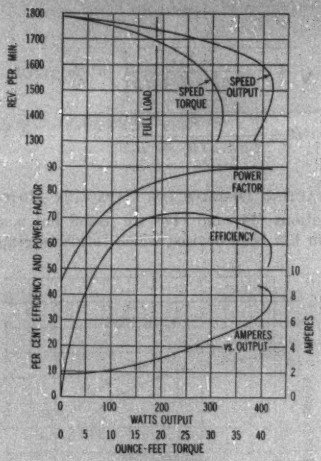

FIG. 7—PERFORMANCE CHARACTERISTICS OF ¼-HP. CAPACITOR MOTOR

(Same constants as Fig. 6)

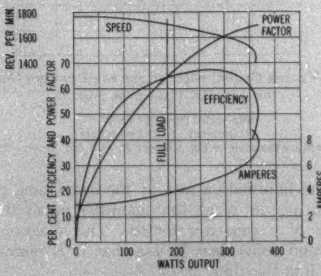

FIG. 8—SINGLE-PHASE PERFORMANCE OF ¼-HP. CAPACITOR MOTOR WITHOUT CAPACITOR

(Calculated curves)
"M" phase only, "S" phase disconnected
(Same constants as Fig. 6)

the unstable nature of the S phase current will be noticed. At high speeds the capacitor current and voltage increase rapidly making it desirable to change connections in order to secure economy and good operating characteristics.

Not only is the magnitude of the S phase current unstable but its angular displacement also varies and with the starting connection the vectors actually cross over so that the second component of torque in Equation (8) becomes reversed in sign and acts to reduce the torque.

Investigation has shown that a high value for a accentuates the instability of the S phase current and for this reason it is often necessary to reduce a in order that the torque on the starting connection at high speeds will be sufficient.

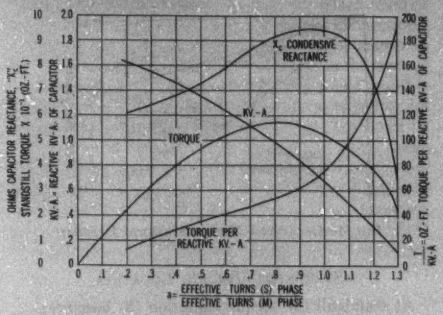

FIG. 9—CAPACITOR MOTOR STANDSTILL CHARACTERISTICS OF ¼-HP. MOTOR

Total current constant—Number of turns on starting winding varied
(Same constants as Fig. 5)

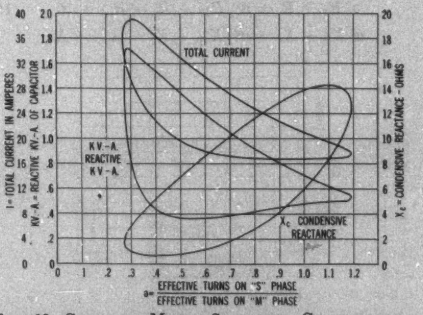

FIG. 10—CAPACITOR MOTOR STANDSTILL CHARACTERISTICS

(Calculated for ¼ hp.)
Constant torque, turns on "S" phase varied
(Same constants as Fig. 5)

For purposes of comparison Fig. 8 is inserted to show the calculated performance of the same motor with the S phase disconnected and operated as a single-phase induction motor.

DOUBLE FREQUENCY TORQUE

The alternating torque of a capacitor motor is given by Equation (10). While it is possible to approach the condition of zero alternating torque over a given range of load by properly proportioning the S

phase and the capacitor, this condition is seldom actually obtained. With a capacitor correctly specified for running operation, the alternating torque is always reduced, generally to less than half that of an equivalent single-phase motor. (See Fig. 12.)

The Design of Capacitor Motors

(a) *Motor Proportions.* Experience shows approximately how much improvement is to be expected in the output and full load efficiency of a capacitor motor due to the action of the capacitor phase. Since both the efficiency and output are affected by a relatively small amount, it is possible to determine approximately the motor proportions as well as the electrical specifications of the M phase and the rotor from single-phase considerations.

(b) *Calculation of the Best Capacitor Phase Proportions.* Since the primary purpose for using a capacitor motor instead of a resistance split phase motor is to secure high starting efficiency, the capacitor phase as well as the capacitor unit specifications should be determined from starting considerations.

The limiting condition on the torque specification varies and for this reason there is outlined a procedure which will apply to all cases with equal facility.

In general, it is found best to fill with copper the slots allotted to the S phase and if this is done the only variable in the motor is the number of S phase turns.

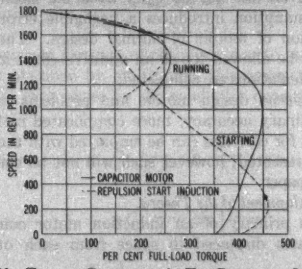

Fig. 11—Torque Curves of ¼-Hp. Capacitor Motor *vs.* ¼-hp. repulsion start induction motor (Average of several makes)

——— Capacitor motor
- - - - - Repulsion start induction

Using the starting diagram it is possible to calculate the best capacitor specification for each of a number of values of a and by plotting a curve of the results *vs.* a the most desirable specification can be determined.

In order to show how the different quantities vary as a is varied Figs. 9 and 10 are included.

c. *Calculation of Limiting Voltage on Capacitor.* The limiting voltage on the capacitor usually occurs at the cut-over speed and it is necessary to make a calculation of the starting connection for that speed in order to determine just what the highest voltage impressed on the capacitor will be.

d. *Verification of Design.* The design having been completed, its performance may be calculated and verified by means of the theory which has been presented.

The large number of variables involved in the design of a capacitor motor is very discouraging to a purely experimental investigation intended to secure the best motor and capacitor proportions. When it is remembered that in an experimental investigation such annoyances as heating, magnetic saturation, and observational errors are encountered, the advantages of a calculational investigation can be appreciated.

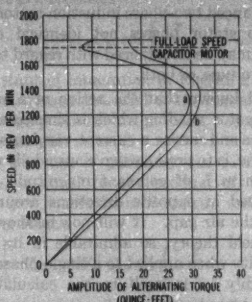

Fig. 12—Amplitude of Alternating Torque *vs.* Speed

(Same constants as Figs. 6 and 8)
(a) ¼-hp. capacitor motor
(b) Same motor operated single phase

As an illustration of the sort of results obtained in practical design Fig. 11 is presented showing a comparison of a ¼-hp. capacitor motor now in regular production with a composite repulsion start induction curve obtained by averaging the values for several of the leading makes of repulsion start induction motors.

Conclusions

The foregoing theory and discussion of the capacitor motor have shown its possibilities as a very high quality motor. It has the following points of merit:

(a) It has excellent torque characteristics, being able to start and accelerate all it can carry;

(b) Its starting current is low, being only about 30 per cent to 50 per cent greater than that of a repulsion start induction motor of equal rating or about ½ that of a split phase motor of less torque;

(c) Its power factor is high, due to the use of the capacitor for power-factor correction after it has completed its duty in starting the motor;

(d) The motor noise is reduced, due to the reduction in double frequency torque and due to the fact that there are no brushes or other rubbing contacts;

(e) The motor is free from radio interference because it has no brushes.

ACKNOWLEDGMENT

The writer wishes to acknowledge his appreciation of the valuable assistance rendered by his associates in making calculations and plotting curves for this paper.

Appendix

THE UNBALANCED TWO-PHASE INDUCTION MOTOR

An unbalanced two-phase induction motor is a motor having two stator phases not identical electrically. The phases may be unbalanced with regard to windings, impressed voltage, or both.

It is well known that the action of a single-phase induction motor winding which is sinusoidally distributed in space and carries a current varying sinusoidally with the time, is to produce two sinusoidally distributed waves of m. m. f. of constant magnitude, gliding at uniform speed in opposite directions around the periphery of the air gap. It will now be shown that the revolving field theory, so admirable in its symmetry and simplicity when applied to a single-phase induction motor, readily lends itself to the calculation of an unbalanced two-phase motor.

The general construction of induction motors shows a number of characteristics peculiar to such machines. They usually have: small and approximately uniform air gaps, uniform slot spacing, symmetrical phase distribution, approximately fundamental sinusoidal distribution of the conductors in each phase, and unsaturated magnetic paths.

Advantage will be taken of these peculiarities to make some customary simplifying assumptions and these will now be listed.

Assumptions.

(*A*) *Perfect sinusoidal distribution of the conductors in each phase.* It can be shown that any induction motor winding can be resolved into an equivalent perfectly distributed fundamental winding plus an infinite number of perfectly distributed harmonic windings.

It can further be shown that except in very special cases all of the fluxes produced by the action of the harmonic windings link negligibly with any other windings except those on the member producing them and that in the case of a quadrature distributed two-phase motor the linkages of the harmonic fluxes are confined to the phase winding producing them. For these reasons, in the case of a uniformly distributed two-phase motor, the effect of the harmonic fluxes may be treated as that of self inductive reactance of the phase winding producing them.

(*B*) *Uniform or quadrature distribution of phases.* The methods of analysis employed in this paper can be used in the study of motors which do not have a quadrature distribution of phases, as, for example, the shaded pole motor; but such discussion does not come within the scope of the present paper.

(*C*) *Infinitesimally small slot openings.* This assumption is made in respect to the air gap only, and is for the purpose of securing a uniform gap and consequently a uniform permeance for the air gap. Correction for the slot openings can be made by the use of "average" gap permeance calculated allowing for the fringings.

(*D*) *Infinite permeability of the iron paths.* The iron paths of most induction motors are not worked above the "knee" of the saturation curve. Below the "knee" a small portion of the total m. m. f. is consumed in the iron paths and most of that which is so consumed is directly proportional to the flux density. For this reason it is possible to approximate with a high degree of accuracy to the conditions which actually prevail by altering the true reluctance of the air gap by an amount equivalent to the reluctance of the iron paths while introducing the assumption that the iron paths are of infinite permeability.

(*E*) *All the air gap flux crosses the gap radially.* With an infinitesimal gap and the preceding assumptions, this is exactly correct. As the gap becomes finite, the assumption is still very closely true for all but the very high harmonic fluxes. The high harmonics induce negligible voltages and, for this reason, with normal induction motors, the above assumption is sufficiently accurate.

(*F*) *Iron loss has no effect upon the motor fluxes.* This assumption introduces a negligible error in the calculation of motors of normal design. The actual loss will be allowed for as an additive effect after the other calculations are completed.

For special design motors and problems requiring extraordinary accuracy, more complicated methods of allowing for core loss can be employed with the theory herein presented; however such will not be considered in this paper.

The Development of the Theory.

If the primary of an induction motor consists of two phases displaced in space from each other by $\frac{\pi}{2}$ electrical radians, the equation for the conductor density of the first or M phase is:

$$\Delta C_M = - \Delta C_{M\,max} \sin \frac{\pi}{\lambda} x \qquad (1)$$

and the equation for the conductor density of the second or S phase may be written:

$$\Delta C_S = - a \, \Delta C_{M\,max} \sin \left[\frac{\pi}{\lambda} x + \frac{\pi}{2} \right] \qquad (2)$$

wherein:

x = the distance around the periphery of the air gap measured in inches from the reference point.

ΔC_M = the conductor density of the first phase per

inch of periphery at the point x distant from the reference point.

a = ratio $\dfrac{\text{effective conductors in second phase}}{\text{effective conductors in first phase}}$

λ = pole pitch in inches.

If Equation (1) be integrated between limits of x corresponding to a positive loop of the wave the relation between the maximum fundamental conductor density and the total fundamental conductors per pole is obtained. If C_M be the fundamental conductors per pole the relation is:

$$\Delta C_{M\,max} = C_M \frac{\pi}{2\lambda} \tag{3}$$

Equations (1) and (2) may now be written

$$\Delta C_M = -\frac{\pi}{2\lambda} C_M \sin \frac{\pi}{\lambda} x \tag{4}$$

$$\Delta C_S = -\frac{\pi}{2\lambda} a C_M \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2}\right] \tag{5}$$

The current in each conductor of phase M will be taken as varying as the cosine of the time angle and the current in phase S will be assumed to lead that of phase M by ϕ time radians, the equations for the two primary currents are

$$i_M = I_{M\,max} \cos \omega t \tag{6}$$

$$i_S = I_{S\,max} \cos (\omega t + \phi) \tag{7}$$

The current per inch of periphery due to each phase may be obtained by multiplying the current per conductor in each phase by the conductor density of that phase.

$$\Delta i_M = -I_{M\,max} C_M \frac{\pi}{2\lambda} \sin \frac{\pi}{\lambda} x \cos \omega t \tag{8}$$

$$\Delta i_S = -I_{S\,max} a C_M \frac{\pi}{2\lambda} \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2}\right] \cos[\omega t + \phi] \tag{9}$$

Equations (8) and (9) may also be written as shown in (10) and (11) as below.

$$\Delta i_M = -I_{M\,max} C_M \frac{\pi}{2\lambda} \left\{ \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x - \omega t\right] + \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x + \omega t\right] \right\} \tag{10}$$

$$\Delta i_S = -I_{S\,max} a C_M \frac{\pi}{2\lambda} \left\{ \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right] + \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2} + \phi + \omega t\right] \right\} \tag{11}$$

In both Equation (10) and (11) the first term in the brackets represents a forward revolving field of current density and the second term a backward revolving component.

The m. m. f. at any point in the air-gap due to the forward revolving component of current of phase M is equal to the integral of the current density at that point. The integration of the first term of Equation (10) with respect to x produces

$$M\,M\,F_{Mf} = \frac{1}{4} I_{M\,max} C_M \cos\left[\frac{\pi}{\lambda} x - \omega t\right] + k \tag{12}$$

The constant of integration in Equation (12) is zero because the average potential of the stator is the same as the average potential of the rotor. In other words, there is no one-pole m. m. f.

The flux per inch of periphery due to the action of $M.\,M.\,F._{Mf}$ is equal to the product of (12) by the permeance per square inch of air-gap section and by the stacking length.

$$\Delta \Phi_{Mf} = \frac{1}{4} I_{M\,max} P\,l\,C_M \cos\left[\frac{\pi}{\lambda} x - \omega t\right] \tag{13}$$

The velocity of the flux is obtained by setting the cosine in Equation (13) equal to a constant and differentiating x with respect to t. The velocity is:

$$V_{\Phi f} = 2 f \lambda \text{ inches per second} \tag{14}$$

Since the velocity of the stator conductors is zero, the relative velocity of the flux with respect to the conductors is also (14).

The voltage per inch of periphery which is generated in the first stator phase by its forward flux is equal to minus* the product of (13) by (4) by the velocity of the flux with respect to the conductors, (14), and by 10^{-8} to convert to volts.

$$\Delta \epsilon_{Mf} = -[2 f \lambda] 10^{-8} \left\{ \frac{1}{4} I_{M\,max} P\,l\,C_M \cos\left[\frac{\pi}{\lambda} x - \omega t\right] \right\} \left\{ -C_M \frac{\pi}{2\lambda} \sin \frac{\pi}{\lambda} x \right\} \tag{15}$$

Equation (15) may also be written:

$$\Delta \epsilon_{Mf} = \frac{1}{4} \pi f P\,l\,C_M^2 I_{M\,max} 10^{-8} \left\{ \frac{1}{2} \sin\left[2 \frac{\pi}{\lambda} x - \omega t\right] + \frac{1}{2} \sin \omega t \right\} \tag{16}$$

If Equation (16) be integrated with respect to x between limits of 0 and λ, the voltage per pole is obtained:

$$\epsilon_{Mf} = \frac{1}{4} \pi f P\,l\,C_M^2 I_{M\,max} 10^{-8} \frac{\lambda}{2} \sin \omega t \tag{17}$$

The impressed voltage equivalent to (17) may be written:

*The minus sign follows from Lenz's law.

$$\underset{Mf}{E_M} = - I_{M\,max} X_m \sin \omega t \tag{18}$$

wherein

$$X_m = 2 \pi f C_M^2 \frac{1}{16} P l \lambda 10^{-8} \tag{19}$$

The quantity X_m is called the magnetizing reactance of the M phase to the forward or the backward field.

It can be shown that the magnetizing reactance for a balanced polyphase winding having m phases is m times as great as (19). Therefore, since the rotor phases are always balanced as far as either the forward or backward fields independently are concerned, and the paths for the rotor fluxes are identical with those for the stator, the magnetizing reactance per phase for a rotor having m_2 phases is,

$$X_{m\phi 2} = 2 \pi f C_2^2 m_2 \frac{1}{16} P l \lambda 10^{-8} \tag{20}$$

If, as has been assumed, the rotor has m_2 phases the equation for the conductor density of the nth rotor phase may be expressed.

$$\Delta C_{2n} = - C_2 \frac{\pi}{2 \lambda} \sin \left[\frac{\pi}{\lambda} x - \left(\frac{n-1}{m_2} \right) 2 \pi - \alpha - (1 - s) \omega t \right] \tag{21}$$

In Equation (21) α is the angle by which the initial rotor phase is displaced forward from the initial stator phase at the time $t = 0$. $(1 - s)$ is the speed of the rotor expressed as per unit of synchronous speed.

The voltage per inch of periphery which is generated in the nth rotor phase due to the forward flux of the first stator phase can be obtained in the same way as the phase M voltage, Equation (15). In this case, however, the conductors as well as the flux are moving and it is necessary to solve for the velocity of the conductors in the same way that the velocity of the flux (14) was determined and to subtract the velocity of the conductors from the velocity of the flux in order to obtain the relative velocity.

The velocity of the rotor conductors is found to be:

$$V_R = (1 - s) 2 f \lambda \tag{22}$$

The relative velocity of the flux with respect to the rotor conductors is:

$$V_{\phi f} - V_R = 2 f \lambda - (1 - s) 2 f \lambda = 2 s f \lambda \tag{23}$$

The voltage per inch of periphery which is generated in the nth rotor phase at a point x distant from the reference point is equal to minus* the product of (23) by (21) by (13) and by 10^{-8}

$$\underset{Mf}{\Delta \epsilon_{2n}} = \frac{1}{4} \pi f s P l C_M C_2 I_{M\,max} 10^{-8} \left\{ \frac{1}{2} \sin \left[2 \frac{\pi}{\lambda} x - \left(\frac{n-1}{m_2} \right) 2 \pi - \alpha - (2 - s) \omega t \right] + \frac{1}{2} \sin \left[- \left(\frac{n-1}{m_2} \right) 2 \pi - \alpha + s \omega t \right] \right\} \tag{24}$$

*See note Equation (15).

The substitution of X_m from (19) and the integration of (24) with respect to x between limits of 0 and λ produces:

$$\underset{Mf}{\epsilon_{2n}} = S \frac{C_2}{C_M} X_m I_{M\,max} \sin \left[- \left(\frac{n-1}{m_2} \right) 2 \pi - \alpha + s \omega t \right] \tag{25}$$

The differential equation for the current in the nth rotor phase is:

$$\underset{Mf}{\epsilon_{2n}} = R_{\phi 2} \underset{Mf}{i_{2n}} + L_{\phi 2} \frac{d}{d t} \underset{Mf}{i_{2n}} \tag{26}$$

wherein:

$R_{\phi 2}$ = resistance per rotor phase

$L_{\phi 2}$ = inductance per rotor phase

With a sinusoidal impressed voltage a sinusoidal current of the same frequency is produced. The equation for the current in (26) may be written:

$$\underset{Mf}{i_{2n}} = \underset{Mf}{I_{2n\,max}} \cos (\beta + s \omega t) \tag{27}$$

Employing (27) and its first derivative with respect to time, Equation (26) may be expressed

$$\underset{Mf}{\epsilon_{2n}} = \underset{Mf}{I_{2n\,max}} [R_{\phi 2} \cos(\beta + s \omega t) - L_{\phi 2} s \omega \sin(\beta + s \omega t)] \tag{28}$$

Assume the following

$$\sin \phi_f = \frac{R_{\phi 2}}{\sqrt{R_{\phi 2}^2 + (L_{\phi 2} s \omega)^2}} \tag{29}$$

$$\cos \phi_f = \frac{L_{\phi 2} S \omega}{\sqrt{R_{\phi 2}^2 + (L_{\phi 2} s \omega)^2}} \tag{30}$$

The substitution of (29) and (30) in (28) produces the following:

$$\underset{Mf}{\epsilon_{2n}} = \sqrt{R_{\phi 2}^2 + (L_{\phi 2} s \omega)^2} \underset{Mf}{I_{2n\,max}} [\sin \phi_f \cos (\beta + s \omega t) - \cos \phi_f \sin (\beta + s \omega t)] \tag{31}$$

Equation (31) may also be written:

$$\underset{Mf}{\epsilon_{2n}} = - \sqrt{R_{\phi 2}^2 + (L_{\phi 2} s \omega)^2} \underset{Mf}{I_{2n\,max}} \sin (\beta - \phi_f + s \omega t) \tag{32}$$

The product of the total inductance per rotor phase by ω is equal to the total reactance per rotor phase at fundamental frequency and since the total reactance is equal to the magnetizing reactance plus the leakage reactance.

$$L_{\phi 2} \omega = X_{m\phi 2} + X_{L\phi 2} \tag{33}$$

The substitution of (33) in (32) produces:

$$\underset{Mf}{\epsilon_{2n}} = -\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}\, \underset{Mf}{I_{2n\,max}} \sin(\beta - \phi_f + s\,\omega\,t) \tag{34}$$

Equations (34) and (25) represent the impressed voltage on the nth rotor phase. Equating the two values for this voltage the following are obtained:

$$\underset{Mf}{I_{2n\,max}} = -\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}} \tag{35}$$

$$\beta = \phi_f - \left(\frac{n-1}{m_2}\right) 2\,\pi - \alpha \tag{36}$$

The equation for the current in the nth rotor phase is obtained by the substitution of (35) and (36) in (27).

$$\underset{Mf}{i_{2n}} = -\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}} \cos\left[-\left(\frac{n-1}{m_2}\right) 2\,\pi - \alpha + \phi_f + s\,\omega\,t\right] \tag{37}$$

The current density in the nth rotor phase is equal to the product of (37) by (21)

$$\underset{Mf}{\Delta\, i_{2n}} = C_2\,\frac{\pi}{2\,\lambda}\left[\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \sin\left\{\frac{\pi}{\lambda}\,x - \frac{n-1}{m_2}\,2\,\pi - \alpha - (1-s)\,\omega\,t\right\} \cos\left\{-\frac{n-1}{m_2}\,2\,\pi - \alpha + \phi_f + s\,\omega\,t\right\} \tag{38}$$

Equation (38) may be written:

$$\underset{Mf}{\Delta\, i_{2n}} = C_2\,\frac{\pi}{2\,\lambda}\left[\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \left[\frac{1}{2}\sin\left[\frac{\pi}{\lambda}\,x - \phi_f - \omega\,t\right] + \frac{1}{2}\sin\left[\frac{\pi}{\lambda}\,x - 2\,\frac{n-1}{m_2}\,2\,\pi - 2\,\alpha + \phi_f - (1-2\,s)\,\omega\,t\right]\right] \tag{39}$$

The total rotor current density is equal to the sum of the densities for all m_2 phases. For a polyphase rotor, which is always used, Equation (39) becomes:

$$\underset{Mf}{\Delta\, i_2} = \frac{m_2}{2}\,C_2\,\frac{\pi}{2\,\lambda}\left[\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \sin\left[\frac{\pi}{\lambda}\,x - \phi_f - \omega\,t\right] \tag{40}$$

The equation for the flux per inch of periphery due to (40) is obtained as was Equation (13)

$$\underset{Mf}{\Delta\,\Phi_2} = -\frac{1}{4}\,m_2\,C_2\,P\,l\left[\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \sin\left[\frac{\pi}{\lambda}\,x - \phi_f - \omega\,t\right] \tag{41}$$

The voltage per inch of periphery generated in phase M by the flux (41) is:

$$\underset{2Mf}{\Delta\,\epsilon_M} = -(2\,\lambda\,f)\,10^{-8}\left[-C_M\,\frac{\pi}{2\,\lambda}\sin\frac{\pi}{\lambda}\,x\right] \left[\frac{1}{4}\,m_2\,C_2\,P\,l\right]\left[\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \cos\left[\frac{\pi}{\lambda}\,x - \phi_f - \omega\,t\right] \tag{42}$$

Equation (42) may be written:

$$\underset{2Mf}{\Delta\,\epsilon_M} = -\frac{1}{4}\,\pi\,f\,P\,l\,C_M\,C_2\,m_2\,10^{-8} \left[\frac{s\,\dfrac{C_2}{C_M}\,X_m\,I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2}}\right] \left[\begin{array}{l} \dfrac{1}{2}\sin\left(2\,\dfrac{\pi}{\lambda}\,x - \phi_f - \omega\,t\right) \\ \dfrac{1}{2}\sin(\phi_f + \omega\,t) \end{array}\right] \tag{43}$$

The voltage per pole is equal to the integral of (43) between limits for x of 0 and λ. After performing the integration, substituting X_m for its value and dividing the numerator and denominator of the first bracket by $s\left(\dfrac{C_2}{C_M}\right)^2 m_2$ the following may be written:

$$\underset{2Mf}{\epsilon_M} = I_{M\,max}\left[\frac{X_m^2}{\sqrt{\left[\dfrac{C_M}{C_2}\right]^4\left[\left(\dfrac{R_{\phi 2}}{s\,m_2}\right)^2 + \left(\dfrac{X_{m\phi 2} + X_{L\phi 2}}{m_2}\right)^2\right]}}\right] \sin[\phi_f + \omega\,t] \tag{44}$$

At this point three new definitions will be made:

$R_2 = \left(\dfrac{C_M}{C_2}\right)^2 \dfrac{R_{\phi 2}}{m_2}$ Phase M equivalent of rotor resistance. (45)

$X_2 = \left(\dfrac{C_M}{C_2}\right)^2 \dfrac{X_{L\phi 2}}{m_2}$ Phase M equivalent of rotor leakage reactance. (46)

$$X_m = \left(\frac{C_M}{C_2}\right)^2 \frac{X_{m\phi 2}}{m_2} \tag{47}$$

Equation (47) follows directly from (19) and (20).

Upon substitution of (45), (46), and (47) in (44) the following is obtained:

$$e_{\substack{M\\2Mf}} = -I_{M\,max}\left[\frac{X_m^2}{\sqrt{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}}\right]\sin(\phi_f+\omega t) \tag{48}$$

The sine term of (48) may be resolved into two components and the equation for the impressed voltage equivalent to the generated voltage written:

$$E_{\substack{M\\2Mf}} = I_{M\,max}\left\{\frac{X_m^2}{\sqrt{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}}\right\}$$

$$\{\sin\omega t\cos\phi_f+\cos\omega t\sin\phi_f\} \tag{49}$$

Equations (29) and (30) defined ϕ_f and it may be defined:

$$\sin\phi_f = \frac{R_{\phi 2}}{\sqrt{R_{\phi 2}^2+(L_{\phi 2}\,s\,\omega)^2}}$$

$$= \frac{\frac{R_2}{s}}{\sqrt{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}} \tag{50}$$

$$\cos\phi_f = \frac{L_{\phi 2}\,s\,\omega}{\sqrt{R_{\phi 2}^2+(L_{\phi 2}\,s\,\omega)^2}}$$

$$= \frac{X_2+X_m}{\sqrt{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}} \tag{51}$$

The values in (50) and (51) will now be substituted in (49)

$$E_{\substack{M\\2Mf}} = I_{M\,max}\left[\frac{X_m^2\frac{R_2}{s}}{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}\cos\omega t\right.$$

$$\left.+\frac{X_m^2(X_2+X_m)}{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}\sin\omega t\right] \tag{52}$$

The total primary voltage impressed upon the M phase because of the forward field of flux set up directly by the M phase and indirectly by the rotor current due to the M phase is equal to the sum of (18) and (52).

$$E_{fM} = I_{M\,max}\left[\frac{X_m^2\frac{R_2}{s}}{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}\cos\omega t\right.$$

$$\left.-\frac{X_m\left[\left(\frac{R_2}{s}\right)^2+X_2(X_2+X_m)\right]}{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}\sin\omega t\right] \tag{53}$$

Equation (53) for the M phase impressed voltage and (6) for the M phase current are of the form of Equations (28) and (27). In Equation (53), by comparison with (28), the coefficient of the cosine term must represent a resistance and the coefficient of the sine term a $2\pi f L$ term or an inductive reactance. The coefficients of the two terms of Equation (53) may therefore be defined as the apparent resistance and reactance of the M phase due to its own forward flux and the forward rotor flux which the M phase induces.

$$R_f = \frac{X_m^2\frac{R_2}{s}}{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}$$

= Apparent resistance due to the forward field. (54)

$$X_f = \frac{X_m\left[\left(\frac{R_2}{s}\right)^2+X_2(X_2+X_m)\right]}{\left(\frac{R_2}{s}\right)^2+(X_2+X_m)^2}$$

= Apparent reactance due to the forward field. (55)

By a similar line of reasoning to that just presented the total primary voltage impressed upon the M phase because of the backward field of flux set up directly by the M phase and indirectly by the rotor current due to the M phase is found to be:

$$E_{bM} = I_{M\,max}\left[\frac{X_m^2\left(\frac{R_2}{2-s}\right)}{\left(\frac{R_2}{2-s}\right)^2+(X_2+X_m)^2}\cos\omega t\right.$$

$$\left.-\frac{X_m\left[\left(\frac{R_2}{2-s}\right)^2+X_2(X_2+X_m)\right]}{\left(\frac{R_2}{2-s}\right)^2+(X_2+X_m)^2}\sin\omega t\right] \tag{56}$$

The apparent resistance and reactance of the M phase due to its own backward flux and the rotor flux which the M phase induces is therefore:

$$R_b = \frac{X_m^2\frac{R_2}{2-s}}{\left(\frac{R_2}{2-s}\right)^2+(X_2+X_m)^2}$$

= Apparent resistance due to the backward field. (57)

$$X_b = \frac{X_m\left[\left(\frac{R_2}{2-s}\right)^2+X_2(X_2+X_m)\right]}{\left(\frac{R_2}{2-s}\right)^2+(X_2+X_m)^2}$$

= Apparent reactance due to the backward field. (58)

The impressed voltages equivalent to the voltages induced in the S phase by the S produced forward and backward fluxes are:

$$E_{fS} = I_{S\,max}\, a^2 \left[R_f \cos(\omega t + \phi) - X_f \sin(\omega t + \phi)\right] \tag{59}$$

$$E_{bS} = I_{S\,max}\, a^2 \left[R_b \cos(\omega t + \phi) - X_b \sin(\omega t + \phi)\right] \tag{60}$$

The apparent impedances of the S phase due to its forward and backward fluxes are, from Equations (59) and (60):

$$R_{fS} = a^2 R_f \tag{61}$$

$$X_{fS} = a^2 X_f \tag{62}$$

$$R_{bS} = a^2 R_b \tag{63}$$

$$X_{bS} = a^2 X_b \tag{64}$$

In addition to the forward and backward field voltages self induced in the M and S phases, there are two additional voltages generated in each due to the forward and backward fluxes set up by the other. Before deriving these mutually induced voltages, advantage will be taken of equations which have been developed to simplify the flux equations.

If Equations (13) and (41) be added and the symbols for apparent impedance substituted, the equation is obtained for the total forward flux per inch of periphery directly and indirectly produced by the M phase.

$$\Delta\Phi_{fM} = \frac{\frac{1}{4} I_{M\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_f \cos\left(\frac{\pi}{\lambda} x - \omega t\right) - R_f \sin\left(\frac{\pi}{\lambda} x - \omega t\right)\right] \tag{65}$$

The corresponding backward flux is:

$$\Delta\Phi_{bM} = \frac{\frac{1}{4} I_{M\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_b \cos\left(\frac{\pi}{\lambda} x + \omega t\right) + R_b \sin\left(\frac{\pi}{\lambda} x + \omega t\right)\right] \tag{66}$$

For the S phase the following flux equations may be written:

$$\Delta\Phi_{fS} = \frac{\frac{1}{4} a I_{S\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_f \cos\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) - R_f \sin\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right)\right] \tag{67}$$

$$\Delta\Phi_{bS} = \frac{\frac{1}{4} a I_{S\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_b \cos\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} + \phi + \omega t\right) + R_b \sin\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} + \phi + \omega t\right)\right] \tag{68}$$

The voltage per inch of periphery generated in the M phase due to the S produced forward flux is equal to minus the product of (67) by (4) by (14) and by 10^{-8}

$$\Delta \underset{Sf}{\epsilon_M} = \frac{2 a I_{S\,max}}{\lambda} \sin\frac{\pi}{\lambda} x \left[X_f \cos\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) - R_f \sin\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right)\right] \tag{69}$$

Equation (66) may be written:

$$\Delta \underset{Sf}{\epsilon_M} = \frac{2 a I_{S\,max}}{\lambda} \left[\begin{aligned} & X_f \left\{ \frac{1}{2} \sin\left(2\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) - \frac{1}{2} \sin\left(\frac{\pi}{2} - \phi - \omega t\right) \right\} \\ & - R_f \left\{ -\frac{1}{2} \cos\left(2\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) + \frac{1}{2} \cos\left(\frac{\pi}{2} - \phi - \omega t\right) \right\} \end{aligned} \right] \tag{70}$$

The integral of (67) with respect to x between limits of 0 and λ is the voltage per pole.

$$\underset{Sf}{\epsilon_M} = I_{S\,max}\, a \left[-R_f \cos\left(\phi + \omega t - \frac{\pi}{2}\right) + X_f \sin\left(\phi + \omega t - \frac{\pi}{2}\right)\right] \tag{71}$$

The impressed voltage equivalent to (71) is:

$$\underset{Sf}{E_M} = I_{S\,max}\, a \left[R_f \cos\left(\phi + \omega t - \frac{\pi}{2}\right) - X_f \sin\left(\phi + \omega t - \frac{\pi}{2}\right)\right] \tag{72}$$

The impressed voltage equivalent to the M phase generated voltage due to the S produced backward flux is found to be:

$$\underset{Sb}{E_M} = I_{S\,max}\, a \left[R_b \cos\left(\phi + \omega t + \frac{\pi}{2}\right)\right.$$

$$- X_b \sin\left(\phi + \omega t + \frac{\pi}{2}\right)\Big] \qquad (73)$$

The impressed voltages equivalent to the voltages generated in the S phase by M produced fluxes are:

$$\underset{Mf}{E_S} = I_{M\,max}\, a \Big[R_f \cos\left(\omega t + \frac{\pi}{2}\right) - X_f \sin\left(\omega t + \frac{\pi}{2}\right)\Big] \qquad (74)$$

$$\underset{Mb}{E_S} = I_{M\,max}\, a \Big[R_b \cos\left(\omega t - \frac{\pi}{2}\right) - X_b \sin\left(\omega t - \frac{\pi}{2}\right)\Big] \qquad (75)$$

Thus far in the discussion no mention has been made of the primary impedance drop in either the M or the S phase. If the primary resistance and self inductive leakage reactance of the M phase are:

R_{1M} = primary resistance of the M phase (76)

X_{1M} = primary leakage reactance of the M phase (77)

and if the primary resistance and leakage reactance of the S phase are:

$a^2 R_{1S}$ = primary resistance of the S phase (78)

$a^2 X_{1S}$ = primary leakage reactance of the S phase (79)

wherein:

R_{1S} = primary resistance of the S phase if it had the same copper and distribution as it does and the same effective conductors as phase M (80)

X_{1S} = primary reactance of the S phase if it has the same distribution as it does and the same number of effective conductors as phase M (81)

the impressed voltages equivalent to the primary impedance drop in each of the two phases are:

$$E_{1M} = I_{M\,max}\,[R_{1M}\, T_{1M} \cos \omega t - X_{1M} \sin \omega t] \qquad (82)$$

$$E_{1S} = I_{S\,max}\, a^2\,[R_{1S} \cos(\omega t + \phi) - X_{1S} \sin(\omega t + \phi)] \qquad (83)$$

The total voltage impressed upon the primary of the M phase is equal to the sum of the voltages impressed:

$$E_M = E_{1M} + E_{fM} + E_{bM} + \underset{Sb}{E_M} + \underset{Sf}{E_M} \qquad (84)$$

The total voltage impressed on the S phase is:

$$E_S = E_{1S} + E_{fS} + E_{bS} + \underset{Mf}{E_S} + \underset{Mb}{E_S} \qquad (85)$$

The solution of Equations (84) and (85) for the currents results in the equations for the primary currents per conductor in each phase of an unbalanced two-phase motor.

Since the solution of (84) and (85) can be carried out more conveniently if the equations be expressed in vector form, steps will be taken so to express them.

If the impressed voltages and primary currents per conductor be defined in vector notation as follows:

$\bar{E}_M$ = vector voltage impressed upon the primary of phase M (86)

$\bar{E}_S$ = vector voltage impressed upon the primary of phase S (87)

$\bar{I}_M$ = vector current per conductor in the primary of phase M (88)

$\bar{I}_S$ = vector current per conductor in the primary of phase S (89)

the following vector equations may be written for the various components of (84) and (85)

$$\bar{E}_{1M} = \bar{I}_M\,(R_{1M} + j\,X_{1M}) \qquad (90)$$

$$\bar{E}_{fM} = \bar{I}_M\,(R_f + j\,X_f) \qquad (91)$$

$$\bar{E}_{bM} = \bar{I}_M\,(R_b + j\,X_b) \qquad (92)$$

$$\underset{Sf}{\bar{E}_M} = \bar{I}_S\, a\,(X_f - j\,R_f) \qquad (93)$$

$$\underset{Sb}{\bar{E}_M} = \bar{I}_S\, a\,(-X_b + j\,R_b) \qquad (94)$$

$$\bar{E}_{1S} = \bar{I}_S\, a^2\,(R_{1S} + j\,X_{1S}) \qquad (95)$$

$$\bar{E}_{fS} = \bar{I}_S\, a^2\,(R_f + j\,X_f) \qquad (96)$$

$$\bar{E}_{bS} = \bar{I}_S\, a^2\,(R_b + j\,X_b) \qquad (97)$$

$$\underset{Mf}{\bar{E}_S} = \bar{I}_M\, a\,(-X_f + j\,R_f) \qquad (98)$$

$$\underset{Mb}{\bar{E}_S} = \bar{I}_M\, a\,(X_b - j\,R_b) \qquad (99)$$

The vector expressions for (84) and (85) are, from (90) to (99) inclusive:

$$\bar{E}_M = \left\{ \begin{array}{l} \bar{I}_M\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] \\ \\ + I_S\, a\,[(X_f - X_b) - j\,(R_f - R_b)] \end{array} \right\} \qquad (100)$$

$$\bar{E}_S = \left\{ \begin{array}{l} \bar{I}_S\, a^2\,[(R_{1S} + R_f + R_b) + j\,(X_{1S} + X_f + X_b)] \\ \\ - \bar{I}_M\, a\,[(X_f - X_b) - j\,(R_f - R_b)] \end{array} \right\} \qquad (101)$$

The equations for $\bar{I}_M$ and $\bar{I}_S$ are:

$$\bar{I}_M = \frac{\bar{E}_M\, a^2\,[(R_{1S} + R_f + R_b) + j\,(X_{1S} + X_f + X_b)] + j\,\bar{E}_S\, a\,[(R_f - R_b) + j\,(X_f - X_b)]}{a^2[(R_{1S}+R_f+R_b)+j(X_{1S}+X_f+X_b)]\,[(R_{1M}+R_f+R_b) + j\,(X_{1M} + X_f + X_b)] - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \qquad (102)$$

$$\bar{I}_S = \frac{\bar{E}_S\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,\bar{E}_M\, a\,[(R_f - R_b) + j\,(X_f - X_b)]}{a^2[(R_{1S}+R_f+R_b)+j(X_{1S}+X_f+X_b)]\,[(R_{1M}+R_f+R_b) + j\,(X_{1M} + X_f + X_b)] - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \qquad (103)$$

Development of Torque and Output Equations.

The force exerted upon a conductor carrying a current across a magnetic field is in a direction perpendicular to both the current and the field and has a value expressed in absolute units of:

$$F_a = B_a\, l_a\, I_a \text{ dynes} \qquad (104)$$

B_a = flux density in lines per sq. cm.

l_a = length of conductor perpendicular to the flux, in cm.

I_a = current in the conductor in abamperes.

In the analysis of a-c. motors it is convenient to express the force or torque of a motor in terms of "Synchronous Watts."

Definition of Synchronous Watts.

A synchronous watt of force or torque is that force or torque which applied at synchronous speed would deliver mechanical energy at the rate of one watt.

The product of the force F_{os} by the synchronous speed in cm. per second, $2 f \lambda_a$, is the power in dynes per cm. per second (ergs per second).

$$P_a = 2 B_a l_a I_a \lambda_a f \text{ ergs per second} \tag{105}$$

In order to express the torque as the power in synchronous watts it is necessary to multiply (105) by 10^{-7}. The performance of this operation together with the substitution of engineering units (inches and amperes) gives the equation:

$$T = 2 B l \lambda f i 10^{-8} \text{ synchronous watts} \tag{106}$$

General Equation for Torque.

The general equation for the torque per inch of periphery may be expressed by substituting $\Delta \Phi$ for $B l$ and Δi for i.

$$\Delta T = 2 \lambda f \Delta \Phi \Delta i 10^{-8} \text{ synchronous watts per in.} \tag{107}$$

The total flux per inch of periphery is equal to the sum of the fluxes produced by both the M and S phase and the total primary current per inch of periphery is equal to the sum of the primary currents of each of the phases.

The equations for the torque per inch of periphery of an unbalanced two-phase motor is:

$$\begin{aligned}\Delta T &= [\Delta \Phi_{fM} + \Delta \Phi_{bM} + \Delta \Phi_{fS} + \Delta \Phi_{bS}][\Delta i_M + \Delta i_S] 2 \lambda f 10^{-8} \\ &= [(65)+(66)+(67)+(68)]\,[(10)+(11)]\, 2 \lambda f 10^{-8}\end{aligned} \tag{108}$$

The integral of (108) with respect to x between limits of 0 and λ produces the general equation for the torque per pole produced by an unbalanced two-phase motor.

General Equation for the Torque of an Unbalanced Two-Phase Motor.

$$T = \left[\begin{aligned} & I_M^2 [(R_f - R_b) + (R_f - R_b) \cos 2 \omega t \\ & \quad - (X_f - X_b) \sin 2 \omega t] \\ & + I_S^2 a^2 [(R_f - R_b) + (R_f - R_b) \cos 2 (\phi + \omega t) \\ & \quad - (X_f - X_b) \sin 2 (\phi + \omega t)] \\ & + 2 I_M I_S a [R_f + R_b] \sin \phi \end{aligned}\right] \tag{109}$$

I_M = effective current per conductor in the M phase.

I_S = effective current per conductor in the S phase.

It will be noticed that some of the torque components are constant while others vary sinusoidally with the time at twice line frequency. Over a period of time corresponding to any number of complete cycles of the line voltage the alternating torques integrate to zero and the average torque is:

$$T_{avg} = [I_M^2 + a^2 I_S^2]\,[R_f - R_b] + 2 I_M I_S a\,[R_f + R_b] \sin \phi \tag{110}$$

The maximum value of the alternating torque is:

$$T_{\sim Max} = \sqrt{[I_M^4 + a^4 I_S^4 + 2 I_M^2 I_S^2 a^2 \cos 2 \phi][(R_f - R_b)^2 + (X_f - X_b)^2]} \tag{111}$$

Output Equation for Unbalanced Two-Phase Motor.

Since the torque is expressed in synchronous watts the output in watts is the product of the average torque by the speed in per unit of synchronous speed minus the friction loss.

$$\begin{aligned} W.O. = \{ & [I_M^2 + a^2 I_S^2]\,[R_f - R_b] \\ & + 2 I_M I_S a\,[R_f + R_b] \sin \phi \}\,(1 - s) - W_f \end{aligned} \tag{112}$$

Input of Unbalanced Two-Phase Motor.

The input can be calculated as the sum of the products of the voltage impressed on each phase by the in-phase current plus the iron loss.

Single-Phase and Balanced Two-Phase Equations.

It is of interest to notice how the current and torque equations behave under the special conditions of single phase and balanced two-phase operation.

When the following assumptions are made:

$$\begin{aligned} E_S &= +j E_M \\ a &= 1 \\ R_{1S} &= R_{1M} \\ X_{1S} &= X_{1M} \end{aligned} \tag{113}$$

Equations (102) and (103) reduce to:

$$\bar{I}_M = \frac{\bar{E}_M}{(R_{1M} + 2 R_f) + j (X_{1M} + 2 X_f)} \tag{114}$$

$$\bar{I}_S = \frac{\bar{E}_S}{(R_{1S} + 2 R_f) + j (X_{1S} + 2 X_f)} \tag{115}$$

which are the current equations for a balanced two-phase motor.

If R_{1S} be made very large (phase S open circuited) the current $\bar{I}_S$ becomes zero and the current $\bar{I}_M$ becomes:

$$\bar{I}_M = \frac{\bar{E}_M}{(R_{1M} + R_f + R_b) + j (X_{1M} + X_f + X_b)} \tag{116}$$

which is the current equation for a single-phase induction motor.

If the assumptions of (113) and currents obtained therefrom be applied to (109) the torque equation becomes:

$$T = 2 I_M^2 (2 R_f) \tag{117}$$

which contains no alternating torque components and is the torque equation of a balanced two-phase motor.

If I_S be made zero the torque equation becomes:

$$T = I_M^2 [(R_f - R_b) + (R_f - R_b) \cos 2 \omega t - (X_f - X_b) \sin 2 \omega t] \tag{118}$$

which is the torque equation of a single-phase induction motor wherein the average torque is:

$$T_{avg} = I_M^2 (R_f - R_b) \tag{119}$$

and the maximum value of the alternating torque is:

$$T_{\underset{Max}{\sim}} = I_M^2 \sqrt{(R_f - R_b)^2 + (X_f - X_b)^2} \tag{120}$$

Equation (118) is particularly interesting in that it shows that one component of the alternating torque is proportional to the average or useful torque of the motor and, therefore, is not controllable by design. The other component is greatest at light loads and is subject to some control by design though an attempt at such control would probably be too costly for practical purposes.

Capacitor Motor Equations.

The equations for current and torque which have been developed for the unbalanced two-phase motor apply directly to a capacitor motor. Since the effect of the primary impedance is indistinguishable from external series impedance, the effect of a capacitor in series with the S phase can be expressed by substituting for $a^2 R_{1S}$ and $a^2 X_{1S}$ the quantities $a^2 R_{1S} + R_c$ and $a^2 X_{1S} + X_c$ wherein:

$R_c + j X_c$ = vector impedance of the capacitor unit (121)

If the above substitution be made and $\bar{E}_M$ be substituted for $\bar{E}_S$, since both phases are connected across the same line, the current equations for a capacitor motor are:

$$\bar{I}_M = \bar{E}_M \left\{ \frac{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)] + j\,a\,[(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)]\}\{(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)\} - a^2 [(R_f - R_b) + j\,(X_f - X_b)]^2} \right\} \tag{122}$$

$$\bar{I}_S = \bar{E}_M \left\{ \frac{[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,a\,[(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)]\}\{[R_{1M} + R_f + R_b] + j\,(X_{1M} + X_f + X_b)\} - a^2 [(R_f - R_b) + j\,(X_f - X_b)]^2} \right\} \tag{123}$$

$$\begin{aligned} \bar{I} &= \bar{I}_M + \bar{I}_S \\ &= \bar{E}_M \left\{ \frac{[R_c + a^2 (R_{1S} + R_f + R_b) + R_{1M} + R_f + R_b] + j[X_c + a^2 (X_{1S} + X_f + X_b) + X_{1M} + X_f + X_b]}{\{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)]\}\{(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)\} - a^2 [(R_f - R_b) + j\,(X_f - X_b)]^2} \right\} \end{aligned} \tag{124}$$

The torque Equation, (109), is unaffected by the introduction of the capacitor and applies directly to a capacitor motor.

Nomenclature

The nomenclature has been systematically arranged by means of subscripts in such a way that the meaning of each symbol is apparent from its subscripts. For example, in Equation (17) of the appendix the symbol $\epsilon_{\underset{Mf}{M}}$ appears. The subscript M indicates that the voltage is generated in the M phase and the underscript Mf indicates that it is due to the forward flux produced by the M phase.

A = "In phase" component of M phase current

a = Ratio $\dfrac{\text{Fundamental conductors on } S \text{ phase}}{\text{Fundamental conductors on } M \text{ phase}}$

$a^2 R_{1S}$ = Primary resistance of S phase

$a^2 X_{1S}$ = Primary leakage reactance of S phase

B = "Reactive" component of M phase current

C = "In phase" component of total current

$\bar{E}_M$ = Vector voltage impressed on primary of M phase

$\bar{E}_S$ = Vector voltage impressed on primary of S phase

F = "Reactive" component of total current

$\mathbf{h}$ = "Reactive" component of S phase current

$\bar{I}$ = Vector total current

I = Effective value of total current

$\bar{I}_M$ = Vector M phase current

I_M = Effective value of M phase current

$\bar{I}_S$ = Vector S phase current

I_S = Effective value of S phase current

$\mathbf{g}$ = In-phase component of S phase current

R_{1M} = Primary resistance of M phase

R_2 = Secondary resistance reduced to M phase

R_c = External resistance in series with S phase

R_b = Apparent resistance to M phase backward field

R_f = Apparent resistance to M phase forward field

T_{avg} = Average value of developed torque

T_{SS} = Standstill torque

$T_{\underset{max}{\sim}}$ = Maximum value of alternating torque

W_f = Watts friction and windage loss

W_i = Watts fundamental iron loss

$W.I.$ = Total watts input

$W.O.$ = Net watts output

X_{1M} = Primary leakage reactance of M phase

X_2 = Secondary leakage reactance reduced to M phase

X_e = External reactance in series with S phase
X_b = Apparent reactance to M phase backward field
X_f = Apparent reactance to M phase forward field
X_m = Magnetizing reactance of M phase
ϕ = Angle by which $\bar{I}_S$ leads $\bar{I}_M$

The following Nomenclature refers to the Appendix:

C_M = M phase fundamental conductors per pole
C_S = S phase fundamental conductors per pole
C_2 = Fundamental conductors per rotor phase per pole
ϵ = Generated voltage
E = Impressed voltage
i_M = Instantaneous value of M phase current
i_S = Instantaneous value of S phase current
l = Axial length of air gap section
$L_{\phi 2}$ = Total inductance per rotor phase
m_2 = Number of rotor phases
n = An arbitrary number denoting a particular phase
P = Permeance per square inch of air gap section
$R_{\phi 2}$ = Resistance per rotor phase
s = Slip in per unit of synchronism
t = Time in seconds
x = Distance in inches measured around periphery of gap
Φ = Flux
ϕ = Subscript denoting "per phase" quantities
ϕ = Angle by which $\bar{I}_S$ leads $\bar{I}_M$ in time
ϕ_b = Compare with Equations (50) and (51)
ϕ_f = See Equations (50) and (51)
λ = Pole pitch in inches
α = See Equation (21)
Δ = A prefix indicating per inch of periphery.

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Discussion

THE CONDENSER MOTOR

(Bailey)

THE FUNDAMENTAL THEORY OF THE CAPACITOR MOTOR

(Specht)

THE REVOLVING FIELD THEORY OF THE CAPACITOR MOTOR

(Morrill)

New York, N. Y., January 31, 1929

C. R. Boothby: Professor Bailey in his paper has discussed the starting and running performance of capacitor motors having an equal weight of wire in both phases. Such a design will require an expensive capacitor and the maximum torque will in general be low.

With these considerations in mind it will be found that if a greater percentage of the total copper is used in the main phase and a small percentage in the capacitor phase a more economical design will be obtained. I have in a manner similar to Professor Bailey's developed for such a winding the equations for starting torque, maximum starting torque, and capacitor required to give maximum starting torque. These equations reduce to the same form as given by Professor Bailey when the assumption is made that the winding resistance and reactance varies with the square of the number of turns. These equations were developed from the fundamental equations as used by Mr. Specht in his paper. The same notation is also used.

$$T = 2.66 \times P \times 10^{-8} \left[\Phi_c I_1 T_1 \frac{K_1}{K_2} \cos \psi_1 + \Phi_1 I_{1c} T_{1c} \frac{K_{1c}}{K_2} \cos \psi_2 \right] \text{ oz. ft.}$$

For cage rotor $K_2 = 0.636$

$$\psi_1 = \varphi_2 + (\varphi_1 + \varphi_{1c} - 90) \quad \psi_2 = \varphi_2 - (\varphi_1 + \varphi_{1c} - 90)$$

$$T = \frac{1.88\, P\, I_1\, I_{1c}\, K\, r_2 \,.\, \text{Sin}\, (\varphi_1 + \varphi_{1c})}{f}$$

Where r_2 = Main phase rotor resistance in terms of the primary.
K = Ratio of effective turns in capacitor phase to effective turns in main phase.

Making substitutions for I_1 and I_{1c} and Sin. $(\varphi_1 + \varphi_{1c})$

$$T = \frac{1.88\, P\, E^2\, K\, r^2}{f} \times \frac{R\, c\, X - R\, (X\, c - C)}{[R^2 + X^2]\,[R\, c^2 + (X\, c - C)^2]}$$

Where E = Line voltage
R and X = Total resistance and reactance of main winding
Z = Impedance of main winding
$R\, c$ = Total resistance in capacitor phase including capacitor
$X\, c$ = Total reactance of the capacitor winding only
C = Reactance of capacitor in ohms

Differentiating the above equation with respect to C setting equal to zero and solving for C we obtain the value of C to give maximum torque OR

$$C_{max}\, T. = Xc - \frac{R\, c}{R}\, (X \pm Z)$$

For the capacitor motor the negative sign should be used and the equation becomes

$$C_{max}\, T. = X\, c + \frac{R\, c}{R}\, (Z - X)$$

Substituting the value of C for maximum torque in the previous equation for torque we obtain the maximum starting torque.

$$T_{max} = \frac{0.944\, P\, E^2}{f} \times \frac{K\, r_2\, (Z + X)}{R\, c\, Z^2}$$

The maximum starting torque is found to be directly proportional to the ratio of the windings and the secondary resistance and inversely proportional to the total resistance in the capacitor phase circuit also a function of the main winding impedance.

The fact that the secondary resistance appears directly in the maximum starting torque equation suggests that this torque may possibly be increased provided the secondary resistance is a small part of the total winding impedance.

From this equation we see that the maximum torque is inversely proportional to the total resistance of the capacitor phase circuits. When a capacitor with a transformer is used it is therefore of importance to keep the total losses in the transformer low. In case of extremely high starting-torque requirements this may fix the size of the transformer required rather than the full load condition. In order to keep the effective resistance

of the capacitor low, it is an advantage to design the transformer with a small number of turns and high flux density as generally the copper loss will be reduced faster than the iron loss is increased. If at normal voltage the flux density is just below the knee of the saturation curve then we may have a condition that when the applied voltage on the motor is increased the starting torque will be decreased. This is due to transformer magnetizing reactance balancing part of the condenser reactance resulting in a reduction in effective microfarads in the capacitor.

In comparing the performance of a standard two-phase motor when operated from a two-phase supply with the performance of the same motor when operated with a capacitor from a single-phase supply it should be noted that only when the power factor of the two-phase motor is 71 per cent can we obtain the two-phase condition by means of a condenser. At other power factors we must be satisfied with either an unbalance in phase current or an unbalance in watts. This may become an important consideration with slow-speed motors having an inherently low power factor.

P. L. Alger: For some years past the utility companies have been more and more insistently demanding lower starting currents for single-phase motors on house-lighting circuits. Lately the manufacturers of domestic appliances, especially refrigerators, have been asking for higher starting torques on their motors. At the same time, the housewife has been insisting on quieter and quieter motors. And recently all of us have with one voice demanded that the motors shall not interfere with radio reception. Finally, the utility companies have come forward and set standards of performance that must be lived up to.

Under all this pressure of demands, the manufacturers have studied many different ways of improving their machines.

The simple resistance split-phase motor cannot possibly meet the torque per ampere now demanded of high-quality motors. The repulsion-start commutator motor is all right there, but it has brushes which make noise and give radio interference, and may require servicing. To make such a motor fully satisfactory, it must be spring-mounted to reduce its vibration, enclosed to reduce brush noise, and provided with a small capacitor to remove radio interference.

For all these difficulties, the capacitor motor appears the solution. By proper design, and by the use of a sufficient capacity, any reasonable starting characteristics, maximum output, and power factor can be secured. Also, the motor vibration is greatly reduced, radio interference is eliminated, and servicing, except of bearings, becomes unnecessary.

The three papers here presented show that, differ as we may in our points of view, the manufacturers now agree that all important characteristics of the capacitor motor can be predicted with reasonable accuracy, whatever the motor speed, and whatever the degree of unbalance.

Although the present high cost of capacitors will limit the application of the new motor for a time, it will undoubtedly find a field of usefulness wherever the value of superior performance is appreciated. I believe, therefore, that these papers mark the opening of a new era in single-phase motor development.

H. C. Louis: As an operating man, I wish to express appreciation of the development of the capacitor motor which has some very decided advantages. It is desired, however, to call attention to some of its limitations. It has many of the advantages claimed for it, but in most of these papers, and in many of the claims, it has usually been compared with split-phase motors.

For example, on the question of starting current, Mr. Morrill mentioned that it has only 30 to 50 per cent more starting current. This difference may be great enough in some cases to make it a bad application.

We had a particular case where a motor was claimed to be noisy by the customer, and we tried to replace it with a capacitor motor. Just this difference of 30 to 50 per cent made it impossible to use it in this particular location because it would have blown fuses.

I should like to ask if it is really an inherent or a necessary part of the design to have 30 or 50 per cent more starting current. In other words, if you compare this with a commutator motor where the brushes remain on the commutator, you will find it does not show up as well, as these have very good characteristics.

The question of condenser life is also raised. Trouble has been experienced with condenser failure. Can these be built to give satisfactory life without excessive cost?

A. Nyman: There is a large degree of misunderstanding on the rating of condensers. Particular condensers suitable for this work would be the paper or the electrolytic condenser.

The electrolytic condensers are of the liquid or the dry type, and the dry type is just beginning to be produced on a scientific basis. The electrolytic condenser has the advantage of very large capacity, but it has a disadvantage at the present time of very high losses. It is very suitable for filtering service on low voltages where there are only small a-c. components and their losses are important. For capacitor motors I do not believe the electrolytic is suitable.

The paper condenser resembles cables with regard to one feature, and that is the life. It is known from cable practise and it has been established with the condenser of the paper insulation type that the life is very much dependent on voltage. The life of a paper condenser varies as an exponential function of the voltage supplied to it.

If you rate the condenser conservatively, the life may be approximately eight years or ten years. If you double the voltage the life will be reduced considerably.

Now, on some capacitor motors the voltage on the condenser is doubled during starting. Therefore, in figuring out a condenser suitable for a capacitor motor you have to take into account the reduced life during the starting period and subtract that part of the reduction of life from the final life of a condenser as determined by the operating voltage.

The only consideration really necessary for a suitable condenser for capacitor motors is that its rating is conservative. A conservatively rated paper condenser will give long service for a capacitor motor.

E. G. Michelsen and L. F. Hemphill: The reversing characteristics of capacitor motors are discussed in Professor Bailey's paper and a test curve is included showing positive torques at slips greater than unity indicating the reversibility of the motor. This invites a discussion as to just what conditions are necessary to produce a positive torque curve.

It is self-evident that a capacitor motor is not necessarily reversible since the value of the condensive reactance used in series with the starting winding may be increased and as this value approaches infinity, the performance of the motor approaches that of a straight single-phase motor. Reversibility is therefore a function of the reactance employed in series with the starting winding.

Fig. 1 herewith shows two complete torque curves over the range of plus to minus synchronism. Curve No. 1 is for a motor using a relatively high condensive reactance wherein the torque characteristic similates that of a single-phase motor and is not reversible except at very low speeds, the effect of the load torque being neglected. Curve No. 2 is the result of employing a considerably lower value of condensive reactance in conjunction with the same motor and shows the motor to be completely reversible at all speeds independently of load torques. The trend of the curves for intermediate values of external reactance may be readily visualized.

The variation of the reversing torque with respect to the value of external reactance suggests a study of this effect at the speed at which it is desired to reverse the motor. The speed corresponding to a slip of 195 per cent has been chosen for this purpose, it being assumed that the motor is to be reversed at full

load speed. Fig. 2 herewith shows the torque curve as a function of the external reactance at this speed. The curve has a resonant characteristic, the maximum torque occurring at a value of condensive reactance somewhat higher than that required to render the starting circuit resonant. The range of reversibility extends from an inductive reactance of 22 ohms to a condensive reactance of 42 ohms for the motor considered. Values of

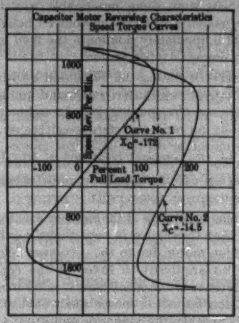

Fig. 1—Calculated for ¼-Hp. Capacitor Motor

Slip varied from "0" to "2"

constants

R_{1M} = 2.02	X_{1M} = 2.79	X_M = 33.4
R_2 = 2.06	X_2 = 1.06	a = 1.18
R_{1s} = 5.12	X_{1s} = 2.31	
R_c = 3	X_c = 14.5.–172	

reactance, either condensive or inductive, in excess of those mentioned result in negative torques and the motor will not reverse if the load torque and friction are neglected but will continue to operate in the same direction but with impaired characteristics. The addition of load and friction torques will, of course, increase the range of reversibility.

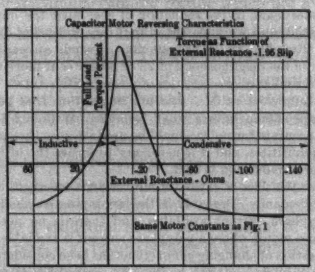

Fig. 2

Certain other factors will also enter to affect the reversing torque such as secondary resistance, the ratio of turns on the two quadrature windings, and the relative weights of copper in the two windings. The effects of these quantities are relatively unimportant compared to the effect of the external reactance and the scope of this discussion is therefore confined to this latter quantity.

The curves shown are based entirely on calculated values, the methods embodied in Mr. Morrill's paper having been employed. Past experience with the use of this method by the writers has led to very accurate results and we have every confidence in the correctness of the values herein given despite the fact that no confirming tests have been made.

We may conclude from the above that the capacitor motor is not inherently reversible but that its characteristics in this regard are dependent upon the design of the motor, the value of external reactance used during the reversing process being the predominant factor. The values of reactance normally used during the starting period with motors designed for heavy starting duty fall in the range of reversibility. The same is not true for motors designed for light starting duty as such motors are very likely to fall outside of the reversible range.

A. G. Oehler: Assuming that the demand for the capacitor motor is proportional to that of the more common types of single-phase motors now in service, what increase in cost can we reasonably expect for this type of motor? Also, to what range of sizes is such a motor applicable?

W. I. Slichter: The motor has had to wait for the development of a reliable capacitor of reasonable bulk and cost. Radio has given much material stimulation to the building of condensers and I have noticed that we are getting much better condensers for our laboratories than used to be the case. If the condenser can be relied upon this new motor will supersede the commutator single-phase motor which is now necessary when much starting torque is required of a single-phase motor, since the brush-lifting and commutator-short-circuiting mechanism is liable to mechanical troubles which cause it to fail occasionally, particularly in those cases where there is no experienced help in attendance.

I believe that the time has come when the capacitor is more reliable than the brush-lifting mechanism.

H. C. Specht: Professor Bailey certainly deserves much credit for presenting a paper that gives a picture of various performance by test with a capacitor motor. Unfortunately, however, all tests and conclusions are based on motors with equal amount of copper in both phases and the same distribution and winding turns of 1:1 and 2:1. This, however, is not the most economical design for the majority of applications.

In making a comparison of starting amperes with a two-phase motor the author apparently adds the currents of a two-phase motor numerically. This, however, does not seem to be fair. It would have been more correct to base such comparison on the amount of copper in the line which is required to give the same line drop. Also the comparison of power factor under running conditions with a two-phase motor is not fair for the same reason. Such a statement might give the impression that the capacitor motor is superior to a two-phase motor. It should be borne in mind, moreover, that the capacitor motor has less pull-out torque and is much higher in cost than a two-phase motor. Consequently there would be no excuse to recommend a capacitor motor in a case where a two-phase circuit is available. In Fig. 2 is shown a reversing switch. This simple arrangement is possible only if both stator windings are identical.

It is stated in the paper that the capacitor voltage reaches its maximum value at resonance. This happens sooner because with the increase in capacity the capacitance in ohms decreases in the inverse ratio and the current increases more slowly.

In Fig. 17 the speed curves of the motor with 311 and 252 μf. cross over at a certain speed and at higher speeds in both directions of rotation the smaller capacity gives more torque. This seems to be due to the great unbalance or the circulating current. I should appreciate it if the author will give his explanation of this peculiar characteristic.

P. H. Rutherford: The cost of the capacitor motor is rather hard to determine because of the fact that there is not a large production at the present time. If the public demands it and there is a large production, then there is a possibility

of making them at somewhere near the cost of the repulsion type of motor.

I don't believe anybody has spoken about the pull-up torque. Really the pull-up torque in refrigeration work is more important than the starting torque. The capacitor motor has a great deal better pull-up torque than the repulsion-induction motor.

B. W. Jones: Assuming that more capacity is used for the starting condition than is used for the running, it follows that a switching operation is necessary. The first question is, is it best to control this switching operation as a function of speed, or as a function of time? Also, assuming that this switching operation is made before the speed has reached the required value, what will be the shape of the speed-torque curve?

P. S. Creager: I have an impression that the field for this motor will be in the small sizes. Here the matter of cost will be important. It seems agreed that the cost of this type will be considerably higher for some time to come than the cost of competitive types. Hence, it would seem that the motor can be sold only on one or both of two bases, (a) freedom from radio interference, a very important matter in many communities, (b) quietness of operation.

Second, the question of starting demand has been mentioned several times. If, as stated above, the field for this motor is in the small sizes, is this question of starting current really important?

This leads me to ask another question somewhat apart from the subject of these papers but suggested by the discussion. Are not the limits of starting demand, etc., as specified by the utilities, too severe? The larger companies seem to be going to combined light and power primaries. In general, that means increased capacity per feeder. It would seem then that a larger starting demand could be handled. In that event, the increased starting demand of the capacitor motor would not appear to be a detriment to its use. Our laboratory is supplied from one of these feeders. So far 50-kv-a. swings have not, to my knowledge, produced complaints.

A more liberal policy in this matter would have large economic significance, to the power company, to the manufacturer, and to the user of electrical energy. This fact alone demands the most careful consideration of the general policy.

B. F. Bailey: The principal question raised in the discussion is that of the cost of the capacitor motor, compared with the cost of other types of single-phase motors. The motor itself is cheap; most of the added cost is in the condenser. The capacitance must be large if considerable starting torque is to be developed. The capacitance needed for running is comparatively small.

As pointed out by Mr. Nyman, electrolytic condensers are cheap, but there is a considerable loss of power in them. By suitable design, these losses have been greatly reduced. They are still too great to allow the use of an electrolytic condenser during normal running of the motor. They are, however, entirely suitable for use during starting. They are built completely sealed, so that they are as dry as a so-called dry cell. In actual service they give less trouble than paper condensers.

Reliability of paper condensers is a matter of manufacturing methods and the use of sufficient dielectric material. As the demand grows the cost will doubtless become less and their reliability will become greater. This is what has happened in the case of the condensers used in automobile ignition.

Another phase of the cost question should not be overlooked. If we consider the *entire* cost of a motor installation, the condenser motor is probably cheaper than the usual repulsion-induction motor. The entire cost includes the transmission lines, transformers, generators, switches, and local wiring. Each of these elements costs less for condenser motors. I have figures from a source in which I had great confidence, indicating that the central station can save from $10 to $20 per motor if condenser motors are used instead of the usual type. These figures are based upon refrigerator motors of the usual characteristics. This saving will more than make up the difference in price between the two motors. The difficulty is that this saving does not directly affect the customer although undoubtedly in the long run he will benefit by it, and yet he must pay the bill in the first instance. If there were some practicable way in which this saving could be passed over to the customer I believe the capacitor motor would immediately dominate the field in many applications, notably domestic refrigeration.

H. C. Specht: In closing the discussion I wish to say a few words to the points of radio interference and quietness in operation which Professor Creager has just brought out. I believe there is practically no radio interference either on the repulsion start induction motor or on the capacitor motor. Only at the start the repulsion induction motor may cause slight radio interference and so will the capacitor motor when the transfer switch operates.

As far as the magnetic noise of the capacitor motor is concerned it may be said that this motor is very slightly quieter than the repulsion induction motor due to the revolving field. However, this should be no argument in favor of the capacitor motor because the difference is too small and it is no more difficult to make the repulsion induction motor quiet enough for domestic service than it is to make the capacitor motor quiet.

The two real good features which speak most favorably for the capacitor motor are the good pull-up torque and the improved power factor and efficiency.

W. J. Morrill: One question has been asked concerning the starting current of the capacitor motor and another concerning the cost. These two questions are interrelated, because the starting-current limit determines the size and the cost of the capacitor. If the power companies decide that certain current limits must prevail, and the device manufacturer needs a certain amount of torque, then the motor manufacturer necessarily has to apply a capacitor which meets those two specifications.

It is difficult to say what will be the ultimate cost of the capacitor motor compared with the cost of its competitive motors, because, as Mr. Rutherford has stated, we have not had sufficient production experience on capacitor motors. There have been great strides made in the reduction of cost since the motor was first initiated, and we hope that the future will bring forth still further. Roughly, at the present time, it would appear the cost should be in the neighborhood of 25 or 30 per cent more than for a motor of similar characteristics of a competitive type.

I was interested in Mr. Specht's suggestion that perhaps a small motor should have a small formula. With small motors the most accurate formulas should be used because they are built in large quantitites, and it is necessary to secure the ultimate in the matter of reduction in cost.

As far as the sizes of capacitor motors are concerned, that is a matter of demand. Probably in the larger sizes we shall always have polyphase current available, and there is no need for a single-phase motor where a polyphase current is available.

The question of changing connections has been considered from a number of angles. It can be done by means of current variation and by means of speed, but it seems the most fundamental change is that by means of speed because the torque characteristic in reference to speed is always the same. Other means can be used for special reasons, but in the usual case, the speed method is the best method.

Line-Start Induction Motors

BY C. J. KOCH*
Associate, A. I. E. E.

Synopsis.—This paper considers the common types of polyphase induction motors suitable for starting at full voltage. The running and starting characteristics of these types are considered as well as the influence of the permissible starting current values upon these characteristics. No attempt is made to describe or consider special types designed to obtain low starting currents by the use of wound rotors, centrifugal switches, or other moving parts.

Importance of Line Start Motors

MANY manufacturers of polyphase squirrel-cage induction motors in the integral horsepower sizes have now added to their other products, a modified form of squirrel-cage motor in which the starting current is reduced below that of the standard squirrel-cage motor. The reduction in starting current is sufficient to warrant the power companies throughout this country to permit the starting of these motors directly across the line without the use of a starting compensator. This type of motor is now sold under a variety of trade names, such as line-start motors, auto-start motors, double-deck motors, double squirrel-cage motors, high-reactance motors, etc.

Because the use of this motor makes unnecessary the purchase and maintenance of a starting compensator, the popularity of the type is assured. The extent to which the standard squirrel-cage motor is already superseded indicates the popularity which the line-start motor will have in the future.

It will be the purpose of this paper to examine the running and starting characteristics of the line-start motor and to compare these characteristics with those of the standard squirrel-cage motor. Power factor and efficiency will be considered as well as the effect of the reduction in starting current on the starting torque and the maximum running torque which are available.

High-Resistance and High-Reactance Motors Relation of Maximum Torque to Starting Current

Line-start motors are of two fundamental types, differing in the manner in which the starting current is reduced below that of the standard squirrel-cage motor. These types may properly be called the high-reactance type and the high-resistance type.

The high-resistance line-start motor may be considered as derived from the standard squirrel-cage motor by a reduction in the total amount of magnetic flux per pole, *i. e.*, by an increase in the number of turns per coil in the stator winding. The starting current will vary inversely as the square of the number of turns per stator coil. At the same time the starting torque and the maximum running torque will be reduced in the same proportion. The starting torque may be increased again to any desired value within limits by decreasing the size of the rotor bars but this leaves the amount of maximum torque unaffected since in an induction motor the maximum torque is independent of the rotor resistance, the latter serving only to determine the slip of the motor at which maximum torque occurs. In the high-resistance type of motor, therefore, the maximum torque and the starting current are quite closely proportional, any reduction in starting current resulting in a corresponding reduction in maximum torque.

In the design of a high-reactance line-start motor, the starting current is limited by increasing the leakage reactance of the motor. This is usually done by providing for each rotor bar a leakage path of comparatively low reluctance. If both the rotor and stator resistance were zero the starting current would be exactly inversely proportional to the leakage reactance. Actually the starting current will not decrease as rapidly as the leakage reactance is increased, due to the effect of resistance, but the departure from proportionality is not wide. In the same way, the maximum running torque which the motor will develop is approximately inversely proportional to the leakage reactance. In the high-reactance type of motor as well as in the high-resistance motor, therefore, the starting current and the maximum running torque are roughly proportional.

It follows that for both types of motors, the minimum starting current for which they can be designed depends upon the minimum value of the maximum torque which is accepted as a standard for general purpose motors. This is a matter for combined opinion to settle.

At the present time 200 per cent maximum torque is conceded to be high enough for general purpose applications. This amount of torque is sufficient to insure that at 90 per cent voltage the motor will be able to sustain satisfactorily an overload of 25 per cent for at least a short period of time and without stalling. Many motor manufacturers have stipulated in the past that standard squirrel-cage motors should be capable of this performance and it seems reasonable, therefore, to apply this same stipulation to any line-start motor designed to replace a standard squirrel-cage motor and compensator.

It is found by experience that high-reactance line-start motors built to have a starting current of 6 amperes per hp. on a 440-volt 60-cycle circuit will

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develop 200 per cent maximum torque with a slight margin and will have efficiencies comparing favorably with those of standard motors. This value of starting current represents therefore a minimum value for general purpose line-start motors.

Only motors for use in special applications where the torque requirements and voltage conditions are definitely known justify any lower value.

Such special applications exist in the driving of centrifugal fans and pumps. In these cases overloading the motor is not a likely occurrence and 200 per cent maximum torque is more than ample to take care of any low-voltage condition. For these cases line-start motors are built with starting currents as low as 4 amperes per hp. at 440 volts. Such motors will develop from 150 per cent to 160 per cent maximum torque and because their maximum torque is low they are usually specifically designated as "limited maximum torque" motors. The starting torque which they will develop is also limited, averaging from 75 per cent to 100 per cent of full-load torque and seldom exceeding the latter figure. Because their torque is low, these motors must be applied with care for a combination of low line voltage and overload would stall the motor.

The maximum torque obtainable with the high-resistance type motor is slightly higher than that of the high-reactance motor for the same starting torque and current. In spite of this it is not practicable to reduce the starting current of the high-resistance type motor below 6 amperes per hp. at 440 volts, owing to the sacrifice in efficiency involved and the resulting increase in heating of the motor.

Limitations of Size of Line-Start Motors

The relation existing between amperes per hp. of starting current and maximum torque given in the preceding paragraphs is approximate only, but within the range of its approximation the relationship is independent of the motor size. If 6 amperes per hp. at 440 volts was an acceptable value of starting current for line-start motors irrespective of size, it would be possible to develop a line of general purpose line-start motors with 200 per cent maximum torque in any size requested. Actually this is not the case.

The most widely accepted rules regulating starting currents for 60-cycle circuits, the so-called "N. E. L. A. rules," are on a sliding scale, the permissible starting currents becoming more stringent for the higher hp. ratings. This imposes a definite limit to the size of a line-start motor which can develop 200 per cent maximum torque and still meet the starting current prescribed by these rules.

The N. E. L. A. rules comprise a set of recommendations drawn up by a subcommittee of the N. E. L. A. in 1923. Although never accepted officially by the N. E. L. A. these recommendations are recognized as good practise by many power companies. The N. E. L. A. rules in this way have grown to be an accepted standard recognized by the motor manufacturer and the power company alike.

Table I is an extract from the N. E. L. A. rules showing the prescribed starting current for three-phase motors on 440-volt 60-cycle circuits. For other voltages and for two-phase service, the rules prescribe starting currents which in each case amount approximately in kv-a. to the kv-a. represented by the currents in Table I.

TABLE I

Horsepower	Starting current 3-phase—440 volts 60-cycles
3	30
5	43.3
7½	58
10	70.5
15	98.5
20	125
30	180
40	190
50	200
100	400

The stringency of the rules at higher horsepowers will be noticed. Thus a 10-hp. motor is allowed a starting current of 7.06 amperes per hp., a 30-hp. motor is allowed 6 amperes per hp., and any motor above 50 hp. is allowed a starting current of only 4 amperes per hp.

What this actually represents may be better appreciated by referring to Table II, which shows the percentage of full-load current which these various starting currents represent. The power factors and efficiencies are taken from the recent N. E. M. A. recommendations for six-pole, 60-cycle, squirrel-cage motors.

TABLE II

Hp.	Eff.	P. F.	Full-load current at 440 volts	N. E. L. A.	Per cent starting current
10	85.5	85	14	70.5	503
30	88.0	88	40	180.	450
50	89.0	89.5	62	200.	322

It is evident from these figures that general purpose line-start motors designed to comply with the N. E. L. A. rules and to have 200 per cent maximum torque are limited to motors of 30 hp. and smaller.

Several manufacturers now list, in addition, line-start motors having ratings above 30 hp. and designed to have a starting current of 6 amperes per hp. at 440 volts. This value of starting current is above the N. E. L. A. recommendation but there are probably places in which such motors might be started across the line without objection, as for example, in central stations.

"Limited maximum torque" motors may be built to comply with the N. E. L. A. rules in any horsepower rating desired.

Normal and High Starting Torque Motors

Line-start motors of two varieties are now built as standardized products in sizes from 5 hp. to 30 hp., differentiated from one another by the amount of starting torque developed. These are known as "normal-torque" motors and "high-torque" motors. Although differing in starting torque, both kinds of motors for the same rating are designed to have the same starting current, *i. e.*, the N. E. L. A. recommendation for that horsepower.

Normal-torque line-start motors may be of either the high-resistance or the high-reactance type but high-torque motors are always of the high-reactance type.

The normal-torque motor has a starting torque of about 150 per cent of full-load torque. This is slightly higher than that developed by a standard squirrel-cage motor when started on the 80 per cent tap of a starting compensator. The normal-torque motor is therefore applicable for general purpose work and is

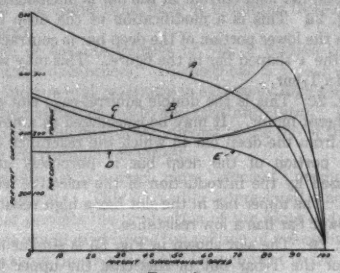

Fig. 1

a. Current curve—Standard squirrel-cage motor
b. Torque curve—Standard squirrel-cage motor
c. Torque curve—High-torque high-reactance motor
d. Torque curve—Normal-torque high-reactance motor
e. Current curve—Normal-torque and high-torque high-reactance motor

intended to be a substitute for the squirrel-cage motor and compensator.

The high-torque motor is designed to have from 225 per cent to 250 per cent starting torque and fills the special torque requirements of such loads as ammonia compressors, certain chain conveyers, and other applications where the torque requirements are severe at starting.

Comparative torque curves of a typical high-torque and normal-torque line-start motor are shown in Fig. 1. Both of the line-start motors are of the high-reactance type. A speed-torque curve of a standard squirrel-cage motor of the same rating is also added for comparison. Both the current and the torque curves are for full terminal voltage. The current curves for the normal-torque and high-torque motors are practically identical and for this reason a single curve has been drawn for the two motors.

Historically, the "high-torque" motor was first of the present line-start motors to find extensive industrial application. Designed as a double squirrel-cage or Boucherot motor with 225 per cent to 250 per cent starting torque, it was first applied industrially as a substitute for the slip-ring induction motor used until then for compressor drive.

At the same time it was found that the starting current of the double squirrel-cage motor, although not as low as that of the slip-ring motor with resistance, was appreciably lower than that of a standard squirrel-cage motor of the same rating. With the formulation of the N. E. L. A. recommendations previously mentioned, the small double squirrel-cage motor was endowed with the further advantage of not requiring a starting compensator. This economy led to the high-torque motor being installed in many places where the starting torque available from the motor was far in excess of the requirements of the load. In other words, in many installations, the attribute for which the high-torque motor was originally designed, that is, its high starting torque, was subordinated to the fact that the motor required no compensator. In these cases of misapplication considerable gains in efficiency and power factor could have been made by designing the motor for a lower starting torque.

There was an evident need for a motor having a starting current as low as that of the high-torque motor and with a starting torque equal to that of a squirrel-cage motor and compensator. Such motors are now available in the "normal-torque" motors.

That the development of the normal-torque motor has come about in this way is somewhat unfortunate. It seems destined eventually to supersede the standard squirrel-cage motor and compensator in many places. At the same time high-torque motors are still applied in many instances where normal-torque motors would do the work. This misapplication is delaying the time when the production of normal-torque motors will be swelled to the point where the manufacturers can invest in them the tool and die equipment needed to bring down their cost to that of the standard squirrel-cage motor of the same rating.

The High-Reactance Type of Line-Start Motor

The majority of both normal- and high-torque line-start motors are of the high-reactance type. The reduction of the starting current below that of the standard squirrel-cage motor is effected by increasing in some way the leakage reactance of the motor. Usually the leakage reactance of the rotor bars is increased, although the same results may be obtained by increasing the reactance of the rotor end rings. For reasons which follow, it is not practicable to increase the reactance of the stator unless a very low starting torque is all that is necessary.

Consider, as an example, a standard squirrel-cage polyphase induction motor having a starting current

of 750 per cent of full load current and a starting torque of 200 per cent of full load torque at normal voltage. If it is assumed that for the particular motor in question the N. E. L. A. starting current is 450 per cent of full load current, the rotor reactance, if increased sufficiently to reduce the starting current to 450 per cent, would as a result reduce the starting torque to

$$\left(\frac{4.50}{7.50}\right)^2 \times 200\% = 72\%$$

assuming the rotor resistance to be unchanged. In order to obtain the 150 per cent starting torque which is considered necessary for a normal-torque motor it would be necessary to increase the rotor resistance by

$$\frac{1.50\%}{0.72\%} = 210\% \text{ or by } 350\% \text{ for a high-torque motor.}$$

If we further assume that the slip at full load was 2.5 per cent for the standard motor, the slip of the normal torque

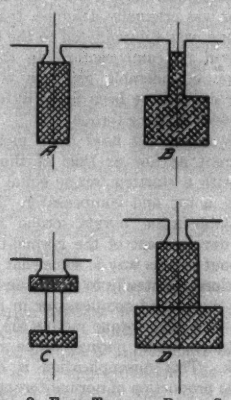

Fig. 2—Four Types of Rotor Slots

motor would be $2.1 \times 2.5\% = 5.3\%$, representing an objectional depreciation of about 2.5 per cent in the full load efficiency of the motor.

It is not necessary, however, to increase the running resistance of the rotor in order to obtain the needed starting torque. When the rotor is at standstill the frequency of the current in a rotor bar is the same as the line frequency. When, however, the rotor has accelerated to its full load speed the frequency of the rotor current is reduced to a very low value. If the rotor bars are so made that with the high frequency existing at starting eddy currents are induced, the equivalent increase in resistance due to these eddy currents appears as additional starting torque although the running resistance of the rotor has not been increased and the full load efficiency is but little affected.

The designer's problem in laying out either a normal-torque or a high-torque high-reactance motor therefore consists in designing a rotor bar or combination of bars which will have sufficient reactance to limit the starting current to the proper value, will give rise to enough eddy-current loss at starting to raise the starting torque to the desired value, will give good running efficiency, and will have as high a maximum torque and full load power factor as is consistent with the other conditions.

Choice of High-Reactance Rotor Slots

The methods for obtaining additional rotor resistance at starting are for the most part well known. Fig. 2 contains schematic drawings of four types of construction which will give the desired results. They are:

Fig. 2A This represents the well known deep bar in which the high reactance of the lower portion of the bar causes the current density to increase at the top of the bar when the frequency is increased, even though the total net load current in the bar is unchanged.

Fig. 2B This is a modification of the deep bar in which the lower portion of the deep bar is contracted to form the T-shaped bar in the figure. This may well be called a T-bar.

Fig. 2C This is the double squirrel-cage slot of the Boucherot motor. It may be considered as a development from the deep bar in which the reactance of the lower portion of the deep bar is purposely greatly increased by the introduction of the selective leakage path. The upper bar in the slot has a high resistance; the lower bar has a low resistance.

Fig. 2D The slot shown in Fig. 2D is similar to the slot for the T-bar but in this case the upper bar is idle, *i. e.*, it is not connected to the end ring and the net current which it carries is zero. At starting, however, the leakage flux crossing the idle bar due to the current in the active bar induces heavy circulating currents in the idle bar.

These four alternatives represent the principal choices open to the designer, although they by no means cover all of the possibilities. Among others, many ingenious arrangements have been devised for inducing eddy currents in the end rings rather than in the rotor bars. This method, although accomplishing the result in high-speed motors, is open to the objection that in low-speed motors the end rings contribute very little to the total resistance of the rotor. In addition, the introduction of any appreciable e. m. f. in the end ring makes it necessary to insulate the rotor bars in the slot in order to prevent short circuiting through the punchings.

Which of the alternatives in Fig. 2 represents the wisest choice? This depends upon the relative magnitude of what may be called the "incremental reactance" of each of these slots. When the motor in question has reached its full load speed and the rotor frequency is very low, the rotor current divides in inverse pro-

portion to the resistance of each individual axial filament of the rotor bar. At standstill, however, the current in the bar redistributes in a different manner and it is a fundamental fact that the leakage inductance of a bar of this kind is lower at fundamental frequency than it would be at the very low frequency obtained while running. Stated in other words, the leakage inductance of the rotor and hence the leakage reactance of the motor as a whole referred to the primary increases as the motor comes up to speed. This increase is the "incremental reactance" of the motor.

Since the maximum torque of the line start motor is decreased below that of the standard squirrel-cage motor by the additional rotor reactance required to limit the starting current, any further increase in the leakage reactance due to the "incremental reactance" means a further reduction in maximum torque. The designer's problem, therefore, consists in designing a slot which will have at standstill the proper leakage reactance, the proper a-c. to d-c. resistance ratio, and will at the same time have a minimum incremental reactance.

Some of the possibilities of Fig. 2 possess certain advantages over others. The deep bar is a poor choice as its incremental reactance is extremely high for very moderate values of a-c. to d-c. resistance ratio. This is due to the crowding of the current to the top of the slot where the reactance is lowest.

The idle bar is a much more "efficient" producer of eddy currents. This is readily appreciated since the idle bar permits the production of the same eddy currents as would exist in the top portion of a deep bar and at the same time restricts the load current to the lower bar where the reactance is high. This type of construction is particularly adaptable to high-speed motors of high horsepower rating where it is desired to increase the torque per ampere at starting as much as possible and at the same time avoid the mechanical complexity of a double-squirrel-cage with double end rings.

The T-bar has a low incremental reactance when it is desired to combine moderate values of resistance ratio with a considerable value of rotor reactance. It is particularly suited to the cast aluminum type of rotor construction and is often used for line-start motors for pump drive under 30 hp. in size and requiring not more than 100 per cent starting torque.

When it is necessary to get high values of resistance ratio with rather low values of rotor reactance the double squirrel-cage slot has the lowest incremental reactance of these four choices.

Besides having a low incremental reactance the double squirrel-cage slot possesses a very important advantage in that all of the bar is effective in providing the double squirrel-cage effect, the projection of the bar at the ends of the motor as well as the portion in the air ducts being just as effective as the portion in the slot. This follows from the fact that any division of current between the top and bottom bars which exists at one point of the rotor slot must exist throughout the bar length.

In the case of the other three alternatives, however, the resistance ratio of the portion of the bar embedded in the slot is higher than the resistance ratio of the rotor as a whole since the end rings and bar extensions contribute a constant resistance independent of frequency, no eddy current effect being present in them. A high resistance ratio is always associated with a high incremental reactance, so that for a given resistance ratio of the rotor as a whole, the advantage of the double squirrel-cage slot is evident. This advantage may be further increased by providing separate end rings for the top and bottom bars.

Fig. 3 shows rotor punchings from a variety of double squirrel-cage line-start motors. The double squirrel-cage in this case is not formed by a pair of bars driven

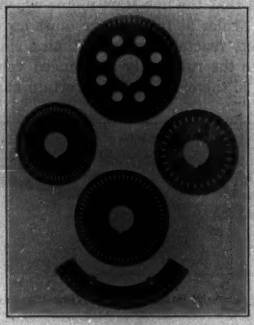

Fig. 3—Rotor Punchings of Line Start High-Reactance Motors

into the slot as in Fig. 2c but in this case the whole slot is cast full of aluminum, the bars being cast integral with the end rings. The resistance ratio, incremental reactance, and leakage inductance at starting are controlled entirely by the shape of the slot.

Fig. 4 shows a double squirrel-cage rotor with cast aluminum bars, the top and bottom bars having separate end rings.

It is not the purpose of this paper to describe the methods by which the characteristics of these bars may be calculated. As a result of calculating a great many combinations, the author has found that the "vector method" of Professor W. V. Lyon[1] involves about the minimum of labor and gives results uniformly in agreement with test.

Effect of Saturation Upon the Starting Current

The starting current of line-start motors is usually measured with the rotor blocked at standstill and with

1. See Bibliography.

either full voltage or a reduced value of voltage applied at the terminals. In the latter case the starting current value at full voltage is found from the value measured at reduced voltage, assuming the starting current and the terminal voltage to be proportional. This proportionality does not actually hold true. The starting current at full voltage may be from 8 per cent to 15 per cent higher than the value extrapolated from reduced voltage readings. This is due to the fact that at full voltage the ampere conductors per slot in both the stator and rotor is higher than at reduced voltage and more saturation of the leakage flux paths occurs. This results in an appreciable decrease in leakage reactance at full voltage. A representative figure may be given for line-start motors, averaged from a large number of tests.

$$\frac{\text{Leakage reactance at full voltage}}{\text{Leakage reactance at full load current}} = 90 \text{ per cent}$$

It follows that the line-start motor which meets the N. E. L. A. rules at full voltage must have about 10 per cent more reactance running than a similar motor which meets the rules only at reduced voltage. The first motor is, therefore, under a certain handicap and

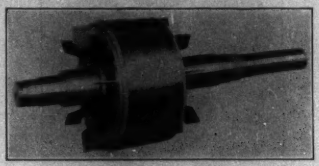

Fig. 4—Rotor from High Torque High-Reactance Motor

has a lower power factor and a lower maximum torque than the second motor.

It might be thought that the maximum torque would be unaffected by saturation since any saturation which occurs at starting would also occur at the maximum torque point of the torque curve, the current at these two points being not widely different. There are two reasons, however, why this is not true.

First: The maximum torque point of the torque curve is seldom approached in actual service due to overload but rather due to a combination of normal load and reduced voltage in which case the current at maximum torque is reduced also.

Secondly: The saturation effect is not an effect proportional to the current but after a certain point is reached it increases quite rapidly. Thus at normal voltage although the current at maximum torque is perhaps as much as 70 per cent of the starting current the actual saturation of the leakage paths at maximum torque may be small.

These two reasons indicate that the maximum torque should be based on a value of reactance measured at standstill at full load current and corrected for the incremental reactance rather than a value found by test at standstill and at full voltage.

The purchaser of a line-start motor is of course interested in knowing the actual starting current of the motor at full voltage, so that the better practise is to design line-start motors to meet the N. E. L. A. rules actually at full voltage. The N. E. L. A. recommendations, however, recognize and accept the measurement by extrapolation from reduced voltage.

Operating Characteristics

Normal Torque High-Reactance Motors

The normal torque high-reactance motor is the line-start motor of the high-reactance type designed for general purpose application and having 150 per cent starting torque.

At full load the leakage reactance of this motor is higher than that of a similar standard squirrel-cage motor. This difference results from the following three causes.

1. The reactance has been increased in order to limit the starting current.
2. Saturation of the leakage paths with full voltage starting.
3. The incremental reactance of the rotor bars.

The total additional reactance represented by these three causes constitutes an additional reactive kv-a. drawn from the line by the motor, the magnitude of which depends on the load. At full load this kv-a. will result in the normal torque high-reactance motor having a power factor from 2 per cent to 4 per cent lower than that of a standard motor of the same size. At reduced values of load, this difference in power factor decreases until at one-half load the power factor of the two motors is very nearly the same. Many motors in actual service operate only a portion of the time at full load and in cases of this kind, the total reactive kv-a. of the line-start motor over a period of time is but little more than that which a standard squirrel-cage motor would require.

This equality in power factor at light loads is only true if the two motors have identical stators and therefore have the same magnetizing current. This latter condition is almost always true, however, between 5 and 30 hp., for four- and six-pole, 60-cycle motors. Eight-pole and slower speed normal-torque motors have generally a higher magnetic flux per pole than the corresponding standard motors and in these cases the power factor of the line-start motor is below that of the standard motor even at the fractional load points.

The efficiency of the normal-torque high-reactance motor is the same as that of the standard squirrel-cage motor. Since its power factor is lower than that of the standard motor, the stator copper loss must be slightly higher, but this additional loss can be counterbalanced by decreasing by a small amount the running resistance of the rotor. This cannot be carried very far, however, without the incremental reactance becoming excessive

and resulting in a prohibitively low maximum torque. In the normal-torque motor it is possible to have the running resistance of the rotor lower than the resistance of a standard motor and still have a margin over 200 per cent maximum torque. The inter-relation which exists between maximum torque and rotor running resistance and therefore rotor copper loss is more important in the high-torque high-reactance motor than in the normal-torque motor and will therefore be considered again later.

High-Torque High-Resistance Motors

The "high-torque" high-reactance motor is the line-start motor of the high-reactance type designed for

FIG. 5—LINE START GENERAL PURPOSE INDUCTION MOTOR 15 hp.—1200 rev. per min.

application to loads requiring a higher starting torque than that developed by the normal-torque line-start motor. It has 250 per cent starting torque as compared with 150 per cent for the normal-torque motor.

The rotor resistance at standstill of the high-torque motor must, therefore, be 167 per cent of that of the normal-torque motor. If the rotor running resistance were the same in the two motors, the high-torque motor would have a very high value of rotor resistance ratio, *i. e.*, $\left(\frac{\text{A-C. rotor resistance}}{\text{D-C. rotor resistance}}\right)$ and the maximum torque would be prohibitively low due to the attendant high incremental reactance. In order to keep the maximum torque above 200 per cent it is therefore necessary in the high-torque motor to have the slip at full load (*i. e.*, the rotor running resistance) higher than in the normal-torque motor. This distinction may be seen in Fig. 1.

It follows that the efficiency of the high-torque motor is lower than that of the normal-torque motor. This difference is from 1 per cent to 2 per cent at full load in sizes from 5 hp. to 30 hp.

The high-torque motor also has a lower power factor than the normal-torque motor. This is partly due to the fact that the incremental reactance is higher. The effect of this is shown in Fig. 1, the maximum torque of the high-torque motor being below that of the normal-torque motor. In many cases the total flux per pole of the high-torque motor must be higher than that of the corresponding normal-torque motor. This entails a further depreciation in power factor.

The running characteristics of the high-torque motor are consistently inferior to those of the normal-torque motor. It is therefore to the purchaser's advantage to use this motor only in cases which actually require the high torque developed.

Normal-Torque High-Resistance Motors

The high-resistance type of motor is distinguished from the high-reactance type by the fact that in the former there is no eddy current effect introduced in the rotor at standstill.

Limitation of starting current is obtained by either reducing the total magnetic flux per pole or increasing the horsepower rating of the frame in question.

The required starting torque is obtained by increasing the rotor resistance and since eddy-current effects are not used the rotor resistance is not altered in going from standstill to full speed. This type of motor has, therefore, an inherently high slip and low efficiency at full load. The speed torque curve of such a motor shown in Fig. 6 indicates a motor very similar to those special high slip motors designed for driving punch presses and other flywheel loads.

The high-resistance type of motor has an advantage in that the full load power factor is quite high but the full load efficiency is correspondingly low and becomes rapidly lower at overloads.

The low efficiency of the high resistance type of

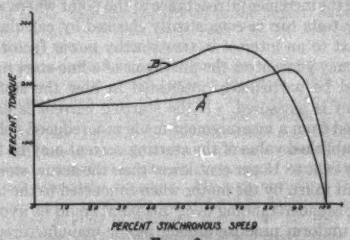

FIG. 6

a. Torque curve—Normal-torque high-reactance motor
b Torque curve—Normal-torque high-resistance motor

motor indicates that this motor will operate at a higher temperature rise than a motor of the high reactance type.

The average accelerating torque is higher than that of a high-reactance type motor having the same starting torque as shown in Fig. 6 where torque curves of the two types are superimposed.

High-torque line-start motors of the high-resistance type are beyond the range of practicability. The high value of rotor resistance which is required results in an objectionably low efficiency at full load. This class of motor is suitable only for application to a-c. elevators, skip hoists, or other installations where the duty is principally one of acceleration.

Testing of Line-Start Motors

The testing of line-start induction motors and the subsequent use of these tests in preparing trustworthy guarantees necessitates precautions not necessary in the testing of standard squirrel-cage motors.

In preparing guarantees for standard squirrel-cage motors, it is common practise to compute the power factor and the conventional efficiency from data obtained from running light and blocked rotor tests. The computation may be done by the use of either the exact equivalent circuit or the circle diagram.

In preparing guarantees for line-start motors of the high-reactance type, however, the leakage reactance obtained from the blocked rotor test is not suitable for use in calculations at full load. It is too low by the amount of the incremental reactance, and power factor guarantees computed from it will be higher than the values actually realized in the motor.

Power factor guarantees of line-start motors should, therefore, be based primarily on carefully conducted wattmeter tests with the motor connected to a dynamometer.

An alternative method of arriving at the power factor consists in calculating the incremental reactance and adding it to the test value of standstill reactance in order to obtain the value of running reactance needed in the equivalent circuit or circle diagram. This method should be considered acceptable only when the individual designer has perfected the technique of calculating the incremental reactance to the point where wattmeter tests can be consistently checked by calculation.

Next to an interest in trustworthy power factor and efficiency guarantees the purchaser of a line-start motor should be particularly interested in how the starting current is measured. If the starting current is extrapolated from a measurement made at a reduced voltage the published value of the starting current may be from 10 per cent to 15 per cent lower than the actual starting current taken by the motor when connected to the lines. Any misunderstanding over this point could be avoided by a uniform practise among motor manufacturers of guaranteeing starting current values actually measured at full voltage.

Conclusions

The principal points brought out in this paper may be summarized:

(1) The field for the line-start general purpose induction motor is large and is steadily expanding.

(2) The line-start motor has necessarily a lower maximum running torque than the standard squirrel-cage motor, the maximum torque being roughly inversely proportional to the starting current.

(3) Because of this lower maximum torque it is not advisable to build line-start motors for general purpose use with a starting current below the equivalent of six amperes per hp. on 440-volt 60-cycle circuits.

(4) Motors in sizes above 30 hp. built to meet the N. E. L. A. rules are not suitable for general purpose applications owing to the low maximum running torque which they develop.

(5) Two classes of line-start motors are available, normal-torque and high-torque motors. The normal-torque motor is for general purpose application. The high-torque motor is intended for a special type of service and its use involves a sacrifice in running characteristics.

(6) From the design point of view there are two types of line-start motors, high reactance and high resistance. As a whole the characteristics of the high-reactance type are superior.

(7) The running characteristics of normal-torque line-start motors compare favorably with those of standard squirrel-cage motors.

(8) Special care is needed in preparing guarantees for line-start high-reactance motors from no load test data due to the change in leakage reactance with speed.

(9) Starting current values should be measured at full voltage rather than increased proportionally from readings taken at reduced voltage.

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Discussion

F. E. Harrell: Although I believe it was not stated bluntly, we can certainly infer from the statements made in this paper, that along with a great many others, Mr. Koch recognizes the fact that regardless of what type of N. E. L. A. motors we consider, the user is penalized in some respect in the performance of that machine due to the present low values of the N. E. L. A. permissible starting currents.

In this connection, I wonder if Mr. Koch can tell us whether the high-reactance type or the double-squirrel-cage motor with the minimum starting current which he mentions of six amperes per hp. for a 440-volt, 60-cycle supply, whether or not that design inherently prevents the speed-torque curve from having a drooping characteristic between the starting point and the maximum torque point.

It has been my general understanding that this one characteristic, namely, that of having some point between starting and maximum at which a torque lower than that developed at starting prevails, has been the thing which has prevented the more general use of this motor, both on the Continent and in America, although it was brought out over there twenty years ago.

It also occurs to me that the name "Resistance Type N. E. L. A. Motor" might cause it to be confused with the punch-press or flywheel load motors which have a high slip, up to 10 or perhaps as high as 15 or 18 per cent, which motors, of course, are practically always for intermittent duty and not for continuous duty. As a matter of fact, I believe that it is just as possible to design a good single-cage N. E. L. A. motor as to design a good double-cage N. E. L. A. motor. The thing which perhaps bears out Mr. Koch's conclusion that as a whole the characteristics of the double cage make it the most desirable type, undoubtedly involves the cost factor, or the manufacturing cost, because it is appreciated that it is entirely possible to meet a given starting current with given performance, in a single-cage motor by increasing the frame size, if not in overall dimensions, then by increasing the $D^2 L$.

There remains, of course, much to be said in favor of the single-cage or so-called high-resistance type of N. E. L. A. normal-torque motor which was not emphasized in Mr. Koch's paper, principally, the higher maximum torque and much higher power factor—in some cases as much as 5 per cent. Also, that existing types of single-cage N. E. L. A. motors, while they do have more slip than standard motors, do not have anything like the slip referred to as placing them in the class with punch-press motors for flywheel loads.

In conclusion, may I say that it is my opinion that Mr. Koch's conclusion that as a whole the characteristics of the high-reactance type are superior, is certainly a highly debatable one, with the balance of argument which might favor the double cage resting principally in the economy of manufacturing.

J. C. Lincoln: In the sixth page of the paper is the following statement: "It might be thought that the maximum torque would be unaffected by saturation since any saturation which occurs at starting would also occur at the maximum torque point of the torque curve, the current at these two points being not widely different. There are two reasons, however, why this is not true."

I can't reconcile that statement with Curve *C* on the third page. The Curve *E* gives the current of the normal torque, and the high-torque high-reactance motor. Curve *E* has a maximum value at the start and it rather rapidly decreases to synchronism. At the start, therefore, we have a maximum current, and according to Curve *C* showing the torque for the high-reactance high-resistance motor, the maximum torque occurs quite near synchronism when the current is quite small.

There is another question that I should like to ask Mr. Koch. The feature of a curve, as Curve *C*, showing a smaller torque between starting and maximum is rather general in double-cage motors, but what is the condition which produces a starting torque greater than the maximum torque?

H. C. Specht: In this paper I missed something and that is in regard to the temperature of the squirrel-cage winding. I believe if the temperature rises very rapidly, which would be the case in only the upper member where the section of the conductor is smaller, that would increase the resistance and drive some of the current down to the lower part of the squirrel-cage winding and thus give somewhat less torque. It seems from the curves that this is probably the case.

P. L. Alger: Mr. Koch's paper gives in a clear way the fundamental facts about the line start motor: that the per cent starting current is necessarily a little more than the per cent maximum torque (as Baffrey[1] has stated), I_s equals approximately

$$\sqrt{4\,T_m^2 + T_s^2}$$

and that the full-load characteristics are necessarily inferior to the normal squirrel-cage motor. Briefly, if a standard squirrel-cage motor has a starting current I, a starting torque T_s, a maximum running torque T_m, occurring at 75 per cent of synchronous speed, and a maximum power factor, P, the most favorable results obtainable in a line start motor of the same efficiency and same dimensions, and having a starting current $k\,I$, are the same starting torque, a maximum torque $k^2\,T_m$, and a maximum power factor $1 - k^2\,(1 - P)$. If the standard motor has a per unit slip, s, at maximum torque less than ¼, the line-start motor can have a greater starting torque, without sacrifice of efficiency, equal to approximately

$$T_s\,[1 + k^2\,(1/2\,s - 2)\,]$$

These relations are only approximate, but they are useful in an appraisal of the relative merits of the motors.

Mr. Koch has thus given us full comparative data on standard and line-start motors. The question we should answer is "What is the proper field for each?" Present practise is, generally, to use standard squirrel-cage motors with full voltage starting in the smallest sizes, line-start motors in the next larger sizes, standard squirrel-cage motors with starting compensators in still larger, and slip-ring motors in the largest, sizes. The division points now come at perhaps 7½, 30, and 300 hp., respectively, in United States practise.

If starting-current limitations could be altogether removed, the line-start motor would probably disappear, while if these limitations are lightened, the median size of line-start motor will increase. It is quite certain that present practise is not ultimately the best, and that the line-start motor must either usurp more and more of the field, or that it will serve simply as a means to the end of introducing full-voltage starting of standard motors, and will ultimately disappear. My own opinion is that the starting-current restrictions can and will be lightened, and that full-voltage starting of standard motors will be used in continually larger, and line-start motors in still larger, sizes.

C. W. Kincaid: The design of these motors is dependent mainly on the locked current allowed by the operating company. Since these are not all the same, values as given by the N. E. L. A. are generally used as a basis.

From Table I, it can be seen that the amperes per horsepower allowed for the various ratings, decrease from 10 amperes at 3 hp. to 4 amperes at 50 hp.

Due to relatively large currents allowed on the small ratings, the standard design can be used. At 6 amperes per hp., a high-reactance motor is necessary, which can be of the deep-bar design if the required starting torque is low, but which must be double-deck if high starting torque is required. Between these limits all kinds will be used to suit requirements.

The maximum torque, which is directly dependent on the locked current, becomes too low for general-purpose motors when the current allowed is below 6 amperes per hp., and in consequence motors above 30 hp. are not made to meet N. E. L. A. current values. If the N. E. L. A. values for ratings above 30

1. *Elektrotech. u. Maschinenbau*, July 8, 1928, pp. 749-754.

hp. were made 6 amperes per hp. instead of 4 amperes per hp. above 40 hp., there would be no limit in the size of the motor that can be built as a line-start motor.

A. M. MacCutcheon: As has been said, in the case of the smaller motors it is relatively easy to meet the N. E. L. A. values with good performance and a suitable starting torque, but as you get up to the larger motors the double-cage motor seems to be at the present time the only answer. But is it correct that the industry should stop in the consideration of the current values as established by the N. E. L. A. and say that is a fixed thing?

Many plants that have their own power systems, or are tied in on the primary side of the distributing system and have their own secondary distribution system for the last five or ten years, have of their own initiative been throwing on to the line the standard induction motor. Mr. R. S. White, of Madawaska, Maine, told me he has been throwing motors as high as 2400 hp. on the line.

I have wondered whether the application of electric motors was not going to be furthered greatly if the power companies would again review this question. They have in the last few years reviewed it and revised it upward. I really believe it should be reviewed again with continued experience and taken upwards still further.

It has been said in the paper that the cost is reduced, but there has been no emphasis particularly placed on the fact that troubles are reduced, that is, provided the motor is so constructed that no trouble is experienced with the motor in so doing. From the users I hear that they have had far, far more trouble with compensators than they have had with squirrel-cage motors.

In the consideration by the power companies of this question of what should be done with motors when thrown on the line and what current should be allowed, I wonder if they have ever considered that they should classify in the plant of the user the type of motor used. If the motor is to be started every five or ten minutes, it seems to me that a very different current should be allowed on that motor from that on a motor which is started once a day or once a week. I think possibly that the power company should consider it from that light and make a classification as to how often a motor is to start, and if it is frequent, I think there should be a lower current value than for those motors which are started seldom. If the limits of current are extended upward, we will then have the simplicity of the straight squirrel-cage motor. I do not think that the double-cage motor is much more complicated, but it is somewhat, and I think it has a somewhat higher cost, but unless there are real good fundamental reasons why we should keep to these present N. E. L. A. values, I think the question should be opened up again.

C. A. Adams: I should like to support what Mr. MacCutcheon said about the revision of the N. E. L. A. allowance upwards.

It just happens that I have been recently operating a single-phase resistance welder connected to a three-phase system where we have loads lasting for a second or two as high as 700 or 800 kv-a. from one phase of the three-phase system, and there is no complaint of system disturbance.

The utility company permits us to take 500 kv-a. regularly. Under these circumstances, it seems to me absurd to retain the present low limits.

Referring now to the speed-torque curves, there are three points worthy of note. First, it is almost impossible, experimentally, to get the points on the unstable or undersides of these curves, since owing to the instability, the rotor is almost invariably either accelerating or decelerating. Second, the reason for the peculiar shape of the curve is fairly obvious. At very low speeds you have a very high resistance and low reactance in the secondary. As the speed increases, the reactance goes up rapidly and the resistance down until you fall on to the high reactance curve which carries you into the normal operating region. Third, what Mr. Specht said about the heating of the secondary is true if the two secondaries are separate. If in this case the speed is held for any length of time at a low point, there is bound to be a considerable increase in resistance, but the amount of this increase depends upon the nature of the material. If the material is copper, the increase in resistance might be considerable, but if it is made of some high-resistance alloy, the increase will be very much less. If, on the other hand, observations of these lower points are made rapidly, there will be little time for change of temperature and resistance. If, however, there is much change in the resistance due to slowness in making the observations, the starting torque, or torque at the low points of the curve, will obviously be affected thereby.

C. W. Franklin: With respect to starting currents, I feel most of the power companies are perfectly willing to allow starting currents up to any value that will not cause objection from customers. That value is variable depending largely on the type of system. We have in New York very large motors started across the line. We also have some 7½-hp. motors that cause appreciable lighting dips when they are fed from combined lighting and power secondaries. The large motor does not give trouble. The 7½-hp. does at times. That is the problem.

In the larger cities there is getting to be a general amount of combining of lighting and power secondaries and we are likely to get motors of many types and classes. Of course, most of them are rather small, but some get up as high as 300 or 400-hp., for cooling systems.

In order to prevent flicker trouble on lighting we have come to the conclusion that increment starting current is the only way. For instance, the lighting at the Roxy Theater requires 320 amperes per phase to start, requiring a final voltage drop of about 7 volts. These 7 volts if applied gradually do not cause any noticeable flicker in light. On the other hand, if we had a compensator slip motor on the line start which would take more amperes you would get a noticeable flicker.

So there are some cases where increment start is desired. That has meant resistance starters to date.

With respect to Mr. Koch's paper, we are highly in accord with the thought that the starting current should be measured with blocked rotor for a final proposition. Any other method may lead to argument. So we, and the N. E. L. A., I think, have set up blocked rotor values so far as the line-starting motor goes. I think it is an excellent way to determine the maximum starting current.

L. L. Elden: The paper on "line-start" motors will be most helpful in assisting those interested in power applications to a clearer understanding of the design and operating characteristics of this particular class of motors.

Written as it is to present the manufacturers' point of view, it gives only limited consideration to the commercial aspects of power supply which more intimately affect the utilities and their customers.

In late years the term "line-start" motors has been applied only to the newer types of motors described in the paper. These have been expected to effect a substantial reduction in the investment required for a given power installation, through the elimination of compensators or other starting devices.

The practise of starting motors across the line is not new. Standard general-purpose motors of all sizes have been so operated for many years in locations where conditions permit the use of liberal starting currents. As the term "line-start" is used, it is not clear why it should not generally apply to any motor which may be started across the line.

Up to within a very recent period, the manufacturers have emphasized the merits of standard squirrel-cage motors for general use, indicating that by concentrating large-scale production on a standardized product, substantial reductions in the cost of such motors could be effected. The introduction of numerous types of "line-start" motors appears to controvert

that theory since these motors have materially complicated the problems of power supply and the choice of motors for a given service. In the event that the production ratios of the two classes of motors are substantially modified, it will be logical to expect a revision of prices upward for squirrel-cage motors.

If on the other hand the relatively high cost of developing, producing, and marketing "line-start" motors in limited quantities was eliminated and an increase in the production of standard squirrel-cage motors thereby secured, it would be reasonable to expect a reduction in the cost of the latter which might even offset the anticipated savings to be secured by the omission of compensators.

Further, with no gain in efficiency or power factor and with an actual reduction in starting and running torque under given conditions, these new-type motors appear to offer only a possible reduction in costs and lower starting currents as a justification for their existence.

Without questioning the merits of the "line-start" motors for use in large power installations and in locations where limitations in starting current are unnecessary, the fact remains that such cases are far in the minority, when compared with the vast number of individual power applications where more or less strict limitations of starting currents must be enforced to conform to existing conditions in local supply systems. In such locations recourse must always be had to compensators or other starting devices, or to certain types of motors having low starting currents in order to preserve satisfactory service conditions.

Long experience in dealing with distribution facilities necessary to serve miscellaneous power applications is convincing to the end that the "line-start" motor has its true application only in large power installations where advantage may be taken of liberal starting currents for individual motors, none of which may be out of proportion to the maximum demands of the entire installation. Under such conditions it may well be that the standard squirrel-cage types of motors would and do perform equally well and with starting currents within permissible limits.

However, where advantage can be taken of such conditions the elimination of the compensator may effect a welcome reduction in costs although the added cost of the heavier wiring required must not be overlooked in weighing the final results.

The author has indicated that the motors described depend materially upon the maintenance of normal voltage and frequency for their best performance and even then their torque may be unfavorably affected. It would be helpful if definite statements could be added to describe the performance of these motors with their limited torque characteristics on network systems when the voltage is 10 per cent or more below rated motor voltage.

Reference is made to the N. E. L. A. motor rules in a number of places in the paper. The statements which appear on the second page in this connection appear to require some modification in order to conform with the facts in the case.

The so-called 1923 N. E. L. A. rules were never authorized and have never been approved by the Association. The record clearly shows that the members of the Sub-committee were unable to reach agreement upon any revision of the 1915 rules then in force, and that the rules prepared in 1923 were merely submitted for discussion. The 1923 rules were never acted upon, owing to lack of approval within the Association.

It having come to the notice of the N. E. L. A. that certain motor manufacturers were in some cases utilizing the unauthorized 1923 rules as one of the important factors controlling the design of new motors, official notice was sent informing the National Electric Manufacturer's Association of the true status of the rules in question. In addition individuals within the association have similarly advised various manufacturers to the same effect from time to time. It would appear that no notice has been taken of these advices.

Recent inquiry within the Association does not support the statement that "these 1923 recommendations are recognized as good practise by many power companies." The contrary appears to be the case since there is no evidence of any use of the 1923 rules or recommendations by any interest other than motor manufacturers.

The only rules that appear to be used in utility operations to any extent are those authorized in 1915. Even these rules have been used by only a limited number of companies, most companies preferring to enforce rules particularly suited to their own local conditions and in most cases limiting starting currents to even lower values than the 1915 rules.

Since the 1915 rules in most cases limit starting currents to values materially below those proposed in 1923, serious difficulties have arisen in many cases between utilities and their customers, when service has been refused for motors until their starting currents have been brought within acceptable limits. In these cases motor representatives invariably refer to the so-called 1923 rules as their understanding of N. E. L. A. requirements.

In view of the wide publicity now given to the unauthorized and unapproved N. E. L. A. recommendations of 1923 and the use which has been made of them in developing new motor designs, it would appear most desirable to modify the text of the paper where the rules are referred to, to convey a true picture of the situation first-hand in the final printing without depending upon a reading of the discussion to disclose the facts.

W. A. Naudain: (communicated after adjournment) The N. E. L. A. rules used by the power companies, which seem to govern the adaptation of this type of motor, do not fill the present needs.

The purpose of these rules is to eliminate fluxation of voltage, and the resultant complaints from other customers on the system. The line fluxation is governed more by the available power supply, and location of the equipment on the system than by the actual values of starting current drawn by connected apparatus. This being true, it appears that this type of motor, in rather large sizes, might be used in metropolitan areas where there is great concentration of load.

With these points in view, and the line-start current allowable being predicated on the specific location and application of the motor, it seems that these rules should be, to a large extent, nullified, allowing the motor designer to design motors having other desirable characteristics, and permitting the ultimate user to benefit.

C. J. Koch: Mr. Harrel and Mr. Lincoln have raised questions regarding the shape of the torque curve of line-start motors.

The shape of the speed-torque curve of the high-reactance type of line-start motor may be varied by changing the relative magnitudes of the resistances and reactances of the bars constituting the double squirrel cage. Thus curves *C* and *D*, Fig. 1, are from two motors having identical stators, the shape of the rotor slot, however, being different in the two cases.

If the double squirrel-cage slot is proportioned to give approximately 150 per cent of full-load torque at starting, for motors of about 50 hp. and below in size, the torque of the motor will increase as the motor comes up to speed as shown by Curve *D*, Fig. 1, reaching a maximum value somewhat above 200 per cent.

If, however, the proportions of the double squirrel cage are changed to give about 250 per cent starting torque as shown by Curve *C*, Fig. 1, the torque of the motor decreases at first as it comes up to speed since the starting torque is now higher than the torque at the maximum torque point. This droop which occurs in the torque curve of the high-torque high-reactance motor is not serious, however, since the motor is designed to have a high starting torque and it should not be forgotten that the speed-torque curve of the load must pass through 100 per cent at full-load speed. I do not recall any instances where a high-reactance motor failed to being its load up to speed after having

once started it. There have been cases where it may have failed to come up to speed due to harmonics but that is a different matter.

Regarding the question raised by Mr. Lincoln of the value of current taken by a motor from the line when passing through the maximum torque point of the torque curve: The current at maximum torque is theoretically about 70 per cent of the starting current. Actually it is less than this due to the increase of starting current due to saturation. Any amount by which it is less than 70 per cent of the starting-current value strengthens rather than vitiates the argument referred to in the paper. Throughout this paper by the term "maximum torque" is meant the highest torque which a motor will carry without suffering a great reduction in speed. Thus in Fig. 1 Curve *C* "maximum torque" occurs at about 85 per cent of full-load speed although the torque at this speed is actually less than the starting torque.

Regarding the effect of increase in temperature on the characteristics of the double-squirrel-cage motor, I do not believe that very large differences of temperature can exist between the top and bottom bars with the cast aluminum type of construction. It has been found, for example, that the starting torque of this type of motor is practically independent of the temperature of the rotor. Increased temperature certainly decreases the eddy-current effect but the increased resistivity normally means more starting torque, so that the one effect balances the other and the starting torque remains practically constant.

Mr. Elden's discussion is most timely and interesting. It was not the intention of this paper to create the impression that the "1923 recommendations" had ever found sanction by the N. E. L. A. or that there was any obligation on the part of the power companies to recognize these recommendations. However, the position which the motor manufacturers have taken in designing "line-start" motors to meet these recommendations is not unjustified. From this paper it will be appreciated that line-start motors designed to have starting currents any lower than the 1923 recommendations would not have desirable running characteristics, particularly overload capacity. The fact that there is a very considerable demand for motors having starting currents at the general level of the "1923 recommendations" indicates that there are many places in this country where the power company will permit the starting of such motors without a compensator. In order to meet this demand, motor manufacturers have designed line-start motors to meet the "1923 recommendations" not because there is any compulsion operating upon the power companies to accept such motors on their lines but because these recommendations comprise the best opinion we have of what starting currents, at a higher level than those prescribed in the 1915 rules, will be acceptable in some places if acceptable at all. The 1923 recommendations therefore have grown to be a standard on which the various motor manufacturers in this country can compete on the same basis in the design of line-start motors. Some standard of this kind is, of course, necessary. The production of line-start motors has burdened the motor manufacturer with an increase in the variety of his product but this increase would be very much more if manufacturers attempted to produce a number of individual groups of motors, each one representing the best balance of power factor, efficiency, and starting current for a particular locality.

No-Load Induction Motor Core Losses

BY T. SPOONER[1] and C. W. KINCAID[1]

Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—A detailed method of calculating the no-load core losses of induction motors is presented whereby each loss component is estimated separately. There is included a discussion of extra losses due to imperfection in workmanship. Calculated and test results on a few commercial machines are given as an indication of the reliability of this method.

IN general the simplest and most reliable method of predicting the core losses in a rotating machine of new design is to estimate them from the test results on machines of similar type. This is the method ordinarily used by experienced designers. This procedure is not adequate, however, in some cases. For instance, if the core losses in a certain type of machine are higher than anticipated, a more detailed analysis is necessary, or if a machine is proposed which has certain radically different features from any existing machine, a calculation of losses, based on fundamentals, is essential if these losses are to be properly estimated. It is our purpose here to consider all of the important factors which produce no-load core losses in induction motors and similar types of apparatus and to give a method of calculating these losses. This method has been used tentatively for several years for special cases and has been found to check the test results fairly well with, of course, occasional exceptions. These exceptions were due, it is felt, more often to errors of test or carelessness in shop construction than to inaccuracies in the method of calculation.

This method is presented primarily for the purpose of stimulating discussion and to obtain suggestions for improving and simplifying the calculations. There are certain weaknesses and elements of uncertainty in the method which we shall point out. Some details of the process have already been presented before the A. I. E. E. Only such portions of these previous papers will be repeated as are necessary to make the calculations and formulas complete.

Types of Loss

This discussion will be confined to induction motors but it will be obvious that the methods can be applied to other types of machines in which both the stator and rotor are slotted and in which the air-gap flux has approximately a sine-wave distribution.

For the purpose of calculation, induction motor losses will be segregated into the following components:

1. Fundamental-Frequency Losses.
   a. Stator core
   b. Stator tooth
2. Pulsation Losses.
   a. Surface (stator and rotor)
   b. Tooth pulsation (stator and rotor)
   c. Copper eddy (stator and rotor)
3. Illegitimate Losses.

The fundamental-frequency losses are those due to the hysteresis and eddy-current losses in the core material corresponding to the applied frequency and are assumed to be the same as would occur under alternating flux, as in a transformer core, for the same maximum induction. This assumption may be open to question as far as the core flux is concerned since this flux has an elliptical rather than an alternating variation but the simpler assumption with corrections, if necessary, for the elliptical field seems to give satisfactory results. It is assumed that the frequency of the main flux in the rotor is so low that the losses are negligible.

The pulsation losses consist first, of the surface losses which are those hysteresis and eddy losses occurring just below the surface of the tooth tops due to the passage of the slots of the other member and are practically the same as the well known pole-face losses.

The tooth pulsation losses are those caused by the high-frequency pulsations of flux extending the whole length of the teeth and a little way into the core due to the reluctance changes in the air-gap as the slots of one member pass the teeth of the other. The distinction between surface and tooth-pulsation losses is somewhat arbitrary but seems to work fairly well.

The no-load copper eddy-current losses are, of course, not really core losses but appear as such by the ordinary methods of test. They are due to the high-frequency slot-leakage fluxes as the result of the momentary changes in the saturation of the teeth. They have both tangential and radial components.

The illegitimate losses are those caused by punching strains in the tooth iron, short-circuited laminations due to burrs and slot filing, bending strains in the iron sheets, and finally to leakage fluxes into the frames, end-bells, and other solid members.

Calculation of Losses

The curves for the calculation of the iron losses will all be based on the specific losses occurring in a good quality of electrical sheet having a silicon content of approximately 1 per cent. For better grades of steel the calculated losses must be decreased accordingly. For instance, if a steel is used having a silicon content

1. Both of Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

of about 2½ per cent the iron losses, other things being equal, will be about 70 per cent of these for the sheet with the lower silicon content. This applies both to the fundamental-frequency and to the pulsation losses. For the same flux density, the copper eddy losses will be higher for the higher silicon material due to the fact that saturation occurs at a lower tooth induction and, therefore, the slot-leakage fluxes are greater.

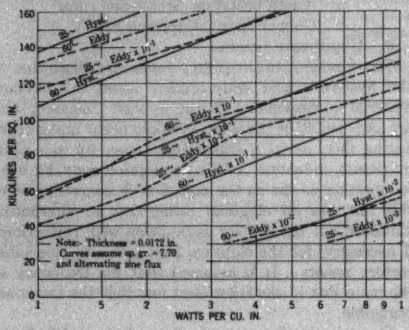

Fig 1—Fundamental-Frequency Core Losses at 25 and 60 Cycles. One Per Cent Silicon Steel

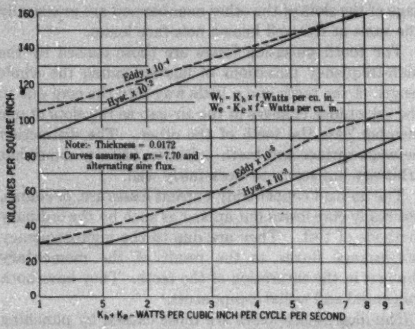

Fig. 2—Fundamental-Frequency Core Losses for any Commercial Frequency in One Per Cent Silicon Steel

Fundamental-Frequency Stator Core Losses

The maximum induction in the core is to be calculated in the usual way by dividing one-half the flux per pole by the core cross-section corrected for space factor, namely: multiply the gross core cross-section by a factor of approximately 0.9 or greater, depending on the smoothness of the surface, thickness of insulation, and density of the particular core material which is used. The specific loss of the core material for the particular flux density as determined from standard core-loss curves for the sheet in question is multiplied by the net volume of the core (gross volume multiplied by space factor); this gives the total fundamental frequency iron losses for the core assuming no illegitimate losses and uniform flux density over the core section.

If the ratio of outside to inside radius of the core is not too large and if the number of poles is not too great, this procedure will give approximately correct losses except as there may be short-circuited laminations or leakage fluxes into the frames due to saturation of the laminated material or other causes to be discussed later.

Some years ago Alger and Eksergian[2] gave a method of calculating the increased iron losses due to non-uniform flux distribution in the core. In order to made these corrections, the hysteresis and eddy losses must be calculated separately. This may be done by the aid of Fig. 1 or 2, the former to be used for 25 or 60 cycles and the latter for other frequencies. In case Fig. 2 is used, multiply the value K_A corresponding to the maximum induction by the frequency and for the eddy loss multiply the value of K_e by f^2. Now refer to Figs. 3

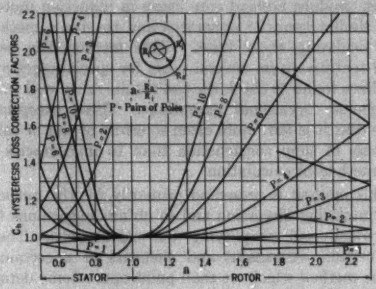

Fig. 3—Corrections to Hysteresis Losses Due to Non-Uniform Flux Distribution

and 4 (due to Alger and Eksergian) and then calculate the ratio of the outside to the inside radius of the core. For the hysteresis losses refer to Fig. 3 and determine the factor c_h corresponding to (a) from the suitable curve, namely: if the machine has 6 poles use curve $p = 3$. The calculated hysteresis loss is to be multiplied by the factor c_h. In the same way the eddy losses are to be multiplied by c_e taken from Fig. 4. These curves were calculated on a basis of constant permeability. Due to the variations of μ with induction, the flux distribution will be different than assumed with a corresponding error in the corrections. The calculated eddy losses will be too small, and the calculated hysteresis losses too large.

Fundamental-Frequency Stator Tooth Losses

The stator teeth of an induction motor are subjected to approximately a sine-wave variation of fundamental-

2. *Induction Motor Core Losses*, P. L. Alger and R. Eksergian, Jl. A. I. E. E., October 1920, p. 906.

frequency flux on which generally is superposed a high-frequency ripple corresponding to the number of slots in the rotor. It has often been assumed that these high-frequency pulsations tend to reduce the fundamental-frequency hysteresis losses. It has been shown,[3] however, that this is not the case and that the fundamental-frequency hysteresis losses are proportional to the maximum flux density whether or not the high-frequency pulsations are superposed.

Referring to Fig. 5, B is the maximum magnetic flux density in the teeth as ordinarily assumed and B_m is the actual maximum flux density. It is necessary, therefore, to calculate the magnitude of these high-frequency flux pulsations in the teeth. This can be done by means of the formulas given in a previous A. I. E. E. paper.[4]

P_s is the stator tooth pulsation in per cent of the maximum tooth flux

P_{r1}, P_{r2}, P_{r3}, and P_{r4} are the rotor tooth pulsations for four cases

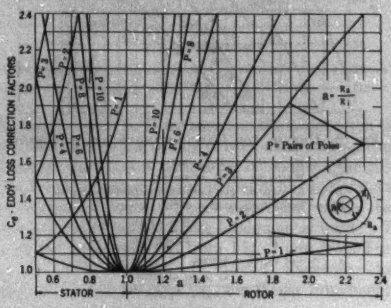

FIG. 4—CORRECTIONS TO EDDY-CURRENT LOSSES DUE TO NON-UNIFORM FLUX DISTRIBUTION

λ_s is the stator slot pitch

λ_r is the rotor slot pitch

t_{ss} is the effective stator tooth width

t_{sr} is the effective rotor tooth width.

Cases 1 and 2 are to be used when the rotor slot pitch is less than the stator slot pitch. Cases 3 and 4 are to be used when the rotor slot pitch is greater than the stator slot pitch.

$$P_s = \frac{\lambda_r - t_{sr}}{t_{ss}} 100 \qquad (1)$$

$$P_{r1} = \frac{\lambda_s - t_{ss}}{t_{sr}} 100 \quad \begin{cases} P_{r1} \text{ cannot exceed } 100 \\ t_{ss} \text{ is greater than } t_{sr} \end{cases} \qquad (2) \quad \text{Case (1)}$$

$$P_{r2} = \frac{\lambda_s - t_{sr}}{t_{ss}} 100 \quad \begin{cases} t_{ss} \text{ is less than } t_{sr} \\ P_{r2} \text{ has a positive sign} \end{cases} \qquad (3) \quad \text{Case (2)}$$

$$P_{r3} = \frac{t_{sr} - \lambda_s}{t_{sr} - \lambda_s + t_{ss}} 100 \quad \begin{cases} t_{ss} \text{ is less than } t_{sr} - \lambda_s + t_{ss} \\ t_{sr} \text{ is less than } 2\,\lambda_s - t_{ss} \end{cases} \qquad (4) \quad \text{Case (3)}$$

$$P_{r4} = \frac{\lambda_s - t_{ss}}{t_{sr} - \lambda_s + t_{ss}} 100 \quad \begin{cases} t_{sr} \text{ is greater than } 2\,\lambda_s - t_{ss} \\ t_{sr} \text{ is less than } \lambda_s + t_{ss} \end{cases} \qquad (5) \quad \text{Case (4)}$$

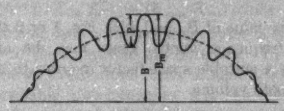

FIG. 5—SHOWING TOOTH FLUX VARIATIONS FOR INDUCTION MOTOR

The effective tooth widths t_s can be determined from Fig. 6. B should be calculated on the basis of net section sine-wave flux distribution and for a tooth cross-section one-third of the distance from the smallest section. These formulas are based on the assumption that the teeth have zero reluctance. Above 70 to 80 kilolines saturation effects begin to be appreciable, namely: the tooth reluctance cannot be neglected. This tooth reluctance causes the percentage pulsations to decrease. Based on theoretical considerations, correction curves have been calculated to take care of the saturation effects. Assuming a pulsating wave having a maximum displacement B, and assuming an average reluctance for the induction-motor stator and

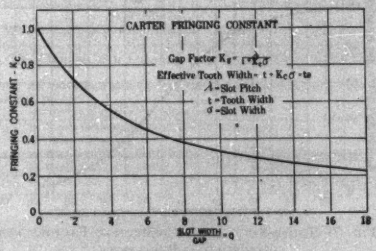

FIG. 6—CARTER FRINGING CONSTANT

rotor teeth and an average length of air-gap, Fig. 7 has been calculated. This gives the percentage, as a result of saturation, of the calculated percentage pulsation as obtained by the above formulas plotted against B. For the case of the induction motor, B is the maximum of the fundamental-frequency of the tooth induction. If there is a considerable departure from the assumed air-gap and tooth reluctances the curve is not very greatly shifted. A smaller relative tooth reluctance due to shorter teeth or a longer air-gap shifts the curve upward.

3. "Effect of Superposed Alternating Field on Apparent Magnetic Permeability and Hysteresis Loss," T. Spooner, *Phys. Rev.*, Vol. 25, Sect. 2, 1925, page 527.

4. *Tooth Pulsations in Rotating Machines*, T. Spooner, A. I. E. E. TRANS., Vol. 43, 1924, p. 252.

To correct the tooth inductions for saturation, then, determine B in the usual way assuming a sine-wave distribution of flux and a net tooth cross-section one-third of the distance from the narrowest part of the tooth. Calculate the percentage tooth pulsation P from the suitable formula. Multiply by the factor K_s determined from Fig. 7. Then the maximum tooth induction is:

$$B_m = B + \frac{B \times P \times K_s \times 10^{-4}}{2} \qquad (6)$$

From this value of B_m and with Fig. 1 or 2 the fundamental frequency tooth losses may be calculated, assuming a suitable space factor in order to determine the net tooth volume.

Surface Losses

Four variables are involved in the calculation of surface losses assuming that the member for which the losses are to be calculated has no slots. This assumption is approximately correct for rotors with nearly closed slots.

The variables are B_a average air-gap induction in kiloline/sq. in. (flux per pole divided by air-gap area per pole).

f_r or f_s tooth frequency; f_r is calculated from the rev. per min. and the number of rotor slots and f_s from the rev. per min. and number of stator slots. It must be understood that for the calculation of rotor surface losses f_s is to be used and for the calculation of stator surface losses f_r is to be used.

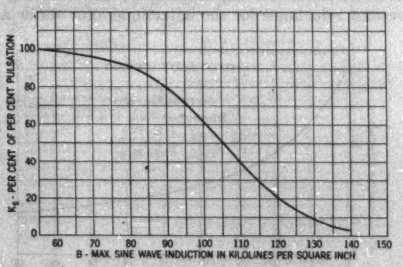

Fig. 7—Tooth-Saturation Correction Factors for Calculating Pulsations

σ_r or σ_s slot widths in inches for the rotor and stator respectively.

q_r or q_s equals $\frac{\sigma_r}{g}$ or $\frac{\sigma_s}{g}$ ratio of slot width at air-gap to single gap width.

The rotor surface losses per sq. in. of surface are then:

$$W_{s_r} = K_{B_a} \times K_{f_s} \times K_{\sigma_s} \times K_{q_s} \qquad (7)$$

where the various K factors are obtained from Fig. 8, the K factors being read from the curves corresponding to the variables B, f, σ, and q.

The total rotor surface loss equals W_{s_r} multiplied by the air-gap area.

If the rotor has partly open slots multiply B_a by the rotor-gap factor K_g as determined from Fig. 6 and the rotor slot and tooth dimensions; also in this case, use the actual area of the rotor tooth tops.

If the rotor slot openings are small, the stator surface losses will be negligible. If the rotor slot openings are fairly wide use B_a as corrected by the rotor gap factor and the formula

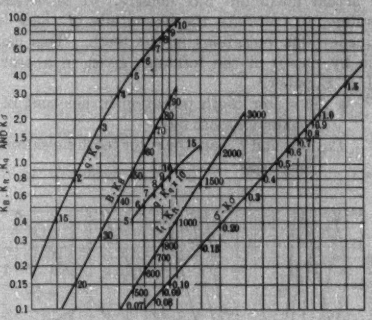

Fig. 8—Curves for Calculating Surface Losses

$W_s = K_B \times K_{fl} \times K_q \times K_\sigma$
W_s = Watts per sq. in. of surface
B = Average air gap induction in kilolines/sq. in.
fl = Tooth frequency in cycles per sec.

$$q = \frac{\sigma}{g} = \frac{\text{Slot width}}{\text{Single gap}}$$

σ = Slot width in inches

$$fl = \frac{\sim \times \text{no. of teeth}}{\text{pairs of poles}}$$

$$= \frac{\text{rev. per min.} \times \text{no. of teeth}}{60}$$

For rotor use no. of stator teeth. For stator use no. of rotor teeth

$$W_{s_r} = K_{B_a} \times K_{f_r} \times K_{\sigma_r} \times K_{q_r} \qquad (8)$$

W_{s_r} is then to be multiplied by the gross area of the stator tooth tops.

The curves of Figs. 1 and 2 are for 1 per cent, 17-mil thick silicon steel. For a medium silicon steel (2½ per cent silicon) reduce the calculated losses by 25 or 30 per cent. For high silicon steel (4 to 5 per cent silicon) use a reduction factor of from 45 to 50 per cent.

Tooth-Pulsation Losses

The method of calculating the magnitude of the tooth pulsations has already been given. Instead of correcting for saturation by Fig. 7, however, correction for the calculated pulsation P, as obtained from the formulas, may be made by Fig. 9 Assuming a straight line relation between induction and percentage pulsation P as effected by saturation, the following formulas may be used instead of Fig. 9.

$$P_S = P\left(1.6 - \frac{B}{100}\right) \tag{9}$$

Fig. 9 and formula (9) are based on theoretical considerations only. Experimental results on actual machines seem to indicate that the following formula more nearly fits the facts for large induction motors.

$$P_S = P\left(1.3 - \frac{B}{165}\right) \tag{10}$$

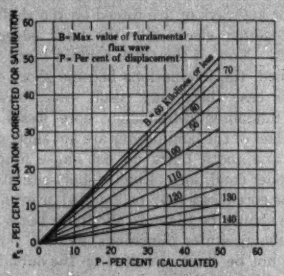

Fig. 9—Tooth-Saturation Correction Curves for Calculating Pulsation Losses

These corrections to P are based on the assumption that the losses vary as the square of the pulsation amplitude. Of course, the effect of saturation increases as the pulsations follow up along the fundamental flux curve toward the maximum value. These saturation effects have been integrated for the whole fundamental wave. The P_S values as determined from Fig. 7 and from Fig. 9 are not very different. If desired, therefore, either value of P_S can be used for calculating fundamental-frequency and tooth-pulsation losses.

The variables for calculating pulsation losses are as follows: f_{st} or f_{rt} as above, where f_{st}, for instance, corresponds to the number of stator teeth but is used to calculate the rotor tooth-pulsation losses.

B_m, the maximum tooth induction, which for the stator teeth may be taken as the same as used in the calculation of the fundamental frequency tooth losses or more accurately should be recalculated using curve 9 or formula (9) or (10). In the latter case:

$$B_m = B\left(1 + \frac{P_S}{200}\right) \tag{11}$$

where B is the net tooth induction $\frac{1}{3}$ from the narrowest section, P_S is the percentage pulsation as corrected for saturation.

$P\,B_m$ the product of B_m and P_S. The tooth-pulsation losses for a wound rotor per net, cu. in. of core material are:

$$W_{rtf} = \underbrace{(10^{-3} f_{rt} K_{Bm} K_P)}_{\text{(Hysteresis)}} + \underbrace{(K_{PBm} K_{frt})}_{\text{(Eddy)}} \text{ watts} \tag{12}$$

The K factors are obtained from Fig. 10, K_{Bm} corresponds to the rotor tooth B_m as obtained from formula (11). K_P is the rotor pulsation constant corresponding to the percentage pulsation as corrected for saturation.

The hysteresis component of this formula was obtained from a fundamental study of the losses of displaced hysteresis loops. The eddy-current component was obtained by many tests on an experimental machine supplied with various slot combinations.

The stator tooth-pulsation losses are calculated as follows when they are appreciable:

$$W_{stf} = (10^{-3} f_{rt} K_{Bm} K_P) + (K_{PBm} K_{frt}) \text{ watts} \tag{13}$$

If the machine has a squirrel-cage rotor instead of a wound rotor the conditions are different. In this case due to the short-circuited turns around the teeth no very appreciable high-frequency flux can flow. Thus the tooth-frequency losses in the rotor teeth will be negligible. As pointed out by Alger and Weichsel[5] some years ago, however, the high-frequency currents which flow in the rotor bars produce a corresponding

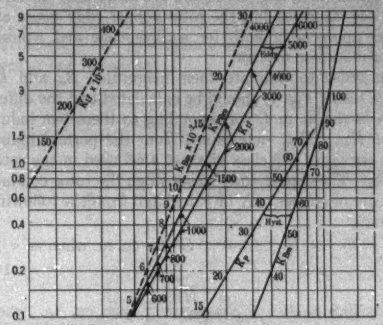

Fig. 10—Curves for Calculating Tooth Pulsation Losses

Note: All inductions and vol. are based on net section assuming a space factor of 90 per cent.

W_{tf} = $(10^{-3} f_t K_{Bm} K_P) + (K_{PBm} K_{ft})$
W_{tf} = Tooth frequency loss in watts per cu. in.
$f\,t$ = Tooth frequency
K_{Bm} = Max. tooth induction factor (B_m is in kilolines per sq. in.)
K_P = Tooth pulsation factor (P is in per cent of B_m)
K_{PBm} = Factor for % tooth pulsation and max. induction product
K_{ft} = Tooth frequency factor

$$f\,t = \frac{\sim \times \text{no. of teeth}}{\text{pairs of poles}}$$

$$= \frac{\text{rev. per min.} \times \text{no. of teeth}}{60}$$

For rotor $f\,t$ use no. of stator teeth
For stator $f\,t$ use no. of rotor teeth

m. m. f. which acts on the stator teeth to produce high-frequency pulsations in these teeth which may be responsible for very considerable losses.

Appendix I gives some experimental data showing this effect quantitatively. The magnitude of the stator-tooth pulsations as a result of the high-frequency rotor currents would be difficult to calculate accurately. Therefore, in the case of squirrel-cage motors the

5. A. I. E. E. Trans., Vol. 44, 1925, p. 160.

expedient has been adopted of calculating the rotor-tooth pulsation percentages as if there were no squirrel-cage windings, then assuming that these are the percentage pulsations which actually occur in the stator teeth due to the rotor high-frequency currents. The rotor-tooth pulsations and therefore the pulsation losses are assumed negligible and the stator-tooth pulsation losses calculated from formula (13) assuming P_s equal to the calculated rotor tooth pulsation percentage. When the number of stator and rotor teeth is not very different, this is a fairly reliable assumption.

Copper Eddy Losses

The cause and nature of these losses have been discussed in a previous paper.[6] They become of appreciable magnitude only at quite high tooth flux densities. These losses are caused by high-frequency slot-leakage fluxes which occur due to the saturation of the teeth. In general the stator no-load copper eddy losses are negligible since the conductors are small and the teeth are less saturated but the rotor copper eddy losses are often appreciable both for squirrel-cage and wound-rotor induction motors.

In order to estimate their magnitude B_m and $B_m - P_s$ (using Fig. 7) should be calculated and the difference in the m. m. f. corresponding to these two inductions H_{max}, H_{min} determined from the normal induction curves of the class of sheet steel used in the rotor teeth. The copper eddy losses when the rotor conductors are of copper may be calculated from the following formula:

$$W_e = 96.8 \times 10^{-10} f_{st}^2 W_{cu}^2 H^2 \text{ per cu. in. when } m < 2. \quad (14)$$

$$W_e = 10.9 \times 10^{-7} \frac{f_{st}^{3/2} H^2}{\left(\frac{t_{cu}}{1.5\,\sigma}\right)^{1/2} W_{cu}} \text{ per cu. in. when } m > 2. \quad (15)$$

where f_{st} is the tooth frequency as calculated from the number of stator slots and the speed of the rotor.

W_{cu} is the radial depth of the individual bars in inches. H is the difference in m. m. fs. between adjacent teeth (in gilberts per cm.)

$$m = 0.148 \sqrt{f_{st} \times \frac{t_{cu}}{\sigma}} \text{ for copper.}$$

t_{cu} = thickness of the copper bars (tangential) multiplied by the number of bars side by side in the slot; σ is the slot width.

In calculating H, of course, H_{min} and H_{max} do not occur simultaneoulsy in adjacent bars but a fair average is to assume that $H = (H_{max} - H_{min}) \times 2/3$.

Illegitimate Losses

These losses have already been mentioned previously and are extra losses in addition to those which would occur in annealed unsaturated cores with perfect insulation between laminations. They are difficult to estimate due to their variability and should be eliminated as far as possible. They are one of the major causes of differences between calculated and test results.

One of the most common types of extra losses in rotating machines appears as the result of eddy currents caused by short-circuited laminations. Short-circuited laminations may be produced when types of construction are used such that the punchings are wedged onto dovetails or keyways or are forced on the shaft. This connects together electrically the back edges of the laminations. Now, if the teeth contain burrs or if the slots are filed, the laminations become short circuited and in this case there may result very considerably increased fundamental-frequency core losses. When the back edges of the laminations are not short circuited slot filing and tooth burrs do not cause much extra losses. Under conditions of badly short-circuited laminations, the core losses may be doubled or even tripled. Great care should be taken to eliminate burrs and slot filings, assuming stator punchings, so that the laminations will not be short circuited on the outside edges. Due to non-uniform flux as a result of these extra eddy currents, the hysteresis as well as the eddy losses may be increased.

For motors with narrow teeth, there may be considerably increased losses due to punching strains. At an induction of 65 kilolines per sq. in., the losses are increased by an amount which can be calculated approximately by increasing the standard loss by a factor equal to 14 divided by the mean tooth width in inches; namely:

$$W_{cor} = W_{std}\left(1 + \frac{0.14}{t}\right) \quad (16)$$

At 100 kilolines the correction is perhaps not over one-half this value and at very high flux densities is very small. If the stampings are annealed after punching there is no such correction.

If the punchings are not flat, bending strains may appear increasing the losses at medium inductions. Above 100 kilolines there would be very little effect due to this cause.

If a motor is operated at very high core inductions leakage flux into the frames, end-bells, and other solid members may cause the losses to go up very rapidly with voltage. In a properly designed machine operated at normal voltage this should be a small factor.

One or all of these factors may be of appreciable magnitude in commercial machines. As a consequence, in calculating the fundamental-frequency losses it is advisable to substitute for the curves of Figs. 1 and 2 others which are determined from actual tests on commercial machines of various standard types. Small induction motors may require one set of curve and large motors another set. The losses,

6. *No-load Copper Eddy Current Losses*, T. Spooner, A. I. E. E. Trans., Vol. XLV, 1926, p. 231.

particularly at the higher inductions, will be appreciably greater for these empirical curves than for those given in Figs. 1 and 2. Such curves are not given here since they would have to be determined by every manufacturer for his apparatus and would be a function not only of design but of shop practise.

Experimental Methods of Separating Losses

The several different kinds of loss which must be considered to obtain the total have been described and it would be very convenient if each one of these several losses could be tested alone or separated from the others. This ideal will hardly be realized, but tests which give a separation of losses with respect to the frequency which causes them can be made.

One method which is applicable only to wound rotor motors was described by R. Richter in the *Elektrotech. Zeitsch.*, January 6, 1921. This method separates the fundamental-frequency losses from the tooth-frequency losses and is carried out as follows: Three saturation curves are taken by varying the applied voltage and reading current and watts. One curve *a* is taken with power applied to the stator and the rotor rotating with short-circuit armature. The second curve *b* is taken with power applied to the rotor collector rings. The stator is short circuited and the rotor is rotating at full speed. The third curve *c* is taken with power applied to either stator or rotor (preferably the stator) and with the other member open circuited. The rotor does not rotate in this case, but should be turned slowly to average the flux due to different positions of stator and rotor teeth.

All curves are corrected to eliminate the $I^2 R$ losses and the friction and windage in curves *a* and *b*. Large rotors usually have a strap conductor and the resistance must be corrected for skin effect when calculating the $I^2 R$ loss in this member.

Curve *a* (corrected) contains losses due to the line frequency in the stator and also all the high-frequency losses. Curve *b* (corrected) contains losses due to the line frequency in the rotor and also all the high-frequency losses. Curve *c* (corrected) contains losses due to line frequency in both rotor and stator and no losses due to tooth frequencies since the rotor is not rotating (practically).

Since curves *a* and *b* are taken from different members in which the voltage is very seldom exactly the same, the losses in the two curves are compared at voltages which give the same densities in the air-gap or any other part of the circuit.

If the curves *a* and *b* are added, the loss found contains the line-frequency losses in both stator and rotor and also twice the high or tooth-frequency losses. If the values from the *c* curve, which contains only the line-frequency losses in both stator and rotor, are subtracted from the sum of curves *a* and *b*, the result is twice the tooth-frequency losses.

In this way, the tooth-frequency losses are separated and if they are subtracted from curves *a* and *b*, the line-frequency losses in the stator and in the rotor may be found separately.

This is the method that has been used in separating the losses in some of the motors shown in the curves.

Another method which is applicable to all types of machines is described by Messrs. Alger and Eksergian.[7] The fundamental idea is covered by their statement: "It is a general principle that whenever any cyclic variations in the permeance of a magnetic circuit are caused by the relative movement of parts of the circuit, all the losses caused by this variation are supplied by the mechanical agency causing the motion."

Applied to the induction motor, this means that the tooth-frequency losses are supplied by torque in the rotor. Now, the rotor can only receive power by running slower than the rotating field, *i. e.*, slipping, and by hysteresis loss in the rotor core and teeth. The slip can be measured very accurately by several means so that this portion of the rotor torque is known, but the hysteresis losses in the rotor core and teeth must be calculated. The hysteresis loss effect is as though it were at line frequency.

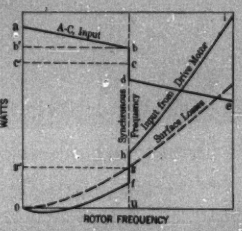

Fig. 11—Loss Diagram for Induction Motor

The no-load slip values decrease as the voltage is increased since the hysteresis loss is increasing and the induced torque due to slip does not need to be so much to overcome the friction and windage and the tooth-frequency losses. It might be of interest to note that in one test we made, the slip decreased to zero, indicating that the hysteresis loss was sufficient to drive the rotor in synchronism.

Another method, requiring a driving motor and also accurate meter readings, makes use of the step in the stator loss curve which occurs when the rotor is driven through synchronism. This has been described by Messrs. Alger and Eksergian[8] and by Messrs. Spooner and Kinnard.[9]

If a constant voltage and frequency are applied to the stator and the rotor is driven by another motor at different speeds, the power taken from the line drops

7. Alger and Eksergian, *loc. cit.*
8. Alger and Eksergian, *loc. cit.*
9. *Surface Iron Losses with Reference to Laminated Materials*, A. I. E. E. Trans., Vol. XLXXX, 1924, p. 252.

and the power taken by the driving motor increases abruptly when the rotor passes synchronous speed. This difference in watts before and after synchronous speed is equal to twice the rotor hysteresis loss.

This can be seen more clearly in Fig. 11.

Curve *a, b, c, d, e* is the stator input. Curve *o, f, g, h, i* is the driving motor output. $I^2 R$ losses and friction and windage losses have been taken out.

b′ c′ is the rotor hysteresis loss and is one-half the step *b d* or *f, h*; *a b′* is the rotor eddy current losses, which at normal speeds is practically zero. *O g′* gives the tooth-pulsation losses and *o c′* corresponds to the line frequency losses.

TABLE I
WOUND-ROTOR INDUCTION MOTORS
COMPARISON OF TESTED LINE FREQUENCY AND SLOT FREQUENCY LOSSES TO CALCULATED VALUES

	Per cent normal voltage	Line-freq. loss		High-freq. loss		Total loss	
		Test	Calc.	Test	Calc.	Test	Calc.
60-cycle...	56.8	1.00	1.36	2.20	2.33	3.20	3.69
12-pole....	79.5	2.10	2.32	3.70	3.55	5.80	5.87
	102.2	4.35	4.83	5.35	5.38	9.70	10.21
	125.0	9.80	9.90	7.40	7.20	17.20	17.10
		0.50	0.50	1.05	0.86	1.55	1.36
60-cycle...	56.8	0.96	1.06	1.94	1.98	2.90	3.04
16-pole....	79.5	1.98	1.97	2.82	3.20	4.80	5.17
	102.2	3.53	3.35	3.57	4.38	7.10	7.73
	125.0	6.03	6.41	4.17	5.83	10.20	12.24
		2.06	2.11	1.65	3.60	3.71	5.71
60-cycle...	56.8	4.67	4.55	6.13	8.35	10.90	12.90
24-pole....	79.5	8.67	8.70	11.63	13.18	20.30	21.89
	102.2	14.65	14.80	16.35	18.55	31.00	33.36
	118.7	19.55	20.15	19.85	21.59	39.40	41.74
	45.5	1.40	1.14	1.50	1.33	2.90	2.47
25-cycle...	60.6	2.31	1.82	2.50	2.11	3.81	3.93
10-pole....	75.8	3.40	2.75	3.50	3.00	6.90	5.75
	90.9	4.80	4.05	4.60	4.06	9.40	8.11
	106.0	7.00	6.21	5.70	5.18	12.70	11.39
	121.1	11.10	10.50	6.60	6.89	17.70	17.39

Checks on other machines, which were not specially tested, show the same general agreement in the total losses.

This method may be used on wound-rotor motors but on squirrel-cage motors the line *a b* is so steep that it is very hard to get satisfactory readings unless the frequency and speed are very accurately controlled.

Test Results

Several large wound-rotor motors have been tested according to the first method for loss separation and the losses have been compared with the losses calculated by the above described methods. The curve for the line-frequency losses was obtained from one of the tested machines and checks well with the loss curve for the grade of iron used. Table I shows the fundamental and the pulsation-frequency losses calculated separately. Table II gives data for other machines without separation of losses. Fig. 12 shows the results graphically for one of these machines.

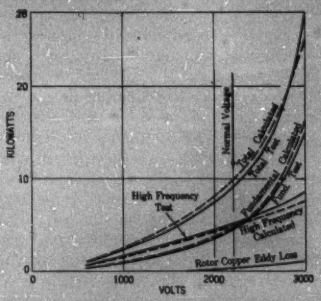

Fig. 12—Wound Rotor Induction Motor

Comparison of calculated and test no-load core losses

In Table III are presented some data on squirrel-cage motors. These checks are not as good as for the wound-rotor machines partly because of the greater difficulties of test. These results were obtained about three years ago. The calculated pulsation losses are probably low because no account was taken of copper eddy-current losses.

TABLE III
SQUIRREL-CAGE INDUCTION MOTORS
COMPARISON OF TESTED AND CALCULATED CORE LOSSES AT NORMAL VOLTAGE

Hp.	Freq.	Line freq. loss		High freq. loss		Total loss	
		Test	Calc.	Test	Calc.	Test	Calc.
250	60	1.98	1.87	1.72	1.22	3.70	3.09
500	60	6.04	6.02	3.93	3.14	9.97	9.16
175	40	1.08	1.07	0.95	0.64	2.03	1.71
125	40	0.82	1.15	1.30	1.27	2.12	2.42
225	40	1.83	1.96	2.09	1.53	3.92	3.49

TABLE II
WOUND-ROTOR INDUCTION MOTORS
COMPARISON OF TESTED AND CALCULATED TOTAL LOSS VALUES

Per cent normal volts	60 cycles 10 poles		25 cycles 4 poles		25 cycles 8 poles		25 cycles 10 poles		25 cycles 34 poles	
	Test	Calc.	Test	Calc.	Test	Calc.	Test	Calc.	Test	Calc.
34.1	2.87	3.50	1.03	12.2	4.8	5.6				
45.5	4.89	5.46	1.68	20.2	11.8	9.9			7.4	6.7
56.8	6.69	7.22	2.37	30.8	16.8	16.1	68.0	50.4	15.1	13.4
68.2	9.55	10.36	4.16	45.3	28.4	26.0	88.6	86.2	24.6	21.7
79.5	12.70	13.36	6.22	61.4	38.0	38.1	137.0	141.	34.9	31.9
91.0	16.58	16.64	8.00	80.6	49.5	49.7	175.0	185.	46.3	44.4
100.0	20.26	19.85	10.66	10.60	63.8	64.2	204.0	222.	53.8	52.8
113.7	25.40	26.70	12.35	12.33	81.2	83.9	258.0	272.	57.9	59.6
125.0	32.00	32.46	14.10	14.67	99.0	98.2	345.0	342.	68.1	75.3

Conclusions

The above described method of calculating no-load induction-motor core losses appears to be rather tedious and complicated and as compared with the older empirical methods this is the case. However, by the use of a suitably prepared schedule the time may be greatly shortened. The time for such a calculation, provided all of the factors which have to be calculated for other purposes are available, is from 30 to 40 min. This is not excessive when dealing with a radically new design or with a type of machine which has given trouble due to high losses. It is hoped that this paper will stimulate discussion and will bring forth other methods for making these detailed calculations.

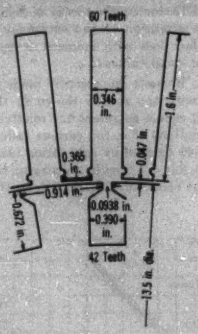

Fig. 13—Special Induction Motor Showing Tooth Slots

Fig. 14—Squirrel-Cage Induction Motor

Stator tooth flux
Normal voltage—no rotor bars

Appendix

In a squirrel-cage induction motor having nearly closed rotor slots, open stator slots, and no rotor windings there would ordinarily be quite appreciable rotor-tooth flux pulsations but only very small stator-tooth pulsations. When, however, the squirrel-cage winding is applied, the rotor-tooth pulsations are damped out by the short-circuited copper turns. The high-frequency currents thus generated in the rotor bars produce a considerable m. m. f. which is approximately equal to the change in m. m. f. applied to the rotor teeth caused by the pulsating reluctance in the gap resulting from the stator slots. This high-frequency m. m. f. produced by the rotor windings causes pulsations of flux in the stator teeth which (for the ordinary induction motor having nearly the same number of stator and rotor teeth) are of the same order of magnitude as the pulsations which would have occurred in the rotor teeth had there been no rotor winding. The stator-tooth frequency will be that calculated from the number of rotor teeth and the peripheral speed.

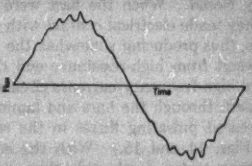

Fig. 15—Squirrel-Cage Induction Motor

Stator tooth flux
Normal voltage—rotor bars but no end-rings

In order to show this, test results were obtained on a 60-slot 4-pole stator fitted with a 42-slot rotor with a copper bar winding and brass end rings. The stator slots were open and the rotor slots nearly closed as shown by Fig. 13. This was a very special machine built for experimental purposes.

An exploring coil was placed around one of the stator teeth and oscillograms obtained of the induced voltage under conditions of normal induction and 20 cycles. This was done for 3 conditions:

1. No bars in the rotor slots.
2. Bars in the rotor slots but no end rings.
3. Bars in the slots and end rings connected.

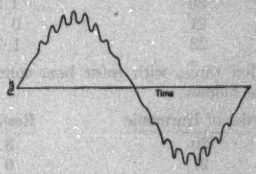

Fig. 16—Squirrel-Cage Induction Motor

Stator tooth flux
Normal voltage—rotor bars with end-rings

Oscillograms of the voltage in the stator-tooth exploring coils were obtained for the three conditions. These oscillograms, of course, show very large high-frequency voltage components.

These oscillograms were then enlarged and integrated in order to obtain the corresponding flux waves. These are shown by Figs. 14, 15, and 16.

These tooth-voltage waves may be integrated automatically if desired by inserting a large inductance in the circuit containing the exploring coil on the stator

tooth and the oscillograph element. This was actually done in this case. Due to the difficulty of obtaining a very large ratio of inductance to resistance the integration was not particularly satisfactory. Since the phase angle of the harmonics with respect to the fundamental was not correct, these oscillograms are not reproduced here.

It will be seen from the curves that, with no rotor bars, the stator-tooth pulsations were very small as would be expected from the fact that the rotor slots were nearly closed. When the bars were inserted in the slots, they made electrical contact with some of the laminations, thus producing somewhat the same effect as would result from high-resistance end rings. As a consequence high-frequency currents circulated around the rotor teeth through the bars and laminations producing increased pulsating fluxes in the stator teeth. (Compare Figs. 14 and 15.) With the end rings in position, these stator tooth pulsations were further increased due to larger high-frequency currents in the rotor bars. (Compare Figs. 15 and 16.)

The frequency of the stator-tooth pulsations should correspond to a 21st harmonic as calculated from the number of rotor teeth. Since, however, these pulsations were in turn proportional to the amplitude of the fundamental-frequency flux, a harmonic analysis should show that the pulsations consisted of a 20th and 22nd harmonic with no 21st. Due to the fact, however, that the variations in slot reluctance at the gap did not follow a sine law and to saturation effects, there were other harmonics present. A harmonic analysis of the flux waves was actually made and for condition one, with no rotor bars, the analysis gave the following results:

Order of Harmonic	Resultant per cent
20	1.8
21	0
22	1.7

For condition three, with rotor bars connected, the analysis is as follows:

Order of Harmonic	Resultant
20	3.3
21	0
22	4.9

The other harmonics have not been included since that would have added nothing of value to the discussion.

Discussion

P. L. Alger: This paper is a logical sequel to Mr. Spooner's extremely well coordinated series of earlier papers. Many years ago Professor C. A. Adams suggested the rational analysis of induction-motor core losses, and his 1909 paper was the real starting point of the long series of investigations which Mr. Spooner has now brought to so successful a conclusion.

Of particular interest to me is the assumption made that the percentage flux pulsation in the stator teeth of a squirrel-cage induction motor is the same as that which would have occurred in the rotor teeth if a slip-ring winding had been employed. Ever since Chapman[1] showed the manner in which rotor flux pulsations set up circulating currents in a squirrel cage, it has been clear that these currents were a cause of extra pulsations in the stator, but the present proposal is the first practical one I know of for calculating them. While a more detailed analysis of these induced currents would be instructive, I think the assumption made is quite satisfactory.

There have been two especially important articles on core losses recently published abroad, a very complete treatise by Dreyfus[2] on core losses in slip-ring induction motors, and a paper by Coe and Taylor[3] on the theory of pole-face losses. The latter paper calls attention to the extremely rapid theoretical increase of pole-face loss with increase in the slot width over air-gap ratio, q, at small values of q.

Professor A. A. Bennett has recently made a careful analysis of this subject, with the results shown in the accompanying figure. The solid curves represent the relative theoretical pole-face losses as functions of q, for various ratios of slot-to-tooth width. These were derived by calculating the maximum, minimum, and average flux densities in the air gap, by elliptic function theory, for various values of q, deriving from these three quanti-

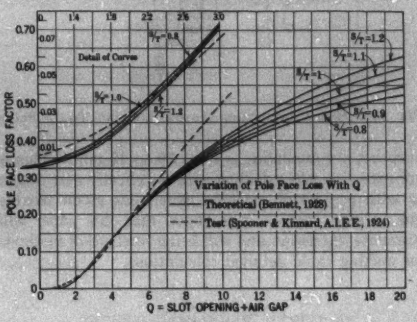

Fig. 1

ties the zero, first, and second order harmonics, A, A_1, A_2, of a Fourier's series representing the flux distribution, and thence determining a relative pole-face loss factor equal to

$$\frac{A_1^2 + 2\,A_2^2}{A_0^2}$$

The A_2^2 term was given double weight in determining the loss factor on the assumptions that the loss increases as the square root of the product of frequency and tooth pitch, and that the neglected higher-order harmonics would amount to about half as much as the A_2 term alone.

The dashed curve in the figure gives the average of the relative pole-face loss factors due to q given by Spooner and Kinnard for four different kinds of laminated steel in their 1924 paper. The loss factors compared being merely relative, the points at $q = 5$ on the two curves were taken as the same. The theoretical curve shows that as the value of q decreases below 1.5 the pole-face loss decreases with extreme rapidity, being zero for all practical purposes when q falls a little below unity. The test

1. *London Electrician*, 1916.
2. *Archiv f. Elektrotech.*, May-July, 1928.
3. *Philosophical Mag.*, Vol. VI., July 1928.

data of Spooner give higher losses than the theoretical at both high and low values of q, but they agree very well in the middle range. The theoretical curve is of particular interest, as it explains how turbine generators, having very small values of q, can use solid steel rotors, while other machines with moderate values of q must employ laminated rotors.

C. A. Adams: The point brought up by Mr. Alger seems to me of vital importance, namely, desirability of developing our rational knowledge of this subject to such a point that we can select slot ratios and other proportions to the end of reducing these losses to a minimum.

We are altogether too apt to hesitate to tackle theoretically a job which appears on the face of it to be very complicated, and I am sure that those of us who have ever persisted, have discovered that you can carry theoretical analysis much farther than was at first obvious, and to very great advantage. As long as we are dependent upon experimental data or upon the cut-and-try method, we are almost helpless when we attempt to solve many of the more intricate problems in this field.

C. W. Kincaid: I might say to Mr. Adams that before we had this method perfected we used one of his formulas. Of course, it was made up for d-c. machines and we made a few modifications. We have used that for years. In fact it is now difficult to convert some of the men to the new method. They still stick to his formula and say it is quicker than the one we are using now.

Insulation Tests of Electrical Machinery

Before and After Being Placed in Service

BY C. M. GILT[1] and B. L. BARNS[2]

Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—The A. I. E. E. Standards have provided high-potential test values as a means of establishing a standard basis of design and construction of insulation. Many purchasers test equipment after installation before placing it in service to be sure that the connections and equipment are in a safe condition for service. It is recommended that a Standard Test Voltage be established that may be used under these conditions.

It also seems to be advisable that suitable rules should be set up governing test voltages that should be used in a maintenance program. Periodic observations of the insulation resistance are suggested as substitute or supplemental tests. If there is a general desire that rules of this sort should be made, it is proper that the Institute should enter this new field of standardization.

* * * * *

THE work of standardization performed by the American Institute of Electrical Engineers has been and is one of the most valuable undertakings that lie within its field of activities. This standardization has been primarily in the nature of establishing rating standards by which equipment of different manufacturers can be compared. While these standards have been based upon what is believed to be typical or average operating conditions they obviously are not standards for operation.

There is a rather decided opinion among many engineers that the Institute should undertake the development of standards for operation in fields in which they can be established.

One of the fields in which it is believed such standards can be developed is that of high-potential tests. These tests are not offered as standards of comparison of equipment nor as acceptance tests for purchasers from manufacturers, except as possibly determined by special contracts, but are intended as operating guides. They are an attempt to crystalize a variety of opinion as to what should be an adequate and safe test to determine the suitability of equipment for service, for those who wish to employ high-potential tests for this purpose.

This paper endeavors to present the subject in two parts, first, a test for new equipment before being placed in service, and second, a high-potential test for occasional check on equipment that is or has been in service. The first part has been prepared chiefly by the first author and the second part by the second author of the paper.

Part I.

Insulation Tests for Initial Operation

High potential testing may be roughly grouped in three general classes, each with a distinct purpose in view, but all with the fundamental object of determining the ability of the equipment to withstand the normal and abnormal potentials that may be reasonably expected under the operating conditions for which the device is designed.

The first group of tests is intended to determine the characteristics of the material or design in question. Such tests may be fundamental in character, such as determining the behavior of material under high-potential stresses, as well as of more immediate use, such as determining thickness of insulation or contours of creepage surfaces.

The tests of the second class are those carried on to discover factory errors in the manufacture of equipment or details of design. In general, the routine factory high-potential tests conducted on equipment at the completion of its manufacture fall into this class. While these high-potential tests are sometimes used as a check on the design when differing from the usual practise, the necessary thickness of insulation and clearance between live parts are so well established for ordinary service that there would be little need for the factory high-potential tests on normally designed equipment supplied by reputable manufacturers, when once the standard of design and over-potential stresses have been established, were it not for defects in material and workmanship that occasionally occur.

The third class of high-potential tests is made to determine the adequacy of equipment for service, to discover whether some accident has happened to the equipment since the factory test prior to its final connection to the lines ready for service or after once having been placed in service whether there has been sufficient deterioration of material or accidental damage to make the equipment unsafe for service. The tests after installation and prior to service commonly include, not only individual pieces of equipment such as motors or transformers, but assembled equipment in generator stations, substations, or customer's vaults and include cables, insulators, circuit breakers, transformers, instrument transformers, etc., as they are connected for placing in service.

The present rules of the Institute covering high poten-

1. Inside Plant Engineer, Brooklyn Edison Co., Brooklyn, N. Y.

2. A-c. Engineer, Canadian General Electric Co., Peterborough, Ont., Can.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

tial tests were primarily adopted as a means of establishing minimum standards of design for ordinary operating conditions and are essentially the result of experience that the equipment so designed will have a reasonable life under normal operating stresses including such overpotential surges as are commonly met with under ordinary operating conditions. The tests were further intended to insure that the insulation met the expectations of design. Over-potential surges of various magnitudes and duration occur repeatedly during the life of the equipment. Some have a higher and some a lower value than the test value used to determine the adequacy of the design and material but while there has been some agitation to increase the standards of insulation and insulation tests for given operating voltages, experience indicates that insulation that will withstand the standard factory test will withstand the ordinary repeated over-voltage surges during what has been considered a reasonable life.

While high-potential tests may have some deteriorating effect on certain classes of insulation, this deteriorating effect is so slight that it is accepted as a price that must be paid in the present development of the art as a means of reducing to a minimum the disastrous effects of failure of equipment in service as a result of defects or accidents that occasionally occur in manufacture and installation. Unfortunately the insulation processes are so different from that of working metals and alloys that a test sample of the material cannot always be taken as a reasonably conclusive proof of the strength and adequacy of the completed article.

Subsequent to the tests at the factory finishing touches are sometimes added to it. It is crated, shipped, and connected ready for service in the final installation and during this handling process accidents to the equipment occasionally occur, which if not discovered prior to placing in service, may result in serious damage to it, other equipment, and to the service rendered by the purchaser. These defects cannot always be discovered by observation. The placing in service of a new, large piece of equipment with its various connections is a nerve-racking undertaking and many of the operating companies believe it essential to take every precaution to be sure that no faults in equipment or connections exist that will result in expensive repairs and serious delays that occur when a fault develops with the tremendous capacity of modern systems feeding into it.

These tests are not ordinarily considered acceptance tests unless the equipment is delivered and erected by the manufacturer, as the equipment commonly passes out of control of the manufacturer when it leaves the factory, but are intended to cover the period and work between shipment and operation.

Obviously, equipment which is assembled on the premises of the purchaser without a complete factory test should receive the same test as though it had been previously assembled and tested at the factory.

As a generally accepted example of this practise, a test is placed on cable before it leaves the factory to discover defects in the manufacture of the cable and after the somewhat severe process of pulling into the ducts, a further test is placed upon it and the joints to be sure that the completed cable is ready for service. In certain respects this condition is somewhat different from other equipment as the test after installation is partly to determine the ability of the cable to withstand the process of pulling into ducts and as damage from exterior causes is somewhat easier to determine by inspection than in more complex pieces of equipment. It is recognized that the process of installation certainly reduces the strength of the insulation at various points and consequently the tests after installation are lower than those at the factory, or rather one might say, the factory tests are somewhat higher than the standards set after installation to make due allowance for the deterioration that results from handling.

Similarly, the various parts of assembled equipment are given individual high-potential tests before they are assembled in the completed machine and the value of the tests of the various parts is higher than that of the completed piece of equipment. The final tests of the completed machine are to insure that no damage has resulted to the individual parts and to place a test upon such connections as cannot be effectively tested before the final assembly. An allowance is made for the deterioration of portions of the equipment and the rules recognize a test on the completed assembly of 15 per cent lower than the tests on the part receiving the lowest individual test.

In spite of the very good reason for testing of equipment before energizing with service voltage, many careful purchasers of equipment forego this final check for one or both of two reasons. First, there is the general feeling that each high-potential test causes some permanent damage to the insulation, and second, that the insulation may not be in as good condition as before leaving the factory due to the accumulation of dust or the absorption of moisture and while safe for operating voltages the high-potential test might be damaging. It cannot be denied that many purchasers do not have the facilities for insuring that the insulation is thoroughly dry and in proper condition for a high potential test and the manufacturers quite naturally question the advisability of a high-potential test of a severe nature under these conditions. Experience has clearly indicated that there have been many pieces of equipment that have absorbed moisture to the point that a high potential test would be somewhat hazardous and yet have operated under load safely and have been dried out by that process.

The question of the voltage strain and damage to the insulation is one on which opinion is very widely separated. That there is probably some slight deterioration with each high-potential strain is commonly believed and one school of thought holds that in the

long run there are more failures and more damage as a result of the cumulative deterioration of equipment in reasonable condition as a result of the repeated strains imposed by tests than occur due to the normal deterioration of insulation and the consequent failures that are not anticipated by test. Certain insulating mediums such as oil and inorganic substances seem to be affected little or not at all by high potential stresses of any number of repetitions provided they are in good condition so that there is no appreciable leakage through them. The actual effect of high-potential stress on fibrous insulation is much more uncertain and whether overvoltage for short periods and within a reasonable value has any appreciable effect upon the insulation is not certain. When the insulation is stressed to a point near that of failure, the matter of time and duration is of great importance in the performance of the material and yet we know that insulation is subjected to brief overvoltage stresses of two or three times normal voltage many times during its life and whether the life of the insulation is reduced appreciably beyond the limit of the normal aging process in service, is not certain. It would seem that an additional application of overvoltage of short duration or of a value reasonably below the breakdown value of the insulation placed upon the equipment before going into service would not have any appreciable effect upon the life of the insulation. Unfortunately, the phenomenon of electrical breakdown of insulation is not clearly understood and it is hoped that out of the various studies that are now being conducted on what actually takes place when insulation fails, will come information that will point to the answer of this much discussed question of repeated tests. The tests reported by F. M. Clark on *Dielectric Properties of Fibrous Insulation*, (A. I. E. E. TRANS., Vol. XLV, 1925, p. 193) indicate that for a certain type of insulation the breakdown voltage is actually increased by a number of previous high-voltage tests, probably due to a drying out process, but that the insulation is permanently damaged by too large a number of overvoltage applications. The further conduct of such tests on various materials and with relative magnitudes of voltage and time will do much to settle this question.

It is obvious that equipment should not be subjected to high potential tests unless the insulation is in such a condition that it may be reasonably expected to withstand the test without damage or failure. The equipment must be carefully gone over to be sure that there are no apparent points of weakness or damaged parts of insulation, to be sure that there is no foreign material in the equipment that might cause a breakdown, that it is free from an accumulation of dust that might result in a failure, and above all one must be sure that the insulation is free from moisture. When equipment is shipped from a factory even though protected to some extent from atmospheric conditions it almost surely will absorb considerable moisture if at all hygroscopic. One would not think of operating or testing an oil insulated transformer that had been exposed to the air without oil for any but the briefest period, without a thorough drying out, and if shipped filled with oil a test of the oil should be made to insure freedom from moisture and high dielectric strength.

Similar precautions are and should be taken before equipment is placed in service even though no high-potential test is made. It should not be considered safe to operate equipment that is not in a condition to withstand safely an overvoltage test. This is particularly true of equipment connected to high-voltage lines where the equipment is quite commonly subjected to overvoltage stresses due to surges or various causes. Low-voltage equipment in which the insulating value of insulation is far beyond that required by the ordinary operating voltage may sometimes be operated with a fair degree of safety under conditions in which it would not withstand a high-potential test without serious danger of damage. Such conditions are rather the exception when considering equipment which a purchaser would ordinarily undertake to test.

Due to the fact that it is desirable not to cause any undue deterioration of the insulation by the test procedure and due to the fact that the equipment may have been subject to some slight deterioration as a result of handling and may have absorbed some slight amount of moisture, and that factory tests have already been made to insure against inherent defects in the insulation, it would seem reasonable that the test before operation should be somewhat less severe than at the factory. It should be searching enough to discover damage of the insulation that may later develop into a serious fault. The reduction of test may be in either time or magnitude of voltage or both and there is a variety of opinion as to what this reduction should be. A number of arguments has been advanced for a reduction of as much as 25 per cent of the original test voltage while others believe that the reduction should not be more than 15 per cent to correspond with the existing standards for the reduction in test voltage of assembled equipment while still other arguments are advanced for maintaining the magnitude of the original factory test voltage but reducing the time of test from one minute to momentary. The British standards recognize the establishment of such a test voltage for this purpose and have set it at 25 per cent below the factory test voltage.

In view of the differences in opinion as to what such a test voltage should be, it would seem desirable for the Institute to broaden its range of standards and include a standard of test voltage that may be used when desired by the purchaser when placing new equipment in service.

This standard is not intended to be mandatory but should be considered as establishing a standard test voltage that may be applied at the option of the purchaser, not as an acceptance test, but to assure him-

self as far as is possible that neither the equipment nor the service will be damaged by some accident that may have occurred to the apparatus.

It would appear that there is a field for such a standard and it is recommended for further consideration that the Institute establish a standard of test voltage that may be used for new equipment after installation of 85 per cent of the factory test voltage of the equipment connected at the time of the test receiving the lowest factory test and for a period of one minute.

Part II

Insulation Tests of Electrical Machinery in Service

For the same reason that it is desirable to set up a standard practise for testing the insulation of electrical apparatus at the time of initial installation, it is also desirable to establish a rule or recommendation as to what tests would be suitable for a maintenance program.

It is true that since the manufacturer is presumably desirous that apparatus of his manufacture should be properly maintained the user should be able to obtain recommendations of this sort from the manufacturer. This has been done to some extent but there is a desire that an agreement should be made regarding suitable tests that would be applicable to apparatus of any manufacture. Such a procedure would be of great assistance in formulating a suitable maintenance program where apparatus of more than one manufacture is installed in the same station on system.

The lack of any semblance of uniformity in practise in this respect has been drawn to the attention of the Committee on Electrical Machinery and after a careful consideration this committee has decided that a discussion of this subject should be presented to the membership of the Institute in the hope that it may lead to an agreement of some sort. Previous attempts have been made to formulate some standard practise or recommendation of this kind and probably the best efforts in this direction have been made by the Apparatus Committee of the National Electric Light Association by whom questionnaires were circulated among members for the purpose of determining their practises and the degree of satisfactory results. The reports of this committee indicate that the practises of the different members are so various that it is not practicable to choose any one as representing a general preference or giving better results than the others. Some pay no attention to making tests, some occasionally make tests to ground at rated potentials or at slightly higher values, while a small percentage regularly use test voltages approximating the values set out in the A. I. E. E. standards for new machinery.

In considering suitable means of checking the condition of insulation for use in a maintenance program it is but natural to turn to the present A. I. E. E. standards for guidance. But the insulation test voltages set up in these standards are designed to insure that the insulation has sufficient dielectric strength to allow for a certain amount of deterioration and still be sufficient to withstand the voltages that may be impressed upon it under operating conditions and in this sense is in the nature of an expected life test. It therefore appears that it should not be expected that after a period of operation in which undoubtedly some deterioration has taken place the insulation should withstand the same test as when new.

If the deterioration of insulation followed a known rate of reduction in dielectric strength under operation conditions it would be a simple matter to set up a schedule covering the expected strength over a given length of time and the corresponding test voltages. In that event tests would serve principally to search out possible mechanical injuries. But the life of insulation is an indefinite term. It contains, however, the element of time and is dependent upon the temperature to which it has been subjected as well as various other

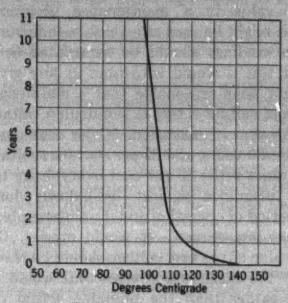

Fig. 1—Insulation Tests of Electrical Machines in Service

conditions. In their paper on *Temperature and Electrical Insulation*,[3] Steinmetz and Lamme offered curves showing the relationship between length of life and temperature. Fig. 1 is a reproduction of their curve for Class A insulation. The curve for Class B insulation is of the same type. In these curves the length of life does not refer to the time in which the insulation will entirely lose its dielectric strength but as explained by the authors it has to do with the time at which the fabric or varnish becomes brittle and loses its mechanical strength. The life of an insulation in this sense may be entirely spent and yet retain approximately its original dielectric strength provided it has not been mechanically injured. Therefore, the term life as used in connection with accepted practises in testing insulating materials is somewhat misleading for it does not have reference to its dielectric property. The useful life of an insulation is not dependent upon the effect of temperature and time alone but also upon various

3. A. I. E. E. Trans., Vol. XXXII, 1913, pp. 79-89.

other conditions which tend to destroy it by mechanical forces. It is reasonable then that the insulation should be expected to last an indefinitely long time if it were protected against mechanical injury barring, of course, the effect of excessively high temperatures which would change the chemical composition or the structure of the insulation. The common causes of mechanical injury are vibration and distortion due to temperature changes and electromagnetic forces. These causes are present in widely varying degrees due to differences in design and conditions of operations and it is, therefore, impractical to formulate a relationship between length of service and dielectric strength to indicate what may be expected of any particular type or class of insulation in a given kind of machine or to define a maximum test voltage that the insulation could be expected to withstand at given intervals.

But supposing that a relationship between time of operation and dielectric strength could be established for certain average conditions, would it be of much practical value in a maintenance program? Conceivably it might be used as a guide to indicate the time when the windings should be reinsulated or replaced and what test it would withstand at any given time. It does not seem reasonable that a maintenance program should call for a test at say double voltage at the end of one year of operation, 50 per cent overvoltage at the end of 3 years, and 25 per cent overvoltage at the end of 5 years if the 25 per cent overvoltage test gives sufficient insurance against breakdown during the ensuing period of operation and if uninterrupted service is of equal importance. It would appear, therefore, that the value of the test voltage should be chosen with respect to the maximum stresses that would occur in operation rather than the probable actual dielectric strength. If, then, operating conditions are defined it becomes possible to set up a value for the test voltage.

It is seldom that machines are required to operate at voltages above 10 per cent over normal. This is especially true of machines connected to low-voltage systems. Important a-c. generators are provided with overvoltage relays to limit abnormal rises of voltage to approximately 25 per cent overvoltage and the neutral point is grounded for relay protection of the winding. Under normal operating conditions the voltage between machine terminals and ground is 58 per cent of the rated terminal voltage and under an abnormal condition of 25 per cent overvoltage this would be increased to approximately 72½ per cent of the rated voltage. An insulation test between winding and ground at rated voltage would impose a stress approximately 38 per cent greater than would occur when the machine voltage is raised to 25 per cent above the rated voltages. Such a test would insure a sufficient margin in the dielectric strength for further service without risk of failure to ground. Such a test would not, however, be a check on the condition of the insulation between phases but by testing one phase at a time with the other two phases grounded the insulation between coils in different phases would be tested at a voltage higher than the operating voltage in proportion to their distance in the circuit from the terminals. The insulation between terminals would not be tested at a higher voltage than the normal operating voltage and there would not be any assurance of a margin in the condition of the insulation between phases.

By separating the tests and making one test to ground and another between phases, two different values of test voltage could be used, each suitable for its purpose without imposing an excessive voltage where a lower test is sufficient assuming that the rated voltage is sufficient for testing between winding and ground a higher voltage than could be chosen to test between phases which would produce the same relative overvoltage stress on the insulation between phases. This result would be obtained by testing between phases at 175 per cent of rated voltage with none of the phases grounded.

To those who are accustomed to associate all insulation test voltages with those that have been prescribed in the A. I. E. E. standards for new machinery, the proposed test voltage between winding and ground appears to be ridiculously low yet it fulfills the requirements. The proposed practise is not new since it has been used by the service departments of at least two of the large manufacturing companies when overhauling electrical apparatus for their clients. Their experience is that this program has produced satisfactory results. On the other hand the author has received many expressions of opinion from engineers who are responsible for the operation of electrical apparatus that the test voltage to ground should not be less than 150 per cent of the rated voltage. Nevertheless many instances have been cited of machines which have given years of service after the insulation has reached such a condition that it would unquestionably have failed at test voltages only slightly higher than normal. It is a commendable practise to maintain apparatus in a first class condition but the greater reliability thus obtained does not always justify the cost. It is to be expected that as the test voltage is increased so is the frequency of repairs increased.

It is generally agreed that high-potential tests of insulation do lasting damage to it and for that reason all unnecessary tests of that nature should be avoided. A maintenance program that would omit high potential tests and substitute some other test that would not damage the insulation is by far the better. While comparatively little experience has been obtained it is believed that the periodic measurement of the insulation resistance of windings would give warning of dangerous conditions that may develop. A program covering the determination of the insulation resistance at regular intervals during the period of operation and high potential tests at times of overhauling when it would be convenient to make repairs should afford a fair degree of

freedom from inopportune shut-downs. The adoption of such a scheme would call for the exercise of much good judgment to obtain satisfactory results and in the absence of sufficient practical experience upon which good judgment can be based it is advisable to set up a recommended practise in applying high potential tests.

In view of the divergence of opinion regarding the value of test voltage that should be used as between rated voltage to ground and 150 per cent rated voltage a compromise might be struck at 125 per cent of rated voltage. In order to use the same form of rule as now used for new machines a constant of 500 volts may be added to this value. Such a rule would then read as follows:

> It is recommended that except for distribution transformers which may offer hazards to life, the dielectric tests of windings of machines in service shall be at 125 per cent of the rated voltage plus 500 volts. When it is convenient to separate the phase windings a further test between phases shall be made at 175 per cent of rated voltage plus 500 volts.

Conclusion

The theme of this joint paper, therefore, is a plea that the Institute enlarge its scope of operations to meet what appears to be a genuine need of the membership at large. This need is two-fold: first, for a basis of insulation testing prior to initial operation, and second, a similar basis for the operating period of this apparatus.

As a basis for consideration and possible acceptance by the Institute, these recommendations have been offered, which are in brief:

> For initial operation—a voltage 85 per cent of factory test for one minute, based on connected equipment receiving lowest factory test, and
>
> For maintenance—a test to ground of 125 per cent rated voltage plus 500; and between phases, 175 per cent plus 500 volts—supplemented by periodic measurements of insulation resistance.

Discussion

J. S. Henderson: The additional standards on insulation tests proposed by Messrs. Gilt and Barns are necessary. Even though the machinery has received insulation tests at the factory in agreement with the present Institute rules, an additional test should be applied after the apparatus is installed as insurance against damage since leaving the factory. In fact, most large pieces of apparatus are tested today after installation, and it is now only a question of what the voltage should be for this final test. I hope there will be a full expression on what is considered an adequate voltage.

The proposed standards for tests on machinery which has been in service are of a different type and are really an operating guide. The value of 125 per cent proposed agrees closely to the recommendation of 60 per cent of the original test voltage which the Westinghouse Company has made for some years.

One other point touched on by the authors is the question of drying out apparatus before making the insulation test. There has been a tendency to wish to omit this drying out, particularly on large turbine generators. It is undesirable to run these machines on short circuit because of possible damage to the steam turbine if it is operated at no-load for any length of time.

Generally, these units are handled with great care and any moisture on the winding is likely to be only on the surface of the end turns and not on the buried part of the coil. This surface moisture may cause failure because of reduced creepage resistance. In general, two courses are open. If the machinery has been shipped and erected in dry weather, the dry-out may be omitted. If, however, the shipment has been made in rainy weather, or if there has been damp air in the station during erection, the recourse can be made to using space heaters under the end windings and the insulation resistance watched until it becomes constant at constant temperatures.

The paper suggests as a test on new apparatus after erection, 85 per cent of the value specified in the A. I. E. E. rules. This is a test for one minute. The question of the length of the test should be carefully considered. Most high-voltage disturbances on a system are of short duration and last a second or less. The probability of failure while making the test, after the machine has once passed the standard high-voltage test, is likely to be due to decreased surface resistance caused by dust or moisture on the surface of the end turns. A high voltage for a second would not establish a carbon streak on the surface while a high voltage for a minute would. For this reason, a test for a minute is many times more severe than a high voltage for a second in service. It would seem, therefore, that the time could be reduced to a half minute and still leave a margin for insurance.

J. R. Dunbar: It is well recognized that new equipment that is assembled on the premises of the purchaser should receive the same tests as though it had been previously assembled and tested at the factory. However, when the purchaser decides to place the machine on commercial load without complete tests it does not seem reasonable to require it afterwards to stand up to the same tests as new apparatus, since there is bound to be a certain accumulation of dust and a consequent reduction of insulation resistance. It would, therefore, seem desirable that the Institute Standards include recommendations for voltage tests for the conditions described. These tests should be lower than the tests on new apparatus, yet higher than the maintenance tests described in the second part of the paper.

Second-hand and repaired equipment requires tests lower than the tests for new apparatus and possibly higher than the maintenance tests. In the case of repaired apparatus, when new coils are installed they should certainly be tested for the full voltage of new apparatus, but it is not to be expected that the complete winding containing old coils would stand up to the full tests. It is possible, in this case, that the maintenance tests would be sufficient, but it seems to me that this point is worthy of further consideration.

In the case of apparatus sold as second-hand, the purchaser naturally cannot expect the same life from the insulation as would be obtained from a new machine, but it would be reasonable to expect tests which would indicate, to a certain extent, the probable remaining life of the insulation. Such tests would have to be higher than the maintenance tests described by Messrs. Gilt and Barns in the latter part of the paper, but lower than the tests on new equipment.

A maintenance test between phases at 175 per cent of rated voltage with none of the phases grounded is mentioned in the paper. On high-voltage transformers with rated voltage of the order of 220 kv., 175 per cent of the test voltage is nearly 400 kv. The testing transformers for this voltage are usually designed with one terminal grounded, so that for high test voltages the proposed test would not be feasible with present testing equipment. Furthermore, it is undesirable during any test to apply voltages to a winding which is floating with respect to ground. In order to reduce the possible voltage stresses to ground where tests are applied between phases, with none of the phases grounded, it would be necessary to use a testing transformer so designed that

the center point of the winding could be grounded, or to use a special grounding transformer. If this is done the voltage stress from either phase to ground would be half the test voltage between phases. It is recommended that additional standards be established to govern all the foregoing conditions.

F. E. H. Mowbray: Mr. Gilt's suggestion that the prescribed test for insulation after assembly in the field be 85 per cent of the normal factory test is, I think, very good, but it seems to me it would apply only where the apparatus had been completely assembled and tested for insulation according to A. I. E. E. Rules, in the factory previous to shipment. I do not quite see how this insulation test could be reduced by 15 per cent, say in case of a large a-c. generator wound on the customer's premises. The customer would naturally expect the apparatus to fit the regular prescribed test.

However, most manufacturers have had cases where, as soon as the apparatus was assembled, it was put in service and then after several weeks or months, the customer would request that the final insulation tests be made. Under this or a similar condition, I should think that the maker would be entitled to some reduction and think that 85 per cent of normal is quite fair. In some cases that have come under my observation, a considerable amount of cement dust and other dust had collected on the windings and it was almost a physical impossibility to get it all out. Under this condition I do not think that the above reduction should cause objection.

Regarding the maintenance tests, I regret to say that I differ with the writer considerably. His proposed scheme means that each time a three-phase generator, for example, is tested, four insulation tests are to be made, *viz:*

(1) Ground test of complete winding
(2) Between phases A and B
(3) Between phases B and C
(4) Between phases C and A.

In the first place a ground test on a large generator taking in the whole winding is likely to give unequal stresses in different parts of the winding due to capacity effect, and requires considerable care in its application and method of control. The average power company is not equipped with the proper apparatus for this test. Aside from this feature the proposed value of 125 per cent of normal rated voltage, plus 500, does not give the different voltage ratings the same per cent test above normal voltage. For example, a 2200-volt machine would get $2200 \times 1.25 + 500 = 3250$ volts or 47½ per cent above normal, while a generator rated at 13,200 volts would get $13{,}200 \times 1.25 + 500 = 17{,}000$ volts, or approximately 28½ per cent. Personally, I think that a flat rate of 30 per cent would be a more fair distribution and the time element should be not more than 15 sec.

With regard to the between-phase tests of 175 per cent of normal + 500 volts, this means that a 2200-volt generator would get $2200 \times 1.75 + 500 = 4350$ volts between phases. In the first case I consider this too high, and secondly it means that a special testing transformer with the middle point of the high-voltage winding grounded, or two testing transformers, would have to be employed. Again this test puts on another insulation test to ground of 2175 volts, repeating the ground test at roughly normal voltage, which in itself is approximately 70 per cent above the normal star voltage, assuming a grounded star. I also doubt very much if the average power company would wish to put a between phase test of from 79 per cent to nearly 100 per cent above normal terminal voltage, on their apparatus. In my opinion, the two tests should be combined and then the number of tests made would be three in number, with capacity and stress conditions identical, *i. e.*, ground two phases at both star and line terminals, and test the other phase to ground, thus getting a ground and between phase test in the one operation, and repeat this for the three phases.

The periodic checking of the insulation resistance while the apparatus is warm is a good check.

In conclusion, I should like to see the last paragraph of the article changed to read as follows:

For maintenance—a test to ground on one phase at a time with the other phases grounded of 130 per cent of normal line voltage for a period of 15 sec.

Generating apparatus which meets this requirement should be in a reasonably safe operating condition.

D. A. McKenzie: While the title of this paper is general, the impression upon reading it is that it applies chiefly to equipment which is expected to operate at moderately high voltages and above. This rather definitely confines its application to the main powerhouse generators as well as to the main station transformers (either step-up or step-down) and this is especially emphasized since the major part of the discussion is centered on fibrous insulation, its deterioration due to handling and due to the absorption of moisture, while still new, as well as its deterioration in service, due to heating from loads, natural aging, and voltage strains impressed during service, etc.

To an operating engineer, the suggestion that new equipment before being placed in service in the station is to be already considered as having deteriorated from factory conditions, is rather startling. While the later editions of the Standards of the A. I. E. E. specify that "Unless otherwise agreed upon, high-voltage tests shall be made at the factory," they also state that the "Commercial tests shall be made with the completely assembled machines," and it would seem that this last clause will have the effect of automatically transferring these tests to the power house of the customer, for the larger machines, which are assembled in place. However, the chief feature in which the purchaser or operator of electrical machinery is interested, is its ability to perform satisfactorily under operating conditions and the factory tests are of value or interest to them only as indicating its probable performance in operation. If this equipment is so designed that it will deteriorate in shipping or handling or installation so that after installation in the customer's powerhouse it will only withstand 85 per cent of the factory test, it would seem advisable to change either the design of the equipment or the method of care till ready for service, or to increase the value of the factory test, since it is believed that the safety factors intended by the Standards should be attainable when the equipment is ready for service. This is especially true since in most cases, these large pieces of equipment are under the care of the manufacturers until being actually placed in service. It would also seem that even while some large pieces of generating equipment might be assembled at the manufacturers shops, the facilities for properly drying it out are not available until it has been erected in its place in the powerhouse and connected up to its prime mover.

It would seem that the operating engineer is interested in having as high a safety factor as possible when the apparatus is to be placed in service, and is desirous of having acceptance tests on equipment made at that time unless there is good assurance that the equipment may be placed in service in equally as good condition as when tested at the factory.

With regard to the matter of testing the insulation of equipment in service, it will be found that most operating engineers are rather averse to running any risk of damaging machines or rendering them unfit for service for even a short time although information regarding the condition or progressive deterioration of machines would be valuable in arranging for reserve capacity.

Some years ago, the matter of making regular or periodic high-voltage tests on the generators of a rather large generating station in the Niagara district was considered. It was found that fairly expensive equipment would be required and moreover, owing to the high load factor on the plant it was necessary to keep all machines in service as continuously as possible and it was therefore not desired to run any risks of breaking down the insulation. Consequently the matter was dropped, without having decided on a ratio of test voltage to operating voltage.

It is, however, highly desirable to have some idea of the condition of the insulation of machines, especially of generating equipment, and more particularly with the large units now in general use.

We have in the Standards, specifications covering the minimum value of insulation resistance for the armatures of generators, together with the statement that while the insulation resistance may afford a useful indication as to whether the machine is in a suitable condition for the application of the high-voltage test, it shall not be considered a requirement of the Standards. They also state that these insulation resistance tests shall, if possible, be made with a d-c. voltage of 500.

This last suggestion seems to point rather definitely to a very convenient and useful instrument known as the "megger."

Now it would seem that in addition to information at a certain definite time, it is almost equally important that relative information on the same subject be secured throughout the complete operating history of the equipment to show the rate of change, if any. Therefore, if some definite measurement could be made on the machinery at monthly, or bimonthly periods, in exactly the same manner each time, it would seem that the rate of change would provide some criterion as to the condition of the equipment, even though any measurement by itself were not definitely accurate.

It is believed that many companies, by the use of meggers, have been able to observe the progressive deterioration of insulation on motors, and it would seem that similar measurements of the insulation resistance of generators would be of somewhat similar value. It might be necessary, however, to use a voltage more comparable with the generator terminal voltage than is obtainable from the megger.

With the advent of the kenetron it should be quite convenient for the manufacturers to supply compact equipment whereby the insulation resistance could be measured at a d-c. voltage approximately equal to, or if necessary greater than the terminal voltage of the machine. Similarly, it might be possible to obtain measurements or information regarding the performance of this same insulation under the application of alternating voltage using equipment which is readily portable. This will limit the voltage to a value which could not possibly result in any injury to the insulation of the machine, but might require special instruments.

Such a device should preferably be portable so that it could be readily moved directly to each machine with no appreciable length of high voltage testing leads or necessity for alterations in existing powerhouses. It should also be of small kv-a. capacity so that its supply voltage could be readily obtained at any part of the powerhouses. It would, of course, be self-contained as regards voltage regulation and complete with all necessary instruments. At present it would seem that a maximum voltage of about 15,000 direct current would be ample for this type of equipment for the normal voltage rating of most generating equipment in service. The opinion of the manufacturers as regards the value of such measurements and the possibility of obtaining the equipment would be of interest to many operating companies.

It is noted in the third paragraph on the fifth page that it is assumed that the neutral point of a-c. generators is grounded for relay protection of the winding. Many large generators, recently installed, are now operating with the neutral completely isolated and this tendency is apparently spreading.

In closing, the application of a high voltage to a winding does not give any definite information regarding the condition of that winding except that it was sufficiently strong at that instant to withstand that stress. It is conceivable that the immediate reapplication of this same voltage might result in breakdown. It would appear that if definite values relating to certain characteristics of the insulation were periodically obtained or measured during the history of the apparatus, and on a sufficiently large number of items of each class of equipment, it should be possible to evolve some law connecting the condition of the equipment with the measurements obtainable.

F. D. Newbury: I should like to discuss very briefly the first part of the paper covering the recommendation that the Institute adopt a standard for insulation test of generators before operation. I think that is an excellent suggestion and the Institute should take such action.

I am not quite so sure of the wisdom of the Institute's taking action regarding the second recommendation, that is, the establishment of standard tests during service. I think that part of the problem has some of the elements of insurance. The frequency of the test and the value of the voltage depend somewhat on the insurance value the operator wishes to take out to guard against breakdown at inopportune times. If he wishes to take out heavy insurance against breakdown, he will make a high-voltage test at frequent intervals. If he wishes to take greater risks, he will adopt a lower test or a longer interval. Judgment on the part of the operating company will, I think, be a controlling factor in the determination of the test program during service. For that reason I question whether the Institute could adopt any single value that would fit all situations and be satisfactory to all operating companies.

P. L. Alger: I believe the recommendations of Messrs. Gilt and Barns are reasonable in all respects. I consider, however, the complication of testing old machines, both phase to phase and phase to ground, undesirable, and I think it preferable to apply a single test of perhaps 1½ times normal voltage on each phase to ground separately, with the other phases grounded.

I should like to make some suggestions about the periodic tests on machines in service. The quality of insulation, after it has been in service for a long time, depends on three things: (1) on the amount of dust and dirt covering the end windings, and (2) on the internal condition of the insulation in respect to moisture and (3) in respect to air spaces. If the leakage current on direct current or the power factor on alternating current is measured periodically, it cannot be used as a guide to the changes in insulation quality, because of the variations in moisture present. If, however, in every case, measurements are made both cold and hot at definite temperatures, the differences between the measurements can be taken as a measure of the leakage current due to moisture, and the low temperature reading corrected for variations in the moisture can then be taken as a direct measure of the insulation quality. An increase in this current with time should then serve as a fairly reliable indication that the insulation is deteriorating. If an inspection of the windings showed the surfaces to be clean, there would then be a strong probability that the insulation was weak.

To separate by test the effects of air spaces in the insulation from other defects, readings can be taken at both high and low voltage at the same temperature. If the leakage currents here are not proportional to the voltages, air spaces are indicated, and hence periodic checks on these measurements will enable the increase of air spaces with time to be watched.

By thus increasing the number of measurements made periodically, I believe it is possible to develop a fairly reliable means of recording the deterioration of generator insulation.

W. F. Dawson: All of these discussions have mentioned the possibility of dirt and moisture in the windings when high-potential test is applied. Of course, everybody who knows anything about high-voltage insulation knows that those things are detrimental and I believe the existing A. I. E. E. rules state that before any high-potential test is made at least the surface condition of winding should be prepared by cleaning. I have seen, particularly in turbine alternators, great quantities of all sorts of dust. It may be dry dust that floats in from the roadway or is sucked in by the ventilation system, or it may be dust contaminated with oil. It may be good clean oil but probably it is not. It is loaded with coal dust and everything that it should not have. Of course, there is no sense in applying a high-

potential test to a winding in that condition. I have seen dust a quarter of an inch thick, not occasionally, but frequently. Any periodic testing ought to be preceded by a thorough cleaning out of the winding.

We have a prescription for cleaning,—first by blowing out, then by washing with gasoline or carbon tetrachloride if gasoline is considered an explosive danger. Clean thoroughly and dry and apply a coat or two of good air-drying varnish and know that the winding is in condition to receive a high-potential test before applying it.

V. M. Montsinger: There have been several discussions on rotating machinery. I should like to say a few words with reference to transformers.

In regard to the first part of the paper, there is no question but that some value or values should be set up; whether it should be 85 per cent or not is a question. We may have a condition where a transformer is shipped a long distance and perhaps collects a little moisture. If that transformer is put in service at normal voltage, the moisture may not do any harm. It may be driven out in two or three days and the transformer would be all right. On the other hand, if it is tested at too near the factory test values, it might break down. For this reason I do not favor setting the test value too near the factory test.

In reference to the second part, I think the conditions are somewhat different in transformers from those in rotating apparatus because we have oil which can be tested periodically. Practically all operating companies test their oil and if they find it is getting low in dielectric strength, measures are taken immediately to increase the dielectric strength and for that reason I doubt if there is as much need for rules for testing transformers periodically while in service as for other types of apparatus.

We have had this question before the committee for some time but have been handicapped because we do not know what the practises are and what the operating companies would like to have. So we were hoping that this paper would produce experiences of the operating companies. If we knew that, we would be in a much better position to judge whether such rules should be incorporated in the A. I. E. E. Standards.

As Mr. Newbury pointed out, there should be rules relating perhaps to the first part but whether there should be to the second part depends considerably on whether there is really any need for it.

C. E. Skinner: (communicated after adjournment) The manufacturers of electrical machinery and equipment have always, and rightly so, taken the responsibility for any damage which may occur to new machinery in the factory or at the time of installation due to the dielectric test, covered by the contract between the manufacturer and the purchaser. The responsibility for the repair of breakdowns which may occur as a result of the test after apparatus is ready for service as proposed by Messrs. Gilt and Barns may, in some cases, be open to question. The responsibility for breakdown after the machinery and equipment has been in service for some time should be distinctly that of the owners or operators. When machinery is returned to the manufacturer for repair there should be a definite understanding as to the dielectric test to be made and the owner of the machinery should be responsible for repairs made necessary by the application of the stated dielectric test. When the owner himself does over-hauling and repairing, he will, of course, assume the responsibility for repairs made necessary by breakdowns caused by the dielectric test applied.

It is suggested that the operators in consultation with the manufacturers should lead in taking the responsibility for the establishment of tests after machines are in service and for the test to be made on repaired apparatus. In the case of repaired apparatus, very much depends on the nature of the repairs, the urgency of the case, etc., as to whether the dielectric tests should be severe or not. A specific historical case will illustrate the point. A large armature was returned to the manufacturer for repair of broken leads. The armature was in bad condition with regard to dirt and after as thorough a cleaning as possible a dielectric test at normal operating voltage resulted in breakdown. Further difficulty was experienced in attempts to repair this breakdown with the final result that a complete new winding had to be put on this armature. The owner of the machine was extremely indignant, both at what he considered excessive time required to make the repairs and at the cost of the same, stating that he required nothing done except the repair of the broken leads. Definite rules or understandings before such repairs are begun would obviate similar difficulties.

With regard to the time of application of test, one minute for high voltage apparatus is entirely too long for tests after installation, and the time of application should be reduced as the voltage of the apparatus is increased. It is very doubtful if a test of more than two or three seconds should ever be applied to transformers of 100,000 volts and above after they have been in service for some time. Such tests have been known to cause damage to the insulation, resulting in breakdown later, and the test is applied to a part of the insulation which is rarely broken down as a result of service.

C. M. Gilt: There seems to be quite a consensus of opinion, on the first part of the paper, that it is advisable to establish some rule for testing and that the standard for tests of new equipment before placing in service should be somewhat less than the present Institute standard. Whether it should be, as some people think, the full voltage test for a minute or 85 per cent for one minute, or 85 per cent for a half minute, or, as I believe the British rule is, 75 per cent for one minute, is a question on which there is a difference of opinion.

It was my intent that this test was not to apply to equipment that was delivered and assembled without receiving the full factory test before shipment. I think most of the operating companies, and I believe most of the manufacturers as well, would prefer to see that equipment receive the full factory test once before placing in service. Where it occasionally happens that equipment is placed in service before it receives the full factory test and is operated for some time, it would seem obvious that the test then made should be at a somewhat lower value, probably at a value subject to special negotiation depending upon the time in which it had actually been in service. Perhaps the 85 per cent test is a reasonable compromise on that.

In regard to the second part of the paper, my opinion is that equipment that will not safely stand a 50 per cent overvoltage test is not actually safe for operation under modern conditions of heavy power supply. I would not recommend that as a regular monthly routine test. As far as our own company is concerned, we have established the practise that, when it is necessary to test equipment that is in service, but not as a regular routine test, we test it at 50 per cent above voltage because we would rather have the equipment fail on test than in service. This of course applies only to apparatus, not to cable tests.

B. L. Barns: The suggestion was made by Mr. Newbury that the test is in the nature of an insurance policy rather than just to satisfy curiosity, and therefore the conditions obtaining in the power system or the importance of uninterrupted operation will determine largely the amount of insurance you want to take out. This might be taken care of by raising the test value and shortening the time. Usually any disturbance that occurs on a system to impose an overvoltage on a machine winding is for a very short period of time. If we hold the voltage test for a whole minute, there is much more chance of doing damage and apparently the one-minute condition does not reproduce what might happen in actual operation.

Messrs. Dunbar and Skinner have mentioned dielectric tests on apparatus that may be returned to the factory for repairs. The establishment of standard test voltages for such occasions

was not in my mind when preparing this part of the paper. I was interested more in the possibility of establishing a recommended practise to be followed by the users of apparatus in a maintenance program and there was no thought of suggesting any responsible interest of the manufacturer in such tests. I agree fully with Mr. Skinner's opinion as to the responsibility for failures resulting from tests made after the apparatus has been in service. As to the establishment of rules for use when apparatus is repaired leaving all or part of the old windings in place, I believe, as has been remarked, that there are so many various conditions and circumstances that such a procedure would be impracticable.

I was interested in Mr. Dunbar's remarks about the desirability of some standards for second-hand apparatus. It was this idea that was first proposed to the Committee on Electrical Machinery. It was suggested that with some standard to govern tests on second-hand machines not only would the purchaser be benefited but perhaps the purchase of used apparatus would be stimulated and the industry as a whole would gain. I canvassed a number of established dealers in second-hand apparatus and was surprised and gratified that many make it a practise to test all machines according to the A. I. E. E. rules covering dielectric tests on new machines.

Messrs. Dunbar and Mowbray have pointed out some objections to the suggested tests between phases. These were suggested for the purpose of obtaining the same degree of assurance regarding the insulation between terminal coils as would be obtained by testing a phase winding to ground. Both tests would have the same margin over the corresponding operating voltage in the case of machines operating with grounded neutrals. The practical difficulties that would be met in applying such tests to high-voltage apparatus constitute a real objection to the proposed procedure. It was intended that this would apply only to apparatus having a rated voltage not exceeding, say, 15,000 volts. As Mr. Montsinger has pointed out, oil-insulated transformers should not be included in these tests since it is possible to determine the condition of the oil by testing a sample.

As for Mr. Mowbray's objection to adding an arbitrary 500 volts to 125 per cent of the rated voltage, it does not appear that this should be more objectionable than the additional 1000 volts that the A. I. E. E. rules specify for new apparatus.

The principal purpose of this part of the paper was not to establish definite values of test voltages for apparatus in service, but rather to present the question of whether it is desirable that the Institute should enter this new field by establishing recommended practises for the maintenance of apparatus in service. If an affirmative decision is reached the choice of test voltage values could then be discussed.

Influence of Temperature on Large Commutator Operation

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Synopsis.—The paper discusses the operating characteristics of large commutators. It is pointed out that many cases of blackened and burned bars are caused initially by a slight roughness which causes irregular commutation and a slight sparking which when once started gradually becomes worse. The requirements of the surface of a high-speed commutator are discussed, showing that radial variations of the order of one ten-thousandth of an inch between adjacent bars may give serious trouble. Some causes of roughness are discussed, showing that temperature is a major factor, but that redesigning the commutator to reduce the temperature does not necessarily cure the trouble, and may actually give poorer performance. Some methods of commutator testing are described with typical records showing the performance of various types of construction.

The question of safe temperature limits is then discussed. It is maintained that the question of the permissible temperature limits is entirely unlike that of insulated windings, since the materials used are not injured by temperatures considerably above the present limits. The permissible limit of a commutator is determined by its mechanical construction. Low temperature limits may lead to illogical designs, since large heat dissipating surfaces are required which may give higher stresses and larger expansion effects, which may be more harmful than higher temperatures. A temperature limit based on operating characteristics would be more satisfactory.

DURING the past five years, considerable experimental work on large commutator performance and characteristics has been under way and many different types of construction have been investigated and compared. Much information of value to designers has been accumulated and this information has led to a clearer understanding of the desirable characteristics of commutators and the influence of operating temperature on performance.

While this work was undertaken primarily to secure information for design, it has yielded information of a more general interest to operating engineers, and it is this latter phase of the subject that will be treated in this paper. Permission to publish the information in this paper was granted the authors on condition that it would be confined to information relating to commutator performance and operation, but even with this limitation, it is believed that the paper will make available to the profession information of considerable value. The paper, as indicated in its title, will deal with the subject from the standpoint of large commutators found, usually, only in power station and substation machinery.

A commutator is fundamentally a device for accomplishing a very rapid switching of the current in the armature coils. A rate of 5000 bars or switching operations per second is very common. The only practical device for this purpose has been the conventional form of stationary brushes, making contact with a rotating surface composed of copper bars electrically insulated from each other. Copper must be used because of its electrical, thermal, and chemical properties, but its mechanical properties are far from ideal, particularly its low elastic limit and tendency to "creep" or slowly yield at comparatively moderate stresses. Mica has exceptional insulating and heat resisting qualities, but its mechanical performance is erratic, and a large amount of research has been required to develop manufacturing methods which give the most desirable mechanical characteristics and uniform quality.

A large variety of constructions has been proposed for holding the bars, but in nearly all commercial designs, the bars are held by some form of shrink ring, giving a radial clamping, or by V rings acting at the ends of the bars. There are many other important features, such as whether the bar is held by clamping against a bush or drum, (drum bound), or by clamping against an arching pressure acting between the bars, (arch bound), or held by a wedging or V binding effect at the ends of the bars (V bound). There are also various means of providing for the thermal expansion.

Because of the nature of the materials and the extremely smooth surface required, the commutator problem is very unlike other problems with which engineers are familiar, so that the problem does not lend itself to theoretical mechanical analysis alone. The theoretical work must be supplemented by a large amount of research work and testing on both the materials and assembled commutators. From this, the best combination of factors for any given conditions can be determined, but this is a design question outside of the scope of this paper. In all types of construction, the heating of the commutator is inevitable because of the mechanical friction of the brushes and the electrical loss at the contact. In most electrical apparatus, the permissible temperature is determined by the temperature at which the insulation is permanently injured, but in large commutators, the materials are not injured by temperatures considerably above the limits commonly applied. There are other very important effects of temperature, but this is ordinarily not in the nature

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of a definite limit fixed by the materials, but rather a limit based on the construction features of the commutator, and particularly its ability to withstand the change in dimensions caused by a change in temperature. For this reason, the authors believe that a different type of temperature limitation should be used for commutators from that used for apparatus where the temperature is limited by the point at which the material is injured. This paper will be devoted to a general discussion of the nature of the temperature limitation of commutators.

The primary function of the commutator is to make contact with the brush. A certain variation in the contact drop is permissible, but, as will be shown later, some commutators reach a condition where the brush cannot transmit direct current to the bars without vicious sparking, even when no commutation is involved. Under such conditions, satisfactory commutation is impossible. With the bars moving at a speed of a mile a minute or more, the commutator surface must be very smooth if the brushes are to maintain uniform contact with all bars.

Commutator bars deflect radially under centrifugal force and temperature. Since these bars are necessarily separated by insulating material, the bars may deflect unequally. If adjacent bars deflect unequally, an abrupt change or bump in the surface is produced. In considering the effect of commutator roughness, it is necessary to distinguish between eccentricity and other gradual irregularities, such as may occur either

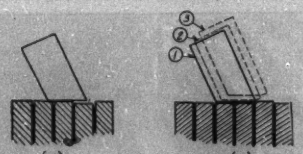


Fig. 1—Action of a Brush Passing Over an Irregular Surface

a—Shows the impossibility of making contact with a low bar

b—Illustrates the tendency of a brush to jump when it strikes a high bar which may be very serious at high peripheral speeds

in a slip-ring or commutator, and these abrupt changes from one bar to the next which are peculiar to commutators. Ordinarily, a brush will follow an eccentricity of several mils without losing the contact between bar and brush, but when the roughness occurs abruptly from one bar to the next, a very slight irregularity will give trouble. For instance, if one bar is lower than the adjacent bars, the brush usually covers several bars and therefore cannot make contact with a low bar. If a bar is higher than adjacent bars, the bar will strike the brush and cause it to bounce away from the commutator surface. Fig. 1 illustrates the action of a brush passing over a low and a high bar.

Requirements of a Commutator Surface

The exact nature of the mechanism by which current is conducted between the bar and brush is somewhat of a controversial question, but at any rate there should be approximate mechanical contact between the bar and brush at least at certain points on the brush surface. Theoretically, there may be a separation even at these points, due to an air film and fine dust particles, but this is very minute. Whatever the exact nature of this contact, the commutator surface must be such that this contact can be maintained uniformly and without interruption. As has been pointed out, a commutator surface is composed of bars separated by insulation,

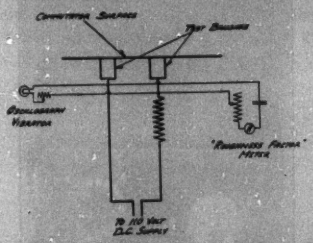


Fig. 2—Wiring Diagram for Brush Drop Test

Power circuits are shown by heavy lines and instrument circuits by light lines. The oscillograph and roughness-factor meter are both shown connected

so that there is the possibility of relative movement between adjacent bars giving an abrupt break in the contact surface. A very minute break in the surface can be absorbed in the surface deflections of the materials in contact, but with a greater variation a movement of the whole brush is required, and if the speed is high this will give an interrupted contact.

Measurement of Commutator Surface Conditions

Any progress in science or engineering is dependent first on knowledge of the facts. Suitable measuring instruments and methods are fundamental to progress. This study of commutator design and performance made very slow progress until a satisfactory device for measuring commutator surface conditions was developed.

Since the condition of a commutator surface may change with speed and temperature, it must be measured at full speed, and operating temperature. The measurement of a variation of the order of one ten-thousandth of an inch between adjacent bars running at a surface speed of over a mile a minute is beyond the range of ordinary instruments. Because of the difficulty in measuring this mechanically, and since we are primarily interested in the effect of the surface condition on the brush contact drop, the measurement of this contact drop seems to be the most logical means of determining the surface condition. Fig. 2 shows the wiring diagram for a scheme of doing this which

will be called the "brush drop test." Fig. 4 shows a photograph of a small commutator set up for the test. In this test, two or more brushes are mounted in line so as to make contact with the same commutator bar. Current is passed from one brush to the bar and back through one or more brushes in series with sufficient resistance, so that practically constant current is maintained. The double-contact drop is recorded by an oscillograph. In this way, the surface condition of the commutator can be studied during heating and

Fig. 3—"Roughness-Factor" Meter

cooling and for various speeds. If a timing contact is used as a second record on the oscillogram, the position of any roughness of the commutator can be located.

If it is not necessary to locate the bars giving trouble, but only to determine the average operating condition of the surface, then the "roughness factor meter" may be used instead of the oscillograph. Fig. 2 shows the wiring diagram for the brush-drop test with both the oscillograph and the "roughness-factor" meter connected to measure the brush drop. Fig. 3 shows a photograph of the "roughness-factor" meter. It consists of a large capacity condenser, variable resistance, and thermal milliammeter all in series. The condenser has sufficient capacity to offer only a negligible impedance to fluctuations in brush drop. For a perfectly uniform contact, the reading is zero, but in case the brush drop fluctuates, the current through the meter is limited by the resistance in series. The product of the resistance in ohms by the current in amperes is called the "roughness factor." It is the r. m. s. value of the fluctuations in brush contact drop in volts and is a measure of the surface condition of the commutator. With a good surface, it should be 0.15 or less, while with a bad surface, it may rise to several volts.

This method of measurement does not, of course, give bar movements in inches and in the case of eccentricity, a movement of several mils may give no indication, but since we are concerned with the uniformity of the contact drop, it does indicate the things in which we are interested. Because of its sensitivity, reliability, and simplicity, this test has been extensively used to study commutator surface conditions. Usually, the commutator is ground and the test brushes mounted. The commutator is then heated by narrow belts lined with maple blocks. These are so arranged that they do not strike the surface at points where the test brushes will bear. This method gives the surface heating and temperature distribution closely approximating service conditions. It also gives additional heat on any bar which tends to become high combined with considerable mechanical vibration, so that it is a very severe test which should disclose any trouble likely to develop in service.

This test shows the condition of certain parts of the surface. The roughness will almost never be the same along the full length of the bar. Since the test can be used to show the condition of any part of the surface, it can be used to study the nature of the bar deflections causing the roughness.

Brush Action on a Rough Surface

The action of a brush in passing over an irregular surface is of importance mainly in designing the "brush rig" which is another major problem, but it must also be studied in connection with the commutator problem. For this purpose, a rotating copper disk was made with

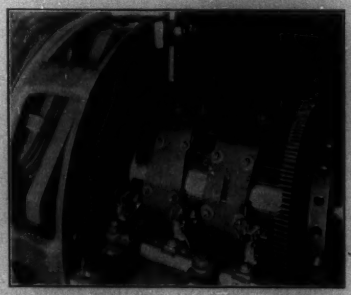


Fig. 4—Commutator Set up for Brush Drop Test

a surface slotted to represent an undercut commutator. In addition, four larger slots approximating the width of a commutator bar were cut in the surface. These four slots were then fitted with bolted-in fillers which could be adjusted by means of shims to give any desired surface roughness. By using gold leaf as a shim, an adjustment of a fraction of a ten-thousandth of an inch could be obtained. Brushes were mounted to run on this surface. The contact condition was determined by recording the contact drop with the oscillograph. Using an ordinary brush, the average contact condition

would be given and by dividing the brush into electrically insulated sections, the contact condition at any part of the brush could be studied. Fig. 5 shows the rotating disk, the brush mounting, and a device for recording brush movement.

These tests showed that for a given commutator surface condition, a wide variety of contact conditions could be obtained, depending on the brush material and surface condition, particularly for very slight irregu-

Fig. 5—Apparatus for Studying the Action of a Brush on a Rough Commutator Surface

larities in the surface. Fig. 6 shows three records of the brush voltage drop for brushes passing over exactly the same point in the surface, which, in this case, was a very slight roughness. This shows how with a very slight roughness, it is possible to get a brush condition which will pass over the roughness with almost no variation in voltage drop, while a slightly different brush condition will give a serious variation in the brush drop. With a somewhat greater roughness, the best brush condition would still give a serious fluctuation in brush drop and considerable sparking.

The surface speed is one of the most important factors

Fig. 6—Contact Drop Obtained with Four Different Brush and Brush Holder Arrangements

The cut was traced from oscillograms showing contact drop passing over the same surface roughness, only the brush and holder being changed

in determining the action of a brush. It is obvious that at very slow speed, a brush can pass over a considerable irregularity without losing contact with the surface. In Curve *A*, of Fig. 7, the fluctuation in brush contact drop is plotted against speed. The roughness or bump in the surface was made by raising the segment representing a bar 0.00015 in. above the rest of the cylindrical surface. The speed was changed without disturbing the brush so that the curve shows the effect of speed alone. This curve could be repeated for the given brush condition, but the result could be changed materially by a change in brush, brush holder, or by merely a different brush wearing surface as was illustrated by Fig. 6. Curve *B* of Fig. 7 is similar to Curve *A*, except that a different brush was used and the roughness or bump was 0.0007 in. high instead of 0.00015 in. These curves show how, for a slight roughness, the performance may be entirely satisfactory at one speed and give serious trouble at a slightly higher speed. Another effect of the higher speed not shown by the curve is the increased duration of the period of poor contact, as the peripheral speed is increased. This duration of poor contact tends to increase nearly with the square of the peripheral speed, which shows the importance of designing for the minimum peripheral speed.

These results show that the brush contact drop changes quite abruptly from the usual drop of about one volt, to a different type of current conduction giving a

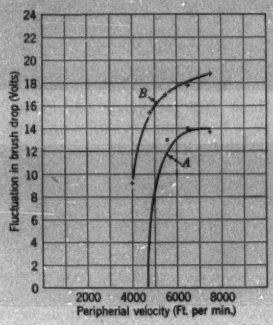


Fig. 7—Curves Showing Fluctuations in Brush Drop Plotted Against Speed

A—Roughness consisting of one bar raised 0.00015 in.
B—Roughness consisting of one bar raised 0.0007 in.

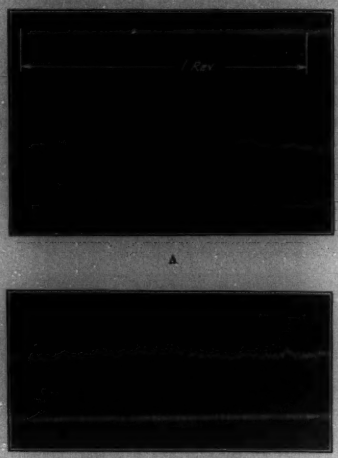
voltage drop of several volts, which is accompanied by a serious burning of the surface. The exact magnitude of the roughness which can be permitted before the brush drop is seriously disturbed is evidently a function of the surface speed and brush condition. However, for the purpose of discussing the accuracy required in a high-speed commutator, it can be considered to be of the order of one ten-thousandth of an inch for an abrupt variation from one bar to the next.

Effect of Roughness on Operation

Commutation is a complex problem, involving the electrical characteristics of the machine, the brushes, and the commutator surface, so that it is difficult to tell just what is the cause of any given trouble. It is particularly difficult to analyze trouble caused by the surface condition, since the cause of the trouble does not necessarily produce a noticeable effect at once.

A commutator may develop a certain roughness at a high operating temperature. This roughness will

usually exist only over a part of a given bar. Some brushes will maintain contact with the given bar, and these brushes will suddenly be overloaded.[2] This may cause only a very slight sparking at first, but this gradually produces a burning or smutting of the bar which still further disturbs the brush action until a serious sparking results. In other cases, particularly with natural graphite brushes, a slight initial roughness may cause unequal wear of the copper, finally resulting



A

B

Fig. 8—Effect of Overspeed

A—Brush drop record at normal speed
B—Brush drop record at 25 per cent overspeed

in a flat spot so that the commutator must be ground. In either of these cases, the effect may continue, even though the roughness which really caused the trouble has disappeared.

This paper is confined to the subject of the commutator surface. It should not be inferred that we consider this to be the only source of trouble, although it is particularly troublesome because it is one of the most difficult to control.

Causes of Roughness

Since the commutator bars must be supported through mica insulation, any load yielding of this mica would disturb the alinement of the bars, resulting in a rough surface. Because very little information has been available concerning the properties of mica, it has been blamed frequently for many troubles due to other causes. The requirements of commutator mica are very severe and its properties very different from most engineering materials, so that much research work has been required to develop a satisfactory material for the purpose.

Commercial mica must be built up using mica flakes cemented together by some form of bond. It must be possible to mold the material, and yet it must withstand high pressures in the commutator. In order to support all bars equally, it must be manufactured very accurately to the required thickness and at the same time have a high elasticity or "springiness" to absorb slight variations in the expansion of the metal parts. When the commutator is first assembled, even with V mica manufactured very accurately to thickness, a slightly plastic characteristic is desirable when first heated to allow the mica to conform to the machined surface, but it must quickly harden or set to a permanent condition, for if slippage or yielding of the mica takes place after the commutator is in service, a rough surface will result. For this reason, the compression of mica under the influence of temperature and pressure is very important. To determine these properties, a special mica testing machine was developed in which any temperature and

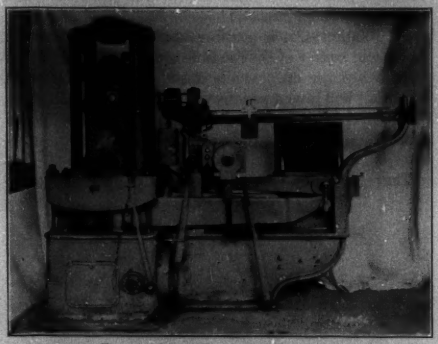


Fig. 9—Mica Testing Machine

Used to study compression of mica under combined pressure and temperature

pressure can be automatically maintained and the compression of the mica accurately measured at any time during the test. Fig. 9 shows this testing machine, which consists of a standard 100,000-lb. capacity Riehle testing machine with automatic control added for maintaining a constant load in the test sample, and with a small electrically-heated oven for automatically maintaining the sample at any desired temperature. Special measuring devices were installed which indicate the compression of the sample at any time during the test.

2. For a brief discussion of the effect of a sudden change in load on a brush, see Arnold and la Cour, "Die Gleichstrommaschine," Par. 79.

These tests have made it possible to determine quickly if a given bond and grade of mica are suitable for the purpose. For instance, a new synthetic bond was recently developed which had desirable characteristics for some applications. It was considered for commutator use, but the compression tests showed that in its present form, it was not suitable for commutator V mica, mainly because if left soft enough to have any ability to conform to the machined surface, it would not season to a stable condition in a reasonable length of time.

On the basis of this kind of tests, it has been possible

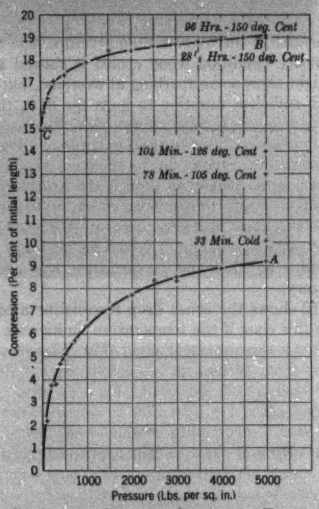


FIG. 10—COMPRESSION-PRESSURE CURVE FOR A TYPICAL SAMPLE OF MICA

O—A shows initial compression curve cold
Points A to B show compression at intervals of time during heating under 5000 lb. per sq. in. pressure
B-C shows compression curve of seasoned mica

to develop a built-up mica plate which will quickly season to a stable condition and yet have satisfactory molding characteristics. Fig. 10 shows a compression curve for a sample of V mica. It shows a large initial compression cold and a large additional compression when first heated, but practically no additional yielding after the first 24 hr. of heating. The compression-pressure curve of the seasoned mica is also shown.

Upon first thought, centrifugal force will usually be considered to be the most serious cause of commutator roughness. A long bar may deflect several mils under centrifugal force. Since this deflection varies with the fourth power of the distance between supports, a slight change in the support of adjacent bars such as might be introduced by irregular mica, would give a considerable variation in deflection, which would give a rough surface. Again, in a bad design, there is the possibility of a slow "creep" or yielding of the copper, which is not the same in all bars, leaving the surface rough. However, tests have shown that it is relatively easy to make a commutator that will remain perfectly smooth at all speeds up to a reasonable overspeed, *if tested cold*. Fig. 8 shows "brush drop" records for a typical commutator, first at normal speed and then at overspeed. In this case, the speed was increased 25 per cent, giving 56 per cent increase in centrifugal force and bar deflection without producing measurable roughness.

The effect of temperature is usually far more serious in commutator operation than centrifugal force. There are so many possible effects that it is particularly difficult to provide for all of them. One example of the effect of temperature is the difference between the expansion of the copper bar and the supporting parts. The steel parts have a lower coefficient of expansion, and in addition run cooler than the copper bars. This results in a differential expansion, which, if not properly provided for in the structure, may lead to serious results. As a quantitative example, if a copper bar at 50 deg. cent. rise is held between steel supports 15 in. apart and having a temperature rise of 20 deg. cent., the expansion of the copper is $(15 \times 50 \times 0.0000166) = 0.0124$ in. and of the steel parts is $(15 \times 20 \times 0.000011) = 0.0033$ in. or a difference in expansion of 0.0091 in., which, if not provided for in the structure, will result in serious stresses and distortions. For example, in the ordinary design in which the bars are held by V rings, this difference in expansion results in a large change in the bolt force holding the V rings together. This produces a distortion of the V rings and usually a bowing or bending of the copper bars. This same difference of expansion acts in a circumferential and in a radial direction. In some cases, this difference in expansion may result in a sliding action of the bar on the mica. It might seem that the solution of the problem is a low temperature rise. However, a low temperature rise requires a large surface for heat dissipation. For any given design, this usually requires a longer bar. The deflections causing roughness usually are of the nature of a bending or bowing of the bar which increases with the second to the fourth power of the bar length, depending on the type of support. Then, since the deflection increases as a power of the length while the temperature decreases only with the inverse of the first power of the length, for any given type of construction the roughness may be increased, rather than decreased by lowering the temperature rise.

Unequal temperatures between adjacent bars are particularly serious, since unequal expansion is a direct result and in almost every case, the temperature throughout the bar will not be uniform, giving an inherent warping of the bar. There are many possible causes of unequal heating. For example, a slight

smutting or burning of the bar will change both the contact drop and the brush friction. Any sparking will result in considerable energy lost as heat at that point. Any roughness disturbs the brush drop, giving unequal heating. The roughness produced by a given unequal heating can be reduced by certain mechanical constructions, but its complete elimination is practically impossible. Again a reduction in the temperature of the commutator as a whole will not necessarily reduce the roughness. Decreasing the temperature by increasing the cooling area has almost no effect on the temperature difference produced by a given unequal heating. The low thermal conductivity of the mica between bars retards temperature equalization, so that temperature difference for a given unequal heating can be reduced by increasing the area of the mica; that

A

B

Fig. 11—Ideal Performance

A—Brush drop record after heating commutator to 140 deg. cent.—115 deg. cent. rise)

B—Brush drop record after cooling to room temperature

is, by increasing the bar length, but the deflection resulting will vary as a power of the length, so that for a given type of construction the resulting roughness will be increased rather than decreased by increasing the bar length.

In addition to the temperature effects mentioned, there are various mechanical causes of roughness, such as vibrations and other slight movements or shifting of structural parts. Mica slippage is decidedly influenced by temperature; but, as explained under "seasoning," in a properly designed commutator, the slippage and set of mica can be made to take place during the seasoning by heating the commutator somewhat above the highest operating temperature. The vibrations and other mechanical movements are usually independent of temperature and therefore not affected by the temperature limit.

Types of Commutator Performance Under Influence of Temperature

I. Ideal. The ideal commutator should stay perfectly smooth under all operating conditions. Fig. 11 shows a brush drop test for a commutator of 23 in. active face length (26½ in. total bar length) and 5500 ft. per minute peripheral speed. It was heated to 140 deg. cent. (115 deg. cent. rise) using a belt lined with maple blocks and tested while cooling. Records at 115 deg. cent. rise and when cooled to room temperature show that a temperature rise far above present limits did not produce any measurable roughness in this commutator.

II. Fair. A commutator which becomes rough when heated to the overload temperature, but returns to a smooth condition when cooled is classed as fair. This might show some sparking at overload, but if this is not of sufficient duration to burn the bars, the performance should be satisfactory when the load is reduced. If, as has been pointed out, the overload is of long duration, the bars may be burned or smutted by the sparking, so that even though the load is reduced and the bars return to a smooth condition, the burned bars will still cause sparking. However, if the burning is not too severe, a period of very light load may polish the surface so that the performance will again be satisfactory.

Fig. 12 shows the brush drop records for a commutator in this class if the temperature limits were somewhat above the present limits. Fig. 12A shows the condition of the ground surface before heating; Fig. 12B after heating to 135 deg. cent., Fig. 12C after cooling to 105 deg. cent.; Fig. 12D after cooling to 70 deg. cent.

III. Bad. A commutator which when heated either to an operating or even a slight overload temperature, becomes rough and fails to return to a smooth condition on cooling is bad, because the surface must be re-ground to put it in a satisfactory condition. Fig. 13 shows a test on an old type construction which became seriously rough at 100 deg. cent. and this roughness only partially disappeared on cooling.

Commutator Seasoning

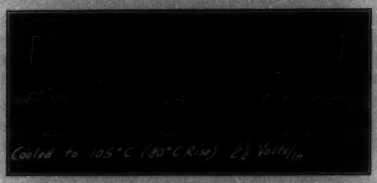
Practically all large commutators, if tested immediately after assembly and before any heating treatment, would show the performance just referred to as bad. This is due to the nature of the copper and mica, both of which tend to yield when first subjected to stress and temperature. The gradual yielding of the copper and mica under stress and temperature until finally a stable condition is reached is referred to as seasoning.

In a low-speed commutator, a sufficiently stable condition may be reached by merely repeated heating and tightening. In a commutator with higher rotational stresses these stresses must act on the commutator for a considerable time during heating and cooling before a stable condition may be reached. Originally,

this was done by loading the machine with the brushes set off neutral to give excessive heating. Now, methods have been developed for seasoning the commutator, before it is even assembled on the machine. In some

A

B



C

Cooled to 70 deg. cent. (45 deg. rise) 2½ volts/in.

D

FIG. 12—FAIR PERFORMANCE

A—Brush drop record after grinding
B—Brush drop record after heating commutator to 135 deg cent. (110 deg. cent. rise)
C—Same except cooled to 105 deg. cent.
D—Same except cooled to 70 deg. cent.

cases, this may be accomplished by merely rotating in an oven. In other cases, a more severe method known as "block seasoning" may be required. This consists in heating the commutator by the friction of belts lined with maple blocks. This gives the surface heating and temperature distribution approximating operating conditions, combined with mechanical vibrations.

The most obvious effect of seasoning is to eliminate the yielding of the mica, but in the best type of high-speed commutator, the copper must be permitted to yield slightly to allow it to conform to any irregular mica surface. Due to the peculiar nature of copper, this yielding will take place only partially during assembly. The rest will take place as a slow creep during

A

B

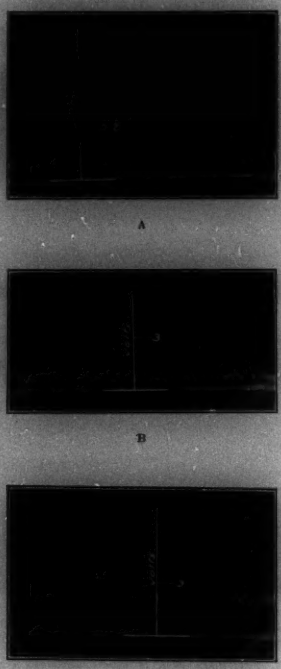
FIG. 13—BAD PERFORMANCE

A—Brush drop record after heating to 100 deg. cent. (75 deg. cent. rise)
B—Brush drop record after cooling to room temperature

the seasoning period. Tests have shown that the mica now used reaches stability in about 30 hours of seasoning at 150 deg. cent., while the copper may in some cases require 150 hours even when accelerated by overspeed and added building force.

Fig. 14 shows the results obtained by block seasoning. Each record shows the roughness produced by a heating and cooling cycle. In the first test (Fig. 14A), taken after a short period of block seasoning, the surface was very rough so that only a small current could be used on the test brush. The roughness is shown by the irregularity of the contact drop. The second test (Fig. 14B) shows an intermediate condition. In the

test after the final block seasoning the surface remained almost perfectly smooth so that full current was used on the test brush giving the higher normal contact drop evident in Fig. 14c. The roughness is indicated by the irregularity of the contact drop and this irregularity is almost independent of the test current used. The number and value of the "peaks" in the oscillogram are the significant factors, rather than the average value of the voltage drop. Even after all of these pre-



A

B

C

Fig. 14—Effect of Seasoning

A—Commutator block seasoned for a short period and then ground smooth while cold, heated for 10 hr., and record taken after cooling to room temperature

B—Given additional period of block seasoning, ground smooth while cold, heated, and record taken when cold

C—Same as "b" except after an additional period of block seasoning

cautions, there may be some minute bar shifting during the first few months of operation, so that more maintenance is to be expected during this period than after the commutator has been in service for some time. A large commutator is one of the few mechanical devices which improve with use during early years of service.

Safe Temperature Limits

In the windings of electrical apparatus, fibrous insulation is used either exclusively (Class A), or as the mechanical support of mica insulation (Class B). In these cases there is a definite temperature, which if it be exceeded will rapidly and permanently injure the insulation, and the Institute practise of a definite temperature limit is logical. In commutators, however, mica is the only insulating material, so that the materials in the commutator would not be injured by temperature far in excess of the present limits. The insulation of adjacent windings would also be unaffected on account of the cooling effect and temperature drop introduced by commutator necks. There is, nevertheless, a definite temperature limit for any given construction, but this limit is based on the temperature at which the surface fails to remain smooth, rather than on the effect of temperature on insulation. Since this limit does not depend on the materials used, but rather on the type of construction employed, the present A. I. E. E. Standards should not logically apply to commutators.

A rational type of commutator design and construction may have a safe limiting temperature of 125 to 150 deg. cent without developing surface roughness, whereas other types of construction may be unsuited to limits as low as 75 deg. cent. The ability to withstand repeated temperature cycles without developing roughness determines a commutator's safe limiting temperature. The problems involved are entirely mechanical in nature, and the practise of the present rules in setting an arbitrary low-temperature limitation does not meet the requirements of the situation.

The present standard of a temperature-rise limit and load test is inconclusive. It is ordinarily made at a low-ambient temperature and does not show what trouble might develop from the additional total temperature obtained with the higher-ambient temperature permitted by the Standards, and it cannot usually be of sufficient duration to discover effects that may gradually accumulate due to very slight initial mechanical disturbances. The present practise of setting a temperature limit as a criterion for a mechanical problem is objectionable in that it results in design practise which actually encourages the development of operating trouble which it purports to prevent. Low-limiting temperature such as the present limit of 60 deg. cent. rise on converter commutators require large heat-dissipating surfaces on the commutator, which frequently require extra long commutators, one or more sets of ventilating vanes, and other characteristics which lead to high stresses, large expansion effects, etc., that may be more detrimental to operation than a higher limiting temperature. A physically large commutator having high mechanical stresses and high surface speed that will operate at a low temperature when newly ground and in good mechanical operating

condition, may well be a much less satisfactory commutator to the operating engineer than a physically smaller commutator that under the same conditions of loading, operates at a considerably higher temperature, but stays dead smooth under its maximum operating temperature range. Adherence to the present standards forces the designer to use a highly stressed commutator with high peripheral speed, that may be far more detrimental to good operation and low maintenance costs than would be the higher temperature rise resulting from a smaller lower stressed commutator.

A better measure than the load test, although scarcely practical in many cases, would be a repeated cycle test with the maximum ambient temperature. Satisfactory performance would be indicated by no appreciable increase in sparking with successive heat cycle tests. A less expensive, and therefore more feasible test for determining the permissible operating temperature, would be a "brush drop" or equivalent test, which could be made at the maximum temperature and would have the sensitivity to show any slight roughness which might in time give trouble.

Discussion

C. Richard Soderberg: While there has always been a feeling that the majority of troubles in large commutators are of a mechanical nature, the general subject of commutation is too difficult and too imperfectly understood to enable the electrical designer to state with certainty whether or not the electrical performance of his machine is satisfactory. The brush-drop test, described in this paper, removed this difficulty, and I have no hesitation in stating that this method of analyzing mechanical commutator performance constitutes the greatest step forward in the art of building commutators, that has been made for a long time. It is only necessary to point to the fact that it is now possible to arrive at a definite conclusion as to whether a certain commutator is satisfactory or not, from a mechanical point of view, even before it has been assembled with its armature. If any criticism of the paper were warranted, it would be a criticism of their neglect to bring out this point with the emphasis it deserves.

In reading the paper one is struck by the complete absence of the usual controversy about different types of detail construction of commutators.

The results of this paper go beyond these minor differences, and any one can profit by making use of these results, regardless of the type of construction that he may favor.

C. L. Dawes: For a number of years I have been associated with manufacturers of mica segment plate such as the authors describe, and we already have performed a considerable amount of research work on built-up mica plate. There always has been great difficulty in finding any authoritative publications or information dealing in a scientific manner with the subject of segment plate, particularly with the conditions of its manufacture and the subsequent effect on the performance of the plate under operating conditions in commutators.

Hence, I am in perfect accord with the following statement of the authors: "Because very little information has been available concerning the properties of mica, it has been blamed frequently for many troubles due to other causes. The requirements of commutator mica are very severe and its properties very different from most engineering materials, so that much research work has been required to develop a satisfactory material for the purpose."

Then the authors go on to say: "It must be possible to mold the material, and yet it must withstand high pressure in the commutator. In order to support all bars equally, it must be manufactured very accurately to the required thickness and at the same time have a high elasticity or 'springiness' to absorb slight variations in the expansion of the metal parts." It must also have the further property of becoming hard and unyielding after it has been baked in the commutators as otherwise the commutator will yield when in service. Also, in addition, many manufacturers require a high dielectric strength. Most of the foregoing properties of segment plate are controversial.

About ten years ago it was customary to use 10 to 12 per cent of binder which is ordinarily shellac. It was very difficult to reduce the amount of binder at that time because the plate was laid up by hand. It is very difficult to lay up the films by hand with sufficient uniformity and yet have the binder content below these percentages.

With the advent of laying machines the percentage of binder has been gradually reduced until at the present time it is somewhere between 3 and 5 per cent. Many users of plate, ignorant of the physical effects within commutators, began to think that low binder content was a criterion of good quality. Hence it developed in many cases that the manufacturer who could produce plate with the lowest binder content would obtain the business. If there is too little binder within segment plate, it becomes mechanically poor and brittle, and when punched the segments split apart. On the other hand, if there is too much binder, slipping develops in the commutator and the films slide out. With high binder content it is possible to have high dielectric strength and also to obtain the softness and springiness, which is required, and which the authors mention.

In order to obtain information which might assist us in manufacturing plate which would better meet conditions, we have written to a number of manufacturers and users of commutators. Among other questions we requested the temperatures at which they bake their commutators, so that we might adapt our plate to meet such conditions. A number did not reply, and it appeared to us that they did not know. I believe that the Westinghouse Company in one of the bulletins which they send to colleges state that they bake their commutators at 250 or 300 deg. fahr.

One of the manufacturers wrote to us stating that if anybody tells you the exact temperature at which they bake their commutators, they are not telling the truth because they don't know. This has been our experience.

In the smaller commutators such as are used for electric starters and vacuum cleaners, the commutators are sent through an electric oven usually maintained somewhere near 700 deg. fahr. The process is a continuous one and the commutators are sent through at such a rate that they never attain the temperature of the oven. Naturally under these conditions the maximum temperature which the commutators reach would not be known. The temperature and rate of passing through the oven are adjusted until satisfactory results have been obtained.

We find wide variations among the requirements of various commutator manufacturers. Some do not object to 6 per cent binder; others insist on having, say, less than 5 per cent. Sometimes plate is supplied with binder content as low as 3 per cent, but such plate is near the limit of good mechanical qualities. With the authors, I believe that it would be very helpful to all concerned if more exact scientific knowledge of commutator manufacture was available.

The curves in Fig. 10 showing the compression of molding plate are similar to curves which we have been obtaining for some time, not only for molding plate, but for segment plate as well. We have a much smaller and simpler apparatus than shown in Fig. 9. We use a very strong spring, carefully calibrated to obtain the compression, and by using small samples of plate we can subject it to the very high compression per unit

area and in that way measure the change of compression with time at various temperatures.

We have a large number of data giving the effect of the binder content on the compression, but they have not as yet been published since we are not certain as to how near our temperatures and pressure conform to actual practise. I believe that I am correct in saying there is less known scientifically both of the manufacture of commutators and of the actual operation of the commutators than of any one part of electric machines.

The authors, I believe, have accomplished much in the matter of increasing this knowledge by showing us in such a thorough and fundamental manner the reactions which are actually taking place in the commutator when it is under operation, and also in pointing out so specifically what the properties of mica plate should be to make a good commutator.

W. I. Slichter: Mr. Hague made a statement incidental to the paper which struck me as of very great importance at this particular time. That was that the A. I. E. E. rules of temperature limitation contributed to the design of a commutator which was not as good as it should be. As chairman of your committee on the Standards for d-c. rotating machinery delegated to confirm or improve these Standards, I am particularly interested in this because we have in our standards which are about to be submitted to the American Standards Association for final adoption, certain temperature limitations which are from 10 to 15 deg. higher than that adopted by European standards. As a member of the Advisory Board of the International Electrical Commission I am delegated to work to bring these two together and, representing the American engineers, to try to have the European temperatures brought to our higher levels. I am very much interested, therefore, in hearing whether that is a desirable thing to do or, as a result of the work of Messrs. Penney and Hague, it may be the European engineers have the better idea.

E. S. Lee: Professor Dawes' remarks are interesting concerning the available information relative to mica. There is much known about mica which may not have been published. It would take many more printed pages than we now have if everything were published.

I can subscribe to the effectiveness of the method described in the Hague-Penney paper of obtaining physical characteristics of mica at elevated temperatures, as we have used this method considerably during the past several years.

F. D. Newbury: Commutator construction has been and probably still is more of an art than a science. If the present paper has any value it is in placing this difficult branch of machine construction on a little more scientific and certain basis.

Mr. Soderberg in his remarks mentioned the difficulty of separating the mechanical and electrical performance characteristics of commutators. Those who have been responsible for the entire performance, whether it be mechanical or electrical, factory or design, know how serious the argument as to responsibility for poor performance may become, and the means described in this paper for testing a commutator solely with regard to its mechanical operating characteristics, I feel, is an achievement of prime importance.

I should like to say a word about the Standards aspect of this situation. In our present standards and, in fact, in the standards of the entire world, as far as I know, the only requirement from the standpoint of operation of commutators is the one of limiting temperature rise. The determination of that limit has been based wholly on the safe limiting temperature of insulation. While this limit is important, it is of only minor importance as compared with the limitation of smooth operating surface. The paper is timely from this aspect of the situation on account of the present discussion in Europe concerning temperatures of commutators. Our temperature limits are somewhat higher than European limits as Professor Slichter points out.

The point that I would emphasize is that our present Institute limit may be too high or it may be too low but whether it is one or the other is purely accidental because the limit has been based on the wrong criteria. It is a mechanical and not an insulation problem, and the test should be a mechanical test, not a temperature test. Temperature of commutators becomes important only because temperature may be related to smoothness. This paper presents a means of making a mechanical test and that, in turn, makes possible a rational standard. I don't mean that we are ready to include in our standards such a mechanical measure of commutator performance, but if others will use the means here described or some similar means, we will accumulate experience that will make a rational standard a possibility for the future.

A. M. MacCutcheon: I wish to emphasize the value of this paper. It is very interesting and helpful to all who are working with commutators.

I agree with Mr. Newbury in his feeling that the permissible temperature of commutators has not been established on a scientific basis.

I have often noted that good commutation accompanies a cool commutator. In particular, I recall one case where the current density was very high, the mechanical operation was not good, and yet the commutation was excellent. In this instance, the commutator was very cool, having less than 40 deg. rise.

G. W. Penney: Mr. Dawes raised the question of the temperature at which commutators are baked and quoted one manufacturer as saying that no one knows this temperature. This is probably true in the case referred to of small commutators passing through an oven in a continuous process, although even here the actual commutator temperature could be measured. However, a statement such as the one just referred to should not be taken to mean that the manufacturer of large commutators does not know the temperature at which his commutators are baked. In our practise large commutators are baked in a steam heated oven equipped with recording thermometers. Then in order to determine the difference between the actual commutator temperature and that of the recording thermometer, thermocouples have been embedded in commutators of various sizes and time temperature curves taken so that by comparing these results with the recorded thermometer temperatures for any other commutator the actual commutator temperature can be very closely approximated.

To answer Mr. Dawes' question, our practise is to bake large commutators at from 135 to 150 deg. cent., although this does not mean that all commutators should be baked at this temperature since the correct temperature for a given line of commutators depends on the binder used in the mica, on the kind of solder used, on the hardness of the copper, and other factors. I do not know where the figure of 250 to 300 deg. cent. came from. It might refer to an oven temperature in some process connected with small commutators but it obviously could not refer to the actual temperature of large commutators since pure tin solder is used and this would flow out at 230 deg. cent. and furthermore, a temperature of 250 to 300 deg. cent. would rapidly anneal the copper.

The remark is frequently made that experience shows that high operating temperatures give trouble and therefore there should be a low temperature limit. However, an investigation of such cases usually shows mechanical roughness which is a function of the temperature such as that described in the paper but where redesigning for lower temperature may not be as successful as redesigning to reduce the roughness at the same temperature. Brush trouble is another serious question but here again there apparently is no fundamental temperature limit.

One test of the temperature question is to use an external fan and blow a large quantity of air at high velocity against a commutator which is giving trouble so as to give a lower operating

temperature. In many types of trouble due to mechanical roughness this should improve the operation, but if the commutator is mechanically smooth this should not improve the surface. In order to determine if there was a temperature limitation of some other nature besides mechanical roughness a commutator which was apparently mechanically smooth but which still would not commutate well was cooled by an external fan but this gave no noticeable change in the performance. This is another indication that temperature of itself is not a limitation.

So long as the troubles are not a function of any given temperature and so long as there is apparently no fundamental temperature limit (within the operating range) the practise of fixing an arbitrary limit is hardly more logical than fixing an arbitrary limit to the speed for a machine of a given rating or an arbitrary limit to the current density that may be used in the winding. As Mr. Newbury has pointed out, this does not say whether the actual operating temperature should go up or down but merely that it should be determined on a more logical basis.

Effect of Transient Voltages on Power Transformer Design

BY K. K. PALUEFF*

Associate, A. I. E. E.

Synopsis.—When subject to transient voltage excitation, local concentration of voltage takes place in an ordinary transformer in which the capacitance charging current of the coils to ground is supplied through the winding, or the ratio of inductance and capacitance of the various parts throughout the winding is not constant. Calculations and tests of voltage distribution in the winding, caused by the impact of (a) damped high-frequency oscillations, and (b) unidirectional traveling waves, are given. In order to make the analysis clearer, the transformer winding is considered as a network of inductances and capacitances, and this term "network" is used throughout the paper. Certain simplified and typical networks are considered.

Transformers having one terminal grounded, such as are used in three-phase star connection, particularly in high voltage systems, are frequently built with the insulation graded to other windings and ground, in the order of the normal frequency voltage stress. The danger of such a practise is shown in power transformers which are subject to transient overvoltage, since voltage oscillation in the winding may raise the voltage to ground at intermediate points above the terminal voltage, unless the design of the winding eliminates oscillation.

The theoretical and experimental data given show that the distribution and magnitude of voltage stresses existing during recognized standard insulation tests are essentially different from stresses created by transient voltages. This permits the construction of transformers that would satisfactorily pass standard insulation tests but at the same time would not be suitable for average service.

A new type of a transformer called "non-resonating," for use on grounded neutral systems, is described.

In transformers of this type, voltages of all frequencies distribute uniformly along the windings, as the possibility of internal voltage resonance is eliminated by a proper balance of distributed capacitance and inductance of the winding.

This is accomplished principally by means of conducting surfaces (shields) placed outside of the winding and connected to its line terminal.

The action of the shields is similar to that of the shielding ring on an insulator string. It neutralizes the effect of the capacitance current from the inside surface of the winding to ground, by supplying to every point of the winding a "charging" current equal to the "discharging" current of that point to ground. The application of the shield in some cases reduces the local stresses to one-eightieth.

Up to the present time, the total capacity of this new type of transformer exceeds half a million kv-a.

If the problem of designing the transformer insulation were limited to the requirements of normal frequency dielectric stress, it would be relatively simple.

It is not, however, the normal voltage stresses which, in high-voltage transformers, require the most careful consideration to predetermine the amount, kind, and arrangement of the insulation, but the transient stresses set up by abnormal conditions on the circuit.

The necessity for extra insulation, above that required to meet the A. I. E. E. test (or any other recognized standard rules) is due to two facts:

1. Transformers, even of high voltages, are subject to transient voltages many times the normal circuit voltage to ground. Records show from 10 to 15 times normal voltage to ground even on 220-kv. systems.

2. Most transient voltages are high-frequency oscillations, or are traveling waves lasting a number of microseconds, and the ordinary transformer does not permit these voltages to be uniformly distributed throughout the winding. This is because the transformer winding is not a pure inductance, but also contains distributed capacitance.

To determine the effect of the above considerations on the transformer design two alternative designs of 20,000-kv-a., 220,000-volt transformer were prepared. Both alternatives were to satisfy identical operating characteristics as efficiency, reactance, potential test, etc.

The first design was made in accordance with the A. I. E. E. Standards, as well as transient voltage requirements developed by the years of experience and incorporated into practise for at least ten years.

The second design was made in accordance with A. I. E. E. Standards only.

The comparison revealed that first transformer cost 40 per cent more than the second and has 75 per cent more of active material than the second.

It is important to note that the second transformer would withstand successfully not only the tests called for by A. I. E. E. Standards but also all the tests called for by any other recognized standard rules including those which specify so called "surge or impulse test." This is because such tests impose on transformers transient voltage of an amplitude negligible in comparison with those experienced in actual service.

In 1919, L. F. Blume and A. Boyajian presented to the A. I. E. E. a paper on *Abnormal Voltages within Transformer Windings*.[1] In this paper, with certain approximations, fundamental laws governing voltage transients in transformers were explained and expressed in mathematical form.

In 1922, J. M. Weed,[2] in a paper entitled *Prevention of Transient Voltage in Windings*, before the A. I. E. E.,

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1. A. I. E. E. Trans., Vol. XXXVIII., p. 577.
2. A. I. E. E. Trans., Vol. XLI., p. 149.

29-34

described a principle for the construction of non-resonating windings, in which line voltages of all frequencies are distributed uniformly throughout the windings.

With the assistance of these two papers, and much additional study and experiments, a new type of transformer has been developed for operation with solidly grounded neutral, which is believed to be better adapted to resist stresses caused by voltage transients than any type used heretofore.

This new type differs from the ordinary concentric winding, core type transformer, by the presence of conducting surfaces enveloping part of the outside (high-voltage) winding. These surfaces are called "shields." They are connected to the line terminal and properly insulated from the rest of the winding and ground. The shields, being on the outside of the winding and some distance away from it, do not affect its construction, and therefore ordinary disk coils, that are typical of power transformers, are used in "non-resonating" transformers. The capacitance of shields to the winding is so proportioned that voltages of all frequencies and wave shapes distribute uniformly throughout the winding. Therefore, the voltage stresses, produced by the transients at various parts of such a transformer, are very materially smaller than those found in transformers of the conventional type. In some cases, application of shields reduced stresses to one-eightieth.

The object of this paper is to describe the effect of transient voltage on ordinary as well as "non-resonating" transformers.

However, in order better to understand the principles involved, first an attempt will be made to describe non-mathematically, the chief characteristics of transient voltage phenomena.

Part I

Transient Voltage Phenomena

A. Equivalent Networks

1. Elementary Circuit. Most engineers, who deal with electrical phenomena of commercial frequencies only (25 to 60 cycles), picture to themselves inductance as a spiral wire, and capacitance as two parallel plates, and therefore are not apt to recognize these characteristics when they are disguised in different geometric forms. Furthermore, again due to everyday experience with low frequencies, they are accustomed to think of some apparatus (such as transformers and choke coils) as pure inductances, and of others as pure capacitances (parallel plates, synchronous condensers), or as resistances. To discard these faulty notions is the first step toward the understanding of transient phenomena.

If $a\,b$ (see M, Fig. 1) is a piece of conductor in air, and direct current is applied across the conductor, a current I will be established in it. If we take any two points on the conductor, like a and b, $a_1\,b_1$, etc., we find that there is a difference of potential between them $e\,e_1\,e_2$, etc.

As the conductor is surrounded by a dielectric (air), these potential differences will establish lines of dielectric stress, as shown in Fig. 1. The current I, flowing through the conductor, will of course create lines of magnetic stress around the conductor. The lines of dielectric stress can be represented by minute condensers $c_1\,c_2\,c_3$ and the magnetic flux by minute inductances. As no conducting material is entirely resistanceless, the resistance must also be included, and it can be shown in a conventional manner.

Thus we see that a straight piece of conductor, no matter how small, and regardless of its material, is not a pure resistor, but is equivalent to a complicated network of elementary condensers, inductances, and resistances.

Even a conducting sphere immersed in a dielectric with potential applied between any two points of its surface is also equivalent to a network of all three elementary constants. Also two parallel plates with terminals $a\,b$ are equivalent to a network of all three elementary circuits.

Depending on the size of the conductor and the

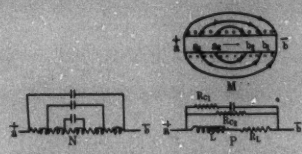

Fig. 1—Equivalent Circuit of a Simple Conductor

nature of the problem, different elementary distributed constants can be lumped together, and thus converted into a simpler (but for a given problem, equivalent) network of a pure condenser shunted with a pure inductance, which inductance is connected in series with a pure resistance (N in Fig. 1).

When it is desired to take losses in the condenser into account, two more pure resistances are added, as shown at P Fig. 1. Their ratio depends on the characteristics of the dielectric and the shape of the condenser plates.

The conclusion is that pure constants do not exist in practise; therefore, whenever a part of a circuit acts as pure inductance, capacitance, or resistance, it simply means that at the frequency of the phenomenon under consideration, the characteristics of this particular constant dominate.

B. Sustained and Damped Oscillation

1. Elementary Circuit. To simplify further discussion, let us neglect the resistances of elementary network, Fig. 2, and thus assume the elementary conductor $a\,b$, as represented by a pure concentrated capacitance C in parallel with a pure concentrated inductance L.

With voltage E of a given frequency f impressed

across terminals $a\ b$ of elementary circuit Fig. 1, current I will flow through the terminals.

$$\text{Current } I = i_1 - i_c$$

where i_1 is current in the inductance
where i_c is current in the condenser

As current is proportional to the susceptance of the circuit, I = susceptance of inductance + susceptance of the condenser. Taking the susceptance of the inductance as positive, and of the condenser as negative, we have:

$$\text{Susceptance of inductance} = + \frac{1}{2\,\pi\,f\,l}$$

$$\text{Susceptance of condenser} = - \ 2\,\pi\,f\,C$$

Fig. 4 shows graphically the change in susceptances due to change in frequency. Curves L and C illustrate susceptance or current in the inductance and condenser respectively. Curve A represents combined susceptance of the two, connected in parallel, or the current through terminals $a\ b$. This curve is obtained by

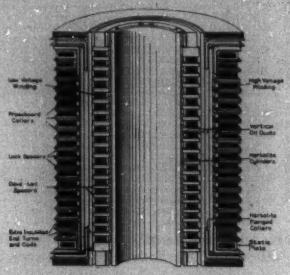

FIG. 2—CROSS-SECTION OF COMPARATIVELY HIGH-VOLTAGE POWER TRANSFORMER WINDING

subtracting the respective values of curves L and C respecting their signs. As reactance is inverse to susceptance, the reactance of the circuit is represented by curve Z.

Judging the circuit by the voltage and current at terminals $a\ b$, we conclude that at zero frequency (direct current) the entire line current is flowing through inductance and the circuit acts as a pure inductance of value L. With rising frequency, the current in the inductance diminishes and condenser current is created. However, as this condenser current is much smaller than the inductance current, the line current is still inductive but of smaller value than for inductance L alone. Thus the circuit at low frequency acts as pure inductance but greater in value than L. This apparent increase in inductance of the circuit increases with increase in frequency until near a certain frequency f_1 the circuit acts as practically infinite inductance. At frequency f_1 the inductance current is exactly equal to the condenser current, and therefore there is no current in the line and the network acts as an open circuit. At a frequency slightly beyond f_1 the condenser current is greater than the inductance current, therefore the line current is leading and the circuit is in effect a pure condenser but much smaller than condenser C. With further increase in frequency, the inductance current

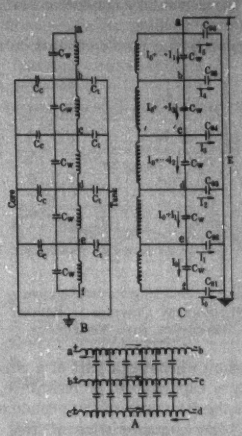

FIG. 3—TRANSFORMER EQUIVALENT NETWORK

A. Network of adjacent turns
B. Network of entire winding.
C. Simplified B.

falls down rapidly, while the condenser current steadily increases, and the line current increases as if the circuit were that of a pure condenser, increasing in size with frequency but always remaining smaller than C. Finally at infinite frequency, the current in the inductance becomes zero, and the entire line current flows

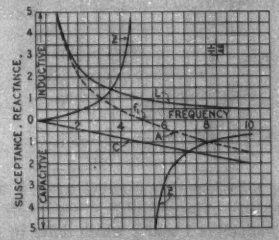

FIG. 4—ELEMENTARY CIRCUIT. CHANGE OF REACTANCE AND ADMITTANCE WITH FREQUENCY

through the condenser, and the circuit acts as a pure condenser of value C.

The conclusion is that the elementary circuit, under sine wave excitation at frequencies between zero and f_1, acts as a pure inductance ranging in value from L to infinity and at frequencies between f_1 and infinity acts as a pure condenser ranging in value from zero to C.

It is common knowledge that the frequency f_1, the critical or natural frequency of the circuit, depends only on the product $L\,C$ and is inversely proportional to the square root of this product—

The values of L and C depend on the physical characteristics of the elementary conductor and of the surrounding dielectric medium.

The conclusion is that pure constants do not exist in

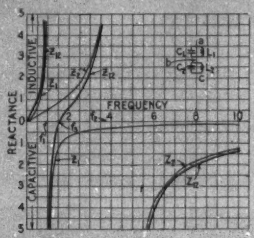

FIG. 5—TWO ELEMENTARY CIRCUITS IN SERIES. CHANGE OF REACTANCE WITH FREQUENCY

practise, therefore whenever a part of a circuit acts as pure inductance, capacitance, or resistance, it simply means that at the frequency of the phenomenon under consideration, the characteristics of this particular constant dominate.

It follows that one and the same elementary circuit may behave as a short circuit, an inductance, an open circuit, or a capacitance, depending upon the frequency of the applied voltage.

2. *Two Elementary Conductors in Series.* Let us

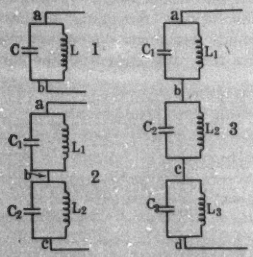

FIG. 6—ELEMENTARY CIRCUITS NO. 1, 2, 3

take two conductors 1 and 2 which differ in L and C as well as in their respective products. The variation of the reactance of each conductor with frequency can be shown as on Fig. 5 (curve Z_1 corresponds to conductor 1 and curve Z_2 to conductor 2).

If these conductors are connected in series then the combined reactance Z_{12} for any frequency will be equal to the algebraic sum of Z_1 and Z_2 for that frequency, and the share of the total voltage E_{ac} that will appear across each conductor will be equal to the proportion that the reactance of this conductor bears to the total reactance Z_{12}.

Examination of curves Z_1, Z_2, and Z_{12} reveals the following:

At $D\,C$ or low frequency, both conductors act as pure inductances of L_1 and L_2 in values and divide line voltage in direct proportion to these values.

The reactance of circuit No. 2 rises slowly with frequency. The reactance of No. 1 rises very rapidly and becomes infinite at f_1. For these reasons at frequencies approaching f_1 a greater and greater portion of the line voltage appears across conductor No. 1, until at f_1 the entire line voltage E_{ac} concentrates across $a\,b$. This is shown by curve No. 1 of Fig. 7.

Between frequencies f_1 and f_2, conductor No. 1 acts as a condensive reactance, while conductor No. 2 is still an inductive reactance, and the voltage across the two parts of the circuit is 180 deg. out of phase.

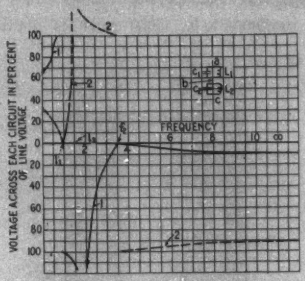

FIG. 7—VARIATION OF VOLTAGE DISTRIBUTION WITH FREQUENCY BETWEEN TWO ELEMENTARY CIRCUITS

At a certain frequency f_3, the reactance of both becomes numerically equal, and the total reactance of the circuit becomes zero, therefore any finite voltage will create an infinite voltage across each one. This rise of voltage to infinity, however, is not instantaneous with the closing of the circuit. It is linear with time. The amplitude of each succeeding half cycle increases by the amplitude of the line voltage.

At any frequency above f_2 both conductors act as pure condensers, gradually increasing in size with increasing frequency until at infinite (or very high) frequency they reach their actual values C_1 and C_2.

The line voltage divides between them in inverse ratio of the values of their effective capacities for the corresponding frequency.

As at f_2 and somewhat higher frequency, conductor No. 2 acts as an extremely small condenser in comparison with No. 1, the whole voltage concentrates across it. (Curve No. 2, Fig. 7.)

For example, taking a network where:

$$\frac{L_1}{L_2} = 2; \quad \frac{C_1}{C_2} = 0.1$$

The dielectric stress from a to b and b to c, at different frequencies, will be as follows:

Frequency range	Dielectric stress in per cent of line voltage	
	a to b	b to c
Direct current or very low.....	66.6	33.3
Natural frequency of $a\,b = f_1$...	100	0
Natural frequency of $b\,c = f_2$...	0	100
Natural frequency of $a\,c = f_3$...	infinity	infinity
Infinite or very high.....	9	91

This tabulation shows that to design apparatus properly it is necessary to know the range of frequency to which it may be subjected as otherwise its life may be unduly short.

3. Three Dissimilar Elements in Series. A circuit composed of three dissimilar conductors in series has

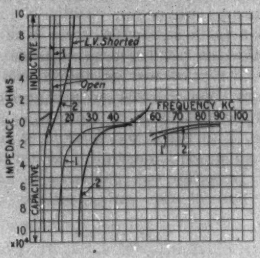

Fig. 8—High Frequency Impedance Test on Power Transformer B with Shield Removed

No. 1 L. V. open
No. 2 L. V. shorted

characteristics similar to that of two conductors. It has, however, three frequencies of current resonance f_1, f_2, and f_3, corresponding to three products $L_1\,C_1$, $L_2\,C_2$, and $L_3\,C_3$. At each of these frequencies the entire line voltage concentrates across the corresponding part of the network.

The network also has two frequencies of voltage resonance f_4 and f_5 (f_4 is between f_1 and f_2 and f_5 between f_2 and f_3). At f_4 voltages $a\,b$ and $b\,d$ are infinite and opposite in phase. At f_5 voltages $a\,c$ and $c\,d$ are infinite and opposite in phase.

The very important similarity to a two-conductor network lies in the fact that at very low frequency the line voltage is distributed in direct proportion to inductances L_1 L_2 and L_3 as if condensers were not present, while at very high frequency it is distributed in inverse proportion to capacitances C_1 and C_2 and C_3 as if inductances were not present.

It is easy to foresee that each addition of a different element to the circuit will bring one new frequency of current resonance and one new frequency of voltage resonance.

C. Traveling Waves

1. General. The effect of a traveling wave on the circuits considered above, is so essentially different from the effect of a sustained oscillation that it justifies special discussion.

There must be distinguished two principal types of traveling waves—infinite, or the effect of which is equivalent to that of infinitely long waves, and finite waves. A traveling wave created by lightning is typical of a finite wave.

For the present we will assume that the surge impedance $\sqrt{\frac{L}{C}}$ of the circuit, connected to the

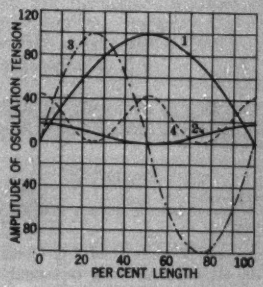

Fig. 9a—Oscillation of Elastic Rod

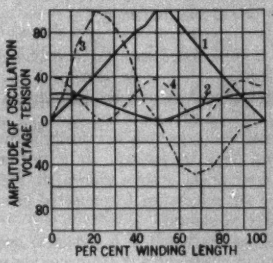

Fig. 9b—Oscillation of Transformer Winding

terminals of the network under consideration, is negligibly small compared with the surge impedance of the network.

In such a case a traveling wave emerging from the external circuit undergoes practically complete reflection from the terminals of the network. Thus the amount of energy lost by the wave to the network, from the moment the front of the wave arrives at the terminals to the moment its tail leaves them, is negligible in comparison with the total energy of the wave.

In the case of complete reflection, the steepness of the front and of the tail of the voltage wave and its

crest value at the point of reflection (the terminals of the network) is twice as great as that of the traveling wave; while the lengths of the front, crest, and tail of the voltage wave are equal to the lengths of the corresponding parts of the traveling wave (Fig. 10).

Under these conditions, the investigation of the influence of the traveling wave No. 1 of Fig. 10 on a network is reduced to finding the behavior of a network with voltage wave No. 2 maintained at its terminals.

2. *Infinite Traveling Wave.*

a. Capacitance in Series with Inductance. Analysis

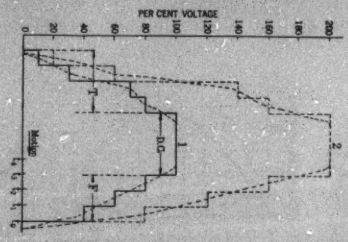

Fig. 10

No. 1 Traveling wave
No. 2 Voltage wave produced by traveling wave No. 1 at point of complete reflection

shows that in case of a circuit of Fig. 11 being subject to impact of an infinitely long rectangular wave of $+E$ volts, the voltage of the condenser will oscillate with amplitude E and frequency $f = \frac{1}{2\pi\sqrt{LC}}$ about a point $+E$ volts above zero and its maximum voltage will be $+2E$.

A mechanical analogy may be of assistance here.

It is well known that mass and elasticity correspond to inductance and capacitance, velocity corresponds to current, and force to voltage. It is further obvious that the mechanical system shown in Fig. 11 corresponds to an inductance L connected in series with capacitance C as shown.

Let it be assumed that E is the force necessary to hold mass L in position No. 2 (D inches above position No. 1). When a force applied to L is brought from zero to E in an extremely gradual manner so that the applied force reaches E at the same moment that L attains position No. 2, no oscillation will take place during the transient. This happens because the increase in force is controlled in such a way that the applied force exceeds the pull of the spring by an infinitesimal amount until the value of E is reached.

However, should force E be applied abruptly, at the end of the first elementary time interval, L will move a distance d, the pull of the spring will be $-E\frac{d}{D}$ and the total force on L will be the difference.

$$E - E\,d/D = E\,(1 - d/D)$$

Mass L, therefore, will behave for the next time interval as a free body under the influence of force $E\left(1 - \frac{d}{D}\right)$ and will accelerate proportionately to this force and store a certain amount of energy in kinetic form.

During the following interval, mass L will be under the influence of $E\left(1 - 2\frac{d}{D}\right)$, that is of a smaller force, and its acceleration will diminish. With every following interval, the force and hence the acceleration will grow smaller, but the velocity will be steadily increasing and therefore the total storage of kinetic energy.

At a certain time t_1, a pull of the spring will be exactly equal to E, and total force acting on L will be zero. At this moment L will occupy position No. 2, (that is, final position for a case where force E is applied so gradually that no oscillation takes place). The acceleration will become zero and the velocity will reach its maximum value. Under an abrupt force E, the kinetic energy accumulated by L during its travel from position No. 1 to No. 2 will force it to continue its motion in an upward direction. At the end of the elementary time interval, following t_1, L will experience a pull of the spring greater than force

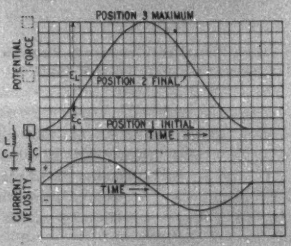

Fig. 11—Oscillation of Inductance in Series with a Condenser Due to Infinite Rectangular Wave Excitation

E, and therefore the total force acting on L will be

$$E - E\left(1 + \frac{d}{D}\right) = -\frac{d}{D}E$$

That is, the force will change its direction and become downward, which will cause a certain retardation. With every time interval following, the negative force will steadily increase, and so will the retardation. At the moment t_2, the force acting on L will become $-E$, the retardation will reach a maximum, and the velocity zero. By this time, all the kinetic energy absorbed by L on its travel from position No. 1 to No. 2 will be transferred to the spring. As this transfer followed

exactly the same law as the absorption, only in reverse order, it naturally will be accomplished in the same time t_1, and the body will travel the same distance D.

Thus L will arrive at position No. 3, a distance $2\,D$ away from No. 1 after $2\,t_1$ seconds. At the same moment, the spring being expanded to $2\,D$ will be under strain $+\,2\,E$, and the mass will find itself under the action of $-\,E$. Now $-\,E$ acting on L will cause a downward motion of L which will be exactly the same function of time as the upward motion.

Thus L will oscillate between positions No. 1 and No. 3 about No. 2, subjecting spring C at regular intervals to a stress equal to $2\,E$. The oscillation about position 2 will be sinusoidal. The frequency of oscillation will be $f = \dfrac{1}{2\,\pi\,\sqrt{L\,C}}$.

In practise, oscillation is always followed by a loss of energy, and therefore it dies out with time and the system comes to rest.

In case the force or voltage is applied an indefinitely long time, this new state of rest coincides with the one about which the oscillation took place.

In the example it is position No. 2. We shall refer in the remainder of this paper to this state of distribution of force of a system as a "final" one. The state of distribution of force in which a system finds itself immediately after the application of force will be termed "initial."

We can formulate a conclusion from the above example as follows: In case a circuit is subjected to abrupt and steady voltage and its initial voltage distribution is different from the final voltage, a transient will take place. This transient will be oscillatory if losses in the circuit are relatively small. If the circuit has only one natural frequency the oscillation will be at that frequency and its amplitude equal to the difference between initial and final states.

b. Elementary Circuit. Considering again, as in case of sustained oscillation, an elementary circuit consisting of inductance in parallel with capacitance, we find that with a rectangular voltage wave applied no oscillation will take place because the initial and final states of voltage distribution are identical, as full voltage appears across condenser and inductance at the very beginning of the phenomenon and remains constant as long as the wave lasts.

c. Two Elementary Circuits in Series. Referring to Fig. 6, from a review of the behavior of this circuit under sustained oscillation, we conclude that at very high frequency the voltage will distribute between $a\,b$ and $b\,c$ as though inductance $L_1\,L_2$ were absent. This means that the voltage will distribute inversely as the capacitance. As the front of a rectangular wave acts as a quarter of a cycle of infinite frequency, it is obvious that at the first instant the voltage will distribute inversely as the capacitance, as shown below.

The initial voltage across $a\,b$, $E_{abi} = E\,\dfrac{C_2}{C_1 + C_2}$

The initial voltage across $b\,c$, $E_{bci} = E\,\dfrac{C_1}{C_1 + C_2}$

In the numerical example of Fig. 12.

The initial voltage across $a\,b = E\,\dfrac{1}{10 + 1} = 0.09\,E$

The initial voltage across $b\,c = E\,\dfrac{10}{10 + 1} = 0.91\,E$

In reviewing a sustained oscillation, we concluded that at very low frequencies or direct current, the line voltage will divide between $a\,b$ and $b\,c$, as though capacitance were absent; that is directly proportional to the inductances L_1, L_2.

As the effect of the crest of a rectangular wave of infinite length on the circuit is identical to that of direct current, it follows that the final voltage distribution will be directly proportional to the inductances as shown below:

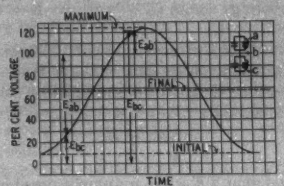

FIG. 12—OSCILLATION OF TWO ELEMENTARY CIRCUITS CONNECTED IN SERIES DUE TO INFINITE RECTANGULAR WAVE EXCITATION

$$E_{abf} = E\,\frac{L_1}{L_1 + L_2} \qquad E_{bcf} = E\,\frac{L_2}{L_1 + L_2}$$

in numerical example of Fig. 12.

$$E_{abf} = 0.66\,E \qquad E_{bcf} = 0.33\,E$$

Thus initial state

$$E_{abi} = E\,\frac{C_2}{C_1 + C_2} \qquad E_{bci} = E\,\frac{C_1}{C_1 + C_2}$$

and final state

$$E_{abf} = E\,\frac{L_1}{L_1 + L_2} \qquad E_{bcf} = E\,\frac{L_2}{L_1 + L_2}$$

Examination of Fig. 12 shows that the maximum voltage across C_1 is numerically equal to the final voltage of C_1 + a voltage numerically equal to the amplitude of the oscillation, which is the difference between the final and the initial state.

Thus

$$E_{abm} = \frac{L_1}{L_1 + L_2} \quad E + \left(\frac{L_1}{L_1 + L_2} - \frac{C_2}{C_1 + C_2} \right) E$$

and across C_2

E_{bcm} = final voltage of C_2 + voltage numerically equal to amplitude of the oscillation

$$= \frac{L_2}{L_1 + L_2} E + \left(\frac{L_1}{L_1 + L_2} - \frac{C_2}{C_1 + C_2} \right) E$$

In the numerical example

$$E_{abm} = [0.66 + (0.66 - 0.09)]\, E = 1.23\, E$$

$$E_{bcm} = [0.33 + (0.66 - 0.09)]\, E = 0.90\, E$$

With L_2 and C_2 negligible in comparison with L_1 and C_1 the final voltage across $C_1 = \frac{L_1}{L_1 + L_2} E = E$.

The amplitude of oscillation

$$= \left(\frac{L_1}{L_1 + L_2} - \frac{C_2}{C_1 + C_2} \right) E = E$$

and maximum voltage $E_{abm} = 2\, E$.

The final voltage across $C_2 = \frac{L_2}{L_1 + L_2} E = 0$.

The amplitude of the oscillation is E (the same as for C_1) and maximum voltage $E_{bcm} = E$. It is obvious that if on the contrary L_1 and C_2 are negligible in comparison with L_2 and Q_2, then

$$E_{abm} = E$$

$$E_{bcm} = 2\, E$$

It is evident that $2\, E$ is the maximum possible voltage that may be created by an infinite rectangular wave across either part of the circuit.

Elimination of Transients

As such an oscillation creates voltage across some part of the circuit in excess of the final value, it may be harmful and therefore undesirable.

As the cause of all transients is the difference between initial and final states, it follows that the elimination of this difference means elimination of the transients. In this case it can be accomplished by making

$$\frac{L_1}{L_1 + L_2} = \frac{C_2}{C_1 + C_2},$$

which is equivalent to having

$$\frac{L_1}{L_2} = \frac{C_2}{C_1} \text{ or } L_1, C_1 = L_2 C_2.$$

To secure this relation it is sufficient to be able to choose the proper value for one of four constants. Referring to Fig. 15 it is seen that with such a relation of constants, the division of line voltage between two parts of the circuit is identical for the entire range of frequencies. The effective reactances change from inductive to capacitive at one and the same frequency f_1. Therefore their voltages are always in phase and hence at no frequency can either of them exceed line voltage.

For the same reason at no frequency can combined impedance Z_{12} become zero. Such a circuit therefore is incapable of voltage resonance.

d. Three-Element Circuit. To visualize the tendency of transient characteristics as the complexities of the network increase, we will further review the behaviour of three dissimilar elementary circuits connected in series, under the influence of an infinite rectangular voltage wave.

From considerations previously outlined, the initial and final voltages for each branch of the circuit can be written from mere inspection of Fig. 6. Taking C_0 and L_0 for capacitance and inductance from a to d

Initial voltages

$$E_{abi} = \frac{C_0}{C_1} E; \quad E_{bci} = \frac{C_0}{C_2} E; \quad E_{cdi} = \frac{C_0}{C_3} E$$

Final voltages

$$E_{abf} = \frac{L_1}{L_0} E; \quad E_{bcf} = \frac{L_2}{L_0} E; \quad E_{cdf} = \frac{L_3}{L_0} E$$

In case some or all initial values are different from final

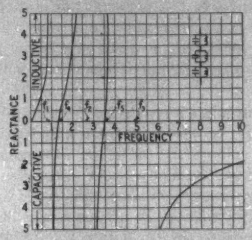

Fig. 13—Variation of Reactance with Frequency. Three Elementary Circuits Connected in Series

values, transient phenomena will occur in the form of oscillations of points b and c.

Fig. 13 indicates that this circuit is capable of voltage resonance at points b and c at frequencies f_2 and f_4 respectively.

With point b oscillating at its natural frequency f_2, the voltage between b and d will be pulsating with that frequency, and therefore will cause the voltage between c and d to pulsate at the same frequency with amplitude depending upon the ratio of reactances of $b\,c$ and $c\,d$. On the other hand, point c occupies in the circuit a position similar to b and therefore is equally capable of oscillating at its own natural frequency f_4; in which case the voltage between a and c will pulsate at that frequency and so will point b.

Thus point b is in a state of free oscillation at its own natural frequency f_2, and of forced oscillation at frequency f_4; similarly point c is in a state of free oscillation at frequency f_4 and of forced oscillation at frequency f_2.

The amplitude of oscillation depends upon the difference between initial and final voltage distribution, and also upon the ratio of reactance between points a and b, d and c, c and d, at frequencies f_2 and f_4.

In case $\frac{C_1}{C_2} = \frac{L_2}{L_1}$, point b has no natural frequency and the circuit oscillates at the natural frequency of point c. In case $\frac{C_2}{C_3} = \frac{L_3}{L_1}$, the natural frequency of point c disappears and the circuit oscillates at the natural frequency of b.

Elimination of Transients

It is obvious that should constants be adjusted so that

$$\frac{C_1}{C_2} = \frac{L_2}{L_1} \quad \frac{C_2}{C_3} = \frac{L_3}{L_2}$$

the initial and final voltage distribution will be identical, no oscillation will take place. Such a circuit will be incapable of voltage resonance.

3. Infinitely Long Traveling Wave with Slanting Front.

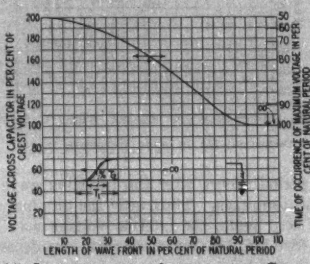

Fig. 14—Inductance in Series with Capacitance

Voltage across capacitor as the function of the slope of the front of an infinite wave

In the case of a system consisting of a spring and a mass, it was found that a given steady force E subjects the spring to a stress of either $2\,E$ or E depending on whether it is applied suddenly or gradually.

This suggests that the maximum voltage stress created in a given network by a traveling wave depends on the steepness of the wave front.

The actual front of the wave may start to rise very slowly (say up to 10 per cent of the crest value), then fast (say up to 90 per cent of the crest value), and then slowly again. If the length of a given front T, in Fig. 14, is measured from the very beginning of the front up to the very crest of it, a wrong impression may be gained of its effect on the circuit. It was decided therefore to consider as the slope of the front of the wave the slope of the portion of the front lying between 10 per cent and 90 per cent of the crest value. The effective length of the front becomes T_2 as shown.

To illustrate the relation between the steepness of the front of the infinite wave and the maximum internal voltages created thereby across the capacitance part of the circuit, the curve of Fig. 14 was prepared for the case of an inductance in series with a capacitance.

As in such a case, the rate of change of internal voltage depends only on the front of the wave and the natural frequency of the network, the length of the front is expressed in per cent of the natural period of the network.

The lapse of time from the moment of impact of the wave to the moment the maximum voltage is reached is also given and expressed in per cent of the same period.

Fig. 14 shows that in case of inductance in series with a capacitance, there is a critical or minimum length of the front which does not produce oscillations or voltage across condenser in excess of its final value, which is the line voltage E. The same rule holds true for any other circuit. The critical length of the front depends only on natural frequency of the circuit and in case of the circuit of Fig. 14 is equal to its natural period.

On account of the above characteristics, a given front may be qualified as a very steep one for a circuit of low natural frequency, and as a very slanting one for a circuit of high natural frequency.

4. Finite Rectangular Traveling Wave.

a. General. When a transient phenomenon is the result of several causes, it may be solved by the method of superposition.

By this method, the effect on the circuit of each cause is found under the assumption that the circuit is not subjected to the effect of other causes. Then to find the actual values of currents or voltages at a given point, simultaneous values produced by all the causes are added algebraically. This is mathematically correct only where circuit constants are independent of absolute values of current and voltage.

A finite rectangular traveling wave of L miles in length (or T seconds in duration) and of E volts can be looked upon therefore as a result of two infinite rectangular waves; one of $+\,E$ and the other of $-\,E$, traveling in the same direction with the same velocity as the finite, but with positive wave L miles (or T seconds) ahead of the negative one and with its front coinciding with the front of the finite wave, Fig. 16.

The effect of such a finite wave on a circuit is identical therefore, with the effects of the two infinite waves, determined independently and superimposed one on the other.

Transients due to waves of the same magnitude but of opposite signs are, of course, identical in form, but 180 deg. out of phase.

b. Inductance in Series with Condenser. It was found that the voltage across a condenser, due to a positive rectangular wave, will change with time, as shown by curve No. 1 on Fig. 16, as the negative wave arrives at the terminal of the circuit t seconds later,

the condenser voltage created by it will be as shown by curve No. 2.

By superimposing curves No. 1 and No. 2, we derive curve No. 3 as the condenser voltage wave due to a finite wave of L miles (or t seconds). In a similar way, the voltage wave across an inductance may be determined.

To determine the effect of traveling waves of different lengths on condenser voltage, wave No. 2 should be slid along the abscissa to corresponding positions with respect to wave No. 1, and then combined with the latter.

Very often in practise it is not the shape of the voltage across a given part of a circuit that is important, but rather the crest value of the transient. Inspection of Fig. 16 indicates that the abrupt ending of the wave cannot increase the voltage across the condenser

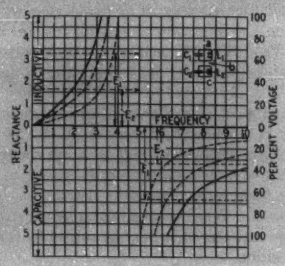

Fig. 15—Variation of Reactance with Frequency and Distribution of Line Voltage Between $a\,b$ and $b\,c$, where $\frac{L_1}{L_4} = \frac{C_3}{C_1}$

beyond $2\,E$ (as wave No. 2 is always subtracted from No. 1). It should be remembered, that $2\,E$ is the maximum condenser voltage that can be produced by an infinite wave. It also shows that the crest value will reach $2\,E$ unless the wave is shorter than half of the period of the natural frequency of the circuit, that is unless

$$t < \frac{T}{2}$$

or as

$$T = \frac{1}{f} = 2\,\pi\,\sqrt{LC}$$

$$t < \pi\,\sqrt{LC}$$

The curves of Fig. 17 show the relation between the length of a traveling wave (expressed in per cent of natural period) and the value of the crest voltage produced thereby across the condenser expressed in per cent of voltage produced by the traveling wave across the terminals of the circuit. (It should be remembered that due to assumed complete reflection of the traveling wave from the terminals of the circuit, the voltage of the traveling wave is half of the terminal voltage E.) The maximum voltage across the inductance occurs at the moment of collision of the traveling wave with the circuit, and therefore is independent of the length of the wave. Its value equals terminal voltage E.

Examination of this curve shows that the crest voltage produced by waves, shorter than half of the

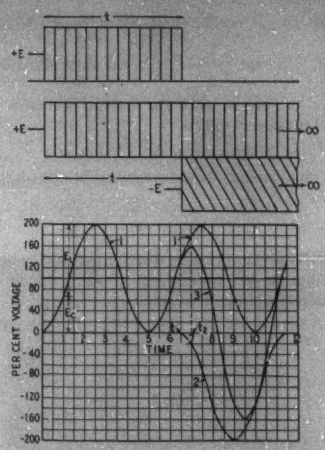

Fig. 16—Inductance in Series with Capacitance

Oscillation due to finite rectangular wave

natural period occurs after the traveling wave leaves the circuit terminals. Thus, after the terminal voltage becomes zero again, the internal voltage may be still rising. (See Fig. 16, part of the actual condenser wave between t_1 and t_2.)

Examination of curve No. 3, Fig. 16, shows that

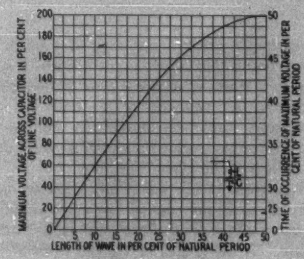

Fig. 17—Effect of Length of Rectangular Wave on Maximum Voltage across Capacitor

abrupt ending of the finite wave does not produce any sudden change in this circuit as the oscillation that follows is still sinusoidal but the axis of oscillation changes from a level $+\,E$ volts above zero potential to zero potential. Depending upon the length of the

wave, this may result in increase of the amplitude of oscillation, and it may become equal to twice line voltage in case the length of the wave is an odd multiple of $\frac{1}{2}T$ as can be seen from Fig. 16 if wave No. 2 is allowed to be shifted to the right, so that its beginning coincides with crest of wave No. 1.

c. Two Elementary Circuits in Series. Analyzing in a similar manner a network consisting of two dissimilar elementary circuits in series, we obtain the voltage waves shown on Fig. 18. The wave differs in character from the wave of Fig. 16. In the case of pure inductance in series with pure capacitance, it was found that the shape of a condenser voltage wave can never have a rectangular component, as it is controlled by the natural frequency of the circuit. In the case of the circuit under consideration, however, a voltage wave across either part of the circuit contains a rectangular component equal to the initial voltage of the respective part, which coincides with the front and tail of a finite wave. This occurs because of the presence of paths of pure capacitance from point *b* to both terminals.

The oscillation, after the rectangular wave is re-

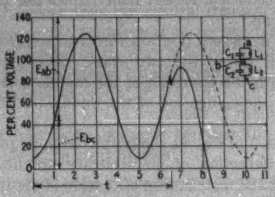

FIG. 18—TWO ELEMENTARY CIRCUITS IN SERIES

Oscillation due to finite rectangular wave of *t* seconds

flected and leaves the terminals, is similar to that of the previous circuit. It also takes place about the line of zero potential, and its maximum amplitude may be twice the amplitude of the preceding oscillation.

It may be $2E$ (as in previous circuits) only if $\frac{C_1}{C_2}$ and $\frac{L_1}{L_2}$ is so large or so small that C_1 and L_2 or C_2 and L_1 can be neglected, but then the circuit becomes identical with the preceding one. It should be remembered, of course, that in this circuit, capacitance must be very small to be neglected while inductance must be very large.

d. Three Elementary Circuits in Series. A three-element circuit acts similarly to a two-element circuit. The presence of two natural frequencies produce transients (following the end of the traveling wave) of considerable complexity. The crest voltage across any one part may be equal to practically double line voltage if the "initial" and "final" voltage distributions are very different.

5. Traveling Wave of any Shape.

In practise, rectangular waves do not exist, and therefore the effect of deviation of wave shape from the rectangular must be considered.

A traveling wave of any shape, as well as a terminal voltage wave produced by the traveling wave, can be assumed to consist of a certain number of short rectangular waves, all of the same sign as the "main" wave, following one another in unbroken sequence, with their amplitudes and lengths corresponding to that part of the main wave with which they coincide in time. (See Fig. 10.) The voltage produced by such a wave at any point of the circuit is obtained by superimposing the simultaneous values of voltages produced by all the elementary waves at that point.

Part II.

ORDINARY TRANSFORMER

A. EQUIVALENT NETWORK

At operating frequencies, transformers act as inductances in series with some resistances. But, as was shown above, no apparatus or any part thereof can be free of capacitance,—therefore the capacitance must be present in the transformer.

In Fig. 2, a cross-section of a transformer is shown schematically.

In case of a potential difference existing between turns, turn insulation is under voltage stress, but any dielectric acts as a condenser if it is subjected to voltage stress; therefore insulation between two adjacent turns, combined with parts of conductor surfaces facing each other, comprises a condenser. Therefore, each turn has capacitance to turns on its four sides.

As there is a continuous metallic path from any point of one turn to any point of the next, in case of potential difference existing between turns, some current will flow along the conductor and produce magnetic flux.

Thus adjacent turns are equivalent to parallel conductors with distributed constants. (Fig. 3.) If the phenomenon under consideration permits the distributed constants to be lumped, the turn can be represented by a concentrated inductance in parallel with concentrated capacitance.

Between core and turns, located on the inside surface of the winding, the capacitance effect is present on account of the potential difference between the parts. The same is true of turns on the outside surface of the winding and the tank.

Instead of taking a turn as an element of an equivalent network, some problems permit the use of much larger portions of the winding for this purpose.

Taking each separate coil (disk or pancake) as an element, a transformer equivalent network becomes as shown in Fig. 3.

The network shown in this figure lacks negative induc-

tance links representing mutual inductance between various parts of the winding. They were omitted so as not to obscure the more essential members of the network. Therefore, actually this circuit is not equivalent to a transformer at all, but their behaviors under similar conditions, while quantitatively different, are sufficiently alike qualitatively to justify the use of such a network for getting a rough idea of the action of a transformer under transients. Whenever calculated results of a transformer's behavior are given in this paper, the effect of mutual inductance is properly taken into account.

B. SUSTAINED AND DAMPED OSCILLATIONS

1. Transformer Winding.

There is a similarity between circuit No. 3 of Fig. 6 and C of Fig. 3, and therefore it can be expected that circuit No 4 has a certain number of natural frequencies of current resonance, and "sandwiched" between them, natural frequencies of voltage resonance.

It should be expected, therefore, that a curve of variation of the reactance of circuit No. 4 of Fig. 3 with frequency should be similar to that of circuit No. 3.

The result of a sustained high-frequency impedance test on a large power transformer is shown on Fig. 8. It substantiates the above expectation. As the resistance component cannot be eliminated in test, the impedance at voltage resonance frequencies is not zero but is equal to the effective resistance of the circuit at that frequency.

Similar to circuit No. 3, at different voltage resonance frequencies infinite voltages, if resistance were absent, could be created between different points of the winding and its terminals, as well as between some of the points. The location of these points in the winding is different for different natural frequencies.

At frequencies of current resonance, no infinite voltages are possible, but the entire line voltage may appear across a fraction of the winding, this fraction growing smaller the higher the frequency.

In case the winding is uniform in its construction, the constants of the equivalent circuit will be uniform, and a definitite relation between the magnitude of succeeding voltage resonance frequencies can be expected.

Examination of the test results of Fig. 8 show that, if the lowest voltage resonance frequency is taken as the first, and the next for the second, etc., the frequencies increase very nearly as the second power of their order. The lowest natural frequency is considered as the fundamental, the higher frequencies as its harmonics; therefore the frequency of a given harmonic is equal to the fundamental times the second power of the harmonic order. This law holds for harmonics of a lower order. The frequency of harmonics of very high order is $f = \frac{n}{2\sqrt{LC}}$ where C is the capacitance of one end of the winding to another through the body of the winding as if there were no shunt capacity to ground. L is the inductance of the transformer and n is the order of the harmonic.*

2. Mechanical Analogy.

It is unfortunate that a mechanical analogy to even such a simple circuit as C is so complicated that it is far easier to visualize the behavior of the circuit than to see the analogy between the two systems.

A simple mechanical system, used below, to illustrate the phenomenon of transient voltages taking place in a transformer, is not therefore analogous to a transformer, but it is hoped that apparent qualitative similarity justifies its use.

The equivalent transformer circuit of uniformly distributed inductances and capacitances is comparable to a long, very elastic rod with considerable distributed mass.

If one end of the horizontal rod is fixed, and the other is made to oscillate in a vertical direction with a frequency natural to the rod, the latter will bend into a single arc, first in one direction and then in another. With very small amplitude of oscillation at the free end, the amplitude of the middle will increase with each half cycle, and soon will exceed many times the amplitude of the free end.

As a rod, in its bent position, forms an arc, the greatest tension will be at the two ends. The tension will decrease with the approach of the middle where it will be zero, as the elongation of the middle portion of the rod is zero.

Curve No. 1 of Fig. 9A shows the rod in its position of maximum deflection of an arbitrary amplitude, and curve No. 2 the distribution of tension along it at that moment. Curves Nos. 1 and 2 of Fig. 9B show respectively the voltage to ground of various points of the winding, and voltage between adjacent points along the winding of a power transformer. These values were obtained by test, under excitation of damped oscillations of the fundamental natural frequency of the transformer.

By doubling the frequency of oscillation (2nd harmonic) of the rod, the latter will be made to bend during its oscillation in S form, Curve No. 3 of Fig. 9A, and the tension along the rod will be as shown by Curve No. 4 on Fig. 9A.

Curves Nos. 3 and 4 of Fig. 9B show the voltage distribution obtained by test on the same transformer under excitation of damped oscillations of its second harmonic (frequency of which is about four times the fundamental).

The rod oscillating with a frequency of its third harmonic will bend in three halves of a wave, and the maximum tension of individual elements will again be shifted to the new parts.

A transformer winding excited with third harmonic

*This is in agreement with formulas given in the A. I. E. E. paper by L. F. Blume and A. Boyajian, referred to above.

(the frequency of which is approximately nine times fundamental) will give a curve of voltage distribution which also forms three halves of a wave.

The nth harmonic therefore will "bend" the voltage distribution curve in nth halves of a wave, producing high stresses between adjacent elements of the winding at each node and high voltage to ground and line terminals at each anti-node.

Considering a transformer with isolated neutral, we may compare it with the same rod having one end oscillated and the other one free. It is obvious that at its fundamental natural frequency, it would bend into a half arc, into an arc and a half at its 2nd harmonic, and into $(2n + 1)$ halves of an arc at its nth harmonic. The waves of simultaneous voltages along the winding of an isolated neutral transformer follow the same law.

3. Natural Frequencies Experienced in Service.

Examination of voltage distribution test data, for various frequencies of damped oscillations given in the paper, reveal that no part of the winding is free of voltage concentration, as long as the natural frequency of oscillation of the circuit, external to the transformer, cannot be kept outside the range of natural frequencies of the transformer.

It so happens that the natural frequency of circuits connected to the transformer in service range between the same limits as do the natural fundamental and harmonic frequencies of transformers.

For instance, in case of an arc-over of an insulator on a transmission line, as the frequency of oscillation of line depends only upon distance between the transformer terminal and the arc it follows:

Distance between the transformer and the arc in miles											
0.10	0.20	0.4	0.8	1.0	2.0	4.0	8.0	10.0	20.0	40.0	80.0	100.0
Natural frequency of oscillation in kilocycles												
470	235	117.5	58.7	47.0	23.5	11.75	5.87	4.7	2.35	1.175	0.587	0.470

The fundamental natural frequency of transformers range from about 1000 to 60,000 cycles, and their harmonics of practical importance reach 750,000 cycles.

4. The Cause of the Existence of Natural Frequency of Voltage Resonance in a Transformer.

In studying the behavior of circuits Nos. 1, 2, and 3, we found that the voltage resonance occurs due to dissimilarity of the circuit elements. We also found that, if the reactances of all adjacent parts are alike at all frequencies, no voltage resonance can take place, and the voltage will distribute uniformly between all the elements (like $a\,b$, $b\,c$, $c\,d$ of Fig. 6).

Referring to network No. 4 of Fig. 3 of a uniform winding, we find that while inductances L, as well as series or internal winding capacitances C_w, are all alike, the capacitances C_s, from the surface of the winding to ground (the "shunt" capacitances) in spite of their being alike among themselves, make reactances of elements $a\,b$, $b\,c$, $c\,d$ differ from one another.

Should capacitances C_s therefore be removed, or their effect on the winding neutralized in some manner, the reactance of all the elements of network No. 4 will become one and the same; and as such a circuit is incapable of voltage resonance, voltage of all frequencies will distribute along it uniformly.

This conclusion is of fundamental importance.

From the charcteristics of circuits 1, 2, and 3, it also follows that once the effect of shunt capacitance C is neutralized, the voltage resonance can be eliminated not only by making all corresponding constants alike, but also by making them satisfy the relation:

$\frac{L_1}{L_2} = \frac{C_2}{C_1}$; $\frac{L_2}{L_3} = \frac{C_3}{C_2}$; etc.; then voltage at all frequencies will divide in direct proportion to inductances or inversely to capacitances.

5. Effective Reactance of a Transformer.

Similarly to circuits 1, 2, and 3, transformer effective reactance, as it appears to the external circuits, varies in magnitude and sign with frequency as curve of Fig. 8 shows.

The important fact is that the ranges of frequencies at which a power transformer acts as a capacitance increase with the frequency, while the inductance range grows narrow; therefore the sum of all capacitance ranges is much greater than that of inductance.

C. TRAVELING WAVES

1. Infinite Rectangular Traveling Wave.

a. Initial Voltage Distribution. It was shown above that a transformer, similar to circuits No. 2 and No. 3 of Fig. 6, has two parallel paths from one terminal to another, one a pure inductance and the other a pure capacitance. This means that at very high frequency the current will follow the capacitance path and at low frequency the inductive path.

A sufficiently steep front of a traveling wave corresponds to a quarter of a cycle of a very high frequency, and therefore its current will follow the capacitance path alone. This means that during the rise of terminal voltage from zero to crest of the wave no current will flow along the conductor, and the transformer will act as though every turn is disconnected from the adjacent one, that is, as a pure condenser. This state is called here the initial or electrostatic.

Referring to Fig. 19, it is seen that under such a circumstance, a transformer winding represents a mass of dielectric surrounded with metal surfaces of core and tank, and with conductors distributed throughout its cross-section. The cross-section of the line turn being very small in comparison with its distance from tank and core, will cause the lines of dielectric field to crowd in its neighborhood in a way similar to that shown. The rest of the conductors "floating" in this dielectric field will assume potential corresponding to the equipotential surface with which they coincide. This means that by far the greater part of line voltage will appear across the insulation in the neighborhood of line turn.

Instead of picturing the electrostatic field distribution during the impact of a steep front wave, the voltage concentration at the line end may be explained in the following manner.

The capacitance path of the transformer network contains a number of equal condensers connected in series and a number of condensers connected in shunt with different portions of the winding as shown in the

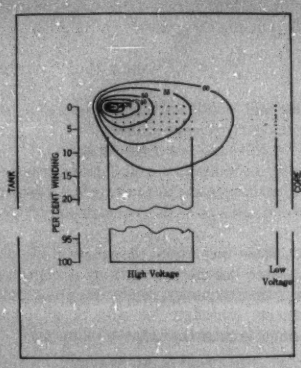

FIG. 19—INITIAL DIELECTRIC FIELD IN TRANSFORMER WINDING WITHOUT STATIC PLATE DUE TO STEEP WAVE FRONT

(Equipotential lines in per cent of applied voltage)

equivalent circuit of Fig. 3. This sort of a condenser system is similar to that of a string of suspension insulators, where the capacity from one hardware to the next corresponds to the series capacity C_w between coils of the transformer and capacitance from hardware to ground corresponds to the shunt capacity of the transformer coils C_s to ground.

The fact of voltage concentration at the line end of a string of insulators is well known. It is caused by the presence of the shunt condensers, because the current of the shunt condenser must flow through the series condenser as shown on Fig. 3. Therefore, starting at the ground end each succeeding series condenser C_w carries more current than the preceding one. As all series condensers are alike it is obvious that the magnitude of their voltages will correspond to magnitudes of their currents.

The concentration of voltage will be the greater, the greater the shunt capacitance in comparison with series capacitance.

Tests, as well as calculations, show that in practise the voltage difference between adjacent turns at the line end, caused by the front of the wave, ranges between 10 and 30 per cent of the crest value of the line voltage and between adjacent coils (disks or pancakes), at the line end, may exceed 60 per cent. In case only a few coils comprise the entire winding these values become particularly high. If the length of the winding is taken for abscissa and voltage between the line terminal and various parts of the winding for ordinate, the curve of electristatic distribution will have a shape similar to those of Fig. 20, which were obtained by test on three power transformers.

b. Voltage Distribution during Oscillation. Assume for the time that the traveling wave is infinitely long and therefore its crest acts on the transformer network (Fig. 3) as d-c. voltage and thus chooses the inductance path for its current. The permanent or "final" voltage distribution will depend therefore on the ratios of the elementary inductances of the network (as with circuits 2 and 3). Under the assumption of uniformity of the winding all elementary inductances are alike, and therefore the final voltage distribution

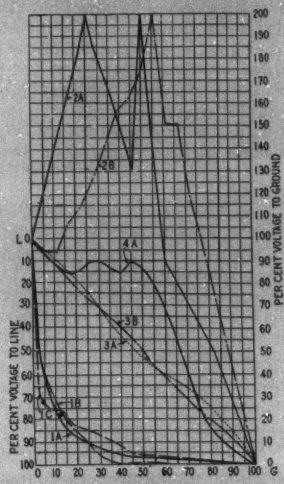

FIG. 20—VOLTAGE STRESSES MEASURED IN TRANSFORMERS UNDER TEST

Transformers tested: (A), core type; (B), core type; (C) shell type.

Non-Shielded

Curves 1A, 1B, 1C: Initial or electrostatic voltage distribution with traveling wave on transformers *A*, *B*, and *C* respectively (without entrance bushing). (Dots indicate measured voltages on *C* with entrance bushing).

Curves 2A and 2B: Envelope of voltages in transformers *A* and *B* produced by damped oscillations.

Curve 4A: Envelope of oscillations caused by traveling wave on transformer *A*.

Shielded (Non-resonating)

Curves 3A and 3B represent envelopes of all voltages caused by damped oscillations from 3 to 1000 kc. as well as steep front traveling waves on transformers *A* and *B*, respectively.

will be perfectly uniform and appear as the straight line *L G*, Fig. 21, while the initial distribution was as shown by curve 1 of Fig. 21.

The difference in "initial" and "final" states is apparent and will cause a transient state which will consist of a number of sinusoidal oscillations of various frequencies and amplitudes superimposed on one another. As long as voltages at the terminals remain constant or change abruptly, these frequencies and amplitudes

depend only on transformer constants, as it were with the simpler circuits.

It was found above with simple networks, that when subjected to infinite wave of steep front, they oscillate at all their natural frequencies of voltage resonance. Transformers follow the same law.

In the chapter on sustained oscillation it was pointed out that the transformer possesses a fundamental natural frequency and a number of its harmonics. At these frequencies, standing voltage waves are created

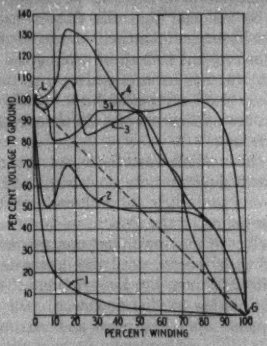

FIG. 21—MAXIMUM VOLTAGE TO GROUND DUE TO RECTANGULAR WAVE. TRANSF. A.

No. 1 initial distribution (tested)
No. 2, 3 due to waves of 1.5 and 4.6 long (calculated)
No. 4, due to waves infinitely long (calculated)
No. 5, due to waves 5 miles long (tested)

along the winding composed of a number of halves of a wave. Their number corresponds to the harmonic order for transformers with grounded neutral.

It was also shown with simple circuits that the axis of oscillation is the final state of a given point in the winding, while its amplitude is the sum of the amplitudes of all natural frequencies of the circuit.

The initial state is shown by curve No. 1 of Fig. 21. The final is a straight line $L\,G$. Therefore, a point of the winding 20 per cent from the line terminal will oscillate about a point of intersection of line $L\,G$ with ordinate corresponding to the 20 per cent point. This part of the winding, therefore, starts to oscillate with an amplitude of 68 per cent of line voltage. As the oscillation contains various frequencies of different amplitudes, the rise above the point of oscillation may not be equal to the original amplitude, but may be somewhat higher or lower than that, depending upon the remoteness of the given point of the winding from the line terminal as well as the high frequency characteristics of the transformer.

The curves of Fig. 22 show results of calculation of the potential above ground of the middle point of the winding caused by a traveling wave. Curves 2, 3, 4, and 5 of Fig. 21 show the maximum voltage above ground attained by different points of the winding due to various traveling waves (Curve No. 5 is obtained by test with a wave approximately five miles long, all others are calculated).

It may be easier to visualize the behavior of a transformer under traveling wave excitation if it is again compared with the elastic rod (or here an elastic ribbon uniformly loaded with weights may better appeal to the imagination). If the ribbon were at first in a horizontal position with one end fixed, and the other end pulled upward perpendicularly to its length to a new position, with abruptness corresponding to duration of the front of the traveling wave, the ribbon would be bent in a shape very similar to the initial voltage distribution (Curve No. 1, Fig. 21).

Should the point of application of force be held at rest at its new position (which corresponds to the crest of the terminal voltage wave being held constant), the final state of the ribbon would be a straight line $L\,G$ (similar to the final voltage distribution along the winding).

Thus the initial and final states of the ribbon are different, and therefore it must oscillate about its final state. During its oscillation different points will rise above their final position.

The distances of various points during their oscillation, from their original positions preceding the application of force, correspond to the voltage from winding to ground, while their distances to a line drawn

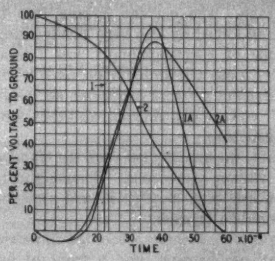

FIG. 22—EFFECT OF TAIL OF WAVE ON OSCILLATION AT MIDDLE POINT OF WINDING OF POWER TRANSFORMER

No. 1, and 2 voltage waves at transformer terminals
1 A, and 2 A voltage to ground at the middle of winding

from the new position of the "line" end parallel to the original position of the ribbon corresponds to the voltage between winding and line terminal.

2. *Traveling Wave with Slanting Front.*

a. Initial Voltage Distribution. From the discussion of initial voltage distribution due to rectangular front, an impression may be gained that a very steep front is necessary to create a purely electrostatic field in the winding, and therefore the concentration of voltage at the line end of the winding cannot be ex-

perienced in practise where rectangular waves do not exist.

It is clear that the higher the natural frequencies of the transformer are, the steeper must be the necessary front to produce a purely electrostatic field, since the transformer with higher natural frequencies responds more quickly to transient voltages.

In a given transformer, therefore, the concentration of voltage at the line end, due to the wave-front, is smaller for a longer wave-front, generally speaking.

This can be seen by considering an extremely long

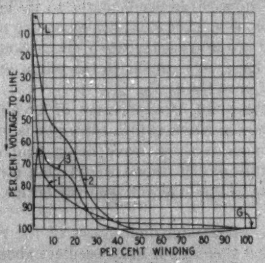

Fig. 23—Effect of Slanted Front on Initial Voltage Distribution. (Transf. A.)

No. 1 for one microsecond front (test)
No. 2 for five microseconds front (calculated)
No. 3 initial voltage stress after bushing arcs-over at the crest of five microseconds front (calculated)

front which causes a correspondingly slow rise of terminal voltage. The current flowing into the transformer, being forced by this very slowly rising voltage, is of course of correspondingly low frequency and hence will choose the inductance path of the transformer equivalent network producing no "electrostatic" or "initial" voltage distribution, and therefore no voltage concentration.

Here again, the analogy between the transformer and an elastic ribbon is obvious. Should the ribbon be pulled upward very slowly, it will remain straight during the entire transient.

As some traveling waves, recorded in service by the cathode ray oscillograph, had a front of five microseconds, the voltage produced by such a wave-front was calculated for the power transformer previously referred to and is shown as curve No. 2, Fig. 23. For comparison curve No. 1 obtained by test with a wave of one microsecond front is also shown. It shows that a 5-microsecond wave-front creates extremely high voltage concentration on the line end; therefore the wave front must be very long indeed, to cause no excessive voltage stress between coils and turns at the line end of the winding.

b. Voltage Distribution during Oscillation. Fig. 24 illustrates the effect of the length of the wave-front on voltages created by the oscillation following the initial state.

Curves 1, 2, 3, and 4 show the rise of voltage above ground at the middle of the transformer winding caused by voltage waves at the transformer terminals, as shown by curves 1a, 2a, 3a, and 4a respectively.

There is practically no difference between the maximums of curves 1 and 4, in spite of the great difference between the exciting waves 1a and 4a. It should be noted that 4a wave has a front of 30 microseconds.

3. Finite Traveling Wave.

The effect of a finite traveling wave on a transformer is again analogous to its effect on the ribbon.

Let it be assumed that the ribbon was suddenly pulled from its original position No. 1 to its new position No. 2, as before, but instead of holding the "line" end in position No. 2 indefinitely, after a certain lapse of time t it was suddenly pulled back. This would correspond to the application of a finite rectangular wave of t seconds long.

Let t be the moment when the entire ribbon is moving in an upward direction, the downward pull will bring the "line" end to the original position immediately, but some other points of the ribbon, remote from the line end, will still be moving in an upward direction, on account of their inertia, until the downward pull will be transmitted to them along the ribbon. The velocity of the propagation of this downward pull along this ribbon depends upon the mass and elasticity of the

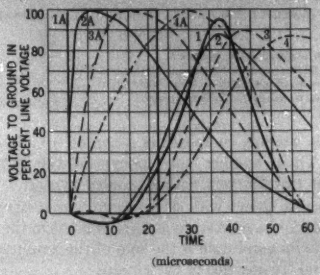

Fig. 24—Effect of Slanted Front on Oscillation of Middle of Winding of Transformer B

1 A, 2 A, 3 A, and 4 A—voltage at transformer terminals
1, 2, 3, and 4—voltage at the middle of winding

ribbon, and is smaller the greater either of these constants.

It is clear that if t (the length of the wave) is longer than the time t_1 that is necessary for a given point of the ribbon to reach its highest position, then it will reach it; but if it is shorter than t_1, then it may or may not reach it, depending on the remoteness of the point from the line end and the ratio of t_1/t.

As the downward pull was assumed to be very sudden, the method of super-position suggests looking at it as an application of an infinite negative wave of steep front at moment t. Such a wave will cause its own initial voltage distribution, which being negative must

be subtracted from the potential that the respective points possessed just before t. This reveals that a traveling wave with steep tail, such as may be created by the arc-over of a bushing or a string of insulators, may set up "initial" stresses near the line end of the winding identical to those produced by the steep front, and even greater if the end of the wave should coincide in time with high potential to ground of the part of the winding near line end.

Curve No. 3 of Fig. 23 shows voltage distribution along transformer winding at the moment of arc-over of the transformer bushing. The arc-over is assumed to occur at the crest of five microseconds front.

Fig. 22 shows the change of voltage with time at the middle point of the winding of the same transformer and how it is affected by the shape of the tail of the wave. It shows that while the maximum voltage produced by a rectangular wave and a wave of the same crest, but with long tail, may be practically the same, the long tail delays the fall of the voltage to ground potential. These curves also show that internal voltage may be rising for a very long time after the potential across the transformer becomes zero. For instance, with a rectangular wave 22 microseconds long, at the time the transformer terminal voltage becomes zero, the voltage to ground of the middle point of the winding is 25 per cent of the crest of the wave and is steadily rising for the next 15 microseconds, until it reaches 98 per cent of the crest.

Part III

Transient Voltages in Practise

The data on voltage transients experienced in service accumulated by various investigators can be summarized as follows:

1. Lightning

Taking the crest value of normal operating voltage from line to neutral as reference voltage E, the order of magnitude of transient voltages was found to be:

Lightning traveling waves:

Rarely.....15 E
Often.....10 E

Minimum steepness of the front, actually recorded by oscillograph five microseconds; estimated from Lichtenberg figures from 1 to more than 100 microseconds. The tail of the waves, recorded by oscillograph, exceeds 40 microseconds.

2. Oscillation

By switching, and other sudden changes of the network, moderately and highly damped oscillations are produced. They rarely reach 8 E, quite often 5 E, and very often 3 E.

3. Damping

The damping increases with magnitude of transient voltages, evidently due to corona loss.

4. Upper Limit of Transient Voltages

Laboratory experiments, completely substantiated by service records, show that on most of the existing transmission systems transient voltages reach values beyond the dielectric strength of the line insulation. Except near the center of long spans, therefore, the dielectric strength of the line insulation becomes a practical limit to the magnitude of transient voltages possible on the line. The effect of ground resistance of towers or poles, and of time lag in the discharge of lines to ground, must be taken into account and may raise the voltage limit.

5. Conclusions

In view of the behavior of transformers under transient voltage excitation, and the high magnitude of these voltages, the construction and the distribution of the insulation throughout the transformer winding is dictated not by stresses set up in the transformer, under the standard A. I. E. E. potential test, of low frequency, but by the law of distribution of voltages in the winding caused by various transients actually observed in service.

The following table shows the order of magnitude of voltage stresses produced between different elements of a transformer winding having one end permanently grounded. Column I gives stresses produced by the standard "Induced Voltage Test." Column II gives the dielectric strength necessary to withstand service transients produced in a uniform winding.

In the numerical example, Column III, the minimum values (in kv.) are given for a typical transformer to be operated on a 220-kv. system.

As a voltage of the same magnitude, but of a different order of frequency, has a materially different effect on a dielectric, all stresses and strength are expressed in values of equivalent 60-cycle one-minute test.

The voltages in Columns I and II are given in terms of effective value E, which is the normal operating line to neutral voltage of the transformer.

	I	II	III
High-voltage line end to low voltage and ground	3.46 E	3.46 E	440 Kv
Any other point between line and ground end p% away from ground end	p E	3.46 E Unless p is less than 10%. Some reduction is permissible for p less than 10%	
Turn ins. of line coil..	100 to 600 V.	0.63 E	80
Turn ins. near line coil	100 to 600 V	Gradually reduced from 0.63 E to 0.20 E	
Turn ins. in the main part of the winding..	100 to 600 V.	0.20 E	25
at ground end.....	100 to 600 V.	0.35 E	44
Coil to coil at line end..	0.04 E	1.3 E	165
Coil to coil near line end	0.04 E	Gradually reduced from 1.3 E to 0.65 E	
Coil to coil in the main part of the winding.	0.04 E	0.65 E	82
near ground end...	0.04 E	0.88 E	110

The difference between values of the first and the second columns is responsible for the difference in cost and volume of 20,000-kv-a. 220-kv. transformers referred to in the Introduction.

Part IV

Protection Against Transient Voltages

1. Non-Resonating Transformers.

a. Theory. It was shown above that the cause of non-uniform voltage distribution along transformer windings was due to the presence of shunt capacitance (C_s in network 3 of Fig. 3), because due to this shunt capacitance, damped (or sustained) oscillations of a series of frequencies, applied to the terminals of such a network, cause different parts of it to get in voltage resonance and produce over-voltages shown by the curves (2 *A*, 2 *B* of Fig. 20).

If the shunt capacitance were to be removed, the circuit would consist of a chain of identical elements ($L\,C_w$). As was shown, such a circuit has no series natural frequency. It will distribute the voltage of all frequencies uniformly.

In the case of traveling waves, the same shunt capacitances were found to be responsible for internal over-voltages, as the "initial" and "final" voltage distributions were different because the charging current of all shunt capacitances was supplied through the series capacitances, causing concentration of voltage at the line end at the time of impact of the wave.

With shunt capacitances removed, there would be no shunt current to be supplied by any series capacitances, and as they are all alike the initial voltage would be uniform. The final voltage distribution also will be uniform, as all elementary inductances are alike, and therefore there will be no transient.

Thus with shunt capacitances removed, the voltage will be distributed between all elements uniformly, no matter what the shape of the terminal voltage wave is, whether it is ocillatory or unidirectional, of infinite frequency or direct current. But the removal of shunt capacitances is physically impossible. There must always be ground potential surfaces and surfaces of the winding facing them. If the entire winding should be screened from the ground by metal sheets interposed between the winding and the grounded surfaces, it would do no good as long as the potential of the screens is not fixed, and if they are connected to any part of the winding, including the line end, they would create a new system of shunt capacitance and the matter would be worse than before.

Fortunately, as J. M. Weed[2] pointed out in his A. I. E. E. paper on *Prevention of Transient Voltage in Windings* in 1922, the removal of shunt capacitances is not necessary; what is necessary, is the neutralization of their effect on series capacitances. The neutralization of their effect on series capacitances simply means prevention of the flow of their charging current through the latter.

Referring to *A* of Fig. 25 on the left side of the transformer equivalent network a new system of shunt condensers C_p is shown. This system, however, is not connected to ground as C_s are, but to the line terminal.

It is obvious that values of each capacitance (C_p) can be selected so that, with all inductances open circuited, and with the voltage uniformly distributed throughout the winding, the current through respective C_P to a given element will be exactly equal to the current through the shunt capacitance C_S of that element to ground. In such a case the current through all series capacitances will be equal and the voltage across them alike. This would hold true, of course, at all frequencies.

If all inductances now are connected as before, and the circuit subjected to impact of a rectangular traveling wave, the voltage distribution, as was shown, will be purely electrostatic.

As the protecting condensers are so proportioned that the voltage divides equally between the series capacitances, the initial voltage would be a straight line (as shown by *L G* of Fig. 21), the final distribution is also

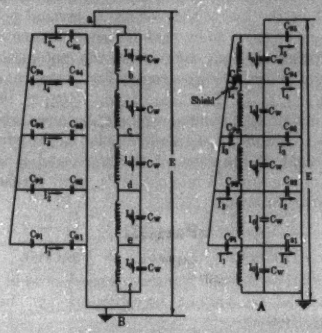

Fig. 25—Equivalent Network of Non-Resonating Transformer

a straight line (as all *L*'s are alike),—therefore there can be no transient.

As there is no exchange of current at any frequency between the two circuits at the point of the contacts *a, b, c, d, e,* we have a perfect right to think of them as two independent circuits connected in parallel—one consisting of all the shunt capacitances, and the other of series capacitances and inductances. (*B* of Fig. 25.)

The first circuit is a pure capacitance and therefore cannot have any voltage resonance frequency. The second circuit, consisting of like links $C_w\,L$, also can have no voltage resonance frequency. Therefore there can be no voltage resonance frequency for the two circuits connected in parallel. That is, the reactance-frequency curve of this circuit cannot cross zero line at any frequency, and must be similar to curve of elementary circuit No. 1 of Fig. 6 (see *Z* of Fig. 4).

b. Test Data. On Fig. 26 impedance *versus* frequency curves are given. They were obtained by test with sustained oscillation on a large power transformer

which was mentioned before. Compare this with Fig. 8 obtained on the same transformer before the effect of its C_w's was neutralized by means of C_s's.

Extensive tests were made on some power transformers with damped oscillations of frequency ranging from 3 to 1000 kilocycles, as well as with impulse waves of various shapes.

The results as summarized on Fig. 20, curves 2*A* and

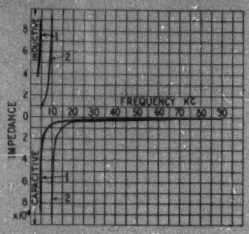

Fig. 26—Impedance-Frequency Curve. Results of Test on Non-Resonating Power Transformer B. 1, L. V. Open. 2, L. V. Shorted

2*B* show the maximum voltage to ground produced by damped oscillation of various frequencies on transformers *A* and *B* respectively before they were equipped with shields, and curves 3*A* and 3*B* after they were so equipped.

Curves 1*A* and 1*B* show the initial voltage distribu-

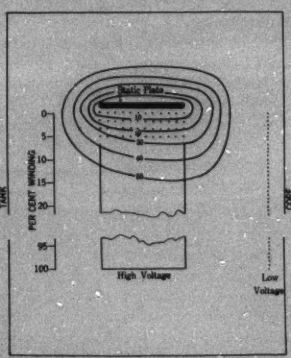

Fig. 27—Dielectric Field Due to Steep Wave Front in Transformer Winding Equipped with Static Plate (Calculated)

tion of the same transformers at steep wave-front. Initial voltage distribution, after they were equipped with shields, is shown by the same curves 3*A* and 3*B*. Curve 4*A* shows the maximum voltage to ground due to a traveling wave obtained on transformer *A* before it was shielded, which became curve 3*A* after it was shielded.

The difference in stress, as shown by these curves, in the new and the conventional type of transformer, is so great that it seems unnecessary to make any additional comments in its regard.

If the behavior of an ordinary type of transformer can be likened to that of a very flexible rod, or even a ribbon, the behavior of the "non-resonating" transformer is analogous to that of a perfectly rigid rod, which with one end fixed and the other oscillating does not bend at all, no matter what character of oscillation is applied.

In practise, system of condenser C_p is simply a smooth metal surface or a small number of such surfaces properly spaced and insulated from ground and the windings, and connected to the line terminal of the protected winding. These surfaces are called "shields."

They are located on the outside of the high-voltage windings, so as not to interfere with its usual construction. Fig. 29 shows a power transformer of the non-resonating type. The non-resonating type so far

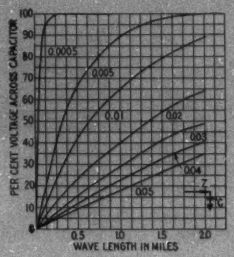

Fig. 28—Effect of Capacitor in Shunt with Transformer on Maximum Voltage of Traveling Waves of Different Lengths (Capacitances are Given in Microfarads)

has been adopted for more than half a million kv-a. of transformers.

2. Static Plate.

A device, known as the static plate, which has been in use for many years, has a shape usually similar to a coil of the winding that it protects. It is a continuous conducting surface with smooth edges placed adjacent to the line coil of the winding and connected to the line. It is insulated with varnish cambric cloth wrapped around it.

The object of this plate is to reduce the concentration of the initial voltage due to steep wave-front on the line end of the winding. It does this by virtue of its relatively large surface, as contrasted with that of one turn, thus changing the electrostatic field set up by the front of the wave from the one shown on Fig. 19 to that shown on Fig. 27.

From examination of Figs. 19 and 27 it will be clearly seen that the voltage concentration at the line end, as well as the beneficial effect of the static plate, depends entirely upon the geometric proportions of the winding.

It is clear that while the static plate reduces stresses

materially near the line end, it cannot prevent oscillations and voltage resonances of the winding.

3. Effect of the Transformer Bushing.

It has sometimes been stated that the capacitance of the transformer entrance bushing has an appreciable effect in slanting the wave-front and decreasing the voltage concentration within the winding.

The capacitance of the bushing is in shunt with the transformer, and is of the order of 0.0002 microfarads, which corresponds to the capacity of 0.15 of a mile of transmission line. This value is so negligibly small that it can have no effect on the transformer behaviour.

Curve No. 1*C* of Fig. 20 was obtained by tests on a power transformer which, in this case, happened to be of the shell type without any bushing. It was subjected to the impact of a traveling wave transmitted to it over a circuit having a surge impedance of 500 ohms. The tests were then repeated with the bushing in place and connected, and the points marked by dots were obtained. These tests show that the bushing had no effect on the voltage distribution. It did not slant the wave-front sufficiently to cause any reduction in initial voltage distribution.

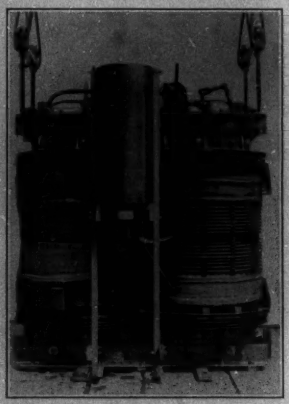

Fig. 29—Non-Resonating 20,000-Kv-a. 132,000-Volt Transformer

A condenser used for protection of a transformer against transient voltages, must be so large that it slants the wave-front to an amount sufficient to reduce the initial voltage concentration appreciably, which, as has previously been shown, requires a wave-front greater than some 40-50 microseconds, and it must absorb the entire energy of the wave without requiring a voltage in excess of the breakdown of the apparatus.

Fig. 28 gives the relation between the size of the condenser and the maximum voltage that waves of various lengths will produce across it. The curves were obtained with surge impedance of transmission line assumed to be 500.

Higher voltages will be obtained with smaller surge impedance.

4. Inductance Coil.

a. Choke Coil. In the light of the relation of various circuits discussed in the paper, it appears that a choke coil alone, in series with a transformer, may do more damage than good.

The choke coil, while its reactance also varies with frequency on account of its small capacitance, acts as an inductance up to relatively very high frequencies (several hundred kilocycles).

Consider a transformer, having a high-frequency reactance similar to the one shown in Fig. 8, connected to a transmission line through a choke coil.

At a frequency of 45,000 cycles, the transformer acts as a condenser, while the choke coil still acts as an inductance. Should the line arc to ground about a mile (0.96 of a mile) away from the transformer, the line will oscillate at 45,000 cycles (see table above), and the voltage across the transformer will rise above the high-frequency voltage applied across the transformer and the choke coil. Should 45,000 cycles happen to be the natural frequency of capacitance of the transformer and of the inductance of the choke coil combined, the voltage across transformer may become many times the line voltage. And if this frequency, in addition, is near one of the natural frequencies of the transformer (in the case of transformer of Fig. 8, 47,000 cycles is its second harmonic), very high internal voltage may be created with the amplitude of the line disturbance of comparatively small magnitude.

The danger of resonance is often underestimated due to a faulty impression that for the creation of overvoltages of dangerous magnitude the frequency of applied voltage must be exactly equal to the natural frequency of the circuit under consideration.

Examination of reactance *versus* frequency curves of transformers and other circuits given in this paper shows that in many cases the reactance curve approaches and leaves the zero line very gradually, which means that a correspondingly wide band of frequencies produce a very substantial rise of internal voltage. In our example the choke coil still would cause a substantial increase of transformer transient voltage over line transient voltage, with variation of the distance between the transformer and the point of line fault, or variation of the natural frequency of choke coil plus transformer of some 40 per cent.

In case a traveling wave strikes a choke coil, in series with a grounded transformer, initially a considerable part of its voltage will appear across the choke coil,

while finally practically the entire voltage will appear across the transformer. Therefore, an oscillation will take place.

As the natural frequency of the point of the contact of the choke coil and the transformer is generally very high, it means that transformer voltage will rise above the line voltage unless the traveling wave is very short (see Fig. 17). For instance, with the natural frequency being 45,000 cycles, the traveling wave must be shorter than four microseconds (or 4000 ft.). Most of the waves, recorded in service, were found to be considerably longer.

b. Current Transformer. A current transformer acts very much like a choke coil on account of its inductance, and therefore should always be bridged on the line side with a "by-pass" in the form of a gap with resistance in series or by other suitable arrangement.

c. Current Limiting Reactor. The current limiting reactor, being similar to a choke coil, also may cause rise of transient voltage across the transformer terminals. Very interesting tests, substantiating the above, were described by F. W. Peek[4] in an A. I. E. E. paper on *Effect of Transient Voltages on Dielectrics—III* in June 1923.

It was recommended there to shunt reactors with some voltage limiting device, as for instance carborundum resistors.

For simplification of the discussion, the effect of the surge impedance of a transmission line, resistance, and dielectric loss of a transformer on transient phenomena within the transformer, was neglected in this paper. As the surge impedance of transmission lines is measured in hundreds of ohms, and that of transformers in thousands of ohms, neglecting the former does not affect the phenomena within limits of practical interest. The theoretical, as well as experimental data so far in possession of the writer, indicate that very little help in reducing the transient voltage stress can be obtained from the damping characteristics of the transformers. This is particularly true for transformers of large sizes. On account of the limitation of space, the damping effect of losses could not be discussed in this paper at length.

I wish to acknowledge here that in the preparation of this paper the interest and assistance of Mr. F. F. Brand proved to be invaluable.

The valuable assistance of Mr. J. H. Hagenguth in the preparation of data is also recognized.

Discussion

J. F. Peters: This paper is a very interesting review of the subject of voltage distribution within transformers. When it comes to applying the fundamentals to an actual transformer it is necessary either to use a rather rough approximation or to introduce other conditions that may be a serious hazard. The windings of all high-voltage power transformers consist of a stack or stacks of pancake coils in some form or other. These coils are connected in series either as indicated by Fig. 1 or by Fig. 2 herewith. If connected according to Fig. 1 it is obvious that Mr. Palueff's Fig. 3 does not give a correct representation of the capacitance from windings to core and tank, and a shielding on the outside of the coils cannot offset the capacitance to core. If the coils are connected according to Fig. 2, his representation in Fig. 3 is correct, but it is then necessary to carry the series connections through the oil ducts between coils where the space is necessarily limited and therefore produce somewhat of a hazard.

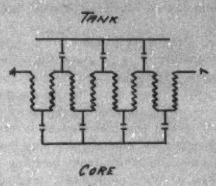

Fig. 1

In any structure that has a number of electrostatic capacitances connected in series with parallel capacitances connected in at various points along the structure, the voltage distribution will not be uniform throughout the structure. This principle has been recognized and applied to line insulators. The principle applies equally well to stacks of coils. The distribution of voltage along a string of insulators is a function of its dimensional ratio, length to diameter. Long strings have poor distribution. The same conditions exist in transformer windings. Where the stack is long and narrow the initial voltage distribution is relatively poor while where wide coils are used, thus giving a short wide stack of coils, the distribution is relatively good without the addition of shields. Where the initial distribution is relatively good the amplitude of oscillation between initial distribution and final distribution is relatively small.

Since transformers of different construction differ tremendously in physical proportions one may find it highly desirable to provide shields to improve the distribution in some transformers where in other transformers it may be quite inadvisable to provide shields.

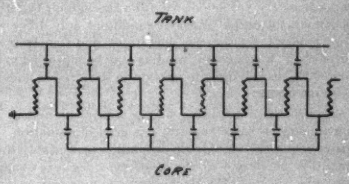

Fig. 2

F. F. Brand: I think everyone will admit that the transformer is in general a highly reliable device, but a small percentage of failures still takes place. With the growth of high-voltage networks, and the increasing power concentration in extra high-voltage systems, a reliability practically equivalent to that for metropolitan service is now required in even the highest-voltage systems so that transformers of even greater reliability are now necessary.

It is probably safe to say that the majority of transformer failures are caused by the transformer being subjected to transient over-voltages.

The author in his paper has given a fairly complete analysis of what happens in the ordinary transformer winding when it is subjected to transient over-voltages such as are caused by lightning or switching surges.

A practical transformer designer realizes that voltage failures are caused by oscillations within the transformer winding. These cause high local voltages to appear, by the concentration of voltage on the end turns when lightning strikes the transformer. In a few rare cases actual breakdown of the insulation to ground occurs on the end of the winding caused by excessive voltage applied to the transformer or breakdown to ground in the main body of the winding caused by the oscillation building up a high voltage to ground.

The difficulty with which the transformer designer is confronted in the ordinary transformer is to know the value and position of the local voltages within the transformer winding, because these depend on two things, first, the constants of the transformer itself, and secondly, the frequencies as well as the voltage of the transient at which the transformer is subjected. Over these latter, the designer has no control. Any means, therefore, which would prevent oscillations from building up within the transformer winding at once eliminates uncertainty as to the amount and distribution of insulation throughout the winding which is necessary to withstand service conditions. All that is necessary is to know the maximum voltage and to some extent the time during which the transformer will be subjected to this voltage.

The object of the paper is not only to describe and analyze the voltage distribution throughout the transformer winding under widely varied conditions of wave form and frequency of transients, but to describe a new type of transformer winding which is non-oscillating. It is perhaps to be regretted that the author did not explain the design of this new and much improved type of transformer more fully.

Any practical design of transformer which can reduce local stresses within the winding in the order of 10-to-1 to 100-to-1, is a tremendous step forward in the design of such apparatus, and must result in greatly improved reliability. At once, it places the operation of the transformer under transient conditions beyond the realm of conjecture and makes the customary factory low-frequency test a real measure of the ability of the transformer to withstand lightning and other transients.

There is a relation between the 60-cycle strength of insulation and its strength under very short-time transients, but it is somewhat dangerous to consider this ratio as applied to completed apparatus since the voltage distribution throughout the winding of such apparatus under transient conditions may differ widely from that under low-frequency conditions, and the stress which various parts of the apparatus winding may be subjected to under transients will bear no definite relation to the stresses under low-frequency operation.

For a transformer operating with a neutral grounded, it has been customary in some cases to use graded insulation. It is obviously dangerous to grade the insulation appreciably towards the neutral if the design of the transformer permits oscillations to occur, which might raise the voltage to ground or to the low-voltage winding, at some intermediate point in the winding, very much above the proportion of the terminal voltage stress which the intermediate point has been designed to stand. Certainly if the voltage at intermediate points of the winding can rise higher than the terminal voltage, graded insulation in the ordinary transformer is unsafe if the transformer is to be subjected to high transient overvoltages such as are common on overhead lines.

Graded insulation has been successfully used in testing transformers, but in these the surges which are set up are initiated at the voltage at which the testing transformer is operated, and probably do not give voltages much above the normal rating of the transformer.

With power transformers, however, subject to lightning, the transient overvoltages may be many times normal voltage to ground that the transformer is designed to withstand at low frequency. It is true that the time lag of the insulation in resisting short-time transient voltage comes into play. This may be sufficient to prevent failure of a transformer having uniform insulation to ground through the winding unless the terminal voltage is really excessive. It may not be sufficient to cause failure if the insulation is graded.

The ability to produce a transformer which has a straight-line voltage distribution from line terminal to ground no matter what transient it is subjected to, thus places graded insulation in a class of apparatus which can be safely used, since the value of the stress to ground at any point in the winding is the same proportion of terminal voltage during factory test and during service.

That the means of rendering the transformer non-oscillating can be applied to a standard type of winding which has proved highly reliable in service is indeed fortunate, since it eliminates the possibility of the new type of transformer being considered experimental. As a matter of fact, the shielded arrangement can be added to a standard disk-coil transformer without any change in the winding at all.

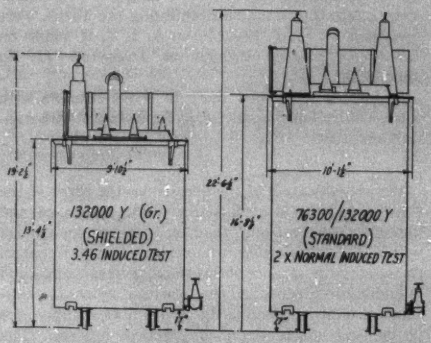

Fig. 3

With the new-type transformer, it is only necessary to set up some protective means to limit the value of the voltage applied so that the insulation of the terminal and of the winding to ground is safe under these conditions. Other than this, lightning or switching surges have no effect on the transformer.

Philip Sporn: In connection with the practical results of the problems discussed by Mr. Palueff, it may be of interest to see just how some transformers designed in accordance with the principles brought out in the paper appear.

Fig. 3 herewith shows the outlines of two transformers installed in the same station having exactly the same rating and being used for exactly the same purpose. The one on the left shows a transformer rated at 21,000 kv-a. single-phase, of the shielded design. The one on the right shows a transformer of the same rating non-shielded. The first is designed for grounded-neutral operation, and the second for either grounded or isolated operation. It is interesting to know, however, that the induced voltage tests on the first transformer were 3.46 normal, and on the second, only twice normal. Based on that, the estimated impulse strength of the two transformers is the same, but both the designers of the transformers and we with them believe that actually the shielded transformer will stand a considerably higher impulse than the non-shielded transformer. Fig. 3

shows the great difference in size between the two units. These transformers have been installed at the Philo plant of the Ohio Power Company.

In a paper which I presented before the Institute last year on the question of rationalization of transmission system insulation strength, I pointed out what Mr. Palueff has brought out forcibly in his conclusions, that the present standards of the Institute are not being followed by the manufacturers in the construction of apparatus involving high voltages, particularly in specifications with regard to high-voltage insulation, and that before a definite and rational solution of the problem can be obtained, it is essential that the 60-cycle strength and test be divorced from and made independent of, the impulse strength and test.

The investigations and work carried out by Mr. Palueff make such a step much more practicable and feasible than seemed possible at that time.

F. W. Peek, Jr.: Mr. Palueff's paper presents the results of a transformer research that has been carried on for a number of years. During this research it was discovered that lightning impulses or transients applied to the terminals of a transformer may cause very high voltages at any point in the winding. For example, the full lightning voltage applied to the transformer terminals may appear across a very small portion of the winding on the ground end of the transformer. This, during the early work, seemed remarkable because the transformer was generally thought of as a very high inductance and high inductance was supposed to prevent the penetration of high-frequency or steep waves. As a matter of fact the transformer does consist of a high inductance and the current flowing through this inductance causes even distribution of voltage along the winding for operating frequencies and for very low-frequency transients.

However, there is also in combination with this inductance, capacitance. This capacitance plays a negligible part in the distribution of voltage at low frequency. At high frequency, however, the capacitance becomes the controlling factor in voltage distribution. In all conventional transformers, of either shell-type or core-type design, the arrangement of the capacitance is such as to cause a very uneven distribution of voltage,—generally a very high concentration of voltage on the line end. This uneven capacitance distribution is caused principally by the capacitance of the windings to the grounded core and tank.

It follows that when a steep impulse is applied the inductance at the first instant acts exactly as an open circuit. The capacitance is the controlling factor and causes the voltage to pile up on the line end. Eventually as time goes on this uneven distribution of voltage must even up and reach the steady-state condition as determined by the inductance. However, the transition from the initial distribution to the steady-state distribution cannot occur instantly but must do so through an oscillation. A few microseconds after the impulse is applied the voltage swings past the steady-state line and reaches a maximum above the line in excess of the distance it was below the line at the first instant. This causes a very high voltage concentration in some other part of the winding which makes the insulation problem extremely different in the conventional transformer.

An entirely new transformer, a *non-resonating transformer*, has solved this problem. The prevention of these transient localized stresses has been accomplished in a remarkably simple manner. The electrostatic voltage distribution has been made to correspond to the inductance voltage distribution. When an impulse is applied it instantly takes the steady-state distribution because it instantly takes the capacitance distribution and this corresponds to the inductance or steady-state distribution. Therefore there can be no oscillation but each section of the winding takes its share of the applied transients. The voltage distribution is the same for all frequencies, all transients, and all impulses and corresponds to the 60-cycle distribution. To accomplish this it has not been found necessary to go to a freak design. In fact, all the features of the well tried standard designs have been retained.

It may well be asked if these extremely high and dangerous voltage concentrations can be caused by transients that are likely to occur in practise. The very beautiful oscillograms that Mr. Palueff has secured on large power transformers connected to transmission lines answer this quite decidedly. These Dufour cathode ray oscillograms show dangerous voltage distributions not only for lightning impulses of very steep wave front but also for lightning voltages of slanting front and extremely long duration. They also show that the shield reduces local stresses in many cases as much as 100 to 1.

One very important point in favor of the non-resonating transformer is that the 60-cycle test is a true measure of the insulation value of the transformer under the lightning conditions incidental to operation. This follows only in this type of transformer because the lightning distribution and the 60-cycle distribution correspond. This, of course, is not the case in the conventional transformer.

Mr. Palueff's paper and cathode ray oscillograms show that a high series inductance coil is more likely to be a hazard than a help during lightning disturbances unless the inductance is shunted by the proper value of resistance. This information, which is of great importance to operating engineers, while not new, is very definitely confirmed by the oscillograms.

V. M. Montsinger: I wish to discuss this paper from a somewhat broader standpoint than that of the distribution of voltage within a transformer when subjected to lightning. This phase of the question is covered very well in the paper itself. I will discuss the various steps in our recent progress towards the mastery of lightning as relating to protection against it under service conditions and at the same time point out how the non-resonating transformer fits into the trend of the times and helps to complete the picture.

Our somewhat more than usual progress made within the past two or three years can be attributed to the attack on three problems, namely, (1) the study of lightning voltages in the field by means of the surge-voltage recorder and the cathode ray oscillograph; (2) the proper correlation of transformer insulation with the factors that govern or limit the lightning voltage received by the transformer under service conditions; and (3) the building of a practical transformer that eliminates unpredictable and dangerous internal high-frequency voltages.

It is not necessary to comment on the progress made in the first problem as the status of this work is well known to Institute members through the series of recent papers.

As to the second problem, let us go back a few years and see what our practise has been with reference to the design and application of transformers from the standpoint of lightning.

Previous to 1925 the A. I. E. E. Rules' only requirement of the amount of insulation to be used in a transformer was that it withstand one-minute 60-cycle test of twice the rated circuit voltage plus 1000 volts. This was supposed to be entirely satisfactory under service conditions and I will add it was generally satisfactory for the line insulation used up to this time. A few operating companies used lightning arresters, but certainly not all.

The above refers, of course, to transformers used on isolated or ungrounded systems. When it became the general practise of large operating companies to ground solidly the neutral of the system, the demand grew that transformers operating on this kind of a system need not be insulated for twice the rated circuit voltage but for something less. At this stage there was serious argument as to whether the transformer should be tested for twice line to neutral voltage or some voltage between this value and twice line-to-line voltage. This argument must have been based on the premises that there was no danger from lightning voltage but that twice normal operating voltage was the sole criterion. It was finally agreed to take a value half way between

twice line-to-neutral and twice line-to-line, which is equal to 3.46 × line-to-neutral voltage. Taking the average of 2 and 3.46 gives us 2.73,—hence, the derivation of the well known 2.73 transformer.

The A. I. E. E. Rules in 1925 adopted the 2.73 transformer as standard for solidly grounded systems. No reference was made to the amount of adjacent line insulation used although by this time it had become quite general practise to over-insulate some transmission lines to cut down outages.

The other point which shows that the section adopted by the Institute in 1925 on reduced and graded insulation was not quite correct is that, according to these rules, a transformer with solidly grounded neutral and where the only dielectric test is the induced voltage test, can be built with the insulation between the high-voltage windings and low-voltage winding graded strictly proportional to the 60-cycle or induced voltage stress along the coil stack. It is well known and has again been shown in Mr. Palueff's paper that this is not a safe practise to follow where lightning is encountered.

It can be said in defense of the adoption of the 2.73 design, however, that the first grounded high-voltage systems of the 220-kv. class in this country were in localities where lightning voltage is very infrequent, namely, in the Pacific Coast States. The 2.73 transformers which have been in service on these systems for several years have given a very good account of themselves, largely, no doubt, because of the absence of severe lightning rather than because the neutral was grounded.

Experience has shown that while this practise is satisfactory when applied to sections where lightning is not severe, it is not always safe when applied to sections where lightning is very severe. As a matter of fact there is no reason why it should be safe unless the transformer insulation is properly coordinated with the line insulation, because it is an entirely wrong application and one that was not intended when the 2.73 transformer was first introduced into practise.

Standard Rules for coordinating the transformer insulation with the line insulation used has been under preparation by the A. I. E. E. Subcommittee of the Electrical Machinery Committee during the past two or three years, and at the present time this problem is pretty well in hand. At least, we are on the road to a better understanding of the proper relationship between expensive high-voltage apparatus and the factors that limit lightning voltages received by the apparatus.

So far then we have made two important steps, namely, (1) obtained a better knowledge of what lightning is, and (2) learned how to apply the apparatus more intelligently to conditions as existing in service. This, of course, has no bearing on what the lightning will do after it is received into the apparatus. It has been well known for years that dangerous internal voltages far in excess of the 60-cycle stresses can be built up within the windings. The big problem, however, was how to prevent it in a practical way. In other words, the next problem was to make the lightning behave in a well ordered manner. Once this was done the design of transformers could be placed on a better basis.

The development of the non-resonating transformer, therefore, will not only make it possible to arrange the insulation in a more scientific manner but will enable us to demonstrate for the first time by low-frequency tests the ability of a transformer to resist high-frequency voltages found in service, something which is not possible with the ordinary transformer. It can readily be appreciated that the non-resonating transformer fulfills a long felt want. It marks one more important step in our progress.

L. F. Hickernell: The Consumers Power Company, Michigan, has in operation approximately 70 transformer units connected to 500 mi. of metallically connected 140-kv., 60-cycle steel-tower transmission line and 18 units connected to 250 mi. of 140-kv., 30-cycle line. The line insulation for the most part consists of 10 units spaced 4¾ in. These transformers have all been purchased according to A. I. E. E. Standards in so far as dielectric tests are concerned.

On the basis that the impulse strength of the transformers is 10 times their rating, the theoretical impulse flashover strength, based on the Pittsfield wave, of the transformers, bushings, and line insulators is as follows:

	Impulse flashover kv.
Transformers	1400
Bushings	1390
Insulators	1200

To the best of my knowledge, there has not been a single case of failure of the insulation from line to ground due to lightning. There have been, however, several failures of the turn insulation.

These experience data are offered for the consideration of the Transformer Subcommittee of the Electrical Machinery Committee who have recently submitted a proposal on the "Relation of Transformer Insulation to Line Insulation" for inclusion in the A. I. E. E. Standards.

Judging from our experience, I am inclined to believe that line-to-ground insulation is only a small part of the story and that further attention and study should be directed toward the turn insulation, revising, if necessary, the present induced test value now contained in the Standards.

W. M. Dann: (communicated after adjournment) It is well known that in a device like a power transformer, having in effect a number of electrostatic capacities in series and a number of parallel capacities connected in at various points, the voltage distribution following a surge will not be uniformly distributed throughout the windings. However, the actual distribution will naturally be different in different designs of transformers because of different arrangements of condensive elements.

The phenomenon of uneven voltage distribution has been recognized in the case of high-voltage line insulators made up of a number of disks and it seems to me that the practise in shielding long strings of line insulators constitutes an analogy which is of interest in studying the shielding of the windings of a high-voltage transformer.

The voltage distribution over a long string of insulators without grading rings is not uniform, the end units taking more than their proportionate share of the voltage. This is the reason that many people advocate the use of grading rings on long strings of insulators because they actually do help to improve the voltage distribution. However, no one proposes to use grading rings on short strings of insulators where the voltage distribution is inherently better, because the distribution is satisfactory and the addition of rings would be not only unnecessary but also objectionable.

Mr. Palueff makes the point that for very high frequencies the voltage is distributed over a circuit made up of inductances and condensers (for example, a transformer winding) just as if the inductances were not present. The analogy between strings of insulators of differing structure and stacks of coils in transformers of differing structure is, therefore, a true one and stacks of coils in transformers of different structures will be subject to the same differences in voltage distribution that are brought about by structural differences in strings of insulators.

The longer and narrower the stack of coils the poorer will be the voltage distribution from one end of the stack to the other. When the stack is quite long compared to the width of the coils, as shown in Fig. 2 of Mr. Palueff's paper, it may become desirable to introduce the otherwise objectionable grading rings just as in the case of long strings of insulators, in order to improve the voltage distribution.

In structures of the shell type where the coils are much wider and where the length of the group of coils is very much less in proportion to the width, just as in the case of the short string of insulators, a shielding structure would be not only unnecessary

but objectionable. This has been fully demonstrated in tests within the past few years.

Transformers in general are already much more complicated than they used to be and it would seem that voltage distributors or shields ought to be used only when they are necessary to bring about a reasonable voltage distribution.

K. K. Palueff: As Mr. Peters pointed out, the connection of the inside turn of a coil to the outside turn of the next, as shown on Fig. 2 of his discussion, is indeed a hazard. This was recognized by us about twenty years ago, and we have not used it since that time. It was replaced by the so-called twin-section coil with connections between coils, as shown on Fig. 1 of Mr. Peters' discussion.

Mr. Peters suggested that, while shields may bring the potential of the outside turns of the stack to proper values, they cannot do that to the inside turns, which are removed too far from the shields to be affected by them. This is correct only if the coils are very wide, say 10 or 14 in., such as are used in the transformers with which Mr. Peters is most familiar. This width is one of the reasons that make it extremely difficult to produce a non-resonating transformer of shell type construction.

With concentric windings, the coils are narrow and the shield consists not only of a metal surface on the outside of the winding, but also of a static plate at the line end of the stack (see Fig. 1 of the paper), where the effect of which Mr. Peters was speaking is greatest. These features are responsible for the test results shown by curves 3*A* and 3*B* of Fig. 20.

I fear that Mr. Peters is attaching too much importance to the influence of the interconnection of coils on transient voltage concentration. While the interconnection of coils has a decided effect on the voltage distribution produced by the oscillation in the winding ensuing after the impact of a traveling wave, it has very little effect on the "initial" voltage distribution, during which the transformer acts practically as if the turns were disconnected from each other. In such a case the windings, shown by Figs. 1 and 2 of Mr. Peters' discussion, act very similarly.

I am glad to find that Mr. Peters' impulse tests of voltage distribution in the shell-type transformer agree with the results of my tests on transformers of the same type, as shown by Curve 1*C* of Fig. 20. It is unfortunate that Mr. Peters did not show us the voltage to ground that he found for various points of the winding during these tests, as these values are of vital importance to a transformer designer. However, his results are sufficient to enable one to state (in accordance with the principles described in the paper) that the voltage to ground, of at least half of the winding near the ground end, did rise far above its normal share of the applied voltage, (*i. e.*, above the straight line of uniform distribution).

This statement is based on the following considerations:

a. Mr. Peters' tests show a severe voltage concentration at the line end;

b. This means that the initial voltage distribution is of a character shown by Curve 1 of Fig. 21;

c. The final voltage distribution, of course, is a straight line *L G* of the same figure;

d. The difference between the initial and final voltage distribution requires a transient state;

e. The damping factor of power transformers, being small, allows this transient state to be oscillatory. Therefore, various points during the oscillation will rise above the respective points of their final state (which lie on line *L G*).

f. The rise above the final state depends on the shape of the wave as explained in the paper. (See Fig. 21.)

Voltages produced in the winding by damped oscillations of the transmission line are cumulating. They may reach, therefore, as high a magnitude as that of lightning voltages, in spite of the fact that the amplitude of the damped oscillation of the line is roughly half of that of unidirectional lightning wave. For this reason, I believe Mr. Peters' underestimates the hazard of this class of transients. (See Fig. 20.)

Transient voltage stresses between turns on the line end of the winding increase with an increase of turn insulation, at a rate depending upon the size and shape of the conductor and of the winding. The strength of turn insulation increases with its thickness, usually at a different rate. Increasing the turn insulation at the line end, therefore, may not increase the reliability of the winding at all. This is particularly true for wide coils. Therefore, the strengthening of the winding against lightning stresses, by merely increasing the insulation, is not as simple a matter as may appear from Mr. Peters' remarks.

In his discussion of my paper, Mr. Dann drew an analogy between transformer windings and insulator strings.

Realizing that "an analogy is a good servant, but a poor master," I caution the reader that in several places in my paper, the analogies used were only approximate.

While the capacitance circuit of a transformer winding may be compared with that of an insulator string for an elementary discussion of its behavior under high-frequency excitation, such a comparison is entirely inadequate for a quantitative analysis. Where it is desired to determine the effect of the configuration of

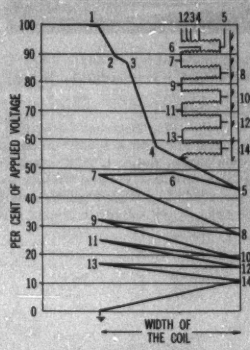

Fig. 4—Initial Voltage Distribution in Stack. 10 In. Long with Wide Coils (10 in.)

the winding on the phenomenon, the separate coils (disk or pancake) cannot be looked upon as solid metal plates comprising condensers C_w of Fig. 3 of the paper, but must be considered as rather a complicated network shown on *A* of Fig. 3 where *a b* is the first turn, *b c* is the second turn, *c d* is the third, etc.

For this reason, the voltage concentration across the line coils increases with increase of its width. If each coil acted as a solid plate, the voltage would decrease.

For the same reason, the relative shortness of the shell type transformers does not exert the beneficial effect expected by Mr. Dann.

Fig. 4 herewith gives a result of lightning tests on a very short stack, (10 in.) with wide coils (10 in.).

Fig. 19 of the paper shows that, with wide coils, during initial voltage distribution, the voltage drop across the line coil is approximately 55 per cent of the total applied voltage. Curve 1*C* of Fig. 20 shows that 65 per cent of the applied voltage appeared across 2.5 per cent of the shell type winding stack while curves 1*A* and 1*B* of the same figure, show that same voltage appeared across 7½ per cent of the stack of the concentric type of transformer. Thus, in this particular case, the voltage concentration in the shell type transformer happened to be three times as great as in the other. If it is remembered, as Mr. Dann

points out, that a shell type stack is substantially shorter than a concentric type stack. it means that the voltage stress per unit length in shell type transformer is even more than three times as great as in concentric type transformers.

That the severe concentration of voltage in shell type transformer does take place is also substantiated by tests by Mr. Peters. That the severe oscillations are produced by a single lightning wave is illustrated by the cathode ray oscillogram, Fig. 5, herewith.

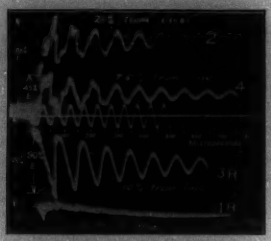

Fig. 5—Oscillation Produced in Shell Type Transformer by Single Unidirectional Wave

I should like to call to Mr. Hickernell's attention that, in the paper, only a transformer with a solidly grounded neutral is discussed. In such a case, the A. I. E. E. standard insulation test, consisting of a so-called induced voltage test with the neutral end of the winding grounded, allows grading of major insulation in the order of the voltage to ground existing during this test. It is this grading of the major insulation that is criticized by the author.

In case of a transformer operating with neutral isolated or with the high-voltage winding connected in delta, the insulation test consists of not only an induced voltage test, but also of a high-potential test between the high-voltage winding and ground. The latter test, subjecting the entire high-voltage winding to full test voltage to ground, allows no grading of the major insulation. No trouble should be expected with major insulation of such a transformer.

As far as I know, the Consumer's Power Company of Michigan, referred to by Mr. Hickernell, operates with isolated neutral and with most of the transformers connected in delta on the high-voltage side. The absence of trouble with the major insulation on that system, therefore, has no bearing on the conclusions suggested in the paper.

I am glad to find that Mr. Hickernell's operating experience emphasizes the importance of coil and turn insulation. On the seventeenth page of the paper, the table of insulation allowances and the test voltage stresses is given. These allowances were used for many years, and show that where the transient stresses are particularly high, the strength of the turn insulation may be more than 800 times the turn-to-turn stress produced by the A. I. E. E. test.

In a non-resonating transformer the insulation between coils and turns is practically the same as shown in the above table, but the stresses due to the elimination of internal oscillation are reduced in the order of from 10-to-1 up to 100-to-1, and thereby very much greater reliability is secured.

I am glad indeed to find that Mr. Sporn, an operating engineer of very wide experience, shares my confidence in the reliability of non-resonating transformers.

It is hoped that the series of cathode ray oscillograms obtained on a power transformer with a transmission line will be published in the near future. Fig. 5 of this discussion is one of those oscillograms.

Transient Analysis of A-C. Machinery

BY YU H. KU*
Associate, A. I. E. E.

***Synopsis.**—This paper shows how the Heaviside operational methods may be applied to determine the transient currents that are produced in synchronous and induction machines by some sudden alteration of their electric circuits. The observed and computed values of the transient currents have been plotted for a number of cases. There is also a table giving the names of the principal investigators who have written on this subject and the methods of analysis that they have employed.*

DURING the last fifteen or twenty years the problem of calculating the transient currents in a-c. machinery has become increasingly important. Many notable papers on this subject have appeared in technical literature both in this country and abroad.

In the analysis of these transient problems it is necessary to make some assumptions, either in regard to the nature of the phenomena or in regard to the characteristics of the fundamental physical quantities involved. The earlier authors, such as Steinmetz and Boucherot, followed the first method, while the later authors, with few exceptions, have followed the second method. The important physical quantities involved are the resistances and inductances of the stator and rotor windings, and the characteristics of the magnetic circuit that enter into the determination of the eddy-current and hysteresis losses. It has been customary to assume that the resistances and self-inductances of the stator and rotor windings are constant, and the mutual inductance between any two windings varies with the cosine of their relative angular displacement. These assumptions in regard to the inductances are practically equivalent to assuming that the air gap is uniform and the flux in it, sinusoidally distributed. Whenever eddy-currents have been considered they have been assumed to exist in an electric circuit of constant resistance and self-inductance.

It seems appropriate to present at this time a brief summary of the technical literature by the principal investigators of these transient problems, indicating the type of solution and the essential features of the methods they have employed. Those authors who adopted the foregoing or other reasonable assumptions in regard to the characteristics of the fundamental physical quantities, and have carried out the mathematical analysis with rigor, are said to have given an exact solution. All other solutions are classed as approximate. Thus all cases where the resistances have been neglected or have been taken only partially into account, as, for example, in determining values for the damping coefficients, have been classed as approximate, even though such solutions may have constituted very important and valuable contributions to this subject.

These transient problems are readily grouped in three general classes. In the first the circuits on each side of the air gap are symmetrical, in the second the circuits on but one side of the gap are symmetrical, and in the third the circuits on neither side of the gap are symmetrical. Exact solutions for problems in the first two classes can be obtained by operational methods, but those in the third class do not readily lend themselves to mathematical treatment, except in one special case.

In the accompanying table is given the summary of the methods employed by the principal investigators.

TABLE I

CASE	DIAGRAM OF CONNECTIONS	AUTHOR OF SOLUTION		TYPE OF SOLUTION	ESSENTIAL FEATURES
III		Steinmetz	1909	Approximate	"Flux Theory"
		Boucherot	1911	Approximate	"E.M.F. Theory"
		Berg	1915	Approximate	For Special Case
		Blondeau	1915	Exact	$r_a/L_a = r_f/L_f$
		Doherty	1918	Approximate	"Constant Flux Linkage Theorem" Discussion
		Shimoda and Ito	1922	Approximate	First Neglected.
		Obata	1924	Exact	Mathematical
III[1]		Doherty and Hickin	1927	Approximate	Double-Reaction Method and Constant Linkages Theorem
II		Boucherot	1911	Approximate	"Constant Flux Linkage"
		Laffoon	1924	Approximate	
		Franklin	1925		Doherty's Method
II		Steinmetz	1909	Approximate	
		Arnold	1913	Approximate	
		Blondeau	1915	Approximate	Boucherot's Method
		Dreyfus	1916	Exact	"Vector" Method
		Lyon	1923	Exact	"Vector" Method or Method of "Revolving Vectors".
		Laffoon	1924	Approximate	
		Franklin	1925	Approximate	
III		Karapetoff	1925	Approximate	Neglect Resistances
I[2]		Dreyfus	1912	Exact	Mathematical
I[2]		Rudenberg	1925	Exact	Dreyfus' Vector Method
		Beikis	1927	Exact	Operational
I		Doherty and Williamson	1921	Approximate	"M.M.F.'s. Consistent and Const. Flux Linkage" "Revolving Vectors"
		Lyon	1923	Exact	
		Dreyfus	1917	Exact	Vector Method
II[2]		Beikis	1927	Exact	Operational

1 Salient Pole Rotor 2 Symmetrical Field Excitation

One of the principal advantages of the Heaviside operational method is that the magnitudes of the transient currents are directly determined by means of the expansion theorem without recourse to the solution of simultaneous equations which is a feature of other methods of analysis. This advantage alone justifies the use of the method.

After the present investigation had been completed

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the Heaviside operational solution by Bekku[1] was brought to the writer's attention. Bekku considered only the case of an alternator with symmetrical excitation and so in this slight respect his solution differs from that presented by the author. In addition the author believes that such a powerful method deserves to be more widely known than would be possible from Bekku's publication in a foreign journal.

The following assumptions that have been made conform with the general practise. First, it will be assumed that the stator windings are symmetrical and that the rotor is turning at constant speed. Second, it will be assumed that the resistances and self-inductances of the windings are constant and that the mutual inductance between any two windings is proportional to the cosine of their relative angular displacement. Thus the effect of frequency upon resistance, the effect of saturation upon inductance, and the effect of harmonics in the distribution of the air-gap flux density are neglected. It should be observed that these assumptions in regard to the inductances cannot be realized except when the machine has a cylindrical rotor. It must be noted particularly that the solution as here presented does not apply unless the circuits on at least one side of the air gap are symmetrical. For example, the method does not apply to the sudden single-phase short-circuit of an alternator with the usual type of field excitation.

Briefly this solution consists in applying first the Stokvis-Fortescue method of symmetrical-phase components to reduce the polyphase relations to equivalent single-phase relations; second, an operational method by which the equations that apply to the stator and rotor circuits and that recognize their relative angular velocity are transformed so that they become similar to the equations that apply to circuits which have no relative rotation; third, the Heaviside operational method, including the principle of superposition as applied to static circuits, is employed to determine the integration constants in the solution.

For the sake of brevity it is only shown how this method of analysis can be applied to the problem of the simultaneous polyphase short circuit of an alternator. Several other cases have been calculated and the results are compared with the measured current wave forms. The agreement between the observed and computed values is quite satisfactory.

Fundamental Equations

With the foregoing assumptions relative to the character of the resistances and inductances, superposition of currents is possible and the principle underlying Thévenin's[2] theorem can be used in determining the currents which flow when the terminals are suddenly short circuited. Thus the current in any phase after the short circuit occurs can be considered as the sum of two components: first, that current which would exist if no disturbance occurred, and second, the additional current that would be produced by suddenly applying to the terminals an alternating e. m. f. equal and opposite to the potential existing there before the short circuit.

If the short circuit takes place under steady conditions, the first component is the normal steady-state current of the phase, and can be readily obtained by the conventional schemes of steady-state analysis.

A short circuit suddenly reduces the terminal potentials to zero. The net effect of this on the circuit currents is equivalent to suddenly applying e. m. fs. to the terminals which will nullify the effect of the potentials existing there. The current in any phase resulting from these suddenly applied e. m. fs.—all other potentials being considered zero—is the additional component mentioned above. This principle is convenient to use because it reduces the short-circuit case to the conventional operational calculus problem—

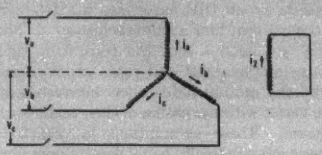

Fig. 1—Connection for Sudden Application of E. M. Fs. to Alternator Terminals

that of finding the response of a circuit to a suddenly applied e. m. f.

The operational equations for the alternator's response to the suddenly applied terminal e. m. fs. can be written as follows, reference being made to Fig. 1:

$$(R_1 + L_1 p)\, i_a + M\, p\, [i_2 \cos n\, t] = v_a 1 \qquad (1)$$

$$(R_1 + L_1 p)\, i_b + M\, p \left[i_2 \cos \left(n\, t - \frac{2\,\pi}{3} \right) \right] = v_b 1 \qquad (2)$$

$$(R_1 + L_1 p)\, i_c + M\, p \left[i_2 \cos \left(n\, t - \frac{4\,\pi}{3} \right) \right] = v_c 1 \qquad (3)$$

$$(R_2 + L_2 p)\, i_2 + M\, p \left[i_a \cos n\, t + i_b \cos \left(n\, t - \frac{2\,\pi}{3} \right) + i_c \cos \left(n\, t - \frac{4\,\pi}{3} \right) \right] = 0 \qquad (4)$$

Where

R_1 = resistance of stator, per phase.
R_2 = resistance of field.
L_1 = synchronous self-inductance of stator, per phase. It is the self-inductance of one

1. S. Bekku, "Sudden Short-Circuit of an Alternator," *Researches of the Electrotechnical Laboratory*, No. 203, June 1927, Tokyo, Japan.

2. L. Thévenin, "Sur un Nouveau Théorème D'Electricité Dynamique," *Comptes Rendus*, Vol. 97, 1883, pp. 159-161. While this theorem was written for the steady state, its extension to the transient case can be readily made.

stator phase, plus the mutual effect of each of the other stator phases.

L_2 = self-inductance of field.

M = maximum value of the mutual inductance between one phase of the stator and the field winding.

i_a, i_b, i_c = instantaneous values of stator currents in phases a, b, and c.

i_2 = instantaneous value of field current.

v_a, v_b, v_c = instantaneous values of the stator terminal potentials in phases a, b, and c.

n = relative angular velocity between windings of stator and field.

p = $\frac{d}{dt}$, the time differential operator.

1 = unit function. In operational calculus notation this means a time function which is discontinuous at $t = 0$, being zero before and unity thereafter.

Although the e. m. fs. which are applied are equal and opposite to the terminal potentials, when considered with respect to the assumed positive direction of current flow in the armature winding, they have the same direction as the original e. m. fs. in the phase.

Simplifying the Equations

If the stator potentials are balanced, the e. m. fs. which are applied will also be balanced, so

$$v_a + v_b + v_c = 0 \tag{5}$$

With no connection to the neutral, the currents in the stator must satisfy the relation

$$i_a + i_b + i_c = 0 \tag{6}$$

It is seen from these relations that Equation (3) is obtainable from (1) and (2), and hence need not be considered further.

Also by means of (6) it is seen that Equation (4) can be reduced to

$$(R_2 + L_2 p)\, i_2 + \sqrt{3}\, M\, p \left[i_a \cos\left(n t - \frac{\pi}{6}\right) + i_b \cos\left(n t - \frac{\pi}{2}\right)\right] = 0 \tag{7}$$

The equations for solution now consist of (1), (2), and (7).

Introduction of Symmetrical Components

By introducing the Stokvis-Fortescue scheme[3] of symmetrical component analysis, the phase currents can be resolved into their positive- and negative-sequence components. Writing these relations for the currents in phases a and b,

$$\left.\begin{aligned} i_a &= i_+ + i_- \\ i_b &= i_+ a^2 + i_- a \end{aligned}\right\} \tag{8}$$

where a and a^2 are respectively $\epsilon^{j\frac{2\pi}{3}}$ and $\epsilon^{j\frac{4\pi}{3}}$.

The e. m. fs. applied to phases a and b can be similarly resolved, so

$$\left.\begin{aligned} v_a &= v_+ + v_- \\ v_b &= v_+ a^2 + v_- a \end{aligned}\right\} \tag{9}$$

and the induced e. m. fs. due to mutual induction can be written

$$\left.\begin{aligned} M\, p\, [i_2 \cos n t] &= e_+ + e_- \\ M\, p \left[i_2 \cos\left(n t - \frac{2\pi}{3}\right)\right] &= e_+ a^2 + e_- a \end{aligned}\right\} \tag{10}$$

If the cosine terms in (10) are written in their exponential form, it is seen that

$$\left.\begin{aligned} e_+ &= \frac{M}{2}\, p\, i_2\, \epsilon^{jnt} \\ e_- &= \frac{M}{2}\, p\, i_2\, \epsilon^{-jnt} \end{aligned}\right\} \tag{11}$$

Equations (1) and (2) can now be rewritten so as to include the relations of Equations (8), (9), and (10). They become

$$(R_1 + L_1 p)\,(i_+ + i_-) + (e_+ + e_-) = (v_+ + v_-)\,1 \tag{12}$$

$$(R_1 + L_1 p)\,(i_+ a^2 + i_- a) + (e_+ a^2 + e_- a) = (v_+ a^2 + v_- a)\,1 \tag{13}$$

These can be solved to obtain

$$(R_1 + L_1 p)\, i_+ + e_+ = v_+ 1 \tag{14}$$

$$(R_1 + L_1 p)\, i_- + e_- = v_- 1 \tag{15}$$

and upon insertion of the expressions for e_+ and e_- there results

$$(R_1 + L_1 p)\, i_+ + \frac{M}{2}\, p\, i_2\, \epsilon^{jnt} = v_+ 1 \tag{16}$$

$$(R_1 + L_1 p)\, i_- + \frac{M}{2}\, p\, i_2\, \epsilon^{-jnt} = v_- 1 \tag{17}$$

This transformation of (1) and (2) into (16) and (17) has resulted in considerable simplification for now one equation has been obtained which contains only positive-sequence terms, and another which contains only negative-sequence terms.

When the relations of (8) are substituted into (7), the latter becomes

3. L. G. Stokvis, "Sur la Création des Harmoniques 3 dans les Alternateurs par Suite du Déséquilibrage des Phases," *Comptes Rendus*, 159, 1914, pp. 46–49. L. G. Stokvis "Analysis of Unbalanced Three-Phase Systems," *Electrical World*, 65, 1915, pp. 1111-15. R. E. Gilman and C. L. Fortescue, "*Single-phase Power Service from Central Stations*," A. I. E. E. Trans., Vol. XXXV, 1916, pp. 1329-1347. C. L. Fortescue, "*Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks*," A. I. E. E. Trans., Vol. XXXVII, 1918, pp. 1027-1115.

$$(R_2 + L_2 p)\, i_2 + \frac{3}{2} Mp\,(i_+ \epsilon^{-jnt} + i_- \epsilon^{jnt}) = 0 \quad (18)$$

Equations (16), (17), and (18) are now in form to be readily handled by operational methods.

REDUCTION TO STATIONARY COUPLED-CIRCUIT PROBLEM

There is an operational transformation formula by means of which an exponential time function ϵ^{-kt} forming part of the operand of an operational expression can be shifted around its operator to the position of a multiplying factor. The compensating change in the operator to show that this has taken place is made by replacing p throughout with $p - k$. This process can be expressed operationally as follows:[4]

$$\frac{1}{Z(p)}\, \epsilon^{-kt} f(t)\, 1 = \epsilon^{-kt} \frac{1}{Z(p-k)} f(t)\, 1,$$

where $1/Z(p)$ is the operator, and $\epsilon^{-kt} f(t)\, 1$ is the operand, or time function, before the shift is performed.

The converse of this process is possible also, and an exponential ϵ^{-kt} can be shifted into the operand if the operator is changed by replacing p with $p + k$. This transformation affords a convenient means of simplifying the alternator equations still further before attempting to evaluate them. If Equation (17) is multiplied by ϵ^{jnt}, and Equation (16) by ϵ^{-jnt}, and then these exponentials shifted to the operands, the two equations become

$$[R_1 + L_1(p + j\,n)]\, i_+ \epsilon^{-jnt} + \frac{M}{2}\,(p + j\,n)\, i_2 = v_+ \epsilon^{-jnt}\, 1 \quad (20)$$

$$[R_1 + L_1(p - j\,n)]\, i_- \epsilon^{jnt} + \frac{M}{2}\,(p - j\,n)\, i_2 = v_- \epsilon^{jnt}\, 1 \quad (21)$$

By this transformation two linear equations with constant coefficients have been secured which can be combined with Equation (18) to solve for the three variables i_+, i_-, and i_2. Thus the alternator equations, written with a variable mutual inductance parameter to care for the relative motion between circuits, have been reduced to those of the equivalent coupled circuits having windings which are stationary.

Solving equations (18), (20), and (21) simultaneously, the following operational expressions for i_+ and i_- are obtained:

$$i_+ = \epsilon^{jnt} \frac{1}{\sigma L_1 D(p)} \Big[(G_+ + H_+)\, v_+ \epsilon^{-jnt} - H_- v_- \epsilon^{jnt} \Big] 1 \quad (22)$$

$$i_- = \epsilon^{-jnt} \frac{1}{\sigma L_1 D(p)} \Big[(G_- + H_-)\, v_- \epsilon^{jnt} - H_+ v_+ \epsilon^{-jnt} \Big] 1 \quad (23)$$

where

$$\sigma = 1 - \frac{3}{2} \frac{M^2}{L_1 L_2},$$

$$D(p) = p^3 + \Big[h_1 + \frac{h_1 + h_2}{\sigma} \Big] p^2 + \Big[n^2 + \frac{h_1}{\sigma}(h_1 + 2h_2) \Big] p + \frac{h_2}{\sigma}[n^2 + h_1^2]$$

$$= (p - p_1)(p - p_2)(p - p_3)$$

$$h_1 = \frac{R_1}{L_1}, \text{ and } h_2 = \frac{R_2}{L_2}$$

$$G_+ = (p + h_2)(p + h_2 - j\,n)$$

$$G_- = (p + h_2)(p + h_2 + j\,n)$$

$$H_+ = \frac{1}{2}(\sigma - 1)\, p\,(p - j\,n)$$

$$H_- = \frac{1}{2}(\sigma - 1)\, p\,(p + j\,n)$$

The e. m. f. applied to phase a is of the form

$$v_a = V_m \sin n\,t = \frac{V_m}{2j}\epsilon^{jnt} - \frac{V_m}{2j}\epsilon^{-jnt} = v_+ + v_-$$

so

$$\left.\begin{aligned} v_+ &= \frac{V_m}{2j}\epsilon^{jnt} \\ v_- &= -\frac{V_m}{2j}\epsilon^{-jnt} \end{aligned}\right\} \quad (24)$$

If these values are inserted in Equations (22) and (23), and all constant factors moved to a position before the operator,

$$i_+ = \frac{V_m}{2j}\epsilon^{jnt} \frac{1}{\sigma L_1 D(p)} [G_+ + H_+ + H_-]\, 1 \quad (25)$$

$$i_- = -\frac{V_m}{2j}\epsilon^{-jnt} \frac{1}{\sigma L_1 D(p)} [G_- + H_- + H_+]\, 1 \quad (26)$$

It is apparent from these results that the effect of the shifting step has been to remove the rotational factors from the operand to the position of multiplying coefficients. At this point it becomes the conventional operational problem of evaluating the result of an operator operating on the unit time function.

Equations (25) and (26) have been derived on the assumption that the short circuit occurs at the instant of zero potential in phase a. As the disturbance can occur at any time in the cycle, a more general form of the equations can be written with the angle $n\,t$ replaced by $n\,t + \theta$. The equations then become

$$i_+ = \frac{V_m}{2}\epsilon^{j(nt+\theta)}\epsilon^{-j\frac{\pi}{2}} \frac{1}{\sigma L_1 D(p)} [G_+ + H_+ + H_-]\, 1 \quad (27)$$

4. O. Heaviside, "Electromagnetic Theory," Vol. II, p. 294. V. Bush, "Operational Circuit Analysis," (Wiley), 1929, Ch. VIII, Sec. 10.

$$i_- = \frac{V_m}{2} \epsilon^{-j(nt+\theta)} \epsilon^{j\frac{\pi}{2}} \frac{1}{\sigma L_1 D(p)} [G_- + H_- + H_+] 1 \tag{28}$$

The factors $1/j$ and $-1/j$ have here been written in their exponential form $\epsilon^{-j\frac{\pi}{2}}$ and $\epsilon^{j\frac{\pi}{2}}$.

Evaluation by Heaviside Expansion Formula

Equations (27) and (28) have been reduced to the ordinary form of operational equations, and there

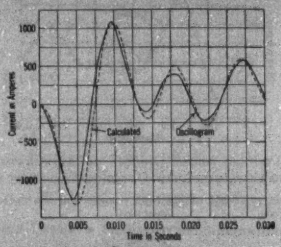

Fig. 2—Polyphase Short Circuit of Alternator at No-Load Current Transient in Phase A

The full lines represent oscillograph records and the dotted lines the calculated values

remains the evaluation of operational expressions such as

$$\frac{A(p)}{B(p)} 1 = \frac{G_+(p) + H_+(p) + H_-(p)}{D(p)} 1. \tag{29}$$

A convenient way of doing this is by means of the Heaviside expansion formula in the form[5]

$$i = \sum_{k=0}^{k=n} \frac{A(p_k)}{F(p_k)} \epsilon^{p_k t} \tag{30}$$

where $F(p_k) = [(p - p_0)(p - p_1) \ldots (p - p_{k-1})(p - p_{k+1}) \ldots (p - p_n)]_{p=p_k}$ in which, for our case, $p_1, p_2 \ldots \ldots p_n$ are the roots of $D(p) = 0$, and $p_0 = 0$ because the operand in (29) is the unit function.

The result of using this on Equation (27) is the series of terms

$$i_+ = \frac{V_m}{2} \epsilon^{j\left(nt+\theta-\frac{\pi}{2}\right)} \frac{1}{\sigma L_1} \left[\left\{ \frac{G_+ + H_+ + H_-}{(-p_1)(-p_2)(-p_3)} \right\}_{p=0} \right.$$

$$+ \left\{ \frac{G_+ + H_+ + H_-}{p_1(p_1 - p_2)(p_1 - p_3)} \right\}_{p=p_1} \epsilon^{p_1 t}$$

$$+ \left\{ \frac{G_+ + H_+ + H_-}{p_2(p_2 - p_1)(p_2 - p_3)} \right\}_{p=p_2} \epsilon^{p_2 t}$$

$$\left. + \left\{ \frac{G_+ + H_+ + H_-}{p_3(p_3 - p_1)(p_3 - p_2)} \right\}_{p=p_3} \epsilon^{p_3 t} \right] \tag{31}$$

and a similar solution can be written for (28).

When Equations (27) and (28) have been finally interpreted into functions of time it is found that i_+ and i_- are conjugate quantities, so that from Equation (8),

$$\left.\begin{aligned} i_a &= i_+ + i_- = 2(i_+)_{real} \\ i_b &= i_+ a^2 + i_- a = 2(i_+ a^2)_{real} \end{aligned}\right\} \tag{32}$$

Consequently the actual numerical work is reduced to the interpretation of i_+ from (27) by means of the expansion formula given in (30).

Application to Polyphase Short Circuit Under Load

When the short circuit occurs under load, the angle θ does not correspond to the rotor position angle as in the no-load case. This can be seen by consideration of the excitation e. m. f. E and the terminal potential V in Fig. 3. The terminal potential lags behind E by an angle

$$\alpha = \beta - \theta.$$

Under these conditions

$$\left.\begin{aligned} v_+ &= \frac{V_m}{2j} \epsilon^{j(nt+\theta)} \\ v_- &= -\frac{V_m}{2j} \epsilon^{-j(nt+\theta)} \end{aligned}\right\} \tag{34}$$

the same as before, but the exponentials in Equations (22) and (23) are now $\epsilon^{j(nt+\theta+\alpha)}$ and $\epsilon^{-j(nt+\theta+\alpha)}$. This

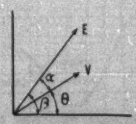

Fig. 3—Relative Position of Excitation E. M. F. and Terminal Potential at Instant of Short Circuit Under Load

results from the fact that the e. m. fs. due to mutual induction in (11) depend upon the position of the rotor, so under load conditions they take the form

$$e_{ac} = \frac{M}{2} p\, i_2 \epsilon^{-j(nt+\theta+\alpha)},$$

and the exponentials involved in the shifting process correspond. When these are combined with the values for v_+ and v_- given in (34), there results

$$i_+ = \frac{V_m}{2j} \epsilon^{j(nt+\theta+\alpha)} \frac{1}{\sigma L_1 D(p)} \left[G_+ + H_+ \epsilon^{-j\alpha} + H_- \epsilon^{j\alpha} \right] 1$$

5. L. Cohen, "The Heaviside Expansion Theorem," *Jour. Franklin Inst.*, 194, 1922, pp. 765-770. V. Bush, *loc. cit.*, Chap. VII, Sec. 4.

$$= \frac{V_m}{2} \epsilon^{j\left(nt+\theta-\frac{\pi}{2}\right)} \frac{1}{\sigma L_1 D(p)} \left[G_+ + H_+ + H_- \epsilon^{2j\alpha}\right] 1 \tag{35}$$

$$i_- = \frac{V_m}{2} \epsilon^{-j\left(nt+\theta-\frac{\pi}{2}\right)} \frac{1}{\sigma L_1 D(p)} \left[G_- + H_- + H_+ \epsilon^{-2j\alpha}\right] 1 \tag{36}$$

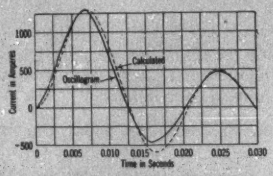

FIG. 4—FIELD CURRENT TRANSIENT CORRESPONDING TO CASE OF FIG. 2

The full lines represent oscillograph records and the dotted lines the calculated values

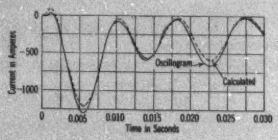

FIG. 5—POLYPHASE SHORT CIRCUIT OF ALTERNATOR HAVING UNITY POWER FACTOR LOAD. CURRENT TRANSIENT IN PHASE a

The full lines represent oscillograph records and the dotted lines the calculated values

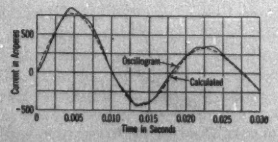

FIG. 6—FIELD CURRENT TRANSIENT CORRESPONDING TO CASE OF FIG. 4

The full lines represent oscillograph records and the dotted lines the calculated values

For the particular case of $\alpha = 0$, these two equations reduce to those of the no-load case given in (**27**) and (**28**). The expressions for i_+ and i_- are again conjugate, so it is necessary to evaluate only i_+, and the phase currents are given again by Equations (**32**).

Curves showing comparisons between experimental and calculated phase currents for short circuits occurring under loads having power factors which are unity, lagging, and leading, are given in Figs. 5, 7, and 9. Calculations for these are given in the Appendix.

FIELD CURRENT TRANSIENTS

The expression for the field current transient can be obtained by solving Equations (**18**), (**20**), and (**21**) simultaneously for i_2. For the general case of short circuit under load

$$i_2 = \frac{3}{2} \frac{M}{L_1 L_2} \frac{V_m}{\sigma} \frac{p}{D(p)} [n \cos \alpha + (p + h_1) \sin \alpha] 1 \tag{37}$$

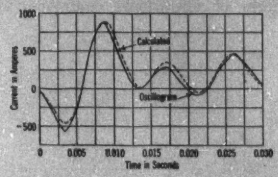

FIG. 7—POLYPHASE SHORT CIRCUIT OF ALTERNATOR HAVING LAGGING POWER-FACTOR LOAD. CURRENT TRANSIENT IN PHASE a

The full lines represent oscillograph records and the dotted lines the calculated values

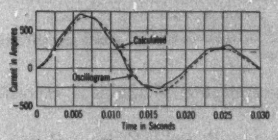

FIG. 8—FIELD CURRENT TRANSIENT CORRESPONDING TO CASE OF FIG. 6

The full lines represent oscillograph records and the dotted lines the calculated values

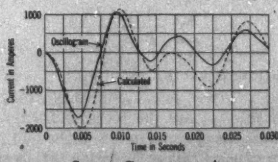

FIG. 9—POLYPHASE SHORT CIRCUIT OF ALTERNATOR HAVING LEADING POWER-FACTOR LOAD. CURRENT TRANSIENT IN PHASE a

The full lines represent oscillograph records and the dotted lines the calculated values

When the short circuit takes place at no load, $\alpha = 0$, and the expression simplifies to

$$i_2 = \frac{3}{2} \frac{M}{L_1 L_2} \frac{n}{\sigma} V_m \frac{p}{D(p)} 1 \tag{38}$$

Since the evaluation of (**38**) is not difficult, in the no-load case it is often convenient to solve for the field transient first. Since the field transient consists of a sum of terms, there will be a term in the stator transient corresponding to each; so it can be expressed by

$$i_a = \sum_{k=1}^{k=3} \epsilon^{j(nt+\theta)} \left\{ \frac{-M(p_k + j\,n)}{L_1(p_k + h_1 + j\,n)} i_2(p_k) \right\}_{real} \tag{39}$$

The field current transients to correspond to the above cases of short circuit at zero load, and at loads of unity, lagging, and leading power factors are given in Figs. 4, 6, 8, and 10. Calculations for these are given in the Appendix.

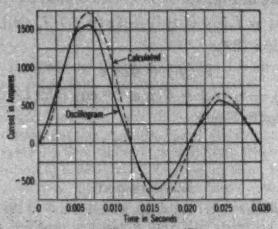

FIG. 10—FIELD CURRENT TRANSIENT CORRESPONDING TO CASE OF FIG. 9

The full lines represent oscillograph records and the dotted lines the calculated values

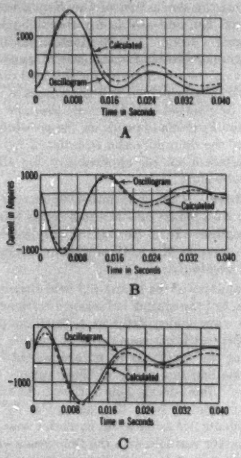

FIG. 11-A, B, C—POLYPHASE SHORT CIRCUIT OF AN INDUCTION MOTOR. STATOR CURRENTS

The full lines represent oscillograph records and the dotted lines the calculated values

ADDITIONAL APPLICATIONS

In addition to the above applications to the alternator, the author has applied this scheme of analysis to the short circuit of the induction motor, both the balanced and unbalanced cases being considered. The case of sudden voltage reduction at the induction-motor terminals was also studied. Curves giving a comparison between calculated and experimental results for two of these cases are added in Figs. 11 and 12.

ACKNOWLEDGMENT

The author wishes to acknowledge his indebtedness to Professor V. Bush and Dr. B. A. Behrend for their suggestions and encouragement, and particularly to Professor W. V. Lyon and Mr. M. F. Gardner, who have revised the paper for publication. Credit is also due to those former graduate students of the Massachusetts Institute of Technology whose theses were the source of helpful experimental results.

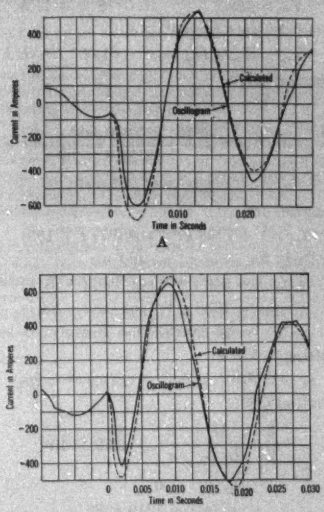

FIG. 12-A, B—SUDDEN VOLTAGE REDUCTION AT INDUCTION-MOTOR TERMINALS. STATOR CURRENTS

The full lines represent oscillograph records and the dotted lines the calculated values

Appendix

NUMERICAL DATA AND CALCULATIONS

Machine Constants

$R_1 = 0.0232$	$R_2 = 0.138$	$M = 8.34 \times 10^{-3}$
$L_1 = 5.66 \times 10^{-3}$	$L_2 = 19.3 \times 10^{-3}$	$n = 377$
$h_1 = 4.10$	$h_2 = 7.13$	$\sigma = 0.046$
		$\frac{1}{\sigma L_1} = 3840$

General Calculations

$D(p) = p^3 + 248\,p^2 + 144 \times 10^3\,p + 220 \times 10^6 = 0$

gives the roots: $p_1 = -169$

$$p_2 = -39.6 + j\,359 = -361\,\underline{/-83.7}$$

$$p_3 = -39.6 - j\,359 = -361\,\underline{/83.7}$$

$G_{\bullet}(0) = 2690\,\underline{/\mp 89.4}$ ‖ $H_{\bullet}(0) = 0$

$G_{\bullet}(p_1) = 66{,}500\,\underline{/\pm 66.4}$ $H_{\bullet}(p_1) = 33{,}300\,\underline{/\pm 66.9}$

$G_{+}(p_2) = 14{,}200\,\underline{/-58}$ $H_{+}(p_2) = -7{,}500\,\underline{/-59.3}$

$G_{-}(p_2) = -265{,}000\,\underline{/7.93}$ $H_{-}(p_2) = 127{,}000\,\underline{/9.4}$

$G_{+}(p_3) = [G_{-}(p_2)]_c$ $H_{+}(p_3) = [H_{-}(p_2)]_c$

$G_{-}(p_3) = [G_{+}(p_2)]_c$ $H_{-}(p_3) = [H_{+}(p_2)]_c$

$F(0) = 22.1 \times 10^6$ $F(p_2) = -98.8 \times 10^6\,\underline{/76.5}$

$F(p_1) = -24.6 \times 10^6$ $F(p_3) = -98.8 \times 10^6\,\underline{/-76.5}$

Short Circuit with no load

$$V_m = 188 \qquad \theta = -71.3$$

Initial steady-state current = 0

$$(G_{\bullet} + H_{\bullet} + H_{\bullet})\,\rho_1 = -61{,}000\,\underline{/\mp 89.5}$$

By Equation (27):

$i_+(p_1)$

$$= \frac{188}{2}\,\epsilon^{j(377t-71.3-90)}\,\frac{3840 \times (-61{,}000\,\underline{/-89.5})}{-24.6 \times 10^6}\,\epsilon^{-169t}$$

$$= 894\,\epsilon^{-169t}\,\epsilon^{j(377t+109)}$$

By Equation (32):

$$i_a(p_1) = 1790\,\epsilon^{-169t}\cos(377\,t + 109)$$

and similarly for roots p_2, p_3, and p_0:

$$i_a(p_2) = 949\,\epsilon^{-39.6t}\cos(736\,t - 51.1)$$

$$i_a(p_3) = 1038\,\epsilon^{-39.6t}\cos(18\,t - 88.7)$$

$$i_0(0) = 88\cos(377\,t + 109)$$

For plotting in Fig. 2:

$$i_a = i_0(0) + i_a(p_1) + i_a(p_2) + i_a(p_3)$$

By Equation (38):

$$\frac{3}{2}\,\frac{M}{L_1 L_2}\,\frac{n}{\sigma} = 93.9 \times 10^4$$

$$i_2(0) = 0$$

$$i_2(p_1) = 1210\,\epsilon^{-169t}$$

$$i_2(p_2) + i_2(p_3) = 1290\,\epsilon^{-39.6t}\cos(359\,t - 160)$$

For plotting in Fig. 4:

$$i_2 = i_2(p_1) + i_2(p_2) + i_2(p_3)$$

Short Circuit with Unity Power Factor Load

$$V_m = 125 \qquad \theta = 180 \qquad \alpha = 46.7 \qquad \beta = 227$$

By Equations (35) and (32):

$$i_a(p_1) = 437\,\epsilon^{-169t}\cos(377\,t + 47.7)$$

$$i_a(p_2) = -585\,\epsilon^{-39.6t}\cos(736\,t + 115)$$

$$i_a(p_3) = 637\,\epsilon^{-39.6t}\cos(18\,t + 161)$$

i_{as} (total steady-state current) $= 84.3\cos(377\,t + 47.3)$

For plotting in Fig. 5:

$$i_a = i_{as} + i_a(p_1) + i_a(p_2) + i_a(p_3)$$

By Equation (37):

$$i_2(p_1) = 296\,\epsilon^{-169t}$$

$$i_2(p_2) + i_2(p_3) = 796\,\epsilon^{-39.6t}\cos(359\,t - 112)$$

For plotting in Fig. 6:

$$i_2 = i_2(p_1) + i_2(p_2) + i_2(p_3)$$

Short Circuit with Lagging Power Factor Load

$$V_m = 99 \qquad Z_L = 2.52\,\underline{/84.3} \qquad \alpha = 2 \qquad \beta = -35$$

By Equations (35) and (32):

$$i_a(p_1) = -930\,\epsilon^{-169t}\cos(377\,t - 34.5)$$

$$i_a(p_2) = 499\,\epsilon^{-39.6t}\cos(736\,t - 13)$$

$$i_a(p_3) = -544\,\epsilon^{-39.6t}\cos(18\,t + 126)$$

$$i_{as} = -81\cos(377\,t - 34.4)$$

For plotting in Fig. 7:

$$i_a = i_{as} + i_a(p_1) + i_a(p_2) + i_a(p_3)$$

By Equation (37):

$$i_2(p_1) = 628\,\epsilon^{-169t}$$

$$i_2(p_2) + i_2(p_3) = 676\,\epsilon^{-39.6t}\cos(359\,t - 158)$$

For plotting in Fig. 8:

$$i_2 = i_2(p_1) + i_2(p_2) + i_2(p_3)$$

Discussion

Louis Cohen: Mr. Ku made effective use of the Expansion Formula in his study of a very interesting problem, and he has also brought in the use of the "Operand Shift" principle. He has of course used the formula correctly, but in shifting the operand the impression is created that in applying the formula to a-c. problems it is always necessary to shift the operand and operate on unity function. This of course is not at all necessary. In my paper on the derivation of the Expansion Formula referred to by Mr. Ku, and also in my recent book "Heaviside's Electrical Circuit Theory," I have given a modification of the Expansion Formula suitable for the case of applied alternating voltages by which the solution can be obtained directly without resorting to the shift principle, and then operate on unity function. It may be pointed out, however, that the successful use of the Expansion Formula depends on the possibility of evaluating the roots of the determinantal equation $Z(p) = 0$, and this imposes a limitation on its applicability for the solution of engineering problems.

M. F. Gardner: There is a number of points of mathematical interest in Dr. Ku's paper, one of which seems to me to warrant particular attention. It is the use of the principle of superposition in the solution of a system which contains a parameter which is not a constant.

The self parameters of the stator and field circuits are assumed to be constants, but the mutual inductance between these circuits is assumed to vary sinusoidally as a function of time. The principle of superposition is considered applicable to the system, so the effects of the various e. m. fs. are solved for separately and an algebraic addition made of these results to obtain the actual response. Since discussions of the superposition principle are generally presented from the standpoint of circuits with constant coefficients, its application in such a case as this where there is a parameter variable with the time needs some investigation. While the use of this principle under such conditions is frequently made in engineering analysis, there does not seem to be available in engineering literature any mathematical demonstration that such applications are legitimate.

When the parameters of the circuit are fixed, that is when they are independent of time or circuit performance, and the case of impulsive response is barred, the principle of superposition can be expressed by the definite integral.

$$i(t) = \frac{d}{dt}\int_0^t A(t - \lambda)\,e(\lambda)\,d\lambda. \qquad (1)$$

In this $A(t - \lambda)$ is formed from $A(t)$ which is the characteristic

of the circuit as embodied in its response to a suddenly applied unit continuous e. m. f. when the circuit is initially in equilibrium. The integral then expresses the response $i(t)$ to an applied e. m. f. $e(t)$ of arbitrary form in terms of the same circuit's response to the unit e. m. f.

When the parameters of the circuit are not all fixed, but include some which are varying with time, this integral no longer has any significance because there is now no characteristic $A(t)$ for the entire circuit which is independent of the time at which it is determined. Such circuits can still be treated, however, for in general they can be divided so as to separate the variable portion from the invariable portion.[1] The differential equation for this case can then be written in the operational form

$$i(t) = \frac{e(t) - f(p, t)\, i(t)}{\bar{Z}(p)} \tag{2}$$

where $f(p, t)$ represents the operator formed from the variable portion, and $Z(p)$ from the invariable portion. The first is expressed in general form to show that it may be a function of derivatives and integrals as well as time. The differential equation written in this modified form shows that the effect of the variable part of the circuit has been accounted for by the substitution of a fictitious e. m. f. equivalent to the impedance drop in that portion. This combines with the applied e. m. f. to produce the current that flows in the fixed part of the network. Since the superposition theorem holds for the fixed portion, it can be written

$$i(t) = \frac{d}{dt} \int_0^t A(t - \lambda)\, [e(\lambda) - f(p, \lambda)\, i(\lambda)]\, d\lambda \tag{3}$$

and we now have an integral equation. It should be noted that $A(t)$ is the characteristic of the invariable part of the circuit only, and the problem has been expressed as the application to this part of two related e. m. fs.

Now let us extend this idea to the case where there are several electromotive forces impressed in the circuit. Consider that a voltage $e_1(t)$ is applied. The response is given by

$$i_1(t) = \frac{d}{dt} \int_0^t A(t - \lambda)\, [e_1(\lambda) - f(p, \lambda)\, i_1(\lambda)]\, d\lambda \tag{4}$$

Had a voltage $e_2(t)$ been applied instead, an expression of similar form would have been obtained for its response $i_2(t)$. Now consider that $e_1(t)$ and $e_2(t)$ are applied in the network simultaneously, and write the resulting current as $[i_1(t) + i_2(t)]$. There then is obtained

$$[i_1(t) + i_2(t)] = \frac{d}{dt} \int_0^t A(t - \lambda)\, [(e_1(\lambda) + e_2(\lambda)) - f(p, \lambda)\, (i_1(\lambda) + i_2(\lambda))]\, d\lambda. \tag{5}$$

On the other hand if the above results, $i_1(t)$ and $i_2(t)$, of applying the two voltages separately are added and combined by transformation into one integral, the resultant expression is identical with (5).

The transformation necessary in making this step is

$$f(p, t)\, i_1(t) + f(p, t)\, i_2(t) = f(p, t)\, [i_1(t) + i_2(t)]. \tag{6}$$

This is legitimate because $f(p, t)$ is a linear operator, and obeys the distributive law although not the laws of algebra in general. It is interesting to note at this point that it is the lack of this property in functional operators, such as $f(p, i)$, that prevents a similar treatment of networks containing parameters which are dependent upon the currents and voltages present,—cases in which the superposition principle is known to be invalid.

Since it is seen that the result of adding the effects of two voltages applied separately to a circuit having time-varying parameters is the same as that obtained when these voltages are applied simultaneously, it justifies the splitting of a voltage into its parts and solving for the effects of these parts separately when such parameters are present. Since this is precisely the principle of superposition, it is seen that this principle can be legitimately applied when there are parameters present which are variable provided their change is one with time only.

1. J. R. Carson, "Theory and Calculation of Variable Electrical Systems," *Phys. Rev.*, Vol. 17, Feb. 1921, pp. 116-134.

Two-Reaction Theory of Synchronous Machines

Generalized Method of Analysis—Part I

BY R. H. PARK*

Associate, A. I. E. E.

Synopsis.—*Starting with the basic assumption of no saturation or hysteresis, and with distribution of armature phase m. m. f. effectively sinusoidal as far as regards phenomena dependent upon rotor position, general formulas are developed for current, voltage, power, and torque under steady and transient load conditions. Special detailed formulas are also developed which permit the determination of current and torque on three-phase short circuit, during starting, and when only small deviations from an average operating angle are involved.*

In addition, new and more accurate equivalent circuits are developed for synchronous and asynchronous machines operating in parallel, and the domain of validity of such circuits is established.

Throughout, the treatment has been generalized to include salient poles and an arbitrary number of rotor circuits. The analysis is thus adapted to machines equipped with field pole collars, or with amortisseur windings of any arbitrary construction.

It is proposed to continue the analysis in a subsequent paper.

* * * * *

THIS paper presents a generalization and extension of the work of Blondel, Dreyfus, and Doherty and Nickle, and establishes new and general methods of calculating current power and torque in salient and non-salient pole synchronous machines, under both transient and steady load conditions.

Attention is restricted to symmetrical three-phase† machines with field structure symmetrical about the axes of the field winding and interpolar space, but salient poles and an arbitrary number of rotor‡ circuits is considered.

Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy

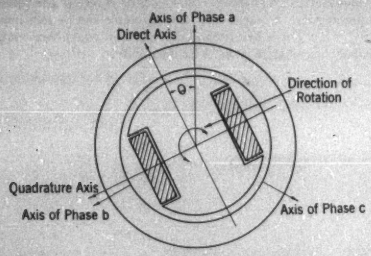

FIG. 1

currents in the armature iron are neglected, and in the assumption that, as far as concerns effects depending on the position of the rotor, each armature winding may be regarded as, in effect, sinusoidally distributed.[3]

A. *Fundamental Circuit Equations*

Consider the ideal synchronous machine of Fig. 1, and let

i_a, i_b, i_c = per unit instantaneous phase currents

e_a, e_b, e_c = per unit instantaneous phase voltages

ψ_a, ψ_b, ψ_c = per unit instantaneous phase linkages

t = time in electrical radians

$$p = \frac{d}{dt}$$

Then there is

$$\begin{aligned} e_a &= p\,\psi_a - r\,i_a \\ e_b &= p\,\psi_b - r\,i_b \\ e_c &= p\,\psi_c - r\,i_c \end{aligned} \tag{1}$$

It has been shown previously[3] that

$$\begin{aligned} \psi_a &= I_d \cos\theta - I_q \sin\theta \\ &\quad - \frac{x_0}{3}[i_a + i_b + i_c] - \frac{x_d + x_q}{3}\left[i_a - \frac{i_b + i_c}{2}\right] \\ &\quad - \frac{x_d - x_q}{3}[i_a \cos 2\theta + i_b \cos(2\theta - 120) \\ &\qquad + i_c \cos(2\theta + 120)] \\ \psi_b &= I_d \cos(\theta - 120) - I_q \sin(\theta - 120) \\ &\quad - x_0 \frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3}\left[i_b - \frac{i_c + i_a}{2}\right] \\ &\quad - \frac{x_d - x_q}{3}[i_a \cos(2\theta - 120) + i_b \cos(2\theta + 120) \\ &\qquad + i_c \cos 2\theta] \\ \psi_c &= I_d \cos(\theta + 120) \\ &\quad - I_q \sin(\theta + 120) - x_0 \frac{i_a + i_b + i_c}{3} \\ &\quad - \frac{x_d + x_q}{3}\left[i_c - \frac{i_a + i_b}{2}\right] \\ &\quad - \frac{x_d - x_q}{3}[i_a \cos(2\theta + 120) + i_b \cos 2\theta \\ &\qquad + i_c \cos(2\theta - 120)] \end{aligned} \tag{2}$$

where,

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†Single-phase machines may be regarded as three-phase machines with one phase open circuited.

‡Stator for a machine with stationary field structure.

[3]For numbered references see Bibliography.

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29-33

I_d = per-unit excitation in direct axis
I_q = per-unit excitation in quadrature axis
x_d = direct synchronous reactance
x_q = quadrature synchronous reactance
x_0 = zero phase-sequence reactance

As shown in the Appendix, if normal linkages in the field circuit are defined as those obtaining at no load* there is in the case of no rotor circuits in the direct axis in addition to the field,

Φ = per-unit instantaneous field linkages
$= I - (x_d - x_d')\, i_d$

where,

I = per-unit instantaneous field current

$$i_d = \frac{2}{3}\{i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)\} \tag{3}$$

On the other hand, if n additional rotor circuits exist in the direct axis there is,

$$\Phi = I + X_{f1d} I_{1d} + X_{f2d} I_{2d} + \ldots + X_{fnd} I_{nd} - (x_d - x_d')\, i_d$$

where,

I_{1d}, I_{2d}, . . etc., are the per-unit instantaneous currents in circuits 1, 2, etc., of the direct axis, X_{f1d}, X_{f2d}, . . . etc., are per-unit mutual coefficients between the field and circuits 1, 2, etc., of the direct axis.

Similar relations exist for the linkages in each of the additional rotor circuits except $x_d - x_d'$ is to be replaced by a term x_m. However, since all of these additional circuits are closed, it follows that there is an operational result

$$I_d = I + I_{1d} + I_{2d} + \ldots + I_{nd} = G(p)\, E + H(p)\, i_d \tag{4}$$

where E is the per-unit value of the instantaneous field voltage, and $G(p)$ and $H(p)$ are operators such that

$$G(o) = 1 \qquad G(\infty) = 0$$
$$H(o) = 0 \qquad H(\infty) = x_d - x_d''$$

x_d'' = the subtransient reactance[2]

It will be convenient to write $H(p) = x_d - x_d(p)$ and to rewrite (4) in the form,

$$I_d = G(p)\, E + [x_d - x_d(p)]\, i_d \tag{4a}$$

If there are no additional rotor circuits, there is, as shown in Appendix I,

$$\Psi = I - (x_d - x_d')\, i_d$$
$$E = T_0\, p\, \Psi + I$$

where T_0 is the open circuit time constant of the field in radians.

There is then,

$$G(p) = \frac{1}{T_0 p + 1}$$

$$x_d(p) = \frac{x_d' T_0 p + x_d}{T_0 p + 1}$$

*This definition is somewhat different from that given in reference 2.

If there is one additional rotor circuit in the direct axis there is,

$$\Psi = I + X_{f1d} I_{1d} - (x_d - x_d')\, i_d = \frac{E - I}{T_0 p}$$

$$\Psi_{1d} = X_{11d} I_{1d} + X_{f1d} I - x_{m1}\, d\, i_d = \frac{-I_{1d}}{T_{01d}\, p}$$

which gives,

$$G(p) = \frac{[X_{11d} - X_{f1d}]\, T_{01d}\, p + 1}{A(p)}$$

$$x_d(p) = x_d - \frac{T_0 T_{01d}[X_{11d}(x_d - x_d') - X_{f1d} x_{m1d}]\, p^2 + [(x_d - x'_d)\, T_{01d} + x_{m1d} T_0]\, p}{A(p)}$$

where,

$A(p) = [X_{11d} - X_{f1d}^2]\, T_0\, T_{01d}\, p^2 + [X_{11d}\, T_0 + T_{01d}]\, p + 1$

If there is more than one additional rotor circuit the operators $G(p)$ and $x_d(p)$ will be more complicated but may be found in the same way. The effects of external field resistance may be found by changing the term I in the field voltage equation to $R\,I$. Open circuited field corresponds to R equal to infinity.

Similarly, there will be

$$I_q = [x_q - x_q(p)]\, i_q \tag{5}$$

where,

$$i_q = -\frac{2}{3}\{i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)\} \tag{3a}$$

$$x_q(o) = x_q,\ x_q(\infty) = x_q''$$

So far, 10 equations have been established relating the 15 quantities e_a, e_b, e_c, i_a, i_b, i_c, ψ_a, ψ_b, ψ_c, i_d, i_q, I_d, I_q, E, θ in a general way. It follows that when any five of the quantities are known the remaining 10 may be determined. Their determination is very much facilitated, however, by the introduction of certain auxiliary quantities e_d, e_q, e_0, i_0, ψ_d, ψ_q, ψ_0.

Thus, let

$$i_0 = \frac{1}{3}\{i_a + i_b + i_c\} \tag{3b}$$

$$e_d = \frac{2}{3}\{e_a \cos\theta + e_b \cos(\theta - 120) + e_c \cos(\theta + 120)\}$$

$$e_q = -\frac{2}{3}\{e_a \sin\theta + e_b \sin(\theta - 120) + e_c \sin(\theta + 120)\} \tag{6}$$

$$e_0 = \frac{1}{3}\{e_a + e_b + e_c\}$$

$$\psi_d = \frac{2}{3}\{\psi_a \cos\theta + \psi_b \cos(\theta - 120) + \psi_c \cos(\theta + 120)\}$$

$$\psi_q = -\frac{2}{3}\{\psi_a \sin\theta + \psi_b \sin(\theta - 120) + \psi_c \sin(\theta + 120)\} \tag{7}$$

$$\psi_0 = \frac{1}{3}\{\psi_a + \psi_b + \psi_c\}$$

then from Equation (1) there is

$$e_d = \frac{2}{3}\{\cos\theta\, p\,\psi_a + \cos(\theta - 120)\, p\,\psi_b + \cos(\theta + 120)\, p\,\psi_c\} - r\, i_d$$

$$e_q = -\frac{2}{3}\{\sin\theta\, p\,\psi_a + \sin(\theta - 120)\, p\,\psi_b + \sin(\theta + 120)\, p\,\psi_c\} - r\, i_q$$

$$e_0 = p\,\psi_0 - r\, i_0$$

but,

$$p\,\psi_d = \frac{2}{3}\{\cos\theta\, p\,\psi_a + \cos(\theta - 120)\, p\,\psi_b + \cos(\theta + 120)\, p\,\psi_c\}$$

$$-\frac{2}{3}\{\sin\theta\,\psi_a + \sin(\theta - 120)\, p\,\psi_b + \sin(\theta + 120)\, p\,\psi_c\}\, p\,\theta$$

$$= e_d + r\, i_d + \psi_q\, p\,\theta$$

$$p\,\psi_q = -\frac{2}{3}\{\sin\theta\, p\,\psi_a + \sin(\theta - 120)\, p\,\psi_b + \sin(\theta + 120)\, p\,\psi_c\}$$

$$-\frac{2}{3}\{\cos\theta\,\psi_a + \cos(\theta - 120)\,\psi_b + \cos(\theta + 120)\,\psi_c\}\, p\,\theta$$

$$= e_q + r\, i_q - \psi_d\, p\,\theta$$

hence there is

$$e_d = p\,\psi_d - r\, i_d - \psi_q\, p\,\theta \tag{8}$$

$$e_q = p\,\psi_q - r\, i_q + \psi_d\, p\,\theta \tag{9}$$

$$e_0 = p\,\psi_0 - r\, i_0 \tag{10}$$

Also it may be readily verified that

$$\psi_d = I_d - x_d\, i_d = G(p)\, E - x_d(p)\, i_d \tag{11}$$

$$\psi_q = I_q - x_q\, i_q = -x_q(p)\, i_q \tag{12}$$

$$\psi_0 = -x_0\, i_0 \tag{13}$$

Equations (8) to (13) establish six relatively simple relations between the 11 quantities e_d, e_q, e_0, i_d, i_q, i_0, ψ_d, ψ_q, ψ_0, E, θ. In practise it is usually possible to determine five of these quantities directly from the terminal conditions, after which the remaining six may be calculated with relative simplicity. After the direct, quadrature, and zero quantities are known the phase quantities may be determined from the identical relations

$$\begin{aligned} i_a &= i_d \cos\theta - i_q \sin\theta + i_0 \\ i_b &= i_d \cos(\theta - 120) - i_q \sin(\theta - 120) + i_0 \\ i_c &= i_d \cos(\theta + 120) - i_q \sin(\theta + 120) + i_0 \end{aligned} \tag{14}$$

$$\begin{aligned} \psi_a &= \psi_d \cos\theta - \psi_q \sin\theta + \psi_0 \\ \psi_b &= \psi_d \cos(\theta - 120) - \psi_q \sin(\theta - 120) + \psi_0 \\ \psi_c &= \psi_d \cos(\theta + 120) - \psi_q \sin(\theta + 120) + \psi_0 \end{aligned} \tag{15}$$

$$\begin{aligned} e_a &= e_d \cos\theta - e_q \sin\theta + e_0 \\ e_b &= e_d \cos(\theta - 120) - e_q \sin(\theta - 120) + e_0 \\ e_c &= e_d \cos(\theta + 120) - e_q \sin(\theta + 120) + e_0 \end{aligned} \tag{16}$$

Referring to Fig. 2, it may be seen that when there are no zero quantities, that is, when $e_0 = \psi_0 = i_0 = 0$, the phase quantities may be regarded as the projection of vectors $\bar{e}$, $\bar{\psi}$, and $\bar{i}$ on axes lagging the direct axis by

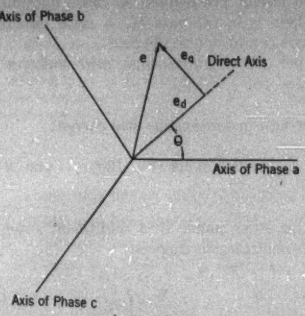

Fig. 2

angles θ, $\theta - 120$ and $\theta + 120$, where taking the direct axis as the axis of reals,

$$\bar{e} = e_d + j\, e_q$$

$$\bar{\psi} = \psi_d + j\,\psi_q$$

$$\bar{i} = i_d + j\, i_q$$

If we introduce in addition the vector quantity,

$$\bar{I} = I_d + j\, I_q$$

the circuit equations previously obtained may be

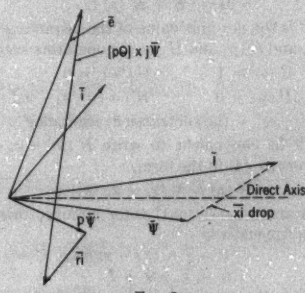

Fig. 3

transferred into the corresponding vector forms,

$$\bar{e} = p\,\bar{\psi} - \overline{ri} + [p\,\theta]\, j\,\bar{\psi}$$

$$\bar{\psi} = \bar{I} - \overline{xi}$$

where, $\overline{xi} = x_d\, i_d + j\, x_q\, i_q$

Fig. 3 shows these relations graphically.

B. Armature Power Output

The per-unit instantaneous power output from the armature is necessarily proportional to the sum

$e_a i_a + e_b i_b + e_c i_c$. By consideration of any instant during normal operation at unity power factor it may be seen that the factor of proportionality must be 2/3. That is,

$$P = \text{per-unit instantaneous power output}$$
$$= 2/3 \{ e_a i_a + e_b i_b + e_c i_c \}$$

Substituting from Equations (14) and (16) there results the useful relation,

$$P = e_d i_d + e_q i_q + e_0 i_0 \tag{17}$$

C. Electrical Torque on Rotor

It is possible to determine the electrical torque on the rotor directly from the general relation,

{Total power output} =
{mechanical power transferred across gap}
+ {rate of decrease of total stored magnetic energy}
− {total ohmic losses} (18)

However, since this torque depends uniquely only on the magnitudes of the currents in every circuit of the machine, it follows that a general formula for torque may be derived by considering any special case in which arbitrary conditions are imposed as to the way in which these currents are changing as the rotor moves.

The simplest conditions to impose are that I_d, I_q, i_d, i_q, and i_0 remain constant as the rotor moves. In this case there will be no change in the stored magnetic energy of the machine as the rotor moves, and the power output of the rotor will be just equal in magnitude and opposite in sign to the rotor losses. It follows that under the special conditions assumed, Equation (18) becomes simply,

{armature power output} =
{mechanical power across gap} − {armature losses}

$$\text{or, } P = T p \theta - \frac{2 r}{3} \{ i_a^2 + i_b^2 + i_c^2 \}$$
$$= T p \theta - r \{ i_d^2 + i_q^2 + i_0^2 \}$$

Then,

T = per-unit instantaneous electrical torque

$$= \frac{e_d i_d + e_q i_q + e_0 i_0 + r \{ i_d^2 + i_q^2 + i_0^2 \}}{p \theta}$$

but subject to the conditions imposed,

$$e_d = - \psi_q p \theta - r i_d$$
$$e_q = \psi_d p \theta - r i_q$$
$$e_0 = - r i_0$$

It therefore follows that,

$$T = i_q \psi_d - i_d \psi_q \tag{19}$$
$$= \text{vector product of } \bar{\psi} \text{ and } \bar{i}$$
$$= \bar{\psi} \times \bar{i} \tag{19a}$$

a result which could have been established directly by physical reasoning. Formula (19) is employed by Dreyfus in his treatment of self-excited oscillations of synchronous machines.[14]

D. Constant Rotor Speed

Suppose that the constant slip of the rotor is s. Then there is,

$$e_d = p \psi_d - r i_d - (1 - s) \psi_q$$
$$e_q = p \psi_q - r i_q + (1 - s) \psi_d$$

but,

$$\psi_d = G(p) E - x_d(p) i_d$$
$$\psi_q = - x_q(p) i_q$$

Putting

$$p x_d(p) + r = z_d(p)$$
$$p x_q(p) + r = z_q(p)$$

there is

$$e_d = p G(p) E - z_d(p) i_d + (1 - s) x_q(p) i_q \tag{20}$$
$$e_q = (1 - s) [G(p) E - x_d(p) i_d] - z_q(p) i_q \tag{21}$$

Solving gives,

$$i_d = \{ [p z_q(p) + (1 - s)^2 x_q(p)] G(p) E - z_q(p) e_d - (1 - s) x_q(p) e_q \} \div D(p) \tag{22}$$

$$i_q = \frac{(1 - s) r G(p) E - z_d(p) e_q + (1 - s) x_d(p) e_d}{D(p)} \tag{23}$$

where, $D(p) = z_d(p) z_q(p) + (1 - s)^2 x_d(p) x_q(p)$

E. Two Machines Connected Together

Suppose that two machines which we will designate respectively by the subscripts g and h, are connected together, but not to any other machines or circuits, and assume in addition that there are no zero quantities. In this case the voltages of each machine will be equal

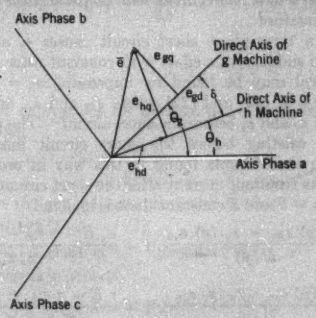

FIG. 4

phase for phase, and it therefore follows that the voltage vectors of each machine must coincide, as shown in Fig. 4.

Referring to the figure it will be seen that the direct and quadrature components of voltage of the two machines are subject to the mutual relations,

$$e_{hd} = e_{gd} \cos \delta - e_{gq} \sin \delta$$
$$e_{hq} = e_{gd} \sin \delta + e_{gq} \cos \delta \tag{24}$$
$$e_{gd} = e_{hd} \cos \delta + e_{hq} \sin \delta$$
$$e_{gq} = - e_{hd} \sin \delta + e_{hq} \cos \delta \tag{25}$$

On the other hand, for currents there will be

$$i_{hd} = -\{i_{gd}\cos\delta - i_{gq}\sin\delta\}$$
$$i_{hq} = -\{i_{gd}\sin\delta + i_{gq}\cos\delta\} \qquad (26)$$
$$i_{gd} = -\{i_{hd}\cos\delta + i_{hq}\sin\delta\}$$
$$i_{gq} = -\{-i_{hd}\sin\delta + i_{hq}\cos\delta\} \qquad (27)$$

F. One Machine on an Infinite Bus

In (E), if machine h has zero impedance, it follows from (**20**) and (**21**) that $e_{hd} = 0$, e_{hq} = bus voltage say $= e$.

Then for machine g there is,

$$e_d = e\sin\delta$$
$$e_q = e\cos\delta \qquad (28)$$

G. Torque Angle Relations

From Equations (**11**), (**12**), and (**19**), there is,

$$T = \frac{I_q\,\psi_d}{x_q} - \frac{I_d\,\psi_q}{x_d} - \frac{x_d - x_q}{x_d\,x_q}\,\psi_d\,\psi_q$$

Then if the rotor leads the vector $\overline{\psi}$ by an angle $\overline{\delta}$ there is

$$\psi_q = -\psi\sin\delta$$
$$\psi_d = \psi\cos\delta$$
$$T = \frac{I_q\,\psi}{x_q}\cos\delta + \frac{I_d\,\psi\sin\delta}{x_d} + \frac{x_d - x_q}{2\,x_d\,x_q}\,\psi^2\sin 2\,\delta \qquad (29)$$

A derivation of this formula for steady load conditions has been previously given by Doherty and Nickle.[9]

H. Three-Phase Short Circuit with Constant Rotor Speed Maintained

Since a three-phase short circuit causes e_d and e_q to vanish suddenly, its effect with constant rotor speed maintained may be found by impressing $e_d = -e_{d0}$, $e_q = -e_{q0}$ in (**22**) and (**23**) where e_{d0} and e_{q0} are the values of e_d and e_q before the short circuit. The initial currents existing before the short circuit must be added to the currents found in this way in order to obtain the resultant current after the short circuit.

With $s = 0$ and E constant there is in detail,

$$i_d = \frac{z_q\,(p)\,e_{d0} + x_q\,(p)\,e_{q0}}{D\,(p)}\,.\,1 + \frac{x_q\,E - r\,e_{d0} - x_q\,e_{q0}}{r^2 + x_d\,x_q}$$

$$i_q = \frac{z_d\,(p)\,e_{q0} - x_d\,(p)\,e_{d0}}{D\,(p)}\,.\,1 + \frac{r\,E - r\,e_{q0} + x_d\,e_{d0}}{r^2 + x_d\,x_q} \qquad (30)$$

The working out of the formulas may be illustrated by consideration of the simple case of a machine with no rotor circuits in addition to the field. In this case there is

$$x_q\,(p) = x_q$$
$$x_d\,(p) = \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}$$

$$D\,(p) = \left\langle \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}\,p + r \right\rangle \langle x_q\,p + r\rangle$$

$$+ \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}\,x_q$$

$$x_d'\,x_q\,T_0\,p^3$$
$$+ [x_d'\,r\,T_0 + (x_d + r\,T_0)\,x_q]\,p^2$$
$$+ [r\,(x_d + x_q + r\,T_0) + x_d'\,x_q\,T_0]\,p$$
$$= \frac{+r^2 + x_d\,x_q}{T_0\,p + 1}$$
$$= \frac{d\,(p)}{T_0\,p + 1} \qquad (31)$$

By the expansion theorem there is, finally,

$$i_d = \frac{x_q\,E}{r^2 + x_d\,x_q}$$
$$+ \sum_1^3 \frac{(T_0\,\alpha_n + 1)\,\langle (x_q\,\alpha_n + r)\,e_{d0} + x_q\,e_{q0}\rangle\,\epsilon^{-\alpha_n t}}{\alpha_n\,d'\,(\alpha_n)}$$

$$i_q = \frac{r\,E}{r^2 + x_d\,x_q} + \sum_1^3$$
$$\frac{\langle x_d'\,T_0\,\alpha_n^2 + (x_d + r\,T_0)\,\alpha_n + r\rangle e_{q0} - (T_0\,\alpha_n\,x'_d + x_d)\,e_{d0}}{\alpha_n\,d'\,(\alpha_n)}$$
$$\epsilon^{-\alpha_n t} \qquad (32)$$

where the summation is extended over the roots of $d\,(\alpha) = 0$ and $d'\,(p) = \dfrac{d}{d\,p}\,d\,(p)$

The phase currents may, of course, be found from

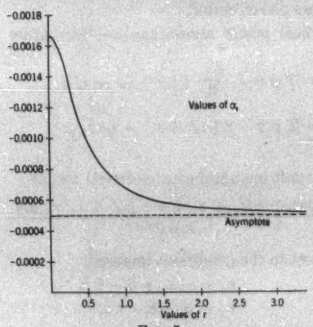

FIG. 5

Equations (**32**) by the application of Equations (**14**). For the particular case

$$T_0 = 2{,}000,\ x_d = 1.00,\ x_q = 0.60,\ x_d' = 0.30$$

the roots α_1, α_2, α_3 of the equation $d\,(p) = 0$, were found to be as shown in Figs. 5, 6, and 7, where

$$\alpha_2 = \alpha_a + \alpha_b$$
$$\alpha_3 = \alpha_a - \alpha_b$$

It will be noted that, as would necessarily be the

case, where $r = 0$, α_1 is equal to the reciprocal of the short circuit time constant of the machine, *i. e.*, for $r = 0$,

$$\alpha_1 = -\frac{x_d}{x_d'}\frac{1}{T_0} = -0.001667$$

while for $r = \infty$

$$\alpha_1 = -\frac{1}{T_0} = -0.000500$$

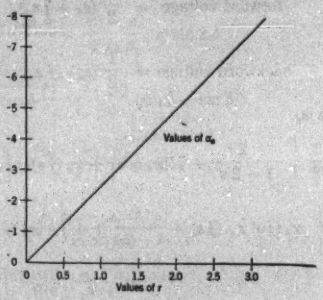

FIG. 6

The root α_a is found to be almost exactly equal to the value which it would have were $T_0 = \infty$, *i. e.*,

$$\alpha_a = \frac{r\,(x_d' + x_q)}{2\,x_d'\,x_q} \text{ approximately}$$

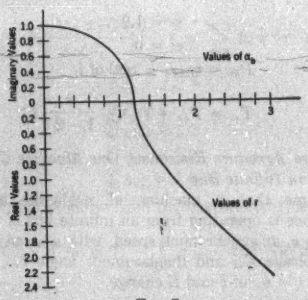

FIG. 7

Thus, in the special case considered this approximate formula gives

$$\alpha_a = \frac{(0.30 + 0.60)\,r}{2 \times 0.30 \times 0.60} = 2.50\,r$$

which checks the result found by the exact solution of the cubic.

I. Starting Torque

On infinite bus and with slip s, there will be, choosing $\delta = \frac{\pi}{2} - s\,t$, and referring to Equation (28),

$$e_d = \cos s\,t$$
$$e_q = \sin s\,t$$

If we now introduce a system of vectors rotating at s per-unit angular velocity there is

$$ed = 1.0$$
$$e_q = -j$$
$$p = j\,s \tag{33}$$

Then from (22) and (23),

$$i_d = \{\, j\,s\,x_q\,(j\,s) + r - j\,(1-s)\,x_q\,(j\,s)\,\} + \{\,[j\,s\,x_d\,(j\,s) + r]\,[j\,s\,x_q\,(j\,s) + r] + (1-s)^2\,x_d\,(j\,s)\,x_q\,(j\,s)\,\}$$

$$= \frac{j\,(1-2\,s)\,x_q\,(j\,s) - r}{r^2 + (1-2\,s)\,x_d'(j\,s)\,x_q'(j\,s) + j\,s\,r[x_d'(j\,s) + x_q'(j\,s)]}$$

$$= \left\{ j\,x_q\,(j\,s) - \frac{r}{1-2\,s} \right\} + \left\{ x_d\,(j\,s)\,x_q\,(j\,s) + \frac{r}{1-2\,s}\,[r + j\,s\,\langle x_d\,(j\,s) + x_q\,(j\,s)\,\rangle\,] \right\} \tag{34}$$

$$i_q = -\frac{[j\,s\,x_d\,(j\,s) + r]\,(-j) - (1-s)\,x_d\,(j\,s)}{r^2 + (1-2\,s)\,x_d\,(j\,s)\,x_q\,(j\,s) + j\,s\,r[x_d\,(j\,s) + x_q\,(js)]}$$

$$= \left\{ x_d\,(j\,s) + \frac{j\,r}{1-2\,s} \right\} + \left\{ x_d\,(j\,s)\,x_q\,(j\,s) + \frac{r}{1-2\,s}\,[r + j\,s\,\langle x_d\,(j\,s) + x_q\,(j\,s)\,\rangle\,] \right\} \tag{35}$$

The expressions for average power and torque then become,

$$P_{av} = 1/2\,[e_d \,.\, i_d + e_q \,.\, i_q]$$
$$T_{av} = 1/2\,[i_q \,.\, \psi_d - i_d \,.\, \psi_q]$$

where the dot indicates the scalar product, or

$$P_{av} = 1/2\,[1 \,.\, i_d - j \,.\, i_q]$$
$$= 1/2\,[\text{Real of } i_d - \text{Imaginary of } i_q] \tag{36}$$

There is in general,

$$e_d + r\,i_d = p\,\psi_d - (1-s)\,\psi_q$$
$$e_q + r\,i_q = (1-s)\,\psi_d + p\,\psi_q$$

$$\psi_d = \frac{\begin{vmatrix} e_d + r\,i_d & -(1-s) \\ e_q + r\,i_q & p \end{vmatrix}}{\begin{vmatrix} p & -(1-s) \\ (1-s) & p \end{vmatrix}}$$

$$= \frac{p\,(e_d + r\,i_d) + (1-s)\,(e_q + r\,i_q)}{p^2 + (1-s)^2} \tag{37}$$

$$\psi_q = \frac{p\,(e_q + r\,i_q) - (1-s)\,(e_d + r\,i_d)}{p^2 + (1-s)^2} \tag{38}$$

$$\psi_d = \frac{j\,s\,(e_d + r\,i_d) + (1-s)\,(e_q + r\,i_q)}{1-2\,s}$$

$$\psi_q = \frac{j\,s\,(e_q + r\,i_q) - (1-s)\,(e_d + r\,i_d)}{1-2\,s}$$

with $e_d = 1.0$, $e_q = -j$

$$\psi_d = \frac{j\,s + j\,s\,r\,i_d + (1-s)\,(-j) + (1-s)\,r\,i_q}{1-2\,s}$$

$$= \frac{-(1-2s)\,j + r\,[j\,s\,i_d + (1-s)\,i_q]}{1-2\,s}$$

$$= -j + \frac{r}{1-2\,s}\,[j\,s\,i_d + (1-s)\,i_q] \qquad (39)$$

$$\psi_q = \frac{j\,s\,(-j + r\,i_q) - (1-s) - r\,(1-s)\,i_d}{1-2\,s}$$

$$= \frac{-(1-2\,s) + r\,[j\,s\,i_q - (1-s)\,i_d]}{1-2\,s}$$

$$= -1 + \frac{r}{1-2\,s}\,[j\,s\,i_q - (1-s)\,i_d] \qquad (40)$$

Thus,

$$T_{av} = 1/2 \begin{bmatrix} i_q\,.\,(-j) + i_q\,.\,\frac{r}{1-2\,s}\,\langle j\,s\,i_d + (1-s)\,i_q\rangle \\ -i_d\,.\,(-1) - i_d\,.\,\frac{r}{1-2\,s}\,\langle j\,s\,i_q - (1-s)\,i_d\rangle \end{bmatrix}$$

$$= P_{av} + \frac{r}{2(1-2\,s)} \begin{bmatrix} (1-s)\,\langle i_q^2 + i_d^2\rangle \\ +2\,s\,i_q\,.\,j\,i_d \end{bmatrix}$$

$$= P_{av} + \frac{r}{2}\,(i_q^2 + i_d^2) + \frac{r\,s}{2(1-2\,s)} \begin{bmatrix} i_q^2 + i_d^2 \\ +2\,i_q\,.\,j\,i_d \end{bmatrix}$$

$$= P_{av} + r\,\frac{i_q^2 + i_d^2}{2} + \frac{r\,s}{2(1-2\,s)}\,(i_q + j\,i_d)^2 \qquad (41)$$

Mr. Ralph Hammar, who has been engaged in the application of the general method of calculation outlined above, to the predetermination of the starting torque of practical synchronous motors, has suggested an interesting modification of formulas (36) and (41), based upon the fact that, since the total m. m. f. consists of direct and quadrature components pulsating at slip frequency, it may be resolved into two components, one moving forward at a per-unit speed $1 - s + s = 1.0$, and the other moving backward at a per-unit speed $1 - s - s = 1 - 2\,s$. Thus from this standpoint half of both the direct and quadrature components will move forward, and half backward. Since the quadrature axis is ahead of the direct it follows that as far as concerns the forward component the quadrature current i_q is equivalent to a d-c. $j\,i_q$, while as regards backward component it is equivalent to a direct component $-j\,i_q$. It follows that the vector amounts of forward and backward m. m. f. or current are

$$\text{forward current} = i_f = \frac{1}{2}\,(i_d + j\,i_q)$$

$$\text{backward current} = i_b = \frac{1}{2}\,(i_d - j\,i_q) \qquad (42)$$

If we define by analogy,

$$\text{forward voltage} = \frac{1}{2}\,(e_d + j\,e_q)$$

$$\text{backward voltage} = \frac{1}{2}\,(e_d - j\,e_q) \qquad (43)$$

There is,

$$i_f = \frac{1}{2}\left\{ \frac{-2\,r}{1-2\,s} + j\,[x_d\,(j\,s) + x_q\,(j\,s)] \right\} + \left\{ x_d\,(j\,s)\,x_q\,(j\,s) + \frac{r}{1-2\,s}\,\langle r + j\,s\,[x_d\,(j\,s) + x_q\,(j\,s)]\rangle \right\}$$

$$i_b = \frac{1}{2}\left\{ j\,[x_q\,(j\,s) - x_d\,(j\,s)] \right\} + \left\{ x_d\,(j\,s)\,x_q\,(j\,s) + \frac{r}{1-2\,s}\,\langle r + j\,s\,[x_d\,(j\,s) + x_q\,(j\,s)]\rangle \right\}$$

$$e_f = 1.0 \qquad (44)$$

$$e_b = 0 \qquad (45)$$

$$P_{av} = e_f\,.\,i_f = \text{real of } i_f \qquad (46)$$

$$T_{av} = P_{av} + r\,i_f^2 + \frac{r}{1-2\,s}\,i_b^2 \qquad (47)$$

J. Zero Armature Resistance, One Machine Connected to an Infinite Bus

Assume that a machine of negligible armature resistance is operating from an infinite bus of per-unit voltage e, at synchronous speed, with a steady excitation voltage E_0, and displacement angle δ_0. At the instant $t = 0$, let δ and E change.

There is,

$$i_d = \frac{E_0 - \psi_{d0}}{x_d} - \frac{1}{x_d\,(p)}\,\Delta\,\psi_d + \frac{G\,(p)}{x_d\,(p)}\,\Delta\,E$$

$$i_q = -\frac{\psi_{q0}}{x_q} - \frac{1}{x_q\,(p)}\,\Delta\,\psi_q$$

$$\psi_d = e\cos\delta$$

$$\psi_q = -\,e\sin\delta$$

From which there is, by obvious re-arrangement,

$$i_d = \frac{E - e\cos\delta}{x_d} + e\,\frac{x_d - x_d(p)}{x_d\,x_d(p)}\,(\cos\delta_0 - \cos\delta)$$

$$- \frac{x_d(p) - G(p)\,x_d}{x_d\,x_d(p)}\,\Delta E$$

$$i_q = \frac{e\sin\delta}{x_q} + e\,\frac{x_q - x_q(p)}{x_q\,x_q(p)}\,(\sin\delta - \sin\delta_0) \quad (48)$$

Then,

$$T = \frac{E\,e\sin\delta}{x_d} + \frac{x_d - x_q}{2\,x_d\,x_q}\,e^2\sin 2\,\delta$$

$$+ e^2\cos\delta\,\frac{x_q - x_q(p)}{x_q\,x_q(p)}\,(\sin\delta - \sin\delta_0) \quad (49)$$

$$+ e^2\sin\delta\,\frac{x_d - x_d(p)}{x_d\,x_d(p)}\,(\cos\delta_0 - \cos\delta)$$

$$- e\sin\delta\,\frac{x_d(p) - x_d\,G(p)}{x_d\,x_d(p)}\,.\,\Delta E$$

But quantities a_{dn}, a_{qn}, α_{dn}, α_{qn}, b_n, β_n may be found such that

$$\frac{x_q - x_q(p)}{x_q(p)}\,.\,1 = \frac{x_q - x_q''}{x_q''}\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}$$

$$\frac{x_d - x_d(p)}{x_d(p)}\,.\,1 = \frac{x_d - x_d''}{x_d''}\sum a_{qn}\,\epsilon^{-\alpha_{qn}t} \quad (50)$$

$$\frac{x_d(p) - x_d\,G(p)}{x_d(p)}\,.\,1 = \sum b_n\,\epsilon^{-\beta_n t}$$

$$x_q'' = x_q(\infty)$$
$$x_d'' = x_d(\infty)$$
$$\Sigma\,a_{dn} = 1.0$$
$$\Sigma\,a_{qn} = 1.0$$
$$\Sigma\,b_n = 1.0$$

It therefore follows from the operational rule that,

$$f(p)\,F(t) = F(o)\,\phi(t) + \int_0^t \phi(t-u)\,F'(u)\,d\,u \quad (51)$$

where,

$$\phi(t) = f(p)\,.\,1$$

that if

$$\delta = \delta(t)$$
$$p\,\delta = \delta'(t)$$
$$\Delta E = \Delta E(t)$$
$$p\,\Delta E = \Delta E'(t)$$

Equations (48) and (49) may be rewritten in the form,

$$i_d = \frac{E - e\cos\delta}{x_d}$$

$$+ e\,\frac{x_d - x_d''}{x_d\,x_d''}\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}\int_0^t \epsilon^{\alpha_{dn}u}\sin\delta(u)\,\delta'(u)\,d\,u$$

$$- \frac{1}{x_d}\sum b_n\,\epsilon^{-\beta_n t}\int_0^t \epsilon^{\beta_n u}\,\Delta E'(u)\,d\,u \quad (48a)$$

$$i_q = \frac{e\sin\delta}{x_q}$$

$$+ e\,\frac{x_q - x_q''}{x_q\,x_q''}\sum a_{qn}\,\epsilon^{-\alpha_{qn}t}\int_0^t \epsilon^{\alpha_{qn}u}\cos\delta(u)\,\delta'(u)\,d\,u$$

$$T = \frac{E\,e\sin\delta}{x_d} + \frac{e^2(x_d - x_q)}{2\,x_d\,x_q}\sin 2\,\delta$$

$$+ e^2\frac{x_d - x_d''}{x_d\,x_d''}\sin\delta\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}\int_0^t \epsilon^{\alpha_{dn}u}\sin\delta(u)\,\delta'(u)\,d\,u$$

$$+ e^2\frac{x_q - x_q''}{x_q\,x_q''}\cos\delta\sum a_{qn}\,\epsilon^{-\alpha_{qn}t}\int_0^t \epsilon^{\alpha_{qn}u}\cos\delta(u)\,\delta'(u)\,d\,u$$

$$- \frac{e\sin\delta}{x_d}\sum b_n\,\epsilon^{-\beta_n t}\int_0^t \epsilon^{\beta_n u}\,\Delta E'(u)\,d\,u \quad (49a)$$

Formula (49a) may be used to determine starting torque and current with zero armature resistance, by introducing $\delta(t) = s\,t$, $\delta'(t) = s$. Thus the average component of torque is found to be,

$$T_{av} = \frac{1}{2}\,\frac{x_d - x_d''}{x_d\,x_d''}\sum a_{dn}\,\frac{\alpha_{dn}\,s}{\alpha_{dn}^2 + s^2}$$

$$+ \frac{1}{2}\,\frac{x_q - x_q''}{x_q\,x_q''}\sum a_{qn}\,\frac{\alpha_{qn}\,s}{\alpha_{qn}^2 + s^2} \quad (52)$$

Since

$$\frac{\alpha\,s}{\alpha^2 + s^2} \text{ is never greater than } \tfrac{1}{2}, \text{ and}$$

$$\Sigma\,a_{dn} = \Sigma\,a_{qn} = 1.0$$

it follows that T_{av} is never greater than

$$\frac{1}{4}\left\{\frac{x_d - x_d''}{x_d\,x_d''} + \frac{x_q - x_q''}{x_q\,x_q''}\right\} \quad (53)$$

Equation (53) thus provides a very simple criterion of the maximum possible starting torque of a synchronous motor of given dimensions, when armature resistance is neglected.

The same formula may also be used to obtain a simple expression for the damping and synchronizing components of pulsating torque due to a given small angular pulsation of the rotor.

Thus if the angular pulsation is

$$\Delta\,\delta = [\Delta\,\delta]\sin(s\,t)$$

and if the pulsation of torque is expressed in the form

$$\Delta T = T_s \Delta \delta + T_d \frac{d}{d t} \Delta \delta$$

there results,

$$T_s = T_{s0} + e^2 \sin^2 \delta_0 \frac{x_d - x_d''}{x_d x_d''} \sum \frac{a_{dn} s^2}{(\alpha_{dn})^2 + s^2}$$

$$+ e^2 \cos^2 \delta_0 \frac{x_q - x_q''}{x_q x_q''} \sum \frac{a_{qn} s^2}{(\alpha_{qn})^2 + s^2} \qquad (54)$$

$$s T_d = e^2 \sin^2 \delta_0 \frac{x_d - x_d''}{x_d x_d''} \sum \frac{a_{dn} s \alpha_{dn}}{(\alpha_{dn})^2 + s^2}$$

$$+ e^2 \cos^2 \delta_0 \frac{x_q - x_q''}{x_q x_q''} \sum \frac{a_{qn} s \alpha_{qn}}{(\alpha_{qn})^2 + s^2}$$

where,

$$T_{s0} = \frac{e I_{d0} \cos \delta_0}{x_d} + \frac{e^2 (x_d - x_q)}{x_d x_q} \cos 2 \delta_0$$

δ_0 = average angular displacement, *i. e.*, total angle $= \delta = \delta_0 + \Delta \delta$.

It can be shown that for the case of no additional rotor circuits, Equations (**54**) are exactly equivalent to Equations (**24**) and (**25**) in Doherty and Nickle's paper, *Synchronous Machines III*. The new formulas herein developed are, however, very much simpler in form, especially since in the case which Doherty and Nickle have treated, there is only one term in the summation; that is, $n = 1$, and α is merely the reciprocal of the short circuit time constant of the machine, expressed in radians.

K. The Equivalent Circuit of Synchronous Machines Operating in Parallel at No Load, Neglecting the Effect of Armature Resistance

Let, δ_a = angle of rotor a and bus
θ_a = angle of rotor a in space

In general, the shaft torque of a machine depends on its acceleration and speed in space, and the magnitude and rate of change of the bus voltage as a vector. If all of the machines are operating at no load and if there is no armature resistance, a small displacement of any one machine will change the magnitude of the bus voltage only by a second order quantity; consequently for small displacements the magnitude of the bus voltage may be regarded as fixed, and only the angle of the bus and rotor need be considered. Furthermore, the electrical torque may be found in terms of (δ) by employing an infinite bus formula. But Equation (**49a**) implies the alternative general operational form,

$$T = \frac{e I_{d0} \sin \delta}{x_d} + \frac{e^2 (x_d - x_q) \sin 2 \delta}{2 x_d x_q}$$

$$- \frac{x_d - x_d''}{x_d x_d'} e^2 \sin \delta \sum \frac{a_{nd} p}{p + \alpha_{nd}} . \cos \delta \quad (49b)$$

$$+ \frac{x_q - x_q''}{x_q x_q''} e^2 \cos \delta \sum \frac{a_{nq} p}{p + \alpha_{nq}} . \sin \delta$$

Therefore in the case under consideration there is for machine a,

$$T_a = \left[\frac{e I_a}{x_{da}} + e^2 \frac{(x_{da} - x_{qa})}{x_{da} x_{qa}} \right] \delta_a$$

$$+ \frac{x_{qa} - x_{qa}''}{x_{qa} x_{qa}''} e^2 \sum a_{nqa} \frac{p}{p + \alpha_{nqa}} . \delta_a \qquad (55)$$

where: e = per-unit bus voltage
I_a = per-unit excitation of machine a, etc.

This equation can be represented by Fig. 8, in which the charge through the circuit represents (δ_a) and the

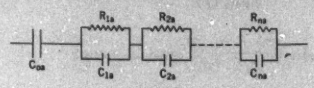

FIG. 8

voltage across the circuit represents the electrical torque of the machine (T_a).

The capacitances and resistances must be chosen so that

$$C_{0a} = \frac{x_{da} x_{qa}}{e I_a x_{qa} + e^2 (x_{da} - x_{qa})} \qquad (56)$$

$$C_{na} = \frac{x_{qa} x_{qa}''}{e^2 a_{nqa} (x_{qa} - x_{qa}'')}$$

$$R_{na} = \frac{1}{C_{na} \alpha_{nqa}}$$

The equation for the mechanical torque is

$$T_{sa} = T_a + M_a p s_a \qquad (57)$$

where:

M_a = inertia factor of machine a in radians

$$= \frac{2 \times \text{stored mech. energy at normal speed}}{\text{base power}}$$

$$= 2 \pi f \frac{0.462 \ W R^2 \left(\frac{\text{rev. per min.}}{1000} \right)^2}{\text{base kw.}}$$

s_a = per-unit speed of machine a

t = time in seconds $\left(p = \frac{d}{d t} \right)$

But, $s_a = p \theta_a$

Thus there is

$$T_{sa} = T_a + M_a p^2 \theta_a \qquad (57a)$$

which corresponds to the equivalent circuit of Fig. 9, in which change $= \theta_a$

$$L_a = M_a$$

The machine operating on an infinite bus can be

represented by the equivalent circuit of Fig. 10, since the condition

$$\theta_a = \delta_a = 0$$

is fulfilled.

Several machines in parallel on the same bus may be

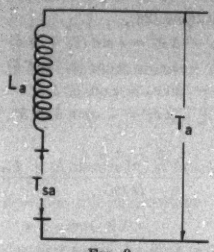

FIG. 9

represented by the diagram of Fig. 11, since the conditions

$\theta_a - \delta_a = \theta_b - \delta_b = \ldots$ (= bus angle in space)

$T_a + T_b + T_c$, etc. = bus power output = 0

A transmission line may be represented by a condenser.

Thus two machines connected by a line of reactance (x) would be represented by the circuit of Fig. 12, where

$$C = \frac{x}{e^2} \tag{58}$$

Shaft torques are, of course, represented by voltages.

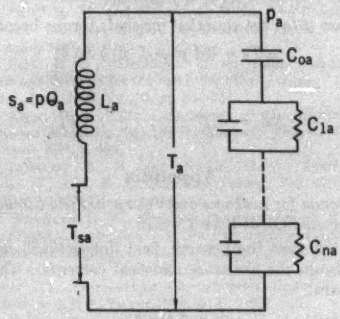

FIG. 10

Mechanical damping, such as that due to a fan on a motor shaft or that due to the prime mover, is represented by resistance in series with the inductance (L) as in Fig. 13. (R) must be chosen equal to the rate of decrease in available driving torque with increase in speed.

Governors and other prime mover characteristics may also be represented by connecting their circuits in the inductive branch of the circuit. Thus a governor which acts through a single time constant may be represented by the circuit of Fig. 14, where

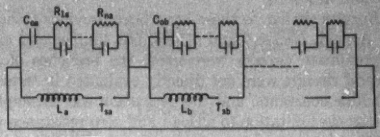

FIG. 11

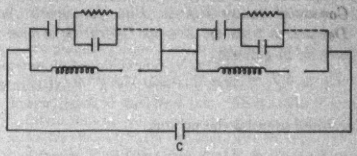

FIG. 12

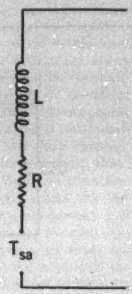

FIG. 13

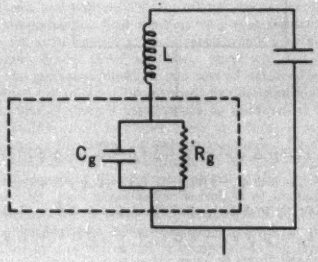

FIG. 14

$$R_g = \frac{1}{\text{regulation}}$$

$$C_g = \frac{\text{time constant of governor in elec. radians}}{R_g} \tag{59}$$

An induction motor is represented by the simple circuit of Fig. 15 and is precisely the circuit of a synchronous machine with only one time constant and $C_0 = \infty$ on account of $I = 0$.

Results similar to these have been previously shown by Arnold, Nickle,[10] and others, but simpler and more approximate circuits were used, the branches of the several circuits were not directly evaluated in terms of machine constants, and the derivation was incomplete in that the limitation to no load and zero resistance was not appreciated.

L. Torque Angle Relations of a Synchronous Machine Connected to an Infinite Bus, for Small Angular Deviations from an Average Operating Angle

There is, in general,

$$T = T_0 + \Delta T = (\psi_{d0} + \Delta \psi_d)(i_{q0} + \Delta i_q) - (i_{d0} + \Delta i_d)(\psi_{q0} + \Delta \psi_q)$$

For small angular deviations,

$$\Delta T = i_{q0} \Delta \psi_d + \psi_{d0} \Delta i_q - i_{d0} \Delta \psi_q - \psi_{q0} \Delta i_d$$
$$= \{\psi_{d0} + i_{d0} x_q(p)\} \Delta i_q - \{\psi_{q0} + i_{q0} x_d(p)\} \Delta i_d \quad (60)$$

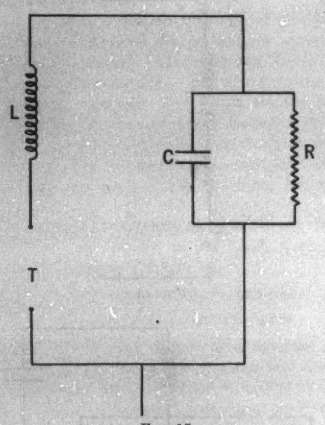

FIG. 15

$$e_{d0} + \Delta e_d = p \Delta \psi_d - r(i_{d0} + \Delta i_d) - (\psi_{q0} + \Delta \psi_q)(1 + p \Delta \delta)$$
$$e_{q0} + \Delta e_q = p \Delta \psi_q - r(i_{q0} + \Delta i_q) + (\psi_{d0} + \Delta \psi_d)(1 + p \Delta \delta)$$
$$\Delta e_d = p \Delta \psi_d - r \Delta i_d - \psi_{q0} p \Delta \delta - \Delta \psi_q$$
$$\Delta e_q = p \Delta \psi_q - r \Delta i_q + \psi_{d0} p \Delta \delta + \Delta \psi_d$$

from which there is

$$z_d(p) \Delta i_d - x_q(p) \Delta i_q = -\Delta e_d - \psi_{q0} p \Delta \delta$$
$$z_q(p) \Delta i_q + x_d(p) \Delta i_d = -\Delta e_q + \psi_{d0} p \Delta \delta$$

$$\Delta i_d = \frac{z_q(p)(-\Delta e_d - \psi_{q0} p \Delta \delta) + x_q(p)(-\Delta e_q + \psi_{d0} p \Delta \delta)}{D(p)} \quad (61)$$

$$\Delta i_q = \frac{z_d(p)(-\Delta e_q + \psi_{d0} p \Delta \delta) - x_d(p)(-\Delta e_d - \psi_{q0} p \Delta \delta)}{D(p)}$$

where,

$$D(p) = z_d(p) z_q(p) + x_d(p) x_q(p)$$

but from Equations (28),

$$e_{d0} + \Delta e_d = e \sin(\delta_0 + \Delta \delta)$$
$$e_{q0} + \Delta e_q = e \cos(\delta_0 + \Delta \delta)$$
$$\Delta e_d = e \cos \delta_0 \Delta \delta$$
$$\Delta e_q = -e \sin \delta_0 \Delta \delta \quad (62)$$

$$\Delta i_d = \frac{-(e \cos \delta_0 + \psi_{q0} p) z_q(p) + (e \sin \delta_0 + \psi_{d0} p) x_q(p)}{D(p)} \cdot \Delta \delta$$

$$\Delta i_q = \frac{(e \sin \delta_0 + \psi_{d0} p) z_d(p) + (e \cos \delta_0 + \psi_{q0} p) x_d(p)}{D(p)} \cdot \Delta \delta \quad (63)$$

$$\Delta T = \frac{[\psi_{d0} + i_{d0} x_q(p)] \left\{ \begin{array}{l} (e \sin \delta_0 + \psi_{d0} p) z_d(p) \\ +(e \cos \delta_0 + \psi_{q0} p) x_d(p) \end{array} \right\} + [\psi_{q0} + i_{q0} x_d(p)] \left\{ \begin{array}{l} (e \cos \delta_0 + \psi_{q0} p) z_q(p) \\ -(e \sin \delta_0 + \psi_{d0} p) x_q(p) \end{array} \right\}}{D(p)} \cdot \Delta \delta \quad (64)$$

say,

$$\Delta T = f(p) \cdot \Delta \delta$$

From (57a) the equation for shaft torque becomes

$$\Delta T_s = (M p^2 + f(p)) \cdot \Delta \delta$$

Thus,

$$\Delta \delta = \frac{1}{M p^2 + f(p)} \cdot \Delta T_s \quad (65)$$

Appendix

Formula for Linkages and Voltage in Field Circuit with no Additional Rotor Circuits

In this case the per-unit field linkages will depend linearly on the armature and field currents. That is, in general,

$$\Psi = a I - b i_d$$

Then if normal linkages are defined as those existing at no load there must be $a = 1.0$.

The quantity b may be found by suddenly impressing terminal linkages ψ_d with no initial currents in the machines and $E = 0$.

By definition there is, initially

$$i_d = -\frac{\psi_d}{x_d'}$$

but also there must be from the definition of x_d'

$$i_d = \frac{I - \psi_d}{x_d}$$

hence there must be an initial induced field current of amount

$$I = \psi_d \left(1 - \frac{x_d}{x_d'} \right)$$

But, initially the field linkages are zero, thus

$$\Psi = \psi_d 1 - \left[\frac{x_d}{x_d'} + \frac{b}{x_d'} \right] = 0$$

hence $$b = x_d - x_d'$$

Similarly, there will be

$$E' = \text{per-unit field voltage}$$
$$= c\,p\,\Psi + d\,I$$

Normal field voltage will be here defined as those existing at no load and normal voltage. This requires that $d = 1$. The quantity c may then be recognized as the time constant of the field in radians when the armature is open circuited, since with the field shorted under these conditions there is

$$(T_0\,p + 1)\,I = 0$$
$$c\,p\,\Psi + I = 0$$
$$\psi = I$$

$c = T_0$ = time constant of field with armature open circuited.

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Discussion

H. C. Specht: I should think Mr. Park's theory could be applied just as well to the so-called synchronous induction motor, that is an induction motor in which the rotor teeth between the poles are cut out for a distance of about one-third of the pole pitch. Such a motor runs at synchronous speed. However, the pull-out torque is much less than that of an induction motor with the full number of teeth.

C. MacMillan: There was one statement in the first page of Mr. Park's paper to the effect that "Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy currents in the armature iron are neglected . . .". And with regard to Fig. 5, Mr. Park remarked that it represented a rigorous solution. Perhaps Mr. Park could give us a little more insight into the effect of taking into account saturation, and give other cases in which certain elements have been neglected with more or less effect upon the final results.

W. J. Lyon: In a paper of this description, certain premises should be chosen and, with these always in mind, the mathematical development should be rigorous. The paper may then be criticized either because of insufficient premises or because of incorrect mathematical development. I believe that the former is the kinder method; it is the one I shall employ.

The premises that Mr. Park chooses are that the field and armature windings are symmetrical, that saturation and hysteresis are neglected and that the armature windings are in effect sinusoidally distributed. I take this last to mean that the air-gap flux due to the armature currents is sinusoidally distributed, for if the armature windings themselves were sinusoidally distributed, there would be produced space harmonics in the air-gap flux distribution due to the saliency of the poles, which, as we all know, would complicate the problem tremendously. In order that the mathematical method used by Mr. Park shall be rigorous, I believe it is necessary to make one further assumption. I think I can best explain this by asking you to consider the result of supplying the field winding with a sinusoidal current while the armature rotates at some speed which may be called synchronous. Under these conditions, there will first be produced in the armature windings two sets of balanced currents each of which will produce 3 component flux distributions in the gap. The first of these is what would be produced if the air-gap were uniform, and is proportional to $1/2\,(x_d + x_q - x_a)$, where x_a equals the armature leakage reactance. The second of these components is proportional to $1/2\,(x_d - x_q)$. The third component is of the same size as the second. Using the values that Mr. Park gives under Section H of his paper, the relative magnitudes of these components would be $(0.8 - x_a)$ and 0.2. The first and second components react on the field, and produce in it a current of the impressed field frequency. These are the components that Mr. Park has recognized, but the third component produces an entirely different frequency in the field, which will then be reflected into the armature and the process will be repeated. That is, in this respect, it is similar to the condition that exists in a single-phase alternator. As far as I am aware, the Heaviside operational method cannot be used to obtain a rigorous solution for the single-phase alternator. In spite of this, I think the objection that I have raised is of no more importance than the effect of neglecting saturation or

hysteresis or the space harmonics in the air-gap flux that are actually produced, but I do think that Mr. Park should have mentioned this point in order that the mathematical work which he builds up should be rigorous. I discovered the necessity of this additional premise about a year ago while working on the problem of the transient short-circuit currents in a salient-pole alternator.

There is another problem of considerable interest that Mr. Park has not mentioned. About two years ago I became aware that Mr. Fortescue's historic 1918 paper could be extended to the transient case by using an operational method. That is to say, any of the problems involving unbalanced circuits that Mr. Fortescue rigorously solved in the steady state can also be solved in the transient state by an operational method.

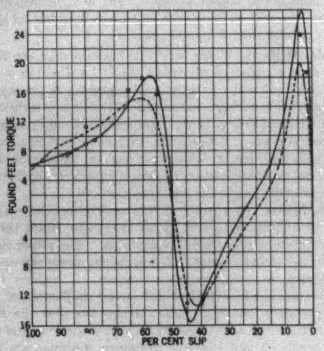

FIG. 1—ELECTRICAL TORQUE OF A H I

Solid Line—Calculated
Dotted Line—Experimental

Note: The six points plotted above were taken experimentally at a temperature of 25 deg. cent.

The balance of the dotted curve was taken at varying higher temperatures

The solid curve was calculated for a temperature of 25 deg. cent., and the points in squares for 100 deg. cent.

W. B. Heinz: (communicated after adjournment) Since the completion of Mr. Park's paper as presented, experimental work has been done which illustrates the worth of the new theory developed.

A small synchronous motor rated 15 hp., 220 volts, 1800 rev. per min. was selected for study. Its squirrel-cage winding was removed in order to simplify the mechanical work of calculation.

This motor was set up with an electric dynamometer, and its torque-slip curve was taken at 110 volts over the entire range of slip from 100 per cent to zero. Power was supplied to it by a special 220-volt sine-wave generator of 375-kv-a. capacity. The constants of this machine as well as of the connecting line and switches were determined, and these constants were included in the calculation of torque in order to simulate as closely as possible infinite-bus conditions.

The torque as indicated by the dynamometer was corrected for windage and friction and for dynamometer errors, the net electrical torque exerted on the rotor thus being obtained.

By the use of measured constants of the test machine and its circuit, a mathematical evaluation of torque was made for a temperature of 25 deg. cent. These constants as determined, on a per unit base of 24.75 kv-a. at 220 volts, are as follows:

$x_d = 1.0$
$x_d' = 0.178$
$x_q = 0.442$
$x_q' = 0.442$
$r = 0.0325$
$T_0 = 159$ radians
Normal torque $= 96.8$ lb-ft.

The calculated curve appears as a solid line in Fig. 1 herewith and near it is the dotted curve obtained experimentally.

During the progress of the test run with the dynamometer no control or regular measurement of the synchronous motor temperature was made. It fluctuated widely with changes in slip, and the only points taken at a known temperature are the ones near the beginning of the run. For the first six, those surrounded by circles, the temperature probably had not risen far from ambient of 25 deg. cent. They check quite closely the calculated curve.

In order to evaluate the effect of temperature change eight points were calculated on the basis of 100 deg. cent. These are shown within squares. They show a better coincidence of slips at which maximums occur, but little improvement in the maximum error. Previous theories have usually neglected the effect of salient poles. The solid line in Fig. 2 shows the calculated curve in this case, that is, for $x_q = x_d$. A comparison of the solid curves in Figs. 1 and 2 shows the large differences between the two calculations.

I. H. Summers: (communicated after adjournment) One outstanding feature of this paper is that the equations are expressed in "per-unit" quantities. This feature presents the important advantage that the quantities involved are those which are known directly from test or from the design of the machine. Furthermore, machines are more readily compared with others of different rating when their constants are known in "per-unit" values.

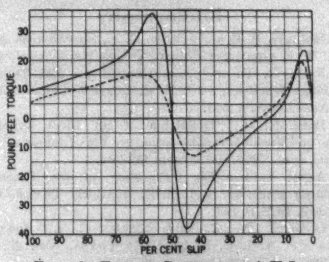

FIG. 2—TORQUE CURVES FOR A H I

Solid Line—Calculated
Dotted Line—Experimental

However, these equations may not look familiar at first sight to those who are more used to dealing in the physical quantities such as ohms, amperes, volts, henrys, and farads. It therefore seems appropriate to point out the correlation between this new method and the more usual one. Consider, for example, Equations (4a) in the paper, together with the two immediately following.

Ordinarily we should have written for a machine with no rotor circuits in addition to the field,

$$(R_f + L_f\, p)\, I_f - M\, p\, [j_a \cos\theta + j_b \cos(\theta - 120^\circ) + j_c \cos(\theta + 120^\circ)] = E_f \qquad (1)$$

In this equation R_f is field resistance, L_f is field inductance, M is mutual inductance between field and any one armature phase in the position of maximum mutual inductance, j_a, j_b, and j_c

are armature phase currents, and E_f is field voltage. All these quantities are in practical units.

Let

$$j_d = \frac{2}{3}\left[j_a \cos\theta + j_b \cos(\theta - 120°) + j_c \cos(\theta + 120°)\right] \quad (2)$$

Divide through by $I_n R_f$ where I_n is that value of field current that causes normal voltage to be generated in the armature at no load.

$$\frac{E_f}{I_n R_f} = (1 + T_0 p)\frac{I_f}{I_n} - \frac{3}{2}\frac{L_f}{R_f}\frac{M_p}{I_n L_f} j_d \quad (3)$$

where

$$T_0 = L_f/R_f \quad (4)$$

Let

$$\left.\begin{aligned} E_f/I_n R_f &= E \\ I_f/I_n &= I \\ j_d/i_n &= i_d \end{aligned}\right\} \quad (5)$$

Where i_n is normal armature current, maximum value. Then

$$E = T_0 p\left(I - \frac{3}{2}\frac{M}{L_f}\frac{i_n}{I_n} i_d\right) + I \quad (6)$$

This equation may be put into the form:

$$\left.\begin{aligned} E &= T_0 p \Psi + I \\ \Psi &= I - \left(\frac{3}{2}\frac{M_{in}}{L_f I_n}\right) i_d \end{aligned}\right\} \quad (7)$$

These results may be compared with those immediately following (4a) in the paper. It is easy to show by methods similar to those used by Mr. Park that

$$\frac{3}{2}\frac{M_{in}}{L_f I_n} = x_d - x_d' \quad (8)$$

where x_d and x_d' are the per-unit direct axis synchronous and transient reactance respectively. Then the equations are the same as those given by the author.

Without this last relation we should doubtless have written the equations as in (7) or even as in (1). But both of these contain cumbersome quantities involving mutual and self inductances. The expression used by Mr. Park is simpler and more expressive and contains quantities which are familiar to most engineers who have to do with synchronous machines.

Throughout this paper, although the problem is inherently complex and some of the demonstrations appear long, the results are relatively simple and in terms adapted for immediate use.

R. H. Park: Mr. Specht is entirely correct in his thought that the present theory could be applied to synchronous induction motors such as those to which he refers. In principle this type of motor is not greatly different from a salient-pole synchronous motor with an amortisseur winding.

In reply to Mr. MacMillan's question, I would point out that it is always necessary to make some assumptions; in fact, even in the simplest problems there invariably exists an enormous number of assumptions most of which are not recognized as such. Therefore a rigorous solution invariably means only "rigorous on the basis of the assumptions." The solutions presented in the paper are rigorously correct in this sense.

Professor Lyon has criticized the assumptions made in the paper. This is due to a very definite misconception as to their character. Thus, referring to the assumption "that the armature windings are in effect sinusoidally distributed," Professor Lyon states that it is necessary to interpret the statement to mean something other than it says. I believe, however, that this is not necessary. Thus he states that were the armature windings themselves sinusoidally distributed, there would be produced space harmonics in the air-gap flux distribution due to the saliency of the poles, which "would complicate the problem tremendously." The reason that it does not complicate the problem is that with a sinusoidal distribution of turns only space fundamental flux produces any armature flux linkages.

This follows from the fact that

$$\int_0^{2\pi} \cos n\theta \cos m\theta \, d\theta = \int_0^{2\pi} \cos n\theta \sin m\theta \, d\theta = 0$$

if n and m are integers and $n = m$.

Although this answers Professor Lyon's objection the impression should not be derived that the theory developed strictly presupposes an exactly sinusoidally distributed winding. The full statement of the assumption referred to is that the armature windings are in effect sinusoidally distributed "as far as concerns effects depending on rotor position." Interpreted in terms of self and mutual inductive coefficients this statement is exactly equivalent to the two assumptions,

(a) that the self inductance of the armature circuits is expressible by an equation of the form,

$$L = L_0 + L_2 \cos 2\theta$$

(b) that the mutual inductance between the armature and any rotor circuit is expressible by an equation of the form,

$$M = M_1 \cos\theta$$

The assumptions which have been made in previous studies, such as those which Mr. Ku in his paper "Transient Analysis of A-C. Machinery" has referred to as "exact," have been precisely the same except that the second harmonic term in the armature self inductance equation has not been considered, while amortisseur and other circuits in addition to the field have been neglected.

Actually the expressions for both armature and rotor self inductance will involve all of the even harmonics of angular position, and the expression for armature mutual inductance will involve all of the odd harmonics of angular position. However, tests on actual machines have shown that in most cases only the zero and second harmonics of armature self inductance, the first harmonic of mutual inductance, and the zero harmonic of rotor circuit self inductance are of primary importance. About the only case in which a consideration of any other harmonics would be of value appears to be in the study of locking torques. With proper design, however, the tendency to lock at other than half speed is slight.

Professor Lyon is, in my opinion, also in error when he endeavors to point out the inadequacy of the theory by means of an example in which alternating current is supplied to a rotating field in a salient-pole machine. In this connection he merely makes certain dogmatic statements without proof, which would not agree with the writer's findings. However, such mere statements or opinions do not constitute a proof, and the writer continues to disagree with them.

As I understand the example given, it corresponds to the application of alternating current to a synchronous machine field winding with a three-phase short circuit across the armature terminals.

From Equations (11) and (22) of the paper there is in this case

$$I_d = \left[1 + \frac{\langle x_d - x_d(p)\rangle \langle p x_q(p) + (1 - s^2) x_q(p)\rangle}{D(p)}\right] G(p) E$$

= total per unit rotor excitation.

Evidently this is of the same frequency as that of the field voltage E. That is, there are no harmonics. Mr. C. A. Nickle has submitted a physical interpretation of this result. Thus he states

"Let $I \cos t$ be impressed on the field which is running at a speed S expressed as a fraction of synchronous speed for the frequency of the current impressed on the field winding. The current $I \cos t$ produces a space fundamental flux $I \cos t$ in the direct axis and space harmonics alternating in time. But since the armature winding is assumed to have sine distribution, these harmonic fluxes can induce no voltage and may be neglected. The fundamental flux, $I \cos t$, may be resolved into two rotating components $I/2$ rotating forward at normal speed with reference

to the pole structure and $I/2$ rotating backward at the same speed. The first component rotates forward with respect to the armature at a speed $1 + S$ and since the armature is short circuited polyphase, a positivo phase sequence current of frequency $1 + S$ will flow in the armature winding. Similarly a negative phase sequence current of frequency $1 - S$ will flow in the armature winding due to the backward rotating component of field flux. The polyphase current i_{1+S} will produce a rotating m. m. f. also i_{1+S} and this m. m. f. rotates forward relative to the pole structure at normal speed. Two fluxes are produced due to dissymmetry of the direct and quadrature axes. These are

$$\frac{i_{1+S}}{2}(x_d' + x_q')$$

rotating forward with respect to the poles at normal speed or $1 + S$ with respect to the armature, and

$$\frac{i_{1+S}}{2}(x_d' - x_q')$$

rotating backward with respect to the poles at normal speed or at $1 - S$ backward with respect to the armature. The first will induce an armature voltage of frequency $1 + S$, and the second a voltage of frequency $1 - S$.

The armature current of frequency i_{1-S} produces an m. m. f. i_{1-S} rotating backward with respect to the poles at normal speed and produces two rotating fluxes. One,

$$\frac{i_{1-S}}{2}(x_d' + x_q')$$

rotates backward with respect to the armature at a speed $1 - S$ and

$$\frac{i_{1-S}}{2}(x_d' - x_q')$$

rotates forward with respect to the armature at a speed $1 + S$. Thus, this current also produces armature voltages of frequencies $1 + S$ and $1 - S$. Therefore, no frequencies other than $(1 + S)$ and $(1 - S)$ need be assumed, in the armature winding and therefore, these are the only frequencies that exist there. Moreover, this implies that only fundamental frequency can exist in the rotor."

Magnetic Analysis†

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Non-member

Synopsis.—This paper presents a brief outline of the development and application of magnetic analysis for the nondestructive testing of iron and steel and their products. The subject is treated under the following headings: Historical development, Experimental results, Practical commercial applications, and Conclusion.

* * * * *

Introduction

THE use of electrical methods for the measurement of non-electrical quantities, such as temperature, displacement, time, etc., is quite common. In the majority of such applications, these methods provide the most convenient and accurate means for making the desired determinations. The time is approaching, if indeed it has not already arrived, when magnetic analysis should be included in the list. The term "magnetic analysis" has been adopted to distinguish magnetic tests made with a view to their interpretation in terms of structure, mechanical characteristics, or soundness of ferrous materials from those made solely for the purpose of determining magnetic properties as such. The term connotes, therefore, not only the process of testing but also the analysis of the results from the point of view of mechanical characteristics. Although the development of such magnetic testing methods has not by any means reached a satisfactory state of perfection, their practicability has been demonstrated sufficiently well at least in a few cases to warrant the expectation that they will ultimately have a much wider application and thus meet the need which is becoming more and more urgent for a reliable nondestructive testing method.

That there is a close connection between the magnetic properties of ferrous materials and their other physical properties has long been recognized, and many investigations have been made for the purpose of discovering the nature of the connection. It is only in recent years, however, that investigations along this line have been undertaken with the definite object in view of developing practical magnetic testing methods for routine inspections and for the study of the phenomena associated with the thermal and mechanical treatment of steel.

It is the purpose of the present paper to trace briefly the development of magnetic analysis, to indicate its present status, and to suggest lines along which further investigation might profitably be directed.

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Historical Development

Pioneers in the development of magnetic analysis were Burrows and Fahy, who, in the latter part of 1911, undertook at the Bureau of Standards under the joint sponsorship of the Bureau and the Pennsylvania Railroad, an investigation on spring steel. This investigation was continued over a period of about five years, during which time its scope was somewhat extended to cover preliminary experiments on the applicability of magnetic analysis to various forms of steel products. Three papers[1, 2, 3] were published giving the results of this investigation. Burrows also published a paper[5] in which the experimental evidence on the relationship between magnetic and mechanical properties recorded prior to 1916 was summarized. From the evidence presented he concluded that there was every reason to believe that the two sets of properties are intimately connected and that practical research and inspection methods could be developed. During this same period other papers[2, 3, 4] were published having a definite bearing on the problem.

In 1918 the American Society for Testing Materials authorized the formation of a technical committee to be known as Committee A-8 on Magnetic Analysis with Dr. Burrows as its Chairman. This committee was composed initially of investigators already working in the field, but from time to time its membership has been strengthened by the addition of members who have subsequently become actively interested in the development of magnetic analysis. The committee has functioned primarily as a clearing house for the interchange of ideas and experience among its members who have carried on individual investigations on various phases of the subject; but it has itself also carried out a number of investigations of a general and fundamental nature.[28, 32, 40, 51] A considerable proportion of the literature available on the subject of magnetic analysis has been contributed by members of this committee, either independently or through the auspices of the committee.

A second major source of technical literature is papers published by the Bureau of Standards. In general, the work of the Bureau has been directed along the more fundamental lines although some work on practical applications has been carried out.

Worthy of special mention is the work of de-Forest.[22, 24, 25, 31, 43] This ingenious experimenter has

confined his attention in general to the development of methods employing alternating currents, and has brought out a number of applications of considerable interest and wide applicability.

Most of the work definitely belonging in the field of magnetic analysis has been done in the United States. There has, however, been some interest in other countries. In this connection the work of Honda[7,58] in Japan, and of Fraichet[29, 30] in France, should be especially mentioned. Honda has employed magnetic methods in the study of structural transformations occurring during heating and cooling, and, with his associates, has published a great many papers on the subject. He is probably the first one to use the term "magnetic analysis" but he employed it, in a limited sense, to denote only the single phase in which his work is preeminent. The work of Fraichet has consisted mainly of the study of relations between magnetic and elastic properties with special reference to the determination of the elastic limit by magnetic means.

Experimental Results

In discussing the results of investigations in the field of magnetic analysis, it will be found convenient to consider the following topics: Fundamental correlation, tests for general quality, detection of flaws, alternating-current methods, and thermomagnetic analysis.

Fundamental Correlation. In his paper published in 1916[5] on the correlation of magnetic and mechanical properties of steel, Burrows[5] enunciated what he believed to be the fundamental principle of magnetic analysis as follows: "There is one and only one set of mechanical characteristics corresponding to a given set of magnetic characteristics, and, conversely, there is one and only one set of magnetic characteristics corresponding to a given set of mechanical characteristics." He goes on to say: "Although there is no evidence to refute the preceding rather broad statement, the utility of this generalization is decidedly limited by the complexity of the relations due to the large number of variables and the lack of sufficient quantitative data . . . Even with these limitations, magnetic testing, in conjunction with mechanical testing, may be expected to be of considerable value in determining mechanical properties." Unfortunately, some have made extravagant claims based on the first part of this statement while overlooking entirely the caution as to the inherent limitations due to the complexity of the relationships involved and the lack of fundamental data. As a rule, such claims are detrimental rather than advantageous, as failure to substantiate them has a tendency to discredit the whole proposition and deter some from entering the field.

In the attempt to discover fundamental correlations between the two sets of properties, it has been the general practise to make magnetic measurements by some well-known testing method on specially prepared samples. The results of these tests are then correlated either with the heat treatment or with the results of mechanical tests subsequently made on the same samples. Such correlation is not easy because the magnetic properties are in themselves complex. Attempts have been made to simplify the problem by using certain critical values, such as maximum permeability, induction corresponding to a specified value of magnetizing force, residual induction, coercive force and the like, as criteria. These attempts have not been very successful and it has been found necessary to consider the magnetic properties as a whole in drawing conclusions. Another lesson that has been learned is that it is generally not safe to draw too definite conclusions from the results of correlating magnetic properties with heat treatment alone, for we can never be assured that some accidental factor has not entered into the heat treatment and thus produced mechanical properties not normally associated with the specified heat treatment. Too much emphasis cannot be laid upon the necessity for proper metallurgical control in correlation investigations. The results of experiments in which this factor has not been adequately cared for must necessarily have but limited value in the quantitative sense.

Space does not permit a detailed discussion of the results of all the investigations recorded in the technical literature. These results, however, are in the main concordant and the following general conclusions may be drawn:

1. Any treatment which alters to a measurable extent the mechanical properties of a piece of steel at the same time appreciably alters its magnetic characteristics, though not necessarily to a corresponding degree.

2. The nature of the effect of various treatments on magnetic properties is generally similar for different types of steel but this is not a universal rule, as notable exceptions have been observed.

3. Although the magnetic and mechanical properties appear to be functions of the same variables,—that is, composition and structure,—there are certain secondary effects, notably that of mechanical strain, which exert an influence on the magnetic properties out of all proportion to their effect on the mechanical properties. This is an important point which should never be overlooked in the search for laws of correlation.

Tests for General Quality. In their final application, tests for general quality must be applied to manufactured products of various shapes and sizes, and for this reason a solution of the problem calls for a considerable amount of ingenuity in the design of apparatus for the testing of irregular shapes. In this type of application it should not be necessary to await the establishment of definite laws of correlation, for if pieces of steel having identical magnetic properties also have identical mechanical characteristics, it should only be necessary to compare the pieces under test

magnetically with one known to have the requisite mechanical characteristics and which may therefore be taken as a standard of quality. A considerable number of investigations has been carried out as to the applicability of magnetic analysis for the testing of various products on this basis. Among the products investigated are cutlery, small tools, ball bearing races and balls, small case-hardened chain, and twist drills.

Encouraging results have been obtained in the majority of cases, but the obstacle in the way of practical application on a commercial basis is usually the effect of secondary influences as noted above. In other words, the test generally fails on the side of safety, but leads to the rejection of an excessive amount of good material. This should not be considered to imply that the outlook is hopeless, but only that the problem is not so simple as it might appear at first glance. The need for a satisfactory nondestructive test is so great that efforts in this direction should not be abandoned until every possibility has been exhausted. And the fact that successful application has been accomplished in even a few cases leads to the conviction that the general problem is not impossible of solution.

The Detection of Flaws. A large proportion of the failures of steel parts in service is due not to poor general quality of the materials but to the presence of hidden flaws or defects. It is for the detection and location of hidden flaws that a reliable nondestructive test is most needed. For this reason, a considerable amount of attention has been given to this phase of magnetic analysis, and at least one successful application has been recorded.[47] A large number of different products such as steel rails, rifle-barrel steel, rifle barrels, ball-bearing races, and welded chain, has been the subject of study. One of the most important problems is that of the test of welds, for the applicability of welding is seriously limited by the lack of a suitable test for quality of the weld.

The usual test for the detection of flaws consists in an exploration for the determination of the degree of magnetic uniformity of the specimen. Abrupt changes in magnetic permeability may be the result of flaws; on the other hand, they may indicate the presence of internal strain or may result from relatively unimportant surface conditions. The problem therefore, at least in part, is to discover some way of differentiating between the indications due to strain and those due to actual flaw. A partial solution has been found[39] in the fact that variations due to strain are less in magnitude under high values of magnetizing force than for low or medium values. Unfortunately, the sensitivity to the effect of real flaws is materially decreased when high magnetizing forces are used. A great deal of work remains to be done on this important problem.

Alternating-Current Methods. For routine inspection on a commercial scale, alternating-current methods appear to have some practical advantages. The quantitative interpretation of the magnetic results is generally more difficult than for the ordinary direct-current methods, due primarily to the phase relations involved. With standardized equipment and arbitrarily chosen limits, however, these difficulties are of minor importance.

During the course of an investigation on high-speed steel carried on by Committee A-8 of the A. S. T.M.,[40] several methods were studied by different members of the committee. Although the results of the investigation left much to be desired in the way of definite correlations, the alternating-current methods developed by deForest[24] seemed to offer the most encouraging field for further study. Two types of inspection have been developed. In the earlier form, the sample under test is compared with a standard by means of an inductance bridge using a separately excited a-c. galvanometer as a detector. By adjusting the phase of the current in the galvanometer field with respect to the induced voltages, it is possible to obtain readings on the galvanometer which depend mainly on the difference in permeability of the two specimens, the difference in hysteresis loss, or various combinations of the two factors. Thus it may be seen that in addition to speed in operation, the method offers great flexibility.

The second method[43] consists of the use of an oscillograph, preferably of the cathode ray type, with which the magnetic differences are shown graphically and appear as closed figures or loops of various shapes and sizes. By adjusting the phase of the time coordinate, it is possible to emphasize the difference in any particular part of the hysteresis loop. This method, first adapted to magnetic analysis by deForest, has been discussed with respect to its more theoretical aspects by Spooner.[44] From the evidence already presented, it would appear that these two types of alternating-current testing provide exceedingly valuable methods for the application of magnetic analysis both in the laboratory and on a commercial scale.

Thermomagnetic Analysis. Thermomagnetic analysis consists of the study of magnetic changes which occur during heating or cooling of a specimen. While not properly classed as a nondestructive test, thermomagnetic analysis should be considered as one phase of magnetic analysis. Its development and application have received little attention in the United States, but in the hands of the Japanese investigator Honda[7, 48] it has proved to be an exceedingly valuable tool for the study of the phenomena associated with the heat treatment of steel. Apparatus for the application of thermomagnetic analysis has been set up recently at the Bureau of Standards and it is hoped to develop further this phase of magnetic analysis.

Practical Applications. The methods of magnetic analysis have found some practical application both in the laboratory and in commercial routine inspection. In the laboratory, the applications have been of two kinds. As a research tool for the study of the constitu-

tion and structure of ferrous materials, it has found use which will undoubtedly be extended. Indeed, it may well be that in this connection magnetic analysis will find its most valuable application. Magnetic tests have been used also in the laboratory in connection with long-time mechanical tests, such as endurance tests and tests upon the effect of elevated temperatures on mechanical properties. In tests of this sort, it is of great importance to be sure that all of the specimens used are initially in the proper condition and that they contain no flaws. Under such circumstances, the rejection of some good material is of less consequence than the acceptance of unsatisfactory material, since the results on such material may be not only valueless but even misleading.

In the realm of commercial routine inspection there are three outstanding examples: the inspection of small case-hardened chain,[22] the testing of small heat-treated forgings,[43] and the examination of steam turbine bucket wheels.[47] The first two are applications of the two types of alternating-current tests developed by deForest for the American Chain Company. The third mentioned method is a d-c. exploration test developed by the General Electric Company with the collaboration of Dr. Burrows.

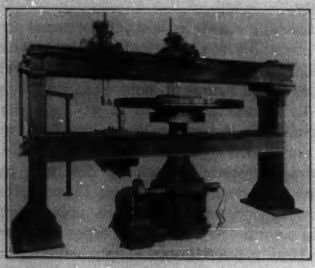

Fig. 1—Magnetic Analysis for Turbine Bucket Wheels.

In the testing of case-hardened chain, the inductance bridge method is used. The phase of the galvanometer field current is so adjusted as to give readings which experience has shown are proportional to brittleness. Readings which lie outside of certain specified limits indicate material which is either too brittle or too soft. This method for the control of the heat treatment of the manufactured product has been in use for over five years, several thousand individual specimens being inspected each day.

For the inspection of small heat-treated forgings, the oscillograph method is utilized. Experience has shown what form of loop indicates a satisfactory condition of the material. Variations from this form, greater than a certain amount, indicate defective material.

One form of apparatus for testing turbine bucket wheel forgings is shown in Fig. 1. The forging under test is rotated on its own axis between the poles of an electromagnet which can be adjusted to any given position on the radius of the circular forging. The magnetic flux passes across the air-gap between the pole tips, in which air-gap the disk is rotated. Any variation in the magnetic quality of the material in this air-gap is indicated by means of a sensitive galvanometer connected to coils on the specially designed pole tips. In most cases, the value of magnetic explorations for the detection of flaws and irregularities is limited by the effects of strain and other obscure conditions. This test, however, is not applied until the disk has been heat treated for the removal of strain, and machined to final dimensions, and hence operates under the most favorable conditions. Although rejections are very few, in view of serious damage and sometimes even loss of life resulting from failure, it is exceedingly important that no defective piece be used in the finished machine.

Conclusions

From the foregoing rather cursory outline of the history and development of magnetic analysis, it is evident that the subject is one of great interest both from the point of view of commercial application and that of scientific investigation. The problem is not a simple one, as it involves not only the complex magnetic properties of materials, but also their composition and structure which determine the mechanical properties. The more rapid development of magnetic analysis has been hindered by a number of factors, some of which may be inevitable, but others of which might be obviated,—at least to some extent.

With relation to the magnitude of the problem in all its phases, the number of workers in this field is very small, and their activities have extended over such a broad area that general progress has been necessarily slow. More workers and better coordination would do much to remedy the situation. One circumstance which may be inevitable, but which, nevertheless, operates to retard development, is the tendency for most workers to cover each discovery and application as fully as possible with patents. The potential new worker in this field, particularly in industrial laboratories, is somewhat uncertain as to his own status, and in this case the tendency is to await further development in the hope that such development as may interest him can be made on a secure basis.

Above all, the need is for more fundamental work. The possibilities in the way of practical applications are definitely limited by the lack of fundamental data. To be of maximum value, such fundamental investigations must be planned and carried out not only from the standpoint of the magnetic phenomena but also from the point of view of the metallurgical principles involved. This involves the closest cooperation between the magnetic investigator and metallurgist. One of the most important phases of the problem is the study of the so-called secondary effects, which greatly influence the

magnetic properties without a correspondingly great effect on the mechanical properties.

The outlook, on the whole, is decidedly encouraging. Magnetic analysis has already proved of use in research on the treatment and structure of steel and for routine inspection on a commercial scale. There seems to be no reason why future progress should not be more rapid, and the hope that magnetic analysis may be found more and more useful as time goes on appears to be amply justified.

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56. Honda, K. and Associates, "Numerous Papers on Thermagnetic Analysis," *Science Reports*, Tohoku Imp. Univ., Sendai, Japan, 1911 and later.

Discussion

B. M. Smith: Mr. Sanford has covered the field of magnetic analysis quite thoroughly, but there is one application which he has not mentioned and which I have found quite reliable. I refer to the determination of the chemical composition of magnetic materials with special reference to the carbon content of iron and steel. In this direction there have been two methods successfully used, one utilizing magnetic saturation and the other, the unsymmetrical hysteresis loop.

Hadfield and Hopkinson presented a paper before the Institution of Electrical Engineers December 8, 1910 showing how the magnetic saturation of iron could be lowered by increasing the amounts of carbon within certain limits, the reduction being in a linear relation to the increase in carbon and sufficiently exact to suggest the determination of the carbon content by magnetic measurements.

Russel J. Eddy, in *Testing*, Jan. 1924 described a "Carbometer" devised by Messrs. Malmberg and Holmstroem which determined the carbon content of steel by utilizing the unsymmetrical hysteresis loop. For percentages of carbon between certain limits the calibration curve is practically a straight line. Experiments which we have conducted along this line have shown the method to be very reliable and quite suited for determining the carbon content of steel at the melting furnace.

During my work of the past 10 years on magnetic analysis I have collaborated with Dr. Burrows and S. L. Gokhale in developing the turbine bucket-wheel test and with others in developing the Durescope. I have, however, found no application more interesting or more promising for further investigation than the application of magnetic analysis to the chemical composition of magnetic materials.

W. B. Kouwenhoven: Last June Dr. J. D. Tebo of the Bell Telephone Laboratories and I presented a paper before the American Society for Testing Materials on a new method of magnetic testing. Briefly, this method involves the applying of a constant d-c. magnetizing field to a specimen and then superimposing a second field which may be alternating or of some other form and determining the change in magnetic characteristics due to the superimposed field. This method we have named an incremental permeability method.

We are continuing this year at Johns Hopkins. I have one man actively engaged on it. The further investigations with this method and the results at present are fairly promising. However, a great deal of work still remains to be done and we need more fundamental data. We need more investigators and cooperation would be welcome.

A. V. de Forest: I should like to draw attention to the very intimate relationship between magnetic measurements and all the unknown properties of steel. In the magnetic method we introduce the possibilities of measuring all kinds of properties that do not come out in any of the formal methods of mechanical testing, and yet we attempt to correlate our magnetic measurements with the results of these exceedingly crude mechanical tests. No wonder it requires quite a long time. Also, no wonder unpleasant experiences occur every once in a while in that a wild kind of material will come along and give the same mechanical tests but different magnetic ones, or vice versa, the same magnetic tests and wild mechanical ones.

I have found it quite desirable to consider the magnetic method as far more fundamental, and very much more subtle than the mechanical tests.

One of the reasons for this is that almost all of the magnetic methods bring in the question of internal stress and yet the internal stress conditions have very little to do with the ordinary measured static mechanical properties. It is necessary to go into fatigue and impact and probably into corrosion and perhaps corrosion fatigue phenomena in order to get reflections of the internal stress conditions which the magnetic is quite able to pick up without any trouble at all.

Perhaps another way of looking at the thing is to consider that both ordinary mechanical static tests and the magnetic tests are measurements on the average properties of large groups of crystalline grains. When we get into mechanical tests of the fatigue variety, perhaps in a long length of wire there will be only one grain, an infinitesimal thickness of material, which will determine the actual life under working conditions of the entire mass of material that we are testing. No static test and no hitherto devised magnetic test can be applied to a single grain or a single section of a large piece. We, therefore, have to study both mechanical and magnetic relationships in the light of average results and not on the single individual specimens. The difficulty is that the mechanical testing of large numbers of specimens is so far almost hopeless. There aren't fatigue-testing machines enough and there isn't time enough to accomplish the correlation.

Electrical Instruments Used in the Measurement of Flow

BY W. H. PRATT[1]

Fellow, A. I. E. E.

Synopsis.—This paper outlines some important flow-measurement problems and points out the character of electrical apparatus that may be used with other equipment for measuring flow.

IN the measurement of fluid flow by electrical means, the electrical devices, though frequently a major portion of the equipment, are essentially auxiliaries. Fundamentally, fluid measurement is a problem in hydromechanics and electrical apparatus is used as a convenient supplement to the mechanical actions which are generally utilized to make manifest the behavior of the fluid. The significance of this statement will appear as we proceed.

Measurements of the flow of water, oil, air, fuel gas, and steam are, as far as numbers of applications, the most important, but there are many other actual and possible occasions for utilizing similar ideas and apparatus. Certain instruments described in the latter part of this paper are as electrical devices, instruments for measuring the relation of two electrical quantities and as such are capable of a wide variety of applications. As here described they are for the measurement of flow.

A broad division of the quantities to be measured may be made according to the conditions of measurement; *i. e.*, 1, measurements of unconfined flow, 2, measurements of flow in confined channels or pipes. The first is illustrated by the measurement of wind velocities and by the determination of the air speed of an aeroplane, the second by the flow of steam in a steam main.

Other divisions may be made following the ideas utilized in effecting a connection between the fluid flow and the electrically responsive measuring devices; for example, we may have an anemometer wheel whose speed response to the fluid flow is used to determine a voltage or a frequency of electrical impulses, or a drop in pressure may be brought about by an orifice or by a venturi tube and the resulting pressure difference communicated to electrically responsive apparatus, or again the change by convection of the temperature of electrically heated wires when exposed to the moving fluid may be the means of establishing a connection with the electrically responsive apparatus.

In measurements of unconfined flow the quantity determined is essentially a velocity or a velocity-density function, and the first stage of measurement is to establish a pressure, a displacement, a motion, or a temperature change which is a function of that velocity or function.

I. If the first alternative is chosen, *i. e.*, if the pressure resulting from the relative motion of the fluid and the measuring apparatus is employed, it may be utilized to change a resistance as for instance that of a carbon-pile, or indeed it is conceivable that through the ministrations of the piezoelectric effect a suitable electrical measurement could be achieved if not directly at least by the use of amplifying devices.

II. Displacements may be used to alter the capacity of a capacitor, to change the inductance of a reactor, or to change the resistance or conductance of a suitable conducting circuit. There are many ways in which electrical effects may be derived from such combination.

III. If the third alternative is chosen the motion will undoubtedly be that of the wind-wheel of an anemometer. From here on to the indication or record the connection may be electrical and may take one of the many forms of electrical speed indicators among which may be mentioned,

1. A magneto, which produces a voltage proportional to speed, connected to a suitable voltmeter.

2 (a) A contact maker which releases electrical impulses at a rate proportional to the speed of the moving member. These may be condenser discharges which thus may release a quantity of electricity proportional to the speed, which may be indicated on a milliammeter acting as a balistic instrument; or the impulses may control an independent source of mechanical motion which may be indicated by the magnetic drag induced by the relative motion of permanent magnets and a body of conducting material or by purely mechanical forms of indication.[2]

(b) Instead of electrical contacts periodic variations of electrical capacity or of inductance may be used to control electrical pulses which in such cases will probably be used in connection with vacuum tubes and electrical oscillations of audio or higher frequencies. The final indications will be proportional to the rate of pulses and the actual apparatus closely paralleling that indicated under (a).

IV. The heat imparted to the surrounding medium

1. Engineer, General Electric Co., West Lynn, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

2. These ideas have been employed for the distant indication of electrical quantities.

29-45

by a wire carrying an electric current is a function of certain properties of the medium itself, of the temperature and surface conditions of the wire, and of the relative motion of medium and wire. If now energy at a constant rate is supplied to the wire and other conditions remain unchanged the temperature of the wire will be a function of the relative motion of medium and wire. By using a wire having a suitable temperature coefficient of resistance the resistance becomes a function of the fluid flow and a measurement of undisturbed flow resolves itself into a measurement of resistance after the necessary constants have been determined.[3]

The measurement of flow of fluids in pipes has much in common with the measurement of the relative motion of an unconfined fluid medium, and a measuring device. The measurement of steam or of air under pressure is generally carried out by recourse to pressure drop through an orifice or by the pressure-velocity relations determined by pitot and static-pressure tubes. In general the quantities of interest are the mass of fluid per unit of time passing the point of observation, and the total amount of fluid which shall have passed between the epoch of start and any chosen times for observation. Generally these measurements are made at pressures fairly near chosen standards; the one is a velocity density function, the other an integration with respect to time of this function.

The electrical apparatus used generally consists of two portions, 1, a container and a body of mercury whose configuration is changed to correspond to the pressure changes communicated to it and which thereby alters, by shorting out resistance or inductively, the conductivity of an electric circuit; 2, an electrically responsive member which will translate the changes of conductance or admittance into indications or integrate an appropriate function of one of these quantities with respect to time.

In the measurement of the flow of liquids, mechanisms quite similar to those used for gaseous flow may be employed. From the hydromechanic point of view the problem is simplified by the absence of change of density.

Electrical power is thus utilized to perform the work incident to indication or integration and the hydromechanical organizations determine the magnitude or other characteristic of the electrical action.

For most of the measurements here discussed alternating current has many advantages over direct current. In any case an instrumentation whose indications are independent within moderate limits of the supply voltage and in the case of alternating current, independent of the frequency changes that may be expected, is much to be preferred as compared with one which must rely on a fixed or automatically controlled source.

When a body of mercury is caused to change its level by being subjected to a differential pressure it can be caused successively to make contacts which thus change the resistance or impedance of an electric circuit, and in so doing no pressure reaction should result. On the other hand a finite and practically a rather limited number of steps is imposed.

If the body of mercury is so disposed that it acts as the secondary of a transformer, a change of level in the mercury surface may be made to change correspondingly the amount of mercury in the secondary path and consequently the characteristics of the primary circuit. Measurements made in the primary circuit will correspond to conditions in the secondary and these in turn correspond to the equilibrium conditions established by the hydromechanical portion of the equipment. For an arrangement of this kind the electro-magnetic reaction between primary and secondary circuit must be limited to an amount that will not importantly change the mercury level. Small changes can be provided for in the calibration and voltage compensation of the instruments.

In the remaining portion of this paper two mechanisms are described both actuated by alternating current. The one is intended for indication on a scale or for recording on a chart flow as determined by a flow meter in which a body of mercury acts as a secondary and thus influences the characteristics of the circuit in which the instrument is connected; the other integrates with respect to time a quantity similarly determined.

1. An instrument that indicates the conductance of an electric circuit.

This instrument indicates the ratio between the power expended in a standard circuit and that of another whose admittance is controlled by a flow meter or other device. It consists of two elements, similar to the electromagnetic system of the ordinary induction watthour meter, arranged to oppose their torques on a common disk. The disk is so modified that the torque developed by one of the elements, or if conditions should require it, of each of the elements, is inter alia dependent on the position, with regard to deflection, of the disk. Thus for a steady indication, the torques acting on the disk are balanced and

$$k_1 E I_1 \cos \phi_1 f_1(\alpha) = k_2 E I_2 \cos \phi_2 f_2(\alpha)$$

$$k_1 \frac{E^2}{Z_1} \cdot \frac{R_1}{Z_1} f_1(\alpha) = k_2 \frac{E^2}{Z_2} \cdot \frac{R_2}{Z_2} f_2(\alpha)$$

$$k_1 G_1 f_1(\alpha) = k_2 G_2 f_2(\alpha)$$

$$\frac{G_1}{G_2} = \frac{k_2}{k_1} \frac{f_2(\alpha)}{f_1(\alpha)} = F(\alpha)$$

in which

α represents the angular position of the moving system

k_1, k_2 are constants

E is the e. m. f. of the circuit

I_1 is the current in the first circuit

I_2 is the current in the second circuit

3. This idea in a modified form is used in the Thomas Gas Meter.

G_1 is the conductance of the first circuit
G_2 is the conductance of the second circuit
ϕ_1 is the lag angle of the first circuit
ϕ_2 is the lag angle of the second circuit

If the constants of the second circuit are fixed and this circuit is used as a standard

$$G_1 = G_2 F(\alpha)$$

If now each value of G is uniquely determined by the quantity we seek to measure, the angular position of the instrument pointer gives an indication of that quantity and a scale may be appropriately marked in terms of it.

2. To integrate a quantity $f(G)$ with respect to time we may utilize an induction watthour meter structure and by changing the proportioning of its part cause the damping effect of the magnetic field from the voltage coil to be large enough to supply all the damping necessary to maintain the speed at a suitable value. As then constituted the torque of the meter will be proportional to the square of the voltage since this torque is determined by the reaction of currents in two circuits in each of which the flow is proportional to the voltage. Also the damping effect is proportional to the square of the voltage, for the magnetic field and hence the Foucault current induced thereby are both proportional to the voltage. The interaction of this field and current produces the damping.

With a simple structure as outlined above a change of frequency would produce an inconsequential change of torque but the current in the voltage coil would vary inversely as the frequency and thus the damping would change inversely as the square of the frequency.

If, however, a capacity is introduced in the voltage circuit, as shown in the diagram, and if its value is so chosen that the resonant frequency of that circuit shall be about double the actual frequency, the current in the potential circuit instead of decreasing in inverse ratio as the frequency will increase with increasing frequency and the damping which increases as the square of the current in the potential circuit can be made to change at the same rate as the resulting change of torque, thus making the speed of the meter independent of the frequency over moderate ranges of frequency. The conditions that make for independence of voltage remain effective. The constants of the circuit are such that resonance from the fundamental frequency of the circuit or its harmonics is precluded.

The first of the two instruments described is clearly one that indicates the ratio between two similar electrical quantities and so can be utilized to measure the ratio between any two quantities that can be represented by suitable electrical quantities, as for instance, temperature. In this case, standard temperature may be represented by a standard resistance having a small or zero temperature coefficient, while the temperature to be measured may be represented by a resistance that varies suitably with that temperature.

Instead of measuring flow as such, two pressure-sensitive devices may be used, one to influence the one, the other the second element of the instrument whereby a ratio of flows may be measured. The ratio of steam delivered to air used might be one application or the measurement of the ratio of steam extracted to total steam used, another.

An instrument could be constructed with two moving systems and associated elements, the two indicators arranged to move in closely parallel surfaces over a single scale in such a way that the reading point should be the intersection between the two indicators. In this way indications of such functions of four quantities as may be expressed in the form $F\left(\frac{X}{Y}, \frac{U}{V}\right)$ could be obtained as a single reading. Non-electrical quantities that can be represented by suitable electrical quantities can be so measured. The only application (of which the author is aware) of such a structure was for the measurement of the product, or ratio of electrical quantities for the measurement of another electrical quantity, namely power factor, $\frac{P}{E\ I}$. The actual relation made use of in the instrument was $\frac{P}{E^2} \cdot \frac{P}{I^2}$ but the occasion may easily arise in connection with other measurements, and a not impossible application may be in the determination of a compound ratio of the flow of several fluids.[4]

Induction instruments of the ratio type are nearly free from one of the drawbacks that characterize certain other forms, notably induction voltmeters and wattmeters, in that the indications result from the equilibrium of two torques that have the same characteristics with respect to temperature. Small residual errors are cared for by a small amount of temperature-sensitive magnetic material in the air gaps.

In the integrating meter, temperature errors are self-eliminating.

It is not expected by the author that this paper more than outlines the nature of some important fluid measurement problems and points out the general character of the electrical apparatus that may be employed. The essence of fluid measurement is contained in the subject of hydromechanics. The electrical apparatus which generally may be advantageously used has characteristics that suggest its usefulness in connection with other types of measurement that may have nothing in common with fluid measurement aside from the electrical apparatus used for giving final indication.

Numerous bibliographical references may be found in the bibliographies relating to the Measurement

4. Pratt patent 719,609, Feb. 3, 1903. Induction elements could replace the electro-dynamometer elements illustrated therein.

of Non-Electrical Quantities by Electrical Means which have appeared in the A. I. E. E. TRANSACTIONS.[5]

Appendix

By way of explanation of the diagrams:

In Fig. 1 the disk is represented as divided by a curved line into two parts. The main portion is of high conductivity material and the irregular boundary of this part is so chosen that the torque of the back

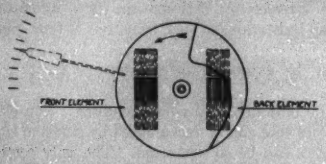

FIG. 1—DISK OF CONDUCTANCE METER

element is a function of the angular position of the disk as well as of the current, voltage, etc., to which that element is subjected. The smaller part of this disk is of high resistance material and its purpose is solely to give mechanical balance to the element.

Fig. 2 gives the connections of the electrical elements

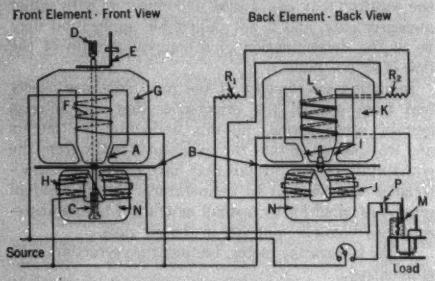

FIG. 2—SCHEMATIC DIAGRAM OF CONDUCTANCE METER

associated with the moving system shown in Fig. 1. In Fig. 2, M represents a flow meter element, a transformer with a mercury secondary.

G, K, N, N..... laminated iron cores

B.....the disk shown in Fig. 1.

L.....the exciting coil of the back element which also serves as a primary to which the circuit including R_1, R_2, and J is a secondary. The resistance $R_1 + R_2$ constitutes the standard electrical quantity with reference to which the indications of the instrument are determined.

5. A. I. E. E. TRANS., XLIV, 1925, p. 258.
A. I. E. E. TRANS., XLVI, 1927, p. 710.
A. I. E. E. Quarterly TRANS., Vol. 47, October 1928, p. 1168.

In Fig. 3, C and K are laminated iron cores. The magnetic flux which crosses the gap between them in which disk E rotates not only produces the torque but also supplies all the damping.

The circuit including B and I is highly reactive, leading, thus with increasing frequency, the current in this circuit increases and the damping, which may be taken as proportional to the square of the current in B, increase at a rate sufficient to compensate for the increases of torque which also increases with the fre-

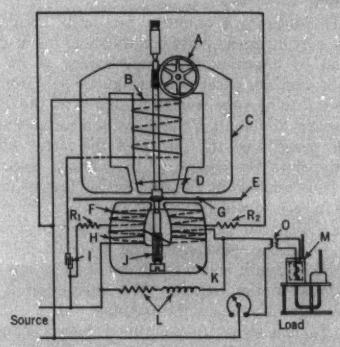

FIG. 3—SCHEMATIC DIAGRAM OF INTEGRATING METER

quency. In the absence of the capacitor I the torque would remain almost unchanged with change of frequency while the damping would vary inversely as the square of the frequency, since the predominant part of the damping is due to the flux from coil B. The

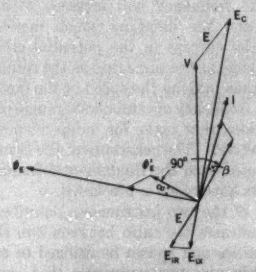

FIG. 4—VECTOR DIAGRAM OF INTEGRATING METER

circuit including coil F and resistors R_1 and R_2 is of high resistance. Its purpose is to supply a small amount of torque to balance out the excitation losses of the transformers O and M. H is the main current winding and L is an adjustable reactive shunt to effect calibra-

tion and provide a small adjustment for the phase angle of the current in coil H.

Fig. 4 shows the vector relation of the quantities involved in the action of the meter shown in Fig. 3.

V represents the voltage impressed on the voltage circuit and is the sum of the two components

E_c the voltage across the condenser

E the voltage across the voltage coil

the latter is the resultant of reactive and energy components as indicated by E_{IX} and $E_{IX} - E_{IR}$.

ϕ_E represents the flux in phase with the current in the voltage coil

ϕ_E' is that portion of the flux ϕ_E that crosses the disk, the phase shifted through the angle α by the light load adjusting plate (not shown)

I represents the current in the current coils of the meter

ϕ_I' the resultant flux due to the current I and current in short-circuited bands around the current poles (not shown in Fig. 3).

Discussion

I. F. Kinnard: Mr. Pratt shows (Fig. 3) a very interesting application of the induction watthour meter to measure conductance. It will be observed that there are no damping magnets in this arrangement, but the damping on the disk is supplied entirely by the potential flux. In other words, the same flux drives the disk and produces damping. It should also be noted that a condenser is used in series with the potential coil in this device in order to correct for frequent variations.

There has recently been developed a modified scheme also utilizing the watthour meter which may be of interest to the Institute and ought to be on record.

The watthour meter, of course, has a moving element, usually an aluminum disk. It has a driving element consisting of a potential winding and a current winding, and the control torque is supplied by a permanent magnet. If we dispense with the permanent damping magnet and substitute instead another electromagnet which is connected directly across the line through a fixed resistor, we have a conductance meter which is very closely allied to the standard watthour meter. As a matter of fact, it has been found possible to get very excellent results using standard watthour meter disks, bearings, and driving elements in this way. It is obvious just to get a physical picture of the way this device works that the only variable that can cause a change in speed of the meter is a change of the conductance of the load. That is, if the potential should vary, both the driving torque and the damping torque varies according to the square of the potential and there is no resultant speed of the meter. There may be many applications in the future whereby this very interesting modification of the watthour meter could be profitably used.

Use of the Oscillograph for Measuring Non-Electrical Quantities

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Member, A. I. E. E.

and

W. B. BATTEN*
Associate, A. I. E. E.

Synopsis.—The electromagnetic oscillograph has proved to be a useful instrument for obtaining recorded measurements of non-electrical quantities. The problems of application involve the translation of mechanical phenomena into electrical changes which are proportional. Several typical applications are described showing how motion, time, stress, pressure, etc., are measured.

ALTHOUGH the electromagnetic oscillograph was developed primarily for, and has found its principal application in, recording electrical quantities in terms of current, voltage, and watts, its inherent characteristics have made it useful for non-electrical applications. This oscillograph offers a means of recording photographically phenomena which are of too short a duration for the eye to watch or for other types of instruments to record. Furthermore, it is frequently desirable to record simultaneously both electrical quantities and mechanical actions of some apparatus. For this, the oscillograph is admirably adapted. However, these quantities must be of such a nature that they can be translated into electric potentials or currents which have a definite relation to the quantities to be recorded.

The element of the instrument is a low inertia galvanometer, the two ribbons of which move with respect to each other in a magnetic field. The motion tilts a mirror and through an optical lever causes a beam of light to be deflected on a screen or film. The motion is proportional to current in the ribbons so that equal increments of current through the galvanometer or equal increments of potential applied to the galvanometer with series resistance are faithfully recorded.

The problem in adapting the oscillograph for other than electrical uses lies in the translation of the phenomena to be recorded, into proportionate electrical values. It is the aim of this paper to show a number of typical applications illustrating how this object was obtained. Many phenomena, such as changes in pressure, vibrations, stresses, and time intervals, give rise to mechanical motion, either linear or angular, resulting in a variation of speed or an acceleration of the body as a whole or in part. If this motion can be made to vary an impedance in an electrical circuit as a function of this motion, the current through the circuit, or the potential across a portion of this impedance, may be utilized to actuate the galvanometer for a record.

Travel Indicator

Where the motion of a part of a machine is to be studied for uniformity, acceleration, rebound, etc., a resistance wire can be placed along a stationary part, and a moving contact attached to the moving part. The contact slides along the wire and changes the resistance included in the galvanometer circuit as in Fig. 1, in which $A\,B$, the resistance, may be either of continuous length or steps in a wound resistor. C is the sliding contact for indicating relative motion between the moving member to which it is attached and the fixed member to which $A\,B$ is fixed. In such a circuit, a constant current flows from a battery through $A\,B$. The resistance of the galvanometer circuit $A\,G\,C$ is made sufficiently high so that changes in the position of C do not alter the potential distribution along the resistor $A\,B$ appreciably. The proportions of current in the two circuits (resistor and galvanometer) can be adjusted to secure any reasonable degree of accuracy.

A device of this kind has proved particularly useful

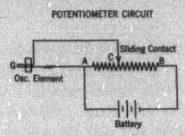

Fig. 1—Potentiometer for Travel Indicator

in studying the travel of circuit breaker parts. As shown in Fig. 2, the resistor wire is wound on the edge of an insulating disk which can be rotated against the action of a special spring. The device is clamped to the frame at a convenient point and a cord, connected to the disk and wrapped around a drum on its shaft, is attached to the moving part to be studied.

Speed Recorder

When a rotational speed record is desired in a form, such as an oscillogram, particularly adjacent to some other record for the purpose of timing, rotation must be translated into an electrical current in the oscillograph element proportional to the speed. A magneto or other constant field generator has a voltage characteristic proportional to the speed, but the usual electric tachometer has a wave form that is very bad, being full of high-frequency harmonics. At various times homopolar generators have been used but they generally

*Both of Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 23-Feb. 1, 1929.

develop current collection troubles and generate a very low potential. A remedy to the magneto trouble can be obtained by using a special oscillograph element, which is equipped with a damping vane and will not respond to high frequencies.

Fig. 3 is a section of an oscillogram recording the operation of two reversing mill motors. The top record shows the speed of one motor as measured with a magneto and oscillograph element having a damping vane. The fifth record from the top shows the speed of a second motor as measured with a magneto and standard oscillograph element. These two records show the

Fig. 2—Circuit-Breaker Travel Indicator

effect of a damping vane on an oscillograph element when making this type of record.

It is sometimes desirable to record minute speed variations of rotating parts as when investigating governor action. This may be accomplished by coupling or belting a small magneto or constant field generator to the machine under test. The magneto or generator is connected in series with a battery, the potential of which opposes and equals the magneto voltage during steady state condition. The oscillograph records the differential potential and can be adjusted to respond to a fraction of one per cent change in speed.

Vibration

If vibratory motion of small amplitude can be made to change the reluctance of a magnetic circuit of proper design an electrical equivalent of the motion can be obtained. This was used in exploration of turbine blade vibration by placing a vibration converter on the end of the turbine shaft. This device is really a small generator with a permanent magnet field. It is so designed that the normal position of a field pole is midway between two armature teeth. As the field moves with respect to the armature, the flux shifts from one tooth to the other and induces a potential in the coils on the armature teeth. In practise, the armature is clamped rigidly to the end of the turbine shaft and hence follows the motion of the turbine rotor. The field of the vibration converter is free to move with respect to its armature and its inertia is such that it does not follow the high-frequency vibrations which may be present in the turbine shaft, though it rotates at the same average speed as its armature.

Timing

Sequence of operation without reference to magnitude of motion or resulting phenomena can be shown nicely by the oscillograph. Two factors are necessary, one an indication of operation, and the other a time scale. If, for example, the relation between the opening or closing of several switches is desired, each unit can be so arranged as to short-circuit a resistor in the galvanometer circuit or insert resistance. At each operation the recording line makes a jump up or down. The time scale may be made by an a-c. wave of known frequency, simultaneously recorded a timing record from a tuning fork or the tick of a watch. In the last case a small mirror mounted on the hair spring deflects a beam of light giving impulses on the film. This is accomplished by removing a small section from the edge of the watch case opposite the hair spring. The mirror, which is about 0.017 by 0.070 in. in size, is fastened directly to the hair spring at a point near the place where the hair spring is secured to the frame. Hence the mirror moves only slightly, this motion causing a small rotational movement about its transverse axis. The light spot reflected from the mirror moves up and down as the hair spring moves. As the light spot passes the slit in the oscillograph film holder, a spot on the film is exposed. Hence the record consists of a series of dots or dashes, such as that shown at the bottom of Fig. 3 for slow speed work. This device is very dependable. It has

Fig. 3—Oscillogram Showing Speed Record with and without Corrected Magneto Wave

the advantage of not requiring a regular oscillograph element, thus leaving an extra element for some other measurement. The mirror affects the speed of the watch only slightly.

Stresses

For most materials the strain from load on a mechanical member is proportionate to stress within the elastic limit, so that a record of stretch or compression can be interpreted in terms of stress. Usually the amount of motion is small, so that slide wire scheme is not applicable. However, other methods such as change in resistance of a pile of carbon disks with small motion

or change in reluctance in a magnetic circuit in combination with suitable coils, are available.

The strain gage is an interesting form for studying railroad track and locomotive side rod stresses. This

Fig. 4—The Strain Gage

can be applied wherever it is desired to obtain oscillographic records of mechanical movements of very small amplitude (a few thousandths of an inch). The gage consists of two stacks of U-shaped sheet-steel punchings

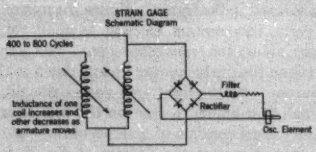

Fig. 5—Strain Gage Circuit

with a coil on each and an armature of sheet-steel punchings as shown in Fig. 4. The open ends of the two stacks of punchings are placed facing each other, with the armature between them, and the gap between the

Fig. 6—Pressure Recorder

armature and U-punchings is sufficient to permit the armature to move with respect to the punchings. In other words, the strain gage records the movement of the point to which the armature is fixed with respect to the part to which the U-punchings are attached. The two coils are connected in series and a fixed current at relatively high frequency (400 to 800 cycles) is sent through them. The oscillograph records the voltage across one of the coils. This potential varies with the distance between the armature and U-punchings. Experience has proved that practically a straight line relation exists between potential across one coil and armature position as long as the armature does not actually touch either stack of U-punchings. The record shows a modulated high-frequency current; hence vibrations having a frequency higher than about 15 per cent of the frequency of the current through the coils cannot be recorded accurately. Inasmuch as the modulated high-frequency wave occupies a large space on the film, it is difficult to record six stress measurements simultaneously without overlapping, giving a record confusing to analyze. To eliminate this condition, the voltage to be recorded is sent through a full-wave copper-oxide disk rectifier, and then through a choke coil before reaching the oscillograph galvanometer. The

Fig. 7—Oscillogram Showing Circuit-Breaker Test with Travel Indicator and Pressure Records

choke coil is of such value that the rectified high frequency is practically ironed out, but is low enough so that it does not affect the envelope curve (40 cycles or less). This arrangement gives a straight line record, the position of the line being varied only by a change in the position of the strain indicator armature or in other words by the stress in the member to which the strain gage is attached.

Pressure Recorder

A great deal of effort has been expended upon the subject of recording transient steep wave front pressures. Strictly speaking, any device in which motion occurs due to the pressure cannot give a true record, for the motion has changed the volume and consequently the pressure. Furthermore, the motion of a piston or diaphram involves time lag. But for ordinary purposes we can be satisfied with much less than the idea. Probably the piezo crystal comes as near being perfect as anything. Certain crystals exhibit electrical changes when subjected to pressure which, however, alter their shape very little. For many commercial uses, how-

ever, such a crystal is far too delicate a device, and the electrical change much too feeble.

The carbon pile is often used but even though a bridge arrangement is employed it is difficult to maintain a constant calibration and it is too delicate for rough handling.

A much better instrument for circuit-breaker pressure records was developed later using a very light-weight gas-engine indicator modified to translate the piston motion into change in resistance. The pointer which is ordinarily used for tracing an indicator card was made part of an electrical circuit and travels over a drum made up of alternate copper and insulating disks, the copper being connected to suitable external resistance units. An inch of travel on the pointer tip covers 20 steps. Thus each step represents a pressure of 1/20 of the piston spring calibration. This makes the record self-calibrating. The pressure record is shown in steps but if these are made of the proper magnitude there is no objectionable lack of smoothness in the curve. The device is very convenient and withstands test floor handling without damage. For protection during tests the recorder is placed in a fireproof metal box and a short length of metal hose is used for the connection to the circuit-breaker tank. It is interesting to relate that at first a two ft. length of heavy rubber hose was used. Records were obtained of explosion pressures long after the phenomenon was over. There was a time delay in propagation of the wave of pressure through the rubber sufficient to cause this. Substituting a rigid metal connection eliminated this.

We have described above a few interesting uses of the oscillograph outside of the purely electrical field. Many more could be cited but these will suffice to show the inherent usefulness and versatility of this instrument.

Bibliography

J. W. Legg, "The Multi-Element Oscillograph," *Elec. Journal*, Nov. 1927.

J. W. Legg, "Six-Element Portable Oscillograph," *Elec. Journal*, Mar. 1925, p. 108.

J. W. Legg, *Portable Oscillograph*, A. I. E. E., July 1920, p. 674.

The following bibliographies previously published contain many references to the application of the oscillograph to non-electrical measurements.

TRANSACTIONS A. I. E. E., Vol. XLIV, 1925, p. 257.

TRANSACTIONS A. I. E. E., Vol. XLVI, 1927, p. 709.

TRANSACTIONS A. I. E. E. Quarterly TRANS., Vol. 47, October, 1928, p. 1168.

Discussion

M. S. Rusher: One who is constantly using the oscillograph probably has found that there are more applications of this instrument to the measurement of non-electrical quantities than to electrical ones. The methods used for adapting it to these measurements will naturally differ greatly and no particular method can be called the best for all cases.

The authors state that the "Travel Indicator" described in their paper, gives any reasonable degree of accuracy with proper adjustment, but it will be found that accuracy by this method is at the expense of considerable power loss in the resistor circuit. The step-by-step method of measuring displacement which Dr. H. L. Curtis called a "displacemeter" in his paper on *Use of an Oscillograph in Mechanical Measurement*, (A. I. E. E. TRANS., Vol. 44, p. 264, 1925) has been used by us for many years with the added feature that the resistance in series with the oscillograph vibrator on different steps varies over a certain range and then repeats itself so that direction of travel and rebound are apparent at a glance. Displacements of greater magnitudes are quickly indicated by noting the number of peaks of a given height on the film the same as when using the long marks only across an ordinary scale. For measuring smaller variations it is necessary to examine the deflections of various heights. Time and distance can be measured with great accuracy by this method.

For measuring rotational speed of motors or generators, a small constant-field generator has been found very satisfactory. The wave shape being comparatively free from high harmonics, it therefore gives a curve sufficiently smooth for the required accuracy.

Vibrations of any parts made of magnetic material or to which a piece of magnetic material can be attached without changing the vibrations appreciably, have been successfully recorded by the oscillograph using the variations in the reluctance of the magnetic circuit through an exploring coil placed near the vibrating part. In order to measure this vibration accurately, the *Vibration Recorder*, described by Mr. A. V. Mershon (A. I. E. E. TRANS., Vol. 45, p. 1007, 1926) was found very useful. This same apparatus has been adapted to the measurement of pressure variations by means of a steel diaphram. This has been used for measuring pressure variations in the draft tube under a large water wheel as well as for obtaining indicator diagrams of the pressure inside an electric household refrigerator unit (*General Elec. Rev.*, Vol. 31, p. 526, 1928).

The applications of the oscillograph for timing purposes have been too numerous and varied to describe here. A portable timing device controlling a vibrating reed or timing vibrator is best adapted for very slow records. The voltage wave of known frequency is satisfactory for the great majority of tests while in some cases where great accuracy is required, the use of a tuning fork such as described by Dr. Curtis in the paper mentioned above is very satisfactory. Measurements have been made of the time required for the explosion of dynamite as easily as that of the time of operation of an oil circuit breaker.

In addition to the above applications to the measurement of non-electrical quantities are those in which the emphasis is placed on recording the variations in such quantities. Sound was recorded with the oscillograph by Mr. L. T. Robinson in 1905 (A. I. E. E. TRANS., Vol. 24, p. 212). This has developed into the present talking-movie recording apparatus.

The oscillograph was also used with a special galvanometer for recording heart beats. This too has developed into a special instrument, the "electro-cardiograph."

When new apparatus of any kind is being developed, it is usually found that the oscillograph can be used to advantage in solving some of the problems which arise.

Béla Gáti: Applying a microphone and an oscillograph, the curves show quite distinctly the difference between a new and a used gasoline engine. In the same manner, the microphone and oscillograph can be used for studying various noises and for determining means for abatement of the noises, for instance in the subway. The oscillograph and its accessories are too expensive as yet for a private engineer. Perhaps the proposed Navy reserve institution could supply our Institute with an oscillograph equipment closet where each engineer could work on his special problem.

The noise of a cruiser can be detected in 12 km. distance, and by the noise a sound-governed torpedo can be released. It is to

the interest of the Navy to support these and similar investigations and researches.

A. V. de Forest: In connection with the oscillograph on non-electrical measurements, I should like to mention further a matter which has been already spoken of, and that is the carbon pile, devised by Peters at the Bureau of Standards. It seems to be working out into an exceedingly flexible mechanism for pressures and motions of all kinds and along with it there is a twelve-element oscillograph that Peters has worked up which is a very compact and pleasant piece of apparatus to use compared with the ordinary oscillograph of some years ago.

That particular field seems to be opening up commercially into all kinds of directions. I have recently seen some figures on the stresses in track bolts made by that method that looked extremely suggestive of the possibilities of redesigning all kinds of members subjected to unknown stresses.

W. B. Kouwenhoven: One of the previous discussors has mentioned the electrocardiograph, which is used by medical men for studying the heart. Doctors are using electricity more than ever in their work. At Hopkins I often receive the query from our Medical School, "Can we do this electrically?" I think that it behooves the electrical profession to help them whenever possible, and to aid them in properly applying electricity to their work.

One of the latest non-engineering electrical measurement problems that I have worked on is the determination of the effect of diet on teeth. A poor tooth is more porous than a sound one and is more quickly permeated by a solution. The teeth of animals that have been fed special diets are extracted, and tested as follows: The nerve space of the tooth to be tested is filled with mercury; which serves to make electrical connection to the inside, and the tooth is then partly immersed in a conducting solution. It is connected in one arm of an a-c. bridge and its resistance is measured at certain intervals of time. The decrease of resistance with time of immersion is more rapid with poor teeth than with good ones.

G. D. Floyd: The measurement of non-electrical quantities affords an opportunity for combining more than one quantity on the same element of an oscillograph, the reason being that in most of these measurements of non-electrical quantities, the unidirectional electrical quantity is recorded on the oscillograph. In this case it is possible to take an alternating quantity and place on the same element the unidirectional quantity, giving what one might call a bias to the combined measurement and detracting nothing from either quantity. I think this point is important and could be made use of in a large number of cases.

Perry A. Borden: The use of the oscillograph for work other than the actual determination of current and voltage wave-forms has received a great stimulus in the advent of an instrument as flexible as that used in the paper under discussion. Those not so fortunate as to have at their disposal an instrument giving six or more simultaneous records are frequently handicapped, and are sometimes forced to resort to peculiar expedients to make best use of the equipment at hand. Such an instance is recalled in the case of tests performed some years ago in the investigation of performance of a remotely controlled hydroelectric station. Here it was required to obtain as many simultaneous records as possible with an instrument having only three vibrators; and it became necessary to "crowd" one vibrator, to the extent of having it perform three distinct duties, the record on the film afterwards being "unscrambled."

The vibrator in question was energized by a battery current upon which were superimposed oscillations set up by a tuning fork, thus establishing a standard of time. The main current was periodically interrupted by a contactor attached to the shaft of the machine under test, thus giving a count of the revolutions of the unit, while the magnitude of the whole current was regulated through a rheostat attached to the gate mechanism of the governor. There was thus obtained a record in which the height from the zero line represented gate opening, while the periodic breaks in the line showed revolutions of the generator shaft; and the ripple introduced by the tuning fork completed the data from which could be plotted the desired curves of velocity against gate opening, leaving the remaining vibrators free for the recording of electrical phenomena.

C. A. Mead: In a recent meeting of the Iron and Steel Institute, W. B. White of the American Steel and Wire Company gave a very interesting discussion of his research work in connection with steel wire, piano wire principally. That has caused considerable interest in connection with piano tuning and the loading of steel wire, etc. He has taken some very interesting oscillograms showing that the touches of various pianists differ, the same note being struck by different artists not being reproduced identically.

F. J. Schlink: One very simple method of obtaining the necessary variation of resistance for use in recording by the oscillograph, has not been mentioned: that is, a rheostat, made by winding a fine enameled wire on a cylinder with adjacent turns touching, and then removing the enamel insulation along one element of the cylinder by the use of a fine abrasive like crocus paper. In this way steps of resistance as small as two or three thousandths of an inch can easily be obtained without the necessity of piling up thin conducting and insulating plates which then have to be laboriously calibrated as to the distance from contact to contact. In the method described, the enamel wire can very easily be wound so that successive turns are in physical but not electrical contact, and no calibration is necessary beyond determining the diameter of the wire over the insulation. Such a device was described by the speaker in the *Journal of the Optical Society of America* and *Review of Scientific Instruments*, June 1925, p. 661. Very little shop work is necessary, and a rheostat of this type can be prepared ready for connection to the oscillograph with perhaps an hour's work.

Moreover, the contact surface being very perfect, depending upon the finish of the cylinder on which the wire is wound, motions can be recorded at extremely high speed and in some oscillograms taken in this way contacts were made at rates as high as 5000 per second with perfect legibility in the oscillograph record. It would be interesting to see how small a contact interval could be obtained by the use of wire especially drawn for the purpose and winding facilities devised for the particular problem. It may be that steps of one thousandth of an inch could be made available by this method with special wire, a simple device to assure a light uniform winding tension, and some manipulation skill.

F. G. Graf: I might mention a very interesting application of the osiso to the oil industry. The osiso elements are situated throughout the area in which a pool has been located and by setting off charges of dynamite and making records of the variations in the vibrations in the ground, the extent of the pool has been very accurately plotted. I think that as this application becomes better known and is studied further, considerable money will be saved not only to the oil industry but probably to other mining industries.

W. B. Batten: The travel indicator described in our paper gives a true position—time curve on the oscillogram. This, in our opinion, is preferable to any device which varies in steps, and we use the step method only when the motion is small as in the case of the pressure recorder. A power consumption of five or ten watts in the resistance wire has never offered us any difficulty.

There is no constant-field generator suitable for measuring rotational speed which, with the exception of homopolar machines, gives a good wave form. It is not a question of accuracy but of appearance.

On the pressure recorder described in the paper the use of disks was adopted because of the difficulty in obtaining satisfactory contact conditions when the pointer moved directly over the turns of a rheostat.

Study of Noises in Electrical Apparatus

T. SPOONER* and J. P. FOLTZ*
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The objections to unnecessary noises in electrical apparatus are becoming so insistent that the manufacturers are making every effort to reduce them. In order to attack the problem intelligently it is first necessary to determine the amplitude and frequency of the components of the complex sounds. The first requirement then is a simple portable sound analyzer. Descriptions are given of two resonance types which have been developed. There is included also a short discussion of the laws of sound variation and sound units. Finally there are discussed various applications of the sound analyzer to the study of noises in gears, motors, generators, induction regulators, loud speakers, vacuum sweepers, and the like.

* * * * *

Study of Noises in Electrical Apparatus

Introduction. Psychologists have found that noise reduces human efficiency. This effect may be very considerable even for the most phlegmatic individuals. Moreover, most of us object to unnecessary noises. In recent years there has arisen a very insistent demand for quieter electrical apparatus. This applies particularly to household apparatus such as refrigerator motors, vacuum sweepers and the like. In order intelligently to attack the problem of reducing noises, it is first necessary to study them in detail and resolve them into their components.

That the human ear is very inadequate for such purposes is well known. Two examples from our own experience will serve to illustrate this. A few years ago nine loud speakers, practically alike, were mounted at one end of a long hall with an observer stationed at the other end. This observer was unable to tell with certainty whether or not one of these loud speakers or all nine were operating. Ordinarily the human ear can detect much smaller differences in sound volume than this but the illustration serves to show that under certain circumstances the ear is a very poor judge of sound intensity. The second illustration has to do with the estimate of frequency. A certain member of the Laboratory Staff has a very good sense of absolute pitch. If a note is struck on the piano this individual can usually tell what note it is and give the corresponding frequency in cycles per second. There was under test in a noise proof room a compressor motor which was very noisy, the frequency of this sound corresponding to the number of stator teeth. The above mentioned observer was asked to listen to this motor and make a guess concerning the frequency of the noise. He stated that the motor had two frequencies, one at about 350 and another at 450 cycles per second. As a matter of fact there was only one frequency of any magnitude and this was 900 cycles per second. This shows that one's judgment of frequency may be very poor for unaccustomed sounds. Many have known of cases where an inspector has rejected a machine one day because of noise and the next day passed the same machine without any change in the apparatus having been made in the mean time. Such a thing may happen because the room noises were greater on the second day than on the first or perhaps simply because the inspector had had a good breakfast or for some other reason was in an amiable frame of mind.

Theory. The sensation of loudness is a function of the sound energy but is not directly proportional to it. For a given frequency the loudness is approximately directly proportional to the logarithm of the r. m. s. sound pressure. This is a form of Weber's law; namely,

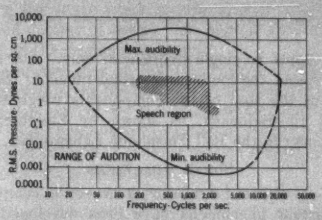

Fig. 1—Maximum and Minimum Limits of Audibility

Loudness (sensation) $= K \log_{10}$ pressure (stimulus) where K is a function of the frequency. The r. m. s. pressure is a function of the frequency (f) and amplitude (a) thus:

$$p = 41.2\ (2\ \pi\ f\ a)\ \text{dynes/cm.}^2$$

Fig. 1, due to the Bell Telephone Engineers,[1] shows the maximum and minimum limits of audibility as a function of sound pressure and frequency. The ordinary speech range is given by the cross hatched portion. Of course the minimum limit of audibility curve is an average for many observers and corresponds to perfectly quiet conditions. With ordinary room noises present the limit would be raised very considerably.

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1. Fletcher, "Useful Numerical Constants of Speech and Hearing," *Bell System Technical Journal*, Vol. IV, July, 1925.

Above the maximum audibility limit the sensation is that of feeling or even pain.

In order to obtain arbitrary loudness units which may be used for comparing various sounds, the audibility range has been divided into ten equally spaced sections (double log paper), each section corresponding to ten units. Such curves are shown by Fig. 2 as a function of r. m. s. pressure and frequency. These

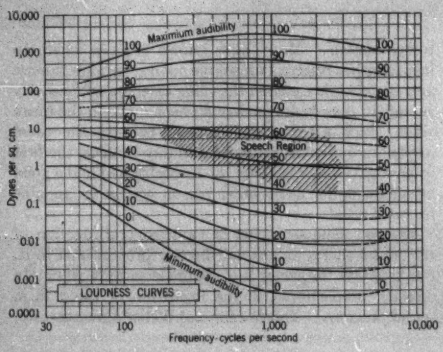

FIG. 2—LOUDNESS UNITS FOR COMPARING VARIOUS SOUNDS

curves apply to single frequencies only. For a composite sound the loudness is proportional to the logarithm of the sum of the components weighted with respect to frequency, as shown by Steinberg[2] and Kingsbury[3] in the Bell Telephone Laboratories.

SOUND ANALYZERS

Various devices for measuring the sound components of recurrent phenomena have been developed. The analyzers used for the investigations about to be de-

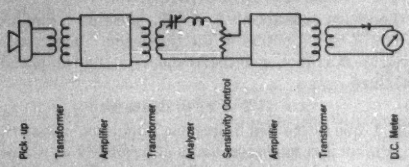

FIG. 3—FIRST NOISE ANALYZER

scribed were of the simple resonance type. Other analyzers using different methods and usually much more complicated have been built and used where it was not essential that the analyzers be portable.

2. J. C. Steinberg, "The Relation between the Loudness of a Sound and Its Physical Stimulus," *Physical Review*, Second Series, Vol. 26, pp. 507-23, October, 1925.

3. B. A. Kingsbury, "A Direct Comparison of the Loudness of Pure Tones," *Physical Review*, Second Series, Vol. 29, pp. 588-600, April, 1927.

Figs. 3 and 4 show diagramatically two types of analyzers which have been constructed. Referring to Fig. 3, the pick-up consists of an electro-dynamic microphone feeding into a step-up audio-frequency transformer, vacuum-tube amplifier, and step-down transformer. The resonant circuit of the analyzer consists of a fixed inductance and variable condenser. The voltage across the non-inductive resistance with a variable tap is fed

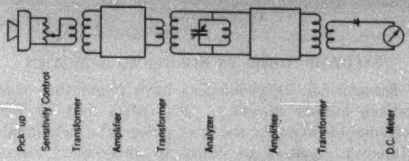

FIG. 4—SECOND NOISE ANALYZER

into a second amplifier, a step-down transformer, and a d-c. milliammeter and copper oxide rectifier. Obviously if a sound having a frequency component corresponding to the setting of the resonant circuit impinges on the microphone a reading will be obtained on the d-c. meter. If the analyzer circuit is not in resonance it will offer such a high impedance that the particular frequency in question will not produce sufficient current in the analyzer circuit to produce an appreciable drop in the non-inductive resistance. The second type of analyzer (Fig. 4) is similar to the first except that the sensitivity control is transferred to the microphone circuit and the analyzer circuit is arranged differently. In this case the resonant arrangement provides practically a short circuit for the secondary of the second transformer except when tuned to the incoming fre-

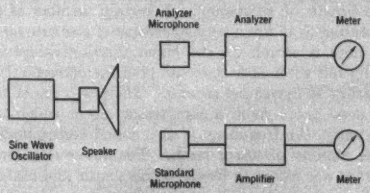

FIG. 5—METHOD OF CALIBRATING ANALYZERS

quency. When the tuning corresponds to a certain component of the sound frequency, the analyzer circuit offers a high impedance to this frequency and an appreciable voltage is applied to the input of the second amplifier. The rectifier for this second analyzer makes use of a vacuum tube rather than a copper oxide rectifier. The second analyzer gives a meter reading which varies very considerably with frequency for a given sound pressure. This is no serious disadvantage, however, since the analyzers are calibrated over the whole range of frequency.

The method of calibrating is as follows: (see Fig. 5) An electro-dynamic loud speaker capable of going to low frequencies is excited from a variable frequency sine wave source of voltage. In front of the loud speaker is placed the electro-dynamic pick-up of the analyzer to be calibrated. Beside the analyzer pick-up is placed another very carefully built electro-dynamic pick-up, the constants of which are known; namely, for a given sound pressure the generated voltage can be calculated. This standard microphone is connected through an amplifier to a milliammeter. This amplifier combined with the milliammeter is calibrated between input volts and milliammeter readings. Since the r. m. s. sound pressure is directly proportional to the generated voltage the pressure can be estimated directly from the reading of the milliammeter. Next a sine wave a-c. voltage of a known frequency is applied to the loud speaker element, the analyzer adjusted for a maximum reading by moving the variable condenser and the condenser setting and the meter reading noted. This gives an amplitude and frequency point on the calibration curve of the analyzer. This is repeated for various frequencies and amplitudes until the complete calibration curves have been obtained.

In connection with noise studies it is often necessary to measure mechanical vibrations as well. For this purpose an electrical phonograph pickup specially adapted for the purpose, together with an amplifier and osiso, has been used. Such apparatus has been developed by Mr. J. W. Legg.[4] An apparatus of this kind can be readily calibrated for amplitude. If a two-element osiso is used, a study of vibration phase angles as well as frequency and amplitude may be made. This frequently makes it possible to trace the noise further to its source by finding the particular surface from which the disturbance is radiated. The mode of vibration of the surface can also be determined and then it is possible in a large number of cases to arrive at the direct cause as well as to prescribe a remedy.

Applications

We shall now proceed to describe certain typical applications for the analyzers which have been made.

I. *Gears.* One of the first applications made of the noise analyzer was for the purpose of studying electric railway motor gear noises. These noises are of three general types:

a. Ringing noises which are the same as those which appear when a gear is struck with a hammer.

b. Tooth frequency noises.

c. Hobbing noises which are a result of imperfections in the gear cutting machine.

The ringing noise occurs only at no-load. This can readily be noted, frequently, when the power is shut off and a trolley car is running down hill. The frequency is usually of the order of from 1000 to 2000 cycles per second. If the gear is struck with a hammer when stationary its ringing frequency may be determined by means of a noise analyzer. The frequency of pinions is usually so high as to be above the audio range.

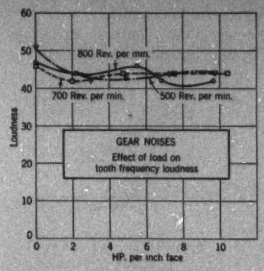

Fig. 6—Gear Noise Tests

This ringing noise can easily be eliminated by a method devised by Mr. R. E. Peterson which will be described in a paper soon to be published by him.

When the gears are running there is given forth a noise having a frequency which is proportional to the gear speed and number of teeth. The pitch is usually much lower than the ringing frequency.

If the master gear on the gear cutting machine has imperfections in the teeth, which sometimes occurs, the gears which are cut by this machine will have a frequency when running corresponding to the rev. per min. and number of teeth on the gear cutting machine. This is known as the hobbing noise.

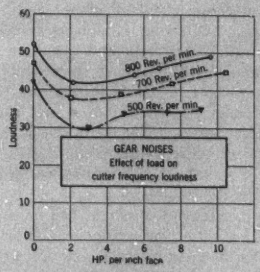

Fig. 7—Gear Noise Tests

If a set of gears is put in operation and the various frequencies are determined by the analyzer, it is very easy to determine the gear, or gears, responsible for the noises because the ringing and tooth frequencies are already known or can easily be determined. Any other frequency must be due to imperfections in the cutting of the teeth. Figs. 6, 7, 8, and 9 show some typical gear

4. J. W. Legg, *Expansion of Oscillography by the Portable Instrument*, A. I. E. E. Jour., Vol. 42, pp. 106-11, February, 1923. J. W. Legg, "Oscillography," *Elec. Jour.*, Vol. XXIV, No. 6, pp. 268-72; No. 7, pp. 341-46; No. 8 pp., 398-405; June-August, 1927.

noise tests made on commercial somewhat imperfect gears as a function of load and frequency.

Electric Motors and Generators. There are several causes of noise from rotating electrical machines as follows:

a. Tooth frequency noises (stator and rotor).
b. Windage (fans).
c. Natural period of frames.
d. For single-phase motors, variations in torque and forced frame vibrations.

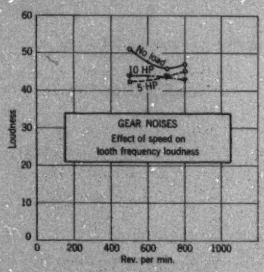

Fig. 8—Gear Noise Tests

In induction motors having rotor slots there is nearly always an appreciable amount of vibration which is proportional to the number of stator and rotor teeth and the rotor speed. These noises are the result of reluctance pulsations which produce vibrations of the teeth and sometimes the frame. Often these noises are very considerably increased as the load becomes larger. An interesting case of magnetic noise recently came to our attention in connection with a salient pole synchronous machine. This machine produced a very considerable noise having a frequency corresponding to the number of stator teeth and the rotor speed. An analysis showed that the pole width and stator slot pitch were such that at one instant practically three teeth were under the pole and an instant later when the pole had advanced one-half a stator slot pitch practically four teeth were under the pole. This gave rise to very appreciable reluctance pulsations between the rotor and stator. This condition probably would not have produced any appreciable noise if it had not been for one further unfortunate circumstance. The pole pitch was such that at a certain instant several adjacent poles were subjected to approximately minimum reluctance conditions and then an instant later to maximum reluctance. This resulted in a very considerable frame vibration having the frequency noted above.

When motors are supplied with ventilating fans it is quite usual to obtain appreciable noise which is a function of the motor speed and the number of blades on the fan. This is known as the siren frequency and often consists not only of the fundamental but also of harmonics. Other portions of the rotor also may produce siren frequencies.

It occasionally happens that a machine may be operated at such a speed that some frequency component may be very close to the natural period of the frame or some other portion of apparatus. In this case noises may be produced which are considerably in excess of those which occur at some slightly different speed. A slight change in speed will reveal such a condition.

Single-phase, 60-cycle motors almost always have appreciable 120-cycle noises. These are due to torque variations or to frame vibrations. Also not infrequently harmonics of 120-cycles may appear.

In studying noises from small motors where the sounds are of no great magnitude and the microphone is placed close to the motor, care must be taken to shield the microphone from stray fields or to keep it sufficiently far removed from the apparatus under test. Otherwise, odd harmonics of the voltage will appear in the analyzer which are not caused by the sound.

Induction Regulators. For 60-cycle regulators sound analyses so far as we have observed will always show that the frequency consists mainly of a 120-cycle component. In the past it has usually been thought that when a regulator was particularly noisy the causes were dissymmetry in the magnetic fields and loose bearings. Recently some three-phase regulators were studied to determine the source of the noise by means of the vibration apparatus mentioned above. The phono-

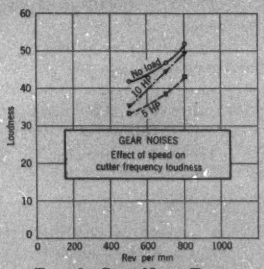

Fig. 9—Gear Noise Tests

graph pick-up and amplifier were connected to one element of an osiso, a second element being operated from the exciting current of one of the phases in order to supply a phase reference point. Vibration measurements on the rotor and various parts of the stator disclosed the fact that the rotor movements were much less than the stator vibrations and there were no appreciable torque vibrations. It was disclosed that the stator punchings and frame assumed an elliptical form, the ellipse revolving with the rotating field.

Obviously then the method of minimizing the noise from this particular type of regulator was to stiffen up the stator punchings and frame.

Vacuum Sweepers. These household devices operate at a very high speed, usually of the order of 10,000 rev. per min. The various principal causes of noise are as follows:

a. Sweeper fan noises.
b. Motor and ventilating fan noises.
c. Unbalance.

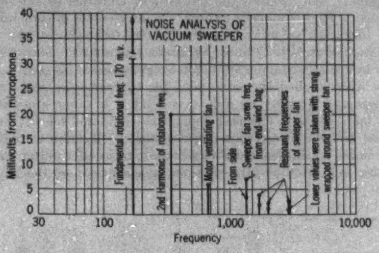

FIG. 10—NOISE ANALYSIS OF VACUUM SWEEPER

Figs. 10 and 11 show noise analysis results on a well-known vacuum sweeper. Fig. 10 gives the noise components as a function of microphone millivolts which are proportional to sound pressures and Fig. 11 gives the same data as a function of loudness units. (See Fig. 2.) It will be seen that in terms of loudness units the higher frequency components are much more important than would be indicated from the sound pressure spectrum, due to the greater sensitivity of the ear at the higher frequencies. It was found by striking the fan, thus causing it to vibrate and measuring the frequencies with the noise analyzer, that the sweeper fan had three natural frequencies corresponding to three different modes of vibration. It will be seen from the spectrum that under operating conditions all of these frequencies appeared to be giving comparatively large noise components. Of course the siren frequency of the sweeper fan was a rather large percentage of the total noise. The motor ventilating fan gave a surprisingly loud noise considering the fact that it was much smaller than the main sweeper fan. The particular sweeper under test was not very accurately balanced; therefore, a very large frequency component of approximately 170 cycles resulted. This corresponds to approximately 10,000 rev. per min. Also there appeared a very appreciable second harmonic of the rotational frequency.

Obviously if an attempt were made to alter this sweeper so that it would be quiet it would be of very little use to reduce any one component to a small value because the total noise could thereby be decreased only slightly. The production of a really quiet vacuum sweeper is a problem of no little difficulty.

Loud Speakers. Since its development in connection with noise problems, the sound analyzer has found other applications. As an example, it may be used for obtaining response curves for loudspeakers. While other methods now in use may have the advantage of greater speed, the use of the analyzer eliminates the necessity of a pure voltage wave for the speaker supply. This is true since the analyzer can measure the pressure of the fundamental of the sound wave produced by the speaker and reject any harmonics whether they are due to harmonics in the supply or are generated in the speaker.

Conclusions. We might enumerate other examples of noise analyses but those given are sufficient for illustration. The quantitative determination of noise is very difficult even with an analyzer due to nodes and sound reflections. Noise often varies very greatly in different locations around a piece of apparatus and it varies with the position of the objects in the room. For the accurate study of noise, the apparatus should be taken to a sound proof room with deadened walls so that there will be little reflection. However, in a large number of instances where only one or two frequency components are important qualitative results are quite sufficient. In other cases when tests must be run under shop conditions only qualitative results can be obtained due to a large amount of reflection as well as interference from the shop noises. In these cases results can be obtained very quickly by means of the analyzer by using head phones in place of the meter ordinarily used to measure amplitude. Then even though the surrounding noise level is very high it is possible to obtain accurate frequency determinations of the noise being studied. Usually if the frequencies of the principal components can be determined the cause or causes of the noise can be assigned and means taken to eliminate or reduce it. Therefore, a qualitative test is very often sufficient to enable the designer to avoid similar conditions in future apparatus.

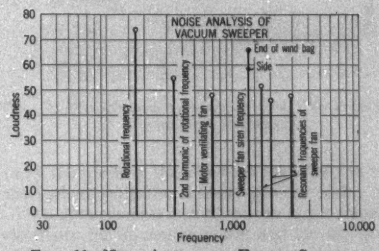

FIG. 11—NOISE ANALYSIS OF VACUUM SWEEPER

We are greatly indebted to Mr. R. E. Peterson who has given considerable assistance and many valuable suggestions in the work on gear noise. We also wish to acknowledge the assistance rendered by Mr. C. F. Royce in certain phases of this work.

Discussion

H. C. Specht: The method described in this paper is for measuring the frequency of noise; however, as manufacturers of electrical machines we are also interested in the amount of noise. To measure the degree of loudness is very difficult as there is no accepted unit for comparison and because of different responses of different ears. It is very difficult to agree on a definite maximum level of noise which may be considered unobjectionable. Also, the loudness depends on the kind of body which is in contact with the noise producer. If the natural frequency of the body is the same as that of the noise, there may result very great amplification.

Unfortunately, the test of noise in regular production is conducted by means of the human ear, which cannot be considered as very satisfactory, as the ears of different persons are not the same and even the sensitiveness of the same person at different times is not always alike. Another difficulty is to build inside a noisy factory building, a noise-proof room. From time to time the room must be opened, at which time the factory noise will enter and it will then take some time before the ears of the tester are again adjusted for testing. It would be far better to have the noise-proof room entirely separated from the factory building, but this, of course, would be undesirable on account of the cost of transportation.

J. P. Foltz: I believe the points mentioned by Mr. Specht were very well taken. We have been confronted from time to time with problems of commercial noise testing and it has been quite a problem to us inasmuch as it is rather complicated. We have to deal with unequal ear sensitivity as regards different frequencies as well as surrounding shop noises. In the case of induction regulators, however, we have used an entirely different principle. We measure the mechanical vibration of the outer surface. That eliminates entirely the interference of shop noises and inasmuch as in that case only one frequency exists, the variation of sensitivity of the ear also is eliminated. It was very easy to devise a commercial test for regulators in that way. We have since applied the same principle to gear cases with some success and we may be able to extend it to other apparatus. The fact is that in enclosed apparatus we do not get any noise unless we have a corresponding mechanical vibration to radiate that noise, so this principle may in most cases offer better possibilities as a commercial test than measurement of the actual noise produced.

Electrical Aids to Navigation*

BY ROBERT H. MARRIOTT†

Fellow, A. I. E. E.

Synopsis.—Electrical and magnetic aids to navigation by water and air are outlined in this paper. The principal subjects discussed are the earth inductor compass, the radio compass, the radio beacon, submarine signals, depth finders, distance finders, channel-marking signals, fog-penetrating lights, height indicators for aircraft, etc. A bibliography is included.

THIS paper points out that there are numerous ways in which electricity and magnetism aid air and water navigation, partially describes and illustrates some of the less familiar aids which are in use or are coming into use, and mentions some aids that may come into use.

Magnetism as an aid to navigation goes back to the more romantic probabilities, history, and tradition of the past. By tradition natural magnetic iron is said to have been known as far back as 2634 B. C., by history,

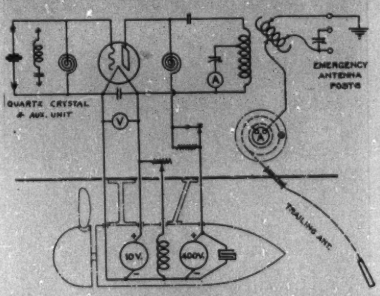

FIG. 1—DIAGRAM OF CRYSTAL-CONTROLLED AIRCRAFT RADIO TRANSMITTER

as far back as 800 B. C. The now poetic name "Lode Stone" was the name of the natural permanent magnet found in Magnesia. The Lode Stone was the first magnetic compass used to aid navigation. The attracting power of the Lode Stone and the name "Magnesia" gave rise to such names as "magnet" and "magnetism." Speaking broadly, experiments with the Lode Stone resulted in the production of electric current, as the steel magnet was first made by contact with the Lode Stone and later used to produce electric current. In other words, the Lode Stone was one of the first instruments used as an aid to navigation and was an ancestor of dynamo-made electricity.

The word "navigation," judging from the Latin words from which it was made, originally related to the moving or propelling of a ship. Now we usually consider that word in connection with the steering of a ship. However, electricity aids under both meanings. Battleships are driven by a-c. motors and submarines are driven from storage battery power by d-c. motors. Electric winches are used to pull vessels into dock. Electrically driven gyroscopes are used to make ice breaking ships roll and to keep yachts from rolling. Electric ignition is used in the propulsion of both air and marine craft.

Electric navigation lights and docking or landing lights are used for both air and marine craft. Almost every form of electrical communication, signaling, and remote control device is used as an aid to navigation. Even loud speakers with audio amplifiers are used to

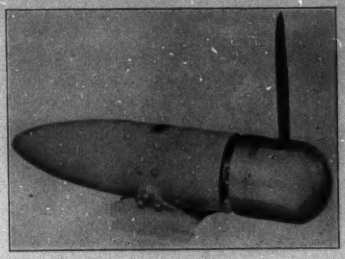

FIG. 2—WIND-DRIVEN GENERATOR WITH DESLAURIERS SELF REGULATING PROPELLER

carry the words of the officer on the bridge to men on shore when navigating a vessel into dry dock.

The gyro-compass is a gyroscope but it depends on electricity for its motive power and for the operation of the control devices that go with it.

Practically all of the electrical comforts used on land contribute to the comforts in the navigation of the modern steamships.

The coming of air navigation is expanding the use of electrical devices for aiding that rapidly growing navigation. The success of air navigation apparently depends largely on the ability of air craft to dodge dangerous storms. To do this the air craft must be fast and the pilots must know where storms are, how violent they are, how much area they cover, how fast they are moving, and what direction they are following.

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More and more information regarding storms is being supplied for air craft by wire line and radio communication. The fundamental principle is to collect weather information from a number of widely distributed observers rapidly, translate all of the information into definite statements of storm conditions and deliver those statements to the pilots, promptly.

Fig. 3—Mechanism of Self-Regulating Propeller

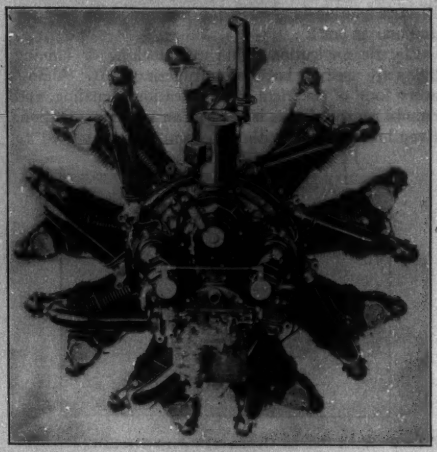


Fig. 4—Generator Geared to an Air-Craft Engine

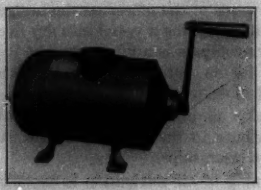


Fig. 5—Hand-Driven Generator for Emergency Aircraft Radio Use

We have repeatedly seen that great feats have been carried out by airplanes and dirigible balloons when severe storms were not met and we have seen great catastrophies when those craft have met storms. The electrical means of communication by wire and by radio applied to the rapid supplying of weather information to the pilots evidently must be one of the greatest aids for successful air navigation.

The devices to be used for that purpose are the radio apparatus on ships, radio and wire apparatus on land, and to some extent the cables under the seas.

The life and property saving ability of the radio S. O. S. call in connection with water navigation forced the use of radio on practically all large ships and for some years the position of icebergs and other menaces to navigation have been regularly reported to ships by the same radio communication that is used for other purposes.

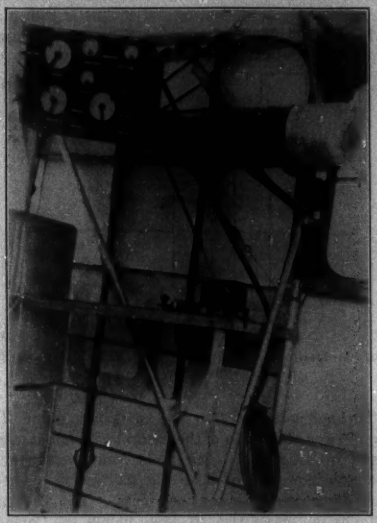


Fig. 6—Radio Cabin of Airplane *Roma*. Retractable Wind-Driven Generator Swung in

The foregoing electrical aids to navigation need no description. The main point about them is that they are numerous examples of the electrical engineer's contributions to navigation. There are some special electrical aids that will bear some description, such as for example, the earth inductor compass, the radio compass, and the depth finder.

There is some relationship between the old magnetic compass and some of our more modern aids. The magnetic compass, the earth inductor compass, the radio compass, and the compass used to follow an energized cable in a channel, are all magnetic flux indicators. The first two indicate the south-to-north

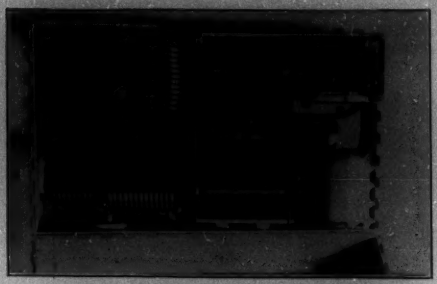


FIG. 7—INTERIOR OF AN AIRPLANE TRANSMITTER

FIG. 8—150-WATT RADIO TRANSMITTER OF AIRPLANE *America*

FIG. 9—RADIO ROOM OF THE *Graf Zepplin*, 1928

Including a 70- and a 140-watt telephone and telegraph transmitter, a short wave transmitter, radio compass, and four receivers

pole direction of the flux lines of the earth. The other two indicate the direction of the side movement of lines of flux set up at some center such as an antenna or other conductor of alternating current and thereby are used to locate that center. The first two indicate

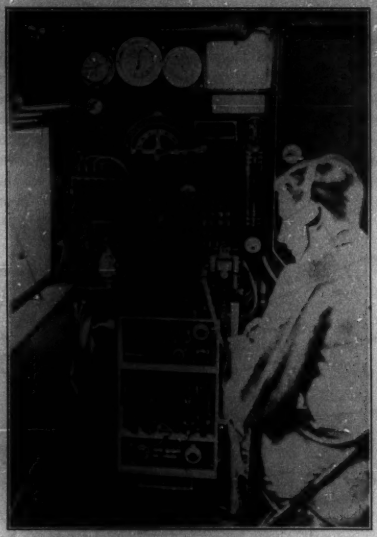
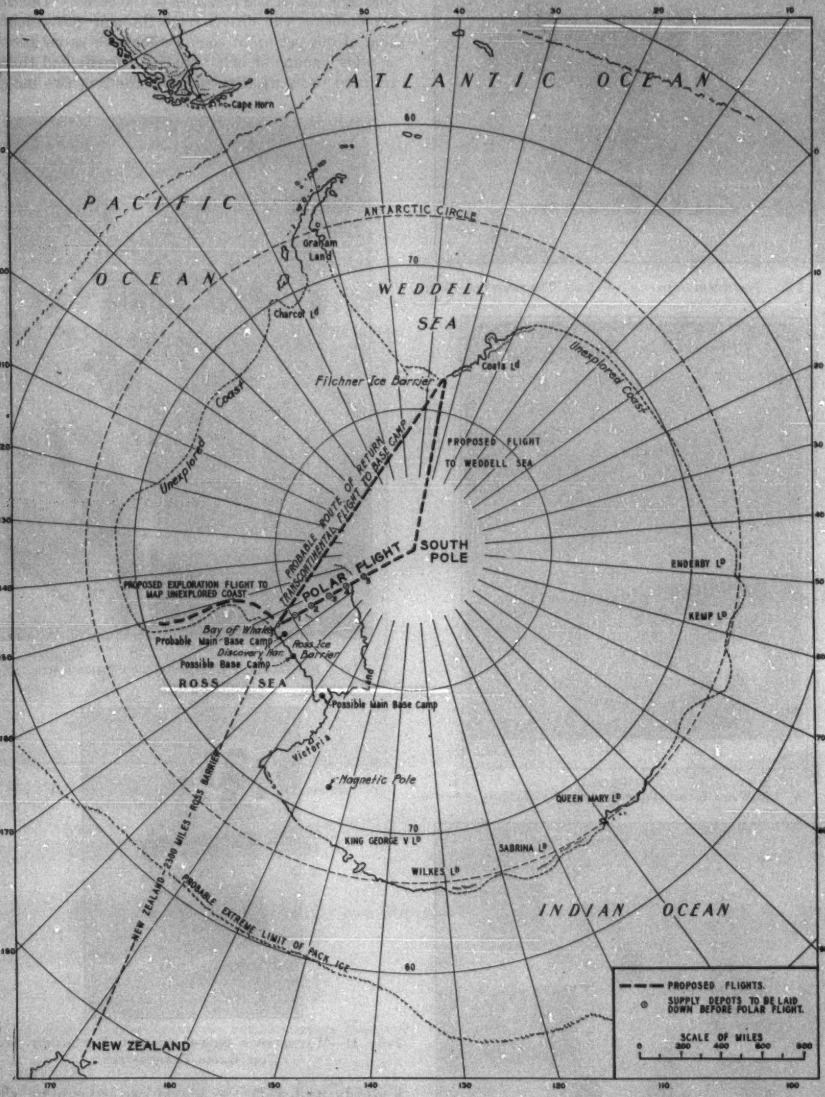


FIG. 10—RADIO INSTALLATION IN A BRITISH AIRLINE

FIG. 11—WATERPROOF SELF-CONTAINED EMERGENCY SPARK COIL RADIO TRANSMITTER

a steady field while the second two indicate an alternating field of flux that is accompanied by an alternating electric field.

The magnetic compass is a polarized iron armature that is attracted into a position which is parallel to



Courtesy of *The Scientific American*

Fig. 12

The Antartic regions showing main bases and routes of exploratory marches and flights of the Byrd expedition. Here, radio and other electrical aids are expected to be of great service in navigating ice, air, and water.

the earth's flux. The earth inductor compass is a dynamo, its poles are the north and south poles of the earth, and its rotating armature consists of loops of wire connected to commutator segments.

Fig. 13—Generator Unit Driven by a Gasoline Engine, for the Advance-Parties of the Byrd Antartic Expedition

This expedition includes in its equipment 33 radio transmitters and receivers for use in navigating the air, water, and ice. Two radio compasses will be used at base locations suitable for triangulation to locate all planes and ice navigating parties. Dry batteries and gasoline engine-driven generators are provided for locations where ship power is not available

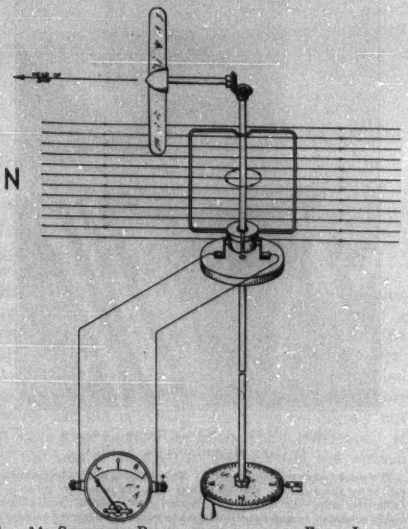


Fig. 14—Schematic Representation of the Earth Inductor Compass

The other two compasses may be said to be of the stationary armature type that are cut by alternating magnetic fields. They consist fundamentally of single loops that may be revolved to positions for cutting from a maximum to a minimum phase difference in the alternating fluxes. When the planes of these loops are at right angles to the direction of the source of the alternating magnetic flux the minimum current is set up, and that indicates the direction of the source.

Earth Inductor Compass

The earth inductor compass is the device that is most closely related to the old magnetic compass. In an airplane the compass must be close to the pilot or navigator where he can see it. But there may be iron nearby which makes the magnetic compass inaccurate. The revolving coil of the earth inductor compass can be located at some distance from interfering iron and the current from it can be conducted to a galvanometer located where the pilot or navigator can see it.

Fig. 15—Earth Inductor Compass Wind Mill and Generator

The earth inductor compass is a d-c. dynamo in which the magnetic field is supplied by the earth. It will be seen by reference to Fig. 14 that a windmill at the top revolves a one-turn armature winding connected to a commutator, to cut the earth's north-to-south field, and that the brushes connect to a meter. Also it will be noted that the position of the brushes can be shifted. The meter is a d-c. galvanometer; therefore if the brushes are set in one position a maximum deflection will be obtained when the windmill is turning the armature rapidly. With the brushes moved to a position at right angles no deflection will result. By revolving the brushes an additional ninety degrees a reverse deflection will be obtained. Since the field is fixed in space, turning the whole instrument produces the same result as shifting the brushes the same amount.

If the device is mounted on an airplane and the

plane is headed in the direction it is to follow and the brush controller is turned to produce no deflection on the galvanometer, there will be no deflection so long as the plane continues in that course. If the airplane turns off course to the left the galvanometer pointer will move to the left and vice versa if the plane heads to the right. Should the airplane be turned completely around, the galvanometer pointer will move to the right when the plane moves to the left. The device is used in the simplest way by starting the plane in the direction it is to go, turning the brush controller to put the galvanometer pointer on zero, and then steering the plane so the pointer will remain on zero.

Fig. 15 shows the horizontal type of self adjusting windmill at the top and the case below which contains the revolving armature, in gimbals that permit the

Fig. 16—Controller of the Earth Inductor Compass

revolving armature to remain vertical by its gyroscopic action when the plane heads up or down. Fig. 16 shows the brush controller and Fig. 17 shows the galvanometer.

While the principle is simple, fine workmanship is required to produce an accurate and strong instrument. For example, flexible shafts are used and the copper commutator with its silver brushes revolves in filtered oil with a wiper on the commutator to remove tiny particles of metal. The armature winding includes 24 sections of many turns each. The armature develops about 1.5 millivolts at 1800 rev. per min.

Radio Compass

A radio compass, direction finder, or goniometer is a radio receiving device which permits determination of the line of travel of waves as received from a transmitting station.

According to the usual explanation, radio waves are composed of two parts,—an electric part and a magnetic part. The electric part moves up and down from the earth and may be imagined to be like the ripples in water that spread out when a stone is dropped into

Fig. 17—Galvanometer or Indicator of the Earth Inductor Compass

Fig. 18—U. S. Naval Radio Compass at Cape Hinchinbrook, Alaska

The second story contains the coil. The platform with railing, above the coil, is for the calibrator and his transit while following the course of the calibrating ship. An underground control cable leads to the transmitter which is located at a distance sufficient to avoid affecting the accuracy of the compass

the water. That is, the electric portion of the electromagnetic waves move out about the same way from the transmitting antenna. The magnetic part also moves out from the transmitting antenna not as an up and down movement but as expanding rings of

magnetic flux which are revolving successively in opposite directions parallel to the earth's surface about the transmitting antenna. The change from a maximum in one direction to a maximum in the other occurs in a sinusoidal manner; therefore if two conductors

Fig. 19—Steamship Type of Radio Compass

vertical to the earth's surface are cut by these flux lines and one of the conductors is one-quarter of a wavelength farther from the transmitter than the other, then the currents set up in them will be 90 deg. out of phase and if those conductors are connected together at the top and bottom, forming a loop, these currents will not neutralize each other completely but will leave some current due to the phase difference. If now the loop is revolved so that both of the vertical conductors of the loop are the same distance from the station there will be no phase difference and there will be no current in the loop. Therefore under ideal conditions when a loop antenna is turned broadside to the radio station no signal is heard and that is the fundamental feature of a radio compass. If the radio waves arriving at that point are always tilted forward or backward the broadside position will not show a zero but a minimum which is usually satisfactory. Also if there are certain fixed conductors in certain directions, then the flux lines arriving from those directions will not be exactly at right angles to the direction of the transmitter and the scale of the compass must be corrected accordingly.

The radio compass must be carefully built, properly located, calibrated under service conditions, and corrected to suit the peculiarities of its location. When these things are done a good degree and sometimes a remarkably fine degree of accuracy is obtainable. Otherwise the radio compass is much like the loop type of broadcast receiver with which many are familiar and with which good or poor directional effects may have been obtained depending on the receiver and the location.

The loop antenna or radio compass is not new. It is sometimes called the Hertz loop. But as the energy in it depends on phase difference, the energy is small

Fig. 20—The Radio Receiver, Indicators, and Hand Wheel of the Steamship Type of Radio Compass

compared to the energy in an open antenna. It was the amplifying ability of the vacuum tube that brought the loop back into use long after the days of Heinrich Hertz.

In the United States the Navy Department maintains radio compass stations at suitable points along the coast lines, Fig. 18. All navigators have the exact

location of these compass stations on their charts. When a ship wants a bearing during thick weather it calls for a navy compass station. A compass station replies and asks the ship to transmit for a sufficient interval for the compass station to get the ship's bearing relative to the compass station. This information is then transmitted to the ship. If two compass stations can get the ship's bearing at the same time, the exact location of the ship can be calculated from the triangle. Otherwise the ship moves by magnetic or gyroscopic compass for a certain distance and then gets another bearing from the one compass station, thus completing the triangle.

These compass stations provide free aid to navigation in peace time and provide the Navy with stations and trained personnel for use in war time. In Europe

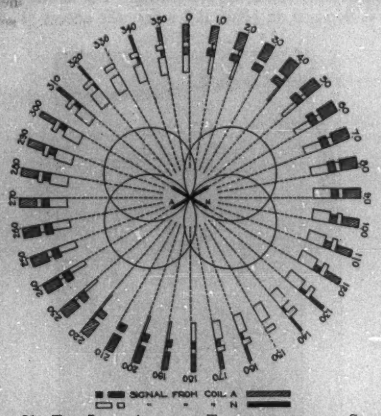


FIG. 21—TWO-LOOP ANTENNA TRANSMITTING THE SIGNALS *A* AND *N*

If the aircraft approaches the transmitter along the lines 0, 90, 180, or 270 equal volume *A* and *B* signals will be received. On either side of those lines one will be louder than the other

similar compass stations are located along the lines of air travel. They keep following the craft in the air and tell them by radio telephone where they are.

Fig. 19 shows a type of radio compass that is used on steamships. The hollow square at the top is a tube containing the turns of the compass loop. That square portion is above the top of the pilot or chart house and is sturdy to withstand weather. The loop is turned by the hand wheel shown below. Fig. 20 shows more plainly the hand wheel, compass scale, and radio receiver. Similar radio compasses of lighter construction are used on large aircraft.

RADIO BEACON

A radio beacon is a radio transmitting station in a fixed geographic location which emits a distinctive or characteristic signal for enabling mobile receiving stations to determine bearings.

Such a beacon may transmit in all directions, or chiefly in one, two, or four directions or it may be directive and revolve, giving different indicating signals for different directions. When the beacon transmits equally in all directions a radio compass is necessary for the receiver to determine the direction of the beacon from the receiver. The direction of a revolving or fixed directive beacon may be determined by the receiver without the use of the radio compass. With a revolving directive beacon the ship's radio operator can tell when he is at a certain angle from the charted beacon he is receiving from because at that angle the signals come to him the loudest and the character of the signals or revolution timing from north or east signal tell him what the angle is. With the fixed directive beacon the receiver gets the maximum signal or characteristic signal when following the path along which the beacon sends the maximum signal or some characteristic signal.

For aviation, low power marker beacons which radiate in all directions but give a characteristic signal are located along air routes. If the flyer sticks to his course he will hear them one after another as he flies near them. Directly over them he will not get a signal from a vertical antenna. The combination of marker beacons, directive beacons, and weather reports by radio telephone can be of great assistance to the flyer, who has no space for long range receiving apparatus and a radio compass.

Fig. 21 represents a directive beacon that sends two signals A and N in Morse code at different angles. When the flyer is on his course he will hear A and N equally well which will make them sound like a dash to him. If he gets off to one side of his course the dot-dash of the A will be noticeable, if he gets off to the other side the dash-dot of the N will be noticeable.

Fig. 22 shows a circuit diagram of one of the types of beacons used. The triangular shaped directive loop antenna are shown at the right. It will be seen that this apparatus depends largely on vacuum tubes for use as radio and audio frequency generators and amplifiers. Such use of vacuum tubes is characteristic of present day direction finding apparatus. It is the vacuum tube that has made possible many of our aids to navigation.

Fig. 23 shows the general appearance of an air-beacon station. Fig. 24 shows the small receiver in shock absorbers, that is used on small aircraft for receiving radio beacon signals and radio telephone messages.

Fig. 25 shows a visual device that may be mounted on the board in the front of the pilot. It operates something like a reed type frequency meter. A beacon station sending two signals at two angles and modulated to the frequencies of the reeds can cause one reed to vibrate farthest when the aircraft is off the

course on one side, the other to vibrate farthest when off the other side, and both to vibrate the same amount when he is on the course.

Submarine Signal

The electrical submarine signal transmitter is a large tuned steel diaphram driven by alternating current. The submarine signal receiver is, in its simplest form, a small diaphram and microphone button, like a telephone transmitter, connected through a battery to a telephone receiver.

The submarine transmitter is, for example, suspended below a light-ship and actuated by groups of short and long signals that are characteristic of that light-ship. All vessels equipped with submarine receivers can pick up those characteristic signals through the water when the light-ship cannot be seen through fog or when a sound could not be heard over the same distance through the air.

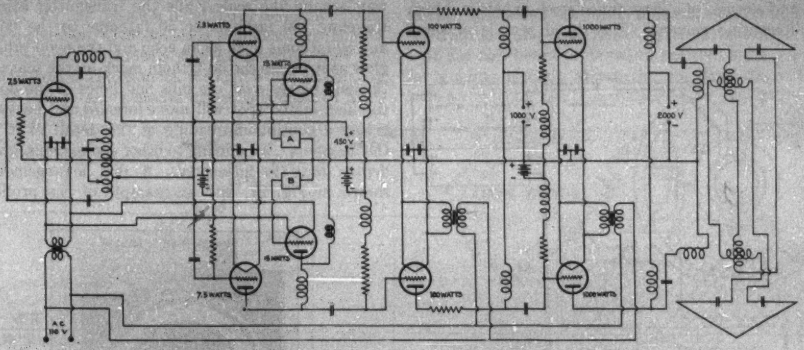


Fig. 22—Circuit Diagram of Modulation Type of Beacon

Fig. 23—Bureau of Standards Directive Radio Beacon at College Park, Maryland

Fig. 24—Airplane Receiver for Radio Beacon and Telephone

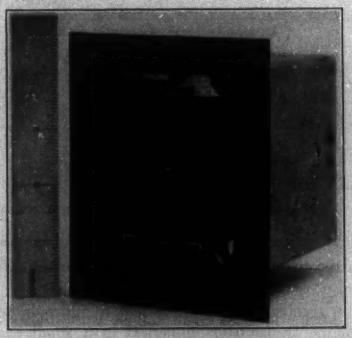


Fig. 25—Reed Type Visual Indicator of the Course

Such signals have been heard in this way as far as 80 miles. There are numerous submarine signal transmitters of the pneumatic bell and electrical types and probably over two hundred steamship lines have equipped their vessels with submarine signal receivers.

Depth Finding

Echo depth sounding, the sonic depth finder, and the fathometer all refer to the same principle, namely, sending a submarine signal from a vessel and receiving on the same vessel the sound or echo that is reflected from the bottom.

The submarine transmitter is attached to the vessel's hull and consists of a large tuned steel diaphram driven by alternating current. The signal sent is of short

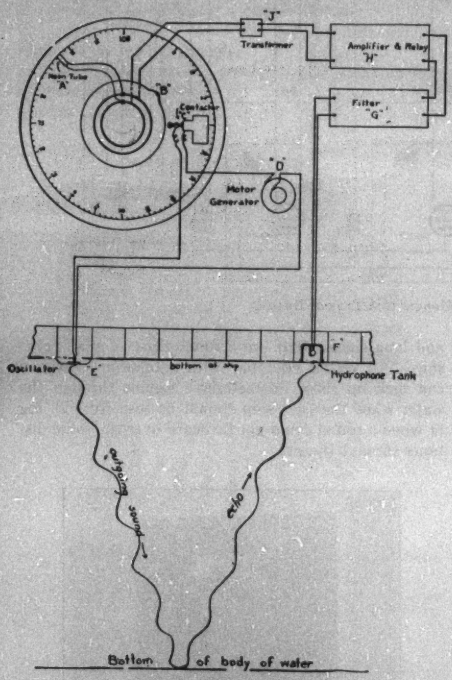


Fig. 26—Schematic Representation of the Mechanics of Depth Finding

duration to prevent its interference with the reflected sound or echo.

The receiver sometimes called a hydrophone is a telephone transmitter immersed in a small enclosed metal tank of fresh water on the ship's hull at some distance from the transmitter, Fig. 26. The telephone transmitter is connected through its source of direct current and a filter for reducing other sounds, to a vacuum tube amplifier which operates a relay. The relay through an electrically energized transformer causes a neon Geisler tube *A* to glow when an echo is received. The motor driven Geisler tube revolves like a pointer back of a scale that is transparent at its edge. The scale is calibrated in fathoms. The Geisler or neon tube pointer makes a revolution in one-quarter of a second and every time it passes zero the transmitter is operated by *B*. Sound travels about 4800 feet or 800 fathoms per second; therefore the neon pointer will travel for one-eighth of a second or half way around the scale while the transmitter sound is traveling 100 fathoms or 50 fathoms to the bottom and 50 fathoms back as an echo and the neon tube will glow as it passes the 50-fathom mark on the scale. As the ship passes over the hills and holes in the bottom the illuminated spot will move forward and backward, giving a continuous reading of the depth as it is at the moment. By shifting to other connections, greater depths may be measured. A dial arrangement is shown in Fig. 27, and an example of the profile of

Fig. 27—The Depth-Indicating Dial

sea bottom that may be obtained in this way in Fig. 28.

Combined Submarine Sound Beacon and Radio Beacon

In this, the submarine signal transmitters and receivers as described above are used in cooperation with a radio beacon transmitter and receiver. For example, one key on a light-ship is used to operate both a submarine transmitter and a radio transmitter at the same time, sending a short signal. The receiving ship's officer listens with head phones that are connected to both a submarine receiver and a radio receiver. He hears the radio signal almost instantly and presses a button that starts a clock. Later he hears the submarine signal and presses the button again, stopping the clock at a figure which corresponds to the number of nautical miles between his ship and the light-ship.

Possible and Probable Electrical Aids to Navigation

There are several electrical schemes which may prove to be valuable for use as aids to navigation,—

schemes that range all the way from those which have been tried and seem to be practical to schemes that may be only visionary.

For example, a device that will enable vessels to navigate a narrow, crooked channel, in fog, has been demonstrated on large transatlantic liners and naval craft by the U. S. Navy Department in New York Harbor. By the device a blindfolded helmsman could follow a channel. The device is comparatively simple. The transmitter includes a single conductor cable laid in the bottom of the channel, through which alternating current of say one thousand cycles is conducted to its outer end which is grounded. The receiver on a steel vessel includes two flat coils of wire hung on the port and starboard sides of the ship, parallel to the ship's sides and connected to two head phones or other indicators. When the keel of the ship is steered directly over the cable the responses from the two coils will be equal. When the ship gets off on the starboard side the greatest response will come from the port-side coil and vice versa, until the ship is brought back to where it should be. A single coil in gimbals can be used on wooden ships, Fig. 29.

One cable carrying a distinctive signal, even speech, can be used for incoming vessels and another for outgoing vessels.

Such a device has been proposed time and again and our Navy spent a great deal of money in demonstrating its practicability. However, the device is not used even in such an important channel as that leading to New York. Its lack of use seems to stand as a criticism of the traditions and principles of water navigation.

It has also beeen proposed to use that same scheme for guiding aircraft across the country. In France such a conductor has been installed on a pole line.

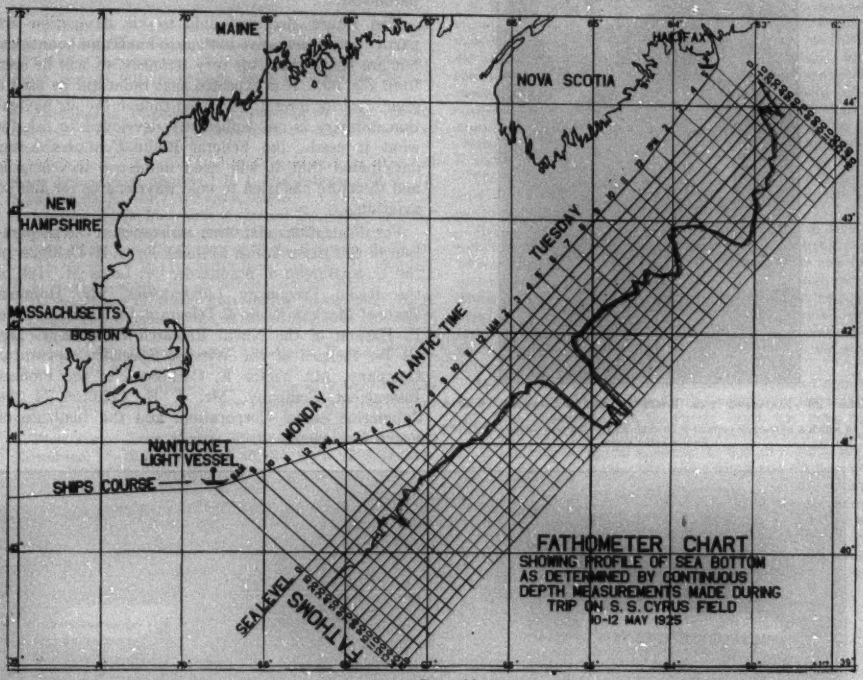


Fig. 28

It is proposed to use several high-power Geisler types of tubes containing neon or other gases producing light which will penentrate farther through fog than ordinary light will penetrate.

Also it is proposed to use the photoelectric cell which is more sensitive than the eye to the light rays that penetrate fog. The effect on the photoelectric cell is to be amplified to operate an indicator such as a galvanometer.

Aviators are provided with barometers to indicate their height above sea level but that is not an indication

of their height above the ground unless they know where they are and know the height of the ground below them. Therefore several devices have been proposed to measure height above the ground.

As aircraft make a great deal of noise it is proposed to produce sound signals on those craft which are higher in frequency than any of the sounds produced by the aircraft and pick up those signals after reflection from the earth, by means of an indicating device tuned to their frequency. That scheme is similar to the depth finder used in water navigation except that higher sound frequencies are proposed.

FIG. 29—LOCATING THE POSITION OF A CHANNEL CABLE

[In which a 1000-cycle current is flowing, by turning and tilting a single coil on a wooden vessel

Another scheme involves the transmitting of radio waves to ground and picking up their reflection on the air craft. The strength of the reflected signal may work out to be the factor for indicating height or the phase relation of the outgoing frequency to the reflected frequency may give the altitude in terms of the radio wavelength.

Over a radio beacon using a vertical antenna the aviator probably could estimate his height by the diameter of the dead area directly above the station.

The variations of the air dielectric of a condenser with altitude has been proposed as a substitute for a barometer.

The future electrical aids to air navigation are particularly speculative both as to kinds and quantities, but are being taken up very seriously, as will be seen from Fig. 30. It is expected that radio will be used a great deal for aircraft and even though the air navigation industry is not sufficiently developed to ask for what it needs, the Federal Radio Commission has anticipated that it will need numerous wavelengths and therefore has tried to hold wavelengths for aids to aviation.

For illustrations and other assistance in the preparation of this paper I wish to thank Dr. J. H. Dellinger of the U. S. Bureau of Standards, Dr. Louis M. Hull of the Radio Frequency Laboratories, Mr. Haraden Pratt of Mackay Radio & Telegraph Co., Mr. Malcolm P. Hanson of the Naval Research Laboratory, Mr. G. W. Pickard of the Wireless Specialty Apparatus Company, Mr. Victor E. Carbonara of the Pioneer Instrument Company, Mr. J. E. Colloton of the Submarine Signal Corporation, and the Institute of Radio Engineers.

FIG. 30—LABORATORY, HANGAR, AND LANDING FIELD NEAR COMPLETION AT BOONTON, NEW JERSEY

Exclusively for research in and development of radio and other aids to air navigation

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Discussion

P. A. Borden: In the depth sounding apparatus which Mr. Marriott mentioned, the telephone receiver attached to the hull of the ship is immersed in a tank of fresh water. I was a little puzzled as to why that should be necessary.

G. D. Robinson: I might venture partially to answer that question. A fluid of the same density as water is used within the tank to provide transmission of the sound to the microphone; with only air in the tank, most of the sound arriving is reflected and almost none would reach the microphone.

E. S. Lee: Of interest on the general subject of the measurement of non-electrical quantities by electrical means is the recent measurement of the shaft horsepower output of the electrically driven airplane carriers U. S. S. *Lexington* and *Saratoga*. Measurements of shaft horsepower output of large ships are usually made mechanically by torsion meters as part of the shaft. Electrical measurement is possible in the electrically driven ships by measuring the input to the motors and multiplying this by the efficiency of the motors. The means are old, but the application is new.

The method was proved several years ago on the U. S. S. *Maryland* where measurements were made with torsion meters and electrical measuring instruments at the same time.

Measurements on the U. S. S. *Lexington* and U. S. S. *Saratoga* were made by measuring the power input into each motor phase with single-phase indicating wattmeters. Ammeters and voltmeters were used for measuring current and voltage values. There being 8 three-phase motors, 24 sets of measuring equipment were necessary complete with current and potential transformers. Calibrations were required to cover a current range of five to one, a voltage range of three to one, frequency and power factor range of two to one.

The extent of the measurement can be ascertained from the fact that the weight of the measuring equipment boxed, ready to ship from Schenectady to the Pacific Coast, was three tons. Fifty men were required to observe all electrical instruments, to make calibrations, and to supervise the test, in order to get all completed results onto the trial board within one-half hour after completion of any test. The highest value of shaft horsepower output measured was 215,581 shaft hp. As far as is known, this is the largest amount of power measured at one time in a single test.

The satisfactory performance of the measurement was a distinct credit to the application of electrical measuring equipment to do this work.

Telemetering

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Synopsis.—The paper, in addition to surveying the field of application of "telemetering" and relating it to "supervisory control," presents several innovations in the types of "telemeters" available. Operating requirements and channels available for the transmission of readings are discussed, and an installation of the varying frequency type of telemeter on the Montaup Electric Company's System is described.

* * * * *

I. Introduction

AUTOMATIC performance has been applied extensively during recent years to the generation, transmission, and distribution of electrical energy. Automatic devices and equipments are very satisfactorily performing duties that were performed only a few years ago by operators, so that a large number of men heretofore performing important but minor duties have been released for more active and responsible positions. As a result, today we find stations under lock and key, visited only for inspection and maintenance. However, it has been found advisable to place these stations under the supervision of a power dispatcher, to provide for occurrences such as require prompt operation of station equipment but do not occur in the ordinary automatic cycle. Automatic supervisory equipments provide the operator with visual indications of apparatus positions, and means for controlling power apparatus located at remote points. One example is that of switching in case of fire in a section fed by a power feeder. Another example is that of switching so as to improve the economy of distribution over an entire power network. To obtain the most efficient operation, the dispatcher must be able to read electrical quantities at various outlying stations. A suggested way of doing this is to give the dispatcher television pictures of the substation instruments when a visual indication is required. While this method falls within the scope of modern engineering solution, it is not at present economically justified.

This paper deals with the telemetering problem of transmitting instrument readings or their equivalents from one point to another, which is one of transmitting approximate indications and records of quantities from remote points and is not one of transmitting quantities for metering and billing purposes.

When readings were required over relatively short distances, the permissible length of the leads of instrument transformers and shunts was sufficient for the transmission of information. However, the distances between the point of metering and the point of indication soon became greater than could be reached by instrument transformer leads, and other methods were sought for transmitting the information. Current readings at a distance were obtained by reducing the transmitted current, as, under this condition, a greater voltage drop could be tolerated without increasing the burden on the instrument transformer. As a result, the interconnecting leads could be considerably increased in length. These improvised methods, however, were not suitable for transmitting all types of readings. The result was the development of special apparatus which could be utilized for transmitting the necessary readings. This class of apparatus is described as "telemetering" equipment.

II. Quantities or Equivalents to be Transmitted

The readings to be transmitted by telemetric means may be classified according to their use as follows:

1. Indicating
2. Recording
3. Demand
4. Integrating

The term "indicating" refers not only to readings transmitted for an instantaneous indication, but also to readings that are employed to actuate control devices such as the tap-changing device on a transformer, or the operating mechanism on an induction voltage regulator. Many such devices are now being operated manually, but probably a great number of them can be operated automatically by suitable telemetering equipment. The term "recording" applies only to readings used to actuate instruments that give a detailed permanent record of operating conditions. The term "integrating" refers to readings used directly to actuate an integrating device totalizing readings over a period of time. The term "demand" refers to readings transmitted to record the short time power demand of a network.

Experience indicates that the following list includes the quantities that are needed most commonly, arranged in order of the frequency of their probable application.

1.	A-c. amperes	7.	D-c. amperes
2.	A-c. watts	8.	D-c. watts
3.	D-c. volts	9.	Volt-amperes
4.	A-c. volts	10.	Frequency
5.	Totalized watt-hours	11.	Power factor
6.	Reactive volt-amperes	12.	Various quantities used in industrial applications

1. All of the General Electric Co.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

III. Operating Requirements

The following enumeration is intended to include all possible variations in operating conditions under which readings may be transmitted.

A. One reading transmitted continuously.

B. One reading transmitted continuously for the control of a device.

C. A plurality of readings transmitted continuously.

D. A plurality of readings distributed to integrating, indicating, and recording instruments, and control devices.

E. A plurality of readings, one at a time at the call of the dispatcher through a supervisory equipment.

F. One reading from each outlying station or a totalized reading from each group of outlying stations.

IV. Channels[2] Available for the Transmission of Readings

In the design of a telemetering equipment, it is necessary to study the channel limitations and to design the terminal equipments accordingly. For economic reasons, it is desirable to have as many readings as possible transmitted through a given channel. The optimum number of channels depends on, among other factors, the distance of a proposed transmission. Some equipments can be adapted to long distance transmission and others only to the shortest distance. The choice depends very largely on economic factors, although there are engineering features that must also be considered. Some methods require very expensive and complicated terminal apparatus. It is out of the question to apply such types of equipment to transmission over very short distances. A terminal equipment that is simple, but requires the use of more than one channel, may be economically practical where the cost of additional channels is not excessive.

There are three important types of channels; multiconductor cables, wire construction, and transmission lines. The first two, cables and wire construction, are essentially metallic connections, though in some instances it is the practise to install insulating transformers to protect terminal equipments from high electrostatically or electromagnetically induced voltages. The third type of channel, the transmission line, requires the use of additional terminal equipment and coupling capacitors. For this reason, the transmission line as a telemetering channel is expensive for short-distances, and is suitable only for longer-distance transmission or for short-distance transmission where it is impracticable to obtain metallic channels. Specific cases will determine the economies of the application.

Important limitations in connection with the use of telephone lines for telemetering indications relate to the allowable current, voltage, and energy levels. If the channel is owned by the telephone company, the restrictions as to energy level are definite because the telephone company must protect its communication channels against excessive interference. Some types of telemetering equipments can be used to transmit readings when a conversation is taking place over the same channel, whereas other types cannot be used in this way because the operating frequency may cause interference.

Telemetering systems employing direct current impulses or high frequency alternating current may in general make use of these channels without seriously interfering with telephonic communication.

V. Terminal Equipments

Several successful types of terminal equipments are described below. The equipments were designed to satisfy the demands discussed in the previous sections of the paper.

A. The Inverse-Current Type. A successful telemetering system, described in previous papers, and in use since 1917 on the Chicago, Milwaukee and St. Paul Railroad, indicates at a dispatcher's office the total power taken from the high tension transmission lines at several points. This system uses the inverse current principle—that is, the current flow in the metering system is inversely proportional to the power taken from the high-tension transmission lines. This scheme of remote indication has proved very satisfactory and reliable.

B. Selsyn Type. One of the simplest systems for remote indication consists essentially of two Selsyn motors. A Selsyn motor is an alternating-current motor with a three-phase stator winding and a single-phase rotor winding. When two or more such motors have their stators connected together and their rotors excited from the same single-phase alternating-current source, the rotors take the same relative angular positions. When one rotor is deflected, the rotors of all the other Selsyns are deflected by the same amount. Thus one Selsyn operating mechanically from an instrument serves as a transmitter, and another Selsyn located perhaps miles away, taking the same position as the transmitter, serves as an indicator. Satisfactory remote indication may therefore be accomplished by having the receiving Selsyn actuate either the pointer of an indicating device or the pen of a recording device.

When it is desired to operate large remote indicators that require more torque than the transmitting instrument is capable of furnishing, a device is used which is essentially an amplifier and is known as an indication transmitter.

Selsyns are well adapted for use as remote indicators at relatively short distances. They are accurate and require little attention. However, the rotors of all the Selsyns in the system must be excited from the same source of alternating current, and three wires are required to connect the stators of the Selsyns. It is essential that the matter of rotor supply voltage be considered carefully, because in some power systems

2. The word "channel" as used in this paper refers to the path over which the readings are transmitted.

the electrical angle between substations a short distance apart may be as much as 20 or 30 degrees.

It has been found that a transmitting Selsyn will actuate a remote indicator through lines that are equivalent to two miles of standard No. 19 B. and S. telephone cable. A remote recorder will operate satisfactorily over half of this distance. If an "indication transmitter" is used, the indication may be transmitted through an equivalent of from 12 to 30 miles of cable, depending upon the type of receiver employed.

C. Rectified Current Type. When it is desired to transmit readings of a-c. amperes and a-c. volts a very simple device, shown in Figs. 1, 1A, and 1B, is available which consists only of an instrument transformer, a rectifier, a transmission channel, and a milliammeter. Transmission by direct current instead of alternating current eliminates all trouble due to pilot-wire inductance and capacitance.

The rectifier transformer used in obtaining the voltage readings is a transformer operated from the secondary side of the main current transformer. The secondary winding is tapped at the middle; this is done in order to obtain full wave rectification of the power taken from the rectifier transformer.

In case it is desired to obtain a measurement of potential instead of current, a rectifier transformer is operated from the potential transformer in a manner similar to that used for the current indication. The voltage across the secondary winding of this rectifier transformer is dependent on the resistance of the load circuit.

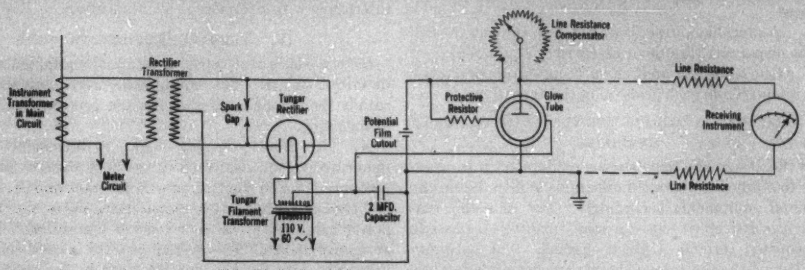


Fig. 1—Rectified Current Telemeter

Fig. 1A—Rectified Current Transmitter

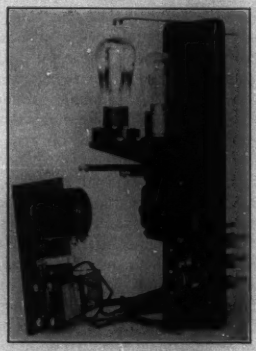


Fig. 1B—Rectified Current Transmitter

The burden that each rectifier transformer imposes on the instrument transformer is approximately 10 volt-amperes, except that for the current transformer the burden is nearly doubled when the protective spark gap operates. Thus, there is very little chance for overloading the instrument transformers with this equipment.

The full-wave rectifier in the secondary of the rectifier transformer is of the hot-cathode type. The rectifier

and metering circuit is as follows: The two terminals of the rectifier transformer secondary are connected to the anodes of the rectifier. The rectifier alternating current passes from the cathode of the rectifier through a pilot wire, a milliammeter at the receiving station, and another pilot wire back to the midpoint of the secondary winding of the rectifier transformer. The filament of the rectifier tube is excited from the secondary of a small transformer connected to the 110-volt a-c. source. In order to obtain long life the filament is allowed to operate continuously. This is because with intermittent operation the life would be shortened by the repeated heating and cooling of the filament.

The combined resistance of the line, receiver, compensating resistance, and the protective resistor of 2500 ohms, is 5000 ohms. Adjacent to the rectifier are located a condenser for smoothing out the rectified current, the current-limiting resistors, and the resistors for the purpose of compensating for line resistance.

The receiver is a standard direct-current indicating or recording instrument of the d'Arsonval type. It is calibrated in terms of the unit being measured.

A protective spark gap integral with the rectifier transformer is connected across the secondary to prevent the voltage across that winding from rising excessively in case of an open circuit.

The principal protection against high voltage on the transmission channel is the neon glow tube; if the voltage across the transmission channel rises excessively as it does on open circuit, the neon glow tube takes the current and causes the voltage to drop. Finally, the circuit from the rectifier to the remainder of the terminal equipment is so arranged that no current can pass unless the neon glow tube is in place. A secondary protection against high voltage on the channel wires or on the transmitter is a disk film cutout. This is connected between the negative and positive wires as shown in Fig. 1. In case the voltage between these two points rises to several hundred volts, the cutout will break down and will short-circuit the transformer. This is what would happen if the neon tube became inoperative, or if the line-resistance compensator became open circuited.

It is evident that when the rectifier type of telemeter is used to determine a-c. voltage, the resistance of the terminal apparatus and channel wires affects the reading at the receiver. Therefore, it is necessary to adjust the line-resistance compensator so that the receiver operates correctly. In the case where a current reading is desired, the line resistance has little effect on the reading, because the current transformer must maintain ampere turns in the secondary to balance the ampere turns in the primary; thus the variation of resistance in the line wires does not seriously affect the calibration of the equipment when used for current readings.

The rectified-current method is subject to a small error if the wave form of the supply is not sinusoidal. This error can be compensated for by a transformer ratio adjustment.

D. The Varying-Frequency Impulse Type. The three types of telemeter discussed above have depended primarily upon current or voltage magnitudes. Another type of telemeter is available which has a different principle of operation in that the readings are transmitted by means of impulses, the magnitude of the quantity being a function of the rate at which impulses are sent.

The transmitter used in this scheme is a meter of the same general type as the watt-hour meter. Since the operation of this system is according to the frequency of the impulses, the transmitter is usually a rotating device which sets up these impulses by means of a contact-making device—usually a small commutator. This commutator is geared to the motor shaft by a pair of gears of such a ratio that with full load (full scale reading) the meter originates approximately two impulses per second. Usually, this gear ratio is 1:1. The frictional load imposed by the commutator and brush is very small and is compensated for in the light load adjustment of the meter. (Note Appendix A.)

If the impulses from the watt-hour meter are used to actuate a speedometer, the speedometer mechanism may be calibrated so that its deflection indicates kilowatts. The actual receiver is not a simple speedometer, but is an analogous device, compensated for the temperature error usually found in speedometers. It is adapted to telemetering as follows: the impulses from the transmitter are received by a sensitive relay. This relay operates a portion of the telemeter known as the "rotary relay," whose function is to translate impulses into intermittent motion. By this means the output shaft of the rotary relay turns through one revolution for each impulse received.

This rotary relay consists of a motor controlled by an electrically operated latch that allows the motor to turn through one revolution each time the latch winding is energized by an incoming impulse. The shaft of the rotary relay is connected mechanically through a helical spring to a system of magnets comprising a flywheel. The flywheel possesses considerable inertia and the spring takes up the intermittent motion, so that the flywheel turns at a practically uniform speed proportional to the average frequency of the impulses received. The magnets induce eddy currents in a copper drum and thereby tend to produce rotation. The rotation of the drum is restricted by a spring, so that the deflection is proportional to the dragging force, and therefore to the speed of the magnets. The deflecting system has some inertia and is elastic, and consequently it must be damped to prevent mechanical oscillations. The deflecting element actuates a pointer or pen to serve as an indicating or recording receiver. This

instrument when operating on a frequency of two impulses per second has a torque of about 2000 gram-millimeters. (Note Appendix B.)

It is very easy to check the operation of either the transmitter or the receiver at will by counting impulses. If the transmitter is operating correctly, the impulses come through with fair uniformity. If the transmitter is not operating properly, either the impulses do not come through, or they come through intermittently. The relation between the frequency of impulses and the receiver deflection is linear.

The same impulses that operate the indicator may also be used to determine both the total kilowatt-

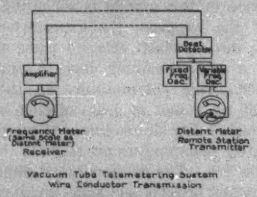


Fig. 2—Vacuum Tube Telemetering System
Wire Conductor Transmission

hours and the demand over a given time interval. The total kilowatt-hours are obtained by operating a counter known as the "distant dial." The demand is determined by operating any of the conventional demand meters by the use of the impulses. Thus this type of telemeter gives an indication of power, an integration of power, and the power demand, from the impulses sent out by a single watt-hour meter. Provision can be made to take account of the direction of power flow.

The summation of two or more readings can be accomplished by adding the impulses from the several transmitters. This is done by the use of a device known as a differential relay. This relay sends out a new series of impulses to operate the receiver according to the sum of the original impulses. The differential relay or impulse-totalizing device consists only of the following apparatus: impulse meters, differential gearing, ratio gearing, a damping device, and a contact device to produce impulses. A differential relay can be designed to supply both additions and subtractions. Such a relay is usually designed for a given number of circuits with various impulse constants.

E. Vacuum Tube Type. Fig. 2 shows the apparatus which is employed. A small condenser is attached to the movement of the instrument whose reading is to be telemetered. This condenser is connected to an oscillator so that the frequency of the latter varies according to the meter position.

A second oscillator, operating at a frequency close to that of the first oscillator, has a fixed condenser, and therefore furnishes a constant frequency. A beat frequency is set up by these two oscillators and is detected by means of a third vacuum tube. This beat frequency is relatively low, and varies in accordance with the instrument deflection.

The beat, or telemetering frequency, is transmitted over the connecting line to the receiving station, where it operates a direct reading frequency meter. The frequency meter may be directly energized by the incoming signal or an amplifier may be provided, the method depending upon the nature of the line, its impedance, and on the energy level permissible.

The frequency meter is furnished with a scale which corresponds exactly to that of the distant instrument. It indicates at all times the position of the remote instrument. The receiving instrument indication depends upon frequency only, and, provided it receives more than the minimum power necessary to operate it, is not affected by changes in signal strength. Thus any changes in the impedance or leakage resistance of the line do not affect the accuracy.

If it is desired to transmit the instrument readings by carrier waves, over special conductors, telephone lines, or high voltage power lines, transmitting and receiving apparatus very similar to telephone communication equipment, which is already familiar in the Central Station field, is employed.

Fig. 3 is a schematic diagram of the general arrangement for carrier current telemetering. The heat

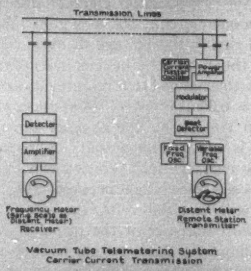


Fig. 3—Vacuum Tube Telemetering System
Carrier Current Transmission

frequency output of the oscillator circuit, instead of the output of a microphone speaking circuit, is applied in the regular way to the modulator. The carrier current generator consists of the usual arrangement of master oscillator and power amplifier. There is thus sent out over the channel a carrier wave of constant frequency modulated by an audio frequency which varies with the magnitude of the quantity being telemetered.

At the receiving end, the carrier is demodulated and the audio signal amplified. The amplified signal is then taken direct to the frequency meter which reproduces the position of the pointer of the distant instrument. It will be specially noted that again the signal

strength—provided it be sufficient to energize the apparatus—has no effect upon the accuracy of the indication.

In cases where it is desired to telemeter more than one reading from a remote station, a continuously operating selector[3] at the transmitting station automatically connects each instrument, one after the other, to the telemetering transmitter for a brief period. Twenty or more instruments may be taken care of in this way. corresponding transmitting instrument is connected to the telemetering apparatus by the transmitter selector. Thus each pair of instruments, one transmitting and one receiving, is given possession of the telemetering channel once in each complete cycle which includes all of the instruments. The receiving instruments employed have pointer movements of the type which retain their position when deenergized. Thus, when operated in the above manner, each instrument indicates the

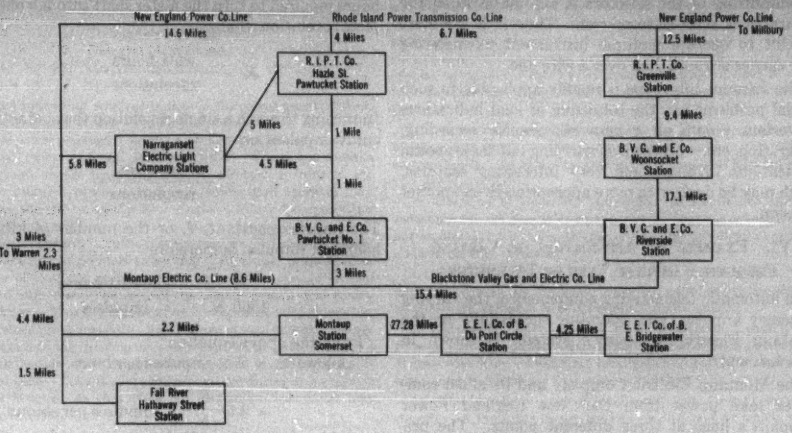


FIG. 4—MONTAUP ELECTRIC COMPANY AND ALLIED UTILITIES

FIG. 5—INDICATING DEVICE VARYING FREQUENCY IMPULSE TELEMETER

At the receiving stations, an equal number of instruments is installed. All receiving instruments are frequency meters identical in construction, but each has a scale the same as the corresponding instrument at the remote station. A selector associated with the receiver is arranged to connect each receiving instrument to the channel during the period at which the

FIG 6.—VARYING FREQUENCY IMPULSE TELEMETER

a. Demand meter
b. Indicating and recording device
c. Transmitting device

reading shown when last connected to the telemetering circuit. If we assume that each instrument is energized for a second, which time is sufficient for it to come to

3. An automatic instrument selecting device.

rest at its correct reading, each reading will be checked about every ten seconds if there are ten instruments. With other numbers of meters, the frequency of correction will vary accordingly.

Clearly it is necessary to synchronize the selector at the receiving station with that at the transmitter. A feature of this scheme which is of special interest is that, unlike many existing methods of synchronizing, it does not require a special synchronizing channel. Synchronizing of the selectors is automatic when the apparatus is started from rest. Thus it is no more difficult to operate multiple instrument telemetering over carrier or radio than over a wire line.

The vacuum tube type is readily applicable to such special problems as: the totalizing of load indications at various points on a network, graphic recording, integrating, etc. Methods of carrying out these special applications involve some very interesting features, which may be dealt with more appropriately on another occasion.

VI. Example of Application of Varying Frequency Impulse Type of Telemeter

An automatic telemetering equipment of the varying frequency impulse type, operating successfully on the Montaup Electric Company's system, is shown in Figs. 4, 5, and 6.

The Montaup Electric Company and its allied companies take power from the New England Power Company's lines at three different points. The primary purpose of the telemetering equipment is to indicate and record at Montaup Generating Station the total power taken from the New England Power Company. It is desirable to know the instantaneous transmission of power between the various systems and to have a record of the fifteen-minute demands.

In detail, the functions of the telemetering system may be listed as follows: (see schematic sketch in Fig. 4).

(1) To record at the Riverside St. Substation the maximum 15-minute demand of the Greenville and Riverside St. Substations.

(2) To indicate and record at Pawtucket No. 1, the total power taken from the New England Power Company at the Greenville Station and at the Pawtucket No. 1 Station.

(3) To record at the Pawtucket No. 1 Station the maximum 15-minute demand of the Pawtucket No. 1 Station.

(4) To record at the Fall River Station the maximum 15-minute demand of the Fall River Station.

(5) To indicate at the Montaup Station, Somerset, the total power taken from the New England Power Company at the Greenville and Pawtucket No. 1 Substations.

(6) To indicate at the Montaup Station, Somerset, the total power taken from the New England Power Company at the Fall River Station.

(7) To indicate and record in the dispatcher's office at the Montaup Station, Somerset, the total power taken from the New England Power Company by the Greenville, Pawtucket No. 1, and Fall River Substations.

(8) To record at the dispatcher's office at the Montaup Station the totalized maximum 15-minute demand of the Greenville, Pawtucket No. 1, and Fall River Substations from the New England Power Company; and to actuate an alarm if this totalized maximum demand exceeds a certain value.

Appendix A

Transmitter Constant

Let us assume the transmitter to be of the watt-hr. meter type with a meter constant K equal to the watt-hours required to turn the meter shaft through one complete revolution; that is,

$$K = \frac{\text{watt-hours}}{\text{revolutions}}$$

In turning through a single revolution the contact sends out N impulses, or

$$N = \frac{\text{impulses}}{\text{revolutions}}$$

The impulse constant, I, or the number of kilowatt-hours per impulse, is therefore

$$I = \frac{K}{1000\ N} = \frac{\text{kilowatt-hours}}{\text{impulses}}$$

From the above equation

$$\text{kilowatts} = I \times \text{impulses per hour.}$$

$$= 3.6 \frac{K}{N} \times \text{impulses per second}$$

Appendix B

Receiver Constant

The impulse frequency that should give the full scale or maximum deflection of the receiver may be determined as follows:

Let the desired full-scale reading of the telemeter be M kilowatts. Then

$$\text{impulse constant } I = \frac{\text{kilowatt-hours}}{\text{impulses}}$$

and,

$$\text{frequency of impulses} = \frac{M}{I} = \frac{\text{impulses}}{\text{hours}}$$

so that,

$$\text{impulses per second} = \frac{M}{3600\ I}$$

Bibliography

Automatic Transmission of Power Readings, by B. H. Smith and R. T. Pierce, A. I. E. E. Journal, Trans., Vol. XLIII, February 1924, pp. 303-310 inclusive.

"Power Limiting and Indicating System of the Chicago, Milwaukee, and St. Paul," by J. J. Linebaugh, *General Electric Review*, April 1920, Vol. XXIII, No. 4.

Discussion

S. C. Jacoby: The Montaup telemetering equipment was installed for two purposes: first, to record the fifteen-minute system demands of each of our member companies, and second, to determine the power received from the New England Power Company System at our points of interconnection.

The equipment used to record the fifteen-minute demands of our member companies is to be used for billing purposes. It has not been in continuous operationg long enough to determine the value of its records. The equipment on our power company connections is used to assist operation. It has been in service about six months. It consists of the following instruments:

1. Two indicating receivers.
2. One graphic receiver.
3. One demand meter.
4. One distance dial.

1. One indicating receiver indicates the power received from the New England Power Company System at Fall River; the other, the power received from the New England Power Company System at Pawtucket and at Woonsocket. These instruments are mounted on the main switchboard directly in front of the governor motor controls so that the switchboard operator may determine the effects on the New England Power Company ties of a change of the Montaup System load or a change of the Montaup Station output. These instruments require approximately ten seconds for the pointers to assume their proper position after a change of load has occurred. An appreciable change of load is noticed immediately because of the ticking sound of the rotary relay release. This ticking sound at first appeared objectionable, but has since proved useful. Any change of load causes a change in the sound of the instrument and is noticed at once, whereas a visible indication alone might not be noticed for an appreciable length of time. This is of special advantage during transmission troubles because, at that time, an immediate indication may avoid an interruption.

2. The graphic receiver indicates and records our total power company takings. This instrument is mounted in the dispatcher's room. An adjustable contact on the instrument operates an alarm when the demand limit is reached so that, in case the operator and the dispatcher do not notice an increase of total takings, the alarm will call it to their attention. To prevent the ticking noise of the rotary relay release from interfering with the dispatcher's telephone conversations, the relay is mounted in the control room and drives the receiver through a selsyn generator and motor. This makes the dispatcher's instrument practically noiseless.

3. The demand meter prints the total fifteen-minute demand of our power company takings.

4. The distance dial integrates the total power received.

Reliability. Outages have been caused chiefly by commutator failure, by the sticking of the differential relay, and occasionally by telephone fuses blowing at Greenville. These outages, we believe, can be prevented as they are not caused by defects in the principle of the varying frequency impulse equipment.

Accuracy. The accuracy of the indicating instruments compares favorably with the accuracy of regular switchboard instruments and is limited chiefly by the transmitting watthour meter. False readings on the indicating instruments are rarely encountered as failure of equipment usually causes the instruments to hunt or to stop altogether.

Advantages of the Telemeter to Operation. The advantages we have obtained by the use of our telemetering equipment are:

1. Improved service by avoiding interruptions. Where systems are connected by more than one tie, there is danger that, if one tie is lost during a disturbance, the increase of load on the remaining ties may cause their loss also. If all ties are lost, an interruption may result, but if the load on the remaining tie is reduced to a safe limit at once, its loss can be prevented. The telemeter provides us with an immediate indication, both audible and visible, of a loss of either major tie at a point where the load on the remaining tie can be controlled.

It is not advisable to have standing orders to pick up load during trouble without the telemeter indication because, in case of local trouble resulting in a loss of load, picking up load on our generators would aggravate the condition and might result in further trouble.

2. Smoother operation. On an interconnected system, correct frequency is not an indication of normal conditions as variations of system load or generation affect the load on the interconnecting tie lines and not necessarily the frequency. The telemeter indicates to the dispatcher whether or not he has anticipated his load correctly and, if not, steps can be taken to correct same without being informed by telephone. This is of special advantage when an unexpected change of load occurs, such as a change due to sudden darkness on one section of our territory.

During normal load change periods, the Montaup operator is instructed to keep the load on the power company ties within certain limits. In this way, excessive exchanges of load between systems is avoided.

3. Checking of switching operations. Much of our routine switching on the power company ties is done on schedule and not on direct order. The telemeter provides our dispatcher with a check on this switching.

4. Determining the source of disturbance. The graphic telemeter, used with the frequency recorder, supplies our dispatcher at once with valuable data concerning disturbances.

5. Economy. Last but not least, economy. Usually, the price of purchase power differs from the price of local generation. It may cost more or less than local generation, but is rarely the same price. There are two reasons for this:

1. The increment cost of local generation varies because it depends on the units in service and on the load at which these units are operating.

2. The cost of purchased power depends on variable conditions on the power company system.

During primary periods, our maximum rate of purchasing power is limited by our primary demand. During secondary periods, it is limited by the capacity of the ties or by the power available. We find that, with the telemeter in service, we can keep 1500 kw. hr. per hour closer to our desired takings than when we depend upon the men at the receiving points watching the load and notifying us of variations by telephone.

To put a commercial value to this, assume a difference of two mils per kw. hr. between our increment cost and the cost of purchased power, the resulting saving will be three dollars per hour. On a basis of 15 hr. per day and 200 days per year, this saving amounts to $9000 per year. This saving is not clear profit, but it does tend to put the telemeter on a paying basis.

Chester Lichtenberg: The subject of telemetering is relatively new; as a matter of fact, it is an art which has only been recognized as such for about four years. Initially the attempt was made to move instruments to a relatively great distance from the shunts or other sources from which they derived power. That means long leads and eventually large leads with the usual voltmeters, ammeters, wattmeters, and the like. When the distances became more than two or three miles, the usual types of instruments were not sufficiently delicate and the usual sources of power such as shunts, current, and potential transformers were not of sufficient rating to generate energy to operate the instruments efficiently. But the main trouble was to get large enough wires for long enough distances. The urge for telemetering came through the centralizing of the control of power in load dispatchers or power directors.

Mr. Jacoby has pointed out one use; such a use was really started about 1912 by the Chicago, Milwaukee & St. Paul

Railway, where the load demand had to be kept to a certain minimum because there wasn't enough water power to furnish all that the railroad required. In the last few years, the demand for telemetering has increased quite rapidly. Centralizing the control of power has reduced the number of operators in urban systems.

During the last four years, the tendency has been to use telephone wires or their equivalent for connecting power stations or substations or the sources of power supply to the central points at which the power directors have been located. The idea has been to use the existing telephone cable systems and, therefore, the amount of energy which might be used has been limited. At first the idea was to use either current or voltage in the usual fashion for transmitting the instrument readings, but it was soon found that inductive interferences and leakage caused an appreciable error. In looking around for other means to transmit instrument readings, it was finally decided that frequency gave the best means of transmitting electrical indications.

The number of wires which may be used for telemetering has sometimes given concern because each wire costs money, either to install or to maintain or to rent. Some systems are now available whereby as many as three instruments may be read continuously over one pair of telephone wires, but there does not seem to have been much demand for such designs. Instead, as Mr. Stewart pointed out, one pair of wires is being used for reading as many as 24 instruments successively,—not continuously, but successively.

There is a number of other systems which have not been described. Those which have been described merely give an idea of this new art of telemetering and present to the central station and industrial engineers as well as to the railway engineers a vision of what they may expect in the way of development so that in laying out new systems and in consolidating present systems they can take advantage of this new art for economical operation.

C. E. Stewart: No mention has been made of the application of telemetering in connection with automatic supervisory equipment or equipment for the remote control and indication of power apparatus. There are several phases of this particular application which are of interest.

The application of telemetering by itself requires an independent channel used for no other purpose except possibly that of telephone communication. The circuit under that condition is a continuous one and the telemetering readings are of a continuous or semi-continuous nature.

When we use telemetering with automatic supervisory equipment, however, it is possible to select and obtain readings from a large number of station circuits. These readings may be transmitted to the dispatcher over the same wires which are used for the supervisory equipment provided a momentary reading is satisfactory. This arrangement is only recommended for momentary readings because it necessarily holds the supervisory equipment on the selected circuit and does not permit it to function in case of the automatic operation of other supervised apparatus.

A preferable method is to use an independent channel for telemetering. The supervisory equipment may then be used to selectively connect this channel to the desired substation circuit. The supervisory equipment is thus left free to perform other functions.

When telemetering of a multiplicity of circuits is required the best practise is to use a supervisory equipment entirely for that purpose. Such an installation is soon to go into service in the city of New York in the Laconia Avenue Substation of the Bronx Gas and Electric Company. The combined supervisory and telemetering equipment operating over four line wires, provides the dispatcher with means of reading the a-c. current and a-c. voltage on twelve power circuits. The dispatcher can obtain these readings with as little effort and as promptly as if he were located in the substation itself.

The rectified-current type of telemetering and the synchronous-selector type of supervisory equipment are used on this intallation. An independent supervisory equipment provides the dispatcher with supervision of the substation equipment.

Totalizing of Electric System Loads

BY P. M. LINCOLN*

Fellow, A. I. E. E.

Synopsis.—The importance of measuring an electric system load at any distance from that load is emphasized. The use of thermal wattmeters in conjunction with thermal couples is suggested. Time of response of such devices is discussed; temperatures attained are also discussed. Characteristics of circuits for transmitting the thermal e. m. f. from the load to the point of measurement are discussed in detail. A list of users together with data pertaining to the measurement-transmitting circuits is given.

WHEN all the power of a given electric system is generated in a single plant, the totalizing of the entire load is relatively an easy problem. A number of methods is known and used for accomplishing the desired result of making available the information as to the total of the load that is being supplied. These totalizers may be of the indicating or the graphic recording type as may be desired.

When two or more power plants are used to supply a given system, and these plants are located at a distance from each other (and this is invariably the case in modern electric systems of any size), the problem of totalizing the entire load of the system becomes more difficult. If to this situation we add the further complication of purchase and sale of energy to one or more contiguous systems—at points perhaps many miles removed from each other—the problem becomes much more complicated. Yet this last situation is exactly where many, if not the majority, of the modern large electric service supply systems find themselves at the present time.

Large electric supply systems are rarely fed from a single power generating plant. Moreover, a consideration of the economics of the situation usually dictates that the various power plants serving a given territory shall be widely separated within that territory. Further, the purchase and sale of power from contiguous territories naturally occurs at the boundaries of a given territory. These purchase and sale agreements between neighboring properties are becoming more and more common. The term "superpower" has been coined to cover this modern tendency and it is generally accepted as being both economic and logical This situation, however, makes more acute the problem of load dispatching.

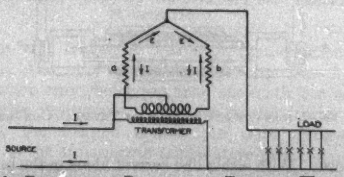


FIG. 1—ELEMENTARY DIAGRAM OF THERMAL WATTMETER

The problem of load dispatching requires that the load dispatcher shall know at all times not only the aggregate load on his own system but also how much load he is supplying to other systems and how much they are supplying to him. When the generating stations are widely separated and when the points of contact with contiguous systems are still more widely separated, as is the case in all large systems, how is the load dispatcher to obtain this essential information?

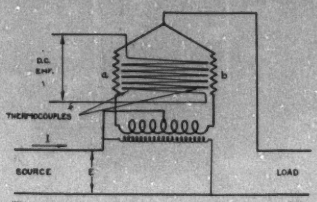


FIG. 2—ELEMENTARY DIAGRAM OF THERMAL WATTMETER USING THERMOCOUPLES AS MEASURING PRINCIPLE

It is the object of this brief paper to indicate one solution of this problem. The fundamental idea in the solution proposed is the same as that in the instrument described by the writer in a paper before the A. I. E. E. in October 1915.[1] Fig. 1 herewith is a reproduction of Fig. 2 of that paper and shows the elements of a thermal wattmeter. In the appendices of the 1915 paper is submitted a mathematical proof that, with circuit connections as shown in Fig. 1, the difference in temperature between resistances *a* and *b* of Fig. 1 is proportional to watts.

In the 1915 paper above referred to, a number of methods is proposed for measuring the difference in temperature between these two resistances *a* and *b*—among them the use of thermocouples. Still another method is described in the present author's paper[2] read before the A. I. E. E. on Feb. 15, 1918. This latter method has been incorporated into a commercial

*Director, School of Electrical Engineering, Cornell University, Ithaca, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

1. *Rates and Rate Making*, P. M. Lincoln, A. I. E. E. TRANS., 1915, p. 2279.

2. *The Character of the Thermal Storage Demand Meter*, P. M. Lincoln, A. I. E. E. TRANS., 1918, p. 189.

demand meter and has found considerable use among public utilities, particularly in Canada. Although this latter method is excellent for use in a demand meter, it does not lend itself readily to a solution of the totalizing problem outlined above. The use of thermocouples, however, does so lend itself in an ideal manner. As shown in the 1915 paper, the difference in temperature of the resistances *a* and *b*, Fig. 1, is proportional to watts. If thermocouple junctions are associated with these resistances, the resulting thermal e. m. fs. are then proportional to watts for each individual thermal wattmeter. By connecting these thermal e. m. fs. in

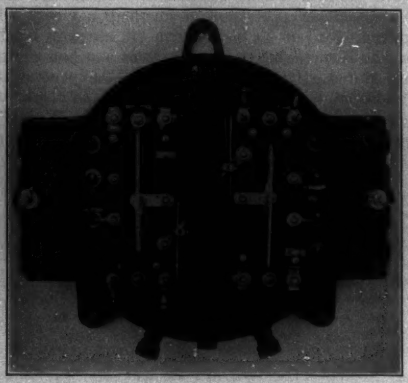


Fig. 3—Thermal Converter with Cover Removed Showing Adjustments

series, and measuring the resultant total, the sum of all of the readings of the individual thermal wattmeters may easily be obtained at any desired point.

It should be noted that the thermal wattmeter shown in Fig. 1 is reversible. If the power flow is in one direction, resistance *a* becomes hotter than resistance *b* by a given amount. If the direction of wattage flow is reversed, resistance *b* becomes hotter than *a* by exactly the same amount—assuming, of course, that the watt value is the same in each case. In one case, therefore, the resultant thermal e. m. f. is in one direction and in the other case in the opposite direction. These thermal wattmeters may therefore be used to subtract a given load from the total as well as add its value; the thermal wattmeter is completely and accurately reversible.

All that is necessary to obtain a total indication is an electrical circuit connecting the thermal wattmeters to be totalized and some means of reading the total resultant thermal e. m. f.—preferably a potentiometer. This briefly is the basis of the method that is now proposed for load totalization. There is a number of points that warrant further discussion.

The Thermal Converter

For the sake of convenience, the instrument incorporating the metering circuit shown in Fig. 1, together with the necessary thermocouples and adjustments, has been called a "thermal converter." Fig. 2 shows such a thermal converter in diagram and Fig. 3 is a photograph of two such thermal converters mounted in a single case and comprising a complete polyphase wattmeter—one converter in each phase. Fig. 4 shows a diagrammatic section of one of these thermal converters. The form and arrangement of parts in Fig. 4 is due to the suggestions of Mr. H. S. Baker, Meter Supervisor of the Hydro-Electric Power Commission of Ontario at Niagara Falls.

The experimental work necessary to convert Baker's suggestion into a successful commercial instrument was carried out by Louis A. Paine of the Lincoln Meter Co. Ltd., Toronto, with numerous suggestions by Mr. Perry A. Borden, at that time an engineer in the laboratories of the Hydro-Electric Power Commission of Ontario, now an engineer with the Bristol Co., Waterbury, Conn.

Referring to Fig. 4, *A* is a heavy metallic plate, brass or copper. Around this plate, and insulated from it by pure sheet mica on the sides and by a grooved bakelite tube top and bottom, are the thermocouples *T*. Over these thermocouples is placed another layer of pure sheet mica and then the heaters *R*. After still another layer of insulation, the plates *P* are applied and firmly clamped together by suitable bolts. The terminal *B*.*B* of the heaters are associated in close thermal contact (but of course electrically insulated from each other), thereby eliminating errors that might be due to thermal conduction between leads and heaters.

Fig. 5 shows in diagram the adjustments that are

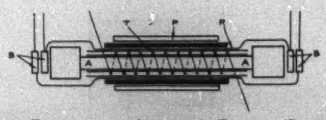


Fig. 4—Diagrammatic Section of Thermal Converter

provided. With either line current alone in the heaters or current from the voltage transformer alone, the two heaters must obviously attain exactly the same temperature. This may be accomplished by proper adjustment of the small resistances labeled "current balance adjustment" and "potential balance adjustment" in Fig. 5. After these adjustments have been made, there is necessary a still further adjustment so that with a given value of watts applied to the thermal converter there will be a given thermal e. m. f. from the thermocouples. It is of course necessary that all of the thermal converters used to totalize a given load shall have the same thermal e. m. f. per watt; this final adjustment is secured by the shunting resistance marked "shunt"

in Fig. 5. The thermal e. m. f. that has been adopted and found suitable in practise is ten millivolts per meter element; that is, with full normal load on the metering element of a thermal converter, the resulting thermal e. m. f. is ten millivolts. For a polyphase unit, where two thermal converters are used (one in each phase), the resulting thermal e. m. f. at normal full load is, of course, 20 millivolts. The relation between watts and the resulting thermal e. m. f. is a perfectly straight line; the thermal e. m. f. is always proportional to watts input. It might be pointed out here that this perfectly straight line relationship is secured by properly designed compensation. It is well known that the thermal e. m. f. of most couples increases slightly with increasing temperature; it is also well known that heat emissivity and heat conductivity decrease with increasing temperature—at least within the temperature range used in this device. By proper combination

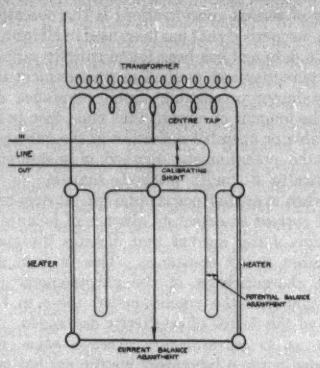


FIG. 5—DIAGRAM SHOWING ADJUSTMENTS ON THERMAL CONVERTER

of these two tendencies complete compensation is secured. Also, the device is relatively free from temperature error, being less than one-tenth per cent per degree centigrade. This temperature error is positive, the thermal e. m. f. per watt increasing slightly as the atmospheric temperature rises.

The total thermal e. m. f. that is produced at the totalizing point is, of course, dependent on the number of thermal converters that are used in series on any particular project, as well as upon the load on each individual thermal converter. There is no limit to the number of thermal converters that may be thus connected in series. The maximum number so far actually used on a single installation is 19 thermal converters used by the Hydro-Electric Power Commission of Ontario to measure the power supplied to the Toronto Hydro Electric System.

TIME OF RESPONSE

No thermal device can respond instantly to the action of the currents that do the heating. This principle is utilized in the thermal demand meter and the sizes of the various parts are so adjusted as to secure the desired time of response. The most widely used time of response for demand meters is ten minutes to reach 90 per cent of the final indication. The thermal converter shown in Figs. 2, 3, and 4 has a very much shorter time, *viz.*, between 8 and 9 sec. to reach 90 per cent of its final reading. The reasons for this much shorter time may be worthy of further discussion.

As shown in Appendix 3 of the writer's paper[1] referred to previously, the instantaneous difference in temperature of two heaters arranged as in Fig. 1 is given by the expression

$$\theta_1 - \theta_2 = \frac{H_1 - H_2}{SE + 2Q}\left(1 - \epsilon^{-\frac{(SE+2Q)t}{M}}\right) \quad (1)$$

where

θ_1 = instantaneous temperature of heater a above environment

θ_2 = instantaneous temperature of heater b above environment

H_1 = rate in gram calories per sec. at which heat is applied to a

H_2 = rate in gram calories per sec. at which heat is applied to b

S = surface area in sq. cm. of a or b (similar)

E = heat emissivity of a (or b) in gram calories per sec. per deg. cent. per sq. cm. of surface

Q = thermal conductivity between a and b in gram calories per sec. per deg. cent. of temperature difference

M = amount of heat in gram calories stored in a (or b) per deg. cent. of temperature rise

t = time in seconds after first application of heat to a and b

ϵ = base of Naperian logarithms

In Appendix 2 of this same paper, it is shown that $H_1 - H_2$ is proportional to watts when the arrangement is that shown in Fig. 1. From Equation (1), it is evident that the instantaneous temperature difference between a and b, Fig. 1, rises along an exponential curve and that it finally becomes proportional to watts, when t becomes infinite. It is further evident that the time of response is governed by the coefficient of t, *viz.*, the quantity $\frac{SE + 2Q}{M}$. The temperature difference between a and b will reach 90 per cent of its final value (assuming a steady load application) when

$$t = \frac{2.302\,M}{SE + 2Q} \quad (2)$$

Fig. 6 shows a photograph of the parts and the assembled element of a demand meter having a time of response of ten minutes (to reach 90 per cent of final value). Comparing Fig. 6 with Fig. 4 and comparing them with reference to the relation shown in Equation

(2), the reasons for the much shorter time of response of the structure of Fig. 4 becomes apparent. The value of M, Fig. 4, is very much less than that of Fig. 6—less than 10 per cent. The value of Q in Fig. 6 is nearly zero, while in Fig. 4 it is comparatively very large. These two factors alone are sufficient to account for the fact that the structure shown in Fig. 4 will respond to the application of power in approximately one-seventieth of the time of that shown in Fig. 6. It is obvious, further, that the time of response of the structure shown in Fig. 4 (or 6) may be modified if desired by modifying the design. However, the 8½ sec. (approximately) time of response of the structure shown in Fig. 4 has been found to be quite suitable for the purposes of a totalizing meter.

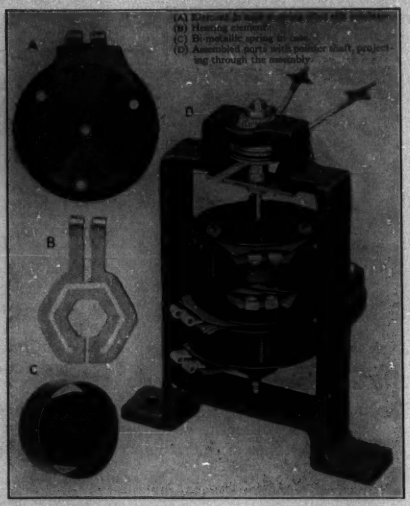


FIG. 6—PARTS AND ASSEMBLY OF THE MEASURING ELEMENTS OF A DEMAND METER (THERMAL STORAGE TYPE)

TEMPERATURE ATTAINED DURING OPERATION

The total number of couples used in series in each thermal converter is usually 21. The microvolts per couple per degree of temperature difference is approximately 42. To attain a total of 10 millivolts at full load, the resulting difference in temperature of the two elements a and b, Fig. 1, is therefore approximately 11 deg. cent. The actual rise in temperature of the hotter element above the surrounding air depends on power factor. The lower the power factor the higher will be the resulting temperature rise, since the total heat dissipated in both elements dictates the maximum temperature rise and this in turn is dictated by the total current flowing in resistances a and b. This total current obviously increases as power factor decreases. With the designs used in practise, the maximum temperature attained at 50 per cent power factor does not exceed 60 deg. cent. above the surrounding air at full load in watts. Since the insulation used is pure mica, there is obviously ample margin for safety in so far as temperature rise is concerned.

CIRCUIT CHARACTERISTICS

There are two characteristics of the circuit connecting the thermal converters with the central point of measurement that should be considered. These are:

1. Line resistance in series with thermocouples.
2. Possibility of stray e. m. fs. in connecting line.

The first of these is of relatively minor importance while the second is of vital importance.

1. Resistance in series with thermocouples.

The importance of the circuit resistance depends on the method of measuring the resulting e. m. f. While other methods might be possible, the only method worthy of serious considerations is the potentiometer. This is the method that has been used in all installations made to date and has been found entirely satisfactory. If the potentiometer method of measurement be used, the maximum permissible circuit resistance depends upon the amount of current that must flow to give an observable or operable deflection in the galvanometer. While the potentiometer method of measurement is theoretically a "null" method—that is, no current flows when exact balance is obtained—it does require a flow of current to detect an absence of balance. It is the value of this current that dictates the maximum permissible circuit resistance. This in turn depends on the characteristics of the galvanometer used in connection with the potentiometer, the e. m. f. available, and the degree of sensitivity demanded.

First as to galvanometer characteristics, what is demanded is not a laboratory instrument but one that can be placed under the supervision of the usual plant operators and give reliable results. Also, in most cases (all cases so far actually installed) a graphic record on a clock or time driven chart is demanded. In the installations so far made, eminent success has been obtained by the use of a recording-potentiometer made available by a well-known American firm.[3] This firm has available a line of recording potentiometers with galvanometer resistances varying from 11 ohms to 2300 ohms. These instruments record the potential of thermocouples on a clock or time driven chart approximately ten inches wide, the chart scale being divided into 100 divisions.

The galvanometers used in connection with these potentiometers are of the D'Arsonval type. This type has the advantage that it may be critically damped—a very important consideration in the successful operation of the recording mechanism. Two styles are available, one having a permanent magnet field in the galvanometer and the other an electromagnetic field. The sensitivity of the latter style is considerably

3. Leeds and Northrup Co., Philadelphia, Pa.

higher than the former. The sensitivities and resistances of these two styles are given in the following table.

TABLE I

Galvanometer resistance ohms	Sensitivity—microamperes per step	
	Perm. mag.	Electro mag.
11	2.	1.2
22	3.	1.8
100	1.6	1.1
1180	0.25	0.17
2300	0.21	

Sensitivity (columns 2 and 3, Table I) may be defined as the amount of current that must flow through the galvanometer to make its pointer just engage the operating lever. Such an engagement will cause the pen to move approximately ¼ per cent of full scale. If this movement is not sufficient to cause the current in the galvanometer to reduce below the engaging point, movement of the pen continues until such a reduction in galvanometer current does occur.

The e. m. f. available to cause current to circulate depends on the balance between the e. m. f. of the thermal converter under measurement and the e. m. f. within the potentiometer itself. As above indicated, the e. m. f. at full load on one polyphase thermal converter is 20 millivolts. Therefore, in case of a departure from perfect balance, the e. m. f. available to cause galvanometer current is that departure's proportion of 20 millivolts. For instance, a one per cent departure from perfect balance—no matter in what part of the scale—would cause one per cent of 20 millivolts, or 200 microvolts, to become available for circulating galvanometer current. To determine the per cent departure to cause operation, we must consider not only the e. m. f. available but also the total resistance. If we assume for the moment that we have zero line resistance, the only resistance to the galvanometer current will be the galvanometer itself and the thermal converters. Each polyphase thermal converter has a resistance of approximately 30 ohms. With these data at hand it is quite easy to determine the per cent departure from perfect balance that will cause each galvanometer listed in Table I to begin to respond. When a single thermal converter is used these percentages are given in the following table.

TABLE II

Galvanometer resistance ohms	Total resistance (1 polyphase thermal converter)	Per cent departures from perfect balance to cause response (zero line resistance)	
		Perm. mag.	Electro mag.
11	41.	0.41	0.25
22	52.	0.78	0.47
100	130.	1.04	0.72
1180	1210.	1.51	1.03
2300	2330.	2.38	

From Table II, the sacrifice in sensitivity caused by any given amount of line resistance when using any given galvanometer combination may be easily determined. An increase in the total resistance (column 2, Table II) will result in exactly the same relative increase in the percentage of departure from perfect balance (columns 3 and 4, Table II) to cause response.

Table II holds for a single thermal converter. When a number of such converters is used in series (as is invariably the case), a much better performance is secured than that indicated in Table II. Each additional thermal converter adds only about 30 ohms to the resistance of the galvanometer circuit,—in most galvanometer combinations, an insignificant proportion. The voltage available from each converter with a given departure from perfect balance is exactly the same whether there be one or a hundred converters in series. Therefore the more converters in series, the less will be the per cent departure from perfect balance to cause response. To take a specific example, let us assume that we have a system using ten thermal converters in series, that we are equipped with the 1180-ohm permanent magnet type galvanometer, that our total line resistance is 1000 ohms. From the above data, it may be easily determined that such an installation would respond to a departure of 0.31 per cent of the total possible load, no matter in what converter that departure took place. If we should add another thousand ohms to the line resistance, the per cent departure would rise from 0.31 per cent to 0.435 per cent; if we eliminated all line resistance it would fall to 0.285 per cent. The electromagnetic style of galvanometer would give considerably better performance.

It is obvious from this discussion that for the usual case, line resistance is of no great moment; the distance between converters may be as great as any possible case will call for, and the conductors may be made as small as practicable and still secure a line resistance that will give entirely acceptable results.

One other point should not be overlooked in determining this line resistance. To obtain successful operation, it is highly important that the galvanometer be damped. Critical damping is secured with certain values of resistance in series. With the line of potentiometers indicated above, these values of resistance for critical damping vary from 25 ohms (galvanometer resistance 11 ohms) to 6000 ohms (galvanometer resistance 2300 ohms). Laboratory tests have shown that these values of resistance may be considerably exceeded (by perhaps 75 per cent), and still secure reasonably good results in so far as damping is concerned.

The methods of determining the maximum permissible value of circuit resistance are indicated in the above paragraphs. The size and character of the conductors to be used will depend on this maximum resistance and also upon the considerations to be discussed in the following paragraphs.

2. Possibility of stray voltage in connecting line.

The matter of stray voltage on the transmitting conductor is of much more importance. However, the only stray voltage that need be seriously considered is a direct (or continuous) voltage. The means of detecting and measuring the resultant thermal e. m. f. consists, as has been mentioned, of a galvanometer of the

D'Arsonval type. Such a galvanometer recognizes direct currents only and a superposed alternating voltage is of no particular moment, unless it becomes unduly large. The filtering out of these superposed alternating voltages may be quite readily accomplished.

Neighboring circuits may affect the transmitting circuit either by induction or by leakage. Induction may be either electrostatic or electromagnetic; either of these forms of induction, however, can induce only alternating e. m. f. into the transmitting circuit and therefore this source of stray e. m. f. is of no particular moment. The same is true of conduction effects so long as the conducted current is purely alternating. If, however, this conduction is of such a nature as to cause a direct-current e. m. f. to appear in the transmitting circuit, an error in the transmitted reading will be caused equal in amount to the proportion that the parasitic e. m. f. bears to the thermal e. m. f. of the thermal converters under measurement. The thermal e. m. f. is, at full load on each thermal converter, 20 millivolts per polyphase thermal converter. The total useful e. m. f. to be measured varies therefore from perhaps one or two millivolts as a minimum to perhaps 500 millivolts as a maximum, depending on the number of thermal converters used and the loads on each. It is obvious, therefore, that a relatively small amount of leakage from an external source would be fatal to the accuracy of this method of measurement. The transmitting circuit *must* be properly protected from leakage. However, with proper construction and proper precautions there is no need to anticipate any difficulty from this source. Perhaps the best proof of this statement is to enumerate existing installations and indicate the length and character of the transmitting lines. This information is given in Table III.

Fig. 7 shows about four hours record of one of these totalizers on the Toronto installation. This particular portion of the chart has been selected to show one of the advantages of this system of totalization, *viz.*, the ability to suppress the zero and thus increase the virtual scale length. On the left hand portion of this chart, the top represents 100,000 kw. and the bottom 250,000; the zero has been suppressed to the extent of 100,000 kw. At the right the zero suppressor has been removed and now the top is zero and the bottom 150,000 kw. With this type of measuring device it is a very simple matter thus to suppress the zero; it can be done to any extent desired.

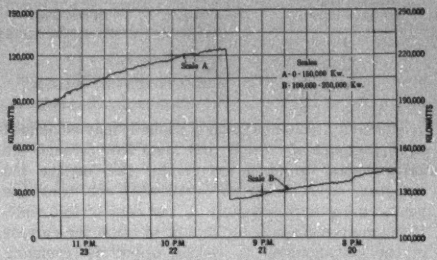


FIG. 7—FOUR HOURS RECORD OF THE TORONTO INSTALLATION SHOWING ZERO SUPPRESSION

TABLE III

Company	Location	No. thermal converters (polyphase)	Total conductor miles	Total resistance ohms	Kind of conductor	Began operation	Max. power measured
Hydro-Elec. Power Commission	Toronto, Ont	19	25	530	Paper ins.—lead covered	Sept. 1926	166,000 kw.
Ottawa Gas and Elec. Co.	Ottawa, Can.	3	3.5	74	Style B telephone wire rubber ins-duplex	May 1926	20,000 kw.
London Public Utilities	London, Ont.	4	1.9	10	No. 10 B and S overhead wires on poles	?	20,000 kw.
Shawinigan Water and Power Co.	Three Rivers, Quebec	4	Within a single station	No data	Inside wiring	Aug. 1927	40,000 kw.
Wayagamac Pulp and Paper Co.	Three Rivers, Quebec	4	0.57	No data	No data	July 1927	8,400 kw.
Shawinigan Water and Power Co.	Thetford Mines, Quebec	3	2.5	53	No. 12 Style B copper weld overhead	Sept. 1927	7,000 kw.
Montreal Light, Heat and Power Company	Montreal, Quebec	2	Within a single substation		Inside wiring	Oct. 1927	No data
Price Bros.	Isle Maligue, Quebec	3	1.5	65	No. 16 style B	?	No data
Port Arthur Public Utilities	Port Arthur, Ont.	4	10	198	Paper ins., lead-covered cables	Oct. 1927	24,000 kw.
Windsor Hydro Elec. System	Windsor, Ont.	5	2.5	20	Underground control cables	?	20,000 kw.

In conclusion, while actual experience with this new totalizing system in practise has been limited to only about three years, nothing has developed to lead to the anticipation of any difficulty. Direct current leakage into the transmitting wires is the only thing that need be zealously guarded against and experience thus far indicates that with proper installation, no fear need be entertained from this source.

Discussion

H. B. Brooks: Mr. Lincoln points out that the potentiometer method of measurement, rigorously speaking, is not a true null method. While this distinction is not usually significant for laboratory work, it may become so for special cases like his, and it should be borne in mind.

In determining the value of total resistance in the galvanometer circuit, one should include the resistance introduced by the potentiometer itself. This may be of importance in the case of relatively few converters and low line resistance.

Table II shows that the galvanometers of higher resistance and better current sensitivity are inferior in performance for the case cited, namely, a single converter of 30 ohms resistance and no resistance in the line. For galvanometers of a given type which differ only in the size of the wire with which the coil is wound, the greatest sensitivity under working conditions will be obtained when the net resistance of the galvanometer coil is equal to the resistance of all the rest of the circuit. Thus for the case of a single converter of 30 ohms resistance, with zero line resistance and negligible resistance of the potentiometer and of the galvanometer suspensions, the galvanometer coil should be wound to have a resistance of 30 ohms. In practise, one cannot always expect to get a galvanometer coil specially wound for each case, but the principle just mentioned will assist in selecting the best stock galvanometer for the purpose.

I should like to suggest an exception to Mr. Lincoln's statement that the only instrument worthy of consideration for measuring the resultant e. m. f. of the converters is the potentiometer. Wherever a graphic record is desired, the recording potentiometer which he mentions is possibly the only suitable instrument available. However, it is bulky and expensive, and where it is desired merely to indicate the load in magnitude and direction of energy flow, an indicating instrument may well be used, if some expedient be provided which will render the indications free from error caused by variations in the total resistance. Methods for doing this were given on pages 540-541 of the TRANSACTIONS, Vol. 39, 1920, and an instrument embodying still another procedure was described by Harrison and Foote in the same volume, pages 371-402.

A simpler method which seems applicable to the case of load totalizing is as follows: A suitable pyrometer millivoltmeter with its scale figured in kilowatts is provided with an adjustable rheostat forming part of its series resistance. The remainder of the series resistance is non-adjustable. The resistance values are such that with the adjustable rheostat set to one-half of its maximum resistance the millivoltmeter reads the total power correctly for an average value of line resistance, and the range of adjustment of the adjustable rheostat permits the maintenance of the proper total value of resistance (of instrument plus converters and line) for all temperature conditions to which the line is exposed. A push button on the instrument, when depressed, inserts additional series resistance equal to this proper total value. If the adjustable rheostat is properly set, pushing the button at any time will reduce the reading to one-half its value, and if the line resistance has changed since the adjustable rheostat was last set, the latter is adjusted until the pushing of the button cuts the reading to one-half.

As to the feasibility of using such an instrument, the first installation given by Mr. Lincoln in Table III, with 25 mi. of conductor in the line, may be taken as an example. The full-load e. m. f. of the 19 polyphase thermal converters in series is 380 millivolts, their total internal resistance is 570 ohms, and the line resistance is 530 ohms. A particular stock switchboard pyrometer millivoltmeter gives full-scale deflection with a current of 200 microamperes. Such an instrument, with its internal resistance adjusted to 800 ohms, would give full-scale deflection if connected to the Toronto line with full load on all the thermal converters. If we assume that the resistance of the line and the converters is all of copper, to take an extreme case, and that the temperature may vary from − 20 deg. cent. to + 40 deg. cent., there will be a variation of 132 ohms on either side of the average resistance at 10 deg. cent., and if this is not compensated, it will affect the readings by 7 per cent. For some rough purposes this might not be objectionable, but it is relatively easy to provide the instrument with the means above described for eliminating this error.

It may seem that a pivoted instrument requiring only 200 microamperes for full-scale deflection is too delicate for the purpose. The instrument taken as an example is made for use in industrial pyrometry where average conditions are at least as severe as in a power plant or substation. In an article in *Die Messtechnik* for April and June, 1928, summing up the status of remote metering, Dr. G. Keinath of the Siemens and Halske Co. states that d-c. indicating instruments may be used for this purpose which give full-scale deflection on as little as 50 microamperes. The matter contained in this article, with some additional matter, is given on pages 166-185 of Vol. II of the third edition of Dr. Keinath's book, "Die Technik der Elektrischen Messgeräte."

F. L. Lawton: It may be of interest to mention that very satisfactory results have been obtained with the thermal converters used by Price Brothers & Company and the Duke-Price Power Company at Isle Maligne, Quebec. These converters are used for totalizing the load of two large paper mills of the former company, the mills being 23 mi. apart. The total load is about 100,000 hp.

Our experience indicates several distinct advantages of this method of totalizing power demands, such as:

(1) Ease and reliability with which an indication of the total load on a group of feeders or transmission lines can be obtained at a remote point.

(2) Flexibility of the method—equally good results can be had at points remote from, or close to, the load-measuring instruments.

(3) High accuracy of the thermal converters. Readings from different groups of converters, totalizing the same circuits, agree very closely.

(4) Absence of moving parts means that the thermal converters can be placed in almost any location, close to the instrument transformers, thus cutting the burden on the transformers, and giving low wiring costs.

(5) The zero suppressor makes possible the recording of occasional loads in excess of the normal range, or increasing the range, with only a moment's work on the part of the station operator.

(6) A very open graphic record is obtained.

Bela Gati: By replacing the resistances and thermocouples with barretter wires, that is, wires 1 to 3 micrometers in diameter, we can get the following results: The Lincoln wattmeter's time response will be reduced to 0.00005 sec. instead of 7 sec. The sensitiveness will be one hundred times greater. I proved this in the *Physik Nalische Zeitschrift* before the War. Telephone current and voltage can be measured with barretter wires; thus the Lincoln thermo-wattmeter could be transformerd into a speech-wattmeter, an instrument which has been sought for many years.

D. A. McKenzie: The theory on which this system is developed has been well treated by Professor Lincoln. However, I might point out some of the special features of this system which appeal particularly to the operating man.

1. The thermal converters contain no moving parts. Since our experience dates back to the first of these devices ever placed in service, we naturally have had some trouble with them but these devices are now an exceptionally reliable commercial product.

2. The scheme is very flexible inasmuch as not only may these converters be connected up to add or subtract different loads but the converters themselves are reversible and are therefore suitable

for use on tie feeders. It is therefore possible to take care of almost any conceivable scheme of system connections or supply.

3. The apparatus in the transmitting station is very simple and requires a minimum of space. Since it requires no regular inspection it may be located wherever it is found convenient to connect into the measuring circuits. In fact the converter itself may be given its regular periodical calibration test from any point where it may be found convenient to connect the testing equipment into the necessary circuits.

4. By suitably arranging the transmitting circuits, it is possible at any central point to obtain the load on any desired station or section, after which the groups may be totalized.

5. An exceptionally high accuracy may be reliably obtained. These devices are regarded by the operators as absolutely accurate. The Toronto Totalizer has a scale spread of 15,000 kw. per in.

Mr. Lincoln dismisses the matter of the transmitting circuit rather summarily, but it is our belief that this part of the equipment requires very careful attention. The London equipment shown as using No. 10B. & S. overhead wires carried on porcelain insulators on the same pole circuit as two of the 13.2 kv. feeders never operated satisfactorily even though special means of protecting this circuit were evolved and applied. As a matter of fact, the recorder was only permanently placed in service after an underground cable connection was obtained. A considerable portion of the difficulties encountered with the Toronto installation is attributed to the fact that the Toronto Hydro-Electric System does not use cables in which the wires are arranged in twisted pairs, which makes the position of each conductor in the cable with reference to all other conductors a matter of somewhat indefinite and variable quantity. Also the transmission of other quantities on other conductors in the cable has caused some interference, though it is believed that this could be considerably reduced by special arrangements. The trouble may be said to be more of electrostatic than electromagnetic interference.

So far the recorder used in connection with this system has been of one type, *viz*, a recording potentiometer of a well known make which, since it had been in commercial use for a number of years, was expected to be suitable for use as it then was. It has been found that great care must be used, as indicated by Mr. Lincoln, in selecting a recorder with a galvanometer whose characteristics are suited to the remainder of the circuit including the transmission circuit and resistance of the converter elements as the damping in this circuit plays a very great part in the character of the record obtained. Due to the slight time lag in the thermal converter, it is naturally to be expected that a somewhat smoother load curve should be obtained from these devices than from direct-acting meters. It might also be mentioned that by careful adjustment and maintenance, it has been possible to obtain a performance from these recorders which is believed to surpass materially the expectations of the manufacturers.

C. G. Brown: I should just like to ask Professor Lincoln whether in his tests on the wattmeter as he showed it, he has any results as to the accuracy of the wattmeter element with, for instance, half voltage and double current.

A. R. Wells: The recording potentiometer as used in the Toronto installation was found to be designed to record a number of temperature indications at widely different portions of the scale. It was found the meter was designed to jump from, say, 10 per cent to 70 per cent of scale for the next indication. That does not suit the case where there are small variations of load, all of which are required to be recorded, so it was necessary to modify the speed of the pen across the paper so that it moves the full width of the paper in approximately 40 sec.

There are three essential adjustments in all graphic potentiometers, (1) zero adjustment of the galvanometer, (2) the adjustment of the slide-wire current, and (3) the adjustment of the pen zero so that it coincides with the chart zero line.

It was considered necessary in view of the importance of the record, to have a record on each day's chart that all of these adjustments were correct. This was accomplished by using the fact that as long as the galvanometer pointer remains at zero the pen will make a line on the chart parallel to side of the chart.

The galvanometer zero is recorded by disconnecting it from the circuit and short circuiting it through a resistance of the proper value for damping.

The correctness of the slide-wire current is recorded by connecting the galvanometer in the usual manner for this test. If the adjustment is correct the galvanometer will be on zero and the pen will make a line parallel to the edge of the paper.

The fact that the pen zero is correct is recorded in the same way as is normal in other graphic meters, by disconnecting the meter from the external circuit, but in this case the instrument must be short-circuited through a resistance.

All of these features have proved their value in service because of the magnitude of the load and because this meter must agree with the sum of the indications of the loads which are totalized.

P. M. Lincoln: There is one point on which I expected some question concerning the method of totalizing which I have described. That was on the relatively low voltages which were generated by the so-called thermal converters. We use, as I indicated in the paper, 10 millivolts per single phase element, which will mean 20 millivolts per polyphase element, where there are two single-phase elements used to measure the total polyphase energy. Twenty millivolts per converter, even with the 23 such thermal converters used on the system of the Hydro-Electric Power Commission, will still make a total of considerably less than one volt at the potentiometer. It might be claimed that if there is any d-c. leakage into such a circuit, the error will be very considerable, which is true. It might be contended that if the individual converter voltages were higher there would be an improvement. I would question such a contention. If there is going to be any d-c. leakage, there will be an error and the error is going to be the ratio that the parasitic voltage bears to the normal voltage. If such a parasitic voltage occurs, it is, I submit, much better to have such a voltage apparent at once to the operator than to have the parasitic voltage be so small a percentage of the measured voltage that it cannot be readily distinguished. I, therefore, submit that it is preferable to have a total measured voltage which is low so that parasitic voltages can be instantly detected, rather than to have the measured voltage higher.

Mr. Brooks has indicated that there are methods of measuring the voltage other than the potentiometer which is described. It has been our experience that in all cases we have come in contact with, a graphic record is demanded and the recording potentiometer which we have been using gives such a graphic method. If we are willing to dispose of the graphic chart and simply wish an indication, there are many other methods which might be accepted and adopted and one that Mr. Brooks has mentioned is certainly one that would be considered if the graphic record were not required.

Mr. Brown asked concerning the accuracy of these thermal wattmeters with other voltages and other currents applied. He asked specifically if we took one of these thermal wattmeters which had been adjusted for normal voltage and normal current and put on half voltage and double the current, would there be any error. I can say no. There is no observable error with a change of such a nature. The observable error is less than can be read. In other words, the thermal meter is just as truly a wattmeter as the ordinary electrodynamic instrument with which we are all familiar.

A New High Accuracy Current Transformer

BY M. S. WILSON[1]

Associate, A. I. E. E.

Synopsis.—This paper is presented for the purpose of describing a new method of compensation, by the use of which the ratio and phase-angle errors of current transformers are materially reduced.

There are several methods by which the ratio and phase-angle errors of current transformers may be reduced. The following are those which are most commonly used:

1. By changing the number of secondary turns, which reduces ratio errors only. This method is universally used and operates satisfactorily under various conditions of burden, frequency, etc.

2. By the use of a larger number of ampere-turns, a large cross-section of core, or both.

An increase in either one of these factors improves the accuracy characteristics of the transformer under the different conditions of burden, frequency, or secondary current.

3. By the use of a high-permeability low-loss steel, which improves the transformer characteristics for moderate values of secondary burdens. The errors increase rapidly under overload conditions.

4. By the use of the two-stage type transformer, which operates satisfactorily under the various conditions of burden and frequency. Slightly special equipment is required, however, for power or energy measurements.

5. By the use of compensating shunts across the primary or secondary windings of the transformer. This method operates satisfactorily only under fixed conditions of burden, frequency, etc.

6. By a new method, in which the secondary turns enclose a section of the magnetic circuit and a short-circuited band encircles the same section. By the use of this method both ratio and phase-angle errors are reduced for the various conditions of burden, frequency, and secondary current.

The change in ratio and phase-angle characteristics between light and full load is considerably reduced, resulting in more nearly straight ratio and phase-angle curves. This is a desirable feature in view of the demand for meter and transformer combinations which must operate satisfactorily over large ranges of current.

The overload characteristics of the current transformer are not injured by the use of this method of reducing the transformer errors.

The usual insulation between the primary and secondary can readily be maintained.

The use of this compensating feature does not in any way impair the strength of the transformer to withstand over-current shocks.

No changes in the physical dimension of the transformer are required by the application of this compensating method.

INTRODUCTION

As is quite well known, the ratio and phase-angle errors inherent in current transformers are due to the exciting current which is required to maintain the flux and supply the losses in the core. There is a number of methods by which these errors may be reduced or compensated for. The following are most commonly used and operate with varying degrees of effectiveness for the various conditions of burden, frequency, etc., to which the transformer may be subjected.

1. The changing of secondary turns. This does not affect the losses, but raises or lowers the ratio approximately the same amount for the different values of secondary current. It has no effect on the phase angle. This method, which is universally used, operates equally well under the various conditions mentioned above.

2. The use of a large number of ampere-turns, a large cross section of core, or both. For a first approximation the errors in a current transformer are inversely proportional to the square of the number of ampere turns and to the cross section of the magnetic circuit, assuming the mean length to remain constant. Therefore, an increase in either one or both of these factors will improve the operating characteristics of the transformer under all conditions. The use of these two methods is limited, however, by the physical dimensions of the transformer.

3. The use of a high permeability low-loss steel.[2] By using a steel which has a higher permeability and lower losses than the ordinary transformer steel, the exciting current may be diminished, thus reducing the transformer errors. A material having these characteristics, however, usually saturates at quite low densities, so that for moderate burdens or overload conditions the ratio and phase-angle errors increase very rapidly.

4. The use of the two-stage type of transformer.[3] This transformer is built in two stages or cores. The function of the second stage is to supply a current which is equal in phase relation and magnitude to the exciting current required by the first stage. This type of transformer works well under any of the operating conditions to which it may be subjected. Somewhat special equipment is necessary, however, for power or energy measurements.

5. The use of compensating shunts across the primary or secondary windings of the transformer. This method of correction operates satisfactorily only for fixed conditions of burden, frequency, etc.

6. A new method has recently been developed. When used in conjunction with the first method as described above, it furnishes a simple and effective means by which the ratio and phase-angle errors of current transformers are materially reduced. This type of correction operates in such a way as to effect an alteration of the flux conditions in the magnetic circuit,

1. Standardizing Laboratory, General Electric Co., West Lynn, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

2. *Current Transformers with Nickel-Iron Cores*, by Thomas Spooner, A. I. E. E., TRANS., 1926, p. 701.

3. *Two-Stage Current Transformer*, by H. B. Brooks and F. C. Holtz, A. I. E. E., TRANS., 1922, p. 382.

so that the product of the flux and secondary turns will vary in a direct proportion to the primary current.

This tends to produce a constant ratio between the primary and secondary currents for different percentages of load. The resulting constant error may then be reduced by the usual method of changing secondary turns.

In the new method of correction, the phase-angle errors of the current transformer are reduced by the use of a lag band on a section of the magnetic circuit. The function of this band is to lag the flux slightly, so as to throw the secondary current nearer in phase opposition to the primary current. The effect of this compensation varies with the percentage of load in such a way that a large correction is obtained in the light load region, where the errors are large, and a smaller correction as full load is approached, where the errors are less. This combination of ratio and phase angle corrector operates well under the various conditions of burden, frequency, and load, as well as under various overload conditions. The transformer is also entirely standard, in so far as the use of external apparatus in either the primary or secondary circuits is concerned.

Details of the New Method of Reducing the Ratio and Phase Angle Errors of Current Transformers

The principle upon which this new type of ratio corrector operates is, in effect, that of automatically

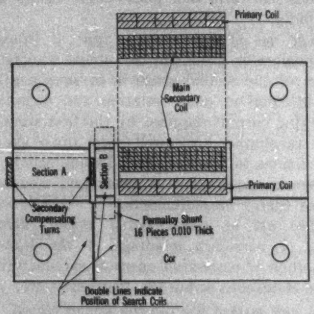


Fig. 1—Sectional View of a Current Transformer

Illustrating location of compensating turns and permalloy shunt. Double lines indicate position of search coils

changing the effective secondary turns as the load increases or decreases.

For example, assume a ratio correction factor of 1.010 for 10 per cent load and of 1.000 for 100 per cent load. This indicates that at 10 per cent load the secondary current will be 1 per cent too small and at 100 per cent load it will be correct. Therefore, if it were possible to increase the number of effective secondary turns by the proper amounts as the current increased from light load to full load, the ratio of the full-load point would be the same as that of the light-load point.

The resulting constant error, which in the case mentioned above would be 1 per cent, could then be eliminated by reducing the number of secondary turns 1 per cent.

In order to determine a method by which the effective secondary turns of a current transformer could be

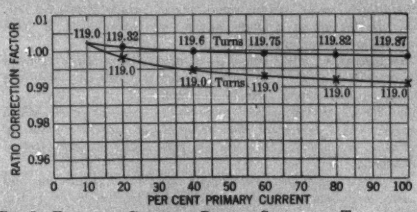


Fig. 2—Effect of Changing Partial Secondary Turns on a Current Transformer

x Standard transformer
● Compensated transformer

automatically changed for different load conditions, tests and calculations were made on a transformer in which the magnetic circuit and secondary turns were arranged as illustrated in Fig. 1. Here will be noted that the secondary windings are arranged in two sections on the magnetic circuit, with the larger number of turns on the first or main section, and the remainder, usually one or two per cent of the total, on the second section. Since all of these turns are wound in the same direction, a correction in the ratio would be obtained if the flux could be shunted around the smaller number of turns for the lower values of secondary current and gradually caused to flow through them in the proper direction as the current increases. This would then have the effect of adding secondary turns as the current or load increased, which in turn would increase the ratio and straighten out the ratio curve. With the proper number of turns on both sections, the five-ampere or full-load point would have approximately the same ratio as the 0.5-ampere point. This is graphically illustrated in Fig. 2.

In order to afford a means by which the amount of flux linking the smaller number of turns could be altered, a shunt was employed which was made up of permalloy strips inserted as illustrated in Fig. 1. (Any material which has a low reluctance at low flux densities may be used for this shunt.)

The secondary turns around section *A* produce a magnetomotive force, the action of which is vital to the operation of this corrective method. This magnetomotive force causes a circulating flux, as illustrated in Fig. 3, which is superimposed on the main or primary flux in sections *A* and *B*. This results in a change in

the relative flux densities of these two sections. Since the circulating flux varies with the secondary current, the relative flux densities in the two sections will also vary. This variation in density will cause a change in the relative reluctances of the two sections so that at higher currents, the reluctance of section B will be considerably larger than that of section A, thus causing a relatively larger portion of the main flux to flow in the latter section.

Therefore, since the proportion of main or primary

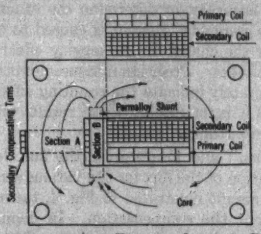


FIG. 3—REDISTRIBUTION OF FLUX IN CORE OF COMPENSATED CURRENT TRANSFORMER

flux, linking the secondary turns on section A, increases with the secondary current, the desired effect of an increase in effective secondary turns will be produced. This tends to maintain a nearly constant ratio for different values of secondary current.

From the arrangement of the turns on section A, and the arrangement of the magnetic circuit, it is seen that it is merely necessary to have a portion of the primary flux change from section B to section A as the secondary current increases, in order that the secondary turns on the larger section may become more effective and increase the ratio of transformation. This change will occur when the flux density of section B is such that the reluctance increases more rapidly than the reluctance of section A. In order to produce this condition, it is necessary to operate section B on the descending side of the permeability curve so that as the secondary current increases, the reluctance of this section will increase at a faster rate than that of Section A, which operates lower down on the permeability curve. The reluctance of the latter section may actually decrease as the flux density increases, due to the increase in secondary current or burden.

Although the ratio errors may be very materially reduced by the use of this type and construction of the magnetic circuit, it was considered impractical from a manufacturing standpoint for use in standard type of current transformers. Consequently, calculations, tests, and flux measurements were made on a transformer in which the parallel magnetic circuits were formed by an opening or hole through the laminations, as indicated in Fig. 4. The flux measurement proved that, with the proper number of secondary turns on the parallel section of the magnetic circuit, it was possible to produce flux conditions in this region which had the same effect on the ratio characteristics as was obtained by the use of the permalloy shunt. This construction proved to be satisfactory both from an electrical and mechanical standpoint and has been adopted by the General Electric Company in the manufacture of transformers using this method of compensation.

It will be noted in Fig. 4 that the secondary compensating turns enclose the narrow section of the magnetic circuit. The results obtained with the turns on either the narrow or wide section of the magnetic circuit are practically identical, providing the proper direction of winding is maintained. The direction of winding should be such that the instantaneous direction of the flux in the wider section of the parallel magnetic circuit, due to the current in these turns, will be opposite to the main flux. If these flux relations are maintained, a change in the main flux from sections S or T to R or U, respectively, as the flux density increases, will produce practically identical results regardless of whether the compensating turns enclose the wide or narrow section of the parallel magnetic circuit.

The flux densities in sections T and U are plotted

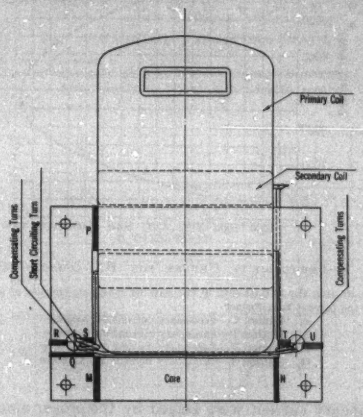


FIG. 4—15,000-VOLT COMPENSATED CURRENT TRANSFORMER

(Letters indicate position of search coils used in making flux measurements.)

on a characteristic d-c. permeability curve for $D^{\circ}X$ steel in Fig. 5, to illustrate the difference in the relative permeabilities of the two sections for 0.5 and 2.5 amperes secondary current.

It will be noted that until the flux densities indicated for both sections with 0.5 ampere secondary current are doubled, there will be but a slight change in the relative permeabilities and consequently the reluctances of the two sections. Thus, under these conditions, the

compensating turns would start functioning to correct the ratio at approximately 1.0 ampere secondary current, as illustrated by the ratio curves in Fig. 7.

It is realized that the flux relations as illustrated on a d-c. permeability curve are not strictly true, but are considered satisfactory from the point of view of a qualitative analysis.

The corrective idea described thus far concerns the reduction or elimination of the ratio errors of current transformers, and has practically no effect on the phase angle. The phase angle for most current transformers, except those having a large number of ampere turns or having a large leakage reactance, are generally positive at lower current values and decrease as the current increases, until at full load rating the phase angle may be zero, or even negative. Since a positive angle indicates that the secondary current reversed leads the primary current, it is evident that if the

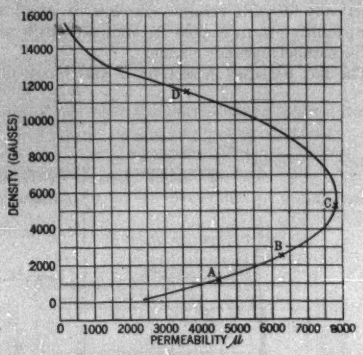


FIG. 5—PERMEABILITY CURVES FOR *DX* (SILICON) STEEL

Illustrating the flux density in the parallel magnetic circuits of a compensated current transformer

A. Section U—Secondary current 0.5 ampere
B. Section T—Secondary current 0.5 ampere
C. Section V—Secondary current 2.5 amperes
D. Section T—Secondary current 2.5 amperes

secondary current were lagged by the proper amount, the phase angle could be reduced to zero. Also, if this lagging effect were diminished as the current is increased, it would be possible under proper conditions to correct for the large phase-angle errors at light loads or low currents and the small phase-angle errors occurring at full load.

The method used to reduce the phase-angle errors involves the use of a lag band which encloses the same section of the magnetic circuit as does the secondary corrective turns. The action of the current in this band causes the main flux to lag slightly, consequently lagging the secondary current and throwing it nearer in phase opposition to the primary current, thus reducing the angle between the two currents. The action of this lag band may be likened to that of the lag band of a watt-hour meter, in which case the potential flux is caused to lag sufficiently to produce the quadrature relation between the flux and voltage of the potential element.

In the case of the compensated transformer the magnitude of the angles involved are so small that it is practically impossible to make flux measurements of sufficient accuracy to determine the exact manner in which the secondary current is caused to lag, also the reason for the change in the lagging effect as the flux density changes. Precision measurements of the phase angle accurately indicate the resulting effect of the lag band, and it is, of course, these results which are of primary importance.

In Tables I and II are given the flux densities and phase relation of the flux for the various sections of the magnetic circuit illustrated in Fig. 4, taken both with and without the compensation. In connection with these data it is interesting to note the effect of the leakage flux on the distribution of the mutual flux in the core. The density in the external member of the magnetic circuit is more than twice that of the inside member. There is also a change in phase relation of approximately 25 deg. between the fluxes in these two sections.

TABLE I

Flux measurements taken on a 1500-ampere-turn, 15,000-volt current transformer (cross-section of core 19.5 sq. cm. mean length of core 50 cm). Secondary burden 0.68 ohms resistance and 980 microhenrys inductance (19.4 volt-amperes at 0.88 power factor). Normal transformer, no secondary corrective turns or short circuited turns. (See Fig. 4.)

Section encircled by search coil	Sec. amperes	Induced voltage in search coils	Angle betw. flux in search coils and sec. current	Total flux in search coils (max. value)	Flux density gausses
P	0.498	0.0224	124°48′	841	42
R	"	0.0228	158°41′	1077	141
S	"	0.0231	159°52′	865	196
Q	"	0.0523	159°32′	1960	100
M	"	0.0546	160°48′	2046	104
N	"	0.0557	160°44′	2085	106
U	"	0.0298	159°44′	1115	146
T	"	0.0240	160°18′	900	204
P	2.506	0.1146	124°57′	4300	219
R	"	0.1447	158°35	5430	715
S	"	0.1204	159°21′	4510	1025
Q	"	0.267	159°5′	10020	512
M	"	0.278	159°49′	10420	532
N	"	0.285	160°19′	10700	547
U	"	0.1525	159°47′	5720	753
T	"	0.1256	159°33′	4710	1070
P	4.98	0.228	125°12′	8530	436
R	"	0.282	158°38′	10560	1390
S	"	0.244	159°46′	9130	2080
Q	"	0.536	159°18′	20100	1025
M	"	0.557	160°3′	20900	1065
N	"	0.572	160°36′	21450	1095
U	"	0.288	160°25′	10770	1417
T	"	0.267	159°44′	10030	2280

TABLE II

Flux measurement taken on a 1500-ampere-turn, 15,000-volt current transformer (cross-section of core 19.5 sq. cm. mean length of core 50 cm.). Secondary burden 0.68 ohms resistance and 980 microhenrys inductance (19.4 volt-amperes at 0.88 power factor). Three secondary corrective turns around Sections S and T. One short circuited turn around Section S. (See Fig. 4.)

Section encircled by search coil	Sec. amperes	Induced voltage in search coils	Angle betw. flux in search coils and sec. current	Total flux in search coils (max. value)	Flux density gausses
P	0.499	0.0263	127°42′	987	50.
R	"	0.0450	209°58′	1685	221
S	"	0.0458	106°32′	1718	390
Q	"	0.0566	157°40′	2132	109
M	"	0.0588	158°35′	2208	113
N	"	0.0608	159°32′	2284	117
U	"	0.248	334°12′	− 9300	1225
T	"	0.306	155°11′	11470	2610
P	2.5	0.1296	128°23′	4860	248
R	"	0.253	206°53′	9480	1246
S	"	0.222	99°33′	8320	1890
Q	"	0.285	158°24′	10680	546
M	"	0.295	159°8′	11060	565
N	"	0.306	160°7′	11460	585
U	"	1.073	346°15′	−40250	5200
T	"	1.365	164°42′	51200	11630
P	4.97	0.249	127°39′	9320	476
R	"	0.512	207°48′	19180	2520
S	"	0.445	98°16′	16680	3790
Q	"	0.556	158°46′	20840	1064
M	"	0.580	159°37′	21720	1107
N	"	0.596	160°23′	22320	1139
U	"	1.208	354°40′	−45300	5960
T	"	1.760	169°48′	66000	15000

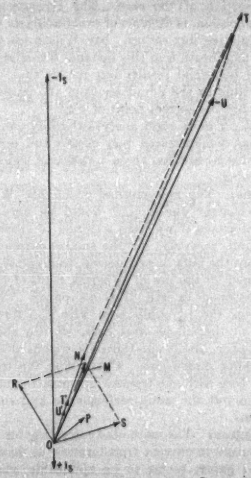


FIG. 6—VECTOR RELATIONS OF FLUX IN A COMPENSATED 15,000-VOLT CURRENT TRANSFORMER

0.5 ampere secondary current

Letters refer to sections enclosed by search coils in Fig. 4. Primes indicate measurements made without compensating turns on sections S or T

A vector diagram illustrating flux conditions in the magnetic circuit of the compensated transformer is given in Fig. 6. It will be noted on this diagram that the flux in Section U for the 0.5-amperes secondary point is more than eight times the amount in the uncompensated transformer for the same section. The sign is reversed which indicates a reversal of the flux

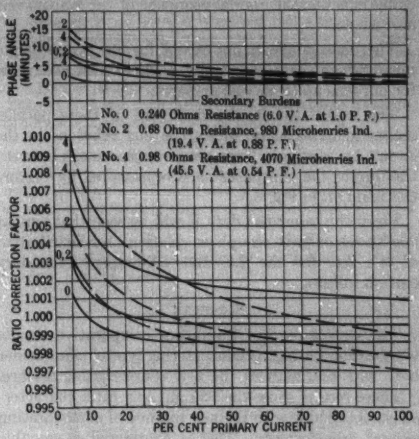


FIG. 7—RATIO AND PHASE-ANGLE CURVES

60 cycles frequency

——— Compensated type, 15,000-volt current transformer

— — — Same transformer without compensation

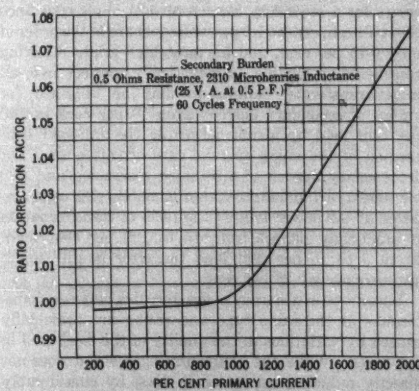


FIG. 8—OVERLOAD CURVE ON 15,000-VOLT, 20-AMPERE, 4:1 RATIO, COMPENSATED CURRENT TRANSFORMER

in this section. This alternation of the flux condition is, as was the case when the permalloy shunt was used, due to the secondary turns on Section T.

It should be noted that inasmuch as the flux measurements were made by the use of the a-c. potentiometer

there are possibly some errors in the data due to wave form distortion. This distortion is confined, however, to the section of the magnetic circuit in the vicinity of the compensating turns, and has no appreciable effect on the normal operation of the transformer as a whole.

The phase-angle errors except for low power factor measurements are practically negligible on 60 cycles, for burdens up to 45 volt-amperes.

The use of the lag band on one of these parallel magnetic paths again alters the flux relations in the circuit by causing a large phase displacement between the flux in the two paths as is represented by vectors $O\,S$ and $O\,R$ in Fig. 6.

From the tabulated data it will be seen that similar conditions exist in the phase relation and distribution of the flux for the 2.5- and 5.0-ampere secondary current values.

Results

The curves in Fig. 7 indicate comparative results obtained on the high grade metering type of transformer both with and without the new compensating feature.

These results show the straight line ratio characteristics, obtainable with the new method of compensating current transformers. This is considered a valuable feature in view of the demand for long range metering equipments, since by the use of this method of compensation the transformer range has been considerably extended in the light load regions. Although the ratio errors for a secondary burden of 0.24 ohms resistance (6.0 volt-amperes at 1.0 power factor) are in the order of only 0.15 per cent, and for most high grade metering considered negligible, a practically perfect over-all meter-transformer calibration may be obtained for values of secondary current from 10 per cent to 100 per cent load by merely adjusting the full-load calibration of the meter to take care of this error. For a secondary burden of 0.68 ohms resistance and 980 microhenrys inductance (approximately 20 volt-amperes at 0.9 power factor) the ratio of transformation is practically perfect from 15 to 100 per cent load. From 5 to 15 per cent load the error is still small, approximately 0.25 per cent.

Variations in accuracy characteristics between transformers of the same type and rating are principally caused by variations in the quality of the steel used in the cores. While the compensating method does not directly reduce these errors, it does, by considerably reducing the greater errors, make these smaller errors more important, and undoubtedly will result in a greater effort to improve the quality and uniformity of the steel used in current transformers.

The overload characteristics for this type of transformer are illustrated in Fig. 8. Tests indicate that there is practically no difference in the overload performance of the compensated and uncompensated transformers of the same type and rating.

The successful operation of protective methods employing differential and balanced relays largely depend on the similarity and uniformity of the current transformers characteristics under overload conditions. Since this compensating method can be applied to transformers of standard construction, both the compensated and uncompensated types can be used interchangeably for this service.

Accuracy tests indicate that there is practically no change in ratio or phase-angle characteristics for changes of 25 deg. cent. in ambient temperature.

The use of this new compensating feature in no way impairs the strength of the transformer to withstand over-current shocks.

The usual insulation value for breakdown between primary and secondary and primary to core can readily be maintained.

Discussion

I. F. Kinnard: The scheme of compensation described by Mr. Wilson was first conceived by him as a simple and direct method of obtaining the effect of variable secondary turns by utilizing the varying permeability of a magnetic shunt.

After much painstaking experimental work he discovered that the scheme not only worked but worked well. He found that the same results could be obtained by dispensing with the shunt and utilizing a parallel magnetic circuit in a small section of the core. The arrangement, although simple in application, is difficult of exact analysis, due largely to the local circulating flux set up. Mr. Wilson has indicated that it is not the modification in this internal secondary burden that produces the beneficial results but rather that the main flux divides according to the admittances of the paths at all times. This is probably the correct point of view.

Undoubtedly a more rigid analytical study can be made but the author has demonstrated very clearly the practical importance of the scheme and has given a clear and logical explanation of its functioning.

J. B. Gibbs: The new method which Mr. Wilson outlines gives very excellent curves and would be very valuable for current transformers if we were confined to silicon steel cores. We have found, however, for the smaller current transformers which make up the bulk of production, that equally good results can be obtained by the use of hypernik cores of proper proportions as mentioned in Mr. Wilson's paper. Comparing the transformer on the hypernik core with the former standard transformer on the silicon-steel core, we use about the same amount of copper, about one-third the weight of core, and considerably smaller dimensions. Comparing the transformer on the hypernik core with the transformer with the new compensation, we get about the same performance and smaller weights and dimensions.

A. M. Wiggins: The method of correcting for the ratio and phase angle errors in current transformers, as described by Mr. Wilson in his paper, seems to be applicable where the characteristics of the core material cannot be used to the same advantage. From the formulas derived by Dr. Agnew in the Bureau of Standards publication, Vol. 7, No. 3, the per cent ratio and phase angle of current transformers may be stated:

$$\text{Per cent ratio} = \left[\frac{n}{r} + \Delta'\left(\frac{w}{V\,A}\right)\right] 100 \qquad (1)$$

$$\text{Phase angle} = \tan^{-1}\left(\Delta''\frac{w}{V\,A}\right) \qquad (2)$$

Where, n = turn ratio, secondary to primary
r = nominal, or name-plate, ratio
w = weight of core material in lb.
VA = volt-ampere burden on the secondary, including external and internal burdens
$\Delta' = P \cos \varphi + Q \sin \varphi$
$\Delta'' = Q \cos \varphi - P \sin \varphi$
P = core loss in watts per lb. of core material
Q = magnetizing volt-amperes per lb. of core material
φ = power factor angle of secondary circuit

Now if in (1) and (2) a combination of design values can be

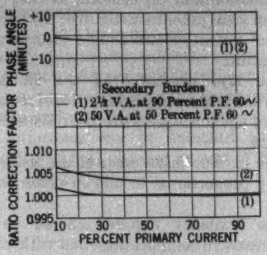


Fig. 1—Ratio and Phase Angle Curves

Nickel-Iron Alloy Core
7500-Volt Design

found such that $\left(\Delta' \frac{w}{VA}\right)$ and $\left(\Delta'' \frac{w}{VA}\right)$ are constant, or nearly so, then the flat, or straight line, characteristic for ratio and phase angle curves obtained by Mr. Wilson can be realized without the compensating windings. Unfortunately, with the present commercial silicon steels this cannot be done, practicably. However, the proper use of a certain nickel-iron alloy results in nearly constant values for both these quantities over a considerable range. This alloy is described in the paper by Spooner, cited in the references in Mr. Wilson's paper.

Finding the correct combination for this condition is more of an experimental proposition than a calculation, as the calculations are at best only qualitatively reliable, due to the difficulty in calculating the effect of the leakage flux. As one example, in a 7500-volt design the combination which seems to give the best results employs 1500 to 1600 ampere-turns and rather high flux densities, approximately 2000 gausses, in the core section under the secondary winding with an external secondary burden of 25 volt-amperes, 0.80 power factor, 60 cycles. The weight of this core is only 3.5 lb. In all cases the flux densities which produce most satisfactory results are from 2 to 4 times those of a similar silicon-steel design, for the same burden, ampere-turns, etc. From this it might be supposed that such transformers would have very poor ratio characteristics at overcurrents, since nickel-iron alloys saturate at lower values than do silicon steels. Such is not the case, however. Figs. 1 and 2 herewith show typical performance curves for a 7500-volt design for both normal and overcurrents. From a comparison of these curves with the ones given by Mr. Wilson, it is evident that almost identical results have been obtained, but by somewhat different methods.

M. S. Wilson: The type of compensation described in the paper furnishes a simple and reliable means of improving the performance of standard current transformers.

There is, of course, a number of different methods by which the transformer characteristics may be improved. The use of a core material which reduces the exciting current obviously will reduce the ratio and phase-angle errors.

Due to the lack of a complete statement as to the secondary burden at which the overload curve presented by Mr. Wiggins was made, a discussion of this curve must be based on certain assumptions. It is the Institute practise to state the secondary burden of instrument transformers in ohms and henries, which are maintained constant regardless of the frequency or percentage of primary current.

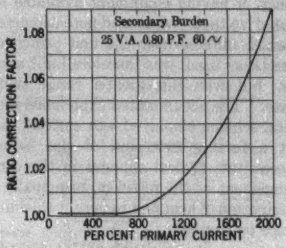


Fig. 2—Overcurrent Curve

Nickel-Iron Core
7500-Volt Design

From Mr. Wiggins' description of the transformer and from the character of the overload curve shown in his Fig. 2, it would appear that the impedance of the secondary burden was adjusted for different percentages of current, so that the volt-ampere value was maintained constant. On this basis the impedance external to the transformer would be 1.0 ohm for 100 per cent current and only 0.0025 ohms for 2000 per cent current, whereas in the compensated type 1 ohm impedance was held constant throughout the entire range of primary current as shown by the curve in Fig. 8 of the paper.

On this assumption the overload curves for the two types are not comparable.

132-Kv. Shielded Potentiometer
for Determining the Accuracy of Potential Transformers

BY C. T. WELLER[1]
Member, A. I. E. E.

Synopsis.—The potentiometer principle is well understood. Its application to the determination of the ratio and phase angle accuracy of potential transformers is not new, but has been limited in unshielded form to about 66 kv. and in shielded form to about 33 kv. The paper describes a shielded potentiometer for 132 kv. which is designed for ultimate extension to 220 kv.

INTRODUCTION

THE first potentiometer used in the Schenectady Laboratory for determining the ratio and phase angle accuracy of potential transformers was described in 1909. This "deflection" potentiometer was rated at 33 kv. and remained in service until 1920. By that time many 66-kv. potential transformers had been built so that it became necessary either to extend the voltage range of the potentiometer or to abandon the early policy of having equipment available for testing at full rated voltage. A second potentiometer rated at 66 kv., which differed from the first principally in the type of resistors used, was therefore built and put into operation; but even this greater range soon became inadequate. The accumulated experience showed that the general scheme employed was entirely satisfactory but that a further extension of the voltage range would necessitate radical changes in the construction and arrangement of the resistors, principally to secure better dissipation of heat and to permit shielding against capacitance to ground.

In order to make further extension possible, the author proposed in 1924 to construct 100,000-ohm resistor units of fairly large dimensions rated at 11 kv. each, and to maintain the respective unit shields at the proper potentials by means of a special auto-transformer (in separate sections) of ample rating with suitable taps. Four of the units and one auto-transformer section constitute a complete 44-kv. group; the number of series groups necessary is determined by the voltage range desired. To the best of his knowledge, the proposed arrangement and method of shielding had not been utilized before. The 132-kv. potentiometer herein described incorporates the proposed features.

OBJECTIVE

A potentiometer was desired, which would (1) cover the range from 6.6 to 132 kv. at first and to 220 kv. by future extension and (2) conform as closely as practicable to an ideal potentiometer over the entire range.

An ideal potentiometer should incorporate the following features:

a. The d-c. resistance of the unit(s) obtained by means of a low voltage bridge should remain constant and should equal the a-c. effective resistance at operating voltage.

b. The current in all parts of the resistor(s) should be exactly proportional to and in phase with the line voltage.

c. The flexibility required should be obtainable without excessive complications and without sacrificing convenience of operation.

SUMMARY

The installation of three 44-kv. series groups and accessories at Pittsfield was completed in 1928. The groups alone occupy about 380 sq. ft. (42 ft. by 9 ft.) of floor space, have a height of approximately 12 ft. and weigh about 65 tons (the first potentiometer complete weighed less than one ton). A "General Description" of the potentiometer is given under that title.

Potential transformers have been tested for accuracy over the range from 6.6 to 132 kv. at 25 and at 60 cycles (see comments under "Test Results").

Compared with the ideal outlined:

a. The d-c. resistance of the units is practically constant and agrees with the a-c. resistance within 0.01 per cent.

b. The current in the units is directly proportional to the line voltage at all points but small phase displacements exist. The "net" phase angle of the current at 60 cycles ranges from approximately $-1'$ (leading) at 6.6 kv. to $-7'$ at 44 kv. in the first group, to $-6'$ at 88 kv. with two groups and to $-4'$ at 132 kv. with three groups; the corresponding corrections have the same signs.

c. Transformers rated between 6.6 and 44 kv. in 1.1-kv. steps (with a corresponding increase in ratio) and between 44 and 132 kv. in 5.5-kv. steps can be readily tested; this includes practically all standard ratings. Odd ratings differing less than 4 per cent from those indicated can usually be tested without special adjustments, but other odd ratings require an adjust-

1. General Engineering Laboratory, General Electric Company, Schenectady, N. Y.

2. *Electrical Measurements on Circuits Requiring Current and Potential Transformers*, by L. T. Robinson, A. I. E. E. TRANS., Vol. 28, 1909, p. 1005.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

29-43

ment of the resistance. The ratio (correction factor) is read directly (or by inter-polation) for standard ratings but must be calculated for odd ratings; the phase angle is calculated from instrument deflections in all cases.

The complications are not excessive and the operation is reasonably convenient; provision is made also for operating with the mid-point instead of with the lower end of the potentiometer grounded.

The twelve resistor units and the three auto-transformer sections or shielding transformers are respectively interchangeable.

Conclusions

The accuracy obtainable is sufficient to permit certifying results to within 0.1 per cent in ratio and to within five minutes in phase angle over the entire range; these limits are in accord with our best previous practise.

The potentiometer can be extended to 220 kv. without making radical changes in any of the groups whenever the demand justifies it.

between the total potentiometer resistance in use and the normal known fraction thereof is the same as the marked or nominal ratio of the transformer under test (60:1 or 60,000:1000 ohms for a 6600:110-volt transformer).

In Fig. 1b, which applies only to the determination of the ratio between E and E_S, a "balance" is obtained by interpolation between the 0.1 per cent steps (not indicated) in the fractional resistance; the corresponding ratio accuracy (ratio correction factor) is read from a dial scale; the correction due to the phase angle γ is usually negligible (equals only $+ 0.015$ per cent for a γ of ± 1 deg.).

The fractional resistance is made up of a 950-ohm "fixed" resistor and a 100-ohm "dial" resistor with one-ohm steps and a contact arm. Since the normal resistance value is 1000 ohms, this gives a range of ± 5 per cent in 0.1 per cent steps in the drop voltage E_D. Each 0.1 per cent step is marked directly in "ratio correction factor,"

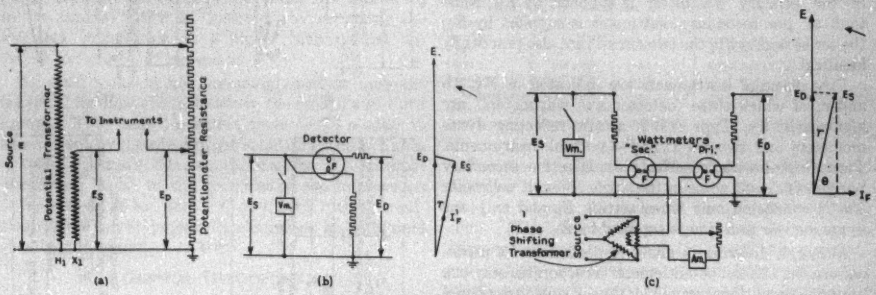


Fig. 1—Principles of Operation of "Deflection" Potentiometers

(a) Ratio and phase angle
(b) Ratio only
(c) Phase angle only

Principles of Operation

The principles of operation of our "deflection" potentiometers, which utilize deflections as well as null or zero instrument readings in comparing voltages, have been described[3] but are outlined again here. These principles will be apparent from a study of Fig. 1.

In the general diagram Fig. 1a, which applies to the determination of the ratio and the phase angle between E and E_S, the primary H_1 winding of the potential transformer under test and the potentiometer resistance at the proper tap are connected in multiple and supplied with rated voltage E from a grounded sine wave source. The transformer secondary X_1 voltage E_S is compared with the voltage drop E_D across a known fraction of the potentiometer resistance in use.

The potentiometer resistance chosen is 9.09 ohms per volt (1000 ohms for each 110 volts). The ratio

$$\left(\text{or } R.\,C.\,F.;\ \text{this is the } \frac{\text{true ratio}}{\text{marked ratio}} \text{ or } \frac{T\,R}{M\,R}\right).$$

The field F of the ratio detector is supplied (with I_F') by E_S, while the moving element is supplied by the vector difference between E_S and E_D; series resistors are used in both detector circuits. If a balance between E_S and E_D, as shown by a zero detector reading, is not obtainable on any step in the "dial" resistor, interpolation between any two adjacent steps is readily made by means of the two corresponding deflections. The ratio between the total potentiometer resistance in use and the fractional resistance at the "balance" point is evidently the true ratio of the potential transformer under test ($T\,R = M\,R \times R.\,C.\,F.$). The correction necessary for occasional large transformer phase angles γ is due to the fact that γ causes the hypotenuse E_D of the voltage triangle to exceed slightly the leg E_S in magnitude.

3. L. T. Robinson, *Loc. cit.*

In Fig. 1c, which applies only to the determination of the phase angle between E and E_S, the phase angle accuracy (γ, in minutes) is obtained by the two-wattmeter method; the phase of the excitation I_F is shifted until the "primary wattmeter" reads 0, when the "secondary watts" W_S divided by the volt-amperes equals $\cos\theta$ or $\sin\gamma$; that is:

$$\text{Sin}\ \gamma = \frac{W_S}{E_S I_F}$$

γ is read from a table of sines; the correction for the phase angle Δ (not indicated) of the potentiometer resistance in use is applied. A positive γ (E_S lags E) is indicated in the figure.

The fields F of the two practically identical wattmeters are supplied in series (with I_F) from the secondary of the phase shifting transformer, the primary of which is supplied from the main polyphase source. The field current I_F is set at a convenient value as shown by the 10-ampere ammeter Am. The moving element of the primary wattmeter is supplied by E_D, while that of the secondary wattmeter is supplied by E_S; the series resistors in the two circuits are also practically identical.

Five essential instruments are indicated in Fig. 1b and c, of which three (detector and wattmeters) are high-sensitivity, Type A L-2, astatic reflecting dynamometers and two are Type P-3 portable instruments. These instruments constitute an effective secondary burden of 0.5 volt-ampere; the single 150-volt voltmeter Vm is connected only when setting E_S and only one dynamometer at a time is supplied by E_S.

Although differing in many respects from its predecessors, the 132-kv. potentiometer incorporates only one important additional principle; that of shielding against capacitance of the resistance circuit to ground.

Effect of Capacitance

All potentiometer resistance used has been of manganin wire non-inductively wound and immersed in oil. Troublesome resistance changes and appreciable inductance were thus eliminated.

In the first potentiometer (33-kv.), the 0.010-in. manganin wire was wound on mica cards. The capacitance of the card sections to the grounded metal oil tank and the "offsetting" capacitance between the sections were both small.

In the second potentiometer (66-kv.), the 0.010-in. manganin wire was wound on brass tubes. Owing to the comparatively large amount of metal used and to the doubled-voltage range, the capacitance of the tube sections to the tank was comparatively large. The "offsetting" capacitance between the sections was also larger than before. The importance of these capacitances (particularly the former) at high voltages is indicated by the fact that at 60 cycles the "net" phase angle (and correction) at 22 kv. was + 3′ (lagging) while at 66 kv. it was + 51′.

In outlining the effects of capacitance, it is convenient, although not exact, to consider the distributed "internal" capacitance between resistor sections and the distributed "external" capacitance between the resistor sections and the grounded tank as independent "lumped" capacitances.

The "internal" capacitance between resistor sections acts as a shunt to the sections and causes the current to lead the voltage slightly. In Fig. 2a, the distributed capacitance between sections is represented by one capacitor C, which is shunted across part of the potentiometer resistance. The effect is indicated in the

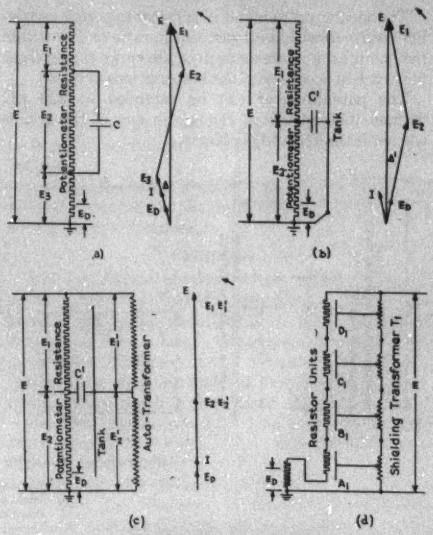


Fig. 2—Effects of Capacitance

(a) "Internal" capacitance between sections
(b) "External" capacitance to tank (ground)
(c) Elimination of effect of (b) by means of an auto-transformer
(d) Schematic arrangement of 44-kv., Group No. 1

vector diagram. The current I in the circuit and the voltage drop E_D in the fractional resistance lead the supply voltage E by the angle Δ (the fractional resistance is actually separate from the other resistance and has a small "net" positive phase angle Δ' of its own—this has been omitted for clearness). The negative angle Δ represents the potentiometer phase angle and correction.

The "external" capacitance between the resistor sections and the grounded tank acts as a shunt to the grounded end of the potentiometer resistance. In Fig. 2b the distributed capacitance to the tank is represented by one capacitor C', which is shunted across the grounded half of the potentiometer resistance.

The effect is indicated in the vector diagram. The current I in the circuit leads the supply voltage E but the voltage drop E_D lags E by the angle Δ', which is now positive.

It will be evident that if the potential of the tank is raised to that of the mid-point of the potentiometer resistance, no current will flow in the capacitor C'. In Fig. 2c this is accomplished by means of an auto transformer with a middle tap. In the vector diagram, I and E_D are now in phase with E.

This, in general, is the scheme adopted for the 132-kv. potentiometer. The schematic arrangement of the first 44-kv. group is indicated in Fig. 2d. The tanks of the four resistor units are maintained at mid-unit potential by means of taps on the auto or shielding transformer; the fractional and accessory resistances are taken out of and are separate from the first unit. The "net" phase angle is leading ($-7'$ at 60 cycles) at the highest resistance tap (44 kv.) due to the effect to the "internal" capacitance; at lower taps this is partly offset by the effect of the "external capacitance to the tank(s) which is "inserted" by supplying the shielding transformer on a higher than normal tap (at 6.6 kv. $\Delta = -1'$ instead of $-7'$).

All unit tanks in use are maintained at mid-unit potential for taps above 44 kv. in the second and third groups. The "basic" negative angle ($-7'$) is offset to some extent by the effect of the small direct capacitance of the resistance circuit, (principally the external connecting leads) to the grounded group area screen ($\Delta = -6'$ at 88 kv. and $-4'$ at 132 kv.); this capacitance effect can be practically eliminated by additional shielding, if desired.

General Description

The 132-kv. shielded potentiometer combines potentiometer resistance suitable for alternating current with shielding transformers and other special auxiliary equipment. It is conveniently arranged for use, in connection with standard power-supply equipment, in determining the ratio and phase-angle accuracy of potential transformers rated from 6.6 to 132 kv., 25 to 60 cycles and 25 to 1000 volt-amperes, respectively.

The potentiometer is shown schematically in Fig. 3a and b and directly in Figs. 6 and 7. The three 44-kv. series operating groups, each consisting of four 100,000-ohm resistor units and one shielding transformer, occupy about 380 sq. ft. (42 ft. by 9 ft.) of floor space and have a maximum height of about 12 ft., 3 ft. of which is due to the Herkolite insulating cylinders. The group area contains about 900 sq. ft. (52 ft. by 17.5 ft.); part of the grounded wire mesh, which acts as an electrostatic screen for the groups, is buried under the floor, while the remainder forms a fence 20 ft. high on the four sides of the area. The supply voltage is brought in from the testing area through high voltage bushings. The metal and glass operating booth is about 16 ft. square and 8 ft. high. It contains the fractional and accessory resistances and all instruments and control equipment. Approved safety devices are used wherever necessary.

Resistors. The potentiometer has a total resistance of 1,200,000 ohms and is rated at 0.11 ampere. The resistance is divided into three 400,000-ohm operating groups, which are subdivided into four 100,000-ohm units. From the construction standpoint the 100,000-ohm interchangeable units are basic; they are made up of ten 10,000-ohm sections, which are in turn made up of twelve 833.3-ohm tubes each. The arrangement and ratings are summarized in Table I. A "Detailed Description," which includes the fractional (normal value—1000 ohms) and accessory resistances, is given under that title.

TABLE I
ARRANGEMENT AND RATINGS OF POTENTIOMETER RESISTORS

Item	Rating		Number Required	
	Ohms	Kv.	Per Step	Total
Tube	833.3*	0.092	12 per Section	1440
Section	10,000†	1.1	10 per Unit	120
Unit	100,000	11	4 per Group	12
Group	400,000	44	3 for 132 kv.	3
3 Groups	1,200,000	132		

*Approximate adjustment—rated at 0.11 ampere (9.09 ohms per volt) nominally, 0.125 ampere maximum.

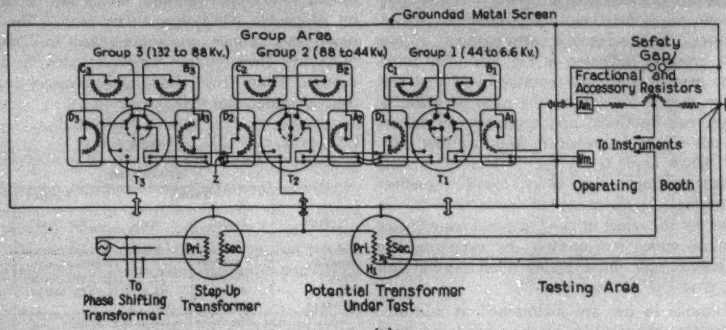
†Adjustment correct within 0.02 per cent.

Shielding Transformers. The three interchangeable shielding transformers (one for each 44-kv. group) are rated as follows: 25 cycles–75 kv-a.–50,000: 500/500 volts. (The maximum voltage rating for the three primary windings in series is 150 kv.) The secondary and tertiary windings are considered as equalizing windings as their function is to equalize the flux density and voltage distribution in the transformers when either two or three are operating in series; the connections are indicated in Fig. 3a and b. Each transformer has eight primary taps and nine bushings, one of which supports the somewhat complicated tap switch. The four unit tanks are connected to four of the bushings and from them to the proper transformer taps by means of the switch and thus maintained at predetermined potentials.

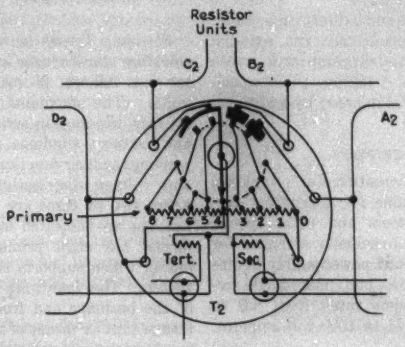
Operation. In operating, the total resistance used in each case is obtained by setting the tap switch on the 100 per cent tap in each of the 100,000-ohm resistor units up to the unit in which the final setting is made; all or only part of the resistance of this unit may be required (ten equal steps are obtainable with the tap switch but only the 100 per cent and 50 per cent taps are, in general, used above group No. 1); the tap switch is set on the zero tap in each of the 100,000-ohm units beyond the unit in which the final setting is made. The correctness of the setting is then checked by a bridge measurement; this procedure is independent of the number of units used, since all units are connected in series; however, connection is made to the upper end

of the "highest" group involved. The connections for 600,000 ohms or 66 kv. are indicated in Fig. 3a; the connection Z between the resistance and shielding transformer circuits is opened for the bridge measurement; the 0.2-ampere ammeter (Am) gives an approximate indication of the operating current.

The shielding transformers are excited magnetically only when one or more resistor units in their respective groups are in use. In group No. 1, transformer T_1 is excited on tap No. 6 (nominally 33 kv.) for voltages between 6.6 and 30.8 kv. to obtain a minimum phase angle; taps No. 7 and 8 are used for higher voltages up to 44 kv. One lead between the tertiary of T_1 and the secondary of T_2 is opened (similarly between T_2 and T_3); the other lead assures electrostatic excitation of group No. 2 (and No. 3) at line voltage; the tap switch of T_2 (and T_3) is set on the zero tap. Transformer T_3 is also excited magnetically for voltages between 44 and 88 kv. and transformer T_2 for voltages between 88 and 132 kv.; the "highest" transformer tap used in either group No. 2 or group No. 3 has, in general, the same nominal voltage rating as the corresponding tap (100 per cent or 50 per cent only) of the "highest" resistor unit in use. The correctness of the shielding transformer tap setting is checked in all cases by comparing the reading of a 600-volt voltmeter (Vm) supplied from the secondary of T_1 with the reading of a 150-volt voltmeter supplied from the secondary of the potential transformer under test; the proper ratio between the two readings is predetermined for each marked or nominal potential transformer ratio. The connections for 66 kv. are indicated in Fig. 3a and b.



(a)



(b)

Fig. 3—132-Kv. Potentiometer—66-Kv. Connection (Schematic)

(a) General outline
(b) Shielding transformer T_3 in Group No. 2

The distribution of voltage in the resistor units and in the shielding transformers for the 132-kv. connection is summarized in Table II. In this case the indicated current in the resistor circuit would be approximately

0.11 ampere, while the ratio between the secondary voltages of T_1 and of a 132-kv:110-volt (1200:1 ratio) potential transformer under test would be approximately 440:110 or 4:1.

After obtaining the proper settings as outlined, the desired secondary burden is set (by means of dial switches) and verified by instrument readings. The ratio and phase angle readings are then taken as indicated under "Principles of Operation."

TABLE II
DISTRIBUTION OF VOLTAGE AT 132 KV. IN THE THREE 44-KV. GROUPS

Designation			Shielding Transformer Tap		
Group No.	Resis. Unit	Shield Trans.	No.	Kv.	Connected to
			0	0	Ground
			1	5.5	Tank of 1st Unit (A_1)
	A_1		2	11 *	—
			3	16.5	Tank of 2nd Unit (B_1)
1	B_1	T_1	4	22 *	—
			5	27.5	Tank of 3rd Unit (C_1)
	C_1		6	33 *	—
			7	38.5	Tank of 4th Unit (D_1)
	D_1		8	44 *	Tap No. 0 of T_2
			0	44	Tap No. 8 of T_1
			1	49.5	Tank of 5th Unit (A_2)
	A_2		2	55 *	—
			3	60.5	Tank of 6th Unit (B_2)
2	B_2	T_2	4	66 *	—
			5	71.5	Tank of 7th Unit (C_2)
	C_2		6	77 *	—
			7	82.5	Tank of 8th Unit (D_2)
	D_2		8	88 *	Tap No. 0 of T_3
			0	88	Tap No. 8 of T_2
			1	93.5	Tank of 9th Unit (A_3)
	A_3		2	99 *	—
			3	104.5	Tank of 10th Unit (B_3)
3	B_3	T_3	4	110 *	—
			5	115.5	Tank of 11th Unit (C_3)
	C_3		6	121 *	—
			7	126.5	Tank of 12th Unit (D_3)
	D_3		8	132 *	Line

*Voltage at corresponding resistor unit; the resistor and transformer circuits are joined at "Ground" and "Line" only.

Detailed Description

The principal items described are: (1) 100,000-ohm resistor units, (2) fractional and accessory resistors, (3) operating panel and burdens, (4) shielding transformers, and (5) connections; a partial list of auxiliary equipment is included under (6).

1. *100,000-Ohm Resistor Units*

Construction details of the units are shown in Figs. 4, 5, 6, 9, and 10. Each unit is rated 11 kv., 0.11 ampere; its principal parts are: (a) 833-ohm resistor tubes, (b) 10,000-ohm resistor sections, (c) woodwork, (d) tap changing switch, (e) tank, (f) No. 10-C. Transil oil, (g) top shields, and (h) Herkolite supporting cylinders. The total of 100,000 ohms is made up of ten accurately adjusted (within 0.02 per cent) 10,000-ohm sections, each of which is made up of twelve approximately adjusted 833-ohm resistor tubes.

a. 833-Ohm Resistor Tubes. Several 833-ohm resistor tubes and one 10,000-ohm resistor section are shown in Fig. 4. Each resistor tube is made up of perforated sheet brass; it is 12 in. long with an outside diameter of 1.25 in. Treated paper is used for insulation between the tube and the first layer of wire and also between the first and second layers of wire. The two layers each have the same number of turns of 0.011-in. silk-covered manganin wire; the layers are connected in series (through the tube) with their respective magnetic fields in opposition (the small residual inductance is "offset" by the capacitance between layers and to the tube so that the "net" phase angle is very small). Three different black insulating compounds are used for impregnation; the compounds become oil-proof after suitable baking.

b. 10,000-Ohm Resistor Sections. Twelve tubes are assembled with insulating spacers, binders, and tie

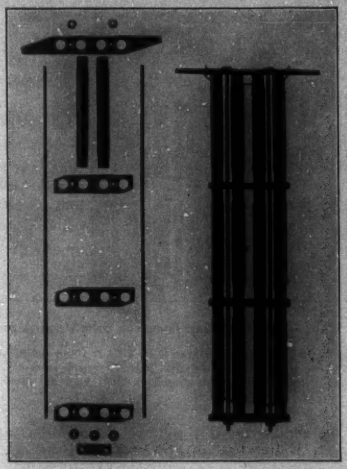


Fig. 4—Parts and Assembly of a 10,000-Ohm Resistor Section

(Approximately 36 in. x 8 in): 833-ohm resistor tubes; insulating spacers, binders, and tie rods

rods to make up a section approximately 36 in. by 8 in. The top binder, which contains the terminals, is made long enough to permit suspending the section between two parallel horizontal wooden supports. The section contains about the minimum insulating material consistent with proper insulation between tubes, as it is important to keep the overall dimensions as small as possible in order to minimize "leakage" and capacitance to the tank (see Preliminary Work). The arrangement of the section and its supports facilitates the dissipation of heat. The sections are interchangeable.

c. Woodwork. The treated woodwork is shown in Fig. 5. The framework for supporting the ten resistor sections is arranged to permit a zigzag assembly of the

sections, which gives the maximum insulation and the minimum capacitance between adjacent terminals and tubes of opposite polarity; the sections can be readily removed.

The framework is supported by the cover, which is approximately 5 ft. by 3.5 ft. The cover also supports the tap changing switch; the bushing for the switch

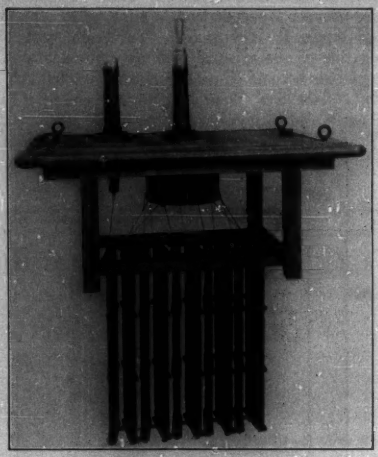


FIG. 5—100,000-OHM RESISTOR UNIT UNTANKED

With treated wood bushings cover (5 ft. x 3.5 ft.) and resistor supports; tap-changing switch; ten 10,000-ohm "zigzag" resistor sections. Welded steel tank (5.5 ft. deep) and two top shields (3.5 ft. high) not shown

and the similar bushing for the "zero" resistance tap extend through the cover. The edges of the cover project slightly beyond the sides of the tank; the edges are cut to a 1-in. radius and are covered with sheet copper which extends back about 6 in. to assist in shielding the resistor sections and in preventing corona: the remaining surface is protected by a sheet of treated pressboard.

d. Tap Changing Switch. The tap changing switch is shown in Fig. 5; it consists of an operating handle, position indicator, shaft and contact arm supported by the bushing (and cover); eleven contact points are mounted on the lower edge of an insulating cylinder which is supported by the cover and slotted to increase the creepage distance between the contact points. The number of sections in series can be readily changed from 0 to 10 by means of the switch.

e. Tank. The tank is shown in Fig. 10; it is approximately 5 ft. long, 3.5 ft. wide, and 5.5 ft. deep and is made up principally of sheet steel with welded seams. Four special internal lifting lugs, each of which has two holes to receive the top shields and a tapped hole for the cover "grounding bolt," and the angle pieces for supporting the cover are welded to the inside top edges of the tank. The outside corners and lower edges of the tank have a 2-in. radius to prevent corona up to 250 kv.

The tank contains the No. 10-C Transil oil and acts as a radiator of heat and as an electrostatic shield for the resistor sections on five sides. The "grounded" copper edges of the cover act as a partial shield for the top of the sections.

f. No. 10-C Transil Oil. The oil acts as a heat storage and transfer medium and as an insulator for the resistors; by its use, creepage between the resistor tubes or sections is reduced while corona is eliminated. There are about 530 gallons of oil in each tank; the oil level is just below the tap switch contacts.

g. Top Shields. The two top shields for each resistor unit are shown in Fig. 9; they are approximately 4.5 ft. and 3 ft. long, respectively, and extend about 3.5 ft. above the cover; they are made up of 1.75-in. diameter steel tubing welded at the joints; there are three parallel and two vertical bars; the latter set into the holes in the tank lifting lugs and are readily removable.

The two shields are located on the most exposed sides of each unit in the group and help to shield the top of the resistor sections and the terminals (bushings) and also the leads connecting the units. The diameter of the tubing is sufficient to prevent corona up to about 160 kv.

h. Herkolite Supporting Cylinders. The Herkolite cylinders are shown in Fig. 10; four cylinders, each 36 in.

FIG. 6—132-KV. SHIELDED POTENTIOMETER: THREE 44-KV. GROUPS, GROUP AREA AND SCREEN; OPERATING BOOTH; TESTING AREA

long with an outside diameter of about 18 in., support each unit and insulate it from ground and from adjacent units; the complete unit weighs about 2.8 tons.

2. Fractional and Accessory Resistors

A 2600-ohm adjustable resistor is shown in Fig. 8. There are two of these resistors; one is used as the 950-ohm "fixed" resistor, which, together with the 100-ohm "dial" resistor (not shown), makes up the fractional

resistance; the other is used as an accessory resistor to make up the difference between the fractional resistance and the 2500 ohms (approximate) short-circuited in the first 10,000-ohm resistor section of the first unit A_1.

The general scheme of operation adopted necessitates "bringing out" the fractional resistance from unit A_1. The approximate value chosen (2500 ohms) makes possible a special maximum fractional resistance of

Fig. 7—Interior of Operating Booth: Operating Panel, Instruments, and Burdens; Pier for Reflecting Dynamometers; Field Control Panel

2000 ohms (instead of the normal 1000 ohms) for use in interchecking between potentiometer resistance taps and in testing special potential transformers with 220-volt secondaries.

Each resistor consists of fifteen resistor tubes with the necessary taps; the corresponding terminals are conveniently arranged to give resistance values from "zero" to 2611 ohms in 0.1-ohm steps. The tubes, which are of insulating material, are wound differently from the 833-ohm tubes. There are two "opposing" windings of manganin wire in one layer; the two windings are connected in multiple; this scheme permits tapping the joint winding at any point.

The container consists of an oil-filled glass jar set in a box of treated wood; the box is covered with sheet brass which is grounded. The purpose in view was to provide suitable "heat insulation" so that the heating of the 2600-ohm resistors would simulate that of the 100,000-ohm resistor units. The two 2600-ohm resistors are interchangeable.

3. Operating Panel and Burdens

The operating panel is shown at the left in Fig. 7. The switches at the right of the panel control the connections to the instruments (the switches on the horizontal panel on the pier are supplementary). The current in the potentiometer resistance circuit is read on the ammeter at the top. The center handwheel is attached to, but insulated from, the contact arm of the 100-ohm "dial" resistor (not shown); the ratio correction factor (or R. C. F.) for the "dial" step in use appears in the small slot at the right of the handwheel. All of the dial switches at the left control the connections to the secondary burdens. Most of the portable instruments in the tray are used in verifying the settings of the burdens.

The burdens obtainable range from "zero" up to about 1000 volt-amperes at 110 volts at unity power factor and also at about 0.05 power factor lagging at both 25 and 60 cycles; this range of values is obtainable in one volt-ampere steps. Part of the inductive burdens can be seen below the instrument tray; the coils are tapped; there are air gaps in the cores, which operate at comparatively low flux densities; separate sets of coils are used for each frequency.

4. Shielding Transformers

The shielding transformers are shown in Figs. 6, 9, and 10. Each transformer is rated 25 cycles–75 kv-a.–50,000 : 500/500 volts. It is convenient to consider it first as a straight auto transformer.

As indicated in Fig. 3b, the primary winding has eight taps; the "zero" tap is permanently connected to one bushing and the middle tap (No. 4) is connected to another bushing; these single conductor bushings are connected to the "joint secondary lead" (ground when operating group No. 1 only) and to the transformer tank, respectively; the tank is therefore maintained at mid-winding potential. The other four

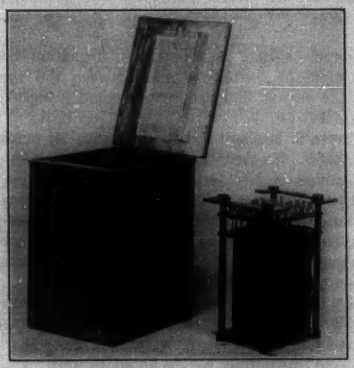


Fig. 8—2600-Ohm Adjustable Resistor and Shielded Container (for fractional or accessory resistance)

similar bushings are connected to four contacts of the tap changing switch and to the four resistor unit tanks, respectively; the corresponding switch contacts are connected to taps Nos. 1, 3, 5, and 7 respectively.

A larger bushing supports the handwheel and shaft of the tap changing switch; this bushing is connected to the "joint tertiary lead" (supply line when operating group No. 1 only). The switch connects this lead to

any of the taps, and to the tanks of all resistor units not in use; it also connects the tanks of all units in use to the respective "mid-unit" winding taps.

It is evident that the secondary and tertiary windings would not be necessary to assure equal voltage distribution if group No. 1 only was to be operated, as indicated in Fig. 2d; they are essential, however, when operating more than one group; they require twin conductor bushings and heavy connecting leads (one "joint" connecting lead also carries the primary current). The three windings are interleaved to obtain

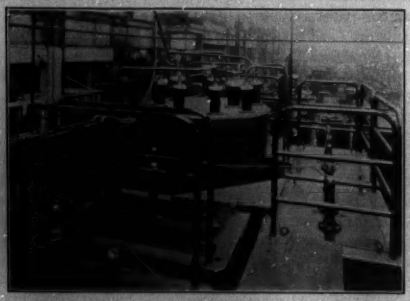


Fig. 9—Upper Part of 44-Kv. Group No. 3: Resistor Units; Shielding Transformer; Top Shields; Connections; Treated Wood Floor Segments

minimum leakage reactance; the insulation between the windings themselves and between the windings and the core is designed to withstand a high-voltage test of 101 kv.

The tank is about 7.8 ft. deep with an outside diamter of 5.3 ft.; the seams are welded; it contains about 925 gallons of No. 10-C Transil oil. The nine high-voltage bushings constitute the only projections from the tank; both edges of the tank have a radius of about 2 in. to prevent corona up to 250 kv. The core and windings are supported from the cover; the lifting lugs are screwed in only when required.

The two concentric Herkolite supporting cylinders are 36 in. long; the outer has an outside diameter of about 49 in. while that of the inner is about 13 in.; the complete transformer weighs about 10 tons.

5. Connections

The various connections are shown in Figs. 6 and 9. The minimum distance between any live part of the three 44-kv. groups and the grounded group area screen is 3 ft., while the minimum distance for any connection (or top shield) is 4 ft.; these comparatively wide spacings assist in reducing capacitance and in preventing corona, both of which are functions of distance: the spacing dimensions given do not, however, apply to the leads through the screen.

The connections between the 100,000-ohm resistor units are made with 0.375-in. diameter copper tubing; this comparatively small diameter is sufficient to prevent corona as the leads are fairly well shielded. The connection Z in Fig. 3a between the resistor and shielding transformer circuits is made with a flexible lead; this lead is shown at the extreme left in Fig. 9 in group No. 2; it is disconnected when group No. 3 is in use.

The connections between the unit tanks and shielding transformer bushings are made with 1.25-in. diameter copper tubing; this also applies to the connections between the shielding transformer bushings. This diameter is sufficient to prevent corona except at a few exposed points in group No. 3, where a diameter of 1.75 in. is used (the same as for the top shields).

The connections between unit A_1 and the fractional resistance in the operating booth and between the secondary winding of shielding transformer T_1 and the 600-volt voltmeter (one lead grounded) are made with 0.375-in. diameter copper tubing.

The connections between the tertiary of one shielding transformer and the secondary of the next are made with apparatus cable consisting of 1600 strands of

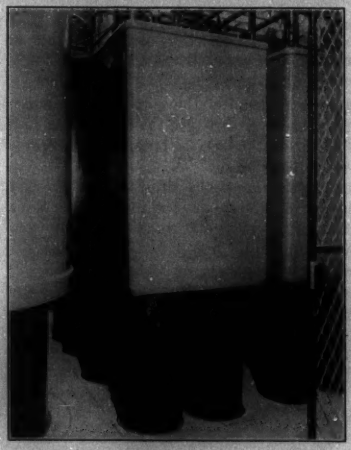


Fig. 10—Lower Part of Groups: Tanks of Resistor Units and Shielding Transformer; Herkolite Insulating and Supporting Cylinders

No. 24 wire (646,400 cir. mils); to obtain minimum reactance the cables are tied together. The cables or leads from each transformer are brought to terminals mounted on a post insulator supported by the end top shield of the group. Connections between busbar "group" terminals are made with suitable links; the connections are shown in Fig. 9; one link is disconnected when group No. 3 is not in use.

The connections between the group and testing areas are made with high-voltage single-conductor bushings; 1.25-in. diameter flexible leads are used in both areas for 44 and 88 kv., respectively, while a 1.75-in. diameter lead is used for 132 kv.

6. Auxiliary Equipment

Part of the auxiliary equipment is shown in Figs. 6 and 7; in general, this equipment is referred to in connection with other equipment which is not shown.

The power supply equipment includes a motor-generator set with suitable control panels and four single-phase step-up transformers. The set consists of a d-c. motor and a three-phase generator rated 60 cycles, 100 kv-a., 550 volts. The largest step-up transformer is shown in Fig. 6 in the testing area; it is rated 50 cycles, 100 kv-a., 220,000: 500/1100 volts. The load drawn by the complete potentiometer at its maximum voltage rating (150 kv.) is about 23 kw. at 25 cycles and 21.5 kw. at 60 cycles.

All voltages in the range of the potentiometer are dangerous to life. All entrances to the testing and group areas are provided with contacts operated by the gates. It is impossible to close the supply oil switch, if any of the gates are open; conversely, opening any gate when voltage is "on" will open the oil switch. The contacts are "supplemented" by several red lamps which light when voltage is "on;" the shielded lamps shown in Fig. 6 at the top of the group area screen are to warn the cranemen when voltage is "on."

An automatic temperature control system is installed in the operating booth. The practically constant temperature assures uniform accuracy for resistance measurements as the Wheatstone bridge is located in the booth.

Preliminary Work

An accuracy guarantee of within 0.1 per cent in ratio and within 5 minutes in phase angle means that the accuracy obtainable with the equipment under actual test conditions must be well within those limits in order to provide a reasonable "margin of safety" in the guarantee to cover slight differences between test and operating conditions. This in turn means that the sources of error in the equipment must be thoroughly investigated and that errors of the order of 0.01 per cent and 0.5′ are important.

An absolute accuracy better than within about 0.01 per cent cannot be expected for d-c. resistance values obtained by the best methods available; this limit must be somewhat increased for direct measurements with a high grade Wheatstone bridge. However, a much greater degree of comparative accuracy can be obtained by measuring the difference between two nearly equal resistors or between two nearly equal conditions of the same resistor. The latter method, which is also applicable to a-c. measurements, was therefore used whenever possible in investigating the sources of error.

The following investigations or tests on the completed equipment, preliminary to using it for routine testing, are outlined: leakage, heating and capacitance of units; characteristics of shielding transformers; effect of incorrect tank potentials and of corona and capacitance on operation (special conditions); determination of phase-angle corrections.

Leakage.

The relation between the d-c. resistance of the units obtained with a low-voltage bridge and the a-c. effective resistance at operating voltage was determined indirectly by investigating the corresponding characteristics of the insulation resistance between the different parts of the units.

The insulation resistance between the resistor circuit and the tank of one 100,000-ohm unit was determined (a) with a 1000-volt, 5000-megohm megger and (b) by calculation based on the change in the resistance of one unit, when part of it was shunted by the insulation resistance or leakage circuit the unit was compared with a similar unshunted unit at 11 kv., 60 cycles. The approximate values obtained are: (a) 5000 megohms, (b) 800 megohms; the ratio between (a) and (b) is in the order of 9:1 ((a) taken from Table III).

The results of a separate investigation indicated that the excess "leakage" with a-c. does not occur in the No. 10-C Transil oil, but is due to the presence of the solid insulation in the oil; this is one of the principal reasons for using a minimum of solid insulation in the oil.

The difference between the d-c. resistance of the units and the a-c. effective resistance, due to the effect of the combined leakages through the insulation between the different parts, does not exceed 0.01 per cent.

Heating.

A 36-hour heat run was made on the complete potentiometer at 150 kv., 60 cycles, principally to determine the change in resistance and the temperature rise of the 100,000-ohm resistor units.

The maximum resistance change (decrease) in any unit was found to be about 0.02 per cent; the average change was slightly less. This change does not necessitate a corresponding correction in the (ratio) test results, since the result appears as a ratio between the total potentiometer resistance and a fraction thereof.

The results of d-c. resistance measurements with different connections of the twelve units are given in Table III. The maximum difference (1669 ohms) obtained between "isolated" and "series" resistance measurements is only a fraction of the resistance of one section (10,000 ohms), so a "series" measurement assures the correctness of any desired tap setting. The insulation resistance changed from about 7000 to 5000 megohms per unit during the heat run.

TABLE III
D-C RESISTANCE MEASUREMENTS ON 12 UNITS IN HEAT RUN

Method	Ohms			Connections
	Before	After	Change	
Bridge	1,200,198	1,199,968	−230	Units isolated—sum of 12
Bridge	1,199,838	1,199,309	−529	12 in series—ungrounded
Bridge	1,199,513	1,198,299	−1214	12 in series—grounded
Megger	600×10^8	400×10^8	-200×10^8	12 in multiple to tanks

The maximum temperature rise in degrees centigrade indicated in the different parts of the units is as follows: oil at top, 20 deg.; side of tank near top, 15 deg.; side of tank near bottom, 7 deg. The maximum rise indicated in any part for the first eight hours was less than 15 deg.

The results of a separate investigation indicated a maximum rise of 6 deg. above the oil for the 833-ohm resistor tubes at the same current (0.125 ampere); this temperature rise occurs soon after applying voltage but in this case also, no (ratio) correction is necessary.

Capacitance.

The capacitance between the parts of the resistor units was determined by one or more of the following methods: capacitance bridge; calculation from the results of a volt-ampere test; calculation based on the change in the phase angle of one unit when part of it was shunted by the "capacitance circuit" (part of the "*b* test" under "Leakage"). These methods gave similar results; the choice of methods depended on the magnitude of the capacitance to be determined and on the normal operating voltage.

The capacitance between the resistor circuit and the tank and top shields of each 100,000-ohm unit is about 250 $\mu\mu$ f. This capacitance supplements the capacitance between sections in producing the basic negative "internal" phase angle Δ of about 8 min. at 60 cycles (see Fig. 2). This negative angle would, in general, be obtained whenever the unit tanks in the first group are at mid-unit potential (as for the 44-kv. connection), except for the fact that it is partly offset by the positive "external" phase angle Δ' of about 1 min. in the fractional resistance; the "net" phase angle Δ under this condition is there about -7 min. When the tank of the first unit is grounded (a special condition), its "net" phase angle Δ' becomes about $+1$ min. which is practically the Δ' of the fractional resistance; in other words, the "internal" and "external" phase angles of the first unit (about 8 min. each) offset or cancel each other when the tank is grounded.

The capacitance of the three groups to ground is about 1400 μ f. All or part of this capacitance, depending on the number of groups in use, constitutes a load on the shielding transformers in use and tends to produce phase displacements between the primary voltages and the line voltage.

The small direct capacitance of the potentiometer resistance circuit (principally the external connecting leads) to the grounded group-area screen is difficult to determine by the methods outlined because of the "intervening" tanks and top shields; however, it produces a positive shift in the potentiometer "net" phase angle Δ of about 3 min. from 44 kv. (-7 min.) to 132 kv. (-4 min.) at 60 cycles.

Shielding Transformers.

Since the three shielding transformers practically constitute parts of one auto-transformer, it was important to determine whether or not any appreciable discrepancies existed between them. Impedance, core loss, and ratio and phase-angle tests were made on each transformer and the transformers were checked against each other to determine the agreement between taps. The agreement between the transformers is very close in all respects. Some of the results are given in Table IV.

TABLE IV
TEST DATA ON SHIELDING TRANSFORMERS
Rated 25 Cycles–75 Kv-a.–50,000 : 500/500 Volts

Characteristic	Average Amount	Cycles	Volts	Amps.	P. F.	Winding
Short-circuit impedance	1.37%	60	685	1.50	0.29	Prim. to sec.
Short-circuit impedance	1.57%	60	7.85	150	0.27	Sec. to ter.
Coreless	0.9 kw.	60	50,000	0.048	0.37	Primary
Coreless	1.44 kw.	25	50,000	0.116	0.25	Primary

The equivalent load imposed at 132 kv., 60 cycles by the capacitance to ground under operating conditions is about 2.3 kv-a. This is based on the assumption that the load is concentrated in one capacitor at the 66 kv. tap (the estimated corresponding load at 220 kv. is 13.8 kv-a.).

This load does not appreciably affect the phase angle of shielding transformer T_1; that is, the phase angle of T_1 is practically the same at 44 kv. with only one group in operation as at 132 kv. with all three groups in operation, indicating that the three series primary voltages are in phase with the line voltage. However, calculations indicate that it requires only small phase angles between the shielding voltages and the line voltage to affect appreciably the potentiometer accuracy.

Tank Potentials.

The effect of incorrect tank potentials was determined by testing a 132-kv. potential transformer at 132 kv., 60 cycles (a) under normal conditions and (b) with the supply connected to tap No. 7 (instead of No. 8) on shielding transformer T_3. This raised all tank potentials, the range being from 5.5 kv. on resistor unit D_3 to 0.24 kv. on resistor A_1. (It also changed the secondary voltage of shielding transformer T_1 from 440 to 459 volts.) The differences between the results obtained under (a) and (b) are: $+0.16$ per cent in ratio correction factor ((b) $>$ (a)) and $+33'$ in phase angle ((b) lags (a)); the signs of the differences are in accordance with expectations. This special condition indicates that the effect of incorrect tank potentials is cumulative.

Corona.

The effect of corona and added capacitance (of the resistance circuit) to ground was determined by testing a 132-kv. potential transformer at 25 and at 60 cycles (a) under normal conditions and (b) with six of the eight top shields removed from the units in each group. The differences between the results obtained under (a) and (b) are: − 0.005 per cent in ratio correction factor ((b) < (a)) and − 2.8′ in phase angle ((b) leads (a)) at 25 cycles and − 0.024 per cent and − 6.7′, respectively, at 60 cycles; the signs of the differences are in accordance with expectations. This special condition gives an indication of the efficiency of the top shields.

Phase Angle Corrections.

Phase angle corrections were determined in the past by testing several single and double-ratio (series—multiple primary) potential transformers on two or more potentiometer taps each, starting at a comparatively low voltage tap where the effect of capacitance to ground was inappreciable. (This was supplemented by calculating the transformer phase angles from impedance and core-loss data.) This procedure was also used for the present potentiometer and necessitated the design of a special multi-ratio potential transformer rated 25 cycles, 132 kv.

In order to obtain an independent check on the phase angle of the first unit (A_1), a "standard of phase angle" was made up and (A_1) was compared with it. It consists of a 50,000-ohm cord-wound "working" resistor in five shielded sections; the shields are maintained at the mid-potentials of the respective sections by means of taps from a similar parallel "shielding" resistor. The phase angle calculated from the dimensions is of the order of 0.1′ at 60 cycles. The resistance of the different taps in both resistors can be readily determined and the connections to the shields verified, thus assuring sustained accuracy.

The results obtained by the two methods agreed closely.

Test Results

Representative ratio and phase angle results obtained to date with the new potentiometer are not included as it is not the purpose of the paper to discuss the characteristics of potential transformers. It will be evident, also, that a tabulation of test results, although undoubtedly of interest in that connection, is not necessary to demonstrate the soundness of the general proposition that all potential transformers should be tested for accuracy at full rated voltage; this procedure is now possible up to 150 kv. maximum at from 25 to 60 cycles.

Acknowledgment

The development, design, construction, and testing of a previously untried combination of equipment of such magnitude, to operate under conditions for which little useful data were available, would obviously have been impossible without the cooperation and the suggestions of a large number of individuals.[4] Their assistance is greatly appreciated and the author regrets that a more direct acknowledgment is not possible here. It is a pleasure, however, to acknowledge particularly the assistance of Mr. C. W. LaPierre.

Discussion

W. B. Kouwenhoven: The development of the 132-kv. shielded potentiometer or voltage divider described by C. T. Weller is very interesting. A divider of this range is a real step forward.

It is a well known fact that, when a large resistance is placed across a high-potential alternating voltage, the currents flowing in the resistor are not in phase with the voltage. In the section of the resistor at the high-voltage end, the capacity currents are the largest and the voltage drop in that section leads the applied voltage in phase. In the grounded section of the resistor the current lags behind the applied voltage.

In order to bring the current in the resistor into phase with the applied voltage it is necessary to shield the resistance units and to apply the proper potential to each shield as pointed out by Mr. Weller.

The characteristics of such a shielded circuit may be studied by the use of Maxwell's telegraph equations. Orlich was the first to do this.

If such a resistor consists of but a single section or unit, then the solution of the equations gives the proper potential of the shield or ⅓ of the applied voltage. This will bring the current at the grounded end into phase with the applied voltage.

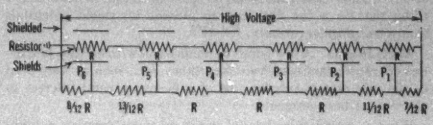


Fig. 1

If the resistance is divided up into a number of sections, each having the value of resistance R, then the theoretical values for the potentials on the shields as determined from Maxwell's equations are as shown in the accompanying Fig. 1.

Except in the two end sections the potentials applied to the shields are equal to the voltage at the midpoint of the resistance unit guarded. Actually no difference in phase between the high voltage and the current in the grounded section can be detected if the potential of each shield is brought to the midpoint of the resistance it guards.

Theoretically such a resistance voltage divider should have its resistance in the form of a straight wire running centrally through its shield. Such a construction is not practical and instead resistance cards are used in practise. These are so arranged, however, as to reduce the capacity C of the author's Fig. 2A to a minimum. In the construction adopted by Mr. Weller this capacity is fairly high.

A source of error will be present in the voltage divider if the phase angle of the voltage applied to the shields differs to any extent from the drop in the corresponding resistance unit. The value of the voltage applied to the shields may be represented by the relation $P_1 (1 + j \phi_1)$ for the first shield and by similar relations for the other shield potentials. The permissible value of the angle ϕ_1 depends upon the resistance of the unit R, the

4. Principally in the General Engineering Laboratory at Schenectady and in the Pittsfield Works Laboratory and the General Transformer Engineering Department at Pittsfield.

capacity between the unit and its shield C' (Fig. 2b of the author's paper) and 2π times the frequency. The larger the values of R and C' the smaller the angle ϕ_1 for a given percentage of error. I should like to know the value of ϕ_1 that is present in Mr. Weller's voltage divider.

In 1912, while a student at the Technische Hochschule at Karlsuche, Baden, Germany, I designed and built a voltage divider of this type with a maximum range of 28 or 30 kv. I did not use wire-wound resistances, but instead I used the so called manganin solution. This solution consists of 121.1 grams of mannitol, 41.2 grams of boric acid, and 0.06 grams of potassium chloride per liter of distilled water. This solution has a constant resistance over a temperature range of 5 deg. cent., and it has a high specific resistance. I made my resistances by placing it in Jena glass tubes each 1 meter in length and of 1.08 mm. inside diameter. Such a tube had a resistance of 24,000,000 ohms. The shields consisted of pieces of spouting and the capacity between a shield and its resistance unit was 1.25×10^{-12} farads. Similar construction was used in the resistor for providing shield potentials. The divider was used in checking a number of potential transformers and proved very satisfactory. It is inexpensive to construct.

A few years ago I constructed a voltage divider for use at Johns Hopkins University. In that instrument I used wire wound resistance cards. It is in almost daily use in our cable investigations.

Dr. Silsbee at the Bureau of Standards has thoroughly investigated the wire wound resistance divider type and has published his results in a Bureau of Standards paper.

Mr. Weller's voltage divider has the greatest voltage range of any divider that I know of. His construction is novel and I think that he is the first man to provide his shield potentials from a transformer primary.

A. R. Hand: A few points in connection with certain properties of transformer oils, resistance tubes, etc., such as were used in the new equipment, may be of sufficient general interest to warrant further comments.

When a series of resistance cards or tubes forming a high-resistance unit are used submerged in oil, as in an a-c. potentiometer circuit or potential divider, it is essential that an oil of high leakage resistance and low power factor be employed to keep at a minimum such stray currents as may leak off through the oil between the various parts of the resistor unit between which a difference of potential exists. As there has been considerable discussion at various times concerning the nature and extent of a-c. losses in dielectrics, such as oil, and as there were few data available on losses in transformer oils at moderate room temperatures and low frequencies, a series of direct measurements was made on various samples of oil at ordinary room temperatures and commercial frequencies to secure first hand information as to the losses and power factor likely to be encountered. A capacitor consisting of two 30-in. metal disks spaced ½ in. apart and provided with a shielding ring to minimize "fringe effects" was used. This capacitor had an air capacitance of approximately 300 $\mu\mu$f. and was used with r. m. s. voltage of 5500 volts between plates. The currents, voltages, and losses involved were measured on astatic reflecting electrodynamometers. A partial tabulation of test results is given in Table I. It is estimated that the measured losses and power factors are accurate to within approximately 20 per cent.

It was found that

1. The energy losses and power factors of high-grade oils were extremely low when the oil was new and uncontaminated. d-c. resistivities of about 60 mega-megohms per cu. cm. and power factors of about 0.0002 were noted.

2. An appreciable increase in losses with exposure of the oil to air and ordinary handling was evident. A given oil, however, appeared ultimately to settle down to a fairly stable electrical state. Oil which had been in use for some time showed practically constant resistivity and power factor at a given temperature.

3. With No. 10 Transil oil where it was possible to check the a-c. leakage resistance against the d-c. resistance (measured on a 1000-volt, 5000-megohm megger) it was found that the a-c. and d-c. leakage resistances were practically the same.

4. Treatment of No. 10 C Transil oil with Fuller's earth resulted in a temporary decrease in dielectric losses, but the improvement was short lived and in the course of several weeks both the treated and untreated oils showed practically identical properties.

It was concluded that No. 10 C oil, without special treatment, was suitable for use in the tanks of the resistor units because it embodied both high a-c. resistivity and moderate cost.

The accurate determination of high a-c. leakage resistances (of the order of 500 megohms) and small capacitances (of the order of 100 $\mu\mu$f.) under test conditions simulating actual high-voltage conditions, presents some rather interesting problems. Direct measurement with instruments such as the Evershed megger and capaciometer, or laborious calculations from physical dimensions, are often both impracticable and of questionable accuracy. It was found that the modified potentiometer scheme shown in Fig. 2 herewith served quite satisfactorily and yielded results of a good degree of accuracy.

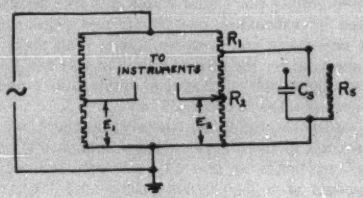


Fig. 2

Two similar 11-kv. potentiometer circuits were connected in parallel across a power source of the desired frequency. Suitable highly sensitive instruments were arranged to permit the accurate comparison of the relative magnitudes and phase angles of the respective fractional voltage drops E_1 and E_2. In practise, two 11-kv. units and two 1000-ohm fractional resistors were used to form the potentiometer circuits. One of the two 11-kv. 100,000-ohm units was provided with a tap lead at the 60,000-ohm point. The measured change in the voltage drop E_2 with respect to E_1 produced when a test resistance or capacitance (or combination of both) was shunted across the 60,000-ohm tap furnishes a ready basis for calculating the constants of the test piece. The change in magnitude ("ratio") of the voltage drop E_2 defined the "equivalent shunted resistance" of the shunting test piece, while the observed change in phase displacement defined its capacitance. It can be shown that the equivalent shunted resistance R_s, commonly known as the leakage resistance, is represented closely by the formula

$$R_s = \frac{R_1 R_2}{(R_1 + R_2)(P - 1)}$$

where P is the ratio of the magnitudes of the fractional voltage drop E_2 before and after connecting the test piece. It can likewise be shown that the value of shunted capacitance C_s is represented closely by the formula

$$C_s = \frac{(R_1 + R_2) \tan \theta}{\omega R_1 R_2}$$

where θ is the measured change in phase angle between the voltage drops before and after connecting the test piece, and ω has its customary meaning of $2\pi f$.

The above method is directly applicable to the measurement of the insulation resistance and capacitance between the resistance circuit of one unit of the potentiometer described by Mr. Weller and its containing tank when the unit is used as the test piece. It was determined that the a-c. leakage resistance had a value of approximately 800 megohms and the capacitance a value of approximately 250 $\mu\mu$f. The resistance of the leakage paths between various parts of the units are obtainable in the same manner. By combining the several leakage resistances, a fair estimate of the net effective leakage resistance shunting the metallic resistor circuit can be made. Calculations show that the a-c. effective resistance of the resistor circuit should deviate less than 0.01 per cent from the true d-c. resistance because of these effects.

High-grade manganin wire has long been recognized as a very satisfactory material for accurate "working standards" of resistance both because of its low temperature-resistance coefficient and its permanency of resistance after once being properly aged. Tests on representative samples of high-grade 11-mil manganin wire having a resistivity of 45 microhms per cu. cm. show an average resistance decrease from 25 deg. cent. to 35 deg. cent. of less than 0.01 per cent. Permanency of resistance (and calibration) of the high-resistance potentiometer circuit is thus assured for ordinary variations in operating temperatures. The choice of a relatively large working current (0.11 ampere) has been justified in that it has permitted the use of readily available materials without sacrificing accuracy; that is, the effects of leakage currents in particular have been relatively unimportant.

The employment of cylindrical tubes rather than the usual thin flat cards as forms on which to wind the resistance wire possesses a distinct advantage in that there are no sharp bends around edges where the wire may be weakened and easily broken in two. One of the chief disadvantages in the use of the old-style card-wound resistances for this purpose has been found to be repeated outages after several years' operation, due to the breaking of the resistance wire at such bends.

It may be of interest to point out that a new 44-kv. shielded potentiometer similar to the first group of the 132-kv. potentiometer described by Mr. Weller has been set up to replace the present 66-kv. unshielded potentiometer in the laboratory at Schenectady within the past year.

R. L. Ten Broeck: Some interest has been shown concerning the phase-angle calibration of the 132-kv. shielded potentiometer. The circuit used is shown in Fig. 3 herewith. The standard of phase angle was connected in parallel with the first unit A_1 of the potentiometer, but, in addition to other changes from the normal 11-kv. connection, the tank of A_1 was disconnected from the shielding transformer and grounded. The current through the working circuit of the standard went directly through the moving coil of the secondary dynamometer wattmeter. A resistance a equal to the resistance of the potential (moving-coil) circuit was put in series with the guard circuit to keep the shielding correct. The errors due to the inductance of the moving coil of the wattmeter and to the capacitance to ground of the lead from the "standard" to the wattmeters were calculated to be considerably less than 0.1 min. and were disregarded.

The maximum voltage that could be applied to the "standard" was 750 volts, so that in order to improve the sensitivity of the potentiometer at that low voltage, the drop voltage E_d point was changed to include 2000 ohms instead of 1000 ohms and the series resistance for the "primary" dynamometer wattmeter was reduced from 25,000 to 5000 ohms.

A common current was put through the current coils of the two wattmeter dynamometers and the phase angle between the currents in the potential circuits measured in the same manner as is explained under "Principles of Operation" in Mr. Weller's paper, except in this case the voltage E is the total voltage; this gave the phase angle of the primary circuit.

The phase angle of the secondary circuit was next measured with the working circuit of the standard in series with moving coil of the primary dynamometer and voltage E_s applied across the secondary leads as in Fig. 4 herewith.

The special correction for the first unit A_1 is then found by subtracting the phase angle of the secondary circuit from that of the primary circuit. This correction applies to the first unit with tank at ground potential, the fractional resistance of the potentiometer, 2000 ohms, and the series resistance for the primary dynamometer, 5000 ohms. Several potential transformers were tested for phase angle under this condition and then retested with the normal 11-kv. connection; that is, with the tank

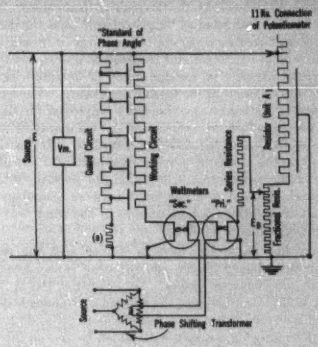


FIG. 3

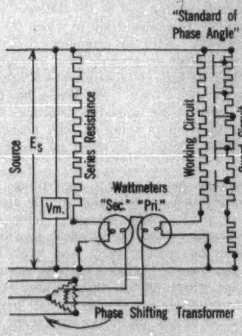


FIG. 4

excited from the shielding transformer, the fractional resistance 1000 ohms, and the series resistance for the primary dynamometer 25,000 ohms. The average of the differences between these tests was added to the original "special" correction to obtain the correction for the first unit or the 11-kv. connection under normal conditions.

The corrections for the other voltage connections or taps of the potentiometer were found by interchecking with convenient potential transformers. An 11-kv. potential transformer was tested on the 11-kv. tap A_1 which had just been calibrated and then on the 22-kv. tap B_1. The difference between the results of these two tests added to the correction for the 11-kv. connection gave the correction for 22-kv. potentiometer connec-

tion. The other corrections were obtained in a like manner. A 22-kv. potential transformer was tested on the 22-kv. tap and on the 44-kv. tap D_1; a 44-kv. potential transformer was tested on the 44-kv., 55-kv. A_2, 66-kv. B_3, and 88-kv. D_2 taps; a 66-kv. potential transformer was tested on the 66- and 132-kv. D_4 taps; and an 110-kv. potential transformer was tested on the 88-kv., 110-kv., and 132-kv. taps. This gave the correction for the 132-kv. tap in four or five steps from the first unit.

G. Thompson: Mr. Weller's paper on the 132-kv. shielded potentiometer represents the logical and perhaps the ultimate development of the pioneer work done some 20 years ago by Robinson and Craighead at the General Electric Company, by Sharp and Crawford at the Electrical Testing Laboratories, and by Agnew and Silsbee at the Bureau of Standards. In those days, 22 kv. was regarded as "high-voltage" and the discussions which followed Institute papers on methods of measuring ratio and phase angles of instrument transformers were devoted more to the relative merits of various kinds of detectors of balance than to errors inherent in the testing apparatus. The resistance-train potential divider was universally employed for the testing of potential transformers because of the comparative ease with which it could be constructed and measured. It was felt that the purity of the component resistance units did not affect the precision of the measurement provided the portion of the resistance train corresponding to the low-tension or secondary voltage of the test transformer is constructed in the same way as that across the high-tension winding.

As operating potentials became higher and the resistance trains correspondingly longer, the shunting capacitances began to assume troublesome values. In Scientific Paper No. 516 (June 16, 1925), Dr. Silsbee of the Bureau of Standards described a shielded resistor for use with transformers up to 30 kv. It is interesting to note the concern with which he viewed the difficulties of constructing a resistance train for higher voltages. In Silsbee's resistor, the shields for the component units of the resistance train were maintained at the different potentials by means of an auxiliary resistance train to which the shields were connected at intervals. With this arrangement the currents due to capacitance of shields to ground flow through the guarding train producing, for high voltages, a non-uniform distribution of voltage along the guard train. Mr. Weller has neatly solved this problem by substituting tapped transformers for the auxiliary train, the transformers being of such capacity that the displacement currents produce no effect on the distribution of voltage.

Those of us who are called upon to test voltage transformers looked with considerable apprehension at the size of the apparatus described in Dr. Silsbee's paper. What shall we say then to an equipment having the physical dimensions of that which Mr. Weller has constructed? And when it is further considered that the General Electric Company has only just now arrived at a point where 132-kv. transformers can be tested, when 250-kv. transformers are already a very real possibility, one wonders whether or not the testing equipment ultimately is not going to call for more space than the building in which the transformers are manufactured.

It would seem then that it behooves us to look about to see if some other method is available possessing the same accuracy as the resistance-train method but involving a smaller expenditure of money, consuming less power, and occupying less space than the imposing apparatus here described. In the paper referred to above, Dr. Silsbee rightly points out that ratio and phase angle have no physical dimensions but represent only ratios of alternating vector quantities which can be compared by null methods using two impedances in series across the high-tension terminals of the transformer. These impedances may be resistances, inductances, capacitances, or combinations of the same. The condenser represents an impedance which can be constructed with a purity equal to that of a resistance. It can be very effectively shielded from stray fields. It may be built for any voltage now commercially contemplated, will occupy much less space, requires comparatively little kilovolt-ampere capacity in the supply transformer, and probably represents a much smaller investment than the corresponding resistance train.

The Doyle-and-Salter transformer bridge for the measurement of the phase defect angle of a cable, described before the Institute in May of 1926, gives the ratio of two condensers in terms of the ratio of transformation of the transformer supplying the high-voltage. This bridge can be reversed and the ratio of transformation determined from the ratio of the two capacitances

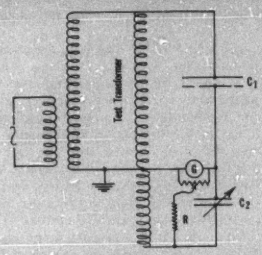


Fig. 5

(see Fig. 5 herewith). Also the method which these two writers described for determining the relative phase defect angle of the two condensers can be used to determine the relative phase displacement of the primary and secondary voltages of the transformer.

In April, 1927, B. G. Churcher of the Metropolitan Vickers Company of Birmingham, described a similar circuit which had been used up to 66 kv. in testing potential transformers at the factory of that company. Churcher used a mutual inductance excited through a non-inductive resistance from the secondary winding of the test transformer to measure the relative phase displacement. (See Fig. 6 herewith.) The Churcher method of measuring phase angle is somewhat superior to that of the

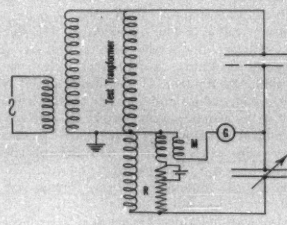


Fig. 6

Doyle-Salter bridge, in that it is possible to adjust the circuit conditions so that phase angle may be read directly from the graduations of the mutual inductor. Neglecting phase displacements in the two condensers the ratio of the transformer is equal simply to C_2/C_1. Neglecting the self-inductance of the primary winding of the mutual inductor, the phase angle is given by $\tan \varphi = (\omega M/R)(1 + C_1/C_2)$.

The refinements which are necessary to secure results of the highest accuracy with the capacitance train method are in no way burdensome. The high-tension condenser, C_1 should be preferably a no-loss air condenser; the "low plate" should have

a guard ring which in most cases can be connected directly to ground; the troublesome capacitance between the "low lead" and ground is thereby avoided. The low-tension condenser C_1 can be a good variable mica condenser in which the units of the principal dial have been adjusted to the marked values to better than 0.1 per cent. The phase angles of such a condenser are usually smaller than 5 min. If desired, exact balance can be secured with the aid of a small variable air condenser connected in parallel with the mica condenser. The self-inductance of the primary coil of the mutual inductor can be compensated by means of capacitance shunted across a part of the series resistance in the usual manner. The simple equations given above are then valid.

Furthermore, it is not necessary that C_1 should be an air condenser provided its phase defect angle at various voltages is known. At the Electrical Testing Laboratories we have had good results using one of our glass tube "standard dielectric load" condensers for C_1. The phase angle of these loads is of the order of 3 or 4 min. and is constant over a wide range of voltage. For precise work the phase defect angles of C_2 can be determined and the necessary correction applied.

The detector in the above circuit has a potential different from ground potential by the voltage induced in the secondary coil of the mutual inductor. If this is liable to introduce error in precision measurements the potential of the guard about the "low" lead and the detector may be brought to that of the test circuit by means of a second mutual inductor in the manner described by Churcher.

Two difficulties are met in the use of the capacitance-train method. If C_1 is fixed in value and must be used for a wide range of transformer ratios, C_2 may be inconveniently small for low-ratio transformers and inconveniently large for high ratios. However, the cost of constructing several air condensers to cover voltage ranges from, say, 30 kv. upwards, would not be prohibitive. The second difficulty is the determination of the capacitance of C_1. If the condenser is of a suitable geometric shape and well shielded its capacitance may be computed. We have used our large air condenser with a movable ground plate for C_1 and then determined the capacitance for the particular test condition either by the substitution method at low voltage using a bridge with resistance arms (Wien Bridge), or by calibration using a standardized voltage transformer of a lower voltage rating.

It would seem, therefore, that the capacitance-train method possesses characteristics which ought to make it very useful for testing voltage transformers from perhaps 20 kv. and upwards. Our experience has led us to set about 25 kv. as the point at which we change from the resistance train to the capacitance train. However, there may be features which we have overlooked and I should be very glad to have Mr. Weller tell us his opinion of the capacitance train method and the reasons why his company felt it desirable to make the investment represented in the apparatus described in his paper.

F. B. Silsbee: The construction of the truly monumental piece of apparatus which is described in this paper is a notable contribution to the art of precise measurements. It is pleasing to see that the attainment of accuracy is valued so highly by the electrical industry that a manufacturing corporation is justified in investing the large sum of money which this equipment must have cost solely for the purpose of maintaining the accuracy of its products at the high level demanded by its customers.

A detailed study of Mr. Weller's paper shows that the results obtained with his equipment are in good accordance with the theoretical relations governing the performance of such shielded resistors, which were published by the Bureau of Standards some time ago.[1] Thus Mr. Weller gives as the capacitance constant C of one of his units 250 $\mu\mu$f. From Equation 45 of Scientific Paper 516 one would expect the change in phase angle of the first unit when the potential of its tank is changed from 5.5 kv. to 0 to be $\frac{\omega r c}{4} = 7.9$ min. Mr. Weller observes a change of 8 min. The value of the phase angle when the tank is grounded indicates that the constant A is equal to $C/32$. This relation is quite reasonable if much of the distributed capacitance between the resistor and the tank is located near the ends of the resistor.

For the experiment in which the supply voltage was connected to tap 7 instead of tap 8, the equations would indicate a change in phase angle of 30 min. The observed change was 33 min. The fact that the effective ratio was also changed by this procedure, implies that the tank potentials must have been slightly out of phase as well as abnormal in magnitude during this test. This effect is only one-sixth as great as the change in the in-phase component and is presumably the result of the unequal excitation of the auto transformers.

It would be interesting if more data were available on the regulation of the auto transformer primary circuits since it is this property which determines their ability to maintain the proper tank potentials in spite of the capacitance currents to ground.

It should be pointed out that the use of transformers to maintain the shield potentials has greatly extended the possibilities of the shielded type of resistor since the disturbing effects of capacitance are similar to those which would exist in a set-up in which the guard circuit had an impedance equal to the leakage reactance of the transformer (here perhaps 1400 ohms), while the load imposed on the supply circuit corresponds only to that of the open-circuit impedance (about 3×10^8 ohms). The gain in this case therefore may be by a factor of nearly 2000 in impedance and according to the 4th power relation indicated on page 511 of Scientific Paper No. 516 should permit the construction of shielded resistors for voltages nearly 7 times as high as can be handled by equipment in which there is no interaction between sections of the guard circuit.

C. W. La Pierre: (communicated after adjournment)

Fractional Resistors. The chief requirements of the fractional resistors are stability in the non-reactive resistance and convenience in obtaining taps. Residual inductance introduces no phase-angle error in the final result as its effect forms a part of the corrections, experimentally determined.

In winding these single-layer resistances with two wires in opposite directions but connected in parallel it is necessary to have two crossings per turn. For convenience in manufacture these crossings are made at one point instead of at two points diametrically opposite. An inevitable rotation of the crossing point about the spool occurs as the winding progresses so that one winding has effectively 6 or 8 turns per spool more than the other. The residual inductance due to these turns is negligible, especially since they have a wide spacing.

The taps are brought out to four sets of binding posts, each set arranged similar to a dial switch. A central binding post with connecting links to any of the other posts takes the place of the rotating member of a dial switch. The panel containing these connections is beneath the surface of the oil.

Shielded Resistor Phase Angle Standard. The five-unit 50,000-ohm shielded resistor for use as a standard of phase angle was inspired by a study of Dr. Silsbee's Bureau of Standards Paper No. 516 on "A Shielded Resistor for Voltage Transformer Testing." Dr. Silsbee demonstrates in a complete manner the effectiveness as well as the limitations of shielding by means of a resistance guard circuit. It is clear from a study of the quantitative relations involved in the shielded phase-angle standard that the shielding must be effectively perfect. The residual phase angle is due therefore entirely to the constants of each unit.

1. Bur. of Standards Sci. Paper No. 516 "A Shielded Resistor for Voltage Transformer Testing," by F. B. Silsbee, Dec. 1925.

The total capacitance of each resistance unit to its shield is approximately 26 $\mu\mu f$. By distributing this capacitance in accordance with the dimensions an approximate value for its effect may be obtained. The effect so determined at 60 cycles is less than 0.1 min. The card wound resistances have an inductive effect of less than 0.1 min. at 60 cycles as determined by approximate calculation. Since the two offset, the resultant comes out to be 0.01 or 0.02 min. Due to possible errors in computation the resultant has little or no value in considering the final angle. However, the small magnitude of the individual components is important in showing that an unlikely error in computation would have to be present in order to result in a phase angle larger than from one- to two-tenths of one minute.

Accuracy of the Equipment. The methods used in comparing the various points have been described by Mr. Ten Broeck. The magnitude of the errors involved in obtaining the phase angle of the first 11-kv. unit are fairly well understood to depend largely upon the reference standard. In transferring this result to the higher voltages a great deal depends on the precision of the results (differentiated from accuracy) or what might be termed more descriptively, their consistency.

Naturally in the course of comparing the various taps a great many data were obtained. From these I have arbitrarily chosen the results from the minimum number of tests necessary to obtain the phase angle of the 132-kv. tap in terms of the shielded resistor. The first step consists in comparing the shielded resistor with the first 11-kv. unit direct. The second to fifth steps compare the 11-kv. and 22-kv. taps and so on to 44, 66, and 132 kv. by means of potential transformers. Each comparison is made with five points at five different voltages. With no attempt at weighting the points in accordance with the variations in sensitivity at the different voltage, the average deviations from the mean of the five points in each step were (1) 0.053 min., (2) 0.043 min., (3) 0.084 min., (4) 0.043 min., and (5) 0.026 min. Of the 25 points four deviations from the mean of over 0.1 min. occurred, the maximum being 0.134 min.

The values obtained from all of the data available, agreed very closely with the values obtained with this single set of data. The differences ranged from a maximum of 0.2 min. at the 44-kv. tap to a minimum of 0.016 min. at the 132-kv. tap. These results are for 60 cycles. It should be emphasized, however, that the values just given do not represent the accuracy but are merely indicative of the precision of the results.

Corresponding results for ratio are not available for presentation. They are much more difficult to obtain due to the necessity of changing the value of the fractional resistance in going from tap to tap. The transfer thus depends upon resistance measurements which we have not been able to make to a degree of accuracy comparable to the sensitivity of the equipment. By an increase in the dynamometer excitation a sensitivity of 0.001 per cent for ration may be reached without any difficulty. The resulting burden on the transformer in test requires 2.5 volt-amperes instead of the usual output of 0.5 volt-ampere.

In connection with resistance measurements formulas have been derived for the correction for leakage when measuring a large number of units in series. These formulas are for one terminal of the resistance grounded. (The shields of each unit are grounded for all resistance measurements.)

$$\Delta R = \frac{R^2}{R_L}\left(\frac{4N^2 - N}{3}\right) \text{ (ohms)}$$

ΔR = error in resistance

where $2R$ = the resistance of one unit

R_L = the leakage resistance between the resistor of one unit and its shield

N = the number of units in series

For the bridge and measured circuit completely insulated from ground except of course through the leakages in each unit

$$\Delta R = \frac{2R^2}{R_L}\left(\frac{N^2 - N}{6}\right) \text{ (ohms)}$$

A correction for ratio has been derived for use in tests involving comparatively large phase displacements. For ordinary use of the equipment the correction is negligible. The true ratio may be expressed for practically all cases by

$$K = \frac{R_t}{R_2} \times \frac{\cos\theta_2}{\cos\theta_t} \times \frac{\cos(\gamma - \theta_d - \phi_{fe})}{\cos(\theta_2 - \theta_d - \theta_t - \phi_{fe})}$$

where K = the true ratio

R_t = the total resistance

R_2 = the fractional resistance

θ_t and θ_2 = the corresponding phase angles

θ_d = Resultant angle of the detector potential coil circuit and the fractional resistance R_2.

ϕ_{fe} = The phase angle of the excitation applied to the detector field (dynamometer type of instrument)

γ = Phase angle of the potential transformer.

In the exact correction, the last term is much longer.

Extension to the High Voltages. Referring again to Dr. Silsbee's equations it is evident that such equipment as this paper describes has also a definite voltage limit beyond which it cannot be extended. In the design this limit was set at 220 kv. nominal (250 kv. maximum). The tests with the equipment so far completed indicate that no particular difficulty will be encountered in such an extension. It is quite likely, however, as anticipated, that for the most accurate work some flexibility will be sacrificed, and it may be necessary to use only the fifth group in its entirety. This, however, is not an important item.

The limiting factor in the extension lies of course in the leakage inductance and in the resistance of the shielding transformer windings. Establishing definite values for the effective inductance of the windings is complicated chiefly by the three windings and to a lesser extent by the comparatively large capacitance between windings.

Measurements of the capacitance between primary and secondary or tertiary indicated approximately 0.0048 microfarad with the tank connected to the midpoint of the primary. The measurements between secondary and tertiary windings indicated a capacitance of approximately 0.004 microfarad. (Measurements were made between two windings at a time and the effect of the third was not considered.) With the proper connections of secondary and tertiary in the final equipment, this results in a concentration of excitation current in the central portion of the primary or main winding.

Various impedance measurements were made on each winding with one or both of the others short-circuited. Measurements were also made on portions of the primary with the remainder or total short-circuited. In 15 such measurements under all conditions the phase angles ranged from 72.5 to 74.2 deg. with the average around 73 deg. The constancy of the phase angle indicates that for these short-circuit conditions at least the leakage inductance of each winding is a constant. Based on these results and the resistance measurements, the impedance of the primary at 60 cycles becomes approximately $55 + j\,180$ ohms and the impedance of the secondary and tertiary each $0.0081 + j\,0.0265$ ohms.

Such impedance values will result in a slight displacement of the shield voltages for five groups in series. The ratio error will be appreciable but less in magnitude than our present guarantee. A small correction will probably be necessary. This error is due to a displacement in the phase of the shield potentials.

For five groups in series it is expected that the phase-angle correction may change sign but not be larger in magnitude than those now occurring.

C. T. Weller: Professor Kouwenhoven's question in regard to angle ϕ_1 can be answered by stating that it is negligible in

group No. 1 because of the small "capacitance load" and this is apparently true also for three groups as inferred in the paper under "Shielding Transformers;" Mr. LaPierre has calculated its approximate "overall" effect for five groups (220 kv.).

Mr. Thompson has described a "capacitance" potentiometer or voltage divider and has asked why we did not adopt capacitance in preference to resistance because of its relatively smaller size and lower cost. We investigated the possibilities of capacitance for this purpose about three years ago and arrived at the following conclusions:

(a) Capacitors of the types and ranges required were not at that time in the same class as resistors from the standpoints of stability and accuracy. These views were confirmed, in general, by Dr. H. L. Curtis of the Bureau of Standards;

(b) The impedance of resistance at different frequencies is constant while that of capacitance is inversely proportional to the frequency. This tends to amplify harmonics which are usually objectionable in precision testing equipment;

(c) We felt that we knew a great deal more about resistors than we did about capacitors for potentiometer purposes.

However, it is obviously desirable to keep the size and the cost of testing equipment at a minimum and it is hoped, therefore, that other laboratories will proceed with the necessary development work so that "capacitance" or "impedance" potentiometers will be available which are comparable with the 132-kv. shielded potentiometer in flexibility, stability, and accuracy as well as in voltage range.

Dr. Silsbee has shown that the results obtained are in general agreement with results calculated from the formulas in his Scientific Paper No. 516 of the Bureau of Standards. In this connection it should be noted that the capacitance and phase angle constants given for the 132-kv. potentiometer are approximate. He has given a multiplying factor of about 7 for the extension possible by means of shielding transformers; this would bring the maximum voltage to approximately 220 kv. He has concluded from the difference in ratio results that the shielding transformer primary (or shield) voltages were slightly out of phase with the line voltage, due apparently to "unequal excitation" of the shielding transformers, in the special test in which the supply voltage was connected to tap No. 7 instead of No. 8 on shielding transformer T_1. It is interesting to note that no similar difference in ratio was obtained between any "odd voltage" connection of the potentiometer (such as the 110-kv. connection for testing a 132-kv. potential transformer) and the nearest "standard voltage" connection (such as the 132-kv. connection for the same transformer), although a similar "unequal excitation" of the shielding transformers exists with all but the 132- and 88-kv. connections in groups 2 and 3; however, the unit tanks are at "mid unit" potentials for all standard voltages in groups 2 and 3. It has been suggested that the difference is due to the fact that the voltages across the various resistor units were out of phase with the line (and shield) voltage in the special test.

The limit for the extension of the voltage range of the potentiometer is set from a practical standpoint by the relation between the resulting potentiometer corrections and the "certificate guarantees" for the results of ratio and phase angle accuracy tests; it is preferable that the corrections should not greatly exceed the "guarantees."

The most important features of potentiometers for determining the ratio and phase angle accuracy of potential transformers are stability and flexibility; these insure precision and speed in testing. Our various potentiometers have incorporated these features. It is desirable, of course, to have the corrections as small as possible and in this respect the 132-kv. potentiometer represents an advance over its 66-kv. predecessor. It should be noted that the present range (up to 150 kv. maximum at from 25 to 60 cycles) takes care of practically all potential transformers now manufactured, including those for 220-kv. circuits.

A Precision Regulator for Alternating Voltage

BY H. M. STOLLER[1] and J. R. POWER[1]
Member, A. I. E. E. Non-member

Synopsis.—A recently developed precision voltage regulator for use with alternating current is described. It will maintain its output voltage constant to within 0.03 per cent over an input voltage range of 10 per cent and a load range of from zero to full load. This regulation is accomplished by means of a small transformer inserted in one of the lines which boosts or bucks the impressed voltage by the required amount. The transformer is controlled by a vacuum tube circuit acting through an inductance bridge.

Introductory

MODERN industrial conditions are making it more and more imperative that the voltages used for testing purposes be held within precise limits. In some cases the requirements have far exceeded the regulation obtainable by the commonly used types of regulators.

This paper will describe a vacuum tube type of a-c. regulator which will maintain the output voltage within 0.03 of 1 per cent over the extreme range from maximum load minimum input voltage to minimum load maximum input voltage. This regulator is employed in the inspection of telephone equipment and is designed for operation on a line whose voltage fluctuates between 85 and 90 volts, 20 cycles.

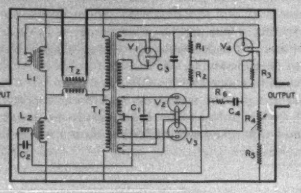


FIG. 1

For the great majority of tests, the line voltage limits are sufficiently precise, but there are frequent cases where a much higher degree of precision is necessary. It is therefore advisable to use a regulator of small power rating situated at the test position rather than at the power source because of line drop and because it would be uneconomical to regulate the entire alternator output so precisely.

Apparatus

The circuit employs a small transformer whose secondary is in series with the line so that any voltage induced in it either aids or bucks the line voltage. The primary voltage supplied to this transformer is derived from a bridge circuit. Referring to Fig. 1, this circuit consists of the two halves of the primary winding of the power transformer T_1, the fixed retardation coil L_1, the variable retardation coil L_2, and the output transformer T_2. In the variable arm L_2 of this bridge, the inductance is varied by the commonly known method of changing the degree of magnetic saturation of the reactor by means of a direct current in an auxiliary winding. For this purpose a three legged core with a winding on each leg is used (Fig. 2). The two outer windings consist of an equal number of turns and are connected in series so as to aid each other in producing flux in the outer legs of the core. As they are very carefully balanced no flux is set up in the middle leg by these windings. Any flux set up by the middle winding divides equally between the two outer legs. If, then, a direct current is sent through the middle winding it tends to saturate the entire core and so reduce the impedance of the outer windings to alternating current. In the present instance the ratio of turns of the d-c. winding to the a-c. winding is large. By this means a direct current of a few milliamperes can control the impedance of a reactor capable of handling several amperes of alternating current.

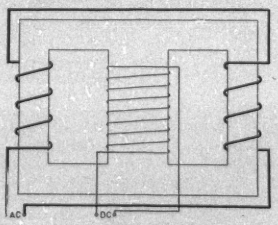


FIG. 2

The important feature of the fixed retardation coil (L_1, Fig. 1) is that it must be so designed that it will not saturate over the working range. This condition is met by the use of a large air-gap. An auxiliary winding used for compensation is placed on the same core and is connected in the regulating circuit.

The two element vacuum tube, V_4, is the primary source of control. It is supplied with an excess of plate potential (from the rectifier tube V_1) so that the space current is entirely controlled by filament emission.

1. Both of Bell Telephone Laboratories, 463 West St., New York, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28–Feb. 1, 1929.

29-26

The filament is made of tungsten[2] and is 0.001 inch diameter so as to have small time lag between current and temperature. This filament is connected in series with a resistance across the voltage output terminals so that a change in voltage produces a proportional change in filament current. A change of 0.1 per cent in filament current will cause about a 2 per cent change in the space current or, in other words, the tube has an effective amplification factor for current of about 20.

The tubes V_2 and V_3 are amplifier tubes supplying

Fig. 3

rectified current to the d-c. winding of the inductance L_2.

The plate-voltage supply for these tubes is derived from the transformer T_1 which is connected across the output circuit of the regulator. Thus the tubes have a constant source of potential which is essential to satisfactory operation of the circuit.

The assembled apparatus is shown in Fig. 3.

Principle of Operation

Referring again to the circuit in Fig. 1, the secondary of the transformer T_2 is in series with the line. Its primary is connected across the mid-points of the bridge formed by the transformer T_1 and the retardation coils L_1 and L_2. When the impedance drop across L_2 is equal to that across L_1, the bridge is balanced, and no potential is applied to the output transformer and none, therefore, is induced in the line. Under this condition the output voltage is equal to the input voltage except for the impedance drop in the transformer. If the impedance of the variable reactor changes, this balance is disturbed and a voltage is induced in the line that either bucks or boosts the line voltage. As described above, the impedance of this reactor is a function of the direct current through it. This current is controlled by the vacuum tube circuit.

If the input voltage should increase, the instantaneous effect would be an increase in output voltage. This would cause the filament current of the regulator tube to increase and, as a result, its space current would increase at a much greater rate as shown in Fig. 4. The space current flowing through the coupling resistance R_3 would change the balance of the resistance bridge of which the tube and this resistance form one side. This change is in such a direction as to bring the potential of the junction between the tube and the resistance R_3 nearer to that of the junction between the resistances R_1 and R_2 which form the other side of the resistance bridge. By this means the negative bias on the grids of the amplifier tubes is reduced and their output increased. This increased output flowing through the winding of the variable reactor decreases its impedance which changes the balance of the inductance bridge and either reduces the aiding voltage supplied to the line by the transformer T_2 or increases the bucking voltage, depending upon previous conditions. This change in induced voltage will bring the output voltage almost back to its original value. In practise the operation of this sequence of events is sufficiently rapid to prevent the transient increases in output voltage from attaining noticeable proportions except on a very sudden and severe change in input voltage.

This reaction to an increase in input voltage would restore the output voltage almost, but not quite, to normal except for the action of the compensating auxiliary winding on L_1. With the aid of this winding the required increase in filament current through tube V_4 is secured without raising the output voltage at all. This effect is secured by making use of the increased voltage across L_1 due to the changed condition in the inductance bridge.

By using the proper number of turns in the auxiliary

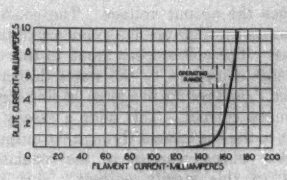


Fig. 4

winding, the voltage may be kept constant or even be caused to rise. This feed-back effect does not cause hunting because there is a slight time lag in its action due to the high inductance of the d-c. winding on the variable retardation coil L_2.

In the case of load variation, the action of the regulator circuit is similar to its operation with voltage variation, an increase of load causing the same action as a decrease in applied line voltage and a decrease of load the same action as an increase in applied voltage.

2. The tungsten is pure and contains no thorium or other materials having thermionic activity.

There are two pieces of auxiliary apparatus that contribute to the precision of the regulator. They are the capacity C_1 and the a-c. by-pass circuit composed of the capacity C_4 and the resistance R_8. The by-pass circuit smooths out the pulsating current through the coupling resistance R_3. This pulsation is due to the cooling of the regulator tube filament between peaks of the alternating current through it. As there are two peaks per cycle the pulsation will be at twice the line frequency. If these pulses were in phase with the line voltage they would be permissible as the resultant pulsation in the grid potential of the amplifier would be in phase with its plate voltage and an acceptable output could be obtained from the amplifier. This phase relation does not exist, however, on account of the time lag introduced by the heating characteristics of the regulator tube filament. If capacity alone is used in the filter an oscillation is set up that causes the output voltage to hunt or, with an excess of capacity, the regulator is sluggish in its reaction to transients. In order to prevent these effects a small capacity is used and a resistance placed in series with it. The effect of this resistance is to damp the oscillation to such an extent that hunting is prevented but without causing a marked decrease in the filtering action.

The capacity C_1 aids the regulation with respect to load. The cause of a change in output voltage with a change in load is two fold. The first is a change in the impedance drop across the winding of the output transformer in series with the line. The other (the more important cause) is that the line current in the secondary of this transformer causes a corresponding current to flow in the primary. The path of this current is from the line through the fixed retardation coil, the primary of the output transformer and half of the primary of the power transformer to the other line. This current changes the balance of the bridge in such a direction as to reduce the output voltage. The condenser C_1 is placed across the high-voltage secondary of the power transformer. The volt-ampere input to this capacity is made large compared to the watts drawn by the transformer T_1 and results in a large leading component in its primary current. This adds a steady leading component to the line current through the output transformer. Any output current of the regulator adds vectorially to this steady current. This vectorial addition decreases the importance as viewed from the bridge circuit of all load variation on the regulator, especially lagging, and unity power factor loads. At the low frequencies employed in this case it was not considered probable that the leading component of the load would exceed 25 per cent of the rated full load of the regulator and the operation under these conditions was found to be satisfactory. Other important functions of this capacity are that of power factor correction for the entire regulator and the by-passing of harmonics in the amplifier circuit.

Performance Characteristics

The output characteristics of the regulator are shown in Fig. 5. The curves show the output voltage plotted against the input voltage with the load on the regulator as a parameter. These curves are plotted in per cent of normal voltage instead of in volts because the normal voltage may be adjusted over a range of several volts by means of the variable resistance R_4.

The regulator is not critical to frequency over a range of about $\pm$ 10 per cent of the normal frequency. Also the normal frequency can be adjusted over a range of about 30 per cent by adjustment of the by-pass circuit C_4 and R_8 and of the inductance L_1.

The distortion of wave form due to the regulator is slight. In the worst condition, that of low input voltage, the harmonics introduced are barely noticeable when viewed on a cathode ray oscillograph, and were estimated to be less than one-fourth of 1 per cent.

The limits of the regulating range with respect to input voltage are due largely to amplifier characteristics.

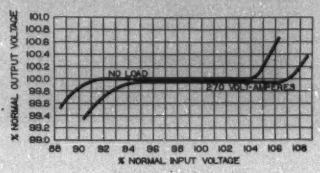


Fig. 5

The upper limit is fixed at the point where the grids of these tubes draw current. The lower limit is due to the ineffectiveness of a change in grid potential near the cut-off value and also to the insensitivity of the variable reactor at low values of direct current.

Fig. 5 shows clearly the effect of a load on the regulator. It will be noted that this effect consists entirely of a shift of the regulating range and does not impair the regulation within this range. As a result, the maximum load that the regulator can carry depends upon the input voltage.

It is estimated that the life of the tubes should be about 3000 hr. It is necessary to readjust the resistance R_4 about once in 100 hr. of operation in order to compensate for the gradual reduction in the size of the filament of V_4 due to evaporation of the tungsten. The other apparatus in the circuit has a practically indefinite life.

It should be noted that no auxiliary sources of power supply are required for the plates or filaments of the tubes so that the circuit is well adapted for practical shop testing purposes.

Discussion

The precision obtainable from a regulator of the type described is determined by the following factors:

1. The intrinsic precision of the voltage measuring device. (In this case, the emission characteristic of the tungsten filament of tube V_4).

2. The amplification between the measuring device and the controlling means.

3. The use of a suitable compensating means to offset the small change in output voltage which would otherwise be required to actuate the regulating circuit.

As regards the first factor, the tungsten filament gives a good intrinsic precision. If operated at a low emission it will give a long tube life.

The second factor is a matter of balancing cost against the value of the precision and range attained. In the circuit of Fig. 1 only one stage of amplification is employed. Another limiting factor in amplification is the tendency to hunting of the regulator. This tendency is reduced by reducing the time elements in the circuit between the measuring means and the control means. In this circuit the time factors are the heating lag in the tungsten filament, the time required to change the charge on the condenser C_4 across the grid circuit of tubes V_2 and V_3 and lastly, the time required to change the current in the highly inductive d-c. winding of L_2.

The third factor of compensation is essential in order to realize the full benefit of the circuit. For example, the circuit described would show a precision of only 0.70 of 1 per cent without compensation as compared with 0.03 of 1 per cent with compensation. The means of compensation must be such as to avoid impairing the stability of the circuit. This is most readily accomplished by introducing a small time element between the operation of the regulator and its compensation.

It may be concluded therefore that the limiting factors in the design of this type of regulator are economic rather than physical. By employing more amplification and suitably reducing the time factors in the circuit, it appears that a still higher degree of precision or a wider operating range could readily be obtained.

The circuit could of course be equally well designed to operate on 60 cycles or any other frequency of that order; in fact, the higher the frequency, the smaller and cheaper the apparatus would become.

Discussion

F. A. Byles: The voltage-regulator equipment described by Messrs. Stoller and Power is not only applicable for high grade laboratory service but may be very useful along many other lines. For many years the saturated-core transformer principle has been used to advantage in connection with various regulators. Not only has the vacuum-tube control been found to be very accurate, although rather complicated and expensive, but various vibrating-contact mechanical controls have also been found to be very satisfactory when used in combination with saturated core transformers.

I cannot say that I fully agree with the statement made in the first paragraph of their paper regarding the commonly used types of regulators.

A standard vibrating contact regulator will not usually give sufficiently accurate results for precision tests and measurements. However, with a few refinements such as jewel bearings, special windings, and resistors, a very fine degree of regulation may be obtained. I recall instances where I have noted regulation within 0.1-volt limits; that is, 0.05 volts either side of normal and continuing within these limits for days at a time on a 110-volt circuit.

In 1920 two installations were made where great accuracy was desired; one, in connection with nitrogen-fixation developments, and the other, for holding constant voltage for pipe insulation tests. Each of the regulators was designed to use buck-and-boost elements similar to those described in Messrs. Stoller's and Power's paper. The d-c. saturating windings were controlled by means of a very accurate magnetic mechanism with vibrating contacts. The results obtained came up to our expectations, not only from the standpoint of accuracy but also from the standpoint of rapid recovery under varying load conditions.

One of these regulators was designed to regulate a single-phase feeder circuit having a maximum capacity of 150 amperes at 110 volts. The voltage variation on the non-regulated incoming line was approximately 10 volts, the frequency varying between 58 minimum and 62 maximum.

A vacuum-tube regulator embodying the saturated-transformer principle for obtaining buck-and-boost regulation was built in our laboratory several years ago for meter testing. The precision results obtained with it augment the statements made in the paper under discussion.

Research and developmental work has been done, during the past few years, using similar principles of control, in connection with mercury-pool thyratron regulators and precision speed regulators, which has also proved the possibilities along this line of research.

Due to the comparatively high first cost, complicated construction, and the fact that vacuum tubes for this service are still in a more or less developmental stage, I have found that regulators of this general type have a somewhat limited application.

J. R. Power: In informal comment on this paper the use of a three-element tube as a rectifier has been questioned. This was used to reduce the number of types of tubes required. The rectifier tube is now identical with the amplifiers. There has also been comment on the fact that the grid of this tube is tied to the filament instead of the plate. This was done because of the high voltage across the tube during the reverse half wave. As it works in a high-impedance circuit the effect of the increased tube impedance due to this connection is negligible.

The regulator as shown in Fig. 3 is rather heavy and bulky. This is due to the fact that the frequency is low and is variable. It is designed for operation on from $16^2/_3$ to 20 cycles. If the frequency were increased the coils, of course, would become smaller and if the frequency were constant broadly tuned circuits could be used in some places to reduce still further the size of the apparatus. Another very important factor in the size and cost of the regulator is the range of input voltage. A very good index to the capacity of this type of regulator would be the product of the permissible input voltage variation and the maximum current. In this case such a rating would be 10 volts $\times$ 3 amperes or 30 regulating volt-amperes.

Bessel Functions for A-C. Problems

BY HERBERT BRISTOL DWIGHT*

Fellow, A. I. E. E.

Synopsis.—Tables of Bessel functions of zero and higher orders are given, for use in problems of skin effect and proximity effect in conductors carrying alternating currents. Formulas for deriving numerical values of the functions are given, in the form of series for large and for small values of the argument. The series are complete with their general terms.

The application of the tables to the calculation of the skin effect resistance ratio of an isolated tubular conductor is described and an example is given.

THE behavior of alternating current in a round conductor or in groups of round conductors is calculated by means of Bessel functions, except for the case of infinitely thin tubes. Such problems are usually known as skin effect and proximity effect problems. The division of current between conductors connected in parallel, or in other words, the circulating currents in parallel conductors, the distribution of current over the cross section of the conductors, the watts loss in each conductor, the resistance drop, and the reactance drop, are characteristics which are desired to be known and which can be calculated for round wires.

Such problems occur in electrical engineering where heavy alternating currents of more than about 1000 amperes at 60 cycles are involved. When it is remembered that generators for power stations sometimes have a rating of 5000 amperes or more, and that electric furnaces up to 50,000 amperes capacity are built, it is evident that these problems occur in practical engineering. Small conductors carrying currents at radio frequencies require calculations similar to those for large conductors carrying 60-cycle current.

In this paper is presented a collection of available values of Bessel functions which are used in the classes of problems described.

In Table I are given values of ber x, bei x, ber′ x, bei′ x, ker x, kei x, ker′ x, and kei′ x. These are Bessel functions of the first and second kinds, order zero and argument $x\,i\sqrt{i}$.

The values of ber x, bei x, and ber′ x and the values of bei′ x from $x = 0.1$ to $x = 6.4$ were published by Professor A. G. Webster[9] in 1912. The corrected values of bei′ x from $x = 6.5$ to $x = 10$ were given in the same publication in 1916, and the tables of ker x, kei x, ker′ x, and kei′ x in 1915, by H. G. Savidge.[12] The writer has corrected the published value of bei 8.9 which contained a typographical error. Table I was published in *A Precise Method of Calculation of Skin Effect in Isolated Tubes.*[19]

Table II contains values of Bessel functions of the first kind and of higher orders than order zero. They were published in *Proximity Effect in Wires and Thin Tubes.*[20] They are used in problems of proximity effect of groups of wires or cables. See Bibliography, Nos. 14, 18, 21, 22, and 24.

Table III contains values of Bessel functions of the second kind and of higher orders than order zero. This table has not previously been published. The greater part of the values were calculated by W. H. Hastings[23] as thesis work in Massachusetts Institute of Technology in 1927. This table is used in calculating proximity effect in tubes.

It is sometimes required, in problems which arise, to have values of Bessel functions not given in the tables. The argument of the required function may be intermediate between those of the tables, or it may be outside the range of the tables. For such cases, formulas for deriving the values are required, and these formulas will now be given. It should be remembered that this paper deals with Bessel functions of argument $x\,i\sqrt{i}$, for problems in a-c. distribution. There are, of course, many other formulas and tables for other types of argument. For some intermediate values of x, the tables[9] of differences may be used.

For comparatively small values of x, the following series can be used:

$$\text{ber}\,x = 1 - \frac{\left(\frac{1}{2}x\right)^4}{(2\,!)^2} + \frac{\left(\frac{1}{2}x\right)^8}{(4\,!)^2} - \ldots + (-1)^{s-1}\frac{\left(\frac{1}{2}x\right)^{4(s-1)}}{(2s-2)\,!\,(2s-2)\,!} + \ldots \tag{1}$$

$$\text{bei}\,x = \frac{\left(\frac{1}{2}x\right)^2}{(1\,!)^2} - \frac{\left(\frac{1}{2}x\right)^6}{(3\,!)^2} + \frac{\left(\frac{1}{2}x\right)^{10}}{(5\,!)^2} - \ldots + (-1)^{s-1}\frac{\left(\frac{1}{2}x\right)^{4s-2}}{(2s-1)\,!\,(2s-1)\,!} + \ldots \tag{2}$$

$$\text{ber}'\,x = -\frac{2\left(\frac{1}{2}x\right)^3}{(2\,!)^2} + \frac{4\left(\frac{1}{2}x\right)^7}{(4\,!)^2} - \frac{6\left(\frac{1}{2}x\right)^{11}}{(6\,!)^2} + \ldots + (-1)^s\frac{2s\left(\frac{1}{2}x\right)^{4s-1}}{(2s)\,!\,(2s)\,!} + \ldots \tag{3}$$

$$\text{bei}'\,x = \frac{\frac{1}{2}x}{(1\,!)^2} - \frac{3\left(\frac{1}{2}x\right)^5}{(3\,!)^2} + \frac{5\left(\frac{1}{2}x\right)^9}{(5\,!)^2} - \ldots$$

*Professor of Electrical Machinery, Massachusetts Institute of Technology, Cambridge, Massachusetts.

For numbered references see Bibliography.

29-112

Table I

Bessel Functions of Order Zero

x	ber x	bei x	ber' x	bei' x	x	ker x	kei x	ker' x	kei' x	x
0	+1.0	0	0	0	0	+ ∞	−0.785 398 2	− ∞	0	0
0.1	+0.999 998 435	+0.002 500 000	−0.000 062 500	+0.049 999 974	0.1	+2.430 474 0	−0.776 850 6	−9.980 959 3	+0.145 974 8	0.1
0.2	+0.999 975.000	+0.009 999 972	−0.000 499 999	+0.099 999 167	0.2	+1.733 142 7	−0.758 124 0	−4.922 948 3	+0.222 926 8	0.2
0.3	+0.999 873 438	+0.022 499 934	−0.001 687 488	+0.149 993 672	0.3	+1.337 218 6	−0.733 101 9	−3.319 865 2	+0.274 292 1	0.3
0.4	+0.999 600 004	+0.039 998 222	−0.003 999 911	+0.199 973 334	0.4	+1.062 633 9	−0.703 800 2	−2.353 009 9	+0.309 514 0	0.4
0.5	+0.999 023 464	+0.062 493 318	−0.007 812 076	+0.249 919 631	0.5	+0.855 682 5	−0.671 581 7	−1.819 799 8	+0.333 303 8	0.5
0.6	+0.997 975 114	+0.089 979 750	−0.013 498 481	+0.299 797 507	0.6	+0.693 120 7	−0.637 449 5	−1.456 538 0	+0.348 164 4	0.6
0.7	+0.996 248 828	+0.122 448 939	−0.021 433 032	+0.349 562 345	0.7	+0.561 378 3	−0.602 175 5	−1.190 943 2	+0.356 300 8	0.7
0.8	+0.993 663 142	+0.159 896 330	−0.031 988 023	+0.399 146 758	0.8	+0.452 882 1	−0.566 367 6	−0.987 335 1	+0.359 042 5	0.8
0.9	+0.989 751 357	+0.202 265 363	−0.045 538 565	+0.448 465 538	0.9	+0.362 514 1	−0.530 511 1	−0.825 564 7	+0.357 443 2	0.9
1.0	+0.984 381 781	+0.249 556 040	−0.062 445 733	+0.497 396 511	1.0	+0.286 705 2	−0.494 994 6	−0.694 603 9	+0.352 369 9	1.0
1.1	+0.977 137 973	+0.301 731 269	−0.083 081 791	+0.545 807 563	1.1	+0.222 844 5	−0.460 129 5	−0.585 905 3	+0.344 531 0	1.1
1.2	+0.967 629 156	+0.358 704 420	−0.107 805 842	+0.593 523 499	1.2	+0.168 945 6	−0.425 163 6	−0.494 643 2	+0.334 473 9	1.2
1.3	+0.955 324 747	+0.420 435 066	−0.136 972 169	+0.640 338 102	1.3	+0.123 455 4	−0.393 291 8	−0.417 227 4	+0.322 711 8	1.3
1.4	+0.940 075 057	+0.486 733 934	−0.170 928 324	+0.686 008 176	1.4	+0.085 126 0	−0.361 664 8	−0.351 055 1	+0.309 641 6	1.4
1.5	+0.921 072 184	+0.557 580 062	−0.210 011 017	+0.730 350 674	1.5	+0.052 934 9	−0.331 395 6	−0.294 181 6	+0.295 608 1	1.5
1.6	+0.897 891 139	+0.632 725 677	−0.254 544 638	+0.772 739 922	1.6	+0.026 029 9	−0.302 565 5	−0.245 114 7	+0.280 903 8	1.6
1.7	+0.869 471 237	+0.712 037 292	−0.304 838 207	+0.813 104 947	1.7	+0.001 691 1	−0.275 228 8	−0.202 681 8	+0.265 777 2	1.7
1.8	+0.836 721 794	+0.795 261 955	−0.361 182 125	+0.850 926 931	1.8	−0.014 696 1	−0.249 417 1	−0.165 943 4	+0.250 435 5	1.8
1.9	+0.797 524 167	+0.882 122 341	−0.423 844 516	+0.885 736 950	1.9	−0.029 661 4	−0.225 142 2	−0.134 128 2	+0.235 065 7	1.9
2.0	+0.751 734 183	+0.972 291 627	−0.493 067 125	+0.917 013 613	2.0	−0.041 664 5	−0.202 400 1	−0.106 601 0	+0.219 807 9	2.0
2.1	+0.698 685 003	+1.065 383 161	−0.569 060 755	+0.944 181 289	2.1	−0.051 106 5	−0.181 172 6	−0.082 823 4	+0.204 789 7	2.1
2.2	+0.637 600 457	+1.160 900 944	−0.652 000 244	+0.966 955 614	2.2	−0.058 358 8	−0.161 436 7	−0.062 357 3	+0.190 113 7	2.2
2.3	+0.568 048 926	+1.258 528 975	−0.742 018 947	+0.983 806 691	2.3	−0.063 670 5	−0.143 135 7	−0.044 747 7	+0.175 861 8	2.3
2.4	+0.489 047 772	+1.357 485 476	−0.839 203 721	+0.994 423 643	2.4	−0.067 373 5	−0.126 241 5	−0.029 712 3	+0.162 106 9	2.4
2.5	+0.399 968 417	+1.457 152 044	−0.943 583 409	+0.998 488 847	2.5	−0.069 688 0	−0.110 696 1	−0.016 929 8	+0.148 895 4	2.5
2.6	+0.300 092 090	+1.556 877 774	−1.055 131 815	+0.994 262 944	2.6	−0.070 525 7	−0.096 442 9	−0.006 135 8	+0.136 268 9	2.6
2.7	+0.188 706 364	+1.655 742 407	−1.173 750 173	+0.981 458 365	2.7	−0.070 973 6	−0.083 421 9	−0.002 904 3	+0.124 235 8	2.7
2.8	+0.065 112 108	+1.752 850 564	−1.299 264 112	+0.958 965 456	2.8	−0.070 296 3	−0.071 570 7	+0.010 399 0	+0.112 874 8	2.8
2.9	−0.071 391 179	+1.847 781 116	−1.431 311 114	+0.926 559 305	2.9	−0.068 602 7	−0.060 825 5	+0.016 534 2	+0.102 136 2	2.9
3.0	−0.221 380 249	+1.937 586 785	−1.569 846 632	+0.880 482 334	3.0	−0.067 029 2	−0.051 121 9	−0.021 476 3	+0.092 024 1	3.0
3.1	−0.385 531 455	+2.022 839 042	−1.714 104 430	+0.822 297 688	3.1	−0.064 678 6	−0.042 395 5	+0.025 373 8	+0.082 592 2	3.1
3.2	−0.564 376 430	+2.101 573 388	−1.863 616 954	+0.749 923 691	3.2	−0.061 984 8	−0.034 583 3	+0.028 360 3	+0.073 772 2	3.2
3.3	−0.758 447 012	+2.172 310 131	−2.017 589 996	+0.662 139 131	3.3	−0.059 032 9	−0.027 619 7	+0.030 555 4	+0.065 579 4	3.3
3.4	−0.968 038 995	+2.233 445 750	−2.175 445 573	+0.557 999 903	3.4	−0.055 932 6	−0.021 446 3	+0.032 088 2	+0.057 988 1	3.4
3.5	−1.193 595 180	+2.283 249 957	−2.336 059 130	+0.435 296 178	3.5	−0.052 639 3	−0.016 002 6	+0.032 988 6	+0.050 982 1	3.5
3.6	−1.435 305 322	+2.319 863 555	−2.498 252 527	+0.293 662 421	3.6	−0.049 315 6	−0.011 231 1	+0.033 408 7	+0.044 539 4	3.6
3.7	−1.693 259 984	+2.341 297 714	−2.660 778 962	+0.134 686 760	3.7	−0.045 971 7	−0.007 076 7	+0.033 403 0	+0.038 636 4	3.7
3.8	−1.967 423 273	+2.345 433 061	−2.822 163 850	−0.052 216 621	3.8	−0.042 646 9	−0.003 480 7	+0.033 040 0	+0.033 248 0	3.8
3.9	−2.257 599 466	+2.330 021 882	−2.980 743 427	−0.259 654 097	3.9	−0.039 373 61	−0.000 410 81	+0.032 380 46	+0.028 348 32	3.9
4.0	−2.563 416 557	+2.292 690 223	−3.134 653 964	−0.491 137 441	4.0	−0.036 178 85	+0.002 198 40	+0.031 478 49	+0.023 910 62	4.0
4.1	−2.884 305 732	+2.230 942 780	−3.281 521 353	−0.745 166 860	4.1	−0.033 084 40	+0.004 385 82	+0.030 381 79	+0.019 908 04	4.1
4.2	−3.219 470 533	+2.142 167 987	−3.419 951 224	−1.031 262 169	4.2	−0.030 107 58	+0.006 193 61	+0.029 132 42	+0.016 313 67	4.2
4.3	−3.567 910 563	+2.023 647 069	−3.546 519 744	−1.343 251 997	4.3	−0.027 261 77	+0.007 661 27	+0.027 767 30	+0.013 003 84	4.3
4.4	−3.928 306 621	+1.872 563 796	−3.658 765 306	−1.683 250 947	4.4	−0.024 556 89	+0.008 825 62	+0.026 318 68	+0.010 243 31	4.4
4.5	−4.299 088 553	+1.686 017 204	−3.753 851 326	−2.052 534 662	4.5	−0.021 999 88	+0.009 720 92	+0.024 814 54	+0.007 715 43	4.5
4.6	−4.678 356 937	+1.461 036 536	−3.828 010 348	−2.452 012 698	4.6	−0.019 595 03	+0.010 378 86	+0.023 279 08	+0.005 492 26	4.6
4.7	−5.063 885 587	+1.194 600 797	−3.878 239 793	−2.881 799 197	4.7	−0.017 344 41	+0.010 828 72	+0.021 733 06	+0.003 549 76	4.7
4.8	−5.453 076 175	+0.883 056 554	−3.900 599 216	−3.342 181 300	4.8	−0.015 248 19	+0.011 097 40	+0.020 193 91	+0.001 798 78	4.8
4.9	−5.842 942 443	+0.525 146 811	−3.891 060 511	−3.833 085 297	4.9	−0.013 304 90	+0.011 209 53	+0.018 676 61	+0.000 415 22	4.9
5.0	−6.230 082 479	+0.116 034 382	−3.845 339 473	−4.354 146 518	5.0	−0.011 511 73	+0.011 187 59	+0.017 193 40	−0.000 819 98	5.0
5.1	−6.610 653 357	−0.346 663 218	−3.758 900 943	−4.904 640 985	5.1	−0.009 864 74	+0.011 052 01	+0.015 754 36	−0.001 860 79	5.1
5.2	−6.980 346 403	−0.865 839 727	−3.626 966 748	−5.483 504 900	5.2	−0.008 359 11	+0.010 821 28	+0.014 367 57	−0.002 726 05	5.2
5.3	−7.334 363 435	−1.444 260 151	−3.444 527 187	−6.089 232 022	5.3	−0.006 989 28	+0.010 512 06	+0.013 039 35	−0.003 433 49	5.3
5.4	−7.667 661 337	−2.084 516 693	−3.206 347 194	−6.719 855 073	5.4	−0.005 749 13	+0.010 139 38	+0.011 774 46	−0.003 999 69	5.4
5.5	−7.973 596 451	−2.788 980 155	−2.907 031 958	−7.372 913 383	5.5	−0.004 632 16	+0.009 716 31	+0.010 576 33	−0.004 446 16	5.5
5.6	−8.246 575 962	−3.559 746 593	−2.540 959 318	−8.045 364 552	5.6	−0.003 631 56	+0.009 254 96	+0.009 447 17	−0.004 769 28	5.6
5.7	−8.479 372 252	−4.398 579 111	−2.102 401 197	−8.733 575 532	5.7	−0.002 740 38	+0.008 765 72	+0.008 388 18	−0.005 000 41	5.7
5.8	−8.664 491 263	−5.306 844 040	−1.585 512 696	−9.433 251 539	5.8	−0.001 951 58	+0.008 257 74	+0.007 399 67	−0.005 145 84	5.8
5.9	−8.793 666 753	−6.285 445 623	−0.984 382 394	−10.139 388 963	5.9	−0.001 258 12	+0.007 739 02	+0.006 481 21	−0.005 216 80	5.9
6.0	−8.858 315 963	−7.334 746 541	−0.293 079 967	−10.846 223 584	6.0	−0.000 653 04	+0.007 216 49	+0.005 631 71	−0.005 223 92	6.0
6.1	−8.849 080 413	−8.454 495 269	+0.494 289 242	−11.547 178 908	6.1	−0.000 129 53	+0.006 696 06	+0.004 849 57	−0.005 176 37	6.1
6.2	−8.760 062 474	−9.643 739 286	+1.383 522 213	−12.234 815 078	6.2	+0.000 319 05	+0.006 182 75	+0.004 132 75	−0.005 082 53	6.2
6.3	−8.588 792 593	−10.900 736 825	+2.382 248 300	−12.900 776 902	6.3	+0.000 693 12	+0.005 680 77	+0.003 478 86	−0.004 951 05	6.3
6.4	−8.276 249 873	−12.222 863 128	+3.489 851 325	−13.535 756 059	6.4	+0.001 016 83	+0.005 193 58	+0.002 885 28	−0.004 788 03	6.4
6.5	−7.806 890 928	−13.606 512 001	+4.717 382 012	−14.129 423	6.5	+0.001 278 080	+0.004 723 992	+0.002 348 995	−0.004 600 032	6.5
6.6	−7.328 687 885	−15.046 992 991	+6.067 462 487	−14.670 413	6.6	+0.001 483 446	+0.004 274 219	+0.001 867 130	−0.004 392 632	6.6
6.7	−6.649 176 464	−16.538 424 538	+7.544 164 266	−15.153 856	6.7	+0.001 653 215	+0.003 845 947	+0.001 436 521	−0.004 170 782	6.7
6.8	−5.815 515 115	−18.073 623 609	+9.150 973 359	−15.543 406	6.8	+0.001 777 204	+0.003 440 398	+0.001 053 999	−0.003 936 844	6.8
6.9	−4.814 536 200	−19.643 992 365	+10.890 503 759	−15.847 109	6.9	+0.001 855 512	+0.003 058 385	+0.000 716 382	−0.003 700 681	6.9
7.0	−3.635 293 580	−21.239 402 580	+12.764 502 500	−16.041 489	7.0	+0.001 922 022	+0.002 700 365	+0.000 420 510	−0.003 459 509	7.0
7.1	−2.257 144 280	−22.848 078 597	+14.773 723 174	−16.109 484	7.1	+0.001 950 901	+0.002 366 486	+0.000 163 267	−0.003 218 283	7.1
7.2	−0.673 695 370	−24.456 479 797	+16.917 584 856	−16.037 856	7.2	+0.001 956 620	+0.002 056 820	−0.000 054 966	−0.002 981 917	7.2
7.3	+1.130 799 552	−26.049 183 639	+19.194 204 342	−15.792 207	7.3	+0.001 940 312	+0.001 770 454	−0.000 247 403	−0.002 744 978	7.3
7.4	+3.187 549 952	−27.620 770 623	+21.600 120 353	−15.367 001	7.4	+0.001 907 373	+0.001 507 429	−0.000 406 628	−0.002 516 671	7.4
7.5	+5.454 962 184	−29.115 711 867	+24.130 124 710	−14.757 162	7.5	+0.001 856 568	+0.001 266 568	−0.000 538 787	−0.002 295 904	7.5
7.6	+7.999 382 494	−30.548 262 965	+26.777 064 473	−13.875 324	7.6	+0.001 800 431	+0.001 047 959	−0.000 646 478	−0.002 083 300	7.6
7.7	+10.813 065 478	−31.882 362 359	+29.531 637 360	−12.762 551	7.7	+0.001 731 326	+0.000 849 790	−0.000 732 165	−0.001 884 234	7.7
7.8	+13.908 911 71	−33.091 539 670	+32.382 176 399	−11.372 739	7.8	+0.001 654 654	+0.000 671 373	−0.000 798 170	−0.001 688 855	7.8
7.9	+17.287 127 04	−34.135 428 983	+35.314 428 983	−9.677 223	7.9	+0.001 573 564	+0.000 511 564	−0.000 846 677	−0.001 504 150	7.9
8.0	+20.973 955 611	−35.016 725 165	−38.316 725 165	−7.660 318	8.0	+0.001 485 834	+0.000 369 584	−0.000 879 724	−0.001 336 313	8.0
8.1	+24.956 880 800	−35.667 980 514	+41.352 754 078	+5.285 490	8.1	+0.001 396 782	+0.000 244 032	−0.000 899 210	−0.001 176 567	8.1
8.2	+29.245 214 796	−36.061 119 681	+44.415 316 208	−2.529 555	8.2	+0.001 306 386	+0.000 133 902	−0.000 906 891	−0.001 027 888	8.2
8.3	+33.836 407 69	−36.159 400 616	+47.472 024 583	+0.821 498	8.3	+0.001 215 743	+0.000 038 090	−0.000 904 388	−0.000 890 168	8.3
8.4	+38.738 422 961	−35.919 529 530	+50.492 416 438	+4.231 841	8.4	+0.001 125 797	−0.000 044 481	−0.000 893 190	−0.000 763 309	8.4
8.5	+43.935 872 751	−35.297 700 300	+53.441 618 430	+8.289 519	8.5	+0.001 037 349	−0.000 114 902	−0.000 874 656	−0.000 646 733	8.5
8.6	+49.423 084 977	−34.245 760 640	+56.280 822 496	+12.832 116	8.6	+0.000 951 070	−0.000 174 175	−0.000 850 022	−0.000 540 398	8.6
8.7	+55.136 932 906	−32.714 319 308	+58.966 717 374	+17.883 887	8.7	+0.000 867 511	−0.000 223 306	−0.000 820 407	−0.000 443 513	8.7
8.8	+61.200 735 224	−30.651 367 879	+61.451 354 116	+23.485 444	8.8	+0.000 785 248	−0.000 263 248	−0.000 789 819	−0.000 356 543	8.8
8.9	+67.468 740 848	−28.002 857 538	+63.681 960 575	+29.598 302	8.9	+0.000 710 249	−0.000 294 910	−0.000 750 159	−0.000 278 127	8.9
9.0	+73.935 729 857	−24.712 783 168	+65.600 770 999	+36.299 384	9.0	+0.000 637 164	−0.000 319 153	−0.000 711 231	−0.000 208 079	9.0
9.1	+80.576 411 145	−20.723 569 533	+67.144 889 467	+43.589 976	9.1	+0.000 568 055	−0.000 336 788	−0.000 670 745	−0.000 145 903	9.1
9.2	+87.349 952 678	−15.976 414 197	+68.246 178 268	+51.488 534	9.2	+0.000 503 046	−0.000 348 579	−0.000 629 326	−0.000 091 093	9.2
9.3	+94.208 443 358	−10.411 661 917	+68.831 185 381	+59.935 547	9.3	+0.000 442 203	−0.000 355 236	−0.000 587 517	−0.000 043 145	9.3
9.4	+101.096 359 718	−3.986 283 324	+68.821 113 743	+69.011 850	9.4	+0.000 385 540	−0.000 357 420	−0.000 545 789	−0.000 001 559	9.4
9.5	+107.950 031 581	+3.410 573 282	+68.131 840 035	+78.683 888	9.5	+0.000 333 029	−0.000 355 743	−0.000 504 544	+0.000 034 158	9.5
9.6	+114.597 114 473	+11.786 984 139	+66.672 989 017	+88.940 424	9.6	+0.000 284 904	−0.000 350 768	−0.000 464 122	+0.000 064 485	9.6
9.7	+121.256 066 253	+21.217 531 810	+64.353 071 386	+99.762 855	9.7	+0.000 240 168	−0.000 343 010	−0.000 424 806	+0.000 089 887	9.7
9.8	+127.535 651 521	+31.757 530 896	+61.099 692 033	+111.124 240	9.8	+0.000 199 598	−0.000 332 940	−0.000 386 830	+0.000 110 811	9.8
9.9	+133.454 460 262	+43.459 152 913	+56.719 839 030	+122.988 479	9.9	+0.000 162 731	−0.000 320 983	−0.000 350 379	+0.000 127 684	9.9
10.0	+138.540 465 945	+56.370 458 554	+51.195 358 394	+135.309 302	10.0	+0.000 129 466	−0.000 307 524	−0.000 315 597	+0.000 140 914	10.0

$$+ (-1)^{s-1} \frac{(2s-1)\left(\frac{1}{2}x\right)^{4s-2}}{(2s-1)!\,(2s-1)!} + \ldots \qquad (4)$$

$$\ker x = (0.1159315 - \log_e x)\,\mathrm{ber}\,x + \frac{\pi}{4}\,\mathrm{bei}\,x - \left(1+\frac{1}{2}\right)\frac{\left(\frac{1}{2}x\right)^4}{(2!)^2} + \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\frac{\left(\frac{1}{2}x\right)^8}{(4!)^2} - \ldots + (-1)^s\left(1+\frac{1}{2}+\frac{1}{3}\ldots+\frac{1}{2s}\right)\frac{\left(\frac{1}{2}x\right)^{4s}}{(2s)!\,(2s)!} + \ldots \qquad (5)$$

$$\mathrm{kei}\,x = (0.1159315 - \log_e x)\,\mathrm{bei}\,x - \frac{\pi}{4}\,\mathrm{ber}\,x + \frac{\left(\frac{1}{2}x\right)^2}{(1!)^2} - \left(1+\frac{1}{2}+\frac{1}{3}\right)\frac{\left(\frac{1}{2}x\right)^6}{(3!)^2} + \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)\frac{\left(\frac{1}{2}x\right)^{10}}{(5!)^2} - \ldots + (-1)^{s-1}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2s-1}\right)\frac{\left(\frac{1}{2}x\right)^{4s-2}}{(2s-1)!\,(2s-1)!} + \ldots \qquad (6)$$

$$\ker' x = (0.1159315 - \log_e x)\,\mathrm{ber}'\,x - \frac{1}{x}\,\mathrm{ber}\,x + \frac{\pi}{4}\,\mathrm{bei}'\,x - \left(1+\frac{1}{2}\right)\frac{\left(\frac{1}{2}x\right)^3}{2!\,1!} + \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\frac{\left(\frac{1}{2}x\right)^7}{4!\,3!} - \ldots + (-1)^s\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2s}\right)\frac{\left(\frac{1}{2}x\right)^{4s-1}}{(2s)!\,(2s-1)!} + \ldots \qquad (7)$$

$$\mathrm{kei}'\,x = (0.1159315 - \log_e x)\,\mathrm{bei}'\,x - \frac{1}{x}\,\mathrm{bei}\,x - \frac{\pi}{4}\,\mathrm{ber}'\,x + \frac{\frac{1}{2}x}{(1!)^2} - \left(1+\frac{1}{2}+\frac{1}{3}\right)\frac{\left(\frac{1}{2}x\right)^5}{3!\,2!} + \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)\frac{\left(\frac{1}{2}x\right)^9}{5!\,4!} - \ldots + (-1)^{s-1}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{2s-1}\right)\frac{\left(\frac{1}{2}x\right)^{4s-3}}{(2s-1)!\,(2s-2)!} + \ldots \qquad (8)$$

See Reference 10, p. 210.

The constant 0.1159315 is equal to $\log_e 2 - \gamma$, where γ is Euler's constant, 0.5772157. See Reference 15, pp. 23 and 254.

For higher orders, where n is a positive integer,

$$\mathrm{ber}_n\,x + i\,\mathrm{bei}_n\,x = J_n\,(x\,i\,\sqrt{i}) = u_n + i\,v_n \qquad (9)$$

TABLE II

BESSEL FUNCTIONS OF THE FIRST KIND, OF ORDERS HIGHER THAN ZERO

$$\mathrm{ber}_n\,x + i\,\mathrm{bei}_n\,x = J_n\,(x\,i\,\sqrt{i}) = i^n\,I_n\,(x\,\sqrt{i}).$$

$$\mathrm{ber}_n'\,x = \frac{d}{dx}\,\mathrm{ber}_n\,x$$

x	1	2	3	4	5	6	7	8	9	10	x
$\mathrm{ber}_1\,x$	−0.395 868	−0.997 078	−1.782 64	−1.869 25	+0.359 78	+ 7.462 20	+20.368 9	+32.506 9	+20.719 2	− 59.478	$\mathrm{ber}_1\,x$
$\mathrm{bei}_1\,x$	+0.307 557	+0.299 775	−0.487 45	−2.563 82	−5.797 91	− 7.876 67	− 2.317 2	+21.673 5	+72.054 3	+131.879	$\mathrm{bei}_1\,x$
$\mathrm{ber}_1'\,x$	−0.476 664	−0.720 532	−0.635 99	+0.658 74	+4.251 33	+10.206 52	+14.677 5	+ 5.866 4	−37.108 0	−132.087	$\mathrm{ber}_1'\,x$
$\mathrm{bei}_1'\,x$	+0.212 036	−0.305 845	−1.364 13	−2.792 83	−3.327 80	+ 0.235 45	+12.780 7	+36.882 2	+61.749 0	+ 45.127	$\mathrm{bei}_1'\,x$
$\mathrm{ber}_2\,x$	+0.010 411	+0.165 279	+0.808 37	+2.317 85	+4.488 43	+ 5.242 91	− 0.950 4	−22.889 0	−65.869 2	−111.779	$\mathrm{ber}_2\,x$
$\mathrm{bei}_2\,x$	−0.124 675	−0.479 225	−0.891 02	−0.725 36	+1.422 10	+ 7.432 44	+17.592 4	+25.438 9	+10.134 8	− 66.610	$\mathrm{bei}_2\,x$
$\mathrm{ber}_2'\,x$	+0.041 623	+0.327 788	+1.030 93	+1.975 73	+2.049 97	− 1.454 56	−12.493 0	−32.589 1	−50.963 2	− 28.840	$\mathrm{ber}_2'\,x$
$\mathrm{bei}_2'\,x$	−0.248 047	−0.437 789	−0.286 47	+0.853 82	+3.785 30	+ 8.368 74	+11.015 1	+ 1.300 6	−38.551 6	−121.987	$\mathrm{bei}_2'\,x$
$\mathrm{ber}_3\,x$	+0.013 788	+0.085 612	+0.130 44	−0.282 63	−2.094 35	− 6.430 04	−12.876 5	−15.420 4	+ 3.166 6	+ 72.253	$\mathrm{ber}_3\,x$
$\mathrm{bei}_3\,x$	+0.015 629	+0.144 210	+0.565 38	+1.437 76	+2.454 41	+ 1.901 46	− 4.407 2	−22.575 0	−54.538 7	− 81.423	$\mathrm{bei}_3\,x$
$\mathrm{ber}_3'\,x$	+0.039 433	+0.093 575	+0.072 00	−0.914 09	−2.922 76	− 5.747 81	− 6.249 2	+ 3.979 6	+38.354 6	+104.463	$\mathrm{ber}_3'\,x$
$\mathrm{bei}_3'\,x$	+0.048 634	+0.239 418	+0.636 27	+1.073 55	+0.695 57	− 2.498 96	−11.222 9	−25.707 4	−35.563 4	− 7.513	$\mathrm{bei}_3'\,x$
$\mathrm{ber}_4\,x$	−0.002 60	−0.040 97	−0.193 27	−0.493 10	−0.628 67	+ 0.648 3	+ 6.083 5	+19.094 7	+38.667	+ 46.579	$\mathrm{ber}_4\,x$
$\mathrm{bei}_4\,x$	−0.000 13	−0.008 30	−0.093 02	−0.499 85	−1.727 62	− 4.230 2	− 7.116 9	− 5.288 8	+14.082	+ 70.500	$\mathrm{bei}_4\,x$
$\mathrm{ber}_4'\,x$	−0.010 40	−0.080 56	−0.234 32	−0.323 71	+0.248 34	+ 2.770 0	+ 8.745 2	+17.319 5	+19.140	− 12.148	$\mathrm{ber}_4'\,x$
$\mathrm{bei}_4'\,x$	−0.000 78	−0.024 83	−0.183 52	−0.716 65	−1.834 36	− 3.071 1	− 1.921 9	+ 7.703 5	+34.545	+ 80.465	$\mathrm{bei}_4'\,x$
$\mathrm{ber}_5\,x$	+0.000 19	+0.006 80	+0.058 59	+0.273 08	+0.851 04	+ 1.830 5	+ 2.209 0	− 1.821 3	−18.619	− 58.722	$\mathrm{ber}_5\,x$
$\mathrm{bei}_5\,x$	−0.000 18	−0.004 84	−0.025 54	−0.033 53	+0.211 43	+ 1.475 6	+ 5.242 3	+12.812 8	+21.384	+ 15.193	$\mathrm{bei}_5\,x$
$\mathrm{ber}_5'\,x$	+0.000 97	+0.017 84	+0.104 78	+0.360 76	+0.815 11	+ 1.007 4	− 0.847 2	− 8.623 9	−26.955	− 53.427	$\mathrm{ber}_5'\,x$
$\mathrm{bei}_5'\,x$	−0.000 87	−0.011 00	−0.028 32	+0.046 69	+0.565 64	+ 2.220 0	+ 5.589 6	+ 9.233 7	+ 5.504	− 24.511	$\mathrm{bei}_5'\,x$

TABLE III

BESSEL FUNCTIONS OF THE SECOND KIND, OF ORDERS HIGHER THAN ZERO

$$\mathrm{ker}_n\, x + i\,\mathrm{kei}_n\, x = \frac{\pi i}{2} H_n^1 (x\, i \sqrt{i}) = i^{-n} K_n (x \sqrt{i})$$

$$\mathrm{ker}_n'\, x = \frac{d}{dx} \mathrm{ker}_n\, x$$

x	1	2	3	4	5	6	7	8	9	10	x
$\mathrm{ker}_1\, x$	− 0.740 322	− 0.230 806	−0.049 898	+0.005 351 3	+0.012 737 4	+0.007 676 09	+0.002 743 59	+0.000 322 857	−0.000 355 78	−0.000 322 80	$\mathrm{ker}_1\, x$
$\mathrm{kei}_1\, x$	− 0.241 996	+ 0.080 049	+0.080 270	+0.039 166 0	+0.011 577 8	+0.000 288 35	−0.002 148 90	−0.001 566 975	−0.000 650 05	−0.000 123 52	$\mathrm{kei}_1\, x$
$\mathrm{ker}_1'\, x$	+ 0.887 604	+ 0.287 983	+0.100 178	+0.022 690 0	−0.002 318 3	−0.005 920 41	−0.003 660 46	−0.001 352 336	−0.000 185 34	+0.000 158 19	$\mathrm{ker}_1'\, x$
$\mathrm{kei}_1'\, x$	+ 0.794 742	+ 0.073 632	−0.038 005	−0.036 928 3	−0.018 366 4	−0.005 612 66	+0.000 156 61	+0.000 985 180	+0.000 748 45	+0.000 321 35	$\mathrm{kei}_1'\, x$
$\mathrm{ker}_2\, x$	+ 0.418 03	+ 0.261 472	+0.128 391	+0.048 134 2	+0.011 183 7	−0.001 088 3	−0.002 910 45	−0.001 819 91	−0.000 683 40	−0.000 101 28	$\mathrm{ker}_2\, x$
$\mathrm{kei}_2\, x$	+ 1.884 20	+ 0.309 001	+0.036 804	−0.017 937 6	−0.018 064 9	−0.009 093 7	−0.002 820 51	−0.000 149 65	+0.000 477 20	+0.000 370 64	$\mathrm{kei}_2\, x$
$\mathrm{ker}_2'\, x$	− 0.141 46	− 0.154 871	−0.107 070	−0.055 545 6	−0.021 666 9	−0.005 268 9	+0.000 411 05	+0.001 334 70	+0.000 863 10	+0.000 335 85	$\mathrm{ker}_2'\, x$
$\mathrm{kei}_2'\, x$	− 4.120 77	− 0.528 809	−0.116 579	−0.014 941 8	+0.008 046 0	+0.008 255 2	+0.004 265 37	+0.001 373 73	+0.000 102 03	−0.000 215 04	$\mathrm{kei}_2'\, x$
$\mathrm{ker}_3\, x$	+ 4.887 27	+ 0.298 022	−0.036 451	−0.052 071 1	−0.029 282 9	−0.011 449 9	−0.002 707 2	+0.000 267 67	+0.000 720 5	+0.000 456 3	$\mathrm{ker}_3\, x$
$\mathrm{kei}_3\, x$	− 6.269 71	− 0.886 821	−0.236 018	−0.060 518 2	−0.007 685 2	+0.004 511 5	+0.004 464 6	+0.002 263 32	+0.000 714 8	+0.000 047 3	$\mathrm{kei}_3\, x$
$\mathrm{ker}_3'\, x$	− 16.269 7	− 0.850 418	−0.080 360	+0.017 701 2	+0.022 435 5	+0.012 924 7	+0.005 212 6	+0.001 292 32	−0.000 094 4	−0.000 327 3	$\mathrm{ker}_3'\, x$
$\mathrm{kei}_3'\, x$	+ 17.772 4	+ 1.296 62	+0.300 78	+0.092 108 5	+0.025 293 0	+0.003 405 0	−0.001 977 0	−0.002 029 80	−0.001 059 0	−0.000 347 9	$\mathrm{kei}_3'\, x$
$\mathrm{ker}_4\, x$	− 47.753 1	− 2.774 90	−0.410 62	−0.057 09	+0.007 143	+0.012 375	+0.007 257	+0.002 878 3	+0.000 680 7	−0.000 072 2	$\mathrm{ker}_4\, x$
$\mathrm{kei}_4\, x$	+ 3.981 0	+ 0.940 03	+0.348 52	+0.137 36	+0.049 433	+0.014 000	+0.001 780	−0.001 192 6	−0.001 153 8	−0.000 584 3	$\mathrm{kei}_4\, x$
$\mathrm{ker}_4'\, x$	+191.990	+ 5.966 15	+0.740 16	+0.136 71	+0.020 426	−0.003 344	−0.005 361	−0.003 228 8	−0.001 317 5	−0.000 327 2	$\mathrm{ker}_4'\, x$
$\mathrm{kei}_4'\, x$	− 8.035	− 1.042 25	−0.323 58	−0.131 38	−0.054 819	−0.020 620	−0.006 088	−0.000 814 8	+0.000 516 8	+0.000 322 9	$\mathrm{kei}_4'\, x$
$\mathrm{ker}_5\, x$	+287.76	+10.209 4	+1.467 9	+0.327 07	+0.077 13	+0.012 982	−0.001 719	−0.003 146 2	−0.001 873 6	−0.000 746 0	$\mathrm{ker}_5\, x$
$\mathrm{kei}_5\, x$	+253.88	+ 6.076 6	+0.353 1	−0.052 99	−0.056 32	−0.029 378	−0.011 767	−0.003 455 3	−0.000 417 5	+0.000 324 1	$\mathrm{kei}_5\, x$
$\mathrm{ker}_5'\, x$	−1407.9	−24.236 0	−2.402 6	−0.465 59	−0.117 13	−0.029 468	−0.005 162	+0.000 774 4	+0.001 375 4	+0.000 837 2	$\mathrm{ker}_5'\, x$
$\mathrm{kei}_5'\, x$	−1306.0	−17.818 4	−1.125 3	−0.071 26	+0.026 42	+0.023 332	+0.011 279	+0.005 038 1	+0.001 529 2	+0.000 200 1	$\mathrm{kei}_5'\, x$

(Refer. 14, p. 622).

$$\mathrm{ber}_n\, x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p}}{p\,!\,(n+p)\,!} \left(\frac{1}{2}\,x\right)^{n+2p} \cos\frac{(n+2\,p)\,\pi}{4} \tag{10}$$

$$\mathrm{bei}_n\, x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p+1}}{p\,!\,(n+p)\,!} \left(\frac{1}{2}\,x\right)^{n+2p} \sin\frac{(n+2\,p)\,\pi}{4} \tag{11}$$

$$\mathrm{ber}_n{}'\, x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p}\left(\frac{n}{2}+p\right)}{p\,!\,(n+p)\,!} \left(\frac{1}{2}\,x\right)^{n+2p-1} \cos\frac{(n+2\,p)\,\pi}{4} \tag{12}$$

$$\mathrm{bei}_n{}'\, x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p+1}\left(\frac{n}{2}+p\right)}{p\,!\,(n+p)\,!} \left(\frac{1}{2}\,x\right)^{n+2p-1} \sin\frac{(n+2\,p)\,\pi}{4} \tag{13}$$

$$\begin{aligned}\mathrm{ker}_n\, x + i\,\mathrm{kei}_n\, x &= i^{-n} K_n\,(x\sqrt{i}) \\ &= \frac{\pi\, i}{2}\, H_n^{(1)}\,(x\, i\sqrt{i}) \\ &= \frac{\pi\, i}{2}\,[J_n\,(x\, i\sqrt{i}) + i\, Y_n\,(x\, i\sqrt{i})] \\ &= \frac{\pi\, i}{2}\,(\mathrm{her}_n\, x + i\,\mathrm{hei}_n\, x)\end{aligned} \tag{14}$$

$$\begin{aligned}\mathrm{ker}_n\, x &= -\frac{\pi}{2}\,\mathrm{hei}_n\, x \\ &= (0.1159315 - \log_e x)\,\mathrm{ber}_n\, x + \frac{\pi}{4}\,\mathrm{bei}_n\, x \\ &+ \frac{1}{2}\sum_{p=0}^{n-1} \frac{(-1)^{n+p}\,(n-p-1)!}{p\,!}\left(\frac{1}{2}\,x\right)^{2p-n} \cos\frac{(n+2\,p)\,\pi}{4} \\ &+ \frac{1}{2}\sum_{p=0}^{\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{p}+1+\frac{1}{2}+\frac{1}{3}+\ldots\right. \\ &\left.+\frac{1}{n+p}\right)\frac{(-1)^{n+p}}{p\,!\,(n+p)\,!}\left(\frac{1}{2}\,x\right)^{n+2p} \cos\frac{(n+2\,p)\,\pi}{4}\end{aligned} \tag{15}$$

$$\begin{aligned}\mathrm{kei}_n\, x &= \frac{\pi}{2}\,\mathrm{her}_n\, x \\ &= (0.1159315 - \log_e x)\,\mathrm{bei}_n\, x - \frac{\pi}{4}\,\mathrm{ber}_n\, x \\ &+ \frac{1}{2}\sum_{p=0}^{n-1} \frac{(-1)^{n+p}\,(n-p-1)\,!}{p\,!}\left(\frac{1}{2}\,x\right)^{2p-n} \sin\frac{(n+2\,p)\,\pi}{4} \\ &- \frac{1}{2}\sum_{p=0}^{\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{p}+1+\frac{1}{2}+\frac{1}{3}+\ldots\right. \\ &\left.+\frac{1}{n+p}\right)\frac{(-1)^{n+p}}{p\,!\,(n+p)\,!}\left(\frac{1}{2}\,x\right)^{n+2p} \sin\frac{(n+2\,p)\,\pi}{4}\end{aligned} \tag{16}$$

See Reference 8, paragraph 3.

$$\begin{aligned}\mathrm{ker}_n{}'\, x &= (0.1159315 - \log_e x)\,\mathrm{ber}_n{}'\, x - \frac{\mathrm{ber}_n\, x}{x} + \frac{\pi}{4}\,\mathrm{bei}_n{}'\, x \\ &+ \frac{1}{4}\sum_{p=0}^{n-1} \frac{(-1)^{n+p}\,(2\,p-n)\,(n-p-1)\,!}{p\,!} \left(\frac{1}{2}\,x\right)^{2p-n-1} \cos\frac{(n+2\,p)\,\pi}{4} \\ &+ \frac{1}{4}\sum_{p=0}^{\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{p}+1+\frac{1}{2}+\frac{1}{3}+\ldots\right. \\ &\left.+\frac{1}{n+p}\right)\frac{(-1)^{n+p}(n+2\,p)}{p\,!\,(n+p)\,!}\left(\frac{1}{2}\,x\right)^{n+2p-1} \cos\frac{(n+2\,p)\,\pi}{4}\end{aligned} \tag{17}$$

$$\begin{aligned}\mathrm{kei}_n{}'\, x &= (0.1159315 - \log_e x)\,\mathrm{bei}_n{}'\, x - \frac{\mathrm{bei}_n\, x}{x} - \frac{\pi}{4}\,\mathrm{ber}_n{}'\, x \\ &+ \frac{1}{4}\sum_{p=0}^{n-1} \frac{(-1)^{n+p}\,(2\,p-n)\,(n-p-1)\,!}{p\,!} \left(\frac{1}{2}\,x\right)^{2p-n-1} \sin\frac{(n+2\,p)\,\pi}{4} \\ &- \frac{1}{4}\sum_{p=0}^{\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{p}+1+\frac{1}{2}+\frac{1}{3}+\ldots\right. \\ &\left.+\frac{1}{n+p}\right)\frac{(-1)^{n+p}(n+2\,p)}{p\,!\,(n+p)\,!}\left(\frac{1}{2}\,x\right)^{n+2p-1} \sin\frac{(n+2\,p)\,\pi}{4}\end{aligned} \tag{18}$$

By putting $n = 0$, Equations (1) to (8) are obtained. The function $\mathrm{ber}\, x = \mathrm{ber}_o\, x$, etc.

For large values of x, an asymptotic series for $I_n\,(z) = i^{-n}\, J\, n\,(i\, z)$ is available. (Reference 16, para. 7.23) If n is a positive integer, putting $z = x\sqrt{i}$,

$$\sqrt[4]{i} = e^{\frac{i\,\pi}{8}} \text{ and } \frac{1}{\sqrt{i}} = e^{\frac{-i\,\pi}{4}} = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4},$$

$$J_n(x\,i\sqrt{i}) = \mathrm{ber}_n\,x + i\,\mathrm{bei}_n\,x = i^n I_n(x\sqrt{i})$$

$$= \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\, e^{i\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)}\,[L_n(x) + i\,M_n(x)] \quad (19)$$

where $L_n(x) = 1 - \frac{4n^2-1^2}{1!\,8x}\cos\frac{\pi}{4}$

$$+ \frac{(4n^2-1^2)(4n^2-3^2)}{2!\,(8x)^2}\cos\frac{2\pi}{4} + \ldots$$

$$\ldots + (-1)^p \frac{(4n^2-1^2)(4n^2-3^2)\ldots\{4n^2-(2p-1)^2\}}{p!\,(8x)^p}$$

$$\cos\frac{p\pi}{4}\ldots \quad (20)$$

$$M_n(x) = \frac{4n^2-1^2}{1!\,8x}\sin\frac{\pi}{4}$$

$$- \frac{(4n^2-1^2)(4n^2-3^2)}{2!\,(8x)^2}\sin\frac{2\pi}{4} + \ldots$$

$$\ldots + (-1)^{p+1} \frac{(4n^2-1^2)(4n^2-3^2)\ldots\{4n^2-(2p-1)^2\}}{p!\,(8x)^p}$$

$$\sin\frac{p\pi}{4} + \ldots \quad (21)$$

Then,

$$\mathrm{ber}_n\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\left[L_n(x)\cos\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)\right.$$

$$\left. - M_n(x)\sin\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)\right] \quad (22)$$

$$\mathrm{bei}_n\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\left[M_n(x)\cos\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)\right.$$

$$\left. + L_n(x)\sin\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\pi}{2}\right)\right] \quad (23)$$

The derivative of the Bessel function is given by the reduction formula

$$2J_n'(x\,i\sqrt{i}) = J_{n-1}(x\,i\sqrt{i}) - J_{n+1}(x\,i\sqrt{i}) \quad (24)$$

This gives two series similar to (19), but they can be combined into one series as follows:

$$J_n'(x\,i\sqrt{i}) = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\, e^{i\left(\frac{x}{\sqrt{2}}-\frac{5\pi}{8}+\frac{n\pi}{2}\right)}\,[S_n(x) + i\,T_n(x)] \quad (25)$$

where

$$S_n(x) = 1 - \frac{4n^2+1\times 3}{1!\,8x}\cos\frac{\pi}{4}$$

$$+ \frac{(4n^2-1^2)(4n^2+3\times 5)}{2!\,(8x)^2}\cos\frac{2\pi}{4}$$

$$- \frac{(4n^2-1^2)(4n^2-3^2)(4n^2+5\times 7)}{3!\,(8x)^3}\cos\frac{3\pi}{4} + \ldots$$

$$\ldots + (-1)^p (4n^2-1^2)(4n^2-3^2)\ldots\{4n^2-(2p-3)^2\}$$

$$\frac{\{4n^2+(2p-1)(2p+1)\}}{p!\,(8x)^p}\cos\frac{p\pi}{4} + \ldots \quad (26)$$

$$T_n(x) = \frac{4n^2+1\times 3}{1!\,8x}\sin\frac{\pi}{4}$$

$$- \frac{(4n^2-1^2)(4n^2+3\times 5)}{2!\,(8x)^2}\sin\frac{2\pi}{4}$$

$$+ \frac{(4n^2-1^2)(4n^2-3^2)(4n^2+5\times 7)}{3!\,(8x)^3}\sin\frac{3\pi}{4} - \ldots$$

$$\ldots + (-1)^{p+1}(4n^2-1^2)(4n^2-3^2)\ldots\{4n^2-(2p-3)^2\}$$

$$\frac{\{4n^2+(2p-1)(2p+1)\}}{p!\,(8x)^p}\sin\frac{p\pi}{4} + \ldots \quad (27)$$

$$\mathrm{ber}_n'\,x + i\,\mathrm{bei}_n'\,x = \frac{d}{dx}(\mathrm{ber}_n\,x + i\,\mathrm{bei}_n\,x)$$

$$= i\sqrt{i}\,J_n'(x\,i\sqrt{i}) = e^{\frac{3\pi i}{4}}\,J_n'(x\,i\sqrt{i})$$

$$\mathrm{ber}_n'\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\left[S_n(x)\cos\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right.$$

$$\left. - T_n(x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right] \quad (28)$$

$$\mathrm{bei}_n'\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}}\left[T_n(x)\cos\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right.$$

$$\left. + S_n(x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right] \quad (29)$$

An asymptotic series for

$$K_n(z) = i^n\,\frac{\pi i}{2}\,H_n^{(1)}(i\,z) \quad (30)$$

also is given in Reference 16, paragraph 7.23. From this is obtained

$$\ker_n\,x + i\,\mathrm{kei}_n\,x = i^{-n}K_n(x\sqrt{i})$$

$$= \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-x/\sqrt{2}}\, e^{-i\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)}\,[L_n(-x) + i\,M_n(-x)] \quad (31)$$

$$\ker_n\,x = \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}}\left[L_n(-x)\cos\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}\right.\right.$$

$$\left.\left. + \frac{n\pi}{2}\right) + M_n(-x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right] \quad (32)$$

$$\mathrm{kei}_n\,x = \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}}\left[M_n(-x)\cos\left(\frac{x}{\sqrt{2}}\right.\right.$$

$$\left.\left. + \frac{\pi}{8}+\frac{n\pi}{2}\right) - L_n(-x)\sin\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)\right] \quad (33)$$

$L_n(-x)$ and $M_n(-x)$ are obtained by changing x to $(-x)$ in (20) and (21).

For the derivative, the reduction formula can be used as was done for (25). The two series will combine into one series, as follows:

$$\ker_n' x + i \operatorname{kei}_n' x$$

$$= -\left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} e^{-i\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2}\right)} [S_n(-x) + i\, T_n(-x)] \tag{34}$$

$$\ker_n' x = -\left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[S_n(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2}\right) + T_n(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2}\right)\right] \tag{35}$$

$$\operatorname{kei}_n' x = -\left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[T_n(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2}\right) - S_n(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2}\right)\right] \tag{36}$$

$S_n(-x)$ and $T_n(-x)$ are obtained by changing x to $(-x)$ in (26) and (27).

Bessel Functions of Order Zero, with Large Values of x.

Putting $n = 0$, $J_0(x\, i \sqrt{i}) = \operatorname{ber} x + i \operatorname{bei} x$

$$\operatorname{ber} x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[L_0(x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) - M_0(x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right)\right] \tag{37}$$

$$\operatorname{bei} x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[M_0(x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) + L_0(x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right)\right] \tag{38}$$

where

$$L_0(x) = 1 + \frac{1^2}{1!\,8x} \cos\frac{\pi}{4} + \frac{1^2 \cdot 3^2}{2!\,(8x)^2} \cos\frac{2\pi}{4} + \frac{1^2 \cdot 3^2 \cdot 5^2}{3!\,(8x)^3} \cos\frac{3\pi}{4} + \ldots + \frac{1^2 \cdot 3^2 \cdot \ldots \cdot (2p-1)^2}{p!\,(8x)^p} \cos\frac{p\pi}{4} + \ldots \tag{39}$$

$$M_0(x) = -\frac{1^2}{1!\,8x} \sin\frac{\pi}{4} - \frac{1^2 \cdot 3^2}{2!\,(8x)^2} \sin\frac{2\pi}{4} - \ldots - \frac{1^2 \cdot 3^2 \cdot \ldots \cdot (2p-1)^2}{p!\,(8x)^p} \sin\frac{p\pi}{4} - \ldots \tag{40}$$

$$\operatorname{ber}' x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[S_0(x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) - T_0(x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right)\right] \tag{41}$$

$$\operatorname{bei}' x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[T_0(x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) + S_0(x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right)\right] \tag{42}$$

where

$$S_0(x) = 1 - \frac{1 \cdot 3}{1!\,8x} \cos\frac{\pi}{4} - \frac{1^2 \cdot 3 \cdot 5}{2!\,(8x)^2} \cos\frac{2\pi}{4} - \frac{1^2 \cdot 3^2 \cdot 5 \cdot 7}{3!\,(8x)^3} \cos\frac{3\pi}{4} - \ldots - \frac{1^2 \cdot 3^2 \cdot \ldots (2p-3)^2 (2p-1)(2p+1)}{p!\,(8x)^p} \cos\frac{p\pi}{4} - \ldots \tag{43}$$

$$T_0(x) = \frac{1 \cdot 3}{1!\,8x} \sin\frac{\pi}{4} + \frac{1^2 \cdot 3 \cdot 5}{2!\,(8x)^2} \sin\frac{2\pi}{4} + \frac{1^2 \cdot 3^2 \cdot 5 \cdot 7}{3!\,(8x)^3} \sin\frac{3\pi}{4} + \ldots + \frac{1^2 \cdot 3^2 \cdot \ldots (2p-3)^2 (2p-1)(2p+1)}{p!\,(8x)^p} \sin\frac{p\pi}{4} + \ldots \tag{44}$$

It may be noted that $S_0(x)$ has the same value, term for term, as $L_1(x)$, and $T_0(x)$ the same as $M_1(x)$. This is in agreement with the fact that $J_0'(x) = -J_1(x)$. However, $S_1(x)$ has not the same value as $L_2(x)$.

$$\ker x = \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[L_0(-x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) + M_0(-x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right)\right] \tag{45}$$

$$\operatorname{kei} x = \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[M_0(-x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right) - L_0(-x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right)\right] \tag{46}$$

where $L_0(-x)$ and $M_0(-x)$ are obtained by changing x to $-x$ in (39) and (40).

$$\ker' x = -\left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[S_0(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) + T_0(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right)\right] \tag{47}$$

$$\operatorname{kei}' x = -\left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[T_0(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right) - S_0(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8}\right)\right] \tag{48}$$

where $S_0(-x)$ and $T_0(-x)$ are obtained by changing x to $(-x)$ in (43) and (44).

Since the general terms of these series are given, any desired number of terms can be evaluated. However, the series for large values of x are semi-divergent, and there is a limit to the amount of precision which they will give, though this is usually appreciable only for the smaller values of x. It can be proved that the sum of a given number, greater than n, of terms of the series for I_n or K_n differs from the true value of the function by less than the last term included, and so the series can be used for numerical computation.[6]

Note that K_n is sometimes, particularly in older publications, used to represent a function corresponding to J_n instead of one corresponding to I_n. The term ker, however, has been used to represent only one function, namely, the function whose values are tabulated in this paper.

As a check on the correctness of these series, Equations (1) to (18) have been used to calculate the values for $x = 1$, with $n = 0$ and $n = 5$, and Equations (20) to (48) have been used to calculate the values for $x = 10$, with $n = 0$ and $n = 5$. The calculations were carried out to several significant figures, and they agreed with the independently obtained tabulated values.

Other series for large values of x, of which the general terms were not given, have been published by H. G. Savidge.[12] In his formula for η, the term

$$-\frac{21}{128\sqrt{2}\,x^3}$$

is evidently a typographical error, since it should be

$$+\frac{21}{128\sqrt{2}\,x^3}$$

in order to agree with his preceding calculation on p. 108, and also with his previous publication of the first terms of the same series in 1910.[7]

The values of Bessel functions of higher orders in Tables II and III are not near enough together to give intermediate values by interpolation. They were obtained from the values of functions of order zero by the following reduction formulas:[20]

Bessel Function Reduction Formulas for Functions of Higher Orders than Zero

Special

$$u_0 = \text{ber}\, x \tag{49}$$

$$v_0 = \text{bei}\, x \tag{50}$$

$$u_1 = \text{ber}_1\, x = +(1/\sqrt{2})\,(u_0' - v_0') \tag{51}$$

$$v_1 = \text{bei}_1\, x = +(1/\sqrt{2})\,(u_0' + v_0') \tag{52}$$

$$u_2 = -\frac{\sqrt{2}}{x}\,(u_1 - v_1) - u_0 = \frac{2\,v_0'}{x} - u_0 \tag{53}$$

$$v_2 = -\frac{\sqrt{2}}{x}\,(u_1 + v_1) - v_0 = -\frac{2\,u_0'}{x} - v_0 \tag{54}$$

$$u_2' = -(1/\sqrt{2})\,(u_1 + v_1) - \frac{2\,u_2}{x} = -u_0' - \frac{2\,u_2}{x} \tag{55}$$

$$v_2' = +(1/\sqrt{2})\,(u_1 - v_1) - \frac{2\,v_2}{x} = -v_0' - \frac{2\,v_2}{x} \tag{56}$$

General

$$u_{n+1} = -\frac{n\sqrt{2}}{x}\,(u_n - v_n) - u_{n-1} \qquad n \neq 0 \tag{57}$$

$$v_{n+1} = -\frac{n\sqrt{2}}{x}\,(u_n + v_n) - v_{n-1} \qquad n \neq 0 \tag{58}$$

$$u_n' = -(1/\sqrt{2})(u_{n-1} + v_{n-1}) - \frac{n\,u_n}{x} \qquad n \neq 0 \tag{59}$$

$$v_n' = +(1/\sqrt{2})(u_{n-1} - v_{n-1}) - \frac{n\,v_n}{x} \qquad n \neq 0 \tag{60}$$

These formulas are also suitable for Bessel functions of the second kind by changing u to ker and v to kei.

A practical problem using values from Table I is the calculation of the skin effect resistance ratio and reactance ratio of an isolated tubular conductor. The general solution of this problem was given by Heaviside.[3]

Let μ = permeability of the material of the tube (assumed = 1 for copper).

$\omega = 2\,\pi\,f$

f = frequency in cycles per second

ρ = resistivity of the material of the tube in absolute units (= 1724 for copper of 100 per cent conductivity at 20 deg. cent.).

If the conductor is a cable, with a rope core or hollow center, ρ is not the value for solid copper, but should be determined from the resistance per 1000 ft. of the cable and from its diameter.

$i = \sqrt{(-1)}$

$$m^2 = \frac{4\,\pi\,\mu\,\omega}{\rho}$$

r = outside radius of tube in centimeters

q = inside radius of tube in centimeters

R' = effective value of the resistance per cm. of the tube at frequency f

X' = effective value of the reactance per cm. of the tube at frequency f, due to flux inside radius r.

R = resistance for direct current per cm. of tube.

$$\frac{R' + i\,X'}{R} = i\,\frac{m\,r}{2}\,\frac{(r^2 - q^2)}{r^2}$$

$$\frac{\left[(\text{ber}\, m\,r + i\,\text{bei}\, m\,r) + \dfrac{(C + i\,D)}{(A + i\,B)}\,(\text{ker}\, m\,r + i\,\text{kei}\, m\,r)\right]}{\left[(\text{ber}'\, m\,r + i\,\text{bei}'\, m\,r) + \dfrac{(C + i\,D)}{(A + i\,B)}\,(\text{ker}'\, m\,r + i\,\text{kei}'\, m\,r)\right]} \tag{51}$$

$$\text{where}\quad \frac{C + i\,D}{A + i\,B} = -\frac{(\text{ber}'\, m\,q + i\,\text{bei}'\, m\,q)}{(\text{ker}'\, m\,q + i\,\text{kei}'\, m\,q)} \tag{52}$$

For the derivation of these formulas, see Reference 19. The right-hand part of Equation (51) can be found as a numerical complex quantity. The real part is equal to R'/R, the skin effect resistance ratio of the tube, and the unreal part gives the change in inductance of the tube due to frequency.

The following example was given in Reference 19:

Example. Find the skin effect resistance ratio of a tube in which $t/d = 0.20$ and $m\,t = 3$, t being equal to $(r - q)$, the thickness of the tube in centimeters, and d being equal to $2\,r$, the outside diameter of the tube.

$$\frac{r-q}{2\,r} = 0.2$$

$$r - q = 0.4\,r$$

$$q = 0.6\,r$$

$$m\,(r-q) = 3$$

$$m\,r = 3/0.4 = 7.5$$

$$m\,q = 4.5$$

From Equation (52) and Table I,

$$\frac{C + i\,D}{A + i\,B} = -\frac{\text{ber}'\,4.5 + i\,\text{bei}'\,4.5}{\text{ker}'\,4.5 + i\,\text{kei}'\,4.5}$$

$$= \frac{3.753681 + i\,2.052635}{0.02481454 + i\,0.00771543}$$

$$= 161.387 + i\,32.5401$$

From Equation (51),

$$Z'/R = i\,3.75 \times 0.64$$

$$\frac{(5.454962 - i\,29.115712 + 0.258938 + i\,0.264977)}{(24.130125 - i\,14.735602 - 0.012244 - i\,0.388061)}$$

$$= i\,3.75 \times 0.64\,\frac{(574.136 - i\,609.403)}{810.397}$$

$$= 1.80475 + i\,1.70031$$

Therefore, $R'/R = 1.80475$

This is seen at once to check with the curve for $t/d = 0.20$, reading at $m\,t = 3$, in Fig. 3, of Reference 13.

To check the result with Fig. 1 of Reference 17, we have

f/R_{dc} (in ohms per 1000 ft.)

$$= \frac{\pi\,(r^2 - q^2)}{\rho}\;\frac{f \times 10^9 \times 0.3937}{12000}$$

and

$$m^2\,t^2 = \frac{8\,\pi^2\,f\,(r-q)^2}{\rho}$$

where the dimensions are in centimeters and ρ is in absolute units.

Therefore,

$$\sqrt{f/R_{dc}} = \sqrt{\frac{9 \times 10^9 \times 0.3937 \times 4}{8\,\pi \times 12000}} = 216.8$$

The value of R'/R_{dc} read from the curve is seen to be 1.805.

In order to save calculation, one may often use to advantage the curves published in Fig. 3, Reference 13, and Fig. 1, Reference 17, for obtaining skin effect ratios of isolated tubular conductors.

A problem involving the Bessel functions discussed in this paper which occurs very frequently is that of finding the skin effect resistance and reactance ratios of isolated round wire. A very complete table of these ratios is given in Bibliography 5, Table XXII. These tabulated values may be used with comparatively good accuracy for cables which do not have rope cores.

It should be remembered that, as previously stated, if the curves or tables are used for cable, ρ is not the value for solid copper, but should be determined from the resistance per 1000 ft. of the cable, and from the cable diameter.

It is desired to make acknowledgment of valuable suggestions from Mr. John R. Carson regarding the form of the Bessel function series.

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Some Notes on Transformer Design

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Member, A. I. E. E.

Synopsis.—A method for determining the most economical proportions for transformers is suggested. The method takes into account the densities of flux and of current in addition to the dimensions taken into account in previous discussions of this problem. Various shapes of transformers are considered.

WITH certain conditions which must be fulfilled there is some particular design of any machine which is the cheapest to build. These relations are usually so complicated that no attempt is made to predetermine the conditions or dimensions of the cheapest product except on the basis of experience and empirical constants. There is, however, among electrical apparatus, one, the transformer, which having no moving parts is so simple in its construction and relations, that considerable work has been done in the line of attempting to predetermine mathematically the shape and dimensions that give the lowest cost. Most of the work published has been done in England.[2] These methods have been carried further at the University of Illinois during the last several years with the help of several workers, principally Mr. J. K. McNeely, Mr. L. B. Archer, who has worked out several of the forms and graphs, and Mr. E. E. Perkins, who has done much of the checking and drawing. More variables have been brought into the equation and the necessary curves for numerous types of transformers have been worked out.

It is of course still necessary to make use of numerous empirical constants, most of which depend on the manufacturing conditions obtaining where the transformer is to be brought out. It seems that none of these should be particularly difficult to secure in any particular place and under a definite set of conditions. It has been hardest to find any accurate data on heating which can be used in a general method.

The method of design consists in setting up an equation for the cost of the active material of a transformer including labor in terms of the dimensions, and differentiating this equation for a minimum. Restraining equations for power and losses contain also the terms for densities of flux and current which are interrelated. It is in this development, due to Professor McNeely, that the density variables formerly assumed have been included in the equations and curves. The solution of these equations is rather tedious but their solutions may be reduced to curves once for all from which the necessary values may very readily be taken, and the result for any given case may be very quickly found.[3]

The flux density has been made an independent variable on which the heating depends. As the approximate value for this density is known from experience, for any given type and make of transformer but little work needs to be done to obtain dimensions giving suitable surface for heat radiation. An accurate knowledge of the magnetic quality of the iron to be used is of course presupposed and all the dimension proportions of the design are contingent upon these constants.

It may here be noted that for any flux density there is a fixed relation between the iron losses and copper losses for the cheapest transformer but that the best relation of flux and current densities is not the same for different types of transformers.

While it may never be possible or desirable to build any transformer exactly as figured by the following formulas, they should offer a ready means of checking up on how near a design comes to being the cheapest shape and proportions and to give a method which shows in advance just what should be striven for to give the cheapest design when the unit costs of iron and copper and the unit costs of winding, stacking, and insulating have been determined.

Every method of design has certain limitations and this is no exception. For instance, this method does not allow starting with a certain predetermined efficiency or reactance and working out the design with those factors specified. If, however, a flux density and desired core loss are decided upon, the best proportions are fixed and may be obtained by trial from the curves.

There develops necessarily an indefinite number of solutions, one for every density and that one giving the allowable heating must be picked out. Since the limiting temperature may first be reached in either the iron or the copper, a cheaper practical design may be obtained by varying the density either of flux or of current from that shown by the mathematical formula to give the least cost. Evidently a transformer having an infinite density of flux and current and therefore infinite losses would require no material and therefore be the cheapest to build.[4] If it were possible to place in the equation an expression fixing any quantity by a temperature limit, one solution only would result. This has not been found practicable. But if a basis of watts per square inch of surface can be used an

1. Assistant Professor of Electrical Design, University of Illinois, Urbana, Illinois.

2. Fleming and Hansen, 1903, and others at later dates. See bibliography.

3. Since the writing of this paper Professor McNeely has published a bulletin using similar methods of obtaining a minimum cost transformer. Bul. 87, Eng. Exp. Sta., Iowa State College.

4. The converse of this is not true though it has so been stated in print. A transformer of infinite dimensions does not give zero losses.

29-111

analytic or more easily a graphic solution can be obtained. In regard to reaction it has been often pointed out that this characteristic may be varied within wide ranges without altering the core dimensions and small changes of core to adjust for exact reactions desired may be made with very small increases in cost, since fortunately the curves of cost at the minimum point are very flat.

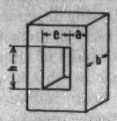


Fig. 1—Core Type

The theoretical derivation of this method for determining the dimensions of a transformer of minimum cost follows: As in practically all electrical machine design, the starting point is the fundamental e. m. f. equation.

$$E = 4.44\, f\, \varphi\, N\, 10^{-8} \qquad (1)$$

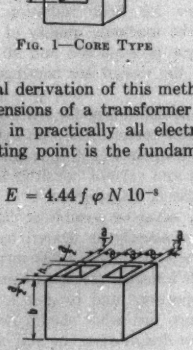


Fig. 2—Shell Type

where N = turns, E = voltage, f = frequency, φ = flux Multiply by I, the current in the coil.

$$E\, I = 4.44\, f\, \varphi\, N\, I\, 10^{-8} \qquad (2)$$

Since there is a difference between the ampere-turns of the primary and secondary sides, we shall use the mean value of $N\, I$. But this difference being usually very small, its neglect would make only a slight difference.

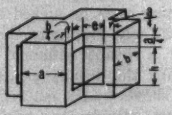


Fig. 3—Cruciform Type

Substituting for φ and $N\, I$ their values in terms of densities and areas, we have,

$$\text{kv-a.} = P = 0.001\, E\, I$$
$$= 2.22\, f\, \beta\, (\text{stacked iron area})\, F_i\, s\, h\, e\, F_c\, 10^{-8} \qquad (3)$$

F_i and F_c being the space factors for iron and copper respectively; β is kilolines per sq. in.; s is current density in kilo-amperes per sq. in.; $a\, b\, h\, e$ are dimensions for which see Figs. 1 to 7.

This "stacked iron area" is

$a\, b$ for forms 1 and 2
a^2 for form 3
$16.395\, p^2$ for form 4

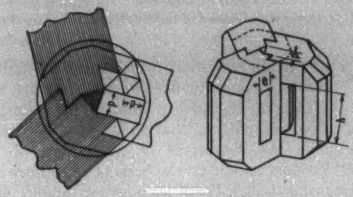


Fig. 4—Ring Coil Type and Cross-Section of Core

$0.618\, a^2$ for form 5
$0.668\, a^2$ for forms 6 and 7 (4)

For forms 5, 6, and 7 a is taken, for convenience, as the inside diameter of the insulating tube carrying the winding. (See Figs. 5, 6, 7.)

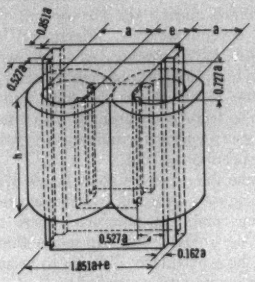


Fig. 5—Cruciform Core—Core Type Transformer

In each case k_s is so chosen that,

$$P = k_s\, \beta\, s\, a\, b\, h\, e \qquad (5)$$

in several cases $b = a$ and in the case of the ring coil type $a\, b$ becomes p^2.

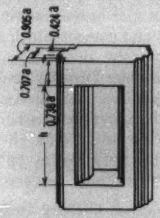


Fig. 6—Double Notched Core Circular Coil—Core Type

The losses in terms of volumes can be expressed thus:

$$L = L_i + L_c = b_1\, \beta^2\, k_i\, F_i\, V_i + c\, s^2\, F_c\, V_c \qquad (6)$$

($b_1\, \beta^2$ being a convenient form of expressing watts loss per lb. of iron—see under symbols, b_1)

The cost of active material,

$$C = C_i k_i F_i V_i + C_c k_c F_c V_c \qquad (7)$$

where C_i and C_c are the costs per pound and k_i and k_c the weights per cu. in. of iron and copper respectively.[5]

In every case, the volumes may be expressed in terms of constants and the four (or three) linear dimensions. The equation is then a third power equation and it will be noticed that this expression does not contain either β or s.

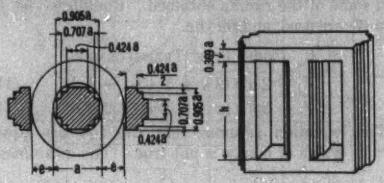


FIG. 7—DOUBLE NOTCHED CORE CIRCULAR COIL—SHELL TYPE CROSS-SECTION OF DOUBLE NOTCHED CORE CIRCULAR COIL—SHELL TYPE

The problem is to find the minimum value of C. To do this we may write:

$$\frac{\partial C}{\partial a} = 0 \quad \frac{\partial C}{\partial b} = 0 \quad \frac{\partial C}{\partial h} = 0 \quad \frac{\partial C}{\partial e} = 0 \qquad (8)$$

There are then six unknowns, the two densities and four linear dimensions, and six equations. In all cases the equations can be reduced to these forms, even if it be decided to increase the area of the yokes or outside legs, in order to decrease densities in these parts. This will only bring a further numerical constant into the quantity. In this discussion the flux density has been assumed uniform in all parts of the iron circuit.

As V_i and V_c will be a different function of $a\ b\ h\ e$ for each type of transformer, a separate solution for each type must be obtained. For the actual mathematical work involved see Appendix.

As might be anticipated these equations solve out as ratios. The values of h/e and b/e are obtained as functions of a/e or p/e. These are most easily determined by drawing curves between a/e or p/e and the other two ratios.

The proper proportioning of iron and copper, *i. e.*, the value of the ratio a/e or p/e will evidently depend on R the cost ratio of iron and copper and the ratio of iron and copper losses. That is, the shape of transformer will depend, not on actual values, but only on the ratios.

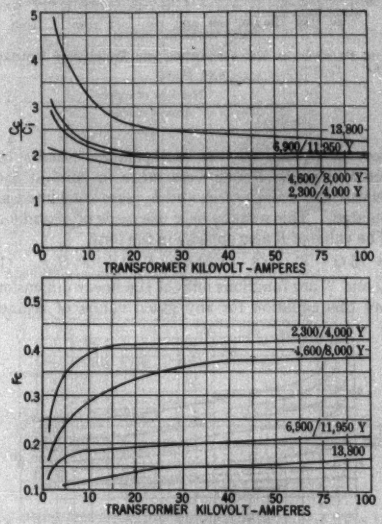
During the solution of the differential equations the quantity

$$(a_0 \beta^{1.8} + b_0 \beta^2)/(.8\, a_0 \beta^{1.8} + b_0 \beta^2)$$

appears. This comes of course from the use of the Steinmetz formula for iron losses.

$$L_{1b} = a_0 \beta^{1.8} + b_0 \beta^2$$

and is for any quality of iron a definite function of β only. This quantity is called r which quantity later



UPPER HALF

RATIO OF COPPER COST TO IRON COST OF TRANSFORMER

$$\frac{C_c}{C_i} = \frac{\text{Cost per lb. of covered copper including winding cost}}{\text{Cost per lb. of assembled core}}$$

LOWER HALF

COPPER SPACE FACTOR OF TRANSFORMERS

$$\text{Approximate space factor } (F_c) = \frac{\text{Area of copper}}{\text{area of window}}$$

proves to be the ratio of iron to copper loss under the conditions of minimum cost. See Appendix, page 10.

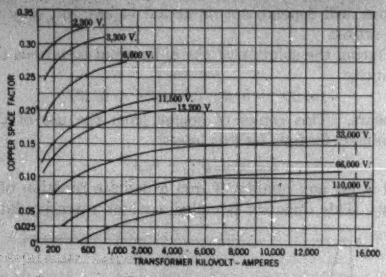
$$\therefore r = \frac{k_6 A}{k_7 B} = \frac{L_i}{L_c} = 1.25 - b_0/(3.2\, a_0/\beta^{.4} + 4\, b_0) \qquad (9)$$

for the minimum cost transformer (but not for others) for any value of β.

5. It should here be noted that the costs must include the labor of putting the iron and copper in place in order to give a minimum cost transformer as manufactured. This necessitates getting unit costs,—costs per pound—at the factory concerned. The shape depends upon the necessary space required for the winding; therefore F_c must be a function of the voltage and of the type of insulation used in the factory. Also C_c must include insulation and labor costs and is therefore also a function of voltage. C_i, the cost of stacked iron per pound, will evidently vary with the proportions of the core, but as these proportions vary only through a narrow range in the region of transformers as now built, unit costs so obtained are quite accurate enough. It will develop further on, and in fact is quite evident by inspection, that the shape and proportions as regards costs are a function of the ratio of the costs and not of the absolute costs. Therefore curves of the ratios of C_c to C_i for various voltages and sizes are sufficient.

Such curves as obtained for certain average conditions from two manufacturers are given.

The curve, which was drawn from the data furnished by the manufacturer of the iron, did not show a_0 and b_0 to be constant as would have been the case if the Steinmetz Law were exactly true. The curves for b_1 and r presuppose an accurate curve showing the relation between flux density per square inch and watts loss per pound of iron. If a curve be used which gives constant



SPACE FACTORS OF TRANSFORMERS FOR DIFFERENT VOLTAGES AND SIZES

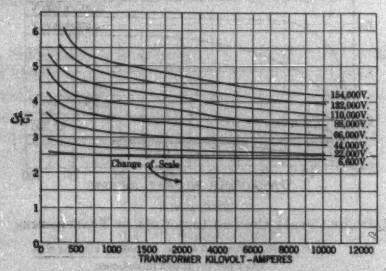
$$\text{Space factor} = \frac{\text{Net area of copper}}{\text{Area of window}}$$

values of a_0 and b_0, the mathematical results here given still hold. Points were taken in pairs at short intervals and values of a_0 and b_0 were calculated and tabulated. This was the only use made of a_0 and b_0.

The value of R also appears in the form

$$R = (5\,Q - 8)/T\,(8 - 3\,r\,Q) = F_c\,C_c\,k_c/F_i\,C_i\,k_i \qquad (10)$$

Q and T are functions only of the linear dimensions. From this equation for any given values of R and r



RATIO OF COPPER COST TO IRON COST OF TRANSFORMER

C_c = Cost per lb. of copper including insulation, winding, and assembly
C_i = Cost per lb. of assembled core

values of a/e and p/e may be obtained. Though this is a rather complicated function it may be reduced, by dividing numerator and denominator by e^4, to terms containing only the ratios a/e (or p/e) b/e, h/e, r, and constants. By means of the curves for b/e and h/e referred to above it may be reduced further to terms only of a/e (or p/e) and r and constants. Therefore, for any values of R and r curves may be drawn from which a/e (or p/e) may be found. (In actual computation it is easier to assume the value of a/e and compute R)

$$L_c = s^2\,c\,F_c\,V_c \qquad \text{whence}$$

$$s^2 = L_c/c\,F_c\,V_c = L_i/r\,c\,F_c\,V_c \text{ see (9)} \qquad (11)$$

and therefore s for the minimum cost transformer is also a function of β.

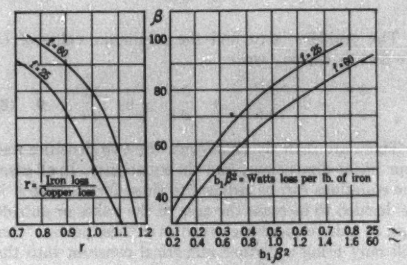
As an example, the simple core type transformer may be used to illustrate certain of the quantities involved and some of the results obtained. Equations (5), (6), and (7) are used, and for the,

1. CORE TYPE TRANSFORMERS

$$V_i = 2\,a\,b\,(2\,a + h + e) = 2\,A \qquad (112)$$

$$V_c = 2\,h\,e\,(a + b + e) = 2\,B \qquad (113)$$

This equation assumes that the coil has square corners which of course is not true. The mean length of turn might be written as $2\left(a + b + \pi\,\frac{e}{4}\right)$ instead



VALUES OF r AND $b_1\,\beta^2$ ARE FOR APOLLO SPECIAL ELECTRIFIED SHEET STEEL, SIZE 29–30¼ x 108 (1923)

of $2\,(a + b + e)$. This would be slightly too small due to the bulging out of the coil along the sides a and b. The actual value lies between the two but the longer value has been used in the calculations.

Equation (5) becomes

$$P_1 = 2.22\,F_c\,F_i\,f\,\beta\,s\,a\,b\,h\,e\,10^{-5} \qquad (114)$$

Equation (6) becomes

$$L_1 = L_i + L_c = 2\,k_i\,b_1\,\beta^2\,F_i\,a\,b\,(2\,a + h + e) + 2\,c\,s^2\,F_c\,h\,e\,(a + b + e) \qquad (115)$$

and Equation (7) becomes

$$C = 2\,F_i\,C_i\,k_i\,a\,b(2\,a + h + e) + 2\,F_c\,C_c\,k_c\,h\,e(a + b + e) \qquad (116)$$

Putting the total derivatives of these three equations each equal to 0 and solving we finally obtain

$$h/e = (3\,a + 2\,e)/(a + e) \qquad (117)$$

$$b/a = (4\,a + 3\,e)/(2\,a + e) \qquad (118)$$

$$R = k_i/k_4 = (5\,Q - 8)/T(8 - 3\,r\,Q) \qquad (119)$$

From these equations the curves for a/e, b/e, and h/e have been computed.

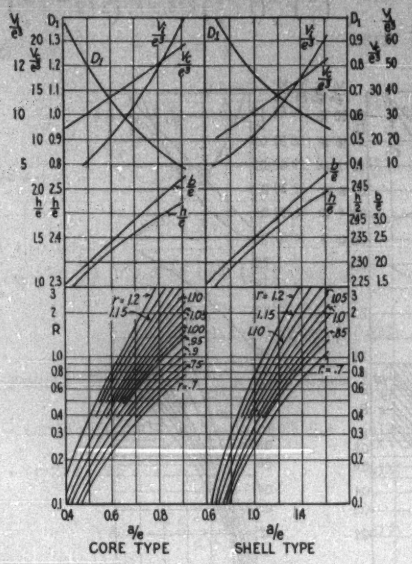
From (6) and (9)

$$L_1 = (1 + r)\, L_e = \left(1 + \frac{1}{r}\right) L_i \qquad (120)$$

whence

$$L_1^2 = (1 + r)\left(1 + \frac{1}{r}\right) L_e\, L_i \qquad (121)$$

Squaring (114) and dividing by (121) into which the



values of L_e and L_i have been substituted from (115) gives,

$$\frac{a\,b\,h\,e}{(2\,a + h + e)\,(a + b + e)}$$

$$= \frac{4\,(1 + r)\left(1 + \frac{1}{r}\right) c\, b_1\, F_e\, F_i\, k_i\, P_1^2}{k_3^2\, L_1^2} = S\, b_1\, P_1^2 / L_1^2 \qquad (122)$$

Where

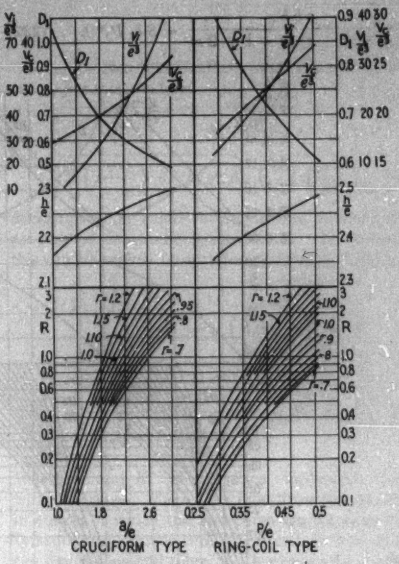
$$S = 4\,(1 + r)\left(1 + \frac{1}{r}\right) c\, F_e\, F_i\, k_i / k_3^2 = 16\, c\, F_e\, F_i\, k_i / k_3^2$$

since for the quality of iron and the densities used it is found that $(1 + r)\left(1 + \frac{1}{r}\right)$ does not vary from 4 by more than 1 per cent, *i. e.*, when r varies from 0.8 to 1.25. This is a less error than that in the iron loss coefficient which is a determining factor.

Substituting into (121) values of L_1 from (115) and using the identities

$$a = \frac{a}{e}\, e, \qquad b = \frac{b}{e}\, e, \qquad h = \frac{h}{e}\, e$$

the equation may be solved for e in terms of one unknown β and the ratios a/e, b/e, and h/e. The resulting equation is



$$e = \sqrt[8]{\frac{S\, P_1^3\, [a/e + b/e + 1]}{4\left(1 + \frac{1}{r}\right)^2 k_i^2\, b_1\, \beta^4\, F_i^2\, (a/e)^3\, (b/e)^3\, (h/e)\, [\,2\, a/e + h/e + 1]}} \qquad (123)$$

By using this equation we may determine e for a few assumed values of β. Having e, the other dimensions and the losses are readily found.

This allows a somewhat different method of attack upon the problem from that usually employed, eliminating all cut and try features. A design is obtained for each assumed value of β and this requires but one substitution in the equations.

All the other types of transformers are solved for in exactly the same manner.

2. Shell Type Transformers

$$V_i = 2\, a\, b\, (a + h + e) = 2\, A \qquad (212)$$

$$V_e = 2\, h\, e\, (a + b + 2\, e) = 2\, B \qquad (213)$$

As in the core type here also the coils are assumed to have square corners.

Using these forms for the volumes and substituting as before,

$$h/e = (3\,a + 4\,e)/(a + 2\,e) \qquad (217)$$

$$b/e = (2\,a + 3\,e)/(a + e) \qquad (218)$$

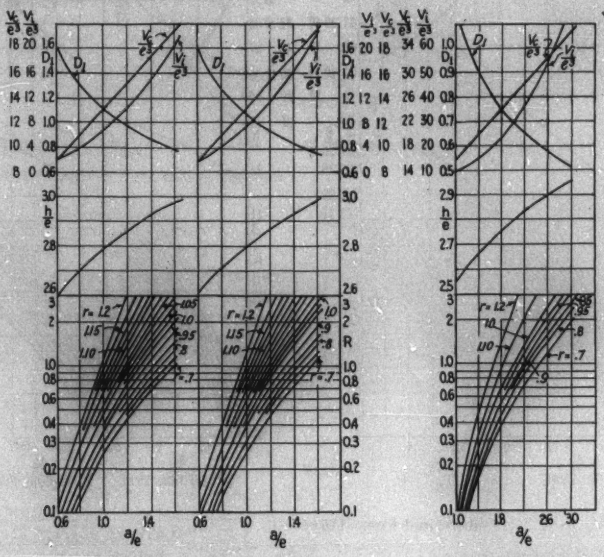
R has the same general form but the quantities Q and T are of course different. This is true in all subsequent types, and therefore the equation for R has not been repeated.

$$\frac{a\,b\,h\,e}{(a + h + e)\,(a + b + 2\,e)} = S\,b_1\,P_1^2/L_1^2 \qquad (222)$$

We have one less unknown quantity and a simpler reduction with less equations on that account.

$$\frac{a^2\,h\,e}{(a + e)\,(2\,e + 2\,h + a)} = A\,S\,b_1\,P_1^2/L_1^2 \qquad (322)$$

$$e = \sqrt[8]{\frac{S\,P_1^2\left(\frac{a}{e} + 1\right)}{\left(\frac{a}{e}\right)^3 \frac{h}{e}\left(1 + \frac{1}{r}\right)^2 k_i^2\,b_1\,\beta^4\,F_i^2\left(2 + 2\,\frac{h}{e} + \frac{a}{e}\right)}} \qquad (323)$$



Cruciform Core, Core Type — Double-Notched Core, Circular Coil—Core Type — Double-Notched Core, Circular Coil—Shell Type

when S has the same value as before, whence,

$$e = \sqrt[8]{\frac{S\,P_1^2\,(a/e + b/e + 2)}{4\left(1 + \frac{1}{r}\right)^2 k_i^2\,b_1\,\beta^4\,F_i^2\,(a/e)^3\,(b/e)^3\,(h/e)\,(a/e + h/e + 1)}} \qquad (223)$$

3. Cruciform Type Transformer

$$V_i = a^2\,(2\,e + 2\,h + a) = A/2 \qquad (312)$$

$$V_c = 4\,e\,h\,(a + e) = 2\,B \qquad (313)$$

$$h/e = \frac{8\,e + 5\,a}{4\,e + 2\,a} \qquad (317)$$

$$a/e = b/e \text{ and } a = b \qquad (318)$$

4. Ring Coil Type Transformer

$$V_i = 32.8\,p^2\,(h + e + 3\,p) = 32.8\,A \qquad (412)$$

$$V_c = h\,e\,(15\,p + \pi\,e) = B \qquad (413)$$

Here again there is a slight difference from actual volume for the sake of simplicity.

$$h/e = 2 + \frac{6.63\,p}{7.50\,p + \pi\,e} \qquad (417)$$

p will be taken as the equivalent of $a = b$ but it should be noted that this is a new definition for a and b and not the one used in the other types.

$$\frac{p^2\,h\,e}{(15\,p + \pi\,e)\,(h + e + 3\,p)} = S\,b_1\,P_1^2/L_1^2 \qquad (422)$$

$$e = \sqrt[8]{\frac{S P_1^2 \left(15 \frac{p}{e} + \pi\right)}{\left(1+\frac{1}{r}\right)^2 k_i^2 b_1 \beta^4 F_i^2 32.8^2 \left(\frac{p}{e}\right)^6 \frac{h}{e}\left(\frac{h}{e}+1+3\frac{p}{e}\right)}} \quad (423)$$

5. Cruciform Core, Core Type Transformer

$$V_i = 1.236\, a^2 (h + e + 1.727\, a)^* = 1.236\, A \quad (512)$$

$$V_c = \left(\frac{\pi}{2}\right) h\, e\, (2\, a + e) = \frac{\pi}{2} B \quad (513)$$

$$\frac{h}{e} = \frac{2\, e + 3.5905\, a}{e + a} \quad (517)$$

$$\frac{a^2 h\, e}{(2\, a + e)\,(h + e + 1.727\, a)} = S\, b_1 P_1^2 / L_1^2 \quad (522)$$

$$e = \sqrt[8]{\frac{S P_1^2 \left(2 \frac{a}{e} + 1\right)}{\left(1+\frac{1}{r}\right)^2 k_i^2 b_1 \beta^4 F_i^2 1.527 \left(\frac{a}{e}\right)^6 \frac{h}{e} \left(\frac{h}{e} + 1 + 1.727 \frac{a}{e}\right)}} \quad (523)$$

6. Double Notched Circular Coil, Core Type Transformer

$$V_i = 1.3356\, a^2 (h + e + 1.738\, a)^* = 1.3356\, A \quad (612)$$

$$V_c = (\pi/2)\, h\, e\, (2\, a + e) = \frac{\pi}{2} B \quad (613)$$

$$h/e = \frac{2\, e + 3.607\, a}{a + e} \quad (617)$$

$$\frac{a^2 h\, e}{(2\, a + e)\,(h + e + 1.738\, a)} = S\, b_1 P_1^2 / L_1^2 \quad (622)$$

$$e = \sqrt[8]{\frac{S P_1^2 \left(2 \frac{a}{e} + 1\right)}{\left(1 + \frac{1}{r}\right)^2 k_i^2 b_1 \beta^4 F_i^2 1.785 \left(\frac{a}{e}\right)^6 \frac{h}{e} \left(\frac{h}{e} + 1 + 1.738 \frac{a}{e}\right)}} \quad (623)$$

7. Double Notched Circular Coil, Shell Type Transformer

$$V_i = 1.3356\, a^2 (h + e + .869\, a)^* = 1.3356\, A \quad (712)$$

$$V_c = \pi\, h\, e\, (a + e) = \pi B \quad (713)$$

$$h/e = \frac{4\, e + 3.607\, a}{2\, e + a} \quad (717)$$

*For dimensions of notched core giving maximum iron section inscribed in a circle, see Bohle and Robertson—"A Treatise on Transformers," p. 169.

$$\frac{a^2 h\, e}{(a + e)\,(h + e + .869\, a)} = S\, b_1 P_1^2 / L_1^2 \quad (722)$$

$$e = \sqrt[8]{\frac{S P_1^2 \left(\frac{a}{e} + 1\right)}{\left(1 + \frac{1}{r}\right)^2 k_i^2 b_1 \beta^4 F_i^2 1.785 \left(\frac{a}{e}\right)^6 \frac{h}{e} \left(\frac{h}{e} + 1 + .869 \frac{a}{e}\right)}} \quad (723)$$

It can be seen that in all the cases the equation for e can be put into the form

$$e = \sqrt[8]{\frac{S P_1^2 \frac{U}{e}}{K \left(1+\frac{1}{r}\right)^2 k_i^2\, b_1 \beta^4\, F_i^2 \left(\frac{a}{e}\right)^3 \left(\frac{b}{e}\right)^3 \left(\frac{h}{e}\right) \left(\frac{Z}{e}\right)}}$$

Substituting the values for S, which is, in every case, a product of constants, space factors, and frequency, this can be broken up into four factors as follows.

$$e = \sqrt[4]{P_1} \cdot \sqrt[8]{\frac{1}{F_c f^2}} \cdot \sqrt[8]{\frac{10^{10}}{\left(1 + \frac{1}{r}\right)^2 b_1\, \beta^4}}$$

$$\cdot \sqrt[8]{\frac{K' \frac{U}{e}}{\left(\frac{a}{e}\right)^3 \left(\frac{b}{e}\right)^3 \cdot \frac{h}{e} \cdot \frac{Z}{e}}} = A_1 \cdot B_1 \cdot C_1 \cdot D_1$$

Where K and K' are numerical coefficients differing with the type. These four quantities can be plotted in curves of which the first three are the same for all types of transformer core, the fourth is dependent on the dimensions and therefore there must be a separate curve for each type.

A_1 is evidently a function of P only and can best be obtained from the slide rule direct

B_1 requires a separate curve for each f. Each curve is a function only of F_c

C_1 for a given frequency is a function of β only

D_1 is a function of a/e or p/e only for any one type.

Procedure in the Design of a Minimum Cost Transformer

When the curves have been drawn as indicated in the previous discussion, the quality of iron and the values of space factors and unit costs being known, the procedure in the design of a minimum cost transformer will be as follows:

Given—Type, kilovolt-amperes, voltages, and frequency, and unit costs.

To find—Dimensions, Losses, Costs.

Assume β at values near those known to give allowable heating.

Calculate—L_i, L_c, C, and S for each value of β and draw curves using β as abscissas.

To find these quantities the dimensions and then the volumes must be obtained and these in turn depend on several quantities, as indicated in the previous discussion.

Assume—$F_i = 0.9$ (or other such value as the factory test may indicate).

F_w, C_c/C_i and r are to be taken from curves or tables.

F_c the space factor for copper, as is evident from the equations, takes care of all space in the window not occupied by copper. This includes space for ventilating ducts.

C_c the cost per pound of copper in place and including insulation, and also

C_i the cost per pound of iron core assembled, must be given by the cost records of the concern making the apparatus.

r can be determined only from the loss curves of the particular quality and thickness of iron to be used. See pages 823 and 824.

R the cost ratio of a cubic inch (gross) of copper to iron

$$= \frac{C_c F_c k_c}{C_i F_i k_i}$$

Having determined these quantities a/e or p/e may be taken from the curves and with this value b/e and h/e are obtained.

$$e = \sqrt[4]{S\ P_1^2 \times f_1\ (\beta) \times f_2\ (a/e)} = A_1 \,.\, B_1 \,.\, C_1 \,.\, D_1$$

See page 7 and preceding formulas.

P_1 = kv-a. + $L_1/2000$ (see page 2) but L is to be determined from the volumes, and will actually vary with each value of β. However, $L_1/2000$ is always very small compared to P_1, usually not exceeding 2 per cent, and it may therefore be assumed with sufficient accuracy for all practical purposes. In fact it may generally be neglected.

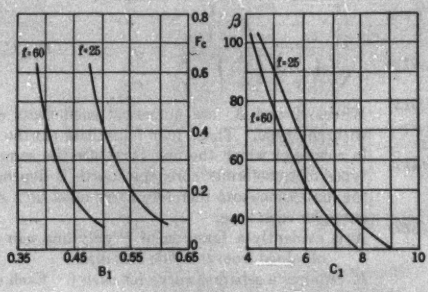
V_i and V_c see formulas given under the various types $L_i = 0.271\ b_1\ \beta^2\ F_i\ V_i$ (see curves of iron loss and explanations of b_1, p. 833).

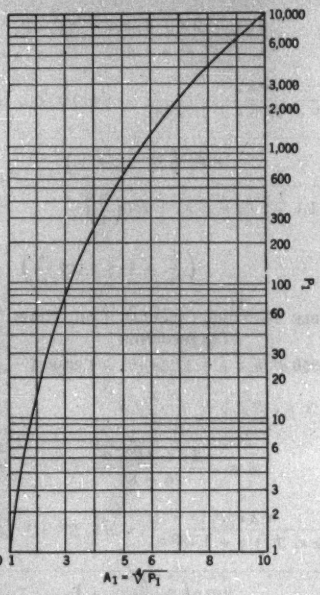
$L_c = L_i/r$ for the minimum cost transformer. In other calculations the following formula must be used.

$$L_c = s^2\, c\, F_c\, V_c$$

$$s = \sqrt{L_c/0.82\, F_c\, V_c}$$

From the various dimensions and losses obtained as above it is easy to plot curves for "watts per square inch of surface" of both iron and copper losses. From these, according to the constants determined for any form and make, the proper density may be picked out to give the working temperature desired.





P_1 = Mean Kv-a. of Pri. and Sec. Windings
A_1 = A Factor Used in Determining Dimension e
B_1 and C_1 are Factors Used in Determining the Dimension e

Distribution Transformer

It will be found that the minimum cost transformer works out so that the iron loss and copper loss are nearly equal and therefore the maximum efficiency comes at full load. This may be quite satisfactory for a "power" transformer but is not the proportion desired for a "distribution" transformer which is to be on the line continuously but loaded only a small part of the time. Here it is of first importance to have the core losses as small as possible and on the other hand a considerable copper loss is not so serious a disadvantage.

Evidently the core loss may be decreased by picking out a low density. This will increase cost and decrease the temperature rise. This is simply underrating the transformer to gain efficiency.

If it is permissible to increase copper loss, material and costs may be saved by increasing s and decreasing β, to a point where copper temperature limits further increase of s. But better, the amount of iron may be decreased and the *amount, i. e.*, the number of turns, of copper increased in proportion. In this way L_1 may be practically constant, and the heating and temperature also, if β and s are kept as before. This method proves a means of decreasing core loss very materially with but small increase in cost and with practically constant total losses provided the changes are made as indicated below. And such a transformer is quite comparable with the minimum cost transformer, as output, losses, and temperature rise are alike.

Since the product φN must be constant and β and s are to be kept constant the product $a\,b\,h\,e$ must be constant. Call this value M, as before $L_i/L_e = k_6 A/k_7 B$ and $C = k_4 A + k_5 B$. Taking the differentials of P and C, as heretofore, putting the differentials of L_i/L_e in place of L' in matrix (7) (see Appendix p. 13) by similar methods of elimination the first four rows of matrix (8) may be obtained just as in the original solution for the minimum cost transformer. From these rows the relation of the ratio a/e to b/e and h/e were obtained. Therefore, these same curves still held with the condition of fixed β and s, which of course might have been anticipated.

As an example of the procedure and of the values obtained, let us consider a 25-kv-a., shell type, 2300-volt, 60-cycle distribution transformer.

If our curves are drawn from the proper manufacturing conditions as to cost, iron, and space factors, we may take at once from the curves the following values:

$F_i = 0.9 \quad F_e = 0.41 \quad C_e/C_i = 1.7$ whence $R = 0.918$ for $\beta = 75$

$\tau = 1.02$ and $b_1 B^2 = 1.11 \quad A_1 = 2.25 \quad B_1 = 0.402 \quad C_1 = 5.03 \quad a/e = 1.23$

$b/e = 3 \quad h/e = 2.38 \quad D_1 = 0.665 \quad V_i/e^3 = 34.3 \quad V_e/e^3 = 29.5$

From these values we immediately obtain:

$e = A_1 B_1 C_1 D_1 = 3.02 \quad V_i = 944 \quad V_e = 812$ whence $L_i = 255$ and $L_e = L_i/\tau = 250$.

Therefore we may take approximately,

$a = 3\frac{3}{4}$ in. $b = 9$ in. $h = 7\frac{1}{4}$ in. and $e = 3$ in. and $a\,b\,h\,e = 730$

This is all that is required to determine a minimum cost transformer if we know that at 75,000 lines the heat may be properly dissipated.

If now we wish to reduce the iron loss to one-half of the copper loss, the values again taken from the curves are $a/e = 0.76$ $b/e = 1.93$ and $h/e = 2.273$. Since we are keeping the densities the same

$a/e \times b/e \times h/e \times e^4 = 730$ whence $e = 3.84$ in. say $3\frac{7}{8}$ in.

Getting the other values (except L_e) as before, we would find the iron loss 190 watts. The total losses would be increased about 10 per cent. Other types than the shell type, with one exception, would show less change of total loss in this method of keeping densities constant.

If iron costs 10 cents per lb., then the cost of the active material in the minimum cost transformer would be \$41.25 and for the distribution transformer, with 25 per cent less core loss \$44.40, an increase of $7\frac{1}{2}$ per cent.

To determine then the best proportions for any transformer becomes a very simple matter and the increased cost for any change from these dimensions is readily found.

It works out that for a given density the least expensive transformer is that having the iron divided into the most parts in the yoke, the core type being the most expensive. It is possible, however, to use a different density in the different forms as the allowable heating may be different.

On the same basis the cruciform type has also the lowest core losses and total losses but the shell type the greatest. The differences in general are small.

Since in simple core and shell type transformers with rectangular cores it always works out to be more economical to have sections where b is greater than a, it seemed reasonable to think that an elliptical coil with notched core fitted in would be cheaper than a round coil. Core and shell types of this shape were worked out and show a decreased cost as expected. It is not large, in the neighborhood of 3 per cent, and would probably never pay to give up the advantages of the round coil.

In this connection, however, it may be worth while pointing out an advantage which, so far as the writer knows, has been utilized by only one concern. It was shown years ago that the maximum iron section would be obtained in a circular coil where steps of the notches both ways were equal, that is, with a cross section symmetrical about two axes at right angles. Therefore, it might naturally be assumed that this is the most economical shape to make the core. This is not true, however, for the same reason that an elliptical coil and core is more economical than circular, the mean length of the iron circuit is decreased. By slightly increasing the diameter of the circular coil, the same amount of iron may be put in the core section in a shape giving a material decrease in dimensions a with a corresponding increase in other dimensions and therefore a material decrease in the iron volume. In fact, by careful adjustments practically the same results may be obtained as with an elliptical coil. In addition, this gives better cooling, for the surface of iron core perpendicular to the lamination is materially increased. The relative amounts of iron and copper for the minimum cost circular coil and elliptical coil transformer, however, is not the same, and a type using a circular coil and flattened core is cheaper if proportions are used nearer those of the elliptical coil type.

The complete mathematical solutions of a number of forms are given in the appendix.

Appendix

Mathematical Solution of the Equations for Finding a Minimum Cost Transformer

On p. 822 the following six equations in six unknowns were given:

$$P_1 = k_3 \beta s a b h e$$

$$L_1 = L_i + L_e = b_1 \beta^2 k_i F_i V_i + c s^3 F_c V_c$$

$$\frac{\partial C}{\partial a} = 0 \qquad \frac{\partial C}{\partial b} = 0 \qquad \frac{\partial C}{\partial h} = 0 \qquad \frac{\partial C}{\partial e} = 0$$

The partial derivatives of C are taken in order to solve for minimum values with the other two equations giving restraining conditions. The two equations are necessary since fixing the size (P_1) introduces two new variables (β, s). Indeterminate multipliers will be used. To do this the total derivative of P_1 is multiplied by λ_1 and that of L_1 by λ_2. For convenience let

$$P_a' = \frac{\partial \beta}{\partial a} \qquad L_a' = \frac{\partial L}{\partial a} \qquad C_a' = \frac{\partial C}{\partial a}, \text{ etc.}$$

$$\lambda_1 (P_a' d a + P_b' d b + P_h' d h + P_e' d e + P_\beta' d \beta + P_s' d s) = \lambda_1 d P_1 = 0 \quad (1)$$

$$\lambda_2 (L_a' d a + L_b' d b + L_h' d h + L_e' d e + L_\beta' d \beta + L_s' d s) = \lambda_2 d L_1 = 0 \quad (2)$$

$$C_a' d a + C_b' d b + C_h' d h + C_e' d e = 0 \quad (3)$$

Adding these three equations there results:

$$(\lambda_1 P_a' + \lambda_2 L_a' + C_a') d a + (\lambda_1 P_b' + \lambda_2 L_b' + C_b') d b + (\lambda_1 P_h' + \lambda_2 L_h' + C_h) d h + (\lambda_1 P_e' + \lambda_2 L_e' + C_e') d e + (\lambda_1 P_\beta' + \lambda_2 L_\beta') d \beta + (\lambda_1 P_s' + \lambda_2 L_s') d s = 0 \quad (4)$$

Since there are six variables and two fixed conditions, four of the variables are independent; a, b, h, and e may be selected. This leaves β and s as dependent variables. λ_1 and λ_2 may be so chosen that the coefficients of $d\beta$ and ds vanish identically.

That is,

$$\lambda_1 P_\beta' + \lambda_2 L_\beta' = 0 \text{ and } \lambda_1 P_s' + \lambda_2 L_s' = 0$$

Since b_1 varies with β the iron loss in watts per lb. must be expressed in the form of the Steinmetz equation, or

$$L_i = (a_0 \beta^{1.6} + b_0 \beta^2) k_i F_i V_i \text{ before differentiating.}$$

Then

$$\lambda_1 k_3 s a b h e + \lambda_2 (1.6 a_0 \beta^{.6} + 2 b_0 \beta) k_i F_i V_i = 0 \quad (5)$$

$$\lambda_1 k_3 \beta a b h e + 2 \lambda_2 c s F_c V_c = 0$$

Multiplying the first equation by β and the second by s, and subtracting the second from the first,

$$\lambda_2 (1.6 a_0 \beta^{1.6} + 2 b_0 \beta^2) k_i F_i V_i - 2 \lambda_2 c s^2 F_c V_c = 0 \quad (6)$$

$$(0.8 a_0 \beta^{1.6} + b_0 \beta^2) k_i F_i V_i = c s^2 F_c V_c = L_e$$

$$(a_0 \beta^{1.6} + b_0 \beta^2) k_i F_i V_i (0.8 a_0 \beta^{1.6} + b_0 \beta^2)/(a_0 \beta^{1.6} + b_0 \beta^2) = L_e$$

$$L_i = r L_e$$

where r is the reciprocal of the last two terms on the left hand side of the above equation and equals the ratio of the iron loss to the copperl oss. (See p. 823).

From Equations (1), (2), and (3) the following matrix is set up for determining the relations among the variables which will satisfy the equations.

$$\begin{vmatrix} P_a' & L_a' & C_a' \\ P_b' & L_b' & C_b' \\ P_h' & L_h' & C_h' \\ P_e' & L_e' & C_e' \\ P_\beta' & L_\beta' & 0 \\ P_s' & L_s' & 0 \end{vmatrix} = \begin{vmatrix} k_3 \beta s b h e & k_3 A_a' + k_7 B_a' & k_3 A_a' + k_3 B_a' \\ k_3 \beta s a h e & k_3 A_b' + k_7 B_b' & k_3 A_b' + k_3 B_b' \\ k_3 \beta s a b e & k_3 A_h' + k_7 B_h' & k_3 A_h' + k_3 B_h' \\ k_3 \beta s a b h & k_3 A_e' + k_7 B_e' & k_3 A_e' + k_3 B_e' \\ k_3 \beta s a b h e & (k_7)_\beta' A & 0 \\ k_3 \beta a b h e & (k_7)_s' B & 0 \end{vmatrix} \quad (7)$$

From this matrix any three rows may be selected to form third order determinants, each of which gives a relation among the variables. Not all of these are independent relations and the useful ones must be found by trial. Rules for the reduction of determinants can be applied to this matrix in order to simplify the calculations. Divide the first column by P_1. Multiply the second column by k_5 and subtract from it k_7 times the third column.

Multiply the third column by k_6 and subtract from it k_4 times the second column. There results:

$$\begin{vmatrix} 1/a & (k_6 k_5 - k_7 k_4) A_a' & (k_6 k_5 - k_5 k_4) B_a' \\ 1/b & (k_6 k_5 - k_7 k_4) A_b' & (k_6 k_5 - k_5 k_4) B_b' \\ 1/h & (k_6 k_5 - k_7 k_4) A_h' & (k_6 k_5 - k_5 k_4) B_h' \\ 1/e & (k_6 k_5 - k_7 k_4) A_e' & (k_6 k_5 - k_5 k_4) B_e' \\ 1/\beta & k_5 (k_7)_\beta' A & - k_4 (k_7)_\beta' B \\ 1/s & k_5 (k_7)_s' B & - k_4 (k_7)_s' B \end{vmatrix} = \begin{vmatrix} 1 & a A_a' & a B_a' \\ 1 & b A_b' & b B_b' \\ 1 & h A_h' & h B_h' \\ 1 & e A_e' & e B_e' \\ 1 & \frac{k_5 2 L_i}{\Delta r} & -\frac{k_4}{\Delta} \frac{2 L_i}{r} \\ 1 & \frac{k_5}{\Delta} 2 L_c & -\frac{k_4}{\Delta} 2 L_c \end{vmatrix} \quad (8)$$

The transformation to the matrix on the right is made by multiplying the rows by a, b, etc., and dividing the second and third columns by $(k_6 k_5 - k_7 k_4)$. Let this latter expression be equal to Δ and from (5) and (6) it can be shown that $\beta\ (k_6)_\beta' A = 2\ L_i/r$ and $s\ (k_7)_s' B = 2\ L_c$. For the ring coil transformer p is used instead of a and b, and for some of the other types, a alone is used.

Core Type Transformer

From page 14 of the symbols: $A = a\,b\,(2\,a + h + e)$

$B = h\,e\,(a + b + e)$

$A_a' = b(4a+h+e)$ $\quad A_b' = a(2a+h+e)$ $\quad A_h' = A_e' = a\,b$

$B_a' = B_b' = h\,e$ $\quad B_h' = e(a+b+e)$ $\quad B_e' = h(a+b+2\,e)$

Form a third order determinant using the first three rows of the last matrix given above.

$$\begin{vmatrix} 1 & a\,b\,(4a+h+e) & a\,h\,e \\ 1 & a\,b\,(2a+h+e) & b\,h\,e \\ 1 & a\,b\,h & h\,e\,(a+b+e) \end{vmatrix} = \begin{vmatrix} 0 & 2\,a & a-b \\ 0 & a+e & -(a+e) \\ 1 & h & a+b+e \end{vmatrix} = 0$$

The transformation to the second form is made by dividing the second and third columns by $a\,b$ and $h\,e$ respectively and then subtracting the second row from the first and the third from the second. Expanding the second order determinant in the upper right hand corner, it is found that:

$$2\,a(a+e) = -\,(a-b)\,(2\,a+e)$$

From which $\qquad b/a = (4\,a + 3\,e)/(2\,a + e)$

From the first, second, and fourth rows:

$$\begin{vmatrix} 1 & ab(4a+h+e) & ahe \\ 1 & ab(2a+h+e) & hhe \\ 1 & ahe & he(a+b+2e) \end{vmatrix} = \begin{vmatrix} 0 & 2a & a-b \\ 0 & 2a+h & -(a+2e) \\ 1 & e & e+b+2e \end{vmatrix} = 0$$

$$-2a(a+2e) = (a-b)(2a+h)$$

This result, using the relation just found between b and a, reduces to:

$$h/e = (3a+2e)/(a+e)$$

From the first, second, and fifth rows:

$$\begin{vmatrix} 1 & ab(4a+h+e) & ahe \\ 1 & ab(2a+h+e) & hhe \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & \frac{k_5}{\Delta}\frac{2L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & 2a^2b & he(a-b) \\ 0 & abZ-\frac{k_5}{\Delta}\frac{2L_i}{r} & hhe+\frac{k_5}{\Delta}\frac{2L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_5}{\Delta}\frac{2L_i}{r} \end{vmatrix} = 0$$

$$2a^2b\left(bhe+\frac{k_4}{\Delta}\frac{2L_i}{r}\right)$$

$$= he(a-b)\left[ab(2a+h+e)-\frac{k_5}{\Delta}\frac{2L_i}{r}\right]$$

The solution is somewhat lengthy, although perfectly simple algebraically. The result is:

$$\frac{k_5}{k_4} = R = \frac{a^2(4a+3e)(2a^2+6ae+3e^2)(6a^2+3ae-e^2)}{e^2(3a+2e)(6a^2+6ae+e^2)[2(2a^2+6ae+3e^2)-3r(2a^2+3ae+e^2)]}$$

By dividing numerator and denominator by e^7, R is made a function of a/e and r. By substituting values of these variables into the equation, values of R can be obtained and curves drawn to show the relation of a/e to R for various values of r. (See Curve Sheet.)

Shell Type Transformers

$A = ab(a+h+e)$ $A_a' = b(2a+h+e)$ $A_h' = a(a+h+e)$ $A_e' = A_a' = ab$
$B = he(a+b+2e)$ $B_a' = B_h' = he$ $B_e' = e(a+b+2e)$ $B_b' = he(a+b+4e)$

Following the same procedure as was used for the core type, there results from the first three rows of (8), p. 830:

$$\begin{vmatrix} 1 & ab(2a+h+e) & ahe \\ 1 & ab(a+h+e) & hhe \\ 1 & ahe & he(a+b+2e) \end{vmatrix} = \begin{vmatrix} 0 & a & a-b \\ 0 & a+e & -(a+2e) \\ 1 & h & a+b+2e \end{vmatrix} = 0$$

$$-a(a+2e) = (a-b)(a+e) \text{ or } b/a = (2a+3e)/(a+e)$$

From the first, second, and fourth rows of (8):

$$\begin{vmatrix} 1 & ab(2a+h+e) & ahe \\ 1 & ab(a+h+e) & hhe \\ 1 & ahe & he(a+b+4e) \end{vmatrix} = \begin{vmatrix} 0 & a & a-b \\ 0 & a+h & -(a+4e) \\ 1 & e & a+b+4e \end{vmatrix} = 0$$

$$-a(a+4e) = (a-b)(a+h) \text{ or } h/e = (3a+4e)/(a+2e)$$

From the first, second, and fifth rows of (8):

$$\begin{vmatrix} 1 & ab(2a+h+e) & ahe \\ 1 & ab(a+h+e) & hhe \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_5}{\Delta}\frac{2L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2b & (a-b)he \\ 0 & abZ-\frac{k_5}{\Delta}\frac{2L_i}{r} & hhe+\frac{k_5}{\Delta}\frac{2L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_5}{\Delta}\frac{2L_i}{r} \end{vmatrix} = 0$$

$$a^2b\left(bhe+\frac{k_4}{\Delta}\frac{2L_i}{r}\right)$$

$$= he(a-b)\left[ab(a+h+e)-\frac{k_5}{\Delta}\frac{2L_i}{r}\right]$$

$$\frac{k_5}{k_4} = R = \frac{a^2(2a+3e)(a^2+6ae+6e^2)(3a^2+3ae-2e^2)}{e^2(3a+4e)(3a^2+6ae+2e^2)[2(a^2+6ae+6e^2)-3r(a^2+3ae+2e^2)]}$$

Cruciform Type Transformers

$A = 2ab\left(2h+2e+\frac{a}{2}+\frac{b}{2}\right)$ $A_a' = 2b\left(2h+2e+a+\frac{b}{2}\right)$ $A_h' = 2a\left(2b+2e+\frac{a}{2}+b\right)$ $A_e' = A_b' = 4ab$
$B = he(a+b+2e)$ $B_a' = B_h' = he$ $B_e' = e(a+b+2e)$ $B_b' = h(a+b+4e)$

From the first three rows of (8):

$$\begin{vmatrix} 1 & 2ab\left(2h+2e+a+\frac{b}{2}\right) & ahe \\ 1 & 2ab\left(2h+2e+\frac{a}{2}+b\right) & hhe \\ 1 & 4abh & he(a+b+2e) \end{vmatrix} = \begin{vmatrix} 0 & \frac{a}{2}-\frac{b}{2} & a-b \\ 0 & 2e+\frac{a}{2}+b & -(a+2e) \\ 1 & 2h & a+b+2e \end{vmatrix} = 0$$

$$\left(\frac{a}{2}-\frac{b}{2}\right)(a+2e)-(a-b)\left(2e+\frac{a}{2}+b\right) = 0$$

$$(a-b)(2a+2b+6e) = 0$$

Since $(2a+2b+6e)$ is a positive real number, $(a-b)$ must equal zero and $a = b$. Using this relation $A = 2a^2(2h+2e+a)$, $B = 2he(a+e)$, $P_a' = 2k_3\beta\varepsilon ahe$,

$A_a' = 4a\left(2h+2e+3\frac{a}{2}\right)$ and $B_a' = 2he$

Then from the first, third, and fourth rows of (8):

$$\begin{vmatrix} 2 & 4a^2(2h+2e+3a/2) & 2ahe \\ 1 & 4a^2h & 2he(a+e) \\ 1 & 4a^2e & 2he(a+2e) \end{vmatrix} = \begin{vmatrix} 0 & 2e+3a/2 & -(a+2e) \\ 0 & (h-e) & -e \\ 1 & e & a+2e \end{vmatrix} = 0$$

$$-e(2e+3a/2)+(h-e)(a+2e) = 0$$

$$h/e = (5a+8e)/(2a+4e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & 4a^2h & 2he(a+e) \\ 1 & 4a^2e & 2he(a+2e) \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_5}{\Delta}\frac{2L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & 4a^2(h-e) & -2he \\ 0 & 4a^2e-\frac{k_5}{\Delta}\frac{2L_i}{r} & 2he(a+2e)+\frac{k_5}{\Delta}\frac{2L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_5}{\Delta}\frac{2L_i}{r} \end{vmatrix} = 0$$

$$4a^2(h-e)\left[2he(a+2e)+\frac{k_4}{\Delta}\frac{2L_i}{r}\right]$$

$$+2he^2\left(4a^2e-\frac{k_5}{\Delta}\frac{2L_i}{r}\right) = 0$$

$$\frac{k_5}{k_4} = 4R = \frac{2a^2(a^2+9ae+12e^2)(3a^2+2ae-4e^2)}{e^2(5a+8e)(a+e)[2(a^2+9ae+12e^2)-3r(a+2e)^2]}$$

Ring Coil Type Transformers

$$A = p^2(h+e+3p) \quad A_i' = p(2h+2e+9p) \quad A_o' = A_i'' = p^2$$
$$B = h\,e\,(15\,p+\pi\,e) \quad B_o' = 15\,h\,e \quad B_i' = e(15\,p+\pi\,e) \quad B_i'' = h(15\,p+2\,\pi\,e) \quad P_o' = 2\,k\,\beta\,s\,p\,h\,e$$

From the first, third, and fourth rows of (8):

$$\begin{vmatrix} 2 & p^2(2h+2e+9p) & 15\,p\,h\,e \\ 1 & p^2 h & h\,e\,(15\,p+\pi\,e) \\ 1 & p^2 e & h\,e\,(15\,p+2\,\pi\,e) \end{vmatrix} = \begin{vmatrix} 0 & 2e+9p & -(15\,p+2\,\pi\,e) \\ 0 & h-e & -\pi\,e \\ 1 & e & 15\,p+2\,\pi\,e \end{vmatrix} = 0$$

$$\pi\,e\,(2\,e+9\,p) = (h-e)\,(15\,p+2\,\pi\,e)$$
$$h/e = 1 + \pi\,(2\,e+9\,p)/(15\,p+2\,\pi\,e)$$
$$= (43.3\,p + 12.57\,e)/(15\,p + 6.28\,e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & p^2 h & h\,e\,(15\,p+\pi\,e) \\ 1 & p^2 e & h\,e\,(15\,p+2\,\pi\,e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & p^2(h-e) & h\,e\,(-\pi\,e) \\ 0 & p^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & h\,e\,(15\,p+2\,\pi\,e) + \frac{k_5}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$p^2\,(h-e)\,[h\,e\,(15\,p+2\,\pi\,e) + 2\,k_4\,L_i/\Delta\,r]$$
$$+ \pi\,h\,e^3\,(p^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{32.8}$$
$$= \frac{p^2\,(45\,p^2 + 77.1\,p\,e + 18.85\,e^2)\,(135\,p + 15\,p\,e - 6.28\,e^2)}{e^2(42.3\,p + 12.57\,e)(15\,p + 3.14\,e)[2(45\,p^2 + 77.1\,p\,e + 18.85\,P^2) - 3r(45\,p^2 + 33.85\,p\,e + 6.28\,e^2)]}$$

Cruciform Core, Core Type Transformer

$$A = a^2(h+e+1.727\,a) \quad A_i' = a\,(2\,h+2\,e+5.18\,a) \quad A_o' = A_i'' = a^2 \quad B = h\,e\,(2\,a+e)$$
$$B_o' = 2\,h\,e \quad B_i' = e\,(2\,a+e) \quad B_i'' = 2\,h\,(a+e) \quad P_o' = 2\,k_3\,\beta\,s\,a\,h\,e$$

From the first, third, and fourth rows of (8), p. 830

$$\begin{vmatrix} 2 & a^2(2h+2e+5.18\,a) & 2\,a\,h\,e \\ 1 & a^2 h & h\,e\,(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \end{vmatrix} = \begin{vmatrix} 0 & 2e+5.18\,a & -(2\,a+2\,e) \\ 0 & h-e & -e \\ 1 & e & 2\,(a+e) \end{vmatrix} = 0$$

$$(h-e)\,(2\,a+2\,e) = e\,(2\,e+5.18\,a)$$
$$h/e = (3.59\,a + 2\,e)/(a+e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & a^2 h & h\,e\,(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2(h-e) & -h\,e^2 \\ 0 & a^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & 2\,h\,e\,(a+e) + \frac{k_5}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$2\,a^2\,(h-e)\,[h\,e\,(a+e) + k_4\,L_i/\Delta\,r]$$
$$+ h\,e^2\,(a^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{0.787}$$
$$= \frac{a^2\,(1.73\,a^2 + 6.32\,a\,e + 3\,e^2)(5.18\,a^3 + a\,e - e^3)}{e^2(3.59\,a + 2\,e)(2\,a+e)[2(1.73\,a^2 + 6.32\,a\,e + 3\,e^2) - 3\,r\,(1.73\,a + e)\,(a+e)]}$$

Double Notched Core, Circular Coil, Core Type

$$A = a^2(h+e+1.728\,a) \quad A_i' = a(2\,h+2\,e+5.214\,a) \quad A_o' = A_i'' = a^2 \quad B = h\,e\,(2\,a+e)$$
$$B_o' = 2\,h\,e \quad B_i' = e\,(2\,a+e) \quad B_i'' = 2\,h\,(a+e) \quad P_o' = 2\,k_3\,\beta\,s\,a\,h\,e$$

From the first, third, and fourth rows of (8), p. 830

$$\begin{vmatrix} 2 & a^2(2h+2e+5.214\,a) & 2\,a\,h\,e \\ 1 & a^2 h & h\,e\,(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \end{vmatrix} = \begin{vmatrix} 0 & 2e+5.214\,a & -2\,(a+e) \\ 0 & h-e & e \\ 1 & e & 2\,(a+e) \end{vmatrix} = 0$$

$$2\,(h-e)(a+e) = e\,(5.214\,a + 2\,e) \text{ or } h/e = \frac{3.61\,a + 2\,e}{a+e}$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & a^2 h & h\,e\,(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2(h-e) & -h\,e^2 \\ 0 & a^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & 2\,h\,e\,(a+e) + \frac{k_5}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$2\,a^2\,(h-e)\,[h\,e\,(a+e) + k_4\,L_i/\Delta\,r]$$
$$+ h\,e^2\,(a^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{0.85}$$
$$= \frac{a^2\,(1.74\,a^2 + 6.35\,a\,e + 3\,e^2)\,(5.21\,a^2 + a\,e - e^2)}{e^2(3.61\,a + 2\,e)\,(2\,a+e)\,[2\,(1.74\,a^2 + 6.35\,a\,e + 3\,e^2) - 3\,r\,(1.74\,a + e)\,(a+e)]}$$

Double Notched Core, Circular Coil, Shell Type

$$A = a^2(h+e+0.869\,a) \quad A_i' = a(2\,h+2\,e+2.607\,a) \quad A_o' = A_i'' = a^2 \quad B = h\,e\,(a+e)$$
$$B_o' = h\,e \quad B_i' = e\,(a+e) \quad B_i'' = h\,(a+2\,e) \quad P_o' = 2\,k_3\,\beta\,s\,a\,h\,e$$

From the first, third, and fourth rows of (8), p. 830

$$\begin{vmatrix} 2 & a^2(2h+2e+2.607\,a) & a\,h\,e \\ 1 & a^2 h & h\,e\,(a+e) \\ 1 & a^2 e & h\,e\,(a+2\,e) \end{vmatrix} = \begin{vmatrix} 0 & 2e+2.607\,a & -(a+2\,e) \\ 0 & h-e & -e \\ 1 & e & a+2\,e \end{vmatrix} = 0$$

$$(h-e)\,(a+2\,e) = e\,(2.607\,a + 2\,e)$$
$$h/e = (3.607\,a + 4\,e)/(a+2\,e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & a^2 h & h\,e\,(a+e) \\ 1 & a^2 e & h\,e\,(a+2\,e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2(h-e) & -h\,e^2 \\ 0 & a^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & h\,e\,(a+2\,e) + \frac{k_5}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_5}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$a^2\,(h-e)\,[h\,e\,(a+2\,e) + 2\,k_4\,L_i/\Delta\,r]$$
$$+ h\,e^2\,(a^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{0.425}$$
$$= \frac{a^2\,(0.869\,a^2 + 6.35\,a\,e + 6\,e^2)(2.61\,a^2 + a\,e - 2\,e^2)}{e^2(3.61\,a + 4\,e)(a+e)[2(0.869\,a^2 + 6.35\,a\,e + 6\,e^2) - 3\,r\,(0.869\,a + e)\,(a+2\,e)]}$$

Lighting Transformers

In order to determine the dimensions of a transformer having minimum cost and a lower iron loss than the one selected on the basis of allowable temperature rise, β and s will again be selected as the dependent variables and will be held constant. The equations for the power and for the ratio of iron loss to copper loss will now be

the restraining conditions. Taking the derivatives as in the previous development (p. 830), the following equations are obtained.

$$P_a' \, d\,a + P_b' \, d\,b + P_h' \, d\,h + P_e' \, d\,e = 0$$
$$r_a' \, d\,a + r_b' \, d\,b + r_h' \, d\,h + r_e' \, d\,e = 0$$
$$C_a' \, d\,a + C_b' \, d\,b + C_h' \, d\,h + C_e' \, d\,e = 0$$

Since β and s are constants, the use of indeterminate multipliers is not necessary. A matrix can be set up as before using the coefficient of the differentials in the above equations.

$$\begin{vmatrix} P_a' & r_a' & C_a' \\ P_b' & r_b' & C_b' \\ P_h' & r_h' & C_h' \\ P_e' & r_e' & C_e' \end{vmatrix} = \begin{vmatrix} k_3\beta s b h e & k_3(A_a' B - A B_a')/k_7 B^2 & k_4 A_a' + k_5 B_a' \\ k_3\beta s a h e & k_3(A_b' B - A B_b')/k_7 B^2 & k_4 A_b' + k_5 B_b' \\ k_3\beta s a b e & k_3(A_h' B - A B_h')/k_7 B^2 & k_4 A_h' + k_5 B_h' \\ k_3\beta s a b h & k_3(A_e' B - A B_e')/k_7 B^2 & k_4 A_e' + k_5 B_e' \end{vmatrix}$$

Since $r = \dfrac{k_6 A}{k_7 B}$ $\dfrac{\partial r}{\partial a} = (k_6 A_a' k_7 B - k_6 A k_7 B_a')/k_7^2 B^2$

$$= k_6(A_a' B - A B_a')/k_7 B^2, \text{ etc.}$$

Divide the first column by $P = k_3 \beta s a b h e$ and multiply the second column by $k_7 B^2/k_6$.

$$\begin{vmatrix} 1/a & A_a' B - A B_a' & k_4 A_a' + k_5 B_a' \\ 1/b & A_b' B - A B_b' & k_4 A_b' + k_5 B_b' \\ 1/h & A_h' B - A B_h' & k_4 A_h' + k_5 B_h' \\ 1/e & A_e' B - A B_e' & k_4 A_e' + k_5 B_e' \end{vmatrix} = \begin{vmatrix} 1/a & (k_5 B + k_4 A) A_a' & (k_5 B + k_4 A) B_a' \\ 1/b & (k_5 B + k_4 A) A_b' & (k_5 B + k_4 A) B_b' \\ 1/h & (k_5 B + k_4 A) A_h' & (k_5 B + k_4 A) B_h' \\ 1/e & (k_5 B + k_4 A) A_e' & (k_5 B + k_4 A) B_e' \end{vmatrix}$$

The matrix on the right is derived from the one on the left by adding A times the third column to k_5 times the second and by subtracting k_4 times the second column from B times the third. Divide the second and third columns by $k_5 B + k_4 A$ and multiply the rows by a, b, h, and e respectively.

$$\begin{vmatrix} 1 & a A_a' & a B_a' \\ 1 & b A_b' & b B_b' \\ 1 & h A_h' & h B_h' \\ 1 & e A_e' & e B_e' \end{vmatrix}$$

This matrix is in the same form as the first four rows of the right-hand member of matrix (8) p. 830 and will yield the same values of b/a and h/e, as the ones found for the various types in the preceding pages of this Appendix. Thus the same curves between a/e and b/e, h/e can be used for new assumed values of a/e, which are to be used in reducing the iron volume and loss.

Transformer Design Notes

Symbols.

a = width of iron core in inches, (see Figs. 1, 2, 3) or the diameter of the circle circumscribing notched cores, (see Figs. 5, 6, 7)

a_0 = a constant of the Steinmetz formula for iron loss. Its product with $\beta^{1.6}$ gives watts hysteresis loss per lb. of iron

A = a letter replacing a combination of symbols. See p. 834

$A_1 = P_1^{\frac{1}{4}}$

b = depth of iron core in inches except in Ring Coil Types, see Figs. 1-7.

b_0 = a constant of the Steinmetz formula for iron loss. Its product with β^2 gives watts eddy current loss per lb. of iron.

b_1 = a coefficient by which β^2 is multiplied to obtain watts iron loss per lb. of iron: determined empirically from the iron loss curve for various values of β. (See p. 823.)

B = a letter replacing a combination of symbols. See p. 834

$B_1 = \sqrt[4]{1/F_c f^2}$ see p. 827 also see curve

c = resistance, in microhms, measured from face to face of a copper cube of one inch edge = (0.82)

C = cost of the active iron and copper in the transformer

C_c = cost in cents per lb. of copper (wound in place with insulation)

C_i = cost in cents per lb. of iron (stacked in place)

$C_1 = \sqrt[8]{10^{10} / \left(1 + \dfrac{1}{r}\right)^2 b_1 \beta^4}$ See p. 827 also see curve

D_1 = see p. 834, and curves

e = width of window (See Figs. 1-7)

E = high-tension volts

f = frequency in cycles per second

F_c = space factor of copper (see curves)

F_i = space factor of iron (taken as 0.9)

h = height of window (see Figs. 1-7)

I = high tension current in amperes at rated kv-a.

k_i = weight in lb. of one cu. in. of iron net = (0.271)

k_c = weight in lbs. of one cu. in. of copper net = (0.321)

k_3, k_4, k_5, k_6, k_7 letters replacing combinations of symbols. See p. 834

L = total loss (in watts)

L_c = watts, copper loss

L_i = watts, iron loss

L_1 = any definite numerical value of L

N = high tension turns in series

p = a dimension of Ring Coil Transformer. See Fig. 4

P = mean kv-a.

P_1 = any definite numerical value of P

Q = a letter replacing a combination of symbols. See p. 834

r = L_i/L_c

R = ratio of cost of a cu. in. of copper to one of iron (gross)

s = current density in kilo-amperes per sq. in.

S, T, U = letters replacing combination of symbols. (See p. 834)

V_c = gross volume of copper in cu. in.

V_i = gross volume of iron in cu. in.

Z = a letter replacing a combination of symbols. (See p. 834)

β = flux density in core (kilo-lines per sq. in. of net iron section)

φ = total flux in core.

Symbols and Their Equivalents

1. *Core Type.*

$A = a\,b\,(2\,a + h + e) = V_i/2$
$B = h\,e\,(a + b + e) = V_c/2$

$$D_1 = \sqrt[8]{\frac{3.38\,U/e}{\left(\frac{a}{e}\right)^3\left(\frac{b}{e}\right)^5 \cdot \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 2.22\,F_i\,F_c\,f\,10^{-5}$
$k_4 = 2\,F_i\,C_i\,k_i$
$k_5 = 2\,F_c\,C_c\,k_c$
$k_6 = 2\,k_i\,b_1\,\beta^2\,F_i$
$k_7 = 2\,c\,s^2\,F_c$
$Q = 1 + (1 - h/Z)/(1 - b/U)$
$S = 16\,c\,F_c\,F_i\,k_i/k_3^2$

$$T = \frac{e}{a}\,\frac{h/Z}{b/U} = B/A$$

$U = a + b + e$
$Z = 2\,a + h + e$

2. *Shell Type.*

$A = a\,b\,(a + h + e) = V_i/2$
$B = h\,e\,(a + b + 2\,e) = V_c/2$
D_1 = Same form as core type
k_3, k_4, k_5, k_6, k_7 = Same form as core type
Q, S, T = Same form as core type
$U = a + b + 2\,e$
$Z = a + h + e$

3. *Cruciform Type.*

$$A = 2\,a\,b\left(2\,h + 2\,e + \frac{a}{2} + \frac{b}{2}\right) = 2\,a^2(2\,h + 2\,e + a) = 2\,V_i$$

$$B = h\,e(a + b + 2\,e) = 2\,h\,e\,(a + e) = V_c/2$$

$$D_1 = \sqrt[8]{\frac{13.5\,U/e}{\left(\frac{a}{e}\right)^6 \cdot \frac{h}{e} \cdot \frac{Z}{e}}}$$

k_3, k_5, k_7 = Same as Core Type
$k_4 = 0.5\,F_i\,C_i\,k_i$
$k_6 = 0.5\,k_i\,b_1\,\beta^2\,F_i$

$$Q = 1 + \left(1 - \frac{4\,h - a}{Z}\right)/(1 - a/U)$$

S = Same as Core Type

$$T = \frac{e}{a}\,\frac{h/Z}{b/U} = \frac{e\,h\,U}{a^2\,Z} = B/A$$

$U = a + b + 2\,e = 2\,(a + e)$

$$Z = 2\left(2\,h + 2\,e + \frac{a}{2} + \frac{b}{2}\right) = 2\,(2\,h + 2\,e + a)$$

4. *Ring Coil Type.*

$A = p^3\,(h + e + 3\,p) = V_i/32.8$
$B = h\,e\,(15\,p + \pi\,e) = V_c$

$$D_1 = \sqrt[8]{\frac{0.000382\,U/e}{\left(\frac{p}{e}\right)^6 \cdot \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 36.4\,F_i\,F_c\,f\,10^{-5}$
$k_4 = 32.8\,F_i\,C_i\,k_i$
$k_5 = F_c\,C_c\,k_c$
$k_6 = 32.8\,k_i\,b_1\,\beta^2\,F_i$
$k_7 = c\,s^2\,F_c$

$$Q = 1 + \frac{\left(1 + \frac{1.50\,p - h}{Z}\right)}{(1 - 7.50\,p/U)}$$

$S = 131.1\,c\,F_c\,F_i\,k_i/k_3^2$

$$T = e/p\,\frac{h/Z}{p/U} = B/A$$

$U = 15\,p + \pi\,e$
$Z = h + e + 3\,p$

5. *Cruciform Core, Core Type.*

$A = a^2\,(h + e + 1.727\,a) = V_i/1.236$
$B = h\,e\,(2\,a + e) = 2\,V_c/\pi$

$$D_1 = \sqrt[8]{\frac{11.21\,\frac{U}{e}}{\left(\frac{a}{e}\right)^6 \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 1.372\,f\,F_i\,F_c \cdot 10^{-5}$
$k_4 = 1.236\,F_i\,C_i\,k_i$
$k_5 = 1.5708\,F_c\,C_c\,k_c$
$k_6 = 1.236\,b_1\,\beta^2\,k_i\,F_i$
$k_7 = 1.5708\,c\,s^2\,F_c$
$S = 7.768\,c\,F_c\,F_i\,k_i/k_3^2$
$U = 2\,a + e$
$Z = h + e + 1.727\,a$

6. *Double Notched Circular Coil, Core Type.*

$A = a^2\,(h + e + 1.738\,a) = V_i/1.3356$
$B = h\,e\,(2\,a + e) = 2\,V_c/\pi$

$$D_1 = \sqrt[8]{\frac{8.89\,\frac{U}{e}}{\left(\frac{a}{e}\right)^6 \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 1.482\,f\,F_i\,F_c \cdot 10^{-5}$
$k_4 = 1.3356\,F_i\,C_i\,k_i$
k_5, k_7 = Same as cruciform core, core type
$k_6 = 1.3356\,b_1\,\beta^2\,k_i\,F_i$
$S = 8.4\,c\,F_c\,F_i\,k_i/k_3^2$
$U = 2\,a + e$
$Z = h + e + 1.738\,a$

7. *Double Notched Circular Coil, Shell Type.*

$A = a^2\,(h + e + 0.869\,a) = V_i/1.3356$
$B = h\,e\,(a + e) = V_c/\pi$

$$D_1 = \sqrt[8]{\frac{17.78\,\dfrac{U}{e}}{\left(\dfrac{a}{e}\right)^4 \dfrac{h}{e} \cdot \dfrac{Z}{e}}}$$

k_3, k_4, k_5 = Same as double notched core type above

$k_6 = 3.1416\, F_c\, C_c\, k_c$

$k_7 = 3.1416\, c\, s^2\, F_c$

$S = 16.78\, c\, F_c\, F_i\, k_i / k_3^2$

$U = a + e$

$Z = h + e + 0.869\, a$

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Recent Developments in Telephone Construction Practises

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Synopsis.—*In this paper are described recent developments in telephone construction practises which react to preserve the integrity of the sheath of lead covered cable, thus decreasing insulation troubles due to moisture seeping through armor breaks and warding off serious service interruptions.*

* * * * *

Introduction

GROWTH of the telephone plant in the last few years has naturally followed the tremendous increase in the use of this method of communication. The outside plant which has been built to satisfy these increased requirements is very largely either aerial or underground cable.

The extension of the toll cable network, wherein several hundred telephone and telegraph messages are carried within a lead-antimony cable sheath of approximately 2⅝-in. diameter, and the development of exchange cables in sizes up to 1818 pairs of wires, have necessitated new construction practises intended to lessen service interruptions caused by the opening of this lead sheath.

In this paper a description of recent developments in the construction of telephone outside plant in so far as they affect maintenance and service reliability will be presented. They are the result of development work throughout the country and it is felt that they represent a decided advance in building plant which will be subject to fewer troubles and appreciably less interruptions to service.

The paper is divided into three sections: First, the development of gas pressure testing which has made it possible to determine the location of sheath openings before they result in failure. Second, recent advances made in the methods of erecting aerial cable, which it is hoped will eliminate bowing, buckling, and other movements of the cable which in time cause a fracture of the sheath. Third, the application of catenary construction to telephone cables which has resulted in the possibility of locating cable routes over rivers and rugged terrain which, a few years ago, could not have been followed except where the cables could be attached to bridges or other crossing structures.

Gas Pressure Testing

General. Sheath breaks occur in both aerial and underground cable plants and can usually be attributed to one of the following:

Aerial Cable

Ring cuts
Breaks due to buckling and bowing
Bullet holes
Splicing defects
Manufacturing defects

Underground Cable

Breaks at duct entrances
Breaks due to poor arrangement in manholes
Snagging cable while pulling
Electrolysis
Foreign companies' workmen
Splicing defects
Manufacturing defects

With the extension of the toll cable network, it became necessary to provide a definite plan of preventive maintenance by means of which the above potential troubles might be eliminated.

The plan for eliminating these defects was set up along the following lines:

1. Gas pressure testing
2. More care in racking in manholes
3. Educating employes in the handling of cable
4. Extensive electrolysis surveys
5. Frequent manhole inspections
6. Filling sleeves with paraffin (in the case of exchange cables)
7. Frequent inspection of aerial routes
8. Close observation of work being done by foreign companies' workmen along routes paralleling telephone plant
9. Pulling and splicing cable at such tensions as will insure the elimination of trouble due to bowing
10. Erecting strand at proper tensions in order to procure proper sags
11. Engineering on such a basis as to warrant cable's being free from trees, fire hazards, and other sources which might cause interruptions.

The introduction of gas under pressure into a cable is not a new departure, but the kind of gas and the apparatus for applying it have undergone considerable development and improvement in the past three years.

Briefly, the earlier methods consisted of injecting compressed air, which had been dried with calcium chloride, into the cable, and watching the previously

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29-56

installed pressure gages for a drop in the internal sheath pressure which would indicate a hole or holes in the sheath.

In using this method on telephone cable considerable difficulty was experienced in removing the moisture from the air completely enough to insure that the serviceability of the insulation would not be affected. As a result of this experience it was deemed necessary to find a gas of a lesser moisture content and this led to the use of extra dry carbon dioxide for this purpose. While from a moisture standpoint this gas was well suited for testing purposes, its physical characteristics were such as to curtail, to a certain degree, it use by the field construction forces.

A special dried nitrogen gas has come into widespread

Fig. 1—Small Cylinder Gas Pressure Equipment

Showing regulating equipment valves and awl.

use for testing telephone cables. This gas is extremely dry and has the advantage of requiring very little associated equipment to regulate its flow into the cable.

Equipment. Inasmuch as oil pumped nitrogen has almost universally supplanted carbon dioxide in cable testing work, we will, in this paper, describe only equipment and tests which use this gas.

Two types of cylinders are used for supplying gas to the field forces. The larger size is the usual 55-in. cylinder which the manufacturer uses to send the gas from his plant. These contain approximately 15 lb. of gas at a pressure of 2000 lb. per sq. in., their total weight being 150 lb. Containers of this type are usually employed in large construction projects where there are thousands of feet of cable to be tested and where it is usually convenient to haul them to the job by means of trucks.

A cylinder of this type will test approximately 6000 ft., or one loading section of full sized cable.

In order that small amounts of gas may be available for testing single splices, as is the case when a splicer has worked on an existing cable, a small 25-in. cylinder, (see Fig. 1) containing sufficient gas to test eight or nine 4½-in. sleeves, is used. These tanks are filled

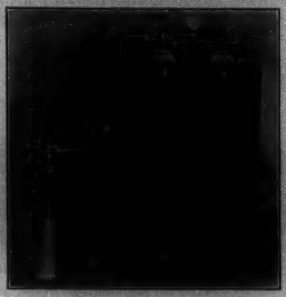

Fig. 2—Manifold Equipment

For transferring gas from large to small cylinder

from a battery of the larger 2000-lb. cylinders, until their internal pressure reaches 1600 lb. per sq. in., as shown in Fig. 2. The small tanks are located at the splicer's supply base and are ordered by him in much the same manner as he would order sleeves, solder, etc.

The regulating equipment for both cylinders is practically the same. This apparatus is also shown in Fig. 1 and consists of a regulating valve for controlling the ingress pressure, a 50-lb. gage for reading this pressure, and a 3000-lb. gage for registering the cylinder pressure. The scale of the latter gage is also calibrated in cubic feet at atmospheric pressure in order to indicate the amount of gas in the tank at all times. The

Fig. 3—Portable Pressure Testing Equipment

rubber hose provides a means of conducting the gas from the tank into the cable.

In the splice is installed a 30-lb. gage and an ingress stem, both designed so as to screw into a hole made with an awl. Each of these items is equipped with a rubber gasket so as to make an air-tight fit with the sleeve of the cable, without employing the use of solder.

A carrying case has been developed in which is packed equipment to be used in placing the cable or splice under pressure and inspecting the metal work after this has been done, (see Fig. 3). The only other equipment necessary to carry on these tests is carried by the splicer as part of his regular kit.

Single Splice Tests. In the case of single splice tests, two holes must be made with the awl, one near each end of the sleeve. These are made at this point as it is here that the possibility of injuring the wire work

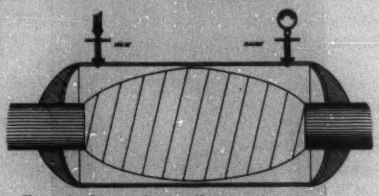

Fig. 4—Single Splice Testing Arrangements

Valve and gage locations

is a minimum. The valve stem and 30-lb. pressure gage are then screwed into these holes. The splice will then look like Fig. 4.

The regulator equipment is then attached to the small cylinder and the hose connected to the valve stem in the splice.

Before the gas is turned on, the entire sleeve and both joints are covered with a heavy coating of soapsuds. After this has been done, the splice is ready to receive the gas. The cylinder valve is then released and the regulator set until the ingress pressure reaches 40 lb., the rate at which the gas is allowed to flow into the cable.

Before the sleeve is placed on the splice, the conductors and muslin are boiled out with paraffin heated to a temperature of 375 deg. to 390 deg. fahr., in order to remove any moisture which may have been deposited on the conductors by perspiration from splicers' hands or manhole dampness. This paraffin usually runs back under the armor of the cable for a distance of six or eight inches. When cold, this paraffin forms a temporary dam which holds the gas in the splice long enough to make the test.

When the pressure within the splice reaches 5 lb. as indicated on the screw-in pressure gage, the gas is cut off at the cylinder. The sleeve and joints are then examined with the magnifying mirror with a view to finding bubble formations which indicate leaks. In the case of large leaks, it is often possible to hear the hiss caused by the escaping gas.

If no leaks are found, the gage and valve stem are removed, the gas permitted to escape, and the holes carefully soldered shut. Should a leak be detected in the joints, they are rewiped and the tests repeated. All splices which are tested by means of gas are stamped "G. T." (gas tested) in order that we can check from time to time on the effectiveness of this type of maintenance.

On new cable projects where long lengths of cable are to be tested, the usual procedure is to splice the cable in loading sections of 6000 ft. and to test these sections as a unit. After all electrical testing has been completed and before the loading is cut in, the ends of the sections are capped and gages installed at each end. At the loading point where the two ends of different sections appear, valve stems are fastened into the lead caps and a large cylinder of gas is connected to the cable by means of a regulator and two-way hose connection. The set up for this type of test would be quite similar to that shown on Fig. 5.

After the equipment is connected to the cable, the gas is turned on and its flow regulated in exactly the same manner as outlined for the single splice tests. Nitrogen is permitted to flow in at a 40-lb. rate until the meters at the far extremities of the section register 5 lb., at which time the tank is disconnected and the pressures at all meters recorded as soon as possible.

In making tests of this nature it is possible to classify the defects to be found into two groups; those at the point of splicing which includes all exposed cable in the manhole in the case of underground cable; and those which fall in the sections between the splices or which are between the manholes (in the duct line).

Troubles of the first type are detected by soaping of sleeves in the sections under test. Special attention

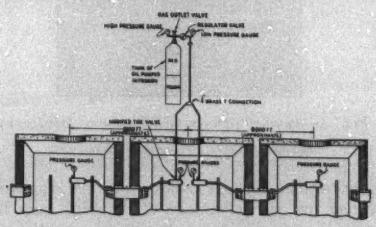

Fig. 5—Section Testing

Method of setting up equipment when pressure testing long sections of cable

is given to seams, wipes, and bends. Leaks, of course, are indicated by the formation of bubbles.

In the case of sectional defects more elaborate means of detection are employed. After the gas has been disconnected from the cable, the readings at the point of application will be considerably higher than those at the ends of the section; but within three hours the pressure should equalize throughout the sections. It is usually customary to leave the gas pressure in the cable over night when practicable. The ability of the

cable to hold the pressure is a positive indication that the sheath is in good condition. Inability to get a pressure reading at the end of the section under test is an indication of an extremely large leak, which must be repaired before the test can be continued.

If the reading at one end is appreciably different from that at the other, or if they are equal but lower than the average at the time the tank was disconnected, it is probable that one or more defects exist, and that they are in the duct line in the case of underground cable, or in the section between the splices in the case of aerial cable.

Several methods of locating this type of trouble have been used with fair degrees of success.

On aerial cable where it is possible to ride the entire section under test, the first step is usually to cover the section with the intention of hearing the hiss caused by escaping gas and to inspect the cable from the ground

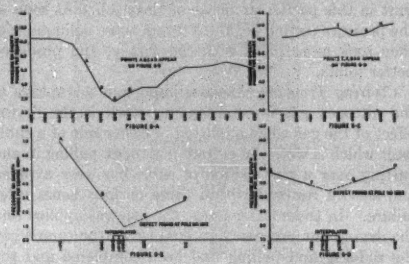

FIG. 6—GRAPHIC METHOD OF LOCATING SHEATH BREAKS

Typical pressure curve for section of aerial cable having two sheath breaks

Note: One 200-cu. ft. cylinder of nitrogen is sufficient to test a 6000 ft. section of cable

for any bends, kinks, or flaws which would be soaped to locate the trouble.

The next step would be to install a gage in each splice in the section. As full size cable is usually put up in 750-ft. sections, there would be seven splices in a 6000-ft. section in which meters would have to be installed. It is necessary that the valve test points be equally spaced along the cable. This would make nine points at which the pressure of the cable can be read.

The pressure readings taken to the tenth of a pound are then recorded on cross-section paper laying off the pressure as ordinates and the test points as abscissas.

Theoretically, were the cable to be free of defects, these points would lie in the same straight line. The presence of defects would be indicated by low points on the curve.

The curves shown in Fig. 6 are an example of the method of locating trouble in a ten-mile section. From curve 6-*A* low spots or defects exist in the vicinity of poles 980, 785, and possibly near 530. Since the condition at pole 980 is most clearly defined, the defect at that point will be considered first.

The plan used in the location is to extend the converging slopes of the curve at the low point and the defect will be found close to the location at which the converging lines intersect. Thus the readings at poles 1109, 1046, 980, and 915 shown in Fig. 6A are laid off on a larger scale on Fig. 6B. Lines drawn through these points intersect near pole 1012 and the trouble was found in a leaky sleeve at that point.

In Fig. 6C the defect was indicated between poles 1728 and 1646. The pressure readings at poles 1800, 1728, 1646, and 1568 are laid off on Fig. 6D and the extended lines on this curve intersect at pole 1695. The defect was found to be a bad sleeve at pole 1693.

This same procedure is used in locating defects in underground cable where the sheath break occurs in the duct line. The defect will be located between the manholes at which the lowest pressures are recorded.

After the defect has been located between two manholes, the splices at those points are filled with paraffin, care being taken to heat the armor so as to have the paraffin run back into the cable and fill all voids. When this wax cools, it forms a dam, and is reasonably effective in preventing the flow of gas during a short period. This length of cable is then again pumped up and a check test made by means of comparative pressure readings. Should these readings again fall, the section is replaced and all electrical and capacity unbalance tests remade at which time the gas pressure tests are repeated.

Still another method of locating sectional defects on underground cable is to segregate the defective sections by means of the paraffin dams just described. The first dam would be placed at the final capacity unbalance splice with gages fitted into the armor on each side. The two half sections are then pumped up and the pressure will fall in the half section having the sheath break. This half section is then redivided into two sections by placing a dam in its midpoint, and so on until the leak is narrowed down to one duct length between two manholes. The trouble is cleared in the same manner as mentioned in the previous case.

Constant Pressure Testing. So effective has this system of pressure testing been that quite a number of the more important toll cables have been placed under constant pressure.

After the cable has been cut into service, all lateral taps and both terminal ends are sealed with dams or plugs to prevent the escape of gas.

In addition to plugging the cable at the ends and lateral taps, intermediate plugs are installed at approximately ten-mile intervals, in order to sectionalize the cable for use in locating sheath breaks. Valve stems are located at the loading splices which give them a 6000-ft. spacing.

In testing underground cable on this principle, the stems are located above the water line of the manholes

and connected to the cable by means of copper tubing.

When the section is ready, gas is introduced into the cable at the usual 40-lb. rate at each of the valve stems along the ten-mile section. By injecting from 180 to 200 cubic ft. of nitrogen for each mile of cable, it is possible to gage by the cubical content registered on the regulator equipment just how much gas should be put in to result in an equalized pressure of 15-20 lb.

After the gas has been allowed to equalize, which in this case requires several days, the pressure reading at each point is taken with a gage made to fit over the valve stems, and is plotted on cross-section paper. As before, the low points are then taken as indications of sheath trouble. When the trouble has been narrowed down to one section, the methods described for sectional defects are used to locate the trouble.

In order to have a means of detecting loss of pressure after the section has been inflated for some time, electrical indicating gages associated with an alarm circuit are placed in each ten-mile section. The alarm circuit is usually a non-loaded pair stubbed out to a terminal, the tail of which has been plugged. The gage and terminal are placed in a box on the pole and the gage connected to the cable by copper tubing. This line is equipped with a cock for shutting off the gas when working on the gage. The alarm is set so as to operate when the cable's internal pressure falls below 12 lb. In the central office or repeater station the alarm circuit is terminated with a relay whose local contacts operate a lamp signal or bell. When the alarm operates, its contacts automatically cross the pair to which the gage is attached.

The testboard man takes a loop resistance measurement to the cross and knows immediately what ten-mile section is in trouble. He then dispatches a cable man to make the readings at the mile points in the section in trouble and by the method of plotting pressures he finally locates the trouble.

Alarm gages are inspected and tested once a month to insure their operating in the event of lowering pressure.

Pressure Testing Results. The results of gas pressure testing have been very gratifying, both from a maintenance and cost standpoint. The number of troubles found by the use of gas varies with the type of plant tested, *i. e.*, underground or aerial, new or old cable.

The rather limited results procured so far show about 6 per cent of the splices on underground cable to be defective. Of this number 3 per cent have been attributed to poor tinning of the joint, cracks, and blow holes. The other 3 per cent showed porous joints due to the use of cold or coarse metal. On new aerial cable work, preliminary tests have shown 4 per cent of the splices tested were found to leak.

These figures are averages, and may be exceeded in some cases while in others fewer troubles will be found. Considering the large number of splices occurring in, say, a 100-mi. length of cable, the importance of insuring the integrity of the wiped joints in order to avoid potential failures in long toll cables, is readily apparent.

It is expected that the number of troubles found will be materially reduced as the splicing and engineering forces are, in some cases, actually able to determine what causes the defects. One of the things which have been noted so far is that in a great many cases the joint at one end of the sleeve is good, and at the other end is porous. This is undoubtedly due in most cases to the fact that the solder was not sufficiently hot when the second joint was wiped.

It is of interest to note that on a recent underground toll cable project, 16 mi. in length, there were 19 troubles found in the splices and two in the duct line. Of the 19 defective joints, 18 occurred in the first five miles of cable tested, and only one in the remaining eleven miles. This is, of course, an extreme case, but it illustrates the fact that as the men had never before had their metal work checked, they were rather negligent in this particular phase of their job. As soon as the pressure testing began to reveal these delinquencies they took more pains with the wiping, and produced better joints.

Clearing Trouble. Another important advantage of the nitrogen gas in maintenance work is the drying effect of the gas when admitted in either side of a cable fault which is wet. In certain instances, pulling in and cutting over a new section of cable has been avoided and service restored within three or four hours after failure. In these latter cases, the gas was applied and the drying out process continued up to the time when the rodding and pulling had been completed and the sections ready to cut over.

Drying under these conditions not only restored practically all the telephone service, but materially aided the splicers in toning through the defective section when cutting over. It is reasonably certain that only in those cases where the cable is partially wet can it be dried out completely with gas.

Cases where the cable is thoroughly saturated with water will have to be handled in the usual manner, either by replacing the section or by cutting in a short section of cable. The gas is, however, found to be helpful in many cases in avoiding the replacement of cable.

Aerial Cable Construction

General. New methods have recently been developed for placing and splicing aerial cable which, it is felt, will decrease troubles and lower maintenance costs. Results have shown that the erection of a pole line according to specifications is a comparatively simple problem and few troubles result from faulty construction; but the placing of a suspension strand and cable presents a more difficult problem.

Cause and Theory of Bowing. The causes of bowing in full sized aerial cable supported on 16-M suspension strand have been determined. Measures for the pre-

vention of bowing have been worked out theoretically and have been put into practise in the field with satisfactory results.

Observations locally have coincided with observations of other sections in that bows are continually increasing and decreasing in size under the influence of daily and seasonal temperature variations, thereby causing the cable to move up and down in the rings or in and out relative to the pole. This motion contributes in some measure to ring cuts, chafing of the cable against pole attachments, sheath cracks at wipes, and breakage of terminal taps. It also sets up varying stresses in the cable sheath which change its structure at bowed points and eventually cause it to crack. Fig. 7 is a typical example of a bow which has developed in a large cable near a splice and which will eventually cause a sheath crack at the wipe.

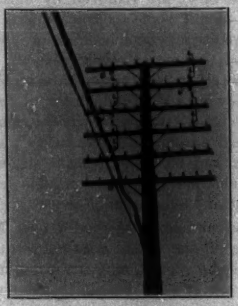

FIG. 7—TYPICAL BOW IN A LARGE CABLE
This condition will eventually cause a sheath break at the wipe

The generally accepted theory of the cause of bowing is as follows:

Due to a difference in the coefficients of expansion of suspension strand and cable, the cable expands more than the strand under the influence of a rise in temperature. The length of strand and cable in any span is assumed to be equal at the time the cable is spliced. As the temperature rises above the splicing temperature, the cable expands faster than the strand and, as it cannot move along the line, compression is set up within the cable. When this compression exceeds the resistance of the cable to bending, the cable bows out of line. The amount of compression and bowing depends upon the extent to which the temperature has risen above the splicing temperature.

Under the influence of a drop in temperature, the cable will contract faster than the strand and when the temperature falls below that at which the cable was spliced, tension is developed in the cable. The tension which will develop in the cable at the minimum temperature of the locality depends on the temperature at which the cable was spliced. If the cable was spliced at a high temperature, the tension developed during extremely cold weather may stretch the cable permanently to some extent. As a result, when the temperature rises again to the original splicing temperature, the cable instead of being once more free of stress, will be under compression and as the temperature rises still further, bowing will occur sooner than it would have if the cable had not been permanently stretched.

Construction Methods to Minimize Bowing. As a result of studies and field trials of bowing in full sized cables supported on 16-M strand, the following construction methods have been recommended for placing new aerial cable so as to minimize bowing:

1. Reduced strand tension.

The first method for the prevention of bowing in new aerial cable is the only one to date which has been included in the specifications for aerial cable construction, namely, to reduce the initial strand tension from 6400 lb. (60 deg. fahr.) to 3600 lb. (60 deg. fahr.). The theory of this method is that a high initial strand tension reduces to a considerable extent the normal (or unstressed) elongation or contraction of the strand with temperature change. Therefore, if the initial strand tension is reduced, the tension existing in the strand when the cable is in place will be reduced. This decrease in loaded strand tension will permit an increased elongation and contraction of the strand with temperature change. As a result, the changes in strand length become more nearly equal to the changes in cable length, and the tendency for the cable to bow is reduced. The initial tension in 16-M strand was not reduced below 3600 lb. because of the greater sags which would result. Field trials which have been under way for several years show that the reduction in strand tension mentioned above has practically eliminated all bowing in those cases where the cable was spliced at a temperature of approximately 40 deg. fahr. or above.

2. Placing strand at correct tension.

In order to permit the field forces to string suspension strand more easily and more nearly at the correct initial tension, an instrument called the "strand dynamometer" has been developed. This instrument may be placed on the suspension strand at any point without cutting or otherwise injuring the strand and will indicate the tension existing in the strand at that point. This instrument consists essentially of a means of deflecting the strand out of line and a means of measuring the force tending to restore the strand to a straight line. This restoring force is proportional to the tension existing in the strand. An indicator dial gives the amount of this restoring force and the actual tension

existing in the strand is obtained from a calibration curve.

3. Placing cable so that it is free from waves.

In order to prevent bowing it is necessary to have the cable the same length as the strand when it is first placed in the rings. Formerly the methods used for placing cable sometimes resulted in a series of small waves developing in the cable, particularly in that section of the cable which is nearest the center of the cable reel. The use of the "aerial cable guide and straightener" has eliminated, in large measure, these waves. This tool consists of a sheet iron shoe terminating in a steel tube of slightly larger diameter than the cable. The cable is pulled over this shoe and through the tube before it reaches the rings. Any waves which may be in the cable as it comes off the reel are thus removed as the cable passes over the shoe and through the tube.

4. Preventing the cable from being spliced while the strand is in a stretched condition.

Splicing is ordinarily done from platforms which support one or two workmen and their equipment. This extra weight will stretch the strand if no measures are taken to prevent it. As a result, after the cable is spliced and the weight of the splicers and their equipment is removed, the strand will contract and some excess cable will remain in that span. This stretching of the strand has been prevented by placing a ladder alongside the splicing point and tying the strand to the ladder before the splicing platform is erected. In this way the ladder is used to support a major part of the weight of the splicers and their equipment and stretching of the strand is largely prevented.

5. Placing tension in the cable.

At temperatures of 40 deg. and below it has been found that the above measures are not sufficient to prevent the cable from bowing when the temperature subsequently rises to the neighborhood of 100 deg. fahr. Accordingly the practise has been developed of placing tension directly in the cable before splicing and holding it there until the splice is completed.

An instrument called the "tension splicing tool" has been developed for placing tension in cables. This tool takes hold of the ends of adjacent reel lengths of cable at the splicing point, pulls them together until the amount of tension required by the existing temperature has been set up in the cable, and holds this tension in the cable until the splice is completed.

The effect of this tension is to reduce the amount by which the cable would otherwise expand or contract and thereby bring the amount that the cable will expand under a given temperature rise closer to the amount that the strand will expand. In this practise the amount of tension placed in the cable varies with the temperature existing at the time the splice is made. For temperatures just under 40 deg. the tension is small, but for temperatures as low as 20 deg. below zero the tension is about 1500 lb. Where this practise has been conscientiously followed, bowing has been prevented in cables spliced at temperatures as low as 5 deg. below zero.

Removing Bows in Old Cable. Two methods have been investigated for removing the bows in cable which has been erected and spliced for some time. They are:

1. Pull the excess length of cable to concentration points and cut it out.

2. Cut suspension strand at intervals, introduce excess strand at these points, and slack off until bows disappear.

Field trials have revealed that bows can be completely removed by either of the above two methods, although the bows removed by Method 1 tend to return in a few years while those removed by Method 2 have not returned in the same length of time.

Where the bows are removed by cutting out the excess cable length, some tension remains in the cable after being pulled to the concentration points but the strand tension is not materially reduced. Therefore, during hot weather, the tension in the cable tends to reduce the bowing while the high initial strand tension tends to promote it. Consequently the results obtained in this manner are not as successful as the results obtained by slacking off the suspension strand at various intervals, which method places the cable in tension and reduces the strand tension simultaneously.

Another advantage of Method 2 over Method 1 is that slacking off the strand does not require that the working cable be opened, whereas in pulling the slack out of the cable it must be cut to remove the excess length. Since the comparative costs of the two methods are approximately the same, slacking off the strand appears to be superior to cutting excess length out of the cable, as a method for removing bows from cables supported on 16-M strand. Cutting the excess length out of bowed cables has its application to those cases where the strand tension is low, the cable is badly bowed, and the existing clearance above ground is such that no material increases in sag can be permitted.

The danger encountered in removing bows by slacking off the strand is that the tension introduced into the old, badly bowed cable may cause sheath breaks.

Analysis of results from a small number of cables indicated that where the cable was not more than four years old there was little danger of causing sheath cracks by slacking off the strand. On cable which was about eight years old when bows were removed, a number of sheath cracks was caused by the resulting tension. An analysis of the sheath cracks caused by tension compared to those which were caused by bowing in another section of the same cable, showed that the sheath breaks caused by tension started at a high rate, but have decreased rapidly in each year since that time. The sheath breaks due to bowing started at a lower rate but have been increasing subsequently. The total number of sheath breaks caused by tension during a

three year period was equal to the number caused by bowing during the same period, but it appeared that from then on sheath breaks due to tension would be considerably less than those due to bowing.

Therefore, in determining whether or not the bows should be removed in a certain cable, it becomes necessary to consider its age and to estimate the length of time that it will remain in service. Where cables are expected to remain in service for a considerable period of time, it is undoubtedly economical to remove the bows, but on cables where the remaining life is limited, an investigation is necessary to determine whether the expenditure involved in removing the bows is justified by the saving due to an eventual decrease in the number of sheath breaks.

The application of these methods has been investigated most thoroughly on cable supported by 16-M strand, and it seems that no difficulties should be encountered in applying these same methods for removing bows to cables supported with 10-M and 6-M strand and the same success should be attained.

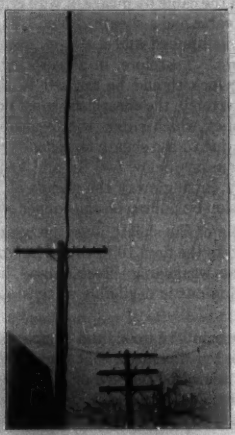

Fig. 8—Bowing in a Comparatively New Cable

This cable was spliced lying loose in the rings and, undoubtedly, tension would have reduced the bowing considerably.

Bowing in Small Cables. Up to the present time, the only results published on the prevention of bowing in aerial cable have been methods which are recommended only for full sized cables supported on 16-M strand. In the plant of the Cincinnati and Suburban Bell Telephone Co. the percentage of cables supported on 16-M strand is small compared to the percentage supported on 6-M strand and, therefore, it has been felt that measures should be taken to prevent bowing of the smaller cables.

The methods used in attempting to prevent bowing in the smaller cables supported on 6-M strand would necessarily be similar to those used on full size cable supported on 16-M strand.

Applying the theory of reduced initial strand tension to 6-M strand, it can be logically deduced that the benefits derived from this method would not be nearly as great when applied to the smaller strands as when applied to the 16-M strand. The coefficients of expansion of the 6-M, 10-M, and 16-M strands are practically the same, and inasmuch as the specification initial strand tension of 6-M strand is approximately 1100 lb. (span—130 ft.; 60 deg. fahr.), reducing this initial tension might not materially affect the change in length of the strand with temperature.

Therefore, applying tension to the cable before splicing has been resorted to since 1920 with reasonable success. Inasmuch as there was no precedent as to the amount of the tension which should be applied to the cables, no specific instructions were issued to the construction forces. Since then, the judgment of experienced foremen has been depended upon and as yet no troubles have ever developed due to an excess of tension in the cables. Another point to be noted is that tension is being applied at all temperatures. The method that is being used to apply tension to the cable, is as follows: The cable is pulled from both ends by means of blocks and tackle and clamped at the ends to the suspension strand by means of grade clamps. After splicing, these grade clamps are removed. To place tension in a cable which pulls around a corner, the cable is blocked out from the pole before applying tension. After the cable is in tension, it is clamped to the strand on each side of the pole by means of grade clamps and the blocks are then removed, thus holding it away from the pole. These grade clamps are never removed.

The trouble encountered due to the exchange cables being in tension, is the difficulty of working in a tight cable when necessary to open it for various purposes. As data become available regarding the minimum tensions which can be applied to the cables and still minimize bowing, this trouble will lessen, since it is a certainty that more tension is now being applied to some of the cables than is necessary. However, after eight years' experience our local supervisors are convinced that the additional difficulty of working in a tight cable is more than compensated by results obtained in the elimination of bowing. However, the difficulty of working in a tight cable when necessary to open it for various purposes may be overcome by the use of the tension splicing tool. This tool may be applied at any point where it is desired to open the cable and after the sheath is stripped off, the conductors may be ballooned to any extent desired by operating the tension splicing tool. Fig. 8 is a typical example of a comparatively new cable which was spliced lying loose in the rings. Probably a small amount of tension would have reduced the bowing considerably. Fig. 9 illus-

trates a badly bowed section of cable, the bowing of which can be partly attributed to construction methods. If tension in the cable is undesirable, the construction forces should at least be required to pull all of the kinks out of the cable when placing it.

Experimental Results. Various other problems and questions have arisen in regard to observations on placing tension in the smaller cables. In order to find answers to these questions and to verify theories

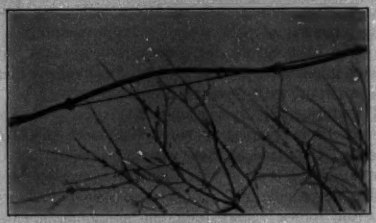

FIG. 9—EXCESSIVE BOWING PARTLY CAUSED BY POOR CONSTRUCTION

The small bows and kinks caused by unreeling the cable were not removed before splicing

regarding the field observations, experimental spans were erected using 50-pair 22-gage cable and 100-pair 22-gage cable supported with 6-M strand. The physical data regarding these cables are:

TYPE B S A (22 ga)

Pairs	Mean outside diameter (inches)	Sheath thickness (inches)	Lb. per ft.
51	0.73	0.070	0.95
101	0.99	0.080	1.58

The suspension strands were erected using the specification initial strand tension of 1100 lb. (span.—130 ft. 60 deg. fahr.) and all tensions were obtained by means of a deflection dynamometer. The problems to which solutions were sought were:

1. Is the total tension applied to the cable transferred to the strand at the grade clamps, thereby increasing the strand tension?
2. Effect of cable tension on the sag.
3. Proportionality of the cable elongation to the applied tension.
4. Effect of the slip of the sheath over the copper when tension is applied.
5. Permanent set.

To determine the increase in the strand tension caused by clamping a cable which is in tension, to the strand at corner poles, tensions were taken with the cable inert and with the cable in tension. The tensions applied to the cables varied from 300 to 750 lb. and in no case were the strand tensions increased by more than 150 lb. The strand tensions on each side of the grade clamp differed approximately by the amount of the stress in the cable. These results indicate that the additional tension placed in the suspension strand, between the clamp and the pole, is negligible.

The effect of the tension in the cable is to reduce the sag. This reduction in sag depends on the tension applied and the span. The sag in a 140-ft. span carrying a 100-pair 22-gage cable was decreased 3½ in. when a tension of 750 lb. was applied.

The sag in a 140-ft. span carrying a 50-pair 22-gage cable was reduced 1¾ in. when 300-lb. tension was placed in the cable.

In any size of cable, elongation is directly proportional to the stress applied within certain limits only. For full sized cable this limit has been set at about 3000 lb. In the smaller cables, there is a tendency for the sheath to slip over the copper when tension is applied at the ends of the cable by means of wire grips. The 100-pair 22-gage cable had a sheath slip of 3/8 in. when 500-lb. tension was applied and a 3¾-in. sheath slip at a 750-lb. tension. Therefore, in attempting to regulate the tension which should be applied to the cable, by means of the stretch, the sheath slip must be eliminated. The discrepancy which enters the measurement of the total stretch due to the sheath slipping over the copper is small at tensions below 500 lb. and can be entirely eliminated by cutting away the sheath at the end and applying tension by pulling on the copper only.

The slipping of the sheath over the copper is practically confined to the final 10 ft. of the pulling end of the cable and the weakening effect caused by the cable sheath thinning out is negligible at tensions below 500 lb.

In locating points of maximum elongation due to the tension applied, it was noted that the sheath slippage is practically confined to the five or six feet of cable on each side of the poles.

The immediate permanent set of the 100-pair cable at 500 lb. tension was negligible but at 750 lb. was 1⅛ in. in a total length of 260 ft. The 50-pair cable showed no immediate permanent set at a 300-lb. tension. The measurement of permanent set in a 100-pair cable of considerable length suspended on strand is liable to considerable error for the following reasons. When the cable is pulled certain portions of it move toward the point from which the pull is made and friction in the rings may prevent this cable from going all the way back to its original position when the pull is released. On full sized cable a 1500-lb. pull placed on a 750-ft. length of cable will cause the cable to come in toward the pulling point about six in. and when this pull is released the cable goes back only about three inches. This remaining three inches is partly due to the fact that the cable has been pulled closer to the rings at poles and closer to the pole at corners, and partly due to the fact that some tension still remains in the cable because contraction of the cable is resisted by ring friction. This tension contributes to the

remaining three inches by the amount of elongation and the decrease in sag due to that tension.

The general conclusion to be drawn from these data is that no physical harm whatever is done to the smaller cables when a tension of 500 lb. or less is applied, and inasmuch as local experience has proved that applying tension to the smaller cables minimizes bowing, it seems that this practise should be more generally followed.

Long Span Construction

General. A "long span" in telephone plant refers to a considerably shorter span than in some other types of construction. This is due to the fact that telephone line long spans differ from others such as power conductors, in that the power wires support their own weight while a telephone cable cannot. Power conductors in such installations are, as a rule, essentially a copper covered steel strand, uninsulated, while telephone cable is formed of a number of small copper wires, each wire being covered with paper insulation and the whole assembly of wires enclosed in a lead-antimony sheath. This combination of numerous small wires and soft sheath naturally causes the entire cable to possess low tensile strength. Thus, while a power conductor may be suspended from steel towers and carry its own weight over long spans, in the case of telephone cable a means of support must be found, outside of the cable itself.

This problem is solved by using long span construction which may take either of two forms: first, the catenary type where suspension guy relieves the cable and its messenger strand of a great part of the load. The suspension guy is attached to the crossing poles on each side of the span and then pulled up until the sag is correct as determined from tables prepared for different spans and cable loadings. The theory of the loaded catenary is not new but its application to the telephone plant is of recent development. The second type of long span construction is simply an elongated form of the usual span wherein a high strength steel messenger strand is used to support the cable which is attached directly to it.

Economic considerations are largely the determining factor in deciding the length of span which can be justified in a given case. Catenary construction is used in our long toll cable plant largely as a matter of convenience and as a means of carrying the cable suspension strand across streams or other barriers continuously without dead-ending it. On the other hand, a long span was built about a year ago and more are projected where the length of the span is in excess of 1000 ft. and in which the cable suspension strand alone carries the entire stress and no catenary construction is used. These particular long span crossings are cable spans inserted in open wire lines at river crossings and the cable terminates at the crossing poles.

Construction of Catenary Spans. After the suspension guy (upper strand) is in place, the messenger is extended across the span and fastened to a point on each crossing pole a normal distance above the ground, the same as on the other poles back from the crossing. The messenger is then attached to the suspension guy at the midpoint by three-bolt clamps and at the quarter points, sixth points, or eighth points, according to the length of span, by hangers of the proper length to make the messenger extend in an approximate straight line across the span, these lengths being given in the tables referred to previously. The rings are then placed on the messenger and the cable pulled through the rings to complete the crossing.

While the curve of the suspension guy is a true catenary before the load in the form of the cable is applied, after loading at a number of points it assumes very nearly the shape of a parabola, modified by the fact that the load is applied to the suspension guy as a series of concentrated loads through the hangers. It approximates a parabola closely enough to make the use of the general span formula for parabolas allowable in making up the tables. This formula is: $d = \frac{wl^2}{8t}$

where d = sag in feet; w = resultant force in pounds per foot due to the combined weight of cable, messenger, suspension strand plus the maximum ice formation thereon acting in a vertical direction, and the wind pressure acting in a horizontal direction: l = length of span in feet; and t = allowable tension in pounds in suspension strand. This formula is for a parabolic curve, which assumption is very nearly true. The tables are also calculated to allow for the work being done at various temperatures, and the proper sag is given for each of those assumed.

Limits. The length of cable span which can be permitted without special construction depends upon several factors, of which the size of cable, length of span, size of suspension strand, and unbalanced loads on the pole at the end of the long span are to be considered. From the standpoint of strength of suspension strand, it is permissible to use spans as long as 200 ft. without special construction in the case of full size cable carried on 16-M strand installed at an initial tension of about 3600 pounds. For long span crossings of full size cables much over 600 ft. in length special consideration may be required such as larger size strand, possibly steel towers, etc. Spans of practically any length can be constructed if the economics of the situation indicate that it would be desirable.

Advantages. Long span construction, under certain conditions, has distinct advantages, especially where there is some extensive obstacle to clear, as a river, or a deep, narrow valley.

The simpler type of long span involving the high strength messenger with the cable attached directly to it has several advantages over the catenary type. Theoretical considerations appear to indicate that a long span without the catenary pick-ups would result

in less buckling of the cable than a span with pick-ups. The simpler type of long span crossing also permits construction with somewhat smaller sags. As a matter of fact, with the same total number of steel strands used slightly smaller sags could be obtained by hanging the strand and cable in one long span supported only at the ends than if catenary construction were used. This is for the reason that the cable suspension strand itself will carry a greater proportion of the weight of the cable in a long span.

In this discussion we have been dealing with spans where the crossing poles have been approximately at the

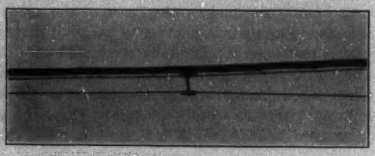

Fig. 10—Hanger Details

Showing horizontal position of cable at pick-up points

same level. Where this is not the case, certain obvious differences will occur, but the principle is the same throughout.

Long span construction—typical projects; Cleves–Elizabethtown cable at Lost Bridge.

The Cleves-Elizabethtown section of the Cincinnati-Lawrenceburg toll cable crosses the Great Miami River at Lost Bridge by a series of spans, each of which is a complete catenary in itself. The total length of the crossing is 759 ft., but this has been divided into three shorter spans of 253 ft. each, corresponding to the number of truss systems forming the bridge itself.

While plans for the construction of the cable crossing were under way, it was observed that vehicular traffic on the bridge set up severe vibration in the structure, and it was agreed that if the proposed cable were attached to the bridge itself, or allowed to come in contact with it in any manner such that this excessive vibration might be imparted to the cable itself it was likely that serious damage might be done to its sheath and insulation from abrasion. The problem was how to make use of the advantages afforded by the bridge crossing at this point, and still protect the cable from injury. A plan was finally worked out whereby it would be supported at two intermediate points of the crossing, these supports to be steel towers approximately 12 ft. high and 6 ft. wide at the base, formed of angle irons bolted together, and each tower resting on a pier of the bridge and bolted securely to the concrete,—the inner side of each tower being about one foot from the outer part of the bridge truss. During construction great care was taken to put the crossing poles on either shore accurately in line with these two towers, so that the cable would maintain an equal distance from the bridge at all points of the span, for if the end poles were out of line toward the structure, the wind blowing down the river might cause the cable to swing against the truss midway of the end spans, and this frequent bumping and rubbing would severely damage the soft armor of the cable in a short time. (Fig. 11.)

As the completed crossing now stands, it is a series of three catenary spans, the cable leaving the southeast shore from a fiber duct about four feet below ground line at the bridge abutment, extending to the tower on the first pier, thence to the tower on the second pier, and from there to a similar fiber duct on the northwest shore, the only contact with the bridge being through the piers, which will transmit little, if any, vibration from the bridge to the towers.

The 6000-lb. messenger strand supporting the cable is secured on each end to an eye-bolt passing through the 18-in. abutment wall, which is prevented from pulling through by a 3-in. channel 12 in. long acting as a washer. The 16,000-lb. suspension guy, which supports the messenger and cable and is secured to the towers

Fig. 11—River Crossing at Lost Bridge

about one foot from their tops, is dead-ended at each shore on a wood pole directly adjacent to the abutment wall and securely fixed in place by a concrete toe-block 6 ft. below the surface of the ground. The upper end of each pole is secured by two guy strands running to a log anchor firmly set in the ground. The fiber ducts carrying the cable pass as close as possible beside the foot of the crossing pole on each shore, to maintain the cable alinement.

The messenger strand is clamped to the suspension guy at the midpoint of each span, and also secure to it at the quarter points by hangers, thus reducing the

greatest actual span of the cable between any two supporting points to 63 ft. 6 in. By this means, very little sag is produced in the cable.

It should be noted here that this project resulted in breaking up a very long span into three shorter spans, each of which forms a complete catenary in itself.

However, the effect of breaking up a cable into shorter spans between suspension points is conducive to buckling rather than tending to reduce it, and it would be better from a buckling standpoint if longer spans between pick-up points were used.

Cable over Licking River at Falmouth, Kentucky. The cable crossing over the Licking River at Falmouth, Kentucky, is a good example of catenary construction which must extend across a long span without any intermediate supports whereby the length might be shortened. It was found necessary to cross the Licking at this point with a 50-pair 19-gage cable, and due to the low, marshy nature of the ground on the north shore, it was decided to set the north crossing pole back from the river a distance nearly one and one-half times the width of the river itself, thus obtaining for the pole a position out of reach of high water and affording an elevation for the cable approximately the same as that on the other poles back from the river. The south bank has a more abrupt slope, and it was only necessary to place the crossing pole back about 100 ft. in this case to obtain the desired elevation.

In this manner local conditions caused a span of over 600 ft. to be built over a stream normally only about 200 ft. wide. This, of course, required long span construction.

The two crossing poles are Class "A" Cedars 45 ft. long, the suspension guys two 16,000 lb. strands, and the messenger a 6000 lb. strand. The messenger and cable are attached to the suspension guys at the sixth points by standard hangers and by three-bolt clamps at the midpoint of the span.

Before the span was built, a mathematical analysis was made of the problem in order to determine the sag of the suspension guys after the load was applied. The basic formula used was the one previously referred to, namely $d = \frac{w l^2}{8 t}$. An extract of the development showing the significant steps follows:

Material Loads.

Material	Weight per foot
Two 16,000-lb. guys	0.88 lb.
One 6000-lb. messenger	0.22 lb.
50-pair—19-gage cable	1.50 lb.
Hangers and rings (approx.)	0.25 lb.
Total material load per ft. of span	2.85 lb.

Ice Loads.

½ in. radial deposit assumed.
No shielding effect considered.

1—6000-lb. messenger	0.23 lb.
2—16,000-lb. guys	1.16 lb.
1—50-pr.—19-gage cable	0.91 lb.
Total ice load per ft. of span	2.30 lb.

Wind Loads

Wind loads figured at 8 lb. per foot of projected area of span with ½ in. radial deposit of sleet covering same.

1—6000-lb. messenger	0.86 lb.
2—16,000-lb. guys	1.92 lb.
1—50-pair—19-gage cable	1.29 lb.
Total wind pressure per ft. of span	4.07 lb.

Resultant force = 6.56 lb. per ft. of span.

Formula

$d = \frac{w l^2}{8 t}$ Where:

t = Tension		16,000 lb.
w = Force per ft. of span		6.56 lb. 1 ft.
l = Length of span		621 ft.—0 in.
D = Sag		?

With two 16,000-lb. strands supporting the entire load

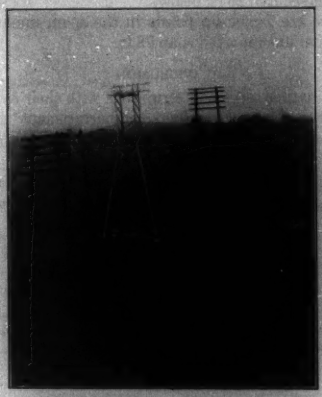

FIG. 12—TYPICAL LONG SPAN RIVER CROSSING
Employing the use of catenary suspension guys supported by steel tower

with a factor of safety of two, the allowable tension in the catenary becomes $\frac{2 \times 16{,}000}{2} = 16{,}000$ lb. $= t$.

$d = 19.8$ ft.—Sag under application of full storm load, (cable, strands, ice, and wind).

Steel Tower Construction. On a recently constructed toll cable line, it was necessary to cross a navigable stream with a long span. This span, which is a good example of steel tower-supported long span construction as shown in Fig. 12, is 623 ft. in length, and the ultimate load on the towers will be very heavy, as it is planned to install eventually one "full size" cable, weighing 7.85 lb. per lineal foot and two "over size" cables, each weighing approximately 11 lb. per lineal foot. The design of the long span for the first cable, which is full size cable already in place, is such as to place an unbalanced load on the tower of about 26,500 lb. under maximum assumed storm loading, and with all cable in place this load will increase to 107,000 lb. under full storm loading. Storm loading of ½-in. radial coating of ice, wind amounting to 8 lb. per square foot of projected area, and a temperature of 0 deg. fahr. were assumed in the design.

To withstand this heavy load, and still give the clearance over the river required by the Government, it was necessary to abandon the use of wood poles, customary for catenary construction over shorter spans and with lighter loads, and erect a large steel tower at each end of the proposed crossing. These towers are 85 ft. high by 30 ft. square, and each rests on four 5-ft. concrete piers. The ends of the suspension strands are attached 5 ft. below the top, and the messenger strand is secured 29 ft. below.

There are 7 pick-up points in the span, making the maximum unsupported span 78 ft.

Conclusion

Eliminating service interruptions with their resultant inconvenience to subscribers, embarrassment of business, loss of revenue, etc., due to cable failures, justifies far reaching measures.

It is felt that definite progress has been made towards building cable plant whose sheath continuity will be undisturbed except from extraneous influences. Further advances can probably be realized by utilizing the ideas presented.

The outstanding features of these methods have been shown to be arrangements for determining the location of sheath breaks before complete failure, minimizing sheath movement through the rings, and new construction methods for long span crossings.

Discussion

C. A. Jaques: I want to ask Mr. Wagner whether they have taken into consideration the possibility of damage to the lead sheath from the 40-lb. internal pressure, and if so, whether care is taken as to the outside diameter of the sheath, or the alloy sheath, or the thickness of the sheath.

E. B. Shand: I should like to ask one question. In bringing up the cables is it necessary to bring them through insulated gas-proof joints?

B. S. Wagner: Forty-pounds pressure, as the maximum internal pressure, has been carefully determined. The lead-antimony alloy sheath that is used on the cable will safely stand pressures below 50 lb. Due to the movement of the cable with temperature variation, and due to movement in handling—even after the cable is in service, of course, either under ground or aerial, it is necessary that it be handled—incipient cracks may be developed inside which will eventually cause trouble with pressure over 50 lb.; so that 40 lb. ingress pressure has been determined to be the maximum safe pressure.

Answering the second question, the wires are brought out into a special terminal which has a pressure-tight cover, so that the entire device is pressure-tight. There are no plugs made, either in the main cable or in the lateral cables, which serve to stub out the wires. The entire arrangement of the alarm device is in a pressure-tight chamber at the present time.

It has been found that it is very difficult to place any kind of a plug in either a main cable or a lateral which will hold its gas. You can pour in hot beeswax or hot paraffin or anything of that sort, and it will plug for some hours, but the difference in the expansion of the copper and the beeswax or paraffin, finally causes a crack between the sheath and whatever is inside of it; so that it has been very difficult to get a dam that would really hold the gas.

Lighting of Airways and Airports

BY H. E. MAHAN[1]
Associate, A. I. E. E.

Synopsis.—*The Department of Commerce under the Air Commerce Act is responsible for the installation, operation, and maintenance of the national airways. The lighting facilities along the airways consist of revolving beacons at 10-mi. intervals and intermediate landing fields approximately every 30 mi. At the intermediate landing fields, in addition to the revolving beacon there are provided an illuminated wind cone, boundary, approach, and obstruction lights. The Department of Commerce is also responsible for the rating of airports, although their establishment, maintenance, and operation are matters of municipal or private concern. The rating of an airport is a measure of the facilities available. A preferred rating for lighting facilities requires the installation of an airport beacon to locate the airport from distant points, an illuminated wind direction indicator, white or yellow lights marking the boundary of the landing area, red lights on all obstructions, green range lights marking approaches, floodlighted hangars or other buildings as a measure of altitude and to illuminate identifying markings, a searchlight for measuring ceiling height and suitable floodlighting for the landing area.*

In view of the regulating authority vested in the Department of Commerce, the practise in lighting airways and airports follows very closely established standards. The floodlighting of the landing area proper offers the greatest opportunity for original thought and ideas. There have developed two schools of opinion regarding the fundamentals of field floodlighting systems. One school favors the use of a single unit or a group of units at one location for floodlighting the field, which system is referred to as the centralized system. The opposing school, advocating two or more light sources placed at different locations about the field, is known as the distributed system. Both systems have in common the use of lighting units delivering a fan shaped beam of light only a few degrees wide in the vertical plane and varying from 45 deg. to 180 deg. in the horizontal plane.

The present lighting facilities are found very satisfactory during fair weather but the problem of aiding the flier during fog still faces the engineer and research scientist as a problem.

* * * * *

LAST year was celebrated the twenty-fifth anniversary of the first airplane flight of the Wright Brothers at Kitty Hawk, N. C. Today the Department of Commerce reports that there are 15,128 miles of airways operating and 1330 established airports in the country exclusive of Army and Navy fields. Of this number 9341 miles of airways, 274 intermediate fields, and 74 other types of fields are lighted for night flying. By July 1, 1929, it is expected to have 11,270 miles of lighted airways. This is truly a remarkable record of progress and a glance at the map (Fig. 1) indicates the present scope of organized air transportation.

Obviously, if such facilities are to be operated at the maximum of efficiency, the service must be continued throughout the twenty-four hours of the day, for it is the distance covered between the end of one business day and the beginning of the next one that gives air travel its greatest advantage over other forms of transportation. In other words, the movement of material and passengers during the night conserves daylight working hours, and the greater the distance that may be covered in this period, the greater the benefit realized. Thus arises the problem of providing adequate lighting that will permit after-dark operations at our airports and along our airways. The author advances no original ideas for the solution of this problem but merely outlines the practise followed at the present time in lighting and marking airports and airways for night flying.

Airways

An airway is defined by the Department of Commerce as "An air route between air traffic centers with landing facilities at intervals equipped with aids to air navigation and communication system for transmission of information pertaining to the operation of aircraft. The term 'airway' may apply to an air-route for either land planes, sea planes, or both." Under the Air Commerce Act of 1926, the Secretary of Commerce has authority to establish and maintain civil airways; operate and maintain intermediate landing fields, lights, signal and radio direction finding apparatus; and other structures and facilities (excepting airports) used as aids to navigation. In fulfillment of this obligation,

FIG. 1—AIRWAY MAP OF THE UNITED STATES SHOWING ROUTES LIGHTED AND TO BE LIGHTED

1. Illuminating Engineer, General Electric Co., Schenectady, N. Y.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

29-52

the Department of Commerce has established along the airways, beacon lights at 10-mi. intervals and emergency landing fields at 30-mi. intervals.

It is desirable to mount the beacons at sufficient elevation to obtain the desired range of visibility and avoid obstructions. A steel tower (Fig. 2) approximately 50 ft. high usually meets requirements. The

Fig. 2—Airway Beacon and Tower

beacon (Fig. 3) consists of a searchlight having a 24-in. diameter parabolic mirror and a 1000-watt, 115-volt incandescent lamp. It is revolved at 6 rev. per min. by means of a 1/6-hp. motor and gearing mounted in the base of the searchlight. Many of these beacons operate in inaccessible places and are without regular attendants. As a safeguard against outages, an automatic lamp changer is employed for replacing a burned out lamp with one held in reserve. A series coil in the lamp changer holds open contact points on normal operation. Upon the failure of the lamp these contact points close, permitting current to flow in a shunt coil the solenoid of which trips a latch swinging the spare lamp into focus.

Power is supplied for airway beacons from commercial circuits where available; otherwise a 2-kw. gasoline driven generator is provided. The beacon is supplemented by two "course" lights consisting of 14-in. projectors equipped with a 500-watt, G-40, 110-volt lamp and giving about a 15 deg. beam spread. One of these course lights faces in each direction along the airway. They are flashed by a mechanism integral with the beacon mechanism and synchronized with the rotation of the beacon so that the course light opposite to the direction of the beacon is flashed. The flashes indicate in code the number of the beacon which in turn indicates to the pilot his position on the course. These course lights are equipped with a red lens where there is no landing field and a yellow lens at intermediate landing fields. Intermediate fields are established at intervals of approximately 30 mi. and are equipped with a beacon, illuminated wind cone, boundary, approach, and obstruction lights.

Airports

The Department of Commerce requires that an airport in order to obtain the highest (A) rating for its lighting facilities, must have the following equipment:

(a) An airport beacon
(b) An illuminated wind-direction indicator
(c) Boundary lights
(d) Obstruction lights
(e) Hangar floodlights
(f) A ceiling projector
(g) Landing area floodlighting system.

Items (a) to (d) inclusive must operate all night.

The function of the beacon is to mark the airport from distant points. In order to accomplish this purpose effectively, a light must be sufficiently distinctive to contrast with surrounding lights; it must have a distribution which makes it visible at all points on the horizon and nearly to the zenith, and be of sufficient beam cp. to carry great distances. A 24-in. revolving

Fig. 3—Airport Beacon Conforming to Department of Commerce Specifications

beacon designed according to government specifications is shown in Fig. 3. The essential details of this beacon have been referred to under the subject of Airways. Any change in the distribution of light away from a symmetrical beam may be obtained by means of diverging lenses which refract the light to the desired beam form.

Beacons at airports are generally located either on a steel tower similar in construction to the tower used on the airways or else on a platform on the roof of a hangar

or other building. In any event, it is desirable to elevate the beacon to a height sufficient for the beam to clear surrounding obstacles and not become a nuisance to the neighborhood.

There are two types of lamps available for use in the beacon, differing in electrical and mechanical characteristics. Both lamps are of 1000-watt capacity, one operating at 115 volts, the other at 30 volts. The latter lamp having a more concentrated filament and operating at a higher temperature, obviously is more appropriate for use with a parabolic mirror and delivers a more definite and intense beam. Both filaments are in a T-20 bulb with a light center of 4¾ in. and a rated life of 500 hr. The beam candlepower with the 30-volt, high-current lamp is approximately 5,000,000; with the 115-volt lamp approximately 2,000,000. In order to carry out the policy of indicating the landing conditions at beacons along the National Airway System, it is recommended by the Department of Commerce that a green flashing auxiliary beacon be placed above

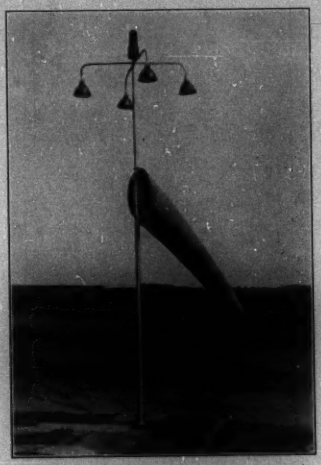

Fig. 4—Wind Cone Mounted on Roof of Hangar Showing Method of Lighting

the principal beacon to indicate that there is an airport at that location.

It is very essential that the pilot wishing to land be apprised of wind conditions on the field. The wind cone is perhaps the most generally used device for giving this information as it not only indicates the direction of the wind but also an approximate idea of its velocity. A method of lighting a wind cone (Fig. 4) consists of four 100-watt lamps in deep bowl porcelain enameled steel reflectors grouped at right angles to each other on two foot arms approximately 6 ft. above the wind cone. An obstruction light is placed on the top of the assembly. It is customary to operate these lights either from the commercial service in the building or from the boundary light circuit. Another type of wind indicator (Fig. 5) is known as a wind tee and consists of a structure simulating an airplane and pivoted to change position with change of wind direction. It is illuminated at night by means of lamps placed on top of the tee and provided with a reflector so as to throw

Fig. 5—Wind Tee

A wind direction indicating device illuminated by means of reflectored lamps on top of tee

the light on the structure and shield the direct light from the eyes of the pilot.

The extent of the field is made known to the pilot by means of light sources placed around the border of the landing area on approximately 300-ft. centers. A type of unit used for this service (Fig. 6) consists of a weather-proof housing and etched clear globe enclosing a 600-lumen (60 cp.) 6.6-ampere series lamp or a 25-watt multiple lamp. Owing to the length of run necessary to mark the boundary of the average airport, it has been found in most cases more economical to adopt the series system of distribution. In this system a 6.6-ampere series circuit regulated by a constant current transformer is distributed by means of parkway cable buried in the earth. The boundary fitting is mounted on a pipe and planted in the ground, the parkway cable being brought up through the pipe to the lamp socket. A film or disk cutout is provided in the socket to maintain the continuity of the circuit in the event of lamp failure. A recent suggestion of the Department of Commerce recommends that boundary lights surmount a cone three feet in diameter and three feet high, painted in alternate bands of chrome yellow and white, separated by a narrow black band.

A green globe is substituted for the clear etched globe used on boundary fittings to serve as range lights for runways or as markers for the most advantageous landing directions. Obstructions on or in the vicinity of the airport are marked with red lights placed at the highest point of the structure. It is also the practise

at some airports to make obstructions visible to the pilot by floodlighting them. This is found particularly advisable in connection with the hangars, for in addition to making the buildings visible, it makes evident their height and in this way gives the pilot a measure of his altitude. Such a system of lighting also provides

Fig. 6—A Form of Boundary Light Equipped with 600-Lumen 6.6-Ampere Series Incandescent Lamp

Fig. 7—A 14-in. Incandescent Searchlight Equipped with a 500-Watt T-20 Mazda Lamp and Used for Determining Ceiling Height

illumination at the aprons of the hangar for the movement of planes and illuminates any signs or markings on the roof of the hangar. The illumination intensity recommended for hangar exteriors by the Department of Commerce is 2.5 foot-candles.

A beam of light has been found useful in determining the height of ceiling. An incandescent searchlight (Fig. 7) equipped with a 500-watt, 110-volt, T-20 projection lamp may be used for this purpose. The beam is directed on the clouds at a fixed angle and by sighting at the spot on the clouds from another point the height of ceiling is determined by triangulation.

Whereas the lighting facilities previously referred to follow rather definite regulations established by the Department of Commerce, a wider range of choice is left to the engineer in selecting a system for the actual

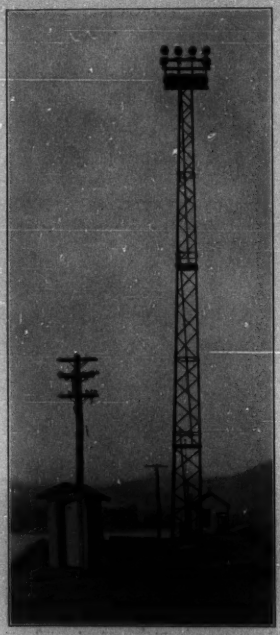

Fig. 8—The Tower System of Lighting Large Areas

illumination of the field. In studying this question, engineers very naturally turned to the pilots for information as to what sort of lighting they found most helpful to them in landing and taking off at night. Much information was obtained but many conflicting opinions were expressed. However, certain fundamental requirements were established and very generally agreed upon. The Department of Commerce requires for a field entitled to an "A" rating a minimum vertical illumination of not less than 0.15 foot-candles over the usable part of the field. This is the illumination measured on a plane at right angles to the field surface and is a measure of the value of illumination

incident upon the vertical sides of obstacles, grass, etc., facing the source of light. This component is, therefore, the illumination which will enable a pilot to see obstacles ahead of him as he taxis along the ground or approaches very near to the ground provided he is moving in the direction of the projection of the light. The component of illumination at right angles to the vertical and called the horizontal illumination is the

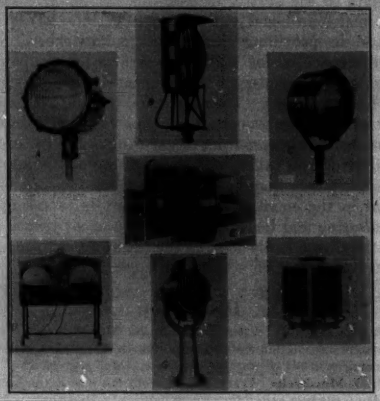

FIG. 9—TYPICAL FORMS OF AIRPORT FLOODLIGHTING EQUIPMENT

A. Incandescent Airport Floodlight. A 24-in. projector with chromium plated reflector and 45-deg. spread lens designed for use with 1500 watt and 3000 watt, 32-volt incandescent lamps.

B. Incandescent Airport Floodlight. Unit equipped with glass reflector designed to deliver a 180-deg. spread of light in the horizontal and limit the vertical spread to a few degrees. Lamp used may be either the 5-kw., 110-volt or 3000-watt, 32-volt incandescent lamp

C. Incandescent Airport Floodlight. Unit equipped with 25-in. mirrored glass reflector and spread convex heat-resisting lens of approximately 40-deg., 50-deg. or 80-deg. horizontal spread. Unit takes either the 1500-watt, 32-volt or 3000-watt, 32-volt incandescent lamp

D. Incandescent Airport Floodlight. Unit equipped with two 5-kw. or 10-kw. incandescent lamps, each in focus of 24 in. parabolic reflector. Beam spread in horizontal plane to about 80 deg. by diverging roundels.

E. Incandescent Airport Floodlight. Unit equipped with 14-3000-watt 32-volt incandescent lamps in the focal axis of a parabolic trough reflector The horizontal spread is 180 deg.

F. Airport Arc Floodlight. Unit equipped with a 150-ampere high intensity arc mechanism in the focus of a 21-element Fresnel lens. The horizontal spread is 180 deg. The 5-kw. or 10-kw. incandescent lamp may also be used with this unit.

G. Airport Arc Floodlight. Unit equipped with an 18-in. glass parabolic mirror and two front door glasses, one with clear glass and the other with 80-deg. spread lens. The spread lens may be swung back and the unit used as a high powered ceiling projector or as an auxiliary beacon. The lamp mechanism is automatic, operating at 55 amperes on a line voltage from 80 to 125 volts direct current

illumination which enables a pilot to see the ground from the air; it makes visible the area on which he is to land. If, therefore, a field is to be clearly visible from the air and at the same time objects and irregularities on the field are to be clearly discernible as the pilot approaches the field at low altitude for landing, the light must be distributed so as to provide adequate illumination in both the horizontal and vertical planes.

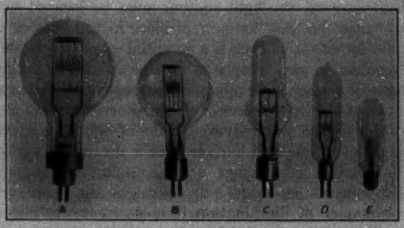

FIG. 10—INCANDESCENT LAMPS USED IN AVIATION SERVICE

A. 10-kw., 115-volt, G-80 lamp for base down burning
B. 5-kw., 115-volt, G-64 lamp for base down burning
C. 3000-watt, 32-volt, GT-38 lamp for base down burning
D. 1500-watt, 32 volt, T-24 lamp for base down burning
E. 1000-watt, 115-volt airway and airport beacon lamp

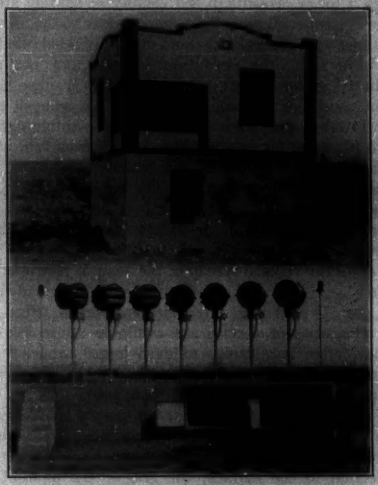

FIG. 11—METHODS OF INSTALLING FIELD FLOODLIGHTING EQUIPMENT

FIG. 12—NIGHT VIEW OF AIRPORT FLOODLIGHTED BY MEANS OF A SINGLE UNIT EQUIPPED WITH TWO 10-KW. INCANDESCENT LAMPS

Furthermore, the light sources must be located so as to cause the least amount of glare in the eyes of the pilot for excessive glare is a potential cause of accidents. Needless to say, simplicity in operation and maintenance together with reliability are essential characteristics of the light sources. Light sources located close to the ground and delivering a fan of light at a grazing angle over the field surface give a maximum vertical illumination component and a minimum horizontal component. In order to increase the horizontal component of illumination it is necessary to elevate the light sources and deliver the light in a more downward direction. An example of this type of lighting is found in the systems in use for railroad yards or other large areas where floodlights are grouped on towers from 75 to 120 ft. high. Such a tower is shown in Fig. 8. This method of lighting has to date been frowned upon for airports largely on account of the hazard which such a tower presents in the vicinity of landing areas. The system employing light sources close to the ground has met with general favor and may be divided into two divisions; *viz.*, the centralized system and the distributed system. In the former system a single large light source or a group of smaller light sources concentrated at one point is employed for obtaining the field illumination; in the latter a number of relatively smaller units placed at two or more locations around the landing area are employed. The equipment (Fig. 9) used for field illumination employs both arc and incandescent light sources and a variety of optical systems for obtaining a fan distribution of light. In all cases the vertical spread of the beam is restricted to a few degrees and the horizontal spread expanded to 45 deg. to 180 deg. Incandescent lamps (Fig. 10) for this class of service are available up to 10 kw. capacity; in fact, lamps of even larger capacity are being built but at the present time there is no commercial equipment in which they are used.

It is interesting to note a trend toward the study of the character of the field surface with respect to night illumination. Obviously, the higher the coefficient of reflection of the surface, the greater the amount of light reaching the observer's eye. Just as our highway executives have found it desirable to mark road boundaries and obstacles with white paint to make them conspicuous at night, so our airport engineers and managers will find that the judicious use of light surfaces and white paint will greatly assist in the discrimination of essential objectives at night.

In locating the field floodlighting equipment, consideration should be given to prevailing winds and the site for the floodlights selected so that ships landing under prevailing winds may land in the same direction as the direction of projection of the light or at right angles to this direction in order that the pilot may not have to face the light source. The topography of the field is also an important consideration if shadowed areas are to be avoided. Modern practise (Fig. 11) provides substantial structures for the field floodlighting equipment with ample provision for housing transformers, control apparatus, etc. A night view of an airport illuminated by a single unit is shown in Fig. 12.

Other aids to the aviator in the form of advertising beacons, roof signs, etc., are daily being added by industry and municipalities. Under the guidance of the Department of Commerce these land marks become helpful to the aviator in following his course. There are many problems still confronting the engineer engaged in providing means for guiding the airman. The menace of fog still offers serious handicaps and attempts to counteract it by means of light have met with little success. The neon light, thought to possess fog penetrating properties superior to other forms of illuminants, appears from recent tests of the Bureau of Standards and others not to have any advantage for this purpose. Some of the largest research organizations in the country are devoting study to means of facilitating and increasing the safety of air travel and we may confidently look to the future for solutions to the problems which now appear most difficult.

Discussion

F. C. Caldwell: I should like to ask Mr. Mahan if he has the latest information with regard to the use of the neon lamps as beacons.

P. D. Manbeck: Is the incandescent lamp or is the carbon arc lamp the best suited for airport lighting? Experiments have been conducted, I believe, with both, and I should like to ask the question, has the carbon arc a field in this work?

A. W. Lee, Jr.: (by letter) I am very glad to see that Mr. Mahan questions seriously the penetrating properties heretofore ascribed to the neon light. I believe its advantages have been greatly exaggerated.

I believe that under the flood lighting of airports another apparently feasible scheme should be mentioned and studied, especially in such ports as have permanent runways, and that is the installation of translucent slabs in the runways themselves, these slabs to be lighted from below by ordinary incandescent lamps.

The majority of aviators consider outline lights on the field of far greater importance than the flood lights,—in fact, many of them make a practise of landing without any flood light whatever. It therefore seems very reasonable that if each permanent runway has scattered throughout its length these translucent rectangles, the pilot will more easily know not only where to set his ship down but when to set it down.

The main point which I should like to see discussed is the statement, "other aids to the aviator in the form of advertising beacons, roof signs, etc., are daily being added by industry and municipalities."

Roof signs with arrows pointing to the field, advertising signs, etc., are very helpful, but there is a growing tendency to establish special revolving beacons on high office buildings and I believe this is one of the most dangerous practises which we have to face.

Even though such a beacon flashes in code or stops once in each revolution and points toward the field, it is obvious that many conditions occur (such as fog, sleet, rain, etc.) which prevent the pilot from hanging around to spell out the beacon's message. If a revolving beacon is to be the standard indication throughout the country that a pilot is on a regular airway or is coming to a registered landing field, all other flashing beacons should be strongly discouraged.

Just two months ago a pilot on one of our contract mail lines missed an office building by a scant few feet. He had seen the flash of a beacon and had mistaken it for a beacon at the city airport, swooping downward after passing it in the same manner that he usually landed at the airport. Three other pilots of the same company have also been misled by this beacon, but have not had such narrow escapes.

If this practise is continued, I feel certain that sooner or later one of our night pilots will fly head on into some office building or smoke stack.

May I ask that this criticism be fully discussed and weighed and that the members present at this regional meeting do everything in their power to persuade the aeronautical branch of the Department of Commerce to disapprove any such future installation.

D. K. Wright: (communicated after adjournment) The writer wishes to take this opportunity to mention a recent development in lamp making which promises to make a big improvement in airport floodlighting. The various incandescent lamps used for this purpose have a rated life of 100 hr. at an efficiency of approximately 24 lumens per watt. The limiting factor which has prevented the operating of these lamps at a higher efficiency has been the blackening of the bulb. The new development which is to be put into effect on April 1 is the placing of a small quantity of specially prepared granular tungsten powder inside the lamp before it is exhausted and filled with nitrogen gas. When the bulb blackens from use the lamp is removed from its socket, inverted, and shaken gently with a circular motion for a few seconds. The abrasive action of the tungsten powder removes the black deposit from the walls of the bulb, leaving the glass bulb clear again. It even improves the lamp, because it will not blacken so readily the next time it is used.

The cleaning of the bulb makes it possible to operate the lamp at a substantial increase in efficiency. Higher efficiency in lamps of this type is of immense value to the floodlighting of landing fields, because the higher efficiency means a more than proportional increase in brilliancy of the filament and this greater brilliancy means, in turn, a beam of higher intensity and greater effectiveness.

The new development is to be incorporated in the 1500- and 3000-watt, 32-volt lamps, and the 5000- and 10,000-watt, 110-115-120-volt, airport floodlighting lamps.

W. T. Harding: (communicated after adjournment) Inasmuch as I have had charge of the development and standardization of night lighting equipment for the past three years for the U. S. Army Air Corps, I should like to bring out several points not clearly developed in the paper, *Lighting of Airways and Airports*.

The Air Corps is not charged with the development of airways equipment as that is a function of the Department of Commerce. Consequently, that part of the paper will be passed over at once, although, as a matter of historical interest, the Air Corps established the first lighted airways in this country.

The Air Corps, however, is vitally interested in the development of airdrome lighting equipment, and has done, and is still doing pioneer work along this line. Almost without modification the requirements for airport lighting as laid down by the Department of Commerce, were based upon those established and used by the Air Corps prior to the Civil Air Commerce Act of 1926. At the present time the major differences are merely in manner of installation as the Air Corps is interested in operation in time of war and is preparing to add auxiliary emergency equipment.

The Air Corps uses a beacon similar to that described merely because at the present time that beacon is the best available. These beacons, however, are essentially fair-weather aids. When needed most they are badly lacking in range and are hard to locate. Several other types have been developed, both incandescent and neon, but so far they have shown no better solution to the problem. The Air Corps and the Guggenheim Association are at present at work upon the problem of fog penetration at Wright Field and on completion of the experiments the problem of development of a beacon for fog will no doubt be in order.

With respect to wind indication, the Air Corps uses the wind tee. The indication of direction at night is made by the lamps themselves, covered with a green globe, instead of reflected light from the sides. From the air it appears as a green "T". Wind velocity is not indicated. The wind cone is not satisfactory even though illuminated internally or externally, as the three-dimensional position of an airplane at night coupled with lack of perspective, may give an incorrect indication.

The boundary light also is of value in landing or taking off, as a pilot may keep a straight course by using the light as a landmark and thus prevent ground looping. The Air Corps uses series circuits with individual lamp transformers for each light or group of lights. By this method the high voltage is kept underground.

With respect to floodlighting, a wide diversity of opinion prevails in the Air Corps. However, standardization has proceeded to a point where two types of installations are being made:

(a) *Primary Fields.* For airdromes where formation landings are made a (centralized) single source unit is used. This floodlight consists of a 180-deg. fresnal lens equipped with a 10-kw. lamp, spherical reflector, and automatic lamp changer and starter. The Air Corps has developed a special housing with automatically operating shutter. Two units are employed on a given field so placed that one or the other is used, depending upon wind direction.

(b) *Secondary Fields.* For airdromes where single ship landings are contemplated, the distributed type of lighting is used.

High mounting of floodlights is not recommended as the beam in misty or foggy weather curtains the ground and makes it hard to discern the exact location when landing. The floodlights should be mounted as low as the contour of the ground permits.

H. E. Mahan: Professor Caldwell has asked about the use of the neon tube for beacons. As far as can be determined, the neon light does not possess any fog-penetrating properties not possessed by the incandescent lamp. The Bureau of Standards has made rather extensive tests in this connection which are reported in the Bureau of Standards bulletin.[1] The form of the neon tube does not lend itself to projection equipment such as the standard form of airway beacon, and where used it generally takes the form of tubes built up to form a cylindrical or truncated conical structure. Its color is distinctive as contrasted with the ordinary lights of a city and to this extent is conspicuous, although similar results may be obtained by the use of color screens on incandescent lamps.

It is often stated that in clear weather the beacon is not needed and in foggy weather it cannot be seen. I think, however, that all agree that the beacon serves a real purpose, for in clear weather its distinctive flashes identify the airport and the airway and it is only in very heavy weather that it is obscured entirely. As far as fog is concerned, it is true that we have not made much progress in overcoming this handicap and there does not appear to be a definite solution in sight. Undoubtedly, the radio beacon is the most promising. A certain amount of success has attended attempts at fog dissipation, but I am afraid this would be very expensive and not altogether a practical solution.

Regarding the incandescent versus the arc lamp, the arc lamp is an excellent light source for any projection equipment and its color is in favor with many pilots. Its principal disadvantage has been that it requires direct current and an operator. The simplicity and flexibility of the incandescent lamp have brought this type of light source into very general favor although some of the larger airports are using arc equipments.

1. *Technical News Bulletin*, 1928, p. 156.

Iron Losses in Turbine Generators

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and

J. F. CALVERT*
Associate, A. I. E. E.

Synopsis.—*In large turbine generators there is no convenient method for determining the iron losses under load. It is the purpose of this paper to show, analytically, the factors influencing the change in iron loss with load. While very definite results are given for a simplified machine, it is not recommended at this time that any change be made in the A. I. E. E. Standards for computing load loss, because there is much to be done before the work can be considered complete.*

This particular problem is typical of many which are entirely suitable for purely analytical study, and should furnish very desirable work for investigators who do not have access to large machines for extensive tests, but who desire to carry on studies related to this type of apparatus. This, as well as similar loss problems, can be studied much more readily by theoretical work and model tests, than from measuring the total losses of an entire machine, because the value sought is never a predominant proportion of the total value measured; and even if satisfactory results be obtained on a particular machine, they are not generally applicable to all such apparatus.

In this article, it is shown that on the basis of the flux distributions, the machines can be most conveniently divided into end zones, which present three dimensional problems and a large central zone, which presents only two dimensional problems. The losses in each part are discussed qualitatively, and the losses in the central zone are obtained quantitatively for an ideal machine in which only the fundamentals of the flux waves are present. It is shown that, contrary to the usual view, these losses do not vary as the square of the generated voltage, but are predominantly affected by the ratio of the slot to total leakage reactance. The results are given in curves. The complications in the problem introduced by the usual commercial designs by non-sinusoidal field forms, phase bands, etc., are discussed qualitatively.

* * * * *

THE losses which occur in the magnetic circuit of a synchronous machine depend upon the weight, magnetic properties, and physical shape of the iron and upon the arrangement and current values of the conductors. The total value of the no load iron losses can be measured with a satisfactory degree of accuracy, but no simple method is available for obtaining a direct measurement of these losses under load conditions. A method for obtaining the rate of loss

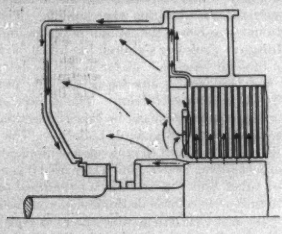

FIG. 1—FLUX DISTRIBUTION IN CENTER PLANE OF THE POLE-ROTOR EXCITED ALONE

at any point in the machine by means of temperature time curves was described by the writers in their paper on *Additional Losses of Synchronous Machines.*[1] Practical results can be obtained by this method, but it requires care and a large number of readings to permit the calculation of the total losses in the iron parts of the machine which is being tested.

The writers are not aware that anyone has presented an analytical study of the magnetic circuit losses which exist in synchronous machines under operating conditions. It is the purpose of this article to illustrate the flux distributions and iron loss conditions for certain principal parts of the commercial machines with the aid of diagrams and to determine analytically the ratio of the magnetic losses which exist under various load conditions to those which exist at no load for the case of an ideal, or simplified generator.

A better conception of the factors producing iron loss in turbine generators can be obtained by investigating the flux distributions in the different parts of the magnetic circuit. The end zones will be discussed briefly and then the central zone in greater detail. Figs. 1, 2, and 3 indicate diagramatically the direction of the flux in the plane passing through the center lines of the magnetic poles. There is a convenient division of the fields into end zones and central zone. In the end zone it is necessary to consider the flux distribution as a three-dimensional problem whereas in the central zone it can be considered as a two-dimensional problem.

It is necessary to decide between just what two radial planes two-dimensional conditions may be assumed. An estimate of this was made by assuming a simplified magnetic circuit as shown in Fig. 4. The magnetization curves for the paths were made as nearly equal to those existing in an actual machine as could be computed. The axial permeances were concentrated at the stator side of the air gap as shown, thus making the assumptions as pessimistic as possible. It was found that when the end leakage flux was something like fifty per cent of the radial air gap flux per packet, the influence of the end zone conditions became negligible four or five packets toward the center. This was thought to be worse than the actual case because effective stepping of the end packets reduces these in-

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1. For references see Bibliography.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

29-49

equalities to a minimum. The fact that the end packets and central packets do not vary greatly in temperature indicates that the penetration is not more than two or three packets under normal conditions. Fig. 5 has been drawn in an attempt to give a three-dimensional picture of these conditions. It is probably reasonable to assume that the end effects are confined to the first three packets on each end and that the large central zone presents only two-dimensional problems.

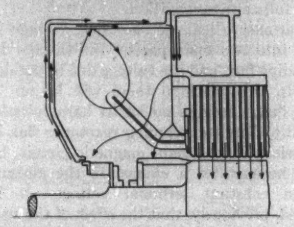


Fig. 2—Flux Distribution in the Center Plane of the Pole, Stator Excited Alone

The flux distribution in the end bell air space, due only to the rotor m. m. f., is influenced predominantly by the rotor retaining ring saturation and to a certain extent by the saturation in the stator iron. The latter causes a leakage flux from the frame to the bell. In this enclosed space there is a revolving field which produces losses in the metal coil braces and somewhat higher local losses in the finger plates. The iron end bells form parallel magnetic paths for the core and carry

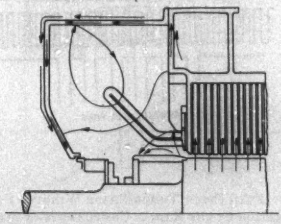
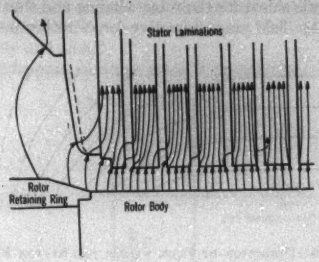


Fig. 3—Flux Distribution in the Center Plane of the Pole, Both Rotor and Stator Windings Excited at the Full-Load, Zero Power Factor, Over-Excited Condition

revolving fields of the distribution shown in Fig. 6. The losses in all of the solid parts are produced by currents which are near the surface of the iron, and consequently depend as much or more on the area than on the volume of material.

When the armature is excited alone there is a rotating flux in the end bell iron and in the enclosed air space. However, these flux distributions do not always maintain a constant shape, and therefore do not behave quite as simple revolving fields. This is due partly to the chording effect of diamond coil end windings, partly to the grouping of conductors in phase bands and to the normal chording of the machine, and partly to the influence of the phase connecting rings.

The flux studies indicate that the armature is probably a more important factor in causing local heating



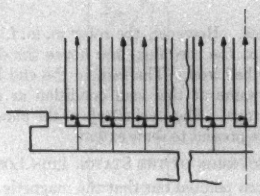


Fig. 4—Paths Assumed in Estimating the Axial Penetration of End Flux

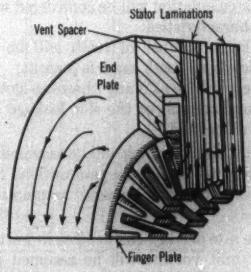


Fig. 5—Flux in the Stator End Parts

in the finger plate than is the field. The radial components of the slot leakage flux exist only at the end of phase bands. If the winding is full pitch (Fig. 7), or if the pitch is an even multiple of the reciprocal of the number of phases (as 1/3, 2/3, 3/3, 4/3, etc., for a three-phase winding) then this radial flux is concentrated in one tooth per phase per pole. This condition may produce high densities and high local temperatures

in some parts of the machine if proper precautions are not taken. For other coil pitches, the radial components of slot leakage flux occur in one tooth per phase per pole for each winding layer, and usually are not factors of importance.

At the zero per cent power-factor over-excited conditions, the armature predominates within the end bell enclosure, because the retaining ring acts as a partial magnetic shield for the rotor winding and thus renders the rotor field magnetomotive forces rather ineffective

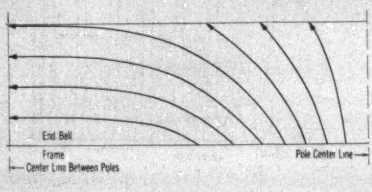


Fig. 6—Direction of Flux within the Stator End Bell Iron

in this space. However, the rotor m. m. f. establishes the gap flux, the core flux, and hence the distribution in the end bell iron. The field in the end bell iron is about the same at this load condition as at no load normal voltage. The local effects of the phase connecting rings are present to some extent.

Discussion of the Stator Iron Losses

It has been pointed out that the magnetic conditions in a turbine generator may be divided most conveniently into two parts. It is the large central part of the stator iron which will be discussed in this article. An ideal machine for calculation will be considered which has the following characteristics:

(1) A large number of teeth will be assumed so that practically no tooth taper is present;

(2) There will be as many phases as slots per pole so that the fundamental of the slot leakage flux may be considered alone; and

(3) A full pitch winding will be assumed.

In addition to the above assumptions for a special machine, the following general assumptions will be made:

(4) The rate of eddy-current loss in watt seconds per second per unit volume will be assumed proportional to the square of the instantaneous value of the resultant flux density in the laminated iron;

(5) The hysteresis loss per cycle will be assumed proportional to the square of the maximum flux density for the conditions encountered in commercial machines; and

(6) No allowance will be made for the redistribution of the flux in the main magnetic circuit due to changes in permeability between no-load normal voltage and full-load normal voltage conditions.

It is intended to consider the factors involved by the variations in the stator teeth and core loss on load as compared with the no-load conditions, due to the fundamental of the cross slot leakage flux and the increase or decrease in the air-gap flux. On all loads which require over excitation, the air-gap flux must be increased to produce a generated voltage equal to the vector sum of the terminal voltage and the armature leakage reactance voltage. Conversely, for practically all under excited conditions the air-gap flux is less at the load than at the no-load full voltage condition.

The armature leakage reactance voltage is usually divided into two main parts,—the voltage due to the end winding flux, and the voltage due to the slot leakage flux. (In this machine there will be no "tooth tip leakage flux," because no higher harmonics have been assumed.) The end winding reactance flux influences the densities in the main magnetic circuit, because it usually necessitates a change in the air-gap flux to maintain constant terminal voltage. It has been explained, previously, that the effect of the end winding flux, as such, can be neglected when considering densities in the central part of the stator iron. The reactance produced by the armature slot leakage flux also necessitates a change in the air-gap flux in order that constant terminal voltage shall be maintained. However, the paths of the latter flux (Fig. 8) are through the same stator iron parts as are occupied by the air-gap flux.

Loss in the Core

It is now possible to estimate the relative losses in the core back of the teeth on the basis of the foregoing

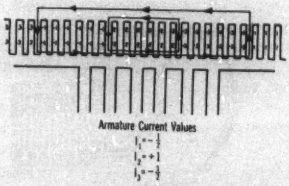


Fig. 7—Full Pitch Three-Phase Winding Showing the Concentration of the Radial Components of Slot Leakage Flux at the Ends of Phase Bands

assumptions. Both the air-gap flux, ${}_1\Phi_a$, and the radial component of the slot leakage flux, $2\,\Phi_s$, have sinusoidal distributions at some circumferential line $d-d$ as shown in Figs. 8 and 9. These components of flux combine to give a resultant sinusoidal flux wave so that on the basis of assumptions 4, 5, and 6, it follows that the ratio of core loss on load to that on no load must be

$$\frac{{}_1W_c}{{}_0W_c} = \left\{ \frac{{}_1\dot{\Phi}_a + \overline{2\,\dot{\Phi}_s}}{{}_0\Phi_a} \right\}^2 \qquad (1)$$

In this case the vector quantity $\overline{2\,\dot{\Phi}_s}$, is in phase with

the maximum radial density due to the cross slot flux.

Loss in the Teeth

The changes in tooth loss with load are much more difficult to describe than the core loss but can be illustrated with drawings. The difficulty is that the air-gap and slot leakage flux distributions do not have the same general shape in the teeth, whereas they did have in the

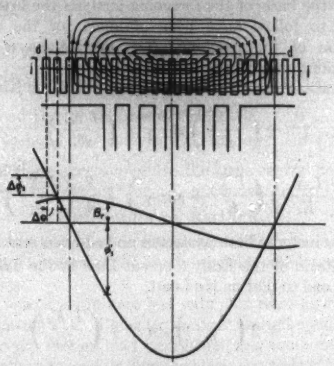


Fig. 8—Assumed Distribution of Armature Slot Leakage Flux

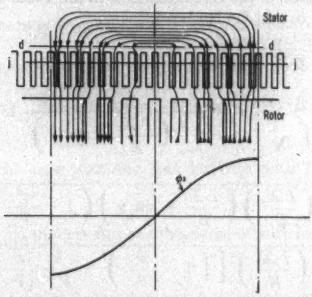


Fig. 9—Assumed Distribution of Air-Gap Flux

core. In the teeth, the armature windings establish both tangential and radial densities while the air-gap field produces only radial densities. See Figs. 8 and 9. Hence the tangential density and the resultant radial density must be known at every point in the tooth throughout one cycle (see Fig. 10 for the component densities at one point in a tooth). Obviously, at every instant of time, the resultant density squared is equal to the sum of the squares of the tangential and the resultant radial densities.

$$\beta_T^2 = \beta_R^2 + \beta_s^2 \qquad (2)$$

The rate of eddy current loss per unit volume per cycle is proportional to the mean square value of the resultant density, $\frac{1}{\pi}\int_{\theta=0}^{\theta=\pi} \beta_T^2\, d\,\theta$, but,

$$\frac{1}{\pi}\int_0^{\pi} \beta_T^2\, d\,\theta = \frac{1}{\pi}\int_0^{\pi} \beta_R^2\, d\,\theta + \frac{1}{\pi}\int_0^{\pi} \beta_s^2\, d\,\theta \qquad (3)$$

Hence the eddy current loss due to the density components displaced 90 deg. in space may be computed separately and added.

The hysteresis loss per cycle is proportional to the

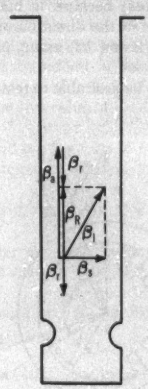


Fig. 10—Tangential, Radial, and Resultant Flux Densities at One Point in a Stator Tooth at One Instant of Time

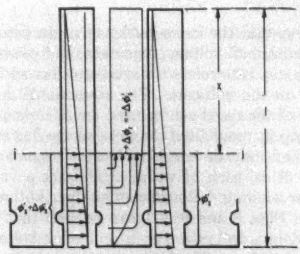


Fig. 11—Tangential and Radial Armature Slot Leakage Flux

square of the maximum density, or β_{Tm}^2. The vectors β_R and β_s may have almost any particular time-phase relation, so that to determine the hysteresis loss a relation between the latter vectors is necessary to determine β_{Tm}^2.

Before proceeding to a quantitative discussion of these densities and the resultant losses, it would be well to

consider the angular or time relations between these values at any one point in a stator tooth. A machine will be assumed which has a counter clockwise rotation of the field. The simple Potier diagram for a non-salient pole generator carrying a reactive load, is shown in Fig. 12, with the density vectors under consideration superposed on the drawing. The radial density due to the air-gap flux, ${}_1\beta_a$, is in phase with this flux. The radial component of density due to the slot leakage flux, β_r, must be in phase with the armature demagnetizing flux, Φ_{dm}. The tangential density, β_s, due to the slot leakage flux is shown leading the radial value, β_r, by 90 electrical degrees; because it has been chosen to consider β_s positive in the direction opposite to that of rotation (see Figs. 8 and 12) and β_r positive in the outwardly radial direction.

Plainly, it would be desirable to relate the densities to

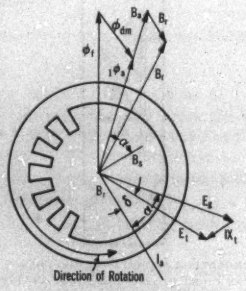


FIG. 12—VECTOR DIAGRAM FOR NON-SALIENT POLE A-C. GENERATOR

voltages so that the loss equations may be expressed in terms of terminal voltage, reactance, and power factor. The first step is to relate the cross-slot flux and the air-gap flux to the voltages. The circumferential line at the base of the teeth will be used for reference. All of the air-gap flux and all of the slot leakage flux cross this line. They may be considered as two synchronously rotating fields, each of which establishes a voltage in the stator winding. Roughly, their distributions are as shown in Figs. 8 and 9. It can be seen that the slot leakage field is less effective in producing voltage than is the air-gap flux. The former enters at the base of the teeth, but does not link all of the conductors; whereas the air-gap flux has the same value at the gap as at the base of the teeth, and links more armature turns for a given number of flux lines. It may be written that,

$$K_1\left(\frac{I\,X_s}{E_g}\right) = \frac{2\,\Phi_s}{{}_1\Phi_a} \tag{4}$$

where K_1 is a constant greater than one. Since a full pitch winding has been assumed, the density due to the cross slot flux varies directly with x (Fig. 11) in every slot. The constant, K_1, is the ratio of the flux turn interlinkages per slot ampere which would obtain for any one slot if all of the existing slot flux linked all of the turns in the slot to the flux turn interlinkages per slot ampere which actually exist. It can be shown (see Appendix) that $K_1 = \frac{3}{2}$.

On the basis of the foregoing analysis the loss equations to follow have been obtained for the central portion of the machine. The derivations of these equations are given in the Appendix.

I. Ratio of the Core Loss on Load to that on No Load.

$$\frac{{}_1W_c}{{}_0W_c} = 1 + 2\left(\frac{I\,X_t}{E_t}\right)\sin\gamma + \left(\frac{I\,X_t}{E_t}\right)^2 - 3\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\gamma\right) + \frac{9}{4}\left(\frac{I\,X_s}{E_t}\right)^2. \tag{5}$$

This includes both hysteresis and eddy current loss.

II. Ratio of the Eddy Current Loss in the Teeth on Load to that on No Load.

$$\frac{{}_1W_{et}}{{}_0W_{et}} = 1 + 2\left(\frac{I}{E_t}\right)\sin\gamma + \left(\frac{I\,X_t}{E_t}\right)^2 - 2\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\gamma\right) + \left(\frac{I\,X_s}{E_t}\right)^2\left(\frac{3}{K_3^2} + \frac{6}{5}\right) \tag{6}$$

III. Ratio of the Hysteresis Loss in the Teeth on Load to that On No Load.

$$\frac{{}_1W_{ht}}{{}_0W_{ht}} = \frac{1}{2}\,\frac{{}_1W_{et}}{{}_0W_{et}} + \frac{1}{2}\int_{\frac{x}{l}=0}^{\frac{x}{l}=1}\sqrt{\left\{\cos^2\gamma - \left(\frac{I\,X_t}{E_t} + \sin\gamma\right)^2 + 3\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\gamma\right)\left(1 - \frac{x^2}{l^2}\right) - \frac{9}{4}\left(\frac{I\,X_s}{E_t}\right)^2\left[\left(1 - \frac{x^2}{l^2}\right)^2 - \frac{4}{K_3^2}\,\frac{x^2}{l^2}\right]\right\}^2 + \left\{3\left(\frac{I\,X_s}{E_t}\right)\left(1 - \frac{x^2}{l^2}\right) - 2\left[\frac{I\,X_t}{E_t} + \sin\gamma\right]\right\}^2\cos^2\gamma}\;d\left(\frac{x}{l}\right) \tag{7}$$

It should be remembered that certain design constants must be involved in the numerical constants of these equations. For instance, the ratio of the iron area for the cross slot flux to that for the radial flux must be introduced, and it depends upon the physical proportions of the machine. These constants are expressed algebraically in the Appendix.

Curves based on the above equations are shown in Figs. 18 to 29 inclusively. Figs. 30 and 31 give the total variations in iron loss with load as functions of power factor. Various ratios of slot to total reactance and two values of total per cent reactance are chosen as shown. Certain ratios for the loss on no load were assumed, as follows:

$$\frac{\text{No-Load Core Loss}}{\text{Total No-Load Iron Loss in the Machine}} = \frac{{}_0W_c}{{}_0W_T} = 0.6$$

$$\frac{\text{No-Load Eddy Current Loss in the Teeth}}{\text{Total No-Load Iron Loss in the Machine}} = \frac{{}_0W_{el}}{{}_0W_T} = 0.1$$

$$\frac{\text{No-Load Hysteresis Loss in the Teeth}}{\text{Total No-Load Iron Loss in the Machine}} = \frac{{}_0W_{hl}}{{}_0W_T} = 0.3$$

All of these curves apply to the large central zone in the stator, and are for four-pole machines.

Discussion of Results

To analyze the curves, it is convenient to consider the loss ratios in terms of the revolving fields which produce the losses.

The variations in core loss with flux were expressed by Equation (1). Consider an over-excited condition. The air-gap flux on load (and hence this component of the core flux on load) is increased over the no-load value, in the ratio of the generated to the terminal voltage. However, the core flux on load is decreased by the vector addition of the slot leakage flux. The slot leakage reactance may be considered $\frac{3}{2}$ times as effective in decreasing core densities, as the total reactance was in increasing these densities. Consequently, if $\frac{I\,X_s}{I\,X_t} < \frac{2}{3}$, the core flux and loss increase with load; if $\frac{I\,X_s}{I\,X_t} > \frac{2}{3}$, the core flux and loss decrease with load; and if $\frac{I\,X_s}{I\,X_t} = \frac{2}{3}$, the core flux and loss remain constant. The first two conditions are reversed if $E_g < E_t$, *i. e.*, at most under-excited conditions. The last condition is true at all power factors, namely, that the core loss does not vary with load when the ratio of slot to total reactance is two-thirds. The 100 per cent power factor case is somewhat unique. The reactance and terminal voltages add at right angles. Hence if the ratio $\frac{I\,X_s}{I\,X_t}$ is either greater or less than $\frac{2}{3}$, the core flux and loss must be greater on load than on no load.

In the teeth there are three components of flux which produce loss. The radial components of slot leakage flux combines with the air gap flux. They add in the radial direction, and give a resultant radial density producing loss at every point. The radial flux components corresponding to the reactance vectors $I\,X_s$ and $I\,X_t$ are about equally effective in producing eddy current loss. When $\frac{I\,X_s}{I\,X_t}$ approaches zero, the loss ratio for the teeth must increase with $I\,X_t$ just as it did for the core. When $\frac{I\,X_s}{I\,X_t}$ approaches unity, there is less variation in loss with load due to the radial density, but the tangential densities produce an appreciable value which must be added arithmetically to obtain the total loss. For any value of $\frac{I\,X_s}{I\,X_t}$, the change in loss on load over no load is a function of $I\,X_t$.

On the under excited conditions, it is well to consider the extreme ratios of $\frac{I\,X_s}{I\,X_t}$. As this ratio approaches zero, the eddy-current loss becomes predominantly influenced by the air-gap flux, and the loss ratio decreases with increase in $I\,X_t$ for fixed values of $\frac{I\,X_s}{I\,X_t}$. As $\frac{I\,X_s}{I\,X_t}$ approaches unity the tangential flux adds very considerably to the density, but the effects of $I\,X_s$ and $I\,X_t$ on the losses due to the radial densities more nearly nullify each other. The aggregate result is an increase in loss with $I\,X_t$.

In considering the variations in the hysteresis loss with load, it is interesting to note that the tangential densities are zero at the instant the radial densities are maximum, if the machine is operating at zero per cent power factor, either over or under excited. Consequently, the hysteresis loss at these operating conditions must be proportional only to the square of the radial densities. As $\frac{I\,X_s}{I\,X_t}$ approaches zero the hysteresis loss ratio must approach the corresponding ratios for the total loss in the core and for the eddy current loss in the teeth. When $\frac{I\,X_s}{I\,X_t}$ approaches unity, the ratio of the hysteresis loss in the teeth differs from the ratio of eddy current loss in the teeth, and from the ratio of total losses in the core. When the current is in phase with the voltage, the tangential density and the resultant radial density attain their maximum at almost the same instant of time for all points in the teeth. The maximum radial density squared is about equal to the sum

of the squares of the maximum radial and tangential densities; and it is nearly proportional to the sum of the squares of the effective values of the latter densities. Thus the variation in the hysteresis loss in the teeth will be practically the same as the variation in the eddy current loss, when the current is in phase with the generated voltage. This is almost the case at the 100 per cent power-factor condition. (Compare Figs. 24 and 28.)

General Conclusions

In the central portion of the ideal generator (which has only fundamental flux waves), the ratios of iron loss on load to those of no load are not only functions of total reactance and power factor, but also of the ratio of slot to total reactance.

The actual loss in these parts is considerably less at operating conditions than is calculated by the usual method wherein the assumption is made that the iron loss varies as the square of the generated voltage. This is illustrated in Figs. 30 and 31. If a machine has a ratio of slot to total reactance of about 0.7, there is practically no variation in iron loss with load.

The quantitative analysis of the variations in iron loss for an actual commercial machine is vastly more complicated than for the ideal generator in the central portion of the stator, and some of the factors cannot be considered at all. (Some explanation of the probable difficulties to be met in the commercial case are outlined toward the end of the Appendix.)

It is possible to make analytical calculations for the loss in certain of the end zone parts. For instance, if the flux distribution in the end-bell iron can be established, a two dimensional problem may be assumed within the metal. However, the effect of the axial flux in the stator fingers, end plates, and first two or three packets, probably can only be properly investigated by tests which will permit the separation of the end zone from the central zone losses. Tests on machines of different lengths could be used if it could be established that the loss per inch axially in the central zone was substantially the same in each test; and not at variance due to building differences.

Acknowledgments

The writers are indebted to Messrs. L. A. Kilgore, A. M. Harrison, and H. C. Myers of the Power Engineering Dept. for valuable theoretical assistance and for calculating data for the curves.

Appendix

Preliminary Derivations.

The constant K_1 was defined by Equation (4) and the accompanying descriptive matter. Referring to Fig. 11 it will be seen that

$$d\,\varphi_s = K_2\,L\,\beta_{sm}\,\frac{x}{l}\,d\,x \tag{8}$$

$$n = N\,\frac{x}{l} \tag{9}$$

$$K_1 = \frac{N\,\varphi_s}{K_2\,L\int_0^l \frac{N\,x}{l}\,\beta_{sm}\,\frac{x}{l}\,d\,x} \tag{10}$$

$$K_1 = \frac{3}{2}\,\frac{N\,\varphi_s}{K_2\,L\,N\,\beta_{sm}\,\frac{l}{2}} \tag{11}$$

but,

$$\varphi_s = K_2\,L\,\beta_{sm}\,\frac{l}{2} \tag{12}$$

Hence,

$$K_1 = \frac{3}{2} \tag{13}$$

And rewriting Equation (4)

$$\frac{3}{4}\,\frac{I\,X_s}{E_g} = \frac{\Phi_s}{{}_1\Phi_a} \tag{14}$$

and since

$$\frac{{}_1\Phi_a}{{}_0\Phi_a} = \frac{E_g}{E_t} \tag{15}$$

then

$$\frac{3}{4}\,\frac{I\,X_s}{E_t} = \frac{\Phi_s}{{}_0\Phi_a} \tag{16}$$

From Fig. 12 it may be seen that

$$E_g = \sqrt{E_t^2\cos^2\gamma + (I\,X_t + E_t\sin\gamma)^2} \tag{17}$$

then

$$E_g = \sqrt{E_t^2 + 2\,(I\,X_t)\,E_t\sin\gamma + (I\,X_t)^2} \tag{18}$$

and

$$\sin\alpha = \frac{I\,X_t + E_t\sin\gamma}{E_g} \tag{19}$$

$$\cos\alpha = \frac{E_t\cos\gamma}{E_g} \tag{20}$$

$$\sin 2\,\alpha = \frac{2\,[E_t\,(I\,X_t)\cos\gamma + E_t^2\sin\gamma\cos\gamma]}{E_g^2} \tag{21}$$

$$\cos 2\,\alpha = \frac{E_t^2\cos^2\gamma - (I\,X_t + E_t\sin\gamma)^2}{E_g^2} \tag{22}$$

$$\sin\gamma = \sqrt{1 - \cos^2\gamma} \tag{23}$$

$$\cos\gamma = \text{power factor} \tag{24}$$

In order to express the losses in terms of voltages it is necessary to relate the voltages and densities produced by the flux. In accordance with Equation (15)

$$\frac{{}_1B_a}{{}_0B_a} = \frac{E_g}{E_t} \tag{25}$$

From Fig. 11

$$B_{sm} = \frac{2\,\Phi_s}{K_2\,L\,l} \tag{26}$$

$$_0B_a = \frac{A_p}{A_i}\,\frac{2\,P}{.636\,\pi\,D\,L}\,{}_0\Phi_a = \frac{A_p\,P}{A_i\,D\,L}\,{}_0\Phi_a \tag{27}$$

Dividing Equation (26) by (27)

$$\frac{B_{sm}}{_0B_a} = \frac{A_i\,D}{A_p\,2\,K_2\,l\,P}\,3\left(\frac{I\,X_s}{E_i}\right) \tag{28}$$

Let

$$K_3 = \frac{A_p\,2\,K_2\,l\,P}{A_i\,D} \tag{29}$$

Then

$$\frac{B_{sm}}{_0B_a} = \frac{3}{K_3}\,\frac{I\,X_s}{E_i} \tag{30}$$

The constant K_3 depends upon the physical proportions of the machine. In the calculations it was found to be nearly constant for two-pole machines, and of another nearly constant value for four-pole turbine generators of modern design. Two average values were used accordingly—one for two poles and one for four-pole alternators.

Quantitative Consideration of the Stator Iron Loss

A. *Core Loss*

Equation (1) stated that the core loss ratio was

$$\frac{_1W_c}{_0W_c} = \left\{\frac{_1\dot{\Phi}_a + \overline{2\,\dot{\Phi}_s}}{_0\Phi_a}\right\}^2 \tag{31}$$

From Fig. 12 it can be written that

$$\{_1\dot{\Phi}_a + \overline{2\,\dot{\Phi}_s}\}^2 = [_1\Phi_a - 2\,\Phi_s \sin\alpha]^2 + [2\,\Phi_s \cos\alpha]^2 \tag{32}$$

(It should be remembered that α is positive when the current lags the generated voltage.)

From the preceding equations

$$\frac{_1W_c}{_0W_c} = \frac{_1\Phi_a^2 - 4\,{}_1\Phi_a\,\Phi_s \sin\alpha + 4\,\Phi_s^2}{_0\Phi_a^2} \tag{33}$$

In order to express this loss equation in terms of voltage and reactance it is only necessary to refer to Equations (15) and (16). Then

$$\frac{_1W_c}{_0W_c} = \frac{E_g^2 - 3\,E_g\,(I\,X_s)\sin\alpha + \frac{9}{4}(I\,X_s)^2}{E_i^2} \tag{34}$$

However, E_g and $\sin\alpha$ are cumbersome to use and by the application of Equations (18) and (19) to the last form it follows that,

$$\frac{_1W_c}{_0W_c} = 1 + 2\left(\frac{I\,X_i}{E_i}\right)\sin\gamma + \left(\frac{I\,X_i}{E_i}\right)^2$$

$$- 3\left(\frac{I\,X_s}{E_i}\right)\left(\frac{I\,X_i}{E_i} + \sin\gamma\right) + \frac{9}{4}\left(\frac{I\,X_s}{E_i}\right)^2 \tag{35}$$

If $\frac{I\,X_s}{E_i}$ and $\frac{I\,X_c}{E_i}$ are expressed as decimals, the formula is greatly simplified for calculation.

B. *Eddy Current Loss in the Teeth*

Equations (3) gave the general relation for the instantaneous densities and the resultant rate of eddy current loss at any one point in the teeth.

In Equation (2),

$$\beta_T^2 = {}_1\beta_a^2 + \beta_s^2 \tag{2}$$

By Fig. 10

$$\beta_T^2 = ({}_1\beta_a + \beta_r)^2 + \beta_s^2 \tag{36}$$

The values must be expressed as functions of θ and x before a quantitative solution can be obtained.

From Equation (8)

$$\frac{1}{K_2\,L}\,\frac{d\,\varphi_s}{d\,x} = \beta_s = B_{sm}\,\frac{x}{l}\cos\theta \tag{37}$$

Referring to Fig. 11 it may be seen that the decrease in cross slot flux below any circumferential line $j - j$ in the teeth must travel radially in the same distance. When the chosen positive directions of flux and rotation are taken into consideration also, it may be written that

$$\frac{d\,\varphi_s'}{d\,\theta} = -\frac{d\,\varphi_r}{d\,\theta} \tag{38}$$

From Equation (37)

$$\varphi_s' = K_2\,L\int_x^l B_{sm}\,\frac{x}{l}\cos\theta\,d\,x \tag{39}$$

$$\varphi_s' = K_2\,L\,B_{sm}\,\frac{(l^2 - x^2)}{2\,l}\cos\theta \tag{40}$$

$$\frac{d\,\varphi_r}{d\,\theta} = K_2\,L\,B_{sm}\,\frac{(l^2 - x^2)}{2\,l}\sin\theta \tag{41}$$

$$\beta_r = \frac{d\,\theta}{d\,A_i}\cdot\frac{d\,\varphi_r}{d\,\theta} \tag{42}$$

$$\frac{d\,\theta}{d\,A_i} = \frac{A_p}{A_i}\,\frac{2\,\pi\,P}{\pi\,D\,L} = \frac{K_3}{l\,K_2\,L} \tag{43}$$

Then,

$$\beta_r = K_3\,B_{sm}\,\frac{1}{2}\left(1 - \frac{x^2}{l^2}\right)\sin\theta \tag{44}$$

Since tooth taper is neglected, and in accordance with Fig. 12

$$_1\beta_a = {}_1B_a\cos(\theta + \alpha) \tag{45}$$

Substituting Equations (37), (44), and (45) in (36),

$$\beta_T^2 = {}_1B_a^2\cos^2(\theta + \alpha)$$

$$+ K_3\,({}_1B_a)\,B_{sm}\left(1 - \frac{x^2}{l^2}\right)\cos(\theta + \alpha)\sin\theta$$

$$+ B_{sm}^2 K_3^2 \frac{1}{4}\left(1 - \frac{x^2}{l^2}\right)^2 \sin^2\theta$$

$$+ B_{sm}^2 \frac{x^2}{l^2}\cos^2\theta \tag{46}$$

The eddy current loss in the teeth on load must be

$$_1W_{et} = \frac{K_e V}{l}\int_{x=0}^{x=l}\left\{\frac{1}{2\pi}\int_{\theta=0}^{\theta=2\pi}\beta_T^2\, d\theta\right\} dx \tag{47}$$

The eddy current loss in the teeth on no load must be

$$_0W_{et} = \frac{K_e V}{2\pi}\int_{\theta=0}^{\theta=2\pi} {}_0B_a^2\cos^2(\theta+\alpha)\, d\theta = \frac{K_e V\,({}_0B_a^2)}{2} \tag{48}$$

The ratio of the eddy current loss on load to that on no load is

$$\frac{_1W_{et}}{_0W_{et}} = \frac{2}{l({}_0B_a^2)}\int_{x=0}^{x=l}\left\{\frac{1}{2\pi}\int_{\theta=0}^{\theta=2\pi} B_T^2\, d\theta\right\} dx \tag{49}$$

Substituting Equation (46) in (49) and performing the double integration indicated by the latter, the following loss ratio results,

$$\frac{_1W_{et}}{_0W_{et}} = \left[\frac{_1B_a}{_0B_a}\right]^2 - \frac{2K_3}{3}\frac{_1B_a}{_0B_a}\frac{B_{sm}}{_0B_a}\sin\alpha$$

$$+\left[\frac{B_{sm}}{_0B_a}\right]^2\left[\frac{2K_3^2}{15}+\frac{1}{3}\right] \tag{50}$$

Substituting Equations (25) and (30) in (50)

$$\frac{_1W_{et}}{_0W_{et}} =$$

$$\frac{E_g^2 - 2E_g(IX_s)\sin\alpha + (IX_s)^2\left(\frac{3}{K_3^2}+\frac{6}{5}\right)}{E_t^2} \tag{51}$$

Also substituting Equations (18) and (19) in (51)

$$\frac{_1W_{et}}{_0W_{et}} = 1 + 2\left(\frac{IX_t}{E_t}\right)\sin\gamma + \left(\frac{IX_t}{E_t}\right)^2$$

$$-2\left(\frac{IX_s}{E_t}\right)\left(\frac{IX_t}{E_t}+\sin\gamma\right)+\left(\frac{IX_s}{E_t}\right)^2\left(\frac{3}{K_3^2}+\frac{6}{5}\right) \tag{52}$$

C. Hysteresis Loss in the Teeth

In the qualitative discussion of the losses it was explained that the rate of hysteresis loss per unit volume was proportional to the square of the maximum instantaneous density. Hence it may be written that this loss on load is

$$_1W_{ht} = \frac{K_h V}{l}\int_{x=0}^{x=l} B_{Tm}^2\, dx \tag{53}$$

and on no load is

$$_0W_{ht} = \frac{K_h V}{l}\int_{x=0}^{x=l} {}_0B_a^2\, dx = K_h V\,({}_0B_a^2) \tag{54}$$

Then,

$$\frac{_1W_{ht}}{_0W_{ht}} = \frac{1}{l}\int_0^l \frac{\beta_{Tm}^2}{_0B_a^2}\, dx \tag{55}$$

From Equation (46)

$$\frac{\beta_T^2}{_0B_a^2} = \frac{_1B_a^2}{_0B_a^2}\cos^2(\theta+\alpha)$$

$$+\frac{_1B_a}{_0B_a}\frac{B_{sm}}{_0B_a}K_3\left(1-\frac{x^2}{l^2}\right)\cos(\theta+\alpha)\sin\theta$$

$$+\frac{B_{sm}^2}{_0B_a^2}K_3^2\frac{1}{4}\left(1-\frac{x^2}{l^2}\right)^2\sin^2\theta+\frac{B_{sm}^2}{_0B_a^2}\frac{x^2}{l^2}\cos^2\theta \tag{56}$$

Substituting Equations (25) and (30) in (56)

$$\frac{\beta_T^2}{_0B_a^2} = \frac{E_g^2}{E_t^2}\cos^2(\theta+\alpha)$$

$$+\frac{E_g}{E_t}\frac{IX_s}{E_t}3\left(1-\frac{x^2}{l^2}\right)\cos(\theta+\alpha)\sin\theta$$

$$+\left(\frac{IX_s}{E_t}\right)^2\frac{9}{4}\left(1-\frac{x^2}{l^2}\right)^2\sin^2\theta$$

$$+\left(\frac{IX_s}{E_t}\right)^2\frac{9}{K_3^2}\frac{x^2}{l^2}\cos^2\theta \tag{57}$$

Let,

$$M = \frac{E_g^2}{E_t^2} \tag{58}$$

$$R = \frac{E_g}{E_t}\frac{IX_s}{E_t}3\left(1-\frac{x^2}{l^2}\right) \tag{59}$$

$$Q = \left(\frac{IX_s}{E_t}\right)^2\frac{9}{4}\left(1-\frac{x^2}{l^2}\right)^2 \tag{60}$$

$$S = \left(\frac{IX_s}{E_t}\right)^2\frac{9}{K_3^2}\frac{x^2}{l^2} \tag{61}$$

Then expanding Equation (57) and substituting Equations (58), (59), (60), and (61),

$$\frac{\beta_T^2}{_0B_a^2} = \frac{M - R\sin\alpha + Q + S}{2}$$

$$+\frac{(M\cos 2\alpha + R\sin\alpha - Q + S)\cos 2\theta}{2}$$

$$+\frac{(R\cos\alpha - M\sin 2\alpha)}{2}\sin 2\theta \tag{62}$$

The maximum value of $\beta_T = \beta_{Tm}$ will occur at the same

value of θ as the maximum value of $\frac{\beta_T^2}{{}_0B_a^2}$. Equation (62) may be considered with θ as the only independent variable and rewritten

$$\frac{{}_1\beta_T^2}{{}_0B_a^2} = A_1 + A_2 \sin 2\,\theta + A_3 \cos 2\,\theta \qquad (63)$$

where A_1, A_2, and A_3 are constants corresponding identically to those in (62).

By inspection the maximum value of

$$\frac{\beta_T^2}{{}_0B_a^2} = \frac{\beta_{Tm}^2}{{}_0B_a^2} = A_1 + \sqrt{A_2^2 + A_3^2} \qquad (64)$$

Substituting these values in Equation (55),

$$\frac{{}_1W_{kt}}{{}_0W_{kt}} = \frac{1}{l}\int_{x=0}^{x=l} \frac{(M - R\sin\alpha + Q + S)}{2}\,dx$$

$$+ \frac{1}{l}\int_{x=0}^{x=l} \frac{1}{2}\sqrt{(M\cos 2\,\alpha + R\sin\alpha - Q + S)^2 + (R\cos\alpha - M\sin 2\,\alpha)^2}\,dx \qquad (65)$$

By substituting Equations (58), (59), (60), and (61) in the first integral of Equation (65), performing this integration, and comparing with Equation (51) it is found that Equation (65) may be written

$$\frac{{}_1W_{kt}}{{}_0W_{kt}} = \frac{1}{2}\,\frac{{}_1W_{et}}{{}_0W_{et}} + \frac{1}{l}\int_{x=0}^{x=l} \frac{1}{2}\sqrt{(M\cos 2\,\alpha + R\sin\alpha - Q + S)^2 + (R\cos\alpha - M\sin 2\,x)^2}\,dx \qquad (66)$$

Substituting Equations (19), (20), (21), (22), (58), (59), (60), and (61) in (66), then

$$\frac{{}_1W_{kt}}{{}_0W_{kt}} = \frac{1}{2}\,\frac{{}_1W_{et}}{{}_0W_{et}} + \frac{1}{2}\int_{\frac{x}{l}=0}^{\frac{x}{l}=1} \sqrt{\left\{\cos^2\gamma - \left(\frac{I\,X_t}{E_t} + \sin\gamma\right)^2 + 3\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\alpha\right)\left(1 - \frac{x^2}{l^2}\right) - \frac{9}{4}\left(\frac{I\,X_s}{E_t}\right)^2\left[\left(1 - \frac{x^2}{l^2}\right)^2 - \frac{4}{K_s^2}\,\frac{x^2}{l^2}\right\}^2 + \left\{3\left(\frac{I\,X_s}{E_t}\right)\left(1 - \frac{x^2}{l^2}\right) - 2\left[\frac{I\,X_t}{E_t} + \sin\gamma\right]\right\}^2 \cos^2\gamma}\;d\left(\frac{x}{l}\right) \qquad (67)$$

The solution of the latter equation is most readily obtained by choosing $E_t = 1$, and expressing $\frac{I\,X_s}{E_t}$ and $\frac{I\,X_t}{E_t}$ as decimals. A number of values of $\frac{x}{l}$ were assumed and the indicated integration performed graphically in obtaining the results shown in Figs. 18 to 29 inclusively.

Discussion of the Losses in the Central Zone in Commercial Machines

In the first portion of the article a general discussion was given of the losses in the various parts of a turbine generator, but it seems advisable to consider the losses for the central zone of commercial machines in somewhat greater detail. The alternators which now will be discussed have trapezoidal no-load field forms and balanced three-phase two-layer windings.

The higher harmonics of the no-load flux increase the

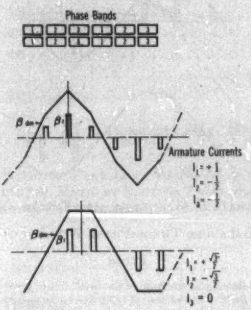


Fig. 13—Armature Magnetomotive Forces and Densities Due to a Balanced Three-Phase Full Pitch Winding

β_T = Radial density at the base of the teeth due to cross slot flux
β_{dm} = Radial density at the base of the teeth due to armature demagnetizing flux

eddy current loss. Neglecting their effect in changing the permeability, and consequently, the flux distribution, their eddy current losses could be computed entirely independently of the fundamental, and these component losses added. The hysteresis loss in the core will be almost proportional to the square of the total flux per pole, because in turbine generators the maximum densities are usually attained at the centerline between poles.

If the no-load field form is a trapezoidal wave composed primarily of a first and a third harmonic, the third harmonic will increase the total eddy current loss in both the teeth and the core, above that which the fundamental alone would produce. This third harmonic will increase the hysteresis loss in the core, because it will increase the total flux; but it will decrease the hysteresis loss in the teeth, because the peak density is less than that which would be produced by the fundamental alone.

In discussing the effect of the armature magnetomotive forces, a full pitch balanced three-phase winding will be assumed first. Fig. 13 shows the winding and the density components at the base of the teeth produced by the armature demagnetizing flux and by the radial part of the cross slot leakage flux. It will be seen that at points between phase bands the actual radial flux density exceeds the fundamental of this wave by an amount which varies with time. The distribution with respect to the stator either in the gap or at the base of the teeth can be expressed very roughly by the equation

$$A = A_4 \cos(\lambda + \theta) + A_5 [\cos(\lambda + \theta)] \cos 6\lambda \qquad (68)$$

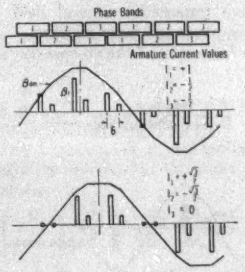


Fig. 14—Armature Magnetomotive Forces and Densities Due to a Balanced Three-Phase Winding of $\frac{8}{9}$ Full Pitch

β_r = Radial density at the base of the teeth due to cross slot flux
β_{dm} = Radial density at the base of teeth due to armature demagnetizing flux

where A, A_4, and A_5 are the amplitudes of the wave and its components.

λ = space angle in electrical radians.

θ = time angle in electrical radians.

The above equation can be rewritten

$$A = A_4 \cos(\lambda + \theta) + \frac{A_5}{2} \cos(5\lambda - \theta)$$

$$+ \frac{A_5}{2} \cos(7\lambda + \theta) \qquad (69)$$

These equations illustrate that both the fundamental and the higher harmonics produce fundamental frequency pulsations in the stator core and that their losses cannot be separated for that part of the machine. (Only the higher harmonics produce pole face losses which, of course, appear in the load loss measurements.) In this case the armature slot leakage flux simply increased A_5 at the base of the teeth above that value which existed at the gap.

It is now possible to consider a two layer chorded winding, because each layer will simply produce the same shaped flux waves, and these must be added with their proper displacement to obtain the total. The distribution at the base of the stator teeth is as shown in Fig. 14. The peculiar fact that the radial densities due to the slot flux produced by the two layers are not equal arises because the slot leakage flux due to the conductors in the base of the slots is greater than that due to those in the top, even though the top and bottom conductors are equally effective in producing gap flux. Hence, the slot leakage flux on entering the

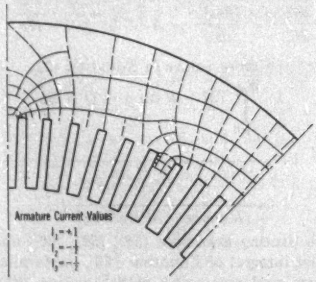


Fig. 15A—Flux in the Stator Core—See Corresponding Current Conditions in Fig. 13

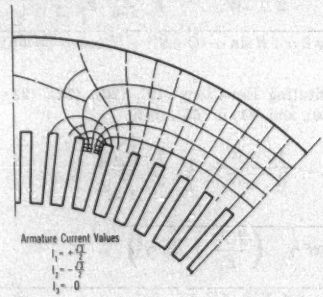


Fig. 15B—Flux in the Stator Core—See Corresponding Current Conditions in Fig. 13

core is found to be out of phase or at least displaced with respect to the armature demagnetizing flux or ampere turns by some angle less than $\frac{\phi}{2}$, where ϕ is the angle by which the winding is chorded.

The flux distribution in the core which the slot flux alone would produce is shown in Figs. 15A and 15B. Fig. 16 shows the distribution in the core based on a sinusoidal distribution at the base of the teeth. These fields are plotted on the basis of uniform permeability.

There are six possible conditions which may establish

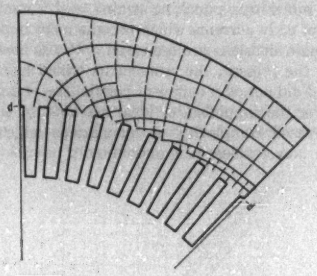


FIG. 16—CORE FLUX ON BASIS OF SINUSOIDAL DISTRIBUTION AT D-D AND UNIFORM PERMEABILITY IN THE IRON

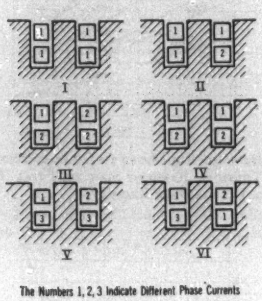


FIG. 17—SIX POSSIBLE M. M. F. CONDITIONS AFFECTING A STATOR TOOTH WHICH CAN BE ESTABLISHED BY A THREE-PHASE ARMATURE WINDING

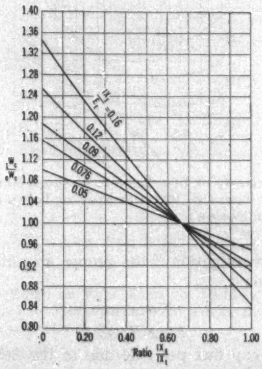


FIG. 18—RATIO OF FULL LOAD TO NO-CORE LOSSES (BACK OF TEETH) *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR OVER EXCITED

the flux cycles for a given tooth (Fig. 17). That is, if the tooth is totally within one phase band, it has one set of flux boundary conditions due to the armature slot flux; if it is influenced by conductors of more than one phase, there are five other slot boundary combinations. In addition to these, the air-gap boundary condition

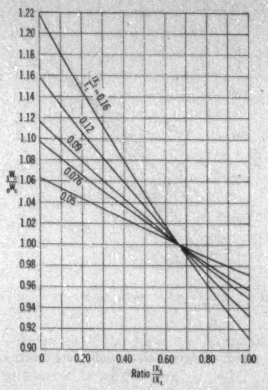


FIG. 19—RATIO OF FULL LOAD TO NO-LOAD CORE LOSSES (BACK OF TEETH) *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.8 POWER FACTOR OVER EXCITED

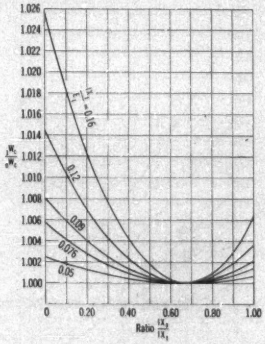


FIG. 20—RATIO OF FULL LOAD TO NO-LOAD CORE LOSSES (BACK OF THE TEETH) *vs.* RATIO OF SLOT TO TOTAL REACTANCE AT 1.0 POWER FACTOR

will vary from tooth to tooth at any fixed load and power factor.

Neglecting tooth taper, these flux distributions and the resultant losses in the teeth may be related to per cent reactance, pitch, etc., but the solution becomes tremendously complicated. The distribution at the base of the teeth may be approximated by the addition of a few more harmonics to Equation (**68**). It will fit

the distribution at a slight distance back of the base of the teeth and be effective for the flux entering about 96 per cent or 98 per cent of the core.

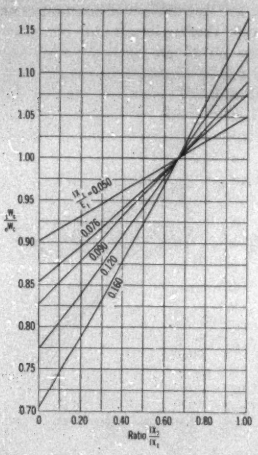


FIG. 21—RATIO OF FULL LOAD TO NO-LOAD CORE LOSSES (BACK OF TEETH) *vs.* RATIO OF SLOT TO TOTAL REACTANCE, 0.0 POWER FACTOR UNDER EXCITED

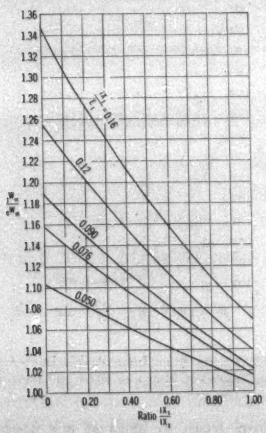


FIG. 22—RATIO OF FULL LOAD TO NO-LOAD EDDY CURRENT LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR OVER EXCITED

These possibilities have been investigated to a certain extent, but the factors which are neglected still are numerous, namely, tooth taper, variable permeability, shorted punchings, punching strains, and a number of sources of eddy currents which become more important as the main densities are increased. It was concluded that for the present, the refinements which have been discussed did not offer sufficient possibilities of improvement over the simplified, or ideal machine, to warrant the increased complexity of calculations which they involve. Nevertheless it is undoubtedly possible to extend this investigation if the proper amount of effort can be put on it.

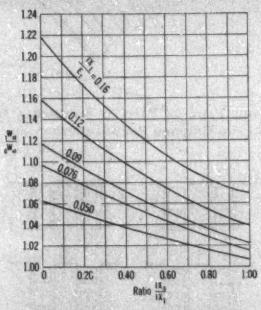


FIG. 23—RATIO OF FULL LOAD TO NO-LOAD EDDY CURRENT LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.80 POWER FACTOR OVER EXCITED

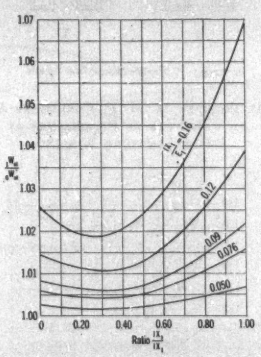


FIG. 24—RATIO OF FULL LOAD TO NO-LOAD EDDY CURRENT LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 1.0 POWER FACTOR

NOTATION

Flux Symbols

Φ_f = air-gap flux per pole due to the field winding only.

Φ_{dm} = air-gap flux per pole due to the armature winding only.

${}_1\Phi_a$ = resultant air-gap flux per pole on load at normal voltage.

${}_0\Phi_a$ = air-gap flux per pole on no load at normal voltage.

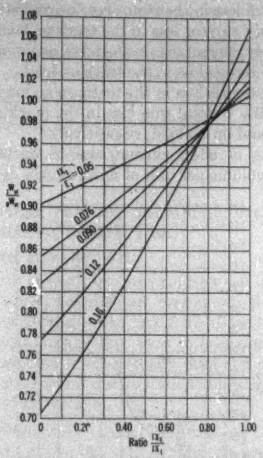


FIG. 25—RATIO OF FULL LOAD TO NO-LOAD EDDY CURRENT LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR UNDER EXCITED

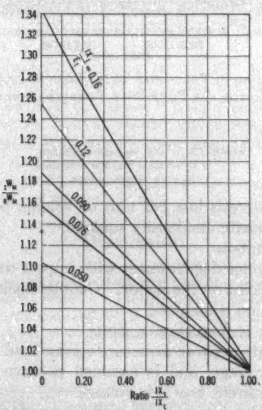


FIG. 26—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR OVER EXCITED

Φ_s = maximum flux across any one slot due to the armature windings.

φ_s = instantaneous value of Φ_s.

φ_s' = instantaneous value of cross slot leakage flux between any circumferential line $j-j$ in the teeth and the teeth tips. (Fig. 11.)

φ_r = one-half the armature slot leakage flux per pole across the line $j-j$.

${}_1\beta_a$ = instantaneous radial density at any point in a tooth due to ${}_1\Phi_a$.

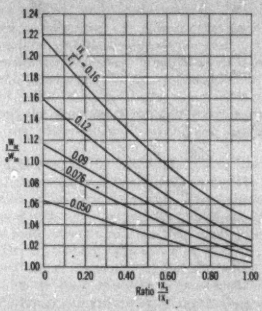


FIG. 27—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.8 POWER FACTOR OVER EXCITED

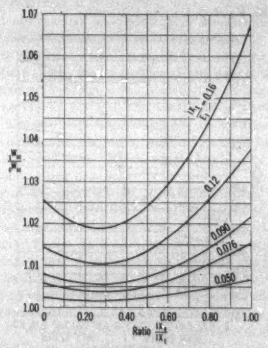


FIG. 28—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 1.0 POWER FACTOR

${}_0\beta_a$ = instantaneous radial density at any point in a tooth due to ${}_0\Phi_a$.

β_s = instantaneous tangential density at any point in a tooth due to the armature cross slot leakage flux.

β_{sm} = β_s at the tooth tip.

β_r = instantaneous radial density at any point in a tooth due to armature slot leakage flux.

β_R = instantaneous resultant radial density at any point in a tooth.

(β_R = vector sum of ${}_1\beta_a$ and β_r)

β_T = instantaneous resultant density at any point in a tooth.

($\beta_T^2 = \beta_R^2 + \beta_t^2$)

${}_1B_a$, ${}_0B_a$, B_{sm}, B_r, B_R, and B_T are the maximum values of

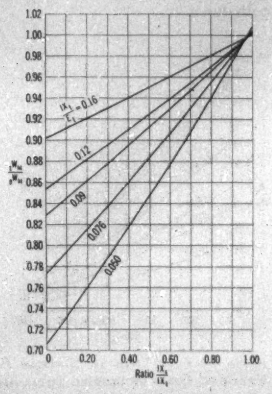


FIG. 29—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR UNDER EXCITED

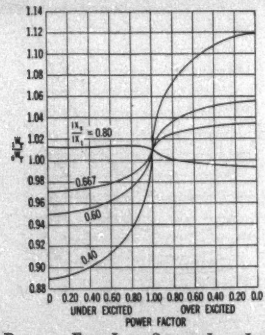


FIG. 30—RATIO OF FULL-LOAD STATOR IRON LOSS *vs.* POWER FACTOR FOR FOUR-POLE TURBINE GENERATOR WITH $\frac{I\,X_t}{E_t} = 0.12$

${}_1\beta_a$, ${}_0\beta_a$, β_{sm}, β_r, β_R, and β_T, respectively.

The radial values are considered positive when outward; and the tangential values positive in the clockwise direction (*i. e.*, opposite to the direction of rotation as later assumed).

Voltage and Current Symbols

(All values are per phase)

E_g = generated voltage.

E_t = terminal voltage.

I_a = full load armature current.

$I\,X_t$ = total armature leakage reactance.

$I\,X_s$ = armature slot leakage reactance.

Loss Symbols

${}_1W_c$ = total loss in the stator core on load.

${}_0W_c$ = total loss in the stator core on no load.

${}_1W_{et}$ = eddy current loss in the teeth on load.

${}_0W_{et}$ = eddy current loss in the teeth on no load.

${}_1W_{ht}$ = hysteresis loss in the teeth on load.

${}_0W_{ht}$ = hysteresis loss in the teeth on no load.

${}_1W_T$, and ${}_0W_T$ = total iron loss in the machine on load and on no load, respectively.

K_e = rate of eddy current loss in watts per unit volume per unit of mean square density.

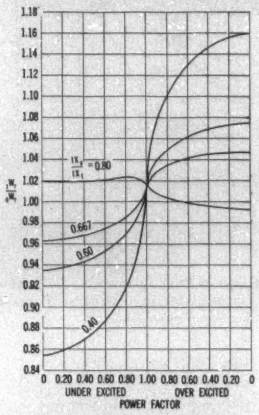


FIG. 31—RATIO OF FULL-LOAD TO NO-LOAD STATOR IRON LOSS *vs.* POWER FACTOR FOR FOUR-POLE TURBINE GENERATOR WITH $\frac{I\,X_t}{E_t} = 0.16$

where IX_s = slot leakage reactance
IX_t = total leakage reactance
E_t = terminal voltage

K_h = hysteresis loss per unit of volume per unit of maximum density squared.

Angular Symbols

θ = angular displacement with respect to some one point on the stator. Counterclockwise rotation is assumed.

(θ = $2\,\pi \times$ frequency $\times$ time)

α = angle between armature current and generated voltage, and is assumed positive when the current lags this voltage.

γ = power factor angle and is assumed positive when the current lags the terminal voltage.

λ = a constant angle as defined in Equation (**68**).

σ = angle by which the armature winding is chorded.

Dimension Symbols and Constants

x = radial distance from the base of the slot toward the tooth tip.

l = total depth of slot.

L = net length of stackings axially.

D = diameter at stator tooth tips.

P = number of pairs of poles.

A_p = total area of cylindrical surface at the tips of the teeth.

A_i = total area of magnetic material in the cylindrical surface at the tips of the teeth.

K_2 = stacking factor.

K_3 = constant for a given machine.

$$\left(K_3 = \frac{A_p}{A_i}\,\frac{2\,P\,K_2\,l}{D}\right)$$

n = number of turns included in one slot in a distance x from the base.

N = total turns per slot.

K_1 = ratio of flux turn interlinkages per slot ampere which would obtain for any one slot if all the existing slot flux linked all of the turns in the slot, to the flux turn interlinkages per slot ampere which actually exist (on the basis of the current distribution assumed in the body of the article).

V = volume of iron in the teeth.

M, R, Q, S, A, A_1, A_2, A_3, A_4, and A_5 are constants which are defined where used.

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Discussion

L. A. Doggett: A number of very interesting points has been brought out in this paper. However, the paper does not present any experimental evidence. I realize that the more complicated the problem, the more difficult it is to get that evidence; but at the same time the more complicated the problem the more important it is to substantiate some of these assumptions that had to be made.

J. L. Beaver: I should like to ask the authors how the lines of flux shown in Figs. 2, 3, 4, and 5 were obtained. I am wondering just how accurate the authors feel those apparently assumed lines of flux distributions are. Has any model ever been used with currents? For instance, I was thinking of a closed circuit carrying currents in some sort of a liquid, in which the flow of amperes might be comparable to flux distribution.

L. A. Kilgore: (communicated after adjournment) In this paper, the authors have made a careful study of flux distribution in the machine under load, and of the resulting losses. They have obtained a definite relation between stator iron loss on load and no-load in relatively simple terms involving only slot reactance and total leakage reactance of the machine.

As they have stated, in order to obtain such relations a simplified machine must be assumed and certain other assumptions made in regard to the relation between losses and densities. The use of a simplified machine is justified only after a careful consideration of the factors neglected.

The chief difference in flux distribution from the ideal machine is the concentration of radial slot-leakage flux in one or two slots per phase per pole for a two-layer winding. These differences are illustrated in the paper. It appears that for the two-layer windings considered, the differences are not great. But for a one-conductor-per-slot machine near $^2/_3$ pitch, as used in low-voltage two-pole turbo generators, the flux distribution in the teeth is very different. Due to interleaving of different phase belts in alternate slots, the densities of slot flux going radially are relatively high, and the resultant stator tooth loss is somewhat higher.

The assumption that the hysteresis loss is proportional to the square of the maximum instantaneous density is consistent with test results on rotating machines. This fact appeared to be in disagreement with the observation of physicists that at higher densities, the hysteresis in a rotating field approached zero.

Recently Dr. Rudenberg, in a series of lectures at the Massachusetts Institute of Technology,[1] and also before the Westinghouse engineers at East Pittsburgh, presented the theory of rotational hysteresis and gave a great many experimental data. Simple rotational hysteresis was found to increase more rapidly than alternating hysteresis at low densities, but to drop off rapidly above 12 kilogauses. In the core of a rotating machine, the tangential and radial components of density are unequal and out of phase, resulting in an elliptical field. The combination of rotating and alternating hysteresis gave an increasing loss at much higher densities than a simple rotating field.

The results of these tests seem to bear out quite well the assumption that the hysteresis varies as the square of the maximum density over the range encountered in rotating machines.[1]

J. F. Calvert: In reply to the questions by Professor L. A. Doggett: I do not believe that there is any possible way of directly measuring the variation in iron loss with load on commercially built machines. However, I should like to make a few observations along this line.

By means of a large number of thermocouples, the rate of change of temperature with time could be measured at various points in the iron when there was a sudden (theoretically instantaneous) change in load. From the measured rates of change of temperature with respect to time and the specific heat of the iron, the change of loss incident to the change of load could be computed at each of these points, for it can be shown that at the first instant of time, all the change in loss at a point must be stored. At one time, we seriously considered this method, but some experience with it showed that the time involved in such measurements would be tremendous. We outlined this method, and our efforts, in a paper on *Additional Losses of Synchronous Machines*,[2] presented before the Institute at New York in 1927. There is an article by M. C. Holmes on *Separation of Stray Losses in Turbine Generators* which appeared in the March issue of the A. I. E. E. Journal (p. 224), describing certain tests made by him. Incidentally, these readings of his seem to check our calculations of the axial penetration of flux from the end with satisfactory exactness. (Compare the discussions of Fig. 8 in Mr. Holmes' paper with that of Fig. 4 in the present paper).

Summing up the measurement by means of time-temperature curves, we may say that it was a possible means of check, but was too lengthy a process for us to use on this job.

1. The M. I. T. Lectures are to be published at a later date.

2. A. I. E. E. Trans., Vol. XLVI, 1927, p. 84.

A number of calorimeter tests on turbine generators has been made by measuring the temperature rise and volume of air passing through a machine under steady-load conditions. Since about 98 per cent of the loss occurring within the frame is dissipated in this manner, it follows that very accurate loss measurements may be made. It is our experience to date, based on quite a number of these tests, some at zero power factor and some at 0.80 power factor, that the additional losses on short circuit are almost equal to those at normal voltage for equal values of armature current. This is what one would expect, if there were little or no change in iron loss with load when operating at normal voltage, and calculations by the method under question indicate just that.

However, the analysis given in this paper is one of simple character, which can be checked step by step. If there are no errors in the analysis, then the results must be as accurate as the assumptions. Assumptions 1, 2, and 3 involve only geometrical considerations and their import may be readily considered and checked. Assumptions 4 and 5 are in agreement with most Epstein tests on commercial iron at the densities used in turbine generators. Bibliography references 4 and 5 add some weight to the latter two points. Assumption 6 is unquestionably an approximation, but if there is little change in core flux between no-load normal voltage and full-load normal voltage, the change in distribution must be slight. This gives some fairly substantial evidence to validate our analysis. We recognize that it is a first step toward the solution of this problem and have called attention to this fact in the synopsis to the paper.

In reply to Professor J. L. Beaver, the basis for the flux distributions indicated in Figs. 1, 2, and 3 are described in detail in Bibliography references 1 and 3. These flux distributions can and are arrived at by purely analytical methods. However, in the case of the end-bell fields, they were first solved experimenally by means of magnetic compasses and iron filings. The use of electrical flow in metal templets to solve flux problems by analogy was described in 1915, in a paper by Professor John F. H. Douglas on *The Reluctance of Some Irregular Magnetic Fields*, A. I. E. E. Trans., Vol. XXIV, Part 1, pp. 1067-1134.

Professor Douglas gives by far the most complete Bibliography on field mapping we know of for articles prior to 1915. We have attempted to add to this for the time since 1915 in the series of articles mentioned in our Bibliography, reference 3.

Figs. 4 and 5 are not intended to be exact, though very considerable efforts were made to achieve accuracy for the calculations, indicated by Fig. 4 and the discussion of that sketch.

In general, I may say that our Bibliography was intended to give the basis for our assumptions and to justify our flux analysis. We realize that we should have given specific bibliographical references in the body of the paper to cover any item which might be open to question.

The Fabrication of Large Rotating Machinery*

BY H. V. PUTMAN[1]

Associate, A. I. E. E.

THE manufacture of machinery is undergoing a change—probably the most significant in history. From very early times man has moulded metal into the things he desired to make. Once moulded, changes in surface or outline were difficult and expensive. And it was the need to simplify this part of the process that led to the invention and development of modern machine tools. Thus, the conventional manufacturing process of pattern making, moulding, and machining was established.

Now this fundamental method of manufacture is changing. Man has learned to handle steel much as a carpenter handles wood. With the gas torch he can take it apart and with the electric arc he can put it together in almost any shape or form desired, eliminating the pattern making and moulding entirely. That it can be done this way more economically than the old way, is due largely to the low cost of steel sheets, bars, and shapes which form the raw material. Strangely enough, it was the electrical industry which contributed most to the development of the steel industry which has made low cost steel possible. And now the low cost of steel is primarily responsible for the radical change taking place in the manufacture of electrical machinery.

Like other manufacturers, the Westinghouse Company has been using structural steel in parts of its produce for many years. In fact, the use of rolled steel in d-c. motor frames by this company, dates back sixteen years or more—antedating even the general use of electric arc welding with the metallic electrode. But the general application of fabricated steel in place of castings in the construction of our larger rotating machinery, is new within the last few years, and the change only now nearing completion.

Before adopting so radical a change as the substitution of fabricated steel in place of castings, it was necessary to make a careful study to determine what economic advantages would be likely to result. Such a change meant the complete redesign of all lines of large rotating apparatus and a complete reorganization of manufacturing facilities. Obviously, these changes involved large expenditures both in the engineering department and the factory, and the decision in favor of such a change was made only after a careful comparison of the cost with the resulting advantages.

In many cases it was found that the use of structural steel permitted little, if any, saving over a good cast design, made with good foundry equipment and machined on modern machine tools. This conclusion cannot, however, be applied generally. Some types of structures showed very appreciable savings while others, especially small ones having high activity, were found to be cheaper when cast.

Even at the same cost, however, the use of fabricated steel may be justified for the following reasons:

1. The possibilities of future cost reduction are greater because the fabrication of steel is a comparatively new art while the cost reduction possibilities of the cast construction have been largely exploited.

2. The elimination of patterns is a clear gain. This is of greatest value in large special machines where only a single unit is made. It applies also to large bedplates where there are few exact duplicates and where, with castings, continual pattern changing was necessary.

3. The fabricated steel design is flexible. A large number of different structures can be made from a limited number of parts by welding them together in different combinations. Minor changes are easily made. Greater strength may be had when required by adding additional structural members.

4. Shorter deliveries are possible with the fabricated steel construction. With the foundry and the pattern problem out of the picture, a shorter and more direct manufacturing sequency is possible. In fact, the whole process can be done in the same shop. Greater concentration of responsibility in the manufacturing organization can be had and this means greater efficiency and quicker production.

5. The saving in weight due to the use of fabricated steel is sometimes a very valuable feature. This is especially true in the case of turbo generators where the saving in weight is greatest. Previous to the use of fabricated steel, a 25,000-kv-a., 4-pole generator had to be shipped disassembled and the armature laminations and coils assembled in the customer's plant. In a recent case, it was possible to ship assembled a 100,000-kv-a. machine with the armature iron and coils in place. This also makes it possible to test these large machines for losses before being shipped, a highly desirable feature.

The saving in weight is also desirable for all machinery used on shipboard. Usually, there is a penalty and bonus on weight written into contracts for such equipment.

6. Fabricated steel structures are often better

*This paper is not intended as a general treatment of the above subject. It presents some of the more interesting experiences of the company with which the author is connected, in changing their designs of rotating machinery to employ fabricated steel in place of castings.

1. Section Engineer, Synchronous Motor Section, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

29-60

structures than were the cast. There are many sales arguments to be advanced in their favor and they are being demanded more and more by the public.

Most of these points were listed in the beginning as economic advantages of fabricated steel before the decision to adopt it was made. Since then, the truth of these statements has been demonstrated many times.

Already, developments in the arts of gas cutting and arc welding are taking place, which improve the quality and reduce the cost. New machines, comparable with our best machine tools, are being built to facilitate the practical application of these arts to quantity production.

The truth of the second and fourth points mentioned above was recently demonstrated in connection with an order for two 5000-hp., 82-rev. per min. horizontal synchronous motors for steel mill drive. These machines were urgently needed, so that short delivery was of utmost importance. Accordingly, a delivery of 90 days was guaranteed on the first machine and the second machine was to be delivered two weeks later. These motors are among the largest synchronous motors, in physical dimensions, ever built for an industrial drive. (See Fig. 1.) These were entirely new, new dies being required for both the armature and pole laminations. Steel had to be ordered special from the mills for the stator frame and for the spider. Incidentally, the slabs for the spider rim, because of their size and unusual length had to be rolled special. When these facts are considered, one may well wonder how such a short delivery is possible.

The first motor was turned over to the test department exactly 82 days after receipt of order, for complete engineering tests. On the 90th day five cars, bearing the first motor, left the works for the customer's plant. Eleven days later another five cars rolled out of the works with the second motor.

This production record was made possible through the use of fabricated steel. There were no patterns to make. The whole manufacturing process took place in one factory under the responsibility of one manufacturing department. A production schedule was formulated and carried out in an orderly fashion. The manufacture of other machines was not interrupted or delayed. A completely equipped fabrication shop was, of course, a necessity, and was available for handling this job.

Some of the special problems encountered by the engineering department in redesigning the various lines of rotating machines are of interest.

One of the greatest problems was to get designers to think clearly and with an open and unprejudiced mind about the fundamental design problems of fabricated steel. Incidentally, much new thinking was done about the old designs in cast iron, and it was discovered cast structures were designed the way they were designed, not because of the requirements of the structure or the machine, but because of the requirements of foundry practise or the machine tools. Such a conclusion seems so obvious as to be almost axiomatic, but a consideration of it helped greatly to establish a new point of view or basis of thinking for our new design work in fabricated steel.

It was determined that all knowledge of cast design should be put aside and even the appearance forgotten in an effort to make new designs with open, unbiased minds. The requirements of each machine or structure should be determined, then a study should be made of the structural material, plates, bars, and beams to see how these materials could be best employed to meet these requirements. Sometimes our knowledge of gas cutting or welded joints was found to be inadequate, and it was necessary to call upon the research laboratory for definite information backed up by actual experiments and tests.

There had to be a definite reason for every step taken in our design work. What was the result? We found our new designs radically different from cast structures and usually much simpler in construction. Any savings were found to depend but little on reductions in weight, but very materially on the simplicity of the design, and its general suitability to the use of structural materials. Structural materials are partially finished materials and a design to be suitable for their use should be so made as to use the rolled surfaces of the plates and bars as finished surfaces in the completed structure. This means that much of the machine work required on cast structures can be eliminated when the fabricated construction is used—provided the design is properly made.

The design of the different lines of fabricated machines is now completed, at least to the point of determining the types of the structures, and it is interesting to see how this point of view has been followed out in the different lines.

The frame construction of the smaller synchronous motors and generators having the core stacked on bolts between two frame rings gas cut from steel plate and left open in the back is now well known. It is a radical departure from the old cast frames in appearance, but it is adapted to the use of structural materials and it fulfills all the requirements of the electrical machine even better than did the cast construction. Fig. 2 shows a typical synchronous motor having this frame construction.

Even the use of the bolt for holding the laminations instead of the old fashioned dovetail was adopted because it was more suited to the fabricated construction. At first it was thought the use of the bolt for holding the core was adapted only to machines with narrow cores, but it was gradually extended to wider cores until now it is used exclusively. The 220-in. core of the 100,000-kv-a. Hell Gate turbo generator was stacked on bolts. Figs. 3 and 4 show pictures of this stator.

In the water-wheel generators the upper and lower brackets are of special interest because of their radical

departure from similar cast structures and their suitability to structural materials. Figs. 5 and 6 show these brackets and Fig. 7 a complete vertical machine.

Another excellent example is the frame construction of the new line of 3600 rev. per min. turbo generators. (See Figs. 8 and 9.) It is built up of plates spaced periodically along the core. These plates support the frame bolts on which the punchings are stacked and at the same time provide the necessary air passages for

Fig. 1—5000-Hp., 82-Rev. Per Min. Synchronous Motor for Steel Mill Drive Assembled for Test

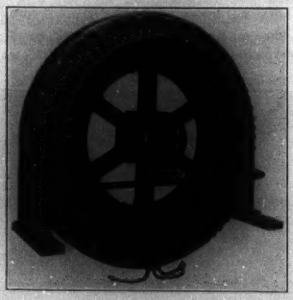

Fig. 2—Typical Low Speed Synchronous Motor Stator with Fabricated Rotor in Place

the re-entrant ventilating system. Except for drilling the end plates, which receive the punching bolts, the whole structure is produced by the torch, arc welder, and the bending rolls and presses in the structural shop. A plate on the back serves as a cover to the air passages and extends over the armature winding on each end where it meets the pressed steel end bells that are bolted to it. This structure is peculiar in that, while it is especially adapted to fabricated steel construction, it can hardly be distinguished from the previous cast structures.

Other examples might be mentioned but one of especial interest is the fabricated commutator spider for large high-speed commutators developed as a part of the

Fig. 3—100,000 Kv-a. Turbo Frame Showing Fabricated Skeleton Construction With Outside Lagging Removed

Fig. 4—100,000 Kv-a. Turbo Generator Frame with Core Assembled and Lagging Partially in Place

Fig. 5—Typical Fabricated Upper Bracket for Vertical Waterwheel Generators

general redesign of the d-c. machines. The special feature in this construction is the support of the front V ring which provides flexibility in the direction of the shaft. The space between the V rings can thus adjust itself to suit the expansion and contraction of the commutator bars. Tests on this construction have

proved its superiority over the cast construction from the standpoint of performance. An idea of the construction can be gained from Fig. 10.

Another problem of the engineering department has been the "appearance problem." While some of the fabricated machines resemble closely their cast iron predecessors, many of them are radically different in appearance. It is a fact that what looks good to one is simply a matter of what he is accustomed to. If one is accustomed to looking at cast machines they look best, but as fabricated machines become more familiar they improve in appearance. What appears radical today seems commonplace tomorrow.

This was especially true in the case of fabricated H beam bed-plates. They produced considerable unfavorable comment at first, because of their appearance, and some customers actually refused to have them. But it is a fact that H beam bed-plates are really better bases than cast bed-plates. The U section of the cast bed-plates, with its smoothly rounded corners, was not dictated by any requirement of the structure, but they were made that way so the pattern could be pulled more easily from the sand. The idea of grouting bed-plates into the top originated with the fabricated bed-plate

Fig. 6—Typical Lower Bracket for Vertical Water-Wheel Generator

Fig. 7—Typical Vertical Waterwheel Generator, 625-Kv-a., 2400-Volt, Three-Phase, 60-Cycle 200-Rev. Per Min.

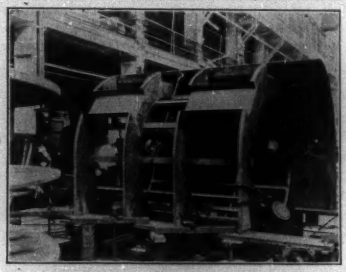

Fig. 8—9500-Kv-a., Turbo Generator Showing Fabricated Frame Construction

Fig. 9—9500-Kv-a. Turbo Generator Assemblor for Test

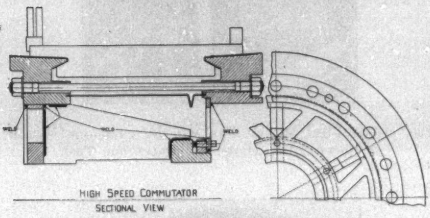

Fig. 10—Sketch Showing Construction of Fabricated Commutator Spider for Large High Speed Commutator

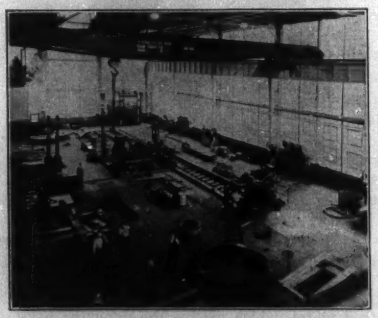

Fig. 11—General View of New Fabrication Shop

and when so grouted, its appearance is equally as good as that of the cast bed-plate. It is just as natural to grout a fabricated bed-plate into the top, as to set a cast base above the floor line. Architects are realizing this, and are designing the floors of power houses and motor rooms with recesses to receive the bed-plates, so that after the grouting is done, the floor covering can be placed flush with the top of the bed-plates. And today, no one hears a word of criticism regarding the appearance of fabricated bed-plates, and very little indeed, regarding the appearance of fabricated machines.

Round corners and sweeping curves are descriptive of cast structures and cast machines. Flat surfaces, square corners, and abrupt lines are equally descriptive of fabricated machines. It is only a matter of time when they will appear equally attractive. In fact, the straight line and the square corner have already come into prominence in the design of modernistic furniture, lighting fixtures, and decorations.

The major problem of the manufacturing department was the construction of a new fabrication shop, so that a brief description of the building and equipment is included. Fig. 11 gives a general view of the shop. The largest piece of equipment is a vertical bending roll for rolling into circles the frames of d-c. machines and spider rims of a-c. machines.

There is also a 1000-ton crimping press for putting an initial bend in the ends of the slabs to facilitate starting them in the rolls. A large furnace which will take slabs 37 ft. long, is provided for heating the slabs preliminary to rolling, and also, for annealing of completed structures. An approach table is provided for carrying the slabs from the furnace, through the crimper and to the bending rolls. The furnace, approach table, crimper, and bending rolls are all controlled from a single pulpit located near the rolling mill.

Another major piece of equipment is a flanging press used for forming the pressed steel end bells for turbo generators. This press has two vertical rams on top of 100 tons capacity each, a smaller horizontal ram, and a lower vertical stripper ram. The end bells shown on the turbo generator in Fig. 9 were made on this press.

One of the most interesting pieces of equipment is the shot blast room. This is a large room capable of receiving the largest completed fabricated structures. It is entirely closed and lined with steel plate. The operator is dressed in a suit and helmet which resembles that of a diver. The floor is a steel grating. The shot drops through the floor and into a hopper where it is picked up pneumatically and recirculated.

Here all fabricated structures are shot blasted before going to the assembly section, every bit of rust and scale is removed so that the surface resembles that of cast aluminum. When the paint is applied to this surface, there is no danger of its chipping, or scaling off, because it has been applied directly on the mother metal—to a surface 100 per cent clean.

Discussion

J. B. Silverman: With reference to these large machines, I wonder if a saving in weight is accomplished in the increased permeability of the material used, that is the structural steel instead of cast iron.

L. L. Bosch: I wonder whether or not pockets are formed in the spokes of the large fabricated machinery, which cause more loss due to air friction.

J. F. Lincoln: (communicated after adjournment) Mr. Putman makes the statement that the application of welding to small machinery does not show a saving, which I am sure he will find with further experience is not true. As a matter of fact the percentage of saving is probably greater when made in the proper way than the saving shown by Mr. Putman in his large application, the reason being that with the design shown by Mr. Putman much gas cutting is necessary, together with the resultant scrap, while in small machinery means have been devised to eliminate all scrap and all gas cutting.

The point that Mr. Putman makes that the hardest job that he had was to get his organization to think in new terms of arc welding, is undoubtedly the keynote to most of the hesitation in adopting this new method. It is necessary if we are to use this new tool, to do new thinking. This new thinking is not difficult, —it is only different.

H. V. Putman: The greater strength of fabricated steel, rather than its greater permeability, has been responsible for the decreases in weight which have been made in changing to fabricated structures. The frames of d-c. machines and the spiders of low-speed a-c. machines have always been cast steel which has a permeability fairly high compared with that of rolled steel. There has been no marked reduction in the weight of these parts.

The use of H-beam spokes in fabricated spiders does increase the windage loss slightly, but we have found it unnecessary to use blowers with such spiders to secure proper ventilation. This increase in windage is probably not as much as would be produced by the addition of blowers to an ordinary spider. Any construction which improves ventilation without increasing cost is, of course, a decided advantage from the design standpoint.

I hope Mr. Lincoln is right in his statement that small fabricated structures can be made to compete in cost with castings. Perhaps we shall find that to be true. It is difficult to make definite statements regarding comparative costs of structures which will hold true for more than one company. What is true in one company may be quite the opposite in another. We have small castings weighing only a pound or two which we produce at about the same price per pound as castings weighing several thousand pounds. We made fabricated steel brush holders for years and they were cheap; but today we produce die-cast holders at a much lower cost.

In conclusion, I should like to say just a word about reliability in connection with fabricated structures. Inspection of fabricated structures is probably not as easy as the inspection of cast structures, because of the difficulty of inspecting the welds. Much depends upon the welder. We take every precaution to make certain that our welders are able to do the work and to do it correctly. They have been trained in our own welding school, and are required to pass rigid tests before they are permitted to weld on our machinery. Each welder is required to stamp every weld he makes with his number so that it is possible to identify every weld on any machine in the shop. Should visual inspection reveal any defect in a weld, it can be traced back to the man who made it. We also occasionally inspect each operator's welding by chipping out a section for examination. This has a tendency to maintain a high quality of work on the part of each individual welder.

Electrical Equipment of Bar Plate and Hot Strip Mills

BY J. B. INK[1]
Member, A. I. E. E.

THE purpose of this paper is to describe the electrical features of the continuous bar plate and hot strip mill recently placed in service by the American Rolling Mill Company at Middletown, Ohio.

Fig. 1 shows general arrangement of mill. Single line diagram, Fig. 2, shows the principal electrical connections.

The new mill produces ingot iron and various grades of steel strip in gages down to 0.093 in. and widths up to 48 in., also plates ¼ in. thick and 60 in. wide in 75 ft. lengths. These products are rolled from 6 in. thick slabs, 39 in. long, and of the width required in the finished product. For ease in handling and storing, the strip steel is coiled on leaving the last stand of the hot strip mill.

The mill consists of 11 stands. The first 7 constitute the "bar mill" and normally reduce the sheet to about ⅜ in. thickness. The last 4 stands are the "hot strip mill." Between stands 7 and 8 are a runout table, transfer, and bar piler for taking off sheet bar or other heavy gage product.

The initial length of slab is 39 in. when it enters No. 1 stand. This length of course increases as the piece progresses through the successive stands. When the length has increased to about 20 ft., as it does in No. 4 stand, it is about equal to the minimum spacing between stands possible with this type mill. Thus, the following stands, to be economically spaced both for

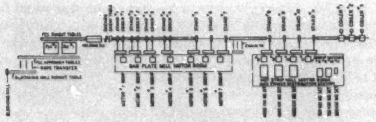

FIG. 1—GENERAL ARRANGEMENT OF BAR PLATE AND HOT STRIP MILLS

conservation of floor area and heat, must be on centers less than the length of the piece, with the result that the piece, after leaving No. 4, is in two or more stands at the same time. This requires very close control of speed, as any change in speed or draft in one mill must be accomplished by a corresponding change in the other stands in the train. In rolling wide strip, wearing of rolls makes frequent changes in draft necessary.

From the above it follows that stands No. 1 to No. 4 are independent as regards speed while No. 5 to No. 7 and No. 8 to No. 11 are inter-dependent.

The independent stands No. 1 to No. 4 are each driven by a wound secondary induction motor and are equipped with fly-wheels. The motor horsepower varies from ¼ to ½ the total rolling horsepower, the balance of the rolling energy being taken from the fly-wheel. The output of the motor is limited by automatic liquid slip regulators which introduce resistance

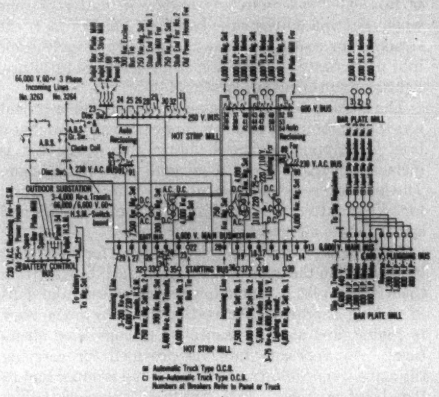

FIG. 2—DIAGRAM OF ELECTRICAL CIRCUITS IN ROLLING MILL

in motor secondary as the motor primary current increases and so permits the fly-wheel to give up stored energy.

The characteristics of the first 4 stands are as follows:

Motor	Hp.	Speed Synchronism	Fly-wheel hp. sec.	Gear ratio	Mill rev. per min.	Roll diameter	Roll ft./min.
1	800	514	30,000	35:1	14.42	32 in.	121
2	800	514	30,000	35:1	14.42	32 in.	121
3	1000	600	40,000	29:1	20.18	32 in.	169
4	1200	720	40,000	29:1	24.3	32 in.	203.5

Motors driving stands 1 to 4 and their control are housed in a ventilated brick and steel motor room. They are equipped with thermocouples in primary windings for checking temperature, thermostatic relays in bearings, and space heaters to prevent "sweating" during shutdown. Primary winding is mica insulated and coil ends are braced to insulated steel ring supported from stator frame to resist shocks incident to the service.

1. American Rolling Mills Company, Middletown, Ohio.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

The rating and performance data of these motors follow:

All—6600-volt, 3-phase, 60-cycle, 40 deg. cent. rise.

Motor	Hp.	Poles	Efficiency			Power Factor			Per cent slip full load	Max. torque times full load
			½	¾	Full load	½	¾	Full load		
Nos. 1 and 2	800	14	90.8	92.8	93.25	67.8	78.4	82.9	1.13	2.9
No. 3	1000	12	92.4	93.6	94.	74.2	82.3	85.7	1.167	2.62
No. 4	1200	10	91.3	93.	93.6	78.6	85.8	88.9	1.	2.92

These motors are controlled from bar plate mill pulpit, which is similar to hot strip mill pulpit shown in Fig. 3.

Motor primary is connected to main bus by Westinghouse type OE6 oil circuit breaker. For plugging there is provided a Westinghouse type B2 breaker which

FIG. 3—CONTROL PULPIT, HOT STRIP MILL

This pulpit controls four 3000-hp. adjustable speed d-c. motors and approach and run-out tables for this mill.

connects the motor to reverse bus. The reverse bus is connected to main bus by an OE6 breaker. The forward breaker and reverse bus breaker are automatic on overload and undervoltage. Thus one automatic breaker gives undervoltage and overload reverse protection to the four motors.

Oil circuit breakers are truck type having all hot parts enclosed. Forward and reverse breakers are interlocked electrically and mechanically to prevent simultaneous closing.

The motor secondaries are connected to the bottom electrodes of liquid slip regulators, a form of water rheostat in which the electrolite is a solution of sodium carbonate. Three top movable electrodes are connected together and the distance between top and bottom electrodes is varied by the action of a torque motor energized from current transformers in the motor primary circuit. The torque of the torque motor is proportional to the square of the primary current so that it is very sensitive to change in load. When the load exceeds a pre-determined amount fixed by counterweights, the torque motor raises the electrodes, thus increasing the secondary resistance and reducing torque due to primary current to a point where the balance between weight of electrodes and motor torque is re-established. Fig. 4 shows a graphic wattmeter record of input to the a-c. motors. Superimposed upon these curves of motor input are shown the actual rolling hp. Note that the motor furnishes directly only a small part of the energy used in rolling, the fly-wheel supplying the major portion.

The interdependent stands 5 to 11 inclusive are driven by 600-volt d-c. adjustable speed motors. Due to the necessity for close speed regulation, and the fact

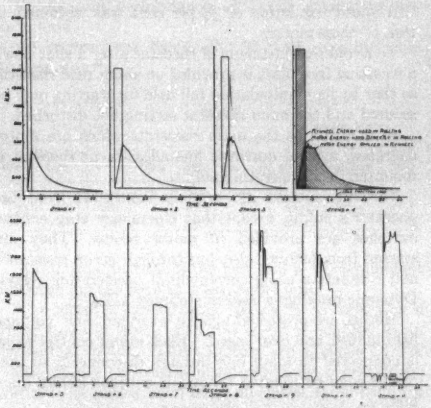

FIG. 4—INPUT TO MILL MOTORS

that the piece is in the stand a considerable length of time, fly-wheels are impracticable on these stands. The motors, therefore, must provide maximum rolling torque.

Characteristics of stands 5 to 11 are as follows:

All 4-high 56 in., working rolls 18 in. diameter, backing up rolls 36 in. diameter, roller bearings.

Motor	Hp.	Speed	Gear ratio	Mill rev. per min.	Roll ft./min.
5	2000	300–500	8.1 :1	37.1/ 61.70	174.7–291.0
6	2000	300–500	6.00:1	50 / 83.3	235.4–392.0
7	2000	300–500	5.32:1	56.4/ 94.1	266.0–443.0
8	3000	180–360	3.02:1	59.6/119.2	280.5–561.0
9	3000	180–360	2.26:1	79.7/159.4	375 –750.0
10	3000	180–360	1.87:1	96.3/192.6	454. –908.0
11	3000	180–360	1.72:1	104.8/209.6	494 –988.0

The seven d-c. 600-volt motors are heavy duty type with low pedestal bearings and fabricated structural

steel bases. The 3000-hp. motors, being very low speed, require forced air ventilation. Motors are equipped with bearing thermostatic relays, over speed devices, and heaters to prevent sweating during shut down. To each shaft is geared a speed indicating magneto. These motors are constant horsepower, speed adjustable by field control. Motors are compounded by use of series exciters. The series exciters consist of generators driven by 5-hp. induction motors. The generator field is excited by ¾ turn of the main motor armature circuit, so that the voltage generated is proportional to the input to the main motor. The generator circuit supplies a series field winding similar to the shunt winding on the main motors. Shunt field excitation is at 250 volts.

Compounding is adjusted to different motor speeds by a rheostat in the series exciter circuit. This rheostat is on the same shaft as the main shunt field rheostat so it is automatically adjusted with the main shunt field.

A speed regulation of ½ per cent was obtained in test of these motors.

To avoid readjustment of rheostat after a shut down a resetting face plate is provided on shunt field rheostat so that by its manipulation full field for starting may be secured and the main rheostat setting not disturbed.

In addition to the main rheostats which are motor operated, a hand operated fine adjustment rheostat is mounted in the operating pulpit.

The d-c. motors are entirely controlled from the operator's pulpit, except that emergency stop control switches are provided in motor rooms. They are started from 600-volt d-c. bus through series resistance and contactors using current limit accelerating relays. Dynamic braking is used in stopping mills.

Motors are protected against overload, low voltage, field failure, and over speed. Each motor control panel is connected to main bus through disconnects so that panels may be isolated for repair without killing main bus.

Rating and performance data on the d-c. motors follow:

All 600 volt—Separately excited at 250 volts, 40 deg. cent. rise.

Horse-power	Poles	Rev. per min.	Efficiency				Efficiency			
				½ load	¾ load	Full load		½ load	¾ load	Full load
2000	10	300–500	At 300 r. p m.	93.2	93.7	93.6	At 500 r. p m.	91.3	92.4	92.6
3000	14	180–360	At 180 r. p m.	92.5	93.85	93.8	At 360 r. p m.	91.9	92.9	93

The mill auxiliaries are with a few exceptions motor operated; 220-volt squirrel-cage induction type motors with magnetic cross line starters are used for all constant speed continuous duty applications such as fans, pumps, and chain tables between mills. These motors are in sizes from 2 to 75 hp. They are standard sleeve bearing motors with drip proof covers. The cross-line starters provide low-voltage protection and have thermal overload relays.

For intermittent duty, on loads having high starting torque and requiring quick acceleration, 250-volt d-c. mill type series or compound motors are used. This includes all cranes, transfer tables, roller tables, except hot strip run-out table; furnace pushers, pinch rolls, and screw downs. The controls for these motors are all of the magnetic definite time limit acceleration type. The control panels excepting for cranes are arranged in groups in dust tight brick rooms. With each group is a spare panel which may be substituted for any control panel in the group by manipulating transfer switches.

Power for operating the new mill is purchased from the local power company at 66,000 volts. Service is over about four miles of double circuit steel tower line from the power company's substation at Trenton, to a step-down transformer station at Middletown, built and owned by the American Rolling Mill Company. The step-down transformer substation consists of two banks of O. I. S. C. transformers 12,000 kv-a. each, connected delta-delta, stepping voltage down to 6600 volts. From the transformers the power is taken 1000 ft. underground on 3-conductor, lead covered, multiple cables to the main 6600-volt distribution bus.

Automatic oil circuit breakers are provided in the line at the 66,000-volt substation at Trenton and at the 6600-volt bus at the mill. Normally a 66,000-volt line, transformer bank, and 6600-volt line form an independent unit. No switching is done at 66,000 volts at the mill. However, disconnects are provided for isolating transformers and lightning arresters and provision is made for adding oil switches when required.

The transformers and lines are of such size that one bank will carry the mill load and the transformer capacity may be increased 50 per cent by adding radiators. Thus even with a large increase in power demand the duplicate sub-station and transmission equipment may be used in case of interrupted service on the line in use.

The main bus for distribution of power at 6600 volts is in two sections. Each section is supplied by one of the transmission units. Feeders are so arranged on the bus sections that one section may be taken out of service for additions or repairs without a complete shut down of the mill. The two bus sections are tied together by a non-automatic oil circuit breaker. The 6600-volt bus and oil circuit breaker structure is of the safety enclosed truck type. Westinghouse type OE6 breakers are used on all main line and feeder circuits. Westinghouse type B2 are used for non-automatic applications such as bus tie, and starting breakers for motor generator sets. A spare truck

breaker of each type is provided and may be substituted for any other breaker.

All 6600-volt circuits are protected by induction type overload relays. The two incoming lines are equipped with balanced reverse power and overload relays. These lines are similarly equipped at the power company's substation and have, in addition, ground relays. By means of this system either transmission line is immediately switched off at both ends if it develops any trouble other than a single-phase ground on the 66,000-volt line. In the latter case the circuit is opened automatically at the power company's substation and an alarm is sounded at the mill to notify the operator to open his switch by hand. In the meantime no harm is done as the transformers are delta connected. To simplify the maintenance of relays a test panel is provided with load rheostat, ammeter, and cycle counter. This test panel is wired to a test bus

Fig. 5—4000-Kw. 600-Volt D-C. 6600-Volt A-C. Motor Generators

These machines supply d-c. power for the d-c. mill motors

and any relay on the system may be transferred to the test bus by means of a "push-pull" type test switch. The testing of relays is a part of the switchboard operator's routine.

Three 4000-kw. 3-unit synchronous motor driven MG sets supply 600-volt direct current for main mill motors; 250-volt direct current for auxiliary motor circuits is supplied by two 750-kw. MG sets, and 250-volt excitation by a 300-kw. MG set. The excitation bus may also be supplied from the 750-kw. MG sets.

Each 4000-kw., 600-volt motor generator set consists of two 2000-kw., 600-volt generators rigidly coupled to a 5800-hp., 85 per cent power factor, 6600-volt, 60-cycle, 14-pole synchronous motor. One of these units is shown in Fig. 5.

The field of each 2000-kw. generator is arranged for 125-volt excitation. The fields of the two generators are connected in series and excited from a 250-volt constant voltage excitation bus. These generators have straight differential and crossed cumulative series fields assuring equal division of the load between the two generators of the set. The three sets are operated in parallel by the use of double pole equalizers between sets.

The two 750-kw., 250-volt generators are driven by 1080-hp., 900 rev. per min., 80 per cent power factor synchronous motors, and the 300-kw., 250-volt exciter by 432-hp., 1200 rev. per min., 80 per cent power factor 6600-volt synchronous motor. The synchronous motor generator sets are started from reduced voltage supplied by either of two auto transformers each having capacity to start all the MG sets at intervals of 15 sec.

The starting auto transformers are oil insulated self cooled with outdoor tanks and bushings. They are located outdoors and protected top and sides with wire mesh.

Transfer of motors from starting bus to running bus and application of excitation is automatic.

Paralleling the 600-volt d-c. machines with wide differences of voltage is prevented by voltage balance relays.

Rating and performance of motor generators are as follows:

All motors are 6600-volt, 60-cycle, synchronous, 40 deg. cent rise.

	Generator			Motor		Set efficiency		
Kw.	Rev. per min.	Volts	Poles	Hp.	Power factor per cent	½ load	¾ load	Full load
4000	514	600	10	5800	85	86.	88.75	89.7
750	900	250	6	1072	80	83.1	86.4	87.7
300	1200	250	4	432	80	84.5	87	87.5

An unusual feature of the new mill is the equipment for handling the strip as it leaves the mill.

The metal is carried from last stand to coilers by a roller table in which each roll is direct coupled to an individual motor. These motors are 3-phase 88/220 volts, totally enclosed squirrel-cage type operating on variable frequency from 24 to 60 cycles, rated from 0.4 to 1 hp., speed range 190 to 480 rev. per min. The rolls are 10 in. diameter so that the speed of the table is from 500 to 1260 ft. per min. The motors are grouped on three circuits so that one circuit may be opened without interrupting operation of the table. When one circuit is opened every third motor stops, the other two being sufficient to handle the material.

The variable frequency current for runout table is furnished by a 24/60-kv-a., 24/60-cycle, 88/220-volt generator driven by an adjustable speed d-c. motor. The table is started from the 220-volt, 60-cycle mill circuit. With this arrangement it is only necessary to provide a variable frequency motor generator of sufficient capacity to run the table. Since the starting current is four or five times running current and has a low lagging power factor, a much larger generator would be required to maintain voltage in starting than is required for running only.

Three coilers take the strip steel from the last stand

of the mill and form it into a coil about 30 in. in diameter. The sheet is at a red heat when it enters the coiler and is cooled by water before being discharged. The coiler is driven by a 75-hp. adjustable speed d-c. motor which drives the coiler rollers through multiple spindle gears. The rollers bear on the surface of the coil and as the coil grows are expanded by a cam driven by a 5-hp. adjustable speed motor. The strip steel is forced into the coiler by a pinch roll. The machine is started automatically when the strip completes a circuit from ground to the surface of an insulated roller.

About 30 sec. time is required to coil, cool, and discharge a strip of 12-gage steel 180 ft. long and 48 in. wide.

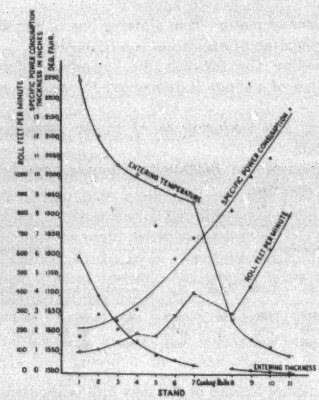

FIG. 6—RELATIVE VALUES AT EACH STAND OF ROLL SPEED, SPECIFIC POWER CONSUMPTION, THICKNESS OF METAL AND ENTERING TEMPERATURE

Specific power consumption equals hp.-sec. per cu. in. of metal displaced

Two hundred and twenty-volt, 3-phase power for auxiliary motors is from two automatic substations located near load centers. Each consists of a bank of three 200 kv-a., 6600/220-volt transformers and two automatic reclosing 1200-ampere, 220-volt, 3-phase feeder panels. A tie connection is run between substations to carry part of substation load in the event of transformer failure.

The 250-volt d-c. system is supplied through automatic reclosing feeders from the 250-volt bus in the power distribution station. This bus is connected by tie lines to two other 250-volt d-c. generating stations in the plant.

Lighting is provided by an overhead system of lights suspended from messenger wire. The messenger wire suspension prevents lamp breakage due to vibration of buildings, and permits arranging units for uniform light distribution.

Illumination values (foot-candles) are as follows:

Bar Plate Mill	3.5
Hot Strip Mill	3.5
Warehouses	3.
Switchboard and Motor Rooms	6

Lighting service is supplied by a bank of three 75-kv-a. transformers, secondaries of which are connected to provide three 220/110 volt circuits. Wires are run open on specially designed supports mounted on lower chord angles of building roof trusses.

Provision is made for emergency lighting so that in case of failure of the 60-cycle service a system of emergency lights is automatically thrown on to storage battery service until the station operator can transfer the main lighting to a bank of transformers connected to the plant 25-cycle system.

Branch circuits are protected by circuit breakers with direct acting time limit feature.

Lighting fixtures are individually fused and are arranged with a disconnecting device for ease of replacement and maintenance.

Both motor rooms are ventilated with filtered air. In the bar mill motor room the air is delivered by a 57,000-cu. ft. per min., 1¼ in. s. p. fan to the basement and thence to the pits under the motors. The air is filtered by two 30,000 cu. ft. per min. rated, rotary type air filters. The system is so arranged that all or part of the air may be recirculated and also may be heated by passing through a group of unit type steam heaters.

The hot strip motor room is ventilated by two 70,000-cu. ft. per min., 1¼-in. s. p. fans which take care of the heat losses from motor generators, and two 65,000 cu. ft. per min., 3-in. s. p. fans which supply the forced air ventilation for the mill motors in this room. The air is cleaned by six 50,000 cu. ft. per min. rotary type air filters and may be recirculated and heated when necessary.

FIG. 7—HOT STRIP MILL MOTOR ROOM

This room also houses the motor-generator sets and main control switchboard

Safety features of the installation include the use of truck type oil circuit breakers in which all hot parts are

entirely enclosed in steel housings making accidental contact impossible. The use of truck breakers avoids disconnecting switches with attendant hazards. Guards and barriers are placed over all bare conductors and moving parts of machines that would otherwise be accessible. High-voltage wiring and apparatus such as 66,000-volt substation buses, arresters, and disconnects are made inaccessible by elevation. Rooms

Fig. 8—600-Volt D-C. Aluminum Bus Structure

In basement of hot strip mill. The supports are ebonized asbestos in welded steel frames

and enclosures containing exposed electrical apparatus are kept locked when possible and arranged so that there is no occasion for unauthorized persons to enter.

All switches, starters, panels, etc., are clearly marked with distinguishing names to avoid confusion in identification.

Conduit containing high-voltage conductors are painted a distinguishing color.

The grounding system is complete in all parts of the system.

Insulating mats are provided in front and rear of all switchboards.

A safety switch is installed in the leads of all motors, at the motors, so that when repairs or adjustments are made to the machines driven by such motors the switch may be opened and so make it impossible for others to start motor.

The power required to roll steel sheets varies with temperature, per cent reduction, speed of rolling, kind of material, and diameter of roll. It is probable that a large part of the input to the old type mill with brass bearings is consumed in friction in the bearings. This fact is demonstrated in comparative tests made on the four high stands of the mill herein described, which are equipped with roller bearings, and another mill making similar product and having brass bearings. In these tests it was found that the specific power consumption in the brass bearing mills was much higher than in the roller bearing mill. This difference increases as the temperature of the piece falls and bearing pressure increases.

Curves of Fig. 6 illustrate the relative value of different variables at each stand of the mill.

The drop in temperature between stands 7 and 8 represents the critical range of ingot iron, between 1922 deg. fahr. and 1562 deg. fahr., in which range the material may not be rolled.

From the accompanying test data the power con-

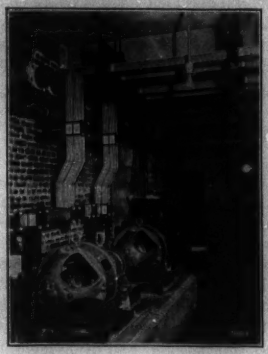

Fig. 9—Series Exciter Set and 600-Volt D-c. Aluminum Bus Structure

sumption per ton of steel rolled is 32 kw-hr., to which must be added the idle friction of mills which make total 40.3 kw. hr. per ton, with a production of 150 tons per hr. To this must be added the power consumption of various mill auxiliaries amounting to about 800 kw-hr. per hour, making a total of 45.6 kw-hr. per ton from blooming mill runout to coil storage.

Discussion

J. W. Bates: There is one question I should like to ask Mr. Gage. Referring to Fig. 6, in regard to the specific power consumption, the increase in the specific power consumption is evidently due to the combined effect of the lower metal temperature and the higher rolling speed. I should like to know if any data were obtained to enable us to know just how much of this increase in power consumption is due to the high rolling speed and how much of it is due to the lower metal temperature.

Referring further to the figure on power consumption of 45.6 kw-hr. per ton, I should like to know if this represents test or operating results and if it pertains to the material mentioned further in the paper, that is, 12-gage sheets, 48 in. wide.

J. B. Ink: Referring to Mr. Bates' question regarding the factors that determine specific power consumption. This power is consumed in fluid friction in the steel and bearing friction in the mill. In the modern mill with roller bearings such as the last seven stands of the mill under discussion, the losses in mill bearings is probably a small percentage of the total. The fluid friction, according to well known laws, varies with the viscosity and is proportioned to the square of the velocity.

In the range of temperature shown, the change in viscosity is not great so we may suspect, until test data upon which to base conclusions are available, that speed of rolling is the predominant factor in fixing the specific power and that this varies somewhere between the first and second power of the rolling speed.

A New Type of Hot Cathode Oscillograph and its Application to the Automatic Recording of Lightning and Switching Surges

BY R. H. GEORGE*

Associate, A. I. E. E.

Synopsis.—*This paper presents a new general purpose type of hot cathode oscillograph, which employs a new electrostatic method of focusing the beam. This oscillograph will operate at any beam potential from 500 to 20,000 volts or more, and at any gas pressure below 30 microns. High photographic sensitivity at medium voltages is attained by the use of a high intensity beam.*

A portable form of the oscillograph for recording lightning surges on transmission lines is described, together with circuits by means of which the lightning surge automatically starts the beam in from ½ to ¼ microseconds after the surge voltage begins to rise from zero. This oscillograph was put into operation on one of the 140-kv. transmission lines of the Consumers Power Company of Jackson, Michigan, on August 27, 1928.

* * * * *

Introduction

With the interconnection of large power systems, and with the rapid developments in the art of radio and carrier current communication, have come new problems many of which involve the study of high frequency phenomena beyond the range of the ordinary Duddell oscillograph. Consequently the importance of solving such problems has stimulated active interest in the development and application of the cathode ray oscillograph,† a device capable of recording extremely high frequency phenomena. As a result there have been developed, within the last decade, some three or possibly four general types of cathode ray oscillographs as follows: the low voltage hot cathode type of the Western Electric Company,[1] the medium voltage hot cathode type of Wood,[2] the high-voltage cold cathode type of Dufour,[3] Norinder,[4] the General Electric Company,[5] and more recently the high voltage hot cathode type of Rogowski.[6]

This paper presents a new general purpose type of hot cathode oscillograph, capable of operating at any potential from 500 volts to 20,000 volts or more, and a special portable form for lightning recording. The object in developing the general purpose oscillograph was to produce one having sufficient flexibility to combine, as far as possible, the desirable qualities of the previous types of cathode ray oscillograph, and yet be simple and reliable in operation.

Salient Features of the New Oscillograph

Some of the outstanding features of the new oscillograph are as follows:

1. The use of a special hot cathode electron gun which makes possible automatic starting and stopping of the beam for recording lightning surges on transmission lines.
2. The use of a new electrostatic method of focusing the beam, effective at any pressure from about 30 microns down to the lowest pressure obtainable with a mercury pump.
3. The entire beam passes through the high voltage anode, thus eliminating the problem of anode heating.
4. Bakelite is used for insulating the high voltage cathode and the deflecting plates, thus reducing the danger of breakage.
5. The deflecting plates are adjustable from outside the vacuum which makes it possible to vary the deflection sensitivity if desired.
6. All parts are readily accessible and easily replaced in case of damage.

General. The method of attack in the design and development of the general purpose oscillograph has been to gain the necessary photographic sensitivity through the use of a high intensity beam at a minimum beam voltage. The principal problem, then, has been one of devising satisfactory means for producing and focusing a high intensity beam over a sufficient range of beam voltages to insure the necessary photographic sensitivity. It is the solution of this problem which constitutes the major contribution of this paper.

Description of the Oscillograph. A general idea of the appearance of the oscillograph may be gained from Fig. 1, which shows a portable outfit designed especially for recording lightning surges. Fig. 2 illustrates the internal construction.

The beam is produced by an electron gun employing a plate voltage of 100 to 1500 volts, which drives a high intensity jet of electrons through a small hole in the plate into a cup-shaped higher voltage cathode where the jet is further accelerated and focused.

The principal object of the electron gun is to protect the filament, although it is made to perform other important functions as may be seen later.

Referring to Figs. 2 and 3, the electron gun consists

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†For a history of the development of the cathode ray oscillograph with complete bibliography see "Measurements in Electrical Engineering by Means of Cathode Rays," by J. T. MacGregor-Morris and R. Mines, *Journal of I. E. E.*, Nov., 1925, Vol. 63, p. 1056.

1. See Bibliography.

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29-61

of an oxide coated ribbon filament at (1), a filament shield at (2), and a positive electrode or plate at (3). The oxide coated filament provides ample electron emission without sufficient light emission to affect the photographic film. Films have been exposed as long as one hour without showing appreciable darkening from the light. The filament shield serves to limit the

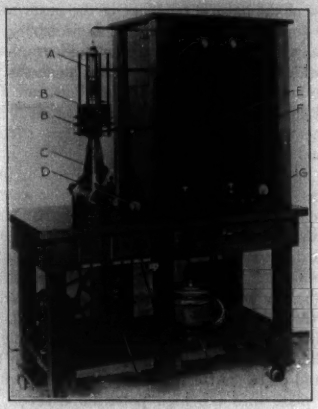

FIG. 1—PORTABLE HOT CATHODE OSCILLOGRAPH FOR SURGE RECORDING

A. Electron gun housing.
B. Terminals of deflecting plates.
C. Oscillograph bell.
D. Vacuum gage.
E. Gun voltage control.
F. Cathode voltage control.
G. Timing oscillator.
H. Vacuum pump.
I. Drying chamber for films.

emitting area of the filament, and to protect it in case of flashover. With the proper spacing between filament and shield, and shield and plate, a converging field can be produced between the filament and plate, which concentrates the electrons from a relatively large area of filament and drives them through the small hole in the plate. As indicated in Fig. 3, the plate protects the filament from the high potential field, and by employing a small hole for the beam to pass through, largely protects it from bombardment by high velocity ions. Arc-over between the gun electrodes is prevented by making the spacings small, and providing for the rapid escape of gases emitted from the hot surfaces.

Aside from the protection of the filament, the use of the gun is an advantage for automatic recording of lightning surges, since the beam may readily be cut off by either removing the plate voltage or impressing a sufficient negative bias on the filament shield. By these means the filament may be held at the proper temperature for good emission, and full voltage maintained on the high voltage electrodes without producing a beam. Then by the proper circuits the transient can be made to apply the plate voltage automatically, or reduce the bias of the shield and permit the beam to form in time to record most of the wave front. As soon as the surge has passed the beam is automatically cut off again.

The high-voltage electrodes are of quite radical design as shown in Fig. 2. The cathode (4) is a cup-shaped piece of metal with the plate of the electron gun forming what would be the bottom of the cup. The cylindrical anode (5) extends inside the cathode,

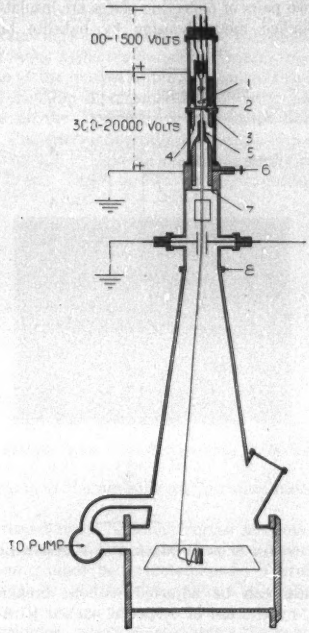

FIG. 2—DIAGRAMATIC SCHEME OF THE PORTABLE OSCILLOGRAPH

(1) Ribbon filament.
(2) Filament shield.
(3) Positive plate.
(4) Cup-shaped cathode.
(5) Cylindrical anode.
(6) Screw for raising and lowering anode.

and can be raised or lowered by means of the knurled head at (6).

The entire beam is permitted to pass through the anode, thus eliminating the problem of anode heating, and at the same time increasing the beam intensity. The spacing between the high-voltage electrodes is small compared to the length of the mean free path of

the electrons at pressures within the operating range, thus preventing a cold or gaseous discharge taking place. The anode extending into the cathode prevents electrons from bombarding the bakelite insulation, and also prevents flickering of the beam on continuous operation by shielding it from the field of surface charges on the insulation.

Provision is made for inserting a diaphram at (7), (Fig. 2), for cutting off the small amount of stray emission caused by a positive ion bombardment of the cathode. If a diaphram is used, the hole in it is made large enough to permit all of the main beam to pass through.

The two pairs of deflecting plates are insulated from the grounded metal housing by bakelite bushings.

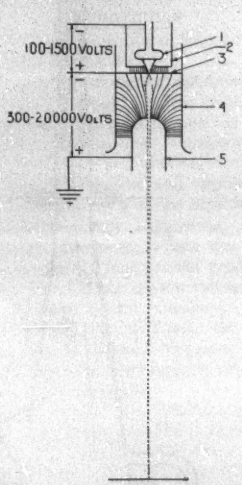

FIG. 3—DIAGRAM SHOWING METHOD OF FOCUSING THE BEAM

The plates can be adjusted without breaking the vacuum, by the use of a special packed joint in the bakelite.

No provision is made on the portable type shown in Fig. 1 for magnetic deflection, but the laboratory type is provided with a section of bakelite at (8) (Fig. 2), to prevent eddy currents when using deflecting coils. A more recent design of portable oscillograph also provides for magnetic deflection.

The general purpose laboratory type of instrument is provided with both a rotating film drum and plate drum. The film drum takes film 5 in. by 30 in. with which the equivalent of a continuous record 140 in. in length can be obtained. The plate holder takes six 4- by 5-in. plates. The portable type shown in Figs. 1 and 2 is made much more compact to reduce volume, and takes 5-in. roll film. The oscillograms are 2½ in. by 5 in., although in a more recent design the oscillograms will be 3½ in. by 5 in. If special long rolls are used, as many as 100 exposures can be taken with one loading.

The Electrostatic Method of Focusing the Beam. The focusing of the electron jet has always been the most difficult problem in cathode ray oscillograph design, because of the natural tendency of the beam to diverge due to the mutual repulsion of the electrons. This is especially true of a general purpose oscillograph employing a wide range of beam potentials, since the positive ion method described by Johnson[1] does not function well at beam potentials much in excess of 2000 volts, and the magnetic method becomes unsatisfactory at potential much below 10,000 volts.

The most important feature of the oscillograph just described is the electrostatic method of concentrating and focusing the beam, which is effective at any beam potential from 500 volts to at least 20,000 volts.

Referring to Fig. 3, the beam from the filament (1) is concentrated and brought to a focus at the hole in the plate (3), by properly proportioning the spacing between the filament (1), the filament shield (2), and the plate (3), with respect to the size of the hole in (2), to produce a rapidly converging field between the filament and plate. After passing through the small hole in the plate into the high potential cathode (4) the beam diverges at first as indicated by the dotted lines, Fig. 3, due to the momentum received from the converging field in the gun.

The cathode (4) and the anode (5) are so shaped that the high potential field converges toward the anode. Therefore the electrons, in falling through this field, are not only accelerated in the direction of the anode, but are given a component of velocity toward the center of the beam. If this inward radial component of momentum of the electrons is great enough to overcome their force of repulsion the beam can be brought to a focus. It is therefore apparent that the distance from the cathode at which the beam will come to a focus depends upon the diameter of the beam when it starts to converge, hence the divergence of the beam on entering the cathode. This fact also necessitates the use of a large hole in the anode.

Since the time through which the mutual repulsion of the electrons in the beam acts, depends upon the velocity of the beam and consequently upon the beam potential, it is necessary to be able to vary the radial component of the high potential field in order to focus the beam. This is done by raising or lowering the anode by means of a rack and pinion operated by the knurled head (6), Fig. 2.

In practise, the anode is raised and the plate voltage of the gun increased to focus a high-voltage beam, and on the contrary the anode is lowered and the plate voltage reduced for a low voltage beam. The reason for raising the plate voltage when focusing a high

voltage beam is to produce the same divergence of the beam on entering the cathode in spite of the higher potential gradient there. However, focusing is not critical to plate voltage and it is only with large changes in cathode voltage that it is necessary to change the plate voltage. The initial size of the beam is not so important for low voltage beams since focusing is assisted somewhat by the presence of positive ions.

Application of the Oscillograph

A. With Continuous Beam.

A steady beam can be produced and maintained for hours, if necessary, at any beam voltage from 500 volts

Fig. 4—Voice Record Taken with 2500-Volt Beam

to 20,000 volts, depending upon the deflectional and photographic sensitivities required.

The continuous beam permits study of the various forms of Lissajous figures, such as power loss loops

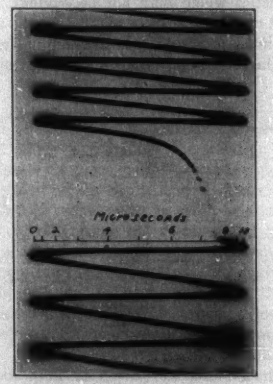

Fig. 5—High Voltage Impulse Superimposed on a 50,000-Cycle Timing Wave. 10,000-Volt Beam

including magnetization and dielectric loss curves, vacuum tube characteristics, frequency checking, etc. Another important use is in making long records of medium frequency phenomena where much greater photographic sensitivity is required than can be obtained with the ordinary oscillograph, and where it is necessary to obtain records free from distortion due to the mass of the recording element. A few examples are, studies of the operation of radio and carrier current apparatus, radio interference from corona, and testing voices for talking movies.

Fig. 4 represents a section of a voice record taken with the laboratory oscillograph at a beam voltage of approximately 2500 volts. The timing is indicated by the dashes which occur at intervals of 1/60 of a second. The diameter of the beam was less than 1/2 millimeter.

Fig. 5 is an oscillogram of a high-voltage impulse super-imposed on a 50,000-cycle timing wave. Fig. 6 is a 500,000-cycle wave having an amplitude such that the beam was traveling at the rate of 100 kilometers (62 mi.) per second when crossing the zero axis. Both oscillograms were taken with a 10,000-volt beam.

B. Recording Lightning Surges on Transmission Lines.

It was at the request of the Consumers Power Company of Jackson, Michigan that the writer under the auspices of the Engineering Experiment Station of Purdue University undertook the design of a cathode ray oscillograph for recording lightning surges on transmission lines. This cooperative project resulted in the

Fig. 6—500,000-Cycle Wave Taken with 10,000-Volt Beam

construction of the portable oscillograph illustrated by Fig. 1.

The recording of lightning surges presents a very difficult problem, in that the time at which a surge will occur cannot be predetermined. Therefore the oscillograph beam must either remain on continuously, or means must be provided whereby the transient, to be recorded, automatically starts the beam. The first method appears more logical since the time available to get the beam under way, by the latter method, is of the order of a fraction of a microsecond. However, there are practical difficulties encountered with the first method which may warrant the use of the second method. With the first method there is the difficulty when using a cold cathode of producing an intense beam which will hold its focus for any great length of time. And with either the hot or cold cathode oscillograph, the fogging of the film, due to bombardment by electrons scattered from the main beam by collision with gas molecules, presents a serious problem. By the proper regulation of vacuum and electrode temperature,

Dr. Norinder[4] has solved both problems in a satisfactory way, but the equipment required is quite elaborate.

As a result of the difficulties encountered with fogging when employing a continuous beam, attention was turned to means by which the transient could be made

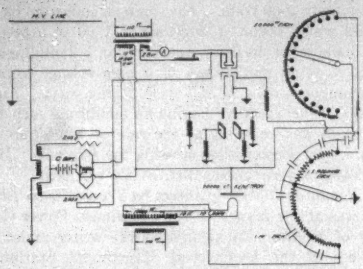

Fig. 7—Circuit for Automatically Starting Beam by Switching Plate Voltage of Electron Gun

to automatically start the beam in time to record the wave front. A method had already been devised in Germany, whereby the beam was initiated by the transient and the impulse to the deflecting plates delayed by passing it through a cable. However, with this method the important question arises as to whether the impulse

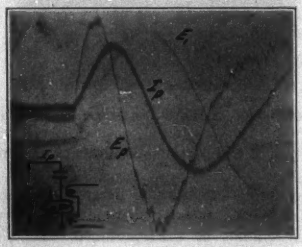

Fig. 8—Time Relation Between Applied Surge, Current, and Voltage Drop across Inductance in the Equivalent of Pick-up Circuit

has suffered distortion while passing through the cable. Since the beam of the oscillograph described in this paper can be cut off either by removing the plate voltage of the electron gun or by impressing a negative bias on the filament shield, it was evident that if a circuit could be devised for rapidly applying the plate voltage, or reducing the shield bias, the problem of automatically starting the beam might be solved.

Fig. 7 represents the circuit diagram for the portable oscillograph including a circuit for automatically initiating the beam by applying plate voltage to the electron gun. With this circuit the plate or beam current of the electron gun must pass through one of the two vacuum tubes inserted between the gun voltage supply and the filament. Therefore the beam can be stopped by applying a sufficient negative bias to the grid of the vacuum tubes to prevent the flow of plate current. Then in order to initiate the beam it is necessary to apply a small positive impulse to the grid of one or both tubes, after which the time required for the beam to build up depends upon the rate at which the electron gun can be charged as a condenser.

The circuit for producing the positive impulse on the

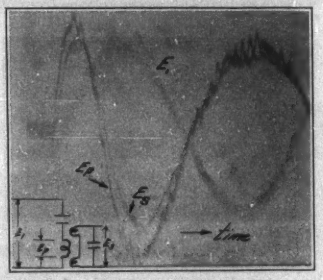

Fig. 9—Time Relation Between Applied Surge, Voltage Drop Across Inductance, and Terminal Voltage in Equivalent of Grid Circuit

grid of one of the vacuum tubes has some desirable characteristics which may not be evident. The first important requirement to be met is that the surge to be recorded must produce a positive impulse on the grid of one of the tubes regardless of the polarity of the surge.

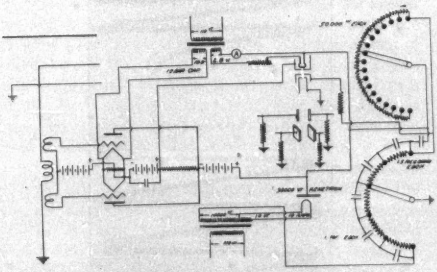

Fig. 10—Circuit for Automatically Starting Beam by Controlling Filament Shield Potential

This, of course, is most readily accomplished by an inductively coupled circuit which also has the advantage that the antenna of the dividing condenser can be insulated from the high-voltage direct current of the oscillograph. A rather unexpected advantage of the inductively coupled circuit comes from making the inductance such that the inductive reactance of the

antenna circuit is small compared to the capacity reactance. This permits the charging current in the antenna circuit to reach its maximum well in advance of the maximum voltage of the surge, and since the voltage drop across the inductance is proportional to the rate of change of current, it therefore reaches its maximum ahead of the current. The inductance is, however, the primary of an air core transformer, therefore the voltage induced in the secondary or grid circuit also reaches its maximum ahead of the current in the primary. In order to check the theory and determine the time relation of the secondary terminal voltage, *i. e.*, the grid voltage, oscillograms were taken of the voltage and current relations in a similar low frequency circuit when the equivalent of a slow surge was applied. Fig. 8 shows the time relation between

Fig. 11—Lightning Generator Impulse which Automatically Started Beam

the applied voltage, current, and voltage drop in the inductance. Fig. 9 indicates the relation between applied voltage, the voltage drop across the inductance, and the voltage in the equivalent of the grid circuit. In studying the oscillograms it must be remembered that in actual magnitude the voltage drop across the primary, and the induced voltage of the secondary are small compared to the surge voltage, although they are shown to be of about equal magnitude by the oscillograms. Fig. 9 shows that the capacity load corresponding to the grid capacity causes quite an appreciable phase shift in the secondary terminal voltage with respect to the primary voltage but that the shift with respect to the surge voltage is small. In order to make sure that the phase relations were not being affected by the burden of the oscillograph elements, the burden was greatly increased but without apparent effect.

An improved form of the circuit shown in Fig. 10 is employed when using a negative bias on the filament shield to stop the beam. Although this type of circuit is faster than the type shown in Fig. 7, it is liable to cut off too quickly on the tail of the wave unless special means are employed to hold the beam on. It has not been definitely decided which circuit is superior in practise, as either circuit will initiate the beam in half a microsecond or less from the time the surge voltage begins to rise from zero. Since klydonograph records indicate that lightning surges may reach a maximum in from one to ten microseconds, this circuit is capable of getting the beam under way in time to record practically

Fig. 12—Installation of Cathode Ray Oscillograph on 140-Kv. Line of the Consumers Power Company

the entire wave. The circuit may also be used for laboratory work to supplant a switch or sphere gap for turning on the beam if wave fronts of not less than one microsecond are employed.

Fig. 11 shows a record taken of a lightning generator wave in which the beam was automatically started by the surge. The surge was applied at right angles to a 50,000-cycle timing wave.

The portable hot cathode oscillograph, Fig. 1, employing the automatic beam starting feature, was put into operation on a 140-kv. line of the Consumers Power Company of Jackson, Michigan at their Black-

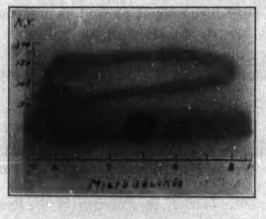

Fig. 13—Lightning Surge Recorded on 140-Kv. Line of the Consumers Power Company

stone Substation on August 27, 1928. Fig. 12 shows this installation.

Fig. 13 shows a 170-kv. surge which occurred during a distant lightning storm. The surge was hardly sufficient to fully turn on the beam and as a consequence it is very poorly focused.

A full description of the installation and results will

probably be presented by the Consumers Power Company following the 1929 lightning season.

This oscillograph is entirely self-contained with the exception of the power supply which is taken from a 110-volt 60-cycle source. The power required is approximately 700 watts. The vacuum is produced by a rotary oil pump although a mercury pump will probably be added for use during the next season. The volume of the oscillograph has been reduced to a minimum so that with well dried film the oil pump alone will produce a workable vacuum in approximately three minutes. The film holder is daylight loading and takes a standard No. 104 film. A six exposure film is sufficient for eight oscillograms, although provision is made for sufficient film to take 100 exposures with one loading by the use of special film.

Development work is being continued along the following lines:

1. Further increasing the beam intensity.
2. Focusing still higher voltage beams.
3. The production of a linear time axis for lightning records.
4. Making the recording of lightning surges more nearly automatic.

Conclusion

As a result of a long series of investigations a general purpose type of cathode ray oscillograph has been developed which is applicable to a wide variety of uses in the fields of science and engineering. A special form of this oscillograph has been developed and put in use for recording lightning and switching surges on high-voltage transmission lines.

It is felt that this oscillograph will prove particularly useful in laboratories where a wide variety of work is handled as well as in field investigations.

The author wishes to express his sincere appreciation to Professor C. Francis Harding for his continued interest and encouragement as well as making the work possible, to Mr. J. W. Raleigh for valuable assistance in the early stages of the work, and to Mr. J. R. Eaton of the Consumers Power Company for many valuable suggestions.

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Discussion

W. L. Everitt: I wonder if this new type of electrostatic focusing increases the life of the filament, so that, if it is made up in a permanent tube such as the Western Electric Company's No. 224 A type, a considerably longer life would be obtained than is now possible with the present method of using a small filament worked at a high current.

Is the filament narrow as it is shown or is it fairly wide?

Is there any dispersion due to collision with the molecules of gas after it has passed through this second focusing slot No. 5?

R. H. George: As to the life of the filament, I believe that with an apparatus sealed off the life would be reasonably long, since the filament is well shielded from positive ion bombardment and the apparatus could be operated at a much harder vacuum than the Western Electric oscillograph.

The life of the filament, *i. e.*, the oxide coat, has not been particularly long with our oscillograph, the maximum being 30 to 40 hr. operation. However, the filament can be recoated a number of times. The principal difficulty with filaments has not been caused by positive ion bombardment, but by the action of oil vapors which lower the emission. The emission can be restored usually by heating the filament in the presence of oxygen, providing it has not previously been heated to excessive temperatures. This difficulty would not be encountered with filaments sealed off in glass apparatus.

The filament shown in the diagram is quite exaggerated in size. It is made in the form of a ribbon 1/16 in. wide and the emission taken from the flat surface, which more than covers the hole in the filament shield.

The film is turned up by means of a metal shaft which passes through a packed joint in the oscillograph housing. The vacuum-proof joint consists of a cotton packing on the high-pressure side of which is a cavity containing heavy grease. With this type of joint the shaft can either be rotated or moved endwise without affecting the vacuum. This same type of joint is employed for the shutter lever and the deflecting plates.

The film drum employed when recording low-frequency phenomena is driven through a conical joint with a special grease seal. Speeds as high as 500 to 600 rev. per min. can easily be obtained without affecting the vacuum.

There is some dispersion of the beam after it passes through the last plate, but it is not appreciable at normal operating pressures, *i. e.*, pressures below 10 microns.

It has recently been found that some of the apparent dispersion was due to the fact that gas liberated from the metal of the electron gun was not escaping with sufficient rapidity around the high-voltage anode, and as a result electrons were being liberated by bombardment of the gas, giving rise to electrons of varying velocity. The design has been altered to take care of this trouble.

Fused Arcing Horns and Grading Rings

Design, Construction, and Operating Experience on 66,000-Volt Transmission Lines of the Union Gas & Electric Company

BY PHILIP STEWART[1]

Associate, A. I. E. E.

Synopsis.—*This paper considers the use of fuses on insulator strings of high-voltage overhead conductors, to interrupt the arc at times of flashover before the line relays operate to disconnect the circuit. Consideration is first given to the original development of this idea, in which a fuse was connected between the line conductor and an arcing ring, attached to the second insulator unit. When an excessive voltage occurs on the conductor to ground, there is a flash between a two-pronged horn on the top insulator unit and the ring. The circuit is completed through the fuse, which immediately opens, breaking the arc. Further consideration is given to a later development of the principle, in which two expulsion type fuses replace the two-pronged horn at the top of the insulator string and the arcing ring is placed at the conductor end of the string. Data are presented from tests and from experience on about 100 mi. of 66,000-volt circuit, of the Union Gas and Electric Company, Cincinnati, Ohio.*

Introduction

THE general practise up to this time is to isolate faults at the ends of feeders or transmission circuits in trouble. This is done by means of oil circuit breakers actuated by relays. Lines and circuits so equipped immediately go out of service in case of faults. The fused insulator string is a device for isolating faults right at the point where the fault occurs, interrupting fault current only, and not interrupting the useful service of the line or circuit.

It is now generally considered good practise to provide some arrangement of arcing or grading rings or horns, at all insulator strings of important high-voltage transmission circuits. Experience and tests point to this means of preventing cascading and possible shattering of insulators by flashovers caused by transient voltages on the line conductors to ground. The use of such devices reduces the time the line may be out of service, due to flashed insulators. The development of the fused device for insulator strings is an attempt to further reduce the interruptions, especially those of short duration caused by the line switch opening to clear the surge flashover and subsequent short circuit which may last for only a few seconds.

One of the two generating plants in the Cincinnati territory is located at Columbia Park, on the Ohio River, about 20 mi. west of Cincinnati. This plant is operated as a base load station at as nearly full load as possible at all times. The major part of the load from Columbia to Cincinnati is carried over four 66,000-volt transmission circuits carried on two double circuit tower lines. These circuits are connected to Terminal Switching Station, north of Cincinnati. From Terminal Station, two 66,000-volt circuits connect to each of three major distribution substations. Two circuits also connect to the system of the Dayton Power and Light Company.

It is very important for the operation of the entire system that these circuits be kept in service without interruptions. It is especially so on the circuits from Columbia to Terminal Station. When these lines were built, the insulator strings were equipped with flux controls. Six insulator units were used in each string. The first year's operation was good, but the disastrous results of insulator flashovers and line outages were evident.

Original Development

The first arrangement using fused arcing rings consisted of a string of seven insulator units and was provided with two horns at the top and a split oval ring below the fifth unit. Fuse clamps were mounted on the ring and conductor clamp, thus shunting the two lowest insulators with a fuse. With the fuse in, the line is insulated with five units between the line and ground. After the fuse has been blown, the insulator will consist of seven units. In this way the flashover distance will be increased and it was thought that in case a second surge originated near this point before the fuse was renewed, the flashover would occur at the next adjacent point of support.

Tests were made with this arrangement to observe the performance in clearing flashovers on the 66,000-volt system. All 66,000-volt lines were energized into Terminal. The tests were made on a line from the main bus. The insulator string was shunted by a piece of small copper wire in order to start the flashover. The line was closed in on the shorted insulator by means of the station oil circuit breaker. An oscillograph was used in order to obtain the short-circuit currents and voltages. An average short-circuit current of 2970 r. m. s. amperes was obtained, with 9000 volts, phase to neutral, during short circuit. The 10-ampere 37,000-volt carbon tetrachloride fuses cleared in one cycle, in each case clearing before the relays operated to open the oil circuit breaker.

Tests were made in the high-voltage laboratory of the General Electric Company. In these tests it was established that a power arc can be produced by fusing

1. Union Gas & Electric Company, Cincinnati, Ohio.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

29-50

a fine copper wire connected between the arcing rings of an insulator string. This method of starting an arc was used throughout.

Tests were also made to investigate the performance of the fused rings when subjected to artificial lightning; first with a ring made of 1½-in. pipe, then a strap iron ring, and in the third case the horns were supplied with hemispheres. These three tests gave approximately the same flashover voltages and arcing characteristics, thereby indicating that for this short string, grading is not so necessary. The average flashover voltage determined in these tests was 530 kv. maximum, which is the same as the flashover voltage between needle points placed the same distance apart. This indicates that the voltage distribution on the comparatively short string of insulator units does not affect the flashover of the parallel gap. However, on longer strings, grading is necessary, and the multi-gap effect of the units must be overcome.

Further tests determined the effect of artificial lightning discharge on the fuse alone. Ten discharges of approximately 6000 amperes (maximum) had no apparent effect upon the fuse, due to the exceedingly short time during which the current was flowing. This shows that the fuse is probably blown by the follow-up current and not by initial current due to the lightning discharge.

Operating Experience

In the spring of 1927, fused arcing rings were installed on two of the Columbia—Terminal circuits, and four of the shorter lines, leaving Terminal, totaling approximately 65 mi. of circuit so equipped. During an eight month period succeeding this installation, there were six cases of the fuses blowing and clearing without

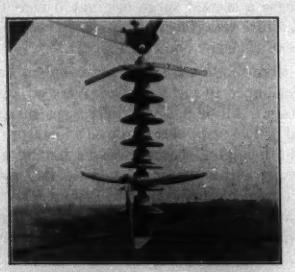

Fig. 1—Original Development of Fused Arcing Ring Using Carbon Tetrachloride Fuses

an interruption to service. There were three cases in which the fuses failed to clear and the circuit breaker opened, interrupting service on the line. During this period, ten fuses were blown. The three service interruptions were caused by the destruction of the fuses at the flashover points.

It is noted that the fuses functioned properly 77 per cent of the time and succeeded in preventing service interruptions 67 per cent of the time.

All failures of the device were due to failures of the fuse itself, caused by follow-up surge voltages. One case noted during the preliminary tests probably ex-

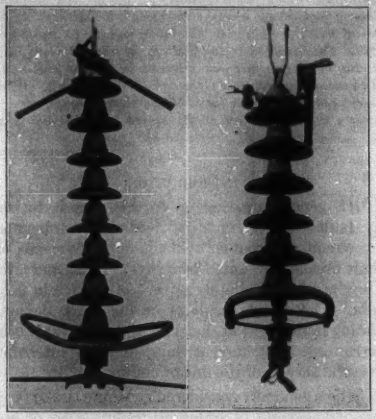

Fig. 2—Later Development of Fused Insulator String Using Expulsion Type Fuses as Arcing Horns

plains these failures. A blown fuse was inserted and an impact of very high voltage was applied. The arc followed a path from the conductor over the metallic part of the fuse within the glass tube, puncturing the glass, and passing over the outside to the ring and to the horn and ground. In actual practise, a dynamic arc would follow this first arc, causing destruction of the fuse.

Development Using Expulsion Type Fuse

The operation of the fused arcing rings, using carbon tetrachloride fuses, (Fig. 1) was successful in preventing a number of service interruptions on the two Columbia—Terminal circuits on which they were installed. In considering the installation on the two remaining circuits, a more economical arrangement of the device was desired. First, the added cost for the extra insulator units shunted by the fuses did not seem to be justified by the first year's operation. Second, the carbon tetrachloride fuses were more expensive than expulsion type fuses. Therefore, the next development was an arrangement using two expulsion type fuses in place of the horn at the top of the insulator string, (Fig. 2) and a ring at the conductor end.

Tests were made on various makes of expulsion type fuses, some the standard products of different manufacturers and some specially designed by the Union

Gas and Electric Company. These tests were made at Terminal Station with the system set-up similar to that of the previous tests. In each case the fuse operated correctly to break the short circuit. The time required with a few exceptions was one-half cycle or less.

Further tests to determine the conductivity of the fuses after they had been blown established that there was practically no conducting material deposited in the fuse holder.

The diameter of the ring and the position of the fused horns were determined from tests made with the impulse generator of the General Electric Company so that flashovers on the assembled string occur from the end of the fuse to the ring without cascading.

In addition to more economical construction, the present assembly has an advantage over the original in that two fuses are employed. It was established by tests that both fuses will not blow at the same time, but one will remain in good condition. When one fuse is blown the arcing distance on that side is greater than on the side of the good fuse and second flashovers go to the tip of the good fuse. Therefore, the device will function for two consecutive surges originating on the same section of line before replacement of the fuses is necessary.

OPERATING RECORD WITH FUSED ARCING HORNS

Line	Year	Interruptions	Flashed units	Type of fuse	Interruptions prevented	Fuses blown
1761	1926	3	30	none		
	1927	2	6	S. & C.	4	9
	1928	0	0	S. & C.	1	1
1762	1926	4	14	none		
	1927	1	2	S. & C.	2	4
	1928	0	0	S. & C.	1	1
1763	1926	1	0	none		
	1927	4	18	none		
	1928	0	0	Expulsion	1	1
1764	1926	2	6	none		
	1927	3	24	none		
	1928	0	0	Expulsion	1	1
1261	1926	1	0	none		
	1927	0	0	S. & C.	0	0
	1928	1*	0	S. & C.	*	2
1262	1926	3	6	none		
	1927	0	0	S. & C.	0	0
	1928	1*	6	S. & C.	*	1
861	1926	0	0	none		
	1927	0	0	S. & C.	0	0
	1928	0	0	S. & C.	2	3
862	1926	0	0	none		
	1927	0	0	S. & C.	0	0
	1928	2	0	S. & C.	0	3

*Flashover at substation.

Upon operation, a bright metal clamp at the out end of the fuse is blown off and the condition of the fuses can be determined by patrolmen's visual inspection from the ground.

Operation in 1928

In the early part of 1928, the fused horns and grading rings were installed on the two Columbia—Terminal circuits not previously equipped with fused arcing rings. During this year there were no interruptions to service on any of these four circuits. Although there were comparatively few lightning storms, four fuses were found blown, indicating that they had been effective in preventing service interruptions.

For two years operation, there have been seventeen cases of flashovers on lines equipped with fused horns or arcing rings. Of these cases twelve interruptions have been prevented. This is an elimination of 70 per cent of service interruptions.

The accompanying table shows the operation record for the 66,000-volt tower lines during three years of service.

Conclusions

1. Arcing rings are essential equipment on important high-voltage transmission lines. Without arcing rings, many insulators are flashed and shattered due to cascading of the string. Shattered insulators mean a line outage for several hours caused by an abnormal condition existing only a fraction of a second.

2. Fuses in the flashover circuit interrupt and prevent the flow of follow-up current. Since the fuses function in approximately one-half cycle and protective relays are ordinarily set to operate in not less than 35 to 45 cycles, the short circuit will be cleared before the oil circuit breaker operates.

3. The fuse is not blown by the current of the initial surge, but by the current of the dynamic arc.

4. The use of fused insulator strings on the high-voltage circuits of the Union Gas and Electric Company has been a large factor in a considerable improvement in service and operating records.

5. Tests and operating experience, as well as economic considerations, show several advantages of the assembly using fused horns at the support end of the string, over the original assembly using a fused ring at the conductor end.

6. The adoption of fused horns and grading rings for important high-voltage lines has proved to be another means by which service and operation may be improved.

Discussion

R. L. McCoy: Mr. Stewart's very interesting paper on the development of the fused grading shield deals with its application to 66-kv. transmission line. The next thing which comes to mind, is how far can its application be extended to higher voltage lines. It is conceivable that its widest field of usefulness should be found on 110-kv. and 132-kv. lines. We have investigated this quite thoroughly, and there is now a fuse available which we believe will make this device practical for lines at these voltages. We have tested this very carefully in the laboratory and it worked perfectly. Of course, the final answer lies in field operation, and we are making arrangements for some small field installations during the coming lightning season which we hope will give some experience by the end of the season, so that there will be a definite basis for recommendations on 110- and 132-kv. lines by next Fall.

S. M. Hamill, Jr.: I think that Mr. Stewart has been a little modest in what he said about the fused grading rings. I notice the article states, "An average short-circuit current of 2970 r. m. s. amperes was obtained, with 9000 volts phase-to-neutral during short circuit." That, of course, is the drop from the point in the transmission line to what approximates the substation neutral, or rather what approximates a very good ground at the substation, and is not the actual phase-to-neutral voltage, open circuit, which was approximately 38,000 volts instead of 9000.

The second point I should like to bring up is that it is impossible to set the induction-type relays low enough so that they will clear with the fuses. In other words, we are not hindered in the relay settings by these fuses. We can set our relays at the minimum setting, namely, about 9 cycles, and still have the fuses clear ahead of them. As a matter of fact, all our low-setting relays are set at this valve on those transmission lines, which are protected by Mr. Stewart's fuses. While Mr. Stewart emphasized a very important item from the point of view of transmission-line maintenance, namely, the prevention of damage to insulators, I think that the most important item is the prevention of the line outage itself.

C. L. Fortescue: We at the Westinghouse Company have been working along similar lines, our idea being to incorporate into the insulating string the characteristics of a lightning arrester. This is really not in itself fundamentally a new idea. For instance, in wood poles we have an action due to the wood pole insulation, that is very similar to that in an arrester. If the lightning strikes the line and the wood pole functions as an insulator, so long as the pole is not ruptured the dynamic arc will not follow the flashover, and you have something very much like an arrrester action.

Of course, the trouble with a wood pole is that it is liable to rupture, due to a heavy lightning stroke, and of course in that case you get an outage. If we could incorporate in the string itself the properties of an arrester, we would have something that would be nearly ideal, because when the lightning stroke hit the line, the resulting voltage of the surge being too high for the insulator, it would function as an arrester and it would absorb the surge. The characteristic of an arrester would cut off any dynamic following current, and we would go right ahead without any interruption.

O. S. Clark: It should be of interest to know that, in connection with the statements of Mr. Fortescue regarding the performance of lightning on wood structures, one of our companies has a line in the construction of which is incorporated a large amount of wood insulation, and that in a number of cases in which the wood material has been ruptured by a lightning discharge, the service has not been interrupted.

In connection with the design of the fused horn and grading ring described by Mr. Stewart, the laboratory work that we have done, we believe, shows that it is possible to apply the device to any form of transmission-line insulation. The problem for any type of insulator and for any length of string is an individual one and it must be worked out, I should think, in an experimental laboratory before any conclusion can be reached. While the device is applied, at this time, to suspension strings only, there is no doubt that, if economically desirable, its application can be worked out for lines using insulators of the pin type.

There is one question that very likely occurs in the minds of many in connection with the double fused horn and grading ring assembly, and that is the possibility of flashes occurring at both fuse tips simultaneously. Laboratory tests have shown that in a very small percentage of the cases when the assembly was sparked over by the impulse generator, this did occur. The probability of such occurrences is too low to consider in its application.

H. C. Don Carlos: In discussing Mr. Stewart's paper, I wish to comment particularly on his conclusion No. 1 wherein he states that "arcing rings are essential equipment on important high-voltage transmission lines."

The Hydro Electric Power Commission has in operation approximately the following mileages of high-voltage lines which are equipped with suspension insulators.

220 kv.—25 cycles—	203 circuit miles
110 kv.—25 cycles—	1125 circuit miles
110 kv.—60 cycles—	200 circuit miles
Less than 110 kv.—25 cycles—	127 circuit miles
Less than 110 kv.—60 cycles—	194 circuit miles

None of these lines is equipped with arcing rings and our flashovers on the 110-kv. 25-cycle system have been at the rate of 1.25 and 1.53 per 100 mi. of circuit during 1927 and 1928 respectively. I believe that there has been no case in which a string of insulators has been damaged to the extent that the line would not go back into service.

The 220-kv. transmission line was put in service October 1, 1928, and, of course, has not yet passed through a complete lightning season. We have therefore no data as regards the performance of this line throughout a complete season, though there was at least one rather severe lightning storm last fall in the district traversed by this line, and also a few this spring. While we have had four outages on this circuit, these have all been due to foreign interference and as yet there has been no evidence of a flashover occurring on this circuit. We therefore do not anticipate any serious trouble from lightning.

As regards our 60-cycle, 110-kv. lines at the head of the Great Lakes supplying Port Arthur and Fort William, about 70 circuit-miles have been in service eight years without a single flashover, and the remaining 90 circuit-miles, which have been in service 4½ years, have experienced a few flashovers, but in no case has an insulator string been damaged to such an extent that the line would not go back into service immediately.

While these lines can be returned to service immediately, there are in some cases one or two units in the string which are damaged, but these are located during line patrol and replaced at our convenience.

While there is no doubt that grading ring should, from a theoretical consideration, be of advantage in reducing the tendency of arcs cascading over the insulators, it would seem that if these are not properly mounted they may cause more trouble than they eliminate. The necessity, or desirability, of grading rings with present transmission voltages is a debatable question, but if the transmission voltage continues to increase it will undoubtedly be necessary to employ some means of improving the potential gradient over the insulator string so as to keep the voltage stress on the units next to the line below their flashover value. Practically, it is believed that wind conditions existing during flashovers are fairly effective in keeping the arc at a sufficient distance from the insulator strings so that in the majority of cases, no serious damage is done to the insulator string.

I am certainly interested to know that the Union Gas and Electric Company considered the expense of increasing their insulator string from six to seven units (one illustration shows eight units) and equipping them with this elaborate fuse protection as justified. It would seem that it would be very difficult to get a comparison of the performance of these circuits on a satisfactory basis to prove that these fuses were of value. For instance, would the service not have been improved to the same extent by adding the one or two units per string without equipping the strings with the grading rings and fuses? It must be expected that there will be many more flashovers on the reduced string than there would have been on the string without this equipment and such as might have occurred might not have resulted in service interruptions.

While no information is given regarding the four circuits between Columbia Park and Cincinnati Terminal (20 mi.) except that they were carried on two separate tower lines, it would seem quite reasonable to expect absolutely continuous service between these points if suitable relay protection were applied. However, if the relay equipment is opening the breakers in 35 cycles (0.58 sec.) it would seem that this is quite adequate protection and should be fast enough to ensure against permanent damage to the insulator in the event of a flashover. It is presumed that the relay protection has not been changed coincident with the change in insulator. No information is given about the grounded sky-wire protection applied to these circuits, nor is any mention made of the condition of the system neutral. We have found that a grounded sky wire properly applied has greatly improved conditions as regards flashovers on several of our lines and this is quite definitely standard practise with us now. Our 110-kv., 25-cycle system operates with a resistance-grounded neutral while the 60-cycle systems have the neutrals dead grounded. It might also be mentioned that we have not found a very heavy percentage of cases where trouble on one circuit of a tower line is communicated to the other circuit on the tower.

Philip Stewart: I might say briefly that the trouble we experienced was due to flashovers on two or three circuits at one time. In one case we had complete shutdown of all the circuits due to flashover on three circuits. I think there is considerable difficulty in comparing service records and requirements of one company with those of another company. You have to take into account the number of lightning storms, and perhaps there is quite a little difference in the various localities that have been mentioned.

Operating Experience with the Low-Voltage A-C. Network in Cincinnati

BY F. E. PINCKARD[1]
Associate, A. I. E. E.

Synopsis.—*This paper describes briefly the operating experience and problems which have resulted from the installation of a four-wire, 120/208-volt secondary network system with combined light and power mains. The problems with reference to network equipment, voltage regulation, and operation of consumers' equipment are stated with their subsequent solution. The results obtained from fifteen months operation of this system indicate that, with reasonable care in layout and selection of equipment, no operating problems of major importance will be met.*

* * * * *

Introduction

When the a-c. low-voltage network system was inaugurated in Cincinnati, a comparatively slight amount of published data was available concerning actual network operating experience, and as might be expected, the operation of this system introduced a number of problems concerned with the design and operation of equipment, also with the success which could be attained with the application of the network system to various types of customers' equipment. During the period of fifteen months since the first network installations were placed in service, these problems have been treated in various manners and with varying degrees of success, and in this paper an attempt will be made to present the most interesting of these together with the steps taken for relief.

The a-c. network system here was designed ultimately to replace the existing three-wire d-c. Edison network in the congested business district of the city, a section approximately one mile square, with a peak Edison system load of 24,000 kilowatts. The ultimate layout of the system provides for the installation of eight 13,200-volt feeders with a transformer vault located at each intersection in the network district and between intersections where the load warrants, with an ultimate capacity of approximately 40,000 kv-a. Transformer banks of 300- and 450-kv-a. capacity with 10 per cent impedance and secondary mains of 400,000 cm. cross section were made standard, using a four-wire, 120/208-volt star connected secondary with combined light and power mains. Single line diagram showing schematic arrangement of the system is shown in Fig. 1.

The original installation consisted of two high-voltage cable feeders with transformer banks interconnected by secondary mains as required to serve certain new loads in the network district. As other new loads have developed and as a certain amount of changeover from direct current to alternating current has been effected, extensions have been made as necessary, at all times working toward the ultimate layout of the system. Fig. 2 shows the two-feeder network as operated at the present time. As the load increases and as additional vaults are installed, this system will be increased to a three-feeder, then a four-feeder, and

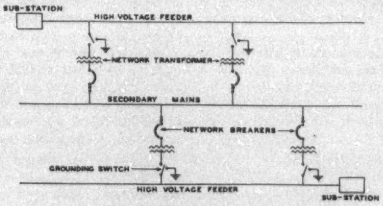

Fig. 1—Single Line Diagram Showing Schematic Arrangement of Low-Voltage Network

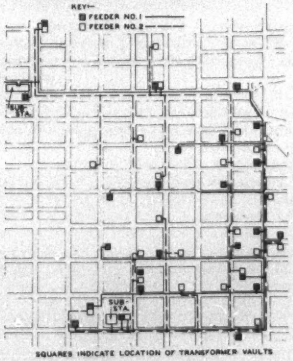

Fig. 2—Diagram of Two-Feeder Network Operated at Present Time

ultimately to an eight-feeder layout. The four-feeder installation which completes the symmetrical layout of the network is shown in Fig. 3.

Network Equipment Problems

With the slight amount of load served by the first

1. Electric Distribution Dept., The Union Gas and Electric Co., Cincinnati, Ohio.

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network installations, it was natural that the first problems to be encountered were concerned with the network equipment and particularly with the automatic network circuit breakers. It should be mentioned here that a number of the problems encountered was the result of the extremely small load on the network at the start and is of importance, not so much from the standpoint of service, but rather from the experience gained in the operation of the equipment under this condition.

The first problem arose from the operation of various elevators connected to the network. In one new building where unit multi-voltage type elevators were installed and where the lighting load was at the time rather small, it was found that at certain times the network breakers would open due to a power reversal. Investigation of this condition revealed that this type of elevator, or any type of elevator using regenerative breaking, might feed a sufficient amount of power into the network to trip the breakers which were set for a

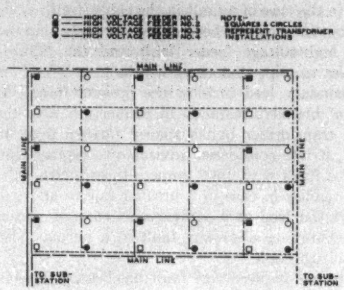

FIG. 3—SCHEMATIC DIAGRAM OF FOUR-FEEDER NETWORK LAYOUT

reversal of approximately 5 amperes, which is considerably less than the energizing current of the transformers. Tests showed that the magnitude of the reversal at times reached a value of 15 kw., or more than one-third of the normal demand of the elevators. In order to correct this condition, heavier magnets were installed on the three-phase relay with some slight success and later the gradual increase in the lighting load served to absorb the reversal.

Later, additional work in connection with this same problem in other parts of the system led to further adjustment in the contact springs of this relay, so that the current required for tripping was just below the magnetizing current of the transformer. Although not working perfectly, this served to eliminate practically all complaints, which ordinarily would have continued throughout the time the transformer banks were lightly loaded.

After the previous problem had been investigated, a severe pumping of breakers under light load conditions was noted. This condition could not be attributed to power reversal, and a further investigation indicated that it was due to the slight phase angle difference between the two primary feeders which originated at different substations. This pumping effect should have been prevented by the operation of the single-phase breaker relay which is designed to control operation of the breaker on phase differences. A check into the operation of this relay indicated that the relay potential elements were reversed.

While the correction of the connections eliminated pumping, it resulted in a number of the breakers on one of the feeders remaining open a considerable part of the time.

When a load of even moderate size was thrown on and off intermittently in the vicinity of one of these breakers, severe voltage variations were noticed due to the breaker opening and closing. Since satisfactory service could not be rendered under those conditions, one of the feeders was extended to a generating station which reduced the difference in phase angle between the feeders from 2 deg. to practically zero, and eliminated this trouble.

The large number of breaker operations caused both by elevator reversals and by pumping due to phase angle difference, and later by variations in load, resulted in considerable attention to the mechanical features of the breakers themselves. A number of failures of breaker trip and closing solenoids was experienced, and in order to insure satisfactory operation, it was found necessary at this time to make a complete inspection of breakers as often as twice weekly. The number of breaker failures caused a great amount of consideration of breaker design and construction, particularly with respect to breaker life and to accessibility for repair.

The failures could be divided into two classes: failure to trip due to faulty trip mechanism, or to poor adjustment; and the burning out of closing coils on solenoid-operated breakers due principally to the tendency of the auxiliary contactor switches to weld closed, leaving the closing coil energized after the breaker had closed.

The failure to trip was solved primarily by the replacement of the old trip mechanisms with a new type developed by the manufacturer. In the case of closing coil failures, it was considered advisable to install some type of fusing in both the auxiliary and the main closing circuits, especially since it was so difficult to replace breaker parts due to the inaccessibility of the breaker when mounted in a submersible case. Tests were made on various types of fuses to secure the proper fusing characteristics, and finally link type thermal cutouts were installed so as to protect both the auxiliary and the main closing solenoids as shown in Fig. 4.

The idea of similar protection in the trip circuit was abandoned since it was considered undesirable to jeopardize the tripping function in any way. The

difficulty of replacing parts on submersible breaker or of removing a submersible breaker from the case without cutting the secondary leads resulted in providing that the panel supporting the breaker parts should be divided into three parts, the upper and lower parts supporting the breaker terminals and the middle panel supporting the operating mechanism, this last so designed as to permit easy removal from the case for maintenance or repair.

As a check on the condition of the trip mechanism of all network breakers, each primary feeder was opened daily during light load period, which gave an immediate indication of the failure of any breaker to open. It probably served to keep the operating mechanism in a more flexible condition as well.

The behavior of some of the network breakers in service showed the necessity of more rigid laboratory tests before placing them in service than had been made before. With this in mind, additions were made to the acceptance test procedure which resulted in considerably less trouble in the field.

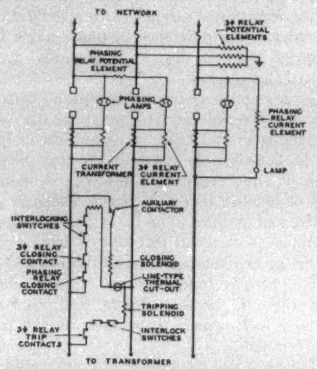

Fig. 4—Simplified Diagram of Solenoid-Operated Network Breaker

The laboratory tests are made primarily to reveal any mechanical or electrical defects due to manufacture or damage in shipment, and are used at the same time to check operating adjustments before the breakers are placed in service. These tests include inspection of electrical and mechanical parts, adjustment of settings of closing and tripping features, adjustment of contact pressure, and finally a detailed adjustment of three-phase and single-phase relays to operate correctly at the specified settings.

After the breakers are placed in service, certain routine tests are made in connection with the network inspections. These tests include checks on the operation of the tripping solenoid and of the closing mechanism, air pressure tests to assure the tightness of the submersible cases, and also tests of voltage drop across breaker contacts to indicate faulty contacts or breaker overload. No attempt is made to adjust network relays in service but a schedule is maintained whereby all relays on the system are returned to the laboratory for test at least once every three months. Better facilities for test are available in the laboratory than in the field, and it is considered advisable to make all tests on this important piece of equipment under the best possible conditions.

One function of the network which is of primary interest is its ability to burn clear any secondary faults. Due to the fact that the secondary mains installed here were composed entirely of new cable, the experience with secondary faults has been limited. In the few cases which have been observed, the faults have all cleared with no noticeable effect on network service. One of these faults occurred in a splice, a very severe test of the ability to clear, and even in this case the action was entirely successful, although the time of clearing was evidently prolonged much more than in the case of a fault in the cable itself.

Attention has also been devoted to assure in the case of a high-voltage feeder fault and the failure of a breaker to trip, that there will be a sufficient amount of secondary feed to blow the breaker fuses. In the case of breakers installed in connection with the 300 kv-a. transformer banks spaced farther than 450 ft. apart, it was considered advisable to replace the 2000 ampere fuses with 1500 ampere fuses which will blow more readily in case of a breaker failure and will still permit the feed of a sufficient amount of power into the network to clear any fault in a normal length of low-voltage cable.

Only one high-voltage fault has been experienced on the network, in this case the failure of a transformer bushing. The oil circuit breaker opened on current to ground and all breakers cleared successfully, but the fault is of some interest due to the difficulty in locating it.

The usual method of locating high-voltage cable faults has been to break down the insulation resistance at the fault, to circulate about 30 amperes direct current through the conductor and back through the lead sheath, then to locate the fault by means of a millivoltmeter used to indicate the direction of flow of current in the sheath. An attempt to apply this method to the location of the network fault proved unsuccessful, however, due to the large number of return paths for sheath current since the cable sheath is grounded to the transformer case and the case in turn is grounded to the neutral. In attempting to follow this method, a number of conflicting and misleading results was obtained.

Another method of locating primary network fault has since been devised, and tests on this method have been quite satisfactory. Essentially this procedure is to break down the fault and circulate about 30 amperes

direct current through the faulted conductor with return path through the sheath as given in the previous method. The sheath current on the faulty primary cable is then neutralized in the various manholes by current supplied by a storage battery. The direction of the fault is shown by the effect of the fault current in the conductor on a compass needle.

Diagrams of these two methods are shown in Fig. 5.

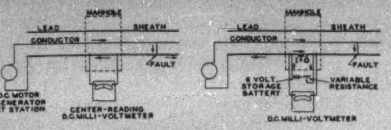

FIG. 5—(a) USUAL METHOD OF LOCATION OF HIGH-VOLTAGE CABLE FAULTS
(b) METHOD OF LOCATION OF HIGH-VOLTAGE NETWORK FEEDER FAULTS

REGULATION

Regulators were not installed on the primary feeders until some time after the network was in service. Two single-phase regulators, connected open-delta and mechanically interconnected, were installed on each feeder. A slight difference in secondary phase voltages noticed soon after they were in service was remedied by

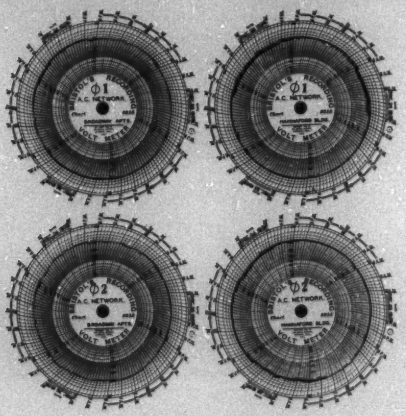

FIG. 6—VOLTAGE CHARTS FROM REPRESENTATIVE NETWORK SERVICES

mechanical adjustments in the gearing between the two regulators on one of the feeders. Because of the possibility of causing pumping and also because of the various types of load connected, no attempt was made to regulate to any one point on the secondary, but rather to regulate to imaginary points approximately half way through the secondary of the transformers, using the same compensator setting on each feeder.

Although the load both in kw. and reactive kv-a. had divided fairly evenly between the feeders with the regulators not in service, it was found that when in service, the kv-a. load on one was at times more than double that of the other. Tests showed that there was little difference in phase angle between the voltages impressed on the regulators, and also that changes in compensation failed to improve matters. It was finally discovered that one set of regulators was not operating properly due to an improper internal connection in the regulators.

When this was corrected, the compensation was arrived at by trial and during the comparatively short time since this correction has been made their operation has been satisfactory.

It may be of interest to note that there is no connection either mechanically or electrically between the two sets of regulators, one set of which is on a feeder from a generating station, the other on a feeder from a substation fed from this generating station and supplying both d-c. load and a-c. load through 4000-volt feeders.

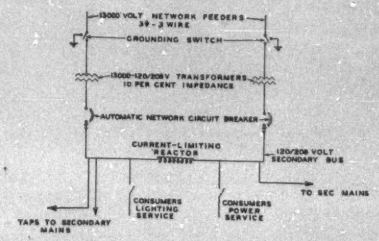

FIG. 7—LINE DIAGRAM OF DOUBLE TRANSFORMER VAULT INSTALLATION

Four general types of load are supplied by the network: manufacturing, hotel, office building, and theater. The charts shown in Fig. 6 are typical of voltage on consumers' premises, and indicate that regulation on the above basis gives very uniform voltage throughout the system in spite of the various types of load served.

There have been no disturbances on the network since the regulators have been in operation, hence no data have been obtained as to their behavior under such conditions.

Another problem of regulation has been to prevent the starting current taken by large motors from affecting the lighting service. The results obtained by the use of reactors in the secondary bus in large buildings requiring more than one transformer bank indicate that it is practical to eliminate all effect whatever on the lighting voltage. A single line diagram of this plan is shown on Fig. 7.

This solution naturally could be used only where

more than one transformer bank was installed. In other cases requiring only a single transformer, it was found necessary to take other steps to reduce the starting current of the motors. This has been accomplished by reconnection of the starters. It has resulted in requiring a longer starting time, but in no case have there been complaints on this account.

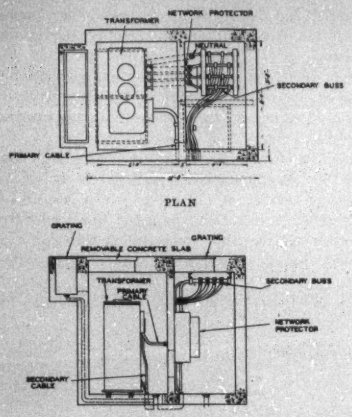

FIG. 8—STANDARD SIDEWALK VAULT FOR INSTALLATION OF ONE THREE-PHASE TRANSFORMER

It is felt that the requirements on starting current for motors connected to the network should be more stringent than for other parts of the system, and regulations are contemplated requiring the use of increment type of starters on motors on which the starting current is apt to cause light flicker.

The use of this type starter on 200-hp. motors connected to compressors has resulted in no noticeable variation in the lighting voltage.

OPERATING PROBLEMS ON CONSUMER'S EQUIPMENT

Since 120 volts for lighting had been standard on the distribution system in Cincinnati for a number of years, there was no reason for deviating from this for the network, especially since it offered more nearly rated three-phase voltage for motors.

It is recognized that a motor operating with 75 per cent or less of rated load will give just as satisfactory service when operated on 208 volts as when operated on 220 volts and as had been expected, the greater part of the motor installations to be changed to the network service were found to fall in this class.

Auto-transformers have been installed for heating units rated at 220-240 volts and, in a few cases where contracts made with customers prior to the installation of the network contained clauses requiring 240 volts, three-phase service for power. However, it is no longer the practise to install them for new loads.

DESIGN AND CONSTRUCTION DETAILS

Figs. 8 and 9 show the construction details of sidewalk vault which are typical of a majority of the installations here. In the case of the double vault, the breaker compartment is located within the building wall, while the single vault is located entirely within the sidewalk. The transformers are of the type ordinarily used for low-voltage networks with grounding switch on the high side having line open, ground, and closed positions. A holding coil energized by the secondary of the transformer prevents the operation of the switch when the transformer is energized. Both open and submersible type network breakers are used, the former when installed in buildings, the latter in sidewalks. Transformers are of the submersible type in all cases.

Transformers first installed had the grounding switch in the transformer case. In those now used, it is

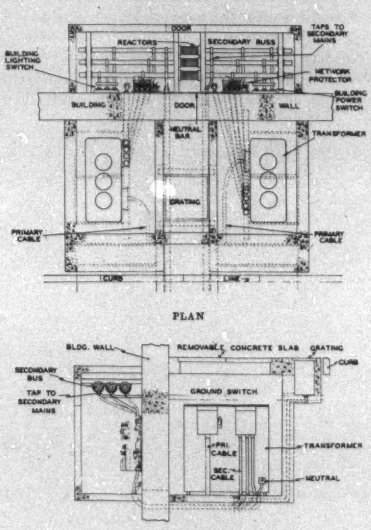

FIG. 9—STANDARD SIDEWALK VAULT FOR INSTALLATION OF TWO THREE-PHASE TRANSFORMERS

contained in the high-voltage pothead. This change resulted in a considerable reduction in size and weight of the transformer, and in addition allows such work as change of oil to be done by throwing the grounding switch to the open position instead of keeping open the primary feeder as was necessary in the case of the original transformers.

While some single-phase transformer banks have been installed, the simplicity of installation of three-phase transformers has made them much more desirable.

From the standpoint of accessibility, both for installation and removal of equipment, and for operation, it has been the experience here that sidewalk vaults are preferable to those in buildings, especially where there is no direct connection between the building vault and the outside.

In order to prevent the necessity for phasing out from substations where additional transformer banks are being cut in, the practise of marking each phase in every splice by means of various colored tapes has been resorted to. This has resulted in a considerable saving of the time required for any such work on primary feeders. It also renders unnecessary any phasing whatever in replacing a faulty length of primary cable.

Equipment Characteristics

At the time the network breakers were purchased, they were available in both the holding coil and shunt-trip type. The latter type was selected because of the possibility that voltage disturbances of a general nature on other parts of the system might cause unnecessary outages of the network due to the inability of the holding coil type breakers to remain closed on voltages lower than 35 per cent of normal. It was later found that this choice prevented a number of these outages, which while possibly not serious, were certainly not desirable from the standpoint of rendering first-class service to consumers.

In the selection of the amount of transformer impedance necessary, it was felt that this should be the maximum commensurate with satisfactory regulation. While the use of high impedance transformers reduces the size of secondary mains allowable due to limiting fault currents, this can be overcome by the installation of more than one set of mains where necessary, which is, in most cases, the most economical method to use in building up the secondary system. Transformer impedance of 10 per cent such as has been in use in Cincinnati has worked out very satisfactorily both from the standpoint of regulation and even distribution of load between transformer banks.

Conclusion

As has been stated before, the difficulties encountered were not of major importance and investigation as to their causes showed that the remedies to be applied were after all comparatively simple. They did serve, however, to bring out many of the main features to be considered in the layout of the system and in the selection and maintenance of the equipment.

It has been the aim here to furnish a-c. service to the downtown district comparable in reliability with that of the Edison system. Since there have been no interruptions in service other than local ones during the fifteen months that the low voltage network has been in operation, it is felt that this has been accomplished.

Discussion

L. L. Bosch: Mr. Pinckard's paper describes a two feeder a-c. network system which is probably the most difficult to operate. Should a fault occur on one feeder while the other is cut out of service, the system fails. Mr. Pinckard and his entire operating personnel have operated this type of system for fifteen months without an interruption. I think that they are to be highly complimented.

Since that time a third primary feeder has been added to the network, which materially reduced the operating hazards. During the fifteen-month period, the connected transformer capacity has been increased from an initial of 2000 kv-a. to a present of 15,000 kv-a.

The mechanical troubles with the a-c. network equipment which Mr. Pinckard described, I believe, show that the a-c. network is reliable rather than the contrary which might be believed at first. First, the mechanical troubles were numerous and difficult to overcome, but yet they did not cause an interruption to the system. In other words, the network is designed such that one, two, three, or any reasonable number of protectors may fail simultaneously without an interruption to the system's service.

It is believed that the question of starting currents of the motors on the a-c. network system should be given more consideration. In the d-c. system, it was necessary to have starting apparatus for the protection of the motor itself and of the connected mechanical apparatus. Many customers have come to the conclusion that since the a-c. motors are self-starting, nothing else need be considered, and that starting apparatus is unnecessary. They neglect entirely the question of line-voltage regulation. Motor starting currents on the network, as described by Mr. Pinckard, have been limited to increments of 100 amperes per ½ sec. This is calculated to give a voltage dip incident to starting the motor not greater than 2 per cent.

The quality of service delivered from this network indicates that this rule is justified.

H. C. Graves, Jr.: (communicated after adjournment) The difficulties Mr. Pinckard has experienced are typical and can be attributed to the breaker control equipment, and should not reflect on the a-c. low-voltage distributing system. Since the control and protective equipment is the weak link in the system, and since improvement here is most desirable and necessary, a brief recapitulation of the major faults emphasized by this article appears desirable.

1. Regeneration, or feed back through transformers causes incorrect breaker opening.
2. Small phase-angle differences between feeders, or regulation between feeders, will cause incorrect breaker opening.
3. System swings or synchronizing surges will cause incorrect breaker opening.
4. High-tension faults will frequently cause power flow through the network to cause incorrect breaker openings.
5. Regulators, both single-phase and three-phase, may cause incorrect openings. Regulator compensation must be carefully made and maintained. The hunting of regulators during system faults may frequently cause breaker operation.
6. Tripping relays must have characteristics suitable to trip on both lagging bank excitation current and leading cable charging current of very small values, and still give correct operation on current values of approximately one hundred times this value for both phase-to-phase and phase-to-ground faults on both high- and low-tension sides of the transformers. This is a severe design problem.

7. Tripping relays must maintain a delicate setting with little attention under very adverse operating conditions. Field tests are practically impossible. Necessarily delicate contacts fail because of the delicate settings and severe operating conditions.

That the equipment operates even as well as outlined by Mr. Pinckard reflects credit on the careful operation and maintenance of the equipment as well as on the ingenuity of the designers.

I should like to ask one question. Is it possible to select a secondary fuse for the network transformers that will blow under all conditions of transformer faults and yet will not operate under any condition of secondary faults such as the trouble in the splice mentioned in the paper? It would appear to me that numerous system conditions could exist which would prevent the proper discriminating action of a fuse.

F. E. Pinckard: Relative to the point brought up by Mr. Bosch on starting currents of motors connected to the a-c. network, while it is necessarily true in the case of a combined set of power and light mains that flicker is apt to occur, this matter has caused comparatively little trouble. It is also one that has a tendency to disappear as the system capacity increases.

As to the selectivity of the secondary fuse for the transformers and network breakers in Mr. Graves' question, this is accomplished by having three or four sets of mains feeding from each transformer bank. In case the network breaker failed to clear, the combined capacity of the mains should be sufficient to feed back enought current to blow this fuse. With the reverse condition, that is, a fault on the mains, it is assumed that this would occur on only one of the sets at a time, in which case the fuse capacity should be great enough to clear the fault.

Quick-Response Generator Voltage Regulator

Field Tests Made With Oscillograph

BY E. J. BURNHAM[1], J. R. NORTH[2], and I. R. DOHR[3]

Associate, A. I. E. E. Member, A. I. E. E. Non-member

Synopsis.—This paper describes elaborate and severe field tests made on high-speed excitation equipment used with a 5385-kv-a. turbo generator. Complete oscillograms and curves showing the performance of this equipment are included together with a discussion of the results obtained.

* * * * *

DURING 1928, high-speed excitation equipment was provided for a 5385 kv-a., house turbo generator which is used for supplying power to essential auxiliaries at the Saginaw River Steam Plant of the Consumers Power Company. Elaborate field tests were made on this regulating equipment, constituting the most severe tests made on a machine of such size, and the results are indicative of those which may be obtained on the larger size generators.

Description of Installation

The Saginaw River Steam Plant, one of the largest steam generating stations on the system of the Consumers Power Company, will have, when completed in the near future, a total of five generators and two house generators, giving a generating capacity of approximately 140,000 kw. The No. 1 house turbo generator on which these tests were made is a General Electric unit rated 5385 kv-a., 3500 kw., 0.65 power factor, 2500 volts, 3600 rev. per min., 60 cycles and has a direct connected exciter rated 33.5 kw., 250 volts, four-pole, shunt wound. The house generators are connected to the 480-volt station auxiliary busses through 6000 kv-a., 2400-480-volt transformer banks. The 480-volt station auxiliary busses are connected to the main generating bus through 24,000-480-volt transformer banks.

The house turbo generators supply power to the more essential station auxiliaries consisting of boiler feed pumps, draft fans, circulating pumps, condensate pumps, etc. All motors used with these auxiliaries are started on full voltage and the larger ones, of 400 hp., are of the squirrel-cage type. The house generators may be operated in parallel with or isolated from the rest of the system.

A General Electric type FA-1 high-speed generator voltage regulator is provided to control the excitation of the No. 1 house generator. This regulator is equipped with a three-phase torque motor control and rheostatic follow-up features.

1. General Electric Co., Schenectady, New York.
2. Stevens and Wood, Incorporated, Jackson, Michigan.
3. Consumers Power Company, Jackson, Michigan.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

Requirements

This high-speed excitation equipment was installed to accomplish the following:

(a) To accurately control the a-c. voltage under all conditions of picking up and dropping load without disturbing the operation of the other station auxiliaries operated from the same machine.

(b) To obtain a high speed of excitation response in connection with this generating unit in order that even the largest induction motors (400 hp.) used for station power service could be satisfactorily started at full voltage with minimum disturbance.

(c) To obtain first hand information concerning the operation of this type of quick response regulator, particularly regarding its speed of operation, precision, and stability of operation.

The range of excitation control on the generator is from 2300 volts, no-load and cold fields, to 2500 volts, full rated kw. load at rated power factor and hot fields. This range is necessary to provide adequate voltage on the 480-volt station auxiliary bus under various load conditions and allows for the regulation of the 2400-480-volt transformer bank. It corresponds to a range of 460 volts at no-load to 490 volts at full load.

Design of Regulating Equipment

In order to meet the above requirements, a quick-response excitation system was provided consisting of an exciter with a high speed of voltage build-up and a high-speed regulator. The exciter is self-excited and has a speed of voltage build-up of approximately 1000 volts per second over the operating range.

Fig. 1 shows a simplified diagram of connections of the house generator excitation system and the FA-1 regulator. The regulator equipment consists of a main control element and an auxiliary control equipment. Figs. 2 and 3 show illustrations of this regulating equipment. The main control element consists of a d-c. system and an a-c. system and includes two lever arms, one controlled by a three-phase torque motor and the other by a d-c. magnet. A contact on the a-c. lever arm floats between two contacts on the d-c. lever arm. The torque motor is energized from two potential transformers which are connected to the three phases of the alternator, thus giving three-phase control. The d-c.

29-62

magnet is connected across the armature of the exciter and provides the necessary anti-hunting feature and also the vibrating of main contacts, which will be described later.

The auxiliary control equipment includes two high-speed contactors, L and R (see Fig. 1), both of which are normally deenergized and arranged in such a manner that the contacts of L normally short circuit field reducing rheostat l in the exciter field circuit and the contacts of R are normally open but connected across the motor-operated field rheostat f and an additional field raising rheostat r in the exciter field circuit.

Quick response in the regulator is obtained by use of a spring dashpot connected to the a-c. lever arm of the main control element. The springs in the dashport allow either main contact to close without first moving the diaphram in the oil. With this arrangement, the torque motor may close the main contacts without

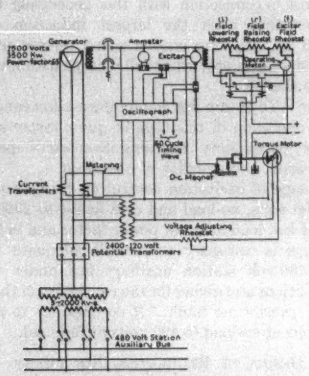

FIG. 1—DIAGRAM OF REGULATOR AND TEST CONNECTIONS SAGINAW RIVER STEAM PLANT

waiting for the movement of the diaphram in the oil dashpot. This dashpot tends to stabilize the operation of the main contacts. This spring dashpot can also be applied to the vibrating type of regulator to obtain quick response.

The anti-hunting feature is provided by means of the d-c. coil on the main control element. This d-c. element, which is connected directly across the exciter armature, opens either the upper or lower main contact, whichever the case may be, after such contact is once made by the torque motor and the exciter voltage has changed the necessary amount.

OPERATION OF REGULATING EQUIPMENT

With the a-c. voltage of the alternator normal, the main control element is in equilibrium and the regulating equipment remains at rest. A decrease in a-c. voltage causes the torque motor to close the upper main contact, thus energizing contactor R, which in turn short-circuits rheostats r and f causing an increase in excitation. Upon an increase in a-c. voltage, the lower main contact closes, thus energizing contactor L which inserts rheostat l in the exciter field circuit to decrease the excitation.

For a decrease in a-c. voltage, the upper main contact

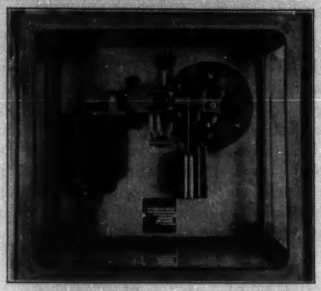

FIG. 2—FRONT VIEW OF REGULATOR MAIN CONTROL ELEMENT

and contactor R will open and close, thus functioning as a vibrating type regulator until the a-c. voltage returns to normal and the main element is in equilibrium. In the same manner, an increase in a-c. voltage will cause vibration of the lower main contact and contactor L until the a-c. voltage returns to normal.

Continuous operation of the high speed contactors is

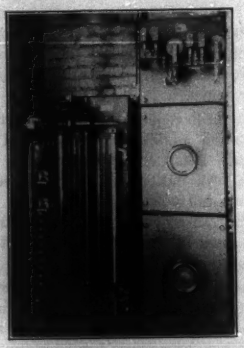

FIG. 3—FRONT VIEW OF AUXILIARY CONTACTOR PANEL AND MOTOR OPERATED RHEOSTAT

kept to a minimum by means of the rheostatic follow-up feature which functions as follows: Each time that either high-speed contactor, L or R, is operated, an auxiliary contact energizes the motor of the motor-operated rheostat in the correct direction to bring the a-c. voltage back to normal. During a time when the

a-c. voltage is low, the arm on the motor-operated rheostat is notched along to cut out resistance each time the upper main contact vibrates and the contactor *R* opens and closes. This follow-up action continues for a short time until the rheostat arm is in the correct position to take care of the new excitation requirements. When this correct position is reached and the a-c. voltage is back to normal, the main control element is in equilibrium and the regulating equipment is again at rest. During a time when the a-c. voltage is high, the equipment functions in a similar manner, opening and closing contactor *L* and notching the rheostat arm along to cut resistance in the motor operated rheostat.

Field Tests on Regulating Equipment

An extensive series of field tests was conducted on this regulating equipment to enable it to be adjusted under actual service conditions and to obtain detail information regarding its operation. These tests consisted of suddenly picking up or dropping large amounts of load with the excitation of the generator under regulator control, and also with fixed excitation. Complete oscillographic records were obtained as well as graphic voltmeter charts and readings on indicating meters. The operation of the regulating equipment when the generator was paralleled with the rest of the system was also recorded.

Preliminary Adjustments. A series of preliminary tests was made for the purpose of observing the operation of the regulating equipment, making the necessary preliminary adjustments and checking its operation. Fig. 1 shows the connections used in making the test. The effect of the regulator on the generator voltage was recorded by means of an Esterline quick-trip graphic voltmeter and readings were taken on the switchboard indicating instruments. The sequence of operation of the various elements of the regulating equipment was noted and the field regulating rheostats were adjusted to the proper value of resistance. The effect of various adjustments was studied by suddenly applying load to the generator.

TABLE I
OUTLINE OF TESTS

Tests	Excitation	Generator		Test conditions
5,7	Regulated	Isolated	Carrying no load	Load applied (2 motors)
6,8	Fixed	Isolated	Carrying no load	Load applied (2 motors)
9	Regulated	Isolated	Carrying load	Load applied (2 motors)
10	Fixed	Isolated	Carrying load	Load applied (1 motor)
11	Fixed	Paralleled	Carrying load	Load dropped
12	Regulated	Paralleled	Carrying load	Load dropped

Final Tests. The final series of tests consisted of throwing a large amount of load on the generator with the regulating equipment in service and with fixed excitation, and under the conditions of load and operation shown on Table I.

Load was suddenly applied by starting one or two 400-hp. motors simultaneously at full voltage. Starting two motors simultaneously was equivalent to a momentary load of approximately 7500 kv-a. or approximately 130 per cent generator rating at approximately 35 per cent power factor lagging. The motors came up to speed in a very short time, approximately one second with the regulator in service and two seconds with fixed excitation, and their load demand dropped rapidly to a low value as soon as they came up to speed. Load was suddenly dropped by operating the generator paralleled with the system and carrying load, then suddenly opening the generator oil circuit breaker.

Oscillograph records were taken of generator voltage, generator armature current, generator field current, exciter voltage, exciter field current, and 60 cycle timing wave. Fig. 1 shows the connections used during the tests.

Test No. 7
Generator isolated and carrying no-load
Regulator in service
Started No. 3 and No. 6, 400-hp. motors

Test No. 8
Generator isolated and carrying no-load
Regulator not in service
Started No. 3 and No. 6, 400-hp. motors

Fig. 4—Oscillograms of Tests Nos. 7 and 8

Results of Tests. Figs. 4, 5, and 6 show representative oscillographic records obtained during the tests and Fig. 7 representative graphic voltmeter charts. The oscillograms have been carefully analyzed and the results are shown by curves in Figs. 8, 9, 10, and 11.

Discussion of Results

From the curves, Figs. 8 to 11 inclusive, the results obtained with and without the regulator can be compared for the different test conditions.

As previously pointed out, the starting of two 400-hp. motors at full voltage constituted a load of approximately 7500 kv-a. at approximately 35 per cent power factor lagging. The inrush current to the motors upon

starting is practically all wattless, therefore the duty upon the generator is severe as far as maintaining a-c. voltage is concerned because of the highly demagnetizing action of the armature current. The maximum momentary value of inrush current was the same with fixed excitation as with the regulator in service and was approximately 145 per cent of the generator current rating when starting two motors. It was approximately 100 per cent of the generator current rating when starting only one motor. With fixed excitation this current continually decreased in value after the first instant. However, with the regulator in service it was

Fig. 5—Oscillograms of Test No. 9

Generator isolated and carrying load
Regulator in service
Started No. 3 and No. 6, 400-hp. motors

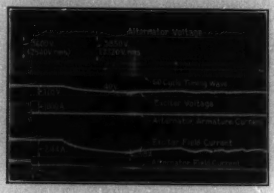

Fig. 6—Oscillograms of Test No. 12

Generator paralleled with system and carrying load
Regulator in service
Dropped load

maintained near its maximum value until the motors approached full speed.

It will be noted that the instantaneous inrush of motor-starting current causes an initial increase in generator field current and an instantaneous drop of generator voltage, these values being the same whether the regulator was used or not. With fixed excitation the exciter voltage remained constant and the generator field current after its initial rise gradually decreases to its original value. With the regulator in service the exciter voltage rapidly increased to its maximum value and increased the generator field current to restore normal a-c. voltage.

When two motors were thrown on the line, the generator voltage dropped instantly approximately 8 per

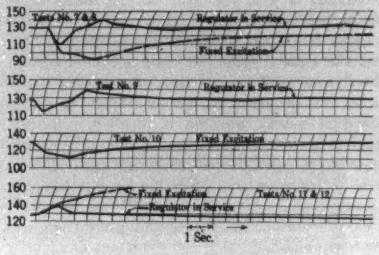

Fig. 7—Graphic Voltmeter Charts

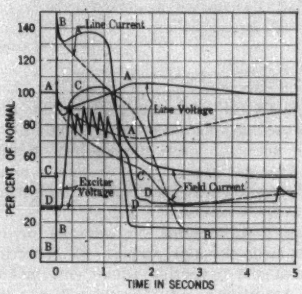

Fig. 8—Curves Showing Results of Tests Nos. 7 and 8

Generator isolated and carrying no load started two 400-hp. motors
———Regulator in service
- - - -Fixed excitation

A. Line voltage 2500 volt = 100 per cent
B. Line current 1245 amperes = 100
C. Field current 110 amperes = 100
D. Exciter voltage 250 volt = 100

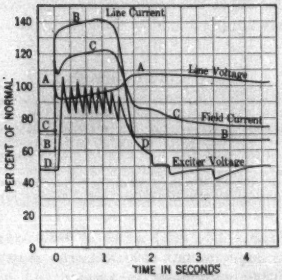

Fig. 9—Curves Showing Results of Test No. 9

Generator isolated and carrying load started two 400-hp. motors
Regulator in service

A. Line voltage 2500 volt = 100 per cent
B. Line current 1245 amperes = 100
C. Field current 110 amperes = 100
D. Exciter voltage 250 volt = 100

cent. After the initial dip of generator voltage there was a decided difference in the voltage curves obtained with the regulator in service and with fixed excitation. The maximum dip in generator voltage and the duration of the dip with fixed excitation was longer and much more severe than was the case under regulator control. Under regulator control the generator voltage was quickly raised to normal, but the sudden drop in line current when the motors come up to speed caused the

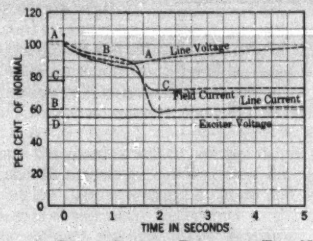

FIG. 10—CURVES SHOWING RESULTS OF TEST NO. 10

Generator isolated and carrying load started two 400-hp. motors
Regulator not in service

A. Line voltage 2500 volt = 100 per cent
B. Line current 1245 amperes = 100
C. Field current 110 amperes = 100
D. Exciter voltage 250 volt = 100

voltage to rise above normal. The regulator was then called upon to adjust the excitation to reduce the voltage to normal. The vibrating action of the regulator is plainly shown by the rapid rise and fall of exciter voltage on the curves. At the time the regulator is vibrating, the frequency of the vibrations is about the same as would be expected with an ordinary vibrating type regulator.

Fig. 8 shows the results with and without the regulator in service under the condition of starting two motors at a time when the generator was carrying no load. The initial increase in generator armature current caused the generator field current to increase instantly from 48 per cent to approximately 100 per cent of normal full-load value. The generator voltage correspondingly decreased instantly to approximately 92 per cent of normal value. With fixed excitation, the generator voltage continued to decrease in value for about two seconds, at which time it reached a value 72 per cent of normal. From this point the generator voltage gradually returned to normal.

The effect of the generator carrying load when starting two motors can be seen by comparing Figs. 8 and 9. The line current did not momentarily increase to such a high value under load conditions as under no-load conditions. However, the duty was more severe under load conditions, as indicated by the duration of the disturbance and the fact that the regulator was not able to raise the a-c. voltage to normal as quickly, even though the speed of exciter voltage build-up was greater. The increased speed of build-up was due to the excitation being at a higher value initially.

Fig. 11 shows the effectiveness of the voltage regulator in limiting the rise of generator voltage upon the dropping of load.

Table II gives a summary of the test results and shows the successive operating intervals of the different portions of the excitation system from the instant the generator load changes until the generator voltage is restored to normal, the speed of voltage build-up of the exciter and the generator, and the limits of exciter and generator voltages reached during the tests.

The successive operating time intervals represent the time in cycles required for the individual portions of the excitation equipment to operate. It will be noted that the main contacts of the regulator closed in approximately 2.5 cycles after the load changed. The auxiliary raised contactors closed in approximately four cycles after the main contacts closed and the exciter voltage built up to its maximum value in an average time of approximately nine cycles after the auxiliary contactors had closed. The corresponding operating time intervals with the regulating equipment decreasing the excitation are somewhat longer. As shown by Test No. 12, the auxiliary lowering contactor

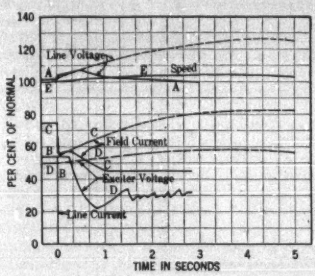

FIG. 11—CURVES SHOWING RESULTS OF TESTS NOS. 11 AND 12

Generator paralleled with system and carrying load
Dropped Load
——— Regulator in service
- - - - Fixed excitation

A. Line voltage 2500 vo't = 100 per cent
B. Line current 1245 amperes = 100
C. Field current 110 amperes = 100
D. Exciter voltage 250 vo't = 100

closed in 14 cycles, and the exciter voltage decreased to its minimum value in 37 cycles after the auxiliary contactors had closed.

The above time intervals are of interest as regards the operation of the various component parts of the regulating equipment, but from an operating standpoint, the total time interval elapsing from the instant the load changes until the generator voltage is restored to normal is of prime importance. Referring again to Table II, it will be seen that with the regulator in

TABLE II
SUMMARY OF TEST RESULTS

Test No.	Test conditions			Successive operating time intervals (cycles)					Speed of build-up (Volts per sec.)		Exciter voltage		Generator voltage		
							Total‡								
	Excitation	Generator	Load	Main contacts	Aux. contactors	Exciter voltage	(A)	(B)	Exciter	Gen.†	Normal	Extreme	Normal	Min.	Max.
5	Regulated	Isolated No load	Picked up	2.0	3.0	9	65	660	930	300	80	236	2540	2240	2620
6	Fixed	Isolated No load	Picked up	....	....	....	1200	1200	....	−290*	76	76	2530	1870	2530
7	Regulated	Isolated No load	Picked up	2.0	3.0	11	70	720	1040	270	80	236	2520	2270	2740
8	Fixed	Isolated No load	Picked up	....	....	....	1200	1200	....	−500*	76	76	2540	1800	2540
9	Regulated	Isolated Carrying load	Picked up	2.0	2.0	7	83	600	1200	225	125	272	2540	2250	2660
10	Fixed	Isolated Carrying load	Picked up	....	....	....	720	720	....	− 170*	130	130	2630	2210	2630
11	Fixed	Paralleled Carrying load	Dropped	....	....	....	....	....	....	225	120	120	2660	....	3240
12	Regulated	Paralleled Carrying load	Dropped	3.0	14	37	110	720	− 140*	− 110*	128	40	2540	....	2720
13	Regulated	Isolated No load	Picked up	2.5	8.0	....	....	....	....	150	....	....	2540	2390	2600

NOTE: *Indicates rate of voltage decay.
†Approximate rate during time motors coming up to speed.
‡A Time interval refers to point when generator voltage first returned to normal.
‡B Time interval refers to point when generator voltage became stable at normal value.

service the generator voltage first returns to the normal value in an average time of approximately 81 cycles or about 1.5 seconds. As previously mentioned, the generator voltage overshot and finally became stable at the normal value after a total average time interval of 570 cycles or 9.5 seconds. With fixed excitation, the voltage returned to normal after an average time interval of approximately 20 seconds. It was rather hard to determine at just what point the generator voltage became stable at the normal value, but the above values are relative.

The speed of exciter build-up was very fast, considering that the exciter was self-excited, being of the order of 1000 volts per second, but it will be noted that the speed of exciter voltage build-down is very much slower. Additional resistance has been connected in series with the field lowering rheostat, since these tests were made, to increase the speed of exciter voltage decay. However, it should be pointed out that from an operating standpoint the speed of voltage build-up and decay of the exciter is of secondary importance to the speed of generator voltage response. The generator voltage increased, as shown by Table II, at an average rate of approximately 250 volts per second during the time the motor was coming up to speed.

Conclusions

The following conclusions may be drawn from the results of these tests.

(a) The quick response excitation equipment is effective in reducing the magnitude and duration of a-c. voltage surges due to sudden increase or decrease in load.

(b) From an operating standpoint, the over-all speed of a-c. generator voltage response is of prime importance rather than exciter speed of build-up. In other words, the total time interval from the instant of load change to the time when the a-c. voltage is restored to normal is most important.

Discussion

F. C. Hanker: When considering the selection of the proper size of auxiliary generator for a large station, the excitation scheme is of prime importance and the information presented by the authors is of considerable use to the application engineer. In the case of the auxiliary generator, continuity of service under varied conditions is very important.

Two years ago a similar set of tests was made by Messrs. Anderson and Monteith at the Richmond Station of the Philadelphia Electric Company, and the results were presented before the Institute. More information of this sort is needed to show the requirements of operation to be met by the apparatus.

The problem can be attacked either by having machines with good inherent regulation that will operate satisfactorily when heavy inductive loads, such as starting auxiliary motors on full voltage, are applied, or by the use of quick-response excitation which will tend to maintain the flux and thus allow only the armature reactance drop which is usually not serious. The degree to which the flux is maintained will depend on the layout of the excitation system.

The regulator which the authors describe is in reality what the Westinghouse Company term an "exciter rheostatic" regulator. This type of regulator has been used by the Westinghouse Company for the past several years. The first application was in the station of the Niagara Falls Power Company with normal speed of response of the exciter voltage. It has since been applied on a number of generators and condensers in connection with quick-reponse excitation, and in the last year or so also where the flux was built up at a very high rate.

The inherent speed of the regulator elements becomes a factor in the application where high-speed excitation is used. With the type of regulator under discussion, larger parts have to be moved than with a vibrating type of regulator; therefore greater forces must be applied. This brings up the question of whether contactors of the normal design will meet the operating conditions satisfactorily. We have found it necessary to use specially designed contactors for this service.

R. M. Carothers: The authors describe a certain and specific regulator problem, point out how that problem was attacked and the success with which it was met. The regulator they describe is not the vibrating type but is a regulator for customers who prefer this type of regulator, and also for places where the vibrating type of regulator is not required.

I might add that it is not the only type of regulator that can be termed "quick-response." The vibrating type of regulator with certain modification is also a quick-response voltage regulator.

H. L. Weinstein: All the test results shown in the paper were taken on the high side of the transformer bank, and the impedance of the bank was not given, so it is hard to say what effect the drop had on the low-voltage bus, but it is possible by maintaining voltage on the high side of the bus to have a varying low side due to the varying power factor of the motors as they come up to speed.

I should like to ask a question about this regulator. In the paper presented by Mr. Carothers and Mr. Nichols at the St. Louis meeting, the lever system contained in addition to the d-c. coil an a-c. coil. Is this a commercial form of regulator or has it been abandoned?

A. C. Monteith: (communicated after adjournment) This paper adds considerably to the meager amount of information available on the question of starting auxiliary motors, and it corroborates the work of Mr. Anderson and the writer which was presented in 1927 under the subject *Auxiliary Power at Richmond Station*. The conclusion drawn at that time was that a voltage regulator was very desirable.

In attempting to make a comparison between the so-called special regulator used by the authors, and the standard Westinghouse regulator used at Richmond Station, the following figures have been tabulated from the two papers on the subject. These are taken from Fig. 8 in the paper under discussion and Fig. 10 of the former paper:

Machine	Kv-a. rating	Reactance	Max. load	P. F.	Load per cent rating	Voltage	
						With rog.	Without
Saginaw.....	5385	6% App.	7500	35%	140	90	72
Richmond...	3333	5.6%	4200	22–35%	125	82	55

It is rather difficult to make a direct comparison on account of the factors involved, such as:

1. Regulation characteristics of the generator
2. Characteristics of the load
3. Speed of response of the exciter
4. Characteristics of the regulator

but nevertheless, the following conclusions are indicated:

(1) A detailed comparison of the regulation characteristics of the generators is difficult without complete details of the two ma-

chines. From the data available, it is believed that the inherent reactance is approximately the same in the two generators. The indications are that the Saginaw machine had somewhat better regulation since 140 per cent load dropped the voltage only 28 per cent without a regulator, while 125 per cent load on the Richmond machine dropped the voltage 45 per cent.

(2) In the case of the Saginaw machine, two 400-hp. motors were started, coming up to speed in one second with a regulator, and two seconds without a regulator. In the case of the Richmond tests, the motors consisted of one 500-hp., squirrel-cage motor driving a circulating water pump; two 150-hp., squirrel-cage motors driving ventilating fans; and two 125-hp., wound-rotor motors driving draft fans. The 500-hp. motor had approximately 22 per cent power factor starting current. The ventilating fans had in the neighborhood of 30 per cent and the draft fans were higher due to having a secondary resistance in the rotor. The last mentioned "CW" motor starting currents were small compared to the other three motors. Also the 500-hp. pump motor came up to speed in 1.4 sec. with a regulator, and in over 2 sec. without a regulator, while it took the four fans considerably longer to come up to speed. The tests indicate approximately 17 sec. with a regulator.

(3) The Saginaw exciter had a speed of response of 1000 volts per sec., while the Richmond machine had 100 volts per second speed of response of exciter terminal voltage.

(4) The characteristics of the regulator will have considerable bearing on the results obtained. The prime requisite is to put as much excitation into the machine as practical to offset the demagnetizing effect of the lagging current in the generator. The regulator used for the Richmond tests was a standard Westinghouse type vibrating regulator. When the load was picked up, the contacts closed and remained closed until normal voltage was restored, permitting the exciter to build up to its maximum or ceiling voltage. This is not the case for the regulator used for the Saginaw tests. Referring to Fig. 8 of the paper, it will be observed that after about 0.3 sec., the regulator contacts opened, arresting the rise of the exciter voltage in spite of the fact that the terminal voltage of the generator is very near its lowest value. Because of these inherent characteristics of the regulators, for a given exciter the former will supply more excitation than the latter.

It is seen from the above that in the case of the Richmond machine, the regulator was very effective, even with normal speed of response of exciter voltage as compared to the higher speed of response of exciter voltage on the Saginaw machine. For the tests tabulated, the Saginaw machine returned to normal in 70 cycles, while the Richmond machine voltage was normal in 96 cycles. The fact that it took the motors longer to come up to speed and that the voltage drop was greater would readily account for this difference in time.

Otto Naef: (communicated after adjournment) It is not often that actual data and test results on the operation of regulating devices are made public, and the authors are to be congratulated for the thoroughness with which they have analyzed and presented the test results obtained with the General Electric quick-response generator voltage regulator.

This regulator is of the combined rheostatic and vibrating type. The combination of these two elements in one regulator is indicative of the general trend in regulator design to get away from vibrating contacts with their attendant wear, and to replace them by something which gives more reliable operation. The question naturally arises, is it necessary to use vibrating contacts at all; it is possible to design an automatic regulator, which has quick response, without vibrating contacts?

I shall try to answer this question by showing a few test results obtained with a regulator built on the purely rheostatic principle, but distinguished from its slow-acting predecessor by its high speed of operation and powerful over-regulation.

The responsiveness of an automatic regulator may be expressed by the time, measured in cycles, which elapses from the moment a voltage change occurs until the regulator has adjusted itself to the position of maximum or minimum excitation, whichever may be required to bring the voltage back to normal. This time is generally a function of the magnitude of the voltage change acting on the regulator. That the regulator described in the paper has this same characteristic is indicated by the test figures presented. When switching on load, the instantaneous voltage drop was 8 per cent, and it required 6.5 cycles for the auxiliary contactor to close, of which 2.5 cycles were consumed in the main contacts. During this interval of 6.5 cycles, before the regulator would act, the generator voltage would drop another 3 to 4 per cent, the total drop being therefore 11 to 12 per cent. When dropping load, the paper states that it took 14 cycles for the auxiliary contactor to open, a considerable increase as compared to the first test. But the instantaneous voltage change acting on the regulator was also considerably smaller than before, as shown by the oscillograms. The influence of the magnitude of the voltage change on the responsiveness of the regulator is thus clearly demonstrated.

Before showing the test results obtained with the rheostatic type regulator it will be necessary that I explain briefly how it operates and in which way the tests were carried out.

Fig. 1 herewith shows a schematic diagram of the regulator, which is manufactured by Brown Boveri and is built in a large size, with four sectors, and in a small size, with two sectors. It is

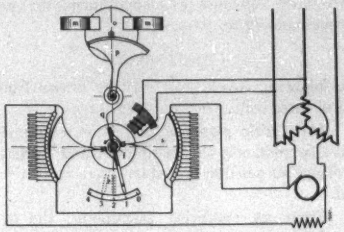

Fig. 1—Diagram Illustrating the Regulating Process

entirely self-contained, requiring no additional apparatus, relays, etc. The diagram explains the mechanism of the sectors which cut resistance in and out of the shunt field circuit of the exciter. Instead of gliding, the sectors roll over the contacts, thus doing away with gliding friction. The reactions from the two or four sectors are accurately balanced and in no position of the moving system do they exert a torque on the regulating spindle. The position of the regulating system is indicated by the position of the pointer. In position "Zero" all resistance is cut out. In position "4" all resistance is in. These two positions therefore correspond to maximum and minimum excitation, respectively. The diagram also shows how the regulator is connected to the generator whose voltage is to be controlled. The electric torque is provided by a powerful torque motor. The mechanical counter torque is obtained by spring tension. *o* and *p* is the anti-hunting device, consisting of an aluminum disk rotating between two permanent magnets. The disk is coupled to the regulating system by a soft spring, and therefore does not resist quick response of the regulator to sudden changes in voltage.

The tests to determine the responsiveness of the regulator were made years ago on a standard four-sector regulator. For this purpose the voltage element was connected to a 60-cycle constant potential source through a resistance absorbing 11 per cent of the voltage. The voltage at the regulator terminals was 110 volts. By short circuiting the resistance by means of an auxiliary switch

the voltage across the regulator terminals could be raised 11 per cent instantaneously. Different starting positions of the pointer were investigated, and the time required by the regulator to move from the starting position to position 4, corresponding to minimum excitation, was measured by connecting a battery and resistance across the regulator rheostat, and oscillographing the current flowing in the circuit.

The oscillogram represented in Fig. 2 herewith shows the regulator at rest in position 1.5, which generally corresponds to approximately full load on the generator. The test conditions were therefore similar to those that would exist if full load were thrown off a generator, resulting in a momentary voltage rise of 11 per cent. The curved line represents the variation of the current through the regulator resistances, the horizontal lines are calibration lines and show the current flowing through the sectors at

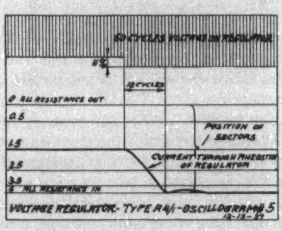

Fig. 2

intermediate positions. It will be noted that the time required by the regulator to begin motion is less than one cycle. After 6 cycles the regulator has completed half its course, and after 12 cycles it reaches its extreme position at 4, corresponding to minimum excitation. For starting positions below or above 1.5, the number of cycles required by the regulator to reach the end position 4 was found to be correspondingly larger or smaller.

It is to be noted that in its effect on the field current, the 12 cycles required by the rheostatic regulator to reach its end position is approximately equivalent to 6 cycles required by a vibrating type regulator to close the contacts of its auxiliary relay. This is due to the fact that the rheostatic regulator starts at once to cut the field resistance in or out, whereas the vibrating regulator has no effect on the field current during the first few cycles.

The above tests, which were made under conditions similar to those disclosed in the paper, as far as the regulator is concerned, indicate that the responsiveness of the rheostatic type of regulator compares very favorably with that of the vibrating type. There remains one difference, however, between the two types of regulators, which has some influence on the rate of generator field current build-up or decay. Whereas the regulator described in the paper begins to vibrate when a change in voltage occurs, and maintains the exciter voltage at an average value below the extreme minimum or maximum value, the rheostatic type of regulator keeps the exciter voltage at its very minimum or maximum until the generator voltage has been restored almost to normal. The rheostatic type of regulator therefore tends to produce a quicker response of the generator.

The purpose of this discussion was to show that the rheostatic principle is as applicable as the vibrating principle to quick-response automatic voltage regulators such as are demanded by the modern requirements of system operation.

E. J. Burnham: Mr. Hanker mentioned the contactors and inquired whether they were special. These contactors were special to the extent that they were made high-speed, that is, ordinary contactors were lightened so that they would vibrate much faster than the standard.

Regarding Mr. Weinstein's question I might say that the a-c. coil which is added to the left-hand member is replaced in this regulator by a spring dash-pot on the a-c. lever. In other words, the same thing is accomplished by this spring dash-pot, and is, you might say, a commercial form of the quick-response feature.

The tabulation cited by Mr. Monteith might indicate that the gain between 1000 and 100 volts per second is not nearly as great as the gain between 100 volts per second and constant excitation. This, of course, is natural. With a normal ceiling exciter it is questionable whether even an infinite volt-per-second rate would produce an appreciable gain over the 1000 volt-per-second arrangement. In other words the gain to be obtained is less and less pronounced as the volt-per-second build-up is increased.

Mr. Monteith also states that if the Westinghouse regulator were used in place of the one described, it would have supplied more excitation and implies that this is an advantage. Extensive tests such as those described in the two papers and discussion given at the St. Louis Regional Meeting, March 1928 (A. I. E. E. Trans., Vol. 47, pp. 944-974) have shown that in most cases it is not desirable to continue increasing the excitation until normal voltage is reached, when maximum benefit and stability are considered.

J. R. North: With regard to the question about the location of the potential transformer during the test, the idea was to get the actual results at the terminal of the generator. In this installation the load is served from the 189-volt bus and under normal conditions the regulator is controlled by potential transformers connected directly to the 480-volt bus.

Street Railway Power Economics
on the Cincinnati System

BY J. A. NOERTKER[1]
Associate, A. I. E. E.

Synopsis.—*The Cincinnati Street Railway Company has, within the past few months, completed the rehabilitation of its entire power system. The system now consists of 19 full automatic synchronous converter substations upon which has been superimposed a complete system of supervisory control and remote metering. Papers have been presented by Frank W. Peters and Harley L. Swift covering the details of this installation. (See bibliography.) This paper discusses the economic factors involved in the selection of equipment and the design of the system. Part I points out that the most important economic consideration is service and presents a method for evaluating this factor. Part II discusses the design of feeder circuits with particular reference to the limitations of Kelvin's Law. Part III discusses system design with reference to the economics involved in the location of substations, and in the selection of control equipment. A general method for conducting extensive system studies is implied. Part IV discusses system load shifting characteristics and Part V points out the advantages of supervisory control.*

I. Introduction

THE management of a modern street railway system, especially one operating under a "service at cost" franchise, should and does aim to provide a service that most nearly meets the requirements of the average car-rider which implies the fastest schedule speeds consistent with safety, and ultimate economy. As a matter of fact, ultimate economy as it concerns both car-rider and company is one of the most important considerations.

The car-rider is interested in continuity, cost, and quality of service. The primary factor affecting the quality of service is speed of car operation, and some idea of the magnitude of this factor may be had by considering that in a city of the size and character of Cincinnati the total annual time spent by the car-riding public in transportation is approximately 50,000,000 hr.

It is practically impossible to determine definitely a value for this time; however, assuming a rate of fifty cents per hour, the annual amount will be $25,000,000, which amount is of sufficient magnitude to urge a careful analysis of all factors affecting the speed of car operation, in the interest of both the car-rider and the management. Among the many factors affecting the speed of car operation the following are outstanding:

1. Topography
2. Traffic congestion and control
3. Number and duration of stops
4. Rates of acceleration and retardation
5. Motor characteristics
6. Trolley voltage.

Two of these factors, motor characteristics and trolley voltage, are directly under the control of the railway engineers. In selecting car equipments, it is necessary to assume average conditions of the above factors. The proper voltage has also usually been determined for average conditions and in the design of the power system the effort has been to furnish this average voltage over the entire system. It is not certain, however, that an average voltage will fulfill all conditions in the most effective manner. Since the primary function of the power system is to furnish the most economical and satisfactory trolley voltage for the operation of the cars, it is of the first importance to determine what voltage or voltages are the most economical and satisfactory for various operating conditions and then provide such voltage through the means of variable converter characteristics and transformer taps.

While poor voltage conditions may result in loss of time by passengers, aside from evaluation of this time, it can be shown that for a city the size of Cincinnati the annual variable cost of power supply subject to manipulation on which there is no practical check is approximately $180,000.00. This is made up of carrying charges on feeders, conversion loss, demand loss, energy loss, and carrying charges on that portion of the conversion equipment required to supply the feeder loss. It is evident from this fact alone that a thorough study of the power system is justified.

In an effort to determine the proper voltages for the operation of cars under various conditions, the following curves have been plotted from the data derived from speed-time and power-time curves of the latest type cars of the Cincinnati Street Railway Company.

Fig. 1 shows the minimum time required to make various runs at 600 volts and the per cent increase in time at lower voltages. It will be observed that for runs of less than 200 ft. the minimum time required to make the run is approximately the same. It should also be noted that for the 1200-ft. run on 6 per cent grade there is approximately a 6 per cent increase in the minimum time required when the trolley voltage is decreased from 600 to 500 volts.

Fig. 2 shows the energy consumption required to make various runs at several different voltages. The curves show that increasing the voltage on these runs materially increases the energy consumption. They

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29-48

also show that the increased energy consumption corresponding to a definite increase in voltage is greater for the short runs.

Fig. 3 is derived from Figs. 1 and 2 and presents the same information in a more convenient form for a 300-ft. level run and a 1200-ft. run on 6 per cent grade. From a careful consideration of the data presented in

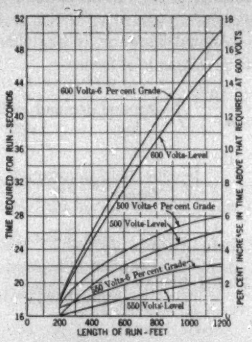

FIG. 1—MINIMUM TIME REQUIRED TO MAKE VARIOUS RUNS (WITHOUT COAST) AT 600 VOLTS, AND PERCENT INCREASE IN TIME AT LOWER VOLTAGES

Car Data—Motors—4 W. E. No. 510 E. hp. 35
Gear Ratio—13.69
Size of Wheels—26 in.
Wt. of Car and Load—19.36 tons
Acceleration—1.5 mi. per hr. per sec.
Retardation—2.0 mi. per hr. per sec.

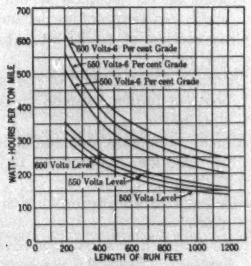

FIG. 2—ENERGY REQUIRED TO MAKE VARIOUS RUNS (WITHOUT COAST) AT VARIOUS VOLTAGES

Car data—same as Fig. 1

Figs. 1, 2, and 3, it is obvious that the trolley voltage is an important factor in determining the speed of operation and energy consumption. Taking into consideration the evaluated time of the car-rider, the platform expense, and the cost of energy, it is possible, for any particular run, to determine the most economical operating voltage.

Fig. 4 shows the most economical trolley voltages for a 300-ft. level run and a 1200-ft. run on 6 per cent grade, with platform expense and evaluated time of car-rider taken at $10.00 per car hr. and energy cost at the car taken as 1.5 cents per kw. hr. The curves

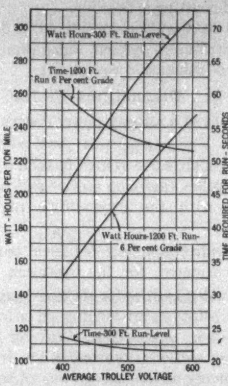

FIG. 3—TIME AND ENERGY CONSUMPTION REQUIRED (WITHOUT COAST) FOR 300-FT. LEVEL RUN AND 1200-FT. RUN ON 6 PER CENT GRADE

Car data—same as Fig. 1

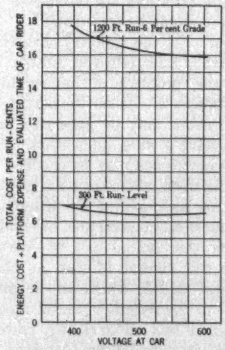

FIG. 4—TOTAL COST FOR SEVERAL RUNS (WITHOUT COAST) AT VARIOUS VOLTAGES

Car data—same as Fig. 1
Energy Cost—1.5 cents per kw. hr.
Platform expense and evaluated time of car-rider $10.00 per car-hr.

show that for congested areas where runs are short the proper voltage may be considerably below 600 volts and that for long runs on heavy grades the proper voltage is above 600 volts.

Inasmuch as increased car speed has been considered

the desirable result of increasing trolley voltage, no consideration has been given to the effect of coasting. It can be shown, however, that if schedule speeds are kept the same, increase in voltage will usually result in decreased energy consumption at the car since higher speeds are reached more quickly with higher voltages and increased coasting is possible.

The above data will be considered in detail as they affect the distribution system, substation layout, and the selection of substation equipment.

II. Distribution System

The functions of a railway power distribution system are:

1. To sectionalize the system in such a manner that the effect of trolley faults on service will be as small as is consistent with the expenditure required to so sectionalize.
2. To provide the most economical trolley voltage, all factors considered, for the operation of the cars.
3. To provide facilities for shifting load in emergencies as when substations are overloaded or incapacitated.

Feeder Sections. A thorough study for the determination of feeder sections involves, in the first place, the setting up of every advantageous way of sectionalizing. Then, with a knowledge of the cost, together with a knowledge of the frequency, duration, and cost (both as to platform expense and evaluated time of carrier) of feeder faults, the distribution engineer can determine the most economical feeder sections. In this connection, the extensive use of outdoor sectionalizing circuit breakers should be considered.

Feeder Sizes. For the majority of cases, the application of Kelvin's Law offers a simple and effective solution to the problem of determining the most economical feeder size. It has the particular advantage that its conclusions may be presented in the form of curves which have a wide application. Further, inasmuch as the conclusions may be checked there is little danger of using them where they will not be applicable.

Kelvin's Law as modified by Kapp is as follows:

> "The most economical area of conductor is that for which the annual cost of energy wasted is equal to the annual interest on that portion of the capital outlay which can be considered proportional to the weight of metal used."

The application of this law to railway power circuits has been presented in a paper by Crecelius and Phillips. (Bibliography.) The following equations and curves present a convenient method for applying Kelvin's Law to extensive system studies. The most variable factor in such a study has been considered to be the cost and value of installed copper.

The total annual cost of a feeder circuit is equal to the annual investment charge plus the annual demand loss charge plus the annual energy loss charge. This may be expressed as a mathematical statement as follows:

$$C_p = (r_1 C_u 10^{-2})(3.67\, m\, D\, 10^{-6}) + \left(\frac{M + r_2 C_s 10^{-2}}{\text{Eff. s. p.}}\right)\left(\frac{I^2\, 11.0\, D\, 10^{-3}}{m}\right) + \left(\frac{e}{\text{Eff. (av.)}}\right)\left(\frac{8760\, I^2\, 11.0\, D\, L\, 10^{-3}}{m}\right) \quad (1)$$

The significance of the various parts of Equation (1) is as follows:

$(r_1 C_u 10^{-2})$—Annual charge on feeder, dollars per pound

$(3.67\, m\, D\, 10^{-6})$—Pounds of feeder

$\left(\frac{M + r_2 C_s 10^{-2}}{\text{Eff. s. p.}}\right)$—Annual demand, dollars per d-c. kw.

$\left(\frac{I^2\, 11.0\, D\, 10^{-3}}{m}\right)$—One hour maximum kw. loss in feeder

$\left(\frac{e}{\text{Eff. (av.)}}\right)$—Energy charge, dollars per d-c. kw. hr.

$\left(\frac{8760\, I^2\, 11.0\, D\, L\, 10^{-3}}{m}\right)$—Annual kw. hr. loss in feeder

where

C_p —Total annual cost of positive feeder circuit (dollars)

r_1 —Rate of return on feeder investment (12 per cent) (Taxes 2.0 per cent; Depreciation 2.0 per cent; Interest 6.0 per cent; Reserve 2.0 per cent)

C_u —Cost of insulated feeder in place (dollars per lb.)

3.67 —Pounds per ft. 1,000,000 cir. mil cable triple braid, weatherproof insulation

m —Cross-section of feeder at substation end (cir. mils)

D —Distance in feet to end of feeder or neutral point of feeder common to two or more stations

M —Annual maximum demand charge (dollars per a-c. kw. metered at 13,200 volts—$12.00)

Eff. s. p. —Efficiency of substation during peak load (0.93)

r_2 —Rate of return on substation investment (15 per cent) (Taxes 2.0 per cent; Insurance 0.5 per cent; Depreciation 4.5 per cent; Interest 6.0 per cent; Reserve 2.0 per cent)

C_s —Cost of substation per d-c. kw. capacity ($40.00)

I —One hour maximum demand (amperes)

11.0 —Ohms per cir. mil ft. 30 deg. cent. 98 per cent conductivity

e —Energy charge on purchased power (dollars per a-c. kw. hr. $0.004)

Eff. (av.)—Average efficiency of substation (0.90)

8760—Hr. in a year

L —Loss factor—the ratio of the average of the squared current demands to the squared maximum (1) hr. current demand. (0.25)

Substituting the above values in the equation for total annual cost and combining terms, the equation reduces to

$$C_p = 0.44 \times 10^{-6}\, C u\, m\, D + \frac{0.32\, I^2\, D}{m} \qquad (2)$$

Equating first derivative to zero for minimum annual cost

$$o = 0.44 \times 10^{-6}\, C u\, D - \frac{0.32\, I^2\, D}{m^2}$$

$$m = \frac{850\, I}{\sqrt{C\, u}} \qquad (3)$$

This is the proper size feeder for minimum annual cost for concentrated load at the end of a feeder of uniform cross-section.

Copper losses caused by a distributed load on a theoretically tapered feeder are equal to one-half those caused by a concentrated load on a feeder of uniform cross-section if this cross-section is the same as that at the station end of the tapered feeder. Since the volume of the tapered feeder is one-half that of the uniform feeder, the investment charge is one-half that shown by the equation for the uniform section with concentrated load.

The annual charge for this type of feeder is

$$C_p = \frac{0.44 \times 10^{-6}\, C\, u\, m\, D}{2} + \frac{0.32\, I^2\, D}{2\, m} \qquad (4)$$

and the cross-section for minimum cost becomes

$$m = \frac{850\, I}{\sqrt{C\, u}} \qquad (5)$$

Also, since it can be shown mathematically that the copper losses caused by a distributed load on a feeder of uniform cross-section are equal to one-third those caused by a concentrated load, the annual charge for such a feeder is

$$C_p = 0.44 \times 10^{-6}\, C\, u\, m\, D + \frac{0.32\, I^2\, D}{3\, m} \qquad (6)$$

and the cross-section for minimum annual cost becomes

$$m = \frac{850\, I}{\sqrt{3}\ \sqrt{C\, u}} = \frac{490\, I}{\sqrt{C\, u}} \qquad (7)$$

If these values of m are now substituted in Equations (2), (4), and (6) for total annual cost, the following mathematical expressions will be obtained for minimum annual cost.

For concentrated load uniform cross-section

$$C_p\ (\text{min}) = 0.00075\, I\, D \sqrt{C\, u} \qquad (8)$$

For distributed load tapering cross-section

$$C_p\ (\text{min}) = 0.000375\, I\, D \sqrt{C\, u} \qquad (9)$$

For distributed load uniform cross-section

$$C_p\ (\text{min}) = 0.000434\, I\, D \sqrt{C\, u} \qquad (10)$$

The maximum voltage drop to the end of feeders and the average voltage drop at the load for minimum annual cost is

For concentrated load, uniform section

$$E_d\ (\text{max}) = \frac{I\, D\, 11.0}{m} = 0.013\, D \sqrt{C\, u}. \qquad (11)$$

$$E_d\ (\text{av.}) = \frac{I\, D\, 11.0}{m} = 0.013\, D \sqrt{C\, u}. \qquad (12)$$

For distributed load, tapering section

$$E_d\ (\text{max}) = \frac{I\, D\, 11.0}{m} = 0.013\, D \sqrt{C\, u}. \qquad (13)$$

$$E_d\ (\text{av.}) = \frac{I\, D\, 11.0}{2\, m} = 0.0065\, D \sqrt{C\, u}. \qquad (14)$$

For distributed load, uniform section

$$E_d\ (\text{max}) = \frac{I\, D\, 11.0}{2\, m} = 0.0112\, D \sqrt{C\, u}. \qquad (15)$$

$$E_d\ (\text{av.}) = \frac{I\, D\, 11.0}{3\, m} = 0.0075\, D \sqrt{C\, u}. \qquad (16)$$

It is of particular interest to note that the proper size feeder for minimum annual cost, Equation (3), is directly proportional to the load and does not depend on the length of the feeder. All of these equations and the curves presented below are applicable to the positive feeder circuit on both single and double trolley systems. Values for annual cost and voltage drops on the complete circuit of the double trolley system are twice that given for the positive feeder in the above equations. Some additional calculations are necessary to obtain these figures for the single trolley system.

The following curves have been drawn with the idea of expressing the results of the above equations in a more convenient form.

Fig. 5 illustrates how the factors of Equation (2) vary as the size of the feeder is increased. The curves have been plotted for 1000-ampere load concentrated at the end of a uniform cross-section feeder 1000 ft. long.

The curve "Per cent Return" has been plotted with the idea of showing the return on additional investment that would be required to build up an undersize feeder to the size corresponding to minimum annual cost as determined by Kelvin's Law.

Fig. 6 presents in a convenient form the results of applying Equations (3), (5), and (7) to a 1000-ampere load. From this figure the proper size feeder for minimum annual cost of any standard type railway feeder may be determined, provided the values of all factors used in solving Equation (1) are true for the particular feeder in question.

Fig. 7 shows the minimum annual cost of various standard types of railway feeders for 1000-ampere load, 1000-ft. length. Since the minimum annual cost is directly proportional to the ampere load and the length, the minimum annual cost for any other load and length can easily be determined.

Fig. 8 shows the maximum voltage drop to end of feeder and average voltage drop on feeder for the feeder size corresponding to minimum annual cost. These curves have been plotted for 1000-ft. length, and since the voltage drop is directly proportional to the length, the voltage drop for any other length can easily be determined. These curves are of particular value in checking the application of Kelvin's Law.

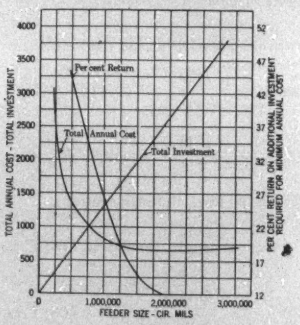

FIG. 5—TOTAL ANNUAL COST AND INVESTMENT FOR A UNIFORM CROSS-SECTION FEEDER 1000 FT. LONG CARRYING A 1000 AMPERE LOAD CONCENTRATED AT THE END

Copper at 18 cents per pound installed

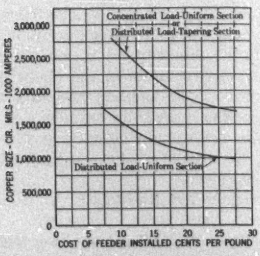

FIG. 6—FEEDER SIZES FOR MINIMUM ANNUAL COST (KELVIN'S LAW)

Reconcilement to Kelvin's Law. After feeder sizes have been determined by Kelvin's Law, the resulting average feeder voltage should be checked against the most economical voltage required for the operation of the cars. If the average voltage at the load as determined by Kelvin's Law is considerably above or below that required for the most economical car operation as shown by curves similar to those of Fig. 4 for the average runs on the section in question, it will be necessary to plot graphs of Equation (2) (total annual cost against average voltage at the load). This graph should then be superimposed and added to the appropriate curve on Fig. 4. The resulting graph will indicate the point of overall minimum cost. The corresponding feeder size can then be read directly from the curves. The following examples illustrate the procedure:

Example 1. Consider a section 7200 ft. long having an average grade of 6 per cent and average length of runs of 1200 ft. The average one hour peak load of 1000 amperes is made up of 12 cars moving up-grade. Ninety-two cars carrying average loads of 60 persons per car and moving up-grade will traverse the section during the two-hour evening peak.

To provide emergency tie-in facilities the feeder is of uniform section. Loss factor 0.25.

Substation voltage 600 volts.

From the standpoint of system stability, it has been found inadvisable to increase substation voltage above 600.

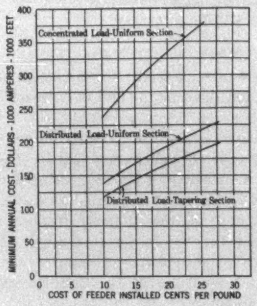

FIG. 7—MINIMUM ANNUAL COSTS (KELVIN'S LAW)

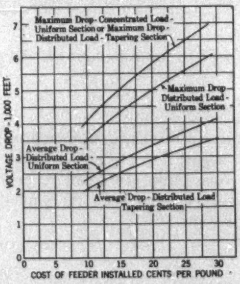

FIG. 8—MAXIMUM AND AVERAGE VOLTAGE DROP ON FEEDERS DESIGNED FOR MINIMUM ANNUAL COST (KELVIN'S LAW)

Cost of feeder installed $0.18 per lb.

Value of car-time $10.00 per car hour.

Cost of energy per d-c. kw. hr. $0.015 (used for estimating cost of energy consumption at car).

Weight of car 19.36 tons.

Required to determine the proper size feeder.

From Figs. 6, 7, and 8, according to Kelvin's Law the proper size feeder is found to be 1,150,000 cir. mils corresponding to an annual minimum cost of $2664.00, and to an average voltage drop of 46 volts; which corresponds to an average trolley voltage of 554. Referring to Fig. 4, it is found that the most economical voltage for the operation of cars is above 600 volts. Therefore, it is evident that Kelvin's Law is not applicable for this case. Fig. 9 illustrates the solution of this problem.

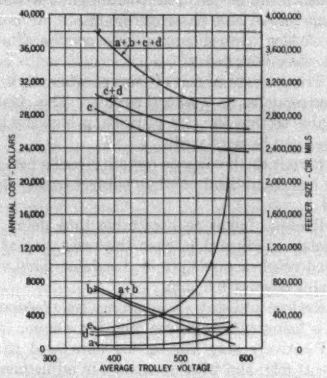

FIG. 9—DETERMINATION OF FEEDER SIZE—EXAMPLE (1)

Curve *e* Feeder Size
Curve *a* Annual investment charges on feeder
Curve *b* Annual loss in feeder
Curve *c* Annual platform expense and evaluated time of car rider
Curve *d* Annual cost of energy consumption at cars

The data required for plotting curve *e* of Fig. 9 have been derived from Equation (**16**) for average voltage.

The data required for plotting curve *a* have been derived from the equation

$$C_a = 0.44 \times 10^{-6}\, C\, u\, m\, D$$

The data for plotting curve *b* have been derived from the equation

$$C_b = 0.32\, \frac{I^2 D}{3\, m}$$

The data for plotting curve *c* have been derived from the expression

$$C_c = (300 \times 92 \times 6 \times 10)\, \frac{t}{3600}$$

where t is the time in seconds required to make a 1200-ft. run as derived from Fig. 3.

The data for plotting curve *d* have been derived from the expression

$$C_d = \left(300 \times 92 \times 19.36 \times \frac{7200}{5280} \times \frac{W}{1000} \times 0.015 \right)$$

where W is watt-hr. per ton mile from Fig. 3.

The curve $(a + b + c + d)$ of Fig. 9 shows that the proper feeder size to secure minimum annual cost is 1,500,000 cir. mils.

Example 2. Consider the feed to a loop in the congested area. The cars operate over a level section 3000 ft. long. The lengths of runs average 300 ft.

The average one hour peak load of 1000 amperes is made up of 20 cars moving over the route.

Three hundred and twenty cars, carrying average loads of 60 persons per car, will traverse the section during the two-hour morning and evening peaks.

A feeder of uniform section feeds the loop from a substation approximately 4000 ft. from the center of the load; therefore, the load will be considered as concentrated at the end of a 4000-ft. uniform cross-section feeder.

Loss factor 0.25.

Substation voltage 560 volts.

From the standpoint of system stability, it has been found inadvisable to decrease full load substation voltage below 560 volts, this voltage being obtained through the use of shunt converters.

Cost of feeder installed $0.18 per lb.

Value of car-time $10.00 per car-hr.

Cost of energy per d-c. kw. hr. $0.015 (used for estimating cost of energy consumption at the car).

Weight of car 19.36 tons.

Required to determine the proper size feeder.

From Figs. 6, 7, and 8, according to Kelvin's Law the proper size feeder is found to be 2,000,000 cir. mils, corresponding to an annual minimum cost of $2720 and to an average voltage drop of 44 volts which corresponds to an average trolley voltage of 516 volts. Referring to Fig. 4, it is found that the most economical voltage for the operation of cars is 490 volts. Inasmuch as Kelvin's Law provides a higher voltage than can be economically utilized, it is apparent that it is not applicable for this case. Fig. 10 illustrates the proper solution of this problem.

The data required for plotting curve *e* of Fig. 10 have been derived from Equation (**12**).

The data required for plotting curve *a* have been derived from the equation

$$C_a = 0.44 \times 10^{-6}\, C\, u\, m\, D$$

The data for plotting curve *b* have been derived from the equation

$$C_b = 0.32\, \frac{I^2 D}{m}$$

The data for plotting curve *c* have been derived from the expression

$$C_e = (300 \times 320 \times 10 \times 10)\frac{t}{3600}$$

where t is the time in seconds required to make a typical run as derived from Fig. 3.

The data required for plotting curve d have been derived from the expression

$$C_d = \left(300 \times 320 \times 19.36 \times \frac{3000}{5280} \times \frac{W}{1000} \times 0.015\right)$$

where W is watt hr. per ton mile from Fig. 3.

The curve $(a + b + c + d)$ shows that the proper size feeder to secure minimum annual cost is 1,700,000 cir. mils.

The above examples have been worked out for a double trolley system. A somewhat similar procedure may be followed for a single trolley system.

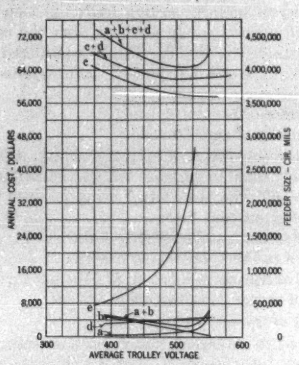

FIG. 10—DETERMINATION OF FEEDER SIZE—EXAMPLE (2)

Curve e Feeder size
Curve a Annual investment charges on feeder
Curve b Annual loss in feeder
Curve c Annual platform expense and evaluated time of car rider
Curve d Annual cost of energy consumption at cars

Tie Feeders. From the study of minimum annual cost of various types of feeders it is found that the most economical type of feeder for the usual distributed railway load is the uniformly tapering section. If the substation units and the high tension power lines connected to each unit were 100 per cent reliable, there would be no occasion to use any other type. However, since such reliability has not been obtained and since power outages represent such a tremendous loss to the public, the Cincinnati power system has been laid out with the idea of carrying the peak load service at reduced voltage with any one substation out of commission. This can only be done through the extensive use of tie feeders, which in many cases has lead to the installing of uniform cross-section feeders. A comparison of Equations (**9**) and (**10**) shows that the total annual cost of the uniform cross-section feeder is 15.7 per cent higher than that of the tapered section. In actual practise, due to the impossibility of obtaining a theoretically tapered section, this percentage is somewhat less. It is considered, however, that the advantages of the uniform cross-section feeder are worth the increased cost. This fact can be proved conclusively through the evaluation of power outages mitigated by the use of tie feeders.

Feeder Control Equipment. For systems similar to the Cincinnati power system where the ill-effects of substation outages have been lessened by the use of a large number of small stations and extensive tie feeders, the circuit breaker and its control circuit become the most important element in the system. Some conception of the expenditure justified to secure effective feeder control equipments may be had by considering that a single outage, during the peak, of 30 minutes' duration on the feeder used in Example 2 would result in a time loss of 2400 passenger-hours.

The feeder control equipments on the Cincinnati system were designed to:

1. Trip on rate of rise of current when such rate of rise corresponds to a short circuit. This feature is adjustable so that the rate of rise at which the breaker will open can be regulated.
2. Permit the breaker to remain closed when subject to any legitimate load or when subject to any decrease in load.
3. Reclose, following a suitable time delay, which time delay is adjustable when the resistance of feeder and load reach a predetermined value. This value is adjustable down to 0.3 ohm.
4. Reclose and stay closed on multiple feeders when feeder is being energized from another source.
5. Positively stay out on 0.4 ohm load with bus voltage at 620; and with no change in adjustment, will positively reclose on 1.0 ohm with bus or feeder voltage at 350 volts.
6. Completely isolate the feeder when the power dispatcher locks it out.

In addition, the supervisory equipment superimposed on the full automatic feeder equipment has been designed so that:

1. The power dispatcher will have indication as to whether the breaker is open or closed.
2. The power dispatcher will have indication as to whether the breaker is under automatic or supervisory control.
3. The power dispatcher can reclose the breaker on a short circuit, but if short circuit conditions exist, the breaker will re-open. In connection with this feature an anti-pumping circuit is used.
4. Failure of supervisory cable will cause feeder equipment to revert to automatic control except when a feeder circuit is locked out.

The circuit breaker is of the latched-in type and both closing and tripping coils are operated from the station control battery. It incorporates in its design a magnetic blow-out for extinguishing the arc.

III. Conversion System

The study of railway power system design, with particular reference to the economics involved in the use of automatic substations, has been covered in considerable detail by committees of the A. E. R. A. These reports present both analytical and practical methods of considerable interest and value. In particular, they emphasize the importance of a thorough economic study of the distribution system and point out the desirable effects of decreasing voltage in congested areas. However, no definite methods are presented for determining the economic effect of the distribution system and trolley voltage on power system design.

The foregoing parts of this paper represent an effort to evaluate these factors so that they can be considered in relation to the other parts of the power system. The following discussion is an attempt to show how these and other factors have affected the location and number of substations on the Cincinnati system.

The immediate cause of the general rehabilitation of the system was the economy that the Street Railway Company could effect by purchasing power instead of operating its own generating plant. Inasmuch as this meant the replacement of all 25-cycle conversion equipment, amounting to one-half of the total installed capacity, an extensive study embracing the entire system was decided upon. The company had had in operation, for approximately six years, three full automatic single unit, 1500-kw. substations. The fine performance records of these equipments were a large factor in determining the selection of automatic control for the entire system.

Several topographical peculiarities of Cincinnati have considerably affected the power system design. (1) There are no large areas of uniform load density; (2) numerous heavy grades exist on which it is particularly advantageous to maintain the highest possible voltages; and (3) after leaving the congested area, the car lines extend radially out from the center of the city an average distance of ten miles with only one intersecting cross-town line. These factors tend to make any application of analytical methods very difficult. On the other hand, it is apparent that such a lay-out will immediately suggest a number of probable substation locations, especially after a careful study of voltage, load, and sectionalizing requirements has been made.

The general procedure for making the study of the system is given below:

Data. Of fundamental importance in making this study was the preparation of maps drawn to scale showing in detail the following data.

1. Location and routing of all cars operating during peak periods.
2. Length and location of grades.
3. Most desirable trolley sectionalizing, together with corresponding load in amperes on the various sections, and the most suitable average trolley voltage for peak periods.
4. Location of existing feeder copper.

Size of Units. Inasmuch as a large number of 60-cycle 1500-kw. units was already in use on the system, it was decided to extend the use of this size in so far as was feasible. However, after a few preliminary calculations it was obvious that a smaller size unit would be desirable for certain outlying sections, and a tentative set-up was made using 750-kw. units for these sections.

Reliability. Since the new 60-cycle conversion equipment would be more susceptible to system disturbance than the old 25-cycle equipment, and since it was decided to provide a service superior to that which had been provided by the old equipment, the principle was tentatively adopted that the loss of any single-unit station or any one unit in a double-unit station should not seriously affect transportation service. It afterwards developed that with a few minor changes in the preliminary system design this principle could be economically extended so as to include the loss of any complete double unit station.

Capacity for Expansion and Reserve. In general the expected one-hour integrated load was taken as approximately 80 per cent of the nominal substation capacity. This allowed about 20 per cent for expansion and unforeseen increase in load and figured on using the 50 per cent overload capacity of the converters to furnish the reserve required to meet the principle outlined above. In connection with expansion, the possibility of operating a proposed Rapid Transit System had a definite bearing. This resulted in some minor adjustments in location and in the providing of buildings large enough to house additional substation capacity.

Possible Locations. After the required data had been assimilated and fundamental principles decided upon, all possible substation locations were determined by inspection and a few preliminary calculations. The use of the system of moments was of particular value in the selection of tentative locations.

Zoning Restrictions and Available Sites. In Cincinnati the existence of some rigid zoning ordinances had considerable effect on the locations of substations. A survey was made as to the availability of sites in the permitted building zones.

Existing Feeder. It was evident that the considerable expense involved in re-arranging the feeder system would have a definite bearing upon substation location, and that as much advantage as possible should be taken of the feeder already installed.

Electrolysis. In the rehabilitation of its power system the railway company had to consider the possibility of changing to single trolley operation. From the first, it was realized that the extensive use of small single unit substations would be a definite factor in the mitigation of electrolysis when single trolley operation should be adopted. Discussing the development of automatic substations, the 1921 Report of the American

Committee on Electrolysis (p. 46) calls attention to the above fact in the following statement.

> "The increased savings attending this development will undoubtedly increase the number of substations which can economically be installed on both interurban and city systems, and if full advantage is taken of these economies, the feeding distances will be reduced to such an extent as to greatly reduce stray currents generally."

A specific instance supporting this statement was presented in a paper before the A. E. R. A. (Bibliography.)

Comparison. A general consideration of the data, principles, and limitations outlined above resulted in materially reducing the number of possible substation locations. As a matter of fact, few alternate schemes remained to be considered in detail. The total annual cost of the following factors, together with a consideration of system reliability, flexibility, and electrolysis conditions, were compared for each of the remaining schemes:

1. Service conditions
2. Distribution investment
3. Distribution loss
4. Equipment investment
5. Realty investment
6. Conversion loss
7. Maintenance and operating cost.

High-Tension Lines. No consideration was given to the high-tension system until after the substation layout had been determined. The high-tension system is usually one of the first considerations, but since the Cincinnati power system contemplated the purchase of power delivered at its substations and since the local power company had a number of strategically located high-tension distribution centers, it was thought that the high-tension system would not materially affect the ideal substation lay-out. Except in a few instances this proved to be the case. The negotiations with the power company developed the fact that considerable economy could be affected in the high-tension distribution system if the railway company would agree to the joint use of high-tension lines. In the case of a number of single unit stations this was agreed to, with the understanding that these lines would be protected with the most effective relay equipment that could be obtained. In the majority of cases these lines are parts of loop systems which are sectionalized at the railway substations. Inasmuch as the railway power system was designed so as to permit the loss of any one substation without seriously impairing the service, the above agreement was regarded as sound.

Final Determinations. The most economical substation lay-out as determined by the consideration of the foregoing factors involved the use of one 300-kw. single-unit automatic substation, five 750-kw. single-unit automatic substations, eleven 1500-kw. single-unit automatic substations, one 3000-kw. two-unit automatic substation, and one 4500-kw. three-unit manual station. In the final determination, it was decided that for the service under consideration, a slower speed machine would be more desirable than the 1200-rev. per min. standard 750-kw. units. Inasmuch as there was little difference in the cost and efficiency between the slower speed 750-kw. machines and the standard 1000-kw. machines and since the outlying territory in which these machines were to be located might be subject to considerable expansion, it was decided to use the standard 1000-kw. units.

The final calculations showed that it would be impossible to maintain reasonable service in the congested areas if the three-unit station should be entirely lost. It was, therefore, decided to tranfer one of these units to an adjacent single-unit station. From the standpoint of system stability, since the surrounding units would all be protected by the use of load shifting equipment, it seemed absolutely essential that the central station should also be protected. It was, therefore, decided to use full automatic control over the entire system.

Converter and Transformer Characteristics. The engineers in selecting the car equipments must necessarily base the application upon an average trolley voltage and average operating conditions throughout the system. If an average trolley voltage is furnished, the car equipments will perform in a more or less satisfactory manner. However, the operating conditions for any street railway car vary between wide limits. Fig. 4 shows the most economical voltages for a 300-ft. level run and a 1200-ft. run on 6 per cent grade. From this it will be seen that trolley voltages for cars operating under normal service conditions vary over a wide range.

It is evident that the railway power system should, in so far as possible, provide the most satisfactory and economical trolley voltage. Since adjustment of voltage through the manipulation of the distribution system does not provide an economical method for varying voltages, it is up to the substation engineers to provide for this either by making use of adjustable machine characteristics, suitable transformer taps, or both.

There is no argument as to the superior performance of shunt converters from the standpoint of substation operation and system stability, but it must be recognized that shunt converters cannot be used in outlying territories, or on heavy grades, except at the sacrifice of car speed. In congested areas, advantage can be taken of the superior operating characteristics of shunt converters with the additional advantage of power saving brought about through the furnishing of more economical trolley voltages. Inasmuch as the advantageous load shifting characteristics of the shunt converter may become a matter of importance on any part of the system, it will be a definite advantage to retain the possibility of shunt operation even on those parts of the system where compound operation is normally considered most effective. For the same reasons, it

will be desirable to provide transformer taps with the idea of varying d-c. bus voltage. Two distinct advantages resulting from this practise are (1) the providing of a more satisfactory trolley voltage, and (2) the providing for emergency load shifting.

It may be of interest to note that all of the new converters on the Cincinnati power system may be operated full shunt, full compound, or at any intermediate degree

Fig. 11—Interior of Single Unit Automatic Substation

of compounding. All new transformers are provided with tap-changing devices in the high-tension windings, so that the d-c. bus voltage may be varied in steps of 20 volts from 620 to 540 volts. It is expected that the high-tension power furnished by the power company will vary little from the 13,200-volt standard.

Fig. 12—Exterior of Automatic Substation

Manual vs. Automatic Control. Among the important factors that determined the type of control selected for the Cincinnati power system were maintenance and operating costs, cost of equipment, reserve capacity, efficiency, reliability, improved service, protection of equipment, and load shifting characteristics. Based upon six years' operating experience with three 1500-kw. single unit automatic substations, it was determined that a fair annual charge for maintenance and operating expenses on the automatically controlled equipment would be $1100 for a single-unit station and $1800 for a double-unit station. The corresponding charge for manual equipment was determined to be $3400 for a single-unit station and $5200 for a double-unit station. Annual savings creditable to automatic control, therefore, are $2300 for a single-unit station and $3200 for a double-unit station.

In the consideration of new single-unit substations, the investment chargeable against the use of automatic control was taken as the difference in cost between automatic control and the corresponding manual equipment. In the case of the rehabilitated station, however, the investment charge against automatic control was taken as the total cost of the new automatic control equipment less the scrap value of the superseded manual control equipment.

Fig. 13—Supervisory Control Room

For new single-unit stations, the difference in cost between automatic and manual equipment was found to be $13,000.00, which at 15 per cent amounted to an annual carrying charge of $1950.00. For rehabilitated single-unit stations the annual carrying charge at this per cent amounted to $2700.00; for new double-unit stations the annual carrying charge amounted to $3900.00, and for rehabilitated double-unit stations to $5400.00. From these figures, it is apparent that automatic control has a decided economic advantage only in the case of new single-unit stations.

Due to load shifting characteristics and thermal protective devices, automatic equipment can be safely subjected to heavier loads than could the corresponding manual equipment. Hence, it can be seen that less reserve capacity is required and that efficiency may be increased. The reserve capacity required for double-unit stations is particularly affected.

The greatly increased number of electrical devices required for automatic control makes this control equipment more susceptible to faults and failures. The only mitigating circumstance is the fact that this disadvantage is well recognized and as a consequence the

automatic equipment is usually more liberally designed and is more carefully installed, inspected, and maintained. Since the functioning of the automatic control equipments represents the best possible procedure in the handling of all normal and emergency operating conditions and since the sequence is arranged so that the control will perform as intended or not at all, it is evident that a more reliable operation will result than could be expected from a manually operated equipment. Thus, the possibility of material damage due to an incorrect manual operation is eliminated.

A definite improvement to service, due to automatic control, is the increased speed with which service is restored after having been interrupted by the failure of the high-tension power supply or by faults on the d-c. feeder system.

Since the bearing, thermal, and ground devices on the automatic control equipment provide protection superior to that possible with manual control, this must be considered an advantage.

The advantage of the load shifting equipment provided as part of the automatic control is one of the most outstanding benefits and should be given considerable weight in selecting control equipment for any railway application.

Buildings. Inasmuch as the general policy of the Cincinnati Street Railway Company has been to provide substation buildings that would be a credit to the community in which they were located, the full economic benefits usually considered possible through the use of automatic control have not been realized. This policy, however, has been instrumental in developing a considerable feeling of goodwill towards the company.

A-C. vs. D-C. Control. The selection of the type of control (alternating or direct current) is an important factor in the design of the railway power system, and should be given considerable attention.

IV. Load Shifting Characteristics

One of the most interesting problems connected with the design of the railway power system is that of determining the proper load shifting characteristics of the various substations. This became a particularly important factor on the Cincinnati system when the principle was established that the loss of any one substation should not seriously affect service conditions. The extensive use of tie feeders and high capacity load shifting resistance was instrumental in materially reducing the reserve capacity that would ordinarily be required to meet this condition. The first step in the determination of the ohmic values of load shifting resistances was to lay out the distribution system and determine the equivalent resistances between each station and its adjacent stations.

The shifting in load occasioned by a drop in voltage can be expressed by the following formula:

$$I = \frac{E}{R} \tag{17}$$

This equation may be proved mathematically correct for substation loads distributed in any way.

The ohmic values of the load shifting resistance were calculated from the following:

$$R_r = \frac{E}{1.5\,I_c} = \frac{R\,I}{1.5\,I_c} = \frac{R\,(1.9\,I_c - 1.5\,I_c)}{1.5\,I_c} = 0.266\,R \tag{18}$$

Where

I —Ampere decrease in load

E —Decrease in substation voltage or voltage drop in resistance

R —Equivalent resistance of tie feeders to adjacent stations

R_r—Ohmic resistance of one step of load shifting resistance

I_c—Full load current of converter

The above formula assumes that the proper ohmic value of a single step of resistance is that necessary to reduce the load on the converter from 190 per cent to 150 per cent. It is obvious that this assumption may not be generally applicable.

On the Cincinnati power system two equal steps of load shifting resistance were used. These were of sufficient thermal capacity to carry a 150 per cent load on the converter for 20 min.

A consideration of Equation (17) in connection with the use of shunt converters indicates the feasibility of eliminating some or all of the load shifting resistance in those stations where a low value of equivalent resistance can be obtained.

Load Shifting Resistor Relay Equipment. Since the cutting-in of load shifting resistors always results in a considerable loss of power, in impaired service, and in the possibility of shutting down the equipment on account of overheated resistors, it is of vital importance that this equipment does not cut-in unless this operation is absolutely essential to prevent overheating of the converters. In this connection, the proper application of relay equipment associated with the cutting-in of load shifting resistance is of particular importance.

V. Supervisory Control

Full Automatic System without Supervisory Control. There are conditions such as those brought about by storms and fires that cannot possibly be met by automatic control. These usually consist in the necessity for deenergizing trolley lines so as to eliminate the possibility of accidents.

In addition to these limitations, there appears to be a number of situations which are not met by the present automatic equipment. Such a situation is the inability of the low voltage time delay starting equipment to meet all required conditions. In order to promptly meet emergencies such as the loss of an adjacent substation, it is evident that the time delay starting relay must be set for short time. If this adjustment is made, because of fluctuating railway loads, it will lead to much unnecessary starting and stopping of the con-

version equipment resulting in higher maintenance cost and decreased efficiency. If the time is lengthened so as to correct this condition, prompt operation in case of trouble is sacrificed. It may be possible to meet this situation by a more effective control, but so far as is known, this has not been done. A situation similar to the above is the proper operation of equipment for the providing of night owl service. Another case in which it is difficult to secure the proper functioning of automatic control is one where trolley voltage is adjusted by varying the machine operating characteristics and substation voltage, in which case the load on the converter may not determine the proper point at which the converter should be shut down.

Functioning of Supervisory Control. The primary functions of supervisory control are (1) to provide necessary checks on operation of substations, (2) to operate the system at the point of maximum efficiency, and (3) to change the normal automatic functions so as to better meet emergencies.

Inasmuch as the entire 600-volt feeder system is tied together so that the loss of any one substation during the light load periods has little effect on the movement of cars, it is apparent that without some sort of indicating system, there is no check on substation functioning at this time. As most faults occurring in automatic substations develop or become apparent during the starting operation, it is of particular advantage to have a check at this time so that the faults may be remedied before the peak period.

Since the load demands on a street railway system are approximately the same from day to day, it is possible to calculate the most efficient way in which to operate the system. This will usually call for a definite schedule for the various units. The maximum efficiency and the minimum maintenance will result from keeping the number of starting and stopping operations as low as possible. This may be accomplished most effectively by the use of supervisory control.

In addition to providing the means for meeting fire and storm emergencies, the supervisory system can often be used to meet emergency overload conditions. These are usually taken care of by automatic equipment through the cutting-in of the load shifting resistance which, as before stated, results in power loss, impaired service, and the possibility of overheated resistors. Improved service will frequently result from dropping tie-feeder loads since this may permit the reclosing of the resistance shunting contactors.

Remote Metering. As applied on the Cincinnati power system, the remote ammeters provide the dispatcher with a continuous check on the operating conditions of the converters both when starting and running. The ammeter, although calibrated to read in d-c. amperes, really measures the current in the a-c. side of the converter. The load readings are essential if the dispatcher is to shift loads during emergency overload conditions. The recording volt meters provide a continuous check on that most important operating factor, substation bus voltage.

Conclusions

Inasmuch as topography, arrangement of the city, density of population, and traffic conditions have such a definite bearing on the distribution of load and car operation, it is impossible to draw conclusions that are generally applicable to all railway systems. In any city, however, the comprehensive design of a power system must consider as one problem the effect of trolley voltage on car speed, the distribution system, the location of substations, and the selection of conversion equipment.

(a) On any railway system the effect of trolley voltage on service is a most important consideration and merits a more extensive study than has yet been given to it.

(b) Kelvin's Law with certain limitations is generally applicable to the design of railway feeders and offers a relatively simple solution to the problems of the determination of proper feeder sizes.

(c) Converters and transformers designed to give the greatest possible flexibility as to operating voltage should be selected. Compound wound converters capable of being operated as shunt machines are most suitable for the railway power system, the flat voltage characteristics being desirable for grades and outlying territories and the shunt characteristics being more desirable for congested areas.

(d) The use of automatic control results in the most economical and reliable system.

(e) With a complete automatically controlled system, it is necessary to have some sort of check on substation operation. Supervisory control, in addition to providing such a check, furnishes a method for more efficient normal and more effective emergency operation than is possible with full automatic equipment acting alone.

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Discussion

T. H. Schoepf: There is one point in the assumptions of fixed values which have been established by the author that I question the correctness of, and that is the fixed charge of 12 per cent in the one instance and 15 per cent in the other, wherein 2 per cent has been included for taxes and 2 per cent for a reserve fund. As a matter of fact the higher you select the total of that percentage, the smaller your cable becomes.

If the size of the cable is figured by the formula as established by the author and established by others on the Kelvin law, the cable is a little larger than common sense dictates, and usually it is trimmed down to some extent.

Those who have had experience with tax commissions I think know how they make up taxes. A budget is submitted to the tax commission of how much the state requires. They look over the companies they think can afford it, and they say, "This is your tax; this is your valuation based on the tax," and that is what you pay. So computing the property by the amount of this cable doesn't make any difference in the tax.

I think the whole percentage should be lower than 12 and 15 respectively. However, as a matter of fact the property here was figured out by the method Mr. Noertker has exposed. It was modified in some places where judgment dictated modification was required, but we were particularly concerned with the tie-feeders, and it is very gratifying to every one to see how completely the scheme laid out by this system works out in practise to accomplish the result aimed at.

J. C. Bailey: In comparing the installation of 1000-kw. capacity converters with 750-kw. capacity, and basing the capacity of these machines on approximately 80 per cent of the required normal substation capacity, it is of interest to note that the 740-kw. machines will operate at approximately 75 per cent of its rated load, and the 1000-kw. machine operates at approximately 50 per cent of its rated load.

The result is a loss in the over-all efficiency of 0.5 to 1 per cent in the operating expense per kilowatt hour. On this basis it is a question, due to the increased operating cost and carrying charges of the larger sized machines, whether there is an apparent saving in the distribution and general operating cost of the system.

An interesting feature on the rehabilitation of the Cincinnati Street Railway system was the adoption by the power company of the total synchronized metering scheme. The instruments on the market and available at the time were subject to variations in supply voltage and frequency, and in order to keep 19 instruments an instrument had to be selected in which the operation would not be affected by variations in supply of voltage. The totalized metering scheme was discussed and the cost was considered prohibitive, and so it was finally decided to adopt the use of the Landis and Garr mexagraph recording demand and kilowatt hour meter.

This instrument is built of two sections consisting of the standard watt hour meter element and recording power demand mechanism.

Briefly the outstanding features which may be mentioned here are: it will operate in synchronism regardless of whether the power is on or off as there is a 50-hr. reserve drive in the main spring for operating the mechanism with the potential removed from the motor. The power demand is recorded by means of a silver-tipped stylus moving over a specially prepared paper, and third, the minute adjustments provided for resetting the charts on demand periods within 15 sec.

In regard to the question of a-c. against d-c. control, I should like to state that our company has done considerable investigation work in regard to the merits of both types, and finds that the d-c. type of control for automatic substations is superior to the a-c. system. The maximum continuity of service is assured, as the control bus is not subject to the usual variation in the supply of voltage, while the station is carrying a load or coming into service with d-c. voltage condition, such as may occur under a general system disturbance.

J. A. Noertker: Colonel Schoepf questioned the allowance of 2 per cent for reserve. I have never found any place where such an item has been included, but in Cincinnati the street-railway properties are limited to a 6 per cent return on our investment because of the franchise, and I would hesitate to recommend any increase in feeder or any other scheme that would possibly show only a 6 per cent return on the investment; therefore, I included 2 per cent additional just to be sure that installing this feeder was a good investment. I did that rather than consider the thing only from the common sense point of view and put in a little smaller or a little larger feeder. It is at least definite.

Mr. Bailey raised the question of 1000-kw. *versus* 750-kw. units. A 750-kw. converter is a 1200-rev. per min. machine, and there is no question but that it is subject to higher maintenance than the 1000-kw. machine. In addition to that the places in which we installed the 1000-kw. machines were outlying territories, and although we couldn't at this time really justify more than a 750-kw., we felt that there may be some possibilities of future growth in those locations; so we put in the 1000-kw. machines. At the present time we are undoubtedly standing a little greater loss than if we had applied the correct capacity, but this is probably offset by reduced maintenance.

Automatic Mercury Rectifier Substations in Chicago

BY A. M. GARRETT[1]

Associate, A. I. E. E.

Synopsis.—Two automatic mercury-vapor rectifier substations having 3000-kw. 5000-ampere, 600-volt rectifiers are described in this paper. These rectifiers are the largest installed in this country. The reasons for the selection of this type of converting equipment instead of the rotary converter are enumerated. Information is given on the auxiliary apparatus and the control arrangement.

THE placing in service recently of two mercury-vapor rectifier substations in Chicago, automatic in operation, marks the fourth of a series of rectifier installations in this vicinity. The application of approximately 10 rectifiers to steam road electrification service, as well as city traction supply, has perhaps made the Chicago District the representative locality for this new type of converter.

The decision to make the latest units automatic in operation was based on two fundamental reasons,

FIG. 1—3000-KW. MERCURY ARC RECTIFIER AT MAYPOLE AVENUE SUBSTATION

first, to determine whether the so called adaptability of the rectifier to automatic operation would be borne out by practical experience, and second, to demonstrate comparative performance with the automatic operation of the synchronous or rotating converter. Both types serve the same class of load supplying 600-volt energy to the elevated and surface lines systems of Chicago.

Because of the number of rectifiers of the iron tank type originally installed in Chicago or vicinity, the subsequent wide-spread interest in their performance, and the changes and additions in the latest units, making them more applicable to American practise, a word in explanation of the decision to use this type of unit for railway service is necessary.

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Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

Although from an operating point of view this decision is based upon a performance not equal as yet to that of the synchronous converter, improvements recently made in rectifiers, together with removal of handicap due to size, are some of the factors which lead us to believe the difference in reliability has practically disappeared.

The usual advantages generally understood to be characteristic of the rectifier are: a converting device simple in design and construction; practically no moving or wearing parts; low maintenance and inspection costs; absence of renewal expense and higher efficiencies. In addition, the advantages which are outstanding for rectifier installations when located in metropolitan areas are the use of water for cooling purposes instead

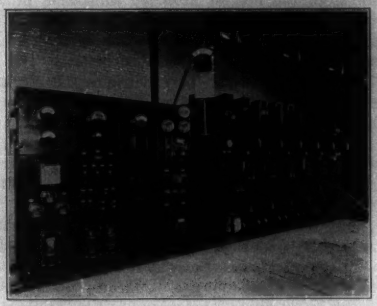

FIG. 2—CONTROL AND FEEDER SWITCHBOARD, MAYPOLE AVENUE SUBSTATION

of air, absence of noise and vibration, and the elimination of massive foundations and ventilating duct construction.

The increased number of facilities required with the installation of the modern converter of the rotating type has become a problem and a matter of concern from an operating point of view, as well as that of investment. Whether located in leased space below the street level in a commercial building, or in a substation building of the conventional type generally found in the outlying territory, the present synchronous converter must be provided with enclosures to which are

29-55

connected extensive space consuming air intake and exhaust systems, air cleaning and blower equipment, part of which is fitted in and around massive foundation work. These facilities or a modification of them must be provided for ventilation of the unit or for noise mitigation, or for both. The use of the mercury rectifier appears to be the answer to these problems which cannot be solved satisfactorily when the older type of converter is used.

The two rectifiers which are of the largest ampere size installed to date in this country, were manufactured by the American Brown-Boveri Electric Corporation. This capacity is obtained from a single cylinder arranged with 12 water cooled anodes. The rating of unit is 3000 kw. 5000 amperes, 600 volts, with 50 per cent overload capacity for two hours and 200 per cent load for one minute. Although nominally known as a 600-volt outfit, a no-load tap arrangement permits a range of voltage on the output side of the rectifier of 575 to 650 volts in steps of six full capacity taps. The rectifier which receives its a-c. energy from oil insulated self-cooled transformer is operated from the three-phase 12,000-volt, 60-cycle grounded neutral system of the Commonwealth Edison Company. The rectifier unit together with the 600-volt feeders are fully automatic; the 12,000-volt switching equipment is remote controlled from the nearest attended substation in the territory. The 12,000-volt supply to the rectifier substation is furnished in two lines arranged in a loop formation from the same attended substation.

The pump equipment for exhausting the air and gases from the cylinder is different from previous units in that two sets of pumps are used instead of one, and the mercury vapor pumps are separated from the rotary vacuum pumps, and are mounted upon the rectifier cylinder making a shorter and more effective piping arrangement with the condensing chamber.

Because the mercury rectifier is essentially a static conversion unit, devices necessary to prepare it for automatic operation are relatively few in number.

The starting of the unit is controlled by a time clock which can be set at any predetermined time to place in motion the customary control equipment commencing with the master control relay which functions in sequence to close the a-c. oil breaker, strike the ignition and excitation arcs within the cylinder, start the cooling water, and close the d-c. breaker connecting the unit to the 600-volt bus and the pick-up load that is available. Ordinarily the total time for this operation is less than one minute. To shut down the unit the master relay trips the a-c. oil breaker which through auxiliary contacts cuts off the excitation and trips the d-c. breaker. To the a-c. breaker operation is added the function of reclosing the three trials at reclosing before locking out after the third attempt.

The pumping equipment, which is used to maintain vacuum within the cylinder, operates independent of any of the other automatic functions of the unit and the control for this equipment is in operation at all times whether the rectifier is connected to the system or not, except at such times that the unit may be taken out of service for repairs or other reasons. This control consists essentially of a hot wire gage operating on the Wheatstone Bridge principle, arranged so that two of the arms of the bridge are subject to the gas pressure within the cylinder, while the other two are exposed to the pressure of the atmosphere. The difference in resistance due to difference in pressure causes a current to flow through a vacuum meter which is calibrated to indicate the vacuum directly in millimeters of mercury. This meter also carries contacts which control the action of the mercury and rotary pumps placing in operation the mercury pump only when there is a high vacuum, both mercury and rotary pumps when a medium vacuum exists, and operates the lockout relay taking the rectifier off the system when low vacuum is indicated.

The amount of water supplied to the rectifier is regulated in accordance with the load demands through the registering of the temperature of the discharge water upon a thermostatic device which in turn controls the solenoid operated water valves.

Briefly the entire installation is protected against overloads, short circuits, overheating of the rectifier cylinder, failure of the water supply, failure of the auxiliary power supply, and high-voltage surges as follows:

Overload relays of the induction type located on the high-voltage side of the transformer protect against overload and short circuit conditions. This action is selective with the d-c. breaker.

A series of continuous overloads which may occur below the setting of the overload relays and cause undue heating of the rectifier, if persisted in long enough, is guarded against by means of a thermal relay. The rectifier is further protected against overheating by a temperature relay which gives a visible warning when the temperature of the cylinder reaches 60 deg. cent. and at 65 deg. cent. locks out the substation. Protection is afforded to the equipment in case the cooling water to the vacuum pumps fails, fuses in the supply circuit to the pumps blow, or the service of the supply circuit is interrupted or fails.

Protective resistance and spark-gaps connected to the anode leads mitigate the effect of high-voltage surges.

An 8000-ampere Westinghouse carbon breaker provides the switching device for the output side of the unit, as well as protecting the rectifier against short circuit, overload, and reverse current conditions.

Further protection as to overload and short circuit conditions on the d-c. side of the rectifier including selectivity with the Westinghouse breaker is provided

by the General Electric moderate speed breakers with which each feeder is equipped. The breakers are automatic and reclosing in action.

To reduce the amount of scale forming matter which may deposit in the water passages around the cylinder due to roiled or muddy water, the cooling water is first passed through sand filters. It is expected that the amount of cleaning of the water chamber will be materially reduced.

The interior of the substation building is designed to meet the requirements of this type of conversion unit, its auxiliaries, and switching equipment. Generally the arrangement is similar to that of a rotating unit except for the absence of those features mentioned before. The exterior architecture of the building presents a structure pleasing in outline and creditable in appearance to the neighborhood.

The length of time the two units have been in service is much too short to gain any information as to operating performance. It may be said, however, that the two rectifiers were placed in service just prior to the time the heavy winter load occurs on the traction systems, and although subject to only a short seasoning period, the units have shown good performance up to the first of January, carrying at times integrated loads almost equal to the rated capacity.

Discussion

Otto Naef: About half of the rectifier substation equipments furnished by the company with which I am associated are designed for automatic operation. The combined output of those automatic stations, however, is smaller than that of the manually controlled stations, which indicates the general trend of using automatic control for a large number of small, decentralized substations and reserving manual control for the larger, more centrally located substations. Among the automatic rectifier substations I wish to mention especially one which is designed as a portable unit and has all the equipment mounted on a standard railroad car.

The two new 3000-kw. rectifier substations in Chicago are outstanding by the size of rectifier used, each tank having a nominal rating of 3000 kw. at 600 volts. This is considerably more than has been thought possible until quite recently. The Chicago rectifiers are the first ones of this large size which were put in operation in this country and I think the operating company which installed them has rendered a very important service to the art by trying them out on their important system and obtaining valuable operating data that will be of interest to other users of converting equipment.

That the engineers have confidence in these large rectifiers is indicated by the fact that up to the present time more than 45 rectifier cylinders of this size have been ordered here and abroad, of which nearly 30 are already in operation. Among the outstanding installations in this country and Canada I wish to mention a repeat order of 4 more rectifiers for the Chicago District and an electrochemical application in a large zinc-smelting plant in Western Canada, where 6 rectifiers will be installed, having an aggregate capacity of 16,800 kw. These latter rectifiers, while having equal output as the rectifiers now in operation in Chicago, have considerably less weight and a smaller diameter, a development which has been made possible by the good results obtained with those already in service.

J. A. Noertker: I should like to ask Mr. Garrett whether rectifiers can be designed so as to have a flat voltage characteristic, and if they are so designed, whether this characteristic can be changed so as to have a shunt characteristic.

On our Cincinnati system we have found it to be advantageous to have a flat voltage characteristic in our outlying territories so as to maintain a scheduled speed. At the same time there is the possibility of these stations becoming permanently overloaded, and this could be rectified to a certain extent if we could later change this to a shunt characteristic, which we intend to do.

T. H. Schoepf: I should like to know the size of the sand filter mentioned in the paper. Probably the water in Chicago has no mineral deposit in it that would cause deposit in the pipes that would go through a sand filter bed.

Also, in an outlying district where the rectifier may be out of service as much as 12 to 24 hours, what provisions do you make to prevent the water from freezing, and is the filtered water used over again?

George Swallow: I think it is remarkable that the total installed capacity of the mercury arc rectifier for railway service has increased practically 100 per cent in the last two years, and I believe it is the consensus of opinion of the operating companies that the rectifier is now well on its way toward replacing the rotating type of equipment, due mainly to the better efficiency.

I don't think Mr. Garrett mentioned in his paper just what the efficiency is, and he might be able to give us some idea of the actual gain made under regular operating conditions at 600 volts. It is evident that at 1500 volts the efficiency is so much greater that rotating equipment cannot compete with the rectifier on the efficiency basis.

There have been figures made and tests carried out on the Illinois Central system on various rectifier substations, and it has been shown that the actual gain in efficiency has been about 17 per cent on a capacity factor of about 12 per cent with 3000-kw. units.

A. M. Garrett: Mr. Naef's contribution has brought out the point that the problems in Chicago may be a little different from those in other places. We have had problems where the space has been so valuable that we could not include in some installations the ventilating devices I mentioned in the paper. We had for years the problem of commutation on the existing converters of the rotating type. Most of them were brought to a satisfactory condition. We even went so far as to look into the question of the design of the rotating unit to determine whether we couldn't thereby improve the commutation. With the advent of the first 60-cycle converters in 1916 we experienced trouble with unstable machines. They have since been improved so that the later types are better and not so susceptible to the swinging loads that accompany heavy drafts of power serving the traction systems. We have also had trouble with flashing of converter commutators, which in some instances has put the substation out of service for some considerable time.

Flashovers are reproduced in the rectifier in the form of arc backs, but in many cases the unit can be switched right back into service. After a flashover no inspection is necessary unless the vacuum has dropped, and that doesn't happen very often. The rectifier has this advantage, it can be switched back immediately but this cannot be done with the rotary converter. Generally, at least, an inspection must be made and if there is a flashed commutator the unit is sometimes out for hours.

Mr. Naef, I believe, spoke of four units now on order. These together with others will total some 18,000 kw. for service in Chicago, and will replace existing 25-cycle converters so that the supply to the load will be changed over from 25-cycle generators to 60-cycle generators, thus working toward a unified frequency of 60 cycles, standard in the Chicago district. We find that it is a rather happy combination in an old 25-cycle converter substation, to install a 60-cycle rectifier in it, using the rectifier as a base-load machine and the converters for peak purposes only.

The efficiency is not so much greater than it is with the 60-

cycle converter as far as the actual plant efficiency of the substation is concerned, but we do have a machine that operates very nicely as a base-load unit. We gain a little in efficiency and saving in energy cost.

Mr. Noertker wanted to know about the voltage characteristics. I can't say exactly as to that. I would rather leave this to some representative of the manufacturers, but I don't believe you can get an absolutely flat characteristic unless you install load-ratio control equipment connected to the transformer winding on the high side.

Regarding the matter of vacuum, if it cannot be restored and the unit must be opened there will be a considerable loss of time. In some severe cases of trouble such openings may run as high as 75 to 100 hr., especially if it is necessary to re-form. If the unit is opened only for inspection of certain parts or possibly to clean it in case the mercury has become dirty, the time out of service is not so long. It then becomes only a matter of the time actually taken to disassemble and reassemble, followed by a short period thereafter to bring the vacuum back by reforming. This is because all materials have been thoroughly seasoned. If a unit needs a new anode or a new insulator, or any other new iron material, considerable time is necessary to lay the machine out of service, take it apart, and put in the new parts, clean it thoroughly internally, and reform until the new material has seasoned properly.

Colonel Schoepf mentioned the size of the water filter. It is about 8 ft. in diameter and stands about 7 ft. high. If it is in some outlying district where the water pressure may be low, possibly a pump arrangement may be needed. In the last installation we put in a pump, but we have not used it as yet.

The possibility of water freezing in cooling passages has to be guarded against. One manufacturer, who has a unit in a substation with low load, during the cold period in the morning in the winter months provides heaters in the water passages to keep the temperature up to a safe limit. Otherwise there will be trouble in starting the arc. We experienced such trouble with one unit during the past winter.

We don't use the water over again in Chicago, but waste it. In the Public Service Company territory, at the Vollmer Road Substation, there is no city water system, and water is drawn from a well and recirculated through surface coolers cooled in turn by a fan.

Mr. Swallow mentioned gain in efficiency. We did gain some efficiency in the case of the rectifiers on the Illinois Central System. That load, however, is a different one from city traction loads, as it is highly fluctuating, and the efficiency of the rectifier is much higher at quarter load or around quarter load, so there is a gain when light loads of considerable duration are placed on the rectifier. Where there are many units in the single substation, however, approximately this same gain may be obtained with rotating converters by cutting them in and out in accordance with the load demands.

Arc Welding of Steel Buildings and Bridges

BY FRANK P. McKIBBEN[1]
Non-member

THE art of welding structural steel by electricity is developing rapidly and this year witnesses many applications of this process to construction and to reinforcement of bridges and buildings. The electric arc furnished designing engineers with a new tool which has been widely adopted not only in the fabricating of bridges and buildings, but also in many factories for welding parts of machinery and for assembling structural steel sections as substitutes for castings.

With proper supervision of design and workmanship one can secure safe construction as is evidenced by the existence of over sixty welded buildings in which no failures have been recorded. For a new type of construction this is truly a remarkable record, but in this connection, we must not forget that welded construction in its early stages has received far more experimentation, more careful supervision in execution, than did concrete or riveted work at the corresponding stages of their development.

Among the most important matters now receiving attention are: revision of building codes, preparation of specifications for welded buildings, accumulation of cost data, training of designers, the qualification of welders and inspectors, and additional tests of welded joints.

Building Codes

The recent movement to change fiber stresses in steel construction from the 16,000- to the 18,000-pound basis has resulted in the revision of various municipal building codes. Among political divisions now actively engaged in revising their codes are New York, Philadelphia, Pittsburgh, Chicago, Lansing, Schenectady, several cities in California, and the State of California, and it is hoped that in these revisions due consideration will be given to the advantages of welding and that enabling provisions will be incorporated therein.

The American Welding Society's Committee on Building Codes is now drafting a code for welding, and until its report is published, cities may either adopt an enabling ordinance without any attempt to specify details of welded steel construction, or on the other hand may draft codes as complete as is now possible to obtain, giving the technique of welding and its application to steel buildings.

An example of an enabling article of general nature is as follows:—"Welding may be substituted for or used in connection with those methods already prescribed in the building code of the City of ..... for assembling the component parts of and connecting together various beams, girders, lintels, trusses, columns, or other structural steel used in building construction; provided, that welding be done by competent workmen in accordance with designs and under supervision of a competent architect or engineer skilled in this work; and under regulations or specifications recommended by the Bureau of Building Inspection of the City of ..... and adopted by the Common Council of the City of ....."

In addition to an enabling article similar to that outlined above, a specific code should include articles pertaining to definitions, materials, apparatus, permissible unit stresses, workmanship, qualification of welders, proportion of parts, and the protection of steel.

Specifications for Arc Welding Steel Buildings

Architects and engineers are finding that specifications concerning arc welding are not easy to secure, and so many are the inquiries regarding this subject that there has been included in the Appendix of this paper a proposed set of specifications based on experiences gained with four welded steel building frames, and prepared after conferences with several prominent structural engineers and welding specialists. These specifications contain nine sections under the following headings: 1–General Application, 2–Definitions, 3–Materials, 4–Apparatus, 5–Permissible Unit Stresses, 6–Workmanship, 7–Qualification of Welders, 8–Proportion of Parts, and 9–Protection of Steel.

Cost of Welding

Judging from the welded trusses of Building No. 1 of the General Electric Company's West Philadelphia plant the cost of welded trusses should be below that of any other method of connecting truss members. In that case a very great reduction was found in weight of welded trusses as compared with riveted trusses designed to perform the same function. A 58-ft., 6-in. welded truss weighed 5000 lb. as compared with 6800 lb. for a riveted truss, a saving of 1800 lb. or 36 per cent of the welded truss. A 78-ft. welded truss weighed 9200 lb. versus 13,200 lb. for a riveted truss, a saving of 4000 lb. or 43 per cent of the welded truss. While in buildings with a large number of trusses a saving is effected by welding, I doubt if very great reductions in costs result at present from use of welding in buildings consisting entirely of beams and columns. But where a large number of plate girders is used in a structure the probabilities are that welding will materially lessen costs, especially where automatic machine welding is applicable.

However, the architect or engineer has a very good way at his command for ascertaining whether he can reduce costs with welding. All he need do is to ask for

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29-57

alternate bids between riveted steel and welded steel; a method recently adopted in several instances.

Qualifications of Welders

On important building or bridge work it is not sufficient to employ welders upon their claims of experience in welding. While their claims may be perfectly correct and made in good faith, nevertheless it is true that a welder may be capable of securing excellent results on one class of work and yet be unable to perform well the work at hand. Each man should be tested by at least one of the three methods mentioned in the specifications in the Appendix.

A tendency exists to exaggerate this matter of tests for qualification of welders. It is certainly desirable to have good welders, but over zeal in this test business is not wise because it tends to create scepticism regarding safety of welds, to unnecessarily increase the cost, to restrict welders with rules not applied to other workmen, such as riveters and mixers of concrete, and to create unnecessary municipal regulatory bureaus.

Recently, it has been suggested that all welders in a certain city be licensed by the municipal Bureau of

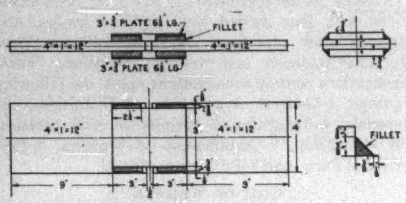

Fig. 1—Test Specimens for Fillet Welds

Building Inspection. I hope this method will not be adopted in any city. All that is needed is that the engineer or architect in charge of a welded steel building frame have on the work a competent inspector who qualifies the welders by any or all of the methods set forth above. If some municipal regulation is necessary it would seem better to have the contractor for the steel submit evidence that the welders he proposes to use on the building in question meet the qualification tests. But from my experience in this matter, I think no municipal regulation should be adopted other than those commonly applied by codes to other types of construction.

Of the test methods, the tests to determine the longitudinal shearing value, Fig. 1 (Section 7, Article 3 of the specifications), is the best but most expensive for ordinary building construction involving trusses, beams, girders, and columns. The test specimen shown in Fig. 1 is designed in such a manner that when the 10 in. of fillets reach their ultimate shearing strength the unit tension in main and splice plates will not materially exceed the elastic limit. In fairness to the welders they should be given an opportunity before making the test specimens of familiarizing themselves with the type of electrode and current density to be employed.

It is desirable to test the welders in this manner from time to time during the progress of the work. The accompanying Table I gives the results of such tests for five welders at the Trenton welding shop of the American Bridge Company during the progress of welding the trusses for West Philadelphia Building No. 1 of the General Electric Co.

Table I shows a maximum ultimate longitudinal

TABLE I

TESTS ON TEN SPECIMENS TO DETERMINE VARIATIONS IN WORKMANSHIP OF FIVE SHOP WELDERS. LONGITUDINAL SHEARING VALUES ARE A MEASURE OF COMPETENCY

Number of specimen	Welder's number	Length of fillets total inches	Ultimate load pounds	Ultimate per linear inch pound	Average per linear inch pounds
3 A	3	10	132,670	13,267	
3 B	3	10	120,700	12,070	12,669
1 A	1	10	127,850	12,785	
1 B	1	10	120,630	12,063	12,424
2 A	2	10	120,570	12,057	
2 B	2	10	113,930	11,393	11,725
4 A	4	10	115,510	11,551	
4 B	4	10	111,580	11,158	11,355
5 A	5	10	105,660	10,566	
5 B	5	10	101,370	10,137	10,352
			Average	11,705	

Experience of Welders Tested in Table I

Welder Number	Experience
1.	Welding school but no previous experience.
2.	No welding school but previous experience.
3.	Welding school and previous experience.
4.	No welding school but previous experience.
5.	Welding school but no previous experience.

shearing stress per linear inch of fillet of 13,267 lb., and a minimum of 10,137 lb., with an average of 11,705 lb. As the minimum shearing section of a ⅜-in. fillet one in. long is 0.266 sq. in., the above figures correspond to ultimate shearing stresses per square inch of minimum section as follows: maximum 49,900, minimum 37,100, average 44,000 lb. per sq. in. respectively. Moreover, when the fillets were stressed to the breaking loads the average unit tension in the two 5-in. by ½-in. splice plates as well as the main 6-in. by ⅞-in. plates were materially less than the elastic limit in each specimen. It will be noticed that the test specimens used in these tests were slightly different from those given under qualifications of welders, Fig. 1, Section 7, Article 3, of the specifications given in this paper.

Table I reveals two interesting facts, though of course it must be remembered that only ten specimens are included. First, the highest result 13,276 lb. per linear in. of the best welder is 3130 lb. or 30.8 per cent above the lowest value of the least efficient welder. The

maximum is about 14 per cent above and the minimum 14 per cent below the average value. Second, although a considerable variation exists between values of the best and poorest welders, a conspicuous similarity is shown between the two test results of each individual welder.

Inspection

As in riveted steel and in reinforced concrete construction, the conservative engineer provides inspection of welded work both in shop and field. It is the duty of the inspector to see that sizes and lengths of steel sections are correct, that welders are qualified, that the proper current and electrodes are used, that the current values are changed to suit the various sizes of fillets and thicknesses of steel, that the sizes and lengths of fillets are deposited to conform with those specified on drawings, that all parts conform to drawings as to size and position. The inspector should be required to make daily reports as to quality and quantity of work performed.

A good inspector can learn by visual inspection a good deal about a fillet. Rounded edges denote lack of penetration of fillet into parent metal, *i. e.*, into parts being welded; one short and one long side of a triangular fillet indicates that the wire electrode has been held at an incorrect angle of welding; a crater at any point in a fillet other than the end is evidence that the arc has been broken and the fillet not continuously laid; numerous gas holes on the surface of a fillet indicate too long an arc and a lack of penetration; a current, shown by the ammeter on the welding machine, too great for thin plates or too light for thick ones, is undesirable and the inspector should see that the welders adjust their machines to obtain the current suitable for the thickness of the material being welded.

In comparing the lengths of fillets actually deposited in the shop with those designated on the drawings it has been my experience that generally the total measured deposits exceed the total compiled from drawings, but now and then an individual fillet may be either greater or less than designated. For example, at the welding shop of the American Bridge Company at Trenton the total fillets on one welded truss aggregated 633⅛ linear in. as compared with 598 in. specified. And while the maximum excess of actual length of an individual deposit over that specified reached as much as 52 per cent, in two other fillets deficits of 8.8 per cent and 10 per cent respectively occurred. These deficits are not due exclusively to the welder, but sometimes may be attributed to inaccuracies on drawings where a specified length of fillet is impossible to execute.

Tests on Welded Joints

Although sufficient test data pertaining to the strength of welded joints are available to enable engineers to produce safe designs for ordinary joints there is still need for additional tests covering several types of joints which will be used more freely in buildings or bridges when sufficient design data are obtainable.

Tests may be classified under three divisions:

1. Longitudinal shear on fillets.
2. Transverse shear on fillets, involving also simultaneous tensile or compressive stresses on some planes within the fillets.
3. Tensile and compressive strengths of welds in butt joints.

1. Longitudinal shearing values of fillets have been sufficiently well established for safe design of ordinary connections, but several important problems in connection with this shear have not been studied sufficiently. These are:

a. Relation between longitudinal shearing strength and the length of fillets. Our knowledge of longitudinal fillets is as complete relatively as it is of the shearing strength of a group of fillets. But the exact distribution of the shear along the fillet has not been determined. Several investigations indicate that the strength per linear inch lessens as the fillet becomes longer. We do not know the laws of distribution of stresses over a group of rivets; *e. g.*, given a lap riveted joint with five rivets in a row parallel to the line of stress; are the stresses on the rivets equal, or does each end rivet in the row carry more stress than the central rivet, if so, how much more? And so it is with a fillet; the end inch has a different stress from the central inch but how this intensity varies we know not. If we can learn this we will know more about welds than we know about rivets.

b. Relation between longitudinal shearing strength and the size of the fillet. A square inch of weld metal in a ¼-in. fillet may have a given longitudinal shearing strength. Will the strength per square inch of a ½-in. fillet be the same? Several series of tests indicate that the smaller the fillet the greater the strength per unit of area, while others do not show this characteristic.

c. Relation between longitudinal shearing strength and the number of layers used in depositing the fillet. Here again there is conflicting evidence, because some tests seem to indicate that the unit strength for a fillet of given cross-section increases if the fillet be deposited in two layers instead of one, while other tests fail to substantiate this.

2. Transverse shear, involving also simultaneous tensile or compressive stresses on some planes, exists in a fillet such as that across the top flange of an I beam welded to the side of a column. If the top of the beam is in tension the fillet is subjected to a transverse stress which apparently varies from pure shear on the horizontal side of the fillet to pure tension on the vertical side thereof. I say apparently, because the exact location of planes of principal stresses needs study.

3. Tensile and compressive strengths of welds in

butt joints have been given a great deal of attention. The former is useful in designing joints in pipes and cylindrical tanks, while the latter is of value in planning joints such as butt splices in some compression members such as at top chord splices. Many tests are added daily to the very considerable amount of available data of this nature.

A Series of Tests to Determine the Longitudinal Shearing Strength of Fillets

In May 1927, the General Electric Company made and tested at Rensselaer Polytechnic Institute a series of 48 welded specimens to ascertain the longitudinal shearing strength of fillet welds; results are given herewith in Tables II and III, and summarized in Table IV.

These two series of tests were designed to give ultimate longitudinal shearing values of ¼-in. and ⅜-in. fillets with triangular cross-sections without stressing the main and splice plates to the elastic limit. One-half the specimens were tested in tension and one-half in compression, the latter specimens being shorter than the former. All welding was performed by one operator, who did the work by hand, with current of 200 amperes, 3/16-in. electrode, arc voltage 20.

Examination of Table IV indicates that shearing values as determined on the tensile pieces are lower than those given by the compression pieces:—for example, 10,899 and 13,681 lb. per linear inch respectively for ¼-in. fillets, the difference being 2782 lb. or 26 per cent of the tensile average; while for ⅜-in. fillets the corresponding values are 13,354 and 16,126, a difference of 2772 lb. or 21 per cent of the tensile. The results in general seem to establish no relation between unit shearing strengths and lengths of the fillets,

TABLE II
LONGITUDINAL SHEARING STRENGTH OF FILLETS
Specimens Tested in Tension

Specimen number	Fillets: Size inch	Fillets: Length each fillet inches	Fillets: Number of linear inches each side	Ultimate loads pounds: Total	Ultimate loads pounds: Per linear inch of fillet
1 T—1	3/8	3	12	173,300	14,442
—2	3/8	3	12	164,100	13,675
—3	3/8	3	12	154,400	12,867
Average				163,933	13,661
2 T—1	3/8	4	16	212,000	13,355
—2	3/8	4	16	216,100	13,506
—3	3/8	4	16	191,600	11,975
Average				206,593	12,912
3 T—1	3/8	5	20	276,580	13,829
—2	3/8	5	20	268,300	13,415
—3	3/8	5	20	262,100	13,105
Average				268,993	13,450
4 T—1	3/8	6	24	317,000	13,208
—2	3/8	6	24	316,900	13,204
—3	3/8	6	24	330,300	13,763
Average				321,400	13,392
5 T—1	1/4	3	12	139,500	11,625
—2	1/4	3	12	131,200	10,933
—3	1/4	3	12	122,300	10,192
Average				131,000	10,917
6 T—1	1/4	4	16	176,740	11,046
—2	1/4	4	16	163,420	10,214
—3	1/4	4	16	164,860	10,304
Average				168,340	10,521
7 T—1	1/4	5	20	232,300	11,615
—2	1/4	5	20	227,620	11,381
—3	1/4	5	20	226,560	11,328
Average				228,827	11,441
8 T—1	1/4	6	24	257,720	10,738
—2	1/4	6	24	262,100	10,921
—3	1/4	6	24	251,800	10,492
Average				257,207	10,717

TABLE III
LONGITUDINAL SHEARING STRENGTH OF FILLETS
Specimens Tested in Compression

Specimen number	Fillets: Size inch	Fillets: Length each fillet inches	Fillets: Number of linear inches each side	Ultimate loads pounds: Total	Ultimate loads pounds: Per linear inch of fillet
1 C—1	3/8	3	12	185,000	15,417
—2	3/8	3	12	190,100	15,842
—3	3/8	3	12	212,300	17,692
Average				195,800	16,317
2 C—1	3/8	4	16	268,700	16,793
—2	3/8	4	16	259,600	16,225
—3	3/8	4	16	263,100	16,444
Average				263,800	16,487
3 C—1	3/8	5	20	318,300	15,915
—2	3/8	5	20	319,500	15,975
—3	3/8	5	20	310,200	15,510
Average				316,000	15,797
4 C—1	3/8	6	24	391,700	16,321
—2	3/8	6	24	370,400	15,433
—3	3/8	6	24	383,000	15,958
Average				381,700	15,904
5 C—1	1/4	3	12	180,000	15,000
—2	1/4	3	12	200,000	16,667
—3	1/4	3	12	182,000	15,167
Average				187,333	15,611
6 C—1	1/4	4	16	217,600	13,600
—2	1/4	4	16	225,700	14,106
—3	1/4	4	16	226,200	14,138
Average				223,167	13,948
7 C—1	1/4	5	20	251,700	12,585
—2	1/4	5	20	231,000	11,550
—3	1/4	5	20	250,600	12,530
Average				244,433	12,222
8 C—1	1/4	6	24	302,700	12,613
—2	1/4	6	24	332,900	13,871
—3	1/4	6	24	296,300	12,346
Average				310,633	12,943

TABLE IV
SUMMARY OF TABLES II AND III GIVING ULTIMATE LONGITUDINAL SHEARING STRENGTH OF FILLETS

	Length of fillet inches	Average ultimate longitudinal shearing strength per linear inch of fillet pounds: Size of fillet 1/4 in.	Size of fillet 3/8 in.
For specimens Tested in Tension	3	10,917	13,661
	4	10,521	12,912
	5	11,441	13,450
	6	10,717	13,392
	Average	10,899	13,354
For Specimens Tested in Compression	3	15,611	16,317
	4	13,948	16,487
	5	12,222	15,797
	6	12,943	15,904
	Average	13,681	16,126

although the unit values of ¼-in. fillets tested in compressive specimens do decrease as the lengths of fillets increase, while others fail to show this trend.

Consider now the ultimate shearing strength per square inch of the ⅜-in. fillets as determined by the tensile specimens. For a ⅜-in. triangular fillet one inch in length, the net shearing area is 0.266 sq. in., which divided into 13,354 lb. (Table IV) gives a unit shearing stress of 50,200 lb. per sq. in. on the minimum section, commonly called the throat, *i. e.*, the section passing through the apex of the fillet's cross-section and perpendicular to its hypothenuse.

Fig. 2—Showing Building with 56-Arc-Welded Roof Trusses of General Electric Co. at West Philadelphia, Pa.

The longitudinal shearing value of 3000 lb. per linear inch of a ⅜-in. fillet has been quite commonly used in designing welded joints of buildings. In the light of the tests herewith presented let us examine this working unit. Considering for a moment the ultimate values for ⅜-in. fillets as determined by tensile specimens it is seen that the lowest in Table II is 11,975 lb. per linear inch for specimen 2 T—3 and that is the only value below 12,000 lb. per linear inch. The unit working stress of 3000 lb. per linear in. has, therefore, a factor of safety of four. If we consider the average of all the tests on tensile specimens, 13,354 lb., Table IV, the 3000 gives a factor of safety of 4.4. Obviously, the factor of safety for the shearing values based on compression tests is much greater than that just determined.

This commonly accepted safe permissible value of 3000 lb. longitudinal shearing strength per linear inch for ⅜-in. fillets, divided by 0.266 (the throat distance) gives 11,300 shearing stress in pounds per square inch.

Three Buildings in Which Arc Welding was Used

During the past year the General Electric Company has completed a building in West Philadelphia, Pa., one in Bridgeport, Conn., and one in Pittsfield, Mass. in all of which the steel is connected principally by arc welding.

Welded Trusses in West Philadelphia Building

The West Philadelphia building, Fig. 2, consists of a head house 78 ft. wide by 171 ft. long with vertical clearance of 43 ft. below bottom chords of trusses, of one aisle 59 ft. by 474 ft., and of a second aisle 79 ft. by 474 ft. These aisles and the head house comprise a building approximately 138 ft. by 552 ft. Supporting the roof over these various parts are trusses of the Pratt type with horizontal chords in the two main aisles, and with a sloping top chord in head house. Each chord is an 8-in. H section with web horizontal, while each diagonal consists generally of two small channels with their backs lying in contact with, and on the outside of the vertical flanges of the H chords to which the channels are welded by fillet welds. Ordinarily each vertical member of each truss is a 7-in. I beam which fits into the trough of top and of bottom chords, to the vertical flanges of which it is welded. These direct connections of web members to chords obviated the use of gusset plates at truss joints.

In any building consisting of a main aisle and a transept, especially where there is a traveling crane in the latter, the truss at the intersection of transept and head house is somewhat difficult to arrange. In this building a truss 17½ ft. deep spans the main aisle, supports a portion of the roof load, the clearstory, a roof truss in the head house, and a head house runway girder spanning the 59-ft. aisle. On account of its depth this truss was shipped in parts, assembled lying flat on the ground, welded and erected. All other trusses were completely welded in the shop, and after erection with sufficient bolts to hold them in position were welded to the supporting columns.

Fig. 3—Details of Column Anchors

This building has in addition to ten-ton bridge cranes in head house and in the 59-ft. main aisle, several smaller bridge and wall cranes.

Welded Trusses at Bridgeport, Conn.

The Bridgeport building code is a flexible one in that it authorizes the Building Commission to permit the use of new materials if after investigation the Commission is satisfied that the new type is safe and satisfactory. After a study of tests data of welded joints the Commission granted a permit for the General Electric Company's welded steel building which connects two existing buildings.

Fig. 4—Welded Trusses Erected in Position between other Buildings

This is a one story structure with roof trusses of 62 ft., span supported on side wall columns. The interesting details here are the use of T-shaped top and bottom chords, the truss end bearings at the tops of columns and the splice at center of top chord. By cutting the web of an H section along its center thus forming two T's for chords to which web members are welded directly, all gusset plates were eliminated. Beams can be cut in this manner either by cutting with the oxy-acetylene flame or by punching. At the bearing of vertical web of this top chord T on the base plate which in turn rests on the column cap, stiffener plates are shop welded to the T-web.

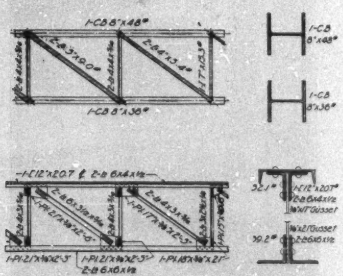

Fig. 5—Welded and Riveted Typical Panels and Chord Sections Pratt Type Truss

As the roof of this building slopes downward from the center line towards the ends of the truss the top chord is given a similar contour, thus necessitating a splice in the top chord at center. This is accomplished by a butt splice with a single V in the horizontal flanges of the T, while the vertical web is spliced on each side by a leg of a web member connected by fillet welds. As the roof trusses were fabricated completely in the shop by welding, the field welding consisted principally of connecting the top and bottom chords to the columns.

Welded Factory Building at Pittsfield, Mass.

This structure is two stories high in part and three stories high elsewhere; length 280 ft.; width overall 60 ft., with approximately 40 ft. between centers of outside wall columns. The interest here lies entirely in the structural details, among which may be included the following:

a. Connection of a 27-in. main floor girder to the side wall column by a shop welded angle bracket upon which the girder rests and by field welding the girder web to face of the column as well as field welding the top and bottom flanges to the face of the column. The ends of these 27-in. girders were milled.

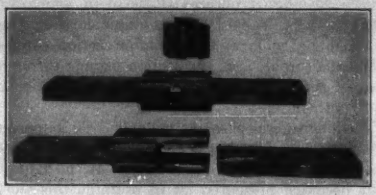

Fig. 6—Tests on Welded Structural Steel Showing Compression and Tension Specimens after Failure

b. Connection of main 18-in. floor beams to web of 27-in. main floor girders by resting the beams on heavy angle brackets shop welded to girder webs. These seat angles were designed to take the loads to which they were subjected.

c. The use of flat bars as stiffeners welded to the vertical webs at the center of 27-in. main floor girders where these rest upon short columns extending from foundation to second floor level only.

All shop and field welding on the above buildings was performed by single operator motor driven generator sets. For depositing ⅜-in. fillets on metal ⅜ in. or more in thickness, the current was about 175 amperes; 3/16-in. electrodes; arc voltage 20; with smaller current values for smaller fillets on thinner steel.

Appendix

Proposed Specifications for Arc Welding Steel Buildings

Section 1. General Application

1–1. Arc welding may be substituted for or used in connection with rivets or bolts, or other methods prescribed in the Standard Specifications for Structural Steel for Buildings adopted by the American Institute of Steel Construction as amended to date, for connecting together, or assembling the component parts of beams, girders, lintels, trusses, columns, or other structural steel used in building construction.

Section 2. Definitions

2–1. *Arc Welding.* The process of joining steel parts in the molten, or molten and vapor states by the electric arc without the application of mechanical pressure or blows.

2–2. *Arc Weld.* A weld in which welding heat is generated by an electric arc formed between the base metal and the electrode, or between two electrodes, with or without the use of hydrogen or other equivalent gas.

2–3. *Electrode.* A specially prepared metal wire or rod used as a terminal in an electric circuit to produce an intense heat by means of the arc, and which may or may not supply the molten steel required for joining the parts to be welded.

2–4. *Filler Metal.* Metal which is added to a weld by the melting of an electrode or a welding rod.

2–5. *Weld Metal.* Material composing a weld.

2–6. *Base Metal.* Material to be welded.

2–7. *Fillet Weld.* A weld, at the intersection of two surfaces approximately at right angles to each other in which the weld metal has a triangular cross-section, its two sides being fused with the original surfaces of the base metals, and its third side being exposed. The size of a fillet weld is expressed in terms of its length and width, the length being the effective length exclusive of crater, the width being the length of the side of the largest isosceles triangle contained in the cross-section of the weld metal.

2–8. *Butt Weld.* A weld joining the edges of a butt joint in which two members are brought together edge to edge but not necessarily in contact, one surface of each member being in the same or parallel planes.

2–9. *Continuous Weld.* A weld of unbroken continuity used for strength or tightness or both.

2–10. *Intermittent Weld.* A strength weld consisting of a series of short welds in line, alternating with unwelded spaces.

2–11. *Tack Weld.* A short weld used not for strength, but for purposes of assembling only.

Section 3. Quality of Materials

3–1. *Structural Steel.* Steel used in structures covered by these specifications shall be of the grade designed as "Structural Steel for Buildings" as specified by American Society for Testing Materials, Serial No. A9 as amended to date.

3–2. *Electrodes and Welding Rods.* Wire used for electrodes or welding rods shall be commercial mild steel wire made for this purpose, of uniform homogeneous physical structure; free from irregularities in surface, from hardness, segregation, foreign matter, oxides, pipes, seams, or other defects.

3–3. Diameter shall not vary more than 0.003 in. above or below nominal values.

3–4. In the hands of an experienced welder, electrodes and welding rods shall show good welding qualities in flat, vertical, and overhead positions, and shall pass through the welding process without any unusual characteristics.

3–5. Chemical compositions shall conform to the following limits for uses specified:

	Wire grade A	Wire grade B
Carbon	0.13 to 0.18%	Not over 0.10%
Manganese	0.40 to 0.60%	0.25 to 0.45%
Phosphorus	Not over 0.045%	Not over 0.045%
Sulphur	Not over 0.045%	Not over 0.045%
Silicon	Not over 0.06%	Trace

Wire of Grade A shall be used for hand arc welding, and grade B for automatic machine arc welding.

Section 4. Apparatus

4–1. Arc welding apparatus shall be of standard designs and products of reputable manufacturers.

4–2. Welding current shall preferably be direct current supplied by generators driven by electric motors or engines having suitable automatic speed regulation. Whether for use in hand or machine welding, generators shall be adjustable for different current values and shall deliver constant current at suitable voltage for each setting of the regulator.

Section 5. Permissible Unit Stresses

5–1. Welded joints shall be proportional so that combined dead and live loads, and impact, if any, shall not cause the stresses therein to exceed the following amounts in *pounds per square inch*:

Shear on minimum section of weld metal	11,300
Tension on minimum section of weld metal	13,000
Compression on minimum section of weld metal	15,000

5–2. Maximum fiber stresses due to bending shall not exceed the values prescribed above for tension and compression respectively.

5–3. Stresses in welded joints due to wind only, and combined stresses due to wind and other loadings, may exceed by 33⅓ per cent the values prescribed above; provided, the section thus obtained is not less than that obtained if the wind force be neglected.

5–4. In designing welded connections allowance shall be made for bending stresses due to eccentricity, if any, except that in designing fillet welds no allowance

need be made for the bending couple formed by resultant stresses on the two fused sides of the weld.

Section 6. Workmanship

6–1. Surfaces to be welded shall be reasonably cleaned, by wire brushing, chipping or hammering, of loose scale, rust, paint, or other foreign matter; except that a thin coat of linseed oil need not be removed before welding.

6–2. Welding operators shall submit evidence of experience in welding structural steel with the kind of welding to be used on the proposed work, and show ability to make flat, vertical, and overhead welds of uniform quality conforming to requirements of these specifications.

6–3. After being deposited welds shall be brushed with wire brushes, and shall show uniform sections, smoothness of weld metal, feather edges without overlaps, freedom from porosity and clinkers. Visual inspection at edges and ends of fillets and butt joint welds shall indicate good fusion with and penetration into base metals.

6–4. Arcs for electric welding shall be of proper lengths to produce good welds.

6–5. *Current Values.* The following figures are approximations only, intended as a guide to the operator who shall select the size of electrode and adjust the electric current to produce good homogeneous welds of above characteristics:

Electrode diameter in inches	Amperes hand welding	Corresponding plate thickness in inches
1/16	50–75	Up to 3/16
3/32	75–100	Up to 1/4
1/8	100–150	1/8 and over
5/32	150–200	1/4 and over
3/16	175–225	3/8 and over
1/4	200–250	3/8 and over

The same size electrode may be used with various thicknesses of plate, but thicker plates require heavier currents.

6–6. Current values for machine welding may exceed those specified above for hand welding but machine welding shall if required meet qualifications for welders hereinafter specified under Section 7.

6–7. In assembling and during welding, the component parts of a built-up member shall be held by sufficient clamps or other adequate means to keep parts straight and in close contact.

6–8. In welding, precautions shall be taken to minimize "locked-up" stresses, and distortion due to heat.

6–9. Before welding new steel to old, both shall be free from paint and other foreign matter, except that a thin coat of linseed oil need not be removed.

Section 7. Qualifications of Welders

7–1. *Butt Welds.* Welders may be required to weld four sample butt joints each made of two pieces ½ in. by 9 in. by 12 in. to form a piece about 12 in. by 18 in., prepared with double V described hereinafter; of these, two sample plates to be welded in the flat position and two in the vertical position with the joint located vertically. Each sample plate shall be machined to reduce the joint to the thickness of the base metal. From each sample plate, standard two in. tensile test specimens shall be made and tested in tension. The average tensile strength of each group of two sample plates shall not be less than 45,000 lb. per sq. in., and the tensile strength of the lowest in each group shall not be less than 40,000 lb. per sq. in.

7–2. *Lap Welds.* Welders may be required to weld four sample lap joints each made of ½-in. by 6-in. by 8-in. plates clamped one on the other with the 8 in. edges lined up but with the 6-in. edges offset ½ in. A full ½-in. fillet is then to be made along one 6-in. edge in accordance with specifications hereinafter described for lap joints. Upon completion of the welds and after cooling, the specimens shall be torn apart and broken by wedging at the unwelded 6-in. edges; and the fractured metal of specimen broken through the weld shall reasonably show:—bright, dense, even textured, crystalline or fibrous weld metal, irregularly torn and void of irredescent colors; good fusion of weld and base metals; good penetration into right angle corner of the fillet.

7–3. *Fillet Welds.* Welders may be required to weld three sample test specimens of size shown herewith by Fig. 1; each specimen consisting of two main plates 4 in. by 1 in. by 12 in. placed in line with their inner ends separated by ½ in., making the over-all length 24½ in. These places are to be connected by two splice plates, each 3 in. by ¾ in. by 6½ in. long, symmetrically clamped to the main plates, one above and the other below the two main plates, and each welded in a flat position to each main plate by two 2½ in. by ⅜ in. triangular fillets with their craters filled; each of these fillets is to be started at outer ends of the 3 in. by ¾ in. by 6½ in. splice plate and continue 2½ in. along the 6½ in. dimension towards the center, thus leaving a 1½ in. space between the inner ends of the fillets; the specimen is then to be inverted and the four remaining fillets deposited also in the flat position. In depositing the weld metal, craters shall be filled to full cross-section of fillets. Electrodes of 3/16 in. diameter shall be used. These specimens are to be tested in tension to ascertain the longitudinal shearing value of the ⅜-in. fillet welds; and the fractured specimens shall be examined for penetration into and fusion with base metals, freedom from gas holes or slag inclusions in fillets. The three specimens shall show at least an average ultimate longitudinal stress of 42,000 lb. per sq. in. and the lowest in the group shall show at least 38,000 lb. per sq. in. of minimum section of fillet.

Section 8. Proportion of Parts

8–1. *Plate Girders.* Girders shall be proportioned preferably by their moments of inertia but where the approximate method of flange design is used the web equivalent may be taken as 1/6 the web area, provided no holes exist in the web.

8–2. Web splices shall consist of a plate welded on each side of the web.

8–3. Stiffeners may be angles or flat bars; welded to the top and bottom flanges and also to the web by continuous or intermittent welds designed to transmit the stresses. Stiffeners at bearings or other points of concentrated loading shall be fitted to the flanges.

8–4. Connection of component parts of girder flanges to each other and flanges to webs shall be by continuous or intermittent fillet welds designed to transmit the stresses. Intermittent welds shall be not less than one and one-half inches long exclusive of crater, spaced in the clear not exceeding 16 times the thickness of thinnest piece connected, nor more than four inches in the clear.

8–5. Plug or slot welds may be used to fasten cover plates to each other and to other members.

8–6. *Beams.* Continuous beams and girders designed in accordance with generally accepted engineering principles may be used, provided that welded connections be properly proportioned to resist bending moments to which they are subjected.

8–7. Welded joints supporting non-continuous beams shall be designed with due regard for secondary bending stresses induced by deflection of the beams.

8–8. When the toe of a seat angle supporting a beam is to be welded and is not truly formed, but is rounded due to worn rolls or other causes, either a preliminary fillet shall be deposited to fill the open space, after which the full fillet required in the design shall be desposited, or the rounded edge of the toe shall be sheared square to receive the full fillet required.

8–9. *Trusses.* In tension members connected by fillet welds the gross cross-sectional area shall be considered as resisting the stresses.

8–10. For tension or compression members having symmetrical cross-sections the fillets shall either be arranged symmetrically about the axis of the member, or else proper allowance shall be made for unsymmetrical distribution.

8–11. For members of unsymmetrical cross-section, like angles for example, lengths of fillets shall be determined by taking moments about the gravity axis of the member.

8–12. *Columns.* Built-up or rolled steel columns shall contain no material, except filler plates, whether in the body of the column or used as lattice bars or stay plates, less than ¼ in. in thickness.

8–13. Ends of columns shall be faced to plane surfaces at right angles to their axes. Column splices, whenever practicable, shall consist of splice plates connected to the columns with proper amounts of fillets, and if filler plates are required they shall be connected with proper amounts of fillets. When sections of columns to be spliced are such that splice plates cannot be used, connections may be formed of plates and angles or other shapes designed to distribute the stresses properly. In all cases, column splices shall be so arranged that by bolts or otherwise the various tiers of a column can be accurately alined before welding, and shall be of sufficient strength to carry erection stresses.

8–14. Fillets connecting component parts of a built-up column shall be either continuous throughout the length of the column, or intermittent arranged as follows: Continuous fillets extending at each end of the column for a distance equal to the least width of column; then intermittent fillets one and one-half inches long spaced not exceeding four inches in the clear for the remainder of the column's length.

8–15. Lattice bars and tie plates, where used, shall be welded with fillets to secure strengths equal to those of the rivets in lattice bars and tie plates connected by rivets as specified in Standard Specifications for Structural Steel for Buildings adopted by the American Institute of Steel Construction and as amended to date.

8–16. *Welding and Bolting.* For welded structures, connections of main members carrying live loads producing impact, and connections of members subject to alternate stresses, shall be welded.

8–17. For welded structures, component parts of all columns, girders, and trusses shall be welded with fillet welds.

8–18. For welded buildings over four stories high all column splices, and all connections of beams and girders to columns, or within three feet thereof, shall be welded.

8–19. Unfinished bolts may be used in the shop or field for connections of small structures used for shelters and for connections of secondary members of all structures, such as purlins, girts, door and window framing, lining up bracing, and filling-in beams in floors. In buildings two or more stories high, sufficient bolts shall be used to connect properly all members during erection.

8–20. So far as possible all joints shall be designed to eliminate overhead welding; and arranged so that the welds will be in shear or direct compression.

8–21. Tension butt welds shall not be used in main truss members, but may be used in secondary members, wind bracing connections, beam and girder connections.

8–22. To the calculated lengths of each weld ½ in. shall be added to allow for the crater.

8–23. Fillet welds of lengths less than four times their width shall not be figured as part of any connection.

8–24. *Butt Joints.* The edges of base metal of members ¼ in. or more in thickness, transmitting stress by means of butt joints, shall be beveled with either single V or double V joints, or their equivalents.

8–25. Steel for single V shall be beveled not less than 30 deg. on each edge to form an open angle of not less than 60 deg., and the joint shall be backed-up in an approved manner.

8–26. Steel for double V joints shall be beveled not less than 37½ deg. on each side of each piece to form open angles of not less than 75 deg.

8–27. In all single V and double V butt joints the beveled edges shall have initial free distances of ⅛ in. to 3/16 in., and steel ¼ in. to 7/16 in. thick shall be

welded in one layer, steel ½ in. to 11/16 in. in two layers, and ¾ in. to 1 in. in three layers.

8–28. All above joints shall be somewhat reinforced but not to exceed 20 per cent for single V nor 15 per cent for each side of double V joints.

8–29. *Lap Joints.* All lap joints in ¼ in. to 7/16 in. steel shall be fillet welded in one layer, ½ in. to 11/16 in. in two layers, and ¾ in. to 1 in. in three layers. The contour of the cross-section of fillet shall be as near as practicable to a triangle with equal sides, and with minimum side not less than the prescribed dimension of cross-section of fillet.

8–30. Weld metal deposited in two or more layers shall have each layer brushed with a wire brush or otherwise cleaned before subsequent layers are deposited.

8–31. *Drawings.* Drawings shall show clearly all welds, including the length and width of all fillets both for shop and field weldings.

Section 9. Protection of Steel

9–1. Structural steel before being welded shall not be painted. If it is to be welded only in the fabricating shop and subsequently erected by bolts or rivets it shall receive in the shop one coat of acceptable metal protection after shop welding is finished, and one coat after erection. Steel to be field welded shall receive one coat of linseed oil after shop welding is completed; and after being erected and welded it shall receive two coats of acceptable metal protection.

9–2. No paint shall be applied to steel surfaces which are to be encased in concrete.

Discussion

S. M. Hamill: I should like to know how the welder tells when he has reached the proper depth of the fillet.

F. C. Caldwell: I should like to bring up the question of reliability of the electric weld and also the question as to whether the welder can be depended upon to produce as reliable work as the old fashioned riveter.

I. S. Campbell: What difference exists in the strength of weld made by the electric arc, the oxy-acetylene, or atomic hydrogen?

R. M. Easton: I should like to ask Professor McKibben for the requirements for inspectors on structural welding. It has been the practise in most structural work that is fabricated by means of rivets, to require that certain standards, which have been developed for the testing and inspection of rivets, be observed. I don't know of any means by which we could look at a weld or test a weld and tell positively whether it is a good or a bad weld.

M. M. MacDonald: I should like to ask Professor McKibben if he could enlighten us any in reference to the welding of the bearings on worn shafting, such as armatures for instance, or on other equipment and working parts of that sort, as to just what results have been obtained with electric arc welding on cast iron as applied to perhaps motor frames or parts of that nature.

J. F. Lincoln: (communicated after adjournment) Mr. McKibben in his Appendix makes certain recommendations which are somewhat open to question.

His specification on electrodes draws rather closely the chemical content of the steel which seems rather unnecessary. Why should it be so necessary to hold the content of the rod closely when the steel that is to be welded may vary in these respects very widely? Why should carbon and manganese be held to a comparatively low point in the welding rod when the amount of each one that is in the weld will be very greatly reduced by passing through the arc? If it is necessary for the carbon in the weld to be held to less than ten points, it would seem to me to be equally necessary for the carbon in the structural steel to be held to the same point, which would mean that at least half of all structural steel would be rejected.

Concerning Mr. McKibben's recommendations as to electrode diameters, currents to be used in them, and thicknesses of plate on which they are to be applied, some question may be properly asked. It is difficult to understand why a 1/16-in., 3/32-in., or 1/8-in. electrode can find any economic use in the welding of structural steel. The 5/32-in. and 3/16-in. electrodes may find a rather limited use in overhead welding, but 1/4-in. and 5/16-in. electrode should be used in doing at least 95 per cent of all structural-steel welding as the art is now known. The currents which should be used, should be in the case of 1/4-in. electrodes from 250 to 350 amperes, and on 5/16-in. electrodes from 300 to 400 amperes, that is for the usual electrode. Special coatings have been devised which will materially increase these currents. In this way the cost of welding and the ease of welding can both be materially bettered.

F. P. McKibben: Replying to Mr. Hamill's question, the welder can secure proper penetration (depth of weld) by watching the molten crater during welding. After completion of the weld the degree of penetration is determined by observing the depth of crater at the end of the weld; the thoroughness of penetration, by inspecting the edges of the weld. A sharp feather-like edge indicates good, and a rounded edge poor penetration. If a poor fillet is found it should be cut out and rewelded, just as a poor rivet must be replaced by a good one.

Replying to Professor Caldwell's question, by qualifying welders and by controlling the technique of welding, that is, the electrodes and current density, arc welding of structural steel is as reliable as other operations in building construction and I have long since ceased to worry about the results. The absence of any failure in the 60 or 70 welded buildings which have been constructed indicates sufficient reliability of control.

Replying to Professor Campbell's question, I would use in designing a welded structure the same allowable unit stresses for oxy-acetylene as for arc welding or atomic-hydrogen welding, namely, for shear on fillets 11,300 lb. per sq. in. of minimum section; for tension, 13,000 lb. per sq. in., and for compression 15,000 lb. per sq. in., both applied to minimum sections of the weld.

Replying to Mr. Easton's question, in the shop fabrication and field erection of welded buildings with which I have been connected, inspectors have been capable welders who first qualify the welders in accordance with Section 7 of the specifications given in the paper. They then inspect the work in progress and after completion as outlined in the paper under the heading Inspection.

Arc welding has been used on nearly all the welded buildings thus far constructed.

Replying to Mr. MacDonald's questions, it is common practise throughout the country to repair worn shafting and other parts by depositing welded metal using either oxy-acetylene, arc, or atomic-hydrogen processes. The most interesting case of this I have seen recently was in an iron-ore mining region where worn flanges on heavy ore cars were being replaced by automatic arc-welding machines which deposited welded metal to replace the original flange contour. Flanges, shafting, and other worn parts are frequently repaired in this manner; often a very considerable economy results. Arc welds in cast iron may or may not be successful. Frequently the results are excellent, then again are disappointing; by preheating or by use of special electrodes success is often secured when by ordinary direct welding failures occur.

Oscillographs for Recording Transient Phenomena

BY W. A. MARRISON[1]

Associate, A. I. E. E.

Synopsis.—Oscillographs developed for recording transient phenomena are described which obtain automatically records of amplitude, wave form, frequency, duration, and time of any electrical disturbance for which they are adapted. Two instruments are described for recording very short or very long transients; these may be used in combination. At power frequencies satisfactory records may be made on film or sensitized paper with a two-watt lamp. The instruments and their performance are illustrated by photographs and oscillograms.

* * * * *

IN this paper oscillographs are described which were developed primarily for recording transient phenomena of which the time of occurrence is neither known nor subject to control. The specific apparatus described was designed primarily for recording transient inductive disturbances in communication lines from neighboring power circuits. When the design of this apparatus was begun, there was no satisfactory way for determining the duration, frequency, or wave form of such disturbances, although apparatus was available by means of which the approximate magnitude of such transients could be determined, and, by constant supervision, the time of their occurrence. It was with the idea of determining part or all of these factors automatically in a single record that the oscillographs to be described were developed.

Transients in general may be of various types. They may have components in a large range of frequencies; they may occur in a large range of amplitudes and may be very long or very short or intermittent. Attention was directed toward recording devices which would obtain records of any disturbances in excess of a predetermined magnitude regardless of the time of occurrence. For practical reasons it was necessary also to give attention to the cost of operation, the power consumed, and the amount of servicing in operation.

To meet these requirements two somewhat different types of oscillograph were developed. One is capable of making records of short duration having uniform resolution throughout. By its use the wave shape of the first half cycle of a transient is recorded as clearly as that of any subsequent wave. The other instrument makes long continuous records and may be arranged to record a disturbance of any reasonable duration. The former instrument makes records in polar coordinates on a sheet of film rotating in its plane and will be called a "polar oscillograph." The latter records in rectangular coordinates on long strips such as motion picture film and will be called a "continuous-film oscillograph."[2]

Features Common to Both Oscillographs

Since these oscillographs were designed for recording the same sort of phenomena and for operating under somewhat similar conditions they have a number of features in common.

As in most oscillographs the optical system consists of a light source, a mirror capable of being vibrated about an axis in its plane with an amplitude proportional to the signal to be recorded, and a lens system, including the mirror, to form an image of the light source on light sensitive film moved in a direction perpendicular to the plane of vibration of the mirror.

The light source consists of a concentrated filament flashlight bulb placed as close as possible to a pinhole aperture in such a way that the aperture, as viewed from the vibrator side, appears to be completely filled by the lighted filament. No condensing lens is used because of the small size of the bulb which permits the filament to be brought close to the aperture. The filament is brighter than its image and the use of a condensing lens in this instance would waste light unnecessarily by reflection and absorption, and would make the optical system larger.

The vibrator is of the moving-iron balanced armature type similar to a driving element frequently used in loud speakers. The armature is attached by means of a stiff rod to a mirror free to vibrate about an axis in its plane in such a way that, as the armature of the element vibrates, the mirror vibrates at a relatively large angular amplitude. With this type of vibrator it has been possible to employ a mirror half an inch in diameter and still retain a satisfactory frequency range and sensitivity. The large mirror makes it possible to use either less sensitive film or a less intense light source than ordinarily would be required for a given recording frequency and film speed. With a half inch mirror it has been found practicable to use a lamp requiring only about two watts for recording on par-speed film.

A plane mirror is used on the vibrator, and a single meniscus lens mounted on front of it serves, in virtue of the reflection, as a symmetrical lens in forming an image of the pinhole on the film. When greater resolution along the time axis is required than can be obtained with this simple system, a cylindrical lens with short focus is placed in the light path just in front of the film.

Each oscillograph is equipped with a camera for the purpose of photographing a clock on the oscillogram

1. Bell Telephone Laboratories, Inc., New York, N. Y.
2. This instrument is also known as the "movie oscillograph."

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

to indicate the exact time of occurrence of the disturbance recorded. Any other information it is desired to associate with the records made by a particular oscillograph may be recorded photographically along with the clock. In several cases calendar clocks have been employed indicating the day and the month, and indicating whether the time is a. m. or p. m.

A schematic diagram of the optical system of the

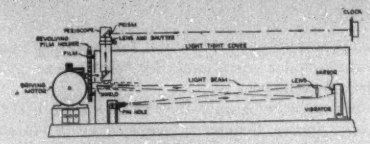

FIG. 1—ESSENTIAL ELEMENTS OF POLAR OSCILLOGRAPHS

recorder and of the camera, as used in the polar oscillograph, is shown in Fig. 1. Some of the mechanism essential for operation is omitted for the sake of clearness.

One of the calendar clocks used in conjunction with these oscillographs is shown with cover open in Fig. 2. A space is left below the clock face for a card on which may be written identifying or other information relative

FIG. 2—CALENDAR CLOCK USED WITH OSCILLOGRAPHS, DOOR IS OPEN TO SHOW LAMP SOCKETS

to records that may be obtained. Lamps are mounted within the cover to illuminate the clock when necessary.

Both oscillographs are equipped with automatic devices which enable them to make records of transients for which they are intended without the attention of an operator. These automatic features will be described in some detail in the following discussion of the individual oscillographs.

One part, however, a high-speed "line-relay," is common to both. It consists of a pair of high-speed polar relays, the windings of which may be connected into a line in such a way that, depending on the polarity, one or the other will be operated by any pulse of sufficient magnitude. The relays may be biased so that they operate only on pulses in excess of any given magnitude. They are connected so that when they do operate they remain operated until reset by some external means. Contacts on both relays are connected in parallel to the apparatus to be controlled so that a single positive or negative pulse will put that apparatus in operation. The time elapsed between the arrival of a pulse and the closing of the operating contact is

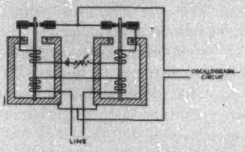

FIG. 3—CIRCUIT OF HIGH SPEED LINE RELAY

less than 0.01 second. A schematic diagram of the line relay is shown in Fig. 3.

MAIN FEATURES OF POLAR OSCILLOGRAPH

A polar oscillograph is shown in Fig. 4 with the light-tight cover removed to show the optical system. The film is held in a standard film holder in a rotating

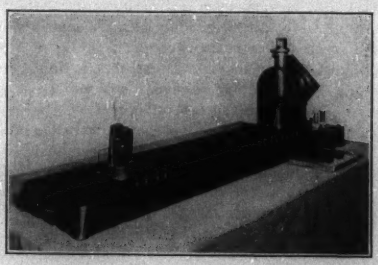

FIG. 4—POLAR OSCILLOGRAPH, SHOWING FILM ROTOR, PERISCOPE, LAMP HOUSING, AND VIBRATOR

member at the extreme right of the picture. The use of standard film holders facilitates loading in daylight as in an ordinary camera. The film is rotated by a small motor geared to the rotating member. The rotating member is separated from the remainder of the oscillograph by a circular light trap which permits free rotation while shielding the film from external light. The circular light trap used is illustrated in Fig. 1 and in Fig. 5. With this arrangement films may be exposed for days at a time under ordinary light conditions without appreciable fogging.

The flashlight lamp is housed in the small light-tight

box on the base just to the left of the film rotor. Excessive scattering of the light is prevented by a small tube, with a diaphram near the end, directed toward the vibrator mirror. The vibrator and mirror and the lens of the optical system are mounted on the base near the end opposite from the film mounting.

The chief value of this oscillograph lies in its ability to record with good resolution from the very beginning of a transient, regardless of the time at which it occurs, and regardless of the angular position of the film at which it begins. To accomplish this, the lamp is lighted continuously during the time a transient is expected and a narrow shield is placed in the light path of just sufficient width to prevent light from reaching the film when the vibrator is at rest. In this way fogging of the film is prevented during the time when no current is flowing into the vibrator but a record is made of any disturbance of sufficient magnitude to move the spot off the shield. A record made

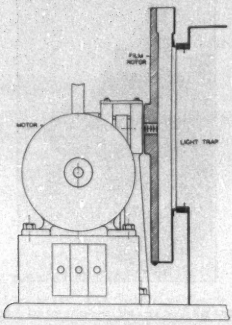

FIG. 5—SCALE DRAWING OF ROTATING LIGHT TRAP

in this way appears like an ordinary oscillogram except that a narrow, clear space is left where a zero line is usually obtained. This is illustrated in Fig. 6A.

In the absence of a shield the film soon becomes so badly fogged that an oscillogram made upon it is useless. Fig. 6B shows the fogging obtained with an exposure of one minute on the zero line without a shield.

If it is desired to record only disturbances in excess of a certain magnitude, the width of the shield may be increased so that no exposure occurs until the vibrator moves at more than a predetermined amplitude. In Fig. 7 a shield for this purpose is shown which may be adjusted to cover a portion in the center of the record from the width of the spot to about half an inch, or removed from the field entirely. As may be seen from the illustration this is accomplished by moving the shield in guides about the axis of film rotation.

There is a disadvantage in using a very wide shield of the type described. If, for example, a disturbance

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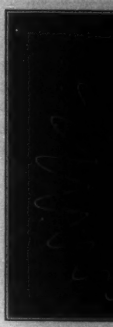

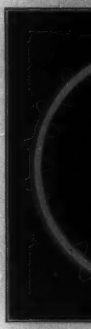

FIG. 6—OSCILLOGRA
OF A LIGHT

speed line relay, w
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great enough to be recorded, the wave is hidden by the shield and iced from the record are the peak y, and time. This difficulty can however, by attaching a shield to electromagnet so that it can be light path when the magnet is gnet may be operated by the high

A

B

ms Illustrating the Use and Omission Shield over the Zero Line

hich is adjusted to operate when the s a certain amount, in this case the moves the light spot at an amplitude dth of the shield. The oscillogram as made with a removable shield of adow of the shield is indicated in the ut does not appear during the re- illogram. For convenience in inter- a zero line is automatically recorded the recording of the oscillogram.

In order to reduce fogging due to stray light a large shield is placed between the film and the light source, having a vertical slit just large enough to allow the vibrating beam of light from the mirror to pass through to the film. This device reduces the fogging due to stray light to a small fraction of what otherwise would be obtained. The vertical slit can be seen in Fig. 7

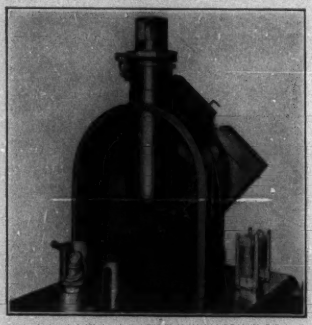


Fig. 7—Shields to Prevent Fogging on Polar Oscillograph

behind the variable shield. A slight corona-like fogging on either side of the circular shadow of the shield, as shown in Fig. 8, is obtained after a few hours of exposure. In practise a film may be exposed for

Fig. 8—Oscillogram Illustrating the Use of Removable Shield

twenty-four hours or more without the fogging becoming so serious as to obscure a record.

To avoid confusion due to overlapping records a device is used to stop recording after one complete revolution of the film. It is put in operation at the beginning of a transient by the high-speed line-relay and allows recording to continue for one complete revolution, regardless of the angular position of the film at which it begins.

This device is shown on the oscillograph in Fig. 9. A vertical shaft driven at half the speed of the film carries a magnetic clutch fastened rigidly to it and a commutator which idles on the shaft except when engaged by the clutch. This commutator has one insulating segment and one conducting segment, each of angle about 180 deg. Contacts, controlling the current to a relay winding, are normally on the insulating segment of this commutator, but when the high-speed line-relay operates, the magnetic clutch is energized and the commutator is rotated until the contacts touch the conducting segment, thus operating the relay. One contact on the relay automatically releases the magnetic clutch, preventing further rotation of the commutator. Another contact interrupts the current going to the light source. Since the filament of the

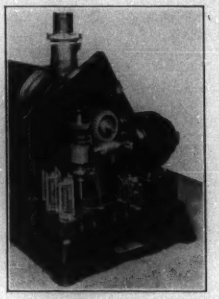


Fig. 9—Device on Polar Oscillograph to Limit Recording Time

lamp is small the light is extinguished in a very short time and, of course, recording is stopped immediately.

After the exposed film has been replaced, and it is desired to put the oscillograph in operation again, the clutch and commutator are restored to their original condition by operating a key which energizes the clutch magnet and releases it again after one-half revolution of the vertical shaft.

If it is desired to make a record covering more or less than one revolution it can be arranged simply by changing the gear ratio between the film shaft and the vertical commutator shaft. The film speed may be changed either by changing the gear ratio between the motor and the film driving shaft or by varying the speed of the motor.

The camera for photographing a clock is shown in Fig. 4. It consists of a lens and shutter, shown at the top, and a periscope consisting of two right angled glass prisms mounted in the vertical tube. The periscope places the image in the center of the film and since there

are two reflections the image will be the same as if none were used. The shutter is equipped with an automatic release which is operated a definite time interval after the beginning of an oscillogram, the time being determined by means of a slow acting relay or by a sequence switch. It is desirable to stop the film from rotating before photographing the clock which may be

Fig. 10—Continuous-Film Oscillograph With Covers Removed

done by means of the same relay that turns off the lamp, or by means of a sequence switch.

Main Features of Continuous-Film Oscillograph

A picture of the continuous-film oscillograph is shown in Fig. 10. It differs from the polar oscillograph mainly in the form in which records are obtained. As pre-

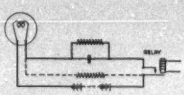


Fig. 11—Circuit for Lighting Lamp Quickly

viously stated, records are made in rectangular coordinates on a strip of film and may, therefore, be of any length depending only on the length of film available and on the size of the storage magazines. The oscillograph shown makes records on motion picture film or sensitized paper of the same width. The film is stored in standard magazines for motion picture film holding up to 200 ft. It is advanced by means of a motion picture sprocket driven through gears and a magnetic clutch from a variable speed motor. The optical system is practically identical with that used on the polar oscillograph.

With an oscillograph of this type it is not practicable to allow the film to be moving all the time on account of waste of film and the maintenance difficulties. In order to avoid this and still make it possible to begin recording soon after the beginning of a disturbance, the motor and associated gears are left running the whole time during which a transient may be expected, and, when a disturbance occurs, a quick acting magnetic clutch engages the film driving shaft with the motor which puts the film in motion very quickly. The line relay lights the oscillograph lamp at the same time. The whole recording mechanism may be put in operation within 0.02 sec., thus insuring a good record of any but a very short transient.

Normally the lamp would require several hundredths of a second to become lighted to full brilliancy if operated at normal voltage. However, with a voltage several times normal and by the use of the circuit shown in Fig. 11, it is possible to bring it to full brilliancy within 0.01 of a second without danger to the lamp. When the circuit is closed by the relay the condenser is charged suddenly to the applied voltage, the charging current passing through the lamp filament. This current, at the outset, is several times the normal current for the lamp and brings it to full brilliancy quickly. The resistance shunting the condenser has

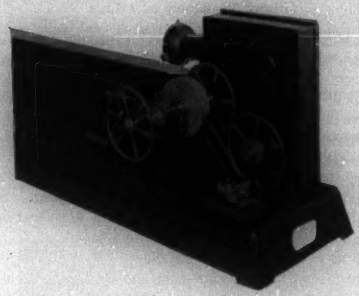


Fig. 12—Quick Acting Magnetic Clutch on Continuous-Film Oscillograph

such a value that normal current flows through the lamp filament in the steady state, so, once lighted, the lamp remains at normal brilliancy as long as the circuit is closed. The lamp may be lighted in even less time if a small current is left flowing through the fila-

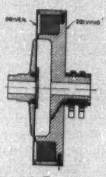


Fig. 13—Sectional Scale Drawing of Magnetic Clutch

ment continuously in order to keep it hot but not hot enough to be luminous. A resistance of suitable value connected as indicated by dotted lines in Fig. 11 will accomplish this result.

The magnetic clutch, while especially designed to operate quickly, accelerates the sprocket and film without shock in order to avoid danger of tearing the film and to reduce wear and tear on the mechanism.

The clutch is shown mounted on the oscillograph in Fig. 12. It is also shown diagrammatically in Fig. 13 to illustrate its construction and operation. The annular coil in the driving member is connected, through slip-rings and contacts on the high-speed relay, to a battery. When current flows in this coil a steel diaphram on the driven member is drawn against

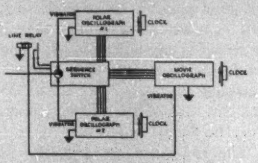


FIG. 14—ARRANGEMENT OF OSCILLOGRAPHS FOR RECORDING ANY TRANSIENT IN A LINE

Two polar oscillographs and one continuous-film with control equipment and clocks are arranged so that a transient of any duration occurring at any time will be recorded with a record of the time of occurrence.

the annular electromagnet, traction being obtained at the outer edge of the diaphram. Due to the small clearance between the diaphram and the electromagnet the diaphram is drawn into contact very quickly, and due to the small moment of inertia of the driven member it is rapidly accelerated to maximum speed. Since this is a friction type of clutch the acceleration is gradual and does not submit the sprocket and film to shock as would a toothed clutch.

The delay in recording after the beginning of a disturbance depends on the time of operation of the high-

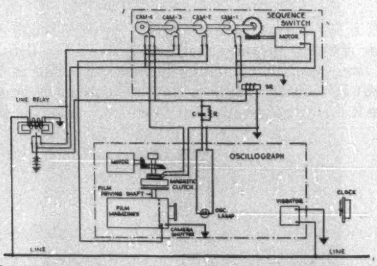


FIG. 15—SCHEMATIC ARRANGEMENT OF CONTINUOUS-FILM OSCILLOGRAPH WITH SEQUENCE SWITCH, LINE RELAY, AND CLOCK, ARRANGED FOR AUTOMATIC OPERATION

speed relay plus that of either the clutch or the lamp, whichever is the longer. The relay requires only a few thousandths of a second to operate and both the clutch and the lamp may be adjusted to operate in less than one hundredth of a second. With this apparatus, therefore, it is possible to record all of a disturbance except that part which occurs during about the first 0.02 of a second. If desired, the lamp can be arranged to operate in considerably less time than the clutch, in which case the first part of the record will not be resolved but will indicate the amplitude of the disturbance which is frequently the most desired information. In the case of 25-cycle or 60-cycle disturbances the maximum of even the first half cycle may be recorded in this manner.

The film driving mechanism is arranged so that the film may be advanced at any speed in a wide range, from a few inches per minute to about a foot per second. This is accomplished by means of a set of change gears and by changing the speed of the driving motor, or by both in combination.

As with the polar oscillograph, a camera is included for the purpose of recording the time automatically on the oscillograph film. This camera may be seen in Fig. 10. It is the same as that on the polar oscillograph with the exception that only one prism is used. This has the advantage, when recording is done on paper, that the image obtained through a lens and a single reflection is not reversed. The shutter is equipped with an automatic release that can be associated with slow acting relays or a sequence switch

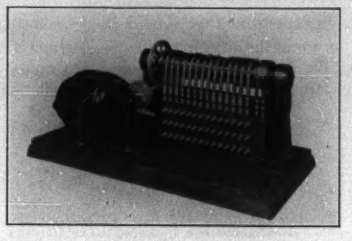


FIG. 16—SEQUENCE SWITCH

to take care of photographing the clock on the right portion of the film.

OPERATION

There are a great many ways in which the oscillographs described above may be used. An arrangement is described in which two polar oscillographs and one continuous-film oscillograph have been used in conjunction for studying transients which are likely to occur at any time during long continuous periods. It was desired to determine the magnitude with considerable accuracy and at the same time to determine the time of occurrence, duration, frequency, and wave form.

The arrangement of oscillographs is shown diagrammatically in Fig. 14. One of two polar oscillographs is connected in the circuit being investigated so that it is in condition to record the first part of any transient that should occur. A high-speed line relay associated with it is arranged to put in motion the sequence switch which takes care of a number of operations consisting chiefly in starting the continuous-film oscillograph, in substituting the second polar oscillo-

graph for the first after a certain small time interval, and in operating the camera shutters at the proper times.

Used in this way the polar oscillograph obtains a record having considerable resolution of the first part of a transient while the other oscillograph obtains a record of the complete transient with the exception of the first few cycles which are, however, obtained on

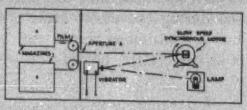


FIG. 17—SCHEMATIC DRAWING OF SAMPLING OSCILLOGRAPH

This oscillograph records one wave out of a number at regular intervals, say one cycle in sixty, with considerable resolution in order to record slow variations in wave form.

the polar machine. The sequence switch is adjusted so that the continuous-film oscillograph is put in operation before the polar machine stops recording so that, with the two records, complete information of the disturbance may be obtained. Two polar oscillographs are used in this way so that, in case two transients occur close together, a record of one of them will not be lost during the time required for reloading. The sequence switch connects the line to the spare polar oscillograph automatically and gives the operator ample time to make any necessary adjustments on the remaining machine. The arrangement is perfectly symmetrical and either polar oscillograph may become the spare.

An example of the record of a transient obtained with the two types of oscillograph, used in conjunction, is shown in Fig. 18. The polar oscillogram is similar to that shown in Fig. 8. It is obvious that the continuous oscillograph began recording about five cycles after the beginning of the transient while, of course, the polar oscillograph began recording immediately. The long record, however, continues 25 or 30 cycles beyond the end of the polar record and shows the manner in which the transient ended. Space does not permit of showing the clock that was photographed on this record.

When certain factors are known about the disturbances to be recorded the arrangement may be somewhat simplified. If, for example, it is known that any transient to be recorded will be of very short duration, the continuous-film oscillograph need not be used. If, on the other hand, it is known that the first cycle or two of the disturbance will be of no importance in the record, the polar oscillograph may be dispensed with.

The continuous film type of oscillograph offers some decided advantages over the polar type in that a large number of records can be made at one loading. Largely because of this it is possible to make the oscillograph entirely automatic in operation, causing it to record, without any attention whatever, all the transients in a circuit as they occur, until the supply of film is exhausted. Such an oscillograph may be left permanently connected into a circuit in which transients are expected, and at the end of any period the film that has been advanced into the "exposed" magazine will show on development records of the magnitude, frequency, and wave form of the disturbances and of the time of occurrence of each.

For automatic operation of the oscillograph a sequence switch of some sort must be used to insure that the various automatic operations are performed in the proper sequence. In Fig. 15 a schematic drawing illustrates the essential elements of such an arrangement. When standing by, ready to record a transient, the motor on the continuous-film oscillograph is running but does not engage the film advancing mechanism because the magnetic clutch is normally deenergized. On the arrival of a transient the sequence of operations is as follows:

The line relay operates and locks in operated position in virtue of the bias current through the outer winding being removed by the opening of the back contact. The lamp is lighted through the resistance and condenser combination $R\,C$, and at the same time, the motor is engaged with the film driving mechanism through the magnetic clutch. Since the vibrator is connected continuously to the line, recording begins immediately. Relay $S\,R$ operates at the same time

FIG. 18—SAMPLE OF RECORDS MADE BY POLAR AND CONTINUOUS-FILM OSCILLOGRAPHS USED TOGETHER

that the clutch is energized and starts the motor of the sequence switch which rotates the cams in the direction of the arrow. After a predetermined amount of film has been advanced, cam 3 rebiases the line relay and restores it to the original non-operated condition, (unless the disturbances on the line continue beyond this time). This releases relay $S\,R$ but the cams con-

tinue to turn since the contact operated by cam 1 is in parallel with that on the relay, which allows the motor to run during one complete revolution of that cam. When the line relay is restored the film stops and the lamp is disconnected. After cam 3 stops the recording and resets the line relay, cam 2 operates the camera shutter which photographs the clock on the film at the end of the oscillogram. As the cam shaft

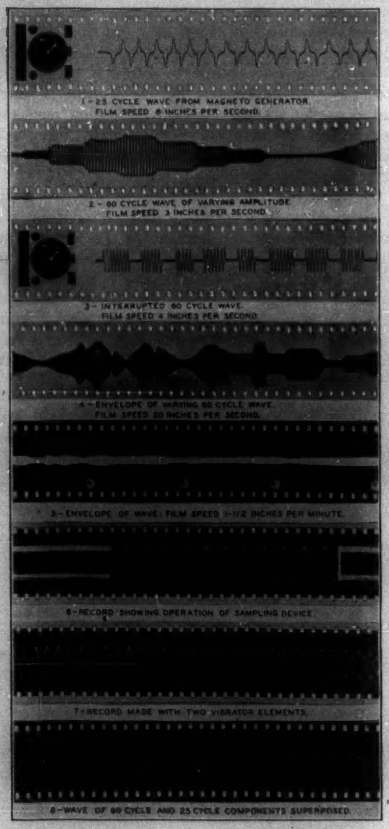


FIG. 19—SAMPLES OF CONTINUOUS-FILM OSCILLOGRAMS

continues to revolve, cam 4 operates the magnetic clutch for a short time without lighting the lamp. This advances the film far enough so that the beginning of the next record will not be superposed on the image of the clock. When the cam shaft has completed one revolution, cam 1 stops the sequence switch motor by opening the contact associated with it.

After this sequence of operations everything is exactly the same as before except that a certain length of film has been advanced into the used-film magazine and a complete record made on it. The length of the record may be adjusted by an adjustment of cam 3.

In case the disturbance lasts until after the amount of film allotted to each record has been used, the line relay will remain operated and the film will continue to be exposed during one whole cycle of the sequence switch. Thus a continuous record can be made of a disturbance, however long, provided there is sufficient film.

Fig. 16 shows a sequence switch that has been used in the arrangements of both Fig. 14 and Fig. 15. With it a great many arrangements of automatically controlled equipment may be set up besides those described, permitting the oscillographs to be used in a great number of ways.

A modification of the continuous-film oscillograph which appears to have some novel and useful features is shown in Fig. 17. It is adapted especially for sampling a wave at regular short intervals instead of making a continuous record or merely a record of unusual disturbances. It involves, in addition to the usual optical system and means for advancing the film, an additional mirror in the light path between the vibrator and the film, rotating synchronously with the current or voltage to be recorded, about an axis perpendicular to both the direction of motion of the film and the axis of the vibrator mirror. The function of the rotating mirror is to sweep the light beam along the oscillograph film past an aperture shown at *A* in such a way that the effective film speed during exposures is many times the actual film speed, and to permit of exposure during only a small part of the total time.

As an example, suppose that the mirror makes one revolution in two seconds and that the wave to be recorded has a frequency of 60 cycles per second. If the distance of the rotating mirror from the film is approximately 8.5 in., one cycle of the wave recorded will be spread over one inch of film. If a rotating mirror with a single facet is used, and if the aperture is just one inch wide, the actual film speed should be one inch in two seconds and every one hundred and twentieth wave will be recorded. If two facets 180 deg. apart are used on the rotating mirror, and if the film speed is doubled, one wave in every 60 will be recorded.

Individual facets on the revolving mirror may be inclined to the axis of rotation in order that successive exposures may be made from different vibrator elements. For example, three facets suitably mounted could be employed to show in succession sample waves of current from the three circuits in a three-phase line, the motor driving the mirror being operated synchronously in phase with the three-phase voltage.

The advantage in this recording method lies in the ability to obtain a good record of slow changes with

good resolution and without the use of a large amount of film.

Another method of sampling which gives a somewhat different kind of information may be used with a continuous-film oscillograph with the usual form of optical system. The film is run at very slow speed in order to obtain normally an envelope of the wave. At intervals the speed of the film is increased to a value sufficient to resolve the wave and show the actual wave shape. A

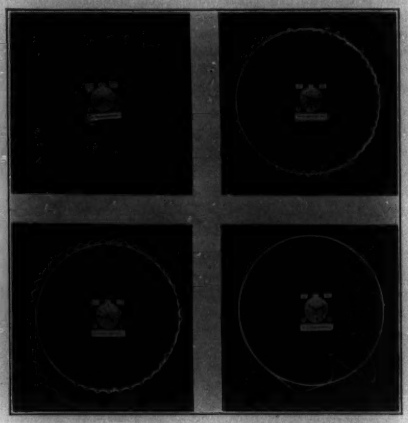


Fig. 10—Samples of Polar Oscillograms

record of the time may be made on the film by photographing a clock at regular intervals, say once a minute. A record made in this way is shown in No. 6, Fig. 19.

Frequently there is an advantage in recording two or more variables simultaneously. A record obtained with two vibrators is shown in Fig. 19, No. 7.

Performance

The limitations of an oscillograph lie mostly in the vibrator and, to a smaller degree, in the optical system and photographic emulsion used. The frequency characteristic of the vibrator up to 800 cycles is quite uniform, permitting records of disturbances having components in this range to be made with little distortion. This range includes the first 13 harmonics of 60 cycles and the first 32 harmonics of 25 cycles.

The vibrator may be wound to have any impedance in a wide range. If wound to have a high impedance it is especially suited for recording voltage waves, and if wound to have low impedance it is better suited for recording current waves. The sensitivity, as usually expressed, in millimeters deflection per milliampere of input, varies approximately as the square root of the impedance of the winding. The ratio of deflection to input is constant over a considerable range due to the balanced structure of the motor element.

As noted previously the oscillographs described are intended for recording in a comparatively low frequency range. In the range given there has been no difficulty in obtaining good records with a two-cp. flashlight bulb. This, of course, is largely due to the large size of the mirror on the vibrator.

Samples of records made with the oscillographs described are shown in Figs. 6, 8, 18, 19, 20, and 21. Those in Figs. 6, 8, and 18 have already been mentioned. Fig. 19 shows eight strip oscillograms and Fig. 20 shows four polar oscillograms illustrating some of the possibilities of these instruments. Fig. 21 is a response calibration, at constant frequency, of a polar oscillograph.

A number of field applications of oscillographs of both types has been made with satisfactory results. In some cases where cooperative studies were being made, the oscillographs have been used for recording transient neutral currents in power systems as well as to record

Fig. 21—Amplitude Calibration of Polar Oscillograph

voltages induced in telephone circuits by power system transients. Experience with the oscillographs in these field installations has suggested a few improvements of a mechanical nature and certain rearrangements of parts to increase the convenience of operation. These changes are now being embodied in a new design. It is hoped that it will be possible in a later paper to describe these features and to give the results of field experience more fully than can be done at this time.

Discussion

Morton Sultzer: Oscillographs of the general type described by Mr. Marrison have been in field use for over two years. I wish briefly to discuss some of the results obtained. I hope at a later date to present to the Institute a more elaborate review of these results.

As stated in the paper, this apparatus was developed for recording transient inductive disturbances in communication lines from neighboring power circuits. In our observations of this

kind of disturbance, there have always been cases where transient disturbances were manifest in communication circuits, for which there was no record of disturbances in the power system to which the communication circuits were exposed, and there have been records of disturbances in power circuits of sufficient magnitude to cause inductive disturbances in neighboring communication circuits, for which there are no records of disturbances in the communication system. An automatic recording oscillograph, equipped with a camera for photographing a clock to indicate the exact time of occurrence of the disturbance, should be a valuable tool in determining what disturbances correspond in the two systems.

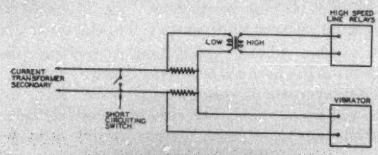


Fig. 1

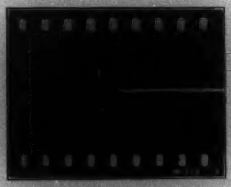


Fig. 2

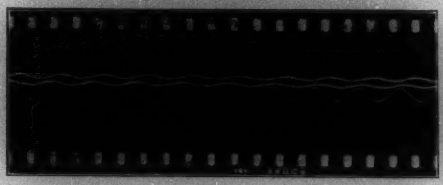


Fig. 3

In certain cases where co-operative studies are being made, oscillographs have been used to record simultaneously the disturbing abnormal in the power system, as well as the disturbance in the communication circuit. In addition to recording transient disturbances caused by abnormal conditions on power systems, this oscillograph has been found valuable in staged tests where the faults were applied to the power system at a predetermined time, and for measuring normal load induction.

Fig. 1 herewith is a diagram showing the connections of the continuous film oscillograph into a power system's neutral ground. In the secondary of the current transformer are inserted two small resistances. The drop across one resistance is supplied to the low side of a transformer, the high side of which feeds the tripping relay circuit. Bridged across the other resistance is the vibrator element of the oscillograph wound for low impedance. By this connection, it is possible to use a small radio-type transformer for operating the tripping relay without causing wave-shape distortion of the resultant oscillograms due to overloading.

Fig. 2 is an illustration of the rapidity of action of the oscillograph, for a surge of this type would probably not result in any breaker operation and therefore would not be called to the attention of the power company. The surge lasted slightly longer than one cycle on a 60-cycle system.

Fig. 3 shows 25-cycle induction from an electrified railroad, illustrating the use of the three-element oscillograph.

Fig. 4 shows oscillograms from stages tests showing the current in the neutral ground circuit of a 60-cycle power system and the resulting induction on communication circuits. These two oscillographs were situated approximately 60 mi. apart at the time of the tests. It will be noticed that communication circuit induction is recorded on two exposed lines by use of a two-element oscillograph, and an inspection of the two strips will illustrate how closely the induction followed the variations in the fault current. After the first six cycles, the second phase was faulted, causing an increase in the fault current, and after 31 more cycles, both phases had cleared. The current and the resulting induction after this point are due to the fact that the fault was placed on one of two transmission lines, working in parallel. After the fault had cleared on the side toward the generating station, it was fed over the non-faulted line until the breakers cleared at the far end.

Fig. 4

Fig. 5

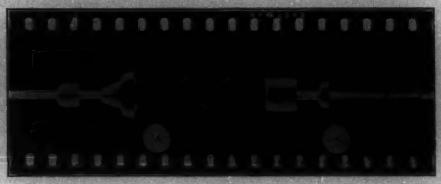


Fig. 6

Fig. 5 shows oscillograms of a 25-cycle calibrating wave. It will be noted that the film in this case is run at a much higher speed than on most of the other oscillograms, the speed in this case being approximately 5 ft. per sec., and in most of the other oscillograms 1 ft. per sec. This speed has been successfully used in staged field tests.

Fig. 6 shows the induction under normal load conditions from a 25-cycle electrified railroad system. This oscillogram was taken at a film speed of about 1½ in. per min., and at 5 min. intervals a short sample of the wave shape was taken by speeding up the film to approximately 1 ft. per sec.

Bushing-Type Current Transformers for Metering

BY A. BOYAJIAN[1] and W. F. SKEATS[2]

Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.—This paper describes a new development in connection with bushing type current-transformer metering circuits. While this development utilizes the two-stage principle and has the same order of accuracy as that of two-stage current transformers, it is different from the conventional two-stage current transformers in that it does not require two-stage wattmeters and watthour meters, but may be used with any wattmeter or watthour meter. The principle and connections of the transformer are explained, and performance curves of a typical unit are given.

* * * * *

Introduction

THE great simplicity, reliability, and low cost of the bushing-type current transformer have led engineers to an effort to improve its accuracy to such a point as will be satisfactory for metering purposes. The introduction of the two-stage principle[3] by Brooks and Holtz, six years ago, marked a satisfactory improvement in accuracy, but the two-stage principle as applied so far has required special two-stage wattmeters and two-stage watthour meters. There has still remained, therefore, the need for a bushing-type current transformer that would have metering accuracy and could be used with any wattmeter or watthour meter. In the development of the arrangement described below, which satisfies this requirement, the two-stage principle has been retained and such additional features have been incorporated as would make it adaptable to single-stage wattmeters and watthour meters.

Circuit of the New Current Transformer and Theory of Its Operation

Fig. 1 illustrates diagrammatically the conventional two-stage current transformer and a two-stage wattmeter or watthour meter. $P_1 S_1$ is the first stage of the current transformer feeding the first-stage current-coil C_1 of the meter W. The current in C_1 differs (vectorially) from the correct secondary current by the exciting current in P_1 divided by the turn ratio of the transformer. The second-stage transformation aims to put into C_2 a current equal to this difference. In the second stage of this current-transformer, P_2' and P_2'' act jointly as the primary; that is, the net ampere turns of P_2' and P_2'' act as the primary ampere turns, inducing a corresponding secondary current in S_2, which flows into the second-stage current-coil C_2 of the meter. The net ampere turns of P_2' and P_2'' are the exciting ampere turns of the first stage and thus the current delivered to S_2 and C_2 represents and makes up for the exciting current error of the first stage. In transforming the exciting current of the first stage, the second-stage current transformer requires an exciting current, so that the exciting current correction by the second stage is not 100 per cent exact. However, since the residual error is the exciting current of the exciting current, it is of second order of magnitude as compared with the error of the first stage or as compared with the error of an ordinary current transformer of the same proportions.

In order that the two stages may perform without interference from each other, it is necessary that the two circuits have no appreciable mutual impedance.[4] One obvious method of accomplishing this is to separate the current coils C_1 and C_2 from each other and provide a separate potential coil for each one to react with.

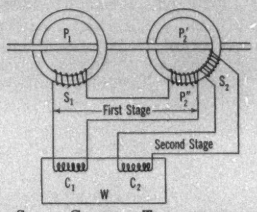


Fig. 1—Two-Stage Current Transformer with Special Wattmeter Having Two Current Coils

This is the practise followed in most of the two-stage current transformer applications, requiring special wattmeters and watthour meters. This disadvantage has been obviated in the present arrangement as follows:

For the proper performance of the two stages, it is not necessary that there should be no mutual impedance between the two circuits at any point, but that there should be no *net* mutual impedance. Accordingly, C_1 and C_2 may have any mutual impedance at any point provided that it is neutralized by an equal and opposite mutual impedance at another point. If C_1 and C_2

1. General Electric Company, Pittsfield, Mass.
2. General Electric Company, Schenectady, N. Y.
3. See *The Two-Stage Current Transformer*, by H. B. Brooks and F. C. Holtz, A. I. E. E. Trans., 1922, p. 382.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

4. By the mutual impedance between two circuits is understood the ratio (vectorial) of the voltage in one circuit produced by current in the other to the current flowing in the other circuit. As far as the authors know, this conception has to date been limited to mutual inductance or mutual reactance, but if the two circuits have a part in common it is obvious that current flowing in the one will give rise to an in-phase voltage in the other due to the resistance of the common part.

29-53

are combined into a single current coil C (Fig. 2) as in any conventional wattmeter or watthour meter, the mutual impedance between the first and second stage circuits is the impedance of the coil C, and therefore it is necessary and sufficient to provide another impedance common to the two circuits and equal and opposite to C. This is accomplished by the auxiliary impedance Z and the transformer T. The impedance Z is sub-

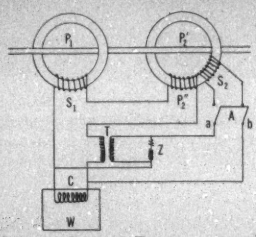


Fig. 2—Two Stage Current Transformer with Standard Wattmeter

stantially equal to the impedance C times the ratio of transformer T, and the transformer T accomplishes the desired reversal of sign so that one can neutralize the other. To make this clearer, assume that the switch A in the second stage is opened. The first stage current flowing through C and the primary T, will naturally circulate a corresponding current in the secondary of T through the impedance Z. If now we trace the second-stage circuit beginning at a and ending at b, through Z and C, we find that there are two

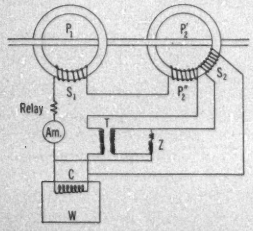


Fig. 3—Two Stage Current Transformer with Standard Wattmeter, Ammeter, and Relay

impedance drops in it, one at Z, the other at C, and that these two are opposite to each other so that no net voltage appears from a to b induced from the first stage current. The equality of the two drops is accomplished by the equality of the two impedances, while the opposition of the drops is accomplished by the fact that the secondary induced current of T must be opposite to its primary current. Thus, the net mutual impedance between the first-stage and second-stage circuits is rendered zero; and, therefore, if the switch A is closed, the second-stage current transformer delivers its current into the current coil C of the meter unaffected by the first-stage current C. Thus, both the first and the second stage currents flow simultaneously in one and the same coil C without any interference.

If Z is vectorially identical with the impedance of C, obviously the exciting current of T will cause the drop across Z to be slightly (vectorially) different from that across C. This error may, if desired, be compensated by making Z proportionately (vectorially) different from C. The magnitude of this error, however, is very small compared with the general improvement accomplished, especially in the bushing-type current transformers; because, first, the auxiliary transformer T is a very low-voltage transformer and can therefore be made far more accurate than the high voltage units

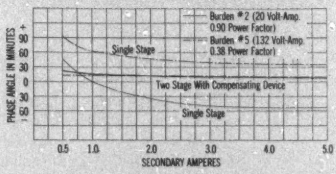


Fig. 4a—Performance Curve

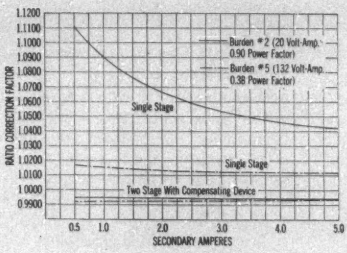


Fig. 4b—Performance Curve

P_1, P_2; and second, the auxiliary transformer T need not be bushing type, but of any high accuracy type, and thus again its error will be almost negligible for the circuit conditions. T is not to be mounted on the bushing but at any convenient point on the switchboard near the meters.

Fig. 3 illustrates the connections of such a current transformer arrangement with a number of instruments. Since great precision is necessary only in the meters and not in the indicating instruments or relay coils, only the watthour meter is shown included in the second stage. This leads to greater accuracy by reducing the burden and exciting-current of the second stage. In such applications, the auxiliary impedance Z must obviously represent the impedance of only that part of the burden which is to be "two-staged." The

auxiliary impedance naturally adds to the burden of both stages, but since it duplicates only that part of the burden which is to be "two-staged," it adds only a small amount to the total.

Performance Curves

Figs. 4A and 4B show performance curves of a 300-ampere turn, 60-cycle unit designed for a 132-kv. bushing. Single stage curves taken on the same unit, and using both first stage and second stage iron are also shown for comparison. The curves were taken for standard burdens No. 2 (20 volt-amperes, at 0.90 power factor) and No. 5 (132 volt-amperes, at 0.38 power factor). It will be noted that errors in phase angle of + 36 min. and − 52 min. at full load have been reduced to + 10 min. and + 8 min. respectively, whereas errors in ratio of 4.2 per cent and 1.2 per cent have been reduced to 0.6 per cent in each case. Moreover, whereas the uncompensated transformer showed much larger errors at low values of current than at high values, the two stage transformer with the compensating device is very nearly as accurate with 0.5 secondary amperes as with 5 secondary amperes.

Table I shows the final correction factor for each burden at 0.5 secondary amperes and at 5.0 secondary amperes and at various power factors.

The table shows that the combined ratio and phase angle error changes from 1.1 per cent at 10 per cent of rated secondary amperes to 0.8 per cent at full rated secondary amperes at 80 per cent line power factor, with additional improvement as the power factor is increased.

TABLE I
FINAL CORRECTION FACTOR

Line power factor	0.5 secondary amperes		5.0 secondary amperes	
	Burden No. 2	Burden No. 5	Burden No. 2	Burden No. 5
0.90 lead	0.994	0.997	0.995	0.995
0.95 lead	0.993	0.996	0.995	0.995
1.00	0.992	0.994	0.994	0.994
0.95 lag	0.991	0.992	0.993	0.993
0.90 lag	0.990	0.991	0.993	0.993
0.80 lag	0.989	0.989	0.992	0.992
0.70 lag	0.988	0.987	0.991	0.992
0.60 lag	0.987	0.985	0.990	0.991
0.50 lag	0.985	0.983	0.989	0.990

Discussion

C. T. Weller: The comparison between single- and two-stage operation suggests a slightly different view of the theory of two-stage operation, which may be of interest:

Referring to the authors' Fig. 1, the first stage of current transformers P_1 and P_2' may be thought of as a frequently used combination of bushing-type current transformers, in which the secondaries of two such transformers are connected in series to obtain better performance. The secondary "ampere-turns" (this term makes the discussion independent of the actual transformer ratio) are less (vectorially) than the primary ampere-turns by the amount of the exciting ampere-turns, which are necessary to force the secondary current through the windings and burden. In general, the secondary ampere-turns differ both in magnitude and in phase angle from the primary ampere-turns, and thus produce corresponding errors in the meter registration.

If, therefore, another set of ampere-turns, which is approximately equal to the exciting ampere-turns, could be superimposed on the secondary ampere-turns, the error would be considerably reduced. This is accomplished by the two-stage transformer in which a tertiary winding (S_2) on part of the core (P_2') is energized by the comparatively small exciting ampere-turns. This winding must, therefore, supply a small part only of the secondary burden. The exciting ampere-turns of the tertiary winding are comparatively large (although of a second order) even when supplying a comparatively small burden; a large burden might render the output negligible.

A comparatively small burden is obtained in the standard two-stage watthour meter by providing a duplicate current coil. The modification described by the authors obviates, as stated, the necessity for the separate current coil. Perhaps the best summary of the results obtained is given in their Table I, in which the "overall" correction factors differ from unity by nominal amounts only for a wide range of line power factors. It can be shown, however, that the accuracy obtainable with the modified method is inferior to that with the standard two-stage method, due principally to the addition of the burden of the auxiliary current transformer T.

P. O. Langguth: The fine performance shown in Figs. 4A and 4B, of the paper, I assume are on the basis of two-staging the watt-hour meter burden only, and that the 20 volt-ampere burden is carried on single stage only.

It would be interesting to know what performances can be expected when applying this transformer to conditions of lower ratio, and higher secondary burden where the complete burden is two-staged, and also if it is necessary to have a special impedance Z and transformer T when it is desired to change meters.

It seems that the space requirements of this type of transformer when used on circuit breakers may be a deciding factor in its application. At first hand it appears that the transformer would be of relatively large proportions when attempt is made to apply it to the lower voltage class of breakers, and probably very little gained over the conventional bushing transformer using a low-loss iron core.

A. M. Wiggins: (by letter) Several years ago the writer tried out a scheme with bushing-type current transformers very similar to that described in the paper by Messrs. Boyajian and Skeats. This scheme consisted of a main transformer and an auxiliary transformer having their secondaries connected in parallel, with an auxiliary burden connected in the circuit of the auxiliary transformer. This scheme was tried out on 50/5-ampere and 200/5-ampere ratio transformers. Very good performance resulted when a proper adjustment of the values of resistance and reactance of the auxiliary burden was obtained. The scheme was objectionable, however, because it was necessary to adjust its auxiliary burden on test for each unit and also because this adjustment was correct for one certain burden only. Apparently the same objections apply to the scheme described in this paper.

About this time the nickel-iron alloy, known as hipernik, had been developed to the point where it could be used in low-ratio designs to give a performance on a par with that of the compensated scheme for small and medium-size ring punchings and for ratios down to 150/5 amperes. However, no reliable tests for metering accuracy have been made on large rings, or for very large secondary burdens. Further experiments with the compensated scheme were abandoned in favor of the much simpler designs using hipernik. Some examples of the performance obtainable with hipernik designs are given in Figs. 1 and 2 shown herewith. Further work is yet to be done with designs employing very large ring punchings.

A. Boyajian: The modified two-stage current-transformer described in this paper is as yet in its developmental stage,

and therefore some of the questions asked cannot be answered with certainty. The device is as yet unavailable in the market.

Referring to Mr. Weller's discussion, his point that the "modified" outfit will be somewhat less accurate than the straight two-stage outfit, may be true, if the compensating impedance is made an exact duplicate of the meter burden, but this auxiliary impedance need not be an exact duplicate of the meter burden.

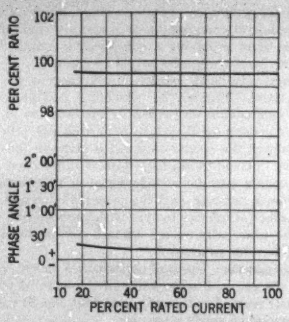


FIG. 1—RATIO AND PHASE-ANGLE CURVES

150/5 A. ring-type current transformer

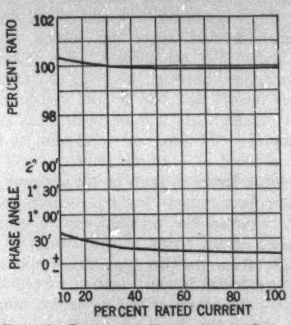


FIG. 2—RATIO AND PHASE-ANGLE CURVES

150/5 A. ring-type current transformer

Answering Mr. Langguth's question, if the meter is changed, the auxiliary transformer T need not be changed, but the impedance Z may well be changed (or adjusted) if the impedances of the two meters are very different.

Referring to Mr. Wiggin's discussion, I may say that there is a world of difference between, (a), an attempt to feed back a variable magnetizing current from an auxiliary fixed impedance, and (b), a scheme to neutralize a fixed mutual impedance by another fixed mutual impedance. When an auxiliary impedance is used to feed back exciting current, the accuracy is seriously affected not only by changes in meter burden but also by changes in load current. It can be adjusted to excellent accuracy at one value of load current (no matter what the primary ampere-turns may be), but when the load current is increased or decreased, large errors may begin to show. The two-stage principle differs from this fundamentally, in that the variable exciting current is transformed in a second stage and fed to the meter burden. There is very little difference in the manner of functioning of the straight two-stage and the modified two-stage outfits.

As to the question of core materials, undoubtedly the better the material, the better is the performance for a given circuit connection. Conversely, for a given quality of material, the two-stage principle will give far better results than the conventional single-stage operation.

Cathode Ray Oscillograph Study
Of Artificial Lightning Surges on the Turners Falls Transmission Line

BY K. B. McEACHRON* and V. E. GOODWIN*
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*A portable impulse generator delivering 400 kv. has been constructed. A cathode ray oscillograph of the cold cathode type has been so connected that impulses incoming over the transmission line to which it is connected will initiate the oscillograph.*

The results of studies on a 5.77-mi. section of a 66-kv. line of the Turners Falls Power Company are given. Impulses of definite forms are applied to the line. The theory of traveling waves is being checked experimentally, demonstrating the existence and magnitude of reflections, both for open ended lines and for lines closed through gaps or a combination of inductance and capacity. A simple description of the theory is given.

Some preliminary results are given for attenuation obtained by successive reflections from either end of the line.

The reduction of the incoming waves by an oxide film lightning arrester is given, the affected wave in one case having a front of six microseconds and in another case a front of 0.5 microseconds.

Further work to be done includes the effect of traveling waves on choke coils, transformers, and ground wires. Additional work is to be done to determine the laws governing attenuation.

As a result of the development of the klydonograph[1] and the surge-voltage recorder,[2] it has been possible to obtain records of lightning and switching surges on transmission lines, and a large number of data has been collected giving information concerning surges appearing on lines both as a result of lightning and of switching. These data[3] have given useful information concerning surge characteristics, such as magnitude, frequency of occurrence, polarity, approximate location of inception (in the case of lightning), and attenuation.

These studies using the Lichtenberg figures have not, however, given much information concerning the travel of waves through the transmission conductors or the effect of the various pieces of apparatus on these waves. Many of the data obtained are contradictory, which is to be expected since the effect of the apparatus on the traveling waves will in many cases depend on the character of the incoming wave, which may have in practise many different shapes. Furthermore, the recorder or klydonograph cannot distinguish between the original wave and a reflection—it is only capable of recording the crest values of transients.

Because of the limitations of this means of determining the characteristics of transients, two cathode ray oscillograph installations were made in the United States during the summer of 1928. The oscillogram[4] taken at Wallenpaupack showed a wave reaching its crest in about eight microseconds and having a total duration of about thirty-five microseconds.

Information of this kind is valuable, since it shows what form impulses due to lightning may take, but to study the reaction of apparatus or lines to natural lightning using cathode ray oscillographs, is both slow and costly. It is practically impossible to obtain a number of approximately identical transients caused by natural lightning, for the purpose of observing the effect of lines and apparatus on the shape of the wave.

It appears that a much more satisfactory way to study this phase of the protection problem, is to apply artificial lightning surges, which may be duplicated at will, and made to appear on the line whenever desired. The authors believe that this paper records the first application of impulses from a lightning generator to an actual high-voltage transmission line. This method of obtaining controlled impulses makes possible the continuation of the studies throughout practically the entire year, without depending on lightning storms which occur during the summer months only. The applied impulses may have various fronts and tails as desired, so that all factors may be studied.

Apparatus

The equipment required consists essentially of cathode ray oscillograph equipment which can be set up and operated in the field, and an impulse generator which is easily moved along the transmission line as desired. It is of course necessary to have power available at both the oscillograph and impulse generator locations. Some form of communication between the men located at the impulse generator and at the oscillograph is desirable. At present it is easier to move the generator than to move the oscillograph; however, as oscillographic apparatus becomes further developed, it may become as truly portable as the lightning generator.

Cathode Ray Oscillograph

No detailed description of the cathode ray oscillograph will be given here as a rather complete explanation was given in a paper[5] before the A. I. E. E. in 1925. The oscillograph used in these studies, while

*Both of the General Electric Co., Pittsfield, Mass.

1. For all references see Bibliography.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

operating on the same principle as the Dufour oscillograph, is of the type described by Lee.[6]

This oscillograph is the cold cathode type which is not suited to continuous operation unless the cathode potential is kept low, which interferes with its ability to record high-speed transients photographically. Therefore, one of the first requirements in the use of this oscillograph for transient registration is, that means must be provided for exciting the cathode at the time of the occurrence of the transient. When using lightning generators in the laboratory this was accomplished readily because one could initiate the transient somewhat after potential was applied to the cathode of the oscillograph. When endeavoring to record lightning surges on a transmission circuit, it is necessary that the incoming transient set off a trip circuit which will excite the cathode and complete necessary connections so that the oscillograph is in operation within perhaps a millionth of a second after the voltage at the oscillograph begins to rise due to the incoming transient.

With a study such as that being described in this

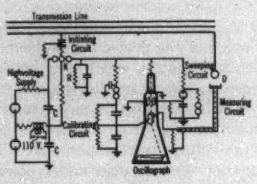


Fig. 1—Circuit for Automatic Initiation of the Cathode Ray Oscillograph by Incoming Transient

paper it is desired to obtain oscillograms of natural lightning transients as well as records of the man-made lightning transients and therefore the circuits are arranged to be operated either way.

The diagram of circuits used for initiation of the oscillograph by the incoming transient is given in Fig. 1. When the oscillograph is about to be used a bank of capacitors C is kept charged to a potential sufficient for the excitation of the cathode of the oscillograph. This potential is applied between the two outside spheres of the three-electrode gap K, the middle sphere of which is held at a mid-potential by connection to the middle of the bank of capacitors. When the potential of the middle sphere begins to shift, due to the incoming transient, sufficient unbalance results so that the gaps arc-over and the potential of the charged capacitors appears across the resistance R. This same potential also appears between the cathode and the anode of the oscillograph, resulting in the establishment of the cathode stream, which is to be deflected by the incoming transient and leave a record of its wave shape on the photographic film.

The potential applied to the cathode is also applied to a capacitor in series with a kenotron and a resistor which controls the rate of voltage rise across the capacitor. The changing potential across the capacitor is applied to a pair of deflecting plates which results in the time scale or sweeping motion of the cathode spot on the photographic film. The rate of voltage change can be calculated but the time scale on the oscillogram is usually checked directly by means of superimposing at right angles the motion resulting from the application of a high frequency potential whose frequency is accurately known. The parallel resistor keeps the deflecting plates at zero potential except when the oscillograph is measuring a transient, while the parallel gap prevents the application of excessive potentials to the deflecting plates. The kenotron is used to prevent the plates from discharging and the cathode beam coming back across the film when the protective gap sparks.

The potential of the incoming transient is divided by the shielded gap D in series with the deflecting plates of the oscillograph which taken together constitute a capacity potentiometer. This circuit is shielded from stray influences by a grounded copper covering and arranged with inherent resistance such that local oscillations are minimized.

The circuit shown to the left of the oscillograph in Fig. 1 is arranged so as to apply to the deflecting plates a transient potential when the sweep has carried the spot nearly across the photographic film. Since the potential applied is known, a means of securing a voltage calibration is obtained which appears on the film as a superimposed rise in potential at the end of the time scale. This calibration circuit can be used or not at will. The use of this circuit does not appreciably affect the measuring circuit since the capacity relations are such that the potential across the calibrating capacity due to the impulse is about one per cent of that across the deflecting plates.

Cathode ray oscillograms taken with the circuit shown in Fig. 1 but without using the calibrating circuit are given in Fig. 7. The oscillograph was initiated by the transient, and the time delay in getting the cathode stream under way is indicated by the gap in the front of the wave. This method of operation of the oscillograph is the one which is being depended upon to secure oscillograms of lightning impulses on the transmission line.

Because this type of circuit loses a portion of the front, the circuit was modified as shown in Fig. 2. In this figure is also shown the circuit for the impulse generator set up to apply impulses to the transmission line.

With this method of operation the potential of the middle sphere of the three-electrode gap at the impulse generator is disturbed when the operator closes switch S. At the same time this applies to the lower conductor of the transmission line a small transient potential which disturbs the potential of the three-electrode gap at the impulse generator, which is held in a charged

condition, with the result that the three-electrode gap sparks over and initiates a discharge of the generator, thus applying to the middle conductor of the transmission line a surge whose characteristics are determined by the constants of the generator and the line.

It is therefore possible for the operator at the oscillograph to initiate a transient at any time which he desires, applied at any point along the transmission

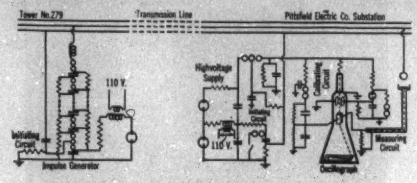


FIG. 2—CIRCUIT FOR APPLYING TRANSIENTS TO A TRANSMISSION LINE

Arranged to pre-excite the oscillograph so that the entire wave-front may be obtained

line, and have the oscillograph already excited by the time the transient arrives, thus securing an oscillogram of the entire wave front. Such an oscillogram is shown in Fig. 10.

THE TRANSMISSION LINE

The transmission line on which these tests are being made consists of a double circuit, and extends from the

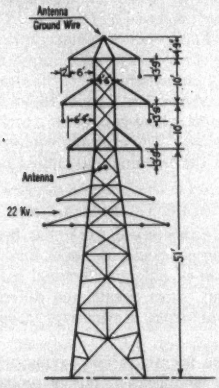


FIG. 3—TYPICAL TOWER

Showing location of line conductors, antennas, and ground wire

Pittsfield Silver Lake Substation to the Cabot Station of the Turners Falls Power Company on the Connecticut River. The total length of circuit is 36.7 mi., passing over rather hilly country in western Massachusetts.

A typical tower is given in Fig. 3, showing the spacing of conductors and the single ground wire on the top of the tower which covers the entire length of line except for a distance 945 ft. on the Pittsfield end and 1400 ft. on the Cabot Station end, where carrier-current wires replace the ground wire. On the Pittsfield end a second carrier wire is installed as indicated in the figure. Two 22-kv. circuits are also carried on the towers from the Silver Lake Station for a distance of one mile. These are also shown in Fig. 3.

The line was built for 110 kv. operation, but is now operating at 66 kv. The 66-kv. line conductors are 2/0-7 strand hard drawn copper and the ground wire is 0.375-in. galvanized steel. The average height of line at the tower is 71 ft. for the top conductor. The line runs through rock, earth, and swamp country, the proportions being about equal on the section of the line used in tests thus far. At five points along the line

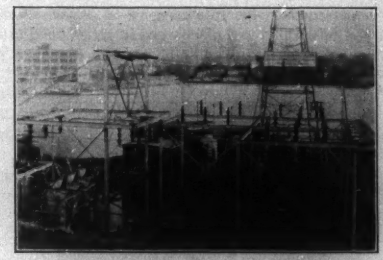


FIG. 4—SILVER LAKE SUBSTATION OF PITTSFIELD ELECTRIC COMPANY

Showing location of oscillograph and terminal equipment

disconnecting switches have been provided. Tests have been made only at Tower 279, which is located in a valley between two hills a distance of 5.77 mi. from the Silver Lake Substation.

CATHODE RAY OSCILLOGRAPH INSTALLATION

The cathode ray oscillograph is installed at the Silver Lake Substation in the smaller of the two buildings seen in the middle left in Fig. 4. Mounted on the roof of this building is the shielded gap which forms a part of the potentiometer for reducing the transients to values suitable for the oscillograph. On the roof of the building just beyond the one housing the oscillograph is mounted a two-way disconnecting switch so that connections to the oscillograph may be made to either side of the choke coil or to other points as desired. The photograph shows the location of choke coils and arresters and gives an idea of the station layout. The transformers at the left supply power to the General Electric Company while the circuit breakers and transformers of the Pittsfield Electric Company are further to the right and do not appear in the photograph. In all of the tests made, the circuit on the left

side of the towers shown in Fig. 4 was used, the disconnects just over the circuit breakers on the right of the photograph were open, so that only the bus work shown in the photograph was in circuit during the tests. Although tests will probably be conducted at a later date with the line energized, all of the tests described in this paper were made with the line dead.

In some of the early work on this line the impulse generator, which was later moved out to Tower 279, was located just beyond the larger of the two small buildings shown in Fig. 4. In every case apparatus such as arresters or water tubes which were connected between line and ground in various tests were always located in the space just in front of the lightning arresters.

Impulse Generator at Tower 279

The impulse generator was originally designed to deliver a crest potential to ground of 300 kv. This was later increased to 400 kv., using 16 oil capacitor units charged in parallel and discharged in series making use of the Marx[7] circuit as indicated in Fig. 2. The

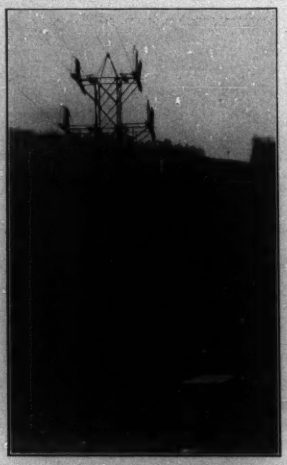


Fig. 5—Tower 279—Showing Portable Impulse Generator

Line in foreground extends to Pittsfield

generator has a capacity of 0.0156 μf., the stored energy being 1250 watt-sec.

Tests have been made with two wave fronts, one a fast wave reaching its crest in about one-half microsecond, and the other a slower wave obtained by connecting an inductance of 0.0018 henrys between the generator and the line. This slower wave rises to its crest in about six microseconds. Practically all of the results to be given in this paper were taken with the slower of these two waves.

In all of the tests described in this paper, unless otherwise noted, the impulse generator has always been connected to the middle conductor of the transmission line, while the lower conductor has been used as a

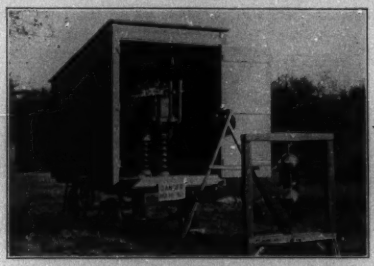


Fig. 6—Portable Impulse Generator at Tower 279

Open circuit voltage is 400 kv.

control wire to initiate the impulse immediately after starting the oscillograph so that the full wave could be obtained on the oscillogram.

Fig. 5 shows the impulse generator at the base of Tower 279 with the connections made, and the disconnects open. The photograph shows the line extending over the hills to Pittsfield and gives a fair idea of the type of country traversed by this line. The middle and the lower conductors of the circuit on the right side of the tower are connected to the impulse generator.

Some idea of the construction of the impulse generator may be obtained from Fig. 6 which shows the inductance coil and a parallel sphere-gap (25 cm.) arcing over on the crest of the wave. The charging equipment is located at the rear of the building and cannot be seen in this photograph.

Line Characteristics

Considerable theoretical work[8] has been done with the travel of perpendicular waves on lines and the effect on these waves of different terminal conditions, such as open circuit, short circuit, and different values of resistance, inductance, or capacity, in combination or alone.

The problem for waves not having a perpendicular front is more complicated, but is probably the only case which occurs in practise, and is therefore of great importance. The reaction of devices such as lightning arresters cannot be calculated and must be determined by test under actual conditions as they exist in practise.

In order that a proper understanding of the principles involved may be obtained by the reader, two oscillograms are given in Fig. 7 which will be discussed in some detail. The circuit diagram is given under each oscillogram. It should be noticed in both cases that

the impulse generator and the cathode ray oscillograph are both at the Pittsfield end of the line. Also in both cases the disconnects are open at Tower 279, 5.77 mi. away. In the upper oscillogram there is no apparatus connected at Tower 279 while in the lower oscillogram a sphere-gap connected between line and ground arced over.

The oscillograms are alike and may be superimposed up to the point where the gap sparked over. Beyond this point the reflection is negative, but otherwise is practically identical with the original reflection. These oscillograms show important characteristics, some of

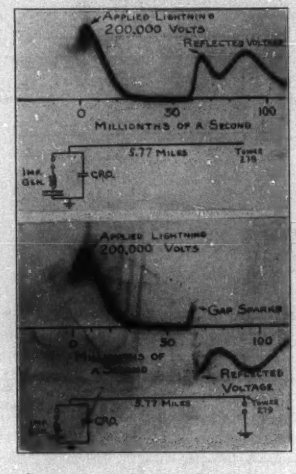


FIG. 7—OSCILLOGRAMS SHOWING INITIAL AND REFLECTED WAVES WITH IMPULSE GENERATOR AND OSCILLOGRAPH BOTH AT SILVER LAKE SUBSTATION

Upper oscillogram open ended line at Tower 279
Lower oscillogram line closed through gap at Tower 279

which should be mentioned before studying the travel of these waves in detail. They show how transients can be duplicated on successive discharges and how beautifully the line repeats its actions when subjected to the same influence. This means that in general if a line reacts differently under one lightning influence compared with another, it is probably due to a difference in the lightning impulse and not due to a line performing differently. These oscillograms also show the initiation of the oscillograph by the transient itself which is the reason for the missing portion of the front of the wave.

TRAVELING WAVES

A proper understanding of the oscillograms in Fig. 7 requires some knowledge of how such waves travel on transmission lines. An explanation made as simple as possible will be given without any attempt to assign any magnitudes to the various factors as the discussion is intended to be illustrative only.

A wave travels along a line by charging up the capacity of the line as the voltage is increasing on the front of the wave and discharging the line capacity as the voltage decreases on the tail of the wave. Such a process requires an increasing current flow through the line as long as the voltage is increasing reaching a maximum when the voltage has ceased to change. In other words, the wave-front of current will exactly correspond to the wave-front of voltage. Likewise, as the voltage decreases in the tail of the wave so also will the current decrease by the amount being discharged from the charged capacity of the line under the tail of the wave. Thus the voltage and current waves will have exactly the same shape both front and tail.

When such a wave is traveling along a line two equal energies need to be considered. One is electromagnetic and is stored in the magnetic field around the conductor due to the transient current, and the other energy is electrostatic and is stored in the electrostatic field between the conductor and ground. If the wave meets with an open circuit at the end of a line all of the energy at the open end becomes electrostatic and the potential at the end of the line is doubled. The returning reflected wave will be identical with the original wave, neglecting losses, at a distance away from the end greater than half the length of the original wave.

If the traveling wave meets a short circuit to ground the electrostatic energy at the short circuit becomes zero and the electromagnetic energy is doubled as is also the current in the short circuit. If the crest current of the traveling wave is I amperes, which value is obtained by dividing the crest potential of the traveling wave by the surge impedance of the line, then the current in the short circuit will be $2\ I$ amperes.

In Fig. 8A is shown a simple method of determining the potential at the end of the line and visualizing, to some extent at least, the phenomenon of reflection. Wave W is traveling toward the open end of the line at X. Another hypothetical wave W' which is an exact reflection, is traveling toward the end of the line from the right. These waves are moved toward and through each other at the same rate and are combined by addition as seen in the case marked (Time II). It will be noticed that the potential at X at the end of the line will always be double, but the value at some other point as K in general will not be double. Thus with the type of waves usually found in practise the double potential occurs only at the point of reflection. If the wave has a flat top then the double potential would extend farther back depending on the duration of the constant potential. Finally the wave marked W' will be found traveling away from the point of reflection.

It should be noted that a cathode ray oscillograph out on the transmission line at a distance greater than the length of the wave would first give an oscillogram like

W followed by the reflection W'. These waves on the line exist both in time and in space. A cathode ray oscillograph at the end of the line would show a wave having the same length but with all of the ordinates doubled. This wave does not appear on the line in space, and is the record of successive voltage changes at the end of the line.

Fig. 8B shows the same wave as in Fig. 8A when the line is grounded. The reflected wave is identical with the original traveling wave except that it is reversed in potential. The wave in space is shown after two different times and finally the reflected wave will appear on the line identical with the original wave but reversed in polarity. The potential at the short circuit is always zero, while out on the line the oscillograph would show first the original wave and then its reversed reflection.

It is important because of the use of apparatus having inductance and electrostatic capacity to consider the effect on traveling waves, first of capacity and inductance alone and then in combination. Therefore, Fig. 8C has been prepared showing the same wave used before, traveling along a line closed through an electrostatic capacity equal to the capacity of the impulse generator. The wave marked "traveling wave reflection" is the wave which would be measured by an oscillograph out on the line a wavelength away from the end. This wave appears in both space and time on the line. The summation of this reflected wave and the original wave gives the potential change across the capacity with respect to time and is what the oscillograph would show when connected across the capacity. Referring again to the reflected traveling wave, if the capacity is made larger, the portion below the zero line will become larger and the portion above the line will be correspondingly decreased. It may be noted that the capacity if small acts almost like an open ended line; if large it will act more like a short circuit, sending back into the line a large portion of the original voltage as a negative reflection, but the potential across itself will be small.

The effect of an inductance is shown in Fig. 8D, using a value of inductance equal to that used in series with the impulse generator. The traveling wave is

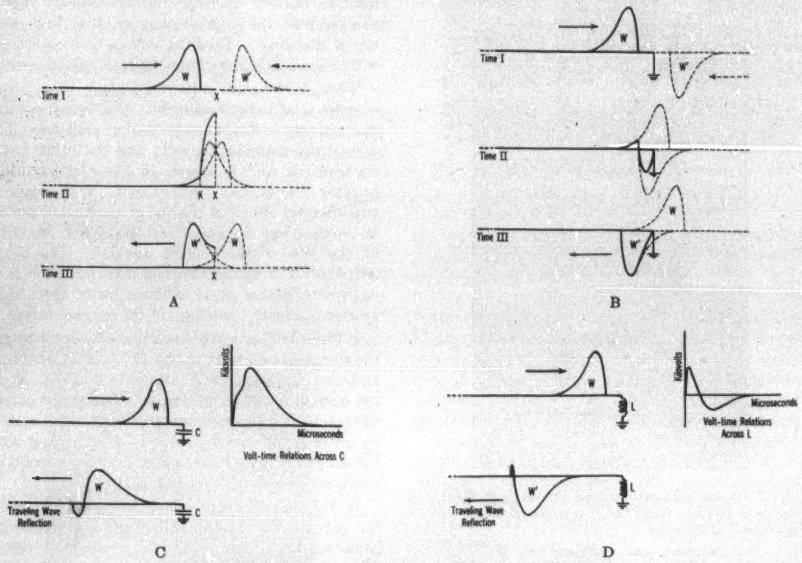


Fig. 8—Reflections for Different Terminal Conditions

Using a wave similar to test wave and L and C equal to the values used in the impulse generator

a. Open ended line
b. Line closed through zero resistance
c. Line closed through capacity equal to that of the impulse generator
d. Line closed through inductance equal to that of the impulse generator

shown and is somewhat similar to the reflected wave for the case with capacity at the end of the line, but is reversed. With inductance, the smaller the value of inductance the more it acts like a short circuit, and the greater the value of inductance the more it tends to act like an open ended line. The voltage variation across the inductance with respect to time is shown in Fig. 8D and is the summation of the original traveling wave and its reflection.

Using the methods similar to those used in Fig. 8, the two conditions seen in the oscillograms in Fig. 7 are set up and sketches of the waves are drawn for the waves on the line and the reflected wave from either end.

In the first tests the impulse generator consisting of series inductance and capacity with several small gaps in series was at the same end of the line as the cathode ray oscillograph, while later the impulse generator was moved to the other end of the 5.77-mi. circuit being studied. Thus, we have oscillograms taken at either end, although the capacity when at the opposite end of the line was reduced to 75 per cent of the capacity when the generator and oscillograph were together at the same end of the line. At the same time the capacity was reduced the potential was correspondingly increased. Fig. 10 is an oscillogram taken with the impulse generator and oscillograph at opposite ends of the line, and the form of the reflection is to be compared with that of the upper oscillogram of Fig. 7.

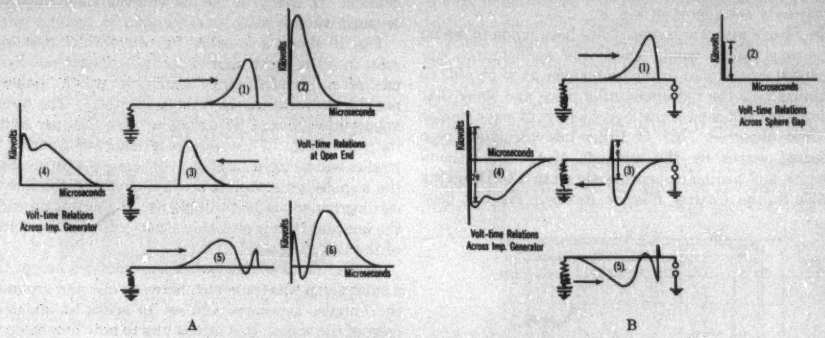


FIG. 9—REFLECTIONS WITH IMPULSE GENERATOR AT ONE END OF LINE

a. With open-ended line
b Line closed through a gap set for e volts

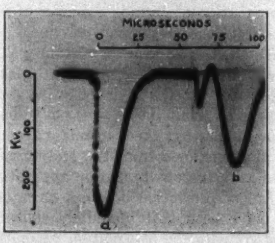


FIG. 10—FULL WAVE 6-8 MICROSECOND FRONT IMPULSE WITH GENERATOR AT TOWER 279

Referring to Fig. 9A, the same impulse discussed in Fig. 8 is seen traveling away from the impulse generator toward the open ended line (wave No. 1). An oscillograph at the end of the line would have shown a volt—time curve such as that shown at the right (2). After reflection, neglecting losses and attenuation, the wave will be the same as the original wave and will now be traveling back toward the impulse generator (3). With the line being considered this distance is 2×5.77 mi. or a time of about 62 microseconds is required for the original wave to return as a reflected wave. In this short space of time our tests have shown that the gaps at the impulse generator are still ionized with the result that the gaps continue to spark and (if the arc resistance is assumed constant) we may consider the impulse generator as inductance and capacity and resistance at the end of the line. The voltage to ground across the impulse generator with respect to time is shown at the left (4). This corresponds to the reflection shown in the upper oscillogram of Fig. 7. This curve is however the sum of the reflected wave (5) from the impulse generator and the reflected wave from the open end (3).

The wave (5) reflected from the impulse generator travels to the open end of the line where a reflection takes place as shown by (6). This may be compared with the reflections seen in Fig. 10.

Thus, except for the change in capacity and voltage of

the impulse generator, losses and attenuation, one-half the sum of the two reflections *a* and *b* in Fig. 10 would give the reflection shown in Fig. 7. The reflection at the impulse generator results in a wave that is longer than the original wave. This lengthening of the wave is due to the storage of energy in the impulse generator and may also be seen in the case of either inductance or capacity considered in Fig. 8.

In Figs. 8 and 9 an attempt has been made to set up conditions which would check the oscillograms, approximately and also in such a manner as to be able to visualize how the reflections take place and show that inductance and capacity will give a form of wave similar to those observed. Mr. O. Brune has calculated these reflected waves by the methods of the operational calculus and his methods are outlined in a forthcoming article in the *General Electric Review*. He has been

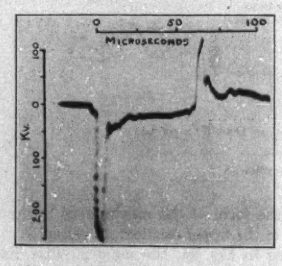


Fig. 11—Chopped Wave-Gap Set at 197-Kv. Crest

able to find quite satisfactory agreement with the oscillograms.

It is possible to break up the original traveling wave in component perpendicular waves and treat each of these separately, adding the results together to find the effect of terminal apparatus on the original wave. This is treated at some length by Rüdenberg,[9] and Carson.[10]

It is interesting to consider the lower oscillogram of Fig. 7. Here the impulse generator and the oscillograph were at the same end of the line and a sphere-gap between line and ground was connected at the far end of the line. This condition is considered in Fig. 9B. Wave No. 1 is the original traveling wave, while No. 2 is the potential time curve across the sphere-gap. Up to the time of sparkover the line is open-ended; after that it is short circuited neglecting the resistance of the arc. Wave No. 3 is the reflected wave which reaches the impulse generator. The volt-time curve of potential rise across the impulse generator is shown in wave No. 4, which corresponds to the lower oscillogram of Fig. 7. Wave No. 5 is the reflected traveling wave traveling toward the gap which is still in an ionized condition. Therefore no potential will appear across the gap and wave No. 5 will be reflected back with reversed polarity.

In all the preceding discussion no account has been taken of losses or attenuation, which must be considered in the actual case in practise.

Chopped Waves

Since in practise traveling waves are frequently cut-off by flashover of insulators, it is worth while to consider the effects of the line constants and apparatus on such waves.

Fig. 10 shows a so-called full wave which rises to its crest in six to eight microseconds as determined by the use of a high-frequency oscillator which makes it possible to measure the front accurately. The wave is apparently affected by corona at about 80 per cent of its crest value. This wave is of negative polarity but studies will be made later with opposite polarity. Since the impulse generator is at the far end of the line the oscillogram shows the voltage rise at the station end of the line and thus is probably close to double the voltage of the traveling wave.

In Fig. 11 all conditions were unchanged except that a sphere-gap was connected between line and ground at the impulse generator and set to spark at about the crest of the wave. It is interesting to note how steep the cut-off is and that the reflection comes back reversed to the station end where the oscillograph is located, which shows that the gap in parallel with the impulse generator is still ionized. The same sloping of the wave due to corona can be seen on the front of the reflected wave. When studying these oscillograms it should always be remembered that they represent voltage changes at the oscillograph with respect to time. From them with proper interpretation the traveling wave can be deduced. All of the tests to be discussed were made with one or the other of the waves shown in Figs. 10 and 11 unless otherwise indicated.

Attenuation

The attenuation on lines is a matter of considerable importance in the protection of lines and station apparatus. If it is known, for instance, that any impulse having a crest voltage up to the flashover value of the insulator for that particular wave will be so attenuated in a certain distance as not to be dangerous to station apparatus, then storms beyond a radius equal to that distance from the station need not be considered as dangerous to station apparatus and thus the problem of protection becomes more definite. It seems to have been quite definitely shown[11] that for dangerous voltages the attenuation depends largely at least on the effect of corona. If this is true then other factors in addition to potential will control the loss. All other factors being equal the corona loss will depend on the potential and for extremely steep waves perhaps on the wave-front.

Tests are being made on the Turners Falls line with the object of determining the factors which control the attenuation, particularly for potentials above the corona voltage. The effect of wave shape, polarity,

maximum potential, ground wires, and other factors will be studied.

Some preliminary results are available on the short section of line worked with up to the present time. Later, the whole length of line (36.7 mi.) will be used which will give somewhat more accurate results than those presented here.

The method of making these tests was to apply at

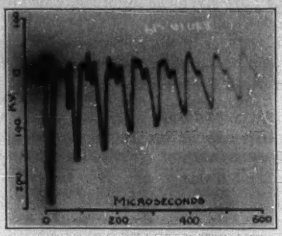


Fig. 12—Successive Reflections, Full Wave

Tower 279 an impulse which reflected back and forth over the line which except for the small capacity of the oscillographic equipment and the station bus capacity was open at the oscillograph and closed to ground through the impulse generator at Tower 279. The first two reflections are shown in Fig. 10. The oscillogram in Fig. 12 shows eight of these reflections which cor-

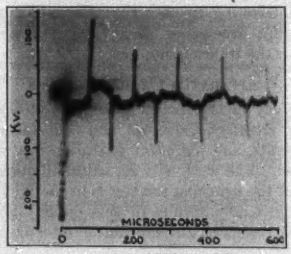


Fig. 13—Successive Reflections, Chopped Wave

responds to a length of line of approximately 85 mi. In Fig. 13 are given the successive reflections with chopped wave similar to Fig. 11 except that the parallel gap was set for 156 kv. instead of the 197 kv. setting used in Fig. 11.

In Fig. 14 the crest values from Figs. 12 and 13 are plotted to show the change in potential with each successive reflection. When discussing this method of studying attenuation, compared with tests on a line of infinite length certain inaccuracies are present which must be considered. In the case of the full wave, Fig. 12, the wave form is very appreciably modified by the impulse generator and there is of course loss in the resistance of the arcs in the various gaps. It is expected that in the future means will be found of getting the generator off the line before the reflected wave returns to the generator.

In Fig. 13 the effect of the constants of the impulse generator do not appear in the reflection since the gap is kept in an ionized condition and acts as a line closed through a small but variable resistance.

There is good reason to believe that chopped waves will attenuate faster than full waves, which is supported by data given in Table I discussed in a following section. When the results are affected by the terminal conditions, it is probable that only the first reflection can be considered.

This method of study of attenuation is open to the objection not only of the effect of the terminal apparatus on wave form but also to the fact that the

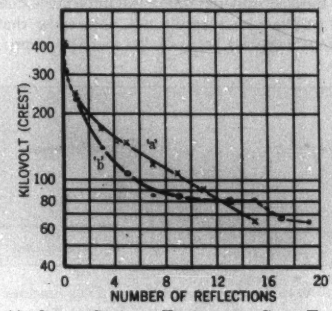


Fig. 14—Curves Showing Variation in Crest Volts with Successive Reflections

a. Full wave
b. Chopped wave

potential may be increased at one reflection point and made zero at another, as is the case in Fig. 13. This will result in different losses due to corona, for instance, which does not represent truly the travel of the wave along an infinite line.

The calculated corona voltage on this line is 110-kv. crest, but the curves in Fig. 14 do not show any close correlation with this value. The ordinates given for the curves represent voltage at the oscillograph which is presumably double that on the line. With curve *b* after the first reflection the traveling wave was not much above the corona voltage, but at the oscillograph on reflection it was appreciably above the corona voltage which may explain the point of inflection in the curve at about 100 kv. For curve *a* increases in potential took place at both ends of the line and the wave was modified by the constants of the impulse generator so that no attempt will be made to draw any conclusion.

As the impulse generator is moved out along the line and successive oscillograms taken for increasing distances, data bearing more directly on the attenuation

problem will be obtained without the complication of the effects of the ends. Increase in potential of the impulse generator above the corona voltage will aid in establishing the laws governing attenuation.

Surge Impedance

Some preliminary work has been done in an effort to determine the surge impedance of the line. When the line is closed through resistance equal to the surge impedance no reflection will take place. By taking oscillograms of the voltage across the resistance its value can be adjusted until the reflection disappears. Two such oscillograms are given in Fig. 15. Fig. 15A shows a positive reflection indicating that the 530 ohms used was too high. On the other hand Fig. 15B with 320 ohms seems to give a negative reflection, indicating too low a resistance.

This method of determining the impedance does not give a very accurate result but does show that the surge impedance lies between 320 and 530 ohms. The

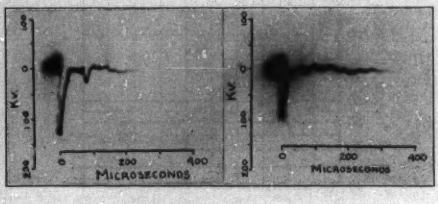


Fig. 15—Volt-Time Curves on Water Tube Resistor Used in Determining Surge Impedance

a. 530 ohms
b. 320 ohms

calculated value of just the middle wire alone and one ground wire is 500 ohms. The effect of the other conductors is to lower the calculated value. The surge impedance may also be lowered by corona, and it is hoped to be able to secure data which will determine this point.

Other methods which will give a better determination of the proper value of surge impedance are being developed and will probably be described in later papers.

With this line it will be possible to determine the effect of stopping the ground wire some distance from the station and also the effect of additional ground wires near the station. The magnitude of voltages and currents will be studied with their relation to ground resistance of the tower footing. Thus, it is expected to be able to give definite information regarding the benefit to be derived from additional ground wires from the standpoint of the effect on the surge impedance. It will also be possible to check the so-called theory of the short circuited secondary with regard to the effect of the ground wire on traveling waves.

Induction With Other Conductors

The induction on the top and bottom wires was measured with sphere-gaps with the impulse being applied to the middle wire. In one case with 216-kv. crest applied at Pittsfield, a potential of 31 kv. was measured between the bottom wire and ground at tower 279, all of the disconnects being closed at tower 279.

In Table I are given the results obtained by applying full waves and chopped waves at tower 279 and measuring the voltages in the respective conductors at the Pittsfield end, using 25-cm. sphere-gaps. The disconnects at tower 279 were open.

TABLE I
INDUCTION ON TOP AND BOTTOM WIRES
IMPULSE APPLIED ON THE MIDDLE WIRE

Potential at Pittsfield	Full wave 205 kv. applied	Per cent	Chopped wave 186 kv. applied	Per cent
Middle wire...	240	100	198	100
Top wire.....	56	23.4	40	20.2
Bottom wire...	45	18.7	29	14.6

No conclusions are being drawn from these results, as they should be considered as being preliminary, but they give approximate results which should be of value. Calculated values based on change in the electrostatic field assuming that the ground is at the surface of the earth give 23 per cent and 21 per cent for the top and bottom wires respectively. These values agree quite satisfactorily with the test results.

It is interesting to note that the carrier wire protective gaps arced-over each time an impulse was applied to the transmission line.

Apparatus Characteristics

The effect on the incoming impulse of choke coils, lightning arresters, bushings and transformers, outgoing circuits, and the effect of station bus work is to be studied. The work with regard to all other apparatus except the lightning arrester is in too preliminary a state to give any results in this paper. Some very interesting results on an oxide film arrester are available.

Oxide Film Arrester

Since voltages comparable with the maximum allowed by the line insulation of six disks were not available it was decided to reduce the size of the arrester to be tested to such a magnitude that it would have a potential applied comparable with what it might get in service.

The crest voltage shown in Fig. 10 for the full wave with the six microsecond front is 270 kv. This voltage represents about 25 per cent of the impulse arc-over of seven disks which is the normal insulation of a 110-kv. line. Since it is desired to have the voltage applied to the arrester in proportion to that which the normal line insulation in service would permit, an arrester should be used having a rating equal to 25

per cent of 110 kv., which is approximately equivalent to a 25-kv. nongrounded arrester which was chosen for the tests.

The oscillogram taken across the arrester is shown in Fig. 16, together with the wave of Fig. 10 without the arrester, as shown by the dotted line. The potential has been reduced by the arrester to 135-kv. crest and its duration has been greatly decreased. The voltage drops almost instantly to 70-kv. crest and subsequently decreases at a rapid rate. Therefore the arrester has changed a wave of dangerous potential and comparatively long duration into one of much shorter duration with a crest voltage safely below the strength of the parallel insulation.

Because of the fact that the arrester has absorbed some of the energy of the surge the reflected voltage is materially decreased. It is interesting to note that the first reflection peak due to the inductance of the impulse generator has not been changed much except

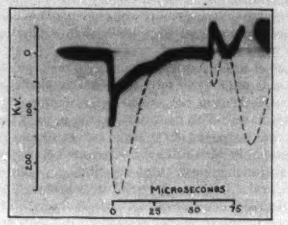


Fig. 16—Arrester Performance Characteristics on 6-Microsecond Front

Dotted curve—impulse at oscillograph without arrester. (See Fig. 10)
Oscillogram—taken across 85 *O F* cells

in sign but the wave after leaving the arrester was sufficiently reduced in energy so that it was unable to charge the capacity of the impulse generator to a very high potential.

Oscillograms showing the arrester performance for a wave of steeper front are given in Fig. 17. This wave is obtained by shorting out the inductance coil of the impulse generator and it is found that the voltage at the oscillograph end of the line rises to a crest value of 285 kv. in an average time of 0.5 microseconds as measured on the time scale of a 100,000-cycle timing wave.

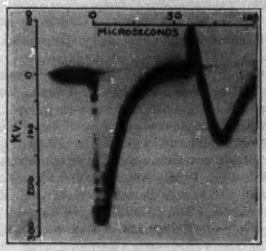
The potential across the arrester as shown by the oscillograph was 145-kv. crest. All voltages measured by the oscillograph were checked with the sphere-gap. It is felt that the two wave-fronts used in these tests are comparable to waves which would be found in practise.

The effect of the arrester on the chopped wave of Fig. 11 has been determined and the characteristics are similar to those of Fig. 16 except that the arrester discharges only for the length of time that the voltage is impressed which is a much smaller time than in the case of Fig. 16. Since the front of the wave is not affected by chopping the tail the potential allowed by the arrester is the same in either case.

Summary

1. Theory of traveling waves is being checked experimentally, demonstrating the existence and magnitude of reflections, both for open ended lines and for lines closed through gaps or combinations of inductance and capacity. A simple discussion of the theory is given.

2. The effect on incoming waves of various pieces



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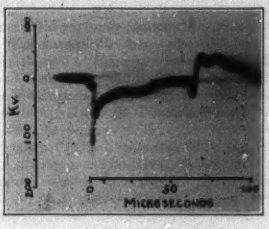

B

Fig. 17—Arrester Performance Characteristic on a 0.5-Microsecond Front

a. Without arrester
b. Across 85 *O F* cells

of apparatus, such as lightning arresters, choke coils, transformer bushings, and the effect of busses or other transmission lines connected to the bus are being studied.

3. The effect of ground wires on attenuation is being investigated which includes the effect of additional wires near the station as well as the effect of stopping the ground wire several towers out from a station.

Acknowledgments

The authors wish to express to Mr. F. L. Hunt and others of the Turners Falls Power Company and to Mr. Granville Whittlesey of the Pittsfield Electric Company their appreciation for the co-operation which

they have given in the investigation and without which the work could not have been done.

The work of the investigation has been carried on by Messrs. E. J. Wade, W. J. Rudge, Jr., O. Brune, and T. Brownlee. These men have all contributed largely in the design and construction on the impulse generator and the successful design of the oscillograph circuits and the operation of the equipment.

The authors express to these men their appreciation for the service they have rendered both in connection with the investigation and the writing of this paper.

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Discussion

C. L. Fortescue: I should like to call Mr. McEachron's attention to the initiating arrangement in Fig. 2. I don't know whether Mr. McEachron is aware that Mr. Harrington, who died recently, first used that circuit, and described it, I think, in the *Physical Review* in 1926, and later on in the *Electrical World*. We ourselves did not use it in the Rankin-Wilmerding tests, but it has been used in laboratory tests by the Westinghouse Company for some time.

I want to say that I think Mr. McEachron's work is of great value, but I don't agree with Mr. McEachron that we can find out all we want to find out about lightning surges from such tests. There are two things we want to do: We want to find out how the line behaves with traveling waves, but we also want to find out how steep a surge we can get from actual lightning, what its value may be if it is not chopped, and how long it lasts and the rate of diminution from its crest value.

Now Mr. McEachron's waves, or for that matter any wave we can place on a transmission line artificially at the present time, will not give the attenuation for a very high lightning wave, for the reason that the potential of a very high lightning wave goes up beyond the corona point of the line, and we have no figures for the attenuation in such a wave due to corona. We suspect that it may be very great.

I may say that we had to stop our Wilmerding-Rankin experiments before we got all the information we should have liked to have had. Last year we installed an experimental station in Tennessee using the new Norinder oscillograph. We have two locations there. This year we are extending those tests to get more authentic information on lightning. We are putting in three other cathode ray oscillograph stations. One is now in operation, or will be in operation in a few days. It is in Northern Illinois. We expect to have one in West Virginia, and another in New Jersey.

In the New Jersey tests we have had the good fortune to be able to obtain the use of a portion of a line which is now in construction, a 220-kv. line, which has the highest insulation of any practical line that has heretofore been made. It will be insulated with 18 units, and we are going to have a portable million-volt surge generator, and a portable cathode ray oscillograph down there to carry out these tests along the line that Mr. Eachron has been doing. We expect to get information on the actual value of the surge impedance of a transmission line, the mutual coefficients between the individual members of transmission lines, and we want to check up on surge impedance of tower footings, and so forth.

In the case of tower footings we measure the ground resistance by various schemes, but we have no assurance that the values we obtain by measurement are the actual surge impedances. We must find that out. This quantity is an important factor in line protection, and by means of the surge generator and the cathode ray oscillograph, as Mr. McEachron has used it, we expect to be able to obtain much important information.

I believe, and I have so expressed myself from time to time, that tests like these described by Mr. McEachron will be far more fruitful of real information than tests made in the laboratory with laboratory models. There are various factors that enter into the problem when using laboratory models which are generally ignored, but which are of vital importance. For example, the model may be reduced to one-hundredth the size of the cloud and actual transmission line, but the other dimensions are reduced in the same proportion, and therefore the time between reflections is reduced to one-hundredth; so that while before you were dealing with quantities like two microseconds between reflections at the tower points, now you will have to deal with quantities of the order of one- or two-hundredths of a microsecond. When dealing with quantities like that, cathode ray oscillograph will follow, but the photographic plate may be too slow; consequently all those values which are of so much importance are masked, and you get an answer which is incomplete and therefore false.

Now doing the work on an actual line in the field, doesn't entail a great deal more cost if you have a line which is in process of construction, and you obtain all these results unmasked. They are the actual facts that occur on transmission lines.

S. M. Hamill, Jr.: I should like to ask Mr. McEachron if the inductive reactance or the capacitance of the oscillograph is large enough to consider when you compare it with the constants of the remainder of the circuit, or is it so small that it is negligible?

Edward Beck: As the authors have pointed out, the way to investigate the behavior of transients in electrical circuits is to make the transients at will and then observe them at leisure with the proper tools. It is equally necessary to find out what kind of transients may occur. Probably many types of transients occur in service, caused by lightning. Possibly all or perhaps only a few of these types are hazardous to insulation. In order to determine the kinds of transients which may be expected on actual lines, tests are necessary such as were commenced last Summer

and described before the Winter Convention of the Institute, particularly in the paper by Fortescue, Atherton, and Cox. This information augmented by data collected in such investigations as described by McEachron and Goodwin will construct a comprehensive picture of the lightning problem. However, this work will take time, particularly as the greater part of the burden must be borne by the manufacturers. The chief beneficiaries of investigations of transient phenomena are the operators, and the solution of their problem could be much accelerated if they would take an active interest in it.

These investigations again indicate the invaluable aid given by the cathode ray oscillograph. Several years ago tests similar to the ones described in this paper were carried on in Germany and described by Binder. The cathode ray oscillograph was not then available and the determination of wave shape was exceedingly laborious and left considerable chance for uncertainties. The oscillograph opened up the possibilities of thorough investigation of these transients.

The authors state in the beginning of their paper that they believe that it records the first application of impulses to an actual high-voltage transmission line. Because of the importance of this work and the comprehensive way in which it is being carried out, it will not detract in the least from its value to call attention to the earlier investigations which have been carried out along these lines. Besides the German work already mentioned, the Westinghouse Company undertook similar work on a 25-kv. line borrowed from the Duquesne Light Company in Pittsburgh in 1926. This work has been mentioned in the paper by Fortescue, Atherton, and Cox, already referred to, and is described in detail, showing oscillograms secured, by A. M. Opsahl in the March 26, 1929 issue of the *Electrical World*. In this investigation the behavior of artificial transients in a line 4.7 mi. long was studied, the effect of protective equipment including choke coils, determined, and the surge impedance of the overhead line accurately measured. Unfortunately this work had to be abandoned because the use of the line was required by the Duquesne Light, but the investigation begun in a preliminary way on the so-called Rankin-Wilmerding line in 1927 will be carried further on a larger scale during the coming Summer.

W. L. Everitt: This study of traveling waves is a very good one, and a method that is going to grow in the analysis of transmission lines. It seems to me that some of the methods which the communication engineer has developed for studying the action of the lines may well be applied to these transients upon transmission lines.

I wonder if any study has ever been made in predicting these wave forms by measuring the effects over a wide band of frequencies, and then trying to relate them to these different wave forms? A steep wave form, of course, means high frequencies are present with a definite phase relation, and with high frequency we have higher attenuation than we do at low frequency and a variation in their velocity, and therefore one would expect a steep wave front to be gradually reduced as it passes along the line.

K. B. McEachron: I think it is clear from the paper that we recognize two problems. One problem is the determination of what lightning is and what kinds of waves may appear as a result of lightning, and the other is the determination of what these waves may do after they get on the line, which is the part of the problem we are investigating. Both problems must be solved.

At the present time in the Pittsfield laboratory, apparatus is set up to catch lightning on the transmission line, and find out what it looks like, and these other laboratories which have been set up are arranged to do exactly the same thing, but we felt that while waiting for lightning storms, which might occur infrequently, we might just as well be busy studying the effect of surges on the apparatus and on the lines.

With regard to the application of voltages above corona limit, the corona limit on the particular transmission line which we are studying is 110 kv. We applied 400 kv. We have at the present moment a million-volt generator, which is entirely portable, which will go very much above the corona limit.

It is interesting to note that three or four years ago a million-volt generator would have required a building much larger than this room. To-day we have a million-volt generator on a truck 18 ft. long and 7 or 8 ft. wide, truly portable. I am glad to give credit to Mr. Harrington for the publication of a circuit similar to the one shown in Fig. 2, as pointed out by Mr. Fortescue. The circuit shown in the paper was developed prior to the publication of Mr. Harrington's work.

The disturbing effect of the cathode ray oscillograph depends on the potentiometer used. The capacitance used is generally small and the disturbing effect not great.

As a result of going from section to section telling this lightning story, I find that the average engineer must have this material explained to him very carefully, or it is not understood, not because it is so difficult, but because it is something with which he is unfamiliar. This paper was therefore written in an effort to make the subject as understandable as possible to all.

Concerning the use of frequency bands, it is fundamental, I should say as a rule in research work, to set up as nearly as possible the actual situation which you want to study. It is true that we might apply to this line or other similar lines, certain frequency bands and obtain certain results. That would give us information, but with regard to the major problem the use of high-voltage transients seems to be about the best method of attack.

I think the reason why the engineer puts transients in one class and 60-cycle phenomena in another, is because he has dealt with 60-cycle phenomena all his life and he is familiar with having the same voltage and the same current here as he has a mile away, but when dealing with transients, a new conception is necessary, since the potential may be zero and 100 kv. at points only 20 ft. apart in the conductor.

Developments in Network Systems and Equipment

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Non-member Associate, A. I. E. E.

Synopsis.—*This paper presents in brief review the development of the automatic network system in the seven years since its introduction. It covers the interesting features of design found in the different systems, the changes that have been made in the design of the apparatus as a result of experience, some of the problems that have been faced in the application of the system, and finally the lines along which thought is now being directed in the future development of the system.*

THERE has been a great deal written in recent years in regard to the a-c. low-voltage network type of distribution system. The various requirements which caused such a system to be evolved, the fundamental principles of the design and operation of such a system, and the equipment used in it have been described in various technical papers. It is felt, however, that it would be interesting at this time to review the growth of the network system and to observe what developments have occurred in its design and to point out what changes have been made in the apparatus used as a result of the experience gained and in order to meet the requirements of various types of systems.

Prior to 1920 several attempts had been made to increase the reliability of the a-c. distribution system by connecting together the low-voltage sides of the distribution transformers. Fuses were generally used as the means of protection for clearing feeder and transformer faults. These fuses had several disadvantages and a more reliable method of protection was very desirable. The first improvement was made by the Puget Sound Power and Light Co. in Seattle,[3] which established a system having the distribution transformers connected to a common secondary through subway oil circuit breakers. These were equipped with relays which caused them to open only on a definite current flow in the reverse direction. The breakers were designed only for hand reclosing.

The Automatic Secondary Network System

At the time when this system was being installed, the United Electric Light & Power Co. was working on the development of what is now commonly called the automatic secondary network system. In this system the energy is supplied through high voltage feeders and is stepped down to the service voltage through what are now called network transformers. A large number of transformers is connected solidly to each feeder without any protective device. The low voltage side of each of these transformers is tied to the secondary cable mesh through a special circuit breaker called a network protector.

The secondary cables are tied together at every convenient point to form a solid network and service mains for consumers are taken from the mesh at satisfactory locations. A single-line diagram of such a system supplied by three interlaced radial feeders is shown in Fig. 1.

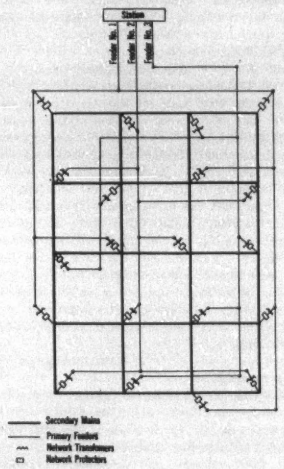

Fig. 1—Diagram of Small Automatic Secondary Network System Supplied by Three Interlaced Radial Feeders

In the United Company's system and in most others a common secondary network is used to supply both power and light. The transformers are connected in star on the low voltage side and have the neutral points solidly grounded and carried out as the fourth wire of a three-phase, four-wire system. Single-phase loads can then be connected between any phase wire

1. General Engineer, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.
2. General Engineer, Westinghouse Electric & Manufacturing Co., St. Louis, Mo.
3. "Merits of A-C. Underground Distribution," M. T. Crawford, *Elec. World*, Dec. 23, 1922.

Presented at the Regional Meeting of the South West District No. 7 of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

29-70

and ground while three-phase loads are supplied from the outside or phase wires.

No protection of any kind is installed for a fault in the secondary cables and trouble of this nature is depended upon to burn clear. In case of a fault on any feeder or any transformer the feeder breaker at the substation opens. At the same time all transformers tied to the faulty feeder are automatically disconnected from the secondary network. This is accomplished by the opening of the corresponding network protectors which open when the current flows through them in the reverse direction. The faulty feeder with its transformers thereby is completely isolated from the remainder of the system. The system will continue to operate, receiving energy from the other feeders. It is so designed that no part will be overloaded when one feeder is out of service.

In order to simplify the operation of the system the network protectors are designed to reclose automatically as soon as the fault is removed and the feeder is energized from the station. To make the control from the station complete the relays are made sensitive enough to cause the protectors to open when only the magnetizing current of the transformers flows through them in the reverse direction. This means that all of the protectors on a feeder can be opened by simply opening the feeder breaker at the station and can be reclosed by closing the station breaker and adjusting the voltage on the feeder so that it is slightly higher than and approximately in phase with the voltage on the network.

After ten years of study and experimental work the United Electric Light & Power Co. installed the first system of this type in the up-town district of New York City in 1922.[4] The principle of operation was entirely new and the equipment which made it possible had to be developed. It is interesting, therefore, to note what changes seven years of operating experience have brought about.

In this country 22 companies now have a system of this type in operation, 6 are installing such systems, 5 have decided upon it, and it is being given serious consideration in at least 20 other localities. The network has been introduced in four foreign cities, one in Cuba, one in Mexico, and two in South America, and is being seriously considered by one other South American and one European city. In all of these systems the fundamental principles of the original design have been retained but the details of the layout have naturally been adapted to meet local conditions.

Features of Various Systems

All of the companies operating or installing an automatic network system, with one exception, have followed the original plan of using the same secondary mains to supply both power and lighting loads. In the Dallas system, polyphase loads are supplied from a three-phase, three-wire secondary mesh while the lighting and other loads are supplied from an entirely independent three-phase, four-wire secondary grid of cables.[5] These secondary networks are connected to different taps on the same step-down transformer banks which are star connected on the low voltage side. The ties from the transformer to each network are through separate network protectors.

Practically all of the companies in this country that have installed an automatic network system have adopted the star connection on the low-voltage side of the network transformers. The Puget Sound Power & Light Co. in changing over from the original system to the full automatic system have still retained the delta connection. In order to balance the loads on the different phases of the feeders the secondary system is divided into sections and the lighting load in each section is supplied from a different phase with its mid-point grounded. The only other exception to the star connected secondary is in the system of the New York Edison Company which retained the two-phase system which it had in operation and established a two-phase, five-wire secondary network. It is planned to change this over gradually to a three-phase four-wire system.

All but one of the systems now in operation use radial feeders. Two systems started operation with a loop feeder arrangement but have since adopted the radial type. The outstanding advantage of the radial feeder is that no high-voltage sectionalizing devices are required on it.

Over 80 per cent of the network systems now in operation in this country are supplied by feeders that come in either the 13.2-kv. or the 4-kv. class and there is just about an even division in the number of systems employing each of these two voltages. One company is operating a network supplied by 27-kv. feeders. Practically every standard size of cable between No. 2 and 500,000 cir. mils is used in the feeders of the different systems but the 4/0 and 300,000 cir. mil sizes stand out as the most popular. Most companies using large size feeders taper them down to smaller sizes in the laterals which connect to the distribution transformers. The 2/0 cable size is by far the most commonly used for this purpose.

The size of cable used to make up the secondary mesh of systems now operating varies from 4/0 up to 500,000 cir. mils. The 500,000-cir. mil size is being used in about half of the systems in this country.

A great deal of thought has been given to the question of secondary cable sizes, particularly with respect to burning faults clear. Operating experience and tests have shown that for sizes up to and including 500,000 cir. mils any type of fault that may develop on three single-conductor cables drawn into the same duct or on one three-conductor cable will burn clear if enough current is available at the point of fault. Very

4. *Underground A-C. Network Distribution for Central Station Systems*, A. H. Kehoe, A. I. E. E. Journal, June 1924, p. 844.

5. "Standard Voltage A-C. Network," John Oram, *Elec. World*, September 1, 1928.

little is known for larger cable sizes. A few systems are using or are considering the use of two sets of cables of a smaller size in parallel instead of one set of 500,000 cir. mil size. The purpose is to reduce the time required to clear a very serious fault. However, most companies do not consider this arrangement necessary.

Three-conductor cable is used for the primary feeders on almost all of the systems. Two of those companies that are using three-conductor feeder cables change these to single-conductor cables before they reach the points at which the network transformers are connected. Single-conductor cable is used to form the secondary mesh of all systems with the exception of one in which three-conductor cable is used and one in which four-conductor cable is found. Single-conductor cable is popular because it minimizes the likelihood of copper-to-copper faults and it is easier to work on it while it is energized.

The most common practise up to the present time has been to use a solid secondary system with the connections from each network protector tied solidly to it. A few companies have, however, used sectionalizing boxes so arranged that it would be possible to divide

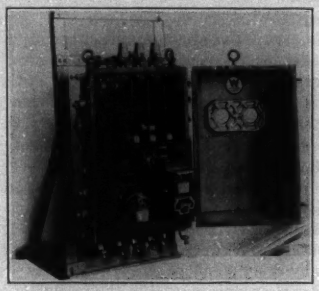

Fig. 2—Network Protector with Control Panel Closed

the secondary mesh were it found necessary to do so. The use of fuses other than those for back-up protection in the network protectors has been avoided by most systems. One company has, however, installed fuses at the end of each street main to facilitate isolating a fault. Besides the cost of installing and maintaining the fuse boxes this scheme has the disadvantage that when a fault develops in a cable it is probable that the fuses on each end of this cable will blow and a block of load will be dropped. Without the fuses the trouble would have cleared at the point of fault and unless this happened to be at a point from which a service was taken off no load would have been lost.

Relatively little use has been made of service boxes in the systems now in operation, the common practise being to splice customer service cables directly to the secondary network cables in the service manholes.

In the construction of transformer manholes, the question presents itself as to whether a fire wall should be used between the transformer and network protector. The arguments in favor of the use of a fire wall are that in case of a transformer failure followed by an oil fire there is no danger of this damaging the network protector and the cables between the protector and the street mains and possibly causing a number of secondary faults. The increased cost of construction is alone responsible for the fact that the fire wall arrangement is not more generally used at present. One system has, however, gone to the exact opposite construction by mounting the protectors directly on the side of the transformers.

It is almost universal practise in subway construction to use only one cable per phase between the transformer and the protector. The use of more than one cable per phase would increase the number of wiped joints needed to bring these cables into the apparatus and would be virtually impossible of accomplishment with the present size and design of standard equipment. As a result the size of cable required between the popular 300-kv-a. transformer and 1200-ampere protector is 1,000,000 cir. mils. This cable is on the low-voltage side of the transformer and should it develop a fault the network protector will open but it is probable that the feeder switch at the station will not open. This section of cable is, therefore, a part of the secondary and since it consists of a size on which a serious fault will probably not burn clear every precaution should be taken to prevent such a fault from developing. One company is using cable with only weather-proof insulation for the connection between the transformer and protector even in manholes that are subject to flooding. The cable is carried along the side wall of the vault and is held in place by porcelain cleats. Since the cables for each phase are spaced about a foot apart there is little danger of a phase to phase fault and the elimination of the lead sheath prevents a fault between it and the copper. The transformers and protectors are equipped with porcelain bushings. In order to insure a water-tight seal at the points where the cables enter the equipment, a soldered joint between porcelain and metal is made by a recently developed process. This same arrangement is being used in some of the vaults of a network now being installed in the South and it is being given serious consideration by another company.

Those companies that are using three-phase transformers in manholes under the sidewalk, design the manhole so that two pavement slabs may be removed to form a hatchway through which the transformer may be lowered into place. The slabs are normally calked in place with pitch and the usual entrance to the manhole is through a round manhole opening.

Network Protectors

The piece of apparatus that has made the automatic network system possible is the network protector. Though there was no extensive experience on which to

base the orignal design, it is worthy of note that with only minor modifications it has stood the test of service for a period of five years. During this period the protectors on one system were called upon to open or close a total of more than 200,000 times and the number of times that they failed to function properly amounted to less than four-tenths of one per cent. It should be remembered that in most of the opening operations included in the above the protector was functioning on the reverse magnetizing current of the transformers. The reports on the operation of the switches on other large network systems show even better results.

About a year and a half ago it was decided to make a complete redesign. A new network protector was brought out which retained all of the operating principles of the original unit but had incorporated in it the features that five years' experience had shown to be desirable. Two views of this type of protector mounted in a subway housing are shown in Fig. 2 and Fig. 3.

In the first place the arrangement of the equipment inside the protector was changed so that it became square in shape and then by replacing the sharp corners of the housing by flat portions the size became such as to require the minimum diameter manhole opening for installation. For sizes up to 1200 amperes the present design of network protector will pass through a 30-in. manhole opening. In keeping with the trend to structural construction, the new type of housing is made of 3/16-in. sheet steel with welded joints in place of the cast iron and aluminum housing that was formerly used. The housing is divided into two equal parts and the front half or cover is arranged so that it can be hinged to the base on either side, giving the maximum flexibility in the location of the protector in the manhole. By making the cover of the housing the same size as the base a large part of the protector is exposed when the cover is swung open which facilitates inspection and repair work.

Slate has been replaced by ebony asbestos as the material used in the main panel in order to reduce the possibility of breakage. The three main poles of the breaker are mounted on one side of the panel and the operating mechanism is mounted on the other side.

The circuit breaker type of construction was retained because of the good service that it had given in the first type of protector. A rather novel arrangement was adopted, however, by means of which the moving contact is made an integral part of the main copper bus and is simply laminated to give the needed flexibility. Both the stationary and moving main contacts are supplied with silver plates to reduce the contact drop. The arcing contacts are made of copper to avoid breakage of the carbon.

The advantages of a motor-operated closing mechanism over a solenoid type in the reduction of the jarring at the time of closing, the reduction of the current required to perform the operation and the consequent reduction in the size of the device required to break this current had been appreciated for some time. This type of device was not used on the original unit, however, because of the lack of a suitable low speed motor mechanism including a trip-free feature to permit the protector to open at any time without retardation due to the action of revolving flyballs. Such a mechanism was developed and the motor was adopted as the standard closing device in the new protector.

The action of the motor is transmitted to the breaker through a system of levers which has been so designed that the breaker is locked closed by the action of a toggle joint being drawn over center. As a result of this, no latching triggers are in motion while the breaker is closing and the danger of the breaker failing to remain closed is minimized. The only latch used in the entire design is in connection with the tripping mechanism and since this is not called upon to act at the end of the closing operation particular care may be

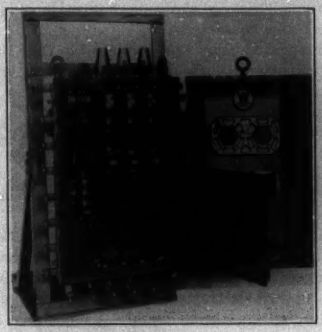

Fig. 3—Network Protector with Control Panel Swung Open

taken to design it for its particular purpose of releasing the toggle joint.

The original network protectors were designed with an undervoltage type of tripping device which had the advantage that the opening of the protector was a mechanical action and no energy was required at the time of tripping. The undervoltage trip gave splendid service on those systems where voltage conditions were steady but had the serious disadvantage that it caused excessive operation of the protectors on those systems which were subject to frequent disturbances of such a nature as to cause the voltage to drop to less than 30 per cent. The shunt trip device is more positive in that it does not cause the protector to open until energy is supplied to it by the action of the master relay. This same feature forms the disadvantage of the shunt trip for use in a network protector where the only energy available for tripping is from the system itself. If a fault were to develop in the transformer the only

voltage that could be impressed across the shunt trip coil would be the voltage drop from the protector to the point of fault plus the drop due to the fault resistance. The shunt trip device was adopted as the standard in the new protector but only after the design of the entire mechanism was made such that as little as 15 volts impressed across the shunt trip coil will supply the energy required to cause positive operation. It is not likely that a condition will ever present itself where less than 15 volts will be available for tripping.

One of the biggest problems confronting those who designed the first network protector was the development of a type of relay in which could be centered the entire control of the protector. The satisfactory manner in which this design was worked out is shown by operating records which indicate that only about 25 per cent of the very small amount of trouble experienced with the original protectors can be charged against the relays. It is true, however, that the design was rather complicated and in redesigning the foremost purpose was to simplify and at the same time retain and improve the operating characteristics.

A three-phase relay was developed to take the place of the three single-phase relays that were formerly used. The three-phase relay has decidedly better closing characteristics than the single-phase type in that it will allow the unit to close in case of a heavy load on one or two phases even though the third phase may be very lightly loaded. The three-phase relay also gives somewhat better performance under certain secondary fault conditions.

The detailed design of the three-phase master relay represents a decided simplification. It has three electromagnets acting on two disks fastened to a common shaft and each electromagnet is equipped with a potential, phasing, and current circuit. A single-pole, double-throw contact arrangement has been adopted which makes it possible to control either a shunt or an undervoltage tripping arrangement. A new and considerably simplified means of adjusting the relay to close its tripping contact on the desired value of reverse current has been introduced in the new three-phase relay. It consists of a straight spring which is fastened to the moving contact support. As the moving contact approaches the stationary "trip" contact the spring comes in contact with an adjustable stop screw and the force acting to close the contacts must be sufficient to overcome the action of this spring. By adjusting the distance through which the contact must travel against the action of the spring, the value of reverse current required to close it may be adjusted.

The current coils of the master relay are supplied by saturating current transformers instead of the reactive shunts that were formerly used. This change, together with the others mentioned above, has made it possible to design a relay having wattmeter tripping and approximately wattmeter closing characteristics and still eliminates the holding coil, adjustable slide wire resistor, fixed resistor, and auxiliary contactor which were required in the first network protector relays. A view of the three-phase relay is shown in Fig. 4.

There are system conditions under which pumping, that is, periodic opening and closing of the network protector, might result with a three-phase relay as well as with the three single-phase relays. These conditions may be caused by the use of three-phase feeder regulators or single-phase induction regulators connected in delta. The pumping may also be caused by differences in the time of opening of the various protectors on a feeder, the charging current of which is considerably higher than the magnetizing current. To prevent pumping under any of these conditions a single-phase relay known as a phasing relay is supplied

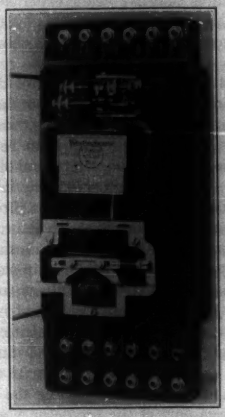

Fig. 4—Three-Phase Network Relay (Glass Cover and Terminal Chamber Covers Removed)

along with the three-phase master relay. In construction it is similar to one element of the master relay, differing from it in that it has only a phasing coil and potential coil and has a single throw contact arrangement. The contact of the phasing relay is in series with the closing contact of the master relay and both must be closed before automatic closing of the protector is initiated. When acting along with the master relay the phasing relay prevents the protector from closing under any conditions where the current that would flow after closing might cause the protector to reopen again and thereby absolutely prevents pumping action.

The entire design of the new protector has been worked out on the basis of maximum accessibility for maintenance and repairs. In the first place everything is mounted on the front of the panel and the entire protector has been built up on the unit principle. Any pole of the breaker may be removed as a unit, the

operating mechanism may be taken off as one piece without disturbing the rest of the apparatus and the tripping device may be removed as a unit. The relays and auxiliary control apparatus are mounted on a hinged steel panel in front of the main apparatus and all control leads are brought to a terminal block on this panel through special flexible wire. As a result this control panel can be readily removed and replaced. The method of connection is shown in Fig. 3. The terminal block of the relay is so arranged that the same set of terminal screws makes the electrical circuits to the various coils in the relay and connects the relay mechanically to the panel. If the terminal screw for any particular circuit in the relay is opened this circuit is broken and may be tested without removing the relay from the panel. This terminal arrangement is shown clearly in Fig. 4. The subway housing is equipped with two non-shatterable glass windows and directly inside one of these is mounted an operation counter and a position indicator while with the aid of a flashlight the fuses and breaker contacts may be observed through the other. Means are provided for electrically isolating the protector without removing the cables but should it be necessary to remove the cables it is possible to do so without breaking the wiped joint and special insulating sleeves have been provided to prevent grounding the energized cables against the housing during the operation of removing them. All of the control wiring has been so arranged that it can be easily traced.

Network Transformers

In the first network systems installed the transformers used were of the same standard type as those on the radial systems. Since that time, however, it has been found desirable to incorporate other features so that there has developed what is commonly called a network transformer particularly designed for service on this type of system.

Since the design of a network system is made up in such a way that a complete feeder may be taken out of service without overloading the remainder of the system under peak load conditions, one of the most important objections to three-phase transformers is removed because it is never necessary to operate a bank in open delta or to make a hurried replacement of a transformer that has failed in order to maintain service. The advantages of the three-phase transformer over three single-phase units are that it is somewhat cheaper in first cost, gives lower losses, takes up less space in the vault, gives a better vault layout in respect to the location of cables and such features, and has some advantages in respect to the mounting of auxiliary apparatus as will be brought out later. It is true, of course, that an opening must be provided in the vault of sufficient size to permit bringing in the transformer and the handling of the larger size unit is considerably more difficult than for the smaller single-phase transformers.

Although only one-third of the companies operating or installing network systems have thus far decided to use any three-phase network transformers, still this number is continually increasing and there is a definite trend toward the use of this type, particularly in those systems where new transformer manholes are being built and it is a relatively easy matter to provide the larger opening required.

The feeders supplying the average network system are interlaced and, therefore, supply transformers located at widely scattered parts of the area networked. It is possible for someone to close in manually a protector located at a point on the feeder remote from that on which a man is working and thereby to connect the feeder to the network. It is, therefore, considered desirable to locate at each transformer bank some device for grounding the feeder which supplies it. A man working on the feeder cable or any of the high-voltage apparatus connected to it can then make sure that the feeder is grounded in the immediate neighborhood of the section on which he is working. The first device brought out for this purpose consisted of a small switch mounted inside the transformer and equipped with an operating handle which extended through a stuffing box to the outside of the transformer tank. This switch was provided with an electrical interlock to prevent the possibility of its being thrown to the ground position with the feeder energized.

As operating experience grew it was found that it would be desirable to have some means of disconnecting each transformer from its feeder as an aid to testing and in order to keep any transformer out of service because of failure or for any other reason without keeping an entire feeder deenergized. This required some type of disconnecting device suitable for opening the circuit only when deenergized. In the first piece of equipment brought out for the purpose the disconnecting feature was incorporated in the grounding switch and made suitable for mounting inside the transformer. It was generally felt, however, that such a switch was too complicated for mounting inside the transformer, particularly for the higher voltages where considerable clearance is required, and for this reason a switch was designed for mounting on the outside of the transformer. The electrical interlock was retained as the means for preventing the feeder's being grounded while energized and in addition to this a mechanical device was arranged on the operating handle of the switch in order to insure still further against incorrect operation. Let it be assumed that the switch has been thrown over to the disconnect position and that while in this position the feeder breaker at the station has been energized. Since the electrical interlock derives its energy from the low voltage side of the transformer, it is ineffective when the switch is in the disconnect position and there is nothing to prevent the switch being thrown to ground even though the feeder is

energized. To minimize the possibility of trouble from this source the mechanical interlock or sequence controller on the operating handle of the switch was so designed as to make it necessary to return to the transformer position from the open position before it is possible to throw the switch over to ground the feeder. Should the feeder be energized the electrical interlock becomes effective as soon as the switch is thrown to the

Fig. 5—Three-Phase Network Transformer Equipped with Grounding and Disconnecting Switch

transformer position and prevents the final operation to ground.

In a three-phase transformer all connections between the phases are made inside the tank and it is only necessary to bring out the three-phase leads. It is very desirable to bring these leads out through a terminal chamber so as to make it unnecessary to get inside the tank to connect or disconnect the transformer. By increasing the size of this terminal chamber it makes a convenient place in which to mount a three-pole switch having the features described. A three-pole, three-position switch mounted in the high-voltage terminal chamber is shown in Fig. 5.

In the case of single-phase transformers the advantages of mounting the switches in the terminal chambers still exist but a somewhat more difficult problem presents itself since it is necessary to mount a two-pole switch on each single-phase unit. Such a switch increases the dimensions of the transformer a considerable amount and the design is, therefore, made such that the entire terminal chamber may be removed from the transformer while it is being installed. However, the taking off and replacing of these chambers does involve a considerable amount of work.

The San Antonio Public Service Co. decided on the use of single-phase transformers in the network system that is now being installed and rather than use a separate switch on each transformer adopted the plan of using a three-pole switch having all of the features previously described but mounted apart from the transformer. The switch itself is oil filled and is arranged with compound filled terminal chambers at the top and bottom. The three-conductor feeder cable is brought into the top terminal chamber while the two cables from each single-phase transformer are brought into the other. The delta connection of the phases is closed in this chamber and three leads are brought from it up to the switch itself. A view of this switch with the front plates removed is shown in Fig. 6.

In all types used thus far the high-voltage grounding and disconnecting switch is oil filled. In the first type mounted on the outside of the transformer tank the common practise was to mount directly below it a compound filled terminal chamber to prevent oil siphoning through the insulation of the cable. The most recent development has been to replace this by a special compound filled pothead wiping bushing such as that shown in Fig. 5.

Tap changers which may be operated by handles brought through the cover of the tank are supplied in network transformers. It has been found desirable in service to have some control of the ratio of the step-down transformers and the tap changer has been adopted as the simplest means of accomplishing this with the transformer disconnected from the circuit.

One of the important questions that presents itself to any company designing a network system is the value

Fig. 6—Separately Mounted Grounding and Disconnecting Switch

of reactance that should be used in the network transformers. The problem is fundamentally one of the operation of transformers in parallel having in mind that in general the more fully the transformers share the loads on the system the more satisfactory will be the operation. The higher the impedance of the transformers as compared to the impedance of the secondary tie cables, the better will be the load division. Since a network is designed so that any feeder and all of

the transformers connected to it may be taken out of service and left out indefinitely, it is rather important that the remaining transformers share the extra load as much as possible. Besides improving the load division the high reactance also has the advantage of reducing circulating currents between feeders which in turn leads to more stable operation of the network protectors, particularly under light load conditions. These two considerations have caused the large majority of companies operating network systems to adopt the policy of using ten per cent impedance at the transformer banks.

The disadvantage of the higher reactance is that it increases the voltage drop. This is not as serious as it might appear to be on first thought, however, because the power factor of the network load is high at the time of full load when the drop is important and the drop caused by the current flowing through the reactance of the transformer is very nearly at right angles to the network voltage. In the average system the drop in voltage due to the addition of 5 per cent reactance will not be more than 1 per cent. However, the possibility of motor starting currents causing flickering of lights is increased.

If a company decides to use transformers having ten per cent impedance the next problem is to determine the most satisfactory manner in which to obtain the increased value. There are three methods that have been used up to the present time. The first method is to have the increased reactance built into the inherent design of the transformer by coil spacing and grouping. In the second method the increased value is obtained by the use of shunt iron paths in the transformer. As a third means transformers having standard reactance may be used and this can be raised to any desired value of the addition of small reactors in the cables coming out from the transformer.

Each of the three methods mentioned has certain advantages. Both the type of transformer having the ten per cent reactance as an inherent part of its design and that obtaining the increased reactance by means of a shunt iron path are self contained units while the third, or the method for which external reactors are required, has two distinct pieces of equipment. This means that the first two types give a unit that is somewhat simpler to install and this is particularly true in view of the fact that it is necessary to break the lead sheath of the cables over which the external reactors are slipped. Both of the first two have the important limitation, however, that the reactance is a fixed value and cannot be changed, while it is a relatively simple matter to change the reactance of the third type using external reactors. If the question were settled as to what is the most satisfactory transformer reactance to use in a network system and it was known that ten per cent at each bank represented the ideal condition, then this point would not be an important one, but this is far from being the case and it now appears that the conditions prevailing on each system and the design of the system will have a decided influence on the choice of transformer reactance. The feature of being able to change the reactance is also found of value in shifting load between banks if, for example, it is found that one transformer is carrying an overload with a feeder out of service while those around it are not carrying full load.

The type of transformer that has an inherent reactance of ten per cent maintains this reactance under short circuit conditions. In other words there is no saturation effect and the impedance drop of the unit when plotted against load forms approximately a straight line. In either of the other two methods, the shunt iron path or the external reactors saturate at about double normal load and have little effect at the time of short circuit. It must be remembered that the chief functions of the increased reactance are to improve load division and to reduce circulating currents. It is felt that for a system having the average load densities it is an advantage to have the added reactance saturate at the time of a short circuit in the secondary system so as to increase the current flowing into the fault and to facilitate burning it clear in the shortest amount of time. It may be that in systems having exceptionally high load densities the currents that could flow might reach such values as to prove destructive to cables, in which case the current limiting feature of the first design would prove advantageous.

Taking all factors into consideration it would appear that the use of transformers having not more than five per cent inherent reactance, with any additional amount desired obtained by means of external reactors, is the most satisfactory arrangement for the system having usual load densities.

Fig. 7 shows the low voltage side of a single-phase network transformer. A rather convenient method of mounting the external reactors is illustrated. These reactors also have applications elsewhere on the system. For example, it is rather common practise where two transformers are located in the same vault to install reactors on the cable bus joining these two transformers and to make these of such a size that they will introduce an impedance which is approximately equal to the impedance of the usual cable tie between banks located in different vaults. This improves the loading of the remaining transformer when one bank in the vault is taken out of service and also minimizes the possibility of trouble due to circulating currents. If the customer's load is split in such a way that the motor load is largely on one bank with the lighting load on the other, the reactor between the two will reduce the effect of motor starting on the lighting.

Problems of Design and Operation

The highest tribute that can be paid to the network system is that it has given the performance expected

of it by the companies that have installed it. There have been problems encountered and a review of some of these is of interest.

With the combined light and power system using the star connection it is impossible to supply the now commonly accepted standard voltages for both power and light. If the voltage to neutral is chosen as 115, then only 199 volts will be available between phases while with 120 volts from line to neutral 208 volts exist between phase wires. A large amount of discussion has been devoted to the relative merits of these two combinations and up to the present no definite decision has been reached. A few fundamental ideas seem to be crystallizing, however, which may point to the future trend. In the first place it is being rather generally

Fig. 7—Single-Phase Network Transformer Using External Reactors

accepted that the nominal voltage to neutral on the network system should not differ from the lamp and appliance voltage elsewhere on the system. The second point that operating experience seems to be establishing is that no very serious troubles are experienced from the operation of a network system, with the improved voltage regulation found in such a system, on either of the reduced values of voltage mentioned above as long as some effort is put forth toward educating customers on the new type of service that they are receiving. There are always some isolated cases of troubles, particularly when changing over apparatus which has been operating at higher voltage to the lower value but these can be readily corrected by minor changes in the apparatus itself or by the use of booster auto-transformers.

Of those companies that are operating star connected combined light and power secondaries, it is found that just about the same number are maintaining 120/208 volts at the customer's service switch as those maintaining 115/199 volts. There are a few companies that are maintaining at the same point a voltage of approximately 117/203 volts. A check was made a short time ago of some of the companies that have been operating the largest networks for a long enough period of time to have obtained some valuable information on the question and it showed that three of the companies maintaining 120/208 volts find it unnecessary to use any boosters in supplying the motor load. The same is true of two of the companies holding 117/203 volts. On five of the large 115/199 volt systems an average of about 40 per cent of the motor load is supplied through booster transformers. In this latter class some companies have adopted a very liberal policy in regard to the application of booster transformers.

Another problem that presents itself is the determination of the size of motor that should be taken on the network system. In general, it might be said that any motor of a size normally found in the downtown area of a city may be supplied from a properly designed network. Special starters must be supplied on the larger motors to divide the starting current into increments that will not cause flickering of the lights. Various companies have adopted different limits in the starting current regulations but the trend seems to be to a limitation of 200 amperes per step at one-half second intervals. Test information might show that higher limitations would give satisfactory results on many systems.

One company started operation with the network feeders brought from the busses of two different stations. It was found that the variation in voltage conditions on the two busses was sufficient to cause excessive operation of the network protectors during periods of light load on the system. The condition was corrected by tying the two stations more firmly together on the high voltage side.

Some cases have been noted of excessive operation of network protectors in the immediate vicinity of elevator motors at the time of light load on the system. This may be caused by regenerative action of the elevator motors or by the relative voltage conditions of the feeder and secondary system being such as to cause the protector to remain open under the normal load being carried by the system but causing it to close as the starting current of the elevator motor lowers the voltage in the secondary grid. In either case the trouble can easily be corrected by raising the reverse current tripping adjustment of the relay. This will probably prevent this one protector from opening on the reverse magnetizing current of its own transformer but it will still trip when the feeder breaker at the station is opened because after all of the other protectors on the feeder have opened the magnetizing

current of all the transformers connected to the feeder will be supplied through the one protector. When a sufficiently high adjustment cannot be obtained with the standard spring a special heavy spring may be obtained.

When the operation of network systems was first started it was felt that where feeder regulators with line drop compensators were used it would be necessary to supply some means of cross-compensation to insure stable operation of the regulators. It was believed that any action of the regulators that would set up a circulating current between the feeders would bring about a cumulative action which would cause some of the regulators to rotate to the full boost and others to the full buck position. To eliminate the possibility of this happening several simple means of cross connecting the line drop compensators were developed and some of these are now in service giving very satisfactory operation. It has since been found that if the line drop compensators are set properly it is possible in most systems to obtain stable operation of the regulators and proper voltage in the secondary system without tying the regulators together in any way. Several systems are now operating the regulators on their network feeders without any cross connections and have experienced no trouble.

Present Trend of Development

Up to the present time only one company, the Knoxville Power & Light Co., is operating an overhead automatic network system of any appreciable size. The question of using networks in such areas has been given serious consideration by other companies and it is believed that decided advances will be made in the development of such systems within the next few years.

The first two of a group of large government buildings are now under construction at Washington. The automatic network system has been chosen as the most satisfactory from an engineering standpoint as well as the most economical type of system that could be used to supply these buildings. Each building will have a secondary cable grid tying together the transformer vaults located in that building and the secondary networks of all of the buildings will be connected solidly together by low-voltage tie cables.

This development is only the forerunner of a widespread application of network systems inside large buildings if the amount of thought being given the problem at the present can be taken as a criterion. Directly connected with this subject is the one of vertical distribution in tall buildings which is being actively considered by two companies at the present time. Preliminary plans call for the location of transformers on several different floors, the group of transformers on each floor being connected into a network which may be permitted to remain independent or may be tied in on the secondary side with the networks on other floors. Along with this development, consideration is being given to every detail of the vault layout and the design of the apparatus to minimize the possibility of a transformer explosion and to localize trouble of any kind.

The a-c. network is being given serious consideration as a system of distribution for large manufacturing plants. In this field such a system should find its greatest application in those factories where the process of manufacture is such that an outage of even a short duration has serious results. A special type of automatic network system is being considered for one manufacturing plant of this type at the present time.

The network system lends itself exceptionally well to the scheme of synchronizing generators at the load. One company that is operating a large network system is now making a thorough investigation of the feasibility of such operation.

The network stands today as a type of system that has stood the test of service and has given the results expected of it. In general, the continuity of service in practically every installation has been as dependable as the source of supply and most thought is now being given to possible means of increasing the dependability of the source.

Discussion

Frank E. Johnson, Jr.: Considerable difficulty has been experienced by the New Orleans Public Service in securing fuses of the proper characteristics for transformer protection. The function of these fuses, as I understand it, is to blow upon failure of the network protector to open for a primary cable fault involving back feed currents of a magnitude large enough to damage the transformers, and also for secondary cable faults (during which the protector is expected to remain closed) where such faults are of sufficient duration or severity to damage the transformers. Although it is necessary that as much current as possible be fed into a secondary fault to clear it, nevertheless this should not be entirely at the expense of the nearby bank, since a transformer failure is likely to involve a feeder outage and hence a reduction in network transformer capacity at a time when it is most needed. It would, therefore, appear advisable for this condition that the nearby bank or banks be properly cleared by fuses.

These two types of faults work against one another in so far as fuse size is concerned, because the ideal fuse to take care of the former fault should blow at current values just beyond the full-load rating of the transformer, while for the latter fault the larger the fuse the better are the results. The problem is further complicated by the different conditions brought out in the paper which exist for short circuits through the various types of inherent or external reactance transformers. In the fuse design it would seem then that preference should be given to primary feeder faults since in a carefully constructed secondary system, utilizing the proper type of cable, little or no trouble may be expected. There have been no secondary cable failures in the New Orleans system. Since various values of fault current will be encountered, a curve should be plotted between fuse current and the fuse blowing time. This curve, for the ideal fuse, would lie just below, but parallel to, the safe thermal-time curve of the transformer the fuse was designed to protect. Unfortunately, due to the great difference in thermal characteristics between the fuse and the transformer, this condition can only be approached. However, since the various makes of network transformers having the same inherent impedance appear to have

approximately the same thermal-time curves, it does seem reasonable to believe that some standard fuse could be devised that would be interchangeable in all manufactured protectors to be used with a given size transformer. At present each new model of protector includes a different type or size of fuse.

A very helpful feature in our network protector operation has been the fine spirit of cooperation that has existed between the manufacturers and the New Orleans Public Service, Inc. Weak design features have been promptly rectified by immediately submitting detailed reports of any trouble experienced to the manufacturers. I think that a similar procedure on the part of all users of network protectors will bring the equipment to the highest state of reliability in the shortest space of time.

I should like to ask if any occasion has arisen where it was desirable that a tap changer on a network transformer be used so that the ratio on a particular bank or banks was different than elsewhere on the system, and if any trouble from circulating currents was then experienced.

In connection with allowable motor-starting currents, I will mention the fact that a 100-hp. slip-ring induction motor, with drum control switch, and used for hoisting purposes, was recently connected directly to the 500,000 cir. mil 1-conductor mains with entire absence of light flicker in the vicinity. The starting current, as read on a well damped ammeter, was 325 amperes. The installed transformer capacity (10 per cent impedance transformers), within a radius of 600 ft. from the building, is 2850 kv-a.

T. J. Brosnan: We have appreciated for some time the desirability of using fuses having the characteristics described by Mr. Johnson for back-up service in the network protectors. A considerable amount of work has been done on the development of such a fuse but as yet the results have not been very encouraging.

There are several companies that have had occasion to use the tap changers to change the ratios of all the transformers on the system but I do not know of any case where some of the transformers have been changed a greater or less amount than the others.

Standard Voltage A-C. Network

BY JOHN ORAM[1]

Non-member

Synopsis.—*The object of this paper is to present a description of the Dallas (Texas) underground network system. This system is of interest since it is directly opposed to the general trend of furnishing non-standard voltages in underground systems. The reasons for its selection and its advantages over the three-phase four-wire system are presented. A brief description of the installation of equipment in vaults and manholes is given. Another feature of this system is the use of standard reactance transformers together with an external reactor, shunted by a fuse, for purposes of overload protection to the transformers. In addition to the regular network this system has a single-wire manhole and vault lighting network which is also used as a control wire for turning on and off multiple street lamps. Another feature of this system is a complete underground telephone system with a telephone outlet in each manhole and transformer vault. This telephone system is also connected to float switches in transformer vaults which automatically notify the trouble office in the event of high water in a transformer vault. The routine tests made in connection with the operation of the network system are described and the method of locating primary faults is given in detail. Complete records are kept of the installation of conduit and cable. Photographic records of construction, operation, and cable failures, etc., are also kept. The record of this network system both as to efficiency and as to continuity of service is excellent.*

* * * * *

THE principal characteristic of the Dallas (Texas) underground secondary network system is that it provides standard 115-volt service for lighting consumers and 230-volt three-phase service for power consumers. This is done by adding an 18-volt secondary coil to the standard 115-volt transformer coil and bringing out three leads on each transformer. With three transformers connected in "Y", the arrangement gives 115 volts for lighting and 230 volts for power service with a common symmetrically grounded neutral.

On account of the high primary voltage used and the relatively short distance of approximately one mile from generating station to load center no feeder regulators are required. The range of voltage regulation is about five per cent from no load to full load and the control of the generating station busbars by Tirrill regulators is adequate for this purpose. Since the major portion of the load follows quite closely that of the underground system and all other lighting service is controlled by individual feeder voltage regulators in substations, no interference with the remainder of the system is caused.

The Dallas network system covers an area of approximately 0.75 sq. mi. and consists of five 13-kv. underground feeders serving eighty-two 300-kv-a. transformer banks which supply two sets of secondary network cables, one being the lighting network and the other the power network. Automatic network units are used between transformers and the secondary network system. The principal load in the underground district consists of office buildings, theaters, department stores, and hotels. The average load density is 20,000 kv-a. per sq. mi. The greatest concentrated load is a hotel which consumes an average of 10,000-kv-a. hours every 24 hr.

The network system at present is designed for a normal maximum loading of 80 per cent on each transformer bank and an emergency loading of 100 per cent. This arrangement will permit the removal from service of any one of the five feeders without overloading the remainder of the system. The normal voltage in the secondary mains is 115 volts for lighting and 230 volts for power. The emergency voltage will average 110 and 220 volts for lighting and power, respectively.

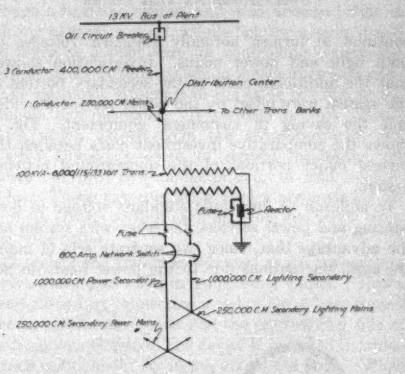

FIG. 1—SCHEME OF DESIGN OF DALLAS UNDERGROUND NETWORK

The system is composed of three-conductor, 400,000-cir. mil 13-kv. primary feeders, 250,000-cir. mil 13-kv. primary mains, 100-kv-a. 8000-115/133-volt transformers, automatic network units, 250,000-cir. mil secondary power cables, 250,000-cir. mil. secondary lighting cables, and a 500,000-cir. mil bare neutral.

The decision to use two sets of mains was made after a careful consideration of investment costs, operating costs, and quality of service to be rendered. It was primarily because of the superior quality of service provided that the seven-wire system was adopted

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Presented at the Regional Meeting of the South West District of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

Previous to the installation of the underground network system the customers of the Dallas Power and Light Company had been furnished with standard 110-volt lighting and 220-volt power service. With the thought of good service and public relations in mind, some secondary network system was sought which would

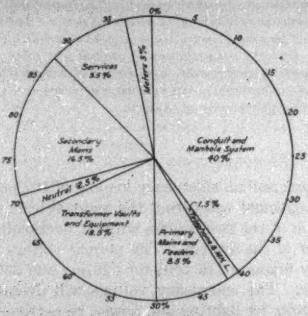

FIG. 2—CHART SHOWING DISTRIBUTION OF COSTS OF DALLAS UNDERGROUND NETWORK SYSTEM

continue to furnish normally standard voltages on both light and power mains. Investigation showed that the additional costs of the secondary portion of the system were offset by the better service provided and the saving in customers' equipment. Fig. 2 shows the comparative investment costs between the several major portions of the underground network system.

In addition to furnishing standard voltage to both lighting and power services, the seven-wire system has the advantage that, since two separate sets of mains are used, fluctuations due to the power loads do not

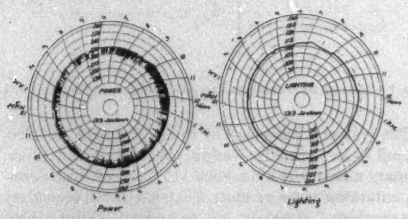

FIG. 3—TWENTY-FOUR HOUR VOLTAGE CHARTS ON POWER AND LIGHTING MAINS TAKEN AT SAME LOCATION

cause flickering of the lights. The absence of flicker on the lighting system may be observed from the voltmeter charts shown in Fig. 3, taken simultaneously on lighting and power services in the same distribution box. There is no limit to the size of motor that may be operated from the network system, and motors up to and including 30 hp. may be started directly across the line without the use of starting compensators. In this way the economy resulting from the simplicity, high performance, and balanced design of the common squirrel-cage motor is retained, together with greater flexibility in application where remote control is desired.

Another feature of this system is that standard reactance transformers are used with an external reactance installed in the neutral leg of each transformer, so arranged that the reactor is normally shunted by means of a fuse, thereby providing good regulation and lower secondary losses during normal operating conditions. When, however, an overload occurs on a transformer bank, due to feeder outage or other abnormal conditions, the fuses blow and thereby insert the external reactance in the circuit and relieve the transformer of part of its load.

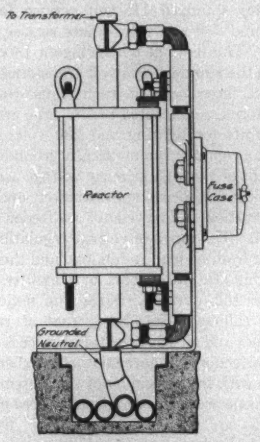

FIG. 4—FUSED REACTOR FOR TRANSFORMER PROTECTION

The transformers are installed in vaults either built by the Dallas Power and Light Company, or if in building basements they are in accordance with Dallas Power and Light Company specifications. In either case the only opening to the vault is through a manhole cover, 20 in. in diameter, built into a removable concrete panel 4 ft. wide, by 5 ft. long, placed in the sidewalk next to the curb. The castings in the panels allow men and small equipment to enter the vault with ease. If necessary to replace transformers or network switches the panel is removed. Each vault is ventilated by two 28-in. square ducts with openings in opposite ends of the vault, one near the ceiling, the other near the floor. Both ducts terminate on the sidewalk next to the curb and are covered with gratings.

The equipment in a transformer vault consists of three transformers, two network switches, primary

cables, secondary cables, three reactors, a condulet each for telephone and lighting service, and miscellaneous equipment such as a ladder, manhole guard, and covering for cable trenches. The primary cables enter the vault in one of the lower ducts and are racked on wall insulators to the transformers. The transformers are placed on T-rails imbedded in the concrete floor. The secondary leads are trained vertically from the transformer terminals to the secondary cable trench in the floor. The secondary power and lighting cables extend in separate trenches to the wall on which the network switches are installed. The 1,000,000-cir. mil secondary cables are extended from the upper

Fig. 5—Typical Double Transformer Vault

terminals on the network switch to the secondary network in the street. The secondary cables are racked separately from the primary cables.

There are at present 84 submersible and 80 non-submersible network switches in operation on the Dallas system. Of the submersible units 72 are solenoid operated and 12 motor operated; all use single-phase relays in air bells which are mounted on the wall of the vault. This arrangement facilitates maintenance and provides better ventilation of the relays. No phasing relays are used on the foregoing units. Of the non-submersible units 78 are solenoid operated equipped with single-phase relays and phasing relays, and two are motor operated equipped with three-phase relays and phasing relays. Few difficulties in the operation of the network switches have been encountered from unbalanced loads and elevator regeneration. These have been overcome by using three-phase relays or otherwise.

Although secondary network system in general consists of 250,000-cir. mil lead covered cables on cross streets or between important loads, additional 500,000-cir. mil cables are sometimes installed. The secondary neutral is grounded in each transformer vault and serves as a primary neutral.

The secondary services are extended to the customers building line in 4-in. fiber conduits, one each for power and lighting services where both are required. The conduits are terminated in a distribution box placed in the outside building wall, or iron conduit of reduced size is carried up the building wall about 12 ft. and terminated with a condulet. A panel is installed in the distribution box and both power and lighting services are connected to this panel. If it is not possible to install the distribution box, the customers' outlets are grouped on the outside wall at the first floor ceiling line. The cables used for services are single-conductor, 600-volt, paper insulated, and vary in size from No. 6 A. W. G. to 500,000 cir. mil.

In addition to the secondary and primary mains in the duct lines there are telephone and manhole lighting cables with outlets in each manhole and vault. The manhole lighting cables are No. 6 A. W. G. and form a secondary network with 115 volts to ground which is used by the workmen for lights, blowers, soldering pots, and other general conveniences. This network is connected to the main secondary lighting network through a fuse and switch, and is so arranged that in addition to serving as a manhole lighting system it will also serve as a pilot wire for controlling a proposed multiple street lighting system in the underground area.

The telephone system is in the same conduit as the manhole lighting cable. It consists of two 26-pair cables which extend from the switchboard in the Trouble Division's office to central locations, and branch out into two-conductor No. 14 lead covered cables. These two-conductor cables are installed on the main streets so that each manhole and vault has a telephone outlet.

The telephone system is also used as a signal system to give warning of water in transformer vaults. This is accomplished by connecting a coil of No. 4 bare copper wire in a porcelain housing to the ungrounded side of the telephone circuit. The device is installed in transformer vaults about six inches above the floor. When water reaches a level of six inches or more the telephone cable is short-circuited and a light signals the operator in the Trouble Office.

In the operation and maintenance of the network system certain routine tests and inspections are made. The vaults are inspected regularly and all dirt, paper, and other accumulations removed from the ventilator pits and floor. Oil samples from all transformers are taken once each month and tested for dielectric strength. The oil level in the transformers is observed and if necessary oil is added. The oil in each transformer is filtered at least once a year regardless of its dielectric strength. Each network switch is inspected weekly and the number of operations recorded. An inspection of network switches is made each month.

They are cleaned and adjusted, and repairs are made if necessary.

To test the operation of network switches, feeders are removed from service once each week during the light load period, by opening the circuit breakers at the Generating Station. If the feeder is not automatically deenergized when the breaker is opened, a trouble man is sent out to see which network switch has failed to open. A signal system installed in each vault saves the trouble man much time in locating the faulty switch. This signal system consists of a 125-volt lamp installed in the ventilator so that it can be seen by the trouble man without getting out of his car. One terminal of the lamp is connected to ground and the other to a lighting phase through an auxiliary switch that closes only when both network switches are open. A signal lamp not burning indicates that a network switch is closed. If faulty the switch is locked open until repaired.

In the location of primary faults special equipment is used. The apparatus is installed at the Generating Station and consists of a 50-kv-a. 440/8000-volt step-up transformer and 8000-volt, 15-ampere reactor. The procedure followed in locating a fault is to manually lock open each network unit on the feeder in trouble, disconnect the feeder at the Generating Station and connect the reactor to the faulty conductor until it is broken down to ground. With a split-core instrument transformer and low reading ammeter the current sent out over the cable is traced to the fault location.

Detailed records of underground installations are kept under five general classifications, *viz.*: (1) Duct Line Records, which are in the form of drawings showing a plan and profile of the duct lines together with the location of all substructures adjacent thereto; (2) Manhole Records, which are also drawings giving the location and details of construction of each manhole; (3) Cable Records, which give the location, size, date installed, and other data pertaining to each cable in each manhole; (4) Transformer Records, which gives the purchase specifications together with a complete history of each transformer; and (5) Service Records, which is a section map of the city showing the location and size of each lighting and each power service.

Records are kept of all equipment failures which includes a detailed description of the failure together with a photograph of the faulty apparatus where possible. Included in these records are cable failures, transformer failures, network unit failures, etc.

During the year 1928 the average efficiency of the underground network system, as calculated from watt-hour meter readings at the Generating Station and those on customers' premises, was 95.5 per cent. The average annual power factor was 0.90 and the load factor 0.50. Approximately 36 per cent of the energy distributed was power and 64 per cent lighting.

The operation of the system since its installation has been successful. The voltage regulation has been good and there have been practically no complaints. The record with reference to continuity of service, like that for voltage regulation, has been satisfactory. The completed network system has been in operation nearly two years and during that time no customer has been out of service. A number of faults on primary cables has occurred, but in each instance the trouble has been automatically and successfully cleared without interruption to service. Secondary cable faults burn clear without noticeable effect on the service.

Automatic Reclosing of High-Voltage Circuits

BY E. W. ROBINSON[1] and S. J. SPURGEON[1]

Non-member Non-member

Synopsis.—*The experience of the Alabama Power Company with the reclosing of high-voltage circuits is described, this covering numerous installations on 22-, 44-, and 110-kv. circuits. No definite reclosing cycles are recommended but instead this paper deals with various typical applications of high-voltage reclosing oil circuit breakers on the Alabama Power Company system.*

Introduction

ALABAMA Power Company operates over a wide expanse of territory where the load density is generally low. About 1922 a large expansion program started which immediately developed the desirability of more primary substations, where the 110-kv. system might be tapped and voltage reduced to 44 kv., and of more switching stations where the existing 44-kv. lines might be interconnected. The cost of attendance at these primary substations and switching stations intervened, however, as a serious obstacle and brought about an active interest in automatic reclosing switching equipment which might allow the substitution of a daily or weekly inspection of such stations for continuous attendance.

General Discussion

No suitable reclosing relays or devices were commercially available at the time so the power company started a series of experiments and developments which resulted in 1922 in the application of a reclosing device to a standard 50-kv. oil circuit breaker controlling a 44-kv. feeder. This device is of interest as being a self-contained mechanism including a motor driven breaker operating device, reclosing relay, and lock-out relay. It required a minimum amount of power, could be operated from any available a-c. or d-c. source, and was applicable to any standard breaker by minor alterations. As originally built the interval between breaker operations was approximately one minute and the device could be set to lock-out after one, two, or three reclosures. The original mechanism was far from perfect but continued in operation for several years.

About this same time various manufacturers put on the market reclosing relays adapted for use with standard breaker operating mechanisms. Among these were the induction type, motor-driven drum type, and motor relay type. These were all more or less unreliable in the beginning, but they, and also the so-called "reclosing device" developed by Alabama Power Company, were rapidly developed and have reached a high degree of reliability. Considerable numbers of each type of reclosing installations were put in service and it is interesting to note than in 1928 all types showed better than 85 per cent of correct operations, and the two preferred types in excess of 95 per cent.

On the Alabama system originally there was a uniform period of one minute between breaker operations. Experience showed that this was satisfactory and subsequently the initial interval was reduced to 45 sec. and in some cases to 30 sec. Later intervals were also reduced in some instances to 45 sec. Originally two reclosures before lock-out were provided for and this schedule has been followed consistently. Two reclosures gave ample time for the controlled line to be restored to service if this were possible and there seemed to be no valid argument for a greater number as it was extremely unlikely that a line would show up bad on two trials and good on the third, while on the other hand, a larger number of reclosures prolonged system disturbances, vastly increased the duty on the breakers, and resulted in increased cost of maintenance and inspection.

The engineers of Alabama Power Company have not been impressed with the necessity nor desirability of a standard reclosing cycle for high-voltage breakers. Instead they have preferred to treat each case separately taking into account the character of the controlled line, peculiarities of the load supplied, possible short circuit duty on breaker, rupturing capacity of breaker for various duty cycles, and class of service to be maintained. In most instances the reclosing cycle is adjustable and in fact is changed from time to time depending on operating conditions. With proper consideration given these factors satisfactory applications will result. This is borne out by the experience of Alabama Power Company where, in a six year period, there have not been in excess of two failures of reclosing high-voltage breakers which might possibly have been avoided had the substations where these failures occurred been attended and breakers non-reclosing.

A better conception of the application of reclosing high-voltage breakers on Alabama Power Company system may be obtained from brief descriptions of various typical installations. Therefore, descriptions and illustrations are included for one or more examples of each of the following typical applications of automatic reclosing high-voltage oil circuit breakers.

a. To plain radial feeders—44 and 110 kv.
b. To 44-kv. networks fed from one source.
c. To 44-kv. networks fed from several sources.

1. Both of Alabama Power Co.

Presented at the Regional Meeting of the Southwest District of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

d. To very long 110-kv. transmission lines.

e. In conjunction with specially designed accessory switching equipment.

These examples follow:

Radial Feeders—44-Kv. and 110-Kv.

The application of automatic reclosing switches to radial feeders is the simplest and most common of all the applications. This type of installation was made on the 44-kv. installation at the North Auburn Primary Substation. All the 44-kv. oil circuit breakers at this station are operated by self-contained, motor driven, automatic reclosing devices. There is also an 110-kv. installation at the Flomaton Primary Substation. Here the breakers are operated by solenoid mechanisms which are in turn controlled by reclosing relays of the induction type. All the circuit breakers installed at the

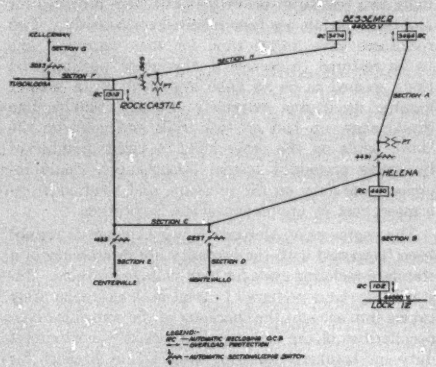

Fig. 1—Blocton Loop Automatic Switching Scheme

North Auburn and Flomaton Primary Substations are arranged for one minute reclosing intervals and lock out after two reclosures. No unusual maintenance problems have been encountered on these breakers.

44-Kv. Network Fed from One Source. The application of reclosing breakers to networks fed from one source presents a more difficult problem, especially when various switching stations are remotely located with respect to the source of power and attended stations. The application becomes even more difficult when it is desired to automatically isolate trouble in relatively small sections of the entire network. A good illustration of this type of network is the Blocton Loop, shown in Fig. 1. This network feeds a great many important customers distributed over a large area. The network is tied together with reclosing oil circuit breakers and numerous automatic sectionalizing switches specially designed for the purpose. These circuit breakers and automatic sectionalizing switches are so arranged and timed that any section of line in trouble is isolated automatically from all the other sections of line and service restored to all the network except the section in trouble. The particular type of automatic sectionalizing switch used in the Blocton Loop network is designed to reduce the number of automatic reclosing oil circuit breakers necessary to secure complete sectionalization, thereby decreasing the initial installation cost. These switches are essentially spring actuated automatic gang operated air-break switches. Each phase includes a high-voltage solenoid connected through an insulating member to an inverse time-limit circuit closing relay. These switches operate in conjunction with special timing relays. Their function is to open a line automatically after it has carried excess current momentarily a given number of times within a given period of time. They are so designed that they cannot open ordinarily when the line is energized, though in some cases it is necessary to add low-voltage relays to make this positive.

The following examples describe the functioning of the Blocton Loop in case of permanent trouble. Transient disturbances are not considered, as they result in no permanent derangement of the Loop.

Assuming trouble in Section *A*, Fig. 1, automatic reclosing oil circuit breakers 3464, 1318 and 4450 open. This "notches up" the timing relays and starts a timing cycle on the automatic sectionalizing switches 1369 and 4491. In one minute all three of the above mentioned oil circuit breakers reclose and open again immediately and the timing relays on switches 1369 and 4491 "notch up" again and start a new timing cycle. After another minute has elapsed, oil circuit breakers 3464, 1318, and 4450 again reclose and again open immediately The timing relays on switches 1369 and 4491 then "notch up" the third time, close their trip contacts and start a new timing cycle. When the trip contacts on the timing relay on switch 4491 are closed the trip circuit is completed through an undervoltage relay connected to a 44,000/110-volt potential transformer and the switch opens. Breakers 3464, 1318 and 4450, which are set for three reclosures (four openings) before lock-out, then reclose. Breaker 3464 opens immediately after reclosing the third time, and locks out, isolating Section *A*. Service is then restored to all other sections through oil circuit breakers 1318 and 4450. Switch 1369 does not open since oil circuit breaker 3474 does not open during the sectionalization and consequently there is always potential on the line between breaker 3474 and switch 1369, thus keeping the trip circuit of 1369 open at the contacts of an undervoltage relay.

Since all the automatic sectionalizing switches used in the Blocton Loop operate the same as those described above, in case of trouble, a detailed explanation of the different cycles of operation of the other automatic sectionalizing switches in the loop is not given in the following discussion but, instead, it is confined to an outline of the breakers and switches which operate to sectionalize trouble in any given section.

When trouble develops in Section *B*, the automatic reclosing oil circuit breakers 4450 and 102 lock-out after the third reclosure (fourth opening) and the trouble is thus isolated. Trouble developing in Section *C* is isolated by oil circuit breakers 4450 and 1318 and automatic sectionalizing switch 4491. Switch 4491 opens after the second reclosure (third opening) of oil circuit breakers 3464, 4450, and 1318. Oil circuit breakers 4450 and 1318 and 3464 reclose again and 4450 and 1318 open and lock-out after the third reclosure (fourth opening), thus isolating the fault in Section *C*. Breaker 3464 restores service to Section *A*. Trouble in Section *D* is isolated by the automatic sectionalizing switch 6237 after the first reclosure (second opening) of breakers 3464, 4450, and 1318. Automatic sectionalizing switch 1455 isolates trouble on Section *E* with exactly the same operation of oil circuit breakers. Trouble on Section *F* is isolated by automatic sectionalizing switch 1369 and oil circuit breaker 1318. Automatic sectionalizing switch 1369 opens after the second reclosure (third opening) of the reclosing circuit breakers 3474 and1318.

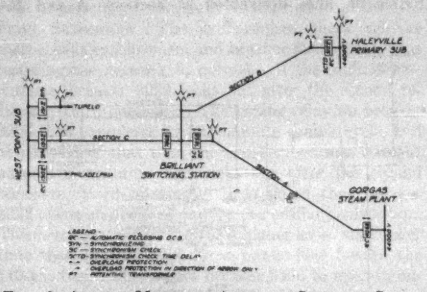

FIG. 2—ALABAMA-MISSISSIPPI AUTOMATIC SWITCHING SCHEME

Breaker 1318 opens and locks out after the third reclosuse (fourth opening) and 3474 recloses and restores service to Section *H*. When trouble develops in Section *G* automatic sectionalizing switch 5033 isolates it after the first reclosure (second opening) of reclosing oil circuit breakers 3474 and 1318. Breakers 3474 and 1318 reclose to restore power to all sections affected except Section *G*. Trouble in Section *H* is isolated by automatic sectionalizing switch 1369 and breaker 3474. Automatic sectionalizing switch 1369 opens after the second reclosure (third opening) of oil circuit breakers 3474 and 1318. Breaker 3474 recloses, opens, and locks out on the fourth opening. Breaker 1318 recloses and restores service to Section *F*.

While this loop is fed from Bessemer and Lock 12 it is considered to have one source of power supply as there is little chance of the two stations being out of synchronism and no provision is made for automatically checking synchronism either at Bessemer or Lock 12.

44-Kv.-Network Fed from Several Sources. The application of reclosing breakers to networks fed from several sources is complicated by the necessity for automatic synchronizing or synchronism checking. Two good illustrations of this type of network are shown in Fig. 2 and Fig. 4.

The Alabama-Mississippi network, shown in Fig. 2, has three sources of power,—Gorgas, Haleyville, and West Point. The power that can be furnished the network from West Point is not sufficient to carry much load in Alabama and the normal power flow is from Alabama to Mississippi. The chief sources of power are from Haleyville and Gorgas.

This network has three special features, all of which are necessary for a complete automatic switching scheme of this kind. These special features consist of an automatic synchronizing device for the control of breaker 1932 at West Point, synchronism check devices at Brilliant on breaker 7858 and at Haleyville on breaker 5122, and a special timing relay at Haleyville on switch 5122. The automatic synchronizing device at the West Point Substation will close breaker 1932 only when there is potential on both the bus and line terminals and then only when both systems are in synchronism.

The synchronism check schemes at Brilliant and Haleyville on breakers 7858 and 5122 consist of a number of relays which allow the breakers controlled to close only when the two sources of power are in synchronism for a period of several seconds. There are other relays used in connection with the synchronsim check schemes which automatically select the breaker across which synchronism is to be checked. For example, if for any reason breakers 4058 at Gorgas and 7858 at Brilliant open, breaker 7858 does not reclose until after breaker 4058 at Gorgas recloses. This scheme then allows synchronism to be checked across breaker 7858 at Brilliant. If, however, Section *B* is deenergized by the opening of breakers 7858, 5122, and 1932, breaker 7858 becomes automatic reclosing and synchronism is then checked across breaker 5122 at Haleyville. The synchronism check scheme on breaker 5122 is further controlled by a timing relay which is the third special feature mentioned above. The function of the timing relay is to prevent breaker 5122 from reclosing until breaker 7858 has restored power to the Brilliant side of breaker 5122, or until breaker 7858 has locked out because of trouble. This timing relay is set to operate for four minutes before closing its contacts. The closing of the timing relay contacts makes breaker 5122 full automatic reclosing. If, however, permanent potential is established on the Brilliant side of breaker 5122 before the four-minute timing cycle of the timing relay is completed, the timing cycle is terminated immediately and the synchronism check scheme functions to close breaker 5122 if the two sources of power are in synchronism. Momentary restorations of voltage across breaker 5122, due to operations of breaker 7858 during trouble, do not affect the timing cycle of

the timing relay nor the functioning of the synchronism check scheme on breaker 5122. All breakers in this network are normally closed.

Assuming a permanent case of trouble occurs in Section A, automatic reclosing oil circuit breakers 4058 and 7858 open and clear the trouble. After one minute, breaker 4058 recloses, but opens automatically again immediately. It then recloses again after one minute, opens, and locks out. This isolates Section *A* as breaker 7858 does not reclose, being prevented from doing so by the synchronism check scheme, on account

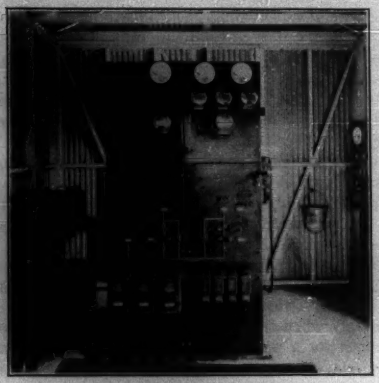

FIG. 3—BRILLIANT AUTOMATIC SWITCHING STATION SWITCHBOARD

of lack of permanent voltage on the Gorgas side. Had voltage been restored, breaker 7858 would have reclosed.

Assuming temporary trouble occurs in Section *B*, breakers 1932, 7858, and 5122 open. Breaker 7858 closes in one minute and restores service to Sections *C* and *B*. Breaker 1932 then synchronizes and closes immediately. The timing cycle on breaker 5122 is terminated as soon as permanent voltage is established from Brilliant. The synchronism check scheme then checks synchronism and closes breaker 5122, thus restoring the network to normal. In case of permanent trouble in Section *B*, breakers 1932, 7858, and 5122 open. Breaker 7858 recloses and opens twice and locks out on the third opening. The timing relay on breaker 5122 completes its four minute timing cycle and at the end of that time breaker 5122 becomes straight reclosing and locks out after the second reclosure. Since breaker 1932 does not synchronize on a deenergized line, the trouble is isolated.

Assuming trouble in Section *C*, breakers 7868 and 1932 open. If the fault is temporary, breaker 7868 recloses and breaker 1932 synchronizes and closes. If a permanent fault, breaker 7868 locks out after the second reclosure (third opening) and breaker 1932 does not close until potential is again reestablished from Brilliant.

Fig. 3 shows the switchboard and relays at the Brilliant Switching Station. It is interesting to note that all the reclosing mechanisms used in this network, except at Gorgas, are of the type originally developed by the engineers of the Alabama Power Company.

The Montgomery automatic switching scheme shown in Fig. 4 employs the synchronism check scheme and a timing relay similar to that installed at Haleyville on breaker 5122, but is much more complex. Almost the entire Montgomery load is carried through the Gas Plant Substation.

Montgomery is normally served from Upper Tallassee and the Montgomery Primary Substation. All breakers in the network are normally closed except breaker 2808 at Montgomery which is not closed except in emergencies. The automatic reclosing oil circuit breakers 2828, 2868, 2822, 3216, and 3210 do not play any part in the operation of the network and are omitted from the discussion.

Breakers 8656 and 8646 at Upper Tallassee open for trouble on lines designated as Sections *A* and *B*.

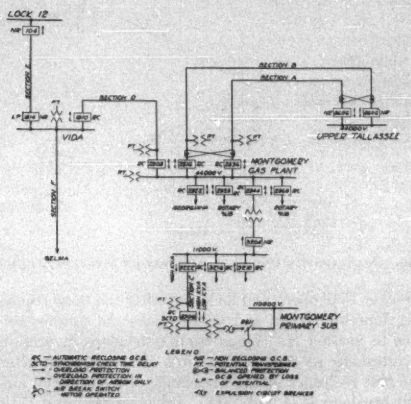

FIG. 4—MONTGOMERY AUTOMATIC SWITCHING SCHEME

Breakers 2836 and 2816 are automatic reclosing and lock out after the second reclosure (third opening). If for any reason potential fails on both the lines from Upper Tallassee, breaker 3204, which is non-reclosing, opens on reverse power. The primary substation will then carry the 11-kv. load in Montgomery. When breaker 3204 opens, the 44-kv. bus is deenergized and breaker 2808 closes after a few cycles and breakers 2836 and 2816 both open if not already in the open position. These provisions are made for isolating the power supply from the primary substation since the entire load of Montgomery cannot be carried from that

source and the source of power from Lock 12 through Vida is not adequate to carry the Montgomery load and the load on all the 44-kv. feeders. When potential is again restored to the Upper Tallassee side of breakers 2836 and 2816, they again reclose. All switching at Montgomery is brought to the attention of the local organization through a signal system, and after the automatic swiching as described above is performed, an operator goes to the station and closes breaker 3204 and opens breaker 2808, thus restoring the station to normal.

Any trouble in the transformer bank at the Gas Plant Substation is cleared by breaker 2844, which locks out after the second reclosure (third opening) and breaker 3204, which is non-reclosing.

A fault in Section *C* is isolated by automatic reclosing breakers 3222 and 8906. Breaker 8906 is equipped with a synchronism check scheme and a timing relay as explained in the above discussion. Ordinarily, if not interfered with by the synchronism check scheme or timing relay, breaker 3222 closes in one minute. If the fault is temporary, the four minute timing cycle of the timing relay on breaker 8906 is terminated as soon as permanent voltage is established from the Gas Plant Substation. The synchronism check scheme then checks synchronism and breaker 8906 closes, thus restoring the network to normal. If the fault is permanent, breaker 3222 locks out after the second reclosure (third opening). The timing relay on breaker 8906 then completes its four minute timing cycle, and at the end of that time the breaker becomes straight automatic reclosing and locks out after the second reclosure (third opening). It is noted that breaker 3222 opens on overload toward the primary substation. This breaker is set to open at a higher kv-a. and longer time than breaker 8906 for a power flow toward the 110-kv. bus. This allows breaker 8906 to open in advance of breaker 3222 in case the 110-kv. line is de-energized, or when trouble occurs in the primary substation. Under these conditions, breaker 8906 does not reclose until power is again applied to the 110-kv. line and then only after synchronism is checked. Breaker 8906 is also opened by overload relays set for relatively high kv-a. and operates only on trouble in Section *C*.

When trouble occurs in Section *D*, breaker 1810 opens on overload. Breaker 2808 immediately closes on failure of potential on Section *D*. If trouble is only temporary, power is restored by breaker 2808, and in one minute, breaker 1810 recloses. An operator then goes to the Montgomery Gas Plant Substation and opens breaker 2808 to restore the system to normal. If, however, trouble persists after the first and second reclosure, then both breakers lock out after their second reclosure (third opening) and the trouble is isolated in Section *D*.

When trouble occurs in Section *E*, breaker 104 opens and clears the trouble. This opens breaker 1814 on failure of voltage. Breaker 2808 then closes and restores service to Sections *D* and *F*.

When trouble occurs in Section *F*, the same operations take place as when trouble occurs in Section *E*, but the operator at Lock 12 is able to restore service to Section *E* when breaker 1814 opens upon failure of bus voltage. When breaker 2808 closes, it provides a new source, so that breaker 1810 may open three times and lock out, leaving Section *D* supplied through breaker 2808.

None of these extensive applications of reclosing breakers has developed any particular or fundamental difficulties in reclosing high-voltage circuits.

Very Long 110-Kv. Transmission Line

A very good example of the application of automatic reclosing oil breakers to very long 110-kv. transmission lines is the Jordan Dam-Hattiesburg line shown in

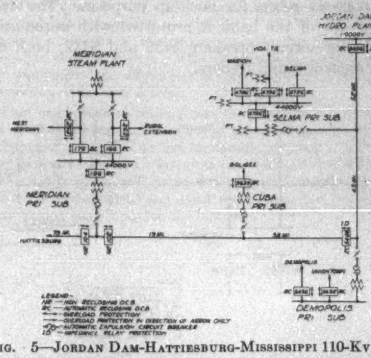

Fig. 5—Jordan Dam-Hattiesburg-Mississippi 110-Kv. Line

Fig. 5. This line is 231 mi. in length, and has four 110-kv. oil circuit breakers in series, two of which are automatic reclosing. It serves a total of seven primary substations, but inasmuch as only four of these are of interest from the standpoint of the application of reclosing oil breakers to 110-kv. lines, only four of them are discussed.

The Jordan Dam hydroelectric plant is the normal source of supply. Breaker 8406 at Jordam Dam is automatic reclosing and is equipped with impedance relays which depend on both the line current and voltage drop in the preceding section of line for their operation. This breaker is also equipped with overload relays for back-up protection.

The Selma Primary Substation includes four 44-kv. automatic reclosing breakers which lock out after their second reclosure (third opening). This station has a source of power from Vida, hence the transformer bank breaker 4756 is equipped with reverse power protection in addition to overload protection. The normal source of power is from the 110-kv. line and the

automatic reclosing breaker 4736 is normally open. In addition to overload relays, this breaker is equipped with reverse power relays for opening the breaker when power flows toward Vida. When the power supply fails on the 44-kv. bus, breaker 4756 opens and breaker 4736 closes and restores power to the 44-kv. bus from Vida, unless there is a fault on the 44-kv. bus, in which case it locks out after the second reclosure (third opening). When voltage is again restored to the source side of breaker 4756 it closes and breaker 4736 is opened, thus establishing normal service to the load. The reclosing oil circuit breakers 4786 and 4726 control straight radial feeders.

Demopolis is 95 miles from Jordan Dam. Breaker 5404, which is in the main line, is automatic reclosing and locks out after the second reclosure (third opening). This switch isolates trouble between Demopolis and Meridian and is equipped with impedance relays and with overload relays for back-up purposes. The low-voltage side of the bank is protected with automatic reclosing oil circuit breakers 5436 and 5432, both of which control radial feeders.

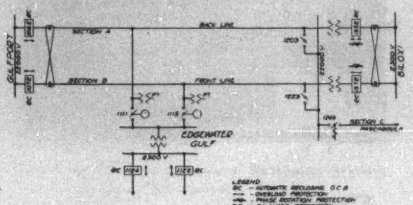

FIG. 6—GULFPORT-BILOXI AUTOMATIC SWITCHING SCHEME

The Cuba Primary Substation transformer bank is protected by one 44-kv. automatic reclosing oil circuit breaker which is set to lock out after the second reclosure (third opening).

Breakers 102 and 104 at Meridian are equipped with overload protection, but no reclosing features. There are several 11-kv. automatic reclosing breakers installed in the Meridian Primary Substation on the radial feeders.

IN CONJUNCTION WITH SPECIALLY DESIGNED ACCESSORY SWITCHING EQUIPMENT

The use of automatic reclosing high-voltage oil circuit breakers in conjunction with specially designed accessory equipment is common on the Alabama Power Company and affiliated systems. Fig. 6 covers an interesting application at Gulfport, Mississippi.

The automatic switching scheme described includes three distribution substations,—Gulfport, Edgewater, Gulf Hotel, and Biloxi,—with two lines connecting Gulfport and Biloxi as shown in Fig. 6. The source of power is Gulfport and the network is designed to eliminate interruptions, in so far as possible, to Edgewater Gulf Hotel and Biloxi. The loop is normally closed at Biloxi, through the two transformer banks and their low-voltage automatic reclosing breakers 1372 and 1352. A transfer bus at Biloxi allows Pascagoula to be served from either the "back" or "front" line through the gang operated air-break switches 1203 or 1223. Normally, switch 1223 is open and switch 1203 is closed. The line to Pascagoula is equipped with automatic sectionalizing switch 1249, which isolates trouble on that line after the "back" line terminal breakers 1352 and 1082 have opened twice within a one and one-half minute period.

The Edgewater Gulf Hotel normally is fed from the "back" line through motor operated air break switch 1111. In case of trouble on this line, switch 1111 opens and motor operated air break switch 1115, which is normally open, closes at the end of 30 sec. and service is restored to the Edgewater Gulf Hotel. When permanent potential is again restored to the "back" line, switch 1111 automatically closes and switch 1115 opens, thus restoring the network to normal again. The Edgewater Gulf Hotel does not suffer an interruption in the transfer from the "front" to the "back" line. The automatic switching scheme at the Edgewater Gulf Hotel does not transfer from a deenergized line to a deenergized line. Neither does it transfer from an energized line to a deenergized line. If, for any reason, both lines become deenergized simultaneously, then the switches correctly transfer the load to the first line to become energized.

The proper sequence of the automatic switching operations of the loop is better explained by assuming individual cases of trouble on the three different lines. If trouble develops on the "back" line, breakers 1082 and 1352 open. In 30 sec. motor operated air-break switch 1111 at Edgewater Gulf Hotel opens and 1115 closes, providing there is power on the "front" line, thus restoring power to Edgewater Gulf Hotel. One minute after their initial opening breakers 1082 and 1352 reclose. If the trouble is temporary, power is thus restored to Sections *A* and *C*. One and one-half minutes after breakers 1082 and 1352 reclose, switch 1111 at Edgewater Gulf closes and switch 1115 opens, thus restoring the loop to normal. If, however, the trouble is of a permanent nature, breakers 1082 and 1352 lock-out after their second reclosure (third opening) and the Edgewater Gulf Hotel load under these circumstances is not transferred from the "front" line to the "back" line until the fault is removed and power is restored to the "back" line.

Assuming a permanent case of trouble on the "front" line, both the automatic reclosing oil circuit breakers 1072 and 1372 act as straight automatic reclosing breakers which lock out after the second reclosure.

When a permanent fault occurs on the line to Pascagoula, the same automatic switching takes place as when trouble occurs on the "back" line except the automatic sectionalizing switch 1249 isolates the trouble after the second opening of breakers 1082 and 1352 and service is restored to the "back" line by the reclosing breakers 1352 and 1082, after which the Edgewater Gulf Hotel is transferred to the "back" line as previously explained.

Conclusions

There is nothing experimental about automatic reclosing of high-voltage circuits. Selection of breakers and choice of reclosing duty cycles should be made separately for each application, based on the various factors outlined. Highly reliable reclosing relays or reclosing devices are now available from various manufacturers. While attendants at stations and substations are made unnecessary by automatic reclosing of circuits, extremely careful and regular inspection and maintenance of breakers and reclosing equipment are essential. This is vastly more important than in the case of attended stations or substations. Even at points where attendants are stationed it is sometimes economical and desirable to install reclosing equipment, either to improve service or to reduce attendance.

Discussion

R. J. Wensley: This paper gives a brief historical résumé of automatic reclosing and to avoid the possibility of a false impression from that history I should like to give just a few facts I happen to know from my own personal experience.

The first use of periodic reclosing for service restoration, so far as I know, was quite a while prior to Mr. Spurgeon's experience. We installed a periodic reclosing equipment at Lancaster in 1916, built under the patents of F. E. Ricketts of Baltimore, Maryland, and a little later another type.

Then in 1919 was developed the Type *G R* periodic reclosing relay which, with very minor modifications, is still the standard of the Westinghouse Company. The first specification of that was made in 1919 for the Guggenheim Copper interests, and in the same year for the Pennsylvania Power and Light Company at Allentown.

The Alabama Power Company and their corps of engineers have done a most excellent job of applying automatic reclosing equipment to complicated networks, and in fact they lead the entire country in such application. It would, however, be unfair to the history of the art to say that the principal manufacturers just started the development of these devices in 1922.

At the present time this motor-driven relay is in very extensive use in this country and abroad. One of the latest developments might be of interest. In this development ordinary d-c. solenoids are employed for opening a-c. breakers. The direct current is supplied by means of Rectox copper-oxide disk rectifiers connected to the a-c. system. This eliminates the more complex a-c. equipment and allows use of the simple d-c. solenoid in connection with the *G R* relay.

That emphasizes something that the principal manufacturers are always striving for,—the use of standard apparatus. The development of special devices for specific purposes is frequently very expensive and when we can take a standard circuit breaker without alteration, and apply a few special pieces to enable automatic reclosing, or any other special function, we feel we have gained something for the industry.

Bare Wire Overhead Distribution Practise

BY M. C. MILLER[1]

Non-member

Synopsis.—An appreciable saving may be made in construction of overhead distribution systems by the use of bare conductors in the place of weatherproof conductors. The reliability of the system will be considerably increased with lower maintenance and operation costs. Bare conductors have practically unlimited life and when removed can be reinstalled as new wire resulting in a reduction of depreciation.

This paper outlines the advantages of using bare wires and gives also advantages of medium-hard-drawn as compared with annealed conductors.

FOR the purpose of furnishing the electrical industry with data concerning the use of bare wire and to assist in choosing between bare and insulated wires for overhead distribution systems, this paper was prepared. It shows relative costs of construction, maintenance, and depreciation with condensed data relative to unit weights and costs, and cost of lines and systems with bare and weatherproof wire.

To give a comparison of cost for conductors of the sizes most frequently used in distribution circuits the following costs per 1000-ft. of bare and weatherproof wires are calculated assuming bare at 20 cents and weatherproof at 22 cents per pound.

No. 6 Solid W. P.	$24.64	No. 6 Solid bare	$16.00	Saving $8.64	35%
No. 4	36.08	No. 4	25.20	10.88	30%
No. 2 Str. W. P.	59.04	No. 2 Str. bare	41.00	18.40	31%
No 1/0	93.28	No. 1/0	72.40	20.88	22%
No. 4/0	176.00	No. 4/0	130.60	45.40	26%

An average saving on a number of distribution systems would be about 25 per cent in cost of copper conductors if bare wire were used instead of triple braid weatherproof wire.

The installation costs of bare wire as compared to triple braid weatherproof do not vary directly with the weight. On short extensions of one or two or three spans the labor costs would be about equal, with some savings up to 15 per cent or 20 per cent on longer lines where the lesser weight of bare wire saves on transportation, splicing and setting up reels, etc.

Construction men who have been installing and maintaining bare copper conductors for lighting and power secondaries, series street lighting circuits and primaries, 2300-volt to 12,000-volt, express a very definite preference for it over the weatherproof wire.

To operate satisfactorily it must be put up right, sagged correctly, and guyed sufficiently, but once put up right it does not stretch and cause excessive sag which periodically must be pulled up or re-sagged, resulting in a large saving in maintenance costs.

No increase in horizontal spacing is necessary as the smaller sag required for bare conductors reduces the possibility of their swinging together. It has been found desirable, however, to provide additional vertical spacing where bare conductors are installed on vertical secondary racks and 12-inch separation eliminates any possibility of conductors getting together. Eight-inch spacing has been used in many places with no report of trouble from this cause. This increased spacing uses up a foot more of pole height but very seldom requires a five foot higher pole.

The life of weatherproof wire is comparatively short and it is often necessary to replace it with new wire after 10 to 20 years in service. The life of bare copper wire is unlimited. When weatherproof wire is removed from service after 10 to 20 years it usually has to be scrapped as the weatherproofing is deteriorated to such an extent that it is not practicable to reinstall it. Bare copper wire removed is worth practically the same as new wire and may be reinstalled. These two features alone will justify the use of bare copper wire.

In many communities there is an ordinance requiring the use of insulated wires and weatherproof wire is often accepted by inspectors in such communities as meeting these requirements. When such ordinances exist steps should be taken to have them changed. The word insulation might be assumed by the courts to mean insulation sufficient to protect against voltage carried by the conductors. This was practical when the industry was in its infancy and many of the existing electrical ordinances were made at that time. Bare wire has been used in many instances even where insulated wire is required. This has been permitted as it is realized that it is impractical to comply economically with such an ordinance and install wires insulated for voltages from 2300 to 13,000 volts.

Comparative estimates for constructing an 11-kv. distribution system in a small town to serve 60 customers show a saving of 9.5 per cent in total construction cost by using bare in place of weatherproof wire. The saving in cost of conductors is 30 per cent, poles 9.4 per cent, and labor 5 per cent. An estimated cost of a mile of No. 4 weatherproof medium hard drawn copper three-phase, 2300-volt line using 35-ft. poles spaced 200 ft. is $1625.00; No. 4 medium hard drawn bare using 35-ft. poles spaced 300 ft. is $1133.00, or a saving of $492.00 or 30 per cent in total cost of bare as compared to No. 4 weatherproof conductors. To summarize the above advantages in using medium hard drawn bare conductors for general distribution construction:

1. Distribution Design Engineer, Texas Power & Light Co., Dallas, Texas.

Presented at the Regional Meeting of the South West District of the A.I.E.E., Dallas, Texas, May 7-9, 1929.

Saving in initial investment cost of conductors may be from 15 per cent to as high as 30 per cent; poles and attachments 5 per cent to 10 per cent; labor 5 per cent to 20 per cent; with a total saving of from 15 per cent to 30 per cent on rural lines and 5 per cent to 10 per cent on complete distribution systems.

Yearly maintenance costs are 15 per cent to 25 per cent less for the reason that conductors do not have to be resagged frequently and there are less outages caused by breaks and conductors swinging together and burning down.

Life of conductors is conservatively three or four times that of weatherproof.

Practically no depreciation occurs in conductors; when salvaged they have practically the same value as new.

Hazard to linemen is not increased as they are necessarily more careful. Hazard to general public is greatly reduced as lines do not break as frequently.

General appearance is greatly improved.

Medium hard drawn has decided advantages over annealed copper wire, having greater strength allowing greater spacing of poles, and considerable savings in construction cost.

The use of bare medium hard drawn copper for overhead distribution systems conductor is worthy of consideration. It has been used on a number of systems in small towns and cities for a sufficient length of time to prove its worth.

Advantages of Medium-Hard-Drawn over Annealed Conductors

In designing or replacing distribution lines another point arises which is worthy of consideration, namely the economy of using medium-hard-drawn instead of annealed wire. Medium hard drawn copper conductors have many advantages over annealed wire, principally on account of the greater strength, being about 25 per cent in No. 1/0, No. 2/0, and No. 4/0 sizes and 35 per cent to 45 per cent in No. 8 to No. 2 sizes. This increase in strength in the smaller sizes allows an increased spacing of poles in urban and rural districts, and in densely built up districts provides an added factor of safety which is very desirable as it affects hazard to the general public as well as reliability of service and maintenance costs.

There is a very slight difference in electrical characteristics, the resistance of medium hard drawn being 1 to 1.5 per cent greater than that of annealed copper wire, which is negligible in distribution system calculation.

Joints in or taps to medium hard drawn copper wire used for overhead lines should not be soldered because annealing of the copper and decreasing its strength would result. There is a number of solderless connectors available. The cost of these installed is usually less than that of a good soldered connection. Various types have been in use for a number of years and no trouble has been experienced from these connectors working loose and causing loose connections. They are more flexible then soldered connections and are of great assistance in replacing conductors, disconnecting taps, services, etc.

The increased working tensions of medium hard drawn copper conductors require more adequate guying to eliminate possibilities of anchors and guys creeping and conductors becoming slack.

Discussion

G. A. Mills: The use of bare conductor for distribution has received a great deal of attention. Mr. Miller and his associated engineers are probably the leaders in this section of the country in the use of bare wire.

I think we have to be careful in the application and rather divide the distribution system into several sections. For instance, rural sections and small towns should have one standard of construction, and the larger communities have another problem. We have found in our larger cities especially, we cannot take advantage of the longer pole space. Narrow lots and the density of houses require a certain pole spacing to take care of the service. Therefore, I fail to see in some of the cases that we can make or save any money on the pole-spacing feature.

I personally am in favor of insulated wire for secondary. I base this on experience in the loss of transformers due to contact between the conductors and on account of the legal requirements, feeling that ordinary weatherproof will offer enough mechanical separation to give insulating value. On primary lines I agree with Mr. Miller, on account of the fact that only under unfavorable circumstances will the insulation give protection in case of a fallen line, which is our greatest hazard.

The companies I am associated with have been experimenting with this problem. They are using a hard-drawn weatherproof wire to try to get half way between the soft-drawn and bare-wire situation. With hard-drawn wire we feel we can maintain our clearances substantially as discussed by Mr. Miller.

We are also experimenting with a lead-oxide covering. It is too early to prophesy what we will find out but according to the theory presented lead oxide will reflect the violet ray which is responsible for the deterioration of the compound which holds the fiber together to make up the insulation and mechanical separation.

J. E. Rose: In the country along the Gulf Coast we have salt foam, shell dust, and dirt of various kinds blowing over the wires and there is no insulation that will hold.

We have been using bare wires for quite a while. We have found the main trouble with the bare secondaries is that small boys occasionally throw pieces of wire into them. We have never had any other case of trouble with bare wire. On our field work of a very temporary nature we are using bare weatherproof triple-braided wire because that will stand up for about six months to a year, and the men are offered some protection by it.

We have used bare wire for all our primary lines. Our men are told that any wire that has no insulation is unsafe and we make them all safe by keeping them bare.

J. B. Thomas: With regard to the advantages and disadvantages of using bare wire, this is primarily an economic problem. We must first, however, be sure that we do not, by the use of bare wire, increase the hazard to either the public or our employes.

Some four years' experience has shown that with proper spacing, proper guying and construction methods in general, you can make bare-wire installation just as safe, in my judgment, as the weatherproof-wire installation can be made. You are working on the very sound principle that a man may take hold of a weatherproof wire thinking that the insulation has value and be seriously injured but the bare wire speaks for itself as being a

thing that must be handled carefully and with discretion. We find that our accidents to employes, as nearly as we can tell, are lessened rather than increased by the use of bare wire and so far, it appears that accidents to the public in general have been materially lessened because the construction is fundamentally stronger and the job generally better done.

Having taken care of safety requirements, then, we come back to the basic economics of the problem and Mr. Miller's paper dwelt largely on the first-cost features. Those are important and we are all working on lowering the first cost of all our construction.

In addition to the first cost there is the cost of upkeep. The serving of the average residence customer results in costs which are principally maintenance, operation, accounting, and service matters so that the cost of energy supplied is a small portion of the total cost. Experience shows that often in the life of an ordinary distribution system built with weatherproof wire you have to go back and pull slack and take up the stretch. You can get around that to a large extent by using weatherproof hard-drawn wire.

Also about every ten years in this section of the country it is necessary to take down a large part of the insulated distribution conductor because the insulation has become loose and hangs down in shreds and carries rain water from one conductor to the other which makes trouble. It is quite general practise to figure on a substantial rebuilding of weatherproof wire distribution systems in, say, ten years. But on a properly constructed transmission line where bare wire of adequate strength is used, there is very little cost arising out of conductor maintenance.

This is a question which has to be worked out carefully and there are many problems. In my organization we have had some four years experience in bare-wire distribution, some in cities of 50,000 and 60,000 people, without substantial troubles. We are installing our distribution systems at lower costs and the very considerable mileage already installed is being reflected in a decreasing curve of the cost per mile of maintaining distribution lines and services.

S. M. De Camp: I should like to ask what effect crystallization would have on hard-drawn wire. Will the wire be crystallized by swinging under the force of the wind?

M. C. Miller: We have had no trouble from that cause so far. We are using the medium-hard-drawn wire rather than the real hard-drawn.

A. C. Scott: It would seem to me that the use of bare copper wire for the primaries might be very advisable but in considering the use of secondaries where there are numerous trees, for example, and opportunities for short circuiting, I should think bare wire would be likely to cause trouble.

In connection with damage suits I have always found it difficult to convince the legal profession that the weatherproof insulation was placed on the wire for mechanical reasons rather than for protection. They always take the position that it should be a complete insulator. I think that if the primaries were universally of bare wire the question that always arises in that respect would be well answered to begin with, as it now is for high-voltage circuits.

M. C. Miller: In some of the towns in which we have distribution systems there is a large number of trees. The secondaries are not far below the primaries and it is necessary to keep the primaries above the trees. So we trim the trees, if necessary, or put in higher poles so we can get the secondaries also above the trees. With secondaries run through trees, especially if they are dense, weatherproofing doesn't last. Where we have gone through small trees, with increased spacing of 12 in. between conductors, the conductors don't get together as readily as weatherproof would. There is a little less slack in them and it is a little harder to pull them together. We haven't had much trouble, but where possible we keep them out of the trees.

Interconnection in the Southwest

BY GEO. A. MILLS[1]
Member, A. I. E. E.

Synopsis.—Interconnected electric service in the Southwest has kept pace with the country as a whole. Eight states are now connected by a continuous transmission system, operated at a voltage of 60,000 volts or more. This system extends 720 mi. from East to West and 810 mi. from North to South. A description of the system is given in this paper.

A few years ago the supply of electrical energy in the Southwest centered around the larger communities, obtaining such supply from steam plants located within the boundaries of the city. The nearby communities were served by moderate voltage transmission lines radiating from these plants.

In the arid sections of the district where condensing water is scarce, oil engines were used extensively as a basis of power generation and for load development.

In the past few years development has progressed with the expansion of existing power plants and the extension of transmission networks around the larger urban areas. In the northern section of the district this development has practically covered a majority of the territory; however, in the southern portion there still exist many scattered small plants and low voltage transmission systems.

The great industrial development which started a few years ago and is now under way, has brought about a change. The small plants and low voltage networks were not only unable to furnish the capacity needed but were unable to serve economically the large industries demanding a power supply. This brought about a consolidation of the smaller properties into larger operating groups, the constructing of higher voltage transmission lines, and the development of larger base load plants.

Such development, in providing an ample supply of economical energy, has stimulated general expansion in all lines of activity such as agriculture, by providing an energy supply for irrigation pumping, gins, oil mills, creameries, condenseries, cheese factories, and general service to the farm; refrigeration; mining, smelting, and quarrying; saw mills, paper mills, and other wood product industries; the oil industries, in the electrification of drilling and well pumping equipment, refineries, pipe line pumping stations and compressor stations; and last but not least, has improved the standards of city life in industrial, commercial, and domestic activities.

The general territorial growth has been so rapid that at the present time the larger operating groups have covered the intervening territory so the border transmission lines have been interconnected, thus bringing about the present interconnected network. These interconnections have been a natural procedure since the operating executives and transmission engineers have in most cases selected similar distribution voltages from sixty to seventy thousand volts with the idea of interconnection in the future. This selection was made necessary on account of the great distances between load centers and the scarcity of favorable plant locations near the load centers, requiring a high distribution voltage to take care of the situation properly.

The skeleton transmission map, Fig. 1, shows the present interconnected lines operated at 60,000 volts and higher voltage. The map also shows the location of points of interconnection between various operating groups and the major power stations.

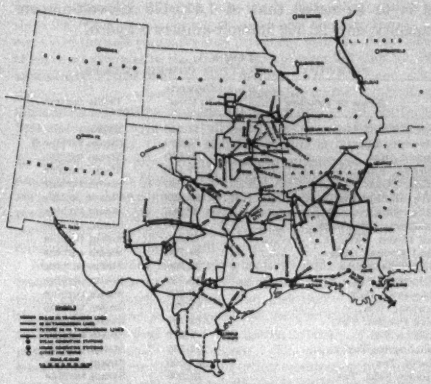

Fig. 1—Interconnected Transmission System of the Southwest

The great distances between power supply points is evident by an inspection of the map. It is to be noted that the distance of continuous interconnected line stretching from Memphis, Tennessee, to the Ozark hydro plant in Missouri, is 1500 miles. The interconnected territory from east to west is 720 miles wide and from north to south is 810 miles in extent.

Interconnection Contracts

The first interconnections made in the Southwest were the result of smaller operating companies purchasing energy from the larger companies having excess prime capacity. The contracts covering these interconnections and the rates used therein were based entirely on the purchase of prime energy. During the

1. Chief Engineer, Central & Southwest Utilities Company, Dallas, Texas.

Presented at the Regional Meeting of the South West District of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

past two years several new interconnections have been made under contracts wherein it has been desirable to utilize the excess capacity of a company which has installed a large power unit ahead of its own customers' requirements. These contracts provide for the mutual benefits of interconnection in meeting operating emergencies and provide for emergency operation of standby plants to hold total plant investment to a minimum cost.

Executives and engineers are fast recognizing the desirability of such mutual contracts since they provide a method of keeping the reserve capacity to a minimum and allow the installation of larger, more economical prime movers, resulting in greater economies and better service to the customer than if the developments were made independently.

Power Supply

Table I lists the principal operating plants in the Southwest connected to one interconnected network. It is to be noted that of 1,415,475 kilovolt-ampere capacity, only 22,500 kilovolt-ampere is hydro.

TABLE I
PRINCIPAL OPERATING POWER PLANTS

Name or location	Capacity in kv-a.	Prime movers
Ozark Beach, Mo	11,250	Hydro
Riverton, Kan	72,000	Steam turbine
Neosha, Kan	50,000	Steam turbine
Wichita, Kan	37,500	Steam turbine
Harrah, Okla	81,250	Steam turbine
River Bank, Okla	28,125	Steam turbine
Byng, Okla	21,900	Steam turbine
Oklahoma City, Okla	12,500	Steam turbine
Bell Island, Okla	17,250	Steam turbine
Sand Springs, Okla	14,700	Steam turbine
West Tulsa, Okla	37,500	Steam turbine
Weleetka, Okla	18,750	Steam turbine
McAlester, Okla	10,600	Steam turbine
Lawton, Okla	11,250	Steam turbine
Lake Pauline, Tex	18,750	Steam turbine
Abilene, Tex	6,250	Steam turbine
San Angelo, Tex	12,500	Steam turbine
Girvin, Tex	22,500	Steam turbine
Wichita Falls, Tex	13,500	Steam turbine
Leon, Tex	38,000	Steam turbine
Ft. Worth, Tex	55,000	Steam turbine
Dallas, Tex	103,000	Steam turbine
Trinidad, Tex	50,000	Steam turbine
Waco, Tex	15,000	Steam turbine
Shreveport, La	37,500	Steam turbine
Texarkana, Ark	9,375	Steam turbine
Sterlington, La	101,850	Steam turbine
Little Rock, Ark	16,500	Steam turbine
Pine Bluff, Ark	13,375	Steam turbine
Remmel, Ark	11,250	Hydro
Memphis, Tenn	67,500	Steam turbine
Jackson, Miss	6,875	Steam turbine
Comal, Tex	75,000	Steam turbine
San Antonio, Tex	36,125	Steam turbine
Victoria, Tex	11,250	Steam turbine
San Benito, Tex	26,250	Steam turbine
Deep Water, Houston, Tex	125,000	Steam turbine
Gable Street, Houston, Tex	35,000	Steam turbine
Neches Plant, Beaumont, Tex	64,750	Steam turbine
Port Arthur, Tex	9,800	Steam turbine
Sabina River, Orange, Tex	9,000	Steam turbine
Total	1,415,475	

The largest and most recently constructed plants in this territory have been laid out with the idea of utilizing the interconnected lines in disposing of the excess energy. In several cases single unit plants were operated, using the interconnected lines to other plants as reserve capacity until the load reached a point justifying a second unit. At the present time four such plants shown in the lists are operating on this basis.

The future power supply will be from the expansion of present base load steam plants located where condensing water is ample and fuel economical, and from future hydro plants in west Texas, northwest Arkansas, southern Missouri, eastern Oklahoma, and south Texas. All of these developments are made possible on account of interconnections since the projects offer capacities in excess of local requirements.

The proposed hydro developments are of unusual

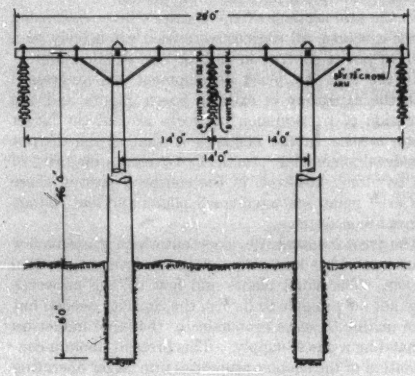

Fig. 2—Typical Structure Used in the Southwest for 66-Kv. and 132-Kv. Transmission Lines

interest since the lakes created will be utilized for flood control and in south Texas the water impounded will be of untold value in providing water for irrigation downstream from the dams. The largest sites are in the Texas Big Bend country of the Rio Grande, miles away from any industrial development.

Line Construction

The present tendency in interconnecting line construction is the use of not less than No. 4/0 conductor supported on H-frame creosoted Southern pine pole structures, spaced for 132,000-volt operation, located on private right-of-way following the shortest path between power supply and load centers or interconnecting points. A typical pole structure is shown on Fig. 2.

Steel towers in the Southwest are only used for special crossings, multiple circuits away from plants and substations, through congested urban areas, and in one or two cases, for double circuit lines.

Operation

Successful interconnection operation between several operating companies under different ownership has been accomplished in the Southwest. The total interconnected network is normally operated in four groups, Arkansas, Louisiana, Mississippi, and eastern Texas in

one group; the balance of Texas and southwestern Oklahoma in a second group; the balance of Oklahoma and northwestern Arkansas in a third group; and Kansas and Missouri in the fourth group. The largest group is that in Texas where seven large operating companies under five different ownerships normally operate connected.

The rapid industrial growth of Texas has been the result of this interconnected system, especially in the arid regions of west Texas where transmission service is necessary to transmit the energy from distant plants located where circulating water is ample and an economic supply of fuel is obtainable.

Future Problems

On account of the great distances covered by single circuit or loop transmission lines and the remote location of base load plants, greater dependence on transmission line service is a necessity. The future engineering development must take into account the delivery of energy to the customer at the same general costs experienced elsewhere by increasing the reliability of the present and future transmission systems as well as the development of more economic plants to overcome the longer transmission distance between power supply and consumption.

The engineers in the Southwest are actively at work on these problems and through studies now in progress are keeping pace with similar work throughout the country to determine the proper construction to render the maximum service.

Future Interconnection

The future interconnections in the Southwest will require greater capacity lines and voltages in the 132,000-volt class to shift the increasing blocks of energy; lines designed with higher flashover voltage characteristics; use of regulating transformers with tap changers which may be adjusted under load and the use of synchronous condensers to deliver the voltage regulation required on such long lines as well as the design of more economic plants, and most important of all, interconnection contracts which will make the maximum economic use of the combined facilities.

The future holds forth more reliable transmission service, increasing number of interconnection points, higher voltage and higher capacity per single transmission circuit, better voltage regulation methods, more steam plants with larger economic units located where circulating water and cheap fuel is ample, large hydro plant developments, and better contractual cooperation in the operation of the combined plants to give the customer the maximum service at lowest possible cost.

Discussion

J. B. Thomas: Perhaps it would be of interest to point out that the density of load on the transmission network in Texas is approximately 1½ kw. for each square mile of the whole territory involved. The cardinal plants listed in Texas aggregate 350,000 kv-a., that is, plants over 5000 kv-a. capacity.

The area of our state is about 254,000 sq. mi. Of course, there are many places of greater load density but with the widely spread load, with limited cooling water, and with fuel supplies not closely spaced, we are primarily up against a transmission problem. Our experience has shown that we can operate in this area to the advantage of customers by cutting down the investment in plants through means of the interconnections which afford reserve sources of power supply and which obviate the necessity of duplicate transmission lines where large areas are dependent almost entirely on transmission-line service.

The determining factor in operating an interconnecting system as large as that of Texas is the good sense and neighborliness of those who own the various properties that interconnect, and it is only by broad constructive thinking and action that this interconnection is made possible, and without the interconnection it would be impractical for this area covered by Mr. Mills' paper to have anything like the power supply available that it now has. We are very proud that in this area a substantial industry can get electric power at whatever point in the area its market and raw material and labor supply indicate that the plant ought to be located.

It occurs to me to mention also that with these long transmission distances, (there are only ten of these larger plants in these 250,000 sq. mi.), our studies show that the availability of service in the area compares very favorably with the availability of service anywhere in the country and is exceeded only by the availability in the metropolitan centers. Out on these long transmission lines where important loads are served, hundreds of miles from generating stations, 99.66 per cent of all the hours that elapse carry with them full electric service on this network.

R. J. Wensley: In Mr. Mills' discussion of interconnection he speaks of the physical and financial advantages, which are unquestioned. It happens that when interconnections become very large and the connection is very intricate, many engineering problems arise. Mr. Mills stressed the need for reliability of service and apparently Texas is getting a very high degree of reliability, but looking for still better.

Many people think that interconnection simply means building transmission lines and connecting them together. There is much more to interconnection than just that. For instance, there is the control of voltage at the various operating centers without causing disagreeable circulating currents or very poor power factor to some of the plants. The manufacturers have been cooperating on the problem of interconnection by building large transformer units with tap changers which can change the voltage ratio under load so that the ideal voltage and power-factor conditions can be secured at every important distribution center. So, in planning for such interconnections do not forget the advisability of voltage control in the transformer units.

Then we have the question of system stability which grows more important with every tie, every increase in the size of the system. There, again, the manufacturers have been cooperating with the development of quick-response excitation systems which will maintain the synchronous operation of generating and condensing equipment so that the systems will not fall out of step under severe surge conditions.

A great deal of very expensive development work is being carried on at the present time. Exciters are being built with totally laminated field structures and voltage ranges up to six or eight times the normal excitation voltage so that tremendous excitation can be thrown on generator fields to secure this kind of stability. So the excitation must be watched to see that adequate means are provided to maintain this desired stability.

Then, tremendous amounts of money are being spent on the development of relay systems to secure effective sectionalization of interconnected systems. It hasn't been very many years since selective timing was considered adequate for all relay practise and the induction disk relay became very prominent because of its ability to secure that selective timing. Today selective timing is a small part of it and the impedance-type

relay and many other forms of new relay methods have been developed to permit such successful sectionalizing of the transmission network. So, wherever you plan to tie together two big systems with one or more ties, do not forget to have a careful relay analysis made because if you don't you are liable to find poorer service after you get the tie put in.

One of the most important things in the operation of a big interconnection is the ability of the dispatching center of each of the systems to be able to communicate freely and dependably at all times with the dispatching centers of the other connected systems. No better means of communication exists than the transmission lines themselves, and there again the manufacturers are cooperating by carrying on extensive developments in the guided radio types of communication systems permitting the use of the transmission-line networks for the carrying of the voice of the dispatchers to distant points. When we first started on this work a few years ago it was thought that all that was necessary was to couple a radio transmitter to a line and then hook on a receiver anywhere and we ought to be able to hear. The phenomena of standing waves, of short spurs with their reflection losses, and differences in lengths of connecting tie lines with the resultant phase angles of the received energy were not thought of, and on many a line when a receiver was hooked on and a transmitter was put on the other end, nothing was received and it wasn't possible to find out why.

We have now reduced the analysis of transmission networks to an exact science, and given the precise measurements of transmission systems we can predict in advance by calculation the exact results to be obtained. We had a case not long ago where we applied a certain type of filter choke to a line supposed to be 13.5 mi. long. The thing wouldn't work. We said that the survey was probably incorrect. We ran a frequency run on the line and said it was 12.3 mi. long. The line was surveyed again and proved to be 12.35 mi. long. The electric survey using the variable-frequency method was more precise than the map the utility company was using. So you can see we have reached a degree of precision of such means of communication as to permit of real service. That is another thing to remember when planning your interconnection. Don't forget the necessity for very dependable communication.

G. A. Mills: There are many things which are going on that cannot be presented in a short paper. Mr. Wensley has brought these to my mind. We don't know yet how we shall solve all of them but we hope to solve them soon.

Electrification of Oil Pipe Lines
in the Southwest

BY D. H. LEVY[1]
Associate, A. I. E. E.

Synopsis.—This paper briefly reviews the history of electricity as motive power for oil pipe line operation and notes some of the reasons for, and the results of, its rapid development in the Southwest. Some attention is paid to the method of serving these loads from transmission lines and the tendency of modern pipe line practise is shown. The last five years have seen rapid advances in the methods of underground transportation of petroleum and the paper reviews some of the present tendencies in design of the modern oil pipe line station.

* * * * *

Introduction

During the last two decades, most of the crude oil produced in the United States has been moved to the refineries, ports, and markets through underground steel pipe lines. The oil was forced through these lines by means of reciprocating pumps driven by steam engines or oil engines.

About three years ago, a number of factors developed which allowed, and in fact required, the extensive employment of electric power for pipe line pumping and this system has now proved highly successful.

The chief factor in this adoption of electric drive was the recent discovery of vast quantities of oil in new fields in Texas and Oklahoma. This required immediate and quick construction and operation of more and larger pipe lines.

Electrical pumping was the solution for several reasons. The proposed pipe lines were to pass through rough and dry territory having few highways and fewer railroad facilities. This condition was favorable to the electrical equipment as it is lighter and more easily transported than Diesel or steam equipment. Furthermore, the available water was generally unsuitable for Diesel engine cooling. Steam for heating the oil was not necessary in the moderate climate in this region. Moreover, electrical equipment had given very satisfactory service in other applications in producing oil.

Considering all these conditions, one pipe line company electrified 150 mi. of 10-in. line and the results were very satisfactory. Later, when the immense Permian Basin fields in southwest Texas were discovered, electrical pumping was installed by several companies operating in that area.

One of the companies electrified 400 mi. of 10-in. line, extending from McCamey, Texas to Healdton, Oklahoma. This line is served by the transmission systems of three power companies and at this writing has been in successful operation for six months, meeting all expectations as to continuity of service and economy of operation. A second company built and electrified 125 mi. of line from McCamey to Del Rio, Texas with the same satisfactory results. A third company extended their lines from Crane through DeLeon, Texas to the Gulf, a distance of 650 mi. This line has 19 stations, 14 of which are completely electric-driven, and the remainder partially so. A fourth company has now under construction 450 mi. of 10-in. line with 11 motor-driven centrifugal pump stations and this line has been in operation since January 1, 1929.

The first pipe line company mentioned is now engaged in building a line from McCamey in West Texas to tide-water at Houston, a distance of approximately 425 mi. According to present information, this line is to be motor-driven.

In most of the cases a specially developed centrifugal pump is used. The centrifugal pump has the advantages of non-pulsating delivery, ability to handle oils of various viscosities, ideal characteristics for booster service, and low first cost.

Several years ago, before this type had been developed for oil-line pumping, the only available centrifugal pumps for the deliveries and heads required were boiler-feed pumps. These were tried and it became immediately evident that they were unsatisfactory. Pump efficiencies were 55-60 per cent and this alone precluded their use in oil line pumping. The centrifugal pump manufacturers attacked the problem from all angles and the so-called "high efficiency" type of multi-stage centrifugal pump came into being. New theories of centrifugal pump design were developed; casings and impellers were redesigned; internal losses were analyzed and reduced as far as possible and the mechanical details received much attention. Stuffing boxes and bearings were redesigned, and from it all, came the present oil line centrifugal pumps with higher efficiencies than had even been hoped for just a few years previously.

As to the electrical features of the typical motor-driven pipe line station, there is no departure from modern substation practises. A wiring diagram of a typical station is shown in the accompanying illustration (Fig. 1). Greater attention perhaps has been paid to the construction of flame-proof installations. Some of the companies favor separating the motor from the pump by means of metal fire walls; others install

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29-64

them all in one room. It seems that personal opinion of the engineers of each company determines the policy to be pursued.

Squirrel-cage induction motors are used in all cases. Motors have been of all standard voltages but the present trend is toward the use of 2200 volts on all

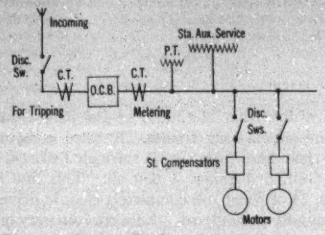

FIG. 1.—TYPICAL OIL PIPE LINE PUMP STATION

Oil line wiring diagram

motors above 50 hp. Oil circuit breakers are usually installed on incoming lines and these are provided with the usual overload, undervoltage, and over-pressure protection.

As to the actual cost of moving oil through pipe lines by means of electric power, it is impossible to give any accurate figures unless all the conditions of service are known. The various factors, such as efficiency of motors, efficiency of pumps, friction of station piping, rate schedules, discharge pressures, etc., all enter into the final analysis.

Below will be found a tabulation of electric pipe line station performance, based on crude oil of 80 Sec. Saybolt Universal viscosity at 60 deg. fahr. and at a discharge pressure of 700 lb. per sq. in.

PIPE LINE "A"
Reciprocating Pumps

Station	Months	Barrels pumped	Total kw-hr.	Barrels per kw-hr.
1	9	7,625,286	1,959,700	3.88
2	9	7,658,579	1,959,500	3.91
3	9	7,633,049	1,829,900	4.17
4	9	7,653,154	1,819,500	4.22

PIPE LINE "B"
Reciprocating Pumps

Station	Months	Barrels pumped	Total kw-hr.	Barrels per kw-hr.
5	6	4,756,768	1,202,900	3.96
6	10	6,538,992	1,634,000	3.98
7	6	4,480,040	1,122,600	4.05
8	10	6,519,459	1,616,100	4.03

PIPE LINE "C"
Centrifugal Pumps

Station	Months	Barrels pumped	Total kw-hr.	Barrels per kw-hr.
1	12	6,761,903	2,618,800	2.58
2	9	7,138,178	1,652,700	4.32
3	12	8,132,583	2,444,600	3.31

To meet the demands on their various generating stations and transmission systems, power companies serving the Southwest have had to make extensive additions, improvements, and extensions. Interconnections have been worked out and the service rendered has been, as a whole, very satisfactory to the pipe line companies.

In laying out transmission lines to serve pipe line loads, the importance of the absolute necessity to maintain continuous service must be considered. Pipe line systems having small ballast or surge tanks at each station do not provide storage capacity sufficient for an appreciable shut-down and a continuous supply of energy must be maintained at all times. To supply this demand, if at all possible, pipe line services should be supplied from a system having a two-way feed, with necessary switching apparatus to quickly sectionalize all transmission line faults.

As a source of constant revenue, pipe line loads are very desirable to the power companies. Revenues are the maximum on account of the high load factor. The average monthly load factor of one of the southwestern pipe lines for the last twelve months has been 86.6 per cent and from present indications it will continue as high for the next four years. These loads are desirable also on account of the ease of service. No extensive distribution facilities are necessary to secure the revenue. In many cases in Texas and Oklahoma, station locations are within sight of the transmission lines. As to the power factor of these loads, if the station is correctly designed, the motors driving the pump should be fully loaded and the power factor should be between 76 per cent and 87 per cent. Certain high-speed centrifugal pump motors, tested under field conditions, showed a power factor of 87 per cent. If the rate schedules contain a power factor clause with a penalty and premium rider, the use of condenser equipment by the pipe line company would be justified in most cases.

CONCLUSION

It is felt that the electrification of oil pipe lines is completely justified by the success of present installations. The time of installation is only half that required for Diesel engine equipment. The operators are easily trained to handle the electrical installations. Their operation on units of a transmission system having several sources of power is just as satisfactory as the present oil engine-driven stations. In general, shut-downs due to transmission line faults are less frequent than shut-downs due to failure of mechanical equipment. Cost of operation compares favorably with oil-engine driven stations. The near future will see many advances in pump construction and in electric pump station design and a consequent increase in over-all efficiencies.

Discussion

W. C. Dreyer: Mr. Levy mentioned that synchronous motors and squirrel-cage motors are used for this application. It seems to me there is a very definite field for the use of the wound-rotor motor. There are operating conditions on pipe lines where they want to run the station at reduced capacity. Much higher efficiency may be obtained at reduced capacity with wound-rotor motors than if the discharge valves of the centrifugal pumps are choked down to reduce the pressure.

Regarding the synchronous motors on centrifugal pumps, I don't believe they will be used as extensively in the future as they have been in the past. When these original major pipe line installations were made there was a very great difference in efficiency between the squirrel-cage motor and the synchronous motor. About the best efficiency on squirrel-cage motors of that size was 91.5 to 92 per cent. During the last year or two, by using silicon iron laminations and special end rings on the rotor, it has been possible to raise the efficiency to about 94.5 per cent. This cuts down the margin of efficiency of the synchronous motor, which was 95.5 or 96 per cent on motors of that size, so that the lower cost and greater sturdiness of the squirrel-cage motor is not compensated for.

On the subject of choice of motors there is a feature that hasn't been mentioned, automatic control of pipe lines. The squirrel-cage motor will be much more adaptable to automatic control when it becomes general, as only a starting operation is necessary. It is a simple matter to operate a station entirely automatically by the pressure in the line or by remote control. Though this is done it will be a long time before the labor at the station will be entirely eliminated. The investment and the combustibility of the fluid will make it very desirable to have one man at a pipe-line station subject to call by an alarm system or some such device.

D. H. Levy: I should like to answer Mr. Dreyer's question about synchronous motors. The nature of the fluid, as he mentioned, is inflammable. The slightest leak at a flange or valve, with 700 lb. of pressure, produces an oil vapor ready to fire. With a standard synchronous motor you have some arcing at the collector rings with attendant liability of an explosion. You also have to have an exciter, increasing the maintenance of another piece of rotary apparatus. You do, however, gain two or three points in efficiency.

To use synchronous motors successfully as applied to direct connected oil pipe line pumps, you should have fire walls between the drive and the driven unit, necessitating two different rooms with packing glands on the shaft between them. You have one operator operating these two stations and he is going to be on one side or the other of the fire wall. He doesn't get a whole view of the situation when he stands at the switchboard or gage board and operating troubles may result, due to the human factor. Our experience has been that we can install induction motors as closely as we want to the centrifugal pump with practically no danger of fire hazard.

Some companies are starting to install control apparatus in separate rooms. That defeats the purposes of inspection or observation. The normal crew of a pipe-line station, electric driven, is four men, chief engineer and three engineers, with no oilers or operators. These stations have no telegraph operators, using the telephone only for dispatching. That leaves one man always on duty.

Our company has gone so far as to try to perfect an automatic control for pipe-line stations controlling centrifugal pumps cut in to the line, by-passed by check valves. One unit is started by a rise in pressure on the incoming line. If the station ahead happens to put another unit on the line to bring up the pressure to 700 lb. (or other final pressure) in turn the pressure will be reflected at the station in question and No. 1 unit will start. It will have a tendency to pull down the suction and increase the pipe-line discharge pressure. This is held for 15 sec., using a timing relay. If conditions are still stable or become so, then No. 2 unit starts and pulls down the suction again to normal. We have four of these in Texas that were not entirely successful on account of the station ahead being a reciprocating station driven by oil-engine units. If one of the oil engines happens to miss fire two or three times, the speed decreases and down goes the pressure and the unit is stopped, then it picks up and comes back again. We try to minimize the surge conditions (at least the starting condition, due to surges), by installing more timing devices. So far we have found these fairly practicable.

The ideal condition would be to have one originating station and a series of booster stations cut in to the lines with only ballast tanks. We do install over-pressure protecting devices on all our electric stations. Prior to the electrification work we had lost many lines due to excessive pressure. If a man happens to switch from one tank to another and forgets to open the valve before he has closed the valve on the line, up goes the pressure to approximately 1000 lb. I have seen as much as 22 ft. of lineal rip in an 8-in. line with consequent loss of time and oil. So we put over-pressure protecting devices set 25 to 50 lb. above normal operating pressure. Pipe lines used for 700 lb. of pressure have an ultimate strength for 1000 lb. The thickness of the walls of the pipe is about ¼ in. Consider the erosion of two or three years and you see how close to the safe limit we operate. The tendency now is to use slightly heavier pipe and higher operating pressures.

As to the use of wound-rotor motors, we would still have the same trouble as to fire hazards. The use of totally enclosed motors prevents the desired inspection. Our company prefers to use the induction motor with automatic and hand starters. We try to limit as far as possible the number of sizes, speeds, and voltages. All except the motors used in oil production work, are now of the standard 2200-volt type.

Lightning Studies of Transformers
by the Cathode Ray Oscillograph*

BY F. F. BRAND[1] and K. K. PALUEFF[1]
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Study of transformers has been under way for some time to coordinate the strength of transformers and transmission line insulation under lightning conditions. This study has taken two forms: first, the transient dielectric strength of the line end of the winding and second, the distribution of transient voltages and therefore the stress caused by them throughout the winding.*

Theoretical studies and spark-gap tests of transient voltage distribution in transformer windings have been previously published by the Institute. Since then an extensive study of transient voltage phenomena has been made by the cathode ray oscillograph on power transformers connected to a short transmission line and subjected to artificial lightning waves sent along the line.

The effects of the transmission line, concentrated inductance, and transformer entrance bushings on traveling waves were studied, as well as the effect of traveling waves of various service conditions in producing internal oscillations in ordinary transformers.

The non-resonating transformer was studied under similar conditions.

A striking agreement between the oscillographic records and theoretical conclusions previously published was found, sufficient to establish beyond any doubt the following conclusions:

1. Very high voltage oscillations occur throughout the entire winding of even a grounded neutral transformer.

2. Points of the winding near the grounded neutral may rise to 95 per cent of the crest voltage of a very short traveling wave (three microseconds long).

3. Entrance bushings have a negligible effect on the shape of the incoming traveling wave.

4. In case of sudden voltage changes, concentrated inductance in series with the transformer, unless by-passed by a suitable device, causes rise of voltage across the transformer terminals as well as internally in the windings.

5. Arc-over of line insulators by a traveling wave produces severe oscillations in a transformer the amplitudes of which are roughly proportional to the arc-over voltage of the line insulators.

6. Grading the insulation between high voltage and low voltage and ground in ordinary transformers with grounded neutral is a dangerous practise when the transformers are subject to lightning.

7. All the above conclusions apply to concentric winding core type as well as interleaved and shell type transformers. From theoretical studies the non-resonating type of transformer has been developed and its action checked by tests. This type of transformer eliminates voltage oscillation within the winding and therefore local concentration of transient voltage.

* * * * *

INTRODUCTION

THIS paper, which deals particularly with the effect of lightning and other transient voltages on transformers, is one part of an extensive coordinated investigation covering all of the various phases of the lightning problem which has been under way for a number of years in the high voltage engineering laboratory at Pittsfield. Other papers of this series on other phases of the problem have already been published.[2]

Part I

Lightning, and to some extent other transient voltages such as are caused by switching surges and arcing grounds, have always been a serious source of failures of high-tension lines and apparatus, including transformers.

Transformers are in general reliable pieces of apparatus; in fact, their reliability is probably higher than that of any other electrical apparatus, but a small percentage of failures does take place and the majority of these is probably due to lightning.

This is not only because surges are frequently of very high voltage, but in regard to their effect on transformers and similar apparatus, it is because lightning voltage waves are of steep front, while switching surges and arcing grounds are often of relative high frequency.

The effect on transformers of high-frequency oscillations and steep front waves, are to some extent similar and a discussion of the effects of lightning in a measure covers the effects of other high-voltage transients.

Previous studies of these phenomena have been either theoretical or tests by spark-gap measurement. Since the cathode ray oscillograph is a new tool which enables one to record values almost instantaneously and obtain complete wave forms of phenomena lasting a few microseconds, it seemed desirable to use this device for such a study.

Part II

LIGHTNING FAILURES OF TRANSFORMERS

Transformer failures sometimes occur directly through the major insulation from high-voltage winding to low voltage or ground, either near the terminal or in the main body of the winding.

Failures are more liable to occur between turns or coils of the same winding or in auxiliary parts connected to the winding, such as taps, tap terminal boards, or ratio adjusters.

Failures which occur through the major insulation near the line terminal are due to excessive voltage applied to the transformer.

Failures occurring internally in the winding or auxiliary parts are due not only to the voltage to which

*With an Appendix, *Present Status of the Cathode Ray Oscillograph*, by H. L. Rorden and J. C. Dowell.

1. Both of the General Electric Company, Pittsfield, Mass.

2. F. W. Peek, *Lightning*, A. I. E. E., TRANS. Vol. 43, p. 1205, Vol. 47, p. 1131; Vol. 34, p. 1857; A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, *Frank. Inst. Jl.*, Feb. 1925.

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29-68

the transformer is subjected but to the nature of the surge and its effect in the transformer.

It is obvious that irrespective of what might be called the internal effect of the surge on the transformer, some limit must be set to the voltage which may be applied to the transformer. If this is not done failure will occur in any transformer.

Part III

Limits of Voltage on Transformer Insulation

To cause breakdown of any insulation requires energy. It is therefore a question of time as well as of voltage. The shorter the time, the higher the voltage required. Since lightning voltage waves last a relatively short time (from a few to possibly 100 microseconds), the voltage required to break down insulation by lightning is always higher than that required for long time duration of the voltage.

The voltage required to cause breakdown by lightning is higher the greater the energy required for a particular kind of insulation. It is fortunate that the oil and most solid insulations require greater energy to cause breakdown than air, and therefore higher lightning voltage for the same 60-cycle strength.

The shape of the insulation, or rather of the dielectric field, has of course a marked effect on the voltage required to cause breakdown.

If the shape of the dielectric field, or in other words, the potential gradient, changes with varying frequency or wave form of applied voltage, it will cause a variation of the ratio of applied voltages to cause breakdown.

To determine the lightning breakdown voltage of even the simplest insulation structure, requires a knowledge of the wave form of the applied voltage. For this reason, data on lightning breakdown by various authorities are not always comparable since the wave form assumed or used for test purposes may vary considerably.

When lightning sparkover of insulators is referred to it is usually based on data published by F. W. Peek, on the insulator sparkover under artificial lightning produced in the Pittsfield Laboratory, for a wave rising to approximately maximum voltage in one-quarter of a microsecond and tapering down to one-half its crest value in five microseconds. This wave form was used long before any actual measurements of lightning wave form were obtained, and it was assumed to correspond to quite severe lightning. The tests were made in such a manner as to utilize the full wave above the half crest value. Below half voltage this wave is not effective because it then corresponds to the 60-cycle value. This particular wave may be said to have an effective length of 5 microseconds. The effects of waves having effective lengths of 5 to 80 microseconds have also been published.[3]

Field tests have since shown that this assumed wave is remarkably close to that obtained under severe conditions.

Based on this wave form, a string of suspension insulators requires a voltage of approximately 1.8 to 2.0 times the 60-cycle dry sparkover voltage, to cause lightning sparkover. Sparkovers caused by natural lightning have given the same range.

Corrugated or petticoated bushings such as are used for outdoor service, require a lightning voltage from 2.0 to 2.5 times the 60-cycle dry sparkover voltage depending upon design.

Oil, and other solid fibrous insulation such as that used in oil-immersed transformers, require from 2.8 to 3.5 times the lightning voltage compared to 60-cycle one-minute breakdown voltage. These ratios are crest to crest value. Effective or r. m. s. values are not used in dealing with lightning waves.

Giving the ratio of r. m. s. 60-cycle one-minute breakdown of oil and solid insulation against crest value, lightning breakdown by the Pittsfield wave results in a value of 4 to 5.

This ratio does not hold good for the ratio of standard factory test voltage at 60 cycles to the breakdown value under lightning of completed transformers due to two causes:

1. The completed transformer has a margin of safety over the factory test. This margin varies with voltage rating, usually being higher in the lower rated voltage apparatus.
2. The ratio holds for a simple insulation structure and does not take into account change of voltage gradient in the transformer with varying frequency.

It is obvious from the different ratio of lightning and 60-cycle strength of different apparatus, that transformers, for instance, comprising oil and solid insulation, may have a lightning strength equal to the strength of bushings, or line insulators having a very much higher 60-cycle sparkover value. For instance, a transformer for a 115,000-volt circuit given a factory test of 231,000 volts, may be stronger under lightning conditions than line insulators having a 60-cycle dry sparkover for 400,000 volts. This conclusion might be correct as regards the insulation to ground of the line terminal end of the winding. The conclusion would not necessarily be correct for the whole transformer due to the effect of the lightning wave form on the distribution of voltage within the transformer winding.

Part IV

Effects of Transient Voltage

If we consider the case of a transformer with one terminal grounded such as Y connected banks with grounded neutral, the normal frequency voltage distribution is a uniform fall of voltage from line to neutral.

In a transformer each turn and coil has capacitance to ground and to adjacent turns and coils.

Under the impact of a traveling voltage of steep front

3. F. W. Peek, Jr., *Lightning*, A. I. E. E. Quarterly Trans., Vol. 48, April 1929.

on account of the inductance of the winding the inductance current cannot be established instantly and therefore only the charging current of the winding capacitance to ground will flow at the first moment.

In a simple way the circuit of capacity elements of the winding (as shown on Fig. 1) can be likened to that of a string of suspension insulators and therefore the flow of charging current establishes voltage distribution

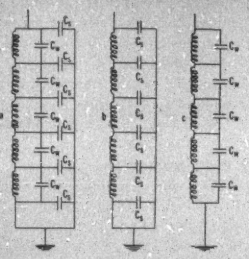

FIG. 1—EQUIVALENT CIRCUIT OF AN ORDINARY TRANSFORMER

a. Complete circuit
b. Circuit effective at low transformer harmonics
c. Circuit effective at high transformer harmonics

along the winding, similar to that of the insulator string as shown by curve 1 of Fig. 2.

This results in a great concentration of voltage on the end coils and turns at the time of impact of the traveling

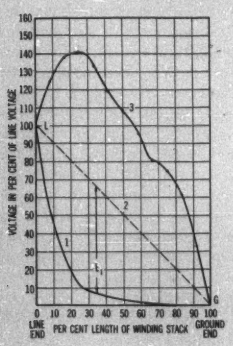

FIG. 2—VOLTAGE PRODUCED BY SINGLE TRAVELING WAVE (CALCULATED)

1. Initial voltage distribution
2. Final voltage distribution (or line of equilibrium)
3. Maximum voltage to ground

wave. Seventy-five per cent of the voltage may be consumed across 5 to 10 per cent of the winding adjacent to the line.

With voltage maintained across the transformer constant for a sufficient time (say 50–100 microseconds), the inductance current will be established throughout the winding and with it uniform voltage distribution, as the inductance current, in contrast to the charging current flowing at the first instant of impact of the wave, is one and the same throughout all the winding. The voltage distribution is shown on curve 2, Fig. 2, and is called final.

Since there is a difference between the initial voltage distribution (see curve 1, Fig. 2) and final distribution (curve 2, Fig. 2) a transient will take place during which the voltage distribution will readjust itself from initial to final value. On account of the low damping factor of transformer windings the transient will be oscillatory.

The voltage of each point of the winding between line and ground terminals will oscillate about the point of its final voltage, reaching maximum voltage to ground as shown by curve 3, Fig. 2.

Part V

MECHANISM OF OSCILLATION

The curve of difference between the initial and the final voltage distribution along the winding, gives the

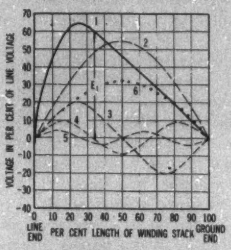

FIG. 3—WINDING SPACE HARMONICS AT THE INSTANT OF IMPACT OF THE TRAVELING WAVE

1. Initial voltage distortion curve obtained from curve 1, Fig. 2 by referring it to line of equilibrium No. 2
2, 3, 4, and 5 are first, second, third, and fourth space harmonics of curve No. 1

initial amplitude of oscillation of different points of the winding and is called the initial space wave or initial voltage distortion curve.

The oscillation following the impact of the traveling wave may be looked on as a collapse of this space wave on line *L. G.* with its inertia carrying it on the other side of this line. The line *L G* therefore is the line of equilibrium.

The initial distribution curve is generally of the character of that shown in curve 1, Fig. 2. With such an initial distribution the initial space wave will consist of the fundamental frequency space wave and various harmonic space waves.

When these are all superimposed on one another the result will be the curve of initial voltage distortion or the initial space wave.

Curve 1, Fig. 3, is the space wave obtained on a 6600-kv-a. transformer and is drawn from the initial voltage distribution curve 1 of Fig. 2. It consists of

fundamental, second, third, and fourth space harmonics of magnitude shown and of seven higher harmonics of smaller amplitude which are not shown.

The frequency of oscillation of the first few harmonics is approximately proportional to the second power of their order.[4] That is, the second harmonic oscillates

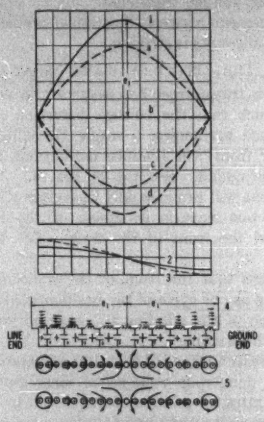

FIG. 4—CURRENT AND VOLTAGE OF THE FIRST OR THE FUNDAMENTAL HARMONIC

1. Simultaneous first harmonic voltage along winding to the line of equilibrium *L G* of Fig. 2, at moment of impact of traveling wave
a, b, c, d simultaneous voltage to line of equilibrium $^1/_8$, $^1/_4$, $^3/_8$, and $^1/_2$ of a cycle of fundamental frequency later
2. Voltage between adjacent elements of winding stack
3. Current in various parts of the winding during oscillation
4. Voltage and current distribution in equivalent circuit *b* Fig. 1
5. Magnetic flux distribution produced by current No. 3

four times as fast as the fundamental and the third nine times as fast. Taking the fundamental frequency space wave shown by curve 1, Fig. 4, and any two points of the winding symmetrically located in respect to the middle point of the winding, the difference in potential of the fundamental frequency of each point and the nearest end of the winding is the same in magnitude but opposite in sign. This potential difference will force current from each point to the respective ends of the winding. Thus the current in the halves of the winding will be in opposite direction.

Curve 2, Fig. 4, shows that the fundamental space wave produces maximum voltage between adjacent elements of the winding, such as turns or coils, at the ends of the winding; therefore the inductance current of oscillation of this wave will be the maximum at the ends as shown by curve 3, Fig. 4. The magnetic fluxes set up by these currents will be as shown on Fig. 4.

Following the same train of thought the curves of voltage distribution produced by other harmonics can be easily delineated. Fig. 5 illustrates the second harmonic.

The examination of these figures reveals that the ratio of the frequencies of the first few space harmonics to that of the fundamental must be nearly equal to the second power of their order.

Where in a transformer series capacitance C_w is not negligible the ratio of frequencies of first three harmonics instead of being 1:4:9 may be 1:3:8 or even 1:2.5:6.5.

The considerations outlined previously and above lead to the following conclusions.

With uniform windings:

1. The impact of a single traveling wave striking the winding causes it to oscillate. The oscillation starts simultaneously throughout the entire winding. It is composed of a number of space harmonics.

2. The higher the harmonic the smaller its amplitude.

3. On account of the symmetry of distribution of harmonics the oscillations cause high voltage not only near the line end but also at many other points including that near the grounded end.

4. The amplitude of oscillation depends upon voltage of the applied wave, the steepness of the initial (or electrostatic) voltage distribution, the steepness of the front of the traveling wave, and the length

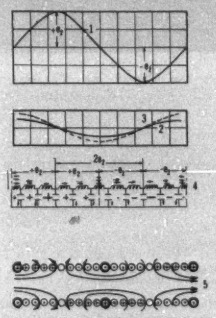

FIG. 5—CURRENT AND VOLTAGE DISTRIBUTION OF SECOND HARMONIC

1. Simultaneous second harmonic voltage to line of equilibrium *L G* of Fig. 2 along winding stack at the moment of impact
2. Voltage between adjacent parts of winding stack at the moment of the impact
3. Current in various parts of the winding during oscillation
4. Voltage and current distribution in equivalent circuit *b* of Fig. 1
5. Magnetic flux distribution produced by current No. 3

of the tail of the wave. The damping factor of the winding is not sufficient to prevent the oscillation.

It is obvious that in non-uniform windings, such as interleaved types, in addition to the oscillation involving the entire winding, local oscillations are present at the points of discontinuity.

4. See *Effect of Transient Voltage on Power Transformer Design*, by K. K. Palueff, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929.

Part VI

Tests by Cathode Ray Oscillograph

In order to check previous theoretical studies of the behavior of transformers under lightning and tests made by spark-gap measurements, a series of tests by an artificial lightning generator in connection with a short transmission line has been made by the cathode ray oscillograph.

Where convenient, several oscillograms were recorded on the same film. In such cases some waves were taken above the zero line, then the polarity of the oscillograph was reversed and the remainder of the waves recorded downward below the zero line. The waves obtained with reversed polarity are marked *R*. The crest value of the wave at the transformer end of the line

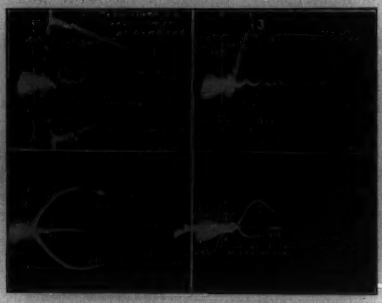

Fig. 6—Effect of Front of Traveling Wave. 6600-Kv-a. Transformer

Top

"Exceedingly steep front" and long tail

1. Voltage across transformer

2, 3, 4, and 5 are voltages to ground at points 97.2, 50, 25, 2.8 per cent away from ground terminal with crest values of 97, 114, 75, and 18 per cent of applied voltage, respectively

Bottom

"Steep front" and long tail

1. Voltage across transformer

2, 3, 4, and 5 are voltages to ground at 97.2, 50, 25, and 2.8 per cent away from ground end with crest values of 93, 64, 24, and 5 per cent of applied voltage

(see Fig. 20) was considered as unity and crest values of all other waves expressed in per cent of this value.

The voltage of any given point of the winding to ground is considered "normal" if it is the same percentage of the crest of the applied wave as it is under normal operating frequency excitation. Excess over this value is called "overvoltage" and is expressed in per cent of the "normal" voltage.

The tests were made on an overhead transmission line over two miles long having a surge impedance of 350 ohms. The following apparatus was used at various times in the tests:

6600 kv-a–110-kv. Circular coil type transformer
1000 kv-a.– 44-kv. Shell type transformer
Air core inductances of 1.0 and 0.24 millihenry
110-kv. solid insulation transformer bushing having a capacitance of 0.00024 microfarads.
Various protective spark-gaps with and without resistance in series.

To date more than 600 records have been obtained representing a very wide range of service conditions affecting the voltage transients.

The agreement between theoretical predictions[4] and experimental results is striking.

The Effect of Wave Front

If a given transformer has electrostatic voltage distribution such that the voltage distortion curve contains n space harmonics, a single traveling wave of rectangular front will produce oscillations containing all these harmonics. See top of Fig. 6. A wave of slanting front will produce only those harmonics whose duration of one cycle is in excess of the length (or duration) of the front of the wave. See bottom of Fig. 6.

This means that the front of the wave must be at least as long as one cycle of the fundamental frequency of oscillation of the winding to cause no voltage distortion in the winding.

For this reason waves of even 100 microseconds front may cause dangerous voltages in large transformers. Fig. 7 illustrates the relation of wave front to maximum voltage to ground at two points in the winding.

It is suggested that the wave front be called "steep" when it produces oscillations of fundamental frequency only, "very steep" when oscillations of fundamental and second harmonic are produced, and "exceedingly steep" when harmonics of a higher order are produced.

With the wide variation of high-frequency characteristics of transformer windings it is obvious that the exact range of the length of the wave front cannot be stated, but appears to be as follows:

"Exceedingly steep."	Of the order of 3 microseconds–0.57 miles.
"Very steep"	Of the order of 10 microseconds–1.9 miles.
"Steep"	Of the order of 50 microseconds–9.5 miles.

Effect of the Length of the Wave

The internal oscillation will reach maximum values to ground at all points of the winding, if the transformer is subject to a traveling wave of exceedingly steep front and of appreciable amplitude for a length not less than the duration of ½ cycle of the fundamental frequency of the winding.

On account of the relation of the harmonics, such a wave will produce severe overvoltage at both the line and the ground quarters of the winding.

Fig. 8 shows the relation between wavelength and voltage to ground of the middle part of the winding.

In discussing wavelengths, it is important to designate them in their relation to the natural frequency of the transformer. Thus, waves may be considered as "very long" if the length is at least equal to a full cycle of the fundamental frequency of the winding, as "long" if at least equal in length to ½ cycle of the

fundamental, as "short" if the length is between ¼ and ½ cycle, and "very short" if less than ¼ cycle.

If expressed in time or length, the order of the various waves would be:

Very long wave	25 to 100 microseconds	5 to 19	miles
Long wave	12 to 50 microseconds	2.3 to 9.5	miles
Short wave	6 to 25 microseconds	1.15 to 5	miles
Very short wave	1 to 4 microseconds	0.29 to 0.76	miles

Effect of the Tail of the Wave

The effect of the steepness of the tail of the traveling wave is similar to that of the steepness of the front,

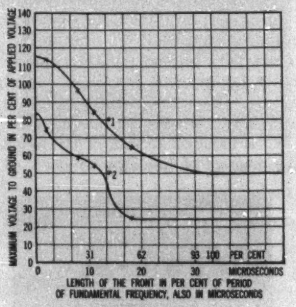

Fig. 7—Effect of Steepness of Front of the Traveling Wave on Voltage to Ground (Test)

1. Point of winding 50 per cent away from ground terminal
2. Point of winding 25 per cent away from the ground terminal

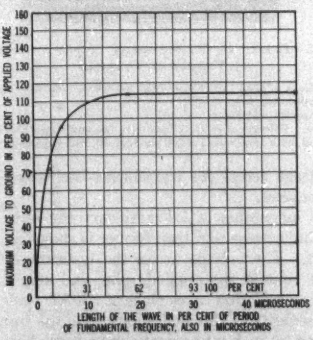

Fig. 8—Effect of the Length of Traveling Wave on Voltage to Ground of Point of Winding 50 Per Cent Away From Ground Terminal (Test)

because the transient effect is essentially the same, whether the terminal voltage suddenly rises from zero to a given value and thereby creates a "positive" transient or suddenly falls from that value to zero and creates a "negative" transient.

However, on account of the fact that at the time the end of the traveling wave reaches the transformer terminals, the winding is in a state of positive oscillation, sudden fall of the terminal voltage superimposes a negative oscillation, and the combination may create local stresses in excess of those caused by a wave of steep front and of slanting tail.

It follows that to prevent internal oscillations from

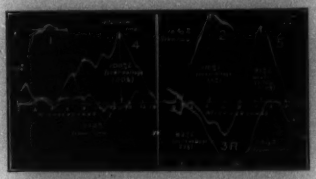

Fig. 9—Voltages at Several Points in Transformer Winding

1. Voltage at transformer terminal produced by traveling wave of approximately five microseconds
2, 3, 4, and 5 are voltages at points 83, 66, 50, and 25 per cent from ground terminal with crest values of 110, 82, 100, and 95 per cent of applied voltage respectively

reaching dangerous values, protective devices must be set so as to reduce the crest value of the incoming wave well below the ultimate strength of the transformer.

As shown by Fig. 6, curve 4 *R*, a long wave with slanting tail produces at a point 25 per cent away from ground a voltage 75 per cent of the terminal value,

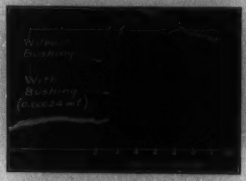

Fig. 10—Effect of Bushing on Shape of Traveling Wave

(To avoid confusion two waves were displaced along time coordinate as shown)

whereas a short wave, Fig. 9, curve 5, produces at the same point a voltage 95 per cent of the terminal value.

Effect of the Bushing

It has sometimes been stated that the capacitance of the bushing has a marked effect in slanting the wave-front and therefore decreasing the severity of voltage distortions. It has been shown previously[4] that very long wave fronts are necessary to avoid voltage distortion.

The bushing capacitance (of even a "high capacitance" bushing) has a negligible effect on the wave front as shown by Fig. 10 and therefore cannot affect the transient phenomena in the transformer.

Effect of the Transformer on the Incoming Wave

The effective capacitance of a power transformer is too small, and its surge impedance is too great to have any appreciable effect on the shape or crest value of the incoming traveling wave.

To illustrate this, a traveling wave was sent along the transmission line and records were made at the far end

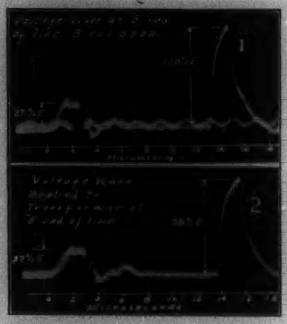

Fig. 11—Effect of Transformer on Incoming Traveling Wave

1. Wave at the end of transmission line with transformer disconnected
2. Wave at the end of transmission line with transformer connected (voltage across transformer)

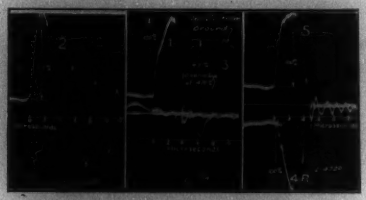

Fig. 12—Effect of Series Inductance in Case of Traveling Wave of Very Steep Front and Tail

A Inductance of 1.0 μ h.
1. Voltage applied across transformer and inductance in series
2. Voltage across transformer. Crest value 115 per cent of the applied wave (Curve No. 1). Frequency of oscillation 333 kilocycles
3. Voltage to ground at point of winding 17 per cent away from the grounded end. Crest value 95 per cent of applied voltage

Note, front of the wave No. 2 is steeper than No. 1. Approximate effective capacitance of the transformer winding 0.00206 μ f.

B. Inductance of 0.24 μ h.

4*R*. Voltage applied across inductance and transformer

5. Voltage across transformer. Crest value 100 per cent of the applied voltage. Frequency 710 kilocycles. Note, front of the wave No. 5 the same as No. 4*R*. Approximate effective capacitance of the transformer winding 0.0019 μ f.

with the transformer first connected to and then disconnected from that end. The wave shape was practically identical in spite of the fact that the waves were only two microseconds long, see Fig. 11. The 2 per cent difference in crest value is well within the limits of error of measurement.

Effect of Inductance in Series with Transformer

Inductances such as current limiting reactor or current transformer if not bridged by a proper by-pass device, enter into oscillation with the effective capacity of a transformer to which they are connected.

Such oscillations may create very severe overvoltage inside of the transformer and cause voltage across the transformer in excess of the crest value of the incoming wave. These oscillations also take place in case of arcing grounds or switching.

Fig. 12 illustrates what happens when a line insulator arcs over at the crest of a 3-microsecond wave. The initial voltage across the transformer without inductance in series and with an inductance of 0.24 millihenry in series are identical.

The voltage from line end of the inductance to ground shown in Fig. 12, curve 4*R*, is recorded downward.

The voltage across the transformer with this inductance in series is recorded on the same film but upward and is identical to that of the incoming wave up to the moment of insulator arc-over. Thereafter

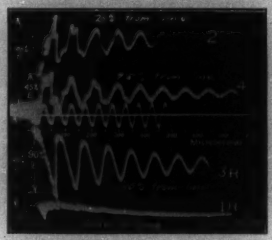

Fig. 13—Effect of Transformer Losses on Damping of Transient Voltages

Oscillations of shell type transformer caused by long traveling wave of eight microseconds front

1*R*. Applied voltage

2, 3*R*, 4. Voltages at points 75 per cent, 50, and 25 per cent away from ground end, with crest values of 106 per cent, 90, and 45 per cent

the voltage across the transformer oscillates at about 710 kilocycles.

With inductance of 1.0 millihenry the wave applied across inductance and transformer is shown in Fig. 12, curve 1. The voltage across the transformer rises to 115 per cent of the applied value and after insulator arc-over, oscillation takes place at 330 kilocycles.

The severity of the oscillation inside the transformer produced by such a transient can be judged by Fig. 12, curve 3. It shows that the voltage of that part of the winding 16.6 per cent away from its grounded end, instead of being at 16.6 per cent above ground, rises to 95 per cent of the applied voltage. Stated in terms of the normal voltage of this point, this is an overvoltage of 470 per cent. This is good evidence of the danger of reducing the insulation between high voltage and low voltage and ground near the grounded neutral.

Inductances of much smaller values have negligible

effect on incoming wave but also enter into oscillation, overstressing the line end of the transformer winding.

Effect of Damping and Type of Transformer

That the damping effect of the transformer is negligible in preventing the high voltages produced by the first few cycles of the oscillation is evident from examination of Fig. 13. It shows the oscillation

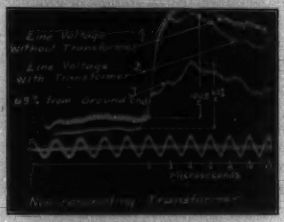

Fig. 14—Non-Resonating Transformer

1. Voltage wave at the end of transmission line with transformer disconnected
2. Voltage wave at the end of the transmission line with transformer connected (voltage across transformer)
3. Voltage at 63 per cent point. Crest value 63 per cent of applied voltage

Note No. 3 a practical duplicate of the shape of No. 2 in spite of the fact that the transformer was out of oil and electrostatic unbalance was created thereby

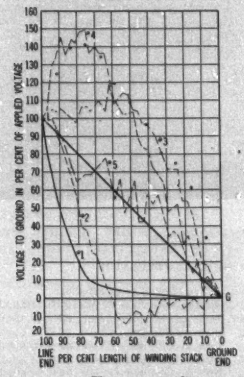

Fig. 15—Simultaneous Voltage to Ground Produced by a Single Traveling Wave (Test)

1. At the instant of impact ("initial" voltage distribution)
2, 3, 4, and 5.—3, 14, 18 and 24 microseconds later respectively

Dots show results of calculation (same as curve 3, Fig. 2) of maximum voltage to ground

in a shell type transformer at various points in the winding obtained with a wave of 8 microseconds front. The rest of the oscillograms given in the paper were made on the core type transformers and also illustrate the negligible effect of damping.

Conclusions

By theory and test, it has been shown that:

1. The initial voltage distribution due to a wave of steep front results in the most of the voltage being concentrated on that part of the winding near the line end.

2. The effect of the concentration of voltage in the end turns under the initial voltage distribution is to

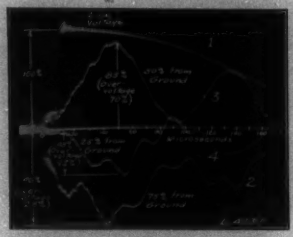

Fig. 16—Oscillations in Shell Type Transformers Due to Traveling Wave of 14 Microseconds Front

increase the potential gradient to ground at the end of the winding due to change in the shape of the dielectric field.

3. At normal operating frequency, steepness of the potential gradient is chiefly due to stress to low voltage winding and ground, the stress between coils being low. Under lightning waves, the stress between coils is increased very much and sharpens the potential gradient. It is as if the ground potential surface had been moved up close within the end of the winding.

The oscillations resulting in adjusting initial to final

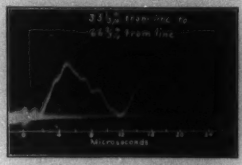

Fig. 17—Harmonic Analysis

Voltage between 33.3 per cent and 66.6 per cent points of winding produced by wave No. 1 of Fig. 6. The principal oscillation is that of second space harmonic because the voltage of fundamental harmonic produces no potential difference between these points and the amplitude of third harmonic is zero for both points.

voltage distribution usually result in higher than normal stress to ground at points intermediate to line and ground.

4. These increases of potential gradient therefore result in lowering the ratio of impulse to 60-cycle voltage breakdown of such a winding to a value lower than the ratio for a simple piece of insulation unless the insulation is properly proportioned throughout the transformer. Thus the line terminal insulation impulse

ratio may vary for different transformer designs depending on the amount of local concentration of voltage near the line end of the winding.

5. If the transformer is subject to transients consisting of a series of damped oscillations such as are caused by arcing grounds, or arcs during the opening or closing of switches, etc., the voltage distribution in the

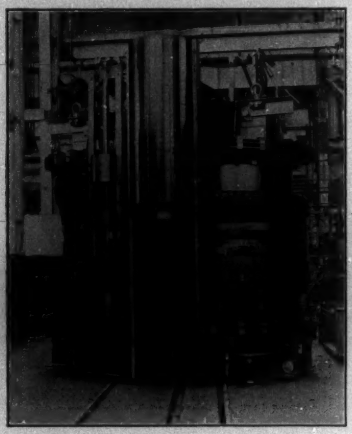

FIG. 18—220-KV. AND 37,000-KV-A. (EQUIVALENT CAPACITY) THREE WINDING NON-RESONATING TRANSFORMER

500-kv. insulation test

FIG. 19—AN EXAMPLE OF A WAVE RECORDED WITH OSCILLATORY SWEEP

Several cycles of single-frequency damped oscillation

ordinary winding becomes a very complex matter depending on the constants of the transformer and the frequency of the applied surge.

6. The oscillations resulting from such surges may cause high local voltages in the transformer winding at many different points, and the position and value of these voltages change with the frequency as well as the amplitude of the impressed voltage.

7. If the frequency of the surge is in the neighborhood of the resonating frequency of the transformer windings as a whole or in parts, voltage to ground as high as twice the impressed voltage of the surge may occur at some point within the winding.

8. If, for grounded neutral operation, the insulation to low voltage winding and ground is graded somewhat in the order of the normal frequency voltage stress to ground, overvoltages due to oscillations may overstress the graded insulation to low voltage or ground at various

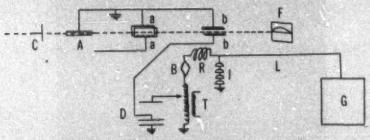

FIG. 20—DIAGRAM OF TESTING CIRCUIT

A and *C* anode and cathode of the oscillograph *aa* "time" and *bb* "voltage" plates of the oscillograph, *F*—Film or screen, *D*—Potentiometer, *T*—Transformer under test, *B*—Transformer entrance bushing, *R* inductance coil, *I*—Line insulator, *L*—Transmission line, *G*—Lightning generator

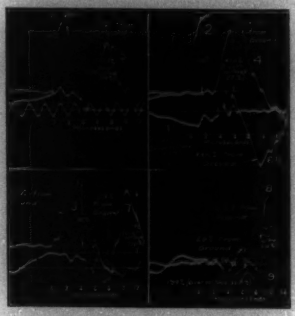

FIG. 21—OSCILLATIONS ALONG WINDING DUE TO SHORT TRAVELING WAVE OF "EXCEEDINGLY STEEP" TAIL, PRODUCED BY SPARKOVER OF LINE INSULATOR

1. The traveling wave applied to transformer
2. Point 97.2% from ground end—Crest 100% of applied voltage
3. Point 83.3% from ground end—Crest 82% of applied voltage
4. Point 75% from ground end—Crest 96% of applied voltage
5. Point 50% from ground end—Crest 62% of applied voltage
6. Point 33% from ground end—Crest 51% of applied voltage
7. Point 25% from ground end—Crest 79% of applied voltage
8. Point 16% from ground end—Crest 79% of applied voltage
9. Point 2.8% from ground end—Crest 13% of applied voltage

points in the winding. Graded insulation in ordinary transformers subject to single waves may therefore be unsafe.

9. It is general practise to reenforce the insulation between turns and coils near the line end of the winding, but the oscillation of the winding as a whole or in parts, may produce very high voltages between turns and

coils at points remote from the line end and cause local failure between turns or coils or across taps, etc., unless the permissible lightning voltage to which the transformer may be subjected is reduced below that set by the terminal insulation alone.

It is apparent that the strength of the transformer winding to resist lightning voltages can be much increased, if the difference between the final voltage distribution and the initial distribution can be eliminated, because:

A. No change of shape of the dielectric field or change of gradient with a given voltage impressed takes place on the end of the winding if the voltage concentration on the end turn and coil is eliminated. It is the same as at normal operating frequency.

B. Overvoltage to low voltage and ground due to oscillations in the winding building up a higher voltage than "normal" with uniform distribution are eliminated.

C. Voltage stresses between turns and coils would be reduced to the ratio of the lightning and normal frequency terminal voltage only.

D. The insulation to low voltage winding and ground could be graded in the order of the normal voltage stress.

E. The terminal voltage stress would therefore be the only lightning voltage limit and this would bear the same ratio to 60-cycle voltage as the impulse ratio of the insulation used alone.

The factory induced voltage test could therefore be made a real measure of the ability of such a transformer to withstand lightning, which is not true with ordinary transformers.

Part VIII

The Non-Resonating Transformer

The reason that the voltage distribution in a transformer under surges is not uniform and that oscillations take place may be briefly stated to be:

Because the electrostatic charging current of the ordinary transformer has to flow through the winding.

If the electrostatic charging current is directly supplied to the various parts of the winding in the proper proportion, then unequal voltage distribution and oscillations will be eliminated.

This principle is the basis of design of the non-resonating type of transformer. At the present time in commercial designs, it is limited to transformers operating with neutral grounded directly or through a moderate impedance.

The electrostatic charging current of the various elements of the winding is supplied by means of carefully rounded and thoroughly insulated metallic surfaces connected to the line terminal, and adjacent to the winding, so that the discharging current of the coils to ground can be supplied directly, without having to flow through the impedance of the winding.

Therefore the proportions of the applied voltage of each part of the winding, given by its position in the dielectric field at any frequency, is the same as the proportion of voltage generated in that part at normal frequency operation. The initial and final voltage distribution is therefore the same and no transient or oscillation can take place.

Thus by properly proportioning the electrostatic characteristic of the transformer, the voltage distribution throughout the winding is made a straight line falling from line terminal to ground irrespective of the frequency or wave shape of the applied voltage.

In practise, the non-resonating winding is uniform and concentric with the low voltage winding. The coils are relatively narrow. Standard circular coils are used, connected together successively at the inside and outside turns, so that cross connections from inside to outside of coils are avoided.

The mechanical design of the transformer follows long established practise, lending itself readily to assembly and handling, and embodies the usual desirable characteristic such as free oil circulation and ability to withstand short circuit stresses.

In the new transformer, voltage stresses between turns and coils under lightning conditions are reduced in the order of from 10 to 1 to 100 to 1. The overvoltage to ground caused by oscillations being eliminated, results in decreasing the stress to ground sometimes as much as 6 to 1. The elimination of local excess voltage between turns and coils reduces the probability of arc-over of tap terminal boards, ratio adjusters, etc. It is well known that the spacing or dielectric strength of taps, ratio adjuster contacts, etc., has to be made much greater than that required for the normal or factory test voltages in the ordinary transformer, due to the possibility of building up very high local voltages during lightning and other transient voltages.

Graded insulation in the order of the normal frequency stress to ground is possible and safe with the new type of transformer, since overvoltage to ground at points in the winding between line and ground ends no longer occurs due to transients.

The proportion of voltage stress in all parts of the winding under all conditions of voltage and frequency remains the same as that during the factory insulation test, which therefore becomes a real measure of the ability of the transformer to resist lightning. This never could be tested correctly before.

In such a transformer, since all voltage stresses are under complete control and are made uniform throughout the winding for all frequencies, the reliability, naturally, will be much greater than in ordinary transformers.

Appendix

PRESENT STATUS OF THE CATHODE RAY OSCILLOGRAPH

on the Measurement of Transients

By H. L. Rorden[5] and J. C. Dowell[5]
Non-member Associate, A. I. E. E.

Since there are several methods of operation of the cathode ray oscillograph, it is felt that the method applied in this case should be briefly reviewed.

In Fig. 20 the cathode beam is made to pass between two pair of deflector plates; "time" plates, *aa*, and "voltage" plates, *bb*, and to impinge upon the fluorescent screen or photographic film enclosed in the vacuum bell of the oscillograph. Since this beam is susceptible to deflection by transverse electrostatic fields, it may be made to sweep across the film if voltage is applied between one of the pairs of plates.

If an oscillating voltage is applied to the "voltage" plates, a vertical straight line will be traced on the film. Should a uniformly rising potential be applied to the time plates which are perpendicular to the first pair, it will sweep the beam in a horizontal direction at a uniform rate, and thus record a voltage-time relation of the voltage wave applied to the first pair of plates. By this method, therefore, the shape of the voltage wave is recorded in rectangular coordinates which of course is the best form of record.

Instead of applying to the time plates a voltage which rises at a uniform rate, some of the investigators are using a sinusoidal sustained oscillation of high frequency which gives an extremely complex record as shown in Fig. 19. Comparison of this record with other records given in the body of the paper, illustrates the superiority of the uniform sweeping method.

Formerly the principal difficulty involved in this method was the comparatively slow speed of sweep obtainable, which allowed a maximum speed of sweeping of 15 to 20 microseconds per centimeter on the film. However, the speed of the sweep has now been increased 200 times so that a wave of a fraction of a microsecond in duration can be spread over several centimeters of the film, permitting detailed analysis of the shape of the wave.

There are two problems peculiar to the study of transformer voltage transients: First, there is the necessity of measuring voltage between two points, both of which are above ground potential. Second, there is the necessity of synchronizing the voltage waves obtained at different points of the winding, produced by identical waves applied several times in succession. Both problems are easily solved. Fig. 17 serves as an illustration of the solution of the first problem, Fig. 15 was made possible by solution of the second. The family of curves was obtained from synchronizing oscillograms of 36 points of the winding and then scaling them at various time intervals as indicated on the figure.

Another recent development is the automatic initiation of the oscillograph, whereby it was made possible to record the wave of natural lightning[3] with the loss of but a small part of the front of even very steep waves.

5. Both of the General Electric Company, Pittsfield, Mass.

Discussion

PROGRESS IN LIGHTNING RESEARCH[1]

(Peek)

LIGHTNING STUDIES OF TRANSFORMERS BY THE CATHODE-RAY OSCILLOGRAPH

(Brand and Palueff)

Dallas, Texas, May 9, 1929

J. F. Peters: For many years it has been recognized that lightning voltages do not distribute uniformly in transformers, that the potential gradients near the terminals may considerably exceed normal operating voltage. To provide for that, the insulation at these places is reinforced. Until recent years we were obliged to depend upon experience, spark-gap measurements, and calculations to guide us in the design of transformers. It is gratifying to find that results obtained by the cathode ray oscillograph check the earlier values obtained by spark gaps so well.

The cathode ray oscillograph has added some very interesting details to our knowledge of voltage within transformers and it, along with other devices, is yielding valuable information on the nature and magnitude of actual lightning voltages encountered, which information is highly important in considering the necessary insulation within a transformer. We find that if the voltage is applied abruptly to a transformer the initial voltage distribution within the transformer is a function of the physical portions of the unit. The initial distribution can be controlled very considerably by properly selecting the proportions of the winding. When the impulse is maintained for a considerable time, 10 to 20 microseconds, the voltage will penetrate into the winding and may overshoot the values beyond a uniform distribution. The amount the voltage overshoots depends to a considerable extent upon the initial distribution which in turn is a function of the physical proportions of the design.

The maximum voltage that can reach a transformer is determined by the line insulation adjacent to the transformer. The flashover voltage value of line insulation varies with the length of voltage application. The line impulse flashover for impulses lasting 2 or 3 microseconds is approximately two times the 60-cycle crest flashover. For impulses lasting 15 microseconds or more the impulse flashover is approximately 1.2 times the 60-cycle crest values. A transformer should be insulated to meet these extreme conditions. The high but short impulses do not produce large oscillations on account of the time being too short. The longer impulses produce larger oscillations but the impulse is lower to start with. This state of affairs should be taken into consideration when determining the proper insulation to use.

S. W. Marshall, Jr.: The developments set forth in the Brand and Palueff paper may well be considered to be classical. The check, by the cathode ray oscillograph, of the calculated initial voltage distribution along the transformer winding caused by a single traveling wave, gives a definite insight as to the "whys" of some transformer failures.

Why is it that the second harmonic oscillates four times as fast as the fundamental?

The paper states: "The impact of a single traveling wave striking the winding causes it to oscillate. The oscillation starts simultaneously throughout the entire winding . . . ." Does this oscillation exist in both primary and secondary windings, or

1. This paper with other discussion of it was published in the A.I.E.E. Quarterly Trans., Vol. 48, April 1929, pp. 436 and 468.

only in the winding conductively connected to the incoming wave?

So much capital is invested in the conventional core-type transformer that I shall confine my concluding questions to this type. Consider a two-winding transformer connected to a transmission line. Is it likely that an incoming wave from the transmission line would flash over the low-tension transformer bushing, or low-tension substation apparatus of normal insulation value? What is the general wave form which is best communicated to the low-tension winding from such an incoming wave?

D. W. Roper: The paper by Messrs. Brand and Palueff is a notable contribution to our knowledge of the effects of lightning transients on transformer windings. It is interesting to note that, as a result of this knowledge, it is possible to alter the design of the transformer so as to make the insulation less vulnerable to lightning disturbances. The paper does not state definitely the sizes or types of transformers investigated, but the general description of the windings indicates that the investigation and perhaps also the modified designs apply to the larger sizes of transformers used on transmission lines. It is hoped that they will later be able to apply the same revised principles of design to transformers of the distribution sizes.

The elimination of lightning as a disturbing factor of electrical supply systems is such a vast and complex problem that it must be solved by cooperation among a great many investigators, including representatives of the manufacturing companies and the operating companies. The problem can best be solved by each investigator applying himself to that particular portion of the problem to which he is best fitted by reason of his circumstances and facilities, and then, on occasions like the present, presenting the results of his investigations to his fellow workers in this field.

The paper is an excellent example of such cooperation. It is also, perhaps, an example of the fact that the manufacturers can do a somewhat better job in solving their portion of this problem than can the operating companies as, with the modern equipment available for laboratory tests, they are able to reproduce in their factories the phenomena occurring in the field and to observe the effects of the phenomena on the apparatus which they manufacture. On the other hand, the operating companies must secure sufficient data so that they can eliminate the lightning as a variable.

Other investigators in the field have reported that there is not only a wide variation in the maximum voltage of such transients, but also a difference in the shape of its voltage wave, and further, that the lack of information on the latter point is probably the reason why it is impossible to reconcile the results reported by the several investigators on the effects of lightning on transmission lines.

The final test of the effectiveness of changes in the design of apparatus connected to electrical supply lines as well as the insulation and the protective devices connected to such lines is to be found in the operating records of the electrical supply companies. Within the past year several companies have reported to this Institute the results of the investigations that have been made on their transmission lines, and it appears probable that a similar paper regarding lightning protection on distribution lines will soon be available. It is the writer's experience that the manufacturers have always been ready and willing to cooperate in the solution of the problems in this connection, and there is no indication whatever that this cooperation is likely to be reduced. In fact, several new investigations have been started within the past year requiring cooperation between the manufacturing companies and the operating companies in order to add to our information on the subject. The increasing number of papers on the subject to this Institute in recent years indicates that the importance of the subject is fully appreciated by all interested parties and that the subject is receiving proper attention, in spite of some very drastic comments to the contrary in the technical press recently.

Edward Beck: During the past few years there have been interesting and valuable advances in the art of cathode ray oscillography. In the last year the Norinder type of automatic oscillograph was brought to this country and used in the field for the recording of natural lightning. The advantages of this type of automatic oscillograph in permitting the complete recording of a lightning transient are well known. The instrument was described in the Fortescue, Atherton, and Cox paper at the last Winter Convention. A new oscillograph of this type has been designed and several have been constructed. This device may be operated as a Dufour, the original laboratory type developed in France during the War, or, by the addition of a chamber containing the Norinder relay scheme, as an automatic instrument for use in the field on transmission lines. This makes the instrument extremely flexible and provides practically the same instrument for those who wish to use it in the laboratory only and those who at times may wish to make automatic records. The addition of the deflecting-plate system comprising the Norinder relay converts the instrument of the Dufour type into one that is entirely automatic within itself, not depending on the operation of external relays to set it in operation on the arrival of a transient.

This latest development in the field of cathode ray oscillographs presents several other novel features, among which is the embodiment in the device of an electron-filtering scheme which removes from the recording cathode beam those electrons which are the chief cause of fogging of the film. This permits the obtaining of a very sharp, well-defined record. The proper use of magnetic fields for focusing the electron beam further assists in the securing of sharp records. The oscillograph case consists of welded steel shells, thereby combining extreme strength with light weight. The deflecting-plate system may be adjusted from the outside through vacuum-tight cone joints. The entire oscillograph may be taken apart and reassembled very easily.

In our work with these oscillographs we are planning to use the high-frequency oscillator scheme for timing the recorded phenomena. In the first work done in this country on an actual transmission line with surge generator and cathode ray oscillograph three years ago, we came to the conclusion that this method of recording is the most satisfactory because it records considerable detail of each part of the transient besides giving a very long time scale because of the doubling back and forth of the timing axis. For instance, using this method of recording, it was possible to obtain in great detail pictures of four and five successive reflections on a 4½-mi. transmission line. While records taken by this means may appear complicated, they are easily intelligible with very little experience. Other methods of timing will produce simpler oscillograms, but it is thereby necessary to sacrifice either detail or length of the record.

J. H. Cox: Mr. Peek has reported the progress and status of the lightning problem from his point of view. He has reported the work being done by his group. I should like to add to the picture and make it more complete by reporting the work being done by the group with which I am associated.

Last year we manufactured three Norinder cathode ray oscillographs and applied two of them in the field on a 154-kv. transmission line of the Aluminum Company of America in Tennessee. As a result of this application one oscillogram was obtained. In the beginning of the work it was decided to use a rotating film drum which provided a maximum of 1000 microseconds per 2½ in. of film. With this time scale the surge which we obtained was somewhat cramped in space since it had a duration of 50 microseconds. This left the time of front somewhat indefinite but a definite value of duration was obtained. In all future work an oscillating time scale will be used with which any desired value of speed may be obtained.

The wave recorded at Tennessee having a duration of 50 microseconds, compares quite closely with that reported by Mr. Peek which had a duration of 40 microseconds. They are of the same order of magnitude. However, it would be dangerous to draw conclusions from only two authentic records of actual lightning. Although, in general, lightning surges may be of this order of duration and shape, they probably vary considerably and a great many more data are needed. In the effort to obtain such data our activities for the present season have been expanded and we will have five Norinder oscillographs in operation, ready to record lightning on actual lines. Two of these are those installed last year in Tennessee; a third is now installed on the 132-kv. line of the Public Service Company of Northern Illinois near Chicago; a fourth is on the 132-kv. line of the American Gas & Electric Company near Charlestown, West Virginia; and the fifth is on the 220-kv. line now under construction of the Public Service Gas & Electric Company of New Jersey. With the activities now going on, undoubtedly more actual records of lightning will be obtained before the season is over.

There are two sides to the lightning problem; one is the determination of the actual lightning surge as generated, and the other is the determination of how it performs on an actual line. The performance of traveling waves has been studied theoretically and has been calculated, but there are certain quantities which must be included in such calculations which are not definite. Such quantities are the value of conductor and earth resistances under traveling wave conditions, the effective size and spacing of the conductors and their images which determine the self and mutual surge impedance, the value of terminal ground resistance, etc. The mutual surge impedance between conductors determines how ground wires act on line conditions, and data on mutual surge impedances under varying conditions are needed. Also, information as to the effect of apparatus on surges must be determined. Mr. Peek has reported that waves as they travel along a line are attenuated rapidly by the effect of corona. This attenuation has been roughly determined by the klydonograph but more accurate information is needed. In view of the small number of actual lightning surges which may be expected at any one point, all of the above factors may be studied much more efficiently by the use of artificial surges.

Mr. Peek has mentioned the work combining a surge generator and cathode ray oscillograph on actual lines as being done by Mr. McEachron of his company. This type of work was first started in this country three years ago in a test on a 4½-mi. length of 33-kv. line between Rankin and Wilmerding, Pennsylvania. Due to the limitations of time, apparatus, and insulation, the amount of actual data secured was limited but many of the above mentioned points were determined under the conditions used and a great amount of experience was obtained which will prove extremely valuable in the more elaborate tests now under way.

As mentioned by Mr. Peek, Mr. McEachron has gone forward extending this type of test to a 66-kv. line of considerably greater length. Naturally he has made his apparatus more flexible for this type of work. We are now planning to extend the work to a 220-kv. installation. The 220-kv. line in New Jersey on which a Norinder cathode ray oscillograph is being installed for the purpose of studying lightning, as mentioned above, being in the construction period and therefore free from the complications imposed by having dynamic voltage and load on the line, presented a rare opportunity for the study of the performance of surges on a line by means of a surge generator and cathode ray oscillograph. Therefore, in this test the cathode ray oscillograph installation has been made portable and a portable 1,000,000-volt surge generator is being added. This makes the entire set-up very flexible as either unit can be moved and connected to any point on the line desired. As a result of this work and that of Mr. McEachron, before the year is out most of the questions outlined above should be settled with considerable accuracy. This will be but a small fraction of the time that would have been required had actual lightning surges been waited for.

Mr. Peek mentioned that direct strokes generally are over the insulation and that the usual surge with which we have to contend on transmission lines is the result of an induced stroke. We see no logical basis for the belief that direct strokes are necessarily so severe as to be hopeless on our higher voltage lines and we have a great amount of evidence which indicates that this is not necessarily so. In fact, from the data we are led to believe that the majority of the surges with which we have to deal on transmission lines and which have been recorded in klydonograph tests are the result of direct rather than induced strokes.

The fact that with three groups working only two surges of actual lightning on transmission lines were obtained last year, stresses the point that lightning surges do not occur frequently at one point on a line and that no large number of records can be expected. This points out the need for greatly expanding the activity in this direction in order to obtain the necessary data in a reasonable time. The only way to obtain data is by measurements on actual transmission lines. It is relatively easy to obtain lightning surges on short antennas since on such conductors high potentials are induced by strokes even though at some distance from the location. However, such records mean very little. The fact that many occur on antennas and extremely few occur on continuous lines determines conclusively that there is a wide fundamental difference between the conditions and that by the time an antenna picture is interpreted in terms of a surge on a continuous line not much reliance can be placed on the interpretation.

It has been suggested that the present activities by the different groups, each working on their own problem, is the best way of solving the lightning problem. I must take exception to this point of view since I believe it is an established fact that nothing is done by scattered individual effort as well as by an organized engineering study and I believe that a real coordinated study of lightning is needed. Much of the study should be done on a greatly expanded scale and a great many more installations are needed. An organized effort should provide more funds and avoid duplication of work. When we consider that $150,000,000 per year is being spent on transmission lines alone, certainly the $500,000 to $1,000,000 per year necessary to do this work properly is amply justified for research of so fundamental a problem. This amount of money is not being spent at present because the whole industry is not cooperating. Only those who have immediate needs, together with the manufacturers, are putting forth any effort and more is needed. It is believed that the necessary cooperation and support would be forthcoming if the work were headed up by some authoritative organization.

F. W. Peek, Jr.: I do not feel that I need go into an elaborate closing discussion since the results of my own work are well covered by my paper. There are a few points, however, that I must clarify.

I am quite in sympathy with the views expressed by Mr. Roper. We have had a coordinated plan of study for the solution of the lightning problem under way for a number of years. Operating engineers have collaborated with us in this plan and have been of great assistance. Scientific and engineering problems are generally solved by those who are extremely interested and enthusiastic rather than by a committee. Our general plan has been to observe lightning in the field, reproduce it in the laboratory and coordinate the results with a mathematical and physical study. The progress of such an investigation is slow at first, but becomes very rapid as data are accumulated and better instruments are developed. Within the last five years "lightning" has been put on an engineering basis. As an instance of recent progress, during the past year 5,000,000 volts have been directly generated in the laboratory; natural

lightning waves recorded by oscillograms have been reproduced and used in research on insulators and apparatus; waves have been sent over lines and their effects in their travels on the insulators, inductance coils, open lines, etc., recorded by the oscillograph; the waves have now been followed into the transformer and Messrs. Brand and Paluefff have shown how high internal voltage concentrations occur in the ordinary transformer and how they can be prevented in the new non-resonating transformer.

One of the discussors has implied that I have expressed the opinion that most lightning failures are caused by induced voltages. It was not my intention to give this impression. I did say that most lightning voltages that happen on transmission lines are caused by induction. Direct strokes to transmission lines undoubtedly occur and may be the main cause of trouble in some of the very highly insulated lines. However, there is also evidence that induced voltages may be very high. This applies particularly to those induced voltages caused by direct strokes in the vicinity of the line. I might also point out here, as I have before, that high induced voltage can only result from a very rapid collapse of the electrostatic field, or what is the same thing, a very rapid discharge of the cloud. The laboratory curves for the lightning flashover voltage of insulators were obtained with all types of waves, since all types of waves must occur on transmission lines. However, the wave used as a standard is the one that gives the same arcover voltage for insulators as natural lightning does under the worst conditions on actual high-voltage lines. These measurements on insulator flashovers afford a check on the effective duration of the waves occurring on transmission lines. The lightning waves already measured are what would be expected from the hundreds of flashover voltages recorded. As brought out in my paper, a total of five oscillograms of waves was obtained last year. A more elaborate program is under way this year and a number of cathode ray oscillographs is connected to lines.

The cathode ray oscillograms shown in the paper by Messrs. Brand and Paluefff, are extremely interesting and important. In fact, it is my opinion that the non-resonating transformer is one of the most important developments that have been made in high-voltage transformers in many years.

K. K. Paluefff: It is gratifying to learn that oscillographic study of transformer oscillation conducted by Mr. Peters during the last few months, confirmed the theory and the oscillographic records presented to the Institute by the present writer at the last Winter Convention.

Thus, the fact that a single traveling wave is capable of producing complex and severe oscillations within the winding, causing it to rise above ground potential far in excess of the uniform voltage distribution, is no longer disputed.

Mr. Peters' present discussion shows that there is still some difference of opinion in regard to the relation between the amplitude of internal oscillations produced by long and short waves.

Mr. Peters states:

(a) Short waves of the order of 2 to 5 microseconds do not produce oscillations which overshoot the uniform voltage distribution.

(b) Long waves, of say 15 microseconds, do produce such oscillations.

(c) If the arcover value of the line insulation adjacent to the transformer is E for the short wave, then for a long wave (15 microseconds) it is 0.60 E.

(d) Thus, generally speaking, the internal voltages, if expressed in volts instead of fractions of applied voltage, produced by a long and a short wave, are about the same and do not overshoot the uniform voltage distribution drawn for the applied voltage of E.

The present writer's views on the above paragraphs are as follows:

(a) Harmonic analysis as well as the oscillographic data, of the relation between length of the wave and the amplitudes of the internal oscillations, indicate that the voltage to ground of approximately one-third of the winding stack near the grounded end, caused by a short wave, like those produced by flashover of line insulators by a lightning traveling wave, overshoot very substantially the uniform voltage distribution. (See oscillogram No. 5 of Fig. 9, and oscillograms 6, 7, 8, 9 of Fig. 21, and also Fig. 8 of the present paper.) These oscillograms show, for example, that a wave cut to a length of approximately 3 microseconds by an insulator flashover, causes the voltage of a point of the winding 16 per cent away from the grounded end to rise to 79 per cent of the crest value of this short wave, instead of being not over 16 per cent as Mr. Peters states.

(b) In this statement, Mr. Peters confirmed the writer's view as expressed in the present and previous papers.

(c) Examination of the oscillographic data presented to the Institute on the variation of impulse ratio of line insulators with variation of the length of the traveling wave, shows that there is only one paper that gives the numerical relation of these factors. This paper was the one presented by Mr. Peek at the Winter Convention. Fig. 11 of this paper shows that if the line insulation arcover voltage for a short wave (5 microseconds) is E, then for a long wave (20 microseconds) of the same front as the short one, it is approximately 0.93 E and not 0.60 as Mr. Peters states.

(d) From the above, it follows that both short and long waves are capable of producing internal voltages far in excess of the uniform voltage distribution. If the amplitudes of oscillation produced by short and long waves are expressed in volts instead of in fractions of amplitude of the applied voltage, then generally speaking, two-thirds of the winding is brought to higher potential above ground by the long waves than by the short. For the other third of the winding, at the grounded end, however, the voltage to ground is high even with short waves.

Mr. Peters further suggests that amplitude of the internal oscillations "can be controlled to considerable extent by properly selecting the physical proportions of the windings." Nothing can be said against such a statement if the physical proportions actually used in power transformers of all types are not taken into consideration.

Very unfortunately, practical designs allow such a narrow variation of the proportion that no radical benefit can be derived from change of the width and the length of the winding stack of any given type of transformer.

It appears that in practical transformer design, the high-frequency characteristics can be substantially modified only by use of electrostatic shields.

I think it would be of considerable interest to the Institute if Mr. Peters shows us the proportions of the winding of a practical power transformer that will give substantial improvement in transient voltage stresses. I think that the statement of quantitative relation between these proportions and the amplitudes of transient voltages would be of particular value as a general statement in such a problem may be misleading.

Answering Mr. Roper's discussion, I should like to say that laboratory transient voltage tests have been made on the entire range of transformers, including distribution, instrument, and power transformers, of a variety of sizes and types. The test data published were obtained, as stated on the fifth page, on a 6600-kv-a., 110-kv. concentric-type transformer, and a 1000-kv-a., 44-kv. shell-type transformer, in conjunction with a transmission line.

Mr. Marshall touches a very interesting phase of the phenomenon. In the original form, this paper contained a rather detailed explanation of the reasons that make the frequency of the successive harmonics of the transformer windings increase faster than their order. In the attempt to shorten the paper, these explanatory remarks were omitted, and only Figs. 4 and 5 were retained in the hope that closer examination of these two figures

by the inquisitive reader would be sufficient to explain this rather unique relation.

The reason for such an unusual relation can be explained as follows:—The oscillation of the first few harmonics takes place essentially between the inductance of the winding and its capacitance to *ground*. And it is these harmonics, therefore, that are principally responsible for the rise of the potential of the winding above ground, after the impact of the traveling wave. For this reason, the equivalent circuit of the transformer winding for the first few harmonics can be simplified by omission of the series capacitance C_w, thereby converting it into circuit *b* of Fig. 1.

Fundamental Harmonic. In spite of the fact that all condensers of this circuit are the same, the simultaneous voltage to ground of a given harmonic is different for different condensers C_s and follows the space waves shown by sine wave No. 1 of Fig. 4.

For instance, the potential difference forcing into oscillation the condensers of the middle of the winding is 54 per cent of the line voltage while it is 38 per cent for condensers at 25 and 75 per cent points and zero for those at the ends (see Fig. 4). For this reason, the effective capacitance of the winding to ground, entering into an oscillation of fundamental frequency, is not equal to C_s (the sum of all condensers of the equivalent circuit, *b* of Fig. 1), but only to a certain fraction thereof, for instance $K_1 C$.

Examination of the fundamental-frequency space wave, shown by curve No. 1, Fig. 4, reveals that voltages between adjacent elements of the winding are maximum at the ends of the winding, and zero in the middle. These voltages are of such polarity that the currents flow in opposite directions in the two halves of the winding, with maximum current at the end of the winding and zero current in the middle. (See 2, 3, 4 of Fig. 4.)

The magnetic flux distribution of fundamental frequency shown on 5, Fig. 4 indicates that the effective inductance for the fundamental harmonic oscillation is a certain fraction, say K_2, of inductance L obtained by connecting two halves of the winding in series and bucking. The frequency of oscillation of the fundamental harmonic, therefore, will be $f_1 = \dfrac{A}{K_1 K_2 \sqrt{L C}}$

where A is a constant independent of the transformer characteristics.

Second Harmonic. As the space wave of the second harmonic consists of two halves of the sine wave shown by wave No. 1 of Fig. 5, each half covers only one-half of the length of the winding and therefore the effective capacitance of each half equals half of the effective capacitance for the fundamental harmonic of the entire winding. In addition, we find that the second harmonic voltage of the line half of the winding is positive and that of the ground half is negative. For this reason, the effective capacitances of the two halves of the winding may be considered as connected in series, thus making the effective *capacitance* of the entire winding for the second harmonic equal to *one-fourth* of the effective capacitance for the fundamental harmonic.

As these elementary condensers discharge into the inductance of the winding, currents will be created as shown by curve 3 of Fig. 5 and by the arrows on the equivalent circuit diagram 4, Fig. 5. The magnetic flux distribution will be as shown on 5 of Fig. 5. Comparison of the current and flux distribution of the second harmonic with that of the fundamental shows that the effective *inductance* for the second harmonic is very nearly equal to *one-fourth* of the fundamental.

It follows that frequency of the second harmonic f_2

$$= \frac{4\,A}{K_1 K_2 \sqrt{L C}} = 4 f_1.$$

In a similar manner, it can be shown that the frequency of the third harmonic is approximately $9 F_1$.

With increasing order of harmonics, the voltage of each quarter of a cycle appears across smaller and smaller parts of the winding. For instance, the full value of the tenth harmonic appears across every 2.5 per cent of the winding. Therefore, the series capacitance between adjacent coils of the winding becomes more important than their capacitance to ground. Hence, for higher harmonics, the equivalent circuit of the transformer winding can be simplified by the omission of condensers to ground C_g of circuit *a* of Fig. 1, converting it into circuit *c* of the same figure, and the ratio of frequency of oscillation will be directly proportional to the first power of the order of the harmonics.

On account of the variation of the ratio of the capacitance between coils, and capacitance to ground for different transformers, in many cases the series capacitance cannot be neglected when calculating the frequency of even the first three harmonics and the ratio of their frequencies therefore, instead of being 1:4:9, may actually be 1:3:7, or even 1:2, 5:6.

As far as the transformation of high-frequency oscillations from the high-voltage winding to the terminals of the low-voltage winding and the bus structure, is concerned, I feel that space limitations will not permit me to answer them completely at the present time. I should like to say, however, that high-frequency oscillations originating on the high-voltage side, after going through considerable modification, appear on the low-voltage side. The wave shape of the low-voltage winding oscillation varies with transformer characteristics as well as with that of the apparatus and bus structure connected to the low voltage. Their magnitudes may, and do, reach, under certain circumstances, dangerously high values, sometimes sufficiently high to arc over the bus structure, or damage the low-voltage network.

The transients that are best communicated from the high side to the low, are those of relatively low frequency. The traveling wave that is best communicated to the low-voltage side is one of considerable length, but of course it appears on the low-voltage side, not as a traveling wave, but as a complex oscillation, the shape of which depends not only on the transformer characteristics and those of the low-voltage network, but also on the shape of the traveling wave.

It generally can be stated, however, that with low-voltage windings located concentric with high-voltage winding, the high-frequency voltages transformed to low-voltage terminals are not as severe as in case of two windings being interleaved.

Flying Field and Airway Lighting

BY H. R. OGDEN*

Non-member

Synopsis.—*This paper discusses methods of lighting airports and airways and describes various types of lamps and equipment used for this purpose. In the information on airport illumination are included beacons, obstruction lights, boundary lights, illuminated wind direction indicators, field and building flood lights, signal lights, and ceiling illumination. The portion on airway lighting covers principal and intermediate beacons and emergency field lighting. An extensive bibliography is included.*

DURING the last few years many fundamental problems of illumination for night flying have been solved by both civil and military organizations. Although the future may see considerable divergence between military and civil lighting practise, at the present time both are similar and in fact commercial aviation has largely followed the trend of military design.

The components of a complete lighting system for an airway and its terminals have gradually increased in number as the need for their existence has become evident through experience. Particularly is this true of the equipment at the terminal airports. Here is a list of those components which should be considered in planning a system for lighting a terminal flying field, given in the order of importance: (1) beacon, (2) obstruction lights, (3) boundary lights, (4) illuminated wind direction indicator, (5) flood lights, (6) building flood lights, (7) signal lights, (8) ceiling projector. The components of a lighting system for an airway will be given later. There is some controversy as to the the relative order of importance of the items listed above,[1, p. 11] but the question is unimportant, for experience indicates that the lack of even the least important item may cause a fatal accident. A night mail pilot with insufficient fog flying experience took off into a low fog, stalled his airplane, and dove into the ground to his death. A positive knowledge of the ceiling through the use of a ceiling projector might have forestalled the flight.

The difficulty of night flying lies not so much in taking off from an airport to fly as in locating an airport and landing safely upon it when once in the air. Some experienced pilots prefer to take off without flood lights so long as the horizon is visible, for the reason that the pupils of their eyes will not have been contracted by any artificial light. A forced landing outside the field immediately after the take-off, in which insufficient altitude has been gained to make a return glide to the landing field advisable, might be more serious than would otherwise have been the case with eyes accustomed to the darkness.

If the problems confronting the night pilot desiring to land are based upon a succession of observations and decisions, we may well attack those problems in the order in which they occur to the pilot.

Airport Beacon. No mariner ever experienced greater satisfaction in identifying a lighthouse than a night pilot feels when he is uncertain of his exact location and finally espies the aerial beacon for which he has been searching the horizon. The majority of night flights must be completed before dawn due to lack of sufficient fuel to remain aloft. A landing field must be located before the fuel is exhausted.

What sort of light can pilots most easily pick up, a white light or colored, a fixed, flashing, or occulting light? The question of a colored beacon was brought up with the development of the red neon light and its comparative value as a beacon is still unsettled, although an official government test[2, p. 2] indicates that in clear weather the range of a neon beacon is no better than street lights. Extravagant claims have been made for the fog piercing quality of the neon light, especially in Europe and England,[4, pp. 2-5] but the marked advantage of this light source has not been proved.[3, p. 4] In view of the fact that even the position of the sun cannot be determined through more than a few hundred feet of fog, human efforts to overcome the fog hazard by artificial lighting are probably wasted so far as beacons are concerned.

Now in regard to the problem of using fixed, flashing, or occulting beacons, this problem was solved for the aviator hundreds of years ago by the navigators of the sea. Flashing or occulting beacons are absolutely essential, not only because they can be located more easily, but because of the prevalence of fixed lights throughout the country which would be confusing to a pilot searching for a fixed beacon light.

A short flash followed by a longer period of eclipse will be referred to as "flashing," while a long period of illumination followed by a short period of eclipse will be referred to as "occulting." The department of commerce recommends a flashing beacon having a flash duration not less than 1/10 second, luminous period not less than 10 per cent, and eclipse period not greater than 10 seconds.[1, p. 11]

The writer once experienced a search for a beacon on a night flight unguided by intermediate route lights. The beacon was a rotating projected beam light of approximately 10 deg. uniform divergence flashing

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1. For references see Bibliography.

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for a duration of about ¼ sec. every 10 sec. It was noted that that period of eclipse was much too long, and the flash much too short, requiring an unreasonable period of concentration upon the light to establish its identity. The subject of the most suitable periods of flashing and eclipse commensurate with a reasonable cost for the beacon might well be made a matter of experiment.

An authority on night lighting equipment states that "..... the periods of illumination of a flashing light should have a duration of at least one second and for similar reasons the period of eclipse in an occulting light should have at least the same duration. On the other hand, it is undesirable that the period of eclipse in a flashing light or the period of illumination in an occulting light should be too long, especially where many other ground lights or other aviation beacons are in the vicinity, as in cloudy weather or bumpy air it is difficult to establish the identity of a beacon having such long periodicity."[7, p. 10]

There are various types of airport beacon in use at the present time. The most common in the United States is the rotating searchlight type with a 24-in. dioptric lens, 1500-watt incandescent lamp, and parabolic reflector. An electric motor in the base rotates the light. A recent development of the light includes a lamp changer mechanism which automatically brings a new lamp into position and incandescence, at the same time giving a warning of the burned out lamp at the control office.[5, p. 2] The beam has a vertical divergence of 3 to 4 deg., and a horizontal divergence of 10 to 40 deg. The greater horizontal divergence increases the length of flash, but decreases its intensity. This difficulty of securing great intensity and length of flash is the only criticism that can be made against this beacon. Even with a 40-deg. horizontal divergence, the length of flash is only 1 1/9 sec. at 6 rev. per min. But the more serious fault of this periodicity is the eclipse of 8 8/9 sec. Equal periods of flash and eclipse would be more suitable, but a beacon furnishing such a light appears to be more costly. A 360-deg. lighthouse lens with revolving reflector would provide such a light.

To secure proper distribution of the lower edge of such a beacon beam along the horizon, some method of adjustment ought to be incorporated in the light, either by an adjustable pivot or an adjustable base. The latter appears to be more practical. The French are using tripod type of mount for this purpose on some of their lights.

The problem of securing visibility of the rotating searchlight beacon over the entire sky is solved in some cases through the use of a vertical slit on the face and transversely along the top of the drum, thus projecting a slender beam to the zenith. The upper edge of a 4 deg. diverging beam, whose lower edge is on the horizon, passes the 1000-ft. level at 3 mi., 3000-ft. at 8 mi., and 5000-ft. at 13 mi. from the source. A pilot flying over the beacon within the inverted cone formed by the upper edge of the beam would not see the main beam during 26 mi. of flight if he were flying at 5000 ft. altitude.

If a water tower is located in the vicinity of the airport, it may be fitted up as a satisfactory short range beacon by painting the wall of the tank white and flood lighting it by means of signboard flashing lights surrounding the periphery. Since such a tower would otherwise have been wired for obstruction lights to safeguard pilots from colliding with it, the cost involved comprises only the lamps, fixtures, signboard flasher, and white paint. A sufficient number of light sources would provide against interruption of the beacon caused by a burned out lamp.

If for any airport lighting system the range of the water tower beacon is not considered sufficient, the tower may still be used to good advantage to mount on its peak any other type of flashing beacon. There are two advantages gained in mounting the flashing beacon on a tower high above the landing field. First, it allows the lower edge of the light beam to be adjusted below the horizontal, in line with the new horizon. This gives greater range of the beacon to a low flying airplane forced to fly near the ground by low clouds. Setting a beacon 100 ft. above the ground increases its range at ground level 13 mi. Second, a flashing beacon near the ground is disconcerting to a pilot about to land. When the intense rays of the beacon strike his eyes, the pupils converge and handicap his vision. To mount an airport beacon high is therefore a distinct advantage.

At the Croydon airport, London, a unique beacon has been tested and been found to possess unusually good visibility in thick atmosphere. It consists of a moving chain passing through a solution of strontium and then through an oxy-acetylene flame. The resultant light is very intense, but its superiority to an electric beacon does not appear to be sufficient to warrant its general adoption. In direct comparison with the strontium beacon, the Croyden airport is equipped with a truncated cone neon beacon, which is reported to pierce a small amount of fog better than any other beacon at the field.[5, p. 4]

The last beacon, and probably the favorite of night pilots, is the lighthouse type of beacon wtih 360-deg. Fresnal lens units. For the same power input, these beacons do not have the range of the searchlight type due to the fact that the light is diffused over 360 deg. as compared to 10 to 40 deg. for the searchlight type. But on the other hand, the lighthouse beacon can be designed to give any type of flash desired. The largest beacons of this type are located in France, one near Paris, and one near Dijon. Each has 1,000,000,000 cp., sufficient to give a range of visibility of 100 mi. in clear weather.

To sum up the present practise in the use of airport beacons, the projected beam type is most common in the United States, the Fresnal lens lighthouse beacon is favored in France,[4, p. 1] while the flashing neon beacon

is extensively used in Germany.[6, p. 3] England, also, is partial to the neon beacon due to the claim for its fog piercing quality. Since the Fresnal lenses are expensive to manufacture, while the neon tubes are expensive to operate, it is evident that European practise in night lighting does not consider cost so important as superiority of equipment.

Obstruction Lights. After a pilot locates his landing field through the aid of the airport beacon, he approaches the field and observes obstructions which he must avoid while examining the landing area, the wind direction indicator, and ships about to land or take off. Crashes which occur through colliding with other airplanes near the ground or with obstacles while still in flight are usually fatal to the occupants. The obstruction lights are therefore very important in any plan for lighting an airport. A careful analysis must be made of the probability of an airplane colliding with any obstacle within a half mile of the field, and other obstacles farther away which are on the direct line of an airway. It is reasonable to assume that no pilot will fly lower than 100 ft. altitude until actually making his approach for a landing. Ordinarily, no obstacles less than 50 ft. high, which do not immediately surround the field, need be lighted. However, each field presents special problems, and no set rule can be made. It should be borne in mind that an airplane glides at an angle never steeper than 1 to 5. This will indicate roughly what structures or natural objects in the vicinity of an airdrome should be considered obstructions. A compromise between ultra safety and cost must always be made. Poor pilots will sometimes hit obstacles in spite of everything that can be done to safeguard them.

The universal practise in Europe and America follows the recommendations laid down at the International Air Convention for marking obstacles with red lights. The Material Division of the Air Corps recommends the use of 50-watt lamps in ruby globes of high light transmission qualities.[6, p. 4] The lights burn steadily, and may be connected with the field lighting circuit as will be discussed later. The lamp fixtures should be so placed that the obstacle light can be seen from the horizontal to the zenith in all directions, and from the ground anywhere on the airdrome. Flag poles and chimneys may be lighted conveniently by the use of unattached fixtures raised to the top by lanyards carried over a pulley at the top. The light can be exchanged at sunrise and sunset with a flag in the case of a flag pole, and can remain constantly in place on a chimney save when lamp replacement or cleaning is necessary.[6, p. 4] Lights too far from the airdrome to make practical a connection with the field circuit can be installed with a separate meter, attached to the circuit of some other party and rent be paid therefor, or, in the event that no power lines are conveniently near, it may be more economical to install an automatic gas light to burn constantly until necessary to replace the gas cylinder at the end of several months.

Boundary Lights. When the pilot of an airplane has observed all the obstructions about a field upon which he intends to land, his next observation is usually to determine the extent and nature of the field. Boundary lights can answer both of these questions. The Department of Commerce has made a careful study of the boundary light problem and has published the results in Aeronautics Bulletin No. 2.[1, p. 12] That bulletin should be consulted in drawing up a plan for an airport lighting system.

The general practise for placing boundary lights is to outline the field completely except along the hangars, outline just the corners of the field, or outline the runways. The last method is the most expensive, and, in the case of raised lights, invites collision from a rolling airplane. However, with lights installed flush with the ground or raised only a few inches, permitting an airplane wheel to roll over without damage, this system has a distinct advantage. It enables the field manager to indicate clearly the best approach and best place to land under various wind conditions by merely lighting the one runway which should be used.

A succession of white neon tubes end to end, spaced intermittently along each side of a runway, enclosed and protected by a concrete trough covered with sidewalk glass, has been suggested for experiment. Some such arrangement has evidently been tried at Croyden, but no report on the results obtained is available. Or a succession of incandescent lamps spaced 100 to 200 ft. apart, and placed in a suitable concrete base, would answer the same purpose, but would not be so visible in heavy atmosphere. Sometime radio is likely to solve the danger of fog flying, and when that time comes some supplementary lighting of the landing runway will be necessary. Two solid lines of red neon tubes may be the solution.

A special plea is made for the design of boundary lights which are fairly indestructible. The present practise of installing a lamp 1 to 3 ft. above the ground on the top of a pipe set in concrete[1, p. 12] has one insurmountable drawback. The installation attracts the attention of curious people, and invites malicious people or undisciplined children to break or pilfer the globes and lamps. No measure short of a constant guard near such boundary lights will protect them from depredation. Landing fields are too large for a manager on one side of the field to be responsible for the protection of the boundary lights on the opposite side. The design of lights flush with the ground will not attract so much attention, and it is believed they can be made fairly indestructible, at the same time fulfilling the purpose for which they are installed.

Another advantage can be gained by using boundary lights flush with the ground. Instead of establishing a row of lights on the very edge of the landing area, such

a row could be placed 300 ft. from any edge along which a telephone or power line was located. Thus, a normal glide to the edge of the boundary lights would insure the pilot's clearing the poles and wires by a safe margin. In addition, lights placed away from the edge of the field would be less conspicuous and less likely to be molested.

In Europe and in England the use of automatic gas operated lights for both obstruction and boundary lights has been found to be the most economical installation,[4, p. 9] but no data on the relative cost of gas and electric operated boundary and obstruction lights in America are available at this writing. Undoubtedly, an electric system which is integral and controlled from the field administration building is convenient from the standpoint of control. From the standpoint of reliability, the individual gas lights are preferable. It is simply the story of not having all the eggs in one basket.

At Croydon airport the gas boundary lights are of the flashing type, 1/10 sec. flash and 9/10 sec. eclipse. This may be an advantage in distinguishing the boundary lights from the obstruction lights, but the practise hardly seems necessary. A more economical distinguishing feature would be the use of white lights for lighting the boundary. In the case of lights flush with the ground, white lights would be necessary to secure sufficient intensity.

Illuminated Wind Direction Indicator. Normal landings are always made into the wind in order to reduce the ground speed of the airplane as much as possible and retain directional control after the airplane is rolling on the ground. To indicate the direction of the wind to the pilot about to land, several devices are in use. For daytime operation, a wind cone of fabric attached at the larger end to an iron ring free to rotate about a vertical axis, and tapering from about 2 ft. diameter at the ring to 1 ft. at the smaller end, has been in use since the beginning of heavier-than-air flying. The first illuminated wind direction indicators naturally were developments of the wind cone. One authority maintains that illuminated wind cones are not satisfactory,[6, p. 5] but a number of internally illuminated wind cones have been in service at a large Air Corps field for nearly a year and have given uninterrupted satisfactory service.[7] Such indicators are unquestionably the cheapest to install and maintain.

Another type which is popular in England and Germany is a system of lights sunk flush with the ground level, so controlled by either a wind vane switch or manual operation in the administration building that an illuminated *T* is visible to the pilot.

The best military practise in America calls for an externally illuminated *T* situated on the ground in front of the operation office of the field. It consists of a skeleton framework built in the shape of a *T*, carrying a vertical fin at the base of the *T* to hold the head of the *T* into the wind, all covered with airplane cloth, and suitably finished.[6, Fig. 6] It is supported horizontally above the ground on a pivot located at the center of gravity. The illumination is provided by two rows of 10-watt lamps located on top of the *T*. The current is transmitted through a brush arrangement on the supporting bearing. For night visibility, the best color for the *T* is white, whereas for daytime visibility the most satisfactory arrangement developed to date is a chrome yellow *T* riding a cement or wood background painted black. A good compromise would be a white *T* on a black background.

The best practise seems to indicate a single wind direction indicator suitable for both day and night, either an illuminated wind cone or illuminated *T*. The latter is visible a greater distance, and therefore is more satisfactory to the flight leader of a formation of airplanes. The former is more economical, and ought to fulfill the needs of most civil airports when properly designed.

Flood Lights. No component of airport lighting systems has received so much attention and been the subject of so much experiment in this country as the airport flood lights. The problems connected with flood lighting have been many. At the same time, an airplane properly equipped for night flying is less dependent upon flood lights than upon several of the airport lighting components generally considered less important. Landing lights now available for installation on airplanes, developed by the Material Division of the Air Corps, make landing at night reasonably safe when the landing field is properly boundary lighted. However, flood lights must be available in an emergency, and flood lights also improve the ability of a pilot to judge his height above the ground.

Since the manner in which a pilot should pass flood lights in landing is an important factor in determining where the lights should be placed, considerable experimenting has been done along this line. Some pilots recommend that the pilot glide down to a landing over the light and parallel to the beam, others that he glide diagonally across the beam from the rear, and still others recommend that a landing be made perpendicular to the beam. However, the last recommendation is based upon the use of a single light source of the 180-deg. Fresnel lens lighthouse beacon type, or a battery of projectors arranged along the side of the field. The disadvantage of the first method is that when the airplane enters the beam near the ground, the pilot is suddenly confronted with the shadow of his airplane extending far out in front of him. This disadvantage becomes less annoying as the path of a landing airplane is changed to a perpendicular to the light beam. No one recommends landing into the glare of a light.

For several years the students at the Air Corps Advanced Flying School have been landing straight down, or diagonally across from the rear of the beam from a 500,000,000 high intensity arc, whose beam had a 40-deg. lateral divergence and a 4-deg. vertical

divergence. No difficulties have been experienced, and the embryo pilots found the landings quite easy. However, recent experiments at Wright Field, the home of the Air Corps Material Division, indicate that even better results can be obtained by greater lateral divergence of the beam and landings made perpendicular to the principal axis of the beam. If the foregoing premise is accepted, then the flood lighting of any field to take care of various wind directions resolves itself into the use of two light sources or two batteries of light sources on adjacent sides of the field. Only the light from one side of the field will be operating at any one time and landings can be made either parallel to that side or diagonally away from it. The arrangement of flood lights is discussed and illustrated thoroughly in the Department of Commerce Aeronautics Bulletin No. 2.[1, p. 14]

The practise followed in the design of flood lights has been in the direction of two entirely different types of light units. Experiments in America for a number of years were restricted, for lack of funds, to the searchlight type. Many of these units were manufactured during the World War, and were available for experiment. A 36-in. drum with a parabolic reflector and a high intensity arc light source was fitted with a dioptric lens which diverged the light beam laterally to a 40-deg. spread. This unit required the constant attention of an operator, the light emitted was not always steady, and when the arc had to be replaced a serious interruption of the light ensued. The first improvement of the unit consisted in substituting a 10-kw. incandescent lamp for the arc. The new light source did not have the same cp. as the arc, nor was the color of the light so good, but the rays were steady, a prompt lamp change could be made, and the quality of the light was satisfactory. The last major improvement was the incorporation of a lamp changer mechanism which automatically brought a new lamp into position and incandescence, and gave a warning signal in the control room when a lamp burned out. As a single narrow beam, single source flood light, this unit is very satisfactory.

However, such a unit lights only a limited portion of the landing area. The advent of formation flying at night will require a wider distribution of light. To secure a greater light spread would require a battery of such lights, the combined overlapping light of which would be much more intense than necessary. To secure a sufficient spread of light at a reasonable cost would require either a new type of single source light with a larger lateral divergence, or a battery of smaller projectors.

European practise invited the attention of American engineers to the 180-deg. Fresnel lighthouse lens flood light, which has been tested comparatively with various types of flood lights[12] and found to be superior for the operation of airplanes in formation. A low mounting of the light, with a sharp cut-off on the upper edge of the beam secured by louvres, produced the most favorable illumination. One objectionable feature of this installation is that irregularities in the surface of the landing field produce shadows.

To overcome this serious objection on irregular fields, a battery of six smaller 24-in. drum projectors with parabolic reflectors, 1500-watt lamps, and 40-deg. dioptric spread lens can be mounted on 10- to 15-ft. standards and spaced 200 to 300 ft. apart along the side of the field. This equipment is much less expensive than the single source lighthouse lens flood light, is preferable for irregular surfaced fields, and is therefore to be recommended for moderate sized airdromes from which airplanes operate one at a time.[12, p. 6] The cost of lighthouse lens units will probably limit their use to military airdromes and major terminals, while the coming host of smaller municipal airdromes will be equipped with batteries of 24-in. projectors. The present equipment which is being manufactured for flood lighting needs only to be installed properly to give entire satisfaction. Many refinements will be made in the future, but at least the fundamental problems have been solved.

Signal Lights. Although the night traffic at municipal airports in this country has not yet reached the volume at which a system of signal lights is necessary, yet the day when such lights will be required is not far off. The Air Corps Advanced Flying School has been using signal lights controlled from a command tower for several years. The installation consists of rows of blue incandescent lamps of small wattage mounted along the entire edge of the two ends of two adjacent hangars. Switches on the command tower permit any number of the four rows to be illuminated at one time. The signals that can be given are limited. Some system will no doubt be developed to use the Morse code. Weather reports will constitute the majority of messages to be transmitted, and the signals should therefore be visible at a high altitude. Perhaps neon tubes will offer clear cut light signals.

It will be seen that the foregoing electric lighting equipment will entail a careful study to work out a satisfactory wiring and control system. The Material Division of the Air Corps has worked out a series circuit to include all of the lights at an airport.[6, p. 18] To insure continuous operation an auxiliary power generator must be included in the equipment. In addition, a safety precaution worth taking is to keep on hand at the administration building several magnesium flares. In case of snow or fog or failure of the electric circuit, these flares would be invaluable in case an airplane should arrive unexpectedly.

Airway Lighting. The components of an airway lighting system are: (1) principal flashing beacons, (2) intermediate beacons, (3) emergency field lighting. Practise in airway lighting in Europe and America has been widely different, but as time goes on, the respective systems are growing more similar. Europe began lighting its airways with widely separated beacons

of tremendous candlepower. Two beacons are 1,000,000,000 cp. each, located at Mount Valerien and Mount Afrique in France. Moderate sized intermediate beacons are located at the terminal airports,[4,p. 30] and still smaller beacons are being installed along the routes. In Germany, the majority opinion favors comparatively low power lights at frequent intervals from 1 to 2 mi. apart.[4,p. 4] On the Berlin-Konigsberg and Berlin-Amsterdam routes are used double series of neon lights which burn continuously night and day. Operation cost of each light is claimed to be about one dollar per month.

American practise began with the installation of beacons spaced close together, though not so close as in the present German practise. And now larger beacons are gradually being used at major terminals to supplement the small beacons.

England has favored the automatic unattended gas lights[4,p. 11] for aerial route beacons, and the Bureau of Aeronautics in our Department of Commerce has installed these beacons on some of the air mail routes where electric power is not available and the lights must operate unattended for long periods of time.

Emergency landing fields are being established at approximately 25-mi. intervals along the air mail routes. Most of the flashing beacons are located at these emergency fields. The lighting equipment in addition to the beacon consists of boundary and obstruction lights, and an illuminated landing *T*.

Future Developments. The impetus which Congress gave to aviation through the establishment of a Bureau of Aeronautics in the Department of Commerce has been very great. The increase in night flying, encouraged by the night air mail and the installation of airway lighting, has been remarkable. Although the fundamental problems of aviation lighting have been solved, much important work lies ahead. As already mentioned there will probably be a continual divergence in practise between the requirements for civil aviation and military aviation. Two factors dictate this divergence: utility and cost. Night flying in the Army and Navy will develop into the operation of groups of airplanes as opposed to airplanes operating singly, so that flood lighting systems will probably be designed along two different lines. Also, military aviation will require mobile lighting units as well as permanent peace time installations, and therefore the development of dual equipment will require larger expenditures for military lighting than for civil. However, as will be noted in this paper, those who are charged with the design and installation of lighting equipment for private, municipal, and federal airports and airways will continue to draw upon the knowledge acquired and the experience gained by the pioneering work of the military services, and particularly in this country by the Material Division of the Air Corps. It is vitally necessary for the military services to pioneer in the development of material which will improve the system of national defense. It is also reasonable that where such development is useful to civil progress, the knowledge gained through the expenditure of public moneys should be extended freely to the public when not inimical to the security of the nation. Such are the conditions surrounding the development of almost all aviation lighting equipment by the U. S. Air Corps. When the business of furnishing lighting equipment to civil enterprises grows to larger proportions we can look for great experimental work to be done outside of the Army and Navy. That such a time will come is certain. The Army has for several years been drawing upon the ideas of commercial airplane designers for improvements in airplanes, and the day is not far distant when very superior aviation lighting equipment will be designed for civil use.

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Electrical Features of the Kansas City New Water Works

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Synopsis.—This paper describes the main electrical features of the new works recently constructed for additional water supply for Kansas City, Missouri. It enumerates the chief features of the power contract under which the city is purchasing its power. It states the type and capacities of the major electrical equipment and touches briefly on the wiring and lighting systems. It does not treat the efficiencies or the costs of the equipment.

* * * * *

UNTIL comparatively recently standard waterworks practise was confined almost exclusively to the plunger type of pumps driven by reciprocating engines. They afforded economic operation over a wide range of conditions, but the floor space requirements and the capital investment were considerable. When it became expedient, in many cases, to install additional pumping capacity in a limited space and with limited funds, the turbo centrifugal pump received recognition. Its successful performance paved the way for electric motor centrifugal pumps which offered still further economies in capital investment. At the time when Kansas City was formulating its decision on the important question of steam-turbine or electric motor centrifugal pumps, standard practise, especially in the case of high-head pumpage, was strongly in favor of the former. In fact, of the thirty-five largest cities of the United States, only two or three used the electric motor as a prime mover, and one of these operated on "off-peak" power using storage reservoirs. Inasmuch as Kansas City is on a direct pumpage basis and cannot avail itself of the benefits of "off-peak" power rates, conditions are not exactly comparable.

Another factor which militated against electric drive in Kansas City was the fact that the city would be totally dependent, for its water, on purchased electric power with all the hazards, real and imaginary, that are involved in seven or eight miles of feeder lines, in addition to other likely causes of service interruptions. In consequence, the electrical layout had to be such as to give maximum assurance of continuity of service but still be kept within limited capital investment since the latter was one of the chief assets of electric drive. Inasmuch as a comparison of over-all costs, that is, fixed and operating charges, between steam and electric drive was made, it was necessary to have low power rates to offset increases in fixed charges.

Power Contract

At this stage it might be interesting to discuss the power contract between the city and the local public utility which contains some unusual features and which had some bearing on the selection of the type of equipment chosen. Some of the outstanding features of the contract are these:

1. That the city shall have first call on the power plants of the company.
2. That no substations shall intervene between the pumping stations and the power plant.
3. That duplicate feeders of very liberal capacity shall connect the stations with the power plant by two separate and independent routes.
4. That no other consumers shall be served by these feeders.
5. That the "off-peak" demands be calculated on a monthly average of daily half-hour peaks rather than on an individual half-hour.
6. That peak demands due to large fires, or emergency due to breakage of large mains, etc., not be considered in establishing the maximum demand.
7. That the peak months of water pumpage—viz.—June, July, and August, not be considered in establishing the maximum demand but be treated as individual months.
8. That a credit or debit be allowed or charged depending on the cost of fuel relative to a certain base price.
9. That a reduction in the demand charge be made based on 80 per cent power factor. Maintenance of 100 per cent power factor reduces the demand charge by 20 per cent.

The following are the rates:

Demand Charge:

$1.56¼ per month per kw., plus $3000.00 per month to cover charges on feeder lines.

Energy Charge:

$0.007 per kw-hr. until the demand at the high-lift station exceeds 2800 kw.

$0.00675 per kw-hr. until the demand at the high-lift station exceeds 4250 kw.

$0.0065 per kw-hr. until the demand at the high-lift station exceeds 5600 kw.

$0.00625 thereafter.

The life of the contract is twenty years but it may be terminated upon the giving of due notice by the city should the company fail to give first-class service.

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29-65

Choice of Motor Equipment

At the low-lift stations the range of head was very wide and since better pump efficiencies could be obtained with a change in speed, the two winding motors received considerable thought. A more careful examination of the head fluctuations disclosed the fact that the extremes in head variation occurred during only 20 per cent of the time. Based on the narrower limits of head variation, it was decided to use synchronous rather than two-winding induction motors. The former had the advantage of better efficiencies during 80 per cent of the time, better power factor at all times, less operating complications, and slightly less cost; while the latter afforded better pump efficiencies for 20 per cent of the time. Besides, synchronous motors would raise the power factor made low by the many small induction motors in the purification works. The power factor clause in the power contract was a determining factor in the decision.

Accordingly, unity-power-factor, synchronous motors were selected for the low head stations and 0.8 power factor (leading) motors for the high head station.

Outdoor Substation Equipment

Power for the low-head stations is bought from the Kansas City Power & Light Company at 13,200 volts and is transformed to 2300 volts in an outdoor substation which is the property of the city. Two feeders, one overhead and the other underground, serve the station by two separate routes. The substation equipment comprises two 2500-kv-a. banks of transformers connected delta-delta. Each transformer is rated 833 kv-a., 13,200/2300 volts, single-phase, 60-cycle. These capacities were chosen on the basis that when overloaded 25 per cent one bank would carry 90 per cent of the ultimate load. Transformers are equipped with oil conservators and thermometers.

The two incoming feeders form a loop and are protected by directional relays. The overhead line is provided with a hairpin loop and oxide film arrester. Each bank of transformers is controlled by an oil circuit breaker on the high-tension as well as on the low-tension side and is differentially protected. A tie-line breaker on the high-tension side provides flexibility while the low-tension side is connected to a bus which is sectionalized by disconnecting switches. From this bus four feeders, two to each station, supply the low and secondary station. Each station is fed from both sections of the bus.

The oil circuit breaker equipment comprises three 400-ampere, 25,000-volt and two 1200-ampere, 15,000-volt breakers, the latter of which are enclosed in switch houses. All breakers are hand operated with d-c. automatic trips. The direct current is furnished by a portable battery located in a meter house and connected to a battery charger.

At this station the power is metered on the high-tension side. Duplicate sets of metering transformers, one for the company and the other for the city, are installed on the structure. Duplicate sets of meters are located on the switchboard in the secondary pumping station. The metering equipment consists of recording demand and power-factor meters and integrating watthour meters.

Low-Lift Station

Water is pumped from the Missouri River to the purification works, about 2000 ft. away. The pumping station in which this initial pumping is done is known as the Low Lift Station. Each of the four main pumps in this station is driven by a 400-hp., unity power factor, synchronous motor operating at 2300 volts, three-phase, 60-cycle, 360-rev. per min. The excitation for these motors is supplied by two 30-kw. motor-generator sets. Each set is capable of furnishing excitation for five 400-hp. motors. These sets may be used for lighting in case of failure of the lighting transformer.

The control for this equipment comprises a fourteen-panel switchboard on a gallery floor 28 ft. above the pump room floor. A mechanical remote-control system through bell cranks and pull rods operates the oil circuit breakers which are supported on a pipe framework and are located on another gallery 12 ft. below the switchboard floor.

The main bus is sectionalized by two oil circuit breakers. In the section between these breakers there is connected a station service feeder which serves power, lighting, and heating transformers. Each outside section of the bus is fed by an incoming feeder, each feeder having the capacity of the entire station. One section serves two motors, while the other serves the remaining two motors and the future unit.

Motor starting is accomplished by means of compensators and a starting bus. This bus is sectionalized by disconnecting switches, each section being served by a compensator. Each compensator is of sufficient capacity to permit of four starting operations within 10 min. without excessive heating of the compensator. In this station there are eleven 600-ampere and four 400-ampere, 7500-volt oil circuit breakers.

Auxiliary power for operating vacuum pumps, sump pumps, motor-generator sets, crane and gate valves in this building as well as the revolving screens in the intake building is obtained from three 25-kv-a., 2300/230-volt transformers connected delta-delta. The lighting transformer is rated at 15 kv-a., 2300/230/115 volts.

Both the intake building and the low-lift station are heated electrically and are served by a 150-kv-a., three-phase, 2300/230-volt transformer. Industrial oven type, 220-volt ribbon element heaters suitably mounted and backed with reflecting and insulating material and provided with grill work in the front are placed along the gallery in a manner similar to steam radiator practise. In most locations two elements are used to form

one radiator but they are individually controlled by local switches so that flexibility in controlling the heat may be obtained. All the transformers in this station are controlled by one oil circuit breaker but are protected by individual S & C fused disconnecting switches.

Secondary-Lift Pumping Station

After the water has been purified it is pumped to reservoirs at the high-head stations. This being the second stage of pumping gives to the station the name of secondary lift pumping station. The pumping capacity of this station is the same as that of the low-lift station and since the heads are about equal, the electrical equipment is a duplication of that at the low-lift station. In addition to the main pumping units and the vacuum pump motors there are three 75-hp., 2300-volt, 1800-rev. per min. motors driving wash water pumps, two 60-hp., 2300-volt, 1800-rev. per min., and two 25-hp., 220-volt, 1800-rev. per min. motors driving house service pumps. The 75-hp. motors are controlled by automatic "across-the-line" contactors and push buttons. Each motor may be controlled from any one of three points. The 60-hp. motors are started across the line by closing the oil circuit breaker. The 25-hp. motors are controlled by starting compensators.

In addition to these pump motors all the power used in the chemical house, chlorine house, filter house, Dorr clarifiers, and the lighting for all these buildings and the grounds are controlled from the switchboard in the secondary-lift station. In the chemical house there are about 150 hp. in motors ranging in size from ¼- to 50-hp. Four 7½-hp. motors operate the Dorr clarifier mechanisms.

The switchboard comprises 21 panels and the system is that of mechanical remote-control, with the main oil circuit breakers and bus located on the lower floor in a switch-room beneath the switchboard. The starting bus, its compensators, and oil circuit breakers are located at the back of the switchboard. The busses are sectionalized in a manner similar to those at the low-lift station, but between the sectionalizing breakers there are two feeders, one for power and one for light. This building is not electrically heated. In the chemist's laboratory, however, there are ovens, sterilizers, refrigerators, distillers, etc. Another small difference between this station and the low-lift station is that the field rheostats for the main motors are electrically controlled in this station. The 36-in. discharge valves in both stations are electrically operated.

There are seventeen 600-ampere and four 400-ampere, 7500-volt oil circuit breakers in this station.

The auxiliary power transformers comprise three 75-kv-a., 2300/230-volt units connected delta-delta. Multiple lighting is served by a 75-kv-a., 2300/230/115-volt transformer, and series lighting by an 8-kv-a. constant-current transformer.

Lighting and Wiring

Lighting in almost all the buildings is laid out to give at the working plane an intensity of six foot-candles exclusive of a liberal allowance for dust and aging of the lighting units. In the filter galleries, this intensity is maintained but over the filters and in the pipe galleries a lower intensity is used. A generous number of wall receptacles is installed. Glass and fancy fixtures are omitted, porcelain enameled steel reflectors being used instead. The installation is substantial and put up to stay for many years. Galvanized iron conduit and condulets are used throughout. Wire is 30 per cent para, double braided, except in damp places where triple braided weatherproof wire is used. Waterproof fixtures are used in such places.

Around the uncovered basins flood lighting projectors, standing two feet above the ground, are used. Thisi s done to reduce the number of bugs which swarm around lights, die, and fall in the water. At each flood light location there are three waterproof condulets, one for connecting the projector, another housing a switch for controlling the projector, and the third, a receptacle. The receptacle permits the concentration of more than one projector at a location during periods of cleaning of the basin. The projectors are fed by armored cable laid from 12 to 18 in. in the ground. At various manholes, lights, switches and receptacles are located and connected to the same cable system. About other parts of the purification works, there are distributed lighting standards for general illumination. Along the roads leading to the pumping station there are similar standards. They are all connected to a series system which is served by a constant current transformer in the secondary-lift station and operated at 6.6 amperes.

Power Cables

All power cables are insulated with varnished cambric and are lead covered where they are installed in pump-room floors or other places that are likely to be damp or flooded with water. Standard 5000-volt insulation is used on 2300-volt circuits and 17,000-volt insulation on 13,200-volt circuits. On 2300-volt circuits 7500-volt potheads are used. The control wiring for the East Bottoms Station is lead covered.

Underground Cable System

Between the outdoor substation and the low- and secondary-lift pumping stations as well as between the secondary station and the chlorine and chemical houses the feeders are three conductor, varnished cambric insulated, lead covered cables installed in fiber ducts surrounded by a concrete envelope. Cables vary in size from No. 6 A. W. G. to 800,000 circular mils. Conduits vary from four to ten ducts. Concrete manholes, properly drained, are installed at frequent intervals.

East Bottoms Stations

The new electrically operated high-head station is

known as the East Bottoms Station and is located about five miles from the purification works. The present main motor equipment consists of two 2000-hp., 0.80 power-factor (leading) 13,200-volt, three-phase, 720-rev. per min. motors equipped with direct-connected exciters. They are provided with temperature detectors and enclosing bell ends. The station is planned for six such motors.

The switchboard comprises 14 panels at present but will ultimately be expanded to 21. The system is that of electrical remote control, some operations being performed by manually operating the control switches while others are automatically controlled by relays and interlocks. The front of the switchboard faces the pump room, while access to the back is had from the switch house. All busses and switching equipment, oil circuit breakers, instrument transformers, compensators, etc., are enclosed in a monolithic concrete cell structure.

The station is fed by two 13,200-volt lines direct from the public utility's power-house bus. Ultimately there will be four such feeders. The main bus in the station is in duplicate but may be made a continuous bus by use of a tie breaker at one end of the bus structure. Indicator lights are located over the disconnecting switch compartments of the cell structure to show the position of the controlling oil circuit breaker.

The oil circuit breaker equipment comprises one 600-ampere, and seven 400-ampere, 15,000-volt units each rated at 14,000 r. m. s. amperes at 15,000 volts, and four 400-ampere, 15,000-volt units each rated at 1500 r. m. s. amperes at 1500 volts. All but two breakers are of the phase per cell type. The breakers are all solenoid operated and the control current is supplied by a storage battery. A small motor-generator set furnishes the trickle charge for the battery.

Motor starting is accomplished by compensator and starting bus as in the other stations but in addition neutral breakers provide a smoother transition from starting to running voltage. Motors have 85 per cent pull-in torque and full voltage is applied before the field circuit is closed.

At this point, the sequence of operations in starting a motor might be interesting. First, the neutral breaker is closed by closing its control switch, which operation automatically closes the compensator breaker. Next, the starting breaker control switch is closed. An accelerating relay controls the next steps. As it closes, it trips the neutral breaker which in turn closes the running breaker. The field breaker control switch is then closed and the compensator breaker control switch, which is the same as the neutral breaker control switch, is opened, which automatically trips the starting breaker. If the field breaker be not closed within ten seconds after closure of the running breaker, the field failure relay trips the running breaker. This time limit has been set on account of hydraulic conditions.

With the closure of the running breaker, a solenoid valve on the automatic check valve is closed. This will be described more fully later. Due to the fact that this solenoid valve closes only after the running breaker is closed, the motor "pulls in" against a closed discharge because the check valve cannot open while the solenoid valve is open. This reduces the required pull-in torque of the motor.

In this station there is a 30-kw. motor generator set which is capable of performing four important functions. It can supply excitation for any one or two of the main motors. It provides a source of direct current for lighting; it can give a full charge to the battery; it provides current for the control circuit while the battery is out of service or being charged.

Auxiliaries are fed from a bank of three 25-kv-a., 13,200/230-volt transformers. Lighting is supplied by a 25-kv-a., 13,200/230/115-volt transformer. Emergency lights are fed from the battery and go on automatically whenever the lighting transformer is out of service.

The description of this station would be incomplete without a word about the automatic stop and check valve. It opens and closes automatically with unbalance of hydraulic conditions in the valve. The necessary unbalance of hydraulic conditions may be controlled by a solenoid-operated valve in the blow-off line of the main valve. The solenoid circuit is interlocked with the running breaker and a control switch on the switchboard is inserted in the circuit. With the closing of the running breaker the solenoid valve closes and permits the main valve to open as soon as the pressure on the pump side of the valve becomes greater than that on the line side. As soon as the running breaker trips out, the solenoid valve opens and the main valve closes. The control switch on the board permits the operator to open the solenoid valve circuit, thereby opening the solenoid valve, which in turn closes the main valve even with the pump operating at normal speed. The operation of the main valve may be noted by the pressure-gage reading. This permits of the checking of one of the most important automatic features of the station without disturbing normal operation.

The use of 13,200-volt motors and electric radiators for building heating are radical steps in waterworks practise. It must be realized that the city is entirely dependent upon electrical power for its water. Moreover, this electric power is being purchased from the local public utility. It is a great tribute to the reliability of electric public utility service and to the electrical industry as a whole.

Meeting Long Distance Telephone Problems

BY H. R. FRITZ[1] and H. P. LAWTHER, JR.[2]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*There have been presented before the Institute numerous papers describing various technical and apparatus developments of value in providing long distance telephone service. Several papers have also appeared covering specific transmission or operating problems or dealing with the advance planning of the telephone plant. Feeling that it might be of interest, particularly to the young engineering graduates, the writers have prepared this over-all sketch of the general problem of actually providing year by year the extensions and additions to a comprehensive network of communication channels necessary to keep pace with a growing public demand for long distance service. Since the writers are most familiar with the area served by the Southwestern Bell Telephone Company the discussion will be restricted to that territory.*

* * * * *

THE Southwestern Bell Telephone Company operates in the states of Missouri, Arkansas, Kansas, Oklahoma, Texas, and a small section of Illinois which is socially and economically associated with greater St. Louis. This is primarily an agricultural region but there are several large manufacturing centers and extensive areas devoted to oil and mineral production. The great difference in climatic and soil conditions which exist over such a wide territory naturally brings about a diversity of agricultural interests. Local, seasonal, peak demands for long distance service occur at the times when crops must be marketed and moved.

Oil, on account of its fortuitous occurrence and the sudden and imperative demand for long distance service attending its discovery, creates the most difficult problems. Through some perverse trick of fate the deposits have been found uniformly in areas otherwise of no great economic importance and having either no or very little telephone development.

Contributing more than any other item to the problem of keeping abreast with the demand for long distance service is the fact that the Southwest is still immature in the social and economic sense and is growing lustily and at a rate which rarely fails to exceed the most optimistic forecasts.

Beside these purely local peculiarities which contribute to the problem there is the generally increasing demand for long distance service on the part of the public. Based fundamentally on the increase of wealth and prosperity, traffic growth has been accelerated by better service and reduced rates. New and improved apparatus and methods become available from time to time—and not the least of the tasks of the field engineer is the more or less continuous modernization of an existing plant.

With this picture of the factors affecting the demand for service, it will be interesting to note the extent and complexity of the long distance network required. The map of Fig. 1 shows the principal towns and cities of the Southwestern area, and the single lines connecting them represent circuit routes. There may be a large number of communication channels on each route, of course. Not all of the cities and towns, and not all of the circuit routes shown are Bell owned and operated. Long distance service must be universal, and considered independently of who owns and operates the plant.

In addition to the network shown on Fig. 1 there is the long distance telephone plant of the Long Lines Department of the American Telephone and Telegraph Company. This department is organized to supply the extreme long distance facilities necessary between the several associate company areas in order that universal service may be realized. This paper does not include consideration of the Long Lines Department's plant or its problem.

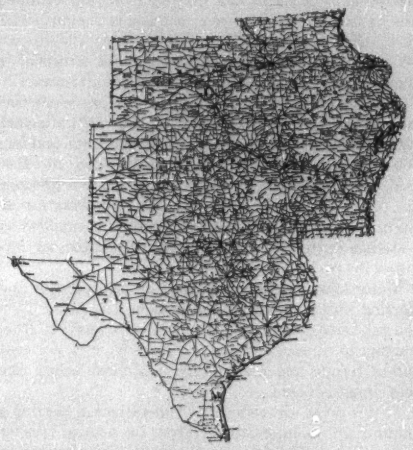

FIG. 1—LONG DISTANCE LINES

Confronted with an extensive existing plant which is meeting the present day requirements, it is the task of the field engineer to plan for continuous extensions, reinforcements, and betterments. The various technical developments that have been made in the art of

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29-63

communication, the engineering practises that have emerged, and the advance planning that has been done must be practically applied and coordinated with a working system.

The fundamental requirements of any link of a system of long distance facilities is that it must talk satisfactorily. It must be capable of transmitting speech easily and without undue effort on the part of the users. To accomplish this there must be:

1. A sufficient volume so that speech may be received with adequate loudness.
2. Freedom from distortion so that speech will be satisfactorily intelligible.
3. Freedom from extraneous interference.

Each circuit together with all of its associated equipment must be designed to comply with these basic requirements. Not only must it furnish adequate transmission as an individual unit but it must also perform satisfactorily as a link of the whole system. In the planning of long distance facilities it is necessary always to envisage each unit of the plant as a part of a nation wide system and to insure its proper performance for any function it may be called upon to play.

In addition to regular long distance telephone communication it is necessary that certain special services be furnished. The radio public demands the chain broadcasting of programs and the frequent picking-up of programs at points remote from the radio transmitting stations. This requires the setting up of very reliable and extremely high quality telephone channels. Extraordinary freedom from distortion and noise is necessary since even with the perfection of modern amplifying and loudspeaking devices the quality of a radio program must exceed that of a simple transmittal of intelligible speech in order that it may prove enjoyable. Also, the radio listener is not in position to interrupt and request the repetition of a misunderstood word or phrase. Fortunately telephone circuits used as an adjunct to radio broadcasting may be operated as one-way circuits and the limiting effects of echoes are thus avoided. The simultaneous operation of telegraph over wire facilities primarily placed for telephone service has long been practised. Telegraph facilities, then, are a by-product of the telephone business, and economy dictates that these be made use of as fully as practicable. The demand for telegraph service is always sufficiently great to call for weighty consideration in planning for telephone facilities. Picture transmission is still before the public for trial. Should a widespread demand develop it will be necessary to set up a network of communication channels surpassing all previous standards as regards constancy, for a very minute deviation during transmission will spoil a picture.

Types of Facilities Used

There are three types of facilities available for providing long distance circuits. These are open wire, carrier current systems (superposed on open wire), and cables. The first two are closely interrelated and are of service in the early and intermediate stages of development, while the last is the final goal of progress at the present stage of the art.

Open wire was the first and is still the main reliance for providing circuits in this country. In a large area of low population density such as that of the Southwest, open wire must constitute the bulk of the plant for a number of years to come. Three sizes of copper wire now find general application. These are designated 104, 128, and 165, the figures indicating the respective diameters in thousandths of an inch. Improved transmission through the use of copper wire larger than 165 is unduly expensive, while sizes smaller than 104 do not possess the requisite mechanical strength for resisting the stresses of storms. Copper open wire circuits possess electrical characteristics so ideal as to endear them to the hearts of transmission engineers. With the aid of suitably spaced telephone repeaters they may be extended to practically any distance. Iron wire has lost its place in a plant designed for universal service. Having when new a transmission efficiency only one-fourth that of the same size of copper, iron wire becomes progressively worse from the day of its installation due to corrosion. This instability renders it wholly unsuited to the application of telephone repeater improvement and thus so seriously restricts its flexibility that it has been determined to eliminate iron wire completely from the long distance plant. The application of the principle of loading to open wire facilities has been rendered totally obsolete on account of the better performance and greater flexbility of non-loaded open wire with repeaters.

Along with its many and manifest advantages open wire construction possesses a number of serious drawbacks. Poles, crossarms, pins, and wire cannot withstand the assaults of exceptional wind and sleet storms, and occasional service interruptions from this source must be expected. By their very nature open wire circuits are of maximum susceptibility to both mechanical and inductive interference. It is thus generally impossible to continue in open wire up to the central office in a city. In such instances it is the practise to terminate the open wire out beyond the city congestion and to bring the circuits into the office over specially designed entrance cables. These entrance cables must often be of considerable length and add materially to the transmission losses. Through the application of the proper systems of loading the cable circuits may be made to match the impedances of the open wire circuits, and thus preserve the full possibilities of telephone repeater operation. With the increasing lengths of telephone circuits which modern service requirements demand, the problem of limiting the cross-talk among open wire circuits has called for greater and greater care in the planning of transposition schemes for them.

For several years now carrier current systems have been available by means of which additional telephone channels may be derived from existing and suitable open wire facilities. For distances beyond certain minimum figures these carrier systems yield telephone channels more economically than possible through the placing of additional open wire. The outstanding usefulness of carrier systems, however, lies in the increased capacity they confer upon an existing open wire structure. It is thus possible to defer heavy expenditures for major relief. The flexibility of carrier systems is apparent. They may be installed speedily to meet an emergency condition, and may be readily recovered and removed to another territory when the emergency has disappeared.

There are two types of carrier telephone systems in general use. One provides three additional telephone circuits above the one furnished by the pair of wires over which it must operate. Under favorable conditions it is economical as compared with stringing of wire for distances as short as 150 miles, and by the use of carrier repeaters spaced at regular intervals it may be extended to any distance. The other provides one additional talking circuit and finds its application with distances between 50 and 200 miles. The three-channel system employs frequencies from 4 to 28, and the single channel system frequencies from 4 to 10 kilocycles. Both of these systems utilize grouped frequency arrangements whereby different frequencies are used to transmit in the two directions.

At the higher frequencies of the carrier systems the attenuation offered by the open wire circuits is greatly increased over that experienced at voice frequencies. When it is remembered that cross-talk effects increase almost directly with frequency and are a function of the energy level differences between the disturbing and the disturbed circuits it will be realized that the application of carrier systems to an open wire lead may present serious difficulties. Transposition arrangements quite satisfactory for voice frequencies are totally inadequate for the carrier range. Special systems of transpositions must therefore be employed which, in addition to making use of extremely elaborate patterns, must be spaced with great precision. The retransposal of existing open wire leads is a large item of expense which must be carefully weighed when the use of carrier systems is being considered.

The necessary entrance cables which complicate voice frequency transmission problems constitute a proportionately more serious problem at carrier frequencies. To reduce attenuation and to limit reflection effects the cable conductors must be suitably loaded for carrier operation. For the frequency range employed by the three channel system loading coils must be placed at 930 foot spacing. In the case of underground construction it is rarely possible to realize this ideal spacing on account of manhole locations. It then becomes necessary to place the loading coils at something less than the theoretical spacing and to build out the short sections to the required capacity by means of special stub cable connected in shunt with the loaded conductors. In order to avoid carrier cross-talk in entrance cable it is necessary to maintain a degree of segregation among the conductors assigned to carrier operation. Where long entrance cables are involved it may well occur that the largest single item of expense in connection with a carrier installation will be the preparation of these for carrier operation.

Despite elaborate preparations in the form of special transposition of the open wire and loading and segregation of entrance cable conductors, it is still not possible to operate carrier systems indiscriminately on any open wire lead. Serious energy level differences and opposite directions of transmission with the frequency groups must be avoided. All the systems on a lead cannot always be coterminous, for a lead may be hundreds of miles in length, while carrier systems over it will serve various intermediate and overlapping lengths. At any intermediate point it may be desirable to introduce a system terminal with its high output level to operate in parallel with other systems whose energy has been greatly attenuated in transmission from distant terminals. Such differences in level have to be minimized either by separation on the pole head or by the introduction of repeaters to raise the lower energy levels. Junctions with other open wire lines and the generally complex nature of the open wire network give rise to situations such that it is practically impossible to realize completely the theoretical carrier possibilities.

With these various restrictions controlling the situation, the average carrier possibilities that can be realized provide for a total of 24 carrier channels on any long four-arm lead. Further exploitation of the remaining carrier potentialities on open wire leads is blocked by the use of phantom circuits which definitely limit the transposition refinements needed to restrict the cross-talk. For important open wire leads consideration is therefore being given to abandoning phantom circuits on all wires of the first four arms, except the pole pairs. A further simplification of the cross-talk problem can be made by reducing the spacing between the wires constituting a pair from 12 to 8 inches and correspondingly increasing the spacing between pairs from 12 to 20 inches. By these arrangements the carriers' possibilities can be increased from 24 to 48 channels, but as 8 phantom circuits are sacrificed the net gain is only 16 talking circuits. It is expected that this refinement will be applicable only to pole lines carrying principally very long haul circuits. Where there are numerous short circuits on a lead, the necessity of terminating wires at frequent intervals destroys the flexibility of the arrangement.

The nature of the long distance telephone development and the growing requirements of the Southwest have furnished a particularly favorable field for the

application of carrier telephone. The distance between many of the more important cities is from two to three hundred miles, and the three-channel system has been made large use of in spanning these. The single-channel system has found equally widespread application. While carrier telephone has been used in the Bell System for about a decade, the installations previous to 1925 were of a transitional nature in the evolution of a commercially practical system. It

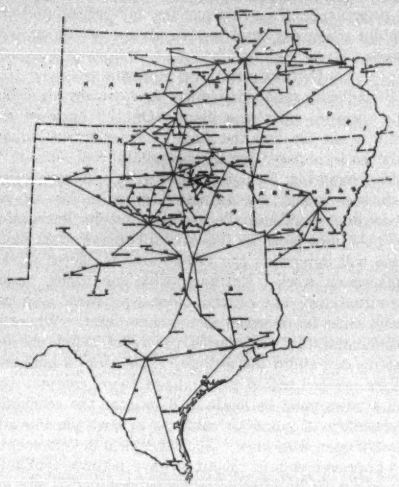

Fig. 2—Carrier Circuits in Southwest

is of local interest to note that with the rendering available of the standardized three-channel system the first commercial installation in this country was placed in service between Dallas and Houston in the summer of 1925. It is also of interest to remark that the first commercial installation following the field trial of the single-channel system which is now standard was placed in service between Amarillo and Borger in the early days of 1927. Borger was a new oil town that had sprung up from nothing overnight. From this auspicious beginning the use of carrier telephone has grown until the present day finds some 35,000 miles of telephone channel obtained by this means in the Southwest. The map of Fig. 2 shows the extent of this development, which represents about one-sixth of that of the entire United States.

From the discussion so far it is clear that with any open wire lead the condition will be reached ultimately when the maximum wire load is in place and the fullest possible use of carrier has been made. It then becomes necessary either to build additional open wire lines or else have recourse to some other means. Where the circuit growth is sufficiently rapid economic studies rule in favor of cables in place of building one or more supplementary open wire lines. In addition to proving in from a strict economy standpoint, cables offer many advantages as a medium for long distance circuits. Increased freedom from interruptions due to physical failure of the plant, lower susceptibility to external interference, and more stable operating conditions are some of the benefits accruing. In the territory of the Southwest the condition is rapidly approaching when cables will be needed between nearly all of the important centers. To meet this situation a cable program has been adopted for the period 1929-1933 which will connect most of these points. This is shown on the map of Fig. 3. Of the cables in the program the sections between Oklahoma City and Tulsa, Oklahoma City and Holdenville, and Dallas

Fig. 3—Toll Cable Program for Period of 1929-33

and Cisco are under way and will be completed during 1929. As it now stands, the program provides for a total of approximately 2600 miles of cable for the five years.

The Tulsa-Oklahoma City and the Fort Worth-Cisco sections of the cables mentioned above will represent a unique departure from the previously standard practises of this country. The usual lead covered cable is to be manufactured and then given a protective coating including several layers of jute and tar and two servings of steel tape armor. A trench is to be dug and the cable simply placed therein and covered up. When it is realized that a standard 750-foot length of this armored cable together with its reel will weigh about five tons and that the route is to be

strictly cross-country, it will be appreciated that some interesting construction difficulties must be met and solved. It is planned to present the story of these installations to the Institute at a later date.

Whether the channels for long distance circuits are derived by means of open wire, carrier systems, or cables, elaborate installations of equipment are required at the circuit terminals and at the intermediate repeater stations. The placing of this equipment and the providing of the necessary building space for housing it constitute major problems. Building additions must be made or even complete new buildings erected, new equipment must be installed and placed in service, and existing working equipment must be relocated to best advantage,—all without interrupting the service. Also, irrespective of the means used to derive the communication channels, arrangements must be made at the terminal offices whereby any two circuits may be connected together and a built-up connection

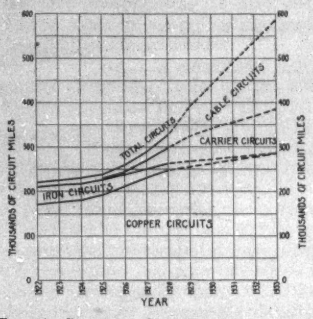

Fig. 4—Growth of Long Distance Circuits

thereby established between two terminals not having direct circuit connection. The problem comes in arranging for such interconnections with the result that built-up connections shall satisfy the same fundamental transmission requirements as are demanded of the individual links. To meet the volume requirement it is essential that a definite gain be inserted whenever two circuits are connected together. The practise of the past has been to provide with repeaters certain of the cord circuits appearing at the switchboard positions which handled switched traffic. The fundamental weakness of this method lay in the fact that the insertion of the proper gain was left in the hands of a very human operator, and there was always the possibility of its being omitted. The latest practise eliminates this element of human fallibility. The switchboard positions are equipped with but one type of cord circuits. When a connection is established between two long distance circuits the proper gain is automatically provided; when a connection is established between a long distance circuit and a local telephone no gain is inserted since none is necessary. The distortion and interference requirements are met for built-up connections simply by causing the individual links to meet sufficiently high standards in these respects so that a chain of four or five in tandem will still yield a satisfactory grade of over-all transmission.

An interesting review of the growth of the various types of facilities used to derive long distance telephone circuits in the Southwest is shown on the diagram of Fig. 4. The rise and fall of iron wire, the abrupt rise to prominence of carrier systems and cables, and the rapidly mounting total are significant of the present trends.

Determining the Procedure

Periodically the field engineer must review his entire long distance telephone plant. On the basis of the years in service and the stage of deterioration, plans must be formulated for the systematic replacement of poles, crossarms, insulators, and wire. In connection with such plans the estimated growths for several succeeding years and the probable future destinies of the pole lines must be taken into consideration, since these will determine the extent and character of the replacement work. Entrance cable conditions, building situations, and central office equipment requirements must be reviewed in the same light. Where it is indicated that even with the relief which carrier systems can afford the ultimate capacity of a pole line will be reached within the next few years, comparative cost studies must be made to determine the economic advisability of going to cable as against placing additional open wire lines. Such technical developments and improvements in the art as have become available recently must be injected as far as practicable into all plans.

From such considerations the field engineer each year evolves a list of projects for the next succeeding five years. This list is submitted to the company executives in order that the necessary financing may be arranged for. The material requirements of this list are submitted to the manufacturers in order that they may be prepared to fill the orders. Such projects as are given approval constitute the definite program of active work for the immediate future. For each approved project detailed plans and an estimate must then be prepared and submitted to the executives for final approval. Upon securing this, the project becomes a construction or installation job, and the engineer's final task is to see that it is executed in accordance with his plans.

In addition to this somewhat orderly program of activity, the years are always spotted with unforeseen situations of an emergency character. A new oil field is discovered at Seminole, a Democratic Convention is announced for Houston, an extraordinarily large cotton crop is realized in Texas, or floods in Arkansas carry away miles of pole miles,—all these call for prompt action with an insistence which cannot be denied.

Conclusion

The writers have prepared this non-technical discussion with the hope that it might prove of some interest to engineers as an indication of the range of problems encountered in this particular field of endeavor. Its purpose will have been accomplished if it has shown something of the zest and interest, something of the wide range of technical acquaintance, and something of the harassment arising from continuously changing conditions which attach to the task of meeting the demand for long distance telephone service in a large area. For it is true that the battle of progress is waged no less actively, and broad technical understanding, initiative, and good judgment are no less necessary in the field outposts than in the more sheltered seclusion of the research laboratories.

Discussion

G. H. Quermann: This paper gives a very complete picture of the problems presented in the design of long distance telephone plant, and although it does not specifically touch on the long lines department's problems, they are in general very similar to those discussed in the paper.

The facilities of the long lines department must of course be so designed to operate satisfactorily with shorter haul toll circuits and local exchange facilities and since they constitute a trunk-line system, which extends practically throughout the United States, they must of necessity be of the highest grade in order to provide satisfactory service on all connections.

With long lines facilities extending over the greater portion of the United States with connections into Canada, Mexico, and Cuba, the furnishing of messsage telephone service together with telegraph service over these facilities for press, government, and commercial private-wire use presents a service problem of no small proportion.

The furnishing of suitable facilities for program transmission for national broadcasting is also one in which the long lines department is vitally concerned.

Picture transmission is now offered between eight cities in the United States, namely, Boston, New York, Cleveland, Atlanta, Chicago, St. Louis, San Francisco, and Los Angeles. These stations are equipped with apparatus for the transmitting and receiving of pictures. St. Louis is the only point in the territory of the Southwestern Bell Telephone Company. The circuit used for this service to the Pacific Coast is at times routed through this territory by way of Little Rock and Dallas.

Open-wire plant at present constitutes the major portion of long lines circuits in this territory. This is not true in the East where the requirements of long distance telephone service has exceeded the capacity that can be economically provided by open-wire plant. There, cable circuits form the major portion of long-distance communicating circuits. However, as has been outlined the situation in this territory is such that the provision of toll cables between the more important centers now appears to be the next practicable means of providing adequate long-distance facilities.

In the territory west of the Mississippi, extensive use is being made of 3-channel carrier telephone systems. At the present time one of the longest carrier systems in service operates through this territory with terminals at St. Louis, Mo. and San Bernardino, Cal. It is 2100 mi. in length and has intermediate repeaters at nine points, including Little Rock, Dallas, and El Paso.

The problem of coordinating the carrier systems operating over an open-wire line is a very important one. As the long-lines carrier systems are frequently operated on lines used jointly with the associated companies, this becomes a mutual problem.

The paper pointed out that in some cases consideration is now being given to abandoning phantom circuits on some of the wires on a lead and reducing the spacing between wires constituting a pair from 12 to 8 in., correspondingly increasing the spacing between pairs from 12 to 20 in., in order to increase the carrier possibilities and obtain a net increase in circuits. The long lines department is now doing on a trial basis, some work of this type in certain sections of its plant extending from Wichita, Kans. to Denver, Colo., Joplin, Mo. to Tulso, Okla., and Oklahoma City, Okla. to Texola, Okla.

C. W. Mier: The paper mentions the difficulties sometimes encountered in meeting the long-distance problems in connection with the discovery of oil. Having been very closely associated with this problem in Oklahoma during the past few years, I should like to tell briefly of the growth in traffic in the vicinity of the famous Seminole Oil Field.

Oil was first discovered in this field about July 1926, and reached a peak production in July 1927. Previous to July 1926, the long-distance telephone traffic out of Seminole amounted to about 800 calls per month and was handled over two circuits by one operator. By January 1927, this traffic had grown to 3688 calls per month, an increase of 360 per cent, and the operating force had increased to 30. This traffic increased by leaps and bounds until in March 1929, when the total out calls for Seminole were 34,574 per month and the operating force had increased to 60. This is an increase of about 4000 per cent in two years and nine months, as compared to an average annual increase under normal conditions of about 10 per cent.

There are at the present time 43 long-distance circuits radiating out of Seminole and, after the completion of the Oklahoma City-Shawnee-Seminole-Holdenville cable, there will be 57 circuits.

The growth in toll traffic at Shawnee, 20 mi. distant from Seminole, has been almost as great. In December 1927, there were 33,205 calls handled out of Shawnee per month. In December 1928, this traffic had increased to 50,247 calls or an increase of about 50 per cent as compared to an increase under normal conditions of about 6 per cent or 7 per cent.

As an indication that the experience of Seminole will be repeated, oil was recently discovered in the vicinity of the town of Maud, located about 17 mi. from Seminole. Until June 1928, this office handled about 868 out long-distance calls per month with an operating force of 5. In July 1928, immediately after the discovery of oil, this traffic had increased to 2467 calls, an increase of 180 per cent, and the operating force consisted of 17 people. In December 1928, the traffic increased to 10,766 calls, an increase of 1200 per cent over June 1928, and in March 1929, this office with an operating force of 47 people was handling 12,851 out calls per month, an increase of 1800 per cent in less than a year.

This, it will be appreciated, throws an unusual task and strain upon the operating forces. That the traffic is being properly handled is indicated by the following figures. The percentage of offered toll calls completed for 1928 was 91.2 for the 39 largest toll offices and 92.4 for all offices in Oklahoma, the highest results yet obtained.

The faster handling of toll calls has resulted largely from the extension of the combined line and recording operating method to all offices in this territory and to its application to an increasingly greater percentage of the traffic handled. Combined line and recording operating provides that the long-distance operator who records the subscriber's call handle it to completion instead of simply recording it and then passing it to another operator to complete. For 1928, the average interval from the time a call was placed until conversation began or a definite report was received was 2.9 minutes. For the last four years the average speed of service was as follows:

1925.....6.4 minutes
1926.....5.7 minutes
1927.....4.2 minutes
1928.....2.9 minutes

It is of interest that the percentage of the total toll traffic on which the speed of handling required more than ten minutes has been decreased from 13.1 per cent in 1925 to 2.2 per cent in 1928.

Through the traffic engineering work much improvement in the handling of this traffic has been realized and still further improvements can be made. Wherever possible, direct circuits are installed, thereby reducing the number of offices through which calls must be switched. Routes quoted for the operators' use are regularly checked to be certain that they are the best available, and through analyses of difficulties encountered on this class of traffic the combined efforts of the traffic and engineering forces are directed toward the routes or situations that are producing the most trouble.

E. T. Mahood: The existing long-distance plant represents a highly complicated, well balanced structure, gradually developed to its present form throughout the years of its building. No radical changes can be made in any one part without disturbing the whole. The fitting in of new instrumentalities has become increasingly difficult as the complexity of the long-distance network has increased.

But what does the future hold? Twenty-five years from now, will we regard the long distance service of today as we now regard that of twenty-five years ago? Of course only the future holds the answer to this question, but if past experience means anything, it certainly indicates that the limits have not yet been reached. Everything seems to point to a continued development of the country, with greater and greater business and social intercommunication. Long-distance communication facilities must keep pace with any such development. What the long-distance telephone plant will then consist of, how it will be used, the character and quality of the service, and the engineering problems involved in providing the necessary facilities challenge the imagination.

L. E. Adler: As I understand the design of the two-way telephone repeater, the gain is chiefly limited by two factors—one, the absorption of a certain amount of gain in the artificial line of network; and the other that after a certain point is reached self-oscillation occurs. What progress, if any, has been made toward the design of two-way repeaters of the full-gain type or the so-called "22-type" in which the limitations are largely removed?

H. R. Fritz: The limitation on the gain of the particular repeater mentioned by Mr. Adler, the so-called 22-type, is controlled by the maximum gain that can be obtained without the system singing. There is a return path through the repeater circuit and the total gain is limited by the best balance that can be obtained between the artificial network and the line.

If this were not a controlling factor, we might add one or more stages of amplification to the present repeater. For four-wire cable circuits in which a separate pair of wires is used for transmitting the voice in one direction and another for transmitting it in the other direction, a one-way amplifier not subject to these limitations is used. It has a maximum gain about twice that of the 22-type, which for other reasons cannot always be profitably used.

Telephone Transmission Networks

Types and Problems of Design

BY T. E. SHEA[1] and C. E. LANE[1]

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*A brief résumé of the nature of telephonic signals is given, showing how the qualities of wave composition which distinguish signals from other electrical waves set the requirements on networks and provide a basis for their design.*

The principal functions of wave filters, equalizers, telephone transformers, line balancing networks, and artificial lines are outlined. In order that these networks may be used in conjunction with other apparatus in the telephone system they must provide efficient transmission, low distortion, good impedance balance, stoppage of longitudinal currents, stable characteristics with current variations, low external coupling, and low reflection coefficient. In addition to these requirements the network must not cross-talk into associated circuits and must have desirable impedance characteristics in the attenuation range of frequencies as well as throughout the transmission range.

An illustration of the use of transmission networks in a typical three-channel carrier telephone system is given describing the functions of the line filter sets, the directional filter sets, band filters, and equalizers.

Some of the engineering limitations on the design and construction of networks are discussed.

* * * * *

THE past thirty years have witnessed many important developments in electrical communication. Looking back, it appears almost as though the various dynamic sciences, which had rapidly grown in scope under influence of Maxwell, Helmholtz, and others, having been for a time pent up, were fairly bursting in their eagerness to spill over into the communication field innumerable facts capable of being put to great practical use. The writings of Heaviside are clear evidence of the transition, pointing to many useful relationships while making important advances in electrodynamics itself.

Subsequently, striking applications of the new knowledge, as typified by inventions and discoveries, went constantly hand in hand with a codification or systematization of the knowledge found useful. Thus, paralleling the origination of the loading coil by Pupin, the thermionic vacuum tube by De Forest, and the wave filter by Campbell, has been the systematizing work of Campbell, Kennelly, Blackwell, and others. Out of the latter process has grown what is often called "telephone transmission theory." One of the most important and striking branches of "telephone transmission theory" is that which relates to the properties of electrical networks used for transmission purposes, called *transmission networks.*

There is not space available here to enter into a complete discussion of the uses to which transmission networks are put*, and for purposes of illustrating such uses we may confine ourselves to certain aspects of long distance telephony. This choice of illustration will serve perhaps better than any other to bring out the physical and economic attractions which the networks can have for engineers.

Long distance circuits are of various kinds. There are open wire,[2] cable, and radio facilities. These impose very different requirements on apparatus used in transmission. From an electrical standpoint, they may provide telegraph, normal telephone (both side circuit and phantom circuit),[3] superimposed carrier current (telephone and telegraph), telephotograph, and radiotelephone transmission.

In Fig. 1 there are shown the routes of some of the more important repeatered[4] open wire circuits of this country. Each route on the accompanying map (Fig. 1) represents not one circuit only but generally a large number of circuits, depending on traffic conditions. In Fig. 2 is a similar diagram of the routes of the long distance carrier telephone and telegraph facilities which are superimposed on open wire lines and cables. Circuits such as these together with long distance cables form the trunk lines, veritably the "backbone circuits," which tie together from a transmission standpoint the lines of the entire Bell System. At the present time such long distance circuits are about half open wire and half cable, with the percentage in cable rapidly growing. Of the open wire circuits about one-fifth are carrier circuits. As examples, the route through Omaha, Denver, and Salt Lake City is

1. Both of the Bell Telephone Laboratories, New York, N. Y.

*A short bibliography is appended.

Presented at the Regional Meeting of the South West District No. 7, of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

2. *Open wire facilities* are those whose circuits employ wires separately strung overhead on pole lines, in contrast to *cable facilities* which use wires from among those provided by a cable. The electrical characteristics of the two types of circuits are generally quite different.

3. *Normal telephone transmission* is that which uses directly the ordinary frequencies of speech and music. In carrier current transmission, a modulation is employed in order to permit using a different range of frequencies for transmission purposes. See section 4.

A phantom circuit is one which straddles two pairs of wires, one pair acting as a conductor in either direction. In this case, each pair is also used for transmission in which one wire of the pair works in one direction and one in the other. Each of the pairs is said to comprise a side circuit and to afford side circuit transmission.

4. *A repeatered circuit* is one which contains repeaters providing amplification for the purpose of overcoming line losses.

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on the "Central Transcontinental Route" and that through Dallas, El Paso, and Phoenix is on the "Southern Transcontinental Route." The economic importance of such circuits is evident.

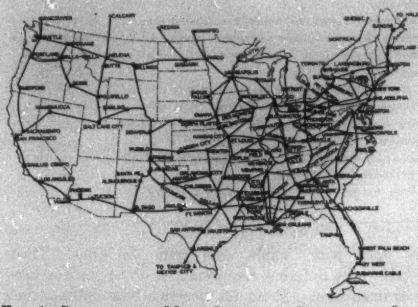

FIG. 1—SOME OF THE MORE IMPORTANT REPEATERED OPEN WIRE CIRCUITS IN THIS COUNTRY

The length of trunk line circuits, together with the high standards of transmission they require, and the desirability of employing them intensively by multiplexing messages, means careful consideration of the fundamental factors influencing transmission and of the numerous functional objectives which component apparatus must attain. This is particularly so if the service provided is to have universal standards, regularity of operation, and low costs of maintenance and operation.

Taking, at a repeater station on one of the routes plotted in Figs. 1 and 2, one side circuit of a pair of intensively occupied phantomed circuits, and schematically outlining its possible transmission channels as in Fig. 3, something may be seen of the orderly functional arrangements of apparatus required. It is not to be supposed that all long circuits provide this number of kinds of message facilities. But on the other hand, they must be considered as potentially capable of being so occupied.

With this situation in mind, we may consider in some detail just what the manifold functions of typical networks are and may illustrate them concretely in a typical system. The illustration will be that of a three-channel (*i. e.*, three-conversation) carrier telephone system. The functions necessarily divide themselves according as they have to do (1) with the primary purposes of employing the networks, and (2) with the conditions to be met by networks as integral parts of systems, working harmoniously with other apparatus. As a prerequisite to such a discussion we shall start with a survey of the ideals of signal transmission, and as a logical closure we shall indicate the directions in which physical limitations are to be found in the construction of networks to have prescribed characteristics.

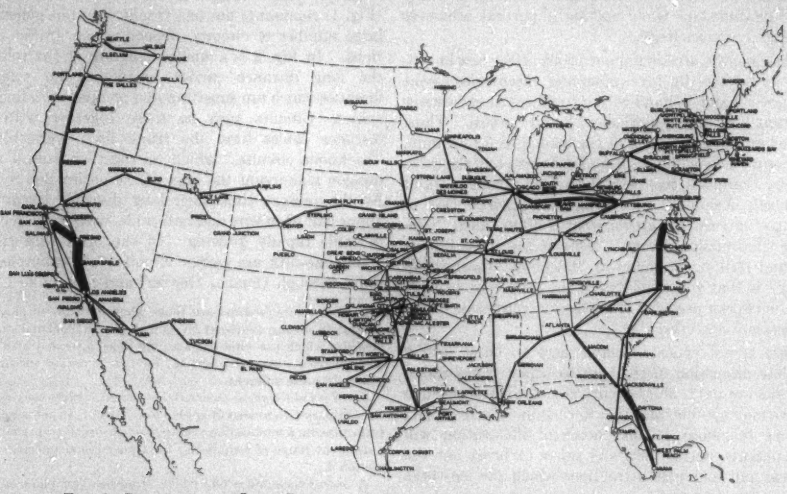

FIG. 2—ROUTE OF THE LONG DISTANCE TELEPHONE AND TELEGRAPH FACILITIES WHICH ARE SUPERIMPOSED ON LOCAL WIRE LINE AND CABLE

The remainder of this paper will therefore be devoted to: (1) signals and their transmission, (2) primary functions of transmission networks, (3) secondary

functions imposed on networks by the necessity of coordinating parts of systems, (4) a specific illustration of their use in a three-channel carrier telephone system, and (5) engineering limitations on network design and construction.

Only the more important types of transmission networks will be mentioned under (2). These are: *wave filters, equalizers, telephone transformers, line balancing (i. e., simulating) networks,* and *artificial lines.* Under (4) the use of but the first two of these types will be shown.

I. Signals and Their Transmission

Signals, whether of systems of telephony (carrying speech and music), telegraphy, or telephotography, have qualities of wave composition which differentiate them from other electrical waves. These qualities determined largely the performance required of networks which aid in their transmission, and thus provide

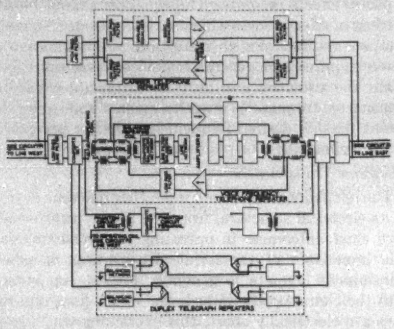

Fig. 3—The Possible Transmission Channels at a Repeater Station

much of the physical basis of transmission network design methods. It is obvious from the purpose and circumstances of signals that they are essentially of a *temporary* character. They are used at a particular time to convey information and they cease as soon as their message has been transmitted. We may consider as pertinent examples of this: articulate speech, music, and telegraph signals.

Consequently, even the most elementary of signals, such as a "single-frequency tone," maintains an approximate steady state merely for a limited period. Such elementary signals can be used to convey only a very small number of different ideas. For the free interchange of ideas through signals we must employ either a complex code of artificial groups of signal elements to represent the characters of a written language, or the much more complex signal elements (vowels, consonants, and notes) required in spoken language and music.[5] In the case of telegraphic signals, because of economic considerations, the duration of and time between pulses is made so short that the transient state predominates, so that the distinguishing characteristics are largely transient in character. In the case of speech and music, owing to changes in expression much of the distinguishing characteristics are also transient in character. To put this in another way, the envelopes of the waves which *modulate with time* the characteristic frequencies which the sounds, if sustained, would tend to possess, have an important bearing on the meaning conveyed by the signal. Preservation of these envelopes is a matter of transient state transmission. But transient states really involve the transmission of steady-state frequencies grouped as *continuous bands* along the frequency scale. *Preservation of the entire content of a signal therefore involves consideration of the transient properties of networks, i. e., involves their steady-state properties over a somewhat wide frequency range.*

The initial and final decay periods of a sound would clearly arise from the exigencies of time,—from the temporary character of sounds and the sequential arrangement of syllables in speech and notes in music. They are, however, attended by psychological peculiarities which have a bearing not only on the superficial meaning of the signals but on the recognition of identity of the speaker (*naturalness* includes this) and on shades or variations of meaning to express ideas pointedly. These peculiarities lead to considerable differences in wave form among examples of the same nominal sound.

In spite of this and of the variations which exist between voices of different persons, it is found possible to evaluate experimentally the relative importance of different frequency ranges in the transmission of sounds and to determine what imperfections of signal waves are permissible, in the aggregate, in transmission. This is usually done by removing, with electrical wave filters, various frequency ranges from a transmission system capable of transmitting the entire audible frequency range. Not only is the importance for the interpretation of speech sounds determined but also the importance for conveying the more subtle and psychological aspects of both speech and music.

The imperfections which may creep into the transmission of signals may be classified under three headings: improper signal *volume, distortion* of signal content, and *interference* by waves foreign to the signals. That is, if waves are received of appropriate intensity, if the important frequency ranges are uniformly

5. The signals corresponding to pictures resolved for transmission purposes are somewhat similar in form to telephone and telegraph signals. On the other hand, depending on the speed at which they are sent, they may require the use of a frequency range greater even than that of speech or music.

In connection with this section see "Transmission of Information" by R. V. L. Hartley in the *Bell System Tech. Journal*, July, 1928.

transmitted, and if nothing but the desired waves are received, the ideals of transmission will have been realized. Waves may be so strong as to cause physiological difficulties in reception, or so weak as not to be recognized. This means that efficiency of transmission is an important factor although, when amplifiers are used, partly an economic one. Signal distortion is of two kinds: *amplitude distortion* and *phase distortion*. Amplitude distortion arises from the transmission of different frequencies with unequal efficiencies and is highly important in all types of signals, but its aggregate importance depends on the importance of the frequency components that are efficiently transmitted. Phase distortion results from different frequencies traveling with different velocities such that their relative arrival times differ from their relative starting times. It is ordinarily less important in the transmission of speech than in other kinds, but the accumulation of phase distortion which may occur on long telephone circuits gives rise to undesirable transient effects. Interference may be of two kinds: It may arise from the introduction of energy into the communication circuit from outside, through electrostatic or electromagnetic coupling, or it may result from the imperfect separation of several simultaneously transmitted signals or from the generation of extraneous frequency components through modulation.

These three types of imperfections should be kept in mind in considering the functions of different types of transmission networks.

II. Principal Functions of Transmission Networks

The principal functions of the more important types of networks may be described as follows:

Wave Filters

It is characteristic of telephonic signals that their transmission requires the use of a continuous band of frequencies whose width may be perhaps 2000 cycles as a minimum and 6000 cycles as a maximum (both approximate). Where, as an example, due to economic factors it is desirable to transmit simultaneously two or more telephonic frequency bands, whether this be done entirely conductively along wires, or partly by radiation through space, discriminating means are required to make the transmission of a particular communication channel efficient to that band, or those bands, of frequencies which are desired, while rendering the circuit highly inefficient at other frequencies. To put this in another way, means are required which will *pass freely* desired bands of frequencies, while *highly attenuating* or extinguishing neighboring undesired bands of frequencies. This is the essential function which the wave filter serves. The frequencies attenuated may, however, be either true signals or unintelligible noises and as a rule comprise both. On the other hand, should disturbing frequencies lie within the frequency band of the desired signals in the same circuit, the two groups cannot be separated by filtering action.

Equalizers

In any telephonic signal which is to be transmitted efficiently, proper audition requires that all component frequencies be treated alike, in so far as efficiency of transmission is concerned, in order that some frequencies may not be unduly emphasized to the detriment of others. This uniformity of efficiency of a band of frequencies is required, of course, primarily as a characteristic of the overall system traversed by the signal. Some parts of a transmission system are inherently not capable of closely uniform transmission. Wave filters, for example, tend to discriminate against frequencies near the edges of their transmission bands due to dissipation of energy in the effective resistances of their elements and to reflection losses at their terminal junctions. Loaded telephone lines behave similarly and unloaded lines have an attenuation which rises gradually with frequency because of the nature of the capacity effects present. The equalizer cannot restore the loss of effectiveness which occurs to some frequency components due to such distortion. Amplification of power requires such generative devices as vacuum tubes. What the equalizer can do is to attenuate efficiently transmitted frequencies in such a manner and to such an extent that all desired frequencies suffer or prosper in transmission alike.

Telephone Transformers

The telephone transformer is a rather different device, in its electrical structure, from the power transformer. Two chief differences in operating requirements exist. The impedance of the terminal apparatus of power transmission circuits is generally low in comparison with the characteristic impedance[6] of the lines and the lines are electrically short, this explaining why "stepping up" voltages on power transmission systems usually leads to increased efficiency. In telephone circuits, on the other hand, the impedance of the terminal apparatus is comparable in magnitude to the characteristic impedance of the lines, and the lines are electrically long. Efficiency of transmission is under the latter conditions governed largely by "matching of impedances" at junction points in a circuit. Consequently the effectiveness of telephone transformers in properly transforming the impedances of a circuit is of primary importance. Secondly, because of the relatively high frequencies employed in telephone work and because a rather wide band of frequencies needs to be transmitted, inherent internal impedances, such as those of winding capacities, become very important and require that the telephone transformer be considered

6. The *characteristic impedance* of a line is one to which the impedance of connecting apparatus should be approximately matched for maximum power output conditions. It is principally determined by the distributed inductance and capacity of the line.

primarily as a network. In the case of transformers working into the very high input impedances of vacuum tubes or providing band transmission at high frequencies, we may say that the frequency range of transmission and the transformation ratio of telephone transformers is as completely dependent on the magnitude of inductance and capacity elements as are the characteristics of wave filters.

Line Balancing Networks

The two-way repeater circuit has essentially the form of an a-c. bridge, in which a balanced condition depends both on the similarity of certain transformer (hybrid coil) windings and upon the equality of impedance of the two halves of the repeater output circuit. In the case of the 21-type (two way, one amplifier) repeater, the two halves are the telephone lines joined by the repeater. In the case of the more stable 22-type (two way, two amplifier) repeater, an output circuit exists at each end of the repeater and each telephone line is matched in impedance against a "dummy" line, or balancing network. The function of this network is solely to present the same impedance to the repeater over a range of frequencies as is presented by the line, *i. e.*, to simulate the impedance of the line. No *through* transmission is required of it. The design of such networks involves a choice of configurations of resistances, capacities, and inductances which bear a close physical relationship to such impedance elements of the line as dominate its total impedance, and a determination of impedance values of the network elements in accordance with network theory.

Artificial Lines

The function of artificial lines is to exhibit some transmission property or properties of a real line, at one frequency or over a range of frequencies, so that there may be compactly constructed a network which will serve for certain purposes involving wave propagation, in place of a real continuous line. The artificial line is generally a network of lumped, discrete impedance elements whose constants are computed from network theory. One of the most common forms of artificial line is the variable *attenuator*, which is used as a basis of comparison in attenuation and transmission loss measurements. It is usually constructed of resistances so that over a wide range of frequencies it displays uniform attenuation. Other forms of artificial line, however, display over a range of frequencies the varying attenuation (and perhaps phase) properties of real lines and circuits and involve properly disposed inductance, capacity, and resistance units. In contrast to the equalizer, they simulate, rather than compensate for, frequency-transmission characteristics.

III. Secondary Functions of Transmission Networks

The functions of transmission networks outlined in section 2 show the ordinary reasons for their employment in communication systems. When so employed, however, since they must fit into the transmission scheme of systems and work harmoniously with other apparatus, they may be faced with numerous additional requirements. Occasionally, the latter duties may even provide the more difficult design requirements. The more important of these possible secondary requirements follow:

Efficient Transmission

This requirement applies particularly to transformers, wave filters, and equalizers. Each of these types of apparatus could fulfill its principal duties while consuming an *undue portion* of the strength of signals entrusted to it. Under some conditions, amplification of signals by the use of vacuum tubes could offset this consumption from a transmission standpoint at an economic sacrifice and the problem would then be one of economic balance; but in other cases the amplification would be objectionable because accompanied by such factors as amplification of noise, impoverishment of circuit balance, and loss of circuit flexibility. Efficient transmission is secured by proportioning materials so as to limit dissipation of energy.

Low Distortion

It is not possible, of course, to secure entirely uniform transmission of all signal frequencies, and indeed refinements of transmission beyond a certain point have in this direction economic limits. To the extent, however, that distortion exists, the quality of signals is impaired. When distortion is excessive, equalizers often provide a desirable way of reducing it, but do so at a sacrifice in over-all circuit efficiency.

Impedance Balance

There is a number of kinds of impedance balance commonly found in communication circuits. All of them are akin to balances in a-c. bridge circuits. They need to be maintained over bands of frequencies. Balancing arrangements offer a kind of selectivity which differs from filter selectivity in that, in general, it keeps apart throughout systems two trains of waves arising from different sources and drawn off to different destinations, instead of separating on a basis of frequency difference waves which have mingled in a common circuit. An example is the balance necessary in a side-circuit, and therefore imposed on transformers or filters located in the latter, in order that currents traveling a superimposed phantom circuit may not interfere with side-circuit transmission. A second kind of balance is the repeater balance, mentioned in section 2, which may lead to requirements on, say, filters located in the output circuits. Here the separation is between incoming and outgoing speech signals using the same frequency range. In other cases, circuit balance is relied upon to prevent interference currents set up between circuit wires and ground from entering the circuit of the former. To obtain a high degree of balance, apparatus located in a balanced circuit must be quite symmetrical in its circuit impedances with respect to the associated circuit from which it is protected by balancing.

Stoppage of Longitudinal Currents

Longitudinal currents are those which travel both sides of a circuit in the same direction as parallel paths and return by some other path, generally a ground circuit. They are objectionable because (1) they tend to enter the transmission circuit whose sides they travel, in this case called a transverse circuit, through irregularities of balance in the latter and to unite with transverse currents; and (2) in many cases they correspond to objectionable voltages of considerable strength. They may be eliminated in two ways: (1) by providing for them a short-circuit path to ground in the longitudinal circuit and (2) by causing virtually an open-circuit gap to occur in the longitudinal circuit, each of the methods being so employed as not to affect transmission in the transverse circuit. Transformers are required by either method.

Transmission in Closely Associated Circuits

When the same wires are used to provide both side circuit and phantom circuit transmission, it is a requirement of apparatus (such as transformers and filters) inserted in side circuits that they shall not impair transmission in the phantom circuit, and vice-versa. This means that apparatus in one circuit must not insert impedance in another circuit and requires close coupling between those impedance elements of networks which are located in series with the line wires.

Stability of Characteristics with Current Variations

If the inductances, resistances, or capacities of a network are subject to variations in value with varying current strength, as is the case, for example, with the inductance and resistance of coils using unstable core materials, two detrimental results are possible. First, the frequency characteristics of the network will be distorted as the current varies from its mean value. Second, a kind of modulation will occur, resulting in the generation of new frequencies whose presence will interfere with interpretation of signal waves traversing the network. Stability of characteristics is, obviously, to be secured by the use of materials whose permeability, resistance, and dielectric constant do not vary with current strength.

Low External Coupling

It is desirable that electrostatic and electromagnetic coupling between a network and parts of its own or other circuits be kept low, that is, that the network be self-contained electrically and receive and supply energy only through its normal input and output terminals. When coupling exists to some other circuit, (1) energy introduced from the latter circuit may interfere with signals or (2) energy derived from the network may cause interference in the circuit to which it is coupled. When coupling exists to other parts of the circuit in which the network is located, (1) impedance balances may be upset, (2) large attenuations, as of a filter, may be nullified by bypass circuits, and (3) circulating currents of considerable strength may be set up when vacuum tubes are involved in the circuit. Electrostatic and electromagnetic shielding permits control of external coupling.

Low Reflection Coefficient

Where networks connect directly or through other transmitting networks to a line, it is desirable that the impedance offered by the network to line currents be such that wave reflections are not set up in the line by impedance mismatches at its ends. Reflections result not only in undesirable transient waves in the circuit in which they originate but cause interference into other circuits and complicate line transposition problems. There is thus usually prescribed for a network an impedance characteristic.

Modulator Circuit Impedances

When networks connect to vacuum tube modulator (or demodulator) circuits, they are required to have certain types of impedance characteristics even in the non-transmitting range of the networks, in order that modulation may be as complete as possible and yet undistorted. In certain modulator circuits the networks must present a high, and in other cases a low, impedance to the modulator if certain frequencies which the tubes tend to supply are to be eliminated.

Parallel or Series Operation

It is a requirement particularly of groups of wave filters selecting signals for a number of communication channels that the impedance of each in the transmitting ranges of the others be such as not to interfere with the operation of the latter. In the case of filters operated in parallel, this means a high shunting impedance; in the case of filters operated in series, a low series impedance. It is possible to meet this requirement because in general numerous configurations of filter elements can be arranged which will meet given transmission requirements but allow wide latitude of choice of impedance characteristics.

A number of considerations should be noted with respect to the above requirements. First, in any given circuit conditions, the requirements can be expressed in numerical measure so that networks can be designed to meet them quantitatively and measured accordingly when constructed. Secondly, the various requirements which might be imposed on a network by circuit conditions, tend for the most part to be conflicting, since they restrict choices of design, and must be evaluated and balanced from an economic standpoint. Finally, the ability of a network to meet the above requirements is subject to physical limitations which will be discussed in section 5.

IV. Illustration of the Use of Transmission Networks in a Communication System

Filters and Equalizers Required in a Typical Three-Channel Carrier Telephone System

As an example of the way in which a number of functional requirements may be simultaneously im-

posed in practise on networks used in communication systems, let us consider the more important of the filters and equalizers which enter into the scheme of operation of a typical three-channel carrier telephone system. The example chosen is that of a "Type C" carrier system, designed for use over relatively long distances and employing the so-called[8] "suppressed carrier, single sideband" method of transmission. It represents, of course, but one set of possible circuit conditions and the numerical constants or characteristic curves which give the required performance of the networks would be greatly influenced by such factors as: grade of transmission desired, gage of wire available, economical distance between repeater stations, and interference conditions. Hence the example should be considered to illustrate principles rather than to define practise closely.

In Fig. 4 are outlined schematically the terminals and one of the repeaters of a Type C system, so drawn as to emphasize the filters and equalizers used therein. The upper half of the diagram relates to transmission from left to right; the lower half to transmission in the opposite direction (and to non-carrier facilities). Beginning at the left-hand end of the diagram, currents derived from the subscriber's line S and traveling to the right, pass through balancing apparatus H which separates on a directional basis incoming and outgoing currents of the carrier system, and thence into a modulator M which raises them in the frequency scale. For each channel the desired sideband is selected by a modulator band filter (for the channel whose modulator is shown, band filter No. 5) and is sent into an amplifier A common to all three channels. Here the strength of the signals is increased so that even the attenuation of a considerable stretch of line may leave them strong enough to override disturbances. The "high pass" directional filter, arranged so as to pass currents in the upper half of the frequency range used by the system, transmits the signals to a high pass "line filter" and this in turn delivers them to the line.

Upon arrival (in an attenuated condition) at a repeater station, the carrier currents are conducted through a high pass line filter to the repeater proper. A separation of high and low frequency carrier currents, corresponding respectively to the two directions of transmission, is effected by directional filters; consequently, currents arriving from the left go through the "input" high pass directional filter, reaching equalizer E_1. The purpose of the latter is to correct for the signal distortion introduced by the line and by directional filters. An amplifier A, next in order, raises the level of strength of the signals of all three channels. Finally, an "output" high pass directional filter leads the amplified signal currents to a high pass line filter and thence to the next stretch of line over which they are to travel, This process may be repeated a number of times, depending on the total distance spanned by the system.

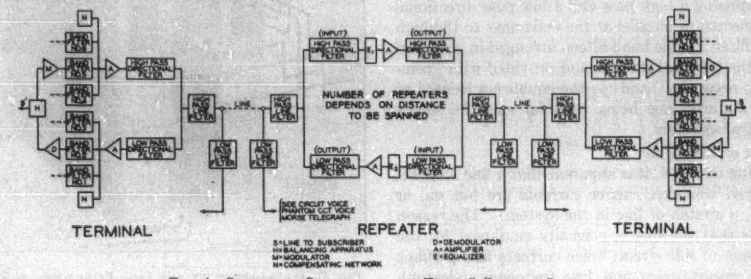

FIG. 4—SCHEMATIC OUTLINE OF THE TYPE C CARRIER SYSTEM

When the signal currents reach the receiving terminal on the right, they are lead successively through a high pass line filter and a high pass directional filter to a terminal amplifier A where the channel currents are strengthened prior to demodulation. Demodulator band filters (No. 5 for the channel shown) select for each channel the desired group of frequencies and reject the others so that the demodulator D may not be hindered in its operation. The latter restores the channel currents to their original position in the frequency scale and delivers them through the balancing apparatus H to the receiving subscriber's line S.

8. The reader is doubtless familiar with the fact that in carrier modulation, as in radio systems, a "carrier frequency" and "two sidebands" are produced. For demodulation and recognition of signals the carrier frequency and one sideband may be suppressed, the other sideband being transmitted and a local carrier frequency supplied at the receiving end of the system. By this method a smaller frequency range is required for communication. For a discussion of methods of operation of carrier systems, reference may be made to *Carrier Current Telephony and Telegraphy* by E. H. Colpitts and O. B. Blackwell, A. I. E. E. TRANS., 1921, which discusses generally the progress which had been made in carrier systems up to that time; also to various later papers which may be found in the *Bell System Tech. Journal*, such as that by H. A. Affel, C. S. Demarest, and C. W. Green, July, 1928, entitled "Carrier Systems on Long Distance Telephone Lines."

Transmission in the reverse direction is accomplished in a similar fashion, except that low pass directional filters, transmitting currents in the lower half of the frequency range of the carrier system, replace the high pass directional filters of the upper half of the diagram.

We may discuss in order the duties of four groups of

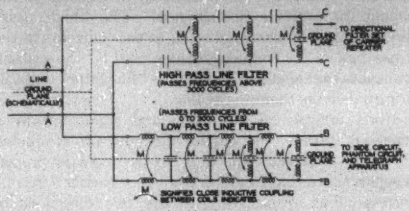

FIG. 5—SCHEMATIC ARRANGEMENT OF A LINE FILTER SET

apparatus: (1) the "line filter sets," each comprising a high pass and a low pass line filter connected in parallel at the ends next to the line, (2) the directional filter sets, each comprising a high pass and a low pass directional filter connected in parallel at the ends next to the high pass line filter, (3) the band filters, arranged in groups of three at the terminal stations and provided with "compensating networks," and (4) the equalizers located at repeater stations, one being used for each frequency range of transmission.

Functions of Line Filter Set

Referring to Fig. 4, it is apparent that a line filter set is employed wherever carrier currents are put on, or taken off, a stretch of line in the system. The reason for this is that the line is normally employed for the transmission of side circuit voice currents and perhaps phantom circuit voice and low frequency telegraph currents as well, and the line filter set provides the apparatus by means of which the carrier currents may be superimposed on the same line without interference.

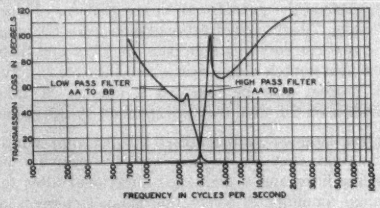

FIG. 6—TRANSMISSION LOSS CURVE CORRESPONDING TO THE LINE FILTER SET OF FIGS. 5 AND 7

The superimposition is accomplished in two steps: first, a low pass filter inserted in the circuit of the normal facilities permits passage of the normal currents but is designed to reject from the normal circuit high-frequency carrier currents; second, a high pass filter transmitting carrier currents is inserted in the carrier circuit so designed as to reject low-frequency currents from the normal facilities. A schematic arrangement of a line filter set is shown in Fig. 5 and the transmission loss curves corresponding to it in Fig. 6. A photograph of a typical line filter set is given in Fig. 7. Fig. 6 shows the reduction in strength suffered by currents because of the presence of the filters between $A\,A$ and $B\,B$, and between $A\,A$ and $C\,C$, respectively. This reduction (called transmission loss) is plotted in *decibels*[9] ($d\,b$) which are logarithmic in character but the corresponding current ratio values are shown by the ordinates at the right of the figure.

We may now observe:

(1) That if telegraph facilities are maintained, the low-pass filter must yield desired telephone current characteristics while carrying substantial telegraph currents.[10]

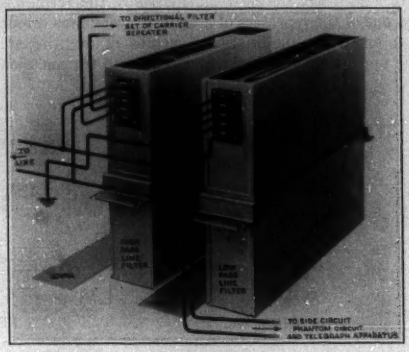

FIG. 7—ILLUSTRATION OF THE LINE FILTER SET OF FIG. 5

(2) That if a phantom circuit is maintained, in one side circuit of which the low pass filter would then be located, the latter must be well balanced with respect to similar apparatus provided in the other side circuit (see Fig. 3) and such close coupling exist between its series impedances that only a small amount of impedance is introduced into the phantom circuit (see Fig. 8).

(3) That the low pass filter must act as a high impedance shunt in the transmitting range of the high pass filter, and that, correspondingly, the high pass filter

9. Current, voltage, or power (magnitude) ratios may be expressed logarithmically in decibels ($d\,b$). The number of $d\,b$ corresponding to a ratio of the magnitudes of two currents I_1 and I_2 is $N_{db} = 20 \log_{10} \left| \frac{I_2}{I_1} \right|$. Voltage ratios are expressed similarly. For two powers P_1 and P_2, however, $N_{db} = 10 \log_{10} \left| \frac{P_2}{P_4} \right|$.

10. The location of the telegraph composite set in Fig. 3 has been changed to eliminate some of these difficulties, so that *the standard location of this composite set is now on the line side of the low pass line filter.*

must act as a high impedance shunt in the transmitting range of the low pass filter, *i. e.*, that the filters must be mutually designed for parallel operation. Impedance characteristics illustrative of this point are given in Fig. 9.

(4) That disturbances of considerable potential, such as those which may occur when telephone lines are exposed to power transmission lines, find the line filter set as the first networks which they would traverse in the system; and the line filter set must have sufficient

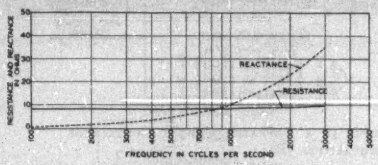

FIG. 8—IMPEDANCE INTRODUCED INTO THE PHANTOM CIRCUIT BY A LOW-PASS FILTER

dielectric strength to withstand potentials not taken care of by standard protection apparatus.

(5) That a good degree of balance, with respect to ground *G*, between the impedances in the respective

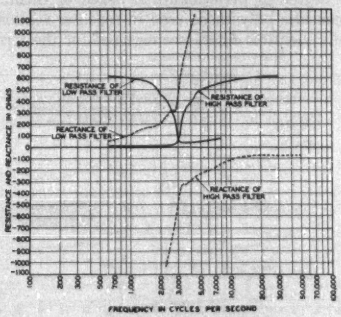

FIG. 9—IMPEDANCES OF LOW- AND HIGH-PASS FILTERS DESIGNED FOR PARALLEL OPERATION

halves of the filters must be provided in order to prevent longitudinal currents from entering the carrier system or the normal circuit.

(6) That only a very small amount of transmission loss can be allowed in the transmission range of each filter, because of the number of line filter sets which currents encounter in a system of considerable length.

(7) That only very low distortion is permissible in the transmission bands, for the reason given under (6).

(8) That, because of the rather rapid changes in the transmission loss characteristics in the vicinity of 3000 cycles, precision of location of this frequency as a separational point requires that the filter characteristics be independent of current values.

(9) That from a standpoint of wave reflections at the end of long stretches of line, the line filter set is in a prominent position and the impedance which it presents to the line (this of course depends upon the other apparatus as well as upon the line filter set) must match

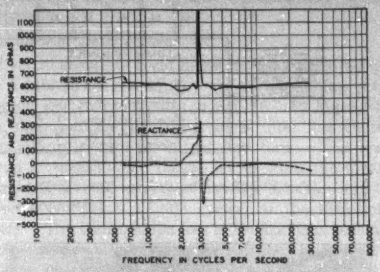

FIG. 10—IMPEDANCE OF A LONG TELEPHONE LINE

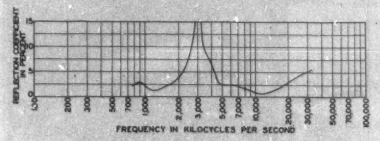

FIG. 11—THE DIFFERENCE BETWEEN THE FILTER AND LINE EXPRESSED AS A REFLECTION COEFFICIENT

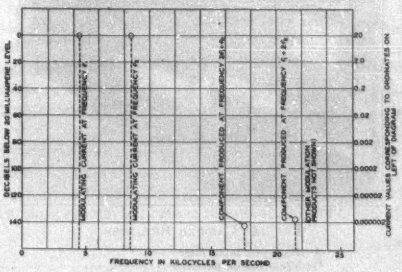

FIG. 12—ILLUSTRATING HOW MODULATION MAY BE GENERATED IN THE LOW-PASS FILTER WHICH WILL PENETRATE THE HIGH-PASS FILTER

the impedance of the line, as illustrated in Fig. 10. The difference between the filter and line impedances of Fig. 10 is expressed as a percentage "reflection coefficient" in Fig. 11 to indicate the portion of arriving waves which will be reflected.

(10) That modulation occurring due to unstable filter elements can give rise to: (a) high frequencies generated in the low pass filter which will be transmitted

as interference by the high pass filter, and (b) low frequencies generated by intermodulation of carrier frequencies in the high pass filter which will penetrate the low pass filter; thus that an additional requirement exists on the stability of the filter characteristics. Fig. 12 shows how modulation current may be generated in the low pass filter which will penetrate the high pass filter.

(11) That a requirement will exist on the degree of interference permissible between the filters of each set and other apparatus of the same or other systems located at the same station. (In this connection, for example, note from Fig. 4 that interference between high pass line filters at the opposite ends of a repeater provides a return path for amplified currents and tends to cause circulating currents or even "singing".)

(12) That when at repeater points a normal voice repeater exists in the circuit in which the low pass filter is placed, an impedance balance will be required between the line filter set and the corresponding balancing apparatus in the repeater.

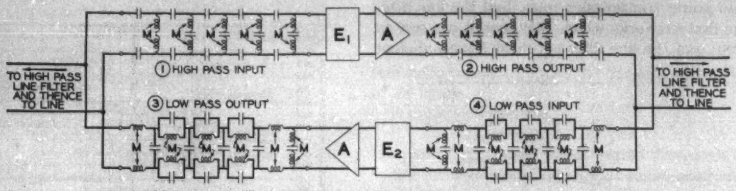

Fig. 13—Configuration of Typical Directional Filters in a Repeater Circuit Containing Amplifiers (A) and Equalizers (E)

Fig 14—Carrier Repeater

A—High-pass input filter
B—High-pass output filter
C—Low-pass input filter
D—Low-pass output filter
E—Equalizer for upper frequency group
F—Equalizer for lower frequency group
G—Amplifier for upper frequency group
H—Amplifier for lower frequency group
I, J, and K—Auxiliary apparatus

Functions of Direction Filter Sets

The functions of directional filters, whether located at terminal or repeater stations, are less numerous in kind than those of the line filters, but a generally more severe degree of performance is asked of them. We need consider only those used in repeater stations, as virtually identical filters are required for the terminal points. Fig. 13 shows the configuration of typical directional filters as employed in a repeater circuit, and Fig. 14 is a photograph of a carrier repeater which includes directional filter sets. The input and output filters are the same in form but differ slightly in values of the network constants.

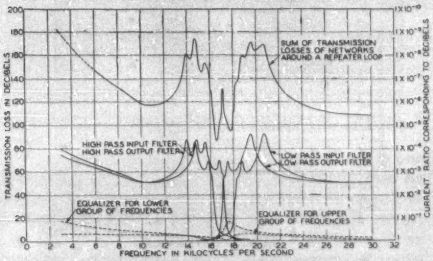

Fig. 15—Transmission Loss Characteristics of the Four Directional Filters Used in a Repeater Circuit. Equalizer Characteristics are Shown in Dotted Form

Transmission loss characteristics of the four directional filters are given in Fig. 15. The sharpness of selectivity displayed may be gaged by comparison with the characteristics of Fig. 6 for the line filters, it being remembered that on a frequency percentage basis a 1000-cycle interval at 17 kilocycles would correspond to

about 175 cycles at 3 kilocycles. From an economic standpoint, of course, it is the *actual* width of frequency interval given up for separational purposes which is of importance, but from a design standpoint *percentage* width is a measure of difficulty.

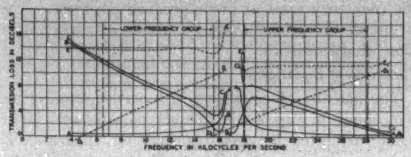

FIG. 16—CHARACTERISTIC OF TYPICAL NETWORK STRUCTURES OF FIG. 20

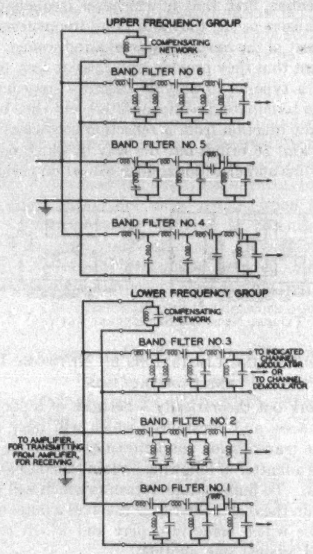

FIG. 17—TYPICAL GROUPS OF BAND FILTERS FOR THE SYSTEM OF FIG. 4

The functions of the directional filters will not be examined in the manner of the line filters, because of lack of space. However, among the most important requirements are those on modulation, interference, distortion, and the combined filter transmission losses around the repeater circuit (safeguarding against "singing").

Functions of Band Filters

Typical groups of band filters for the system of Fig. 4 are exhibited in Fig. 17 and the corresponding transmission loss characteristics in Fig. 18. In Fig. 18 the line filter and terminal directional filter loss characteristics are shown in dotted line form. The lower group of transmission bands lies between the separational point (3000 cycles) of the line filter set and the separational point (17,000 cycles) of the directional filters. The transmission loss in the bands is generally higher than for the directional and line filters, for two reasons; channel currents traverse but two band filters in a system and the loss may be afforded; also it is natural for the loss in filters having fairly narrow transmission bands to be higher than in low pass and high pass filters.

We shall discuss four additional points in connection with the functions of these filters, out of the many which might be selected:

(1) Their parallel operation is aided by certain "compensating networks." In order that each filter may be least affected in its transmission band by the shunting effect of the others, each filter is built out with a special termination at the paralleling end (as if to "load" away the effect of the other filters). Such building-out causes distortion at those *edges* of transmission bands which have no *neighboring* transmission bands in the frequency scale (for example, near 6.5, 15, 18, and 28 kilocycles). To overcome such distortion, the compensating networks are given the impedance values which would be offered by such absent filters in the series.

(2) The band filters are not balanced with respect to a ground system but rather represent only one-half of a

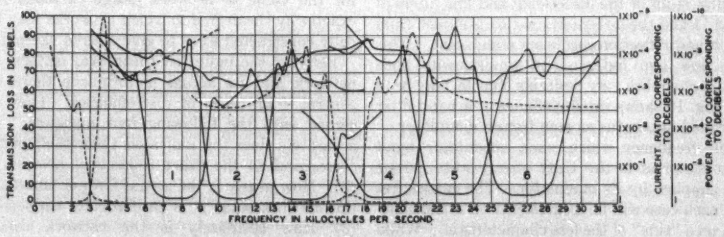

FIG. 18—TRANSMISSION LOSS CHARACTERISTICS OF BAND FILTERS SHOWN IN FIG. 17. LINE FILTER AND TERMINAL DIRECTIONAL FILTER CHARACTERISTICS ARE DOTTED

balanced circuit (a form of "unbalanced" circuit) and a common side of each is maintained at ground potential. This type of circuit is more economical, where it can be employed.

(3) The band filters all possess, at the end remote from paralleling, the same type of termination, *i. e.*, there is in each case an anti-resonant arm, in shunt across the filter. These terminations for band filters correspond to a class of band filter sections which offer desirable impedances in certain frequency ranges to the modulators or demodulators with which, at one end of the system, they work. They do not, however, result in the *least distortion* which might be had in the transmitted bands if modulator and demodulator action had

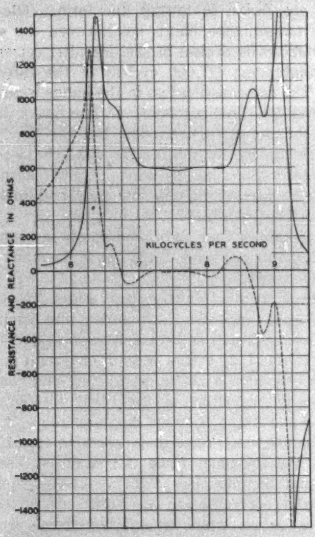

Fig. 19—The Impedance of a Band Filter with Associated Apparatus as Seen from a Modulator

not to be considered. A different type of termination will be found in all of the directional and line filters of Figs. 5 and 13 and the latter termination is chosen from a distortion and reflection coefficient standpoint. This illustrates how two independent requirements both intended to accomplish very desirable objects, may be in conflict. Fig. 19 shows the impedance of a band filter with associated apparatus as seen from a modulator.

(4) One frequency which each modulator band filter must suppress is the truly steady-state carrier frequency supplied by its modulator. The carrier frequency in each case will be located just off a transmission band, on a "side" of the loss characteristic. While the voice currents represented by the transmitted sideband may, from instant to instant, change in amplitude and frequency content, the carrier frequency is fixed in size and frequency location. For its suppression without undue distortion of the wanted sideband, the various transmission bands must be very accurately located in the frequency scale at predetermined positions. This is one of the predominating features of carrier band filter characteristics.

Functions of Equalizers

The position of equalizers in a carrier repeater has been shown in Figs. 4 and 13 and typical network structures, connected between transmitting apparatus Z_S and receiving apparatus Z_R, are illustrated in Fig. 20. Loss characteristics are shown in Figs. 15 and 16. We shall merely add to the previous discussion of carrier repeaters, first, that in whichever frequency group the equalizers occur, they add at all frequencies to the "loop loss" of the repeater networks and second, that to the extent that they provide transmission loss, they are of such a type as to provide desirable impedances in which to terminate directional filters and thus have an important function from a reflection coefficient standpoint. Fig. 16 brings out the way in which equalization for line and directional filter distortion occurs.

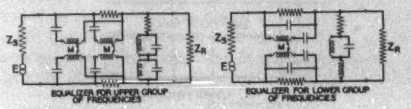

Fig. 20—Typical Network Structures Between Z_S and Z_R

V. Engineering Limitations on Network Design and Construction

To work out theoretically a network of inductances, capacitances, and resistances which will offer certain desirable transmission characteristics over a frequency range, is a matter of following certain theoretical design methods. To build actual networks which will possess and retain these characteristics involves a large number of factors which come into play and which must be balanced against one another.

Accordingly, it is necessary to point out the principal directions in which limitations are to be encountered, for the value of network design methods is limited exactly by the degree to which networks may be physically constructed so as to give desired characteristics.

Between the indicated theoretical performance of a network and its actual performance there enter, in general, four types of discrepancies. They are concerned with the following four questions: (1) How accurately does the indicated theoretical performance correspond to the exact theoretical network chosen? (2) How nearly is the actual form or configuration of the network what it is theoretically supposed to be? (3) How accurately is the network constructed? (4) How stable are the characteristics of the network during operation?

There are two sources of error which affect the theoretical exactness of the computed performance of a network; both have to do with the approximations which are necessary when mechanical aids are used in computation. One lies in computations of the network constants (L's, C's, and R's) from chosen significant frequencies, impedances, or other design bases, and the other in determinations of the characteristics themselves either from the network constants, or from the bases referred to. Discrepancies of either kind are not, when reasonable care is used, ordinarily large enough to be important except in the case of apparatus which needs to be made very accurately.

The form or configuration of a network introduces four important factors leading to discrepancies, *viz.*, (1) interactions between network elements, (2) distributed impedance effects in the elements themselves, (3) admittances from elements to ground, and (4) effects of the wiring system.

Interactions between Network Elements

These arise from the difficulty of confining electrical effects within exact boundaries. Electrostatic and electromagnetic coupling between various pairs of elements give the effect of inserting additional elements in the network. Such effects are most serious when coupling exists between elements differing in potential (relatively) by a considerable amount. An example is the tendency of capacitive coupling between elements at opposite ends of a network of considerable transmission loss to by-pass currents around the network and thus to limit that loss.

Distributed Impedance Effects in Network Elements

It is simplifying from a mathematical standpoint to deal with definite lumped inductances, capacitances, and resistances. But it is only proper to realize that lumped constants do not occur strictly in nature. Rather it is the tendency of circuits, except when specially formed and even then to some extent, to spread out in such a manner as to show distributed impedance effects. Thus the effective impedances of network elements are formed by a summation of differential effects not all of the same kind. What is done, for example, in the design of inductance coils is merely to bring out, or emphasize, inductance in comparison to resistance and capacity, for a certain frequency range and to a first approximation. The character of the impedance, then, must necessarily change with frequency. A coil of wire may be in one frequency range nearly a pure resistance, in another an inductance, and in a third capacity may predominate. By a choice of types of construction, with a view of course to cost, these changes with frequency may be minimized but cannot wholly be eliminated.

Admittances from Elements to Ground

Although interactions between network elements may be reduced by shielding, it should be appreciated that the presence of shields tends to cause admittances to ground from different parts of a network to increase. Even when the best shield has been used there is not, exactly speaking, such a thing as a two-terminal impedance (*i. e.*, one in which current enters and leaves only through a single pair of terminals), unless, of course, one terminal is connected to ground. A condenser, for example, may have not only its main capacity between terminals but a minor capacity from each terminal to ground as well.

Wiring Effects

The impedances of the wiring system of a network are often neglected in theoretical analysis but may be important from several standpoints. Capacity introduced by wiring tends to cause interactions among the network branches, and a ground system of appreciable resistance will not only increase low transmission losses but destroy high transmission losses by providing mutual impedance between input and output terminals of the network.

Accuracy of network construction is dependent, for a given design, primarily on (1) the accuracy of electrical measuring circuits used in conjunction therewith, (2) the fidelity of test conditions, and (3) the care and skill exercised in making adjustments. This assumes that the design is such mechanically as to permit close adjustments to be made. Electrical measuring circuits may be used to measure the constants of individual elements, or the characteristics of groups of elements or of networks as a whole. If the accuracy of the circuits is known, possible deviations in the transmission characteristics may be estimated accordingly. Fidelity of test conditions refers to the condition that elements must be tested with due recognition of the manner in which they are used in networks, and that networks must be measured under the same circuit conditions as used, or allowances made accordingly. How much care and skill may enter into making adjustments is chiefly an economic question.

Stability of network characteristics under operating conditions is largely dependent on the materials employed in construction. Suitable materials are usually limited in number either by economic considerations or by the limitations of engineering knowledge. The chief sources of instability of characteristics are: changes with current variations (either temporary or permanent), changes due to temperature and humidity fluctuations, and a group of changes called "aging" which have to do with releases of strains and fatigue of materials.

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Discussion

S. B. Covey: It is estimated that in the Bell System there are in service more than a quarter of a million of the networks described by Messrs. Shea and Lane. These networks vary from simple resistance windings on small wood spools weighing not more than an ounce to complicated assemblies of coils and condensers occupying space to be reckoned in cubic feet and weighing perhaps as much as 50 lb. Each of the many types of networks must occur at its proper point in the circuit. The providing of a circuit terminal with its full and proper complement of equipment calls for an intricate cross-connection of the office wiring. Changes in circuit layout must be made frequently and the task of executing these changes may be far from simple. Errors in cross-connection contribute definitely to the total number of troubles responsible for improper service, and the rapidly mounting use of these networks cannot but increase this contribution. In connection with the provision of telephone facilities, the present tendencies are toward a greater and greater proportion of the total investment being concentrated in the central office buildings in the form of elaborate equipment. It is estimated that 25 per cent of this equipment is composed of networks. This brings to mind at once that very definite consideration must be given to the matter of space requirements. There are instances where building extensions have been necessary for this purpose alone.

Telephone Circuits for Program Transmission

BY F. A. COWAN[1]

Non-member

Synopsis.—*Networks of telephone circuits which are extensively used in the transmission of programs to broadcasting stations are described in this paper. Certain stages in the development of these networks are considered and the general requirements for satisfactory transmission at the present time are enumerated. The arrangements of the networks as well as the procedures used in setting up and maintaining them are discussed.*

* * * * *

MUCH of the phenomenal growth and present excellence of radio broadcasting has resulted from contributions made by associated branches of the electrical art. Of these contributions perhaps none has had a greater effect than the introduction of the program transmission wire networks, which make chain broadcasting possible. Broadcasting had hardly emerged from the novelty stage before the need for programs presenting music of the highest grade, speeches by prominent people, and descriptions of sports events of sectional or national interest became evident. It was further recognized that in addition to providing programs of this character, it would be desirable to broadcast the programs simultaneously from a number of stations. These conditions established a demand for means of picking up selected program material and transmitting it to broadcasting stations scattered throughout the country. For this transmission, wire lines have proved to be very satisfactory and are in general use for this purpose.

Prior to the general development of radio broadcasting, the Bell System had, incident to other developments, worked out the problems involved in such transmission, and on numerous occasions had set up interconnections of radio telephone stations and public address systems which were substantially the same as those required for broadcast program transmission. The telephone companies were, therefore, able to meet this new requirement for communication service early in the development of radio broadcasting. One of the first occasions of this type was in the fall of 1922, when the description of the Chicago-Princeton football game at Chicago was broadcast through station WEAF, then operated by the American Telephone and Telegraph Company, at New York City. Another of the earlier services was the interconnection of six broadcasting stations, one of which was in Dallas, for the simultaneous broadcasting of President Coolidge's first address to Congress on December 6, 1923. Since that time there has been a steady and continuing growth in this service until there are now furnished by the Bell System more than 30,000 mi. of regularly established program circuits which connect in various combinations and at various times over 125 radio broadcasting stations covering the entire United States. The various stages of this development up to the establishment of a 15 station part-time network in the latter part of 1924 are described in a paper by Messrs. Foland and Rose, published in the January 1925 issue of *Electrical Communications*.

The first multi-station network for which special equipment was provided at key points on a permanent basis was initially called the "red network" for no other reason than that those cities which were to be connected were indicated on a chart by red lines, and a memorandum on the arrangements for the circuits referred to them as the "network shown in red." As other networks came into being, the convenience of a short color name for differentiating between them resulted in a rather general use of this form of designation within the telephone companies, and it was not long before this nomenclature had spread among the various broadcasting companies.

Initially, this network service was confined mainly to evening hours and to sections where telephone message circuits could be obtained but it soon became apparent that more time would be required than could be furnished on this basis, and as rapidly as possible program circuits specially provided for this purpose were made available on a larger scale. The routes of the regularly established program circuits of the Bell System in the United States on March 15, 1929, are shown on Fig. 1. These regular networks are supplemented by special circuits which are established for the transmission to the network control points of programs picked up at the location of events of particular interest or national importance. Also, on certain occasions, several of the chains have been merged into one, thereby forming a network of stations which effectively makes the United States into one large auditorium.

The general transmission requirements of circuits for the satisfactory handling of program material are not greatly different from those for good telephone connections. The nature of the program material is usually such, however, that the specific requirements for a network of program circuits are much more exacting than for the usual types of message circuits.

These differences lie in the fact that for satisfactory transmission of speech from the message traffic stand-

1. Long Lines Dept., American Telephone and Telegraph Co., New York, N. Y.

Presented at the Regional Meeting of the South West District of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

point the primary requisite is that the message shall be readily recognizable and intelligible with naturalness of tone as an important but secondary consideration; whereas the satisfactory transmission of programs for broadcasting purposes requires faithfulness of reproduction of speech, music, and incidental details with intelligibility and naturalness of tone of about equal importance. In order to achieve these effects it is necessary to transmit with reasonable uniformity a wider range of frequencies and volumes than is ordinarily required for message traffic.

For example, a frequency band 2500 cycles in width will, if properly utilized, provide facilities for the transmission and ready interchange of ideas through the agency of easily understandable speech whereas for program transmission with the present type of microphones, amplifiers, and loud speakers a frequency band of between 4000 and 5000 cycles in width is usually required. Most program circuits at the present time transmit frequencies above about 100 cycles and below about 5000 cycles. The normal range of volumes in programs is usually much larger than can be successfully handled by the present type of radio broadcast transmitters and receivers with the result that at the pickup point the amplification is adjusted from time to time to reduce the variations in volume. This compression is such that for average broadcast material the momentary volume peaks vary over a range of 20 to 30 decibels. The program transmission circuits are arranged to transmit satisfactory volume ranges of this order of magnitude. Since much of the character of large symphony orchestras is contributed by variations in volume, it would undoubtedly add greatly to the pleasure afforded the radio listener to reproduce these variations even more faithfully. It is to be expected, therefore, that within the next few years improvements in the broadcasting technique will be considered from various points of view and the network circuits will be called upon to keep pace with such developments.

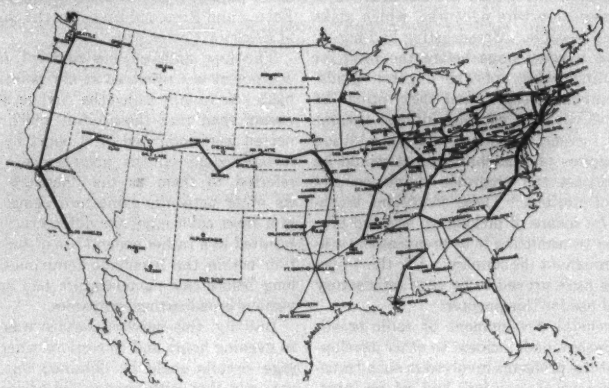


Fig. 1—Bell Sysetm Program Networks in the United States on March 15, 1929

Network	No. of stations	Length in miles
Red — NBC	44	10,800
Blue — NBC	11	3,500
Purple — CBC	42	7,500
Green — ABC	3	1,900
Pink — ABC	5	1,700
Orange — NBC	5	1,700
White — PPA	21	3,800
Brown — DL	3	450
	134	31,350

The wire networks are arranged for the transmission in only one direction in the same manner as the broadcasting stations. This removes the problem of echoes and possible singing which is introduced where two-way transmission is required. It also permits a more simple treatment of the many connections between circuits involved in supplying programs to a large number of points. The networks are set up with main radial circuits over trunk routes from which branches feeding the various stations are taken off. At these branch points the connection is accomplished with the aid of a one-way amplifier arranged to prevent any reaction on the trunk circuit by conditions on the branch circuit.

New York City is the originating point of the red,

blue, purple, and white networks and from it the largest number of wire networks radiates. Atlanta, Cincinnati, Kansas City, Chicago, St. Louis, and San Francisco are also large network centers. The work of properly setting up the veritable web of circuits radiating from and interconnecting these points and properly safeguarding the service, requires the constant attention of a large group of men strategically located and provided with means of rapid intercommunication. To increase the effectiveness of this organization the country has been divided into areas with one of the wire centers listed above designated as the control office for the contiguous area. Telegraph wires are provided connecting each of the repeater offices within a given area with the area control point and the area control offices are in turn connected with New York. By the use of these telegraph wires the reports covering the condition of the circuits and instructions regarding measurements and adjustments required in setting up or rearranging the program circuits are handled. Due to the method used in connecting the branches to the trunk circuits, the various area control offices are able to carry on their work practically independently of the other offices.

To facilitate further the testing and maintenance of the network, the circuits within an area are further subdivided into sections usually about 500 mi. in length. Certain offices are selected and designated as control points for these sections which are known as program circuit units and are established, tested, and maintained as independent units. The units are designated by the names of the two terminal offices and those between the same two points are numbered successively. Complete networks are established by connecting the necessary number of units together. Any two program circuit units over the same route are so designed as to be interchangeable and a spare unit established for emergency or special use may be used to protect the service on several networks.

Information for use by the field forces in establishing and maintaining the networks consisting of data covering the radio stations to be connected and the makeup of the circuits as well as information regarding the transmission features of the set up is prepared and distributed on cards called a "circuit layout record card." The information as to the points at which repeaters are used, as well as the gains to which they should be adjusted, is given on one card while a second card gives detailed information regarding the wires assigned for the circuit unit, the equipment to be connected, and the computed 1000 cycle transmission loss of the line and equipment in decibels. The required transmission equivalent of circuit units of this type is 0 decibel and to obtain this, the sum of the amplifier gains assigned just equals the computed total loss of the line and associated equipment. By making the net loss of all the program units zero the transmission loss of a network is made independent of the number of units entering into its make up.

To illustrate the factors entering into the determination of the proper set up of a circuit unit, the layout of the No. 1 Denver-Omaha program transmission circuit unit which is a typical one, as well as the transmission levels at the various repeater offices, when arranged for transmission Omaha to Denver, is shown in schematic form in Fig. 4. It may be seen from this chart that in addition to making the total gain equal the total loss, the unit is so set up that the gain at each amplifier point just equals the loss in the preceding sections. This results in a uniform level at the output of each amplifier and permits the transmission of the program at a higher average volume level without overloading the amplifiers or introducing interference into adjacent circuits. At each amplifier point there is, in addition to the amplifier transmitting towards the next station, one which is used for monitoring the circuits and making various service observations. An instrument known as a volume indicator is normally connected to this amplifier and is used to check up on the volume of the transmitted program. It is also used sometimes to obtain a quick check of the strength of tones applied for measuring purposes although there is provided for this purpose special precision transmission measuring apparatus.

The line wires in this case are No. 8 B. W. gage

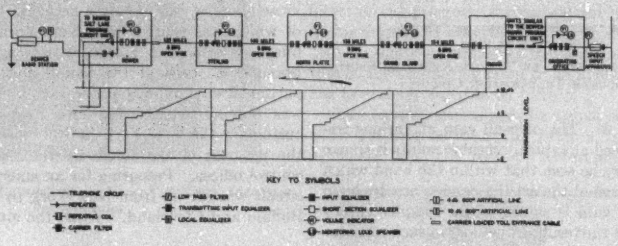


FIG. 2—SCHEMATIC LAYOUT AND TRANSMISSION LEVEL DIAGRAM OF DENVER-OMAHA PROGRAM CIRCUIT UNIT ARRANGED FOR TRANSMISSION FROM OMAHA TO DENVER

copper, and are not loaded, which is the type generally used for program transmission. The incidental cables for entrance into cities are either No. 10 or 13 B. & S. gage and are loaded to have approximately the same characteristic impedance as the line wires, namely, 600 ohms. The cutoff frequency of this cable loading is sufficiently high to permit the operation of the carrier current telephone systems which are connected to these wires. The transmission loss at various frequencies of a typical section of line wires with incidental cables

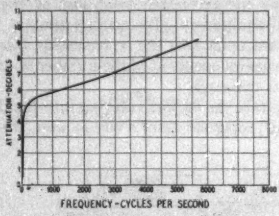


Fig. 3—Line Attenuation of Typical Repeater Section of a Program Transmission Circuit

is shown in Fig. 3. The over-all gain at various frequencies introduced at typical repeater points is shown in Fig. 4. It may be seen that within the band which at the present stage of the art the circuits are designed to transmit, the gain is substantially complementary to the loss so that the resultant net loss in each repeater section for the frequencies within this band is practically uniform. The gain of the repeaters at the higher and lower frequencies relative to 1000 cycles is adjustable so that it may be made to conform to the varying conditions likely to be encountered.

In addition to the constant watch which is maintained through the length of the circuit and particularly at all points where broadcasting stations are connected to the circuits during the transmission of programs, frequent tests are required in order to keep the circuits in shape and prepare them for the periods of use. At the present time complete measurements are made on each program circuit at least once each day and periodic tests of a less comprehensive nature are made at frequent intervals throughout the day. On the complete over-all tests a testing tone of the required strength and having a frequency of 1000 cycles per sec. is applied at the originating point of the section of network under test. At each amplifier point in succession beginning with the originating point the level of the testing current is measured and reported back to the control office. Such changes as are necessary to bring the level to the required value are made and when all points have been covered the frequency of the testing tone is changed and the procedure repeated. In general, measurements are made at only about four or five frequencies covering the band which it is desired to transmit on the daily tests although in some cases it may prove desirable to measure at as many as forty different frequencies. After completing the adjustments at all necessary frequencies the testing tone is removed and the circuit is observed for any noise, crosstalk, or other possible sources of trouble. A test program is then sent over the circuits and reports as to quality are made. To provide for making this last test, the more important control offices are equipped with a high quality phonograph pickup as well as microphones for talking tests.

As has been pointed out in the foregoing, it is necessary for the observers at the amplifier points to be constantly on the lookout for any irregularities which might cause trouble and take such steps promptly as are necessary to clear any troubles observed or forestall any potential ones. It is always essential that this action be coordinated by the control office as the networks are now very complicated. Probably no better example of this could be cited than a situation which arose about a year ago on a network, the trunk circuit of which was routed from New York through Pittsburgh, Akron, and Toledo to Chicago. There were branches at each of the points mentioned. On the particular day under consideration, a very severe storm was moving across Ohio into Pennsylvania. As is customary the New York control office was advised of the progress of the storm by the repeater and subcontrol offices. Preparing for an emergency, a special circuit was set up from New York to Toledo through Buffalo and Cleveland. When the storm hit the line

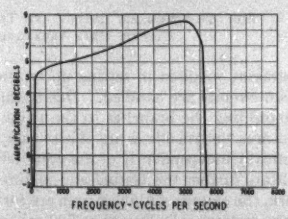


Fig. 4—Over-All Amplification of a Typical Repeater Office on a Program Transmission Circuit

between New York and Pittsburgh in the Pennsylvania mountains the regular circuit began to fail and the one routed through Buffalo and Cleveland was placed into service. To care for the Pittsburgh and Akron radio stations, a branch of the network going south from Toledo was extended on orders from New York, back east to Pittsburgh where it was connected to the section of the trunk circuit which was still intact between Pittsburgh and Toledo. The ever watchful observer at Toledo then found the program coming in satisfactorily on the trunk circuit from the east. Not knowing that this was the very same program that he was receiving over the emergency circuit from New York coming back to him after it had been sent south

from Toledo and to Pittsburgh, he advised the New York control office of the condition and asked if he should take the program from this source now that it was again O. K. Needless to say it was not done since the New York control office was in the fortunate possession of all of the facts.

Another condition which requires careful coordination on the part of the repeater and control offices is the practise of splitting a network at a specified time to provide for the transmission of one program to a certain group of the network radio stations while the remainder receive programs from a different source. These re-

Fig. 5—Part of Equipment at New York for Program Transmission

arrangements may involve the creation of a new point as the temporary originating center, or switching a branch of a network feeding one or more stations from one network to another. Program orders are obtained daily from the broadcasting companies to cover the network set up required for each day's program and these orders are transmitted from New York to the area control office using telephone typewriters. The switches necessary for these rearrangements are then made under the directions of the area control offices. The flexibility of layout resulting from setting the network up in units simplifies the work involved in this splitting and rearranging of networks. It is, however, very necessary that the transmission levels on the networks be carefully watched so that the variations introduced by switching may be negligible.

In the installation of the equipment at repeater stations and distributing centers, care is taken to arrange the circuits so as to provide as great freedom from interruption as practicable and permit the quick rearrangements of circuits which may become necessary during emergencies. The repeaters and associated equipment are grouped together with the telegraph wire terminations and this unit is usually located in a separate room. An example of a part of the equipment arrangements at New York which is typical of the larger wire centers, is shown in Fig. 5. Fig. 6 shows a similar installation at Chicago. One of the features of this installation is a 17-line key switchboard which is used to care for the switching incidental to the rearrangements on networks entering Chicago.

Consideration from an over-all standpoint is essential to the satisfactory performance of any large system. In the case of program networks, this requires among other things that care be taken in making the connections between network and the pick-up as well as the receiving radio station to see that adjustments are made to meet the particular conditions encountered so that the networks may be used to the best advantage. As has been pointed out in the foregoing it is important that the range of volumes in transmitted programs be held within proper limits. The range in volume is a function of adjustments made at the pick-up points but it is necessary to so adjust the network that at the maximum volume, interference will not be introduced into neighboring circuits and that during quiet intervals the program circuits will not be subject to an undue amount of noise or crosstalk from other circuits. This is usually accomplished by adjusting the amplification at the originating point to an average value which has been determined to be correct.

Once having determined the proper set up for a network it is then necessary to obtain a sufficient number of checks upon the over-all performance to ascertain

Fig. 6—Program Transmission Equipment in Chicago Office

that the service is entirely satisfactory. To assist in this there have been developed a number of indexes by which the program circuit performance is rated.

The foregoing discussion has considered program transmission networks as they are today. At this time any attempt to predict the future course of development would probably require rather extensive recasting at some subsequent time. It seems fairly certain, however, that there will be continual improvements in the general broadcasting art and this factor is kept constantly in mind in the design of telephone circuits for program transmission in the future.

Traveling Waves Due to Lightning

BY L. V. BEWLEY*
Associate, A. I. E. E.

Synopsis.—*The purpose of this paper is to describe and analyze the origin and formation of traveling waves on a transmission line induced by lightning discharges, and to investigate their behavior at a transition point where there is an abrupt change of circuit constants. Some of the ground covered is necessarily old and well known, but has been included in the interests of completeness and continuity of treatment. The effect of the rate of cloud discharge and the initial distribution of bound charge on the shape and amplitudes of the traveling waves is brought out. General methods of analysis are formulated and illustrated by practical cases. The assumptions and approximations involved are discussed, and the probable direction of their deviation from fact indicated. Exact and approximate mathematical expressions are derived, and therefrom graphical and tabular methods are outlined. For all of the examples given, line attenuation and distortion are neglected and only the first reflection from a transition point is considered.*

I. Introduction

THIS paper, which deals particularly with the mathematical study of the formation and propagation of lightning waves on transmission lines, is part of an extensive investigation of lightning which has been under way for a number of years under the general direction of F. W. Peek, Jr. Other papers of this series covering laboratory and field studies as well as the effect of lightning on apparatus have already been presented.[1,2,3] While this paper deals essentially with a mathematical analysis of the subject, it has been written in such a way that it is not felt necessary for the reader to go into the mathematics to obtain a knowledge of the results. Such practical aspects as the effects of series induction coils, extra ground wires, etc., on transmission lines are analyzed mathematically and discussed.

II. Origin of Traveling Waves Due to Lightning

When a charged cloud approaches a transmission line, a charge of opposite sign leaks over the insulators and appears on the line as a bound charge fixed in position by the electrostatic field of the cloud. The density σ of bound charge at any point x on the line is proportional to the height h of the line above ground and to the average gradient G between that point and ground, thus:

$$\sigma = \partial Q/\partial x = C \int g\, ds = C\, h\, G$$

where

C = capacitance per unit length at point x
g = gradient along the tube of electrostatic flux through x
ds = element along the tube of flux passing through x.

Since the field of the cloud is not uniform, and since the line may sag considerably at the middle of a span, it is evident that the bound charge will not be uniformly distributed, but may have a wide variety of shapes. The situation is shown in Fig. 1, where R is the resistance of the insulator string at the tower, and C_1, C_2, C_3, represent the increasing capacitance of the line elements as the sag brings the conductors nearer to ground. The density of bound charge is shown by the cross-hatched area. However, the corresponding *potential* distribution when this charge is instantaneously released is quite independent of the variation of capacitance, and is $V = \frac{1}{C}\frac{\partial Q}{\partial x} = h\,G$, thus fully determined by the field of the cloud.

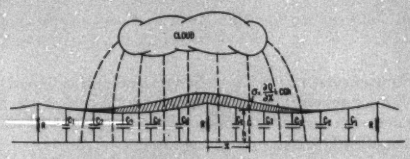


Fig. 1—Distribution of Bound Charge

Measurements by Peek indicate gradients of the order of 100 kv. per foot for heights above ground of the order of a transmission line. However, so far as the author knows, no data are available as to the intensification of the field (if it occurs) during the preliminary or incipient stages of a cloud discharge. Measurements taken on short antennas are not informative. Some idea of the time required for a bound charge to form, may be obtained by calculating the time for a capacitance equal to that of one span of the line to discharge to $(1/\epsilon)$th of its initial potential over a resistance equal to that of a single insulator. Taking $C = 0.003$ mf. and $R = 1000$ megohms, the time is

$$t = R\,C = 3 \text{ seconds}$$

If the preliminary stages of a cloud discharge are completed within 100 microseconds or so, the time is not sufficient for additional bound charge to leak over the insulators.

The measurements taken by Peek[1] on short antennas

*General Transformer Engineering Department, General Electric Company, Pittsfield, Mass.

1. *Lightning—Progress in Lightning Research in the Field and in the Laboratory*, F. W. Peek, Jr., Quarterly Trans., Vol. 487, April, 1929, p. 436.
2. *Effect of Transient Voltages on Power Transformer Design*, K. K. Palueff, A. I. E. E. Quarterly Trans., July 1929.
3. *Lightning Studies of Transformers by the Cathode Ray Oscillograph*, F. F. Brand and K. K. Palueff, A. I. E. E. Quarterly Trans., July, 1929.

29-122

indicate that the final release of the bound charge sometimes takes place in one or two microseconds. Probably much longer periods may occur, but as shown from the theoretical considerations discussed later, these slower rates of cloud discharge cannot give rise to excessive potentials. Physically, a finite rate of cloud discharge is equivalent to the release of the bound charge by small steps (see Fig. 2), which in the limit coincide with the smooth curve of cloud discharge rate. Each of these blocks forms in succession and immediately begins to move away as a traveling wave. But if the time of release is slow enough, some of the blocks

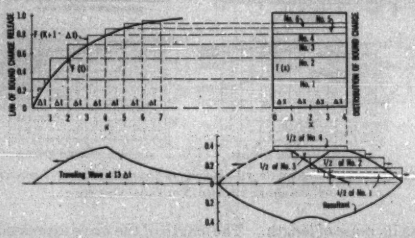


Fig. 2—Graphical Method for Determining Wave Shapes

first formed have completely passed out from under the original distribution before all of the bound charge has been released, with the result that only part of the total number of blocks will overlap and add up by superposition.

Experimental data on the functional rate of cloud discharge are not available. For a strictly mathematical study only the most simple expressions can be handled, but the graphical and tabular methods described elsewhere in this paper are applicable to any rate of cloud discharge which can be drawn as a curve on a piece of paper. The illustrations included are for exponential and straight line discharges. Judging from the meager number of cathode ray oscillograms of potentials induced on short antennas, and working backwards from those of traveling waves caused by lightning, it appears that a fair approximation expressing the release of the bound charge is $(1 - \epsilon^{-at})$. Very rough estimates of the circuit constants of a lightning bolt, also suggest this equation as a possibility. The influence of the function $F(t)$, expressing the law of release of the bound charge, on the shape of the resulting traveling waves is shown in Fig. 3. If $f(x)$ represents the distribution of bound charge, then the curves A, B, C represent the shape of the traveling waves corresponding to the cloud discharging in the same time t, but according to different laws a, b, c.

Neither the shape nor amplitude of a traveling wave is preserved as it moves along the line. This attenuation and distortion may be due in part to the appropriation of a charge from the traveling waves by other cloud fields in the vicinity of a discharge. However, even when such fields do not exist, the line losses soon dissipate the energy of the wave by corona and skin effect. Attempts have been made to formulate mathematically the transient skin effect, but so far as the author knows, nothing has been done in that direction for determining the effects of corona. Although an appendix has been attached to this paper in which the line constants R, L, C, G were assumed to be constant, it is fully realized that they vary over a great range. The resistance increases and the inductance decreases due to skin effect. The conductance increases and there is an effective cyclic increase in the capacitance due to corona. Some degree of approximation may be obtained by arbitrarily attributing different values to the line constants associated with each frequency component, or group of frequencies, in the solution obtained on the assumption of fixed constants. However, the work involved in summing the infinite series to a point of convergency, under such conditions, is prohibitive. There is this value in deriving such expressions,—they show by inspection that the effects of the line loss are to distort the wave shape, reduce the steepness of the front, and depress the crests. Therefore the worst possible conditions are realized by neglecting line losses.

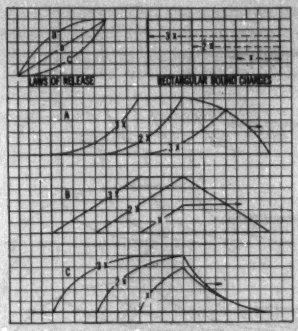


Fig. 3—Effect of the Law of Discharge on the Shape of Traveling Waves

For these reasons the attenuation and distortion of traveling waves have been neglected in this paper. Nor was it considered advisable to consider a multiplicity of reflections at a transition point of the circuit, since each successive reflection is rapidly reduced in severity.

The graphical method, Fig. 2, for obtaining the shape of traveling waves from the curves of bound charge release and initial distribution, and the corresponding tabular method, Fig. 4, are described following Equation (**25**). These methods neglect the line losses, but are otherwise arbitrarily as accurate as desired.

The crest values, shapes, and lengths of traveling waves depend upon the initial distribution of bound charge, but to an even greater extent on the rate of cloud discharge. Fig. 5 shows a set of traveling waves developed from rectangular distributions of bound charge of 1000-, 2000-, 3000-, and 4000-ft. lengths, and released according to the law $(1 - \epsilon^{-at})$; so that the cloud

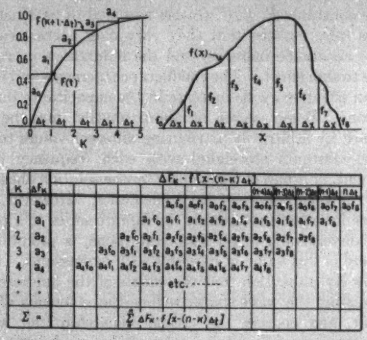


Fig. 4—Tabular Method for Determining Wave Shapes

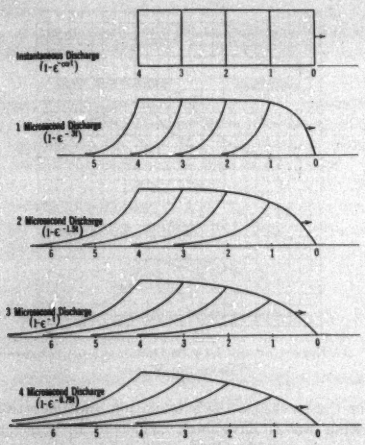


Fig. 5—Traveling Waves from Rectangular Distribution of Bound Charge

discharge was 95 per cent completed in 1, 2, 3, and 4 microseconds. The slower the rate of discharge the lower the crests and the longer the wave. This is a necessary consequence of the fact that all waves originating from a given bound charge must contain exactly the same energy (neglecting line losses) regardless of the rate at which that charge is released. The original bound charge represents the storage of a definite amount of energy, and 25 per cent of this energy must appear in the completely developed traveling potential wave, 25 per cent in the companion current wave, and the other 50 per cent in the pair of potential and current waves moving in the opposite direction. Thus a decrease in crest values must be compensated for by

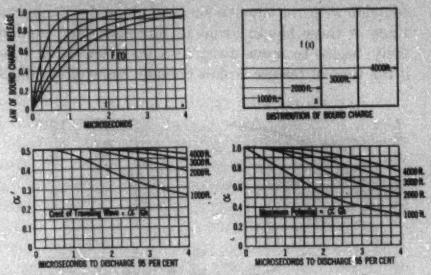


Fig. 6—Influence of the Rate of Discharge and Length of Charge on the Voltage Crests

an increase of length. The length of the wave expressed in microseconds (one microsecond being equal to 1000 ft. of wave travel) is roughly equal to that of the bound charge plus the number of microseconds required for the cloud discharge. The steepness of the wave front is greater, the faster the rate of discharge.

When the length of a rectangular bound charge (in microseconds) is longer than the period of discharge, the crests of the traveling waves reach a value equal

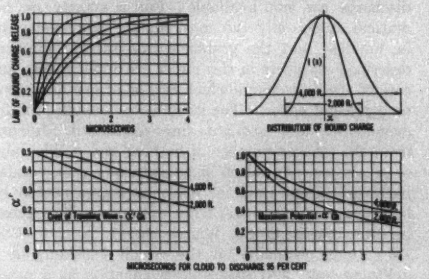


Fig. 7—Influence of the Rate of Discharge and Length of Charge on the Voltage Crests

to $(1/2\,G\,h)$. The maximum potential on the line may reach a value of $(G\,h)$ due to the superposition of the two traveling waves. However, in general these crest values depend on the bound charge distribution and the rate of its release. They are:[1,4]

$$\text{Crest of traveling wave} = \alpha' G h$$
$$\text{Maximum potential} = \alpha G h$$

4. "Lightning," F. W. Peek, Jr., *Franklin Inst. Jl.*, Feb. 1925.

The factors α and α' corresponding to rectangular distributions and exponential release, are shown in Fig. 6. It will be noticed that 100 per cent potential ($\alpha = 1.0$, $\alpha' = 0.5$) occurs only for an instantaneous discharge, or for very long clouds, and falls off rapidly as the time of release is increased, or the length of the cloud shortened. An interesting point is that ultimately the maximum potential and the crest of the traveling wave have the same value. Fig. 7 is a similar set of curves for a bound charge distributed as one loop of $1/2\,(1 - \cos\theta)$. The values of α and α' are somewhat lower, but exhibit the same tendencies.

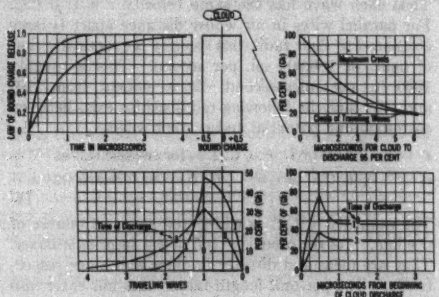


FIG. 8—WAVE SHAPES DERIVED FROM RECTANGULAR BOUND CHARGE RELEASED AT DIFFERENT RATES

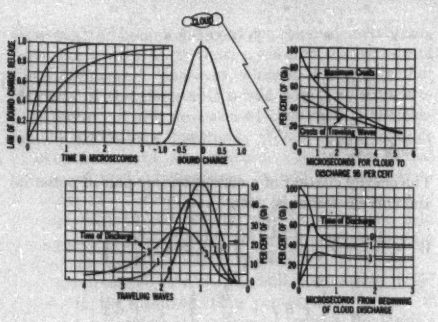


FIG. 9—WAVE SHAPES DERIVED FROM PEAKED BOUND CHARGE RELEASED AT DIFFERENT RATES

Figs. 8 and 9 show the actual wave shapes, and the maximum potential at any instant after the beginning of discharge for a given bound charge released at different rates. While the bound charge distributions in these two figures are quite different, there is a remarkable similarity between the traveling waves which they cause. These traveling waves have been superimposed in Fig. 10. It is this similarity which led the author to state that the rate of cloud discharge was a more influential factor in determining wave shape than the initial distribution of bound charge.

Fig. 11 shows the formation of traveling waves at different stages of their development. This particular set originated from the 2000-ft. peaked bound charge distribution of Fig. 10, released in one microsecond. During the first half microsecond or so, the traveling waves have not fully formed, but are moving out and spreading apart at the same time that they increase in magnitude. Eventually, however, the moving wave crests can be distinguished as they separate out and acquire identity. It is because of this simultaneous growth and separation, that the maximum potential is less than twice that of the traveling wave crests, and it is because of the spreading action that the crests are always less than they would be for an instantaneous cloud discharge.

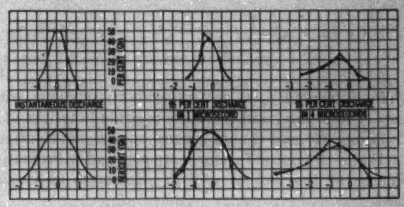


FIG. 10—SIMILARITY OF WAVE SHAPES FROM DIFFERENT DISTRIBUTIONS OF BOUND CHARGE

The mathematical work associated with the foregoing is given in the following section and in the Appendix of this paper.

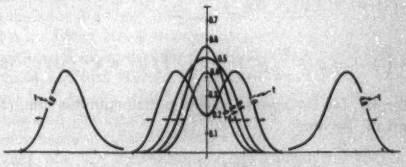


FIG. 11—FORMATION OF TRAVELING WAVES

III. MATHEMATICAL ANALYSIS

General. The differential equations for the potential and current at an unconstrained point on a transmission line are

$$-\frac{\partial i}{\partial x} = (G + C\,p)\,e = Y\,e \qquad (1)$$

$$-\frac{\partial e}{\partial x} = (R + L\,p)\,i = Z\,i \qquad (2)$$

where $p = \partial/\partial t$. Eliminating i between these two simultaneous differential equations, there is

$$\frac{\partial^2 e}{\partial x^2} = Z\,Y\,e = C\,L\left\{ p^2 + \left(\frac{R}{L} + \frac{G}{C}\right)p + \frac{R}{L}\,\frac{G}{C}\right\} e \tag{3}$$

$$i = Y \int e\,d\,x = (G + C\,p) \int e\,d\,x \tag{4}$$

Solving (3) as an ordinary linear differential equation in x,

$$e = \epsilon^{x\sqrt{ZY}} f_1(t) + \epsilon^{-x\sqrt{ZY}} f_2(t) \tag{5}$$

$$i = Y \int e\,d\,x = \sqrt{\frac{Y}{Z}}\left\{ \epsilon^{x\sqrt{ZY}} f_1(t) - \epsilon^{-x\sqrt{ZY}} f_2(t)\right\} \tag{6}$$

where $f_1(t)$ and $f_2(t)$ are integration constants with respect to x only, and therefore possible functions of t. Equations (5) and (6) are general operational solutions to (1) and (2), but in this form are not readily evaluated because of the radical $\sqrt{Z\,Y}$. However, as special cases

$$\sqrt{Z\,Y} = \sqrt{C\,L}\sqrt{p^2 + \left(\frac{R}{L} + \frac{G}{C}\right)p + \frac{R}{L}\,\frac{G}{C}}$$

$$= \sqrt{C\,L}\left(p + \frac{R}{L}\right) \text{ if } R\,C = G\,L \tag{7}$$

$$\sqrt{Z\,Y} = \sqrt{C\,L}\,p \text{ if } R = G = 0 \tag{8}$$

If the conditions of Equation (7) obtain, the line is said to be *distortionless*, and admits of easy solution. If conditions (8) hold, the line is called a *No-loss* line, and is easily solved. By Taylor's theorem, there is the operational equivalent

$$f(t + a) = f(t) + a\,f'(t) + \frac{a^2}{\underline{|2}} f''(t) + \ldots$$

$$= \left(1 + a\,p + \frac{a^2}{\underline{|2}}\,p^2 + \ldots\right) f(t) = \epsilon^{ap} f(t) \tag{9}$$

Hereby, for the special case of a distortionless line, (5) and (6) become

$$e = \epsilon^{\sqrt{CL}\frac{R}{L}x} f_1(t + \sqrt{C\,L}\,x) + \epsilon^{-\sqrt{CL}\frac{R}{L}x} f_2(t - \sqrt{CL}\,x) \tag{10}$$

$$i = \sqrt{\frac{C}{L}}\left\{ \epsilon^{\sqrt{CL}\frac{R}{L}} f_1(t + \sqrt{C\,L}\,x) - \epsilon^{-\sqrt{CL}\frac{R}{L}x} f_2(t - \sqrt{C\,L}\,x)\right\} \tag{11}$$

Thus the potential and current distributions are propagated as traveling waves, and each may consist of a forward wave f_2 moving in the direction of positive x, and a reverse wave f_1 moving in the direction of negative x. Corresponding values of x and t which define the same point on the waves are given by

$$\left.\begin{aligned} (t - \sqrt{C\,L}\,x) &= \sqrt{C\,L}\,\lambda_2 \text{ for the forward wave} \\ (t + \sqrt{C\,L}\,x) &= \sqrt{C\,L}\,\lambda_1 \text{ for the reverse wave} \end{aligned}\right\} \tag{12}$$

The velocity of propagation therefore is

$$\left.\begin{aligned} v &= \frac{d\,x}{d\,t} = \frac{+1}{\sqrt{C\,L}} \text{ for the forward wave} \\ v &= \frac{d\,x}{d\,t} = \frac{-1}{\sqrt{C\,L}} \text{ for the reverse wave} \end{aligned}\right\} \tag{13}$$

Thus each wave has the same velocity $v = 1/\sqrt{C\,L}$. For parallel wires in air whose distance apart is large compared to their radii, this velocity is practically that of light, or 186,000 mi. per second. This is roughly 1000 ft. per microsecond. It is evident from (10) and (11) that the following relationships exist between corresponding potential and current waves:

$$\left.\begin{aligned} e_1 &= \sqrt{L/C}\,i_1 \text{ or } i_1 = \sqrt{C/L}\,e_1 \text{ for reverse waves} \\ e_2 &= -\sqrt{L/C}\,i_2 \text{ or } i_2 = -\sqrt{C/L}\,e_2 \text{ for forward waves} \end{aligned}\right\} \tag{14}$$

The quantity $\sqrt{L/C}$ is called the *surge impedance* of the line. Its reciprocal $\sqrt{C/L}$ is the *surge admittance*. They are measured directly in ohms and mhos respectively. The per-unit-length factor does not enter into their description.

Using (12) for a transformation of coordinates, Equations (10) and (11) become

$$e = \epsilon^{-\frac{R}{L}t}\left\{ \epsilon^{\sqrt{CL}\frac{R}{L}\lambda_1} f_1(x - v\,t) + \epsilon^{-\sqrt{CL}\frac{R}{L}\lambda_1} f_2(x - vt)\right\}$$

$$= \epsilon^{-\frac{R}{L}t}\left\{ f_2(x + v\,t) + f_4(x - v\,t)\right\} \tag{15}$$

$$i = \sqrt{\frac{C}{L}}\,\epsilon^{-\frac{R}{L}t}\left\{ f_3(x + v\,t) - f_4\right\}(x - v\,t)\} \tag{16}$$

The energy content of a corresponding pair of potential and current waves is, using (14),

$$W = W_e + W_i = \frac{C}{2}\int e^2\,d\,x + \frac{L}{2}\int i^2\,d\,x$$

$$= \frac{C}{2}\int e^2\,d\,x + \frac{L}{2}\int \frac{C}{L}\,i^2\,d\,x$$

$$= C\int e^2\,d\,x = L\int i^2\,d\,x = \sqrt{L\,C}\int e\,i\,d\,x \tag{17}$$

Or if preferable to integrate with respect to t

$$W = \int e\,i\,d\,t = \sqrt{\frac{C}{L}}\int e^2\,d\,t = \sqrt{\frac{L}{C}}\int i^2\,d\,t \tag{18}$$

Equations (17) and (18) are continuations of each other through the relationship $d\,t = \sqrt{C\,L}\,d\,x$ from (13). These equations show that the total energy of a pair of traveling waves is divided equally between the electrostatic and magnetic fields. They are useful expressions for checking purposes.

The equations for the *No-loss* line are obtained from those derived above for the distortionless line, by making $R = 0$, thus causing the decrement factors to disappear. In what follows, the losses will be neglected.

Traveling Waves Induced by Lightning. Suppose that the bound charge is distributed as $2\,C\,f\,(x)$, and that the cloud discharges instantaneously. This is equivalent to suddenly impressing on the line a potential $2\,f\,(x)$ at the instant $t = 0$. The current at this first instant is zero. It then follows by Equations (**15**) and (**16**) that

$$i = \sqrt{\frac{C}{L}}\,\{\,f_3\,(x) - f_4\,(x)\,\} = 0 \qquad (19)$$

$$e = \{\,f_3\,(x) + f_4\,(x)\,\} = 2\,f\,(x) \qquad (20)$$

Therefore $$f_3\,(x) = f_4\,(x) = f\,(x) \qquad (21)$$

Thus the sudden release of a bound charge distributed as $2\,C\,f\,(x)$ initiates a pair of exactly similar forward and reverse traveling waves having the same shape, but one-half the amplitude, of the potential corresponding to the fixed distribution of bound charge. The equations for the traveling waves under the conditions of instantaneous cloud discharge, and up until the instant when a transition point is reached, are therefore

$$e = f\,(x + v\,t) + f\,(x - v\,t) \qquad (22)$$

$$i = \sqrt{\frac{C}{L}}\,\{\,f\,(x + v\,t) - f\,(x - v\,t)\,\} \qquad (23)$$

However, the bound charge is not released instantaneously but over a finite time, according to the rate of cloud discharge. Moreover, while it is probable that all parts of the electrostatic field do not collapse proportionately, it will be assumed that such is the case in the following treatment. When more reliable experimental data are available, as to the actual process of a cloud discharge, it will be appropriate to make the necessary change in the mathematical analysis to take care of the way in which the field collapses. As a matter of fact, such a generalization is given in Equation (**41**) of Appendix I.

If the solution to the differential equations $\psi\,(p)$ specifying the behavior of a conservative system is given by $\phi\,(t)$ when the impressed force is Heaviside's unit function 1, then the solution corresponding to any impressed force $F\,(t)$ is given by Duhamel's theorem (also called the Boltzman-Hopkinson principle of superposition). It is

$$\psi\,(p)\,.\,F\,(t) = \phi\,(t)\,F\,(o) + \int_0^t \phi\,(t - \tau)\,\frac{\partial\,F\,(\tau)}{\partial\,\tau}\,d\,\tau \qquad (24)$$

where $$\phi\,(t) = \psi\,(p)\,1$$

For a derivation and discussion of this equation, as well as of five other equivalent expressions obtained by various transformations, see "Electric Circuit Theory," by J. R. Carson. Applying (**24**) to (**22**) there results

$$e = \int_0^t \{\,f\,[x + v\,(t - \tau)] + f\,[x - v\,(t - \tau)]\}\,\frac{\partial\,F\,(\tau)}{\partial\,\tau}\,d\,\tau$$

$$= \lim_{\Delta t \to 0} \sum_{\kappa=0}^{n} \{f\,[x + v\,(n - \kappa)\,\Delta\,t] + f\,[x - v(n - \kappa)\,\Delta\,t]\}$$

$$\{\,F\,[(\kappa + 1)\,.\,\Delta\,t] - F\,[\kappa\,.\,\Delta\,t]\} \qquad (25)$$

Where $n\,.\,\Delta\,t = t$, $\kappa\,.\,\Delta\,t = \tau$, and $F\,(t)$ is the time function by which the bound charge is released.

The application of the integral in Equation (**25**) is limited to analytic expressions for f and F, whose integrals in the above combinations are known. But the summation, the limiting case of which is the exact solution, is immediately applicable to any functions f and F whose graphs are known or assumed. Ultimately, since both of these functions must be found from experimental data, it is advisable to deal directly with the finite summation as an approximation of arbitrary exactness. From it, both graphical and tabular methods can be developed. As a simple and obvious example of the graphical method, refer to Fig. 2, in which the bound charge initially has a rectangular distribution, and is released according to the law $F\,(t)$. Divide the base of $F\,(t)$ into equal time increments $\Delta\,t$ and the base of $2\,f\,(x)$ into corresponding equal space increments $\Delta\,x = v\,\Delta\,t$. For ordinary purposes, when dealing with parallel wires in air, the space and time increments may be taken as equal, if time is measured in microseconds and distance in thousands of feet. At the end of $n = 3$ time increments, one-half of block No. 1 has formed and moved to the right a distance $3\,\Delta\,t$ and the other half to the left a distance $3\,\Delta\,t$. Block No. 2 has moved to the right and left a distance $2\,\Delta\,t$, and block No. 3 a distance $1\,\Delta\,t$. Block No. 4 has just started to form and move out. Only the half blocks which have moved to the right are shown in the figure. It is advisable to subdivide the increments $\Delta\,t$ for the larger steps, as indicated for block No. 1. The traveling component is found by drawing a curve through the feet of the rectangular blocks of which it is composed. The forward and reverse traveling waves at $n = 3$ are shown as heavily drawn full and dashed lines respectively. Their instantaneous resultant has been plotting upside down to avoid interference. This graphical method is very awkward when applied to other than rectangular bound charges. A tabular method, shown in Fig. 4, has been devised as a substitute applicable to complicated bound charges. The schedule is carried through for only one of the pair of traveling waves. The resultant is found by superimposing two such waves displaced by $2\,n\,\Delta\,x$. For periods of time in excess of that required for the cloud to completely discharge, the

traveling waves are fully formed, and thereafter may be moved out as rigid distributions. Formations and waves determined by these graphical and tabular methods have been described in the first part of this paper.

Empirical Equation for Approximate Wave Shape. The combined influence of the initial distribution of bound charge and the rate of cloud discharge, imparts to the traveling waves a characteristic shape. The shapes obtained by the previous analysis check those which have been measured in the field and in the laboratory. An empirical equation which represents this characteristic shape quite accurately, and at the same time is remarkably simple for analytic purposes, is given by

$$e = E\,(\epsilon^{-a\lambda} - \epsilon^{-b\lambda}) \quad \text{for } t > 0. \tag{26}$$

As special cases of this equation there are the various wave shapes illustrated in Fig. 12. Still greater variety of form may be obtained by using

$$e = E_1\,\epsilon^{-a\lambda} - E_2\,\epsilon^{-b\lambda} \tag{27}$$

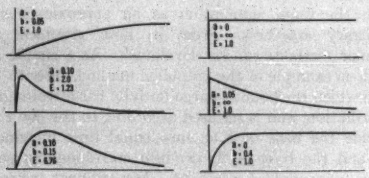


FIG. 12—EMPIRICAL WAVE SHAPES

Given by $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

but for the purposes of this paper only (26) will be applied. The three parameters E, a, b, are sufficient to determine uniquely the *crest, wave length,* and *front* of the wave. The parameters a and b may be *real, imaginary,* or *complex.* The crest, or maximum value of e, occurs when

$$\frac{d\,e}{d\,t} = 0 = -\,a\,\epsilon^{-at} + b\,\epsilon^{-bt}$$

hence at that value of t for which

$$t = t_1 = \frac{\log b/a}{b - a}$$

The instant $t = t_2$ at which the wave has $(100\,p)$ per cent of e is found from the equation

$$e = p\,e_{max} = E\,(\epsilon^{-at_2} - \epsilon^{-bt_2})$$

Hence the crest, wave length, and front of the wave are determined by the three simultaneous equations

$$\left.\begin{aligned} t_1 &= \frac{\log b/a}{b - a} \\ e_{max} &= E\,(\epsilon^{-at_1} - \epsilon^{-bt_1}) \\ p\,e_{max} &= E\,(\epsilon^{-at_2} - \epsilon^{-bt_2}) \end{aligned}\right\} \tag{28}$$

where (e_{max}, p, t_1, t_2) are specified and (a, b, E) are to be solved for. Practically, since (28) is in transcendental form, it is easier to plot a family of curves directly from (26) and to choose the nearest approximation to the desired wave shape by inspection.

Behavior of a Traveling Wave at a Transition Point. When a traveling wave reaches a transition point at which there is an abrupt change of circuit constants, as an open or short-circuited terminal, or a junction with another line, etc., a part of the wave is reflected back, and a part may pass on to other sections of the circuit. The impinging wave is called an *incident* wave, and the two waves to which it gives rise at a transition point are called the *reflected* and *transmitted* waves respectively. Such waves are formed at the transition point in accordance with Kirchhoff's Laws. They satisfy the differential equations of the transmission line, and are consistent with the principle of the Conservation of Energy.

Suppose that the line is closed at the transition point by a general impedance consisting of any arrangement of inductances, resistances, capacitances, and other lines. Let the operational equation specifying this general impedance be written as $Z_0\,(p)$. Let the transition point be taken as the origin of coordinates, and distance along the line away from the point be counted as positive, so that an approaching wave is traveling in the negative direction. By Equation (14) the potential and current incident waves will then have the same sign. Denote the incident waves by (e) and (i), the reflected waves by (e') amd (i'), and the transmitted waves, if they exist, by (e'') and (i''). Then the total potential at the transition point is, using (14),

$$e_0 = e + e' = (i + i')\,Z_0\,(p) = (e - e')\,Y\,Z_0\,(p) \tag{29}$$

where $\quad Y = 1/Z = \sqrt{C/L}$ = surge admittance.

Solving this equation for e' there is

$$e' = \frac{Z_0\,(p) - Z}{Z_0\,(p) + Z}\,e = \text{reflected potential wave} \tag{30}$$

The total resultant wave at the transition point is the sum of the incident and reflected waves,

$$e_0 = e + e' = \frac{2\,Z_0\,(p)}{Z_0\,(p) + Z}\,e$$

$$i = i + i' = Y\,(e - e') = \frac{2}{Z_0\,(p) + Z}\,e \tag{31}$$

In general, $Z_0\,(p)$ may consist of any number of branches in parallel. If one of these branches consists of another line of surge impedance Z_2 connected through a concentrated impedance network $Z_m\,(p)$, then the potential wave transmitted through this impedance to the line is

$$e'' = e_0 - Z_m\,(p)\,i'' = e_0 - Z_m\,(p)\,\frac{e_0}{Z_m\,(p) + Z_2}$$

$$= \left\{ 1 - \frac{Z_m(p)}{Z_m(p) + Z_2} \right\} e_0$$

$$= \left(\frac{Z_2}{Z_m(p) + Z_2} \right) \left(\frac{2\,Z_0(p)}{Z_0(p) + Z} \right) e \qquad (32)$$

Thus if e is known at the transition point as a function of time, then e_0, e', and e'' are determined by solving the above differential equations. In particular, if e is a rectangular wave with an infinite tail, it may be taken as Heaviside's *unit function* 1, and the solution obtained by means of the expansion formula. The solution for a finite wave of any shape is then found from Duhamel's theorem, Equation (**24**).

The application of these general relationships are illustrated in this paper for the wave of Equation (**26**), with the parameters adjusted to give both a rectangular wave with an infinite tail, and a characteristic lightning wave having a 7-microsecond front and a 13-microsecond tail. The differences in behavior of these two waves is sometimes quite striking, and conclusions based on one of them are not always generally applicable to the other. Only two cases will be worked out here in detail. But exactly the same procedure has been followed in deriving all of the results.

1. *Two Lines Connected by an Inductance* (Fig. 22).

$Z_1 = 1/Y_1$ = surge impedance of line No. 1

$Z_2 = 1/Y_2$ = surge impedance of line No. 2

L_0 = connecting inductance

$$Z_m(p) = L_0\,p$$

$$Z_0(p) = Z_2 + L_0\,p \quad \text{and let}$$

$$\alpha = (Z_1 - Z_2)/L_0$$

$$\beta = (Z_1 + Z_2)/L_0$$

Then by Equation (**30**) the reflected potential wave is

$$e' = \frac{Z_0(p) - Z_1}{Z_0(p) + Z_1} e = \frac{(Z_2 + L_0\,p) - Z_1}{(Z_2 + L_0\,p) + Z_1} e = \frac{p - \alpha}{p + \beta} e$$

If e is a rectangular wave with an infinite tail, the solution by the expansion theorem is

$$e' = \left[\left(1 + \frac{\alpha}{\beta} \right) \epsilon^{-\beta t} - \frac{\alpha}{\beta} \right] \text{if } e = 1$$

and by Equation (**24**), if $e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

$$e' = E \left[\frac{a + \alpha}{a - \beta} \epsilon^{-at} - \frac{b + \alpha}{b - \beta} \epsilon^{-bt} + \frac{(\alpha + \beta)\,(a - b)}{(a - \beta)\,(b - \beta)} \epsilon^{-\beta t} \right]$$

From Equation (**32**)

$$e'' = e_0 - Z_m(p)\,i'' = e_0 - Z_m(p)\,Y_1\,(e - e')$$

$$= E \left[\frac{\alpha - \beta}{a - \beta} \epsilon^{-at} - \frac{\alpha - \beta}{b - \beta} \epsilon^{-bt} + \frac{(a - b)\,(\alpha - \beta)}{(a - \beta)\,(b - \beta)} \epsilon^{-\beta t} \right]$$

If $\alpha \to \beta$, $e'' \to 0$ and there is no transmitted wave. Taking $Z_1 = Z_2 = 500$ ohms and $L_0 = 0.000033$ henry, there results for the 20-microsecond wave

$\alpha = 0$, $\beta = 30 \times 10^6$, $a = .1 \times 10^6$, $b = .2 \times 10^6$ and

$$e'' = E \left[\frac{30.0}{29.9} \epsilon^{-at} - \frac{30.0}{29.8} \epsilon^{-bt} + \frac{.1 \times 30}{29.9 \times 29.8} \epsilon^{-30 \times 10^6 t} \right]$$

$$\simeq E\,(\epsilon^{-at} - \epsilon^{-bt}) = e$$

Thus such a small series inductance is entirely ineffective except for very short waves. The entire incident wave is transmitted with negligible change of shape.

2. *Impedance Network of Fig. 25.* The variations shown in (*a*), (*b*), (*c*), and (*d*) of Fig. 25 are in fact mutually convertible combinations, and therefore, under the conditions of a proper interpretation, the same equations apply to each. Moreover, by letting the different constants become infinite or vanish as required, this single set-up contains 24 special cases, of which the majority are described and illustrated in this paper.

Since the surge impedance of a connected line, as Z_2, enters in the generalized impedance $Z_0(p)$ in exactly the same way as a resistance to ground R_2, it is evident that Fig. 25B is the analytic equivalent of Fig. 25A, so that the same equations apply to each when R_2 and Z_2 are exchanged. In Fig. 25C the grounding resistance $R_2 = 0$, and R_0 has been expressed as the resultant of two resistances R' and R'' in parallel. But now again, the form of the generalized impedance is not altered if R'' be replaced by the surge impedance $Z'' = R''$ of an outgoing line. Fig. 25D then follows as a natural consequence.

While this equivalence is quite interesting and allows many combinations to be expressed by a single general equation, yet under many conditions it is quicker, and there is less chance for error, if the result is derived directly from the differential equations of the particular case. In performing a reduction from the general equation applying to Fig. 25, great care must be taken in evaluating the indeterminates which appear for limiting values (zero or infinity) of the constants, and in transforming functions of imaginary variables to function of real variables, etc.

Referring to Fig. 25A, there is

$$Z_0(p) = Z_2 + Z_m(p) = Z_2 + \frac{L_0\,R_0\,p}{R_0 + L_0\,p + R_0\,L_0\,C_0\,p^2}$$

$$e' = \frac{Z_0(p) - Z_1}{Z_0(p) + Z_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \left(\frac{p^2 + 2\,\alpha\,p + \omega_0^2}{p^2 + 2\,\beta\,p + \omega_0^2} \right) e$$

where
$$\alpha = \gamma\,(Z_2 - Z_1 + R_0)/(Z_2 - Z_1)$$
$$\beta = \gamma\,(Z_2 + Z_1 + R_0)/(Z_2 + Z_1)$$
$$\gamma = 1/2\,R_0\,C_0$$
$$\omega_0^2 = 1/L_0\,C_0$$
$$\omega^2 = \omega_0^2 - \beta^2$$

when $e = 1$, the solution by the expansion formula is

$$e' = \frac{Z_2 - Z_1}{Z_2 + Z_1} \left\{ 1 - \frac{2}{\omega} (\beta - \alpha)\,\epsilon^{-\beta t} \sin \omega\,t \right\} \text{ if } e = 1.$$

If the incident wave is given by

$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$, then by Equation (**24**)

$$e' = \frac{Z_2 - Z_1}{Z_2 + Z_1} E \left\{ \frac{a^2 - 2\,a\,\alpha + \omega_0^2}{a^2 - 2\,a\,\beta + \omega_0^2} \epsilon^{-at} \right.$$

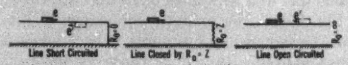
$$- \frac{b^2 - 2\,b\,\alpha + \omega_0^2}{b^2 - 2\,b\,\beta + \omega_0^2} \epsilon^{-bt} + \frac{2}{\omega} (\beta - \alpha)\, \epsilon^{-\beta t}$$

$$\left[\sqrt{\frac{a^2}{a^2 - 2\,a\,\beta + \omega_0^2}} \sin (\omega\, t - \phi_1) \right.$$

$$\left. \left. - \sqrt{\frac{b^2}{b^2 - 2\,b\,\beta + \omega_0^2}} \sin (\omega t - \phi_2) \right] \right\}$$

where $\phi_1 = \tan^{-1}\left(\frac{\omega}{a - \beta}\right)$ and $\phi_2 = \tan^{-1}\left(\frac{\omega}{b - \beta}\right)$

The differential equation for the corresponding transmitted wave by Equation (**32**) is



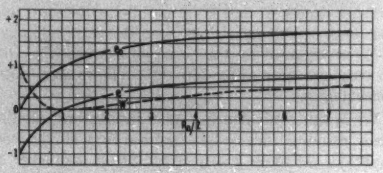


FIG. 13—LINE CLOSED BY A RESISTANCE R_0

$e = f(t) \quad e' = \left(\frac{R_0 - Z}{R_0 + Z}\right) e \quad e_0 = e + e' = \left(\frac{2\,R_0}{R_0 + Z}\right) e$

$$e'' = \left(\frac{Z_2}{Z_m(p) + Z_2}\right)\left(\frac{2\,Z_0(p)}{Z_0(p) + Z_2}\right) e$$

$$= \frac{2\,Z_2}{Z_0(p) + Z_2}\, e = \frac{2\,Z_2}{Z_2 + Z_1}\left(\frac{p^2 + 2\,\gamma\,p + \omega_0^2}{p^2 + 2\,\beta\,p + \omega_0^2}\right) e$$

This differential equation has the same form as that for e', except that in the outside multiplier $(2\,Z_2)$ has replaced $(Z_2 - Z_1)$, and γ has replaced α. The solution, by comparison with that for e', therefore is

$$e'' = \frac{2\,Z_2}{Z_2 + Z_1} E \left\{ \frac{a^2 - 2\,a\,\gamma + \omega_0^2}{a^2 - 2\,a\,\beta + \omega_0^2} \epsilon^{-at} \right.$$

$$- \frac{b^2 - 2\,b\,\gamma + \omega_0^2}{b^2 - 2\,b\,\beta + \omega_0^2} \epsilon^{-bt} + \frac{2}{\omega} (\beta - \gamma)\, \epsilon^{-\beta t}$$

$$\left[\sqrt{\frac{a^2}{a^2 - 2\,a\,\beta + \omega_0^2}} \sin (\omega\, t - \phi_1) \right.$$

$$\left. \left. - \sqrt{\frac{b_2}{b^2 - 2\,b\,\beta + \omega_0^2}} \sin (\omega\, t - \phi_2) \right] \right\}$$

Figs. 13 to 24 inclusive illustrate in a general sort of way the variation in the shape of the reflected and transmitted waves caused by different circuit conditions

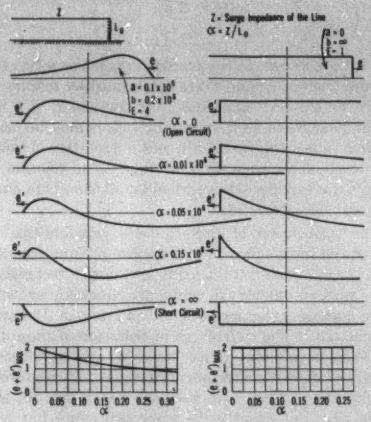


FIG. 14—LINE CLOSED BY AN INDUCTANCE L_0

$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

$$e' = E\left[\frac{a + \alpha}{a - \alpha}\,\epsilon^{-at} - \frac{b + \alpha}{b - \alpha}\,\epsilon^{-bt} + \frac{2\,\alpha\,(a - b)}{(a - \alpha)\,(b - \alpha)}\,\epsilon^{-\alpha t}\right]$$

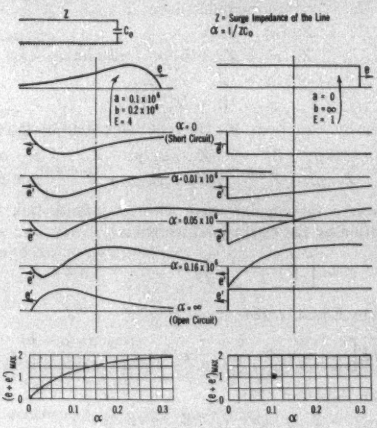


FIG. 15—LINE CLOSED BY A CAPACITANCE C_0

$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$

$$e' = -E\left[\frac{a + \alpha}{a - \alpha}\,\epsilon^{-at} - \frac{b + \alpha}{b - \alpha}\,\epsilon^{-bt} + \frac{2\,\alpha\,(a - b)}{(a - \alpha)\,(b - \alpha)}\,\epsilon^{-\alpha t}\right].$$

at the transition point. Each figure shows a sketch of the circuit under consideration, defines the variable parameters, and indicates the range in shape of the

reflected and transmitted waves corresponding to both an infinite rectangular incident wave and a characteristic 20-microsecond incident wave. The equations of the incident, reflected, and transmitted waves are given below each figure as e, e', and e'' respectively. While these cases are fairly self-explanatory, a few pertinent remarks about each are given below.

Fig. 13. Line Closed by a Resistance R_0. When an incident wave of any shape strikes the end of a line of surge impedance Z closed by a pure resistance, a reflected wave of exactly the same shape will start back on the line. But this reflected wave changes sign as the ratio R_0/Z passes through the critical value $R_0/Z = 1$. If $R_0 = 0$, the line is short-circuited and the reflection is of the same size as the incident wave, but reversed in sign. At $R_0 = Z$ there is no reflection, and the energy of the incident wave is entirely consumed in

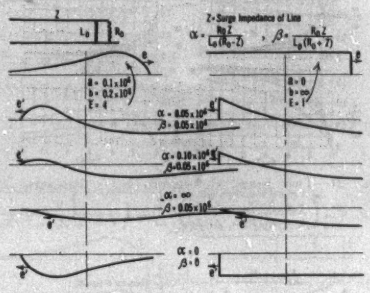


FIG. 16—LINE CLOSED BY R_0 AND L_0 IN PARALLEL

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\,\frac{\beta}{\alpha}\left[\frac{a+\alpha}{a-\beta}\,\epsilon^{-at} - \frac{b+\alpha}{b-\beta}\,\epsilon^{-bt} + \frac{(\alpha+\beta)\,(a-b)}{(a-\beta)\,(b-\beta)}\,\epsilon^{-\beta t}\right]$$

the resistance. At $R_0/Z = \infty$ the line approaches the open circuit condition, and the reflection is of the same size and sign as the incident wave. These three limiting conditions are illustrated in the sketch. The curves show the way in which the reflected voltage e', the total voltage e_0, and the energy content W' of the reflected wave vary with the ratio R_0/Z.

Fig. 14. Line Closed by an Inductance L_0. As L_0 passes through the intermediate values between zero and infinity, the reflected waves are first elongated, and then contracted. Each reflected wave changes sign as it develops. Except for an infinite value of L_0, the total voltage e_0 at the transition point is never double for a finite incident wave.

Fig. 15. Line Closed by a Capacitance C_0. The form of the equations is the same as for the case of the inductance given above, but the sign is reversed.

A capacitance acts as a short-circuit at the first instant.

Fig. 16. Line Closed by R_0 and L_0 in Parallel. While it is impossible by such a combination to completely dissipate the incident wave, yet for values of R_0 of the order of the line surge impedance Z, the reflected wave is considerably reduced in value and spread out.

Fig. 17. Line Closed by R_0 and C_0 in Parallel. The reflections have the same characteristics as for Fig. 15 but the amplitudes have been reduced by the resistance.

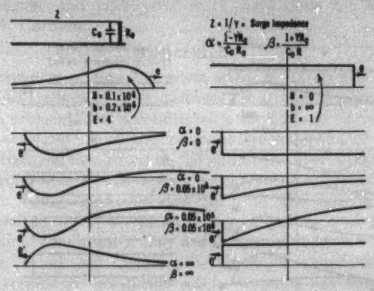


FIG. 17—LINE CLOSED BY R_0 AND C_0 IN PARALLEL

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\left[-\frac{a-\alpha}{a-\beta}\,\epsilon^{-at} + \frac{b-\alpha}{b-\beta}\,\epsilon^{-bt} + \frac{(\alpha-\beta)\,(a-b)}{(a-\beta)\,(b-\beta)}\,\epsilon^{-\beta t}\right]$$

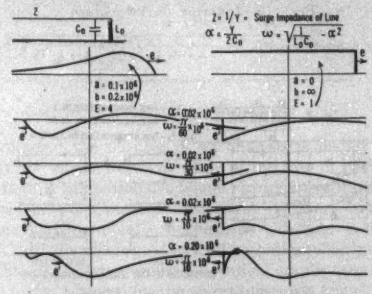


FIG. 18—LINE CLOSED BY L_0 AND C_0 IN PARALLEL

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\left\{-\frac{(a+\alpha)^2+\omega^2}{(a-\alpha)^2+\omega^2}\,\epsilon^{-a} + \frac{(b+\alpha)^2+\omega^2}{(b-\alpha)^2+\omega^2}\,\epsilon^{-bt} + 4\,\frac{\alpha}{\omega}\,\epsilon^{-\alpha t}\right.$$
$$\left.\left[\left(\frac{b\,(b-\alpha)}{(b-\alpha)^2-\omega^2} - \frac{a\,(a-\alpha)}{(a-\alpha)^2+\omega^2}\right)\sin\omega t - \left(\frac{b\,\omega}{(b-\alpha)^2+\omega^2} - \frac{a\,\omega}{(a-\alpha)^2+\omega^2}\right)\cos\omega t\right]\right\}$$

Fig. 18. Line Closed by L_0 and C_0 in Parallel. There is an oscillation in the reflected waves, caused by the oscillatory circuit. By such an arrangement the reflected wave may be either compressed or lengthened out. The development of the wave front in the reflected wave may be made to lag behind several

microseconds,—approximately 6 microseconds in the example shown by the last wave in the figure.

Fig. 19. Junction of Two Lines. When the incident wave reaches the junction, a part is reflected back and a part passes on to the other line. The relative division depends on the ratio of the surge impedances of the two lines. The waves shown are drawn on a *time axis*. If drawn on a *distance axis* the transmitted waves will be contracted in the ratio of the velocities

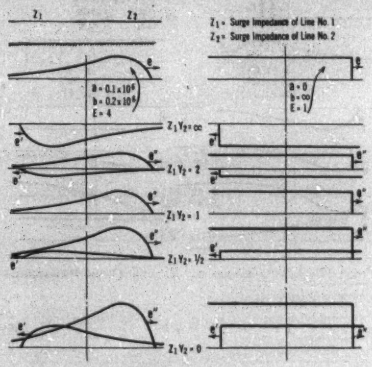


FIG. 19—JUNCTION OF TWO LINES

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt}) \qquad e' = \frac{1 - Z_1\,Y_2}{1 + Z_1\,Y_2}\,e \qquad e'' = \frac{2}{1 + Z_1\,Y_2}\,e$$

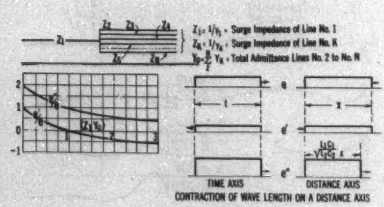


FIG. 20—JUNCTION OF N LINES

$$e = f\,(x + v_1\,t)$$

$$e' = \left(\frac{1 - Z_1\,Y_0}{1 + Z_1\,Y_0}\right) f\,(x - v_1\,t)$$

$$e'' = \left(\frac{2}{1 + Z_1\,Y_0}\right) f\,(x + v_k\,t)$$

of propagation of the two lines, as shown in the next figure. While this case has been illustrated with particular waves, yet incident waves of any form are reflected from and transmitted across a junction between two lines, without change of shape.

Fig. 20. Junction of "N" Lines. Incident waves of any shape are reflected and transmitted at the junction without change of shape. Incident, reflected, and transmitted waves have the same length on a *time axis*, but are contracted on a *distance axis* in the ratio $\sqrt{C_1\,L_1} : \sqrt{C_2\,L_2}$. This contraction is illustrated for rectangular waves, but applies to waves of all shapes.

Fig. 21. Two Lines Connected by a Resistor R_0. As in the previous two cases there is no distortion of wave shape due to impact, but the resistor consumes part of the energy. By making $(R_0 = Z_1 - Z_2)$ it is possible

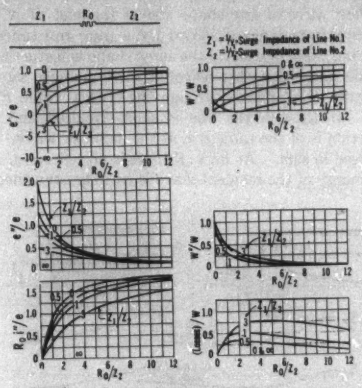


FIG. 21—TWO LINES CONNECTED BY A RESISTOR R_0

$$e = f\,(t) \quad e' = \left(\frac{1 + R_0/Z_2 - Z_1/Z_2}{1 + R_0/Z_2 + Z_1/Z_2}\right) e \quad e'' = \left(\frac{2}{1 + R_0/Z_2 + Z_1/Z_2}\right) e$$

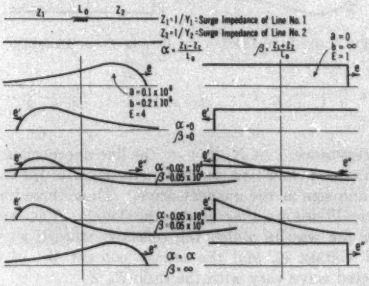


FIG. 22—TWO LINES CONNECTED BY AN INDUCTANCE L_0

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\left[\frac{a + \alpha}{a - \beta}\,\epsilon^{-at} - \frac{b + \alpha}{b - \beta}\,\epsilon^{-bt} + \frac{(\alpha + \beta)\,(a - b)}{(a - \beta)\,(b - \beta)}\,\epsilon^{-\beta t}\right]$$

$$e'' = E\left[\frac{\alpha - \beta}{a - \beta}\,\epsilon^{-at} - \frac{\alpha - \beta}{b - \beta}\,\epsilon^{-bt} + \frac{(\alpha - \beta)\,(a - b)}{(a - \beta)\,(b - \beta)}\,\epsilon^{-\beta t}\right]$$

to wipe out the reflected waves. These curves show how the waves and their energy content vary with R_0, Z_1, and Z_2.

Fig. 22. Two Lines Connected by an Inductance L_0. The reflected wave exhibits the same characteristic as in the case of Fig. 14. If L_0 is large enough, the major

portion of the incident wave can be reflected back, and only a small transmitted wave passed through.

Fig. 23. Two Lines with Shunt Inductance at their Junction. The equations and curves for the incident

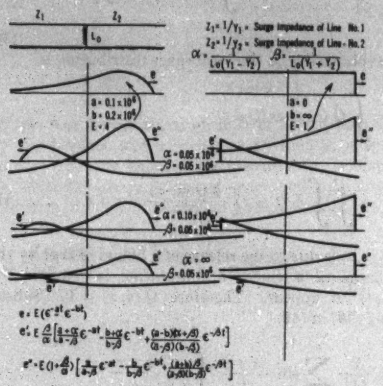


FIG. 23—TWO LINES WITH SHUNT INDUCTANCE AT THEIR JUNCTION

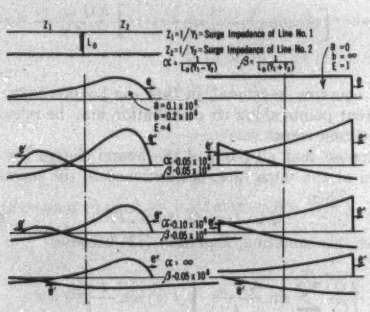


FIG. 24—TWO LINES WITH SHUNT CAPACITANCE AT THEIR JUNCTION

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\,\frac{\beta}{\alpha}\left[\frac{a+\alpha}{a-\beta}\,\epsilon^{-at} - \frac{b+\alpha}{b-\beta}\,\epsilon^{-bt} + \frac{(a-b)(\alpha+\beta)}{(a-\beta)(b-\beta)}\,\epsilon^{-\beta t}\right]$$

$$e'' = E\left(1+\frac{\beta}{\alpha}\right)\left[\frac{a}{a-\beta}\,\epsilon^{-at} - \frac{b}{b-\beta}\,\epsilon^{-bt} + \frac{(a-b)\,\beta}{(a-\beta)\,(b-\beta)}\,\epsilon^{-\beta t}\right]$$

and reflected waves of this case hold identically, when Z_2 is replaced by R_0, to ground.

Fig. 24. Two Lines with Shunt Capacitance at their Junction. The effect of a capacitance in shunt is similar to that of an inductance in series, Fig. 22. The equations and curves for the incident and reflected waves of this case hold identically, when Z_2 is replaced by R_0, to ground.

ACKNOWLEDGMENTS

The author acknowledges with pleasure the encouragement and the many helpful suggestions of Mr. F. W. Peek, Jr., under whose direction this investigation was carried out. The curves, numerical computations, and planimeter work were done by Mr. W. A. Carman.

Appendix

Formation of traveling waves when R, L, G, and C, are all included. The release of a bound charge on a transmission line initiates potential and current waves whose shape and behavior are independent of the terminal conditions at the end of the line, up until the time when the advancing wave fronts have reached those ends. However, the form of the equations expressing these waves must take cognizance of the terminal conditions if they are to remain valid over all time. They will therefore be the more complicated, the greater the complexity of the terminal conditions.

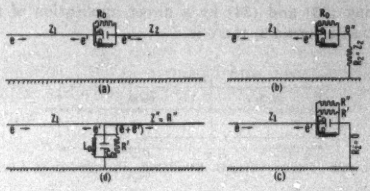


FIG. 25—MUTUALLY CONVERTIBLE NETWORKS

Therefore if the purpose of an analysis is merely to study the effects on wave shape of the rate of release and initial distribution of a bound charge, it is advisable in the interests of mathematical simplicity to arbitrarily choose such terminal conditions as will lead to the most easily handled equations, for regardless of terminal conditions the functions have identical graphs over the time period required for the waves to reach a transition point. Accordingly, for the purposes of this Appendix the line will be considered as short-circuited at each end.

While any mathematical analysis must almost necessarily regard the line constants as the same for all waves, yet these "constants" vary over a wide range depending on the amplitudes and shapes of the traveling waves, the resistance and inductance due to skin effect, and the capacitance and shunt conductance due to corona. The equations derived in this Appendix are based on the assumption that the resistance, shunt conductance, inductance, and capacitance per unit length are perfectly constant. In the final solution, which is an infinite series, it may be feasible to segregate out from the series certain groups of terms and to apply different values of the line parameters to each of these

separate groups. As an approximation, such a procedure is justifiable only when it can be shown that the line constants are primarily functions of the frequency or other parameter with respect to which a segregation has been made. The principal value in deriving equations with the loss factors R and G included, is to be able to notice by inspection of the equations, the direction of their influence on wave shape.

Consider the point charge Q of Fig. 26 suddenly placed at a point x on the transmission line of length l, short-circuited at its ends. The fact that Q is suddenly released may be expressed by Heaviside's unit function $Q\,1$. Let that part of the line to the right of x be designated as Line 1 and that to the left as Line 2. Subscript all associated symbols to correspond. Then the solution to Equation (3), consistent with these conditions is[5]

$$V_1 = \frac{\sinh n\,x}{\sinh n\,l} \sinh n\,(l-y)\,\frac{n}{Y}\,p\,Q\,1 \qquad (33)$$

$$V_2 = \frac{\sinh n\,(l-x)}{\sinh n\,l} \sinh n\,y\,\frac{n}{Y}\,p\,Q\,1 \qquad (34)$$

where $n^2 = Z\,Y = (R + p\,L)\,(G + p\,C)$

Solving (33) and (34) by a direct application of the expansion formula, there is

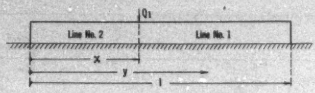


FIG. 26—POINT CHARGE SUDDENLY PLACED ON A LINE

$$V_1 = V_2 = Q\,\phi\,(t) = \frac{2\,Q}{l\,C}\,\epsilon^{-\alpha t}$$

$$\sum_{s=1}^{\infty} \sin\frac{s\,\pi\,x}{l}\,\sin\frac{s\,\pi\,y}{l}\left(\cos\beta\,t - \frac{\gamma}{\beta}\sin\beta\,t\right) \qquad (36)$$

where

$$\alpha = \frac{1}{2}\left(\frac{G}{C} + \frac{R}{L}\right) = \text{attenuation factor} \qquad (37)$$

$$\gamma = \frac{1}{2}\left(\frac{G}{C} - \frac{R}{L}\right) = \text{distortion factor} \qquad (38)$$

$$\beta = \sqrt{\frac{s^2\,\pi^2}{l^2\,L\,C} - \gamma^2} = \text{wave length factor} \qquad (39)$$

Suppose that the charge Q instead of being a point charge suddenly applied, is distributed along a section of the line and is applied gradually over a period of time. The charge on the line is then distributed as a space and time function $Q\,(x, t)$. The amount of charge on the line element $d\,x$ is $Q\,(x, t)\,d\,x$. By Duhamel's theorem, Equation (24), the voltage due to this point charge $Q\,(x, t)\,d\,x$ is

$$d\,V = \left\{ Q(o, x)\,.\,\phi(t) + \int_0^t \phi(t-\tau)\,.\,\frac{\partial\,Q\,(x, \tau)}{\partial\,\tau}\,d\,\tau \right\} dx \qquad (40)$$

The voltage due to the total space distribution is

$$V = \int_0^l Q\,(o, x)\,\phi\,(t)\,d\,x + \int_0^l\int_0^t \phi\,(t-\tau)\,\frac{\partial\,Q\,(x, \tau)}{\partial\,\tau}\,d\,\tau\,d\,x \qquad (41)$$

If $Q\,(x, t)$ is due to the release of a bound charge by the discharge of a cloud, a finite time is required for $Q\,(x, t)$ to appear. Therefore $Q\,(o, x) = 0$. Substituting (36) in (41)

$$V = \frac{2}{l\,C}\sum_{s=1}^{\infty}\sin\frac{s\,\pi\,y}{l}\int_0^l \sin\frac{s\,\pi\,x}{l}\int_0^t \epsilon^{-\alpha(t-\tau)}\left[\cos\beta(t-\tau) - \frac{\gamma}{\beta}\sin\beta(t-\tau)\right]\frac{\partial\,Q\,(x, \tau)}{\partial\,\tau}\,d\,\tau\,d\,x \qquad (42)$$

This equation is general, in that the bound charge at different points along its distribution may be released at different rates.

Suppose that all parts of the bound charge are released at the same rate, proportioned to the function $F\,(t)$. Then

$$Q\,(x, t) = Q\,.\,f\,(x)\,.\,F\,(t) \qquad (43)$$

Under this condition, Equation (42) becomes

$$V = \frac{2\,Q}{l\,C}\sum_{s=1}^{\infty}\sin\frac{s\,\pi\,y}{l}\int_0^l f\,(x)\sin\frac{s\,\pi\,x}{l}\,d\,x \int_0^t \epsilon^{-\alpha(t-\tau)}\left[\cos\beta\,(t-\tau) - \frac{\gamma}{\beta}\sin\beta\,(t-\tau)\right]\frac{\partial\,F\,(\tau)}{\partial\,\tau}\,d\,\tau \qquad (44)$$

As an example take

$$f\,(x) = \frac{1}{2}\left[1 + \cos\left(\frac{l}{2} - x\right)\frac{\pi}{b}\right]$$

for $$\left(\frac{l}{2} - b\right) \le x \le \left(\frac{l}{2} + b\right) \qquad (45)$$

$$F\,(t) = (1 - \epsilon^{-\alpha t}) \qquad (46)$$

These functions are shown in Fig. 7. Substituting (45) and (46) in (44) and performing the integrations, these results

5. "Heaviside's Operational Calculus as Applied to Engineering and Physics," by E. J. Berg, Equations (99) and (100), *General Electric Rev.*, July 1928, Vol. 31, No. 7.

$$V = \frac{2Qa}{lC} \sum_{s=1}^{\infty} \sin \frac{s\pi y}{l} \left\{ \frac{\dfrac{\pi^2}{b^2} \sin \dfrac{s\pi b}{l} \cdot \sin \dfrac{s\pi}{2}}{\dfrac{s\pi}{l}\left(\dfrac{s^2\pi^2}{l^2} - \dfrac{\pi^2}{b^2}\right)} \right\}$$

$$\frac{a-\alpha+\gamma}{(a-\alpha)^2+\beta^2}\left[-\epsilon^{-at} + \epsilon^{-\alpha t}\cos\beta t - \frac{\gamma(a-\alpha)-\beta^2}{(a-\alpha+\gamma)\beta}\,\epsilon^{-\alpha t}\sin\beta t\right] \tag{47}$$

Let the line be measured in units of length such that $b = 1$. Replace y by $y = (l/2 \pm z)$, that is, measure distance from the center of the disturbance. Combine the trigonometric functions of space and time angles. After simplifying and rearranging the equations there is

$$V = \frac{2Q}{lC} \sum_{s=1}^{\infty} \frac{a\left(\sin\dfrac{s\pi}{2}\right)^2}{a-\alpha-\gamma} C(s,k) \left\{ -\epsilon^{-at}\cos\theta z + \frac{\epsilon^{-\alpha t}}{2}\left[S(\lambda_1) + S(\lambda_2)\right] \right\} \tag{48}$$

where

$$C(s,\kappa) = \frac{\pi^2 \sin\theta}{\theta(\theta^2-\pi^2)} \frac{\kappa^2}{\kappa^2+\theta^2} = \text{convergency factor}$$

$$S(\lambda_1) = \cos(\lambda_1) - \frac{\gamma(a-\alpha)-\beta^2}{(a-\alpha+\gamma)\beta}\sin(\lambda_1)$$

= forward wave component

$$S(\lambda_2) = \cos(\lambda_2) - \frac{\gamma(a-\alpha)-\beta^2}{(a-\alpha+\gamma)\beta}\sin(\lambda_2)$$

= reverse wave component

$$\theta = s\pi/l \qquad \kappa^2 = LC\left[(a-\alpha)^2 - \gamma^2\right]$$

$$\lambda_1 = \beta t - \theta z \qquad \lambda_2 = \beta t + \theta z$$

If the cloud discharges instantaneously, $a \to \infty$, and (48) reduces to

$$V = \frac{2Q}{lC} \sum_{s=1}^{\infty} \left\{ \frac{-\pi^2 \sin\theta}{\theta(\theta^2-\pi^2} \right\} \left(\sin\frac{s\pi}{2}\right)^2 \frac{\epsilon^{-\alpha t}}{2}$$

$$\left[\cos\lambda_1 - \frac{\gamma}{\beta}\sin\lambda_1 + \cos\lambda_2 - \frac{\gamma}{\beta}\sin\lambda_2\right] \tag{49}$$

It is of some interest to check Equation (48). The terminal conditions are satisfied as follows:

At $t = 0$, $V = 0$ because the term inside {brackets} vanish

At $t = \infty$, $V = 0$ because the decrement factors vanish

At $z = \dfrac{-l}{2}$, $V = 0$ because the term inside {brackets} vanish

At $z = \dfrac{l}{2}$, $V = 0$ because the term inside {brackets} vanish

Substituting (48) in the differential equation of the transmission line, there is, after simplification

$$-\frac{\partial^2 V}{\partial y^2} + RG + (RC+GL)\frac{\partial V}{\partial t} + LC\frac{\partial^2 V}{\partial t^2}$$

$$= \frac{2Q}{lC}\sum_{s=1}^{\infty} \frac{a\,\epsilon^{-\alpha t}}{a-\alpha-\gamma} \frac{k^2}{k^2 + \dfrac{s^2\pi^2}{l^2}} \sin\frac{s\pi y}{l}$$

$$\left[\frac{s^2\pi^2}{l^2} + RG - a(RC+GL) - a^2 LC\right]$$

$$\left\{ \frac{-\pi^2 \sin\dfrac{s\pi}{2} \sin\dfrac{s\pi}{l}}{\dfrac{s\pi}{l}\left(\dfrac{s^2\pi^2}{l^2} - \pi^2\right)} \right\} = \frac{2Q}{lC} a(a-\alpha+\gamma) LC \epsilon^{-\alpha t}$$

$$\int_0^l \sum_{s=1}^{\infty} \sin\frac{s\pi y}{l} \sin\frac{s\pi x}{l} f(x)\,dx \tag{50}$$

since the term in {brackets} appeared from the integral:

$$\int_0^l f(x) \sin\frac{s\pi x}{l}\,dx$$

of Equation (44), where $f(x)$ is the bound charge distribution. But the term under the integral of Equation (50) is the function $f(x)$ multiplied by the Green's function

$$G(x,y) = \sum_{s=1}^{\infty} \sin\frac{s\pi y}{l} \sin\frac{s\pi x}{l}. \tag{51}$$

A characteristic property of this function is that

$$\int_0^l \sum_{s=1}^{\infty} \sin\frac{s\pi y}{l} \sin\frac{s\pi x}{l} f(x)\,dx = f(y) \tag{52}$$

Therefore

$$-\frac{\partial^2 V}{\partial y^2} + RG + (RC+GL)\frac{\partial V}{\partial t} + LC\frac{\partial^2 V}{\partial t^2}$$

$$= \frac{2Q}{lC} a(a-\alpha+\gamma) LC \epsilon^{-\alpha t} f(y) \tag{53}$$

But $f(x)$ was the initial distribution of bound charge. Hence, over that portion of the line for which $f(y)$ exists, the potential is constrained, and the differential equations do not hold. But beyond the distribution of bound charge the line is not under an artificial constraint, and obeys the differential equation, since $f(y) = 0$ for such points. After infinite time, or for an instantaneous cloud discharge, the right hand side of (53) is zero and then the differential equation is satisfied at all points. But while the discharge is going on, the differential equation does not hold except for points external to the distribution $f(x)$.

Equation (**48**) shows that the potential distribution at any instant may be regarded as consisting of *forward* and *reverse* traveling waves $S(\lambda_1)$ and $S(\lambda_2)$ respectively, multiplied by the same attenuation factor ϵ^{-at}, and a term $(-\epsilon^{-at}\cos\theta Z)$ which exists only over that time interval during which the cloud is discharging. This latter term is fixed in space and vanishes as the cloud discharges. At $t = 0$ it exactly cancels the contributions of the traveling waves, so that there is no potential on the line at that first instant. Thereafter, this fixed distribution of potential falls in value while the traveling waves move out, and its cancellation effect diminishes as the cloud discharge reaches completion. This term has no real or physical existence. It is simply a mathematical compensating function which accounts for the growth and development of the traveling waves during their formation period.

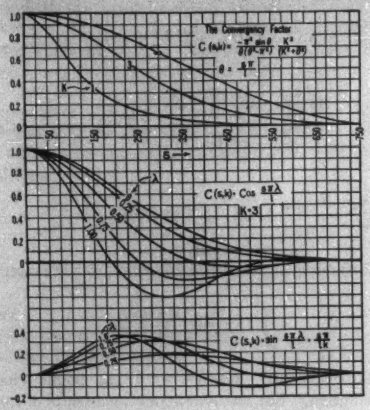


FIG. 27—HARMONIC FACTORS OF THE POTENTIAL DISTRIBUTIONS

Numerical Solution of Equation (**48**). Equation (**48**) converges so slowly, that all odd harmonics up to $(s \simeq 2\,l)$, where l is the length of the line, must be included, before the series may be terminated. This means that a point by point substitution is out of the question. However, the convergency factor $C(s, k)$ is a smooth curve which remains positive over this range of s, and the trigonometric terms do not give an excessive number of oscillations before their contributions become negligible. If $v_{(2s-1)}$ is an individual harmonic of Equation (**48**), and if the magnitudes of successive harmonics are near enough together to allow the construction of a smooth continuous curve, then as approximation

$$V = \sum_{1}^{\infty} v_{(2s-1)} \simeq \sum_{1}^{s \simeq l} v_{(2s-1)}$$

$$= \frac{l}{2}\,(v \text{ average}) = \frac{l}{2}\left(\frac{\text{Area under curve}}{\text{Length of base}}\right) \qquad (54)$$

The procedure then is:

1. Plot curves of $C(s, k) \sim s$ with κ as parameter, Fig. 27.

2. Plot curves of $[C(s, k) \,.\, \cos\theta\, z] \sim s$ with z as parameter, Fig. 27. For each z find the area under the curve with a planimeter, and the average ordinate from (**54**). Multiply these ordinates by ϵ^{-at} for required values of t, and plot the family of curves shown as $D(z, t)$ in Fig. 28.

3. Plot curves of $[C(s, k) \,.\, S(\lambda)] \sim s$ with λ as parameter, and find average ordinate as in 2, above. These values are plotted as $W(\lambda)$ in Fig. 28. This is the fully developed traveling wave (neglecting attenuation).

The potential distribution at any instant is then given by

$$V = \frac{2\,Q}{l\,C}\,\{-D(z, t) + W(\lambda_1) + W(\lambda_2)\} \qquad (55)$$

Combining the waves $W(\lambda_1)$ and $W(\lambda_2)$ at their proper space phase, with the term $D(z, t)$, wave formations such as shown in Fig. 11 were obtained.

It is quite evident that solutions such as (**48**) are of little value for obtaining numerical results. Their application is limited to bound charge distributions and cloud discharge rates which can be expressed by very simple analytic functions, such that the integrations required by Equation (**44**) can be performed. Even when these integrals can be found, and the equation brought down to the form of Equation (**48**), it is a long tedious task to calculate and plot the curves, find their areas with a planimeter, and finally combine the components to obtain the wave formations.

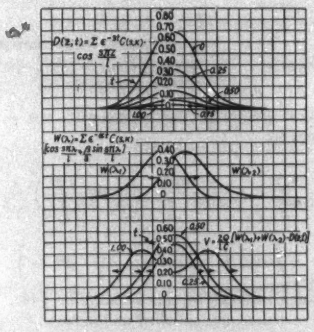


FIG. 28—THE STATIONARY AND TRAVELING COMPONENTS OF THE POTENTIAL DISTRIBUTION

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1929

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Forty-fifth Annual Report, for the fiscal year ending April 30, 1929. A general balance sheet showing the condition of the Institute's finances on April 30, 1929, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year, more detailed information having been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held seven meetings during the year, five in New York, one at Denver, Colorado, and one at Cincinnati, Ohio. The Executive Committee acted upon various matters during intervals between Board Meetings.

Information regarding the more important activities of the Institute which have been under consideration by the Board of Directors, the committees, and the various officers is published each month in the section of the JOURNAL devoted to "Institute and Related Activities."

President's Visits.—President Schuchardt has attended various national conventions and regional meetings, also Section and Student Branch meetings, during the past year.

The following is a list of places visited in addition to many trips to Institute headquarters at New York: Spokane, Wash. (Pacific Coast Convention); St. Louis, Mo.; Madison, Wis.; Atlanta, Ga., (Regional Meeting); Washington, D. C.; Schenectady, N. Y.; Milwaukee, Wis.; Chicago, Ill.; Toronto, Ont.; Cincinnati, Ohio (Regional Meeting); Ithaca, N. Y.; Urbana, Ill. (Joint meeting of Urbana Section and Purdue University and Rose Polytechnic Institute Branches). During May and June the president will attend the Dallas Regional Meeting, Section meetings at Cleveland and Akron, Ohio, and the annual Summer Convention at Swampscott, Mass.

Meetings.—Three national conventions of the Institute were held during the year, namely, the Summer, Pacific Coast, and Winter. Regional meetings under the auspices of the Geographical Districts were held in New Haven, Conn., District No. 1; Atlanta, Ga., District No. 4; Cincinnati, O., District No. 2.

Annual Meeting.—The Annual Business Meeting was held at the Cosmopolitan Hotel, Denver, Colo., on Tuesday morning, June 26, 1928, during the annual Summer Convention. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1928 was presented. The Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1928 and upon the ballots cast for the proposed constitutional amendments as submitted to the membership, under date of April 2, 1928. There was an overwhelming vote in favor of the amendments which related to change in date of annual meeting and various matters of policy and administration.

Summer Convention.—The forty-fourth annual Summer Convention was held at Denver, Colorado, June 25-29, 1928. Four technical sessions were held at which nineteen papers were presented. In addition other sessions were held as follows: the Annual Business Meeting of the Institute, a lecture on "Geophysical Methods of Prospecting," by Dr. C. A. Heiland, and two sessions at which fifteen technical committee reports were presented. The Annual Conference of Officers and Delegates under the auspices of the Sections Committee was held on Monday, June 25; forty-six Sections being represented. The entertainment features included golf and tennis tournaments, a dinner, and an all-day sightseeing trip. About five hundred members and guests attended.

Pacific Coast Convention.—The seventeenth Pacific Coast Convention was held at Spokane, Washington, August 28-31, 1928. At five technical sessions nineteen technical papers and an address were presented. Two student sessions were held at which nine papers were presented. Various social events completed the program. Attendance about two hundred and fifty.

Winter Convention.—The seventeenth Winter Convention was held in New York, N. Y., January 28–February 1, 1929. At nine sessions forty-two technical papers were presented. At another session the Edison Medal was presented to Dr. Frank B. Jewett and a lecture on "Electricity and Chemistry, Team-mates in Progress," by Dr. Harrison E. Howe was given. Numerous inspection trips, a smoker, and a dinner-dance were held. Attendance about two thousand.

Regional Meetings.—Three Regional Meetings were held during the year:—the first in District No. 1 at New Haven, Conn., May 9-12, 1928. Five technical sessions comprising eighteen papers were held, and also a student session with nine papers presented by students. Attendance about five hundred. Second, at Atlanta, Ga., District No. 4, October 29-31, 1928. Five technical sessions were held at which thirteen papers were presented. A general session was held at which two lectures were given and a student session at which five papers were presented. Attendance about six hundred. Third, at Cincinnati, Ohio, District No. 2, March 20-22, 1929. Four technical sessions were held at which seventeen papers were presented; a student session was also held. Attendance two hundred and seventy.

Sections.—While maintaining their interest in purely technical and moderately technical subjects, the Sections have given more attention than heretofore to methods of encouraging their members to participate in the meetings and in public affairs involving engineering. Their responsibilities toward the students and recent graduates have received much attention.

President Schuchardt's deep interest in these phases of Section activity resulted in the arrangement of a considerable number of special programs, in many of which he participated, and his messages in the JOURNAL dealing with the duties and opportunities of Sections attracted wide attention.

The Sections within five geographical districts cooperated in the planning and conduct of the Pacific Coast Convention and three Regional meetings.

The New York Section decided to establish a number of groups in order to extend the activities to a larger number of members, particularly the younger men. The Power Group was organized and others under consideration include Transportation, Communication, and Illumination Groups. The Chicago Section is contemplating the establishment of several groups.

Copies of the first annual report on Section and Branch activities were distributed at the conference of officers and delegates held in Denver during the Summer Convention. It was decided that a further trial of the present plan of preparing these reports should be continued for at least two more years.

As a result of the recommendations of a special committee appointed by President Gherardi early in 1928, the Sections were requested in a letter dated October 31, 1928, to encourage their members who were qualified for transfer to a higher grade to submit their applications.

Many of the Sections held joint meetings with neighboring Student Branches, some programs being supplied entirely by students, and others including both students and practising engineers. Notable among the latter type were several programs designed to assist the students to learn what the industry will expect of them after graduation. Several very successful student conventions were sponsored by Sections.

New Sections were organized at Dallas and Houston, and both have been very active. The formation of a North Carolina Section was authorized by the Board of Directors on March 21, 1929.

Student Activities.—The Branches have shown great activity during the year both in the holding of individual meetings and in participation in special meetings with other Branches and with the Sections.

Conferences on Student activities involved the Committees on Student Activities and Branch Chairmen of seven geographical districts. Also, conferences involving smaller groups of Branches were held in Pittsburgh and at Ohio State University.

Seven Student Conventions were held, three sponsored by District Committees on Student activities, three by Sections, and one by a Student Branch.

The number of talks delivered at Branch meetings by students was considerably greater than during the previous year.

President Schuchardt's messages in the JOURNAL included several of deep interest to students and recent graduates.

Special efforts were made during the year to encourage young men whose periods of enrolment as Students expired and other recent graduates to apply for admission as Associates.

All Counselors were invited to send to headquarters lists of names and addresses of graduates of their schools in the last three classes. A large number of names and addresses was received and sent to the Sections in the territories in which the men are now located. Suggestions regarding methods of forming contacts with these men were sent to the Section officers.

By means of special letter and form, efforts were undertaken to ascertain the principal reasons which prevent former Enrolled Students from applying for admission as Associates immediately after the expiration of their enrolment.

A revised edition of the Suggested By-laws for Student Branches was distributed. Upon recommendations of various District Committees on Student Activities and the Committee on Student Branches, the Board of Directors took favorable action on several matters of importance to the Branches.

New Branches were authorized at the University of Detroit, University of South Carolina, and Cornell University, and all have been active.

Section and Branch Statistics.—

	For Fiscal Year Ending			
	April 30 1923	April 30 1925	April 30 1927	April 30 1929
SECTIONS				
Number of Sections	46	49	52	54
Number of Section meetings held	344	386	431	460
Total Attendance	46,672	49,029	60,708	73,254
BRANCHES				
Number of Branches	68	82	95	100
Number of Branch meetings held	503	548	842	940
Total Attendance	26,893	27,603	42,650	47,408

Meetings and Papers Committee.—The arrangement of programs for national conventions and assistance with programs of regional meetings have been the most important duties of the Meetings and Papers Committee during the year 1928-29. The committee also has reviewed many papers submitted for publication only and given recommendations on their acceptability. The committee members have furthermore assisted the Committee on Award of Institute Prizes in selection of papers for prize awards.

Three national conventions and three regional meetings were held during the year. The technical program

of the 1929 Summer Convention is completed; the programs for the Pacific Coast Convention and the Regional meeting in Chicago are taking definite form and consideration has been given to scheduling certain papers and topics at the 1930 Winter Convention.

One hundred and forty-three papers have been presented during the year including fifteen annual reports of Technical Committees. Many excellent contributions to the information on the theory and practical applications of electricity are contained in these papers. Through the year a large number of high grade papers has been available for use at the various meetings.

The Code of the Meetings and Papers Committee, which is intended as a guide for the activities of the Committee and its members, has been revised. The revised edition includes many policies which have been adopted as a result of the Committee's experience during the last few years.

Publication Committee.—The plan of publishing the TRANSACTIONS quarterly and printing papers in abridged form in the JOURNAL, with complete copies available upon request of those desiring more detailed information, has now been in effect for sixteen months. This period has been long enough to fully demonstrate that the new plan has more completely met the requirements of the membership and further, that it costs less to publish than under the previous plan.

While the new plan has solved many of the problems of the Committee, it was felt by some that the JOURNAL would be more widely read and referred to if in addition to the present contents, special features were added that would be of general interest to the entire membership. For the purpose of obtaining the views of the Institute members on this subject, the Committee published in the January 1929 issue of the JOURNAL, an article outlining in a general way, the proposed changes that have been suggested, and asked the members to express their opinions as to what changes, if any, should be made in the Institute publications.

A large number of replies has been received and many more are expected. From the nature of those received thus far, the Committee believes that the survey will be of material value in that it will not only determine whether any consensus of opinion in regard to JOURNAL improvements exists, but will also give further information as to the attitude of the members towards the new plan in effect since January 1928. Before making an analysis of the results of this survey, the Committee feels that it would be desirable to hear from more of our members, especially from those active in Institute affairs.

Beginning with the Quarterly TRANSACTIONS for 1929 an authors' index of the papers and discussors is being included in the first three issues in order to facilitate the work of looking up references. The last Quarterly of the year will contain the complete authors' index for the year in addition to the usual classified subject index.

Coordination Committee.—The Committee on Coordination of Institute Activities has considered and made recommendations to the Board of Directors upon various matters coming within its jurisdiction under the By-laws. The Committee corresponded with the officers of all Districts and Sections and obtained their views regarding national and regional meetings during the year 1930, resulting in the recommendation to, and adoption by, the Board of Directors, in January 1929, of a complete schedule of national and regional meetings to be held during 1930.

Other matters considered by the committee included qualifications for Student enrolment and organization of Student Branches, procedure in reference to appointment of Counselors of Student Branches, conditions of award for student prize papers, appropriation for duplicating student papers, and proposed revision of various by-laws.

Standards.—The Standards committee has continued its work along recognized lines. There have been no important changes in organization nor in methods of conducting its work during the past Institute year. The Standards Committee has received valuable assistance from a number of the technical committees which have initiated new Standards work principally in revising and amplifying existing Standards. In this connection, the work of the Transformer Sub-committee of the Electrical Machinery Committee deserves special mention. An important new contribution of this subcommittee has been its work on an appendix to Transformer Standards, covering the important subject of service recommendations, with reference to operation at loads greater than rated load when air or water temperatures are lower than standard. This is still in report status, as outlined in the August 1928 JOURNAL.

A new project first called to the attention of the Institute members in the February 1929 JOURNAL, and officially recommended to the Board by the Standards Committee by action at its meeting held May 9, 1929, is the preparation of Test Codes for electrical apparatus. This is an undertaking that may prove equal, in magnitude, to the preparation of the present Institute Standards. The Institute Standards in their present form contain some information and rules for testing machinery and apparatus. There has developed a need, which the Institute has now undertaken to supply, for more complete and detailed Standards of practise covering methods of making tests similar in scope to the Power Test Codes prepared by the Power Test Code Committee of the A. S. M. E.

These two new projects—the transformer service recommendations and test codes—represent important service that, it is hoped, will prove acceptable to the industry.

The more routine work of the Standards Committee is represented by a number of new and revised Standards

adopted and additional Standards approved as American Standards by the American Standards Association. Since May 1, 1928 there have become available eight new or revised Standards pamphlets covering standards and six specifications. Three reports on proposed standards have also appeared.

Three Standards have been approved as American Standards by the A. S. A. There is now a total of sixteen Standards approved as American Standards.

The Institute is at the present time sole sponsor for ten projects and joint sponsor for eighteen projects under A. S. A. procedure. In addition to the direct responsibility represented by these sponsorships, the Institute is co-operating with A. S. A. by being represented on twenty-six additional sectional committees.

Among new Institute Standard projects under way, the following are of unusual interest and importance: Mercury Rectifiers, Switchboards and Switchgear, and Relays.

During the year the Standards Committee has continued its close cooperation in the technical work of the U. S. National Committee of the I. E. C. Following the plenary meeting of the I. E. C. at Bellagio, Italy, in the fall of 1928, the I. E. C. submitted for approval a revision of Publication 34 covering Standards of machinery and transformers. The Institute Standards Committee cooperated in the preparation of the statement of the U. S. N. C., disapproving the adoption of this document by the I. E. C. and in the preparation of the statement of the American proposals to be submitted to a meeting of the Rating Advisors of the I. E. C. to be held in London in the coming summer.

U. S. National Committee of the I. E. C.—The activities of this committee during the past year have been chiefly of a routine character. Important Advisory Committee Meetings have been held during the year. A meeting of the Subcommittee of the Advisory Committee on Nomenclature, for which the U. S. National Committee exercises the secretariat, was held in Paris in April 1928. At this meeting considerable progress was made on the vocabulary which the committee has in preparation.

A meeting of the Subcommittee on Symbols for Telegraphy and Telephony which was held in Berne in April 1928 in conjunction with a corresponding committee of the International Consultative Committee on Long-Distance Telephony resulted in a revised list of graphical symbols for telegraphs and telephones being prepared for submission to the National Committees.

A very important meeting of the Advisory Committee on Steam Turbines (Secretariat—U. S. National Committee) was held at The Hague in May 1928. There were twenty-three individuals from seven nations other than the U. S. and Holland in attendance. Great progress was made in drafting Rules for Acceptance Tests of Steam Turbines. A further meeting of this committee is to be held during the coming summer and it is expected that a set of international rules covering steam turbines will be ready for adoption at the Plenary Meeting in Stockholm in 1930.

It is expected that meetings of the Advisory Committee on Rating of Electrical Machinery and of the Advisory Committee on Instruments will be held next summer. Preparations for these meetings are under way.

U. S. National Committee of the International Commission on Illumination.—The last year has been a very important one for this committee inasmuch as during it occurred the American Meeting of the Commission. The committee was fortunate in enlisting the good offices of the Illuminating Engineering Society in making arrangements for this meeting. The meeting was attended by about sixty delegates from twelve different nations and was held in Saranac, New York, from September 22nd to 28th.

Subsequent to the meeting, the delegates were taken on a tour which included New York, Boston, Philadelphia, Cleveland, Detroit, Chicago, Niagara Falls, concluding at Toronto where the delegates attended the annual convention of the Illuminating Engineering Society.

The Commission has some fifteen technical committees and considered recommendations and reports from the various national committees with a view to formulating recommendations for adoption by the Commission.

The technical papers at this meeting covered a wide variety of subjects, the practical details of lighting practise receiving a large amount of attention. Discussions of street lighting, automobile headlighting, factory and school lighting, residence lighting, aviation field lighting, show window lighting, of a very thorough-going and valuable character and interspersed with other discussions of more specialized topics of interest to the photometrists, etc. To the other technical committees for which the United States National Committee exercises the secretariat were added two new ones; *viz.*, the Technical Committee on Lighting for Aerial Navigation and the Technical Committee on General Practical Applications of Illumination.

The Proceedings of the meeting, now being printed, will constitute a very notable summary of the present state of the art and science of illumination as it exists not only in this country but throughout the world.

The next meeting of the Commission is to be held in 1931 in England.

Committee on Safety Codes.—No new rules for the protection of persons or property against fire accidents and other hazards in connection with electrical installations and equipments were referred to this committee during the year, either from other technical committees within the Institute or from bodies outside of the Institute.

The Institute was represented by one of the members of the Committee on Safety Codes at the annual Safety Congress held in New York City, October 1 to October 5, 1928. The particular committee whose activity seemed to be most directly related to the scope of the A. I. E. E. activity was the Low-Voltage Electrical Committee of the Engineering Section of the National Safety Council.

The National Electric Light Association, which was represented at the meeting, reported that it was tabulating statistics with respect to accidents caused by low-voltage equipment.

The Committee on Safety Codes was represented at the regular annual meeting of the Electrical Committee of the National Fire Protection Association on February 18th and 19th in New York City. Probably the outstanding feature of the meeting was the general agreement reached with respect to certain controversial questions on grounding and wiring methods.

The committee was represented also at the annual meeting of the National Fire Waste Council held in Washington on March 26th, 1929.

Technical Committees.—Reports of Technical Committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the annual Summer Convention and printed in the TRANSACTIONS.

Membership—The additions to, and deductions from, the membership are shown in the following table. In accordance with the established policy of the Institute, emphasis has continued to be placed on the qualifications of candidates. The total membership as of April 30 is 18,133; this indicates a small net loss during the year, which is due principally to the deductions as shown. There is every indication that the development of section, student, and other activities, now in progress, will result in a steadily increasing list of applicants during the coming years.

	Honorary	Fellow	Member	Associate	Total
Membership on April 30, 1928	4	659	3,163	14,439	18,265
Additions:					
Transferred	4	38	219		
New Members qualified	1	4	101	1,238	
Reinstated		1	13	85	
Total	9	702	3,496	15,762	19,969
Deductions:					
Died	1	9	22	46	
Resigned		1	20	328	
Transferred		3	32	226	
Dropped		2	43	1,103	
Membership April 30, 1929	8	687	3,379	14,059	18,133

Deaths.—The following deaths have occurred during the year.

Honorary Member: Ferdinand Foch.

Fellows: William Esty, Valeré A. Fynn, Edwin B. Katte, William Weaver, Jr., William N. Motter, William Pestell, Samuel G. Rhodes, Allen M. Schoen, Edgar E. Stark.

Members: Johan M. Anderson, William F. Bennington, Frederick S. Blackall, Adam Bosch, Charles S. Bradley, George Broughall, Thomas Duncan, Rufus T. Gent, John C. Hatzel, Basil F. Howard, Benjamin N. Jones, Fred A. Jones, Robert C. Jones, Herbert Lloyd, John J. Murphy, John G. Pertsch, Jr., Theodore C. Roberts, Frank R. Shepherd, William L. Simpson, Robert I. Todd, Francis R. Weller, Charles A. Wright.

Associates: Leo O. Ambrose, Philip V. Berger, Harley S. Berry, Paul T. Brady, Wallace L. Brooks, V. Frank Buddeke, John J. Burke, Lester W. Cooper, E. Preston Clifford, Robert L. Crawford, John H. Cuntz, Orin D. Dausman, Earle N. Dillard, Alex. C. Dogherty, Charles N. Fenstermacher, Earl R. Filkins, Arthur L. Freyer, Joseph E. Frisbee, Francis M. Garretson, Jr., George F. Harrington, William S. Hearding, Francis R. Howe, Griffith O. Hughes, Harold B. James, Gunnar Jensen, Samuel J. Keese, James F. Kimbell, Harry R. Knettles, Joseph W. Legg, Clarence R. Lininger, Roy B. MacPherson, Harry C. Martell, Seizo Misaki, Gilbert Mitchell, E. J. Morrissey, Alfred G. Place, Donald K. Prossor, William M. Stratton, Harold P. Thomas, Walter D. Uptegraff, Lauro G. Villanueva, Frank C. Wagner, Charles S. Winston, Cecil F. Wilson, Kyojiro Yosugi, Nelson T. Young.

Board of Examiners.—The Board of Examiners held twelve meetings during the past year averaging about three and one-half hours. A total of 3115 cases were considered, divided as shown in the following table:

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate	1064	
Not recommended	20	1084
Recommended for grade of Member	97	
Not recommended for admission to this grade	22	119
Recommended for grade of Fellow	3	
Not recommended for admission to this grade	2	5
Recommended for enrolment as Students		1612

APPLICATIONS FOR TRANSFER

Recommended for grade of Member	229	
Not recommended for transfer to this grade	19	248
Recommended for grade of Fellow	45	
Not recommended for transfer to this grade	2	47
Total number of applications considered		3115

Institute Prizes.—Four National Prizes and eleven Regional Prizes for papers were awarded during the year, as announced through the Institute JOURNAL. These prizes were for papers presented at Institute meetings during 1927. The National Prizes were presented at the 1928 Summer Convention and the Regional Prizes at various meetings in the respective Districts. The National Prizes consist of $100.00 each and a certificate, while the Regional Prizes consist of $25.00 cash and a certificate.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute each year a scholarship in electrical engineering for each class. The awards are made annually by an Institute committee. The scholarships pay $350 toward annual tuition with provision for reappointment. An award was made for the academic year 1928-9 to Walter L. Lawrence.

Complete details governing prizes and scholarships may be obtained by applying to the National Secretary of the Institute.

Edison Medal.—The Edison Medal, founded by associates and friends of Thomas A. Edison, is awarded annually by a committee consisting of twenty-four members of the Institute "for meritorious achievement in electrical science, electrical engineering, or the electrical arts." The medal for 1928 was awarded to Frank B. Jewett, "for his contribution to the art of electrical communication," The presentation took place at the Winter Convention of the Institute, January 30, 1929.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical, and Electrical Engineers, awarded the twenty-fifth medal to Herbert Clark Hoover, "Engineer, Scholar, Organizer of Relief to War-Stricken Peoples, Public Servant." The medal was presented by the Board of Award at the Executive Mansion, Washington, D. C., Tuesday, April 25, 1929.

Lamme Gold Medal.—The Lamme Medal was founded as a result of a bequest in the will of the late Benjamin G. Lamme, Chief Engineer of the Westinghouse Electric and Manufacturing Company, who died on July 8, 1924. The bequest provides for the award by the Institute of a gold medal (together with a bronze replica thereof) annually to a member of the A. I. E. E. "who has shown meritorious achievement in the development of electrical apparatus or machinery" and for the award of two such medals in some years if the accumulation from the funds warrants.

The Lamme Medal Committee of the Institute has awarded the first (1928) Lamme Medal to Mr. Allan Bertram Field "for the mathematical and experimental investigation of eddy current losses in large slot-wound conductors in electrical machinery." Arrangements are being made for the presentation of the Medal at the annual Summer Convention at Swampscott, Mass., June 24-28.

Commission of Washington Award.—The Washington Award for 1929 was made to Bion J. Arnold "for pioneering work in the engineering and economics of electrical transporation." Mr. Arnold is the second past-president of the A. I. E. E. to receive this honor.

The award is made annually "to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good," by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E., and the A. I. E. E.

Employment Service.—The employment service which the Institute has maintained for many years is conducted as a cooperative bureau in conjunction with a similar service maintained by the National Societies of Civil, Mining, and Mechanical Engineers under the title, "Engineering Societies Employment Service." In addition to the main office in the Engineering Societies Building, New York, offices have been opened in Chicago in cooperation with the Western Society of Engineers and in San Francisco under similar arrangements with the California Section, American Chemical Society, and Engineers Club of San Francisco. It is hoped to continue this development from year to year as conditions warrant. The service is supported by the joint contributions of the societies and their individual members who are benefited. As in the past it consists principally in acting as a medium for bringing together the employer and the employe. In addition to the publication of the "Employment Service Bulletin" in the monthly JOURNAL, weekly subscription bulletins are issued for those seeking positions.

American Engineering Council.—Since the organization of American Engineering Council nearly nine years ago, Council has experienced a steady normal growth. It now comprises twenty-five national, state, and local engineering societies which have an individual membership of over 44,000 professional engineers. Last December the American Society of Civil Engineers voted to become a member of Council, and has since been admitted.

Council represents its constituents at conferences, conventions, and on important national committees, and at Congressional hearings upon matters involving engineering. It also extends valuable individual service to its member organizations. On matters of concern to the entire engineering profession Council has conducted several notable surveys. During 1928 it completed a report on Street Traffic Signs, Signals, and Markings. It is now engaged in a movement through the engineering organizations to have the report presented for consideration of municipal authorities.

Council also, in cooperation with the National

Bureau of Economic Research, completed its portion of the survey on Recent Economic Changes. It has actively supported measures providing for an inventory of our water resources; measures providing for equitable status of sanitary engineering; a more extensive appropriation for topographic mapping; the unification and coordination of governmental public works functions; the establishment of a national hydraulic research laboratory, etc.

Council has continued the publication of a quarterly known as "A. E. C. Bulletin" and also supplies to national member organizations a monthly engineering news letter. The following are some of the committees in active operation: Public Affairs; Publicity and Publications; Flood Control; Government Reorganization; Patents; Power; Research Program; Reforestation; Street and Highway Safety; Communications; Finance; Uniform State Mechanics Lien Act.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical, and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Engineering Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1929 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining, and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc. A lending department is also maintained.

A copy of the annual report of the Engineering Societies Library covering the calendar year 1928 may be obtained by applying to Institute headquarters.

Engineering Foundation.—Engineering Foundation is a trust fund established in 1914 by Ambrose Swasey, of Cleveland, Ohio, by gifts to United Engineering Society as a nucleus of a large endowment "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." The fund has been generously increased through the gifts of Edward D. Adams and others, and also through bequest under the will of the late Henry R. Towne. It is administered by the Engineering Foundation Board upon which the Institute and other national engineering societies are represented. The Board is a department of United Engineering Society.

The Foundation has made appropriations for various research projects and has cooperated in others.

An abstract of the annual report of the Foundation was published in the JOURNAL for May 1929 and a complete report is available in printed form.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has accepted sponsorship and appointed representatives upon a number of new Sectional Committees of the American Standards Association. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report follows:

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON,

National Secretary.

New York, May 22, 1929.

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
OFFICES IN THE PRINCIPAL CITIES OF
THE UNITED STATES OF AMERICA
—AND IN—
LONDON, PARIS, BERLIN, SHANGHAI, MANILA,
MONTREAL, HAVANA, MEXICO CITY

NEW YORK CENTRAL BUILDING
75 EAST 45TH STREET

May 20, 1929.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

We have made a general audit of your accounts for the year ended April 30, 1929, and submit the following exhibits and schedule:

Exhibit

A—Balance Sheet, April 30, 1929.

Schedule 1—Reserve Capital Fund—Securities.

B—Summary of Income and Profit and Loss for the Year Ended April 30, 1929.

WE HEREBY CERTIFY that in our opinion Exhibits A and B, respectively, correctly set forth the financial condition of the Institute at April 30, 1929, and the results of its operations for the year ended that date.

Yours truly,

HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

EXHIBIT A. BALANCE SHEET, APRIL 30, 1929

ASSETS

REAL ESTATE:			
One-fourth interest in United Engineering Society's Land, Building, and Building Equipment, 25 to 33 West 39th Street, (depreciation carried on books of the United Engineering Society)			$493,352.60
EQUIPMENT:			
Library—Volumes and fixtures		$ 41,452.30	
Works of art, paintings, etc.		3,001.35	
Office furniture and fixtures	$28,003.45		
Less reserve for depreciation (including $7,431.79 funded)	12,243.65	15,759.80	
Total equipment			60,213.45
WORKING ASSETS:			
"Transactions," etc.		$ 4,796.00	
Text and cover paper		3,700.10	
Badges		1,931.54	
Total working assets			10,427.64
CURRENT ASSETS:			
Cash		$ 23,273.56	
Note receivable		437.50	
Accounts receivable:			
Members—For dues		22,664.01	
Advertisers		1,179.50	
Miscellaneous		8,081.55	
Accrued interest on investments		2,529.89	
Total current assets			58,166.01
FUNDS:			
Reserve Capital Fund:			
Securities—Schedule 1		$171,370.63	
Life Membership Fund:			
Cash	$ 6,525.41		
Chicago, Burlington & Quincy Railroad Company 4% bonds, 1958, registered, par value, $5,000.00	$ 4,868.75		
Accrued interst	33.33	11,427.49	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 1,097.89		
New York City 4½% corporate stock, 1957, par value, $2,000.00	2,204.05		
New York Telephone Company 4½% bond, 1939, registered, par value, $1,000.00	878.75		
Accrued interest	67.50	4,248.19	
Mailloux Fund:			
New York Telephone Company 4½% bond, 1939, registered, par value, $1,000.00	$ 1,000.00		
Accrued interest	22.50	1,022.50	
Lamme Medal Fund:			
Cash	$ 456.07		
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series "C" bond, 1995, registered, par value, $4,000.00	4,330.00		
Accrued interest	100.00	4,886.07	
Depreciation of Furniture and Fixtures Fund:			
Cash	$ 3,421.79		
Cleveland Union Terminals Company 5% sinking fund series "B" gold bonds, 1973, registered, par value, $4,000.00	4,010.00	7,431.79	
Midwinter Convention Fund:			
Cash		26.70	
Total funds			200,413.37
Total			$822,573.07

LIABILITIES

CURRENT LIABILITIES:		
Accounts payable	$ 9,905.27	
Dues received in advance	2,823.17	
Entrance fees and dues advanced by applicants for membership	519.85	
Subscriptions for "Transactions" received in advance	24.00	
Total current liabilities		$ 13,272.29
FUND RESERVES (NOT INCLUDING DEPRECIATION RESERVE):		
Reserve Capital Fund	$171,370.63	
Life Membership Fund	11,427.49	
International Electrical Congress of St. Louis—Library Fund	4,248.19	
Mailloux Fund	1,022.50	
Lamme Medal Fund	4,886.07	
Midwinter Convention Fund	26.70	
Total fund reserves (not including depreciation reserve)		192,981.58
SURPLUS, Per Exhibit "B"		$616,319.20
Total		$822,573.07

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT AND LOSS FOR THE YEAR ENDED APRIL 30, 1929

EXHIBIT B.

INCOME:			
Dues		*$227,274.03	
Students' fees		10,792.00	
Entrance fees		13,356.00	
Transfer fees		1,315.00	
Advertising		84,790.80	
Journal subscriptions		10,152.92	
"Transactions" subscriptions		20,737.41	
Miscellaneous sales		11,086.45	
Badges sold	$ 4,073.75		
Less cost	3,599.93	473.82	
Interest on securities in reserve capital fund		7,942.29	
Interest on bank balances		1,487.46	
Interest on securities in depreciation reserve fund		250.00	
Total			$389,658.18
EXPENSES:			
Publications:			
Journal	$84,832.06		
"Transactions"	37,316.57		
Year Book	8,668.32		
Miscellaneous	5,537.64	$136,354.59	
Institute meetings		21,300.52	
Administrative expenses		60,803.98	
Sections		34,516.31	
Membership		7,323.40	
Standards		9,826.74	
Finance		400.00	
Code committees		60.00	
Technical committees		163.30	
Edison Medal		156.44	
Geographical districts' expenses:			
Traveling expense:			
Executive committee	$ 1,958.73		
Vice-Presidents	673.46		
Speaker's bureau	107.24		
Best Paper Prize	100.00		
First Paper Prize	100.00	2,939.43	
Branches		10,387.38	
American Engineering Standards Committee		1,500.00	
International Electrotechnical Commission		1,152.18	
Forward		$286,884.27	$389,658.18

TOTAL INCOME—FORWARD		$389,658.18
EXPENSES—(FORWARD)	$286,884.27	
United States National Committee of International Commission on Illumination	300.00	
Louvain Memorial	200.00	
President's special appropriation	668.58	
Board of Directors, traveling expenses	4,454.23	
National Nominating Committee, traveling expenses	893.47	
Institute representatives, traveling expenses	109.73	
Honorary Secretary	4,000.00	
Engineering Societies Library, maintenance	9,334.71	
United Engineering Society assessment	5,237.43	
American Engineering Council	17,673.50	
Engineering Societies Employment Service	1,635.00	
Institute prizes	400.00	
Total		331,790.92
NET INCOME		$ 57,867.26
PROFIT AND LOSS CREDITS:		
Adjustment of "Transactions" inventory	$ 458.59	
Profit on redemption of Cleveland Union Terminals Company 5% sinking fund series "B" gold bond, 1973, registered, par value, $1,000.00 (Depreciation Reserve Fund)	47.50	
Total		506.09
GROSS SURPLUS FOR THE YEAR		$ 58,373.35
PROFIT AND LOSS CHARGES:		
Uncollectible dues and members charges written off	$ 11,628.68	
Furniture, fittings, and equipment scrapped or sold—Net	130.25	
Provision for depreciation of furniture and fixtures	2,092.91	
Total		13,851.84
SURPLUS FOR THE YEAR		$ 44,521.51
SURPLUS, MAY 1, 1928	$599,472.69	
Less transferred to Reserve Capital Fund in accordance with resolution of the Board of Directors	27,675.00	571,797.69
SURPLUS, APRIL 30, 1929		$616,319.20

*Includes $91,325.00 allocated to JOURNAL subscriptions.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Reserve Capital Fund—Securities

April, 30, 1929

Exhibit A.
Schedule No. 1.

	Par Value	Book Value
Railroad Bonds:		
Baltimore and Ohio Railroad Company 4½% convertible gold bonds, due 1933, registered	$ 10,000.00	$ 9,387.50
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series "C" bonds, due 1995, registered	8,000.00	8,940.00
Central of Georgia Railway Company 5½% refunding and general mortgage series "B" bonds, due 1959, registered	5,000.00	5,283.75
Chicago and Erie Railroad Company 5% first mortgage gold bonds, due 1982, registered	1,000.00	1,105.00
Chicago, Burlington & Quincy Railroad Company 5% first and refunding mortgage series "A" gold bond, due 1971, registered	1,000.00	1,010.00
Chicago, Terre Haute & Southeastern Railway Company 5% first and refunding mortgage gold bonds, due 1960	8,000.00	7,940.00
Florida East Coast Railway Company 5% first and refunding mortgage series "A" gold bonds, due 1974, registered	10,000.00	9,818.75
Great Northern Railroad Company 5½% general mortgage series "B" gold bonds, due 1952, registered	10,000.00	9,847.50
St. Louis, San Francisco Railway Company 5% prior lien mortgage series "B" bonds, due 1950, registered	6,000.00	5,497.50
Southern Railway Company 5% first consolidated mortgage gold bonds, due 1994, registered	1,000.00	980.00
The New York Central Railroad Company 5% refunding and improvement mortgage series "C" bonds, due 2013, registered	6,000.00	5,742.50
The Pennsylvania Railroad Company 4½% general mortgage series "A" gold bonds, due 1965, registered	5,000.00	5,130.00
Total (Forward)	$ 71,000.00	$ 70,682.50

	Par Value	Book Value
Total (Forward)	$71,000.00	$ 70,682.50
Public Utility Bonds:		
Consolidated Gas Company of New York 5½% gold debentures, due 1945, registered	$ 5,000.00	$ 5,187.50
Duquesne Light Company, 4½% first mortgage series "A" gold bonds, due 1967, registered	3,000.00	2,970.00
New York Telephone Company debenture 6% gold bond, due 1949	5,000.00	5,525.00
Northern States Power Company 5½% first lien and general mortgage gold bond, series "B," due 1950	10,000.00	10,400.00
Pacific Gas & Electric Company 5½% first and refunding mortgage gold bonds, due 1952, registered	5,000.00	5,137.50
Philadelphia Company secured 5% gold bond, series "A," due 1967	10,000.00	10,000.00
The American Telephone and Telegraph Company 5% gold debentures sinking fund, due 1960, registered	15,000.00	14,625.00
The Detroit Edison Company 6% first and refunding mortgage series "B" gold bonds, due 1940, registered	5,000.00	5,178.13
United Light and Power Company 5½% first lien and consolidated mortgage gold bond, due 1959	5,000.00	4,975.00
Total	$ 63,000.00	$ 63,998.13
Industrial Bonds:		
American Smelting and Refining Company 5% first mortgage 30-year series "A" gold bonds, due 1947, registered	$ 9,000.00	$ 9,085.00
Bethlehem Steel Company 5% purchase money and improvement mortgage 20-year sinking fund gold bonds, due 1936, registered	5,000.00	5,033.75
New York Steam Corporation 6% first mortgage gold bonds, due 1947, registered	10,000.00	10,837.50
The Western Electric Company 5% bonds, due 1944, registered	10,000.00	9,818.75
United States Rubber Company 5% first and refunding mortgage series "A" bonds, due 1947, registered	2,000.00	1,915.00
Total	$ 36,000.00	$ 36,690.00
Total	$170,000.00	$171,370.63

"Synchronized at the Load"

A Symposium on New York City 60-Cycle Power System Connections

I. A Fundamental Plan of Power Supply
By A. H. KEHOE

II. Calculations of Systems Performance
By S. B. GRISCOM

III. System Tests and Operating Connections
By H. R. SEARING AND G. R. MILNE

Review of the Subject

Under our modern conditions it has become a common experience to have certain improvements adopted as necessities, although at some earlier time they would have been classed as extreme luxuries. Electrical service rendered by metropolitan public utility companies in recent years has evolved into one of these necessities due to the change in customs and in living conditions of the American people. In the initial stages of development, while the central station industry was taking the preliminary steps to establish itself in the commercial field, its product being expensive was purchased only where it could be indulged in as a luxury. During this period, methods of utilization of electrical service were very limited, being confined almost entirely to illumination. Such electric systems as existed were small, and the problem of their continuity was not of major importance. Disturbance and even interruption to customers' supply was a regrettable but not a vital incident. Cost of energy was the principal consideration rather than the reliability of supply, particularly where strong competition with other established forms of service was to be expected. As the art developed, many efforts were made to improve the quality of service in order to make a more salable product; nevertheless some interruptions were still experienced and were usually considered a necessary evil. With a much improved, but less expensive product, other competing methods of supply gradually became obsolete, leaving the central station alone in its field with an obligation to render the best possible service. Then interruptions could no longer be considered minor incidents.

Service to districts with high load density, such as the central portion of cities, very soon felt the effect of this requirement, and the demand was usually met by adopting systems of distribution which would give reliable service although at increased expense over other methods of supply, which, while much less costly, could not offer the same degree of reliability. The influence of quality service had its effect on the requirements of these as well as other districts, and the entire elimination of service interruptions has everywhere been given the most serious consideration due to the dependence of the customer on a reliable service that will not cause interruptions resulting in serious inconvenience, economic loss, or possible hazard to life. Also there are many new types of utilization that are constantly being adopted as higher reliability with lower costs makes these available to all. With further reduction in costs, many other new applications are certain to arise. Thus, it follows that low-cost reliable service produces greater utilization which, in turn, makes it imperative to render still better service. This characteristic development is a feature of American life, and its counterpart in some form of material progress will be found to be the basis for most of the changes which have taken place in our modern customs. In the case of metropolitan central station companies, the huge concentrations of power required to provide the services which are a necessity for the normal life and business of the community, make the problem of reliability of supply at low system costs of vital importance to all.

The papers comprising this symposium present a new principle of decentralization of power sources and cover the investigation of a new plan of design and operation of a metropolitan power system. This arrangement is expected to provide the solution to the problem of reliability of supply and low system costs. The underlying ideas have been verified not only by experiments on one system but also by an extensive engineering analysis. Close agreement was observed between calculated and test results, and this should justify the use of the analytical methods outlined in applying the principles involved to the design of other power systems. The fundamental plan is described and the methods and results of calculations made are given. The symposium concludes with system tests and operating experience with the new arrangement of connections.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

"Synchronized at the Load"—I

A Fundamental Plan of Power Supply

BY A. H. KEHOE[1]
Fellow, A. I. E. E.

Synopsis.—The New York 60-cycle system is operating with connections giving parallel paths between generating units only at the load. High values of reactance result for synchronizing circuits between the several generating units, but the usual values are present in the main energy channels between generators and load. Connections are contrasted with two common types of metropolitan power systems; i. e., the "close linked" and "loose linked" systems, and are shown supplemental to multiple-feed network distribution, as the principle of using separate multiple feeds from independent generators connected in parallel only at the load extends the network distribution system to the generating stations. In completely networked distribution, the 120/208-volt mains acting as a short-circuit-proof bus, supply the only synchronizing paths between generators. Substation busses serving radial load are substituted where complete networks do not exist. Calculations, tests, and operating experience indicate synchronizing power sufficient to give stability to all elements of the system not directly affected by a fault. Some advantages derived are: Increased reliability of generating sources, lower interrupting duties on circuit breakers, and reduction in service voltage disturbance.

Connections for Improved System Behavior

THE New York City 60-cycle power system is now operating with an arrangement of connection which gives paralleling paths between generating units only at the load. Each generator while in service on this system is described as being "synchronized at the load." This term refers to the conditions under which each unit is maintained in synchronism, but it does not refer necessarily to the initial synchronizing of one generator with the rest of the system. Fig. 1 shows a simplified diagram of the arrangement. Such a scheme of connections gives high reactance values for the synchronizing power circuits between the several generating units in comparison to the values which are in general use with generators also synchronized at the station bus. Nevertheless, the main energy channels between generators and the load have reactances, in respect to normal capacity, of the same order as those usually experienced with metropolitan underground cable systems. Thus, while this arrangement does not fundamentally affect the normal power distributing characteristics of the system, it does improve system behavior during abnormal conditions in several ways which are discussed in this and in companion papers.[2]

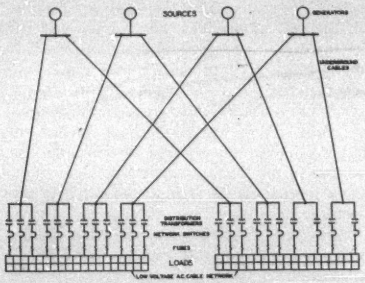


Fig. 1—Schematic Connections for System Synchronized at the Load

Typical Power System Arrangements

Ever since d-c. machines were first operated in parallel, or following this when individual drive alternators were first synchronized, the usual arrangement has been to provide for paralleling as near to the sources of power as possible. For a number of years, machine capacities and system reliability have required some means of obtaining increased reactance values to limit fault currents, and many large station busses have been broken up into sections by inserting current-limiting reactors. Generator leads supplying the bus and feeders from the bus in some cases have been similarly equipped. Such busses usually take the form of a closed loop or ring and are composed of a number of sections, each connected to its adjacent sections through reactances. Another scheme in use is the star arrangement, in which each individual bus section is connected through a reactor to one common bus or star point. Since the amount of reactance as well as its location in the circuit can easily be varied, many different plans of bus layout are in use. In general, however, it has become standard practise to parallel generators in a generating station as close to the source as possible by using some plan of either the "ring" or "star" bus arrangement.

Whenever systems are supplied from more than one generating station, it is customary to use either tie feeders between stations to provide an extension of the paralleling bus, or the system is divided into districts which are normally tied together with relatively low-capacity ties between each district to maintain the system in synchronism as an aid to operation under

1. Electrical Engineer, The United Electric Light and Power Co., New York, N. Y.

2. *Calculations of System Performance*, by S. B. Griscom. *System Tests and Operating Connections*, by H. R. Searing and G. R. Milne.

normal conditions. With the first mentioned arrangement (Fig. 2), that is, with tie feeders between generating stations, the capacity in several stations is used to supply a wide area. Therefore, it is essential that synchronism be maintained not only between all units in one station but also between all stations. This

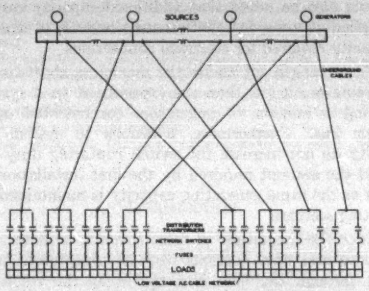


Fig. 2—Schematic Connections Close Linked System

has been designated as the close linked system. With the district arrangement (Fig. 3), or loose linked system, the load is definitely allocated to one adjacent station, wherein it is not essential that it should be maintained in synchronism with the remaining parts of the system. Assuming that the low-capacity ties will open properly on a major fault, this station will operate as an independent unit, alone supplying its particular district.

Characteristics of "Close Linked" and "Loose Linked" Systems

The close linked system requires high rupturing capacity circuit breakers, with increasing rupturing duty as the capacity of the system is increased. The fault protection is complicated, and positive back-up protection is difficult to obtain in case the primary protection fails. During system faults, service voltages are subject to excessive drops due to concentrations of current at the fault. Finally, the entire system service is at stake in case of a series of major faults.

On the other hand, the loose linked system by subdivision of the load, eliminates the likelihood of an entire system shutdown, but it does increase the probability of interruptions to each district. In addition, each station must have some efficient units if economical results are to be obtained. This system, except at district border fringes, cannot obtain the economies and reliability from diversity of a multiplicity of sources of supply. In other words, the loose linked scheme postulates that a power system can become so large that it is either economical, or imperative, to divide it into a number of separate districts power systems in order to obtain reliability.

Improvement in Reliability of Generating Sources

Investigation of the reliability of both the above systems made it appear that a generating station bus system providing parallel synchronizing paths would result in increased reliability to either a close linked or a loose linked system, with either ring or star bus plan. However, such an arrangement increased the switch rupturing duty and the voltage disturbances on system faults. Furthermore, it did not simplify the protection problem.

In 1924, following two years' experience with the Manhattan multiple feed a-c. network system, the author proposed for maximum reliability to operate each generator in parallel with remaining generators only on the low-voltage network.[3] This system uses the load points as a short-circuit-proof paralleling bus for all the units which are "synchronized at the load." In applying the principles established by experience with multiple-feed low-voltage network distribution to a group of generators multipled only on a low-voltage network, it was assumed that if the system were stable, the following desirable conditions would result: (a) highly reliable sources of supply, since they were not affected simultaneously by faults; (b) positive back-up fault protection; (c) minimum fault current concentrations; and (d) minimum service voltage disturbances. The definite evaluation of the synchronizing power available and of the amount necessary to keep sufficient generating sources in step to supply the load under all conditions of system faults presented problems which prohibited the immediate application of such an arrangement. The detailed determinations of these values, both by calculation and test, are described in accompanying papers.

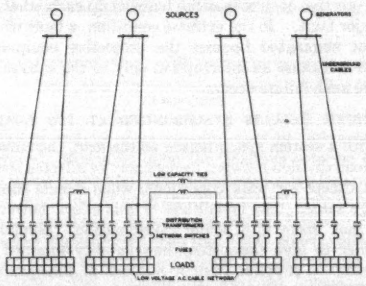


Fig. 3—Schematic Connections Loose Linked System

The synchronizing power for the New York close linked system with this arrangement was found to be adequate. While it is less under normal conditions than that which is obtained between individual units

3. *Underground A-C. Networks*, by A. H. Kehoe, A. I. E. E. Trans., Vol. 43, 1924, p. 848.

synchronized at the sources, yet the effective synchronizing ability at the time of a disturbance is approximately the same as with a close linked system, due to the reduced magnitude of possible faults, and the new system has high reliability due to a multiplicity of independent sources. With a loose linked system the reliability of any district would not be dependent upon the synchronizing ability of the system, but upon that of a single station. For this reason the number of possible independent sources for any district will be less than with a close linked arrangement which is considered preferable to the loose linked system if it is to be "synchronized at the load."

Reliability of Supply Synchronized at the Load

The assurance of reliability of system supply for any one fault is secured by the following considerations: Sufficient generating sources are "synchronized at the load" on a system having ample synchronizing power between the units to produce stable operation between all sources for any single fault regardless of its location, provided the fault is eliminated promptly by protective equipment. So long as enough such independent sources are in service to maintain the load, the loss of the reserve part of the capacity will not result in an interruption, since the remaining capacity is not jeopardized by this loss. The ability of a low-voltage network to eliminate its own faults[4] will provide positive back-up protection even though a reasonable percentage of subsequent failures of protective equipment do not eliminate the original fault. If a system has radial service supplied from a substation, the low-voltage bus, such as a 4-kv. substation bus, will be the last common load point and must be used for synchronizing at the load. In this case, protective devices are relied upon for secondary protection, as there is usually a number of protective devices in series backing up each other for a major fault. In the extreme condition, a fault which is not eliminated because the protective equipment failed will cause an interruption only to the substation where such failure occurs.

System Results Synchronized at the Load

With a system synchronized at the load, the current and load division between feeders may be affected somewhat except for peak conditions, when results should be the same. The magnitude of this effect, however, does not make it of major consideration. Similarly, in order to give the system the reliability obtained by diversity of supplies from a number of independent sources, the load from any generator (or bus section) should scatter to as many different load centers as possible. If this is carried to extremes, the mean length of feeder will be increased unnecessarily. It is pointed out that all units do not have to feed each and every load point, and in the New York case we did not experience any increase in feeder lengths. It should also be noted that additional capacity can be supplied at lower installation costs because of smaller capacity required for electrical connections.

The arrangement makes the maximum short-circuit currents much less than are experienced on a system having a number of generators concentrated on a single bus. Furthermore, additions to system capacity do not increase the switch rupturing duty beyond the amount required by the first installation as long as the same generating capacity is maintained on existing sections.

Since the magnitude of fault currents is materially reduced and since currents from all other units must feed through the load into the fault, the resultant voltage disturbances due to system faults are materially reduced. System service voltages during major faults approximating 60 per cent to 90 per cent of normal are to be expected instead of 20 per cent to 50 per cent of normal.

Conclusions

With ample synchronizing ability inherent in the system to give stability to a number of independent power sources, whether located in one or several different generating stations, generators synchronized at the load can be connected by supply feeders to a number of load points, thus making it impossible for one fault to affect all of the generating capacity; hence, if sufficient reserve capacity is in operation, the loss caused by a single fault cannot affect the reliability. By this method, reliability of supply depends upon having immediately available sufficient spare capacity in operation to compensate for a maximum number of possible major faults which are reasonably likely to occur in the interval required to get spare capacity in operation.

Acknowledgments

Acknowledgment is made to those engineers who between 1924 and 1928 have encouraged the development of the arrangement by stating that in their judgment the synchronizing ability would be ample with the 60-cycle New York City system. In particular, credit for bringing this sytem into operation is due to the Westinghouse engineering staff for its mathematical investigations and conclusions which were proved by test to be highly accurate.

4. *Underground A.-C. Network*, by A. H. Kehoe. A. I. E. E. Trans., 1924. Vol. 43, p. 845. Also, *Extinction of an A-C. Arc*, by J. Slepian, A. I. E. E. Trans., 1928, Vol. 47, p. 1404, and *Theory of the Deion Circuit Breaker*, by J. Slepian, A. I. E. E. Trans., 1929, Vol. 48, p. 523.

"Synchronized at the Load"—II

Calculations of System Performance

BY S. B. GRISCOM[1]
Associate, A. I. E. E.

Synopsis.—*This paper describes the results of calculations made for operating the Hell Gate and Sherman Creek generating stations of the United Electric Light and Power Company synchronized at the load.*

After reducing the numerous branches of the system to a simpler equivalent, calculations were made on the operation of an isolated section. These calculations indicated ample static stability and transient stability under fault conditions representing the maximum which might normally be expected.

Other calculations were made indicating that the entire system, with synchronizing paths completed through the substation low-voltage busses would likewise meet the required stability conditions.

Tentative system design factors, based on the calculations and experience to date, are suggested as a guide to system planning.

This paper shows a practical application of the theories and principles developed in the studies of transmission stability. In the present case a somewhat different object was in view inasmuch as it was desired to substantiate ideas as to the feasibility of the plan of synchronizing at the load, rather than to obtain numerical limits.

* * * * *

Introduction

In general, the principal reason for synchronizing generators at the load is that it gives promise of increasing the reliability of power supply from the generating sources. Having as a background the many papers and technical articles on stability written recently, there is a tendency to associate stability and reliability of service. The subjects are related, to the extent that a system which is capable of reliable and continuous service necessarily possesses stability to a high degree. However, with the usually understood meaning of stability, it is entirely possible to have a stable system which may not be capable of service reliability of a high order.

An example will be used to bring out the conceptions of the term "stability" and "reliability" as used in this paper. Assume that a power system is composed of only one generating station and that all of the generators in the station are connected to one bus. Load is distributed from this bus by means of feeders connected to it through reactors of conventional value, as three or five per cent. This system would be considered stable, with generator and excitation characteristics as are in common use. However, the system might not be considered reliable because it has a vulnerable spot—the single generator bus. Major trouble on the bus, or on any part of the equipment connected to it could result in a complete system outage. Thus, the system is stable but not reliable. If the bus is divided into a number of parts and the sections thus formed are connected by reactors, the combination would be regarded as having a higher order of reliability than the system with a single straight bus. Similarly, if there are two stations, with connecting ties, the over-all reliability is considered as still greater. It will thus be noticed that the insertion of reactance between units of a system has improved the reliability. What has happened to the stability? On a *long distance transmission system* stability is measured in terms of the *ability of the generators to remain in step with the receiving end of the system*, comprising the load and local generating stations. On a more *compact system* in which the load is fed through multiple paths from different generating units, the requirements of stability are met as long as the *generators remain in step with one another*. Therefore, the insertion of reactance not normally carrying current between generators does not affect the static stability of the system as a whole, but decreases the static stability of the individual generating units somewhat. The transient stability (relating to operation during disturbances such as short circuits) may be increased or decreased, depending upon the relations between synchronizing power and fault conditions for the two cases.

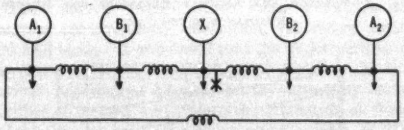


Fig. 1—Ring Bus Connection. Generator Reactance 12 per Cent

An example will be given, showing not only how increased reactance between generator busses affects transient stability, but also that other effects, difficult to evaluate, must be considered in determining the reliability. Fig. 1 shows the assumed layout. The generators are 100,000 kv-a., 12 per cent reactance each, while the connecting reactors may have values of 5 per cent, 15 per cent, and 40 per cent. Due to the symmetrical layout, it is possible to show the effect of a three-phase short circuit on one bus considering two machines only, as *A* and *B*, the short circuited machine going out of step and not affecting the others. Figs. 2,

1. General Engineer, Westinghouse Elec. and Mfg. Co., East Pittsburgh, Pa.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-91

3, and 4 show the voltage and angular relations for reactor reactances of 5 per cent, 15 per cent, and 40 per cent respectively. It should be noted that if the angular oscillation between machines is regarded as a measure of stability, the higher values of reactance afford less stability, but if voltage is also considered, the advantage is definitely on the side of higher reactances. Fig. 5 still further brings out the improvement obtained by the use of high reactance between generating units. The analysis was on the basis of an

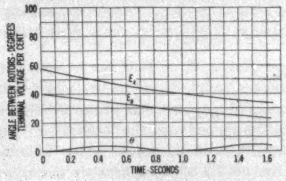


Fig. 2—Voltage and Angular Relations for Reactor Reactance 5 per Cent

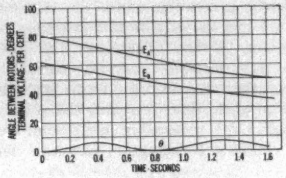


Fig. 3—Voltage and Angular Relations for Reactor Reactance 15 per Cent

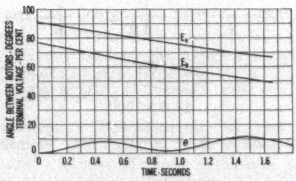


Fig. 4—Voltage and Angular Relations for Reactor Reactance 40 per Cent

isolated system, and as the short circuit deprives the generators of considerable load, the whole system speeds up. The average frequency of the machines for the three cases is shown by Fig. 5 and it will be noted that the higher the reactance between generating units, the less the over-speed. With a single station as indicated, not interconnected with other stations or other systems, this tendency to over-speed is not so serious, but if connected, there is very much more likelihood of pulling out of step from the other stations when the lower values of reactance are used. Thus, judged by angular shifts, a system with generating units tied through low reactance has greater stability than one where high reactances are used, but may have far less reliability and be subject to much greater voltage disturbances.

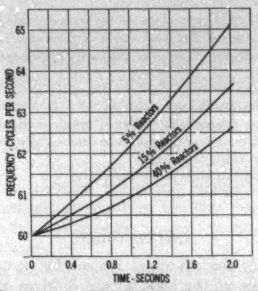


Fig. 5—Variation of Net System Frequency with Three Phase Short Circuit on One Bus of a Ring Bus System

The analysis as described above omitted a number of factors which are difficult to evaluate but which are none the less important. For instance, it was assumed that all of the generators were similar both as to rating, reactance, inertia, and initial loadings and that the governors did not operate; in other words, that everything was symmetrical. Lack of symmetry will cause more difficulty where the generators are tied through low reactance.

Stability Requirements for Satisfactory Operation when Synchronized at the Load

The fundamental plan of synchronizing at the load is described in a companion paper by Mr. A. H. Kehoe. This plan gives a redistribution of reactance on the system so that the reactance between machines and in

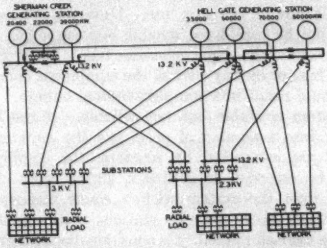


Fig. 6—Synchronized at Generating Station Busses

series with faults is considerably increased. Fig. 6 illustrates in a schematic manner the connections which have been in use in the 60-cycle parts of the generating stations of the United Electric Light and

Power Company and in the system external to them. The generating busses were tied to one another very closely, inside the stations by bus-tie reactors and outside of the stations by cables at generator voltage. The stations themselves were directly tied together by high-voltage cables. Fig. 7 shows the same equipment but synchronized at the load with the reactance between generating units considerably increased by the omission of the bus-tie reactors and inter-station ties, and by splitting the substation high-tension busses. This scheme of connection is only partly equivalent from the stability point of view to a ring bus scheme in which high-reactance reactors are used between busses since the actual ties between units consist of a large number of branches in parallel and because each generator tends to have symmetrical ties with each other generator. In fact, from this point of view, the arrangement is more nearly like the synchronizing bus or Staat connection, but does not have any points where large amounts of energy are concentrated. The stability and reliability of the synchronized at the load

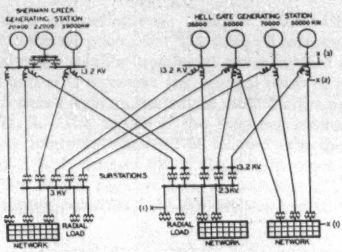


FIG. 7—SYNCHRONIZED AT THE LOAD

arrangement compared to the previous arrangement depends upon the relative amount of change in reactance in series with faults and between generating units. The calculated performance when synchronized at the load is described in this paper, while the test and operating results are described in the companion paper by Messrs. Searing and Milne.

The rigid requirements for power supply in a metropolitan area necessitate a system which has static stability under all conditions of operation, and transient stability under all conditions of operation with the exception that in some cases a disturbance may result in loss of a generating unit, and this is permissible, but it is required that the remaining units maintain synchronism with one another. In addition to possessing the necessary stability, there must be no undesirable hunting between generating units. The layout of the system is such that if stability can be obtained, the requisite reliability of service will automatically be obtained, vulnerable points having been largely eliminated.

By static stability is meant that the generating units shall be capable of carrying the required load under normal conditions without loss of synchronism. With such a closely connected system, it appeared likely that such would be the case, and the analysis of this point was made largely as a formality.

There are three transient conditions to be considered. One condition is a fault located more or less equidistantly from each of the generating units, as (1) on Fig. 6. From the nature of the layout, it is apparent that such a fault affects all of the generators similarly and practically in proportion to the size of the machine, hence, would be almost unnoticed. Another condition is a fault on a feeder close to one generator unit, such as (2), Fig. 7. Practically, this condition resolves itself into a fault on the feeder side of a feeder reactor on one of the generating units. A fault at this location has a tendency to cause the generator nearest it to go out of step. The third transient condition is that of a short circuit on one of the generating bus sections as illustrated by (3), Fig. 7. This condition will almost invariably cause the loss of the generators connected to that bus section. However, it is required that only the remaining generators stay in synchronism with each other.

From the foregoing, it will be seen that the most important considerations from the stability point of view are that all of the generators keep in synchronism for short circuits on the feeder side of a feeder reactor, and that all of the generators except those directly involved stay in synchronism during a bus short circuit. The three-phase short circuit is the most severe, and has therefore been made the basis of study.

Methods of Analysis.

There are, in general, three ways of making an analysis for stability: calculation, laboratory tests, and field tests. It was found that the major portion of the work involved was in obtaining values of the various impedances because of the unusually large number of feeders, and since a determination of these values for the existing and proposed system of the United Electric Light and Power Company was a necessary part of the program, all of the analysis was made by calculation. Later, tests were made to give an over-all check on the derivation of system impedances, machine characteristics, and the electromechanical oscillations set up by short circuits.

The general methods of calculation have been described in several papers before the Institute and details will not be given here. In all calculations, the load was considered as a shunt admittance. This assumption was necessary because of the amount of work which otherwise would have been required, but it is probably fairly accurate for the system under consideration because of the relatively large amount of lighting load. For the static stability analysis, the synchronous reactances of the generators at the initial operating points were used. For the transient analyses, the transient reactances of the generators were used. The effect of variations in the cross flux was not considered.

Decrements of main field flux, and the general method of calculation were about as given in the paper *Studies of Transmission Stability*.[2]

The studies which were made were divided into two groups, one being on a portion of the system which, in addition to lending itself readily to tests, also is of the type representing what appears to be the tendency in the growth of the system. This part of the system was that which was fed mainly by the 120-208-volt a-c. network system. Other analyses were made covering the entire system as it was expected to be when the plan was first placed in operation. The discussion will therefore be divided between the analyses of one generating unit split from the system and operating on the networks, and the entire system with generating units separated at the generating station and synchronizing ties through the low-voltage substation busses.

Partial System Analysis

Reduction of Network. The part of the system analyzed is shown by Fig. 2 in the companion paper *System Tests and Operating Connections, Synchronized at the Load*. In this figure, each small square represents a networked area and consists of hundreds of branches. Due to the extremely large amount of work which would be required, it was not considered feasible to solve each of the networked areas. However, the general structures of all of the networks is about the same. Transformers step down to the network from the high-voltage cables and the total capacity of these transformers bears some relation to the amount of load to be served. The load is located partly along the secondary mains in the streets. The density of load, of course, affects the spacing of the step-down transformers, but considering that each customer must be supplied with energy with a reasonable voltage regulation, it is evident that the total impedance from a high-voltage feeder to the ultimate load will tend to average to some definite percentage value with the load as a base. This refers to the impedance from the high-voltage cables to the network load. From the stability point of view, it is necessary to find the effective impedance through the network, from one generating source to another, it being this value which affects the flow of synchronizing power. To secure reliability in network operation it has been customary to feed a given area from as many independent points as possible so that if the supply from one source is interrupted, the load can be carried from the other sources. The load distribution from the various sources is carefully divided so that each source carries part of the load of practically every section of the area. For this reason, it is logical that the impedance to the flow of synchronizing power should also work out to be a function of the transformer capacity connected to the various sources.

2. Evans and Wagner, A. I. E. E. Trans., Vol. 45, 1926, pp. 51-94.

From the stability point of view, it is usually desired to analyze for the synchronizing power of one generating unit referred to all of the remaining generators as a unit. This requirement suggests obtaining, if possible, a figure of coupling impedance from each source to a fictitious central point, in terms of the transformer capacity connected to the source, or in other words, a star form of representation.

Fig. 8 shows the appearance of typical network. The lines represent 120-208-volt secondary mains and are interconnected wherever they are shown as crossing on the diagram. The network is fed from several different sources, some at 13,200 volts, and others at 3000 volts. Feed-in points are indicated by the trans-

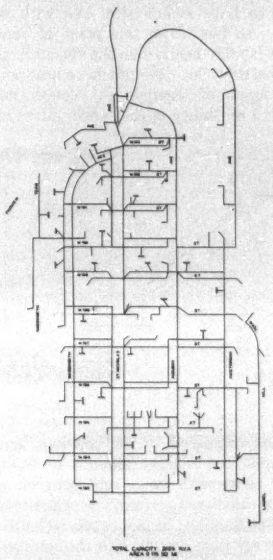


Fig. 8—Fort George Network

former symbol. The high-voltage cables are not shown on the diagram. Considering, however, that each high-voltage cable will supply a number of step-down transformers and that the high-voltage cables may have the same substation bus as a source, the number of independent sources is brought down to from three to six. A typical three-phase and a typical one-phase network were solved and the results considered to be generally applicable to the other networks, according to the connected transformer capacity.

A scale was made to convert distances of the map to reactances, and the reactance diagram of the network was made. The reactances of the transformers were then added at the feed-in points. The network

was then set up on a d-c. calculating board, bussing on the calculating board all of those feed-in points which were supplied from separate and distinct sources. Thus, the entire network would be involved in the calculating board set-up, but there would be only as many terminals to be considered as there were separate sources. Measurements can now be made on the calculating board which will permit reducing the complicated network to the minimum number of branches. In general, any network can be reduced to a mesh in which there is

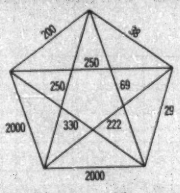


Fig. 9—Equivalent Mesh for Network. Values Given are Ohms at 13,250 Volts

a branch connecting each terminal point with each other terminal point, and this form of representation will give the smallest number of branches that can be used and still represent the original network *exactly*. Fig. 9 shows such an equivalent mesh for the network of Fig. 8. The values of the branches between the feed-in points are obtained as follows: On the short circuit board, connect all of the feed-in point busses but one to the negative bus, as indicated in Fig. 10, and supply potential from the positive bus to the re-

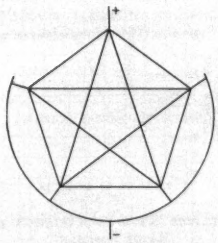


Fig. 10—Method of Obtaining Equivalent Mesh

maining feed-in points. An inspection of Fig. 10 will show that the current which flows from the positive to the negative bus through the network (or in the equivalent mesh which is to represent it) divides between the four feed-in points in proportion to the admittance of the branch from the positive feed-in point to the negative bus; each of the four other feed-in points being at the same potential, no current can flow in the branches connecting them. The admittance from the single feed-in point to each of the other feed-in points may thus be obtained directly from the current reading. If this process is repeated until each feed-in point has been used singly with the others paralleled, two determinations for the value of each branch of the equivalent mesh will have been obtained, thus furnishing a check. The mesh thus obtained is exactly equivalent to the original network.

In order to apply the results obtained from the analysis of two networks to the solution of all of the networks, it is necessary to put the results in terms of the impedance from source to central point, according to connected transformer capacity, as explained previously. There is no exact conversion between the mesh and star forms of network representations for the reason that the mesh is the general form and the star a restricted form which is only applicable when there are certain relations between the branches of the mesh. An approximate formula for the conversion from mesh

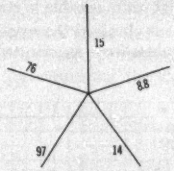


Fig. 11—Approximate Equivalent Star

to star form which has been found to give good results is as follows:

$$Z_1 = \frac{1}{Y_1} - \frac{1}{\frac{(n+2)}{n}\,(Y_2 + Y_3 + \ldots Y_n)}$$

$$Z_2 = \frac{1}{Y_2} - \frac{1}{\frac{(n+2)}{n}\,(Y_1 + Y_3 + \ldots Y_n)}$$

Where Z_1 = impedance of branch from terminal (1) to star point.

Z_2 = impedance of branch from terminal (2) to star point, etc.

Y_1 = Total admittance from terminal (1) to all other terminals in parallel.

Y_2 = Total admittance from terminal (2) to all other terminals in parallel, etc.

n = Number of terminals.

Fig. 11 shows the approximately equivalent star obtained in accordance with the above formulas.

This conversion formula from mesh to star form is empirical and should not be used where it is known that the mesh can be converted to an exactly equivalent star as for instance, where the mesh form has been derived from the star form. Also, there is an exact conversion between mesh and star forms where there are only three terminal points and this conversion should be used in such a case.

A three-phase and a one-phase network were solved in this manner, and due to the fact that the feeds were at 13,200- and 3000-volts, with transformers of different reactance, the results were somewhat different as shown below. The results are expressed as per cent reactance from the high-voltage cable source to the fictitious star point and based on the combined kv-a. ratings of the step-down transformers.

13,200 volts 3 phase	11 per cent
3,000 volts 3 phase	7 per cent
13,200 volts 1 phase	33 per cent
3,000 volts 1 phase	15 per cent

Tests made on the Times Square network as described in a companion paper gave an average impedance of 5.1 per cent and an average reactance of 4.3 per cent not including transformer reactance. This was a 3000-volt, three-phase network, and, adding the transformer reactance of 2.7 per cent gives 7 per cent reactance from high-voltage cable to star point.

After having thus obtained the reactance of the network elements, values were substituted for the small

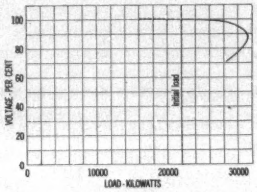


Fig. 12—Voltage-Power Characteristic, Indicating Static Stability

squares of Fig. 2 of *System Tests and Operating Connections*, (Part III of this symposium), in accordance with the transformer capacity connected to the various cables. The effective reactance between the generator to be tested and the remainder of the system, through the networks was found in this manner. Resistance components were then added to the reactances according to what was considered to be the probable ratio of resistance to reactance. Admittances were placed along the connecting impedance between the two generating sources in proportion to the probable electrical distances of these loads from the sources. This gave a static network to which it is only necessary to add the generator reactances to proceed with the analysis.

Static Stability. The synchronous reactance of the generator under test and the remainder of the generators grouped were added at the proper ends of the connecting network, and, with the internal voltages determined in accordance with the initial loadings, completed the data necessary for obtaining a static power-angle diagram. Points were picked off of this characteristic and the generator terminal voltage computed to give the points of the voltage-power characteristic of Fig. 12, this form being more readily interpreted. It will be noted that with an initial load of 22,000-kw. at 100 per cent voltage the static stability analysis indicates that with fixed field excitation, a load of 31,500 kw. could be carried before pullout takes place at 88 per cent voltage. This, of course, does not mean that a higher load than 31,400 kw. could not be carried, but merely represents the approximate margin for a given initial load and corresponding field excitation. This analysis indicates that there

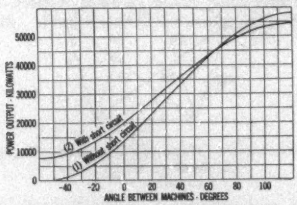


Fig. 13—Calculated Transient Power Angle Diagrams

is ample static stability without the use of automatic voltage regulators.

There was some speculation as to the possibility of hunting, but this factor cannot readily be examined in advance. It was felt, however, that no hunting would occur, and this was found to be the case when tested.

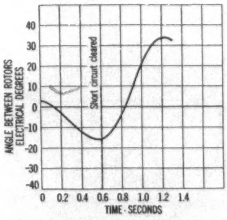


Fig. 14—Calculated Angular Variation in Generator Rotor Position

Transient Stability. The first analysis of transient stability was made for the condition of a fault on a feeder 8000 ft. from the station. Transient power-angle diagrams were obtained by combining the transient reactances of the generators in series with the static network previously obtained, giving the curves of Fig. 13. Curve "1" shows the conditions for normal operation and curve "2" shows the conditions with the feeder fault. The term "transient power-angle diagram" is here used to refer to the variation in power output of the generator as a function of angle, when the time required to move

from one point to another is very short, or in other words, that the internal voltages of the generators are the same as for initial conditions. In making the step-by-step calculations, the decrement of the main generator field fluxes were calculated and corrections made therefor.

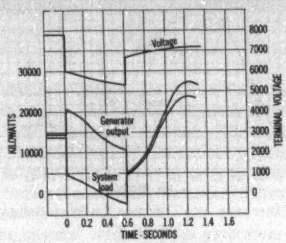


FIG. 15—CALCULATED GENERATOR VOLTAGE, GENERATOR LOAD AND SYSTEM LOAD CORRESPONDING TO FIG. 12

Fig. 14 shows the calculated variation in angular position of the rotors of the generators with a short circuit on for a period of 0.6 sec.

Fig. 15 shows the variation in generator voltage, generator load, and network load during, and for a short period following the short circuit. The network load is equal to the generator load minus the loss in the connecting network, and, in the case of the short circuit, the power lost in the short circuited cable. This figure may be compared with the results obtained on one of the tests and for convenience, these results are reproduced here in Fig. 16. It will be noted that the results show the same general characteristics and some of the differences can be explained by differences in set-up. The

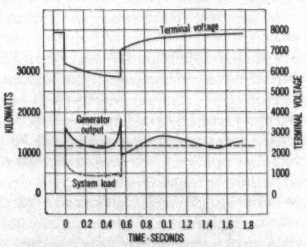


FIG. 16—TESTED CHARACTERISTICS FOR CONDITIONS SIMILAR TO THOSE OF FIGS. 12 AND 13

test length of short-circuited cable was 5000 ft. instead of the value of 8000 ft. used in the calculations, giving a smaller rise in power when the short circuit is applied. The initial load was 11,500 kw. instead of the value of 14,500 kw. used in the calculations. When the circuit breaker opened to clear the fault, there was a considerable momentary power loss in the breaker which was not considered in the calculation. There is also a considerable damping torque generated by the fluxes in the generator and this and the variation of cross-flux were neglected in calculation. In general, however, the conclusions reached by tests and the calculations made prior to them were the same, that is, that with short-circuit currents of the order and duration tested, and with tie impedances as existing at the time of the test, the system would be stable under such transient conditions.

Some approximate analyses and the other tests which were made showed that a short circuit directly outside of the station would be more severe from the stability point of view, but they could likewise be handled.

Comparison of Calculated and Derived Power-Angle Diagrams. During the tests, the set-up was such that the equivalent voltage of the entire system with the exception of the generator under test was available in the same station as the tested generator, so that facilities were available for measuring the magnitude of both

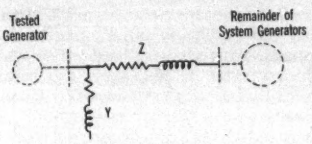


FIG. 17—APPROXIMATE NETWORK EQUIVALENT OF SYSTEM CHARACTERISTICS DURING TESTS

voltages and the phase displacement between them. In addition, oscillographic records were obtained of the variations in power and kv-a. output of the generator and power and kv-a. drawn by the fault, so that the difference gave the power output into the connecting network, from instant to instant, during and after the short circuit. These oscillographic readings were used to obtain an experimental check on the characteristics of the system between the tested generator and the remainder of the system. Fig. 17 shows a general form of static network for representing the system connections, consisting of an impedance between the machines and an admittance at the terminals of the tested generator. The admittance corresponding to the load on the remainder of the system need not be considered because of its relative size. Therefore, considering only the branches Z and Y, the following relations apply:

$$E_s = E_r + Z\,I_r$$
$$I_s = I_r + Y\,E_s$$

hence:

$$E_s = E_r + Z\,(I_s - Y\,E_s)$$

The vector values of E_s, E_r, and I_r are known at various

time intervals; hence, by setting up simultaneous equations in pairs, determinations of Z and Y may be made. The following independent determinations were obtained:

Z Ohms at 8000 volts	Y Mhos at 8000 volts
$0.199 + j\,1.150$	$0.354 - j\,0.073$
$0.204 + j\,1.312$	$0.334 - j\,0.158$
$0.062 + j\,1.118$	$0.271 - j\,0.049$
$0.448 + j\,0.997$	$0.435 - j\,0.215$
Average $0.228 + j\,1.144$	$0.349 - j\,0.124$

These values may be expressed in terms of the generator rating of 25,900 kv-a., and on this basis the impedance becomes 9 per cent resistance and 46 per cent reactance and the admittance becomes the equivalent of 22,000 kv-a. at 94 per cent power-factor lagging (at normal voltage). So far as known, the above represents the first experimental determination of transient load characteristics, expressed in the static network form.

When the transient reactances of the generators were added to the ends of the network thus obtained, the

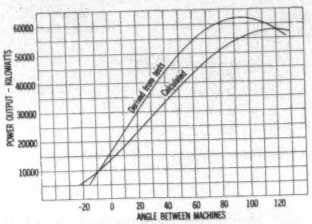


FIG. 18—CALCULATED AND DERIVED POWER ANGLE DIAGRAMS

total impedance to the flow of synchronizing power was found to be 60 per cent on the generator rating. Because of the shunt load, however, the maximum point on the transient power-angle diagram is considerably in excess of the value corresponding to 60 per cent impedance. Fig. 18 shows the power-angle diagram as derived in the above manner, and the calculated power-angle diagram of Fig. 13 is added for comparison.

Fig. 19 gives a comparison of the measured and derived angle time diagrams for test No. 10. The measured curve was obtained from the oscillographic measurements with a small correction for the internal angle of the generator, while the derived curve was obtained from the derived system characteristics, and the measured variations in power output. Considering the manner in which the determinations are interleaved, this figure indicates a fairly good over-all check.

Fig. 20 shows the saturation curve of the generator tested. It is interesting to note that the generator had a short-circuit ratio of 0.76 and despite the fact that there was an instantaneous symmetrical three-phase short circuit of about 2½ times full load on the generator for 1.8 sec., and a relatively weak tie to the remainder of the system, the generator remained in step.

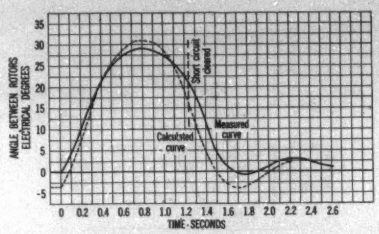


FIG. 19—MEASURED AND DERIVED ANGULAR SWINGS DURING TEST NO. 10

The initial load was approximately 60 per cent of rated load, at rated power factor and no automatic voltage regulators were in use. These figures show quite clearly that *high short-circuit ratios are not required of turbo generators for power supply in relatively closely connected systems.*

ENTIRE SYSTEM

In analyzing the entire generating system of the United Electric Light and Power Company, there are

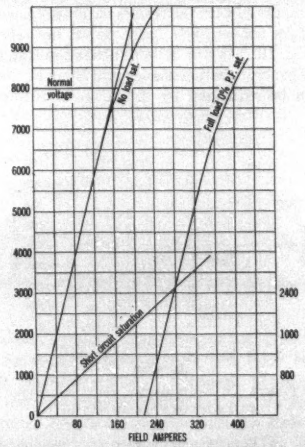


FIG. 20—SATURATION CURVES OF TESTED GENERATOR

two transient stability requirements to be met as well as the static stability requirement. It must be capable of withstanding a three-phase short circuit on one of the generating busses without the other generators losing

synchronism, and it must be able to withstand feeder short circuits. It was not considered necessary to repeat the analysis of static stability in view of the results obtained on the portion of the system previously analyzed.

The proposed initial layout for synchronizing at the load is shown by Fig. 5 in the companion paper by Messrs. Searing and Milne and it will be noted that the synchronizing tie in this case is principally through the substations rather than through the a-c. low-voltage network.

A calculating board set-up was made as in thep revious studies to obtain the equivalent mesh of the ties external to the generating stations. The reactances of the various generators were then added to the mesh, and another calculating board study made for the equivalent mesh including the internal reactances of the generators. The following table shows the per cent reactance between the internal voltage of the generators connected to each bus and the internal voltages of all other generators combined, based on the combined ratings of the generators connected to the bus in question.

Bus section	Per cent reactance
N. E.	30 per cent
S. W.	29 per cent
4	50 per cent
5 and 6	50 per cent
7	50 per cent
8 and 9	52 per cent

These values indicate that the system is at present somewhat unbalanced with reference to the synchronizing power of the various generators. This is due to a temporary local condition in that the feeder capacities at Hell Gate and Sherman Creek are not in proportion to the generator capacities. This condition will gradually disappear as cable rearrangements and additions are made.

Bus Short Circuit. A solid three-phase short circuit was considered as applied to the bus section which gave the largest short-circuit current. Under this condition, the generators operating on this bus section immediately lose all of their load, and over-speed. The resultant effect on the other generating bus sections depends upon how much dissymmetry there is in the layout. With an entirely symmetrical layout, the other generators would tend to speed up in unison with no tendency to lose synchronism with each other. In order to simplify the analysis, the results were calculated on the basis of reducing the system to two equivalent generating groups. One of these groups included those generators on the bus section having the maximum differences from the others, while the remaining generators were grouped together.

Fig. 21 shows the results obtained. It will be noted that the angular oscillation between the generating groups is very small, and that the drop in voltage is quite nominal considering the character of the fault condition. This analysis indicates that the proposed scheme can operate and furnish stable and reliable service with a solid three-phase short circuit on one bus, provided there is enough generating capacity in reserve, and provided that the bus section is isolated in a reasonable time by the opening of feeder switches.

Feeder Short Circuits. A transient stability analysis was next made with a short circuit on the feeder side of a feeder reactor, the reactance of which was such as to

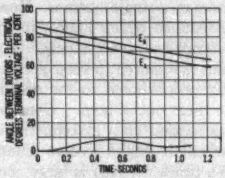


Fig. 21—Effect of Bus Short Circuit with Entire System Synchronized at the Load

produce a short-circuit current of three times the full-load current of the generators connected to the bus. To facilitate calculations, the generators were grouped so that those on the bus having the short-circuited feeder constituted one group, and all other generators the other group. The results of this analysis are shown by Fig. 22. It will be observed that this condition sets up a considerably larger oscillation than the bus short circuit, but that considerably less voltage disturbances are set up. This calculation indicates that short circuits up to three times the current rating of the nearest generators and having a duration up to about two sec. can be handled successfully. However, where possible, both the current magnitudes and duration should be reduced as this will constitute less hazards and give less voltage disturbance.

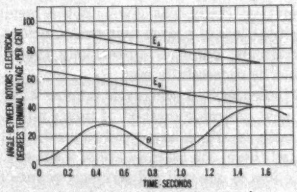


Fig. 22—Effect of Feeder Short Circuits with Entire System Synchronized at the Load

Operation of Interconnected Systems Both Synchronized at the Load. If connected systems had a common load throughout, and the interleaving of tie reactance was carried out in the same manner as can be done on the single system, the operation of two systems interconnected would be exactly the same as one system of

greater capacity. In general, however, there is a relatively small common load between various interconnected systems so that the facilities for thorough interleaving are small. It is therefore apparent that the reactance between two interconnected systems will be considerably higher on a percentage basis than the reactance between the various generators on a single system. Therefore, the synchronizing power which can be developed between two interconnected systems is much lower in proportion to the aggregate capacity of the systems than the synchronizing power available for each of the generators on a single system. However, there is another circumstance which makes it unnecesary to have low reactance between systems. The synchronizing power which is required to keep generators in step is almost directly proportional to the disturbance which tends to force them out of step. Considering a single generating bus section on a system, the magnitude of disturbance which it might receive (such as a short circuit of three times normal) is large compared to its size and a relatively high synchronizing power (or low tie reactance) is necessary. With two systems inter-connected, each of which is laid out so as to restrict short circuits to a *low value* in relation to the *entire system*, a short circuit on one cannot seriously disturb the system as a whole, and therefore the requirements for the flow of synchronizing power between the interconnected systems is minimized.

A study was made of two large systems interconnected by ties having an aggregate capacity of 60,000 kv-a. and 30 per cent reactance. The aggregate connected generating capacities of the systems were 700,000 kv-a. and 400,000 kv-a. It was found that generating capacity carrying 100,000 kw. of load could be dropped from the smaller of the two systems without loss of synchronism. In order for this to be considered as a practical emergency condition, it would, of course, be necessary that a considerable portion of the 100,000 kw. be quickly picked up by spare capacity on the system actually losing the large unit.

No analytical study was given to the results to be expected, if the loss of a large unit was coincident with a bus short circuit, but it is quite likely that this condition could be successfully met.

Fig. 23 indicates in a schematic manner what is probably the most desirable method of interconnecting two systems, both of which are synchronized at the load. As indicated there, the tie lines are of sufficient number so that the loss of one will not seriously reduce the synchronizing power, and the tie-in points are distributed in such a manner that a large part of either system is not affected by a short circuit on one of the tie lines. Furthermore, short circuits on one system will not produce very noticeable effects on the other system.

Tentative Design Constants

From the studies and tests made thus far, the following values are suggested as reasonable for system design.

Synchronizing Ability between Generators. The impedance between the internal voltage of one generating unit and the internal voltage of all other units of the same system in parallel (not including ties to other systems) should lie between 40 and 60 per cent based on the kv-a. rating of the generating unit.

Synchronizing Ability between Systems. The impedance between two systems both of which are synchronized at the load should not exceed 200 per cent on the rating of the smaller of the two systems.

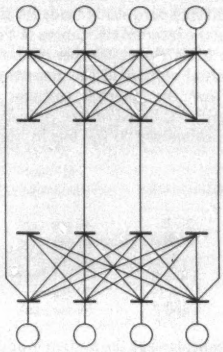


Fig. 23—Schematic Diagram of Interconnected System, Both Synchronized at the Load

Short Circuits. The maximum instantaneous symmetrical short-circuit current on a feeder from a generating unit should not exceed 2½ times the normal current capacity of the generating unit and should not be permitted to last longer than 1½ sec. For simplicity this value should be obtained on the basis of a short circuit with the highest capacity feeder reactor in series with the generating unit on a three-phase short circuit. For higher values of short-circuit current the duration of short circuit must be reduced.

Circuit Breaker Capacity. With the above system proportions, the interrupting duty on a generating unit breaker will not, in general, exceed 1¾ times the generator short circuit current.

"Synchronized at the Load"—III

System Tests and Operating Connections

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Synopsis.—*System test results, operating connections, and experience with the New York City system synchronized at the load are given. Data obtained by test are presented, showing the behavior of the system when generators are operated synchronized only through a low-voltage a-c. secondary network.*

The adaption of this method of connections to the New York City 60-cycle system, including system layout, station and feeder rearrangement, and automatic recording apparatus is described.

Experience with these connections has demonstrated that stable performance of prime movers is obtained, load and wattless control is simplified, oil circuit breaker duty is reduced, and voltage fluctuation at customer's service during system disturbances is materially decreased.

* * * * *

THE New York City 60-cycle system is principally supplied by the generating stations of the United Electric Light & Power Company with interconnections to the Brooklyn Edison Company system. Primary energy is supplied to the United distribution system on Manhattan Island and to the Bronx, Westchester, and Queens districts.

Two stations—Hell Gate with a 60-cycle capacity of 500,000 kw. and Sherman Creek with a 60-cycle capacity of 105,000 kw., supply the systems. In addition to the interconnections with the Hudson Avenue Station of the Brooklyn Edison Company, there are connections to the 25-cycle system through five frequency changers aggregating 185,000 kw.

The 60-cycle distribution load on Manhattan is supplied almost entirely by multiple feed low-voltage networks, having a total connected transformer capacity of 100,000 kv-a. Similar networks are in operation in the Bronx and in parts of Queens. Service to the remaining districts as well as the major supply to these networks is furnished through conventional distribution substations. In network districts, however, the building of new substations has been discontinued and the distribution transformers are supplied at generator voltage.

The high degree of service reliability required by high-load density districts, called for a system superior to the usual radial arrangement. This led to development of the multiple feed a-c. low-voltage network which was first placed in service in Manhattan in 1922. Experience with this system of distribution with rapidly expanding application indicates that the reliability of service is now limited by that of the generating sources.

Operating experience has shown that a bus fault on the conventional ring bus arrangement of generator connections with high capacity interstation ties will cause outage to parts of the system and serious disturbance to the remainder.

The companion papers describe a new arrangement of system connections to the generating sources—synchronized at the load—which applies the principle of multiple feeds upon which the network distribution system is based.

With this arrangement a generator about to be placed in service is first synchronized with an adjacent unit and connected to the bus in the usual manner. After the generator is supplying the load of the bus section to which it is connected, the high-tension connections to the adjacent unit are opened leaving the generator connected only through the low-voltage networks.

In order to check the predictions and calculations and to furnish a basis for operating all generating units synchronized at the load a number of system tests was made.

Experimental Determination of Network Impedance

The proposed arrangement contemplated a system in which the only connections between generators would be those through the secondary network. While the system constants from the generator to the network were susceptible to accurate calculation, there was no information available as to the impedance of the secondary mains system; hence, it was desirable to obtain test confirmation of the calculated values. This test was made on the Times Square Network covering 0.37 sq. mi. with a connected transformer capacity of 11,325 kv-a. supplied by eight distribution circuits. Two of these circuits emanating from a remote substation were out of service during the tests to simplify the determination.

Readings were obtained during light load periods in order to eliminate as far as possible the effect of customer's load. By manipulating the feeder regulator on one circuit while the regulators of the other circuits were kept in a fixed position, values of current, voltage, and power of each circuit and various differential voltages for different regulator positions were obtained, which gave the average impedance.

After subtracting the impedance of the regulators, primary cable, and transformers, the impedance of the network from the transformer terminal points of any

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feeder to an imaginary star point, was found to be 5.1 per cent based on the transformer kv-a. connected to the feeder.

The value of 5.1 per cent agrees reasonably well with a calculated value of 4.6 per cent for a similar case when it is considered that the calculations were made on a different network (Inwood) and the network under test was distorted by having two of its normal feeders out of service.

While this value will vary depending on the design of the network, it is felt that 5.1 per cent represents a high value which may be reduced as the load density, on the network, is increased.

System Tests

Calculations made on an arrangement in which one of the generators was connected to the remainder of the system only through low-voltage networks, forecasted successful operation during normal loading and feeder short circuits on the system connected to this generator. Therefore, arrangements were made to test the performance of one generator at Sherman Creek Station synchronized with the remainder of the system through the Manhattan secondary network.

Due to the limited capacity of network feeders available for this purpose, the coupling between the generator thus isolated and the remainder of the system was small when compared to the generator capacity and therefore, the test conditions were more severe than would be expected under an arrangement where the generator capacity and network interlinkings were comparable. The specific purposes of the test were, to determine the stability of a generator operating under such conditions and subjected to feeder short circuits, check the action of automatic network switches having sensitive reverse power settings, and determine the effect of such operation on the customer's service.

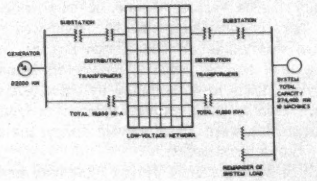


Fig. 1—Schematic Connections for System Test

Two 60-cycle systems, herein known as the "auxiliary system" and the "main system," were set up and isolated one from the other except through their connections at the low-voltage networks as shown schematically in Fig. 1. The auxiliary system consisted of a number of 13.2-kv., 7.8-kv., and 3-kv. feeders through which generator No. 4 at Sherman Creek, which has a rating of 22,000 kw. at 8000 volts with 85 per cent power factor and 12.5 per cent inherent reactance, was connected to the low-voltage networks. The 13.2-kv. feeders of the auxiliary system were connected to generator No. 4 through a bus-tie transformer bank at Sherman Creek. These auxiliary system feeders were connected to the low-voltage networks through distribution transformers, the total installed capacity of which was 19,850 kv-a. The main system consisted of the balance of the New York City 60-cycle system. At the time of the test, there was a total rated generator capacity of 274,400 kw.

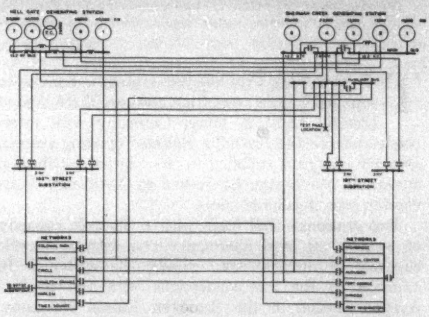


Fig. 2—Detailed Connections for System Test

connected to the main system at Sherman Creek and Hell Gate, most of the load of which was supplied to districts other than the networks involved in the test. These generators which are all 13.2-kv. units were connected to the low-voltage networks through various 13.2-kv., 7.8-kv., and 3-kv. feeders, the 7.8-kv. feeders being connected to the generating station 13.2-kv. bus through bus-tie transformer banks. The main system feeders were connected to the low-voltage networks through distribution transformers, totalling 41,650 kv-a.

A more detailed diagram of the system connections directly involved in the test is shown in Fig. 2.

During the test, the approximate loading was 12,000 kw. on generator No. 4 at Sherman Creek and 218,310 kw. on the generators connected to the main system, of which it is estimated 25,200 kw. was supplied to the low-voltage networks included in the test.

A tabulation of the total capacity of distribution transformers involved is given in Table I.

Neglecting the effect of network load, the impedance between generator No. 4 and the remainder of the system including internal reactances of all machines was approximately 60 per cent based on the rating of generator No. 4; 25,900 kv-a. During the tests, all feeder regulators were on automatic operation. All generators on the system are arranged for manual excitation control.

For each test, the main and auxiliary systems were set up, but remained tied high tension through the two selector switches of generator No. 4 at Sherman Creek. Then the load of generator No. 4 was reduced to a

TABLE I
CAPACITY OF DISTRIBUTION TRANSFORMERS CONNECTED TO LOW-VOLTAGE NETWORKS INVOLVED IN TEST

Secondary network	Type	Installed capacity of distribution transformers in kv-a.		
		Auxiliary system	Main system	Total
Inwood	3 ϕ 4 W	3,375.0	5,287.5	8,662.5
Fort George	1 ϕ 3 W	1,050.0	1,050.0	2,100.0
Fort Washington	1 ϕ 3 W	850.0	675.0	1,525.0
Medical Center	3 ϕ 4 W	1,800.0	1,425.0	3,225.0
Audubon	1 ϕ 3 W	3,050.0	1,425.0	4,475.0
Colonial Park	1 ϕ 3 W	1,250.0	1,800.0	3,050.0
Hamilton Grange	1 ϕ 3 W	1,150.0	3,000.0	4,150.0
Highbridge	1 ϕ 3 W	1,700.0	1,925.0	3,625.0
Harlem	1 ϕ 3 W	1,425.0	4,100.0	5,525.0
Riverside	3 ϕ 4 W	1,500.0	5,100.0	6,600.0
Circle	3 ϕ 4 W	750.0	6,487.5	7,237.5
Times Square	3 ϕ 4 W	1,950.0	9,375.0	11,325.0
	Total	19,850.0	41,650.0	61,500.0

value equal to the sum of the loads on the auxiliary system feeders, after which the two systems were separated high tension by opening the main bus selector switch of generator No. 4. After the various tests had been completed, normal conditions were restored by closing the main bus selector switch of generator No. 4.

On May 7, 1928 the two systems were set up and operated synchronized through the low-voltage networks from 8:52 p. m. to 9:57 p. m. Although no short circuits were applied, the machine showed no signs of hunting, and at all times gave stable operation. During the evenings of May 8, 9, and 11, the two systems were set up and a total of nine three-phase short circuits was placed on the auxiliary system, each short circuit being cleared automatically in a predetermined time. These short circuits were made at various locations over a 7.8-kv. feeder, the last three being applied directly outside the feeder reactors at the generating station. The total phase to neutral impedance, exclusive of generator No. 4, for each short circuit is listed in Table II.

Oscillographic equipment was used at Sherman Creek to measure:

(a) Current and power of short circuited feeder
(b) Terminal voltage of generator No. 4
(c) Voltage of main system at Sherman Creek
(d) Differential between above voltages
(e) Differential watts using current of generator No. 4 and differential volts
(f) Current, power, throttle position, and field current of generator No. 4.

In addition, stroboscopic equipment was used during the tests to view the rotor displacement of generator No. 4 during the disturbances. In order to measure this displacement, a white indicator was marked on the shaft of the generator and was caused to appear stationary by means of a stroboscope which consisted of a neon lamp, a suitable interrupter, a transformer, and a synchronous motor to drive the interrupter. The supply for this synchronous motor was obtained

Fig. 3—Stroboscopic Image on Shaft of Generator No. 4

from the main system. The circumference of the housing about the shaft was marked in such a manner that the displacement of the shaft could be read in degrees. See Fig. 3.

Nine recording voltmeters were installed at various locations on the networks to obtain a record of the voltage dip caused by the short circuits.

The total number of network switches involved in these tests was 649, of which 208 were connected to the auxiliary system and 441 were connected to the main system. During all tests, about 53 men were stationed at various locations to observe network switch and relay performance; 492 locations were supervised by operation counters.

TABLE II

Date	Time (PM)	Duration of short circuit (seconds)	Impedance of short circuited feeder in ohms to neutral							
			Sherman Creek reactors		Feeder cable		Substation reactors		Total	
			R	X	R	X	R	X	R	X
5- 8-28	9:00	0.29	0.018	0.888	0.276	0.207	0.015	0.688	0.309	1.783
5- 9-28	8:15	0.28	0.018	0.888	0.276	0.207	0	0	0.294	1.095
5- 9-28	8:35	0.55	0.018	0.888	0.276	0.207	0	0	0.294	1.095
5- 9-28	8:57	0.92	0.018	0.888	0.276	0.207	0	0	0.294	1.095
5- 9-28	9:15	1.35	0.018	0.888	0.276	0.207	0	0	0.294	1.095
5- 9-28	9:41	1.82	0.018	0.888	0.276	0.207	0	0	0.294	1.095
5-11-28	8:20	0.67	0.012	0.654	0.016	0.023	0	0	0.028	0.677
5-11-28	9:02	1.24	0.012	0.654	0.016	0.023	0	0	0.028	0.677
5-11-28	9:20	1.80	0.012	0.654	0.016	0.023	0	0	0.028	0.677

A summary of the results of all the short-circuit tests is given in Tables III and IV. Detailed results of the short circuit applied just outside the station reactors for 1.24 sec. are given in Fig. 4. The record of throttle position showed no movement and is not given. Complete results could not be given for the short circuit of longest duration, made May 11, 1928 at 9:20 p. m., as no record was obtained of the current and power in the short-circuited feeder. This test was no more severe than the one illustrated, as an inspection of Fig. 4 will show that the unit was past its maximum oscillation during that test.

by a feeder short circuit did not exceed the normal kilowatt load on the generator, network switches of the sensitive reverse power type remained closed even under the most severe condition of feeder short circuit. The voltage on the network was sustained at a value which precluded any network switches of the low-voltage release type from operating prematurely due to low voltage. During the test, a maximum of 1.4 per cent of the network switches connected to the machine under short circuit opened and reclosed. These switches opened because the network load at that point was small at the time of test.

TABLE III

SUMMARY SHORT-CIRCUIT TESTS SYNCHRONIZED AT THE LOAD MAY 7-11, 1928

Date	Time	Duration seconds	Load on gen. No. 4 kw.	*Current in short circuited feeder amps.	Power in short circuited feeder kw.	Voltage drop at gen. No. 4 per cent	Voltage drop on networks per cent		†Max. swing of gen. No. 4 during short-circuit electrical degrees
							Max.	Min.	
5- 8-28	9:00 p.m.	0.29	13,500	2010	3800	16	13	8	4 lead
5- 9-28	8:15 p.m.	0.28	11,500	3120	9400	21.5	16	10	8 lead
5- 9-28	8:35 p.m.	0.55	12,500	3090	9400	28.5	21	11.5	8 lead
5- 9-28	8:57 p.m.	0.92	11,700	3120	9400	28.5	21	13.0	9 lead
5- 9-28	9:15 p.m.	1.35	12,750	3120	9400	31.0	21	11.5	15 lead
5- 9-28	9:41 p.m.	1.82	12,750	3120	9400	31.0	21	12	19 lead
5-11-28	8:20 p.m.	0.67	12,500	4720	..	44.5	33	19.5	26 lead
5-11-28	9:02 p.m.	1.24	12,250	4720	800	45.0	34	19	34 lead
5-11-28	9:20 p.m.	1.80	12,900	4720	..	44.5	33	20	40 lead

*Instantaneous symmetrical r. m. s. value.
†As viewed with stroboscope.

TABLE IV

NETWORK SWITCH OPERATION SYNCHRONIZED AT THE LOAD MAY 7-11, 1928

Date	Time	Switches connected to gen. No. 4	Switches connected to system	Switches under manual supervision	Switches with operation counter	Switches opened during short circuit	Switches showing opening torque	*Switches showing operations on operation counter
5- 8-28	9:00 p.m.	208	441	53	492	2	9	4
5- 9-28	8:15 p.m.	"	"	53	"	0	6	..
5- 9-28	8:35 p.m.	"	"	53	"	1	6	..
5- 9-28	8:57 p.m.	"	"	53	"	1	6	..
5- 9-28	9:15 p.m.	"	"	53	"	3	5	..
5- 9-28	9:41 p.m.	"	"	53	"	4	3	12
5-11-28	8:20 p.m.	"	"	50	"	0	14	..
5-11-28	9:02 p.m.	"	"	50	"	2	12	..
5-11-28	9:20 p.m.	"	"	50	"	2	17	4

*Operations varied from 1 to 8 during the test with an average of 1.6 operations per switch on 20 switches.

Stability

The test results showed that the generator was very stable when synchronized through the low-voltage networks, though the capacity of the connected distribution transformers was less than would be expected under normal system planning. No hunting occurred and even with the most severe short circuit applied just outside the feeder reactors at the generating station for 1.8 sec., the stroboscope indicated a maximum shift of 40 electrical degrees lead between this machine and the remainder of the system.

Network Switch Operation

The tests demonstrated that when the system was operated so that the maximum kilowatt load imposed

System Disturbances

On the networks which were partially supplied by the machine under test, the voltage drop on short circuit varied from 10 per cent to 34 per cent. It should be noted that this drop was much larger than should occur in normal operation because in the test, the network capacity was distributed in such a manner as to give the equivalent of a two circuit network. In a normal set-up where one machine would carry only a third or less of the network capacity, the voltage drop would be considerably reduced.

System Layout for Initial Operation

These tests demonstrated the stability of the arrangement for any disturbance beyond the feeder reactors

and substantiated the general method of analytical treatment.

It was decided to proceed with the layout for operating the entire system synchronized at the load. As there were not sufficient networks to adopt the scheme in its entirety, the 4-kv. substation busses were considered as load points and feeder connections arranged so that the coupling between generating bus sections was provided through transformation and highly re-

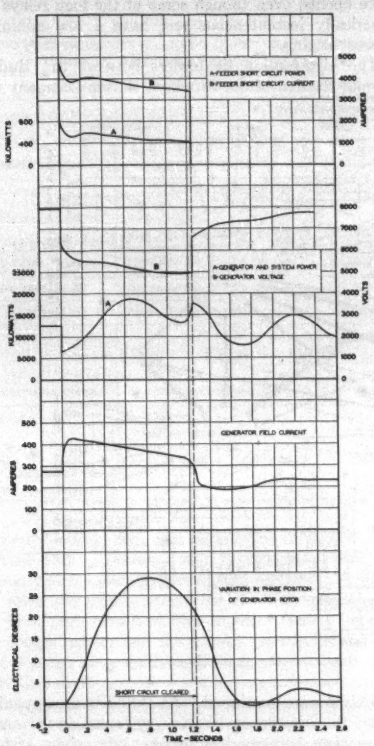


FIG. 4—PERFORMANCE CURVES OF GENERATOR NO. 4 AND SHORT-CIRCUITED FEEDER. TEST OF MAY 11, 1928, 9:02 P. M.

active ties. This was a departure from the original concept, but not from the basic principle, and represents an expedient to be utilized only until sufficient network capacity is available to complete the entire plan.

Calculations made on the proposed connections indicated that under the conditions of a bus short circuit on one section, all generators, except the one short circuited, would remain in synchronism.

It was not desirable, however, to subject the system to a short circuit under this arrangement in order to check the calculations and therefore it was decided to proceed with operation under this plan and provide automatic recording apparatus which would give test data under the normal system disturbances from which the system behavior under a bus short-circuit condition could be predicted.

Nine element oscillographs were installed at Sherman Creek and Hell Gate, arranged for automatic operation. The principal measurements are those of phase angles between each generating group and a common average system load voltage. The internal voltage of each machine is at all times available by means of pilot generators coupled to the shafts of the main units and excited from a separate battery. The average system load voltage is obtained from a number of feeders through the usual line drop compensators. Various other data are recorded to assist in predictions of system behavior.

In planning the system layout for operating synchronized at the load, it became apparent that the ratio of feeder capacity to generator capacity should be approximately the same for all sections; and that any substation or network district should be supplied from different generating station sections so as to maintain diversified feeds under all operating conditions. The first feature is desirable, not only for load division, but in order that each generating section may carry its share of the system wattless kv-a. The second feature is obviously necessary for service reliability. Furthermore, the feeder grouping and capacity must be such that generating units and also stations may be loaded to their economical point.

The general plan of operating the entire system synchronized at the load, contemplated keeping the tie feeders between Hell Gate and Sherman Creek open at all times. Each of the 13.2-kv. bus sections, four at Hell Gate and two at Sherman Creek, would operate as separate sources during the peak period. The 7.8-kv. bus at Sherman Creek would be split in half and the two sections thus formed tied to the 13.2-kv. busses. During the light load period, Hell Gate would operate in two sections and Sherman Creek as a single bus station. It should be realized, however, that these operating arrangements are flexible and Hell Gate has been operated with various grouping of sections. Fig. 5 and Table V show the system arrangement with unit No. 8 (160,000 kw.) in operation. When generator

TABLE V
SYSTEM CONSTANTS

Bus section	Generator capacity kw.	Ratio generator to feeder capacity	*Reactance tie to system per cent
Sherman Creek NE	61,000	0.525	30
Sherman Creek SW	40,800	0.443	29
Hell Gate No. 4	90,000	1.250	50
Hell Gate No. 5-6	160,000	0.904	50
Hell Gate No. 7	40,000	0.556	50
Hell Gate No. 8-9	50,000	0.617	52

*Includes internal reactance of all machines.

No. 9 is placed in operation on section 8-9, load for this unit will be obtained partly by transferring some feeders from other sections, but mainly by new high capacity 60-cycle feeders and a new frequency changer tie to the 25-cycle system.

As Sherman Creek had too much feeder capacity in proportion to generator capacity, some of the feeders serving the Manhattan and Bronx districts were transferred to Hell Gate. In addition to the transfers affected by feeder rearrangement, the four Sherman Creek–Hell Gate tie feeders were utilized to connect a corresponding number of Sherman Creek feeders direct to Hell Gate. At each generating station, some rearrangement of feeders on the various groups was necessary in order to diversify the feeds to any given substation or network district under any of the various generator groupings. Furthermore, under previous operating practise, the incoming feeders were paralleled on the 13.2-kv. busses in some substations. No sectionalizing switches were available to permit connecting feeders directly to transformer banks and it was decided to permanently sectionalize the main bus of such stations. Also two of the Manhattan substations were supplied at 7.8 kv. from a single bus section at Sherman Creek. The growth of network capacity served directly at generator voltage has reduced the load of one of these stations to a value where the station can be regarded as a single source. Although the work has been deferred, it is planned to rewind some of the transformers in the remaining station for 13.2 kv. and obtain the necessary interleaving with feeders direct from Hell Gate.

While copper ties between bus sections were avoided wherever possible, high-tension loop feeders supplying large customers were continued. As these loops are usually fed from substations with reactors at each end of the loop, they are highly reactive. Under subsequent operation, synchronized at the load, no loop switches have opened even though some of the loop relays are practically instantaneous and have a low minimum current setting.

The high-tension tie feeders between the Hudson Avenue Station of the Brooklyn Edison Company and Hell Gate were continued. As stated in a companion paper, the impedance between systems can be much higher than between bus sections of a given system and the reactance value of these ties (112 per cent based on Hudson Avenue) was found to be well within limits.

The Queens district lies between Hudson Avenue and Hell Gate and the increase in load in this district necessitated additional feeders from Hudson Avenue. Accordingly plans were made and work is under way to maintain synchronism between Hell Gate and Hudson Avenue through the 4-kv. busses of the Queens substations and through frequency changers connecting to the common 25-cycle system of the allied companies.

The possibility of a substation operator reclosing on feeders connected to a generating bus section, which by

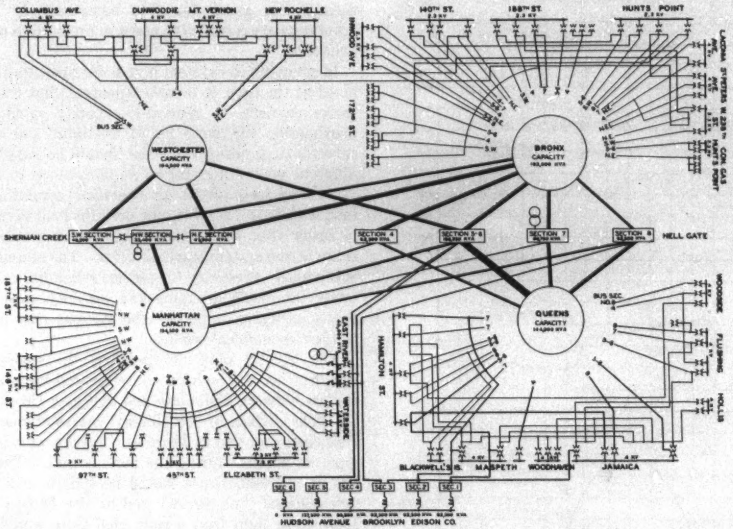


FIG. 5—SCHEMATIC SYSTEM CONNECTIONS—SYNCHRONIZED AT THE LOAD

virtue of a bus short circuit has disconnected itself from the system, is present. Accordingly, neon lamps, arranged for "bright light" synchronizing have been installed in each substation between the potentials of the various incoming feeders and that of the 4-kv. bus.

In order that a unit may be stable under the condition of feeder short circuit the ratio of short-circuit current to generator full load should not exceed a value tentatively set at 2.5. Table VI shows that the short-circuit

TABLE VI
RATIO SHORT-CIRCUIT CURRENT IN FEEDER TO FULL-LOAD CURRENT OF GENERATOR ON SECTION

A—Peak-Load Condition—Six separate generating sections

Generating section	Machines in operation	Short circuit current ratio	
		Normal case	Special case
SW Sherman Creek	Generators No. 5 and No. 6	3.33	5
NE Sherman Creek	Generators No. 4, No. 1 and No. 2 (or No. 3)	2.73	4.05
No. 4 Hell Gate	Generator No. 6	1.4	3.45
No. 5-6 Hell Gate	Generator No. 8	1.1	2.57
No. 7 Hell Gate	Generator No. 7	1.4	3.45
No. 8-9 Hell Gate	Generator No. 9	1.14	2.6

B—Light-Load Condition—Three separate generating sections

Generating section	Machines in operation	Short circuit current ratio	
		Normal case	Special case
Sherman Creek	No. 5 (or No. 6) and No. 1 (or No. 2or No. 3)	3.6	5.1
No. 4 and No. 5-6 Hell Gate	No. 8	1.1	2.57
No. 7 and No. 8-9 Hell Gate	No. 9	1.14	2.6

current ratios are well within limits for all normal cases at Hell Gate. While the limit is exceeded at Sherman Creek, operating experience will tell whether the empirical ratio will require modification. The special cases are for a short circuit on high capacity feeders between the feeder reactor and the step-up transformation and therefore are not considered serious.

With a short circuit on one bus section stable operation is assured if the short-circuit current is divided proportionately among the remaining units. The division of bus short-circuit current under the present operating connections is shown in Table VII. A comparison of Parts *A* and *B* of this table shows that the short-circuit currents drawn from the remainder of the system are smaller when synchronized at the load, and the system disturbance will therefore be materially decreased.

Reduction of Oil Circuit Breaker Duty

One of the advantages of the proposed arrangement was evidenced in the reduction of oil circuit breaker duty. Hell Gate Station was originally planned for five 50,000-kw., 60-cycle units and the main oil circuit

TABLE VII
DISTRIBUTION OF CURRENT WITH BUS SHORT CIRCUITS

A—System synchronized at the load

3 ϕ short at bus section	Instantaneous symmetrical short-circuit current in amperes						
	SW	NE	No. 4	No. 5-6	No. 7	No. 8-9	Total
SW	17,500	3,100	2,400	4,100	1,650	3,450	32,200
NE	3,030	21,300	3,070	4,410	2,100	5,280	39,300
No. 4	2,400	3,100	28,900	6,920	3,250	5,010	43,900
No. 5-6	3,740	4,000	6,170	41,700	6,650	7,900	70,700
No. 7	1,610	2,100	3,220	7,250	23,900	6,430	43,900
No. 8-9	2,730	4,000	3,400	5,730	4,250	48,300	71,300

B—System synchronized at generating busses

3 ϕ short at bus section	Instantaneous symmetrical short-circuit current in amperes						
	SW	NE	No. 4	No. 5-6	No. 7	No. 8-9	Total
SW	17,500	8,650	4,110	6,290	3,480	6,620	46,700
NE	7,960	21,300	4,110	6,880	3,780	7,700	50,900
No. 4	5,240	5,830	23,900	20,600	9,350	17,300	82,200
No. 5-6	6,850	8,220	17,500	41,700	17,100	31,000	121,600
No. 7	5,610	6,580	11,500	24,600	23,900	29,400	102,500
No. 8-9	6,050	7,750	12,300	25,800	16,900	48,300	116,000

breakers which were rated at 1,500,000 kv-a. were of ample capacity to clear a bus short circuit.

This initial layout was greatly exceeded, and one of the problems introduced by the installation of two 160,000-kw. turbines to complete the station, was the fact that the breakers would be unable to clear a bus short circuit which gave a calculated instantaneous symmetrical value of 2,900,000 kv-a.

With the synchronized at the load arrangement, the duty on oil circuit breakers was reduced and one of the immediate advantages aside from increased reliability, was the fact that maximum duty on these oil breakers would still be within their rated capacity. A comparison of feeder breaker duty before and after synchronizing at the load is shown in Table VIII.

TABLE VIII
THREE-PHASE SHORT-CIRCUIT CURRENT
Bus Short Circuit—Section 5-6—Hell Gate

Time	Synchronized at the load	Synchronized at generating busses
Instantaneous symmetrical	70,700	122,000
Instantaneous assymmetrical	124,000	217,000
0.05	85,000	141,000
0.10	70,000	118,000
0.15	62,000	102,000
0.25	56,000	90,000
0.50	53,000	76,000
1.00	47,000	65,000
2.00	43,500	56,000

With a short circuit on a generating station bus section the clearing time of the feeders connected to this section will vary due to the division of current in the system mesh and to different relay settings. Fig. 6 shows the clearing time of both the primary and secondary line of defense with a short circuit on section 5-6 at Hell Gate. These curves assume that the values of current in the feeders remain constant throughout

the entire clearing operation and the actual clearing times would therefore be somewhat less.

Operating Experience

The proposed system layout contemplated generator No. 8 (160,000 kw.) being in service at Hell Gate. Since this generator was not available, it was decided to proceed with the program with modifications, using existing capacity.

The modified arrangement was placed in operation on February 13, 1929, when Hell Gate and Sherman Creek Stations were operated "synchronized at the load" with Hell Gate connected to the Hudson Avenue Station of the Brooklyn Edison Company, through the high-tension tie feeders. Tests were made to determine

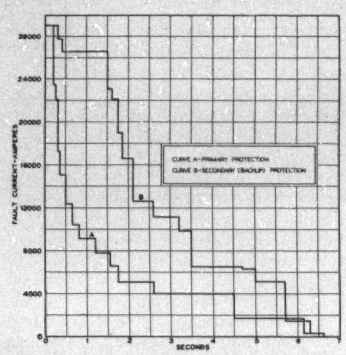


Fig. 6—Clearing Time of Fault Current in Feeders Connecting to System with Three-Phase Short Circuit on Section 5-6 Hell Gate

the extent to which the system wattless kv-a. could be controlled in each of the two generating stations, and in general it was noted that wattless control was more flexible than under the prior arrangement.

The following day, Hell Gate Station was separated into two sections tied together and with Sherman Creek Station, only through the load. The two Hell Gate sections were independently tied to Hudson Avenue.

As the peak load came on, the Hell Gate bus was separated into three sections in the same manner. On March 21, the Hell Gate bus was divided into four sections during peak load: No. 8 generator (160,000 kw.) having been placed in service. Shortly thereafter Sherman Creek Station was also operated in two sections during peak load.

Previous to the new operating arrangement, Sherman Creek, because of its high feeder capacity, was necessarily operated at a lower voltage than Hell Gate to prevent wattless kv-a. overloading. With the set-up of the new system, however, the wattless kv-a. is more easily controlled than formerly. Load may be shifted

TABLE IX
SYSTEM DISTURBANCES WITH FAULTS ON FEEDER 7651

	Per cent voltage variation	
	Synchronized at generating sta. busses	Synchronized at the load
Duration of fault	1.67 sec.	2.1 sec.
Hell Gate dip recorder section No. 7 (13.2-kv. bus)	40	15
Sherman Creek dip recorder (13.2 kv. bus)	15	3.5
Sherman Creek dip recorder (7.8-kv. bus)	23	4
147th Street dip recorder (3-kv. circuit)	24	No record
Elizabeth Street (3-kv. bus)	No record	6.5
45th Street (3-kv. bus)	8.5	7.5
97th Street (3-kv. bus)	8.5	6.5
146th Street (3-kv. bus)	16	6.5
187th Street (3-kv. bus)	11	6
Hamilton Street, Queens (2.4-kv. to neutral)	5	5
Maspeth, Queens (2.4-kv. to neutral)	26	23
Bronx Edison (2.4-kv. feeders)	26	10
Bronx Edison (13.2-kv. loop between 188th St. & 172nd St.)	23	6
Blackwell's Island (13.2-kv. bus)	46	23
Hudson Avenue (26.4-kv. bus)	18	6
Brooklyn Edison—66th Street (6.8-kv. bus)	Over 21	6
Average	20.7	9.0

from one section to another by the turbine throttle, up to the point where a feeder becomes overloaded; after which further load shift must be affected by switching banks from one feeder to another, at the substation.

While the main synchronizing path is at present through the 4-kv. substation busses, a number of generating station feeders connect directly to the network. No difficulty has been experienced with the network switches which are of the sensitive reverse power type.

The voltage of the system is considerably less affected by feeder short circuits under the new arrangement. The disturbances caused by faults on a high capacity feeder showed an average voltage drop of 9 per cent synchronized at the load compared with 20.7 per cent with the previous connections.

All generating units have operated in a steady manner both with normal loading and under a number of system disturbances. Records from the automatic oscillographs recently installed will give quantitative information as to system performance.

Test results have verified the calculations, and operating experience thus far has substantiated the predicted advantages of a system synchronized at the load.

Discussion

G. R. Milne: As the new system "synchronized at the load" has been in operation over four months, a summary of the operating experience to date should be of interest.

During the synchronized-at-the-load period there have been 70 feeder faults causing a voltage dip on a generating bus section varying in magnitude up to 33 per cent, but averaging only 2.5 per cent. The voltage at the load is considerably less affected than with the previous arrangement. Comparison of voltage disturbances at two of the Manhattan substations covering a

thirteen-month period preceding and the four-month period subsequent to the start of operations, showed that the dip was reduced 50 per cent.

While most of the paralleling is at present done on the 4-kv. busses, there are in Manhattan alone 321 network switches on generating-station 13.2 kv. feeders connecting directly to networks. No trouble is experienced with these switches which are of the sensitive reverse-power type provided they are equipped with the standard phasing relay.

Of the 70 feeder faults some were of insufficient magnitude to operate the automatic oscillographs; a total of 46 records has been obtained to date. The largest generator oscillation recorded to date was obtained when a fault occurred on a 12,000 kv-a., 13.2-kv. feeder connected to the N. E. section at Sherman Creek. Generator No. 4 swung a maximum of 8 degrees from its normal position.

In addition to the above feeder faults a phase-to-neutral fault which is worthy of attention occurred on Section No. 7 at Hell Gate on June 21 at 6:45 p. m. At the time of the fault—an insulator flashover—Hell Gate was operating in three sections, section No. 4 with a 50,000-kw. unit, sections Nos. 5, 6, and 9 with a 160,000-kw. unit, and section No. 7 with a 40,000-kw. and a 50,000-kw. unit, and a 35,000-kw. frequency changer. The operator connected a section of a reserve bus to section No. 7, the fault developing immediately on the reserve bus. The fault was cleared by the operator after 10½ seconds. Only one of 12 feeders connected to the section opened at the substation end, and two switches on high-tension customer loops opened but with no outage.

The maximum oscillation of the generators on section No. 7 was 2.5 degrees from their normal position, the oscillation of the generator on section No. 4 being 3 degrees from its normal position. Sherman Creek at the time of this fault was operating as a single bus station and the oscillation of the generators was negligible. The line-to-line voltage was reduced 27.5 per cent on section No. 7 at Hell Gate but only 4.2 per cent on the remaining sections. As the fault was line-to-neutral the corresponding voltage was probably reduced 58.5 per cent. The voltage at the load was affected only to the extent of 8.7 per cent.

E. E. Chilberg: Several months ago the company with which I am associated was asked to investigate the stability of the system arrangement described in the present papers. The results of these studies have provided an independent check on the studies made by Mr. Griscom. In general, the conclusions arrived at and methods of study were practically the same as in Mr. Griscom's work. One difference arose from the fact that in the case of the bus fault Mr. Griscom apparently did not investigate the phenomena occurring after the fault is cleared. Our calculations showed that the most severe oscillation would occur after the clearing of a three-phase bus fault and that on this account such a fault would be more dangerous than a feeder fault.

Another difference was that Mr. Griscom did not consider the effect of the synchronous-induction frequency changers. Our calculations were made both including and excluding their effect. Curves *A* and *C* of Fig. 1 herewith show the decrement in system voltage for the cases of zero and three frequency changers. Evidently these frequency changers greatly improve the voltage conditions during disturbances. This is due to the special characteristics of these machines which are equivalent to a direct transformer tie between the two systems. Were frequency changers of the synchronous-synchronous type employed their effect on system voltage would be much less. Curve *B* of Fig. 1 shows the calculated decrement for three frequency changers of this type.

These differences, however, did not change the final conclusion that the system should operate with stability through all normal disturbances.

In addition to studying the characteristics of the particular system in question, Mr. Griscom has given tentative design constants for any system which is laid out in this way. While this part of the paper undoubtedly represents a valuable effort to facilitate future system layout, we do not believe that it should be accepted too literally, since whether or not any particular degree of synchronizing ability or amount of short-circuit current is acceptable or not depends not only on the expected duration of

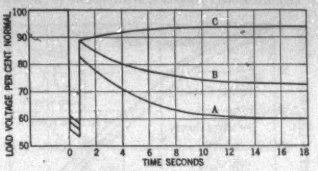


Fig. 1—Calculated Effect of Frequency Changers on System Load Voltage for a Three-Phase Generator Bus Short Circuit

Curve A Without frequency changers between systems
B With three synchronous synchronous frequency changers
C With three induction type frequency changers

faults, but also on the generator short-circuit ratios employed.

In the body of the paper Mr. Griscom states that high short-circuit ratios are not required. While this is probably true in some cases, it should be borne in mind that the use of generators with low short-circuit ratio necessarily tends to reduce the available margin of stability and, therefore, calls for special care in the choice of system layout.

It may be worth while to call attention to the fact that the authors have confined their studies to generators with single windings. The development of generators with double windings has made possible alternative system arrangements. These are illustrated in Fig. 2c and 2d in contrast to the arrangements discussed in the paper which corresponds to Fig. 2a and 2b. It may be seen that apparently the double-winding type of ring bus is a sort of compromise between the pure ring bus of Fig. 2a and the pure synchronizing-at-the-load arrangement of Fig. 2b, while the double-winding synchronizing-at-the-load arrangement might be considered as an extension of the synchronizing-at-the-load principle.

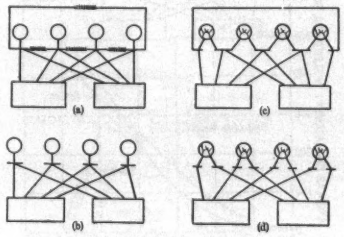


Fig. 2

An important point in connection with the use of double-winding machines is the possibility which they offer of riding through bus faults with stability and without the necessity of dropping off any generators.

This possibility was investigated by calculation. To simplify the calculation equal generators and symmetrical feeder arrangements were assumed. On this account it follows from considerations of symmetry that with a bus fault, generators located symmetrically with respect to the fault will move as a unit. This means that only three angle-time curves need be calculated.

Fig. 3 shows the result of these calculations in the case of the ring-bus arrangement. It will be noted that the angular displacement between generators does not become much greater than 10 or 12 degrees.

Fig. 4 shows the corresponding results for the synchronized-at-the-load arrangement. In this case the maximum displacement angle is around 60 degrees. The greater swing is due to the more unequal distribution of the shock due to the fault.

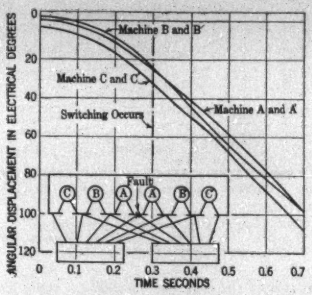


Fig. 3—Calculated Swing Curves for a Three-Phase Generator Bus Short Circuit

Condition previous to fault
Full load on A, A', B, and B'
Two-thirds load on C and C'

From these results it may be seen that with either the ring bus or synchronizing-at-the-load arrangement, the use of double windings permits operation through bus faults without the loss of any generating capacity.

Ability to operate through bus faults is not, however, the only advantage which appears to be offered through the use of double windings. This is illustrated by Table I which compares the single- and double-winding types of machine with respect to, (a) breaker duty, (b) transient drop in system voltage, (c) sustained drop in system voltage, (d) loss of generators, (e) back-up fault protection, (f) load transfer, and (g) sensitivity to feeder faults. By reference to the table it may be seen that in the case of the ring-bus arrangement the only disadvantage in the use of double-winding generators is increased difficulty of load transfer, while definite advantages are shown with respect to breaker duty, transient and sustained drop in system voltage, loss of generators, and back-up fault protection.

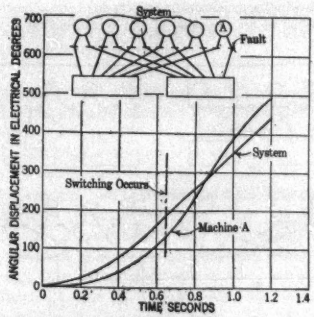


Fig. 4—Calculated Swing Curves for a Three-Phase Generator Bus Short Circuit

TABLE I

	Ring bus		Synchronized at the load	
	Single-winding generator	Double-winding generator	Single-winding generator	Double-winding generator
Breaker duty	High	Intermediate	Low	Very low
Sudden drop in system voltage	High	Intermediate	Low	Very low
Sustained drop in system voltage	High	Low	High	Low
Lose generators ?	Yes	No	Yes	No
Back-up fault protection	Difficult	Intermediate	Easy	Easy
Load transfer	Easy	Intermediate	Difficult	Difficult
Sensitive to feeder faults ?	No	No	Yes	No

In the case of the synchronized-at-the-load arrangement, the difficulty of load transfer and ease of back-up fault protection are equal for the two types of generator, while in all other respects the use of double windings is preferable.

It may, therefore, be concluded that the synchronized-at-the-load type of system arrangement provides an especially attractive field for the application of the double-winding principle.

R. H. Tapscott: The principle of synchronizing generating stations at the load is far-reaching. All systems, however, are not adaptable to this scheme of connections. The 25-cycle system in New York operates with the feeders radial to the converters and synchronized only by ties between stations. The latest unit which will be in operation this fall is a 160,000-kw. double-winding unit. It was selected because it economically obtained high reactance between bus sections without increasing reactance between the load and the system. Benefits were also indicated in limitation of system disturbance and improved stability.

The comparison of the characteristics of these two types of systems just presented is very interesting. The symposium on "Synchronized-at-the-Load" has only been applied to existing systems. The future effect on design with double windings and synchronizing at the load will be interesting, segregation by remote synchronizing carried to the equivalent of the still smaller units by double or multiple windings or the reverse, the larger and more economical capacity units which are coming to all systems subdivided electrically by their windings into units comparable to today's units. This ultimate isolation will increase the reliability and improve economies by reducing the size of the spare unit which will be required in operation.

In selecting the double-winding unit for our 25-cycle system, we have found that we could simplify and also obtain economies in station design by the omission of bus-tie reactors, by oil circuit breakers of smaller rupturing capacity, and by the use of fewer circuit breakers due to the omission of the bus reactors.

The low-tension network system as developed takes advantage of good regulation. If we can do anything with the high-tension feeders to improve regulation, we have improved the network system. With synchronizing at the load and double-winding generators, we can undoubtedly omit feeder reactors even on systems of large capacity and accomplish this result. Possibly this will require the introduction of an impedance device of some type in the neutral, but the usual overvoltage conditions which might result from such devices would not be serious with the system split up into equivalent small units.

I understand a double-winding generator has been built previously but not applied as such. Certainly the independent application of synchronizing at the load and of a double-winding generator is a significant engineering coincidence. I think we have much to expect in the improvement of generating-station and system design in the combination of these two schemes.

E. C. Stone: The plan of system operation explained in this symposium is clearly a definite step in advance. The advantages

that will appeal most strongly to operating engineers are the comparatively high degree of isolation of generating sources and the minimizing of voltage disturbances to service that will result from insulation failures.

The desired results are obtained essentially through a system of connections by which synchronizing circuits between generators are given a relatively high impedance without an increase in the impedance of the circuits over which energy flows from generator to load. A similar principle has been applied to many systems in the treatment of fault currents resulting from grounds. In this case a resistance in the neutral of the power system produces high impedance in the ground fault circuit, thus greatly reducing the disturbances resulting from grounds, but does not increase the impedance of the circuits carrying load.

The power system in New York to which this method has been applied is inherently favorable to the application of the method. It seems evident that the best results will be obtained when the transmission feeders delivering power from the generating plants have a relatively small capacity in relation to the capacity of the generating units, that is, where there is a relatively large number of feeders from each generating unit. Under such conditions, each generator may be tied into a relatively large number of substations, so that the loss of capacity, if a generator fails, is widely distributed over the system. Also, if the capacity of a feeder is relatively small, the voltage disturbance to the system resulting from a short circuit in such a feeder will be relatively small. Another favorable element is found in the fact that practically all of the load is finally distributed at voltages below 5000 so that the relatively high impedance of the substation and network transformers is introduced between the generators and the points of synchronizing. In the example presented, the load of seven generating units is distributed by 80 feeders, equivalent to an average of 11½ feeders per unit.

In some of the other metropolitan systems conditions are quite different. For example, 75 per cent of the entire system load in the Pittsburgh district is supplied over nine 66-kv. overhead transmission lines fed by four generating units, and more than one-half of the load is delivered to the ultimate consumer at 22,000 or 11,000 volts. If the "synchronized-at-the-load" principle were to be applied to this sytem there would be only two transmission lines per generating unit, so that the failure of one would cause a relatively heavy disturbance. With this condition and with such a large proportion of the total load supplied at transmission voltages, it would not be possible to introduce the high impedance available in New York in the synchronizing circuits between generator units. These conditions, it seems to us at the present time, will very largely prevent our making use of the advantages of the "synchronized-at-the-load" principle of system connections. Nevertheless, we are continuing our studies to find a way, if possible, by which the advantages of the new scheme may be applied to our case.

At the present time, the Pittsburgh system is set up on the power station synchronizing bus principle. Each of the two major power plants is divided into two major sections, the only connections between each section being the synchronizing bus connecting the two power house sections and the trunk 66-kv. transmission lines between corresponding section of the different power stations. It will be desirable to increase the number of sections set up on this basis as the capacity of the system grows. The present arrangement, however, is now working out satisfactorily and the serious transmission system short circuits that have occurred since the present set-up was established, have given very little trouble from the standpoint of inconvenience to our customers.

M. T. Crawford: (communicated after adjournment) What is the relay practise on feeders terminating at substation busses? Is conventional power directional equipment installed on the low side of transformers at substation bus connections and overcurrent equipment on an outgoing feeder where it leaves the generating bus section? Where several parallel feeders are from the Sherman Creek N. W. section to the 187th Street substation is balanced relay protection used and how is reliability safeguarded with little or no interleaving in feeder supply to this substation? Is it the practise when off the peak to cut out feeders supplying substations in the same way as feeders direct to networks?

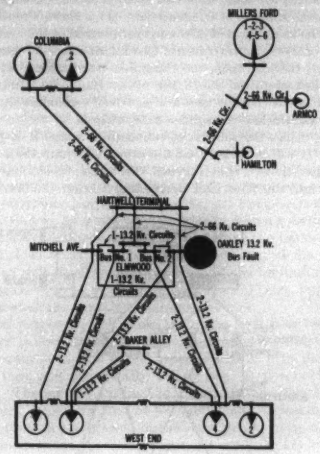


Fig. 5

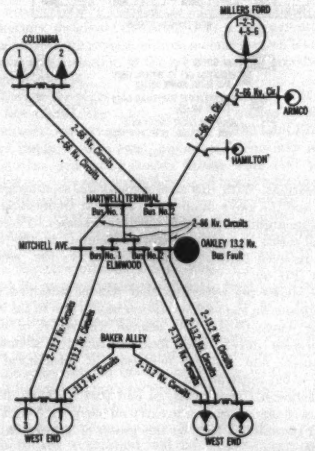


Fig. 6

E. S. Fields: (communicated after adjournment) This discussion of the group of papers on *Synchronized at the Load* pertains to one phase of the subject, that is, segregation of generating capacity. The following remarks refer to the system of the Union Gas & Electric Company, Cincinnati, Ohio.

Fig. 5 herewith shows a skeleton of the system under the present plan of operation, giving the magnitude of short circuit and voltage disturbance on the system associated with a short circuit on a typical 13.2-kv. substation bus (Oakley).

It will be noted that the Hartwell Terminal Substation has all the 66-kv. lines terminating on one bus and that the West End Generating Station 13.2-kv. bus is operated as a closed ring. We have realized for some time two decided operating hazards with the present scheme of connections: (1) a short circuit on the 66-kv. Hartwell bus will affect a great portion of the system load outside of the downtown area of Cincinnati and will remove practically all the load from the Columbia Generating Station, and (2) a short circuit on the 13.2-kv. West End Generating Station bus will affect the downtown area which embodies the Edison System and the low-voltage a-c. network area.

Fig. 6 shows the scheme of connections which it is proposed to use. It will be noted that the proposed connections contemplate operation of the Hartwell Terminal 66-kv. bus in two sections and the West End Generating Station 13.2 kv. bus also

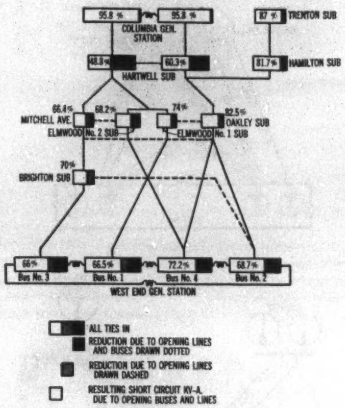


Fig. 7

in two sections. With this arrangement the separate generating stations and the two sections of West End Generating Station will be synchronized through substation busses. These connections will result in a considerable reduction in the operating hazards referred to above with reference to the present connections (Fig. 5).

Fig. 7 shows the reduction that will be obtained in short-circuit current on the various station busses due to the change in operating connections. The change will result in lower oil-circuit-breaker duties, more localized and less severe voltage disturbances, and decided reduction in mechanical stresses imposed on bus construction.

In conclusion I should like to add that we have obtained a large part of the inspiration to carry on our work from a study of the work represented here by this group of papers on *Synchronized at the Load.*

R. D. Evans: (communicated after adjournment) These papers give for the first time, I believe, a discussion of the stability problems of metropolitan power systems. It has been commonly held that on such systems there was no stability problem but only one of reliability of apparatus, lines, cables, and station busses. These papers suggest that there is a stability problem in so far as faults on station busses are concerned. It is indicated by analytical work and tests that systems synchronized at the load can withstand faults on station busses without the remainder of the machines going out of synchronism. This is a rather new conception and makes desirable the use of the broader meaning of stability.

Synchronizing at the load increases stability, not by increasing the normal synchronizing power between machines, but instead by limiting the severity of system faults and particularly by modifying the system layout so that all machines to be kept in synchronism are accelerated or decelerated as a unit. In comparison with bus-reactor schemes, synchronizing at the load decreases the normal synchronizing power, but increases the stability of the system for station bus faults which constitute the limiting conditions. This results in a better balanced design from the stability standpoint and constitutes one of the attractive features of synchronizing at the load.

In Mr. Griscom's paper attention is called to the fact that certain short-circuit tests were made on a machine having a short-circuit ratio of 0.76. These tests are of particular interest in view of certain published empirical formulas which give the desirable short-circuit ratios for machines on metropolitan systems as between 1.0 and 1.25. During the tests the generator was subjected to a short-circuit current of 2.5 times normal, and the duration of the fault was 1.8 sec. The records of the oscillations show that a longer duration would have been tolerable. This severe condition was withstood without instability, even though no voltage regulators were used. The principal discrepancy between the tests and the empirical formulas is, in my opinion, due to the failure to take into account the voltage support obtained from the other machines connected to the system. These formulas are probably derived principally from consideration of steady-state conditions. Subsequently it has become generally recognized that transient conditions are more severe than static conditions. Furthermore, the tests were made for a system synchronized at the load which is more severe for feeder faults than if the system were connected in the more usual way. These tests lead to the conclusion that machines of a short-circuit ratio less than unity are adequate from the stability standpoint on metropolitan systems.

H. Richter: (communicated after adjournment) Despite the greatest ingenuity on the part of system design engineers, the extensive research and utmost care in the manufacture of apparatus and the most ingenious safeguards by the operating staff, it is still a fact that up to the present time the electrical systems supplying the most heavily loaded areas of large cities have not been entirely proof against serious trouble and sometimes partial or complete shutdown.

The same cannot be said of the latest type distribution system in these areas. It has been proved that with the low-voltage a-c. network system employing automatic network protectors continuity of service can be preserved if proper yet simple precautions are taken. This statement is based on the experience started on Mr. Kehoe's system over seven years ago and now being gradually extended to the majority of the larger cities.

It appears that the method outlined in these three papers opens up one possibility of perfect continuity from the major sources of supply. I believe it is unique in employing a secondary line of defense, the automatic network protectors with their back-up fuses. The reliability in service of these two devices together has been shown to be about 99.9 per cent, which is at least the equivalent of station equipment.

In its resulting improvement of voltage conditions, simplification of system, safer operation, and reduced cost, this new method also parallels the similar advantages of the automatic network in distribution engineering. Another similarity is the facility of starting the operation of a system synchronized at the load without difficulty, just as a secondary network may be commenced without scrapping the apparatus of existing systems of other types.

The low-voltage network has made possible a greater use of diversity between loads on feeders than ever before. Synchronized-at-the-load operation will permit this use to be even further extended, due to the feeders from several generating stations being interleaved and picking up load as they pass through the different sections of the city.

Inasmuch as the most favorable distribution system for use with operating synchronized-at-the-load is the low-voltage automatic network, it is conceivable that where this method of operation is contemplated it will constitute one more reason for developing the network system as quickly as possible.

F. C. Hanker: (communicated after adjournment) Power-system engineering has received increasing consideration during the past several years and engineers have been intensely interested in developments that will maintain the necessary standard of service at lower cost. Metropolitan customers have been educated to expect the greatest reliability of the power supply. The three-wire d-c. system with its supporting battery adjacent to the load gave a continuity difficult to equalize. The a-c. low-voltage network portion of the system, introduced several years ago, has proved its reliability equal to the corresponding d-c. network. For further improvement of the complete a-c. system it became necessary to develop a design that would eliminate the hazard of interruption of the supply to the network.

Design, whether of apparatus or of systems, is always a compromise of what in many cases constitutes conflicting factors. The best performance is obtained when the balance of these

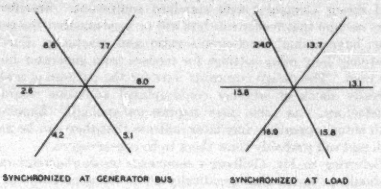


Fig. 8—Reactance between Generating Units Based on Kv-a. Rating of Units of the United Electric Light and Power Company

conflicting factors is secured. In the system layout described in the symposium a much better balance has been obtained in the many conflicting factors. The orthodox layout undoubtedly has a larger factor of safety when individual feeder disturbances are considered. It was vulnerable, however, to bus disturbances. In the "synchronized-at-the-load" layout the margin of stability for feeder faults has been materially improved. This is shown very clearly by the accompanying Fig. 8, where the reactances from generator busses to equivalent star point seem to be much increased by opening the station and station ties. In this way a system has been obtained that can successfully ride through disturbances including bus faults without interrupting service to consumers.

There has been a great deal of analytical work done on system characteristics during recent years, but this has been handicapped by the lack of specific records of operation. In the past these records have of necessity been of the statistical type that have unfortunately left many gaps in the complete story of system performance. The operation of a power system during faults is a complicated phenomenon, influenced as it is by all of the electromechanical factors that are involved in system layouts. In the symposium the section by Messrs. Searing and Milne gives records of the operation when synchronized at the load that will be invaluable in definitely analyzing the performance of the system when it is expanded to the ultimate. Oscillographic records have been made possible by recent developments in this type of equipment and their importance in studying system operation is becoming rapidly recognized. Therefore, the limitations of this type of recording apparatus has made it necessary to simulate actual systems by shop and laboratory set-ups. Unfortunately, all factors of the actual system cannot be duplicated with full accuracy and this has necessitated assumptions that have at times cast doubt on the accuracy of the conclusions. It is to be hoped that similar comparative data will be secured on other types of systems so that accurate comparisons of operating performance can be made.

C. M. Gilt: (communicated after adjournment) Since childhood, we have become accustomed to contradictory advice and typical of the differences in the warnings are "Don't put all of your eggs in one basket," and the opposite, "Put all of your eggs in one basket but don't take your eyes off that basket." One thought is based upon the expectation that we may drop one basket but not both, while the other believes that by putting all of our attention on one basket we can avoid dropping it. To some extent, generating stations and distribution systems have apparently been designed according to both recommendations.

While there is no doubt that some of our designs of electrical systems have decreased reliability, as well as increased costs, by too great diversification and complexity, yet I believe most of us will agree that we have not yet been able to design a station that can be operated without some time "dropping the basket" even with undivided attention. It has therefore seemed the part of wisdom, as well as of necessity because of the limitations in oil circuit breakers, so to sectionalize the busses of large, important stations that all the eggs are not dropped at once.

Our real purpose, which would appear to have been lost sight of at times, is not to keep our generating station busses going but to keep power flowing to every customer with as little likelihood of an outage, and particularly a long one, as is economically possible. The scheme outlined in the papers on *Synchronized at the Load*, recognizes that no bus section and no feeder can be kept free from all outages and carries the diversity of supply from the generator to the low voltage network or substation low voltage busses where substations exist with radial feed from them. The method outlined is most completely applicable to metropolitan areas with generating stations placed in or near a territory of fairly uniformly distributed load; and to a less extent, where generating sources are separated from a concentrated load. The development and operation of the scheme will be watched with a great deal of interest.

In 1922, when the plans for the new 60-cycle system of Brooklyn were formulated, the fundamental importance of diversity from generator to load was recognized, and in 1923 and 1924 when the system began to take form, a similar scheme was installed except that the generating-station busses were tied together through stiff reactors and a synchronizing bus in addition to the 4000-volt substation busses and later the low-voltage network. The substation 4000-volt busses were sectionalized and tied together through 10 per cent reactors and a synchronizing bus which is fed from a spare transformer supplied from a cable ring connecting together various substations. Each feeder bus is supplied by its own transformer and cable from a section of the generating-station busses, no two feeders to any substation coming from the same generating-station section. Ties between generating stations were made at intervening substations, in some cases at the high-tension side of the substation transformers, and in some cases only on the secondary side, so that load was transferred between stations by shifting the substation load from one generating station to the other.

It is expected that when Hudson Avenue Station is complete, no feeders to any substation will be supplied from adjacent generating-station sections. Complete diversification therefore is obtained from generators to substation busses and any generator, generating-station bus, or feeder can be lost without affecting the supply to any customer.

During the early stages of the station, and during the periods of light load, the synchronizing bus has been of real advantage

as a means of transfer of energy from one feeder bus to another, making possible the most economical loading of turbines without tying together of feeder group sections. It also furnished a back-up supply to each generator that was of particular value when the substations were fed by only two or three feeders, and added synchronizing power when the ties through the load were comparatively weak. As the system grows and the relative size of the bus sections decreases, the value of the synchronizing bus decreases somewhat and it must be admitted does increase the interrupting duty on the circuit breakers to some extent. However, it has some advantages, partly associated with its use to transfer power between bus sections making possible the maximum use of feeder copper cross-section and the division of load between generating units and boilers in the most economical manner, and partly as a back-up source of power for each generator. During periods of light load, it permits the reduction in the number of units to a small number without tying together the feeder bus sections and the consequent increase in the extent of any bus disturbance should one take place at such a time.

When the number or size of units exceeds the most extravagant expectations for which the engineers planned, as usually proves to be the case, the synchronizing bus itself may be sectionalized so as to reduce short-circuit duty.

The addition of the next unit at the Hudson Avenue Generating Station will make this necessary and we will either split the synchronizing bus into three or more sections tied together through bus reactors or operate it in two or more distinct sections synchronized through the load whenever the capacity on the bus exceeds about 350,000 kw. or provide the remaining units with double-winding auto-transformers with each winding connected to a separate feeder bus section and through reactors to different sections of the synchronizing bus. The synchronizing bus would then be operated in two sections synchronized through the secondaries of the generator auto-transformers and the load. While data are not yet complete on which a final decision can be made, the last two methods appear to be the most advantageous with the double-winding transformers scheme offering some advantages in stability in case of bus short circuit, though requiring a little more care in the distribution of feeder loads to avoid affecting bus voltages and in this particular case with the size of units under consideration, very slightly increasing the station cost. However, any material increase in the size of units would make the double-winding transformer a relatively less expensive installation.

The papers presented outline a very interesting application of the principle of diversity and separation of supply to the customer. In the type of community and load served, this has apparently been accomplished without increase in investment, and further has permitted a station to grow far beyond the original plans without an expensive change in oil circuit breakers.

S. B. Griscom: It is gratifying to note that the work done by Mr. Chilberg gave results practically the same as those presented in the papers.

Mr. Chilberg is correct in his statement that, in general, the most severe oscillation would occur after the clearing of a three-phase bus fault but this does not necessarily mean that the system is more likely to go out of step after the fault than during the fault. Because of the symmetry, the angular oscillations during a bus fault may be considerably less than the angular oscillations during a feeder fault, but the other system manifestations such as voltage drops would be much more severe.

The curves which Mr. Chilberg shows as illustrating the effect of induction frequency changers are likely to be misleading. They show that, following a short circuit, the load voltage gradually decreases if synchronous frequency changer sets are used, and gradually increases if induction frequency changer sets are used. This result is undoubtedly based on the loss of one generating unit and the assumption that the excitation on the remaining units is not increased. To a certain extent such an assumption might be justified since voltage regulators are not now in use on the systems in question. However, the inferred conclusion seems to be rather far fetched when it is considered that voltage regulators on the generating units will accomplish the same results; in fact, they will bring the voltage up to 100 per cent instead of 93 per cent for the induction frequency-changer set. The fundamental plan of operation calls for the use of sufficient spare capacity to permit the loss of the largest generating unit, which means that there would be sufficient capacity to take care of the wattless kv-a. demands even if a unit is lost during a bus short circuit. It is expected that no serious difficulty will be encountered without the use of voltage regulators, but if experience should show that the voltage should tend to settle to too low a value following the loss of a unit and could not be recovered quickly enough by the station operators, it would seem that the use of automatic voltage regulators would be a much more economical method of accomplishing the desired results. Where frequency changers are necessary for load transfer, the synchronous-synchronous frequency changer set has a distinct advantage in its ability to generate wattless kv-a., and in the fact that the two systems which it interconnects for power flow are still left independent as far as wattless power flow is concerned, thus minimizing the reflection of voltage disturbances from one system to the other.

The tentative design constants were based upon the general scheme of connection of the system of the United Electric Light and Power Company with standard equipment. Specifically, they assume that feeder reactors will be used and that the generator have normal short-circuit ratio and reactance. Further, relatively long relay settings for feeders from generator busses are used. The design constants were for the purpose of making a ready check of whether contemplated additions would be satisfactory. As such, they require no apology. Experience with actual operation may later indicate that they can be modified, and will probably show them to be conservative.

Referring to Mr. Chilberg's comments on short-circuit ratio, it should be noted that practically all of the generators on the United Electric Light and Power Company's system were purchased before any thought was given to stability or short-circuit ratio. Naturally, any new scheme of operation should be able to use these generators, preferably without placing restrictions on other features of system design. Both calculations and tests indicated that operation of the existing equipment synchronized at the load could be accomplished without compromise or special care in the place of system layout. Such being the case, why should generators of higher short-circuit ratio be used? If the system is safe for both feeder and bus short circuits, of what use is an increase in margin of stability? One might equally well argue that any structure in use today should be strengthened because strengthening will give a greater margin, even if years of experience had already demonstrated its safety. On the other hand, higher short-circuit ratio machines mean higher cost and lower efficiency. Further, if it is really felt that additional margin is required, it can be obtained by using voltage regulators, the first degree being to use a slow-speed system and the next degree a quick-response system. The cost and reduction in efficiency would be negligible compared to raising the short-circuit ratio.

Our studies were not confined to single-winding generators, but since the fundamental plan of synchronizing at the load was the prime question, only those phases dealing directly with the fundamental plan were covered in the paper. The scheme shown as 2 d in Mr. Chilberg's discussion would appear to be the most satisfactory method of connecting the double-winding generators into circuit. While the use of a double-winding generator may prevent the generator from pulling out of step during a brief bus short circuit, nevertheless one source of power supply—the short-circuited bus—is lost. Furthermore, the use of double-winding generators does not necessarily reduce the required amount of

spare capacity since trouble on the steam end may necessitate the removal of the whole double-winding generator.

Mr. Tapscott mentions the possibility of omitting feeder reactors in the future by the adoption of the synchronized-at-the-load and the double-winding generator principles. The possibilities of doing this seem to be very good, even without the use of neutral impedances, provided that the time required to clear short circuits can be reduced. In fact, it appears that if the short circuit can be cleared within 0.4 or 0.5 second, three-phase short circuits can be satisfactorily taken care of using single-winding generators. This would, of course, result in larger voltage disturbances than would be the case if feeder reactors were used.

A. H. Kehoe: It is gratifying to find, in the discussion, general agreement with our conclusions that connecting the New York 60-cycle system so it is only "synchronized at the load" has increased the reliability of generating sources, lowered circuit breaker interrupting duties, reduced service voltage disturbances, and simplified the control of system loads.

Mr. Crawford of Seattle has asked about the relaying of certain of the substation circuits shown in the paper by Mr. Searing and Mr. Milne. This is accomplished as follows:

1. Conventional reverse power directional relays are used at the substation.

2. Balanced relays could be used, but standard over-current relays are used on outgoing generating station feeders.

3. The reliability of 187th Street Substation is not safeguarded for generating-station faults. However, the service supplied by this substation is safeguarded by feeds directly to the networks from other station sections; that is, the interleaving is done at the networks and not at the substation in this instance.

4. On off-peak loads, substation transformer banks are eliminated, but feeders remain alive from the generating station to substation

In conclusion, I wish to emphasize that the "synchronized at the load" arrangement embodies a new principle of system design which may be combined with one or more of other well-recognized fundamental plan elements, such as isolated phases, quick-acting circuit breakers, or double-winding generators. Its use does not exclude the others, although it may in some cases make them unnecessary. Adequate system design requires a careful balancing of all such factors.

It is my hope that among those who have discussed the symposium papers or the many others interested in system design, there will be some who will make trial installations of promising system designs, in order that the electrical industry may profit by the improvements which are likely to be obtained in this field.

Automatic Transformer Substations
Of the Edison Electric Illuminating Company of Boston

BY WILLIAM W. EDSON[1]
Member, A. I. E. E.

Synopsis.—*The Edison Electric Illuminating Company of Boston has adopted a policy of numerous relatively small automatic a-c. substations located at the various individual load centers, this policy being enhanced by the development of a new method of automatic control and by the standardization of the station design.*

The rating of a substation is based on the sum of three specially designed transformers each having an inherent 50 per cent overload capacity. Thus, the maximum load is normally carried by the three units operating at their nominal 100 per cent rating, but in emergencies the load can be handled by two banks, the transformer out of service, representing only 33 per cent of the station capacity instead of 50 per cent as in the usual case.

The automatic control consists essentially of load-current relays which switch in or out the follow-up transformers as needed. This operating point for maximum station efficiency equals $\sqrt{\frac{2 \times \text{core losses per bank}}{\text{copper losses}}}$. *Serious trouble on any bank will bring in the succeeding transformer. Each transformer has its own sequence switch so that it may be made leading, follow-up, second following, or manual, irrespective of the set-up for the remaining units. The simplicity of this control equipment is particularly interesting; in fact only the current control and one auxiliary relay are directly chargeable to the automatic control. The circuits and apparatus have several special features promoting safety and reliability.*

The construction of these stations has been standardized by the development of standard "blocks" complete in themselves and suitable for being arranged or added, as desired. Thus, for each transformer section there are four types of blocks—high-tension, low-tension, regulator, and switchboard. Such an arrangement offers the advantages of complete segregation of equipment, flexibility in station layout, efficient provision for growth, standardization of engineering and construction, and the satisfactory use of complete bills of material.

This system of control and standardization of construction has been applied to five new automatic transformer substations in the last two years, and the results have been very satisfactory.

* * * * *

THE rapid growth and expansion of the electrical distribution system of the Edison Electric Illuminating Company of Boston has dictated a policy of numerous relatively small automatic a-c. substations located at the various individual load centers. Such an arrangement results in reduced line and feeder losses, better voltage regulation, improved reliability of service, closer balance between the load and the transformer capacity in service, and in a more efficient building program for the substations.

The value of this policy was materially enhanced by the development of a new method of automatic control which was much simpler in both design and operation than the previous schemes. In the last two years, the adoption of this control, along with the standardization of station design, has been applied to five new automatic transformer substations and the advantages are quite apparent.

Station Capacities

The ratings of these stations are based upon an ultimate of three units, and vary from 5000 to 15,000 kv-a. depending on local requirements, these differences being easily met by selection of the number and sizes of the transformers without detracting from the principles of standardization. Closely associated with this question of unit sizes is the special design of transformers developed some years ago for the Edison Company.

First of all, it is essential that a station be able to carry its full load in event of failure of one of the units. Formerly this contingency was met by installing the three transformers of such size that two at full rating could carry the load. This meant that under normal conditions one unit (50 per cent of station rating) and its switching equipment was out of service, which represents an economic loss.

Under the Edison Company's arrangement this situation is met by selecting a smaller size, such that all three would be used in carrying a full load. At a slight additional unit cost these transformers are given a 50 per cent overload capacity by increasing the copper and the cooling surface, so that if necessary any two could carry the full load without exceeding the normal 55 deg. cent. temperature rise. Of course the efficiency of these units would be reduced somewhat during this overload, but as this is an emergency condition the loss in kilowatt-hours is negligible.

Description of Station

For a 10,000-kv-a. station, for example, there are three 3500/5250-kv-a., three-phase, delta-star transformers having their maximum efficiency at 75 per cent (probable normal load) of their nominal rating of 3500 kv-a.; that is, at 2600 kv-a. Each bank has a high-tension bus section supplied by two 13,800-volt lines, these sections being connected by bus-tie breakers normally closed. The bank supplies a main and auxiliary bus having six 4000-volt, four-wire ungrounded neutral, regulated, automatic reclosing feeders. The low-tension bus-tie breakers are normally closed, but they trip automatically in case of a feeder fault if all

1. Station Engineering Dept., Edison Electric Illuminating Co. of Boston, Boston, Mass.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-76

three banks are in service, thereby reducing the rupturing duty on the feeder oil circuit breaker.

Only two of these banks, however, are installed initially as the station load at first can be carried by one bank in an emergency.

Automatic Transformer Control Design

In designing the automatic operation of an automatic transformer substation the following procedure may be followed: (applies equally well to any type of automatic control)

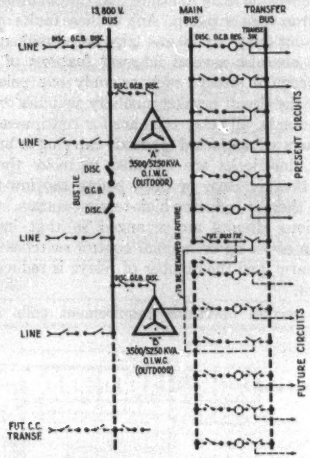


Fig. 1—Single-Line Diagram of Humboldt Avenue Station

1. The size and number of the various units and the one line diagram of the station are first established.

2. A list of the desired operating features is then discussed and agreed upon by the various interested departments of the company. Incidently, as for any type of automatic equipment, this is the real deciding factor leading to successful operation.

The equipment must be able to perform in general the duties of an operator in an accurate, safe, and reliable manner, but there is a surprisingly large number of requirements which may safely be eliminated.

3. The third requisite in the designing of an automatic station is the collecting of the individual relay circuits into a combined diagram which is simple, efficient, and safe. Three items must be kept in mind while studying this completed diagram.

a. Does it meet all of the operating features and control requirements?

b. Are there any short circuits or stray currents under any possible combination of relay or switch operations? Any amount of time spent on this study will be amply repaid when the set goes into operation. Even then, actual service will develop unexpected stray circuits, but these can be mitigated.

c. Can the installation be simplified, such as by assigning several duties to one relay, collecting various circuits to the same set of relay contacts, or by arranging the contacts so that the number of wires leading to an item of equipment is reduced to a minimum?

The schematic diagram is essential as each individual relay circuit can easily be followed and studied.

Some form of sequence chart is also of great help as it serves as a "slow motion" picture of the status at any operating step; that is, it shows the cause and result of each operation.

Operation

The system developed and adopted by the Edison Company performs as follows:

a. For light-load conditions, one transformer carries the load. As the load increases, the other banks cut-in automatically as required. The two properties available to control such operation are the transformer temperatures or the station load. The first is not entirely suitable as the temperature usually lags too far behind the load, it is not independent of the weather conditions, and the present temperature relays are somewhat delicate and difficult to keep in adjustment.

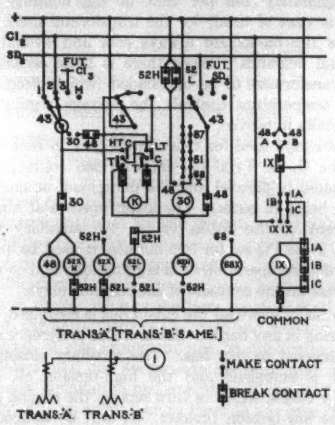


Fig. 2—Schematic Diagram of Transformer Control

The second method (station load) offers better results as the current relays already developed are quite accurate and reliable. Another important advantage is that the transformers can be operated nearer the best over-all efficient point. For example, these cut-in relays can be set at a point where the core and copper losses of one transformer equal the total losses of two transformers. Expressed mathematically this most efficient point

to bring in the second bank equals $\sqrt{\frac{2\,L_i}{L_e}}$ where L_i is the core or zero load losses of one transformer and L_e is its full load copper losses.[2] This point averages

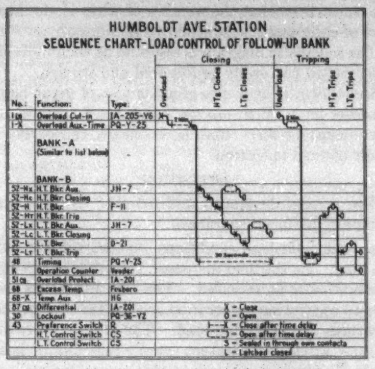


Fig. 3—Sequence Chart—Automatic Starting

approximately 105 per cent of the nominal rating (70 per cent of total) for the transformer used, which means the banks are always cool and available for unusual demands.[2] Since there is little occasion for the transformers to be overloaded (which often occurs with temperature control) the voltage regulation is materially improved.

The relays used for this control (one in each phase) are the G. E. Type IA, the overload contacts being connected in parallel and the underload, or back contacts, being in series. The latter operate at about 90 per cent of the cut-in value. An auxiliary timing relay, Type PQ set for two minutes is used to prevent too frequent operation on a surging load and to reduce the wear on the contacts of the current relays.

In case of overload the cut-in bus is energized, which will bring in any bank or banks whose preference switch is connected to this bus. The auxiliary closing contactor is energized and the high-tension oil circuit breaker closes, which in turn sets up the closing circuit for the low-tension breaker. It will be noticed that when the cut-in bus was energized the protective timing relay No. 48 began to close. By the time it completes its stroke, the high- and low-tension breakers should be closed but if not, or if one drops out due to latch trouble, a tripping circuit is set up and the transformer is locked out.

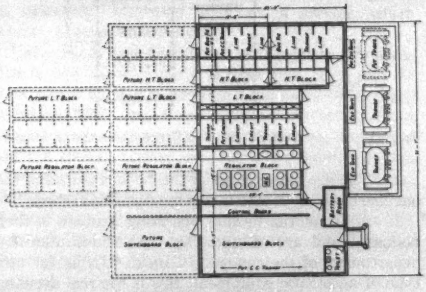
When the station load decreases the shut-down bus is energized. At the same time the cut-in bus is deenergized which allows No. 48 timing relay to open. When it completes its operation the tripping circuit for the high-tension breaker is completed and this breaker opens, followed in turn by the low-tension breaker.

A similar set-up would be used to control the third bank through a second set of cut-in and shut-down busses.

b. The question of safety and reliability assumes special importance in a non-attendant station. These are met in the Edison system by providing each transformer with differential protection relays, excess overload relays, maximum oil temperature relays and an over-all checking relay to insure that the circuit breakers do not drop open or pump. Any of these faults operates the lock-out relay No. 30 and trips both breakers.

There are also several inherent features of design which promote safety; such as, only one polarity is taken to the circuit breaker auxiliary switches or to the relay contacts, all relay coils are for continuous duty, no contacts are subjected to unsafe carrying or breaking currents, interlocks are included to make the high-tension breaker close or open before the low-tension breaker, thereby reducing high-tension voltage stresses, the various circuits are arranged so that improper operation of set-up or manual control switches cannot do any harm, and the number of relays is reduced to a minimum.

c. A second operating requirement calls for the


Future H.T. Block
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Future L.T. Block
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Future Regulator Block
Future Regulator Block
Regulator Block
Control Block
Future Switchboard Block
Switchboard Block
Air Tank
Oil Tank
Oil Tank
Oil

d. Full selectivity is provided by giving each transformer its own sequence switch so that any bank may be made first, leading; second, follow-up; third, alternate; fourth, out. Manual control switches are connected to this fourth position as a matter of uniformity in operation between stations; otherwise they could be omitted. It will be noted that any of the transformers can be operated independently of the others. For example, one may be under manual control and the rest automatic, or if desired, two may be set up to operate in parallel as a group, thereby permitting two smaller banks to operate as a unit and reducing the number of operating cycles.

1. A high-tension block for the two incoming lines, the 13,800-volt bus and its tie breaker, and the transformer high-tension breaker.

2. A low-tension block for the transformer low-tension breaker, the main and transfer busses with their breakers and disconnects, and the 4000-volt feeders.

3. A regulator block for the feeder induction voltage regulators. The battery motor-generator set and the station power and lighting transformers are located in the space opposite the bus-tie cell.

4. A switchboard block for the control switchboard, battery room, toilet, and the constant current lighting transformers if used.

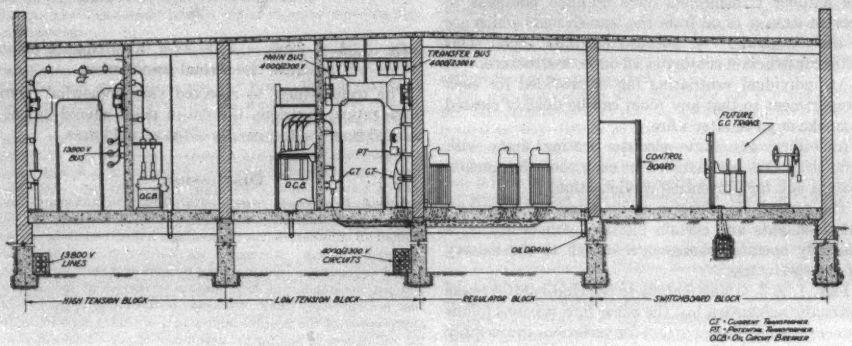

FIG. 5—CROSS-SECTION OF HUMBOLDT AVENUE STATION

MISCELLANEOUS AUTOMATIC FEATURES

The manufacturer's method of automatically reclosing feeder circuit breakers has been revised somewhat to include the safety features and simplicity referred to above.

The battery charging motor-generator normally floats on the control battery, but it will shut down in case of certain faults and will restart when conditions become normal.

A signal is sent over leased telephone wires to the nearest manually controlled station to indicate the opening of any line or feeder circuit breaker, the lockout of any transformer, or a ground on the 4000-volt system. This latter indication is accomplished by means of a CV voltage relay connected between the system neutral and ground.

STATION DESIGN

The construction of these automatic substations has been standardized as far as practicable. The basic feature which permits this standardization of construction is that the station is divided into "blocks," complete in themselves and suitable for being arranged or added as desired.

For each transformer section there are four such blocks:

ADVANTAGES OF THE BLOCK DESIGN

a. Complete segregation, the blocks themselves forming natural fireproof subdivisions.

b. Flexibility in arrangement to meet land or neighborhood conditions.

c. Efficient provision for the growth of the station by the addition of blocks as needed.

d. Standardization of construction, resulting in the simplification of the preliminary load, land and cost estimates as similar information from previous stations is at hand, the betterment of the price and delivery for the purchase of the equipment due to duplication of former orders, and the decrease in the time and cost of the design installation and operation of the station. Incidently, it is an advantage to know in advance that the completed station and its operation will be satisfactory to the various departments concerned as the individual standards have already been tried in service and approved.

e. Satisfactory use of an itemized bill of material, this list being grouped in accordance with the station blocks and giving such information as may be needed during the various stages of engineering or construction.

MISCELLANEOUS CONSTRUCTION FEATURES

Each high- and low-tension block is divided by an

eight-inch concrete fireproof wall so that the oil circuit breakers are in one compartment while the busses disconnects and instrument transformers are in the other. Metal lath and plaster cell walls two inches thick separate the equipment per circuit. The regulator room and breaker cells are drained to prevent spreading of any possible burning oil. There is no reinforcement or other steel work carried over between cells. The ground busses are run on insulators to prevent fault currents passing through the structure.

The cell wiring is flame-proof varnished cambric cable. The high- and low-tension cables running to the outdoor transformers have oil-filled potheads to prevent leakage of oil from the transformers which are of the "industrial" oil pothead entrance type. This latter construction eliminates all outdoor structure.

An individual ventilating fan is provided for each compartment so that any room can be quickly cleared of smoke or gases after a fire.

Individual automatic electric heating units with circulating fans are installed in each room for general heating and for preventing condensation.

The general illumination is by alternating current, but floodlights and certain lighting circuits are on a manually operated change-over switch to the battery for emergency use.

Figs. 1, 2, 3, 4, and 5 apply to the Humboldt Avenue automatic substation but the other four stations follow the same general design except for variations in assembly of the "blocks." For example, in the Arlington Substation the switchboard and regulator blocks are located on the second floor.

The construction and operation of these stations have been very satisfactory and it is expected that this type of design will be continued in future stations.

Appendix

Light station loads can best be carried by one transformer as the core losses of two banks would be considerable. As the load increases, however, the copper losses become excessive as they vary with the square of the load; therefore, a point is soon reached where the second transformer should be cut-in. This point to give minimum over-all station losses is that at which the core and load losses of one transformer equals that of two banks, each carrying half the load, thus:

$$L_i + F^2 L_c = 2\left(L_i + \left(\frac{F}{2}\right)^2 L_c\right)$$

or $$F = \sqrt{\frac{2\ L_i}{L_c}}$$

Where

L_i = zero load or iron loss of one transformer
L_c = copper losses at full load
F = ratio of the load to the rating of one transformer.

The point of maximum efficiency of a transformer is that at which the iron losses equal the copper losses or $L_i = F^2 L_c$

As the average load per transformer may be assumed to be at ¾ its rating the transformer is ordered to be most efficient at this point.

Then: $L_i = \left(\frac{3}{4}\right)^2 L_c$ or $L_i = 0.5625\ L_c$

Substituting in the above formula

$$F = \sqrt{\frac{2 \times 0.5625\ L_c}{L_c}} = 1.06$$

Or $1.06 \times 3500 = 3700$ kv-a. approximate point to cut in the second transformer.

The transformers as received vary somewhat from these ratios; therefore, the cut-in point should be calculated from the test results of the actual losses.

Discussion

A. J. A. Peterson: (communicated after adjournment) Mr. Edson's article describes a development which might well be studied for consideration of the resulting improvements in operation and economy. Automatic transformer substations of the kind described have been in service for approximately nine years, and yet the number of such stations in service at the present time is surprisingly small. This may be due either to the lack of appreciation of the possible advantages of such a station, or to the willingness to pay for the core losses of two transformers at all times, rather than suffer the remote possibility of a slight interruption to service.

Mr. Edson stresses the need for simplicity. This can equally well be stressed in any kind of a station, although it seems of more value in an automatic or non-attended station on account of the absence of constant supervision. In analyzing the possibility for simplicity, it is sometimes surprising what features have been considered desirable, and which may very safely be omitted.

The use of a time-delay relay before carrying out the normal switching operation is open to discussion, first, because under usual conditions, the load cycle is comparatively smooth and does not have many small changes, and second, because the load relays which determine the switching point have their own time delay. In addition, the interlocking of the back contacts of the load relays insures further time delay when units are switched out of service. This performance has been confirmed by several years satisfactory operation in several substations.

The time delay for supervising a switching operation seems to be primarily to insure that the breakers close in and stay latched, as trip-out due to electrical fault conditions is taken care of by the use of lockout relays causing the switch-in of the follow-up transformer. Good design and manufacture, followed by good maintenance, are the best precautions against latch failure. It should be possible, therefore, to assume that the breaker will stay closed on the first attempt, unless there is electrical trouble, and the control scheme can be simplified to permit just one attempt at reclosure, thus guarding entirely against any form of pumping.

Unquestionably, the temperature method of control might be desirable from several points of view if it were not for the difficulties of determining the proper switch-in and switch-out points. As a result, the total station load method seems to be the

simplest and most practical method of transformer control, and gives the further advantage of permitting operation in accordance with the actual transformer economy. It does not, of course, permit increasing the rating of a transformer during cold weather, when this might be necessary without regard to economy.

Full selectivity in transformer sequence, that is, the ability to locate any transformer anywhere in the sequence, independent of the others, may be considered unnecessary. However, it seems equally difficult to obtain any form of selectivity and this full selectivity will probably be used until there is developed some simpler form, whereby only the initial transformer is changed in position and the subsequent or follow-up sequence is the same.

Mr. Edson mentions their revision of the manufacturers' method of automatically reclosing feeder circuit breakers. As the manufacturers usually pride themselves on the simplicity of their control equipment, we should be interested in knowing what changes have been made by Mr. Edson in order to obtain this simplification.

The development of the so-called "block" design of these transformer stations is extremely interesting in its effect on engineering and construction cost and time. This development has been carried still further by the utilization of steel cubicles or compartments for the oil circuit breakers, busses, control panels, etc. These units have sometimes been manufactured separately, but have been completely assembled and tested in themselves so as to reduce the time required for assembly in the substation. In other cases, particularly where smaller oil circuit breakers are used, the complete breaker and switching equipment can be assembled with the control panel in a single unit.

The clear and simple manner in which the advantages of this type of station have been presented should lead to further interest on this subject and will undoubtedly result in further development along this line.

Philip Sporn: (communicated after adjournment) Mr. Edson's paper with its stress on standardization is very interesting to me because for years we have been preaching and practising standards in the matter of design and construction on electric power systems. Properly used and applied standards can reduce by substantial amounts the expenditures necessary on an electric power system, but there is always a danger that standards be made a fetish or be made so rigid and so unresponsive to change that after a while the standards merely block new developments. When that point is reached, of course, it is time to throw away the standards since at that point instead of being an aid to progress the standard becomes a definite hindrance. Put more briefly, it is necessary first that the standards be applied intelligently, and secondly it is necessary that they be revised continually.

Referring to Mr. Edson's paper, there are some questions that naturally present themselves upon its reading. First, I note that the standard transformer is a combination self-cooled, water-cooled transformer. For a number of years now on our distribution systems we have been using a similar combination with the exception of the fact that we have substituted air blast for water. It seems to me that for the distribution transformer which generally carries its peak for a short period of time the air blast is far preferable to water and it ought certainly to be preferable in a location having the low temperatures that are encountered in Boston.

I note that Mr. Edson has installed electric heating in each of his rooms, which is certainly an excellent idea, but I should like to ask why, in a modern substation, advantage is not taken of the development of the storage type of heater. I believe that proper analysis of the capital value of losses by the use of standard heaters will disclose in most instances sufficient justification for the storage-type system. It has of course the further advantage that for an electric utility it means practising what it preaches. If electrical heating is ever going to be anything more than a dream, it will certainly have to be done through some storage arrangement and not through a system that comes on with its demand at a peak period.

The unit room idea is very interesting but it would appear that certain drawbacks to the idea are inherent. For example, it must be very difficult so to treat architecturally a flat and spreading building of the type shown in Figs. 4 and 5 that it will add beauty to the landscape surrounding it; in a residential district this certainly has to be considered today. It is possible too, of course, that such an arrangement would be impossible where land values are high and where such spreading out is impossible.

W. W. Edson: The time-delay features for normal switching and for supervision might safely be omitted as mentioned by Mr. Peterson, but they have been included as these relays are necessary anyway for other purposes.

The overload rating of the transformers questioned by Mr. Sporn is obtained by having sufficient radiator cooling surface and core to allow 150 per cent load to be carried at the standard temperature rise. These modifications are considerably cheaper than the designs calling for water or air-blast cooling. Of course, the efficiency at this overload drops somewhat, but, as explained in the paper, the extra kilowatts lost is not important as this is an emergency condition.

Individual heaters were used for the rooms as they eliminate the need for air ducts or steam pipes running between the various cell compartments and also remove the possibility of condensation or water leaks from steam pipes.

The relatively large area due to this single-story layout is not bothersome as these stations are located where the land values are not high.

I wish to thank Mr. Peterson and Mr. Sporn for their discussions of this paper and am very much interested in the valuable points they have added.

Rehabilitation of Steam Power Plants

BY C. F. HIRSHFELD[1]

Associate, A. I. E. E.

Synopsis.—Certain pertinent history of the recent development in the field of power generation is given in graphic form. The problem of rehabilitation of steam power plants is shown to arise from the great rapidity of this development. While no simple rule can be given to indicate when, if at all, rehabilitation is justifiable, certain guiding criteria which may serve as aids in analysis are stated. These are followed by brief accounts of the various methods available for rehabilitating plants to different extents.

Finally, it is suggested that the possibility of later rehabilitation be taken into account when designing plants for systems in which history or argument indicate that such action is to be expected.

* * * * *

THE exceedingly rapid evolution of the art of generating electric power is known in a general way to all who have more than a casual contact with it. However, even those whose daily business consists of power generation frequently fail to appreciate the speed with which we have advanced or the significance thereof. The graphs shown in Figs. 1, 2, 3, 4, 5, and 6 may serve to visualize the rate of progress. The significance of the development, however, cannot be shown by such curves.

In the first five of these figures the areas within which representative station practise has fallen are indicated by cross hatching and an attempt has been made to indicate what might be called conservative practise or majority opinion by means of a curve drawn through each area. The locations and shapes of these curves must be a matter of personal opinion to a certain extent and no great degree of accuracy can be claimed for them.

The curve given in Fig. 6 is similar to one that has

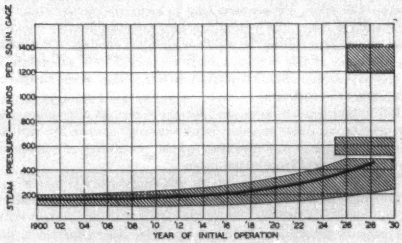

FIG. 1—INCREASE IN STEAM PRESSURES

Shaded areas indicate approximate limits of central station practise

appeared in print several times within the past few years. It represents a remarkable procession in the direction of lower thermal costs. But it appears to be approaching the place where further improvement will be much less rapid. The decreased thermal consumption is, of course, not self justifying because it may have been obtained with more than a compensating increase of fixed charges. Unfortunately, experience is not yet sufficiently extensive to make possible a complete analysis. One can only reason from the reports of the few who have actually built and operated the thermally efficient plants. On such a basis they seem to be justified under certain conditions, at least.

The curve of Fig. 7 shows values to which little attention has been given in the literature of this subject. The increasing output per man hour is in line with the general movement throughout the industrial world.

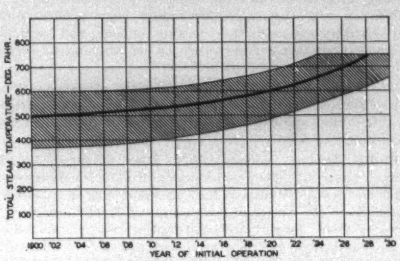

FIG. 2—INCREASE IN TOTAL STEAM TEMPERATURE

Shaded area indicates approximate limits of central station practise

In this case it is brought about largely through improved load factor combined with the use of boilers and turbines of ever increasing capacity per unit. There has also been a conscious effort to design stations in such a way as to facilitate operation and maintenance and thus decrease the number of men required.

From the executive's point of view the rapid advance indicated in these figures has had two opposing results. On the one hand, the total cost of producing power has been decreased. On the other hand, the rate of obsolescence has been high and invested capital has therefore been impaired. The net result of all the forces concerned has been a gradual but consistent lowering of the selling price of power. But the decrease in this price has naturally not been as great as it would have been had it been possible in some magic way to wipe out of existence the older and less efficient plants as more modern ones became available.

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Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-81

The situation from one point of view is shown in Fig. 8. The solid line represents the thermal performance for a typical metropolitan system in which the rate of growth has been rapid. In drawing the line certain liberties have been taken with facts in that the effects of variables other than those here under consideration have been omitted. The dotted line

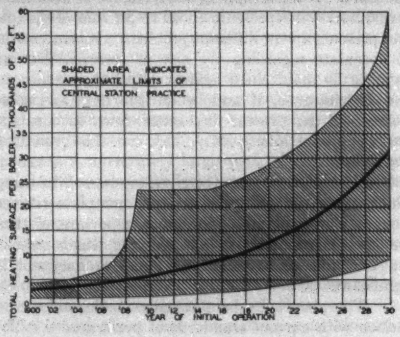

Fig. 3—Increase in Physical Size of Boiler Units

shows what the values would have been if at the time of each improvement all earlier equipment could have been junked and the required capacity obtained with equipment similar to that just installed. One of the problems of the executive of light and power companies is to determine from time to time just how far he is

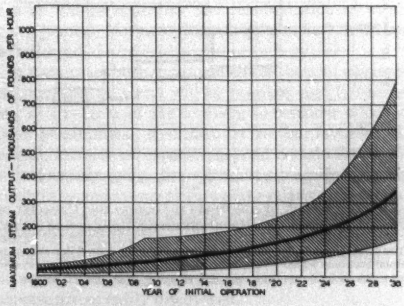

Fig. 4—Increase in Steaming Capacities of Boiler Units

Shaded area indicates approximate limits of central station practise

justified in moving values such as those shown by the solid lines nearer to those shown by the dotted ones. Putting it in another way, the problem is to determine from time to time that balance between production and capital charges which will represent the best compromise for all concerned. It is out of this problem that we get rehabilitation and reconstruction of power plants.

It is self evident that no simple rule can be set as a criterion of the advisability of such operations. There are too many variables of divergent nature. It is of course true that the old simple rule to the effect that in the limit an improvement must at least so reduce production costs as to balance resultant capital costs

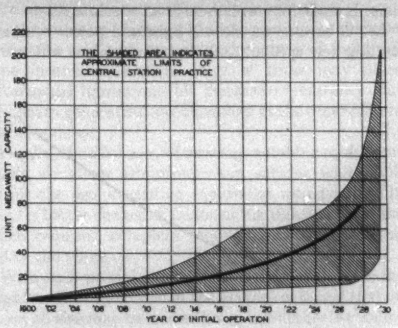

Fig. 5—Increase in Main Unit Turbine Sizes

holds in a general way. But it is equally true that the effect of improvements upon both sets of costs is, in general, so complicated that exact calculation is very difficult if at all possible in any given case, and a universally applicable formula is practically non-existent. Moreover, there are frequently many almost immeasurable although very tangible factors that enter

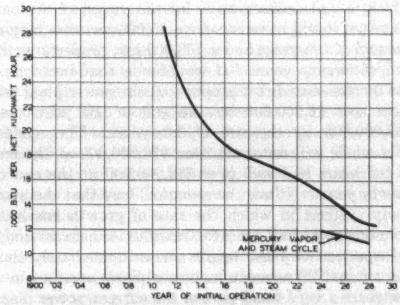

Fig. 6—Thermal Economy of Plants Operating on 50-60 Per Cent Annual Load Factor

the problem, so that judgment must to some extent take the place of exact calculation.

There is one very interesting aspect of this problem which should be mentioned at this point. It is the difference between conditions which arise in the case of rapid and slow growth respectively. It happens that most of the light and power companies of this

country have had a very rapid growth. This is a situation very favorable to the maintenance of high standards and low costs whereas with a lesser rate of growth the problems involved are far more complicated. This can be shown very well by using numbers for an assumed case. It should be noted that the numbers chosen are used for simple illustration only and that no claim is made for correctness in an absolute sense.

Let us assume an electric light and power company starting new with a 25,000-kw. power plant which cost

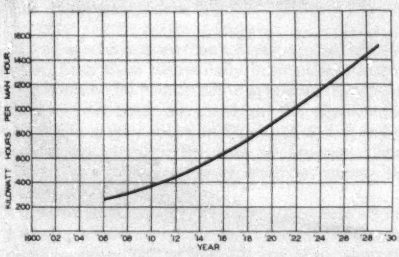

FIG. 7—KILOWATT-HOURS GENERATED PER MAN HOUR

This curve represents operating and maintenance labor for plants characteristic of each year's design practise

$100.00 per kw. of capacity representing a total investment of $2,500,000. Let us also assume that the entire cost of the property of the company, including the plant, is two and a half times the cost of the plant; namely, $6,250,000. If we assume that this company experiences a rate of growth of 10 per cent per year, which is quite conservative for the period of the past 20 years, it will have become a 65,000-kw. company at the end of ten years and a 170,000-kw. property at the end of twenty years. If we assume that investment has increased in direct proportion, the power plant will have cost $6,500,000 by the end of 10 years and $17,000,000 by the end of 20 years. The system as a whole will represent over $16,000,000 at the end of ten years and well over $42,000,000 at the end of twenty years. It may be remarked here that there are many systems in which the rate of growth has been from one and a half times to twice the rate here assumed.

Inspection of these figures shows immediately that the original investment in power plant becomes in a few years a very small part of the current power plant investment and a much smaller part of the total current investment. When it is realized that extensive power plant rehabilitation can frequently be effected while preserving from 30 to 60 per cent of the original investment, it becomes evident that such rehabilitation in a case of the sort assumed need not affect so great a part of the total capital as to represent any particular difficulties with respect to adjustment of the capital account out of reasonably small reserves. For example, if $1,000,000 of investment is to be junked it represents only about 6 per cent of the total investment at the end of 10 years and only a little over 2 per cent at the end of 20 years.

On the other hand, if we assume the opposite extreme, namely, a system which has not grown at all, the old power plant at the end of any given period will represent the same fraction of the total investment as it did at the start. If the scrapping of $1,000,000 of investment is in question it represents about 13 per cent of the total and presents a very different problem. It is interesting to speculate regarding the size of reserves that will have to be maintained against physical deterioration and obsolescence, if and when the light and power systems of the present day settle down to a rate of growth proportionate only, say, to the growth of population.

Consideration may now be given to some of the things which cause rehabilitation or even complete replacement of power plants. The more important of these are listed below with certain remarks which indicate the conditions under which they may or may not be significant.

1. The plant may be thermally inefficient in comparison with current standards of performance. This consideration is generally significant when the plant in question represents a large part of the total generating capacity and therefore a large part of the investment; when it occupies a strategic position in the system with

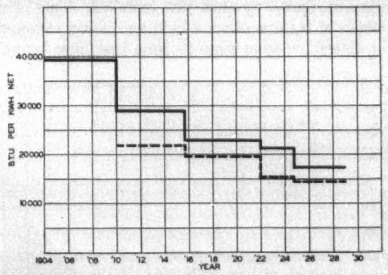

FIG. 8—THE EFFECT OF ADDITIONS TO GENERATING CAPACITY ON THERMAL ECONOMY OF A REPRESENTATIVE GENERATING SYSTEM

The solid line indicates actual conditions while the dotted line shows the thermal economy which would be attained if the older parts of the system were replaced by equipment on a par with the most modern section

respect to receipt of fuel or distribution of load so that its extensive use is required; when it occupies land which has appreciated greatly in value so that the maximum of return must be obtained from it, if it is to be retained; and, in general terms, under any other conditions which make the cost of fuel at this plant a very significant part of the company's annual expenditures.

2. The plant may be inefficient in the use of man power. This consideration is generally not of itself a decisive one but is one that is taken into account in

combination with others. It is serious in cases where labor trouble is to be expected and it may be serious when the plant in question represents a very large part of the total plant capacity.

3. The plant may be inefficient in its use of ground area. This becomes important in the case of land which has appreciated greatly in value, when the plant is particularly favorably located with respect to water or fuel supply and when the load conditions in adjacent territory make greater generating capacity on the site imperative.

4. The plant may be inefficient in its use of invested capital, as by occupying a larger building than necessary, operating boilers at low steaming rates, etc. This consideration is of importance only when the plant represents a moderately large part of the total plant investment or when the location becomes of particular importance for reasons such as those given in preceding paragraphs.

These simple statements should not be understood to indicate that figuratively one can stand off to one side of the problem of rehabilitation and decide offhand with regard to the significance of these controlling factors in any given case. In each case certain rather rough calculations will usually indicate the relative magnitudes of investments, water quantities, and areas and volumes required to obtain certain approximately determined capacities, investments, and thermal performances. Such preliminary calculations will then indicate the impracticability or even the absolute impossibility of certain solutions, leaving a much restricted field for more intense study.

It is practically impossible to set down definite rules or even guides regarding the rehabilitation of steam power plants. A few outstanding facts may, however, be indicated.

In most cases both improvement of thermal efficiency and increased capacity will be possible, and it is necessary at the start of the problem to consider which of these possibilities, if either, is to be favored. In general, increase of capacity beyond certain limits will be attainable only by sacrificing certain thermal possibilities and a clear picture of the relative importance of the two will assist greatly in steering a reasonable course through the maze of things that might be done. Usually the possibility of increased capacity should be given first consideration because certain limits to such increase are so easily discovered.

Thus, it would naturally be foolish to consider increase of capacity if the increased output could not be transmitted or distributed from the site in question. In many cases the available number of routes is limited and these may be found to be used to the limits of their respective capacities. Naturally this is logically the first matter for study.

If the power could be got away if generated, the next logical step is a study of those things which may limit ability to generate on that particular site. These would be primarily the quantity of water available for condensing purposes and the possibility of delivering the required amount of fuel. If all available water is now being used, as determined either by the supply or the size of canals that must be used, increased output can be obtained only as the steam rate can be reduced, or as the vacuum is permitted to become poorer and the thermal efficiency proportionately sacrificed.

If the water and fuel supply appear adequate the area of the site and area and cubical content of buildings come into consideration. Any of these may place unfortunate limitations on increase of capacity but they are in general not so controlling as the other factors just considered.

Finally, the adequacy of existing switching equipment should be given attention. Any scheme which will permit the use of existing equipment would naturally stand a better chance of financial justification than one which required complete rebuilding of the electrical end of the plant.

With such possible limitations clearly evaluated, the most important decision in the average case and that on which all else may be said to hang, is whether the boilers are to be retained or scrapped. If they are to be retained the operating steam pressure is limited to its earlier value except in those cases where a high-pressure unit is installed as an addition to existing equipment. This will be considered later. If the boilers are to be retained there is definite restriction of choice as to what may be done; on the other hand there is the certainty of a smaller expenditure.

There has been much misunderstanding of the significance of high steam pressure. It happens that a very rapid improvement in the thermal efficiency of power plants has occurred during the same time that we have been stepping up steam pressures. As a result, many who are not familiar with all the details have come to assume that the first is entirely the result of the second. This is really far from a complete statement of the facts.

During the same interval, we have increased the boiler room efficiency from values falling between 70 and 78 per cent up to values falling between 84 and 89. A change from 78 to 88 represents a fuel saving of almost 13 per cent.

We have also adopted the regenerative cycle of operation, that is, heating of the feed-water in steps by means of steam bled from the turbine. This represents a saving varying from 5 per cent to 11 per cent when referred to the coal pile.

It should also be noted that we have, in addition, increased the total steam temperature. In a period of about fifteen years we have gone from a total temperature of about 600 deg. fahr. to a total temperature of about 725 deg. fahr. This change of temperature is equivalent to a thermal improvement of between 4 and 5 per cent for the plant as a whole. During all this time the design of the steam turbine itself has been so improved as to result in a material reduction in heat con-

sumption at any given pressure, temperature, and vacuum.

It is therefore apparent that it is not correct to attribute all of the improvement in the thermal performance of the past ten years or so to increasing steam pressures since many other factors have contributed. As a corollary it follows that in the rehabilitation of a station, it may be possible to do much that will result in improvement of the thermal efficiency even though the original steam pressure must be retained.

The decision with respect to the use of higher steam pressure is not entirely confined to its effect upon the thermal efficiency because increased capacity can be obtained in this way through the use of comparatively small high-pressure turbines discharging into the old turbines, utilizing the old condensers. Therefore, while the boiler room investment would necessarily be high this may be balanced by a comparatively small turbine room investment. The use of the old generators would also, in general, make it possible to use the old switching equipment, although there would naturally have to be additions to take care of the generators of the new high-pressure turbines.

Only one case is known to the author in which complete replacement of old boilers with new high-pressure boilers has been decided on. This is a very special case in which new boilers and turbines are being installed in the old buildings for the primary purpose of obtaining greatly increased capacity from a given site of restricted area. If it is a fact that this is the only case of this sort, it may be due to the comparative newness of high pressures. There are, however, several cases in which high-pressure boilers and corresponding turbines have been added to existing stations with the idea that they would carry the base load and thus take maximum advantage of the higher thermal efficiency produced. Such work is more properly regarded as extension of an existing plant than as rehabilitation of an old one.

The majority of cases of rehabilitation have thus far fallen in the class in which the old boilers are retained, thus accepting the limitations set by the original steam pressure. At present, these are the more interesting and probably the more important.

The following methods of attack are available in such cases:

1. The boiler room efficiency may be greatly improved without materially increasing the steam generating capacity. This can be done by the use of more modern fuel burning equipment of one sort or another, by the addition of economizers or preheaters or both, and occasionally by improved baffling of the boilers.

2. The boiler room efficiency may be greatly improved and, at the same time, the steam generating capacity may be increased to a very great extent. It is not at all impossible to double or more than double the steam output of an old boiler room by using improved combustion equipment of greater fuel burning capacity, while at the same time bettering the boiler room thermal efficiency through the use of economizers or air preheaters or both. It will generally be found advisable to use a certain amount of furnace wall cooling in such extreme cases so as to gain the increased output most easily and so as to hold in check the tendency toward increased furnace maintenance. In this connection it should be noted that it is frequently possible to obtain greatly increased furnace volume, as by ignoring existing floor levels and by utilizing modern ash handling equipment which requires small height for its installation.

3. The turbine room efficiency may be increased by raising the steam temperature. This will naturally involve more superheater surface or at least differently located surface. In most cases the use of some sort of radiant superheater in series with the older superheaters will be found advantageous but a careful study of pressure drop through superheaters and connections must always be made. This is of particular importance when the steaming capacity of the boilers is also increased. The adoption of higher superheat will usually involve minor rebuilding of the high pressure end of the turbine and in some cases major rebuilding of that end.

4. The turbine room efficiency and capacity may be increased by several different methods. The most obvious, but also usually the most expensive, is by the installation of new, larger, and more efficient turbines taking steam at higher temperature from a boiler room of increased steaming capacity. When this method is adopted it becomes comparatively easy to introduce regenerative feed heating because the new turbine casings will be supplied with bleeder outlets. The improvement of thermal efficiency obtainable from regenerative heating can therefore be obtained readily. Larger condensers or additions to existing condensers may have to be provided. The old electrical switchgear can sometimes be used by adopting a double winding on the generators so that the capacity per switch is held within the old limitations.

A less obvious but very effective method of obtaining increased capacity and efficiency is found in the compounding of old turbines with new turbines. At first glance it would appear as though this would be impossible without raising the steam pressure but this is not the case. Two old turbines may be rebuilt to receive steam at lower pressure and temperature than they did before and to thus serve as low pressure elements to a single new turbine taking steam at boiler pressure. In such cases the extent to which regenerative heating of feed can be obtained will depend upon the design of the casings of the original turbines; naturally the new turbine can be designed for bleeder nozzles and the exhaust connection between it and the older units can be tapped for feed heating. This general method can be made to yield an increased generating capacity of at least 50 per cent and a marked improvement of over-all efficiency as well.

Even when neither of the methods just considered is available it is still frequently possible to obtain increased output by the rebuilding of old turbines and generators. No exact values can be given but increases of the order of 15 to 20 per cent and occasionally even more can be obtained in this way while at the same time materially improving the over-all thermal performance of the station.

5. Even though it prove economically undesirable to replace or to rebuild the old turbines, it is still frequently possible to obtain marked thermal improvement in addition to that which can be obtained in the boiler room. This can be done through modification of the auxiliary power supply and the heat balance of the station. One is inclined to think of regenerative feed heating as necessarily tied to the bleeding of each main unit. In fact, it is obtained to a certain extent whenever that steam which is used for heating feed-water has done useful work before being used for that purpose. From the thermal standpoint it is most perfectly obtained when the steam has done the greatest amount of work before being used for heating and when the heating is done in the greatest number of temperature steps. Practically two or at the most three steps will ordinarily prove economically the best with moderate steam pressures such as one would expect in plants now being rehabilitated.

The fact that comparatively small turbo generators can now be obtained with steam rates that compare very favorably with those of their larger brothers opens up possibilities of effective regenerative feed heating even when the main units are not equipped with bleeder nozzles. Thus, a new unit equipped for bleeding very great quantities of steam may be used to supply auxiliary energy and such part of the station output as may be necessary to produce the required amount of steam. The steam bled and exhausted from this unit may then be used for regenerative heating of the condensate from all main units. Thermally, the result would be the same as that attainable through bleeding of the main units if the steam rate of the small unit were the same as that of the larger. Practically, the result will generally not be quite as good because of the higher steam rate of the small unit.

Certain disadvantages flow from this sort of a tie between auxiliary power and main bus, because of the ease with which electrical trouble can cascade. However, protective devices and schemes of connection now available make the arrangement reasonably safe and thus worthy of consideration in cases of rehabilitation.

It is worthy of note that rehabilitation of the sort which greatly increases station capacity will generally result in a radical reduction of investment per unit of capacity. It is not at all uncommon to obtain the increment capacity at from four- to six-tenths of the investment characteristic of the original station. On the other hand, the rebuilt station is generally more crowded than the original and operating facility may

TABLE I
STATION A
Yearly Output—Millions of Kilowatt Hours

Generating unit year	No. 1	No. 2	No. 3	No. 4
1	72.7	92.6		
2	155.7	125.6		
3	111.8	130.3		
4	134.3	122.3		
5	118.9	124.7		
6	183.4	103.6	2.2	
7	169.5	76.5	143.3	122.3
8	160.7	119.2	164.8	165.6
9	162.3	111.0	197.3	192.1
10	153.0	135.3	183.0	198.5
11	166.4	129.5	190.2	195.6
12	170.5	79.1	154.5	201.5
13	160.1	76.4	125.2	162.8
14	177.6	63.3	109.7	166.4

TABLE II
STATION B
Yearly Output—Millions of Kilowatt Hours

Generating unit year	No. 1	No. 2	No. 3	No. 4
1	8.6	19.0		
2	90.2	81.9		
3	112.1	115.7	13.6	
4	58.2	86.8	92.5	43.4
5	95.9	79.4	87.0	103.4
6	106.2	102.6	109.1	114.0
7	102.6	107.1	110.7	106.1
8	123.6	121.7	125.9	123.2
9	76.4	75.3	81.6	69.7
10	66.2	66.4	69.7	66.4

TABLE III
STATION C
Yearly Output—Millions of Kilowatt Hours

Generating unit year	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8
1	5.0	3.6	2.2					
2	32.2	47.0	35.4					
3	48.7	44.6	49.0	11.7	4.6			
4	56.5	63.0	64.8	21.1	22.4			
5	71.2	70.6	68.6	18.8	26.2			
6	75.7	77.7	77.0	22.5	26.4			
7	76.3	84.7	59.9	24.9	28.9	54.2	2.2	
8	49.1	63.6	62.8	30.6	29.9	116.3	64.4	
9	48.1	57.2	46.4	39.5	20.1	89.4	102.7	83.7
10	25.1	41.8	35.2	31.9	26.1	84.5	83.9	108.9
11	41.0	30.4	28.5	24.8	42.1	115.8	97.3	112.6
12	25.0	25.1	32.2	29.3	45.6	111.1	118.9	106.1
13	10.1	13.8	15.1	33.9	40.9	103.3	87.3	92.1
14	12.2	11.6	14.9	42.6	43.4	101.4	95.6	79.9
15	21.9	23.3	25.9	32.6	50.7	109.4	99.3	98.0

TABLE IV
STATION D
Yearly Output—Millions of Kilowatt Hours

Generating unit year	No. 1	No. 2	No. 3	No. 4
1	37.1			
2	188.0			
3	206.8	51.9	9.8	
4	188.1	180.2	94.2	33.3
5	202.1	86.4	18.2	179.2
6	177.5	174.0	176.6	171.5
7	173.3	176.8	191.8	162.3
8	157.7	184.1	214.5	175.3
9	184.8	202.3	183.5	180.8
10	160.6	182.0	211.0	179.5
11	169.9	144.2	171.3	129.9
12	148.2	124.8	211.7	110.9
13	183.8	97.7	183.3	106.9

have been sacrificed to a certain extent in producing it. The reduction of man hours per unit of station send out which should follow the increased capacity may therefore be partly counter-balanced through having to use more men than would be used in a new station of similar unit sizes.

It is true that some executive engineers do not accept rehabilitation as economically justifiable in their own systems. Therefore, it is not legitimate in all cases to predicate original station design on the assumption that rehabilitation will be a necessary part of the history of the plant. However, such rebuilding to a greater or lesser extent appears to be becoming more general than it once was and the history of any given system with respect to such practises should be given serious study when designing new plants for that particular system. If it has been one in which rehabilitation has been found economically desirable it is obviously the part of wisdom to design rather generously with respect to ground area, floor area, cubical contents, and clearances, so that if and when rehabilitation becomes necessary there may be the greatest degree of flexibility. It is patent that this consideration should not be used to the disadvantage of the present in order to obtain a questionable advantage in the future. But in the design of any station many decisions must be made on very intangible grounds; the proper values cannot be determined by calculation and the decision must in the end rest upon personal opinion or arbitrary choice. In such cases, it is well to consider the effect of such decisions upon the possibilities of future rehabilitation.

These remarks prompt another line of thought. At the time a station is designed certain economic balances must be struck in arriving at a proper compromise between investment and performance. The factors of useful life and expectable use during that life necessarily enter such calculations. In connection with these there is a fact which is frequently overlooked and which it may be profitable to point out. Few stations are built complete at one time; they are generally built in two, three, or more steps with the process extending over a period of from five to ten years.

While successive steps may be made to embody certain improvements, the general station design and the consideration of operating facility place some real limitations on this process. It follows that such facts ought properly to be taken into account at the time of station design. Thus, if we assume a useful, high output life of, say, ten years as the basis of design and then build serially so that the last unit is placed, say, five years after initial operation of the first, we very materially reduce the period which was assumed as ten years if our other assumptions were correct. The figures given in Table I, II, etc., are taken from real plants and indicate how such things have worked out in some cases. These cases, incidentally, are not at all exceptional.

The comparatively short life at high-load factor of the later units is quite noticeable in several of the tables. Some also show the comparatively short high-load factor life of the entire station, as for example, Table II, which represents a station which was extensively rehabilitated about twelve years after the first unit was put in service.

Discussion

E. S. Fields: I should like to discuss this paper from the standpoint of the practical application of some of the principles outlined in Dr. Hirshfeld's paper to the electric system of the Columbia Gas and Electric Corporation.

The feature of the program involved was to obtain increased capacity from existing stations, rather than any material improvement in economy. The program required considerable construction work within the West End and Columbia Stations of the Union Gas & Electric Company of Cincinnati, Ohio, and the Millers Ford Station of the Dayton Power & Light Company of Dayton, Ohio. The program resulted in an increased system capacity of 105,000 kw. by making changes and additions to equipment within the existing plants and without making additions to the existing buildings.

The history that led to this program was somewhat as follows: Prior to 1927, studies of the system capacity were centered around obtaining new capacity by extension of existing plants or by construction of a new plant in the territory between Cincinnati and Dayton. During 1927, three of the largest power contracts in the history of the company were negotiated which made it necessary to make an immediate decision in regard to the method of obtaining additional generating capacity. At this time the system capacity of the three main stations in Cincinnati and Dayton was 275,000 kw. An estimate of the load growth indicated that in 1928 the system load would be 250,000 kw. and in 1929, 280,000 kw. The largest unit on the system at that time had a capacity of 45,000 kw.

In order to determine the maximum capacity available on the system with reference to these anticipated loads, a check-up was made of the overload capacity of the machines in existing stations. At the same time an examination was made of the system load duration curve and it was noted that the top 25 per cent of the system load, or something over 50,000 kw., was present on the system only 15 per cent of the time. As a result of this consideration the following plan was evolved:

Construction work on West End Station of the company was started in 1916 and the station put into service in 1918. This station consisted of four 25,000-kw. units. A check was made of the overload capacity of these units and it was found that each machine would carry 32,000 kw. The capacity of the units, however, was limited by the capacity of the generators. Consideration was given to rebuilding the generators for a higher capacity and the cost was determined. The cost of rebuilding the turbine for a higher capacity was also determined. As a result of these studies it was decided to rebuild the steam ends of these machines and install new 36,000-kw. generators. In order to install these generators only minor changes were necessary in the machine foundations. In the turbines, new steel diaphrams were installed between the sixth and eleventh stages, designed for increased steam flow. The boiler equipment at this station was not changed but certain changes were made in the auxiliary equipment.

Extraction heaters were installed in connection with each unit in order to heat additional boiler feed water. Steam driven auxiliaries were changed over to motor-drive in order to work into the new heat cycle. The original condensers having 52,000 sq. ft. of surface were large enough for the rebuilt units. No changes were made in the condensing equipment.

The changes in the electrical equipment of the plant were as

follows: New oil circuit breakers, disconnecting switches, and main leads of higher capacity were required for the larger generators. The main station busbars were reinforced throughout the plant by installing higher-strength insulators and additional copper.

The interrupting capacity of the feeder oil circuit breakers was increased. One of the feeder oil circuit breakers was taken out and tested in the manufacturing company's laboratory to establish its rating. The result of these tests indicated that certain things could be done with the breakers in the way of speeding them up, reducing oil head and so forth, to increase their safe interrupting capacity. It developed from an experience at this same time that the feeder reactor installations in the station were unsafe and reactors on all feeders were changed to a mechanically stronger type with a slightly higher value of reactance to reduce the feeders short-circuit currents to the safe capacity of the breakers.

The total increased capacity obtained at the West End Station was 44,000 kw.

The company that manufactured the two turbo generators installed at the Columbia Station suggested that each of these machines be rebuilt from tandem-compound 45,000-kw. units to cross-compound units of 63,000 kw., by abandoning the high-pressure section of the original turbines and installing separate high-pressure turbines to each of which would be coupled one of the generators that was to be removed from the West End units. The new high-pressure turbine proposed with this change would deliver sufficient steam to the low-pressure end of the original machine to produce 49,000-kw. output on the original generator. Under these conditions the new high-pressure unit would deliver 14,000 kw., making the output of the compound unit 63,000 kw. Two of the generators which were removed from the West End Plant were utilized with these new high-pressure units installed at Columbia.

By changing these units from tandem-compound units to cross-compound units an increased capacity of 36,000 kw. was obtained in Columbia Station.

Space was available for foundations so that the high-pressure unit could be installed alongside the original units.

The steam conditions in this plant were originally 600 lb., 725 deg. fahr. In order to provide additional steam, the capacity of the six standard boilers was increased from 160,000 to 220,000 lb. per hr. In the reheat boilers the steaming capacity was increased from 90,000 to 150,000 lb. per hr. Some of the changes necessary in order to obtain this increased capacity of the boilers were as follows: The air-cooled refractory front walls were replaced with water-cooled front walls. New baffling was installed in the drums. The powdered-coal feeders were equipped with new screws of greater pitch to deliver more coal. The induced draft fans were equipped with new runners and new motors of higher speed to get greater capacity. The air pre-heaters were changed from four-pass to two-pass in order to reduce the friction caused by increased air volume. The boiler pressure was increased from 600 to 650 lb. and the temperature increased from 725 deg. to 740 deg. New impellers were installed in the boiler feed pumps, increasing the pressure from 750 to 800 lb. Evaporators were installed to improve the boiler feed-water conditions. A new 70-in. mill was installed in the central fuel-pulverizing plant to supply the increased demand for pulverizing fuel.

In the electrical end of the station, since both high- and low-pressure generators were connected to the same oil circuit breaker, it was only necessary to increase the capacity of sections of the busses, generator disconnecting switches, and current transformer on the original installation and make the installation of the high-pressure generator and its accessories.

The generator oil circuit breakers were originally rated at 3000 amperes. With the high- and low-pressure units operating in parallel through the same oil circuit breaker, the maximum current was 3250 amperes and the space was limited so that larger breakers could not be installed. A test was conducted on the breakers to determine the conditions of heating at this higher current. It was found that the only point of high operating temperature was in the top casting of the switch. This was changed from steel to alloy, and the switch operated satisfactorily at this higher capacity.

At the Millers Ford plant of the Dayton Power & Light Company, there was building space available for an additional unit. A new turbine was purchased and one of the generators removed from the West End Station was installed with this turbine to complete the unit. It was necessary to reconnect the generator used in this installation from 13,200 volts to 6600 volts. This addition increased the capacity of this station 25,000 kw. A portion of this capacity is in effect reserve turbo generator capacity at this time since no work has been done in the boiler room to obtain greater steam capacity. This step has been delayed until the system requires the additional firm capacity.

The program as outlined utilized three of the four generators which were removed from West End Station and the fourth generator now remains on the system as a spare.

W. S. Lee: A power generation paper can be very voluminous and have a great number of plants described therein. This was not the purpose of this paper, but the idea was to impress upon both the designing and executive engineer the importance of rehabilitating a plant only when it was a proper thing to do for service and financial reasons. It also brings before the engineering profession in this day of mergers and larger installations and generating units that we must be prepared to do away with many small units that are operating.

The question of selecting pressures prompts me to suggest that you go into this matter a little more slowly than we have been doing recently. I do not mean to say that I decry high pressures, but high pressures do not fit every job. You can build some large plants with lower pressures, and when you apply the operating cost as well as capital cost, you will find that the high pressures do not act as a panacea for all your troubles.

W. J. Foster: I am reminded of the time when hydraulic generators were relatively more important, you might say, than at the present moment. It has always been possible to do more in the rehabilitation of such generators than of steam turbine generators for the reason that most of those machines were made when the standards required close regulation and operators had to take care of the potential and the load changes by manipulating the rheostats by hand. Since such generators were made with a much higher short-circuit ratio, it has been possible in many cases to redesign the armatures so that the same rotor or field with the same excitation will give a much greater output and it is entirely satisfactory to do this since field regulators will take care of the potential, which we did not have in the old days. We can usually make a good showing on an old hydraulic plant in rehabilitating the generator.

In the case of steam turbines, the older turbines were of much lower speed than is now advisable, so that the unit as a whole has to be replaced, and that, of course, causes the generator to be scrapped.

Mr. Fields speaks of the Columbia Gas & Electric System. After an interval of ten or twelve or fifteen years, there have been in the past and probably will be in the future improvements made by which greater output will be obtained with the same material, that is, increased output of 10, 15, or 20 per cent. Nothing like a 50 per cent increase is to be had there, which is possible in many of the older hydraulic plants.

I think it is good policy on the part of the consulting engineers and engineers of operating companies to insist upon conservatively rated generators in the first place. It is so often found possible by increasing the pressure and the temperature or something connected with the steam end to obtain a greater output. If the generator has been conservatively rated in the first place, it is generally possible to operate it entirely satisfactorily and safely at greater output.

L. W. W. Morrow: I want to compliment Professor Hirshfeld on a very masterly paper on a very difficult topic. Anyone who generalizes about power generation and power cost today is dealing with a hazardous subject. Yet it is the one important topic confronting engineers in the utility field today.

Today we are facing a period of increased investment per unit of output, decreased sales price per unit of output, and a longer time for capital turnover. The power station end of the system, roughly, represents one-third of the investment and one-third of the sale price of the energy we are selling.

It is very significant, as Dr. Hirshfeld shows, that in 1908, our best station practise represented 32,000 B. t. u. per kw-hr.; in 1918, it was down to 18,000. Today it is around 12,000. In other words, for a little handful of coal today, costing one-tenth to three-tenths of a cent, you are getting the use of a kilowatt for an hour. So much for the fuel cost.

What about the capital charge to get that result? On an average, we can say that power station of 1908 and 1918 and 1928 cost roughly, $100 a kilowatt. We can also say that the range today is probably greater than it was in those two periods and under certain conditions is from $65 to $140. Thus power-generating engineers have made an enormous increase in thermal economy and without increasing the capital investment materially. Now we wonder where they are going next.

Dr. Hirshfeld very candidly says it doesn't appear that we are proceeding at the same rate as in the past. When we look ahead at this industry with power production doubling every eight or ten years, which means we are adding now 1,500,000 to 2,000,000,000 kw. of new capacity each year, we begin to see the dollar values involved in making engineering judgments on rehabilitation of existing plants or on new power-station building.

In rehabilitating a plant, we have to take into consideration the period ten years ahead just as we do in building a new station. The same factors enter in. One element Mr. Lee touched on is the system reserve requirements which not only involve the power-generating engineer but the transmission and distribution engineer. One element which is very important is to improve the load factors of these systems. To get more kilowatts out of this capacity, we have to find more uses and more hours of use for the power we have available in these stations.

Another factor that is coming into the picture is this: We hear a great deal of discussion about making power as a by-product of coal some combination of the gas, the chemical, and the power industry. This may come about and may make for some cost reductions—nobody can tell. But people will have to use more chemicals and pay more money for them before that is possible.

The third element is that it may be we will find something beside water to deal with in producing power from fuel. We have worked the water cycle to the limit. We have achieved good results with mercury, and it isn't beyond the scope of possibility that we will use some other fluid and gases.

Sidney Withington: (communicated after adjournment) I should like to emphasize Dr. Hirshfeld's thoughts that the increasing operating economies are to some extent counterbalanced by the increasing capital expenditures represented in modern power-plant design. I believe there is some tendency in certain instances to overlook this in a constant effort to add to operating economies.

Dr. Hirshfeld says that the net result of all forces concerned has been a gradual but consistent lowering of the selling price of power. It would add very much to the paper, I believe, if Dr. Hirshfeld would present figures showing the trend of costs of power during the past few years. It would be necessary, of course, to make corrections for increasing costs of labor per man-hour, and increasing costs of fuel, and possibly a few additional assumptions such as the development of the facilities for burning lower grades of fuel. Such data would, I believe, be of great interest to those concerned with the problem of power production and use. The information might be plotted coordinately with some of the other graphs presented. A segregation of fixed or annual charges and energy, would probably be desirable in order to take into account variations in load factors. After all, it is the final answer which counts and unless that is indicated the whole story is not told.

Philip Sporn: (communicated after adjournment) Dr. Hirshfeld's paper is, I believe, particularly timely in this day of the popularity of the slogan in connection with power systems, such as, "More kilowatts per dollar of investment" and similar slogans. What all of these get down to, is this, that the power producers are spending too much money for facilities to take care of business at the rates they have to offer to get that business. Obviously one way of getting around this situation is not to spend so much money per kilowatt of capacity.

Dr. Hirshfeld has given the entire problem of rehabilitation of steam power plants a very comprehensive, concise summation. I should like, however, to stress three points which have been indicated in the paper.

1. The effect of an existing transmission system on the problem of rehabilitation. The history of development of power systems has very often been one in which as the load developed newer and larger plants were built at various points on the systems, the old ones being left to struggle along as best they could. The transmission system has naturally in all that time kept pace with the growth, that is, the transmission system was developed around the various generating point sites; it is of course the only way it could develop. As the territorial expansion of a system slowed up, and this is true of many of them today, the situation that the engineer was confronted with was one in which a number of the lines built in previous years had gradually ceased to function because the location of the newer stations was such as to favor some other lines. An analysis of the economics of rehabilitating some of the old sites in any case of that sort would very often go far astray unless full weight were given to the combined plant and transmission-line set-up. It will often be found that it is economically desirable and feasible to rehabilitate a plant even though the thermal economy derived from it is not as high as that possible from some other plant for the simple reason that its rehabilitation would put to use a transmission line that might represent two or three million dollars worth of investment.

2. As pointed out by Dr. Hirshfeld, rehabilitation will generally result in radical reduction in investment cost per unit of capacity. It is to be noted that the generating station, through the possibility it offers for rehabilitation, is one of the few places on the electric power system that affords the possibility of a sharp cut in the cost of facilities per kilowatt to be served.

3. On the question of ground area, cubical contents, and related questions which Dr. Hirshfeld brings up as necessary to be settled in the light of the policy to be followed with regard to rehabilitation, I should like to state that I have seen cases where not only would the original design limits with regard to space and area, etc., prove inadequate when rehabilitation time came around but very often showed their inadequacy before the design had progressed to completion. It would seem that this particular question of design could certainly be treated more liberally than would be warranted by a mere consideration of figures produced with the aid of an engineer's scale and slide rule.

Application of Induction Regulators to Distribution Networks

BY E. R. WOLFERT[1] and T. J. BROSNAN[1]
Non-member Applicant for Membership

Synopsis.—This paper considers in a general way the application of automatic induction voltage regulators to network distribution systems. A discussion is given of the usual methods of applying both single- and three-phase induction regulators and their relative merits are considered.

The latter part of the paper takes up various methods that may be used to obtain successful operation when automatic induction regulators are operated on parallel feeders.

* * * * *

ABOUT 80 per cent of the network systems now in operation use induction regulators to control the voltage of the network.

Over 75 per cent of these apply separate regulators to the individual feeders though under certain conditions there are some advantages in regulating the voltage on the bus rather than on the individual feeders. These conditions exist on a network system supplied from one station by feeders of the same voltage rating which carry the entire load supplied by the bus. Because of the interlacing and the fact that they feed into a common secondary system, the load carried by each feeder, particularly at the time of heavy system load, is generally well divided among them. Under these conditions, bus regulation will frequently give satisfactory voltage at the load. Furthermore bus regulation requires the minimum amount of space for regulating equipment and none of the problems involved in the operation of automatic regulators on parallel feeders is encountered.

However, the outstanding disadvantage of a bus regulator is that the entire network system would be without voltage regulation if anything should happen to the one bus regulator. Furthermore the advantages of this type of regulation are considerably minimized if any of the favorable conditions do not exist. For these reasons individual feeder regulators are used in the majority of cases.

Regulator Application

On a three-phase three-wire feeder, one three-phase regulator, two single-phase regulators or three single-phase regulators may be employed, while on a three-phase four-wire feeder, either three single-phase or one three-phase regulator may be used. Any of these will give satisfactory voltage conditions at the load if the bus voltage of each phase and the current carried by each line are balanced.

A three-phase regulator is usually connected to a feeder as shown in Fig. 1. The voltage in all three phases is varied an equal amount by any movement of the three-phase regulator. With the connections shown in Fig. 1 the operation of the regulator is controlled by only certain phases of the feeder. For this reason the results obtained by the use of three-phase regulators are not entirely satisfactory when there is a serious unbalance of either voltage or current in the different phases of the feeder.

The action of a three-phase regulator is illustrated in Fig. 1A. The triangle $A\,B\,C$ represents the voltage conditions existing at the station bus, while the triangle $X\,Y\,Z$ shows the regulated voltage. It may be seen from this that if the three-phase regulators in the different feeders supplying a network are not made to boost or buck the same amount at all times, an angular

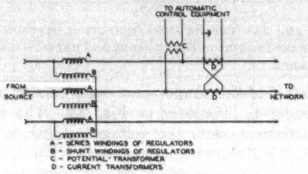

FIG. 1

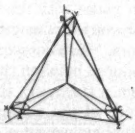

FIG. 1A

FIG. 1 AND 1A—VECTOR AND CONNECTION DIAGRAM OF THREE-PHASE INDUCTION REGULATOR

voltage displacement is introduced between the corresponding line to neutral and line to line voltages in the different feeders. With the first type of network protectors developed, this angular shift of voltage had the serious effect that it was liable to cause pumping, that is, periodic opening and closing of the network protectors. This action is fully explained in a paper by W. R. Bullard.[2] The possibility of network protectors pumping because of this angular voltage displacement has been eliminated in the latest type of protector by the

1. Both of the Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

2. *Operating Requirements of the Automatic Network Relay*, by W. R. Bullard, A. I. E. E. TRANS., Vol. XLV, 1926, p. 1203.

29-115

addition of the phasing relay.[3] The voltage shift, however, still introduces some undesirable system conditions.

The usual connections used for two single-phase regulators on a three-phase feeder are shown in Fig. 2. With unbalanced voltage and current conditions in the feeder, this method will generally give closer regulation than one three-phase induction regulator, but still will not give correct regulation on all three phases.

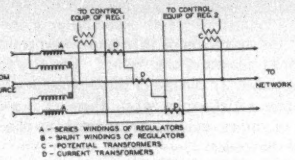

Fig. 2

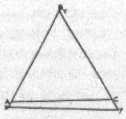

Fig. 2A

Fig. 2 and 2A—Vector and Connection Diagram of Two Single-Phase Induction Regulators on Three-Phase Three-Wire Feeder

The action of two single-phase regulators on a three-phase feeder is illustrated in Fig. 2A. The triangle $A\,B\,C$ again represents the voltage conditions at the bus, and $X\,Y\,Z$ shows the regulated voltage with the two regulators in somewhat different positions. The voltages added to or subtracted from the line voltages $B\,A$ and $B\,C$ are in phase with the line voltages at all times, and are simply varied in magnitude by the movement of the regulators. As a consequence, there can be no angular shift produced between these two voltages in the different feeders. However, the phase angle as well as the magnitude of the voltage in phase $A\,C$ is changed by an unequal movement of the two regulators on the other phases, as is illustrated by the line $X\,Z$. It is entirely possible, therefore, to have an angular displacement of the voltages of this phase in different feeders, and such a shift could introduce the same trouble on the older type of network protectors as that caused by three-phase regulators. To avoid this difficulty, the companies using two regulators connected in open delta on network feeders have mechanically connected together the two regulators on the same feeder which prevents an angular shift of any of the voltages, but at the same time forces all phases of the feeder to accept the same amount of regulation.

If the feeders supplying the network are connected three-phase, four-wire and three single-phase regulators are used on each feeder, these regulators may be applied as shown in Fig. 3. The primary winding of each regulator is connected from line to neutral. Correct regulation from line to neutral can be obtained with the arrangement, no matter how unbalanced the currents and voltages of the feeder may be as long as the corrective factor required is within the range of the regulators.

As illustrated in Fig. 3A, the voltages added or subtracted by the action of the regulators are always in phase with their respective line to neutral voltages, and there is, therefore, a fixed phase relation between these voltages in the different feeders. The line to line voltages may be shifted in phase somewhat due to unequal action of the three regulators, but this has no effect on the voltages being considered since the high voltage sides of the network transformers will also be connected between the phase wires and neutral. It follows, therefore, that three regulators on a three-phase, four-wire system give correct voltage under any unbalanced conditions in the different phases.

Fig. 4 shows the connections used when three single-phase regulators are used on a three-phase, three-wire system. With this arrangement correct regulation will be obtained even though the voltages and currents of the feeder are unbalanced.

The action of the regulators is shown in Fig. 4A. Here again a phase angle shift of the voltage triangle is introduced by the regulators. An angular displacement between the corresponding line to line voltages in

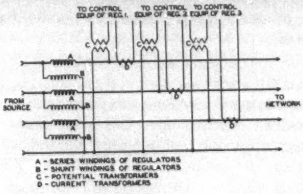

Fig. 3

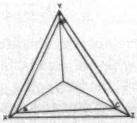

Fig. 3A

Fig. 3 and 3A—Vector and Connection Diagram of Three Single-Phase Regulators on a Three-Phase Four-Wire Feeder

the different feeders can be avoided by mechanically connecting together the regulators in the corresponding phases in the different feeders.

A comparison of the relative first costs of the regulators for different feeder sizes has been worked out for each of the arrangements described, and the results are given in the curves of Fig. 5. In each case the line to line voltage of the feeder has been taken as 4000 and the proper regulators applied to give a regulation of 10

3. "The Network Relay," by J. S. Parsons, *Electric Jl.*, Dec. 1927.

per cent buck or boost. The curves in Fig. 5 have been made up for comparative purposes, and the differences in costs are expressed in per cent rather than in cost figures. The same thing has been done in working out the curves on comparative losses.

The three-phase regulator must have a 4000-volt

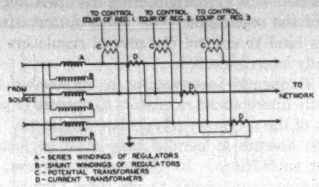

FIG. 4

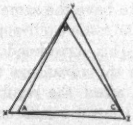

FIG. 4A

FIG. 4 AND 4A—VECTOR AND CONNECTION DIAGRAM OF THREE SINGLE-PHASE REGULATORS ON A THREE-PHASE THREE-WIRE FEEDER

rating for application on a 4000-volt feeder and must have a kv-a. rating of 10 per cent that of the line in order to give the ten per cent regulation. The costs of such regulators for different feeder capacities are shown as Curve *D* of Fig. 5.

A single-phase regulator used on a three-phase four-

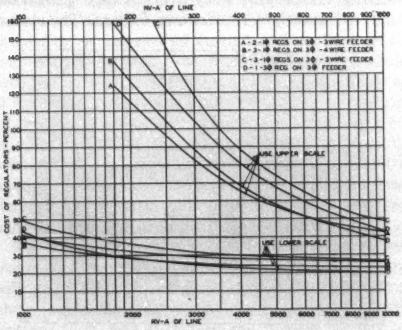

FIG. 5—COST COMPARISON REGULATORS FOR 4000-VOLT FEEDERS

wire feeder is designed for the line to neutral voltage, or in this case 2300 volts. The same total regulator capacity is required to give the range of 10 per cent above and below normal as in the case of the three-phase regulator, or in other words, each single-phase regulator has a rating of 3⅓ per cent of the line kv-a. The cost of three single-phase regulators for such a feeder, plotted in per cent against the line kv-a., is given in Curve *B* of Fig. 5.

The total regulator capacity required to give corresponding regulation is higher when three single-phase regulators are used on a three-phase three-wire system (Fig. 4) than if used on a three-phase four-wire system (Fig. 3). In order to obtain 10 per cent buck or boost of the line-to-line voltage, each single-phase regulator must be rated at 3.8 per cent of the feeder kv-a. The voltage rating of regulators applied in this manner is, of course, the line-to-line voltage. The cost of three such regulators for different feeder capacities is given in per cent in Curve *C* of Fig. 5.

Two regulators, connected in open delta, on a three-phase three-wire circuit must each have a rating of 5.77 per cent of the feeder kv-a., and must be designed for line-to-line voltage. The cost of the two regulators is plotted against feeder kv-a. in Curve *A* of Fig. 5.

The losses of each of these combinations of regula-

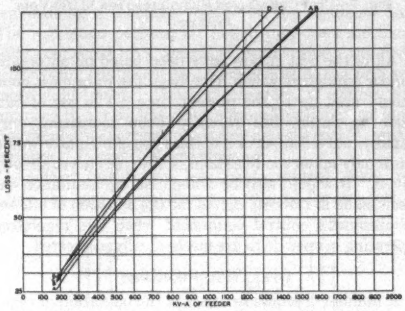

FIG. 6—LOSS COMPARISON REGULATORS FOR 4000-VOLT FEEDERS

tors, expressed in percentage, are shown in the four curves in Fig. 6. The curve letters used in Fig. 6 refer to the same regulator arrangements as the corresponding curves in Fig. 5.

The use of feeder voltages of 11 kv. and 13.2 kv. has become rather common on a-c. automatic network systems. When regulators are supplied on such feeders, it is possible to use high-voltage regulators connected to the feeder in the usual manner, or to use standard 2.4-kv. regulators with shunt and series transformers. The primary windings of the regulators are supplied through the shunt transformers, and the secondary windings of the regulators are connected into the feeder through the series transformers. The question of which type to use is essentially an economic one, and Fig. 7 shows the first cost of the two types. Curves *A* and *B* in Fig. 7 are for three-phase regulators, while Curves *C* and *D* are for single-phase regulators. The comparison is based on regulator ratings only.

The losses of the regulator combination using series

to shunt transformers are usually higher than for the standard high-voltage regulator except in small sizes. The floor space required for the outfit using the larger number of units will be the greater, but this arrangement gives the greater flexibility.[4]

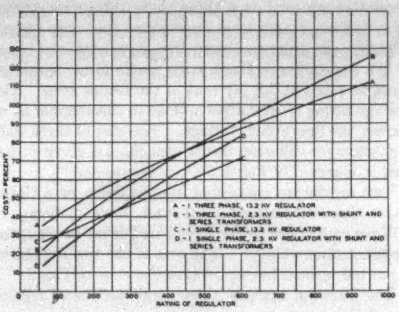

Fig. 7—Cost Comparison Regulators for 13,200-Volt Feeders

All of the cost curves have been based on automatic indoor regulators which have been the ones most commonly used up to the present time. The use of outdoor regulators has recently been increasing very rapidly, but a comparison of this type of regulator would show only minor modifications of the cost curves already given. It should also be noted that no allowance has been made in the cost curves for the omission of one set of automatic control equipment when two regulators operating in open delta are mechanically connected.

Operating Problems

A number of problems arises in the operation of regulators on the parallel feeders supplying a distribution network.

Fig. 11 clearly illustrates that a closed loop is formed by the feeders, the station bus, and the secondary system, and a voltage unbalance will tend to set up a circulating current in this loop. If the regulators are applied in the manner generally followed on the feeders of radial systems, and the line-drop compensators set in the usual manner, it is possible that regulator instability may be encountered. For example, if a small circulating current is set up, the current in one feeder is increased, while that in the other is decreased. The increased current in one feeder acts on the line-drop compensator to cause the regulator to further increase the voltage in this feeder, while in the other feeder the opposite action takes place. It can readily be seen that this may result in a cumulative action, one regulator going to the maximum boost position, the other turning to the maximum buck position.

The problem in brief, therefore, is to maintain satisfactory voltage at the load, and at the same time prevent unstable operation of the regulators.

Parallel Operation

The first and the most reliable means of operating regulators in parallel is by mechanical interconnection. This consists of a shaft connecting the operating mechanism of the regulators. One set of automatic accessories is used to control the several regulators so mechanically interconnected.

It is frequently inconvenient or impossible to mechanically interconnect regulators to operate in parallel, because of the necessary physical arrangement. Regulators to operate in parallel may even be located in different substations. In these circumstances, it has been considered necessary to use some form of electrical interconnection. The mechanical interconnection forces each feeder to have the same voltage, so that if the impedances of the feeders differ, they will not divide the load equally. This corresponds to substation bus regulation, but has the advantage that failure of one regulator does not affect the regulation of the other feeders. The two central regulators shown in Fig. 8 are 344-kv-a., 13,200-volt, three-phase, self-cooled induction regulators, mechanically interconnected and regulating the voltage of two parallel feeders.

One of the first schemes of electrical interconnection used is that shown in Fig. 9. This arrangement is usually referred to as cross-compensation. The diagram shows the circuit for controlling two single-phase induction regulators in open delta on a three-phase, three-wire system, the two regulators of each feeder being mechanically interconnected. The source of potential for the primary relay is the voltage transformer connected directly across its own feeder. The current

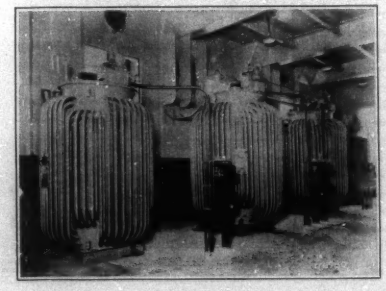

Fig. 8—Three-Phase 13,200-Volt Automatic Induction Regulators Mechanically Interconnected

for line-drop compensation, however, is obtained from the current transformer of the adjacent feeder. Each compensator is set for its own feeder impedance. The action of the auxiliaries is described in the following paragraph.

4. "Regulators for Network Distribution Systems," by E. E. Lehr, *Electric Jl.*, July, 1925.

With an increase of load, each feeder will take a portion of the increase inversely proportional to its impedance at the instant of increase. Thus the feeder of lowest impedance takes the greatest share of the added load. The tendency to increase the voltage of this feeder is not very great, because its compensator is set for low impedance, and the increase in current due to the additional load is small, since it is taken from the adjacent feeder. The feeder having the highest impedance has taken the smallest share of the added load at the instant of increase, but the tendency for the regulator to raise the voltage is greatest, because its compensator is set for high impedance and the current obtained from the adjacent lower impedance feeder is higher than its own current. The tendency of the regulator accessories is therefore such as to cause each feeder to take an equal share of the increase in load. This is one of the requisites of stable parallel operation.

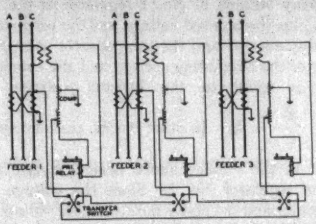

FIG. 9—REGULATORS ON PARALLEL FEEDERS WITH CROSS-COMPENSATION

This method of cross-compensation can only be successfully applied to feeders that have appreciable impedance, since the sensitivity of the control circuit to circulating or unbalanced current is dependent on the compensator setting. For example, assume a feeder with but 5 per cent impedance and the compensator set at 5 per cent. The usual contact setting of the primary relay is $\pm 1\frac{1}{2}$ per cent. Thus it would require a circulating current of 30 per cent full load to cause the primary relay to make contact and cause the regulators to restore balanced conditions. This cross-compensation scheme has been largely superseded by the standard method of connections as will be explained.

Fig. 10 illustrates a circuit developed to secure sensitivity to circulating current. The diagram shows only the control for only one regulator on each three-phase four-wire circuit. Two additional current transformers, $C\ T_1$ and $C\ T_2$, on the parallel feeders are connected in series so that when the feeders are carrying the correct portion of the load, the secondary current of the current transformers simply circulates between the current transformers. If, however, the current in the feeders is unbalanced, twice the unbalanced current will flow through the compensators, since the current transformers have very high impedance to any current other than ratio primary current. The compensators are so connected that a voltage is induced in the primary relay circuit so as to restore normal conditions.

The transformers $C\ T_3$ and $C\ T_4$ supply the current for line-drop compensation in the usual manner. For feeders of low impedance, the current transformer ratios are chosen so as to give a high compensator setting to secure sufficient sensitivity to circulating current. For example, for a 200-ampere feeder having 5 per cent impedance, the current transformer ratio for line-drop compensation would ordinarily be 200 to 5. As previously pointed out, a compensator setting of 5 per cent is not sufficient to secure satisfactory operation. In order to set the compensator at 10 per cent, a 400-to-5 current transformer would be used for line-drop compensation, and the 200-to-5 ratio for the current transformers used for the unbalanced current. It will be seen that this connection forces the feeders to carry current of approximately the same magnitude, or in a definite proportion.

This method has been satisfactorily used with three regulators solidly bussed on both power and load sides, maintaining a constant voltage on the regulated bus. The regulators perform as though they were mechanically connected by a somewhat flexible shaft, the flexibility being controlled by the closeness of the primary relay adjustment. The compensators, of course, are not used to obtain line-drop compensation but to limit the circulating current in this case and the current transformers $C\ T_3$ and $C\ T_4$ are omitted.

Another installation was made several years ago using the connections shown in Fig. 10 for two sets of three single-phase induction regulators on a three-phase four-wire circuit. These regulate two feeders supplying a bank of transformers connected in parallel on both high- and low-voltage sides. This installation has been operating very satisfactorily. The regulators are protected by thermal relays so that in case of failure of any of the auxiliaries to function, the breaker will automatically open the circuit. To date, however, there has

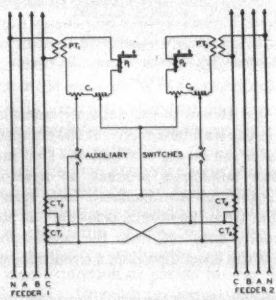

FIG. 10—CIRCUIT FOR LIMITING CIRCULATING CURRENT AND INCLUDING LINE-DROP COMPENSATION

been no occasion for the breakers to open through the action of these relays.

All of the schemes described so far require mechanical or electrical interconnection. Where all regulators feeding the network are located in the same substation, there is no serious objection to electrical interconnection. However, as the size of the network increases new substations may be required. If electrical interconnection is employed, pilot wires must be used between the regulator circuits, which is objectional both from a cost and operating standpoint.

Induction regulators are now being applied to networks in many cases, using only the standard method of connection.

To secure satisfactory operation, certain precautions must be taken and the auxiliaries properly adjusted, and the loop impedance must be sufficiently high to permit an appreciable compensator setting. This has been proved by theory, tests, and actual field practise.

The two regulators shown in Fig. 11 controlling the

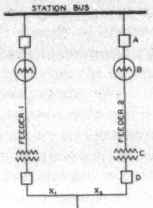

Fig. 11—Single-Line Diagram of Simplified Network System

voltage of the feeders to the network transformers are connected in the standard way. If the compensators of each regulator are set for one-half the loop impedance, the regulator auxiliaries produce no restoring effort when the regulators are displaced from each other an equal amount from the normal position. If one regulator is manually displaced from the normal position by X per cent in the boost direction, a circulating current of $\frac{X}{Z}$ will flow where Z is the loop impedance in per cent. The voltage which acts on the primary relay of the regulator displaced is the regulated voltage $1 + X$ less the voltage of the compensator, which in this case is $\frac{X}{Z} \times \frac{Z}{2} = \frac{X}{2}$. If the regulator is released at this point, it will return to the normal position, since the primary relay voltage is higher than normal. However, suppose the regulator is held displaced and the action of regulator No. 2 examined. The primary relay voltage of No. 2 regulator will then be $1 + \frac{X}{2}$ and it will start toward the buck position. When it reaches X per cent from the normal position in the buck direction, the circulating current will then be $\frac{2X}{Z}$. The primary relay voltage of No. 1 regulator is the normal voltage plus the boosted voltage less the product of the circulating current and the compensator setting. The product of the circulating current and the compensator setting is equal to the boosted voltage, *i. e.*, $\frac{2X}{Z} \times \frac{Z}{2} = X$. In other words, the compensation is equal to the voltage above normal caused by the action of regulator No. 1. Since the voltage of the compensator and the above normal feeder voltage are in opposition in the primary relay circuit, normal voltage is applied to the primary relay and no action takes place. The voltage of No. 2 primary relay is the normal voltage less the voltage below normal plus the compensator voltage. The resultant again gives normal voltage applied to the primary relay.

It should now be noted that if the compensators are set for anything less than one-half the loop impedance, a restoring effort will result as the compensation is less than the voltage above normal caused by regulator No. 1. The primary relay voltage therefore is greater than normal and will tend to cause regulator to return to normal position. If, however, the compensators are set for more than one-half the loop impedance, the compensation will be greater than the voltage above normal caused by regulator No. 1. Thus the primary relay voltage is less than normal and regulator No. 1 will not tend to return to the normal position but will go to full boost position. The opposite action takes place at regulator No. 2 and it will go to full buck position.

Fig. 11 is a single-line diagram of a typical network. The load is shifted from the middle point, as shown, to a point adjacent to the network protector in order to check the stability of operation with different compensator settings. This is accomplished by making X_1 equal to zero and X_2 equal to the impedance of the secondary mains. If the compensators are set to maintain a constant voltage at the load instability will result, since the action of the load current on the compensator of regulator No. 1 is greater than the voltage increase caused by the regulator.

Table I has been prepared from calculations showing the load voltage and division of load current for different compensator settings with a concentrated load, of 100 per cent at 80 per cent power factor with X_1 equal to zero, and varying values of X_2. The table is based on a reactive circuit neglecting resistance, and the power factors given are based on the load voltage. The location of the load chosen in making up this table repre-

TABLE I

Compensator setting Per cent	Load voltage Per cent	Current		Power-factor		Reactance		
		FDR. No. 1	FDR. No. 2	FDR. No. 1	FDR. No. 2	FDR. No. 1	FDR. No.2	X_1
0	96	66.7	33.3	80	80	10	10	10
9	99.5	75.5	27.2	70	98	10	10	10
8	99.0	74.0	28.0	72	95	10	10	10
5	97.7	70.0	30.6	76	87	10	10	10
0	96.4	60.0	40.0	80	80	10	10	5
9	99.5	70.3	33.1	68	97	10	10	5
8	99.1	67.6	34.0	71	94	10	10	5
5	98.0	62.5	37.6	77	85	10	10	5
0	97	50	50	80	80	10	10	0
9	99.7	50	50	80	80	10	10	0
8	99.4	50	50	80	80	10	10	0
	98.5	50	50	80	80	10	10	0
*	99.6	60	40	80	80	10	10	5

*Compensator feeder No. 1 set at 9 per cent and compensator feeder No. 2 set at 13.5 per cent.

sents the most severe condition for obtaining regulator stability. A typical example of such a condition is at start of network operation when one building is supplied by two feeders and two network transformers. The table shows the action, both with and without reactance between network transformers. The table has been made only for single-phase regulators.

It will be noted that as the compensator setting is decreased for a given system condition, there is more nearly an equal division of the reactive component of the load. Better load voltage and division of the load current also prevail when the reactance of the secondary mains is small in comparison to the feeder reactance.

Tests have been made that fully verify all of the results given in the table. Actual operating experience also shows that stable operation of single-phase regulators is obtained and satisfactory voltage maintained in secondary system if care is taken in setting of the relays and compensators.[5]

The operation of three-phase regulators on network feeders differs from single-phase regulators because of the manner in which they change the feeder voltage, (Fig. 1A). The regulated voltage of all the parallel feeders from the same bus will have the same magnitude and phase relation only when the rotors of the regulators have the same relative position. An angular difference in the rotor positions will result in a voltage tending to circulate current. The phase and magnitude of this voltage will depend upon the relative positions of the rotors.

The voltage causing circulating current of two three-phase regulators in parallel, one of which has its regulator voltage $B\,Y$ (Fig. 1A) at slightly less than 90 deg. from $O\,B$ and the other slightly more than 90 deg. from $O\,B$, will be approximately in phase with the voltage $O\,B$. Also any change in magnitude of the voltage causing circulating current will change the regulated voltage $O\,Y$, applied to the primary relay, by approximately the same amount.

However, with the two regulators a few degrees apart but in such a position that voltage $B\,Y$ is nearly in phase with $O\,B$, the conditions are entirely different. In this case, although the voltage tending to circulate current may be of the same magnitude as that in the preceding case, it is nearly at right angles to the voltage $O\,B$.

Furthermore, a change in magnitude of the voltage causing circulating current has no appreciable effect on the regulated voltage applied to the primary relay. Thus the voltage tending to set up a circulating current and therefore the circulating current itself can have a phase relation from practically zero to 180 deg. with voltage $O\,B$. Consequently, satisfactory operation of three-phase regulators in parallel using standard connections may be obtained only when the loop impedance is comparatively high.

The use of electrical interconnection becomes difficult when three-phase regulators are used because of the varying phase relation of the voltage causing circulating current. Mechanical interconnection of the regulators on the different feeders, however, will prove entirely satisfactory. The inherent phase displacement of the regulated voltage by three-phase regulators may be overcome by the use of two three-phase regulators with their secondaries connected in series in such a way that the vector sum of their secondary voltage will always be in phase with the line to neutral voltage. The operation then becomes the same as single-phase regulators.

Summarizing, the type of induction regulators to be applied to network feeders depends largely on the network and feeder characteristics. The feeders and network forming the loop between the regulators may be classed as low, moderate, and high impedance. It is desirable to interconnect mechanically both single- and three-phase regulators on low impedance loops. If single-phase regulators are used on low impedance loops electrical interconnection as shown in Fig. 10 may be successfully used. On moderate impedance loops single-phase regulators may usually be successfully operated without any interconnection while three-phase regulators should be mechanically interconnected. Either single-phase or three-phase regulators will operate satisfactorily on a high impedance loop using the standard operating control circuit.

5. *Operating Experience with the Low-Voltage A-C. Network in Cincinnati*, by F. E. Pinckard, A. I. E. E. Quarterly TRANS., Vol. 47, July, 1929.

The Electrification of the Mexican Railway

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Synopsis.—The Mexican Railway Company, Ltd., locally known as Ferrocarril Mexicano, was the first railway built in Mexico, having been opened to traffic January 1, 1873. The main line runs between Mexico City and Vera Cruz and is 264 mi. in length. There are six branch lines which increase the route miles to a total of 482. The most difficult portion of the line consists of 19 mi. of 4.7 per cent grade between Encinar and Boca del Monte where the table-land is reached.

In 1921, when the road was returned to its owners, following five years of government operation, the property was found to be in an unsatisfactory condition, with operating expenses more than doubled, thereby increasing the operating ratio from 0.51 in 1914 to 0.79 in 1920. Higher wages and new working agreements were largely responsible and continued to become more difficult. The mountain division had about reached its maximum capacity with the existing equipment, making it necessary to consider improvements.

A study of the operating costs of this section was made in 1921 from which it was apparent that the electrification of that section would readily relieve the congestion and make it possible to more than double the capacity of the line and at the same time accomplish a yearly saving of $523,000 in operating expense. The electrification was estimated to cost $2,420,000, thus indicating a return of 21 per cent on the gross investment including electric locomotives, in addition to the increased capacity and many other advantages.

Construction work was started in January 1923 and electrical operation between Orizaba and Esperanza was complete in January 1925. The total cost for the 29 mi. section was $2,427,480.00. Internal disturbances delayed the work several months and reduced the traffic greatly. In March and April 1928 the traffic becomes comparable with that of September and October 1921, for which period the actual traffic records and operating costs for steam operation had been used as a basis for comparison with the estimated cost of an equal traffic with electrical operation.

A comparison of actual operating costs of items affected by electrification for the two periods showed that 10 electric locomotives had hauled 36 per cent greater tonnage in 40 per cent less train hours than had 25 steam locomotives and at 50 per cent of the cost for items affected. When the figures were properly adjusted to compensate for increased tonnage and higher wages the saving indicated was at the rate of $663,348 per year, a return of 26 per cent on the total cost of the electrification.

In the meantime the general results had been so satisfactory that the electrification was extended 35 mi. south to Paso del Macho, making a total of 64 mi. at a cost of $3,606,937.00.

THE electrification of the Mexican Railway (Ferrocarril Mexicano) is of special interest as being the first undertaking of this nature carried out in old Mexico. The initial study of the problems involved was made in 1921 and a contract was signed for the construction work and for the electrical equipment in 1922. The work was completed and electrical operation started in 1925.

The operation has been so conspicuously successful that it would now seem to be of general interest to describe many of the details of construction and to give such operating data as are available.

The main line of the Mexican Railway extends from Mexico City to the Gulf Coast at Vera Cruz and is 264 mi. in length. There are six branch lines, making the total length:

TABLE I
STANDARD GAGE (4 FT. 8½ IN.)

	Miles
Vera Cruz to Mexico	264
Apizaco to Puebla	29
Ometusco to Pachuca	29
Pachuca to Ixmiquilpan	51
	373
Sidings and Second Track	74
	447
NARROW GAGE	
Huajuapam Branch (3 ft.)	56
Zacatlan Branch (2 ft. 6 in.)	33
Huatusco Branch (2 ft.)	20
	109
Total mileage	556

From Mexico to Esperanza the line follows the general contour of the plateau, resulting in frequent reversal of gradients, but none exceeding 1.5 per cent. The elevation at the terminus in Mexico City is 7346 ft. The highest point on the line is near Acocotla, 95 mi. from Mexico, at an elevation of 8320 ft., a rise of about 1000 ft. From this point to Esperanza, a distance of 58 mi., the drop is 240 ft. with the surface undulating as before.

At Boca del Monte (The Mouth of the Mountain), 2.8 mi. west of Esperanza, the plateau ends suddenly and the descent for the following 19.0 mi. to Encinar is very rapid.

At Santa Rosa the line enters the valley of the Rio Blanca and the grade becomes normal on to Orizaba.

From Orizaba to Paso del Macho the drop is considerable though much less rapid than that just described. In the 35 mi. there is a descent of 2466 ft. There are some comparatively long sections of tangent track, and the curves, except in the loop on the Metla Baranca, are not unusually numerous or seriously sharp. Fig. 1 shows a condensed profile of the electrified line.

The section from Paso del Macho to Vera Cruz, 48 mi. in length, is a gradual descent from an elevation of 1560 ft. to sea level, the ruling gradient being approximately 1.7 per cent and the alinement normal.

The promotion, the construction, and the operation of this road, the first to be built in Mexico, have all been in close synchronism with the interesting and tumultuous history of the country. The original charter was obtained in August 1855, but it was 18 years later, or in January 1873, that the road was formally opened to through traffic.

1. General Electric Co., Schenectady, N. Y.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-87

With the comparatively settled conditions which gradually came about with the ascendency of Porfirio Diaz as President, the property became more prosperous and before the beginning of the disturbance leading to his abdication had been brought into splendid condition. However, the revolution of 1910 suddenly brought an end to this prosperity and resulted in the road being taken from its owners for operation by the new Government in November 1914, under which control it remained until August 1916 when it was returned. It was again seized in April 1917 and held until June 1920.

When the property was returned to the owners, the line and equipment were found to be in very poor condition as practically no new apparatus or supplies had been purchased and the stores had been depleted. When the owners took charge of the road in 1920, many of the locomotives and cars were found to be in an inoperative condition with little material or supplies on hand with which to make repairs. New difficulties were also encountered in the nature of strikes which added greatly to the already difficult task of handling the increasing traffic.

In the fall of 1921, the business on the railway had grown so that it became very difficult to get the traffic over the mountain division from Orizaba to Esperanza which had always been the proverbial bottle neck. On October 25 a total of 14 trains, two of which were passenger, an aggregate of 4648 tons, was taken up the mountain, establishing a record for the division and indicating that the economic capacity of the single track line with existing type of motive power had been reached.

In November 1921 a study of the mountain division was begun for the purpose of determining the results that might be expected from the electrification of this congested district. About two months were spent in looking over the line and equipment, studying the conditions and securing operating data and costs. An equal time was required to analyze these data, estimate the cost of electrification, the savings in operating expenses that would result, and to prepare a report which was submitted in April 1922.

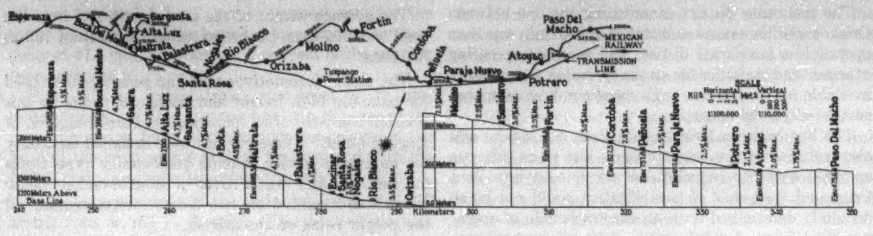

Fig. 1—Plan and Condensed Profile from Esperanza to Paso del Macho, Electrified Section of Mexican Railway

The unsettled conditions that had existed in the country for the preceding 10 years and the taking over of the railroad by the Government between 1914 and 1920 rendered the records for any of this entire period unsuitable for forming a basis for the usual method of estimating the traffic that might reasonably be expected over the line for the following years.

Operating expenses had also been abnormally affected during the interval by substantial increase in wages, as well as by unusually expensive working agreements insisted on by newly formed labor organizations.

The semi-annual reports of the Mexican Railway for the six months ending December 31, 1920, the first period of operation after the return of the property by the Government, compared with a similar period of 1913, the last six months of operation by the owners prior to Government operation, for which the records were available, indicates increased costs as follows:

TABLE II

	1913	1920	Increase Amount	Increase Ratio
Maint. of way and structures	$195,755	$379,652	$183,897	1.94
Maintenance of equipment	245,607	625,256	389,650	2.65
Conducting transportation	720,148	1,563,682	843,485	2.17
General expense	105,120	199,760	94,640	1.90
Total operating expenses	$1,256,630	$2,768,350	$1,511,672	2.20
Gross receipts	2,484,018	3,504,847	1,020,829	1.41
Net receipts	1,227,388	736,497	490,891	.61
Operating ratio	.51	.79	..	..

The comparative costs of the items that would be most directly affected by electrification for the above period were:

	1913	1920	Increase Amount	Increase Ratio
Repairs to locomotives	$ 88,875	$280,463	$191,588	3.15
Enginemen and roundhouse	72,931	236,673	163,742	3.24
Fuel for locomotives	244,202	359,071	114,869	1.45
Train service	80,295	200,008	119,713	2.49
Total	$486,303	$1,076,215	$589,912	2.21

An analysis of the above comparative cost indicated that the motive power in use on the system should receive careful consideration.

The management of the railway suggested that the months of September and October 1921 should be taken as a basis for the study, since the traffic conditions and operating costs during that period were more representative of anticipated future conditions than any other period for which records were available.

The mountain district constituting the line between Orizaba and Esperanza, a distance of 29.5 mi., was then operated as a separate district and accurate operating expenses and statistics for steam operation were readily available for comparison with corresponding estimated costs of electrical operation.

The line between Orizaba and Paso del Macho was also included in the original study but since this was operated with the remainder of the line on into Vera Cruz, and the result of electrification could not be so definitely determined until an exact schedule of operation had been decided upon, only an approximate estimate was made for this section.

Of the 29.5 mi. between Orizaba and Esperanza, the 19 mi. between Encinar and Boca del Monte, which had been given careful consideration in the original survey, again became the determining factor relative to motive power. The actual rise in the 19 mi. is approximately 3500 ft., equal to an uncompensated grade of 3.5 per cent. With the exception of the first 2.5 mi. of this heavy grade just out of Encinar there is practically no tangent track, but continuously reversing curves, many of which are on a radius of 351 ft., equivalent to 16.5 deg.

In locating the line, it was apparently intended to keep the uncompensated grade at a maximum not to exceed 4 per cent and the maximum curve at 16.5 deg. During the study, it was learned that some new consolidation type locomotives that had been recently purchased and were temporarily being used on this division had failed to handle satisfactorily the weight of train they had been calculated to handle on the heavy grade.

The above engines were expected to be capable of taking a trailing load of 250 tons up the mountain grade. In practise, it was found that it was not practicable for them to take more than 175 tons, which represents a coefficient of adhesion of approximately 20 per cent on a grade of 4.7 per cent. Since the profile did not indicate a grade in excess of 4 per cent, or curvature greater than 16.5 deg. at any point, an investigation was made which indicated that there were certain points on the line where the engines stalled from the slipping of drivers and as there was no indication that the condition of the rails at this point was responsible an engineer was sent to check the grade.

It was found that short sections of track from 300 to 500 ft. in length showed actual uncompensated grades as high as 4.4 per cent which when properly compensated for curvature would make the maximum grade 5.24 per cent. The average uncompensated grade for the 1.24 mi. which represented the section of greatest difficulty, was found to be 3.95 per cent, which when compensated for 16.5 deg. curvature at the rate of 0.90 lb. per deg. is equivalent to a grade of 4.7 per cent. Accordingly, this was taken to be the ruling grade for the division in the specifications for electric motive power.

The average weight of the passenger trains over the road was 215 tons, but these trains varied from 165 to 350 tons.

The steam locomotives that had generally been used for handling both freight and passenger trains on this division consisted of 32 four-cylinder Fairlie engines, of English build. These engines were designed especially for mountain service, having pack-saddle type tanks for fuel oil and water over a double-ended boiler which was mounted on two 3-axle swivel trucks, all the weight being on the drivers.

The engines had the appearance of two 3-axle switching locomotives coupled with cab ends together.

The fire-box was in the middle of the engines and contained two oil burners, the cab being over this central portion and the engineer located on one side of the boiler with the fireman on the opposite side. The engines ran equally well in either direction, and having a comparatively short rigid wheel base with all weight on the drivers at once deprived the electric locomotive of three of its usually boasted advantages.

The 32 engines were all of the same general type but of variable ages and weights as follows:

5 weighing, with full tanks	167,428 lb.
12 weighing, with full tanks	216,540
10 weighing, with full tanks	221,000
2 weighing, with full tanks	267,885
3 weighing, with full tanks	305,835

The first five of the above engines were not being used at the time, as ten new locomotives of the consolidation type had just been received and some of these were being substituted temporarily.

The weight of these new locomotives was as follows:

Weight on drivers	170,540 lb.
Weight on guiding axles	21,952
Weight of tender loaded	119,775
Total	312,267 lb.

During the months of September and October 1921 there had been taken over the division a total of 474 trains south bound, with an aggregate tonnage of 108,749 gross metric tons, and 622 trains with aggregate gross metric tonnage of 197,220, north bound. The total hours of elapsed time, or the hours for which the engine and train crews were paid between Orizaba and Boca del Monte where the helper locomotives on the freight trains were usually left off, were 1574 for the south bound and 2756 for the north bound traffic, making the average times for each train over the 25.4 mi.:

	South bound	North bound
Running	2 ft. 30 in.	3 ft. 39 in.
Delayed	0 ft. 49 in.	45 in.
Elapsed	3 ft. 19 in.	4 ft. 25 in.

This represented an average schedule speed of 7.66 mi. per hr., and 5.7 mi. per hr. respectively for the south bound and north bound trains, and an average delay of 47.4 min. per train or 25 per cent of the running time per train.

Practically all freight trains, and most of the passenger trains, required two engines. In the case of the passenger trains, both Fairlie engines were placed at the head of the train with a box-car between and with the freight trains an engine was placed at each end.

The average weight of the trains up grade was 317 metric tons or 350 U. S. tons. The down-grade tons were considerably less, as almost two-thirds of the traffic was north bound.

The average wage of the enginemen and trainmen on the Mexican Railway at the time was approximately as follows, based on an 8-hour day or 100 mi.:

	Pesos per day	Dollars per day
Engineers	12.00	$6.00
Firemen	6.00	3.00
Conductors	11.00	5.50
Brakemen	5.28	2.69

The operating data and actual costs of the steam operation for the months of September and October 1921, were taken as a basis for estimating the saving that might be expected to result from the electrification of the Orizaba–Esperanza district.

Only the items of operating costs which would be most vitally affected were considered. The estimate as submitted in the report follows:

TABLE III

	Steam	Electric	Saving	Ratio steam to electric
Wages of enginemen	$108,892	$ 31,354	$76,538	3.44
Wages of trainmen	85,290	23,191	62,099	3.68
Fuel and power	221,790	150,000	71,790	1.48
Repairs to locomotives	355,248	51,111	304,157	6.95
Enginehouse expense	20,946	5,841	15,105	3.57
Lubricants	16,656	2,921	13,735	5.70
Substation oper. and maint.	..	11,750	..	..
Maint. distributing system	..	9,625	..	..
Total	$808,822	$285,793	$523,029	2.82

The estimated cost of the electrification as submitted was:

10–150-ton electric locomotive units	$1,420,000
1–6000-kw. substation	350,000
30–Route miles distributing system	430,000
Engineering and contingencies	220,000
Total gross capital investment	$2,420,000

The report accordingly indicated a probable saving of $523,029 per year by the expenditure of approximately $2,420,000, or a return of slightly more than 21 per cent on the gross investment which included electric locomotives.

A similar study of the line between Orizaba and Paso del Macho was made at the same time, but as the savings that seemed probable were sufficient for a return only about half that of the Orizaba–Esperanza district, the report recommended that only the latter be undertaken in the beginning.

After a very careful examination of the report by the operating department of the railway, it was approved and recommended to the board of directors.

In October 1922 a contract was made for the required equipment and materials including the general supervision of the installation.

Actual work on the ground was begun in January 1923 and the work was completed and all trains were being hauled electrically by January 1925.

Just when the construction work had been fully organized and got well under way it was almost entirely stopped from the middle of December 1923 to May 1924 by the rebellion which occurred at that time. This interruption caused several months' delay and considerably increased the cost of the work.

The work was done almost entirely with native labor which had to be trained for the work. Three supervising engineers were placed in direct charge of the distribution, the substation, and the locomotives respectively, reporting to the resident engineer of the railway company, in cooperation with the general engineer of the contractor.

The Distribution System

The simple catenary system, with double 4/0 trolley on the main line and single 4/0 trolley in yards and passing tracks, similar to that used on the C., M., St. P. & P., was adopted. Concrete poles were first selected for the supporting structure but it was found that several hundred old 30-ft., 82-lb. rails which had been replaced were on hand and that there was no market for these where the price offered would justify handling and transportation charges.

Some sample poles were made up from these rails by electric welding and tested for deflection and breaking strains which resulted in a decision to use these as far as the rails would supply the demand.

Of a total of 1920 poles made, 405 were concrete costing $45.52 each and 1515 were made from old rails costing $34.00 each. Figs. 2, 3, 4, 5, and 6 show both types of poles and typical examples of the overhead construction. Most of this work was done by contract, the unit price being fixed after sufficient experience had demonstrated a fair price. Bracket construction was used where possible, except where the poles supported feeder wires and it was desirable to avoid crossing these over the tracks too often. In such cases, as between

Boca del Monte and Alta Luz, the feeder cables were kept on the upper side of the track by having the poles continuous on that side, irrespective of the curves,

Fig. 2—Overhead Line in Nogales Station Showing Span Construction with Concrete Poles

Fig. 3—100,000- and 42,000-Volt Transmission Lines Crossing Railroad Tracks

Fig. 4—Overhead Construction on Curve Just Below Cordoba

thus using cross-span supports where the curves were outward from the upper side of the slope. This also prevented interference with the magnificent view from the car windows overlooking the valley.

In addition to the two 4/0 trolley wires two 500,000-cir. mil positive feeders and one 4/0 negative feeder cable were installed from Orizaba to Boca del Monte. The feeders were carried on separate poles at 4 points where the railway made long loops. The advantage thus gained was the reduction of feeder length from 29.5 mi. to 20.4 mi. The single substation for the 29.5 mi. was located near Maltrata and was almost an equal distance from each end of the feeder circuit, 10.2 mi. in length on either end. Only a single 500,000-cir. mil feeder was used between Boca del Monte and Esperanza as the grade here was comparatively light.

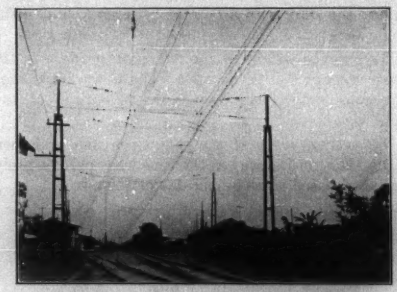

Fig. 5—Overhead Construction at Entrance to Parage Nuevo Yard

Fig. 6—Typical Overhead Construction on Bridge at Rio Seco

A similar feeder was used from the substation to the middle of the Bota loop, making it unnecessary to install any feeders along the railway between Bota and Alta Luz where the curvature was greatest, since this loop was fed from both ends making the equivalent of a double track circuit.

The voltage drop over the line between Orizaba and Esperanza with a 665-ton trailing freight train, also a 325-ton passenger train, is shown in Fig. 7. The drop up the grade with the heavier train was a maximum of

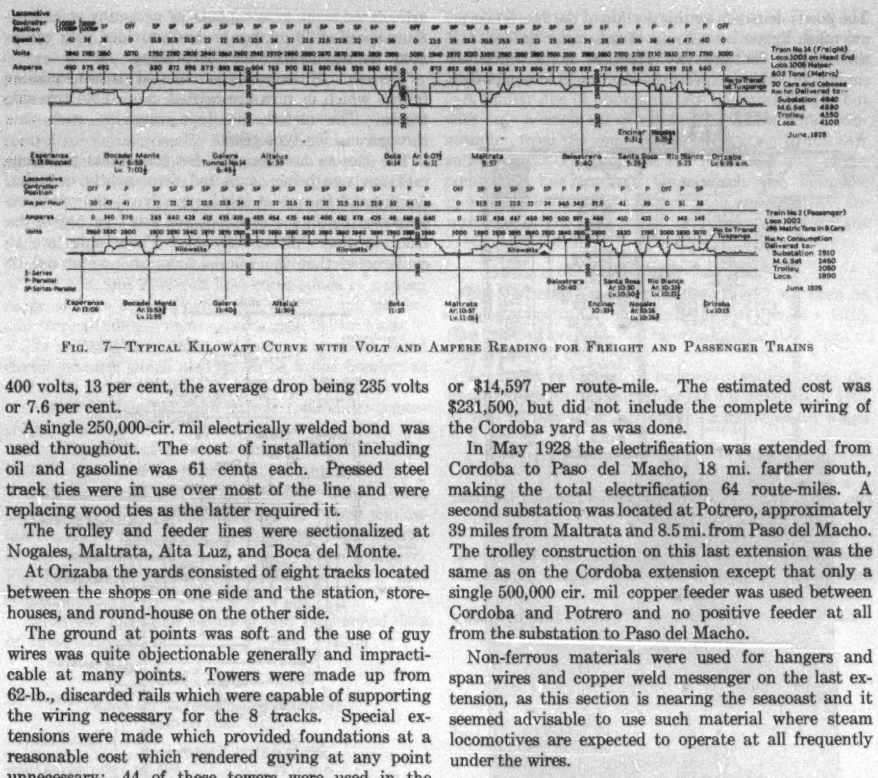

Fig. 7—Typical Kilowatt Curve with Volt and Ampere Reading for Freight and Passenger Trains

400 volts, 13 per cent, the average drop being 235 volts or 7.6 per cent.

A single 250,000-cir. mil electrically welded bond was used throughout. The cost of installation including oil and gasoline was 61 cents each. Pressed steel track ties were in use over most of the line and were replacing wood ties as the latter required it.

The trolley and feeder lines were sectionalized at Nogales, Maltrata, Alta Luz, and Boca del Monte.

At Orizaba the yards consisted of eight tracks located between the shops on one side and the station, storehouses, and round-house on the other side.

The ground at points was soft and the use of guy wires was quite objectionable generally and impracticable at many points. Towers were made up from 62-lb., discarded rails which were capable of supporting the wiring necessary for the 8 tracks. Special extensions were made which provided foundations at a reasonable cost which rendered guying at any point unnecessary; 44 of these towers were used in the Orizaba yard and 33 in the Cordoba yard, the latter costing $102 each. Other lighter towers also made from old rails were used at bridges. The cost of these towers was $75.48 each, Fig. 6.

There are seven tunnels on the first electrification, seven on the Cordoba, and two on the Paso del Macho extensions, a total of sixteen, all but two of which are on curves.

The total cost of the 29.5 mi. of distribution system including poles and fixtures was $462,011 or $15,661 per route-mile.

In 1926 the trolley line was extended from Orizaba to Cordoba, 16 mi. south, making a stub end feed from the Maltrata substation approximately 29 route-miles but feeder cut-offs at loops reduced the feeder length to about 25 mi.

This construction was similar to the original except that rail poles were used exclusively. Two 500,000-cir. mil copper feeders were used almost the entire distance. The total cost of this extension was $233,556 or $14,597 per route-mile. The estimated cost was $231,500, but did not include the complete wiring of the Cordoba yard as was done.

In May 1928 the electrification was extended from Cordoba to Paso del Macho, 18 mi. farther south, making the total electrification 64 route-miles. A second substation was located at Potrero, approximately 39 miles from Maltrata and 8.5 mi. from Paso del Macho. The trolley construction on this last extension was the same as on the Cordoba extension except that only a single 500,000 cir. mil copper feeder was used between Cordoba and Potrero and no positive feeder at all from the substation to Paso del Macho.

Non-ferrous materials were used for hangers and span wires and copper weld messenger on the last extension, as this section is nearing the seacoast and it seemed advisable to use such material where steam locomotives are expected to operate at all frequently under the wires.

Substation Equipment

The original substation located at Maltrata contains two 3000-kw., 3000-volt d-c. three-unit synchronous-motor generator sets for supplying power to the original 29.5 mi. electrification. Fig. 8 shows a plan of the Maltrata substation, Fig. 9 the interior of the generator room, and Fig. 10 a schematic diagram of the wiring of the 3000-volt d-c. generators.

Power is purchased from the Puebla Tramway Light & Power Company at 42,000 volts, 60 cycles. Ample provision is made for housing all of the substation equipment, the transformers and high-tension switching occupying about one-half the building. The other half of the station is occupied by the motor generator sets and switching equipment; each motor generator set is protected by a high-speed circuit breaker and each of the outgoing feeders has also the same protection.

With the extension of the electric zone to Paso del Macho, making the total length of route approximately 64 mi., a second substation was installed at Potrero.

The power demands for this portion of the line, however, are much lighter on account of the easier grades, and this station contains two 1500-kw. units which are similar in other respects to the original equipment.

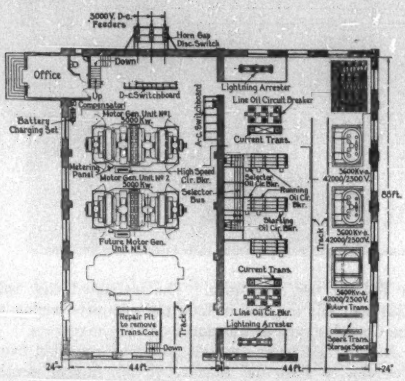

FIG. 8—PLAN OF MALTRATA SUBSTATION SHOWING LOCATION OF APPARATUS

This station is also supplied from the 42,000-volt, three-phase, 60-cycle circuit which forms a part of the power company's network.

Each substation building provides the necessary

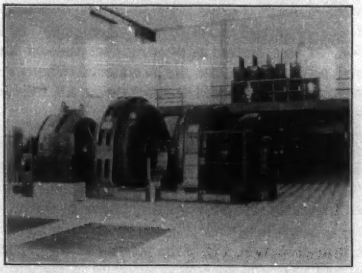

FIG. 9—INTERIOR OF MALTRATA SUBSTATION SHOWING TWO 3000-KW., 3000-VOLT MOTOR-GENERATOR SETS AND HIGH-SPEED CIRCUIT BREAKERS

space and foundations for a third unit when required. The equipment used is standard, being similar to that supplied the Chicago, Milwaukee & St. Paul, the Spanish Northern, and the Paulista Railways.

ELECTRIC LOCOMOTIVES

The initial motive power equipment for the Orizaba–Esperanza section included ten locomotive units suitable for handling both passenger and freight service. These locomotives are of the twin-geared six-motor articulated-truck type capable of operating at a maximum speed of 40 mi. per hr. and provided with equipment for regenerative electric braking.

The cab is of the box type mounted on two equalizer frames which in turn are carried upon three two-axle trucks. The motors are twin-geared to each axle through cushion type gears.

The motors are designed for 1500 volts per commutator, two being connected permanently in series. An unusual feature of this locomotive is the provision for removing the motor wheels and axle from any of the trucks without lifting the cab. These units are so constructed that the wheels, axle, and motor can be

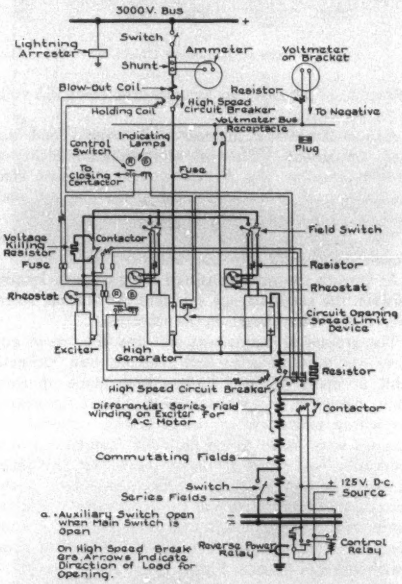

FIG. 10—SCHEMATIC DIAGRAM OF WIRING OF 3000-VOLT D-C. GENERATORS

dropped into the pit after first disconnecting the motor leads or the wheels and axle may be taken out alone by propping up the motor and side frames.

The auxiliary machinery consists of a 3000/1510-volt dynamotor with a 65-volt control generator mounted on the shaft extension; a 1500-volt regenerative exciter set; two 1500/3000-volt motor-driven blowers and two 1500/3000-volt air compressors. The two blowers and the two compressors normally operate in series across the 3000-volt circuit with their mid-points connected to the mid-point of the dynamotor for equalization. The regenerative exciter set operates from the 1500-volt circuit of the dynamotor. The scheme of operation

provides a means for operating one compressor or one blower from the 1500-volt dynamotor bus in case of failure of the other unit. This arrangement also permits of operating the compressors and blowers in case of the failure of the dynamotor.

The locomotives are equipped with electropneumatic control for non-automatic multiple unit operation. A master controller in the engineman's cab at each end is arranged to give a total of 31 accelerating steps, distributed through the three groupings of the motors, a full-field and two reduced-field running steps with each of the three combinations. This gives a total of nine running points which, for a given tractive effort of 50,000 lb. and 2700-volt line, corresponds to a speed range of from approximately 5½ to 26½ mi. per hr. and correspondingly higher speeds with lighter loads.

The main motor circuits are protected by a high-speed circuit breaker which also serves as a line breaker as well as to give overload and short-circuit protection.

Current is collected from an overhead slider pantograph, one of which is located at each end of the cab roof. These collectors have a range of from 15 ft. 5 in. to 24 ft. above the top of the rail.

With the extension of the electrification from Orizaba to Paso del Macho additional motive power was required and for this purpose two locomotives, duplicates of the original design, have recently been delivered. Fig. 11 shows a locomotive attached to a regular passenger train at the Maltrata station.

Below are given the electrical and mechanical data on these locomotives:

TABLE IV

DATA ON ELECTRIC LOCOMOTIVES

ELECTRICAL DATA	
Nominal voltage of system	3000-volt d-c.
Tractive eff., 1 hr. blown (3000-volt)	54,000 lb.
Speed at 1 hr. rating, full field	19 m. p. h.
Total horsepower, 1 hr	2736
Tractive eff., con. 3000-volt full field	48,500
Speed at continuous rating, 3000-volt	19.5 m. p. h.
Total horsepower, continuous	2520
Number of motors	6
Type of motors	GE-278-A-1500/3000 volts
Gear ratio	90/18–5.00
Tractive eff. at 30 per cent tract. coef	92,700 lb.
MECHANICAL DATA	
Track gage	4 ft. 8½ in.
Wheel arrangement	04440
Diameter of drivers	46 in.
Number of driving axles	6
Total wheel base	40 ft. 6 in.
Max. rigid wheel base	9 ft. 2 in.
Width overall	10 ft. 1½ in.
Height over trolley locked down	15 ft. 2 in.
Length inside knuckles	52 ft. 11 in.
WEIGHTS	
Total weight on drivers	309,000 lb.
Dead weight per axle	12,150 lb.
Elec. and air brake equipment	135,000 lb.
Mechanical equipment	174,000 lb.

Transmission Line

A 42,000-volt transmission line 17 mi. in length was constructed from the Tuxpango hydroelectric generating station of the Puebla Tramway Light & Power Co. to the Maltrata substation. Twin lines of 1/0 aluminum cable are supported on steel "A" frames with anchor towers at proper intervals.

A similar line of approximately the same length but with only a single set of conductors is now under construction from the same generating station to the new substation at Potrero. In the meantime the power company's line to Vera Cruz which the new line parallels is being used and switching arrangements are provided in the substation for reciprocal use of the lines when desirable.

Electrical Operation

The Orizaba–Esperanza section has now been in complete operation electrically since January 1925, thus making available the records for four full years of operating data and costs.

The great reduction in business resulting from the unsettled conditions caused by the disturbances of 1924 as well as new labor agreements and increased wages

Fig. 11—Passenger Train with 150-Ton, 3000-Volt Locomotive at Maltrata Station

had tended to make direct comparisons of operating costs look less favorable than they really were. A decrease in the tonnage to be hauled reduces operating costs to an extent but unfortunately not nearly in the same proportion and the capital charges remain practically the same. On the other hand increased wages and higher costs of fuel and materials usually tend to make electrification comparatively more profitable.

The operation has been very satisfactory and successful throughout. No serious difficulties have been encountered with any portion of the equipment. This may be measured fairly by reference to the complaint bill for the usual guarantee period which was less than $3,000.

The general performance of the equipment may best be judged by the costs of maintenance, a fair average of which is given herein.

During the months of March and April 1928 the traffic over the Orizaba–Esperanza section reached a new maximum which was somewhat in excess of that

for steam operation which had been attained during the months of September and October 1921 which was used as a basis in the original study. This offered a favorable opportunity for comparing actual results of electrical operation with that for steam operation, as well as to check the estimates in the report leading to the electrification.

Detailed tabulations of the operating records for March and April 1928 with electrical operation were prepared for comparison with similar records as used in the report for September and October 1921 with steam operation.

A summary of this comparison follows:

TABLE V

	1921	1928	Difference	
	Steam oper. Sept. & Oct.	Elec. oper. Mar. & Apr.	Amount	Per cent of steam
Number of trains run	1,096	1,011	85	108
Number of cars handled	9,238	12,840	3,602	139
Number of tons handled	305,969	416,037	110,068	136
Number of train hours	4,577	2,718	1,859	60
Number of tons per train	279	412	133	148
Number of cars per train	8.43	12.70	4.27	150
Number of hours per train	4.18	2.69	1.49	64
Cost enginemen's wages	$17,982	$14,565	$ 3,417	81
Cost trainmen's wages	14,222	10,721	3,501	76
Cost fuel oil or power	36,965	31,140	5,825	84
Cost locomotive repairs	59,208	5,969	53,239	10
Enginehouse expense	3,391	1,945	1,446	58
Lubricants	2,776	92	2,684	3
Substation operation	..	2,154	2,154	..
Trolley maintenance	..	856	856	..
Total cost items affected	$134,544	$67,442	$67,102	50

Briefly stated, the 10 electric locomotives during March and April handled 36 per cent more tons than was handled by more than double the number of steam locomotives during September and October, with 8 per cent less trains and in 40 per cent less train hours on the road, and at 50 per cent of the corresponding total cost of steam operation for 26 per cent less tonnage handled; thus accomplishing a saving of $67,102 for the two months, or at the rate of $402,612 per year, which alone represents a return of more than 15 per cent on the gross cost of the electrification including electric locomotives. However, this amount does not represent actual saving since the cost of all classes of labor had considerably increased during 1922 and 1923 so that it is necessary to adjust the costs of the steam operation to have them indicate what they would have been if the increased traffic of March and April had been handled by steam and at the higher rate of wages, as was done by the electric locomotives.

Between October 1921 and the end of 1923 and before the electrification had been completed, new contracts were made with practically all labor involved in these operating costs, under the terms of which the rates of pay were materially increased. Enginemen's and trainmen's wages were increased on an average of about 42 per cent, which higher rates were in effect in March and April 1928. The increases were:

	1921	1928	Per cent
Enginemen per round trip	$6.00	$8.88	48.0
Firemen per round trip	2.88	4.44	54.1
Conductors per round trip	5.50	7.20	30.9
Brakemen per round trip	2.69	3.60	33.8

The wages of mechanics and others employed in the repair and care of locomotives were also proportionally increased but last year and the year previous thereto considerable reductions were made in this class of labor, to such an extent that no adjustment for wages in cost of repairs was considered necessary.

The average price paid for fuel oil during the months of September and October 1921 was $1.09 per barrel but the price paid for same in March and April is given as $1.29, an increase of 18 per cent.

In adjusting the cost of steam operation for September and October 1921 for the 36 per cent increase in traffic, it was considered inaccurate to assume that the cost would increase in the same proportion, since the number of trains required to handle the traffic is almost entirely determined by the northbound or up-hill traffic which exceeded the southbound by 32 per cent during March and April. It is also possible to take considerably heavier trains down than up the mountain grade.

The adjustment for increased traffic was consequently made on the basis of the increased number of steam trains that would have been required to handle the northbound business.

The passenger trains run north in March and April numbered 122 as against 89 in September and October, an increase of 33 trains.

The freight tonnage hauled north for the same period was 195,357 and 175,625, respectively, an increase of 19,732 tons. At an average of 330 tons per train which is the exact record for the steam locomotives, 60 additional freight trains would have been required to handle this excess freight tonnage, making a total increase of 93 trains or 14.95 per cent. On an already congested single-track line with infrequent passing tracks, a 14.95 per cent increase in the number of trains would ordinarily considerably lengthen the time per train, and consequently increase the cost of wages and fuel, but since the enginemen and trainmen are paid by the trip if made in ordinary time, and the run being unusually short, no allowance was made for this item. It was considered conservative to assume that all other costs of the items listed would be increased in approximately the same proportion as the number of trains operated, that is, 14.95 per cent.

When the costs of steam operation were adjusted on this basis as outlined, the direct comparison between steam and electric operation for traffic handled in March and April 1928 was:

	Steam	Electric	Reduction	
	Sept. & Oct.	March & April	Amount	Per cent
Enginemen	$30,991	$14,565	$16,426	53
Trainmen	21,722	10,721	11,001	50
Fuel or power	50,139	31,140	18,999	38
Repair locomotives	68,059	5,969	62,090	91
Enginehouse expense	3,898	1,945	1,953	50
Lubricants	3,191	92	3,099	97
Substa. oper. maint.	..	2,154	..	..
Trolley oper. maint.	..	856	..	..
Total	$178,000	$67,442	$110,558	62

In order to directly compare the above with the estimate in the original study on this subject the costs were placed on a yearly basis as follows:

	Steam	Electric	Saving	
			Indicated	Estimated
Enginemen	$185,946	$87,390	$98,556	$76,538
Trainmen	130,332	64,326	66,006	62,099
Fuel or power	300,834	186,840	113,994	71,790
Repairs to locomotives	408,354	35,814	372,540	304,137
Enginehouse exp.	23,388	11,670	11,718	15,105
Lubricants	19,146	552	18,594	13,735
Substa. oper. maint.	..	12,924	..	..
Trolley oper. maint.	..	5,136	..	..
Total	$1,068,000	$404,652	$663,348	$523,029

The above indicated yearly saving of $663,348 in favor of electrical operation is 27 per cent greater than the estimate, but this was readily accounted for by the 36 per cent greater tonnage handled since any material increase in traffic should increase the savings also, and in a greater proportion.

This annual saving represents approximately 26 per cent earning on the gross cost of the electrification of the Orizaba–Esperanza district, which alone should be a very satisfactory result but other more indirect savings which cannot be so accurately segregated, such as wear on wheels and brake shoes, reduction of accidents from broken wheels due to over-heating while braking would, if fully valued, add many hundreds of dollars to the credit of electrical operation.

Perhaps the most valuable unlisted asset in the electrification is the increased capacity of the lines electrified. On March 18, 5381 tons of freight and two passenger trains weighing 741 tons, making a total of 6122 tons, were hauled from Orizaba to Esperanza which was a record for the district. The record day with steam operation was October 25, 1921, when 4008 tons of freight and two passenger trains weighing 640 tons making a total of 4648 tons were handled; 1474 tons or 31 per cent in favor of the electric operation, 14 trains being used in each case.

The total tonnage handled electrically on March 18, north and south, was 9659 and that for October 25, 1921 by steam was 6532. The excess for electric operation was 50 per cent. The train hours electric were 64.6; steam 83.4, a reduction of 18.8 train hours or 22 per cent with an increased tonnage handled of 50 per cent.

By increasing the train weight to 900 tons (metric), for which the line and equipment are capable, it would be practicable to handle over the present line double the tonnage handled on March 18 with no additional facilities except for locomotives, cars, and power.

The ten locomotives now in service were expected to be sufficient to handle a maximum of 4200 tons or regularly 3600 tons in addition to two passenger trains per day in each direction between Orizaba and Esperanza.

This maximum was exceeded by 1181 tons or 28 per cent on March 18 and the average tons per day north bound for the month was 3350 tons but all ten locomotives were used on the maximum day.

The earnings on investment as shown are for the gross investment including the cost of electric locomotives. Had new steam locomotives been purchased these would have been charged as renewals or betterments and since sufficient new steam locomotives to handle an equal tonnage on a mountain district would cost approximately as much as the electric locomotives, it should be quite proper to deduct the cost of the locomotives from the cost of the electrification.

In the case of the Mexican Railway the cost of the electric locomotives was approximately 46 per cent of the gross cost of electrification and usually for heavy grade work this item will average between 40 and 50 per cent.

If the cost of the electric locomotives be deducted from the total cost of electrification the savings from electric operation as shown would be equivalent to an earning of 47 per cent on the net cost.

The entire 64 mi. has been in full operation for about a year, but the records are not yet available for a comparison such as is given for the original electrification and it will not be possible to have them so accurate as long as the 48 mi. on the Vera Cruz end is operated by steam.

The total cost of the 64 route-miles of electrification including electric locomotives was approximately $3,606,937. The orignal study gave an estimate of $4,032,500.

The experience gained from two years' operation in the Orizaba–Esperanza district made it evident that the traffic could be handled with a less number of locomotives than had been considered necessary, so that this item was reduced by approximately 20 per cent.

Discussion

D. C. Jackson: Mr. Cox's paper is a notable one because it shows carefully gathered statistics of comparative cost of steam operation with electric operation over the same route with appropriate coefficients to transform somewhat heavier traffic of the electric period back to the steam period, or vice versa. The only fault with that lies in the fact that the time has been so short that the comparison is on a monthly basis rather than a complete annual basis, although transferred to the annual basis, and it is to be hoped that Mr. Cox may ultimately bring forward a paper that gives an entire actual twelve months of experience. If it is as favorable as his present paper would indicate, it cer-

tainly will be reassuring to the railroad executives and operating people.

One of the difficulties that has existed with respect to electrification of railroads of heavy type, has lain in the fact that the electrical people, and particularly the manufacturers and power companies, have failed to recognize that the railroads have a transportation problem, not a problem of using electrical machinery, and primarily it is one of the great problems of the United States; in fact, it is one of the great problems of the world.

Another difficulty has rested in the fact that data of comparing operating results, including costs, have not been clearly analyzed and made generally serviceable to railroad executives and the railroad-using public. Mr. Cox is starting on a path which is worthy of extension.

Sidney Withington: In connection with Mr. Cox's paper, in presenting the savings accomplished by the railroad electrification, it seems to me it would be more convincing and more valuable if he had also discussed more in detail the effect of modern steam motive power. It is the duty of railroad electrical engineers to do business with hard-headed steam railroad executives and they want to know not only the savings due to electrification in comparison with pre-war equipment, so to speak, but they want to know what the savings of electrification are in comparison with modern steam motive power, equipped with feedwater heaters, super-heaters, and all the other elements which have been developed to increase the efficiency of operation and the capacity of steam locomotives. I think any comparison of the costs of electrification as compared with steam operation should include that basis. Mr. Cox's reference to modern steam locomotives tells only half the story.

Mr. Cox refers to the fact that non-ferrous material is used. I think this is of importance in showing the trend of electrification power-distribution design. The engineers who are responsible for the design of distribution systems are more and more clearly realizing the necessity of installing material that will stay in service, and reducing as far as possible the maintenance problems which of necessity interfere with operation and are interfered with by operation. This is of course of special importance where steam locomotives are operated in the electric zone.

W. B. Potter: Electrification of the Mexican railways as described by Mr. Cox is an excellent example of what may be done to improve the service and at the same time accomplish such reduction in the cost of operation as to show a substantial return on the investment for the new type of motive power. I would call particular attention to the important part which the electric locomotives had in obtaining this economic showing, both as to operation and fixed charges. The cost of maintenance as compared with steam is an appreciable reduction in the operating expense. The relative cost of electric power and fuel for steam locomotive will vary in nearly every case, but the relative maintenance in favor of electric locomotives may always be expected to show an appreciable gain. As affecting the fixed charges, the cost of the electric locomotives is a large item in the total cost of electrification, amounting to 46 per cent in the case of the Mexican Railways and 42 per cent in the Chicago, Milwaukee & St. Paul. The cost of the locomotive has, therefore, an important bearing on the amount to be charged to depreciation, particularly as the locomotive depreciation is at a higher rate than other features of electrification. The cost, maintenance, and depreciation of the substations and overhead construction are relatively of small importance as compared to the locomotives.

J. B. Cox: In reference to Mr. Jackson's criticism it should be explained that the records for several months were gone over carefully, and in the case of steam the two months were selected by the management as being most nearly representative of what they expected in the future and wished to prepare for immediately.

On account of previous conditions of operation only four months of steam operating records were at all suitable for the study, these being July, August, September, and October, 1921. After a general survey of the past records and a careful study of these four months, September and October were selected for the more detailed analysis.

In the case of electrical operation, the months of March and April were designated for the comparison by the chairman of the Board of the Railway, because the traffic over the road during these two months was the heaviest in its history for a like period and was thought to be most desirable for the comparison.

In this instance also the records for January, February, March, and April, 1928, were carefully studied and compared with practically the entire period of electrical operation, precisely as had been done for September and October with steam operation.

Table VI, which is published herewith, gives the operating costs involved for the four months steam operation mentioned, the four corresponding months electrical operation, and the records for 39 months of electrical operation which cover practically the entire period since the original electrification was in complete operation.

The 1928 record includes eight months operating cost of the newly completed divisions between Orizaba and Paso del Macho, or for the entire 64-mile section, and are therefore not directly comparable with the records for 1925, 1926, and 1927. A comparison of the four months steam operation with the corresponding four months electrical operation indicates a total saving of $141,964 for the four months period, or 54 per cent in favor of electrical operation, which is at the rate of $425,892 per year, whereas the two months comparison used in the paper showed a saving of only $402,612 or 50 per cent, before any adjustments were made for increased tonnage and wages in either case. Unfortunately the records of the ton-miles for July and August, 1921, were not recorded in the report, but judging from the locomotive miles and operating costs the increase in ton-miles during the four months electrical operation would be about 30 per cent as against 26 per cent, for the two months period, thus indicating that the two periods would show about the same rate of earnings on the investment.

These records, which in each case have been supplied by the operators and approved by them, should encourage alike the electrical industry and the railroad operating fraternity for a mutual interest in the electrification of railroads.

Practically all the figures presented are taken from official records, used originally in reports made at the request of the management of the railroad company, and are perhaps more accurate and free from adjustments than any others published to date. Space was not available in the paper for these details and therefore explanations have been abbreviated as much as possible. The records may be had in considerably more detail, and undoubtedly will soon be published.

Referring to Mr. Withington's remarks, it may be of interest to state that 30 per cent of the locomotive-miles in September and October were made by the ten new Consolidation locomotives referred to in the paper. No advantages were claimed because of this fact in favor of electrification, for the item of repairs to locomotives which would have been most affected, for the reason that the shop was behind with this work at the time and had continued at the same rate, through the period considered.

It was supposed that the best steam locomotives then available had been supplied. At that time, 1921, the steam locomotive had not nearly reached its present state of efficiency. The records of these new steam locomotives in this instance did not indicate that the growing traffic could be handled by them economically over the single-track line. Double tracking would have been too expensive for serious consideration, about double the entire cost of electrification.

TABLE VI

MEXICAN RAILWAY COMPANY

OPERATING COSTS ORIZABA—ESPERANZA DISTRICT

For 4-Month Steam Operation 1921

	July	August	Sept.	Oct.	Total	Yearly rate
Enginemen	$7,548	$9,625	$8,476	$9,507	$35,156	$105,468
Trainmen	5,745	6,720	6,707	7,508	26,680	80,040
Fuel	19,360	21,570	17,551	19,413	77,903	233,709
Repairs	24,887	27,523	28,832	30,426	111,668	335,004
Enginehouse	2,028	1,427	2,185	1,305	6,945	20,835
Lubricants	1,095	1,287	1,412	1,364	5,158	15,474
Total	$60,672	$68,152	$65,163	$69,523	$263,510	$790,530
1000 ton miles			4,899	5,052		
Locomotive miles	58,844	61,513	57,541	61,822	239,721	719,163

For 4-Month Electrical Operation 1928

	Jan.	Feb.	March	April	Total	Yearly rate
Enginemen	$4,776	$5,517	$7,839	$6,726	$24,858	$74,574
Trainmen	2,850	3,619	6,010	4,711	17,190	51,570
Power	13,740	14,054	14,560	16,580	58,934	176,802
Repairs	2,888	2,534	2,251	3,717	11,390	34,170
Enginehouse	777	756	965	980	3,478	10,434
Lubricants	22	51	47	45	165	495
Substation	869	944	1,052	1,102	3,967	11,901
Distribution	439	269	444	412	1,564	4,686
Total	$26,361	$27,744	$33,168	$34,273	$121,546	$364,632
1000 ton miles	5,062	5,953	8,445	7,420	26,880	80,640
Engine miles	20,780	23,490	32,119	29,256	105,645	316,935

For 39-Month Electrical Operation 1925-1928

	1925 Oct., Nov., Dec.	1926 Year	1927 Year	1928 Year	Total 39 months	Average yearly rate
Enginemen	$9,221	$43,012	$54,992	$80,835	$188,060	$57,865
Trainmen	4,789	25,970	34,193	53,033	117,985	36,303
Power	40,531	163,668	164,840	195,680	564,719	173,760
Repairs	4,493	21,768	36,868	42,514	105,643	32,505
Enginehouse	5,644	11,562	9,998	16,810	44,014	13,543
Lubricants	747	2,173	668	733	4,321	1,329
Substations	4,966	15,130	12,831	18,309	51,236	15,765
Distribution	2,587	9,308	4,510	5,775	22,180	6,825
Total	$72,978	$292,591	$318,900	$413,689	$1,098,158	$337,895
1000 ton miles	9,714	43,856	57,625	99,026	210,221	64,683
Engine miles	37,898	199,424	231,895	355,093	824,310	253,633

In general, where electrification has been compared with the most modern steam power, the initial cost of steam or electric locomotives for equal service has come out about the same. The operating savings effected by electrification were 50 per cent over old steam locomotives, but only about 30 to 35 per cent over modern steam locomotives. Where the cost of the locomotives is about the same in each case, and is about 50 per cent of the gross cost of electrification, the visible return on the capital investment for modern steam power would be at approximately the same rate as that for double the investment required for electrification, but the benefits received from electrification, in the way of improved service and increased capacity, are much greater, and usually will justify the additional expenditure. Much depends on the extent to which it is desired to improve the service and the increase in the line capacity required and the rate at which traffic is increasing.

If all these items are above normal, electrification will usually show an advantage sufficient to make the additional cost fully justified, even when compared with the most modern steam power. Where double tracking and new steam locomotives are necessary, it will be found that electrification will promise a much higher rate of return on the required investment, and in most instances will cost considerably less.

No doubt many such economic mistakes have been made in double tracking in the past, due largely to the fact that the electrical engineer has not been sufficiently experienced in railway operation to be able to present the case convincingly to the railway operating official, whose experience with electric traction has been equally limited.

The power companies have also failed to take advantage of an opportunity to increase their income through a misapprehension as to the effect a railway load might have on their system.

Contact Wire Wear on Electric Railroads[1]

BY I. T. LANDHY[2]

Non-member

Synopsis.—It is the purpose of this paper to present such data on the subject of contact wire wear as the electric operation of four railroads has made available. Design of overhead system and pantographs, lubrication of pantograph shoes, presence of steam locomotives under catenary, ice on contact wire, condition of roadbed, speed of trains, and amount of current collected at pantograph shoe all have a bearing on the rate of wear, the relative importance of each of these factors being a moot question. The four railroads contributing are the New York, New Haven and Hartford, the Chicago, Milwaukee, St. Paul and Pacific, the Pennsylvania, and the Illinois Central.

* * * * *

General Description

THE New Haven's initial electrification in 1907, of approximately 21 route mi. of four tracks from Woodlawn, N. Y. to Stamford, Conn. was extended in 1914 from Stamford to New Haven, Conn., with about 39 route mi. of four tracks.

High-speed suburban and through passenger, and heavy freight traffic in the electrified zone are supplied with 11,000-volt, 25-cycle, single-phase, a-c. power. Speeds up to 70 mi. per hour are attained, the maximum weight of a train being 3800 tons. In addition, there is still a number of steam trains operated, these being mostly heavy-tonnage freight trains. A considerable steam traffic was operated in the electrified zone initially, but since, has been supplanted almost entirely by electric except for that mentioned above.

The contact wire, which is 4–0 grooved of 40 per cent conductivity bronze, normally varies from 16 to 22 ft. above top of rail, the gradient at points of change averaging about 1 per cent. It is supported from an auxiliary 4–0 grooved copper wire at 10-ft. intervals by malleable iron or bronze clips. The contact wire tension varies between 100 lb. and 3800 lb. depending upon the temperature.

The pantograph shoes are of pressed steel, those for the larger locomotives of No. 11 U. S. gage, and for the multiple-unit equipment and smaller locomotives, of No. 14 B. W. G. Normal pantograph pressure on the wire is about 18 lb., increasing to about 25 lb. when the pantograph is traveling down, and decreasing to about 10 lb. when traveling up. Although lubrication was attempted for a time, the shoes are normally not lubricated. The normal maximum current collected is about 200 amperes per pantograph shoe.

Each unit of motive power used in road service carries two pantographs, although normally but one is used at a time. A considerable number of both passenger and freight trains is double headed, which involves two pantograph passes for each movement of such trains. Multiple-unit trains are normally made up with a ratio of one motor car to each two trailers. An average of 25,000 pantograph passes per year per track has been assumed.

The Milwaukee began electrical operation in 1915 of 441 route mi. of line, between Harlowton, Mont. and Avery, Idaho, and in 1919 of an additional 218 route mi. between Othello and Seattle-Tacoma in Washington.

The services operated electrically are through passenger and freight. D-c. power at 3000 volts is supplied the locomotives, about ten per cent of that supplied being returned through regenerative braking.

A ½-in. Siemens-Martin steel messenger supports two parallel 4–0 grooved copper contact wires by means of loop hangers alternately clipped to one and then the other. The height of contact wire above top of rail varies from 17 ft. to 24 ft. 2 in.

Each pantograph consists of two independent steel shoes with hard-rolled copper contact strips. The strips on each shoe are separated to provide a trough between them for the lubricant. The pantograph pressure on the contact wire is 30 lb., making the pressure per shoe equal to 15 lb.

The Pennsylvania's suburban service between Philadelphia and Paoli, Pa., about 20 route mi., was electrified in 1915, and between Philadelphia and Chestnut Hill, about 12 route mi., in 1918.

The inclined catenary type of construction in use between Philadelphia and Paoli consists of a ½-in. steel messenger[3] supporting a 1–0 round copper auxiliary messenger at 30-ft. intervals, the 3–0 grooved phono-electric contact wire being suspended from the auxiliary messenger by clips spaced at 15-ft. intervals, staggered with respect to the hangers. The construction in use between Philadelphia and Chestnut Hill is similar to the above except that the auxiliary messenger is a 2–0 grooved copper wire. Normal contact wire height is 22 ft. above top of rail, the gradients down to lower heights varying between 0.12 and 1.2 per cent.

These electrified tracks are used principally by multiple-unit suburban trains, a relatively small

1. This paper was written in collaboration with Sidney Withington, Electrical Engineer, N. Y., N. H. & H. R. R., R. Beeuwkes, Electrical Engineer, C. M. St. P. & P. R. R., J. V. B. Duer, Electrical Engineer, Pennsylvania R. R., and W. M. Vandersluis, Electrical Engineer, I. C. R. R.

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3. Approximately 40 mi. of the steel messenger has been replaced by ⅝-in. high-tension bronze cable, due to the corrosion of the former by locomotive gases.

29-86

electric locomotive mileage being made for experimental purposes, also. Steam locomotives are used in the electrified territory, too. A-c. power is supplied at 11,000 volts, the multiple-unit cars drawing normally a current of 20 amperes with a maximum of 80 amperes during acceleration. The schedule speed of these trains is high and the station stops are relatively close together so that maximum speeds well up toward 60 mi. per hr. must be attained.

The pantograph, the shoes of which are of mild steel, have been designed for as low an inertia as possible. Normal pressure against the contact wire is 18 lb.

The Illinois Central, in 1926, electrified its suburban service between its Chicago terminal and Matteson, Ill., Blue Island, Ill., and South Chicago, a total of approximately 39 route mi.

The catenary, except for yards, sidings, and crossovers, consists of a 0.81-in. composite copper-copperweld messenger cable supporting, at 15-ft. intervals, a ⅜-in. auxiliary messenger cable which in turn supports two 4–0 grooved copper contact wires. These are in the same horizontal plane, each wire being clipped to the auxiliary messenger at 15-ft. intervals, the clips being staggered with respect to the hangers and clipped to first one and then the other contact wire. In the denser traffic zone, north of the junction with the South Chicago branch at 67th Street, two 3–0, 80 per cent conductivity, grooved cadmium-bronze contact wires are used, each wire suspended at 20-ft. intervals from a ½-in. auxiliary messenger cable, the main messenger being the same. The yard catenary is composed of a 4–0 grooved copper contact wire suspended by rigid hangers from a ⅜-in. phono-electric bronze messenger. Normal contact wire height is 22 ft. with an allowable gradient of 1 per cent down to lower heights to clear overhead obstructions, minimum height being 16½ ft.

D-c. power at 1500 volts is supplied the multiple-unit suburban trains which consist of from two to eight cars. Speeds up to 65 mi. per hr. are attained in order to maintain a fast schedule with fairly frequent stops. The accelerating current averages 770 amperes per pantograph, the normal running current being 200 amperes. There is only a small amount of steam operated traffic, (mostly switching), directly under the wires, but the through passenger and freight tracks are adjacent to the suburban tracks so that locomotive gases are more or less always present.

The pantographs have, in general, galvanized sheet steel bodies faced on the contact surface with hard-rolled copper strips, but a few bodies are of aluminum. The life of contact strips is approximately 10,000 mi. Normal pantograph pressure is 22 lb. All pantographs are lubricated, a trough between the contact strips holding the grease.

The multiple-unit cars operate in pairs, one being a motor car carrying two pantographs, only one of which is normally in use, and the other a trailer.

Discussion

The design of the catenary system is important, aside from its functions of supplying a contact plane for the current collector and a distribution system for the power, in determining the rate of wear of the contact wire or wires. It will be noted that in each of the four systems described special precaution has been taken to obtain flexibility in the contact plane. The line must be flexible enough to insure intimate contact between wire and pantograph and yet not so flexible as to permit the formation of a large wave ahead of the pantograph as it moves forward. The requisite flexibility can best be obtained by staggering the points of support of the contact wire with respect to the messenger, while at the same time damping out the unwanted harmonic wave ahead of the pantograph by means of as high a tension in the contact wire as it will permit. It will be readily seen that a wave propagated ahead of the pantograph has no serious effects as long as its rate of propagation, the speed of the pantograph, and the mass of the contact plane remain of the same relative magnitude; but let any one of these factors change and the smooth passage of the pantograph under the contact wire is interrupted to the mutual detriment of both. Thus, it is always at those pull-offs where the free vertical movement of the wire is restricted and at such points as have a more-than-normal concentration of weight that accelerated contact wire wear is found. A large proportion of any contact-wire system wears slowly and uniformly, yet it is the small proportion of places where the wear is accelerated that require replacement soonest. At gradients in the contact system the inertia of the pantograph causes a greater or lesser pressure on the wire, greater if moving down and lesser if moving up. At a certain pressure, which obviously should be the normal pressure, the wear will be a minimum, any greater or lesser pressure resulting in increased wear; if greater, due to increased friction, and if lesser, due to increased burning caused by insufficient contact.

Pantograph design resolves itself into a judicious composition of the following essentials: (1) sufficient current collecting surface; (2) light weight moving parts; (3) sufficient uniform upward pressure; and (4) freedom from friction in the bearings. The effect of the first will be taken up further on. The necessity of having all moving parts as light in weight as possible is apparent. All changes in grade of the contact wire require that the pantograph follow them, and, in order to minimize the wear, especially at the *change* in grade, the inertia of the pantograph must be small. A sufficient uniform upward pressure must be present to permit of current collection without arcing. Friction in the pantograph bearings may be classed as additional inertia since its effect is the same.

It is probable that if the contact surface of the pantograph were of some metal dissimilar to that in the contact wire a lower rate of wear of the latter would result, on the theory that the coefficient of friction

between dissimilar metals is lower than that between similar metals.

The question of lubricating the pantographs produces a diversity of opinion. The New Haven and Pennsylvania do not lubricate, while the Milwaukee and Illinois Central do. The effect of lubrication *per se* will be taken up at length further on. Just now it may be well to point out that the decision to lubricate or not is probably influenced by the presence under the wire of steam locomotives. The experience of the Milwaukee shows that locomotive smoke and the sand blown from the stack of oil-burning locomotives in sanding the fire-tubes adheres to the grease film on the wire forming an actively abrasive surface. No doubt some similar action would be present in localities where dust or sand storms are prevalent. On lines such as the New Haven's and Pennsylvania's, where a considerable steam traffic passes directly beneath the contact wire, the presence of grease on the wire might increase the wear.

Intrinsically, lubrication plays an important role in reducing wear. Extended measurements and observations on the Milwaukee show that, on long tangents where the contact wire is in the middle of the pantograph the greater part of the time, the lubricant becomes depleted and wear is increased. On the other hand, on curves where the pantograph is wiped across the contact wire due to the chord type of catenary employed, the wear is less, the lubricant from the ends of the pantographs being distributed over the whole surface. A comparison of contact wire measurements shows that there is approximately 30 per cent more wear on tangent track than on curves.

Further evidence of the effect of lubrication is found in tunnels. Here, where the wires are sheltered from sun, rain, dust, and frost, the wear is approximately one-half that on exposed tangent track. Other factors may contribute to this end, but that the shelter of the tunnel is the dominant one is shown by the fact that where water drips from the tunnel roof the wear is greater. The limited experience of the Illinois Central corroborates these observations.

The presence of steam locomotives under the wires, aside from the abrasive deposits left by them, produces a certain corrosion, small though it may be, of the contact surface. Any pitting from this cause removes a small amount of the wire and leaves a roughened surface which is abrasive to a degree. The total effect is probably insignificant.

Ice on the wires separates them from the pantograph, causing small arcs to be drawn. The pits formed by these arcs are substantially the same as those mentioned in the preceding paragraph except that they are larger and hence more injurious. Accelerated wear is in progress at the pitted spots until they are worn smooth again. The kind of service operated is largely influential in determining the total effect of ice deposits on contact wire wear. With frequent suburban service providing less time for the deposits to form the effect is probably of small moment, especially as on the Illinois Central, when there is danger of the formation of sleet all pantographs are raised. With a relatively small number of pantograph passes a day as on the Milwaukee, particularly when the territory served is subject to more or less frequent sleet storms, the wear occasioned by such deposits is important. It may be said, then, that the burning, and therefore the wear, due to ice on the wire is less per pantograph pass where the frequency of the passes tends to keep the surface of the wire clear.

The condition of the roadbed affects the riding qualities of the cars, which in turn affect the pantographs carried by them. A sudden drop due to a low rail joint or frog point may cause the pantograph to leave the wire momentarily, drawing an arc, and producing

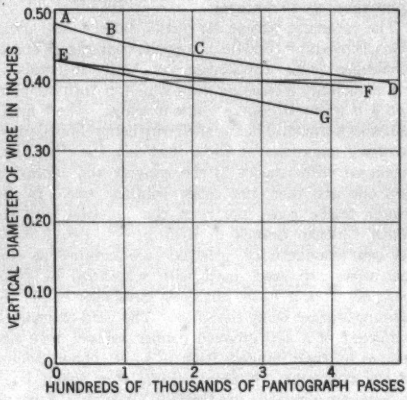

FIG. 1

A–B = 4–0 on C. M. St. P. & P. R. R.
A–C = 4–0 on I. C. R. R.
A–D = 4–0 on N. Y. N. H. & H. R. R.
E–F = 3–0 on I. C. R. R.
E–G = 3–0 on Pennsylvania R. R.

excess wear at the point where the pantograph resumes contact with the wire. The net effect may be said to be the sum of the effects of a change in gradient of the contact wire and the arcing caused by sleet. The wear due to this cause will probably be noticed most in the winter time when the roadbed is frozen and is difficult to resurface, particularly as at this season the pantographs are required to handle an additional electrical load due to the heater current being drawn.

The experience of the New Haven has been that generally the higher the speed of the train, the less the wear of the contact wire. This is borne out by the experience of the Illinois Central in a territory where four tracks carry high-speed express and special trains and two tracks relatively low-speed local trains.

The effect on contact wire wear of the amount of current collected is not so well known. The general impression is that as far as the effect of current alone

is concerned, the greater the amount of current, the greater the wear. Actually, it is difficult to determine the facts from experience because of the obscuring effects of other factors.

In the curves shown in Fig. 1 are given the relations between the vertical diameter of the contact wire and the number of pantograph passes. The curve for the New Haven takes into account all measurements made; that for the Milwaukee those measurements which represent maximum wear on the main line; that for the Pennsylvania only those measurements at places most susceptible to wear; and those for the Illinois Central all measurements.

In conclusion, then, for the purpose of minimizing contact wire wear it is necessary that the contact wire system be flexible, have little or no concentrated weight, and have gradual transitions from one height to another; that the pantographs be light, free moving, and actuated by sufficient pressure to insure good contact with the wire; that the question of lubricating pantographs be decided after considering such other factors as the presence or absence near the wire of abrasives which might adhere to the lubricating film; and that the condition of the roadbed be taken into consideration. Ice on the wires is abrasive and difficult to combat successfully. The speeds of trains and amounts of current collected are variables at best and probably do not affect the rate of wear appreciably.

Discussion

D. C. Jackson: A statistical paper like this one is very desirable because it does have definite influence upon estimates of operating expense from the maintenance aspects.

Sidney Withington: In connection with Mr. Landhy's paper, lubrication of the trolley wire or pantograph shoe is I think one of the fundamental problems in railway electrification which must be solved in the not distant future. It is a question as to whether the trolley wire be lubricated or the pantograph shoe. It is not satisfactory to swab a lot of lubricant on the pantograph shoe when the locomotive or car is at a terminal, and trust that that will stay on the pantograph and properly distribute itself along the trolley wire. This method is likely to lubricate everything including the clothes of the passengers, the road ballast and the car roofs in the vicinity of the terminal, and very little trolley wire after the first few thousand feet. Some better method must be developed.

One very interesting difference in practise in this country as compared with practise abroad, is in the pressure of the pantograph against the wire. We operate in this country with pantograph pressures of 15, 18, and over 20 lb., whereas abroad a pressure of 7 or 8 lb. is common, with the current collections of often about the same magnitude as ours. That is largely due to the design of European pantographs in which is employed an auxiliary bow which follows the slight unevennesses in the contact wire and prevents arcing where there is a sudden change of elevation or a hard spot. Some day I think we will come to some such scheme as that, a moving part on the top of the pantograph which will enable us very considerably to reduce the pressure on the trolley wire.

The material of the pantograph must also be studied much more carefully than it has been thus far. It is possible that chromium plating may have some value in this connection.

In connection with the effect upon the trolley wire of the amount of current which is collected, we are operating a branch line between Providence and Fall River, at 600 volts nominal pressure. The operation there is with the sliding shoe and the current densities are relatively quite high. We have noticed that where the trains leave stations (where the acceleration is taking place and the current is relatively high), the wear on the trolley wire is considerably greater than the wear on the same trolley wire as the trains approach the station where the current is zero, or very slight, but where the speed is about the same as where the trains are leaving the station. This seems to indicate that current density has something to do with the wear.

C. S. Anderson: There is one point which I wish to develop. This is in connection with the effect of the *composition* of the contact wire on its own rate of wear. Some years ago, we departed in many instances from the use of copper contact wires and used bronze contact wires to gain strength and also resistance to wear. However, one must distinguish between the bronzes and choose which type is best. The *tin*-copper alloys, often called phosphor bronze, or silicon bronze, depending on the deoxidizing agent employed, were the first type of bronzes to be used in contact wires. However, in the last year or two, the *cadmium*-copper alloys have been selected for several important lines, and it is of value to note that the results obtained with the cadmium bronzes are better than those obtained with the older tin bronzes.

In grades having equal strength, the conductivity of the cadmium bronzes is much higher and they resist *mechanical* wear as well as the tin bronzes, and they resist *electrical* wear much better. So on operating railroads where these types of wear are always found combined, it is not surprising to see the evidence piling up to show that the cadmium bronzes make more durable contract wires. This situation is evident in the graph on the third page of Mr. Landhy's paper. Curve *E-F* is for cadmium bronze. Note that its slope is decidedly less than the slopes of the other curves which are for copper or for tin bronzes.

In other words, the rate of wear under operating conditions is less for the cadmium bronze than for the tin bronze. This conclusion is confirmed by plotting on the same graph the results of field tests extending over thirty-two months, just completed by the Chicago Rapid Transit Company. Their tests were on cadmium bronzes. Plotting the results of their test we obtain a curve that lies parallel to Curve *E-F* in Mr. Landhy's graph, showing that the slower rate of wear of cadmium bronze is again confirmed.

Finally, it is of value to note that the cadmium bronzes will not become hot-short or brittle if overheated as by a short circuit. This embrittlement is a characteristic of tin not shared by cadmium.

E. L. Moreland: I should like to ask if any comparative studies have been made of the relative wear on the contact wire of two contact shoes as compared with one contact shoe on a pantograph. There is a certain school that apparently is very definitely of the opinion that for a given current collection it is better to have two contact shoes, the theory being that a more continuous contact is maintained with the wire than would be the case with the single shoe.

On the other hand, there are some skeptics who say that a single shoe with light wire pressure would give less wear on the wire, and would give satisfactory current collection on multiple-unit cars where the amount of current to be collected per pantograph is relatively small, particularly at high speeds.

The only comparative tests I know of where equal mileage was run under the same conditions for 5000 miles each way, show that with the double shoe almost as much copper or bronze was lost from the contact strips on each of the two shoes as was lost from the single shoe. In other words, the pantograph with two contact shoes lost almost twice as much metal from the wearing strips as the pantograph with a single contact shoe running under the same conditions. That of course doesn't prove that the wear of the contact wire was in the same ratio, but it at least raises the ques-

tion as to whether the wear on a contact strip isn't in some measure an indication of the wear taking place at the same time on the wire.

My own feeling is that two shoes are probably the soundest for the flexibility they give, but these tests do amplify this question that I already had in my mind before I knew the results of the tests.

J. B. Cox: When the question of a contact device for the pantographs to be used on the electric locomotives for the original electrification of the Chicago, Milwaukee, St. Paul and Pacific Railroad was being considered, early in 1915, it was realized that the duty required would undoubtedly be more severe than had been experienced up to that time.

The Butte, Anaconda and Pacific Railroad had been in electrical operation for more than a year with 5-in. diameter steel tube rollers used for collecting the current from a single 4/0 trolley. It had been found here that while these rollers collected the desired current successfully, there was some sparking at hard spots in the line when only one pantograph was used, but that by using two pantographs in multiple this sparking was practically eliminated. Extensive tests were carried out at the Schenectady factory by rigging up a crank shaft by means of which various forms of contact devices might be caused to slide to and fro under varying conditions and pressures, with and without lubrication. During these discussions and experiments, many suggestions were offered, among which was the use of multiple contact wires with alternating supports.

I suggested the use of a double contact shoe for the pantograph, so interconnected mechanically as to cause the one to be held more firmly against the wire when the other was being forced from the wire by a hard spot or other cause.

Mr. W. B. Potter suggested, "Why not have both the double wire and the double contact, then there can be no reasonable doubt of being able to collect any probable amount of current necessary at any practical speeds that may be desired."

This suggestion was followed out, and special contact shoes designed with due provisions for the lubrication of the contact, which the experiments mentioned had definitely indicated, not only greatly increased the life of the contacts and decreased the wear on the wire, but also added materially to the current-collecting capacity of the pantograph, and reduced the sparking.

The results extending over a period of more than 14 years, have been quite satisfactory, and for similar types of electrification the practise has become almost standard.

Practically the same trolley construction and current collecting device was used on the Mexican Railway electrification.

In almost every instance of electrification, where steam locomotives have continued to be operated under the contact wires for any considerable length of time, there has been objectionable sparking for a few days, but this has always gradually disappeared as the deposit on the wire wore away.

The double contact shoe has been a great advantage in these initial operating periods, and even more useful when the wires become covered with sleet.

Some of the trolley wires on the Mexican Railway had been up more than a year before the electrification was put in operation, and the deposit on the wire from the steam locomotives caused more severe sparking than I had seen before, and it continued for a longer time.

Double-unit locomotives were operated with two pantographs per unit up, thus making a total of four pantographs against the contact wires, which limited the sparking and at the same time hurried the clearing of the wire.

These experiences have led me to feel that there should be no doubt as to the advisability of using the double contact in any instance where heavy currents are to be collected at high speeds.

N. W. Storer: I agree entirely with what Mr. Cox has said about the advantage of the double shoe. The fact of the matter is we had some double shoes running on the Niagara Junction Railway on a 600-volt line while he was still figuring on the roller pantograph on the Butte, Anaconda and Pacific and other lines. They are necessary where heavy currents are to be collected and desirable for smaller currents in order to minimize the possibility of opening the circuit due to the shoe striking hard spots on the line.

W. B. Potter: I fully agree with what Mr. Landhy has said and with the remarks thus far during the discussion. So far as I have observed, it seems quite clear that the wear of the contact wire and collector is much more due to arcing than to the mechanical wear due to rubbing. Continuity of contact is essential if arcing is to be prevented and this can best be accomplished by such flexibility in both the contact wire and collector that they will react upon each other like opposing springs; the contact wire should, further, have no hard spots that might cause the collector to rebound.

Mr. Moreland has referred to the relative merit of the double and single pantograph collector. Both experimental tests and practise seem to prove that the double pantograph better insures the desirable continuity of contact. It will be of interest to mention an earlier installation, that of the Baltimore Tunnel in 1895 where the results were far from satisfactory. The overhead contactor was composed of two steel Z-bars with a plate over the top, the bars being far enough apart to leave a slot in the bottom. This steel structure, which corresponded to the contact wire, was catenary-supported. The collector was a block of bronze in the shape of a shuttle and weighed about 50 lb.; it had a tongue which projected through the slot between the Z-bars and to this tongue was connected the pantograph on the locomotive. The tunnel was rather wet and there was considerable accumulation from drippings within the steel conductor. During the initial operating the shuttle looked like a sort of traveling electric furnace,—ten to fifteen pounds of metal would be melted off during a single trip. The performance was improved by using a collector in the form of a steel scraper. The current was about 1000 amperes, and to collect this successfully without arcing then seemed well nigh impossible. The successful collection of several thousand amperes at 60 to 70 mi. per hr. has now proved to be a much simpler problem than was anticipated.

I. T. Landhy: With two shoes I think I can say from my own experience that you are more sure of contact on the contact wire. I have noticed sometimes that one shoe of the pair will be in contact and the other either barely touching or not at all, and if it were only barely touching with one contact shoe you would get burning along the surface of the contact strips as well as on the contact wire.

An Electrified Railway Substation
Of the Pennsylvania Railroad

BY J. V. B. DUER[1]
Fellow, A. I. E. E.

Synopsis—*This paper contains a description of one of the outdoor substations of the Philadelphia-Wilmington electrification of the Pennsylvania Railroad, recently placed in service. It describes the initial installation of apparatus provided in the substation for the present suburban electrification, as well as the steps taken to accommodate the additional apparatus necessary for the through electrification when it takes place.*

The location and control of the substation are described as well as the types of protective apparatus utilized and the method of operation. Attention is called to certain operating necessities which control the design and location of a substation of this character. The means provided for handling the heavy apparatus into and out of a substation of this character and the facilities for handling, cleaning, and restoring oil to the different pieces of apparatus are described in some detail.

* * * * *

As a preliminary to this description, it is necessary to state that the Philadelphia-Wilmington electrification at its present stage of construction is designed for the multiple-unit suburban service only, but that everything has been so constructed that expansion to care for the handling of through electric locomotive hauled service, both passenger and freight, may be accomplished by adding to the existing facilities, without necessitating the replacement of existing electrical facilities. For complete electrification of main line territory, four single-phase 132-kv. transmission circuits on two tower lines are proposed; with four step-down transformer units at each typical trolley supply substation. The initial installation, for multiple-unit service only, has required but two single-phase 132-kv. transmission circuits, each on its own supporting structures; and at each typical trolley supply substation, but two step-down transformer units.

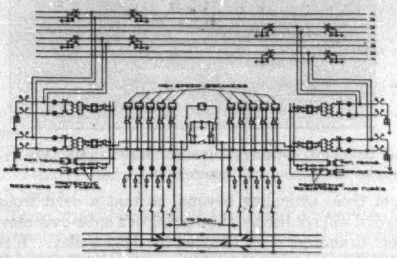

Fig. 1—Trolley Substation—Wiring Diagram

It is then evident that the existing typical trolley supply substation is a structure which at present accommodates two single-phase transmission circuits, allowing for the sectionalization of each of these circuits by air-break motor-operated disconnecting switches. The structure further accommodates a tap from each of these transmission circuits, each of these two taps passing through another air-break motor-operated disconnecting switch to the high-voltage side of a 132/12-kv. step-down transformer, the tap being provided with choke coils and lightning arresters. The low-voltage side of each transformer unit is carried through a suitable two-pole oil switch and the requisite isolating switches to one section of a 12-kv. trolley bus. Ultimately, there will be four transmission circuits, four taps, (one from each circuit), and four transformers; but there will be only two trolley bus sections, as at present, each with two transformers connected to it instead of one.

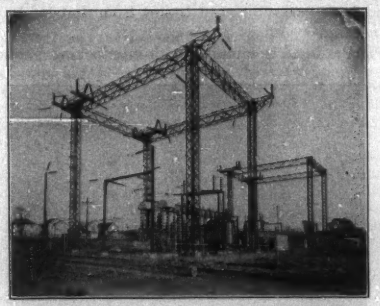

Fig. 2—Typical Trolley Supply Substation, General View

Each bus section consists of a trolley and a rail bus. The two trolley bus sections are connected through a suitable single-pole breaker and the requisite isolating switches. The two rail bus sections are arranged so that they may be connected together through disconnecting switches, and further, so that either section or both may be connected to actual ground if desired.

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Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-80

From the trolley bus section, taps are taken, each of which passes through a single-pole high-speed trolley circuit breaker and the requisite isolating switches to a trolley. The trolleys passing by the substation are sectionalized so that those on one side are electrically separated from those on the other side of the substation. The trolley bus taps from one section of the trolley bus feed the trolleys on one side of the substation, and those from the other section of the trolley bus feed the trolleys

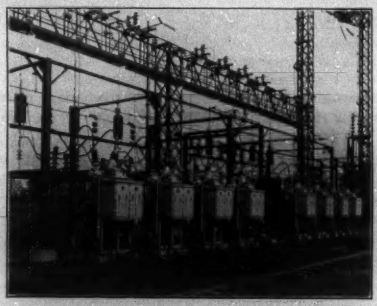

Fig. 3—Typical Trolley Supply Substation—Trolley Bus and Oil Circuit Breakers

on the other side of the substation. Each trolley bus tap is supplied with a choke coil and a lightning arrester and feeds into a different trolley from that fed by any other tap.

The railroad is equipped with interlocking cross-over

Fig. 4—Typical Substation Control Board in Control House

stations for the purpose of crossing traffic from one track to another at frequent intervals. These intervals are such that it is usually possible to locate the substations at certain of the interlocking cross-over stations. At one of these cross-over stations, the cross-overs between tracks are located between signal bridges, usually some 1200 to 2000 ft. apart. There is a signal and switch tower, more generally called an interlocking or block station, with an operator to control the cross-over movements. This tower is connected with the dispatcher's telephone circuit and with the emergency telegraph circuit. There are usually certain signal maintenance men working in the vicinity of the block station. If the telephone circuit of the load dispatcher or power director be run into the block station, and if the block station be equipped with a control board and the necessary signal lamps, the operator can well perform such substation switching operations as the power director may request, keeping the proper record of such operations. Further, the operator can call by whistle or horn such signal maintenance man as may be within hearing and request him to perform such emergency operations within the substation as may become necessary prior to the arrival of a regular substation maintenance man. It is also evident that the trolleys covering the cross-over movements should be independently fed

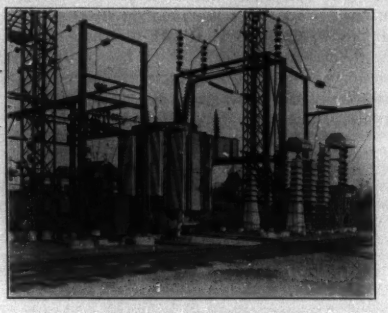

Fig. 5—Typical Trolley Supply Substation—4500-Kv-a. Transformer

from those extending beyond, so that a dead trolley beyond will not hazard the making of a cross-over movement to another track to avoid the dead trolley. If the substation be located near the cross-over station, the cross-over trolleys may be readily fed independently, directly from the substation trolley bus sections. Cross-over stations not at substations require a special bus and switch arrangement to secure this independent cross-over trolley feed feature. We may state, therefore, that in so far as is possible, the typical trolley supply substation is located near a cross-over station and the substation switching operations are controlled from the block station by the block station operator under direction from the power director.

Within the fenced in enclosure of the substation is located a small control house. In this house there are located a control and relay board, a control battery and charging facilities, such signal supply motor-generator sets and switching as may be required, the substation oil treating plant with its purifiers, filters, and valves, and a telephone connected with the power director's circuit.

When the regular substation maintenance men are at work within the substation enclosure, by throwing a switch, they may take over the control of the substation switches from the block station operator's board to the control board in the control house. This operation is made upon direction from the power director and the block-station operator is informed and also receives a signal on his board when the control passes from his board to the control house board, or the reverse.

The entire relay layout for the station is on the control house board. All control operations and signals are at a control voltage of 125 volts, direct current supplied from the control battery. The control circuits are segregated for protection and for the easy location of grounds. Ground detecting apparatus is supplied.

In each substation are two service transformers, one on each section of the trolley bus, which give 230/115-volt, 25-cycle circuits from which the control battery is charged, the oil pumps operated, and the normal lighting obtained. A separate transformer supplies the signal motor-generator set or sets when such are installed at the substation.

Certain emergency lighting is supplied from the control battery.

Into each substation enclosure is run a railroad siding, and the apparatus is so located as to be readily loaded or unloaded to or from cars on this siding by a locomotive derrick or otherwise, as may be necessary.

The relay protection is such as to give the following protective functions:

Trolley high-speed breakers; high-speed impulse for short circuit; normal controlled speed relays for overload.

Each trolley bus section is differentially protected as to all breakers connected thereto for protection against bus faults.

Transformer breakers; reverse power protection for transmission line-to-line faults; differential protection for transformer faults. Voltage relays for transmission line to ground faults supplied from a 66-kv. potential transformer connected from the midpoint of the high-tension winding of the main transformer to ground, the step-up transformers at the station supplying energy to the high-tension transmission having their midpoint grounded through resistance.

Buried oil tanks are provided in each substation for both transformer and switch oil. The piping system is such that breaker oil may be drawn from a dirty oil tank and purified and run into a clean oil tank and run into a breaker. The transformer oil may be run from the transformer through the purifier and back to the transformer, or from the transformer into the tank and thence through the purifier and back to the transformer.

The oil purifier is a combined centrifuge and filter with provision for by-passing either or both, the latter for use when oil is drawn from the clean oil tank and passed to a breaker.

The step-down transformers necessary in our eastern territory for the eventual complete electrification are of 4500-kv-a. normal continuous rating, immediately followed by 150 per cent load for two hours, immediately followed by 300 per cent load for five minutes. Thus, these transformers are much larger than normal commercially rated units of the same nominal continuous rating. They are self-cooled and self-protecting and every attempt has been made to insure serviceability. The number of these units installed initially and ultimately in a typical Philadelphia-Wilmington substation has been previously mentioned as two and four respectively.

The above describes the requirements of the substation layout. The actual physical structure which fulfills these requirements need not always be the same. It depends, naturally, upon the land available and upon other considerations which influence its design.

Typical views of a step-down transformer station are shown.

Discussion

Sidney Withington: It may be of interest to note that on the New Haven, and on some of the other electrified railroads using substations of the same general nature as those discussed, each track on either side of the substation is fed from the same section of the bus, so that if a bus section goes out of service for any reason only one set of tracks is affected, that is, for instance, one continuous east-bound track, the west-bound track remaining unaffected past the substation. In the arrangement discussed by Mr. Duer if one bus section fails all tracks on one side of the substation may be affected unless (as of course is usually the case) there is feed from an adjacent substation.

Another point of interest in connection with the substation design is the high towers employed for the transmission line at the substations. It will be noticed in the illustrations that there is a very high fabricated steel framework for the 132-kv. feeders. There is undoubtedly a reason for that, and I think it would be of interest to bring it out.

An additional item of interest in connection with this substation design is the amount of ground employed. I think it is essential in all high-tension substations, to spread out the equipment so that not only is there plenty of room to allow safe work in an individual unit of equipment, but a failure will affect only a small part of the facility. It is obviously also a great advantage to have everything out of doors for the same reason.

H. C. Cross: Since Mr. Withington has asked two questions, I should like to answer them.

It is true, of course, in the trolley sectionalization scheme which we use that, if one-half of the station bus is lost, we cut the railroad in two, as it were, at that point; whereas Mr. Withington has at least two tracks on which traffic will automatically continue. However, if we do lose half the bus, the differential protection is such that there is practically only a momentary loss, that is, we are only cutting off power between two substations for a momentary time. I confess I do not know from the point of view of flexibility of operation, the exact reasons for the adoption of the scheme we use. It would appear to me to be more or less a question of judgment, in that since differential bus protection is provided there is no question of any serious hazard to the flow of traffic on the railroad.

The other question was about the height of the towers in the substations. If I am not mistaken that was very materially influenced by the striking distance of an arc from an air-break switch located below the 132-kv. transmission wires.

D-C. Railway Substations for the Chicago Terminal Electrification, Illinois Central Railroad

BY A. M. GARRETT[1]
Associate, A. I. E. E.

Synopsis.—Direct-current substations designed to meet the demands of heavy railway terminal service are described in this paper. This direct-current energy is furnished at 1500 volts from seven substations located in the Chicago district and owned and operated by the power companies. In addition to 1500-volt supply for traction purposes, alternating current at 4000/2300 volts is provided for the railway company's light, power, and signal systems.

Approximately 80 per cent of conversion capacity is in synchronous converters and the remainder in mercury rectifiers. Reasons for selecting the latter units are given as well as some of the characteristic features of them.

To meet the rigorous requirements as to voltage and current demands under the agreement with the railway company, the conversion units have a rating of 300 per cent load for 1 minute.

The synchronous converters are of the field control type, and in order to hold the d-c. voltage to definite values for any load within the rating of the unit and to maintain the reactive current of the unit within the safe limits, there is provided, common to all the converters in a substation, a counter e. m. f. regulating set, a 4-circuit rheostat, and a voltage regulator. High-speed air breakers are used on the d-c. side of the units and all 1500-volt feeders.

Another feature of interest is the use of truck mounted enclosed switch units on both the high and low tension sides of the substation. This type was selected in order to eliminate certain operating hazards which exist with fixed type switching arrangement, and to provide the customary factors of accessibility and maintenance.

In operating experience, the substation equipment has met all expectations, the performance of the synchronous converters and high-speed breakers being exceptional, while confidence in the mercury rectifier has not lessened.

Introduction

SEVERAL excellent papers upon the different features of the Electrical Terminal of the Illinois Central System have been presented to the Institute. The purpose of this paper is to describe that group of electrical equipment which comprises the substation, presenting the characteristics of this installation as a type representative of a d-c. application in modern railway practise.

These d-c. substations, owned and operated by the power companies, the Commonwealth Edison Company and the Public Service Company of Northern Illinois, receive the 60-cycle energy for conversion to direct current from the 12-kv. transmission system of the Commonwealth Edison Company and the 33-kv. transmission system of the Public Service Company.

To meet the exacting demands of heavy railway terminal service, the selection of the various electrical units and the arrangement of them in the substation layout followed along tried and proven lines. Yet consideration was also given to some equipment which, lacking at the time experience in the American field, gave promise of satisfactory adaptation to American practise through experience gained with the equipment in foreign use.

The studies, trial investigations, and test set-ups made jointly by the engineering staffs of the railway company, the manufacturers, and the power companies, gave a satisfactory solution of the various problems imposed by the standards of service, and these, coupled with the thought and experience of the power companies in furnishing conversion service for so many years to the traction systems of Chicago, were indispensable considerations in the design of the substation, including not only the major converting and switching units, but the elements of connection as well.

Substation Buildings

There are seven substation buildings in all, two of which are located outside the city in Public Service Company territory with the remaining five within the city limits. In keeping with the policy of both power companies these buildings are pleasing in architecture, substantial in construction, and of a style consistent with the best in the neighborhood. The buildings are one story and basement structures, arranged with more than the average space allotment per unit of equipment. Each bay, except the one for railway light and power equipment, is 24 ft. in width, which provides ample space around the main conversion units. This space is further increased by use of a converter of the consolidated type, in which the transformer and the converter are a single unit.

The use of enclosed switch units of the withdrawal type for both the incoming transmission lines, and the outgoing distribution feeders, requires additional space in front of each unit for pulling out the truck. Under normal conditions, this arrangement provides still more floor space or aisles on each side of the converting row.

Increase in the size and number of auxiliaries required with modern installations has demonstrated that a previous basement height of 12 ft. is inadequate. An increase to 14 ft. was therefore made. Increased headroom is also provided for the operating floor which allowed more liberal window space and pent house facility, thereby improving the light and ventilation.

Conversion Units

To furnish converted energy under the current and voltage stipulation necessary to maintain the high rate of acceleration and high schedule speed demanded by the suburban service on this electrified terminal,

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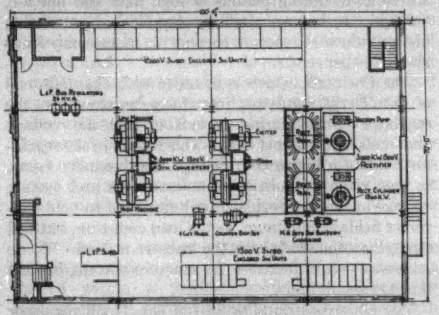
29-102

and to approximate these same duties under extra or congested traffic conditions, were the principal requirements that had to be imposed on the conversion units.

Over 80 per cent of the total conversion capacity is developed from 11 units of the rotating type, the synchronous converters, while the remainder is developed from 4 units of the stationary type, namely, the mercury converter or rectifier.

The decision to use rectifiers can be assigned to several



First Floor Plan—Brookdale Substation of Commonwealth Edison Co.

reasons, principal among them being that this new type had the natural advantage which a unit with no moving parts has over one with rotating parts and wearable and friction surfaces, that it gave every indication through future development and application of meeting the demands placed upon it by American practise. Other reasons included the generally known advantages of the rectifier, high efficiency with fluctuating loads, absence of noise and vibration, low maintenance expense, and elimination of extensive ventilation facilities sometimes required with the synchronous converter. Also in this instance, a considerable amount of reserve capacity permitted the installation of a representative number of rectifiers.

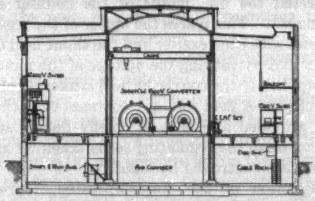
Synchronous Converters

The synchronous converters, which are of General Electric manufacture, are furnished in 3000-kw. sizes with a d-c. voltage of 1500. Each outfit consists of two 1500-kw., 750-volt units operating in series. Each converter set is provided with a 3150-kv-a. air blast transformer arranged as stated before in a compact group by mounting the transformer on the bedplates of the a-c. side of the two machines. This served not only as a space reduction factor, eliminating the usual aisle of transformers lengthwise of the substation, but provides a short and convenient arrangement of a-c. leads between the transformer and the machine collector rings.

To meet the stiff voltage regulation required under the agreement with the railway company, which permits a voltage variation within the narrow limits of 1400 to 1550 d-c. volts, together with the high drafts of current drawn by the system during the maximum rush hour, the converters are designed with a rating of 50 per cent overload for two hours, and 300 per cent load for one minute. They are shunt wound and of the field controlled type whereby the control of the d-c. voltage is obtained by changing the excitation of the unit and providing regulation by means of the reactive current drawn from the line or converter. Because of the reactive current involved, several limits as to rating were established so that overloading by the reactive current or the load current, or by both, would not exist. As stated previously the useful load output varies from 50 per cent overload for two hours to 300 per cent load for one minute, the corresponding rating for the reactive current output varies from 60 per cent reactive current at no-load to 30 per cent for the 50 per cent overload, and 50 per cent reactive for the 300 per cent load of one minute duration.

To keep the reactive kv-a. within the prescribed limits and hold the d-c. voltage to definite values for any load within the rating of the unit, there are provided common to all the converters in each substation, a counter e. m. f. regulating set, a 4-circuit rheostat, and a voltage regulator.

In order to maintain normal voltage conditions on the low-tension secondary connections to the rotary at times when there may be low primary or system voltage,



Sectional Elevation—Brookdale Substation of Commonwealth Edison Co.

or to increase the amount of the secondary voltage during periods of high load, the ratio of transformation between the high and the low side of the transformer may be changed in the ratio of 5 per cent by means of a tapped primary winding that is connected to two oil breakers, one of which provides a normal or 100 per cent voltage on the secondary side and the other connection 105 per cent voltage. That the load may not be interrupted when a changeover is desired, both breakers may be closed for a short interval to permit shifting from one tap to the other. These switch positions are known as the high and the low delta. In addition to this, the transformer is provided with a no-load tap changer on the primary side. Excitation

to the converter fields is furnished through a direct connected exciter operating on the shaft of the low-voltage converter. The exciter provides energy to the shunt fields of both machines at a fixed pressure, and each exciter and the shunt fields of the converters are connected in series to an excitation bus common to all the converter sets in the substation. Also to this bus

12,000-Volt Switchboard (Truck Type—16th Street)

is connected the d-c. motor of the counter e. m. f. regulating set. This places the motor of the regulating set in series with each exciter and the shunt field of each converter and by varying the voltage of the motor, the voltage applied to the converter field is likewise changed or controlled. A field rheostat common to both shunt fields is provided and is used chiefly during starting periods, or when the regulating set is taken off and the converters are controlled by hand.

The function of the motor-operated, four-circuit rheostat and the Tirrill regulator on the d-c. side, is to control the operation of the converter and its regulating equipment under both normal and abnormal voltage conditions on the a-c. supply system in conjunction with the load and voltage conditions on the d-c. side of the unit; and to hold the performance of the unit within the prescribed reactive kv-a. limits over the full range of capacity.

Briefly, these four rheostatic circuits which are known as *A*, *B*, *C*, and *D* circuits, function as follows:

A circuit rheostat, which receives, through a potential transformer connected to 12,000-volt bus an indication of the voltage of the transmission system, controls or determines the entire setting of the rheostat through a rheostat arm that extends over the contacts of the other three circuits. If the voltage of the a-c. system is raised or lowered the arm of the *A* circuit changes, and establishes a new setting for the rheostat. The range of the *A* circuit varies from 15 per cent above normal to 35 per cent below. However, it is not felt these limits in the transmission system voltage will ever be realized.

The *B* circuit limits the minimum excitation of the converter fields so that the permissible reactive kv-a. will not be exceeded when high system voltage and low loads exist.

The *C* circuit operates to limit the amount of reactive kv-a. during periods of high load and low a-c. voltage. It has an auxiliary function of permitting higher values of reactive current on momentary loads above 150 per cent.

The *D* circuit, which is in series with the main coil of the Tirrill regulator, introduces resistance in the regulator circuit during periods of high a-c. voltage and loads below 40 per cent. This results in the regulator holding a higher d-c. voltage than actually exists, so that when a condition of no-load with high system voltage occurs the regulator will not tend to hold converter fields at minimum, as the load comes on, but will strengthen the fields with the increase in load. To do otherwise would increase the reactive current beyond the permissible limit.

The undercompounding coil of the Tirrill regulator provides the proper amount of regulation for the converter. With normal a-c. voltage this regulation is 1452 volts at 300 per cent load and 1525 volts at no-load.

12,000-Volt Enclosed Switch Unit

In those substations where the rectifiers have no regulating equipment, and the synchronous converters are operated in parallel with them, the d-c. voltage of the rectifier is in direct proportion to a-c. voltage and for successful paralleling the converter must have the same characteristics. This is accomplished by inserting a resistance in the undercompounding coil of the Tirrill regulator. By varying the amount of resistance, regu-

lation consistent with the voltage drop of the rectifier is obtained.

The converter which has a speed of 600 rev. per min. is started by the Y-delta method. The unit operates off the 12-kv. bus through 4 oil breakers; the running switch, an automatic breaker of high interrupting duty, the Y and the high and the low delta breakers, the last three being non-automatic and of the ordinary type of frame or panel mounted oil switches.

Each converter is connected to the 1500-volt d-c. bus through 3000-ampere high-speed circuit breakers of the bucking bar type located in both the positive and negative leads. The positive breaker, which is located in a metal enclosure on the operating floor, is arranged to trip in case of a reverse current arising either from faults within the unit, or on the transmission system. The negative breaker, which is the more active of the two breakers, takes care of overloads or short circuits in the forward direction, that is, those overloads or disturbances occurring on the 1500-volt bus or feeder system.

The negative breaker is provided with an automatic reclosing feature in case the overload or short circuit is cleared within a predetermined time. When the breaker opens a current limiting resistor is inserted in the circuit. The voltage drop across this resistor is inversely proportional to the resistance of the external fault. A ground or fault of zero resistance impresses a voltage of 1500 across the resistor. A time delay relay, which comes into action when the breaker opens, causes the negative to remain open and trips the positive breaker if fault condition persists for a definite length of time. This breaker, which is selective with the high speed breakers on the 1500-volt feeders, is located in open frame mounting in the basement in close proximity to the negative leads and bus. Other protective devices consist of a speed limit switch for overspeed protection and a ground or flashover relay, both of which trip all breakers connected to the unit. In addition to the protection against commutator flashing by means of the high-speed breaker, guards and arc chutes or hurdles are placed on the commutator between brush arms to break up and dissipate the flash, thus preventing its spread from bar to bar across the face of commutator.

Mercury Rectifiers

Four of the substations contain mercury rectifiers of the iron tank type. Three of these units operate in parallel with the shunt converters described above. Two of the units are of 3000-kw. size, each consisting of two 1000-ampere, 1500-volt rectifiers operated as a single unit. These sets, which are of Brown Boveri manufacture, have separate transformers and underload tap changers but are connected to a common oil breaker. The water cooled rectifier tank is connected to the double six-phase windings of the transformers. This arrangement provides for 12 anodes which are equipped with air cooled radiators. The water consumption at rated load is approximately ⅓ gallon for 100 amperes of load per minute. The units are designed to carry 50 per cent overload for 20 minutes and 300 per cent load momentarily. Each bowl or rectifier is provided with an ejector or mercury vapor pump and a single stage rotary oil pump, both for the evacuation of gases from the interior of the rectifier. The mercury vapor pump, a stationary device, which operates on the injector principle, is actuated by an electric heater located in the base of the pump.

1500-Volt Switchboard (Truck and Panel Type) 16th Street

The vacuum, or conversely the pressure of gases within the unit, is measured by balancing the gases against a column of mercury known as the McLeod gage. A means of more quickly identifying conditions within the bowl is obtained by the direct reading vacuum meter. This device consists essentially of a hot wire gage operating on the Wheatstone bridge principle arranged so that two of the arms of the bridge are subject to the gas pressure within the rectifier while the other two are exposed to the pressure of the atmosphere; the difference in resistance due to difference in pressure causes a current to flow through the vacuum meter which is calibrated to indicate the vacuum directly in millimeters of mercury.

All seals within the unit are made with mercury, asbestos, or by rubber gaskets. The unit is connected to the 1500-volt busses through a high-speed breaker placed in the positive lead of each cylinder. This breaker is of mechanical latched type, designed to operate on overload or short circuit which may occur in either direction.

The air breakers are interlocked with the oil breakers so that in case the oil breaker opens from overload or short circuit, the air breaker opens also. The rectifier is provided with relay equipment designed to take the unit off the service if excessive temperatures are present within the rectifier cylinder.

The rectifier with the transformers, which are of the

oil-filled and self-cooled type, take the ordinary floor construction customarily found in station and substation structures. However, no conservation of space is found with the rectifier, the floor area occupied by the rectifier being equal to that of the synchronous converter. Absolutely no noise of operation is associated with the rectifier proper. Some noise will be found in the rotary pump but none that cannot be mitigated to any desirable degree.

The 1500-kw. rectifiers are of General Electric manufacture, two 500-ampere cylinders operating in parallel from a single transformer and oil breaker. Each cylinder consists of 6 anodes of graphite composition and is provided with wire wound heaters to keep the anode insulators at proper temperature during periods of external low temperature and low loads that mercury will not condense on them. The water-cooled chamber is supplied with heaters for the same purpose. To reduce possibility of high transient voltages appearing on the rectifier, condensers in the form of power capacitors are connected to one set of three anodes and across the interphase transformer. That the load will divide equally between the bowls, anode reactors are provided. These rectifiers have practically the same other protective features as the 3000-kw. units, except that following the same design provided for the General Electric converter, high-speed breakers of the bucking bar type are placed in the positive as well as the negative lead of the rectifier. These units have a rating of 50 per cent overload for

3000 Kw., 2 Unit Converter (16th Street)

20 minutes and 300 per cent load momentarily. The inherent regulation of the rectifier is approximately 5 per cent.

Switching Facilities

Another feature of interest in these substations, especially those which are supplied from the 12-kv. system, is the design and arrangement of switching equipment on each side of the conversion units. Fundamentally this consists in having the breakers and disconnecting devices truck mounted and enclosed in metal housing.

The use of the enclosed switch unit is not a new idea, having been on the market in various forms for some considerable time, but its application to both high- and low-tension switching in the same substation is somewhat unusual. The point of principal interest in these installations, however, is the application of the switch unit to certain ideas gained from the operating and engineering experience with other switching arrangements. By means of interlocks it prevents that kind of operating hazard which seems to be always with us in the old style of fixed bus and switch structure, namely, that of pulling disconnective switches while under load. The switch unit makes readily accessible for maintenance, repairs, and inspection all of the wearable, adjustable parts. Not only is factor of safety improved but, because of the accessibility, better maintenance and repair work is done and therefore better performance is obtained.

Certain operating practises entirely practical with 600-volt switching equipment are not acceptable when the pressure has been increased to 1500 volts or above. Correction by isolation, through elevation, may not always be desirable or lack of clearance may render the arrangement impractical, therefore the use of the switch unit, with the added advantage of accessibility, is in many cases a solution entirely satisfactory. The advantages accruing to the truck mounted breaker is considerably lessened when it is necessary to unbolt, and in many cases untape, the breaker connection from the fixed conductors within the structure. The use of the automatic disconnect for both the current-carrying circuits and the control circuits does away with the unwieldy, and in many instances dangerous, method of manual disconnection. The principle of pressure-contact disconnects can be used in many truck designs so that danger and bother from overheated disconnect switches is appreciably reduced.

By the enclosure of the individual switches in separate metal compartments, the tendency of the arc to spread in case of breakdown, is greatly reduced over those cases which have occurred in the open fixed type of structures. Confining the arc to one or two immediate compartments is quite different from the trouble experienced in the old design where the entire structure and sometimes the room become involved from one breakdown.

The incoming transmission lines, bus tie connections, and connections to the individual conversion and transforming units within the substation are connected to the 12,000-volt busses through 3-phase, 15,000-volt, 600-ampere, type O.E. 6 oil breaker of Westinghouse manufacture. The breaker, automatic disconnects, and hand-throw disconnects are carried in a panel mounted truck, the panel serving the double purpose of closing the cell compartment and providing the switchboard facility for all instruments, relays, control

switches, and interlocks. The principal features of this type of iron clad switch gear, are that nothing electrical is accessible until the breaker is in the open position and that when the truck is withdrawn live parts are shuttered off or set back in protecting recesses. In addition to these features the switch unit is provided with manual operated knife disconnects which permit all grounding operations to be made through the oil breaker, an operating practise which has been in force in the substations of the power company for many years. The grounding of any line, bus, or unit directly through cable, clamp, and knife switch devices means that the operator when the last connection of the circuit is made to a ground source, must be in immediate proximity of a fault to ground should an operating error be made. The simple method of making any set of disconnects, used in conjunction with a breaker, doublethrow, with the one side connected to a ground bus permits the operator to be at a safe distance when the last link in the ground connection is made (closing the oil switch) and also provides relay protection should an error exist and live circuits be grounded.

This equipment is arranged so that an oil breaker can be partially drawn out of the cell for inspection of the parts, at the same time leaving the control intact for operation of the breaker but cutting off contact with all live high-tension connections. This group of breakers and housings is properly insulated for later installation of a fault bus system should one be found necessary. For the two substations located outside the city and operated by the Public Service Co., the 33-kv. switching equipment and bus including the transformers of the converters and rectifiers is located out of doors adjacent to the substation building. The high-speed air breaker connected to the positive lead of the conversion units as well as all the breakers on the feeders are truck-mounted and enclosed in metal compartments similar to the switching facilities for the 12,000-volt equipment. The 1500-volt bus is also enclosed in same metal housings. The principle and characteristics of this breaker and its application to the 1500-volt distribution system of the railway terminal have been presented to the Institute by Messrs. Monroe and Allen.

In order to avoid the inconvenience of testing high-voltage instruments, the watthour meters which measure the 1500-volt output energy of the converting units are installed in the negative lead of the unit in every instance except one, this being at the Front Avenue Substation where 1500-volt energy is also delivered to the Chicago, South Shore, & South Bend Railway Company.

One bay of each substation contains the transformers, regulating and switching equipment for the a-c. supply to the railway company light, power, and signal systems. Energy is taken off the 12-kv. bus and stepped down to 4000 volts for the 3-phase 4-wire railway light and power system and the 2300-volt for the single-phase signal system. As the signal system is ungrounded, an insulating transformer of equal ratio is provided in the supply to the signal system. All the energy to the light, power, and signal systems is regulated. The switching equipment regulators, methods of installation, etc., are of the customary type found on a-c. distribution circuits. Space is provided in the substation for an extension of this equipment when necessary.

Operating Experience

The synchronous converter without exception has shown a remarkable performance not only in the matter of regulation but under short-circuit conditions in which the number of flashed commutators has been practically negligible. Successful parallel operation with the mercury rectifier has been proved at all ranges of load.

Taking the group of mercury rectifiers as a whole, it cannot be said that our confidence in them has lessened. On the contrary, the operating experience shows that a great number of them could have been installed. For this class of service it has been demonstrated clearly that the use of one rectifier in a substation, operating as a base load unit with the converters reserved for the peak loads, results in a most satisfactory combination. In some instances a certain amount of trouble on the rectifiers can be charged to an overload condition as the earlier type of lacked capacity comparable with that of the synchronous converter operating in the same substation.

In general, the rectifier under normal operation has shown such gratifying results and advantages and with recent improvements made upon the unit and its auxiliaries indicates a reliability practically equal to that of the synchronous converter.

The performance of the high-speed breaker has been exceptional, especially in preventing the spread or reflection of short circuits back into the substation.

The light, power, and signal systems, outside of some trouble initially, have given none.

In common with the opinion given of the operating experience of the entire terminal, as expressed in a previous paper, that "From every point of view the equipment has met all expectations and the complete operation has apparently been successful," the operating experience of the substation equipment has indicated a similar result. Troubles have occurred as they naturally do on systems of this magnitude, but in no instance has anything happened that seemed impossible of solution, and in most cases immediate remedies have been found.

Discussion

D. C. Jackson: I am convinced that the 3000-kw., 3000-volt rectifiers that we are planning to install in the substations of the Lackawanna Railroad are going to prove satisfactory and economical conversion devices. The rectifier has had relatively little service in this country, but large service in Europe. It is an important new instrument in heavy railroad electrification and I think we will hear much more of it. Mr. Garrett speaks of it somewhat cautiously but at the same time indicates a degree of confidence in it which in my opinion is justified.

Sidney Withington: I agree thoroughly that the mercury rectifier is going to be extremely important in connection with any d-c. railway electrification. It would be interesting to know whether any special devices are employed to insure the proper division of load between the rotary converters and the static rectifiers.

W. B. Potter: I believe the rectifier will largely take the place of rotary converters and motor-generator sets on d-c. railways, and to some extent for lower voltages where the conditions are more favorable to stationary apparatus.

I believe the future for railway electrification, about which we have talked so many years, is beginning to look brighter. The steam locomotive engineers are improving the performance of the steam locomotive to such an extent that ere long we may anticipate a demand for even better service than can be furnished by the electric locomotive.

A. M. Garrett: There are eleven or twelve rectifiers in operation now in the Chicago district, and probably double that number immediately outside in Northern Indiana and in Milwaukee.

Professor Jackson raised the question of whether we weren't too cautious in our opinion of the rectifiers. Our experience with rectifiers has shown three things: First, in connection with the phenomena of the electric arc, we have much to learn regarding what takes place inside the unit not only under normal conditions, but abnormal conditions as well. The second point is whether we are reaching a limit in the rectifier as applied to American practise. The rectifier "grew up" in foreign practise and we feel that the systems back of them in Europe, differ from those in this country. Here there is a tendency to hold everything on the bus. In Europe, I understand it is a case of disconnecting to keep the individual lines alive. A third point is whether we can apply improvements to the rectifier that will make it comparable in every respect with the rotating converter.

With rotating converters there are certain problems which have never been solved in connection with brushes and commutators. The rectifier offers a solution to many of those problems. Furthermore, the fact that it is a static machine gives it a marked advantage over a rotating type.

The point was raised of paralleling the rectifier with the converter. In one substation where there is both a converter and a rectifier, where the units might be called upon to operate with a 300 per cent load with voltage limits between 1550 and 1400, it was felt that some special provision should be made for paralleling. A regulator was installed but satisfactory performance can be had without it.

We have two 600-volt rectifiers in operation now in the Chicago district, 3000 kw., 600 volts. They give good service.

At times the adverse of performance of rectifiers has been marked but we believe with certain improvements very good service can be expected from them. For a city traction system where one unit per substation is used and the substations are located frequently along the routes so that load transfer from one to the other may be provided, the rectifier is very well adapted.

The rectifier shows up very well where mitigation of noise is necessary. In residential districts a rotating machine must be enclosed to keep the noise down. The rectifier on that one score alone shows to marked advantage.

Master Reference System for Telephone Transmission

BY W. H. MARTIN* and C. H. G. GRAY†

Member, A. I. E. E. Applicant for Membership

Synopsis.—*The telephone transmission system described here is the Master Reference System of the Bell System for the expression of transmission standards and the ratings of the transmission performance of telephone circuits. The transmitter and receiver elements of this system are reference standards for the ratings of the transmitting and receiving performance of terminal station sets.*

A replica of this reference system, installed in Paris, has been adopted as the Master Reference System of the International Advisory Committee on Long Distance Telephone Communication in Europe. The establishment of these two master systems provides a common reference for the telephone transmission work of the Bell System and the telephone administrations which are members of this International Advisory Committee.

THE Master Reference System for Telephone Transmission, as its name indicates, is to serve as the fundamental circuit in the ratings of the transmission performance of telephone circuits. In describing this system, therefore, it will be advantageous to outline first the general considerations underlying the methods of determining and specifying these ratings and their applications.

The conversions and transfers of energy which constitute the process of telephone transmission result in general in a difference between the speech sounds at the sending end of the telephone circuit and the sounds reproduced at the other end in the ear of the listener. These reproduced sounds may differ from the original in three important respects: their loudness, their distortion or degree to which their wave shape departs from facsimile reproduction, and the amount of extraneous sound or noise which accompanies them. From the standpoint of telephony, the major importance of a difference between the original and reproduced sounds is determined by its effect on "intelligibility," that is, the degree to which the latter sounds can be recognized and understood by the listener when carrying on a telephone conversation. The tolerable departure of the reproduced from the original sounds is limited also by certain effects which are noticeable to the listener before they materially affect intelligibility, such as loss of naturalness.

Measurements of intelligibility are of utmost importance in rating the performance of telephone circuits, but they are unduly cumbersome for direct use in the detailed development and design of telephone circuits and their many parts, particularly where small effects are concerned. It has been desirable, therefore, to handle telephone transmission work in two steps. One natural division is suggested by the statement that intelligibility is a function of the relation between the output and input speech sounds, and of the psychological reaction of the listener to these output sounds. Because of the complex nature of the speech sound waves, however, it has been found more practicable to treat the transmission performance of telephone circuits in the following two parts: (1) the physical performance of the circuit, and (2) the relation between physical performance and intelligibility. The physical performance of a circuit is taken here to cover the transmission characteristics which can be specified in terms of the performance for single frequencies, a number of frequencies being taken to cover the range which is important for the reproduction of speech sounds. These measurements of physical performance cover such things as the response-frequency characteristic of the circuit over the range of speech frequencies, the distortion due to non-linear elements, phase distortion, and the extraneous currents which cause noise. These determinations of physical performance do not include measurements of the speech sounds themselves, nor of the functioning of the talker and listener. This differentiation is advantageous in segregating the studies of speech sounds and of the psychological phases of the work, and permits the design of the operating plant and a large portion of the development work to be carried out on a physical basis.

The determination of the relation between intelligibility and the physical performance of a telephone circuit is a laborious process, because persons play the parts of generators and meters and a number of people must be used in both parts to take into account the normal ranges of their performance. The goal of this portion of the work has been, therefore, to establish suitable relations which will permit the determination of the intelligibility of a circuit by computations which start with the physical characteristics of the circuit. This work[1] has involved determinations of the capabilities of circuits having various kinds of physical characteristics to reproduce intelligible speech, investigations of the nature of speech sounds and hearing, and of people's customs in using the telephone.

Prior to the time when suitable means were available for measuring the physical performance of telephone circuits, and when the kinds of circuits in commercial

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1. For all numbered references see end of paper.

29-110

use were quite similar in their distortion characteristics, the practise was adopted of rating the performance of a circuit by comparing it on a loudness basis with a reference circuit which was adjustable in attenuation, and whose distortion was closely similar to that of the commercial circuits. In such a comparison, a determination is made of the equivalence of loudness or volume of these two circuits by talking alternately over them and adjusting the reference circuit until the sounds coming out of the two receivers are judged to be equal. For the conditions where volume is the important controllable characteristic of telephone circuits, these loudness comparisons constitute a practicable and effective means of indicating the performance of these circuits.

The reference circuit adopted about twenty-five years ago for these loudness comparisons consisted of transmitters, receivers, station sets, cord circuits and a line, of types which were then used commercially. In this reference circuit the line was an adjustable artificial line simulating a No. 19 A. w. g. cable circuit having a capacity per loop mile of 0.054 μf. The amount of cable in this line to give a loudness balance was taken as the rating of the circuit under comparison. This reference system, shown schematically in Fig. 1, has been called the Standard Cable Reference System.

In addition to this rating of the performance of a telephone circuit, this standard cable reference system has had other applications. Certain settings of this reference system were selected as specifying the standards of transmission which were to be provided in the design and operation of commercial circuits for the several kinds of service, such as local and toll. The effect of introducing or changing any part in a commercial circuit was rated in terms of the amount of cable by which it was necessary to change the line of the reference system to produce the same effect on the loudness of the reproduced speech sounds. Likewise, the transmitters and receivers of this reference system were used as reference instruments for the comparison and rating of other transmitters and receivers.

This cable reference system has played a very important and necessary part in the development of telephone transmission, in that it has provided a ready means of rating the performance of the various parts of the system and of any changes, and made it possible to design commercial circuits to provide a predetermined grade of service. The performance of this system was specified by stating the kinds of apparatus and circuits used. The performance of the elements of the electrical portion of the system could be checked by voltage, current, and impedance measurements, but for the transmitters and receivers reliance for constancy of performance was placed primarily upon the careful maintenance and frequent cross comparisons of a group of transmitters and receivers which were specially constructed to reduce some of the sources of variation in the regular product instruments. In this way, reasonable assurance of the performance of the reference system was secured. This system has been widely used both in this country and in other parts of the world, and the performances of the various systems have been kept in accord by frequent circulation of calibrated transmitters and receivers.

As the telephone art has developed, modifications have been found to be desirable in this reference system to make it more suitable for its purpose. Telephone instruments and circuits have been designed and used which have less distortion than existed in the corresponding parts of the cable reference system. For this reason it is desirable to have as a new reference system one with which the transmission over the most perfect telephone circuit or over some less perfect one may be simulated at will. The change of the unit of transmission from the mile of standard cable to the decibel[2] has brought about the need for a change in the line of the reference system.

In selecting a new reference system, it is obviously desirable to eliminate as far as possible the factors

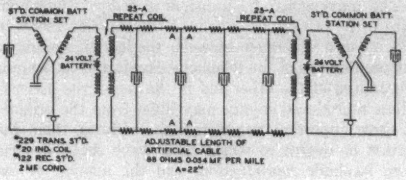


Fig. 1—Standard Cable Reference System

which are not subject to exact measurement, or which may possibly vary with time. For this reason, the elements of the new system have been chosen so that their performance may be definitely measurable at any time, and may remain as far as possible invariable. This applies also to those elements which are provided for insertion in the system when it is wished to produce some distortion which will make more easily possible a loudness balance between the reference system and the circuit under investigation.

A reference system such as that described here, in which the essential elements are so constructed as to reproduce speech with a high degree of perfection, and with which provision may be made for modifying the speech in definite and reproducible ways, affords a convenient means for studying the capabilities of telephone circuits of different physical performances. These investigations, however, are outside the purpose of this paper, which is to describe the new reference system and its application in making volume ratings.

General Requirements

The outstanding conception of the new reference system is that its performance should be suitable to serve as a reference base line for indicating the performance of all telephone circuits and that the trans-

mitter and receiver elements of the new system should provide similar base lines for the performance of electro-acoustic converters. To meet these needs properly the performance of the system and its parts should be capable of being measured and definitely specified in terms of physical quantities. In this connection, there arises a matter which has been the subject of much discussion, namely, as to whether or not the specified performances of the reference transmitter and receiver should be those which are realized when used as telephone instruments. In regard to the transmitter, the difficulty comes in specifying the input when it is placed in front of the mouth of the talker. This is due to the non-uniformity of the sound field from the speaker's mouth, the nature of the waves of speech sounds, and the reflections of these waves from the transmitter. In regard to the receiver, the difficulty is due to determining the output when the receiver is held to the ear. To obviate these, it has been decided to specify the performance of the transmitter in terms of the electrical output for a given pressure on the diaphram of the transmitter and of the receiver in terms of pressure set up in a simple closed chamber for a given electrical input to the receiver. These conditions are definite and reproducible.

The most important requirement, then, for the reference system is that the physical performance of the system and of its component parts should be capable of being measured and definitely specified in terms of physical quantities. If this requirement is realized, a system providing the specified performance can be set up wherever desired. This has been the main criterion in the design of the master reference system for telephone transmission which is described here.

The second main requirement is that specifiable and predeterminable changes can be made with respect to the performance which is selected as the reference. These changes must be capable of varying the relation between the loudness of the reproduced sounds with respect to the initial sounds, the distortion of the wave shape of these reproduced sounds, and also the amount of noise accompanying these reproduced sounds.

For convenience in specifying these requirements, it has been found desirable to impose another requirement, namely, that the system be capable of giving a performance which is as free as possible from distortion and noise. It should be noted that two kinds of distortion must be taken into account: that due to unequal efficiency for sounds of different frequencies and that due to non-linearity causing unequal efficiency for sounds of different magnitudes. This requirement is also of advantage in insuring that the reference system and its parts will have less distortion than any circuit or instrument with which it may be compared. This will permit the simulation of the distortion of such instruments or circuits by the insertion of distortion in the reference system.

For convenience in use, it is highly desirable that the performance of the reference system and its parts be constant for a reasonable time under normal operating conditions.

General Features

The master reference system[3] employs a transmitter and receiver which are capable of a high degree of freedom from distortion. The transmitter is of the condenser type and the receiver is of the moving coil type. Both these instruments are materially lower in efficiency than commercial types of apparatus, but this condition is compensated for by the use of multi-stage vacuum tube amplifiers. These instruments, together with their associated amplifiers, constitute reference standards for converters between acoustic and electrical energy. The third necessary element of a telephone transmission system, namely the line, is provided by a network of resistance elements. Such a line can be made to provide uniform attenuation over a wide frequency range, and can be made to control the magnitude of this attenuation over a large range. This line is taken as giving a reference performance for lines.

The specification of the performance of such a system is based on the principle of the thermophone, which is a converter of electrical energy into acoustic waves by means of the heat generated by the passage of an electrical current through a resistance. From a knowledge of the form and physical constants of this resistance element, of the medium in which it is used, and of the electrical input to the element, the acoustic pressure generated in a chamber of known size can be determined by theoretical considerations.[4] The performance of the condenser transmitter is determined by making its diaphram a wall of a simple closed chamber in which the thermophone is placed. By this means a known pressure wave of any frequency over the range desired can be impressed upon the diaphram of the transmitter. The voltage output of the transmitter for a specified circuit condition is then measured. From this measurement, the ratio of the voltage output to the acoustic pressure on the diaphram is established for that instrument and circuit condition. With the performance of the transmitter thus established, the performance of the receiver element of the reference system is measured by acoustically coupling the receiver to the condenser transmitter, so that the receiver actuates the transmitter, and then determining the relation of the pressure generated by the receiver in the coupler to the voltage input to the receiver. The performance of the line element is determined by well known means. The performance of the whole system can then be expressed in terms of the pressure produced by the receiver with respect to the pressure on the diaphram of the transmitter.

The performance of this system is practically free from distortion for the energies which it is required to handle, and probably materially excels in this respect that of any previous system. With this system, volume relations between output and input sounds can be varied over a wide range with practically no accompany-

ing distortion. In comparing such a system, however, with commercial systems, it is advantageous also to be able to control distortion. This is particularly the case when using the instruments of the master system for rating the volume efficiency of commercial types of transmitters and receivers. To facilitate this, arrangements are made for the introduction into the amplifiers associated with the transmitter and receiver, of networks which may be designed to give a variation of efficiency

Fig. 2—Master Reference System for Telephone Transmission with Associated Calibration Apparatus

with frequency which corresponds to that obtained with commercial apparatus. These networks and their distortion effect can, of course, be definitely specified. The line element of this master system can be replaced by a line or network giving any type of distortion desired. Also, known amounts of extraneous currents to produce noise can be introduced into this circuit without otherwise appreciably affecting its performance. This is accomplished by connecting a relatively high impedance source of voltage of the desired wave shape across a circuit element of relatively low resistance.

This system provides a performance which is definitely known, and which can be varied over a range of volume and distortion. It well meets the requirements and represents a material advance over the standard cable reference system which it now replaces. In order to tie together ratings established in terms of the old system with those of the new system, comparisons of the two have been made to determine the adjustments of the new system which make its loudness performance correspond with that of the old.

Description of Master Reference System

The master reference system with associated calibration apparatus is shown in Fig. 2. This equipment, mounted on steel panels and racks, and arranged as shown, is installed in a room, shielded from acoustical and electrical disturbances, at the Bell Telephone Laboratories in New York City. In Fig. 2 the transmitter, line, and receiver of the reference system are shown on the three racks at the right. The calibration apparatus, consisting of an oscillator, thermophone, vacuum tube voltmeter, and volume indicator are shown on the other three racks. A schematic diagram of the master reference system is shown in Fig. 3.

The reference transmitter consists of a condenser transmitter and associated four-stage amplifier. The condenser transmitter is simple and rugged in construction. It has been available for work of this nature for several years during which time considerable experience has been obtained with it. In this system the condenser transmitter is held in an adjustable mounting and is equipped with a guard by means of which the speaker can keep his lips at a fixed distance from the diaphram. This guard consists of a wire ring 4.7 cm. in diameter, held parallel to and at a distance of 4.1 cm. from the diaphram of the transmitter by three wire supports. A volume indicator with meter visible to the speaker enables him to maintain an approximately constant talking intensity. In the condenser transmitter,[5] shown in Fig. 4 and in cross-section in Fig. 5, a thin highly stretched duralumin diaphram is mounted close and parallel to a steel plate grooved and perforated for air damping. This diaphram and plate form the electrodes of a condenser polarized by a battery through a high resistance. Sound vibrations impinging on the diaphram actuate it and produce variations in the

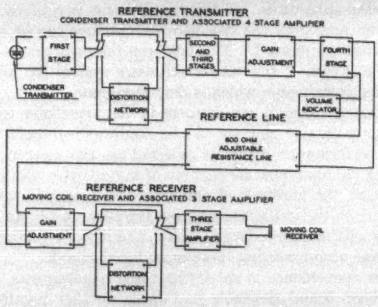


Fig. 3—Schematic Diagram of Master Reference System

capacity of this instrument. The resulting alternating potentials, faithfully representing the sound pressure waves, are amplified by the associated amplifier. Between the first and second stages of this amplifier, provision has been made for the introduction of distortion networks to simulate the distortion of any transmitting system. Between the last two stages are attenuating networks permitting adjustment of the relation between transmitter input and transmitter output over a range of 22 db. in steps of 0.1 db. The output impedance of the amplifier is 600 ohms with negligible phase angle.

The reference line consists of a series of balanced

resistance networks. This line has a characteristic impedance of 600 ohms matching the output impedance of the reference transmitter and the input impedance of the reference receiver, thereby eliminating any reflection effects at these junctions. By means of suitable controls the attenuation may be varied over a range of 101 db. in steps of 0.2 db.

The reference receiver consists of a moving coil receiver and associated three-stage amplifier. Means are provided for adjusting the relation between the

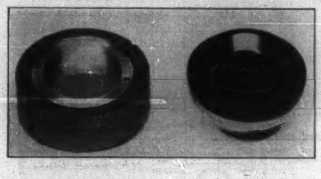


Fig. 4—Condenser Transmitter and Moving Coil Receiver

input to the receiver and its output over a range of 22 db. in steps of 0.1 db. As in the case of the reference transmitter, provision has been made for the introduction of distortion networks to simulate the distortion of any receiving system. In the moving coil receiver[6] shown in Fig. 4. and in cross-section in Fig. 7, a coil of aluminum ribbon, by vibrating relatively to a fixed permanent magnet, actuates a clamped, unstretched, thin duralumin diaphram to which it is attached. Air damping in this receiver is secured by an arrangement somewhat similar to that employed in the condenser

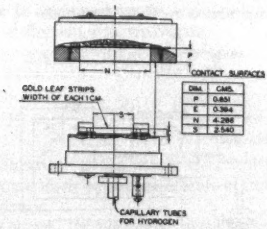


Fig. 5—Schematic Drawing of Condenser Transmitter and Thermophone

transmitter. The structure is simple, rugged, and from the standpoint of freedom from distortion, its performance is comparable to that of the condenser transmitter. The central surface of the diaphram is protected by a meshed wire screen instead of by the more usual type of receiver cap. This construction avoids resonance effects which would otherwise occur in the confined air space between diaphram and cap. The contour of the ear piece, resembling that of the familiar telephone receiver, permits the listener to readily center the receiver on his ear.

In order that an individual part of any telephone circuit may be compared with the corresponding element of the master reference system, the system is arranged so that the reference transmitter, reference line, or reference receiver, may be replaced by the corresponding part to be rated.

Calibration of Master Reference System

To specify adequately the performance of the master reference system, in terms of definite physical quantities, apparatus is associated with the system for making electroacoustic calibrations of the reference transmitter and reference receiver, and electrical calibrations of the circuits. In general, the method employed in these calibrations is to adjust the setting of an attenuator to obtain a deflection on the galvanometer of a vacuum tube voltmeter, equal to the deflection produced when measuring the output of the element under calibration.

Fig. 6—Thermophone in Chamber for Calibrating Transmitter

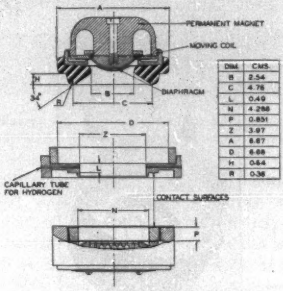
This avoids the necessity for an absolute calibration of the measuring device.

The source of alternating currents, used for calibrating purposes, is an oscillator, capable of producing currents with a harmonic output usually less than 3 per cent of the fundamental.

The measuring equipment used for making the above calibrations consists of a two-stage vacuum tube voltmeter in conjunction with a tuned circuit connected across its input. By means of this tuned circuit, harmonics of the fundamental frequency to be measured are attenuated by at least 20 db.

The calibration of the condenser transmitter is made with a thermophone, the gold leaf thermal elements being shown in Fig. 6. In Fig. 5 is shown a cross-section of a condenser transmitter and a thermophone. Petrolatum is used to form a seal between the instrument and the thermophone block. The air in this small enclosed chamber, formed by the walls and dia-

phram of the condenser transmitter and the thermophone block, is replaced by hydrogen. Since the velocity of sound in hydrogen is approximately 4 times that in air, the frequency range over which measurements may be made before standing wave effects are experienced is extended by the use of this gas. In addition, the efficiency of the thermophone is increased, the constant of diffusivity for hydrogen being greater than that for air. Both alternating and direct currents are passed through the gold leaf thermal elements, the direct current being sufficiently large to make negligible the double-frequency effect resulting when alternating current is supplied to the thermophone. The alternating current passing through the thermal-elements (gold leaf being selected because of its low heat capacity) causes variations in their temperature. Periodic expansions and contractions of the surrounding gas, resulting from the varying heat transfer occasioned by the periodic temperature changes of the thermal



DIM.	CMS.
B	2.54
C	4.76
L	0.49
N	4.298
P	0.831
Z	3.97
A	6.67
D	6.68
H	0.64
R	0.38

FIG. 7—SCHEMATIC DRAWING OF MOVING COIL RECEIVER, COUPLER, AND CONDENSER TRANSMITTER

elements, constitute sound waves of precisely determinable pressure. These sound waves actuate the transmitter and the ratio of the voltage output to the sound pressure gives the transmitter calibration. The process involved is as follows: Referring to Fig. 8.

E_T = Voltage generated by condenser transmitter per bar (one dyne/cm.^{2}).
I_1 = Alternating current through thermophone and attenuator.
P = Pressure developed by thermophone per volt alternating current across it.
V_{VM} = Voltage across voltmeter.
A = Ratio of voltage delivered to 600-ohm load by the transmitter amplifier to voltage impressed in series with condenser transmitter.
V_o = Voltage across attenuator input.
R_1 = Input impedance of attenuator.
R = Thermophone resistance.
N = Attenuator setting in db.
$T\,P$ = Thermophone.
$C\,T$ = Condenser transmitter.

With the switches in Fig. 8 in position "A," the a-c. input to the thermophone and the amplifier gain are adjusted to obtain a convenient voltmeter deflection (V_{VM}). Then:

$$V_{VM} = I_1\,R\,P\,E_T\,A$$

Operating the switches to position "B" and manipulating the attenuator to attain the same voltmeter deflection as before we obtain

$$V_{VM} = V_o\,A \times 10^{(-0.05N)} = I_1\,R_1\,A \times 10^{(-0.05N)}$$

Then:

$$E_T = \frac{R_1 \times 10^{(-0.05N)}}{R\,P}$$

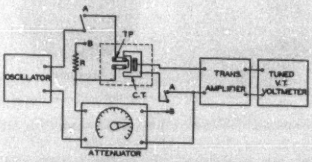


FIG. 8—CIRCUIT FOR CALIBRATING CONDENSER TRANSMITTER

The moving coil receivers are also calibrated in a sealed chamber in an atmosphere of hydrogen by an arrangement shown in cross-section in Fig. 7. A condenser transmitter, calibrated in the above manner, is actuated by the sound output of the receiver under test driven by the oscillator. The process involved is as follows: Referring to Fig. 9,

V_{VM} = Voltage across voltmeter.
V_o = Voltage across attenuator input.
V_R = Voltage across receiver terminals

$$= \frac{V_o\,R_R}{R_R + R_f} \times 10^{(-0.05N_a)}$$

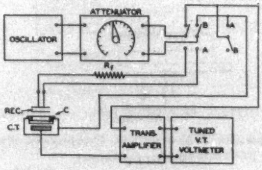


FIG. 9—CIRCUIT FOR CALIBRATING RECEIVER

R_R = Impedance of receiver plus cord.
R_f = Fixed resistance.
P_R = Pressure developed by receiver per volt across receiver terminals.
E_T = Voltage developed by condenser transmitter per bar (one dyne/cm.^{2}).
A = Ratio of voltage delivered to 600-ohm load by the transmitter amplifier to voltage impressed in series with condenser transmitter.
N_a = Attenuator reading with switches on "A."

N_b = Attenuator reading with switches on "B."
C = Coupler (Fig. 7).
$C\,T$ = Condenser transmitter.

With the switches indicated in Fig. 9 in position "A" the attenuator is adjusted to position N_a to produce a convenient deflection on the voltmeter (V_{VM}).

$$V_{VM} = V_O \frac{R_R}{R_R + R_f} P_R E_T A \times 10^{(-0.05N_a)}$$

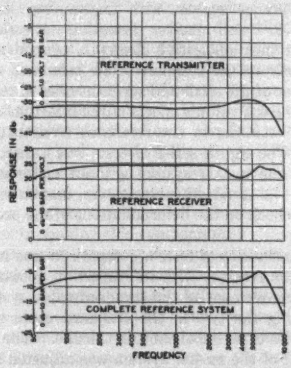


FIG. 10—RESPONSE CHARACTERISTICS OF REFERENCE TRANSMITTER, REFERENCE RECEIVER, AND COMPLETE REFERENCE SYSTEM WITH 0 db. IN THE LINE

Operating the switches to position "B" and adjusting the attenuator to some position N_b to reproduce the above voltmeter deflection V_{VM} we obtain:

$$V_{VM} = V_O A \times 10^{(-0.05N_b)}$$

Then:

$$P_R = \frac{(R_R + R_f) \times 10^{0.05(N_a - N_b)}}{R_R E_T}$$

The calibrations of the purely electrical elements of the circuit are made by measurements of input, output, and impedance.

Fig. 10 shows, for particular amplifier adjustments which are discussed below, the frequency response characteristics of the reference transmitter, reference receiver, and the complete reference system with O db. in the line. The characteristic of the reference transmitter and also of the reference receiver, in each instance, is that of the instrument and associated amplifier combined. However, as the frequency response of each of the amplifiers is uniform within 2 db., from about 50 to 10,000 cycles per second, the curves shown are essentially the calibrations of the instruments determined as described above. The primary purpose of these characteristics is to show the performance of these elements for definitely specified physical conditions. Consequently, no corrections are included in these curves for the effect on the sound field of the speaker when talking into the condenser transmitter. Neither have corrections been applied for the effect of leakage between the listener's ear and the earpiece of the receiver nor for the difference between the volume enclosed in the sealed chamber when making the calibration as compared with the volume enclosed in the ear canal when the receiver is held to the ear. The reference line, consisting of balanced resistance networks, has a uniform frequency response characteristic. The characteristic of the complete system, therefore, except as its level is affected by the attenuation in the reference line, is that of the reference transmitter and reference receiver combined, there being no reflection effects at the junctions of the reference line with either the reference transmitter or receiver.

Fig. 11 shows the effect of introducing distortion networks in both the reference transmitter and reference receiver. The distortion network introduced in the reference transmitter simulates the distortion in the transmitter, station set, and cord circuit, of the standard cable reference system. Similarly, the distortion network introduced in the reference receiver simulates the distortion of the receiver, station set, and cord circuit, of the standard cable reference system. These two networks are designated, respectively, Transmitter Distortion Network No. 1 and Receiver

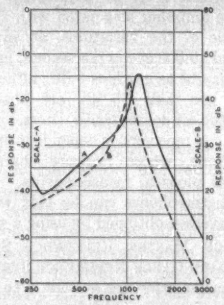


FIG. 11—RESPONSE CHARACTERISTICS OF REFERENCE TRANSMITTER AND RECEIVER WITH DISTORTION NETWORKS
A = REFERENCE TRANSMITTER—0 db.—1.0 VOLT PER BAR
B = REFERENCE RECEIVER—0 db.—1.0 BAR PER VOLT

Distortion Network No. 1. The distortion of any transmitter, receiver, or station set may be similarly simulated by the design and introduction in the master reference system of a proper network. The advantage of such a procedure will be evident to those familiar with the difficulties of making voice volume balances between instruments of widely different frequency characteristics. The volume efficiency of any instrument can also be reproduced by adjustment of the controls in the amplifiers of the reference transmitter or receiver as the case may be. The results of voice

volume balances made with the distorted reference system can be referred to the undistorted master reference system by applying a correction for the volume effect of the distortion network in the master reference system, this effect being determined independently as a rating of the distortion network.

Experience has shown that amplifiers properly designed and constructed remain essentially constant with time. Such changes in gain as may occur because of replacement of vacuum tubes or other apparatus may be readily compensated for by adjustments of potentiometers provided for this purpose. Condenser transmitters are affected to some extent by variations in temperature and barometric pressure. The magnitude of variation at any frequency between 50 and 10,000 cycles per second from these causes may be as much as 2.5 db., although under normal operating conditions such as those experienced in buildings in this climate, the variation is usually not more than 1 db. Since changes of this character are gradual in nature, their magnitude can be readily determined by a thermophone calibration. Corrections for any change in sensitivity of the instrument may then be made by adjustment of the controls in the amplifier associated with the condenser transmitter to maintain the proper gain in the reference transmitter. Similarly, any variations in the response of the moving coil receiver may be compensated for by adjustment of the gain controls in the amplifier associated with this receiver. The magnitude of the variations, at any frequency between 50 and 10,000 cycles per second, in the moving coil receiver may be as much as 3 db., although usually variations of less than 1.5 db. are observed. The calibration of the condenser transmitters, and indirectly of the moving coil receivers, is dependent upon the gold leaf thermophone, whose pressure characteristic is computed from physical measurements. Results obtained with thermophones can be held within about 0.5 db. of the average obtained by using a group.

Comparison of Master Reference System and Standard Cable Reference System

Since the master reference system is to replace the standard cable reference system for volume ratings, the two systems have been compared by means of voice volume balances so that data obtained in the future may be directly comparable in this respect with those obtained in the past. The respective elements of the two systems, as well as the systems as a whole, have been compared. The station set and cord circuit at the sending end of the standard cable system are taken as comprising the transmitting element of that system and, likewise, the corresponding apparatus at the listening end as the receiving element. In these measurements the reflection gain at the junction of the standard cable line and the output terminals of the transmitting element of the standard cable reference system has been taken as part of the transmitting efficiency of that element. Similarly, the reflection gain at the junction of the standard cable line and the input terminals of the receiving element of the standard cable reference system has been taken as part of the receiving efficiency of this element.

In the voice calibration of the master reference system to determine the adjustments which make this system equivalent on a volume basis to the standard cable reference system, eight series of voice tests were made. The purpose of the first five series of these tests was to determine the adjustments which make the master reference system with transmitter distortion network No. 1 and receiver distortion network No. 1 inserted in their respective elements, equivalent on a volume basis to the standard cable reference system. The purpose of the sixth and seventh series of tests was to determine the volume effects of the insertion of these distortion networks in the reference transmitter and receiver. The eighth series of tests was a direct comparison of the master system without distortion networks and the standard cable system, and so serves as an overall check on the determinations of the preceding tests.

In the first series of tests a comparison was made of the reference receiver with its distortion network and the receiving element of the standard cable system by interchanging the two in the standard cable system, with 24 miles of artificial cable in the line. The receiving element of the master system was adjusted so that its sound output was judged to be equal for this condition to that of the receiving element of the standard cable system. In the second series of tests a similar comparison was made of the reference transmitter with its distortion network and the transmitting element of the standard cable reference system. These transmitting elements were connected in turn to a 24-mile standard cable line terminated by the reference receiver with its distortion network, the receiver being adjusted in accordance with the results of the first series of tests. In the third and fourth series of tests the master reference system with the distortion networks in both the transmitter and receiver was compared with the standard cable reference system, the line of the master system serving as the adjustable element. These two series of tests were similar except that 24 miles of standard cable were used in one series of tests and 14 miles of standard cable in the other. The adjustments for the reference transmitter and reference receiver in these tests were those determined from the first and second series. From the results obtained in the third and fourth series of tests, the magnitudes were determined of the reflection gain at the junction of the standard cable line with either the reference transmitter or the reference receiver, and of the volume equivalent of 1 mile of standard cable in terms of db. The reference transmitter and reference receiver, each with its distortion network, were then readjusted in accordance with these data. The fifth series of tests served as a check on the results of the previous four series. In this

fifth series, the master reference system with transmitter and receiver distortion networks and 24 db. in the line was compared with the standard cable reference system with 24 miles of standard cable in its line.

In the sixth series of tests the adjustment of the reference receiver without distortion was determined to make it equivalent on a volume basis to the reference receiver with distortion. The reference transmitter with distortion and the reference line with 24 db. formed the rest of the system during these comparisons. In the seventh series of tests the sound output of the master reference system without distortion was compared to the sound output of this same system with distortion networks in the reference transmitter and reference receiver, the reference transmitter being adjusted to obtain a balance. During these tests 24 db. was kept in the reference line. The final series of tests, serving as a check on all of the above determinations, consisted of a comparison between the master reference system without distortion and the standard cable reference system, the line of the master system being adjusted in making this comparison.

In making voice tests to determine the above settings, the speaker, whose position with respect to the transmitters was kept constant, called standard testing sentences in a conversational tone. A given intensity of calling was maintained by watching the deflections on a volume indicator connected across the output of the transmitting element. An observer, located in a quiet room removed from the systems or instruments under test, determined when the two systems under comparison gave output sounds of equal loudness. Each testing team, consisting of a speaker and observer, made 12 balances. In order to attain a suitable precision in the final results of these tests, a large number of testing teams was used, the number being largest for the tests where the difference in the quality of the output sounds of the two systems under comparison was greatest. For example, for the first series of tests, six teams were used, for the fifth 25 teams, and for the eighth series, where the master reference system without distortion was compared with the standard cable system, balances were made with 37 teams. In all, over 2000 individual balances were made. The standard deviation of the determination for each of the first five series is of the order of 0.5 db. and for each of the last three series about 1 db.

The response characteristics of the transmitter and receiver elements of the master reference system, when adjusted on the basis of the results of the voice tests, to be equivalent on a volume or loudness basis to the corresponding parts of the standard cable reference system, are shown in Fig. 10. The mean values weighted from the standpoint of importance for volume are 0.027 volt per bar for the reference transmitter, 16 bars per volt for the reference receiver, and 0.43 bar per bar for the complete master reference system with 0 db. in the reference line. Further consideration is being given to the values of these response characteristics of the reference transmitter and receiver to be adopted as standards.

The response characteristics of the reference transmitter with transmitter distortion network No. 1 and of the reference receiver with receiver distortion network No. 1, when these elements are adjusted on the basis of the above voice tests to be equivalent on a volume or loudness basis to the corresponding parts of the standard cable reference system, are shown in Fig. 11.

Application of the System

The results of articulation tests over the master reference system when adjusted for optimum volume are practically equivalent to those obtained in direct air transmission in a quiet room. This system and replicas of it, are particularly adapted for use in making articulation studies, since they provide an approximately ideal system with which the loudness of the output sounds can be varied distortionlessly over a wide range and in which distortion networks of various types and controlled amounts of noise can be introduced. In this way the effects on articulation of various kinds of physical performance of a telephone circuit can be investigated.

The master reference system itself will be used chiefly for the important work of rating working standard systems and instruments. These working standards can be simpler than the master system and can be provided in any number required to handle the rating of commercial circuits and apparatus.

The working standard may be of several forms. It can be similar to the master reference system, simplified in its detailed construction but capable of calibration by the means employed for calibrating the master reference system. Such a system would probably find employment in laboratories and factories where the volume of testing is sufficient to justify the use of such apparatus. Another form may include electrostatic or electrodynamic instruments which are not capable of being measured by the calibration equipment of the master reference system. A third form which the working standard may take is that involving the use of transmitters, receivers, and station sets such as have been used in the standard cable reference system. These latter types of working standards can be calibrated by volume comparisons with the master system or with the first type of working standard. They will find their chief field of usefulness at such points as shops for the repair and recovery of station apparatus, where the volume of work is not sufficient to justify the expense involved in maintaining more elaborate working standards.

European Master Reference System

In Europe the recommendation of technical standards for telephony is a function of the Comité Consultatif International des Communications Telephoniques a Grande Distance (C. C. I.), which is composed of repre-

sentatives of the various European telephone administrations. In 1926, at the invitation of the C. C. I., representatives of the Bell System met in London with a committee appointed by the C. C. I. to consider the adoption of a transmission reference system. This committee recommended that the C. C. I. adopt as their master reference system a system essentially the same as the one described in this paper, and that such a system, which would be a replica of one in New York, be installed in Paris in the laboratory of the C. C. I. and be known as the European Master Reference System. This recommendation was adopted by the C. C. I.

Subsequently, some improvements were made in the system, and two duplicate systems, each with its associated calibrating apparatus, have been constructed. One of these is now in the Bell Telephone Laboratories in New York and the other in the laboratory of the C. C. I. in Paris. The C. C. I. further recommended that primary and working standard systems, used in the telephone administrations adhering to the C. C. I., be calibrated in terms of the Master Reference System. The establishment of these two master systems insures the use of a common base line for the expression of transmission standards, and for the ratings of the transmission performance of telephone circuits in the two continents where the telephone system has had its greatest development.

List of References

1. "Speech and Hearing" by H. Fletcher, D. Van Nostrand Co.

2. "Decibel" (db) is the name for the transmission unit which has superseded the "mile of standard cable." W. H. Martin, *Bell System Tech. Jl.*, January, 1929, and A. I. E. E. Jl., Vol. 48, March, 1929, p. 223.

3. A discussion of a preliminary model of this system was given in "A Telephone Transmission Reference System" by L. J. Sivian, *Electrical Communication*, October, 1924.

4. The theory of the thermophone as a precision source of sound is outlined in papers by H. D. Arnold and I. B. Crandall, *Physical Rev.*, July, 1917, and by E. C. Wente in the *Physical Rev.*, April, 1922.

5. The theory and operation of this transmitter are discussed in papers by I. B. Crandall, *Physical Rev.*, June, 1918, and E. C. Wente, *Physical Rev.*, July, 1917, and *Physical Rev.*, May, 1922.

6. This instrument, which is similar in many respects to one described by E. C. Wente and A. L. Thuras in the *Bell System Tech. Jl.*, January, 1928, will be discussed in a future paper by the same authors.

Discussion

F. A. Wolff: The reference standard for telephone transmission described in this paper and the apparatus and method that have been developed in connection therewith will in my opinion stand for many years to come.

In the first place, the apparatus and method meet the prime requirement of standards of measurement in that they give us something that is reproducible. In the second place, they furnish a practically distortion-free basis of reference. That eliminates much difficulty. Through the use of the supplementary distortion networks we are nevertheless able to make ready comparisons between actual telephone transmission of a distorted character and the reference network.

The use of the distortion network is exactly similar to the use of color filters in comparing incandescent lamps differing in the quality of the light which they omit. In general, wherever there is a difference of quality, there is difficulty in making comparisons, because human judgment must enter to such a large extent. Eyes differ, so do ears. That makes it necessary in comparing incandescent lamps of one color with those of another color or those of one efficiency with those of another efficiency, to use color filters. The factor for making allowance for this difference in color is determined by making a very large number of observations by a very large number of individuals in order to refer the result to the average individual and the average eye. So with these distortion networks used by the authors.

I think the Bell System is also to be congratulated because of the development of this method. Its presentation is indicative of the enormous improvements that have been made in the quality of commercial long-distance telephone transmission. Until that quality, which is very high today, far exceeds what it is, I do not think it will be necessary to make a revision of the master reference system for telephone transmission. I am sure that will not be required for a long time.

Electrical Wave Analyzers for Power and Telephone Systems

BY R. G. McCURDY* and P. W. BLYE*
Member, A. I. E. E. Member, A. I. E. E.

Synopsis.—This paper describes two types of electrical analyzers which have been developed for the direct measurement of harmonic components of voltage and current on power and telephone systems. These devices are assembled mechanically in a form suitable for use either in the laboratory or in the field. Both instruments, which differ chiefly with respect to sensitivity and input circuit arrangement, employ multistage vacuum tube amplifiers and two duplicate interstage selective circuits.

The power circuit analyzer is designed to measure harmonic voltages in the frequency range from 75 to 3000 cycles and over a voltage range from 0.5 millivolt to 50 volts. The telephone circuit analyzer operates over the same frequency range and measures harmonic currents as low as 0.05 microampere and voltages as small as 0.005 millivolt. Both analyzers are adapted to measure small harmonic voltages and currents in the presence of the fundamental component and other harmonics relatively large in magnitude.

A number of devices are described which have been adopted for eliminating various sources of error. The paper presents in detail the characteristics of both instruments with respect to selectivity, sensitivity, linearity, balance of input with respect to ground, generation of harmonics, and susceptiveness to stray fields.

* * * * *

INTRODUCTION

THE solution of many problems in electrical engineering, involving currents and voltages of complicated wave shapes can be most practicably carried out by a study of the individual sinusoidal components into which these complicated waves may be resolved. This is particularly true in the case of problems dealing with the inductive coordination of power and telephone systems, since the induced noise in telephone circuits occurs largely at frequencies corresponding to the harmonic components in the current and voltage waves of neighboring power systems.

In recent years, many of the development and research problems in inductive coordination, with which the Bell System has been concerned, have required an accurate and comparatively rapid field method of analyzing complex waves. The analyzers which are described in this paper have been developed to meet the particular requirements of these studies.† The development of these instruments has proceeded in close association with the progress of the field work and modifications were made in the designs from time to time as necessitated by the requirements of the work. This has resulted in the production of two types of analyzers, one of which is particularly adapted for use with suitable instrument transformers and shunts on power circuits, and the other for use on telephone circuits. These have been termed for convenience the "power circuit analyzer" and the "telephone circuit analyzer."

These analyzers have been in active service in the field for some time and have permitted the obtaining of much valuable data as to coefficients of induction between power and telephone systems, the wave-shape of power machinery and systems, and analyses of noise currents on telephone circuits. They are arranged in suitable form for use either in the laboratory or in the field and in some instances have been mounted in specially equipped testing trucks.

Aside from their use in this work on inductive coordination, it is felt that the instruments may have a field in other power and communication problems in which a knowledge of the magnitudes of harmonic voltages and currents is important or where measurements at single frequencies are desired in the presence of extraneous voltages and currents as large or larger than the single frequency it is desired to investigate.

The over-all accuracy of an instrument of this type depends upon the conditions surrounding its use. These include the magnitude of the component being measured as compared to the magnitudes of the fundamental and other harmonics present, the relative magnitudes of the voltage across the terminals of the instrument and the voltage between these terminals and ground, and the severity of stray fields from nearby power circuits. Under average conditions, the analyzers which are described in this paper should give results with an over-all accuracy of within $\pm$ 5 per cent of the quantities measured. By means of these instruments it is possible to make a complete analysis of a complex wave over the frequency range up to 3000 cycles in from 30 to 45 min., depending upon the number of harmonic components present. The working up of the results in terms of volts and amperes consists in multiplying together not more than three quantities. The obtaining of a complete harmonic analysis in terms of volts or amperes should not, therefore, require more than one man-hour. The apparatus is so arranged that practically simultaneous analyses may be made of a number of currents or voltages.

GENERAL

As stated above, two general types of analyzers have been developed, the first, being designed for use on power systems, while the second, a more sensitive

*American Telephone & Telegraph Co., New York, N. Y.

†References to a number of other harmonic analyzers on which information has been published in recent years are given in a brief bibliography attached to this paper.

Presented at Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-77

instrument, is intended primarily for measuring currents and voltages of the magnitudes commonly experienced on telephone systems.

The requirements of both types of analyzers with respect to selectivity and at the same time insensibility to commercial frequency variations are practically the same. Therefore, the same selective equipment, designed to give a response curve which is parabolic in

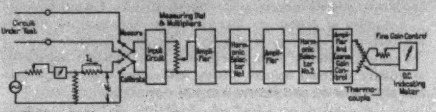


Fig. 1—Schematic Diagram of Analyzer Circuit

form, has been employed in both instruments. Other requirements which have been particularly considered in the development of the analyzers have to do with sensitivity, impedance of input circuits to be provided, insensibility to stray fields, and the limiting of harmonics generated within the instruments due to modulation effects. In both analyzers and in the telephone analyzer in particular, special consideration has been given to the matter of input circuit balance with respect to ground. This is in order that the accuracy of measurement may not be impaired when the voltage across the input circuit is small as compared to the voltage between the input terminals and ground, a condition which frequently obtains on telephone systems. The methods employed in meeting these various requirements and their effectiveness will be discussed in detail below.

Under ordinary conditions the telephone circuit analyzer may be operated satisfactorily on power systems. As will be brought out in the ensuing discussion, however, the power circuit instrument embodies somewhat greater refinement with respect to shielding from stray fields and the limiting of generated harmonics due to modulation. Although the sensitivities of the two instruments are necessarily different and while each instrument includes a number of special features, the general arrangements and methods of operation of the two analyzers are similar.

Electrical Circuit. Each analyzer consists essentially of a multistage vacuum tube amplifier equipped with suitable controls, a measuring dial and associated multipliers, a calibrating circuit, and a sensitive d-c. indicating meter operated by a thermocouple in the output circuit of the amplifier. Selectivity is afforded by means of duplicate tuned circuits inserted between stages of the amplifier. A schematic diagram indicating the general arrangement of the analyzer circuit is shown in Fig. 1.

To assist in visualizing its operation, the analyzer may be considered simply as a combination of a potentiometer and a sensitive selective a-c. voltmeter of infinite impedance, as indicated schematically in Fig. 2. Let it be assumed that a voltage V_c applied to the input of the potentiometer, the sliding contact of which is set at a point C, results in a voltage V across the voltmeter. If the voltage V_c is replaced by a second voltage V_m of the same frequency and if with the potentiometer contact set at a point M, the same voltage V is reproduced across the voltmeter, then from the law of the potentiometer,

$$\frac{V_m}{V_c} = \frac{R_c}{R_m}$$

From this relation, it is evident that the potentiometer might be provided with a voltage scale, graduated in inverse proportion to the resistance, from which, using the voltmeter deflection as a reference, the ratio between two voltages as V_c and V_m might be determined directly. Similarly if the voltage V_c were a known voltage and the scale of the potentiometer were graduated in volts, the point C corresponding to the voltage V_c, the magnitude of an unknown voltage as V_m, might be read directly from the scale.

Referring again to the schematic diagram in Fig. 1, the actual operation of the analyzer circuit is as follows:

For calibration, the input circuit is connected to a source of known single-frequency voltage, V_c, or current I_c, the tuned circuits being adjusted for resonance at the calibrating frequency. The calibrated measuring dial is set to correspond to the magnitude of the calibrating current or voltage. The indicator of the output meter is then brought to a reference position by means of a set of controls which govern the gain of the amplifier. A definite relation is thereby established between the magnitude of the calibrating voltage or current, the indication of the measuring dial, and the deflection of the output meter. The input terminals are then connected to the circuit whose voltage or current wave is to be analyzed. For a complete analysis, the tuned circuits are adjusted progressively over the frequency range to be covered and as the meter responds at various

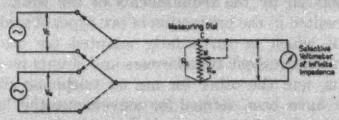


Fig. 2—Equivalent Circuit of Analyzer

frequencies, the tuned circuits are carefully adjusted to give a maximum deflection. The indicator of the meter is then returned to the original reference position by means of the measuring dial and its multipliers. The magnitude of each unknown harmonic is then determined from the product of the setting of the measuring dial, its multiplier, and a small correction factor for the frequency characteristic of the analyzer in case the frequency in question differs from the frequency at which the analyzer was calibrated. The frequencies of the measured harmonics are determined from the

settings of the adjustable condensers in the tuned circuits.

The particular circuit features of the power and telephone analyzers will be discussed in the following sections of the paper.

Mechanical Arrangement. For convenience in transporting and setting up the apparatus in the field, each type of analyzer has been assembled in two units, known as the "amplifier unit" and the "harmonic

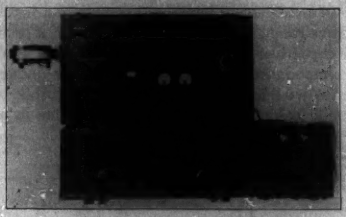


FIG. 3—POWER CIRCUIT ANALYZER AND CALIBRATING OSCILLATOR

selector unit" respectively. The selector units used with both the telephone and power circuit amplifiers are identical. An illustration of a completely assembled power analyzer, together with its calibrating oscillator, is shown in Fig. 3.

The lower unit in Fig. 3 is the harmonic selector, and includes all the equipment making up the two tuned circuits. This apparatus is assembled in a cabinet approximately 33 in. long, 12 in. high, and 11 in. deep, the whole weighing approximately 125 lb. The controls which appear on the panel of the selector unit are dials associated with the adjustable tuning condensers and keys for switching the various tuning inductances and fixed condenser units in and out of the circuit.

The amplifier unit as indicated in Fig. 3, is designed to be mounted directly on top of the harmonic selector. It is assembled in a cabinet of the same length and depth as the selector unit, but is approximately 19 in. high. The amplifier unit weighs approximately 200 lb. The controls which are provided on the panel of the amplifier unit include the measuring dial, a number of keys governing the various measuring circuits used, and miscellaneous equipment incidental to the operation of the amplifier. An ammeter for reading filament current is provided together with a voltmeter which, in conjunction with a number of push button type keys, is used for reading the voltages of the "B" and "C" batteries. The two dials mounted on the upper right-hand corner of the panel provide fine and coarse amplification controls respectively. The vacuum tubes extend outside the front of the panel and are covered by copper mesh which protects the tubes from mechanical injury and also acts as a shield against stray electrostatic fields.

The indicating meter used is of the edgewise type, giving a full-scale deflection on approximately 200 microamperes, and is ordinarily suspended from the left side of the amplifier unit.

The interconnections between the amplifier and selector units are provided with an electrostatic shield as indicated in Fig. 3.

In addition to the equipment outlined above, the analyzer requires an "A" battery of approximately 24 volts, suitable for supplying a current of one ampere, together with a "B" battery of 130 volts from which a current of approximately 40 milliamperes may be supplied.

HARMONIC SELECTOR

A wave analyzer for general use on power and telephone systems must meet two important requirements with respect to selectivity:

1. It must be sufficiently selective to permit the measurement of small harmonics in the presence of relatively large components at neighboring frequencies.
2. The shape of its response curve in the immediate neighborhood of the tuned frequency must be such as to permit small variations in the frequency of the component being measured without resulting in large variations in the magnitude of the analyzer reading.

Two independent tuned circuits operating in tandem,

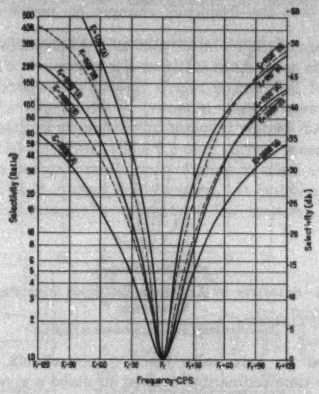


FIG. 4—SELECTIVITY OF TWO SECTIONS OF HARMONIC SELECTOR IN TANDEM

$$\text{Selectivity} = \frac{\text{Response at Tuned Frequency } (F_T)}{\text{Response at Extraneous Frequency}}$$

as employed in the analyzers which are described in this paper, seem particularly adapted to these requirements. The effect of the two circuits in tandem is to discriminate against extraneous harmonic components close to the tuned frequency in proportion to the square of the differences between their frequencies and that to which the circuits are tuned. The response curve obtained with such an arrangement is, therefore, approximately parabolic in form, being relatively flat in the immediate neighborhood of the tuned frequency

and falling off extremely rapidly at frequencies substantially different from that at which the circuits are in resonance.

The maximum selectivity available with two tuned circuits of this type could not be used practicably on commercial systems on account of the errors which would result from small frequency variations. It has been found desirable, therefore, to provide padding resistances which may be switched in series with the resonant circuits to degrade the selectivity somewhat.

The *A* curves in Fig. 4 indicate the over-all degree of selectivity obtained with the padding resistances in series with the two resonant circuits, this being the arrangement commonly used. As may be seen from the 180-cycle *A* curve, it is possible to measure a 180-cycle harmonic in the presence of a 300-cycle component 50 times as great. Since the meter measures the r. m. s. sum of the desired and undesired components, the latter in this case will cause an error of only 2 per cent.

The effect of variations in frequency on the stability of the analyzer reading is indicated by the curves in Fig. 5. These curves are for the normal operating condition, that is, with the padding resistances in series with the tuned circuits. It should be borne in mind that the frequency changes on power systems

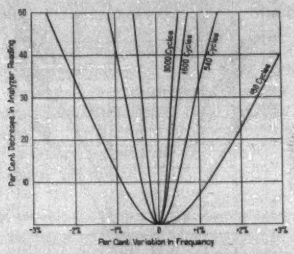


FIG. 5—STABILITY OF ANALYZER READINGS WITH FREQUENCY VARIATION

Two harmonic selector sections in tandem

referred to here take place comparatively slowly. In the time ordinarily required to make a given measurement, therefore, little trouble is experienced from frequency variations.

In cases in which a very high degree of selectivity is required, in the range from 540 to 3000 cycles, and in which the frequency stability of the generating source will permit, the padding resistances may be short-circuited. The degree of selectivity obtained under this condition is indicated in the *B* curves of Fig. 4.

A schematic circuit diagram of a single section of the harmonic selector unit is shown in Fig. 6. As is indicated in this figure each section consists essentially of a series resonant circuit consisting of a fixed inductance and an adjustable capacity. This resonant circuit is coupled to the plate filament circuit of the preceding amplifying tube by a condenser of relatively low impedance (4.5 μf.) which is common to both circuits. The voltage across the inductance branch is utilized as the output of the tuned circuit and is impressed across the grid filament circuit of the following amplifying tube.

In the design of the harmonic selector it was found impracticable to employ a single tuning inductance over the entire frequency range from 50 to 3000 cycles. Such an arrangement would either require a broad and expensive range of adjustable capacities or, if relatively small capacities and large inductances were used, the large impedances of the capacity and inductance branches would render the circuit susceptible to the effects of leakage. Furthermore, variations in ampli-

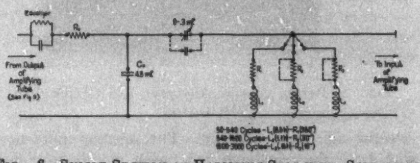


FIG. 6—SINGLE SECTION OF HARMONIC SELECTOR—SCHEMATIC

fication over the range of tuned frequencies would be excessive owing to variations in the losses in a single coil.

An arrangement which has proved practicable provides three tuning inductances of 8.5, 1.1, and 0.15 henrys, respectively, for covering the frequency range as indicated in Fig. 6. Frequencies from 100 to 3000 cycles are covered by adjustable condensers having capacities from 0.0001 to 0.3 μf. The selector is tuned for 50, 60, and 75 cycles by means of fixed condensers which may be connected in parallel with the adjustable units.

The harmonic selector is arranged to give an amplification-tuned-frequency characteristic which is relatively flat over the frequency range from 180 to 3000 cycles. The resistance R_0, Fig. 6, is approximately equal to the impedance of the plate filament circuit of the tube, providing for maximum undistorted output power. Since the reactance of the coupling condenser C_0 is small as compared to the sum of R_0, and the tube output impedance, the current I_0 for a given voltage across the grid filament circuit of the tube is practically independent of frequency. Similarly, since the reactance of the coupling condenser is generally (except at the lower ends of the tuning ranges) small compared to the effective resistance of the tuned circuit, the voltage impressed on the tuned circuit is

$$V_0 = \frac{I_0}{\omega\ (C_0)\ (10^{-6})}, \text{ approximately}$$

which is inversely proportional to frequency. If the effective resistance of the tuned circuit, R_e, were con-

stant the output voltage at any tuned frequency would, therefore, be

$$V_L = \frac{V_0\,(\omega\,L)}{R_s} \text{ approximately}$$

$$= \frac{I_0\,L}{R_s\,C_0\,(10^{-6})} \text{ approximately}$$

and would be independent of the tuned frequency. Actually, of course, the effective resistance of the tuned circuit increases considerably with frequency and the above relation holds over a very limited range of tuned

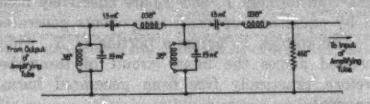


FIG. 7—FILTER FOR R. M. S. MEASUREMENTS—CIRCUIT DIAGRAM

frequencies only. The effect of the increasing effective resistance on the amplification-frequency characteristic of these analyzers is largely neutralized, however, by the insertion of an equalizing circuit consisting of a capacity and resistance in parallel connected in series with the resistance R_0. The remaining effect is taken care of in curves furnished with the instruments in cases in which measured frequencies differ from those at which calibrations are made.

The differences in amplification in the frequency ranges covered by the three tuning inductances are taken care of by independent corrective resistance networks, associated with the resistance R_0, in the three ranges.

In addition to the arrangements for measuring the fundamental and individual harmonics, an additional circuit is provided for measuring the r. m. s. sum of all harmonics between 180 and 3000 cycles. This device includes two sections of modified confluent band filter which may be substituted for the first tuned circuit of the harmonic selector and an equalizer section, for flattening the frequency characteristic in the measuring range, which is substituted for the second tuned circuit. The transmission-frequency characteristic of this arrangement is such as to cut off between 60 and 180 cycles and between 3000 and 5000 cycles, the curve in the range between these points being practically flat. A diagram of the circuit for r. m. s. measurements is shown in Fig. 7.

POWER CIRCUIT AMPLIFIER

In the design of a suitable amplifier unit for use with the harmonic selector in measurements on power systems the following requirements have been particularly considered:

1. Its input circuit including the measuring dial and multipliers must be linear with current and voltage over the range to be measured at any given frequency.

2. The input circuit must be sufficiently well balanced with respect to ground to permit the measurement of small voltages and currents with relatively large voltages existing between the input terminals and ground.

3. Harmonics generated within the instrument itself must be kept at a minimum and if possible should be less than those generated in instrument transformers commonly used on power systems.

4. Its accuracy must not be impaired by pick-up from such stray electromagnetic and electrostatic fields as are commonly experienced in the vicinity of power apparatus.

As stated above, the amplifier unit consists of a vacuum tube amplifier and its associated controls together with a measuring dial and a calibrating circuit. Three input circuits are provided for measuring power circuit voltages, currents, and small voltages across shunts, respectively. The impedances of these three circuits and the sensitivities available are as follows:

Quantity measured	Input impedance	Minimum measurable current or voltage
Voltage	10,000 ohms	0.05 volt
Current	0.2 ohm	2.5 milliamperes
Millivolts	1,000 ohms (Approx.)	0.5 millivolt

At fundamental frequencies the minimum measurable voltages and currents are ten times those given above.

The 10,000-ohm voltage circuit is intended for use across the secondaries of potential transformers or in other locations where the fundamental voltage does not exceed 110-150 volts. The 0.2-ohm current measuring circuit is designed particularly for use in the 5-ampere secondaries of current transformers. It is designed to carry continuously a current of 7.5 amperes. The

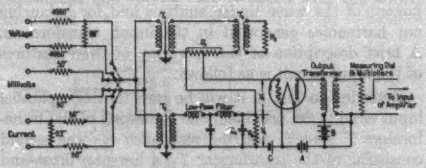


FIG. 8—POWER ANALYZER—INPUT CIRCUIT—SCHEMATIC

millivolts circuit may be used under a variety of conditions and is particularly intended for use with power system shunts. The constants of the three circuits are so arranged that their calibrations are made simultaneously by a single adjustment of the amplifier gain.

A schematic diagram indicating the general arrangement of the input circuits in the power amplifier is given in Fig. 8. These circuits are designed to measure harmonic components over a range of from 1 to 1000 times the current and voltage values given in the above table. Errors due to non-linearity in the input circuits

over this range do not exceed approximately ± 3 per cent at frequencies from 120 to 3000 cycles. Except at the extreme ends of the scale at frequencies below 180 cycles such errors do not exceed ± 1 per cent. Fundamental components of current and voltage can be read up to 7.5 amperes and 150 volts, respectively.

In order to avoid errors due to the effects of relatively large voltages to ground on the input terminals the input circuits are accurately balanced with respect to ground. Transformers whose primary windings are especially well balanced and which include interwinding electrostatic shields are employed between the line terminals and the measuring circuit. Similar precautions are taken to avoid unbalances and stray capacity effects in the wiring external to the transformers. As an example, the degree of balance attained in the input circuits of one particular instrument is indicated in the following table. These figures represent the apparent voltages across the circuit resulting from the effect of one volt between the input terminals and ground, the analyzer being adjusted to measure 60-cycle harmonics:

Frequency (c. p. s.)	Apparent voltage across measuring circuit per volt above ground	
	Current and millivolts circuit	Voltage circuit
180	0.000007	0.0007
540	0.000008	0.0008
1020	0.000013	0.0014
2100	0.000021	0.0022
3000	0.000024	0.0027

In order to avoid the generation of harmonics of appreciable magnitudes within the analyzer itself due to modulation effects it was found necessary to provide special devices for suppressing the fundamental component of the wave under analysis and for balancing out harmonics generated in the input transformers. A brief description of the operation of these features of the input circuit is as follows:

Referring to Fig. 8, it will be seen that two parallel input circuits are provided, one consisting of the transformers T_1 and T_3 and the resistance R_1, the second consisting of the transformer T_2, a low-pass filter, and the resistance R_2. In the first branch, the complex voltage impressed across the primary of the transformer T_1 is reproduced in practically undistorted form across the resistance R_1. In the second parallel branch, the harmonics in the complex wave are effectively suppressed by the low-pass filter, the voltage across the terminating resistance R_2 being of practically pure fundamental frequency. By a proper adjustment of the circuit constants, the voltage across R_2 may be made practically equal to an opposite in phase to the fundamental component of a portion of the complex voltage across R_1. By impressing these two voltages in series across the grid circuit of a vacuum tube, the fundamental component of the complex wave is effectively balanced out.

With the fundamental component eliminated ahead of the first amplifying tube there remain the harmonic components originally in the complex wave and those generated in the input transformers. These latter components, which result in errors in measurements of small harmonics, may be effectively reduced by means of a third transformer T_3 connected as indicated in Fig. 8. By a proper adjustment of the resistance R_3 across the secondary of the transformer T_3 the harmonic components of current in the resistance R_1 generated in the transformers T_1 and T_2 are effectively neutralized by equal and opposite components generated in T_3. The voltage V_1 across a portion of R_1, which is impressed across the input circuit of the tube is, therefore, practically free from generated harmonic components.

These means for controlling the generation of harmonics in the power analyzer have proved very effective. In one particular instrument the generated harmonics observed in the case of a 60-cycle fundamental voltage of 102 volts were as follows:

Fundamental voltage	Per cent generated harmonics						
	2nd	3rd	4th	5th	6th	7th	9th
102 volts	0.07	0.08	0.008	0.02	*	*	*

*Too small to measure—less than 0.01 per cent.

The power amplifier unit is specially shielded against stray electrostatic fields at all susceptible points with copper shielding. Similar shields of annealed iron are provided with all susceptible wound apparatus as a protection against stray electromagnetic fields. A large number of tests carried on in the field indicate that the analyzer is sufficiently well shielded for general use in the vicinity of power apparatus.

A circuit diagram of the amplifier which is used in conjunction with the input circuits described above is shown in Fig. 9. This portion of the circuit is identical in both the power circuit amplifier and the telephone circuit amplifier which is to be described below.

Telephone Circuit Amplifier

The amplifier which has been designed for use with the harmonic selector on telephone systems differs from the power circuit amplifier chiefly in the matter of sensitivity and in the arrangement of its input circuits. Six vacuum tubes are used in the telephone circuit amplifier as in the case of the power circuit unit. In the former, however, added amplification is obtained by the utilization of the first tube as a voltage amplifier instead of as a blocking device as in the power circuit analyzer.

A schematic diagram of the input circuit of the telephone circuit amplifier is shown in Fig. 10. As is indicated in this diagram the input circuit is provided

with three individual transformers by means of which the following circuit arrangements and sensitivities are obtained:

Quantity measured	Input impedance	Minimum measurable current or voltage
Voltages	6 ohms	0.005 millivolt
Voltages	600 ohms	0.05 millivolt
Voltages	120,000 ohms	0.5 millivolt
Voltages	1,000,000 ohms (600 cycles)*	5. millivolt
Power voltages	100,000 ohms	0.05 volt
Currents	600 ohms	0.05 microampere
Currents	1,200 ohms	0.035 microampere
Currents	1,800 ohms	0.03 microampere
Currents	6 ohms	0.5 microampere

*This circuit consists of 0.00025 μ f. in series with the 120,000 ohms of the transformer and measuring dial. Its impedance and sensitivity, therefore, vary with frequency.

The minimum measurable currents and voltages at fundamental frequencies are ten times those tabulated above.

currents. Its impedance is purposely low in order that it may not materially affect the impedance of a circuit in which it may be inserted.

As is indicated in the above, circuits are provided which are practically equivalent to those of the power analyzer. In cases in which somewhat greater susceptiveness to stray fields and somewhat larger generated harmonics can be tolerated, the telephone unit may be conveniently substituted for the power amplifier.

The input circuits of the telephone circuit amplifier have been found to operate satisfactorily over a range of currents and voltages from 1 to 2000 times those tabulated above. In no case have these circuits been found to depart from linearity by more than the specified limit of $\pm$ 3 per cent over this range. In fact, over practically the entire range the maximum error introduced by non-linearity is less than $\pm$ 1 per cent.

The balance of the input circuits with respect to ground is an important feature in the case of the telephone circuit amplifier in view of the large ratios between voltage to ground and metallic circuit voltage which usually exist on telephone lines. Input transformers designed to meet rigid balance requirements and provided with interwinding shields are used in this instrument and special care is taken with the wiring associated with the input circuits to avoid unbalance

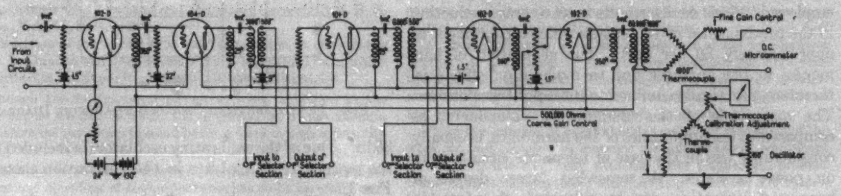


FIG. 9—AMPLIFIER UNIT EXCLUDING INPUT CIRCUITS—CIRCUIT DIAGRAM

Individual gain adjustments are provided in the input circuits so that simultaneous calibrations may be made of the 6-ohm, 600-ohm and 120,000-ohm circuits. In cases in which a number of different quantities are to be measured at a particular frequency this is an important feature since it permits the use of all three circuits with a single calibration.

The 6-ohm voltage circuit is intended for the measurement of voltages across small shunts. The 600-ohm voltage circuit may be used for similar purposes or for measuring the voltage across a metallic telephone circuit. The 120,000-ohm and 1,000,000-ohm circuits are designed for measuring the open-circuit voltages between wires or between line wires and ground, the higher impedance circuit being intended primarily for use on short sections of line of high impedances.

The 10,000-ohm circuit is intended for use on power voltages and is designed to operate at fundamental voltages up to 120. The sensitivity of this arrangement is the same as that of the voltage circuit of the power analyzer.

The 600-, 1200-, and 1800-ohm circuits are designed for the measurement of metallic circuit noise currents. The impedances provided match, approximately, the impedances of common types of telephone circuits. The 6-ohm circuit is intended for measuring longitudinal

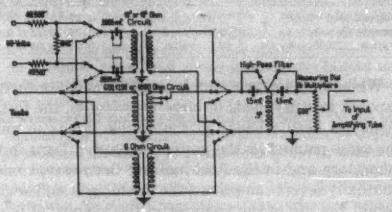


FIG. 10—TELEPHONE ANALYZER—INPUT CIRCUIT—SCHEMATIC

to ground and stray capacities between the line circuits and the measuring dial.

The degree of balance attained in the input circuits is indicated in the following table. The figures given represent apparent voltages across the circuit resulting from the effect of one volt between the line terminals and ground acting on the circuit unbalances.

Input circuit	Frequency	Apparent voltage across circuit
Current (6–ohm)	3000	0.0000015
Current (6–ohm)	540	0.00000035
Current (600–ohm)	3000	0.000012
Current (600–ohm)	540	0.000002
Voltage (120,000–ohm)	3000	0.00029
Voltage (120,000–ohm)	540	0.00002
Voltage (0.00025 μf.)	3000	0.0046
Voltage (0.00025 μf.)	540	0.0042
Power voltages	3000	0.025
Power voltages	540	0.001

As is indicated in the circuit diagram of Fig. 10, a high-pass filter consisting of a series condenser of 1.5 μf. and a shunt inductance of 0.5 henry is included in the input circuit. This filter is effective on systems having a fundamental frequency of 60 cycles or less, in suppressing the fundamental component of the complex wave under analysis. The condenser may be connected on either side of the inductance in order that the filter may be used effectively either in series with a high impedance or in shunt with a low impedance as may be required by the particular measuring circuit employed. This device results in an average reduction in the fundamental of 13 decibels which is equivalent to approximately 80 per cent. The generation of harmonics in the analyzer due to modulation effects is, therefore, practically confined to the input transformers. The magnitude of the fundamental component, as compared to the amplitudes of the harmonics, is usually considerably less in the case of telephone circuits than on power systems. A somewhat larger degree of harmonic generation can be tolerated, therefore, in the case of the telephone circuit analyzer. The generated harmonics in one particular telephone circuit amplifier unit, expressed in per cent of the fundamental component are as follows:

Circuit	Fundamental component	Per cent generated harmonic			
		2nd	3rd	5th	7th
Current (600–ohm)	0.001 ampere	0.28	0.27	0.049	0.006
Current (6–ohm)	0.01 ampere	0.25	0.25	0.05	0.009
Voltage (120,000–ohm)	10. volts	0.25	0.25	0.05	0.007

While the telephone circuit amplifier is not ordinarily exposed to stray fields of large magnitudes, the instrument has been shielded at susceptible points in much the same manner as the power analyzer. Tests in the laboratory and in the field indicate that, except under unusually severe conditions, the shielding is sufficient to permit the use of the analyzer in the vicinity of power apparatus.

Calibrating Oscillator

As discussed above, a source of current or voltage of known frequency is required with the analyzers for calibration purposes. For use in cases in which one of the standard adjustable-frequency oscillators is not available a small portable vacuum tube oscillator has been developed.

The calibrating oscillator employs two tubes, the first functioning as an oscillator and the second as an amplifier as indicated in the circuit diagram shown in Fig. 11. The circuit is arranged to produce frequencies of 180, 540, 1500, or 3000 cycles. These permit calibration of the analyzers in each of the three frequency ranges covered by the harmonic selector and also provide suitable frequencies for checking the frequency charactistics at both ends of each range. The oscillator is designed to give a minimum output of 10 milliamperes in 150 ohms at each frequency. The waveshape of the output at each frequency is such that the r. m. s. sum of all the harmonics is less than 5 per cent of the fundamental.

The equipment making up the oscillator is assembled in a cabinet approximately 13 in. long, 12 in. wide, and 7 in. deep, the complete unit weighing approximately

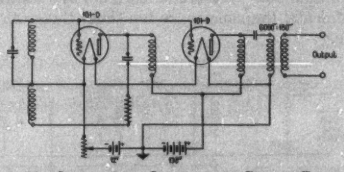


Fig. 11—Calibrating Oscillator—Circuit Diagram

35 lb. One of the calibrating oscillators is included with the power circuit analyzer in the illustration shown in Fig. 3.

The current supplied by the oscillator to the calibrating network is controlled by a potentiometer located in the amplifier unit. The magnitude of this current is determined from the reading of a thermal milliammeter consisting of a thermocouple and the galvanometer provided with the amplifier unit.

Bibliography

The following list of references is by no means intended as a complete bibliography on the subject of harmonic analysis. It is merely to tabulate references to a number of noteworthy articles on the subject which have come to the attention of the authors.

1. "An Electrical Harmonic Analyzer," J. D. Cockroft, R. T. Coe, J. A. Tyacke, and M. Walker, *Jl. I. E. E.*, Vol. 63, Jan., 1925.

2. "Une Méthode Potentiometrique d'Analyze Harmonique des Ondes des Comants Alternatifs des Alternateurs," A. Blondel, *Rev. Gén. de l'Elec.*, Mar. 7, 1925.

3. "An Electrical Frequency Analyzer." R. L. Wegel and C. R. Moore, *Bell System Tech. Jl.*, Apr., 1924.

4. "An Analyzer for the Voice Range," C. R. Moore and A. S. Curtis, *Bell System Tech. Jl.*, Apr. 1927.

5. "Analyzer for Complex Electric Waves," A. G. Landeen, *Bell System Tech. Jl.*, Apr., 1927.

6. *A Simple Harmonic Analyzer*, V. Bush, Jl. A. I. E. E., Vol. 39, 1920, p. 903.

7. *An Electromechanical Device for Rapid Schedule Harmonic Analysis of Complex Waves*, F. S. Dellenbaugh, Jr., Trans. A. I. E. E., Vol. 40, 1921, pp. 451-478.

8. *Another Harmonic Analyzer*, F. S. Dellenbaugh, Jr., A. I. E. E. Jl., Jan. 1923, pp. 58-61.

9. "Harmonic Analyzer for Use on Power Circuits." Serial Report of the Inductive Coordination Committee, National Electric Light Association, Jan., 1928.

Special attention is called to an extensive bibliography on this subject appended to Professor Dellenbaugh's paper of 1921.

Other references are included in a bibliography published by the American Committee on Inductive Coordination in 1925.

Discussion

J. J. Smith: The principal lines along which efforts have been made in the past to develop such analyzers are:- (1) By means of tuned circuits, similar to the analyzer described in this paper, where the frequency which it is desired to measure is tuned and the others are reduced by attenuation. (2) By use of the dynamometer principle where a known current of the frequency which it is desired to measure is introduced in one coil of the dynamometer and the wave to be analyzed is in the other coil. The first suggestion in the literature that I have been able to find of an analyzer of the first type is in a paper by Pupin entitled *Resonance Analysis of Alternating and Polyphase Currents*, A. I. E. E. Trans., Vol. XI, 1894, p. 523.

An analyzer of the second type was suggested in an early paper by Dr. L. Duncan and later developed into a practical form which is described in the paper in reference 1 of the bibliography.

Although the fundamental principles were embodied in the above papers we had to await the development of the vacuum tube before the instruments could be developed to high sensitivity and high accuracy.

In addition to the development of the vacuum tube the proper balance and shielding of such apparatus has to be carefully considered, and the figures given by the authors on p. 1172 show that this has been well taken care of. I have used one of these analyzers for the last few years and experienced no difficulty due to extraneous fields when the apparatus is set up in the vicinity of the usual type of machines tested in the factory. On one occasion, however, when making measurements on a large reactor, trouble was experienced due to pick-up in the instrument (which was set quite close to the reactor). This pick-up, however, was easily detected since it was independent of the multiplier and dial readings. The remedy, of course, was to remove the instrument to some distance from the reactor.

The input circuit described in Fig. 8 of the paper, while allowing high accuracy for standard frequencies such as 25, 50, and 60 cycles, is not always very suitable for work in the factory where it may be desirable to make experiments over a range of intermediate frequencies. I would suggest that the authors give a table showing the probable error for the frequencies in between.

Our experience in using this meter confirms the statement that the analyzer described gives results within ± 5 per cent of the quantities measured, since we have made measurements on a machine and repeated them later under as nearly as possible the same set of conditions and the results agreed within the above order of accuracy.

It may be interesting to point out that with the sensitivity obtained with this analyzer we have gone down to the region where harmonics generated in other parts of our measuring equipment may be a source of error. For instance, in measuring on a high-voltage line it is necessary to use potential or current transformers. On account of the characteristics of the magnetization curve of iron the exciting currents of these transformers will contain harmonics which, although small, will affect the result if the wave which is being measured is very close to a sine wave.

J. O. Coleman: The authors of the paper on electrical wave analyzers deserve considerable credit for the development of the apparatus. I have had occasion in the past five years, in connection with the joint development and research of the N. E. L. A. and the Bell System, to use the analyzers for measurements under many varied conditions. Needless to say, there have been times when we have had considerable trouble in making the required measurements, and part of the time the difficulty was with the analyzer.

Mr. McCurdy mentioned the development since the first analyzer was constructed. I also used the first one. With it a fraction of an ampere in a wire 15 or 20 ft. away, under some conditions, would cause full-scale reading on the meter with nothing connected to the input terminals. With the improved models we have made analyses, in a power substation, over the top of 15 to 20 outgoing feeders with no serious disturbance from them. This, I think, speaks quite highly for the development of the shielding.

The analyzer in the paper appears to be quite a complicated piece of apparatus. One would think that with all the dials and the switches on it, considerable time would be required for measurements. In connection with some of our work, we have made upward of 2000 separate measurements a day.

Another consideration in speaking of balance is that there are two ways of considering ground as far as the analyzer is concerned. If you tie the analyzer shield to the earth, you have to face whatever potentials you are placing on the input coil. The other alternative is to tie the analyzer shield to whatever circuits you are measuring and thereby eliminate the large voltage across the input coil. The analyzer has been used satisfactorily, directly connected to a 2300-volt circuit with its batteries and operator completely isolated from the ground.

There is a number of by-products of anything. In some of the work we had occasion to measure the impedance of equipment at the harmonic frequency with the lower frequencies present. Some equipment may have different impedance at 300 cycles; for instance, when 60 cycles are present from the impedance it has when the 60 cycles are not present. In cases of this character the analyzer can be used strictly as a galvanometer and measurements obtained which would be otherwise very difficult.

L. J. Corbett: (communicated after adjournment) The authors have described the most recent type of mechanical analyzers of periodic e. m. f. and current waves, by which the strength of the components of each frequency may be measured with considerable accuracy,—while it remains constant. The modern developments in amplifiers are made use of in the measurement of very low values of some of the components. In discussing the paper, the writer has no wish to detract from the praise which is justly due to the engineers of the telephone industry and of the power industry whose efforts have produced such an accurate, usable, and strong instrument. Instead he wishes to supplement the paper by adding a word of caution regarding its use.

The waves which are most commonly analyzed by the use of such equipment are those which exist in power circuits, or which may be produced inductively in a second circuit not far removed from a power circuit. The waves in an ordinary power circuit are subject to fluctuations which primarily affect the voltage, and secondarily, the relative proportions of the various harmonics when electrical equipment containing iron magnetic circuits is involved. A third possibility is a slight change in fundamental frequency following load changes, which in turn alters to a far greater degree the frequencies of the higher harmonics. Again, resonance conditions in the power circuit may so favor certain frequencies that the slight change in fundamental frequency in itself changes the relative values of the various harmonics. Speed changes on a 60-cycle system may amount to 0.4 cycle high or low, the return to normal being rather slow and gradual. On the higher harmonics this will amount to several cycles, and from the

curves of Fig. 5 of the paper, the accuracy is low and a re-setting is necessary. In the re-setting, it is evident that conditions are not the same as when the earlier readings were taken, so the readings of the various harmonics taken consecutively are only roughly comparable. Only by simultaneous readings of several mechanical analyzers can dependable readings of a number of harmonics be obtained.

A mechanical analyzer tells nothing of the relative phases of the different harmonics so it does not tell the whole story. Important physical relations are thus slighted, which, if they were known, would give a clearer insight into electric wave phenomena.

Fig. 1—Residual Current on Line Serving Y/Y Bank

It appears to the writer that more use can be made of the oscillograph, possibly in conjunction with the mechanical analyzer. The oscillograph in its present stage of development, is admittedly a rather coarse-reading instrument. However, it does show the relative phases of the lower harmonics quite obviously, and the relative phases of the higher harmonics by somewhat tedious methods of measurement and calculation.

The determination of residual current and voltages in three-phase power systems is the aim of many investigations in which analyzers of the type described in the paper are used. To illustrate some of the physical relations existing which can be shown by the oscillograph, the accompanying oscillograms are presented. At the substation at which the tests were made there were two

Fig. 2—Residual Current on Line Serving Y/Δ Bank

busses, on one of which all lines and transformer banks were first connected. One transmission line at a time was then served by the second bus through the bus paralleling switch. The residual current for each line was obtained from the current transformers of the paralleling switch and was recorded on the middle element of the oscillograph. Above is a 60-cycle timing wave from the station lighting circuit, and below is a constant 180-cycle reference wave obtained between the neutral of a Y-connected set of coils on the 110-volt side of the potential transformers of the first bus, and ground.

The substation at which the tests were made contained only Y-connected transformers with neutrals grounded and without deltas. In Fig. 1 herewith the middle element shows the residual current in a line from the substation serving a station at which a large Y-Y bank with grounded neutral is connected without a delta. In Fig. 2 the same element shows the residual current in a line to a station where the banks are connected grounded Y to delta. In Fig. 3 the line concerned was energized but disconnected at the other end, a combination of leakage and charging current being obtained.

There can also be shown the reversal of direction of flow of such currents due to system changes, as illustrated in a paper presented by the writer before the Pacific Coast Convention of the Institute at Spokane last year.[1] Because of these physical relations which are ignored by the analyzer, it appears that the use of such an instrument as the oscillograph in conjunction with it, would assist materially in many problems.

P. W. Blye: As pointed out by Mr. Smith in his discussion, the input circuits of the power analyzers are provided with fixed values of resistance (R_1, Fig. 6 of paper) and capacity (C_5) to reduce generated harmonics to a particularly low level when the fundamental frequency is 25, 50, and 60 cycles. These three frequencies were selected as those most likely to be encountered in practise. There is no reason, however, why additional values of the resistance R_1 and the capacity C_5 might not be provided for use with other fundamental frequencies, if desirable.

Fig. 3—Residual Current on Idle Line (Leakage and Charging Current Only)

As requested by Mr. Smith, the following table indicates the magnitudes of generated harmonics observed in the case of one particular instrument when fundamental voltages were applied to its input circuit having frequencies different from those for which the input circuit was adjusted. A fundamental frequency of 1 volt was applied to the millivolt terminals in all cases, this being equivalent to 100 volts applied to the voltage terminals.

Input circuit adjusted for	Applied frequency	Generated harmonics per cent of fundamental		
		3rd	5th	7th
60 cycles	55	0.40	0.11	0.06
50 "	45	0.25	0.085	0.05
25 "	40	0.60	0.35	0.07
25 "	35	*	0.33	0.07
25 "	30	*	0.22	0.06

*Analyzer not arranged to be tuned at 90 and 105 cycles.

The following table gives additional data as to the magnitudes of generated third harmonics in a single instrument whose input circuit was adjusted for 60 cycles. In this latter case, the applied frequency was varied in small steps either side of 60 cycles.

1. *Residual Voltages and Currents in Power Systems*, A. I. E. E. Quarterly Trans., Vol. 48, January 1929, p. 92.

Applied frequency	Per cent generated third harmonic
58	0.185
58.5	0.13
59	0.085
59.5	0.052
60	0.04
60.5	0.05
61	0.077
61.5	0.12
62	0.165

Mr. Corbett in his discussion points out the fact that the accuracy of readings taken with analyzers of this type on operating power systems may be seriously impaired due to system frequency variations. Experience with these instruments on a large number of operating systems, both large and small, indicates that relatively rapid variations in the magnitudes of various harmonics under operating conditions are of considerably greater importance than the slower changes due to the frequency variations described by Mr. Corbett. It is possible with an indicating instrument of this type, however, to follow such variations closely and to obtain readings representing average conditions over as long a period of time as may be desirable. During the past few years a large amount of data has been obtained with these analyzers on various types of power systems and, in general, the data have been constant and their accuracy has been satisfactory. The figure of ±0.4 cycle quoted as representative of frequency variations on operating systems is probably intended as an outside figure and is consistent with the authors' experience. However, from observations of frequency recorders on a number of the larger systems it would appear that on such systems frequency variations, on an average, do not amount to more than from ±0.1 to ±0.2 cycle.

As further pointed out by Mr. Corbett, the analyzers described in this paper do not measure phase angles. In general, however, in inductive-coordination studies, for which these analyzers were primarily developed, a knowledge of the phase relations between the various harmonics is of secondary importance. Where such information is desired, it is often possible to obtain satisfactory data at the lower frequencies by means of the oscillograph, as discussed by Mr. Corbett. The oscillograms which he presents demonstrate quite clearly the magnitudes and relative phase angles of several third-harmonic residual currents. However, as he points out the magnitudes and phase angles of the higher harmonic components present can be obtained only by a laborious process of measurement and calculation. Furthermore, if the magnitudes are small the precision obtained is low. It was to obtain information as to the magnitudes of such relatively small but important higher harmonics, as well as the larger lower-frequency components, that the analyzers described in this paper were developed. As mentioned by Mr. Coleman in his discussion, by means of these analyzers as many as 2000 separate measurements of harmonic currents and voltages have often been made in a single day. From the standpoint of time required, the use of an oscillograph in such work would obviously be out of the question.

A New Automatic Synchronizer

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Associate A. I. E. E.

Synopsis.—*The fundamentals of automatic synchronizing are discussed, and the requirements which must be met by the ideal synchronizer are outlined. The design and the principle of operation of two different models of a newly developed automatic synchronizer are described.*

The paper treats the method by which, for any frequency difference within limits of present operating practise, either model of the synchronizer will cause the closing coil of the circuit breaker to be energized in advance of synchronism, by a time equal to that required to close the circuit breaker. The closing coil of the circuit breaker is, therefore, energized when the phase displacement between the voltages of the two systems to be synchronized is such that, assuming the frequency difference between the two systems remains constant during the short time required to close the breaker, the breaker contacts will always be closed at the instant of zero voltage phase displacement.

The reasons for the excellent performance of these automatic synchronizers even when applied to connect systems with very erratic frequencies are outlined.

A series of tests is referred to, showing the superiority of automatic synchronizing in comparison with manual synchronizing, and the results obtained with the new synchronizer models in a generating station with propeller type waterwheels are described.

Fundamentals of Synchronizing

The synchronizing of a-c. generators is one of the most exacting duties required of an operator in a manually operated generating station. When the operator is bringing a new unit on the line he has many duties, and his time is crowded. He must match the voltage of the machine with the line voltage, he must regulate the speed of the prime mover by hand until the frequency difference between the two sources to be paralleled is within the permissible synchronizing range, and at the same time the operator must closely watch the synchronoscope, indicating the phase displacement and frequency difference between the two sides of the breaker to be paralleled, and choose the proper moment to throw the control switch, closing the breaker, so that the relatively slow closing breaker will actually close its contacts at the point of zero phase displacement between the two system potentials.

Due to variations in the speed of the generator and since the frequency of most systems shows variations, the synchronoscope will often not rotate at uniform speed, and this makes the task of the synchronizing operator still more difficult to handle. The operator himself knows that a false move by a fraction of a second may wreck a machine worth hundreds of thousands of dollars, or at least may cause disturbances on the system so severe that important synchronous load connected to it may fall out of step. For this reason the influence of the human uncertainty element is more pronounced in non-automatic synchronizing than in other generating station operations where the result of a faulty move will not be as disastrous.

It has long been the trend within the field of electrical operations to try to eliminate the human element and substitute for it a mechanical device which is not affected by nerves and human emotions, and which always can be relied on to operate at the topmost rate of efficiency and performance. This is even more the case when the mechanical device will do a better job than a skilled operator, and do it cheaper.

Automatic synchronizing stations have been in use for several years. Two different synchronizing methods have been applied, either the self synchronizing method or the automatic synchronizing method. In the self synchronizing method the machine is connected to the system without any field when the frequency of the machine is about 80 per cent of the system frequency. The field is then gradually applied and in this manner the incoming machine is pulled into step. In the automatic synchronizing method the synchronizing is done in the same manner as when manual synchronizing is being used. The automatic synchronizer takes the place of the operator and closes the breaker, while the frequency of the incoming machine is matched with the frequency of line by means of separate speed matching equipment.

The self-synchronizing method has been most commonly applied, especially in applications with erratic frequencies, because the characteristics of the automatic synchronizers previously available were such that it would require too long to synchronize the machine with the line if the frequencies of the two systems were not very stable. However, self synchronizing can usually not be applied when:

1. The capacity of the generator to be connected exceeds $\frac{1}{4}$ to $\frac{1}{5}$ of the capacity of the system.

2. Where important synchronous load connected to the station bus requires that the machine be connected to the system as smoothly as possible.

3. Where the generator to be connected is not equipped with damper windings.

Due to the increased use of the propeller type waterwheels and due to the general interest in automatic stations, a need was felt for a new automatic synchronizer that would synchronize even very erratic machines with the same accuracy and within a shorter time than a skilled operator at his best was able to do.

It may be advisable at this point to mention the

1. Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

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29-120

various factors which are important in any synchronizing application.

When two synchronous generators are to be connected in parallel, the equalizing current between them in the paralleling moment will be zero if, when the breaker contacts engage:

1. The voltages are equal.
2. The phase displacement between the voltages is zero, and
3. The frequency difference is zero between the two systems to be connected.

The voltage equality will have to be taken care of by voltage regulating equipment outside the automatic synchronizer, and will not be discussed in this paper; but the effect of a voltage inequality on the performance of the automatic synchronizer will be mentioned later in this paper.

Zero-Phase Displacement. The equalizing current between the two sources in the paralleling moment will be proportional to the sine of the angle of phase displacement between the voltages of the two systems to be connected, and for smaller angles therefore, (below 45 deg.) the angle at which the breaker contacts engage will give an approximate check on the amount of equalizing current, and consequently the closing angle can be used as a measure of the performance of the synchronizer.

Zero-Frequency Difference. The third requirement mentioned above, the zero-frequency difference, is of a merely theoretical interest, because in the majority of practical applications zero-frequency difference cannot be obtained, at least not within a reasonable time. Consider a case where a generator is to be synchronized with a system. By means of speed matching equipment acting upon the governor of the prime mover the generator is brought up very close to synchronous speed. With an absolutely constant system frequency and with ideal speed matching equipment it would be possible to obtain zero-frequency difference and the ideal synchronizing conditions would then obtain when the phase displacement between the two voltages was zero. In practise, however, the sensitivity of the speed matching equipment is not absolute, and furthermore, even with large systems momentary frequency variations may occur due to load variations. For this reason the frequency difference in synchronizing applications will usually be fluctuating, often within a very wide range not seldom within $\pm$ 1 cycle, and in most applications zero frequency difference will merely obtain in a short instant when the speed of the generator is changing from leading to lagging or vice versa, and consequently an automatic synchronizer designed to close the breaker only when the frequency difference is zero, would not have much chance to complete the synchronizing operation.

Fortunately, however, in actual operating practise exact zero-frequency difference is not necessary. Smaller generators can be safely connected to the line even with frequency differences as high as ¼ cycle per second, while larger units will require a lower frequency difference. It must, however, be required that the breaker contacts be closed at the point of zero voltage phase displacement. The underlying idea of the design of an automatic synchronizer is therefore to provide an apparatus that will close the breaker connecting two sources to be synchronized if the frequency difference between the two sources is below a predetermined value to be chosen for each particular application, and close the breaker at such a moment that, independent of the instantaneous frequency difference, the breaker contacts will always engage at the moment of zero phase displacement between the voltages, or at synchronism.

The factor that complicates the design of an ideal automatic synchronizer is the time element of the circuit breaker. There is always a certain time required from the energizing of the closing coil of a circuit breaker until the breaker contacts close. This time is practically constant for a definite breaker, but varies for different types and sizes of breakers from 0.2 sec. to 0.6 sec. Suppose that a breaker having a time element of 0.5 sec. is being applied to connect two systems, and that a frequency difference of 1/6 cycle corresponding to one revolution of the synchronoscope in 6 sec. is present. To obtain breaker closure at zero phase displacement with this breaker, at lockout frequency difference 1/6 cycle the synchronizer must energize the breaker closing relay at a point 30 deg. in advance of zero-phase displacement.

Suppose now a machine with comparatively small inertia and excellent governor equipment is to be synchronized, and that the frequency difference during one revolution of the synchronoscope change from 1/5 cycle to 1/40 cycle. One-fifth cycle is above the lockout frequency difference and the synchronizer would therefore not close at this frequency difference, but would close on the next beat at a 1/40-cycle frequency difference, and the result would be that the breaker contacts would close 25.5 degrees ahead of synchronism, and since at this point the voltage difference between the two sources to be connected is 43 per cent of the line voltage, considerable equalizing currents will result. To obtain correct closure at 1/40-cycle frequency difference, and assuming that the frequency difference remains constant during the short time of breaker closure, the synchronizer must energize the breaker closing relay at a point 4.5 degrees ahead of synchronism.

From the above considerations, the following requirements to be met by a successful automatic synchronizer can be summed up.

1. The synchronizer must operate only within a predetermined frequency difference. A single adjustment must be provided to vary the synchronizing range covered by the apparatus.
2. The synchronizer must energize the breaker closing relay in advance of zero phase displacement. The amount of advance must be proportional to the in-

stantaneous frequency difference. By one single adjustment it must be possible to vary the rate of change of advance to adjust the synchronizer for breakers with different closing time.

The Design of the New Automatic Synchronizer

In Fig. 1 is shown a front view of the new automatic synchronizer with the front glass cover removed. Two different models of this apparatus are available. The design and appearance of the two models are quite alike, but the principle of operation is different, one model being designed especially for applications with very erratic frequencies.

A schematic outline of the apparatus is shown in Fig. 2, and the following description of the two models

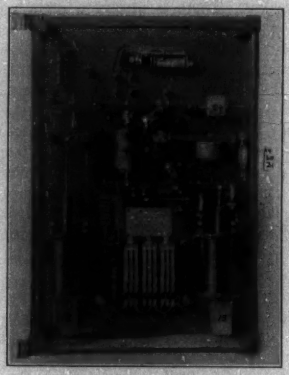

Fig. 1—Front View of the Automatic Synchronizer with Cover Glass Removed

refers to this figure. The models will be referred to as type XY-11 and type XY-12, the latter being the model designed for erratic frequency applications.

The synchronizer consists essentially of two independent coil-core lever systems and four d-c. relays of the telephone type. Referring to Fig. 2 the one lever system is made up of dashpot 12, coil 11, core 10, lever 22, fulcrum 6, springs 19, 20, 21, and lower contact B. When coil 11 is deenergized, the lever system is balanced by means of a counterweight and spring 20 so that lower contact B barely touches upper contact B, when upper contact B is in its highest position. Core 10 is assembled so that when coil 11 is energized the core will be pulled downwards. The downward travel of core 10 is limited by means of a stop post 5 to prevent the springs 19, 20, 21 from being overextended.

Coil 11 is connected in series with coil 14 across the dark lamp beat voltage between the two systems, as can be seen in Figs. 8 and 10, and consequently the voltage across the coil will be zero when the phase displacement is zero, and will be maximum (220 volts) when the phase displacement is 180 deg. For small phase displacements (up to 50 deg.) the voltage difference or beat voltage across the coils will be ap-

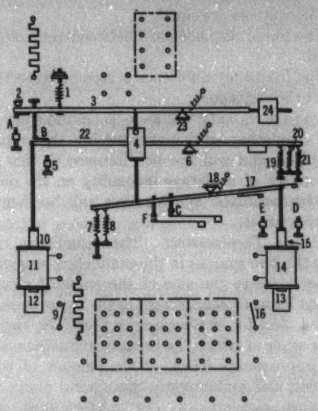

Fig. 2—Schematic Outline of Automatic Synchronizer

proximately proportional to the phase angle provided the voltage waves of the two systems are sine waves, and therefore, assuming that for any impressed voltage across the two coils the ratio between the two impedances is constant, the voltage across coil 11 will be proportional to the angle of phase displacement as shown in Fig. 3.

The pull on core 10 increases when the core travels downward into the coil. To obtain a travel of lower

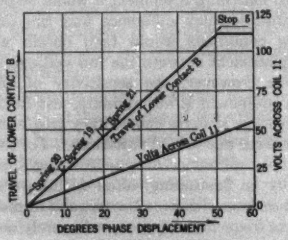

Fig. 3—Variation of Voltage Across Coil 11 and of Travel of Lower Contact B

contact B which is porportional to the angle of phase displacement, springs 19 and 21 are applied to come into action in sequence as indicated on travel curve for contact B in Fig. 3. At 50 deg. phase displacemnt lever 22 reaches stop post 5, and for phase displacements exceeding 50 deg. lever 22 will be resting on post 5;

for phase displacement within 50 degrees contacts *B* will move in proportion to the change in phase displacement.

Dashpot 12 is applied to dampen the mechanical vibrations on the lever system resulting from a high frequency difference. The damping of the dashpot is small, however, and for frequency differences of an order of 1 to 1½ cycles the lever system will be nearly undamped. In order to obtain the proper performance of the synchronizer it is important that the lever system be absolutely undamped for any frequency difference

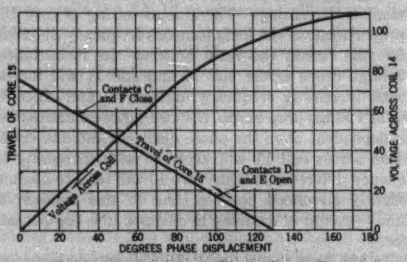

Fig. 4—Relationship between Phase-Angle Displacement Voltage Across Coil 14 and Travel of Core 15

within the applicable synchronizing range. How this feature is obtained through the interaction between the series connected coils 11 and 14 will be explained later in this paper.

Lever system No. 1 (described above) which is lightly damped, provides essentially the means for checking the instantaneous phase displacement between the voltages of the two systems to be connected, while the more heavily damped lever system No. 2 provides the means for obtaining the advance closure of the breaker proportional to the instantaneous frequency difference, and provides the means to prevent synchronizing if the frequency difference is above the permissible synchronizing range.

Lever system No. 2 consists of two lever assemblies, connected by means of the air dashpot 4. Lever 3 supported by fulcrum 23 is balanced by means of spring 1 and counterweight 24 to barely touch stop post 2. Upper contacts *A* and *B* are mounted on lever 3. Lever 17 is supported by fulcrum 18, and balanced by means of springs 7 and 8 against the pull exerted by coil 14 on core 15.

When the phase displacement between the voltages of the two systems is 180 deg., core 15 is attracted by coil 14 and in its lowest position. If the phase displacement is decreased, the pull on the core will decrease, and finally at 130 deg. phase displacement the pull on core 15 is equal to the combined pull of the springs 7 and 8. If the phase displacement is further decreased, core 15 will start to travel upwards. The springs 7 and 8 are so designed that the travel of core 15 will be proportional to the angle of phase displacement as shown in Fig. 4.

Upper contacts *E* and *D* are spring mounted, and for this reason contacts *E* and *D* will not open until the voltage phase displacement is reduced to 100 deg., and because lever contacts *C* and *F* are spring mounted the contacts *C* and *F* will close when the phase displacement is approximately 30 degrees and will remain closed while the core 15 is completing its travel, reaching its highest position at zero-degree phase displacement.

It will be understood that core 15 will tend to assume a position in relation to coil 14, which will primarily be dependent upon the instantaneous phase displacement between the voltages of the two systems to be connected; but the position of core 15 will also be dependent upon the rate of change of phase displacement, or in other words dependent upon the instantaneous frequency difference between the two systems.

The reason for this latter feature is the heavy damping provided by dashpots 4 and 13. Due to this damping the position of core 15 and consequently also lever 17 will be lagging if a frequency difference is present, in relation to the position lever 17 would assume if the frequency difference were zero. The amount of lag when the phase displacement is decreased from 130 deg. towards zero will be proportional to the frequency difference and the effect of this is to reduce the amplitude of travel of core 15 and its con-

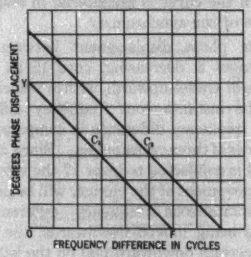

Fig. 5—Closing Characteristic for Contacts *C* for Two Different Adjustments of the Contacts

nected lever 17, by an amount which for low values of frequency difference will be proportional to the frequency difference. Consequently the closing characteristic for contacts *C* will be a straight line as shown in Fig. 5. When the frequency difference is zero contacts *C* may close at *Y* degrees phase displacement. For frequency differences above *F* cycles the contacts *C* will not close during one rotation of the phase-displacement vector. The point *Y* can be moved upwards or downwards by changing the setting of contacts *C*. If upper contact *C* is adjusted downwards in relation to lever 17 the point *Y* in Fig. 5 will move upwards and a

new curve C_2 will obtain. The adjustment of contacts C will not change the slope of the closing characteristic; the slope being entirely dependent upon the damping in dashpots 4 and 13, increased damping will increase the slope, or in other words move point F towards zero while point Y is kept in place.

Contacts F will have a closing characteristic similar to contacts C; while contacts E and D will close at 100 deg. phase displacement when the frequency difference is zero. For increasing frequency difference contacts E and D will close nearer to the point of phase opposition (180-deg. phase displacement) and for a certain definite frequency difference the contacts will not close at all during one phase rotation. The closing characteristic of contacts E and D will not be a straight line. The shape of the curve is immaterial in respect to the performance of the synchronizer.

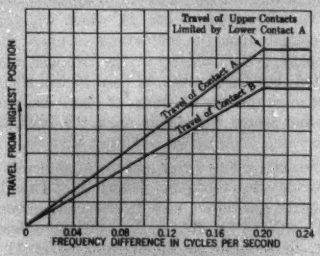

Fig. 6—Travel of Upper Contacts A and B

When the phase displacement between the two system voltages is decreasing, core 15 as previously mentioned will travel upwards with a velocity proportional to the instantaneous frequency difference, as long as the frequency difference is comparatively low, *i.e.*, of an order of three cycles or less. Since the plunger of dashpot 4 is connected to lever 17 the velocity of the downward movement of the plunger will be proportional to the instantaneous frequency difference. When plunger 4 is being moved downwards, the dashpot 4 will exert a certain downward pull on the left hand side of lever 3, and consequently lever 3 will rotate counter clockwise until the pull of dashpot 4 is balanced by the increasing tension on spring 1. The pull of dashpot 4 on lever 3 is proportional to the velocity of travel of plunger 4. The extension of spring 1 is proportional to the tension of the spring, and for this reason the angular movement of lever 3 is proportional to the pull of dashpot 4, furthermore, proportional to the velocity of travel of plunger 4 and consequently proportional to the instantaneous frequency difference between the two systems to be paralleled.

Upper contacts A and B are mounted on lever 3, and referring to above considerations it will be obvious that contacts A and B, when the phase displacement is decreasing from 130 deg. towards zero will assume a position in space, in relation to their highest position, which will be proportional to the instantaneous frequency difference, or in other words, the position of contacts A and B will be the lower the higher the frequency difference is, as will appear from Fig. 6.

The impedances of coils 11 and 14 are equal for any phase displacement angle below 50 deg. if the frequency difference is zero. The impedance of coils 11 and 14 will decrease when the cores travel upwards out of the coils. Since core 15 is heavily damped, the core will be lagging considerably in relation to the momentary voltage phase displacement when the phase displacement is decreasing. The amount of lag of core 15 is proportional to the instantaneous frequency difference; for this reason the ratio between the instantaneous voltage drops across coil 14 and coil 11 will increase with increasing frequency difference, and since the total voltage drop across the two series-connected coils is dependent upon the phase displacement, the voltage drop across coil 11 will decrease with increasing frequency difference, so that core 10 will tend to assume a slightly advanced position, and in this manner, the small amount of damping in dashpot 12 is compensated for.

In Fig. 7 are shown two various closing characteristics for contacts B. When the frequency difference is zero, contacts B will close at the point of zero phase displacement. For any definite frequency difference the contacts will close at a point in advance of synchronism,

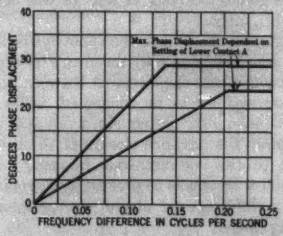

Fig. 7—Closing Characteristics for Contacts B for Various Adjustments of Dashpot 4 and Lower Contact A

the amount of advance being proportional to the instantaneous frequency difference.

By adjusting the amount of damping in dashpot 4, the slope of the curves in Fig. 7 varies, and the synchronizer can thus be adjusted to energize the closing relay of any circuit breaker connecting two systems to be synchronized at such a point in advance of synchronism, that allowing for the time element of the breaker the breaker contacts will close at the point of zero phase displacement, assuming that the closing time of the breaker is practically constant and further assuming that the frequency difference does not change during the short time required to close the circuit breaker.

This method of obtaining breaker closure at the point

of zero phase displacement for any frequency difference within the permissible synchronizing range is applied in both models of the automatic synchronizers. The method in which breaker closure is prevented if the frequency difference exceeds the selected lockout frequency difference, however, is different for the type XY-11 synchronizer than for the type XY-12 synchronizer.

Method of Lockout. The XY-11 synchronizer is arranged so that the breaker will be closed if the frequency difference during the last 90 deg. of its phase angle does not exceed the selected lockout frequency difference, while the XY-12 synchronizer will connect the breaker and synchronize the two systems if the frequency difference at a point 35 deg. ahead of synchronism is lower than the selected lockout frequency difference. Thus it may be seen that the XY-12 synchronizer is especially well adapted for application with very erratic frequencies.

In Fig. 6 was shown the travel frequency characteristic of upper contact A, and it was pointed out that upper contact A when the angle of phase displacement between the two system voltages was decreasing from 130 deg. towards zero would assume a position in relation to its highest position which would be proportional to the instantaneous frequency difference. Lower contact A is a stationary contact mounted on the base of the synchronizer. For a definite setting of lower contact A there will always be a definite frequency difference above which contacts A will engage during one rotation of the phase displacement vector. This feature is used in the lockout scheme for the XY-11 synchronizer as shown in the diagram of connections of the automatic synchronizer type XY-11 (Fig. 8), which is arranged so that whenever contacts A have been engaged it is necessary for the phase angle to exceed 100 deg. once before the synchronizer will again be able to close the paralleling breaker.

A clear conception of the function of the lockout contacts A may be obtained by going through the operating sequence for the various relays of the automatic synchronizer type XY-11 for frequency differences above and below the selected lockout frequency difference.

Assuming that the proper connections to the two systems have been made, and that the master relay connecting the synchronizer to the control circuits close when a frequency difference of 5 cycles exists between the two systems, none of the relays, T_1, T_2, T_3 will be energized because lever 17 due to the damping effect of dashpots 4 and 13 will never come into a position to close contacts D. When the frequency difference is reduced to approximately 3 cycles, contacts D will engage, closing relay T_1 which will seal itself in. Nothing further happens until the frequency difference is reduced to ½ cycle. At this frequency difference contacts C engage at approximately zero phase displacement and relay T_2 is closed and sealed in. At a point 260 deg. ahead of synchronism, contacts E will close and relay T_3 will be energized and sealed in.

Supposing now the frequency difference is lowered to 1/7 cycle and that contact A has been adjusted just not to close at this frequency difference, then the breaker closing relay will be energized at the proper phase advance when contacts B engage, and the breaker main contacts will engage at the point of zero phase displacement. If, however, the frequency difference should happen to increase above the safe synchronizing limit,

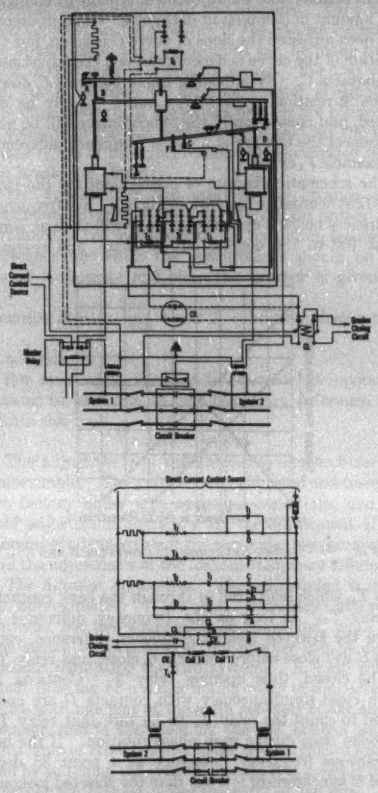

Fig. 8—Diagram of Connections and Schematic Diagram for Automatic Synchronizer Type XY-11

in this case supposed to be 1/7 cycle, then contacts A will engage before contacts B close, relays T_2 and T_3 will be shunted by contacts A and opened, and in this manner synchronizing is positively prevented.

The lockout feature of the XY-12 synchronizer is based on the change in closing sequence of contacts B and C when the frequency difference passes through the lockout frequency difference for which the synchronizer

has been adjusted. In Fig. 9 is plotted the closing characteristic for contacts B and C in the same scale. Considering curves B and C_1 it may be seen that for frequency differences exceeding F_1 cycles, contacts B will close ahead of contacts C; while contacts C will close ahead of contacts B for frequency difference lower than F_1 cycles. The slope of curve B, as previously mentioned, is adjusted to suit the time element of the circuit breaker, and the adjustment is made by changing the amount of damping in dashpot 4. When dashpot 4 has been adjusted the slope of curve C is given, but the location of the curve is dependent upon the adjustment of contacts C. By adjusting upper contacts C downwards in relation to lever 17, a curve C_2 may be obtained, and consequently the synchronizer will be able to connect the breaker at a higher frequency difference, for example F_2 cycles.

The schematic diagram of connections for the type XY-12 synchronizer is shown in Fig. 10. For frequency differences below 3 cycles contacts D will close at some point 100 to 180 degrees out of phase, and energize

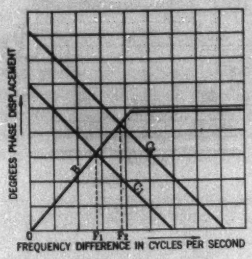

Fig. 9—Closing Relationship Contacts B and C

relay T_1, which seals itself in through the back contacts of relay T_2. As long as the frequency difference is above the selected lockout frequency difference, contacts B will close before contacts C close, and relay T_2 will be closed, thus opening relay T_1. Contacts E which close simultaneously with contacts D are connected to shunt the relay T_2 to be sure that relay T_2 will open once during each phase rotation. If the instantaneous frequency difference at a point 35 deg. ahead of synchronism is lower than the selected lockout frequency difference relays T_2 and T_3 will be closed, provided relay T_1 has already been closed, and hence the breaker closing relay will be energized and the breaker will be closed. The operating sequence under these conditions is: Contacts D close approximately 260 deg. ahead of synchronism, and close relay T_1 which seals itself in across the back contacts of relay T_2.

At a point within 60 deg. phase displacement, dependent upon the frequency difference contacts C close and operate relay T_3 which seals itself in while one pair of the contacts of T_3 parallels the T_2 back contacts in series with relay T_1. When the phase displacement is further reduced, contacts B close at the proper phase advance proportional to the instantaneous frequency difference, and energize relay T_2 which completes the closing circuit of the breaker closing relay so that the breaker will close and synchronize the two systems.

It will appear from above description of the two

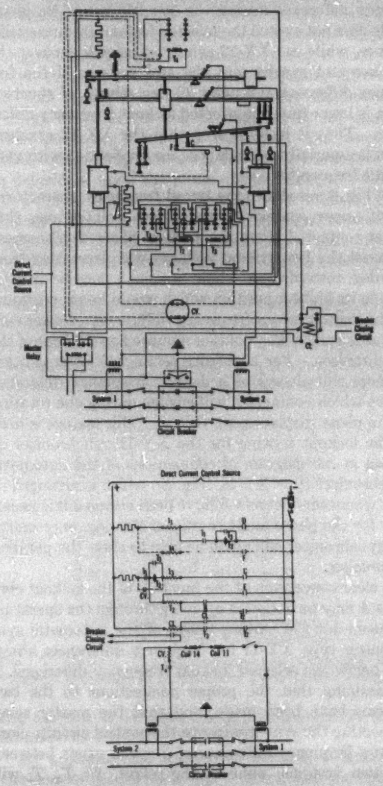

Fig. 10—Diagram of Connections and Schematic Diagram for Automatic Synchronizer Type XY-12

synchronizer models that to make the synchronizer operate it must be required that the voltage phase vector make 1½ revolutions before the synchronizer will be able to synchronize the two systems. For this reason the synchronizers as described above will not be able to synchronize if the breaker to be closed is not the first tie between two systems, since in this case the frequency difference will be zero, and no phase rotation

will obtain. This condition is taken care of by supplying an external relay to be energized by contacts T_4. This relay operates on a-c. voltage and will when energized close its contacts after a predetermined time delay. The relay contacts will open when the relay is deenergized.

Contacts F are adjusted to close within $\pm$ 15 degrees phase angle displacement, and from Figs. 8 and 10 it may be seen that by means of the contacts of the relay marked $C\ V$. the synchronizer will operate if the frequency difference is zero for example, when the synchronizer is used in a line to line synchronizing application. It should be noted that if a frequency difference exists contacts F will not remain closed long enough to close relay $C\ V$., and operation of the synchronizer will therefore be as previously described.

The performance of an automatic synchronizer if the voltages of the two systems to be paralleled are not equal is of interest, since a voltage inequality can be experienced in various types of installations.

As long as a frequency difference exceeding 1/10 cycle per second exists between the two systems the closing

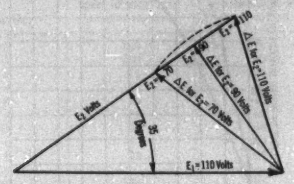

FIG. 11—THE VARIATION OF THE VOLTAGE ACROSS THE SYNCHRONIZER A-C. COILS

When a voltage inequality is present between the two systems while the phase displacement between the voltages is 35 degrees

error is negligible, due to the fact that the position of lower contact B at 35 deg. phase angle displacement will be approximately the same whether the voltages of the two systems are equal or inequal. This is due to the fact that the voltage impressed across the coils will vary inconsiderably when the phase angle displacement is 35 deg., as can be seen from Fig. 11. In this figure are shown the two system voltages E_1 and E_2 at 35 deg. displacement, and also the vectorial voltage difference ΔE for various values of E_2 while E_1 is kept constant.

For a definite voltage inequality there is a minimum frequency difference below which the synchronizer will not operate. This condition is taken care of in application of line to line synchronizing by permanently advancing upper contact A, so that contacts A will barely engage at zero phase angle displacement when the maximum voltage inequality between the two systems obtain. With this adjustment the synchronizer will perform perfectly at the maximum voltage inequality and close the breaker at zero phase displacement; but will close the breaker too early when equal voltages obtain on the two systems. The amount of degrees early closure will be equal to ½ deg. per per cent voltage inequality, and will therefore be negligible in most applications.

APPLICATION

The automatic synchronizer models described above are designed to operate on potential transformers with 110 volts secondary voltage. The average load on each potential transformer is 40 volt-amperes. Taps have been supplied on the resistors in the control circuit of the apparatus so that connections can be made to 48 volts or 125 volts d-c. control source. For 250 volts d-c. control voltage an external resistor is supplied. The back contacts of the circuit breaker are connected in series with the d-c. control circuit, so that the telephone relays of the synchronizer are deenergized when the breaker is closed.

The synchronizer will energize the breaker at a maximum advance of 35 deg. To obtain proper performance of the synchronizer the maximum frequency difference for which the synchronizer should be adjusted to operate and close the breaker is given by equation $F = \dfrac{35}{T \cdot 360}$ where T = breaker closing time in seconds.

For most applications it is desirable for operating reasons to keep the lockout frequency difference well within this limit.

ADJUSTMENTS

The adjustments of the automatic synchronizer are quite simple. The apparatus is calibrated and tested in the factory under true operating conditions, and the field adjustments necessary are the adjustment of the amount of advance to suit the particular breaker applied and the adjustment of the lockout frequency difference.

The Advance Adjustment. This adjustment is made at approximately 1/10 cycle frequency difference between the two sources to be connected, while the line switch is open so that the breaker can be closed without paralleling the two systems. A lamp with a small time lag or a quick acting relay is connected in series with one pair of the main contacts of the circuit breaker, and is energized by the d-c. control source. This lamp or relay is placed close to the synchronoscope and by watching the synchronoscope it is possible to see whether the breaker closes too early or too late. The adjustment of the air valve of dashpot 4 is varied until the breaker contacts close at zero phase angle displacement.

The Lockout Adjustment. The speed of one source is adjusted to give the highest frequency difference at which it is wanted to synchronize, and the lower contact A for type X Y-11 and upper contact C for type X Y-12 is adjusted so that the telephone relays of the synchronizer is just not locked out at this frequency difference.

Comparative Tests between Hand Synchronizing and Automatic Synchronizing

A series of tests was made with the X Y-11 type synchronizer to determine the performance of the synchronizer and compare it with the performance of a skilled operator. A 3000-kv-a., 25-cycle, three-phase motor generator set was arranged to be synchronized with a

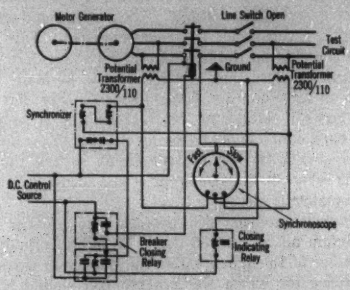

Fig. 12—Connections for Comparative Tests

25-cycle test circuit with the connections shown in Fig. 12. The frequency of the test circuit showed some variations due to load variations; the closing time of the oil circuit breaker used during the test was measured by means of a cycle counter. It was found that the time required from the moment the contacts of the synchronizer closed until the main contacts of the circuit breaker engaged was 0.6 sec. The speed of the motor generator set was regulated by hand to give

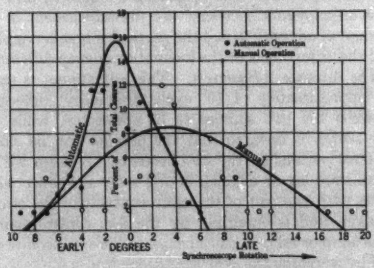

Fig. 13—Results of Comparative Tests

various beat frequencies within the lockout frequency limit for which the synchronizer was adjusted. A quick operating relay (closing time 0.009 sec.) was used as a closing indicating relay and was connected so that it would operate when the main contacts of the circuit breaker engaged. This relay was held in the hand of one of the men conducting the tests, and the position of the synchronoscope pointer was read by him at the instant he felt the indicating relay operating.

With the automatic synchronizer in operation 193 readings were taken, and 67 readings were taken with hand operation.

The result of the tests is shown in Fig. 13, which shows the percentage of the total number of operations for which the circuit breaker contacts would close at a certain voltage phase displacement. The curves show the superior accuracy of the automatic synchronizer as compared with the performance of a skilled operator synchronizing by hand. The results for manual operation show that an operator is liable to close the circuit breaker too late. This would be even more the case for applications with erratic frequency and for such applications the comparison between automatic synchronizing and hand synchronizing would be still more favorable for the automatic synchronizer. From Fig. 13 may be seen that the automatic synchronizer would never close the breaker main contacts at a wider angle than 9 deg., while the operator happened to close

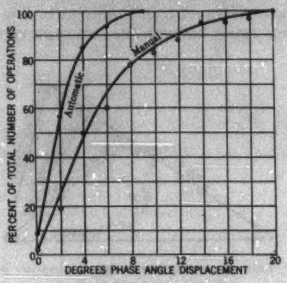

Fig. 14—A Comparison of Automatic and Manual Synchronizing

the breaker as much as 20 deg. too late. The curves do not indicate the fact that automatic closure required much less time than hand closure, since the operator would always wait for smaller frequency differences before he would synchronize; while the automatic synchronizer would close the breaker the first time the frequency difference was below the lockout frequency difference 1/7 cycle.

The same test results as given in Fig. 13 are plotted in a different way in Fig. 14. The curves of Fig. 14 show the per cent of total number of operations for which the circuit breaker contacts close within a certain number of degrees of synchronism. For example, when automatic synchronizing was used the circuit breaker contacts would close within 2 deg. or less of synchronism for 56 per cent of the total number of operations, while with manual synchronizing the operator succeeded in getting the same good results for only 19 per cent of the total number of operations.

Field Tests

Through the courtesy of the Kentucky Hydro Electric

Company a series of tests was made with the two automatic synchronizer types XY-11 and XY-12 at the Hydro Electric Generating Station at Dam No. 7, Kentucky River. This station is equipped with three generators, driven by propeller type waterwheels rated 1000 hp., 150 rev. per min. at 15 ft. normal water head. The generator rating for each unit is 850 kv-a. at 80 per cent power factor, 60 cycles, 150 rev. per min. The inertia of the units is low and due to the inherent characteristic of the hydraulic equipment, the frequency of the generator to be synchronized is very erratic in spite of the excellent condition of the installed governor equipment. The time element of the circuit breakers of the station was measured and found to be 0.25 sec.

The automatic switching equipment of the station was arranged so that any one of the three generating units could be started up by pressing a push button on the switchboard, or by means of supervisory equipment controlled from a distant dispatcher's office. After a unit had been started up the automatic synchronizer was given control when the speed of the generator was high enough to allow the generator voltage to build up to normal. The time interval from the moment when a unit was started up until the synchronizer was given control was practically constant for each individual unit, but showed some variations for the three different units due to the various adjustments of the time delay of the voltage relays connected to give the synchronizer control.

With the XY-11 synchronizer in service 19 different synchronizing operations were made, while 25 different synchronizing operations were made with the XY-12 synchronizer in service. By means of a stop watch the total synchronizing time, from the start of a unit until breaker closure, was measured, and is given in column 2 of Table I. The time required from the start of a

TABLE I
FIELD TEST RESULTS

1	2	3	4	5	6	7
Test No.	Time in seconds from start of unit until breaker closure	Time in second from start of unit until synchronizer was given control	Time in second from energizing synchronizer until breaker closure	Breaker closure degrees phase displacement	Unit No.	Synchronizer model
1	105	40	65	5		
2	240	40	200	0		
3	75	40	35	15		
4	190	40	150	0	3	
5	70	40	30	0		
6	60	40	20	5		
7	70	40	30	5		
8	75	40	35	0		
9	70	40	30	5		
10	135	35	100	0		XY-11
11	130	35	95	5		
12	60	35	25	0		
13	80	35	45	0	1	
14	45	35	10	0		
15	55	35	20	10		
16	95	35	60	5		
17	110	35	75	0		
18	60	45	15	0	2	
19	115	45	70	5		
20	60	45	15	0		
21	120	45	75	15		
22	140	45	95	15		
23	55	45	10	0		
24	55	45	10	5	2	
25	70	45	25	5		
26	75	45	30	5		
27	65	45	20	10		
28	65	45	20	0		
29	45	35	10	0		XY-12
30	45	35	10	10		
31	60	35	15	0	1	
32	65	35	30	0		
33	45	35	10	5		
34	45	35	10	0		
35	45	35	10	0		
36	90	40	50	0		
37	75	40	35	0		
38	70	40	30	15		
39	85	40	45	0	3	
40	80	40	40	15		
41	65	40	25	0		
42	90	40	50	5		
43	65	40	25	10		
44	60	40	20	0		

machine until the synchronizer was connected is given in column 3, and the time during which the synchronizer was in operation is given in column 4. The phase displacement between the voltages in the moment of breaker closure as read on a synchronoscope is given in column 5.

From column 4 of Table I can be found that, with the XY-11 synchronizer in service the average time required to synchronize a machine was 58 sec., while the maximum synchronizing time during 19 operations was 200 sec. With the XY-12 synchronizer in service the average synchronizing time is found to be 30 sec., and the maximum time during 25 operations 95 sec.

During these tests the machine frequency was very erratic; in fact, the synchronoscope pointer was never seen to rotate at uniform speed for one full revolution, and the frequency difference seldom remained within the selected lockout frequency difference (1/5 cycle) for more than 10 sec. Due, however, to the proportional advance incorporated in the synchronizer models, the synchronizer would never close the breaker farther off synchronism than 15 deg. The average closing point for the XY-11 synchronizer, as given in column 5, Table I, was found to be at 2.9 deg. phase displacement, and the average closing point for the XY-12 synchronizer was found to be at 4.8 deg. phase displacement.

The type XY-12 synchronizer was permanently installed in this station, and a series of synchronizing tests was made with the installed equipment to determine the time required to synchronize the whole station to the line. During the tests the push buttons of all three units were pressed simultaneously. Unit No. 1 would then start up. As soon as unit No. 1 reached normal voltage the synchronizer would be connected and the machine synchronized properly with the line. Units No. 2 and 3 would then start up simultaneously, and unit No. 2 was synchronized when the proper synchronizing conditions obtained. When unit No. 2 went on the line the synchronizer was switched over to unit No. 3 which would be synchronized.

During 7 different tests it was found that the type XY-12 synchronizer would connect all three generating units to the line in an average time of 159 sec., while the maximum synchronizing time was found to be 194 sec., and the minimum synchronizing time 119 sec. It should be noted that these data cover the time required to bring all three units up from zero speed to normal speed and to synchronize each unit individually with the line.

Acknowledgment

The writer wishes to express his appreciation for valuable suggestions received from Mr. J. H. Ashbaugh and Mr. H. C. Nycum.

High-Frequency Portable Tools and Equipment

BY C. B. COATES[1]

Member, A. I. E. E.

Synopsis.—This paper presents practical information regarding the application of high-frequency (180-cycle) induction motors with low-resistance rotors to portable electric tools.

In order to compete with existing tools it became necessary to increase the rotor speed above that of 60 cycles, and 180 cycles per second was adopted as commercially practical. Several advantages are shown. There is a smaller drop in speed with this type of motor, than with the d-c. or the universal motor, and because of this, greater power is developed. The life of the cutting tools is longer due to the better speed regulation. Less weight and lower maintenance costs result from this application.

Several new tools for special work are described. The desirable regulation of the frequency converter or motor generator is given as not exceeding 8 per cent.

THE application of the term "high-frequency" to portable tools and equipment is, no doubt, a misnomer in view of the extremely high radio and other frequencies generally understood to come within the scope of this term. As a matter of fact, the frequency now generally employed in so-called high-frequency portable tools is only three times the usual commercial frequency of 60 cycles per second. However, as the name has come to be quite generally accepted as applied to this class of tools, it will be used here.

In order to have a basis for comparison, it is desirable to make some reference to portable tools in general, so that reasons may be shown for this radical departure from the old standards.

Older Types of Tools

Electric drills of the portable type were introduced over 25 years ago and were wound for operation on direct current, to conform to the prevailing conditions in the industrial plants at that time. These tools were generally series wound, but several years later the larger sizes were compound-wound to give better speed regulation, which means higher armature speed at full load with limited free speed and consequently greater brake horsepower.

This was a step in the direction of the high-frequency tool characteristics and it may be of interest, in passing, to note a very satisfactory method of applying compound windings to the larger d-c. drills, as adopted several years ago by one manufacturer of portable tools and still used. Fig. 1 shows the structure of the field and armature of this motor. There are four mechanical poles but only two electrical poles, N and n forming the north pole and S and s the south pole. Series coils are placed on poles N and S and shunt coils on poles n and s. The armature revolves in the direction indicated by the arrow. When the motor is subjected to a heavy load the cross magnetization of the field, caused by the large armature currents, tends to deplete the magnetism in the trailing pole tips of N and S and to build it up in the leading tips of n and s. The gap between the pole faces of like polarity, of course, introduces reluctance into the path of this cross-magnetization. The increased current in the series coils due to the heavy load, builds up the magnetism in the trailing tips N and S and maintains practically sparkless commutation at all loads. Thus we are enabled to operate, under load, at a higher armature speed and develop more power, while the shunt-winding limits the free speed to a safe figure.

A few years after the introduction of the d-c. drill, a-c. drills were developed. These were built to operate on 60 cycles and other prevailing frequencies such as

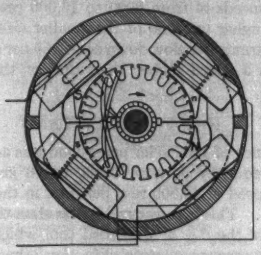

Fig. 1—Compound-Wound D-c. Motor

Special field structure

40 and 25 cycles. They, of course, had good speed regulation, but the weight, due to lower rotor speeds, being limited to 3600 rev. per min. at 60 cycles and correspondingly less for the lower frequencies, was somewhat of a drawback.

About 1908 the now well-known universal drill was introduced and has met with great favor on account of its ability to operate interchangeably at practically the same speeds on either direct or single-phase alternating current of a given voltage. The universal tool is series wound with the typical series motor speed regulation, however.

While the falling off in speed under load was considered to be of advantage in some classes of work, it has now been demonstrated that the sustained speed, as of the induction motor, is generally preferable, due

1. Elec. Engr., Chicago Pneumatic Tool Co., Cleveland, Ohio.

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to the accomplishment of a greater amount of work. This is particularly true of portable grinders as will be shown later.

It is quite obvious that the polyphase induction motor tool has distinct advantages over the direct current and universal tools both as regards speed regulation and lower maintenance cost, due to the absence of commutator and brushes as well as the insulated armature winding. One factor which has retarded its adoption, however, has been its greater weight due to the low rotor speed on commercial circuits of 60 cycles or less. All electric tools d-c., universal, and polyphase a-c. have always had the handicap of weight as compared with pneumatic tools, except at the sacrifice of sturdiness. Nevertheless, many users have put up with the greater weight of the electric tool, on account of the more convenient source of power supply.

In order to reduce the weight of the induction motor tool it is evident that we must bring the rotor speed up and we can do this only by increasing the frequency, since the 60-cycle drills were practically all provided with the minimum number of poles. The synchronous speed of a two-pole 60-cycle motor is 3600 rev. per min., whereas the d-c. and universal tool motors have free armature speeds of from 8000 to 16,000 rev. per min. and even higher in some of the smaller sizes. Under load these armatures slow down nearly 50 per cent which still leaves a speed higher than that of the 60-cycle motor.

High-Frequency Tools

A frequency of 180 cycles per second was adopted for the high-frequency portable tools, which gives a synchronous speed of 10,800 rev. per min. in the two-pole motor used. This is considerably less than the average free speed of the d-c. or universal tool which is very high. This means longer life for the bearings, which are generally of the ball type. The slip under normal full load is 8 per cent to 10 per cent, giving an average loaded speed of about 9800 rev. per min., which is much higher load than the speed of the d-c. or universal tool with its large speed drop. This, of course, means more power with a given size of rotor.

Several factors governed the selection of a standard frequency for these tools. Since it was necessary to obtain spindle speeds for the drills, reamers, and abrasive wheels which would conform to the modern cutting tools, there must be a point where the reduction in weight of the motor due to higher frequencies would be more than offset by the greater weight of the gearing required to obtain the necessary spindle speeds. From this point of view, the desirable frequency was found to be in the neighborhood of 180 cycles per second. Troubles in ball bearings and difficulty of lubrication at high speed is another limiting factor, although improvement in both directions indicates the possibility of still higher frequencies.

Thus the high-frequency tools meet the requirements of intensified production in modern industry where portable tools are crowded to the utmost and in many cases get very little care. The outstanding advantages due to this application are:

I. Greatly increased power for the same weight, due to the higher loaded speeds of the rotor.

II. Reduced weight for a given power output.

III. Practically constant speed at all loads (8 per cent to 10 per cent slip) which results in the accomplishment of more work.

IV. Greatly reduced maintenance cost.

In order to give comparisons with other types, a concrete example of the increased power and better speed regulation of the high-frequency tool is shown in Fig. 2. Here, the actual watts input are plotted as the abscissas against b. hp. in curves No. 1 and 2 and against the spin-

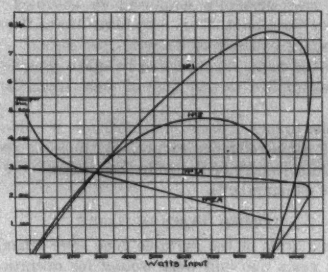

Fig. 2—Comparison of High-Frequency and D-c. Reamers

Curve 1 is brake horsepower of high-frequency reamer
Curve 2 is brake horsepower of d-c. reamer
Curve 1A is speed of high-frequency reamer
Curve 2A is speed of d-c. reamer

dle rev. per min. in curves 1*A* and 2*A*. Curves 1 and 1*A* show characteristics of a high-frequency reamer weighing 50 lb., while 2 and 2*A* are the corresponding curves of a d-c. reamer which weighs 68 lb. or 36 per cent more. The normal load rating of the high-frequency reamer is 5300 watts input at which point it develops 5.35 hp. at 280 spindle rev. per min., or with a slip of 6½ per cent. The normal load rating of the d-c. reamer is 3600 watts at which point it develops 3.5 hp. and 265 rev. per min., representing a slip of 56½ per cent. It will be noted that the reserve power above normal full load of the high-frequency tool is large as compared with the d-c. tool. The armature, exclusive of shaft and the ventilating fan, of the d-c. tool weighs 2 ⅛ times as much as the high-frequency rotor.

Fig. 3 is the detail curve of the high-frequency reamer shown in the previous curve. Note the well sustained speed at maximum hp. and torque, the slip being 15 per cent, also the high-power factor.

A cast rotor of relatively low resistance is used in these tools, which gives close speed regulation. When they were first put on the market, it was thought that

the smaller tools should have considerable speed variation and a rotor wound with bare copper wire was used, making it possible to wind rotors of high or lower resistance as desired. While this practise has been discontinued in favor of the low-resistance cast rotor, it will be interesting to note what was accomplished,

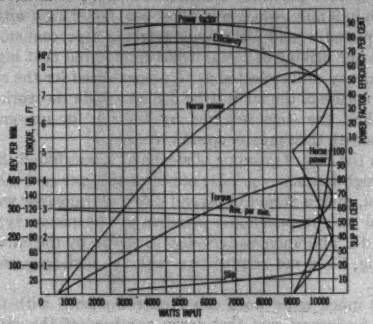

FIG. 3—CHARACTERISTICS OF LARGE HIGH-FREQUENCY REAMER

in the way of speed variation or slip in these small machines.

Fig. 4 shows the characteristic curves of a small drill weighing eight pounds and capable of drilling 5/16-in.

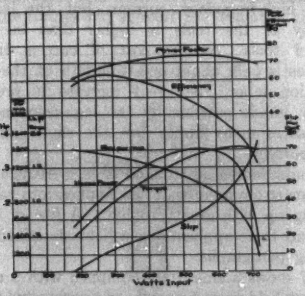

FIG. 4—CHARACTERISTICS OF SMALL HIGH-FREQUENCY DRILL WITH HIGH-RESISTANCE ROTOR

holes in steel. It has a high-resistance rotor. Note that a slip of 75 per cent was reached with the spindle torque very close to maximum. The maximum hp. was reached at about 30 per cent slip. With the low-resistance cast rotor, in the same stator, a slip of 31 per cent is obtained at the same watts input which gave a 75 per cent slip with the high-resistance rotor shown in Fig. 4, and the maximum hp. is developed at 8 per cent slip as against 30 per cent. The maximum hp. is also about 12 per cent greater with the cast rotor, these differences being due to the difference in rotor resistance.

Perhaps one of the most outstanding examples of the advantages of high-frequency tools is to be found in the portable grinder and buffer. Here the great advantage of sustained speed is very pronounced. The cutting wheel can be operated at the proper and most efficient speed without having to allow a high and dangerous free speed, as is the case with other types of electric and some air grinders. In addition to obtaining the most efficient cutting speeds for the wheels, the wheels themselves give practically double the life at the sustained speeds.

Fig. 5 shows the curves of four types of electric grinders, each having the same weight. The spindle, or arbor, rev. per min. are shown as the abscissas plotted against the hp.

Curve No. 1 is of a geared grinder with universal motor and the armature running about three times as fast as the spindle. Note the extremely high no-load speed of 5200 rev. per min., and the speed of 2000 rev. per min. at maximum hp.

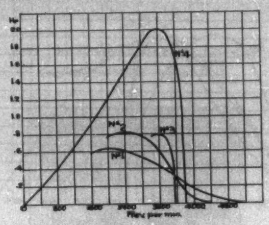

FIG. 5—COMPARISON OF ELECTRIC GRINDERS OF FOUR DIFFERENT TYPES

All these grinders have the same weight
Curve 1—Geared universal-motor grinder
Curve 2—Geared compound-wound d-c. grinder
Curve 3—Gearless 60-cycle induction-motor grinder
Curve 4—Geared high-frequency grinder

Curve No. 2 is of a geared compound-wound d-c. grinder. The effect of the shunt winding is made apparent in the reduced no-load speed. We also have a greater wheel speed at maximum hp.

Curve No. 3 is of a two-pole, 60-cycle induction motor grinder without gearing. The synchronous speed is 3600 rev. per min. Here we have good speed regulation and the absence of gearing is, of course, an advantage in that it permits of a larger motor for the same total weight of the tool.

Curve No. 4 is of a high-frequency grinder geared slightly less than three to one so that the free speed is about 3800 rev. per min. This has good speed regulation and very much more power due to the high rotor speed at full load. This is the only grinder of the group that will successfully carry a 6-in. abrasive wheel at proper speed and with sufficient power behind it. The rubber-bonded wheels which permit of a peripheral speed of 10,000 ft. per min. require a smaller gear ratio

thus increasing the arbor speed, or a larger diameter of wheel may be used with the same ratio.

A comparison of the high-frequency grinder with three types of pneumatic grinders is shown in Fig. 6. Here the arrangement of the curves is the same as in Fig. 5.

Curve No. 1 is of a reciprocating piston type of pneumatic grinder. Note the range of maximum power is at from 1400 to 2200 rev. per min., while the no-load speed is 4000.

Curve No. 2 is of a rotary pneumatic grinder. The load speed is 3000 to 3600 rev. per min., which is an improvement over No. 1, but the no-load speed is 6800 rev. per min., which is excessive for a 6-in. wheel.

Curve No. 3 is of a rotary pneumatic grinder provided with a throttling governor. Here the no-load speed is held down by throttling the air, not only making a safer tool but consuming less air at the lighter loads.

Curve No. 4 is of the same high-frequency grinder as was shown in Fig. 5. The speed is held within

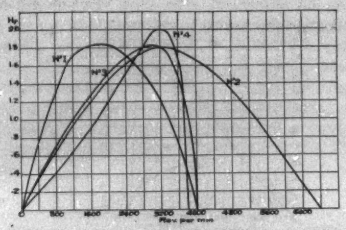

Fig. 6—Comparison of Pneumatic Grinders with High-Frequency Electric Grinder

Curve 1—Reciprocating pneumatic grinder
Curve 2—Rotary pneumatic grinder
Curve 3—Rotary pneumatic grinder with governor
Curve 4—Geared high-frequency grinder, same as in Fig. 5

close limits and there is plenty of power to handle the work. Furthermore, although not shown, the power consumption, as of all electric motors, is less at the lighter loads, while the converse is true of the pneumatic tools except in the case of the grinder in Curve No. 3.

Just as sustained correct speed makes for longer wear of grinding wheels and abrasive disks, so does it also give long life to reamers and drills. In reaming through a considerable thickness of steel when much metal is to be removed, a tool having great speed variation is apt to produce clogging of the reamer at extremely slow speeds and burning when the load is lessened with the reamer still in the hole.

A very striking example of the effect of nearly constant speed in increasing the life of the reamer was observed in a bridge fabricating plant with a 50-lb high-frequency tool having a synchronous speed of 300 rev. per min. The holes were punched 11/16 in. and reamed to 15/16 in. through assembled plates varying in group thickness from 1¼ in. to 3¾ in. Over 3000 linear in. of this reaming was done with the reamer without regrinding. This was at a reaming rate of 3.7 in. per min., including the time of changing from hole to hole.

As an example of reaming 15/16 in. full punched holes in deck plates, on ship work, a high-frequency reamer of 300 rev. per min. synchronous speed reamed 100 holes in 9 min., which was 39 per cent faster than with a compound-wound d-c. reamer weighing 40 per cent more.

In the drilling of metal it has been found that high-frequency drills selected as to proper speed for the size and nature of the holes to be drilled will do the work in one-half the time required by the d-c. or universal drills with their greater speed variation.

In the case of grinding of steel bars it was found that the high-frequency grinder removed 12½ per cent more metal, over a period of several days, as compared with equal weight pneumatic grinders of the piston type. The d-c. and universal grinders as shown by the curves in Fig. 5 are not competitive with the high-frequency type.

So much more work can be accomplished with the high-frequency tool that large numbers of the older types of tools are being discarded. Manufacturers are recognizing the rapidly changing conditions and are willing to scrap the earlier tools to invest in the new equipment necessary, in the shape of frequency converters or motor-generator sets and new wiring in order to gain the advantages of the increased production. The cost of power is somewhat more due to the transformation to the high-frequency current, but the individual efficiency of the tools is higher than in the old types of tools. As the cost of current in many industries is only about one-twentieth of the cost of production labor, the great increase in productive capacity, made possible by the high-frequency tool, compensates many times for the slight increase in cost of current.

The outstanding advantages of high-frequency tools have been so apparent that new applications have been springing up with great rapidity. A few of the tool types and their applications will be described in the following paragraphs.

Fig. 7 shows a small tool equipped with a multiple jaw clutch held apart by means of a spring and put into engagement by means of pressure on the handle. When driving screws or running nuts, the torque developed at the end of the operation causes the abutting faces of the jaws to be forced apart, due to a very definite angle of contact between them.

Fig. 8 shows a nut runner for counterbalanced suspension, equipped with a cushion clutch which is adjustable for various sizes of nuts. This clutch which has multiple jaws, releases at a predetermined load and cannot drop into engagement again until the completion of one full revolution. The jaws are quite shallow and there is considerable angular clearance between the interlocking faces, so that when they drop

in, after having released, the driving faces come together with considerable speed and, due to the yielding effect of the spring which holds them together, they ride up and out of contact with practically no shock to the operator or tool.

Fig. 9 shows a large reamer weighing 50 lb. and

Fig. 7—High-Frequency Screw Driver or Nut Runner with Positive Clutch

handled in most cases by two men. This is capable of 1 $^{5}/_{16}$-in. reaming in steel at a very rapid rate.

Fig. 10 shows a tool for driving screw studs and at the same time indicating their tightness. The tool is carried in a radial arm which is very flexible and serves to take the torque and support the weight of the tool.

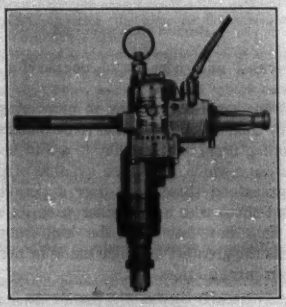

Fig. 8—High-Frequency Nut Runner with Cushion Clutch

It has been the custom in putting in screw studs to drive them part way mechanically and then use a double ended wrench to feel them manually for tightness. This was a double operation and slow.

In the construction shown, the tool is free to turn on ball bearings, around its spindle axis, through an angle of 20 deg. or 30 deg., and this movement is resisted by an adjustable weight and lever arm. A so-called solid chuck is generally used which screws over the upper end of the stud and the tool must be reversed to get it off after the stud is driven home. An automatic depth stop, set for the proper length of projection of the stud from the working surface, releases the driving clutch and stops the tool spindle. The raising of the hand lever at the right engages the reverse clutch and gearing in the tool.

After the tool starts to drive the stud in the tapped hole, the torque developed tends to turn the housing

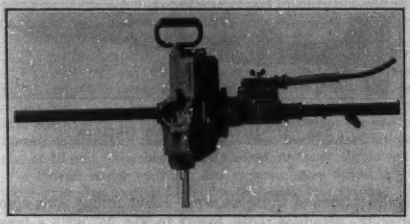

Fig. 9—High-Frequency Reamer

Capacity 1 5/16 in., weight 50 lb.

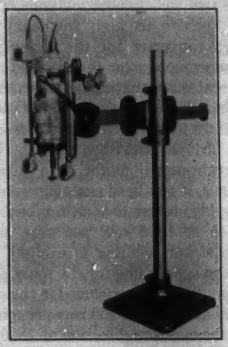

Fig. 10—High-Frequency Stud Setter Mounted on Radial Arm

of the tool against the weight and lever arm shown in Fig. 10 and if this torque is sufficient to lift the weight, an electrical contact is made which causes a green light to appear, indicating to the operator that the stud is of the proper tightness. In order to take care of the danger of getting studs too tight, as in cast iron, a second weight and lever are supplied, cooperating with a red light to indicate when the stud is going in too tightly. The excess torque causes the second weight to be lifted and the circuit for the red light to be completed. Current for the lamps is supplied through a small 4-volt transformer.

The radial arm shown in Fig. 10 is also used to carry nut runners as shown in Fig. 8. This relieves the operator of the weight and all shock of driving, even when the ordinary solid jaw clutch only, is used.

Fig. 11 is a typical portable grinder for foundry and general grinding.

Fig. 12 is a sander carrying a renewable abrasive disk for the sanding of automobile bodies and similar work.

There are many other tool applications which are modifications of those shown. Eight motor sizes comprise the group of tools, Fig. 7 to Fig. 12, with a large number of gear ratios and mechanical combinations.

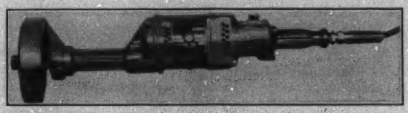

Fig. 11—High-Frequency Grinder

Wheel 6 in. by 1½ in.

Supplying the High-Frequency Energy

There are two well known methods of transforming commercial current to high frequency current, *viz.* the frequency converter and the motor-generator.

The frequency-converter can be used only when the primary current is alternating. As generally employed for operating high-frequency tools it consists of special windings in a slip-ring motor frame, the rotor of which is driven backward, usually by an induction or synchronous motor. In operating high-frequency tools, it has been customary, with a 60-cycle source of supply, to employ an 8-pole frequency converter, driven at 1800 rev. per min. to furnish 180 cycles. The stator winding is excited with 60-cycle energy and the supply line for the tools is connected to the slip-rings. The field produced in the stator winding rotates in one direction at 900 rev. per min. The rotor is turned in the opposite direction at 1800 rev. per min. giving a net speed of 2700 rev. per min. which gives 180 cycles in the eight-pole rotor. As satisfactory tool operation depends on good voltage regulation, it has been the aim of the manufacturers to hold within 8 per cent voltage variation, the full-load voltage being 220. The frequency converter has an inherent regulation and the voltage at a given load varies only with the primary voltage.

The motor-generator or alternator provides an entirely new current, and at 1800 rev. per min., a 12-pole generator will be required to produce 180 cycles. The motor can be operated from either d-c. or a-c. If the latter, an exciter will be required for the generator fields. A regulation not to exceed 8 per cent is also desired with this set but it is possible to vary the voltage either manually or automatically by changing the field current of the generator.

Each type has its advantages and while the simplicity of the frequency converter appeals to many, the possibility of voltage regulation seems more important to others.

The current used for high-frequency tools is three-phase and a cable with four conductors is provided with the tool, terminating in a suitable plug. The fourth conductor is grounded to the housing of the tool and the receptacle in the power line is grounded. This seems to have been satisfactory in the prevention of shocks from grounded windings. It has been suggested that a fourth collector ring be provided on the frequency converter or motor generator to connect to the neutral of the star connected windings and to ground.

Many ways of bringing current to the tools have been devised. The 4-point plug and cable method has been the most popular. Another method, recently offered, consists of four bare copper wires used as trolley wires in an insulated trough or enclosure. These are suspended over the work and a four wheel trolley brings current through a short cable to the tool. The fourth trolley wheel and wire is of course for grounding purposes.

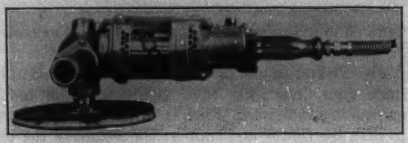

Fig. 12—High-Frequency Sander

Carrying 9-in. abrasive disk

Much hard work was done in pioneering the high-frequency tool, as long tests had to be made to convince the manufacturers of its ability to stand up under the rigorous duty which it seemed capable of performing. It has withstood these tests over a period of several years and those who are familiar with its capabilities cannot help but feel that the high-frequency tool represents the greatest advance made in portable tools since their introduction.

Discussion

A. M. MacCutcheon: I was greatly impressed by Mr. Coates' statement that they get 5.35 hp. out of 50 lb. of material, including the mechanical parts of the tool.

I was particularly interested in Mr. Coates' comments on the source of power. He mentioned that good regulation was very desirable. In that connection, some experiments have been made with a source of high-frequency energy which has come back into use recently because of a demand for very excellent regulation.

The machine might be described as a rotating transformer where alternating current is fed into the rotating member through slipperings and alternating current is taken out through a com-

mutator. If the machine is stationary, it is obvious that the frequency that is fed in is the same as the frequency that comes out. If the machine is operated by an outside source of power at the same speed, that is, the speed which would correspond with the synchronous speed of the winding, direct current is obtained. If 60-cycle energy is fed in and the machine is driven backward, the frequency will be doubled.

This type of machine may be driven by an induction motor. If the winding is a 6-pole winding, with synchronous speed of 1200, but is operated at 900, 15 cycles will be secured.

I have wondered whether there would be a demand for a higher frequency than the 180-cycle.

The regulation of this machine, which may be described as a rotating transformer, is very excellent, at least at low frequency, where I have the tests in mind. Regulation as good as 3 per cent has been obtained on 15-cycle alternation.

I hope that in the A. I. E. E. TRANSACTIONS more and more papers will appear on the application of electricity. I recall that in our own factory about six or seven years ago, although it is a factory manufacturing electrical equipment, air tools were used. In the last five years that has been entirely changed and no air tools are found; they are all electrical.

J. H. Snyder: The induction frequency changer has an advantage over the motor-generator frequency changer in that the motor driving the generator need not have the same capacity as the generator. For instance, in stepping up from 60 cycles to 180 cycles, the generator will provide one-third of the energy through its primary winding and two-thirds of the energy will have to be provided through the driving motor.

That feature of the induction frequency changer makes it in a great many cases less expensive in first cost than the motor generator.

Referring to the regulation of the induction frequency changer, Mr. Coates, I believe, stated that usually a regulation of 8 per cent is specified. That regulation is very good and is not detrimental to the motor when the frequency changer is operated at very high loads.

The commercial motors are at the present time guaranteed for 10 per cent overload and, therefore, if you have any regulation, you are well within the 10 per cent guarantee, so there is no damage possible by operating the high-frequency tool at slightly higher than the rated voltage.

The regulation of a motor-generator type of machine is inherently poorer than that of the induction type of machine; however, as Mr. Coates pointed out, the voltage can be varied by changing the field of the generator to any value desired. This requires expensive automatic apparatus or else an operator.

There is also the question of low power factor in connection with the induction frequency changer, but the amount of power used by the high-frequency tool is usually very small compared to the total amount of power used so that the power factor of the total load is not greatly lowered.

Where large numbers of high-frequency tools are employed the power factor may be raised by the use of capacitors connected to the line.

The present voltage seems to be 220 but there is some demand for an even lower voltage such as 110. Transmitting 110 volts at 180 cycles is quite a problem, if the distribution system is long. It has been suggested that the power be generated at 440 volts or higher and transformed at various points and used at 110 volts. That would have the advantage of using 110 volts on the tool and still transmitting at a higher voltage. The transformer involved in such a case would not be very large. It is, of course, well known that the transmission of power by means of compressed air is very inefficient and I believe that the 180-cycle tool has a higher over-all efficiency than the pneumatic. Also, the maintenance on air lines and hose connections, etc., is rather high.

B. B. Ramey: The portable electric tool moved along rather slowly while we had only the slow-speed motors which made the tools so bulky, but with the introduction of the universal motor, when the size was much reduced because of high speed, the development became very rapid. Now the 180-cycle tool has come in to speed up production still further. That point which Mr. Coates brings out in regard to speeds being maintained is the point around which the greater production and lower cost revolve.

In regard to the universal tool as compared with air, my impression is that the universal tool is not as much handicapped as Mr. Coates' statements seem to indicate. It is my feeling that the universal-motor tool is superior to the air tool in general applications, with the exception of air riveters.

The high-frequency tool is going into industrial plants and taking the place of air equipment rather rapidly, because it can stand up under the heaviest class of service and produce as much work as the air tool, in fact even more. The high-frequency tool is much lower in weight than any other tool.

The ventilation of the high-frequency tool is much better than on the universal tool wherein the speed drops off very rapidly under the heavy loads and the ventilation is greatly reduced.

In regard to the frequency of 180-cycles, I think another fact that accounts for its adoption is that it gives about the best frequency combination that can be got from the standard frequencies of 50 and 60 cycles. Two hundred cycles or higher would give greater motor capacity, but in some cases would involve more expensive gearing to reduce to proper tool speeds. The system of frequency changers, which Mr. MacCutcheon mentioned, is very interesting and I hope something can be developed along that line to help further the use of portable electric tools. My thought is that in order to get the benefits of the high-frequency tool to the small user of portable tools, it will be necessary for this high frequency to be supplied by the power companies.

I believe the loss in the line at 180 cycles amounts to very little in lines less than ½ in. in diameter. Of course, in supplying the power in large quantities so the conductor is very large for 110 volts, it may be necessary to transmit it at the higher voltage and then transform it down.

C. B. Coates: I saw the rotary converter which Mr. MacCutcheon described and it looks as though it has great possibilities. It is very flexible and no doubt can be applied to the driving of high-frequency tools.

In regard to raising the voltage for transmission purposes which Mr. Snyder spoke of, this has not been found necessary, because in the larger installations, sometimes involving as many as 1000 tools, it has been customary to install several frequency converters around the plant in order to economize on wiring. The largest units installed so far are 75 kv-a. The reactance is, of course, three times what it would be for 60 cycles, but by proper arrangement of the wires and wiring the reactance voltage can be kept down to a small value in the 220-volt transmission.

Mr. Ramey's suggestion that there might be a possibility of the power companies supplying high-frequency current for portable tools is very interesting and I am sure this would be welcomed by the manufacturers of such tools.

The tendency has been to install frequency converters or motor-generator sets having excess capacity for the immediate work involved so that there will be ample capacity for future additions. In one of the very early installations in an automobile body plant, a 12-kv-a. motor-generator set was installed to operate 20 tools. It was found later that, owing to the extremely low load (diversity) factor, it was possible to have 60 tools connected to this set. The low load factor was due largely to the great rapidity with which the work was accomplished as compared with the old methods. It has been found practical to install as few as six tools in a plant, in view of the lower maintenance cost and the greatly increased production. Of the installations now in operation about 60 per cent are operated by frequency converters and 40 per cent by motor-generator sets.

Electric Heating Elements

Some Fundamentals Used in Their Design

BY EDWIN FLEISCHMANN*

Member, A. I. E. E.

Synopsis.—*The paper presents the derivation from elementary principles of equations for the design of heating elements. These are then connected with Stefan's law of radiation, and also with a much-used rating curve which has given good results. The reason for the conservatism of this latter curve is shown, and a numerical example worked out for both the theoretical and empirical curves.*

Introductory

THE classical basis, upon which the theory of electric heating design rests, need only be suggested. It is founded on several well-known electrical and thermal laws,—for example, those of Ohm, Wien, and Planck, and the familiar Stefan-Boltzman equation. To be sure, certain data which are based solely upon fortuitous experiment have been useful; but the fundamental relationships must not be ignored. It is the purpose of the present paper to outline briefly some principles in the use of electricity for heat, pointing out particularly those basic ones which govern its skilful employment.

Attention will be focused primarily upon the three vital factors of design and their relation to each other in industrial heating work: the laws of Ohm, the equation of Stefan-Boltzman, and the limitations which govern their use with the materials available for heating elements. The first two canons are more or less well-founded upon theory; the last is substantiated only by experience, but is none the less important at the present stage of the art.

In this paper attention is confined to metallic resistors for high-temperature work (up to 1850 deg. fahr.). It is here that the upper safe value of the operating temperature of the heating unit is approached. At lower temperatures, of course, only part of the heat generated in the elements is carried away by radiation, there being some conduction and considerable convection. At 1850 deg. fahr., the losses by radiation in air are about 90 per cent of the total. In a sealed enclosure, such as a furnace chamber may often be assumed to represent, the heat is all black-body radiation. It is interesting to note that this fact was arrived at simultaneously by Kirchhoff and Balfour Stewart, about 1858. Hence, Ohm's Law and the Stefan-Boltzman Equation together connect electricity and heat completely.

Material for Heating Elements

For temperatures up to 1850 deg. fahr., there is really only one material from which a practical life can be obtained. This is an alloy containing normally about 80 per cent nickel and 20 per cent chromium, with practically no iron. The susceptibility of iron to oxidation endangers the life of the metal at elevated temperatures; so that in the best metals it is carefully avoided. The alloy has the property of forming upon its surface a complicated metallic layer, of which the metallography is still not clearly understood. It is known, however, that this coating effectively prevents further oxidation of the body of the material below 2100 deg. fahr. (1150 deg. cent.). Thus, it assures the life of the element at ordinary heat-treating temperatures.

The resistivity of nickel chromium is consistent and fairly uniform, as indicated by the curves in Fig. 1. Data have been taken from two of the manufacturers'

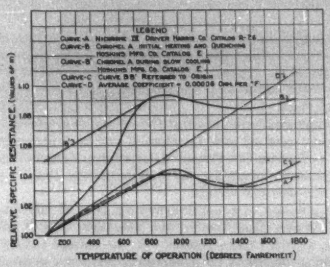

Fig. 1—Resistance of Nickel-Chromium at Elevated Temperatures

catalogs, for material of substantially the same composition. It should be observed that the initial heating and quick cooling of the material give a rise of almost 12 per cent in resistance between cold and 1850 deg. fahr. However, the conditions in a furnace chamber more nearly resemble annealing, since the resistor is seldom cooled very quickly. All the curves are plotted for relative resistances, assuming a specimen of one ohm resistance at 68 deg. fahr. (20 deg. cent.). This material proves to be a satisfactory one for heating units, and it is widely used.

Electric heat is developed in, and by virtue of the properties of the heating element itself, without regard to any extraneous agency, except the supply of electric current. Provision must, therefore, be made for

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sufficient radiating surface on the unit, wherefrom the heat generated in it may be dissipated. It is almost axiomatic that a body to which heat is being supplied will rise in temperature until the losses from its surface just balance the input, provided, of course, that there are no other abstractions of heat from or within the body. Not only must the required amount of heat be generated in the element, but the heat must also be utilized in such a way that the surface temperature of the unit itself will never exceed 2100 deg. fahr. These seemingly elementary facts are the bases of all industrial heating design.

Ohm's Law

The familiar law of Ohm states that

$$E = I R \tag{1}$$

where I = the current in amperes,
E = the impressed voltage, and
R = the resistance in ohms.

The power in watts is the product of E and I, or

$$E I = I^2 R \tag{2}$$

The power may also be expressed in B. t. u., or calories, by application of suitable conversion factors. One watt is equivalent to 3.412 B. t. u. per hour; or one kilogram-calorie per hour equals 1.162 watts. $E I$ is the heating effect of the current.

A variant form of Equation (2) may be obtained by substituting for I its value E/R from (1):

$$E I = \frac{E^2}{R} \tag{3}$$

This gives the heating effect in terms of the voltage and resistance. Both relationships will be found useful in the later stages of the discussion.

The general expression for resistance of a conductor at any temperature is

$$R = m \rho \frac{L}{A} \tag{4}$$

where
R is the resistance in ohms,
m is a constant for any given temperature expressing the ratio of the resistivity at that temperature to the resistivity at some temperature of reference. For nickel chromium, the values of m are shown in Fig. 1.
ρ is the specific resistance, or resistivity at the temperature of reference. In the case of nickel chromium, the value of ρ is 625 ohms per cir-mil-ft., or 0.00004091 ohms per in. of length per sq. in. of cross-sectional area. The latter figure will be used hereafter, because we shall define
L as the length in in., and
A as the cross-sectional area in sq. in.

Substituting the value of ρ in (4), there results for nickel chromium

$$R = \frac{0.00004091\, m\, L}{A} \tag{5}$$

If, now, this value of R be substituted in Equations (2) and (3), they become

$$E I = \frac{I^2 m \rho L}{A} \tag{6}$$

$$E I = \frac{0.00004091\, I^2 m\, L}{A} \tag{7}$$

and,

$$E I = \frac{E^2 A}{m \rho L} \tag{8}$$

$$E I = \frac{24444\, E^2 A}{m\, L} \tag{9}$$

Here are expressions for the wattage generated in a nickel chromium resistor in terms of the voltage, or current, and the physical properties and dimensions of the piece.

Extensions of the theory are now in order. There are several types of resistor cross-sections in fairly general use today. Of these, the most susceptible of mathematical treatment are the round rod and flat ribbon. It will be interesting to consider these shapes in parallel columns. No effort will be made to point out or explain the superiority of one over the other; for, so far as can be determined experimentally, or theoretically, the differences are more hypothetical than actual. Rather will an effort be made to assist the engineer who employs either type in his work to do so intelligently, in the light of fundamental facts.

There is no difficulty in applying the laws to other shapes. The Equations already derived, (4), (6), and (8), are perfectly general. The geometrical simplicity of the rectangular and circular cross section recommends them for this elementary discussion.

The cross-sectional areas in the two cases will be:

Round Rod

$$A = \frac{1}{4} \pi D^2 \tag{10a}$$

where D is the diameter of the rod in in.

Flat Ribbon

$$A = a\, b \tag{10b}$$

where a is the width of the ribbon in in., and b is the thickness of the ribbon in in.

Putting these values in Equations (6) and (7), one obtains for:

Round Rod

$$E I = \frac{4 m \rho I^2 L}{\pi D^2} \tag{11a}$$

Flat Ribbon

$$E I = \frac{I^2 m \rho L}{a\, b} \tag{11b}$$

or, for nickel chromium,

$$E I = \frac{0.0000521\, m\, I^2 L}{D^2} \tag{12a}$$

$$E I = \frac{0.00004091\, m\, I^2 L}{a\, b} \tag{12b}$$

And if these values be substituted in (8) and (9), the following result:

Round Rod — *Flat Ribbon*

$$E\,I = \frac{\pi\,E^2\,D^2}{4\,m\,\rho\,L} \quad \textbf{(13a)} \qquad E\,I = \frac{E^2\,a\,b}{m\,\rho\,L} \quad \textbf{(13b)}$$

$$E\,I = \frac{19195\,E^2\,D^2}{m\,L} \quad \textbf{(14a)} \qquad E\,I = \frac{24444\,E^2\,a\,b}{m\,L} \quad \textbf{(14b)}$$

There are no exceptions to these rules, which show exactly how heat is generated in these two shapes. They are fundamental.

Having determined the manner in which the heat is developed, we may next examine the means whereby it can leave the element; for it is of no value to do useful work unless it reaches the material to be heated. The only way in which heat can leave the unit is through the peripheral surface. The density with which it passes through,—*i. e.*, the wattage radiated or lost by any means from the periphery per square inch, in the terms already used,—will determine precisely the temperature to which the resistor will rise above its surroundings.

The wattage per sq. in. can be related to the physical dimensions of the heating unit itself. If the peripheral surface of the resistor be denoted by S,

Round Rod — *Flat Ribbon*

$$S = \pi\,D\,L \quad \textbf{(15a)} \qquad S = 2\,(a + b)\,L \quad \textbf{(15b)}$$

The wattage per sq. in. of surface will, then, be $E\,I/S$. For convenience, set

$$G = E\,I/S \quad \textbf{(16)}$$

Then, using this value, and, for

Round Rod — *Flat Ribbon*

dividing (11a) and (12a) by (15a), — dividing (11b) and (12b) by (15b),

$$G = \frac{4\,I^2\,\rho\,m}{\pi^2\,D^3} \quad \textbf{(17a)} \qquad G = \frac{I^2\,\rho\,m}{2\,a\,b\,(a + b)} \quad \textbf{(17b)}$$

$$G = \frac{0.00001658\,I^2\,m}{D^5} \quad \textbf{(18a)} \qquad G = \frac{0.00002045\,I^2\,m}{a\,b\,(a + b)} \quad \textbf{(18b)}$$

And, in a similar manner,

dividing (13a) and (14a) by (15a), — dividing (13b) and (14b) by (15b),

$$G = \frac{E^2\,D}{4\,m\,\rho\,L^2} \quad \textbf{(19a)} \qquad G = \frac{E^2\,a\,b}{2\,m\,\rho\,L^2\,(a + b)} \quad \textbf{(19b)}$$

$$G = \frac{6111\,E^2\,D}{m\,L^2} \quad \textbf{(20a)} \qquad G = \frac{12222\,E^2\,a\,b}{m\,L^2\,(a + b)} \quad \textbf{(20b)}$$

These equations go still further and give the number of watts per sq. in. which it is possible to lose from the given shapes of resistors, in terms of their physical dimensions, and either the voltage applied to them, or the current flowing in them. They are extremely useful in design work.

Several simple transformations suggest themselves. One is often interested in the current density in the conductor. The density

$$Q = I/A \quad \textbf{(21)}$$

whence, for

Round Rod — *Flat Ribbon*

$$Q = \frac{4\,I}{\pi\,D^2} \quad \textbf{(22a)} \qquad Q = \frac{I}{a\,b} \quad \textbf{(22b)}$$

and,

$$I = \frac{\pi}{4}\,Q\,D^2 \quad \textbf{(23a)} \qquad I = Q\,a\,b \quad \textbf{(23b)}$$

Substituting this value in

(17a) and (18a) — (17b) and (18b)

$$G = \frac{1}{4}\,Q^2\,\rho\,D\,m \quad \textbf{(24a)} \qquad G = \frac{Q^2\,m\,\rho\,a\,b}{2\,(a + b)} \quad \textbf{(24b)}$$

$$G = 0.000010225\,Q^2\,D\,m \quad \textbf{(25a)} \qquad G = \frac{0.00002045\,Q^2\,m\,a\,b}{a + b} \quad \textbf{(25b)}$$

In short, the wattage per sq. in. of the surface varies with the square of the current density, for any given cross-section.

The voltage per unit length may also be of importance, in its relation to the wattage per sq. in. of surface. This appears very easily from Equation (19). Setting F equal to the voltage per in. of length, and F' equal to the voltage per ft. of length,

$$F = E/L \quad \textbf{(26)}$$

and

$$F' = \frac{12\,E}{L} \quad \textbf{(27)}$$

Since E^2/L^2 appears in both Equations (19) and (20), the substitution will yield,

Round Rod — *Flat Ribbon*

$$G = \frac{F^2\,D}{4\,\rho\,m} \quad \textbf{(28a)} \qquad G = \frac{F^2\,a\,b}{2\,m\,\rho\,(a + b)} \quad \textbf{(28b)}$$

$$G = \frac{6111\,F^2\,D}{m} \quad \textbf{(29a)} \qquad G = \frac{12222\,a\,b\,F^2}{m\,(a + b)} \quad \textbf{(29b)}$$

Or, from (27),

$$G = \frac{F'^2\,D}{576\,\rho\,m} \quad \textbf{(30a)} \qquad G = \frac{F'^2\,a\,b}{288\,m\,\rho(a+b)} \quad \textbf{(30b)}$$

$$G = \frac{42.43\,D\,F'^2}{m} \quad \textbf{(31a)} \qquad G = \frac{84.86\,a\,b\,F'^2}{m\,(a + b)} \quad \textbf{(31b)}$$

Stefan-Boltzman Law

The Stefan-Boltzman equation is an hypothesis guessed at by Stefan[1] in 1879 as a result of one of Tyndall's experiments, and conclusively proved by Boltzman[2] mathematically in 1884, being substantiated by many subsequent investigations. It states that the total radiation from a body is proportional to the fourth power of its absolute temperature:

$$R = K\,T^4 \tag{32}$$

where

R is the total radiation per unit of surface in fundamental units;

T is the absolute temperature on the Kelvin scale; and

K is the constant of proportionality.

the *Smithsonian Physical Tables*. This value has been used throughout the present paper.

For any combination of temperatures, using nickel chromium resistors,

$$G = 3.169\left[\left(\frac{T_1 + 459.4}{1000}\right)^4 - \left(\frac{T_0 + 459.4}{1000}\right)^4\right] \tag{34}$$

This equation expresses the total possible exchange of radiation between resistor and charge, under the temperature conditions stated, with the heated body running unmuffled in the furnace chamber, and takes into account the emissivity of the heater material, which is slightly less than that of a black body.

Frequently, however, the full effect of the radiating

TABLE I

Authority	Value given	Units	K' = value in watts/sq.in./deg. fahr.$^4 \times 10^{12}$
Smithsonian Physical Tables	5.72×10^{-12}	Watts/cm^2/deg. cent.^{4}	3.521
Goodenough[5]	0.1618×10^{-8}	B. t. u./sq. ft./deg. fahr.^{4}/hr.	3.293
Kimball[6]	1.3×10^{-12}	g. cal./cm^2/deg. cent.^{4}/sec.	3.335
Standard Handbook for Electrical Engineers[7]	5.7×10^{-12}	Watts/cm^2/deg. cent.^{4}	3.506
Kurlbaum[8]	5.32×10^{-12}	Watts/cm^2/deg. cent.^{4}	3.264
Woodson[9]	1700×10^{-12}	B. t. u./sq. ft./deg. fahr.^{4}/hr.	3.459

The equation is not very useful in this form, since the total radiation is scarcely ever of much import. Rather is the exchange of heat between hot and cold bodies, or between resistor and charge, in the furnace chamber, of interest. R may, therefore, be expressed in watts per sq. in. of radiating surface, and

$$G = K'\,e\left[\left(\frac{T_1 + 459.4}{1000}\right)^4 - \left(\frac{T_0 + 459.4}{1000}\right)^4\right] \tag{33}$$

wherein

K' is the constant of proportionality;

e is the relative emissivity of the radiating material. For black bodies, $e = 1.00$; for nickel chromuim, $e = 0.9$;

T_1 is the temperature of the resistor in deg. fahr.; and

T_0 is the temperature of the furnace chamber in deg. fahr.

The values of K' are variously given by present-day authorities. The divergence of the figures is indicated by those cited in Table I. An extensive study of all available data on this constant has been made by the Bureau of Standards.[3] The best agreement seems to be upon the figure given by Coblentz,[4] and included in

periphery is not obtained. Proximity of walls or charge, or adjacent strands of the element exercise a blanketing effect, which reduces the liberation of heat at any given temperature.[10] Where the efficiency of radiation is less than 100 per cent, under such conditions as those described by Keene and Luke, Fig. 2 defines the limiting allowable conditions for design of nickel chromium units. It is based upon a maximum allowable element temperature of 2100 deg. fahr. To insure long life of the heaters, their rating must fall below the curve which represents the condition of size, spacing, and radiating efficiency (C) under consideration.

The curves of Fig. 2 apply only to the heat lost by radiation; but that they may be safely employed to indicate the permissible total losses from the element is shown by the dotted curve, which gives the loss of heat from a body at 2100 deg. fahr. by both radiation and convection. Rice[11] has shown that the convection

1. *Sitzungsberichte der Königlichen Gesellschaft der Wissenschaften zu Wien*, Band LXXIX, p. 391, 1879.

2. *Annalen der Physik*, Band XXII, pp. 31 and 291, 1884. This proof is given in English in simple form in *Theory of Heat*, by Thomas Preston, 3rd Ed., p. 566. It is based upon thermodynamics and the electromagnetic theory of light. Since its appearance, the so-called Wein-Planck law has appeared (*Vorlesungen über Warmestralung*, partially reprinted in *Encyklopädie der Mathematischen Wissenschaften*, Band V-3, Heft 2, 1909) and the Stefan Law is also deducible with relative simplicity from this equation, which gives the distribution of radiation in the spectrum by relating it to wavelength, and absolute temperature.

3. U. S. Department of Commerce, Bureau of Standards, Scientific Paper No. 406, entitled *Present Status of the Constants and Verification of the Laws of Thermal Radiation of a Uniformly Heated Inclosure*, by W. W. Coblentz, December 20, 1920.

4. *Proceedings of the National Academy of Science*, Vol. 3, p. 504, 1917.

5. *Mechanical Engineers' Handbook*, 2nd Ed., p. 317.

6. *College Physics*, 2nd Ed., p. 317.

7. Fifth edition, p. 204.

8. Quoted in Preston, *Theory of Heat*, 3rd Ed., p. 564. Later work (1912) gives a value of K' of 3.335. See note 6.

9. A. I. E. E. Journal, December, 1925, p. 1354.

10. A discussion of the relative efficiency of several arrangements of heating units may be found in *Rating of Heating Elements for Electric Furnaces*, by A. D. Keene and G. E. Luke, A. I. E. E. Transactions, Vol. XLV, 1926, p. 475.

11. A. I. E. E. Transactions, Part I, Vol. XLII, 1923, p. 653; Part II, Vol. XLIII, 1924, p. 131.

loss in still air is

$$G' = K'' (T_1 - T_0)^{5/4} \qquad (35)$$

where T_1 and T_0 are the temperatures of the resistor and chamber respectively, in deg. fahr. An average value of K'' for nickel chromium may be taken from those determined by Keene and Luke to be approximately 0.0010795, although this evidently varies widely for different shapes of resistor.

The total losses, upon which the dotted curve of Fig. 2 is based, would then be the sum of Equations (34) and (35):

$$G + G' = 3.169 \left[\left(\frac{T_1 + 459.4}{1000} \right)^4 - \left(\frac{T_0 + 459.4}{1000} \right)^4 \right] + 0.0010795 (T_1 - T_0)^{5/4} \qquad (36)$$

It will be seen that when the limiting curves given embody radiation only, a small factor of safety is obtained. They may, therefore, be used with confidence.

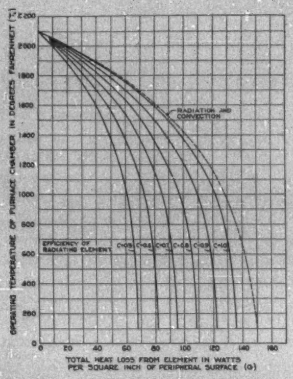

FIG. 2—THEORETICAL RADIATION FROM NICKEL-CHROMIUM RESISTOR OPERATING AT 2100 DEG. FAHR.

COMBINATIONS OF THE TWO LAWS

It is at once evident that in designing resistors, both of the laws must be heeded; and some very useful relationships can be adduced, by substituting the values of G found in Equations (17), (18), (19), (20), (24), (25), (28), (29), (30), and (31) in Equation (34). For this purpose, set $T_1 = 2100$ deg. fahr., in Equation (34), and write

$$G_e = 3.169\, C \left[42.85 - \left(\frac{T_0 + 459.4}{1000} \right)^4 \right] \qquad (37)$$

in which C is the radiating efficiency already defined, as shown in Fig. 2. Then from (17) and (18),

Round Rod	*Flat Ribbon*
$D = 0.7401 \sqrt[3]{\dfrac{I^2 \rho m}{G_e}}$ (38a)	$a\,b\,(a + b) = 0.5 \dfrac{I^2 \rho m}{G_0}$ (38b)
$D = 0.0255 \sqrt[3]{\dfrac{I^2 m}{G_0}}$ (39a)	$a\,b\,(a + b) = 0.00002045 \dfrac{I^2 m}{G_0}$ (39b)

Similarly, from (19) and (20)

Round Rod	*Flat Ribbon*
$D = \dfrac{4 m \rho G_0 L^2}{E^2}$ (40a)	$\dfrac{a\,b}{a + b} = \dfrac{2 m \rho L^2 G_0}{E^2}$ (40b)
$D = \dfrac{0.0001637\, m G_0 L^2}{E^2}$ (41a)	$\dfrac{a\,b}{a + b} = \dfrac{0.0000818\, m L^2 G_0}{E^2}$ (41b)

From (24) and (25)

Round Rod	*Flat Ribbon*
$D = \dfrac{4 G_0}{Q^2 m \rho}$ (42a)	$\dfrac{a\,b}{a + b} = \dfrac{2 G_0}{Q^2 m \rho}$ (42b)
$D = \dfrac{97{,}800\, G_0}{Q^2 m}$ (43a)	$\dfrac{a\,b}{a + b} = \dfrac{48{,}890\, G_0}{Q^2 m}$ (43b)

Equations (28) to (31) inclusive give

Round Rod	*Flat Ribbon*
$D = \dfrac{4 m \rho G_0}{F^2}$ (44a)	$\dfrac{a\,b}{a + b} = \dfrac{2 m \rho G_0}{F^2}$ (44b)
$D = \dfrac{0.0001637\, m G_0}{F^2}$ (45a)	$\dfrac{a\,b}{a + b} = \dfrac{0.0000818\, m G_0}{F^2}$ (45b)
$D = \dfrac{576 m \rho G_0}{F'^2}$ (46a)	$\dfrac{a\,b}{a + b} = \dfrac{288 m \rho G_0}{F'^2}$ (46b)
$D = \dfrac{0.02358\, m G_0}{F'^2}$ (47a)	$\dfrac{a\,b}{a + b} = \dfrac{0.01179\, m G_0}{F'^2}$ (47b)

The values of G_0 for any temperature and radiating efficiency may be taken directly from Fig. 2. To facilitate calculations involving flat ribbon, Table II may be used. It gives values of a, b, $a\,b$, $a + b$, $a\,b\,(a + b)$, and $a\,b/(a + b)$ for a number of standard sizes of ribbon.

For any given temperature, the equations may be still further simplified. For example, most heat-treating furnaces are designed for operation at 1850 deg. fahr. Under these conditions, Equations (38) to (47), inclusive, become,

Round Rod

$$D = 0.00713 \sqrt[3]{\frac{I^2 m}{C}} \quad \textbf{(48a)}$$

$$D = \frac{0.007485\ m\ L^2\ C}{E^2} \quad \textbf{(49a)}$$

$$D = 4{,}473{,}000 \frac{C}{Q^2 m} \quad \textbf{(50a)}$$

Flat Ribbon

$$a\,b\,(a+b) = 0.0000004472 \frac{I^2 m}{C} \quad \textbf{(48b)}$$

$$\frac{a\,b}{(a+b)} = 0.003742 \frac{m\ L^2\ C}{E^2} \quad \textbf{(49b)}$$

$$\frac{a\,b}{(a+b)} = 2{,}236{,}500 \frac{C}{Q^2 m} \quad \textbf{(50b)}$$

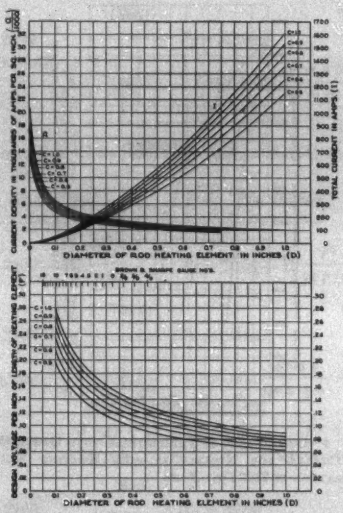

FIG. 3—THEORETICAL DESIGN CURVES FOR NICKEL-CHROMIUM ROD HEATING ELEMENTS IN ELECTRIC FURNACES OPERATING AT 1850 DEG. FAHR.

$$D = \frac{0.007485\ m\ C}{F^2} \quad \textbf{(51a)}$$

$$\frac{a\,b}{(a+b)} = 0.003742 \frac{m\ C}{F^2} \quad \textbf{(51b)}$$

$$D = \frac{1.078\ m\ C}{F'^2} \quad \textbf{(52a)}$$

$$\frac{a\,b}{(a+b)} = \frac{0.539\ m\ C}{F'^2} \quad \textbf{(52b)}$$

For any value of C, these relationships provide complete design data. In the case of the round rod, they may be plotted. Fig. 3 shows Equations (48a), (50a), and (51a). The flat ribbon calculations are readily made with the aid of Table II, the range of sizes making it impractical and inaccurate to plot them.

TABLE II

Width in inches a	Thickness in inches b	B & S gage number	Cross-sectional area in square inches $a\,b$	Semi-perimeter in inches $a+b$	$a\,b(a+b)$	$\frac{a\,b}{a+b}$
$\frac{1}{16}$	0.0640	14	0.00400	0.127	0.000506	0.0316
	0.0510	16	0.00319	0.114	0.000362	0.0281
	0.0400	18	0.00250	0.103	0.000256	0.0244
	0.0320	20	0.00200	0.0945	0.000189	0.0212
	0.0253	22	0.00158	0.0878	0.000139	0.0180
	0.0201	24	0.00126	0.0826	0.000104	0.0152
	0.0159	26	0.000994	0.0784	0.0000779	0.0127
	0.0126	28	0.000788	0.0751	0.0000591	0.0105
	0.0100	30	0.000675	0.0725	0.0000453	0.00862
	0.0080	32	0.000500	0.0705	0.0000353	0.00709
	0.0063	34	0.000394	0.0688	0.0000271	0.00572
	0.0050	36	0.000313	0.0675	0.0000211	0.00463
	0.0040	38	0.000250	0.0665	0.0000166	0.00376
	0.0030	40	0.000188	0.0655	0.0000123	0.00286
$\frac{1}{8}$	0.0640	14	0.00800	0.189	0.00151	0.0423
	0.0510	16	0.00638	0.176	0.00112	0.0362
	0.0400	18	0.00500	0.165	0.000825	0.0303
	0.0320	20	0.00400	0.157	0.000628	0.0255
	0.0253	22	0.00316	0.150	0.000475	0.0210
	0.0201	24	0.00251	0.145	0.000365	0.0173
	0.0159	26	0.00199	0.141	0.000280	0.0141
	0.0126	28	0.00158	0.138	0.000217	0.0114
	0.0100	30	0.00125	0.135	0.000169	0.00926
	0.0080	32	0.00100	0.133	0.000133	0.00752
	0.0063	34	0.000788	0.131	0.000103	0.00600
	0.0050	36	0.000625	0.130	0.0000813	0.00481
	0.0040	38	0.000500	0.129	0.0000645	0.00388
	0.0030	40	0.000375	0.128	0.0000480	0.00293
$\frac{1}{4}$	0.0640	14	0.0160	0.314	0.00502	0.0510
	0.0510	16	0.0128	0.301	0.00384	0.0424
	0.0400	18	0.0100	0.290	0.00290	0.0345
	0.0320	20	0.00800	0.282	0.00226	0.0284
	0.0253	22	0.00633	0.275	0.00174	0.0230
	0.0201	24	0.00503	0.270	0.00136	0.0186
	0.0159	26	0.00398	0.266	0.00106	0.0149
	0.0126	28	0.00315	0.263	0.000827	0.0120
	0.0100	30	0.00250	0.260	0.000650	0.00962
	0.0080	32	0.00200	0.258	0.000516	0.00775
	0.0063	34	0.00158	0.256	0.000404	0.00615
	0.0050	36	0.00125	0.255	0.000319	0.00490
	0.0040	38	0.00100	0.254	0.000254	0.00394
	0.0030	40	0.000750	0.253	0.000190	0.00296
$\frac{1}{2}$	0.0640	14	0.0320	0.564	0.0180	0.0567
	0.0510	16	0.0255	0.551	0.0141	0.0463
	0.0400	18	0.0200	0.540	0.0108	0.0370
	0.0320	20	0.0160	0.532	0.00851	0.0301
	0.0253	22	0.0127	0.525	0.00665	0.0241
	0.0201	24	0.0101	0.520	0.00523	0.0193
	0.0159	26	0.00795	0.516	0.00410	0.0154
	0.0126	28	0.00630	0.513	0.00323	0.0123
	0.0100	30	0.00500	0.510	0.00255	0.00980
	0.0080	32	0.00400	0.508	0.00203	0.00787
	0.0063	34	0.00315	0.506	0.00159	0.00622
	0.0050	36	0.00250	0.505	0.00126	0.00495
	0.0040	38	0.00200	0.504	0.00101	0.00397
	0.0030	40	0.00150	0.503	0.000755	0.00298
$\frac{3}{4}$	0.0640	14	0.0480	0.814	0.0391	0.0590
	0.0510	16	0.0383	0.801	0.0306	0.0478
	0.0400	18	0.0300	0.790	0.0237	0.0380
	0.0320	20	0.0240	0.782	0.0188	0.0307
	0.0253	22	0.0190	0.775	0.0147	0.0245
	0.0201	24	0.0151	0.770	0.0116	0.0196
	0.0159	26	0.0119	0.766	0.00913	0.0156
	0.0126	28	0.00945	0.763	0.00721	0.0124
	0.0100	30	0.00750	0.760	0.00570	0.00987
	0.0080	32	0.00600	0.758	0.00455	0.00792
	0.0063	34	0.00473	0.756	0.00357	0.00625
	0.0050	36	0.00375	0.755	0.00283	0.00497
	0.0040	38	0.00300	0.754	0.00226	0.00398
	0.0030	40	0.00225	0.753	0.00169	0.00299

TABLE II—*Continued*

Width in inches a	Thickness in inches b	B & S gage number	Cross-sectional area in square inches $a\,b$	Semi-perimeter in inches $a + b$	$a\,b(a + b)$	$\frac{a\,b}{a + b}$
	0.0640	14	0.0640	1.0640	0.0681	0.0602
	0.0510	16	0.0510	1.0510	0.0536	0.0485
	0.0400	18	0.0400	1.0400	0.0416	0.0385
	0.0320	20	0.0320	1.0320	0.0330	0.0310
	0.0253	22	0.0253	1.0253	0.0259	0.0247
	0.0201	24	0.0201	1.0201	0.0205	0.0197
	0.0159	26	0.0159	1.0159	0.0162	0.0157
1	0.0126	28	0.0126	1.0126	0.0128	0.0124
	0.0100	30	0.0100	1.0100	0.0101	0.00990
	0.0080	32	0.0080	1.0080	0.00806	0.00794
	0.0063	34	0.0063	1.0063	0.00634	0.00626
	0.0050	36	0.0050	1.0050	0.00503	0.00498
	0.0040	38	0.0040	1.0040	0.00402	0.00398
	0.0030	40	0.0030	1.0030	0.00301	0.00299
	0.2500	..	0.313	1.5000	0.469	0.208
$1\frac{1}{4}$	0.1875	..	0.234	1.4375	0.337	0.163
	0.1250	..	0.156	1.3750	0.215	0.114
	0.1000	..	0.150	1.600	0.240	0.0938
	0.0800	..	0.120	1.580	0.190	0.0759
	0.0700	..	0.105	1.570	0.165	0.0669
	0.0600	..	0.0900	1.560	0.140	0.0577
$1\frac{1}{2}$	0.0500	..	0.0750	1.550	0.116	0.0484
	0.0400	..	0.0600	1.540	0.0924	0.0390
	0.0300	..	0.0450	1.530	0.0689	0.0294
	0.0200	..	0.0300	1.520	0.0456	0.0197
	0.2500	..	0.500	2.2500	1.125	0.222
2	0.1875	..	0.375	2.1875	0.820	0.171
	0.1250	..	0.250	2.1250	0.531	0.118

The equations derived are rigorous in every respect, requiring judgment only in the selection of C, which has already been discussed. They are the maximum allowable ratings in each case and cannot be exceeded. Good practise indicates that it is best to allow considerable latitude for safety.

Empirical Relationships

The equations just derived, and the curves of Figs. 2 and 3, define limiting conditions, which the laws of Nature impose upon industrial heating design. It is to be feared that, in the past, a lack of understanding of these fundamentals has led to a rather haphazard procedure. Guided only by experience and data gathered in the field, the designer has made little or no effort to correlate these with the physical laws.

Strangely enough, however, one empirical relationship which still forms the basis of much design work, approaches remarkably close to the limitations set up by the theory. Yet, when it was first used, no work had been done on the screening effect of nearby objects; it was used because, in almost every case, it gave elements with long life. It is interesting to see why.

The straight line,

$$G = \frac{2100 - T_0}{30} \tag{53}$$

is shown in Fig. 4 (Curve B), in its relation to curve A,—50 per cent radiating efficiency replotted from Fig. 2. It is evident that the line gives an extremely conservative rating, especially at moderate temperatures, even under such conditions as double banking or close spacing of units. It is interesting to look at curve C in Fig. 4, which is plotted from that given by Woodson[9] for the design of heating elements outside the vessels of metal melting pots. Although the curve does not conform to the Stefan-Boltzman law, at any radiating efficiency, it is an effort to express the results of experience in this very arduous type of service. The use of such a rating ought to be confined to work where coiled wire is employed in confined spaces. Certainly Equation (53) provides ample allowances for all ordinary purposes, where the element is free to radiate.

It is interesting to see how the expressions derived for proportioning resistors work out, when substituted in Equation (53). This may be written

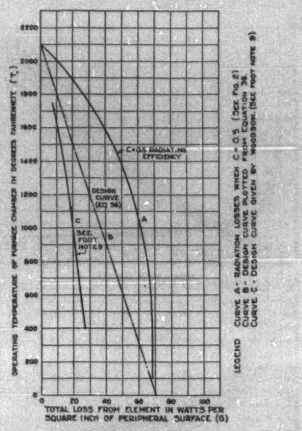

Fig. 4—Comparison of Theoretical with Other Radiation Curves for Nickel-Chromium Heating Element Temperature of 2100 Deg. Fahr

$$T_0 = 2100 - 30\,G \tag{54}$$

Substituting for G its value from (17) and (18),

Round Rod

$$T_0 = 2100 - \frac{120\,I^2\,\rho\,m}{\pi^2\,D^3} \tag{55a}$$

$$T_0 = 2100 - 0.0004974\,\frac{I^2\,m}{D^3} \tag{56a}$$

Flat Ribbon

$$T_0 = 2100 - \frac{15\,I^2\,\rho\,m}{a\,b\,(a + b)} \tag{55b}$$

$$T_0 = 2100 - 0.0006135\,\frac{I^2\,m}{a\,b(a+b)} \tag{56b}$$

Likewise, from Equations (19) and (20),

Round Rod	*Flat Ribbon*
$T_0 = 2100 - 7.5 \dfrac{E^2 D}{m \rho L^2}$ (57a)	$T_0 = 2100 - 15 \dfrac{E^2 a b}{m \rho L^2 (a+b)}$ (57b)
$T_0 = 2100 - 183{,}330 \dfrac{E^2 D}{m L^2}$ (58a)	$T_0 = 2100 - 36{,}660 \dfrac{E^2 a b}{m L^2 (a+b)}$ (58b)

Using Equations (24) and (25) in (53), for

Round Rod	*Flat Ribbon*
$T_0 = 2100 - 7.5 Q^2 \rho D m$ (59a)	$T_0 = 2100 - 15 \dfrac{Q^2 m \rho a b}{(a+b)}$ (59b)
$T_0 = 2100 - 0.00030675 Q^2 D m$ (60a)	$T_0 = 2100 - 0.0006135 \dfrac{Q^2 m a b}{(a+b)}$ (60b)

And from Equations (28), (29), (30), and (31), respectively, for

Round Rod	*Flat Ribbon*
$T_0 = 2100 - 7.5 \dfrac{F^2 D}{\rho m}$ (61a)	$T_0 = 2100 - 15 \dfrac{F^2 a b}{m \rho (a+b)}$ (61b)
$T_0 = 2100 - 183{,}330 \dfrac{F^2 D}{m}$ (62a)	$T_0 = 2100 - 366{,}660 \dfrac{F^2 a b}{m (a+b)}$ (62b)
$T_0 = 2100 - \dfrac{F'^2 D}{19.2 \rho m}$ (63a)	$T_0 = 2100 - \dfrac{F'^2 a b}{9.6 m \rho (a+b)}$ (63b)
$T_0 = 2100 - 1272.9 \dfrac{F'^2 D}{m}$ (64a)	$T_0 = 2100 - 2546.4 \dfrac{F'^2 a b}{m (a+b)}$ (64b)

These relationships give ready means of proportioning heating elements safely for any furnace operating temperature, directly from their physical dimensions and the electrical characteristics of the furnace circuit. As before, the simplification may be carried one step further. At a maximum temperature of 1850 deg. fahr., with T_0 equal to 1850 deg., a very simple set of results appears:

Round Rod	*Flat Ribbon*
$D = 0.01257 \sqrt[3]{I^2 m}$ (65a)	$a b (a+b) = 0.000002455 I^2 m$ (65b)
$D = 0.001363 \dfrac{m L^2}{E^2}$ (66a)	$\dfrac{a b}{(a+b)} = 0.0006813 \dfrac{m L^2}{E^2}$ (66b)
$D = \dfrac{814{,}900}{Q^2 m}$ (67a)	$\dfrac{a b}{(a+b)} = \dfrac{407{,}400}{Q^2 m}$ (67b)
$D = 0.001363 \dfrac{m}{F^2}$ (68a)	$\dfrac{a b}{(a+b)} = 0.0006813 \dfrac{m}{F^2}$ (68b)
$D = 0.1964 \dfrac{m}{F'^2}$ (69a)	$\dfrac{a b}{(a+b)} = 0.09818 \dfrac{m}{F'^2}$ (69b)

To aid in using these equations, some of them, (Nos. (65a), (67a), and (68a)) are plotted in Fig. 5.

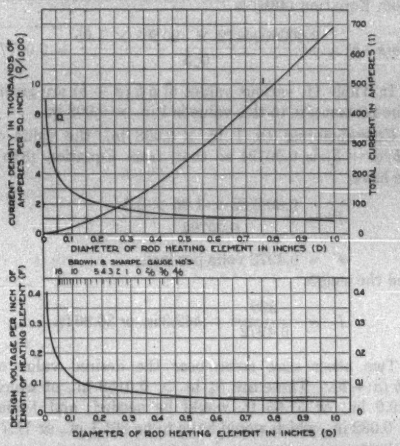

FIG. 5—DESIGN CURVES FOR NICKEL-CHROMIUM ROD HEATING ELEMENTS IN ELECTRIC FURNACES OPERATING AT 1850 DEG. FAHR. (BASED ON CURVE *B*, FIG. 4)

If the designer is interested in more accurate design, he may use the theoretical curves, Fig. 3, instead of the empirical ones just discussed. He must then apply his own factors of safety, a matter which requires considerable experience.

It should be observed that all of the equations in which the expression ρ appears are perfectly general, applying to both metallic and non-metallic resistors, provided the proper value of m is taken for the temperature of operation.

NUMERICAL EXAMPLE

It may be of interest to carry through the design of a heating element by the two methods, to see what difference is obtained.

For example, assume that it is desired to design the heating elements for a 20-kw. furnace, to operate on 220 volts, single-phase with maximum operating temperature of 1850 deg. fahr. and $C = 0.5$.

The current required will be

$$\frac{20{,}000}{220} = 90.9 \text{ amperes.}$$

From the curves, Fig. 3, it is evident that 90.9 amperes is permissible in a diameter of 0.185, to which No. 4 B & S gage is the nearest standard size. Directly beneath, the voltage per in. of length is 0.1459, and the required length is, therefore,

$$\frac{220}{0.1459} = 1508 \text{ in., or } 125.6 \text{ ft.}$$

If the ribbon is to be used, the value of $a\,b\,(a + b)$ from Equation (**48b**) is

$$a\,b(a+b) = \frac{0.0000004472 \times (90.9)^2 \times 1.05}{0.5} = 0.00776$$

In Table II, various values of $a\,b\,(a + b)$ are given. The nearest size to this value is 1 in. × 0.008 in.

Corresponding to 1 in. × 0.008 in., the value of $a\,b/(a + b)$ is 0.00794 so that, from Equation (**51b**), we have

$$F^2 = \frac{0.003742 \times 1.05 \times 0.5}{0.00794}$$

$$F = 0.4972 \text{ volts per inch.}$$

and the length

$$L = \frac{220}{0.4972} = 442.3 \text{ in. or } 36.86 \text{ ft.}$$

Two other sizes come near the desired value of $a\,b\,(a + b)$. These are ¾ in. × 0.0159 in., of which 619.9 in., or 51.65 ft. would be needed; and ½ in. × 0.032 in., which would have to be 861.1 in., or 71.77 ft. long.

Using the curve for I, Fig. 5, it is seen that 90.9 amperes is the allowable current flow in a diameter of 0.258, which is No. 2, B & S gage. The voltage per in. of length may now be found, on the F curve. The value is 0.0744 volts per in. Hence, the length required is 220/0.0744 = 2956 in. or 246.35 ft.

In the case of ribbon, the value of $a\,b\,(a + b)$ can be obtained from Equation (**65b**).

$$a\,b(a + b) = 0.000002455 \times (90.9)^2 \times 1.05 = 0.0213$$

In Table II, the nearest size to this value would be ¾ in. × 0.04 in. ribbon. For this, the value of $a\,b/(a + b) = 0.0380$.

Using Equation (**68b**), we have

$$0.0380 = \frac{0.0006813 \times 1.05}{F^2}$$

$$F = 0.1372 \text{ volts per in.}$$

The total length then is $\frac{220}{0.1372} = 1603$ in., or 133.6 ft.

Another size approaches fairly closely in the table. This is 1 in. × 0.0253, for which $a\,b/(a + b) = 0.0247$ and $F = 0.1702$; whence the required length is 1292 in. or 107.95 ft.

The designer may choose whichever of the elements best suits the space available in the furnace and the manner of mounting. The theoretical values are as follows:

Round Rod	*Flat Ribbon*
125.6 ft. of No. 4 wire	36.86 ft. of 1-in. × 0.008-in. ribbon
	51.65 ft. of ¾-in. × 0.0159-in. ribbon
	71.77 ft. of ½-in. × 0.032-in. ribbon

Conservative practise dictates that the empirical values provide some factor of safety; and, although they involve the use of more metal, they mean lower ratings, decreased element temperatures, greater mechanical strength, and longer life. They are:

Round Rod	*Flat Ribbon*
246.35 ft. of No. 2 wire	107.95 ft. of 1-in. × 0.0253-in. ribbon
	133.6 ft. of ¾-in. × 0.04-in. ribbon.

Conclusion

The derivation of fundamental equations for use in the design of electric heating elements has shown what are the physical limitations of the problem. As a sequel thereto, it has been found that one of the earliest rules-of-thumb gives good factors of safety at all electric furnace temperatures. The use of this relationship is to be commended, in the interest of conservatism, bearing in mind that the best design still relies upon the scanty information available about the material of which the heating elements are constructed.

Discussion

J. Ormondroyd: (communicated after adjournment) Footnote No. 2, p. 1199 in common with all publications on this subject that I have seen, assumes that it is so easy to show the compatibility between the Stefan-Boltzman Law and Planck's Law, that the actual work is never given. It *is* easy to derive the older law from the newer one but most readers feel uncomfortable when they let the mere claim of ease pass for a detailed proof. My comment merely consists of this detailed proof as follows:

Planck's equation giving energy intensity for every wavelength.

$$u_2 = c_1 \lambda^{-5} \left[e^{\frac{c_2}{\lambda T}} - 1 \right]^{-1} \text{ watts per sq. cm.} \qquad (1)$$

where λ wave length in μ

T abs. temp. centigrade

This can be expanded into a series by means of the Binomial Theorem.

$$u_\lambda = c_1 \sum_{n=1}^{n=\infty} \lambda^{-5} e^{-\frac{n c_2}{\lambda T}} \text{ watts per sq. cm.} \tag{2}$$

The total radiation over the whole spectrum is

$$U = c_1 \sum_{n=1}^{n=\infty} \int_0^\infty \lambda^{-5} e^{-\frac{n c_2}{\lambda T}} d\lambda \text{ watts per sq. cm.} \tag{3}$$

Integrating by parts

$$\int_0^\infty \lambda^{-5} e^{-\frac{n c_2}{\lambda T}} d\lambda = \left| e^{-\frac{n c_2}{\lambda T}} \left[\lambda^{-5} \frac{T}{n c_2} + \lambda^{-3} \frac{3 T^2}{n^2 c_2^2} + \lambda^{-1} \frac{6 T^3}{n^3 c_2^3} + \frac{6 T^4}{n^4 c_2^4} \right] \right|_0^\infty = \frac{6 T^4}{n^4 c_2^4} \tag{4}$$

When the limit $\lambda = 0$ is substituted in (4), the indeterminate form $e^{-\frac{k}{\lambda}} \lambda^{-a} = 0 \cdot \infty$ appears. This can be shown to be equal to zero.

From (3) and (4) the total radiation is

$$U = c_1 \sum_{n=1}^{n=\infty} \frac{6 T^4}{n^4 c_2^4} = \frac{6 c_1}{c_2^4} \sum_{n=1}^{n=\infty} \frac{1}{n^4} T^4 \tag{5}$$

$$\sum_{n=1}^{n=\infty} \frac{1}{n^4} = \frac{\pi^4}{90} \tag{6}$$

$$c_1 = 3.7 \times 10^4 \text{ for } u \text{ in watts per sq. cm.} \tag{7}$$

$$c_2 = 14325 \text{ for } \lambda \text{ in } \mu \tag{8}$$

Therefore

$$\frac{6 c_1 \pi^4}{c_2^4 90} T^4 = 5.72 \times 10^{-12} T^4 \text{ watts per sq. cm.} \tag{9}$$

The value of the Stefan-Boltzman Constant given in the Smithsonian Tables is 5.71×10^{-12}.

Edwin Fleischmann: Mr. Ormondroyd's derivation of the Stefan-Boltzman Law from Planck's Equation serves as a valuable supplement, completing the fundamental background of the derivations given in the paper. It explains and substantiates the reference and statement of footnote 2.

Safe Loading of Oil-Immersed Transformers

E. T. NORRIS[1]
Member, A. I. E. E.

Synopsis.—An attempt is made to describe simply and clearly the basis of the temperature rating of oil-immersed transformers and the relations between the temperatures measured in service, the maximum safe output rating, and the national standard ratings.

A description is given of an instrument for indicating the safe output rating of a transformer in service under any conditions of short time or continuous loading.

List of Symbols

A = Temperature of cooling medium—usually air or water.
D = Maximum variation in temperature of the circulating oil.
G = Maximum winding temperature difference.
Ga = Average winding temperature difference.
H = Hottest spot temperature.
h = $H - A$.
K = Constant.
N = Ratio copper to iron loss at full load.
P = Load on transformer as a fraction of full load.
R = Average temperature of the winding by resistance.
r = $R - A$.
T = Hottest oil temperature.
t = $T - A$.

As the load current in an oil-immersed transformer is increased, its temperature rises due to the increase in losses. One limit to the load which may be taken from a transformer is, therefore, set by the maximum temperature permissible.

Basis of Output Rating

The material most susceptible to damage by heat in a transformer is the insulation. The fundamental basis of the rating of the transformer is, therefore, the maximum temperature the insulation will safely withstand. The temperature of the hottest part of the insulation of the transformer (or the hottest spot temperature H) must never be allowed to exceed this safe limit.

The limit applies, of course, to the insulation both in the core and in the windings. Since, however, the core losses are practically constant for all loads, whereas the copper or winding losses increase rapidly with the load, it is the winding insulation which is most affected by the loading, and its hottest spot temperature which determines the safe maximum load.

If it were possible to measure the hottest spot temperature while the transformer was in operation, it would be a simple matter to determine the maximum safe rating and to ensure that it was not exceeded. Unfortunately, except in a few special instances, there are no known practical means of measuring this temperature directly.

As a rule, it is not possible to employ thermocouples or other forms of embedded temperature detector in such a way as not to weaken the transformer insulation and yet to record accurately the true hottest spot temperature, and to be safe in operation. This position has recently been confirmed by a meeting of the International Electrotechnical Commission.

The most useful practical temperature measurements are:

a. The temperature of the hottest oil by thermometer = T.

This temperature is easily measured and a continuous record can be obtained whilst the transformer is in service. It does not, however, follow closely short time load variations.

b. The average temperature R of the windings calculated from the resistance.

This measurement is mainly suitable under testing conditions. It is necessary to disconnect the transformer in order to measure the resistance, so that readings cannot be obtained during operation. The temperature measured is the average temperature of the whole winding.

To determine the maximum safe rating of a transformer from these two temperatures, it is necessary to know the relations between these temperatures and the hottest spot temperature.

Standard Output Ratings

It is important to distinguish between the true thermal rating of a transformer and its rating in accordance with a standard specification.

The true thermal rating is determined by the maximum safe hottest spot temperature. It is not a constant for a given transformer, but may have any value depending on the conditions of loading and cooling.

The primary purpose of a national standard specification is to provide a basis for the comparison of transformers of different makes and types. The standard rating must be so defined that it can be determined easily and accurately in practise, and is, therefore, expressed in terms of the observable temperatures R and T. It is based on certain definite loading and cooling conditions so as to be a constant for a given transformer.

The manner in which the observable temperatures R and T are derived from the hottest spot temperature for standardization purposes, is explained in Pamphlet No. 1 of the A. I. E. E. Standards.

1. Chief designer, Ferranti, Ltd., London, England.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-103

Typical values of these observable temperatures after continuous rated load for the standards of three countries are given in Table I. The standards specify both these limiting temperatures and state that neither may be exceeded.

TABLE I

Country	Self-cooling		Forced-cooling	
	R	T	R	T
U. S. A.	95	90	80	..
Britain	95	90	90	75
Germany	105	95	..	..

The kv-a. output of a transformer depends on its temperature rise above the temperature of the cooling medium. The output can, therefore, be increased:

a. By raising its working temperature until the maximum safe hot spot limit is reached.

b. By reducing the temperature of its cooling medium. Hence, to standardize the output rating of a transformer, the temperature rise must be specified. A complete standard specification includes the two observable temperatures R and T together with the temperature of the cooling medium.

Typical values of the cooling medium temperature A together with the resulting temperature rises are given in Table II.

TABLE II

Country	Self-cooling			Forced-cooling		
	A	r	t	A	r	t
U. S. A.	40	55	50	25	55	..
Britain	40	55	50	25	65	50
Germany	35	70	60	..	..	..

The cooling medium temperature A is the temperature of the ambient air in self-cooled transformers, and of the inlet water in water-cooled transformers.

A transformer complying with a standard specification will have temperature rises measured in oil and by resistance not exceeding these limits, but not necessarily reaching the limits.

The relations between T R and H for any given transformer depend upon the thermal design, and may vary widely for transformers of the same standard rating but of different types or makes. The design conditions are frequently such that, in order that R may be within the specified limits, the value of T must be lower than the maximum the specification allows.

A British Standard rated transformer, for example, with an oil temperature rise less than 50 deg. cent. at full load, is not necessarily running below its true standard rating since the temperature rise by resistance may be up to the full standard limit.

The loading and cooling conditions chosen by all the standardization authorities are extreme. The British and American standards, for example, assume full load continuously, and an ambient air temperature of 40 deg. cent. continuously for self-cooled transformers—operating conditions which are rarely encountered in practise.

An apparent advantage of this basis is that the standard rating is very conservative and may safely be worked to in almost any climate and under any loading conditions.

It has become general practise for operating engineers to consider the standard rating as being the maximum safe rating under normal working conditions. Transformers much larger than is necessary are therefore installed.

A relatively minor disadvantage of this position is waste in transformer plant installed and in exciting kv-a. and losses.

A more serious result is that operating experience cannot distinguish between transformers of good and bad thermal design. Undesirable opportunity is thereby given to the unskilled or unscrupulous manufacturer, and lessons of experience which are essential to progress in design and construction are lost to the industry.

It is suggested that the use of hottest spot temperature indicators be encouraged in standard specifications; that distinction be made between the true thermal rating and the standard rating, as described earlier in the paper; and that alternative standard ratings be recognized covering more practical operating conditions.

Temperature Distribution in a Transformer

The main parts of any type of oil-immersed transformer are the iron core, the windings, the tank, and the oil. During operation, heat is generated in the iron core and in the copper of the windings.

Considering only steady-load conditions, the heat must be transferred to the cooling medium air or water. The oil serves as the transference agent. Heat flows by conduction to the surface of the material into the oil and thence by thermal transference to the cooling medium.

The oil circulates in a closed path between the transformer and the cooling medium, rising in temperature as it collects heat from the transformer and falling in temperature as it delivers it to the cooling medium. This variation in oil temperature D has a large effect on the thermal design of the transformer. For a given specific heat of oil there is a definite relation between the temperature difference D, the quantity of oil in circulation, and the heat losses being dissipated.

The oil circulation path is shown diagrammatically for three types of cooling in Fig. 1, but is not, of course, actually so directly and clearly defined as in the figures. In self-cooled transformers the oil circulation depends on the tank dimensions and winding arrangement. In water-cooled transformers it can be controlled to some extent by varying the cooling conditions, and in forced oil-cooled transformers the circulation is completely under control, and D, for instance, can be given any desired value.

Winding Thermal Conditions

A transformer winding with oil flowing through it is shown diagrammatically in Fig. 2. The temperature T of the oil leaving the windings is here assumed to be the same as the temperature of the hottest oil. This is so in most practical cases. Where the difference is appreciable, however, an average value can be assumed which will give sufficiently accurate results for the present purpose. The same conditions apply to the increase of oil temperature D.

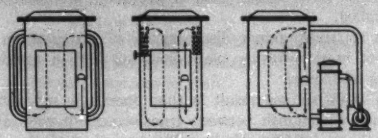

Fig. 1

The temperature of the oil at the bottom of the windings will be $(T - D)$.

The relation between the winding temperature at any point and the adjacent oil temperature depends largely upon which of the following conditions holds:

1. The direction of oil flow is at right angles to the conductors in the winding.
2. The direction of oil flow is parallel with the conductors in the winding.

Condition (1) applies to most core type and some shell type transformers.

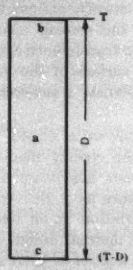

Fig. 2

Condition (2) applies to many shell type transformers.

Under condition (1), considering only a steady continuous load and assuming the windings are fairly symmetrical, the hottest spot temperature of the insulation at any point such as a or b in Fig. 1 will be a constant value G above the adjacent oil temperature. The hottest spot temperature of the copper, and, therefore, of the adjacent insulation at c, for example, will be $(T - D + G)$.

The hottest spot temperature H of the copper for the whole winding will be at some point b and such that

$$H = T + G \tag{1}$$

This temperature H must not under any conditions of loading exceed the safe limit, *e. g.*, 105 deg. cent. for I. E. C. rating.

Under condition (2) the thermal gradient in the winding in the direction of the conductors will be short-circuited by the copper, since the thermal conductivity of copper is approximately 2000 times that of the winding insulation. The temperature of the winding will be constant in the direction parallel to the flow of the oil. The winding temperature difference will not then be constant as in the previous case. At the top of the windings it will be $(H - T)$ and at the bottom $(H - T + D)$. If G is the value at the top of the windings, then:

$$H = T + G$$

as in Equation (1).

The temperature R measured by increase of resistance of the windings will be the average temperature of the copper in the windings taking the average in directions both normal and parallel to the flow of oil.

Under condition (1) its equivalent is the temperature of the copper at some point near the center of the winding such that:

$$R = T - K D + G_a \tag{2}$$

For condition (2) the temperature of the copper is constant at all points in a vertical direction. Assuming G_a is the average value of the winding temperature difference at the top of the windings, then:

$$R = T + G_a \tag{2a}$$

Comparison of this relation with Equation (2) shows that if the flow of oil is parallel to the conductors, K becomes zero.

It should be noted, however, that the direction of oil flow cannot in practise be parallel to the conductors for the whole of the winding. The temperature relations for condition (2) will, therefore, apply to part only of the winding, and for all types of transformers K will be greater than zero.

Equation (2) gives the relation between R and T in any oil-immersed transformer. G_a is the average value of the winding temperature difference taken in a radial direction through the coil. It is usually within 5 deg. cent. of the maximum value G.

The constant K varies between 0.1 and 0.6 depending on the thermal design.

As previously explained, most national standards fix upper limits for both R and T. The designer of the transformer has, therefore, so to adjust D and G_a that neither of these limits is exceeded.

The value of D itself varies widely from 1 deg. cent. to 15 deg. cent. for forced-oil-cooled transformers, and from 5 deg. cent. to 30 deg. cent. for self-cooled and water-cooled transformers. For an A. I. E. E. or B. S. rated self-cooled core type transformer in which both oil and

winding temperatures are at their limits, Equation (2) becomes, assuming $D = 20$ and $K = 0.4$

$$95 = 90 - 8 + Ga$$

$$\text{or } Ga = 13 \text{ deg. cent.}$$

In many cases a cheaper and better design could be made if Ga were larger than 13. For a self-cooled transformer D cannot be varied much, so that Ga can only be increased by reducing the hottest oil temperature T. The revised figures might be:

$$95 = 80 - 8 + Ga$$

giving $Ga = 23$ deg. cent. instead of 13 deg. cent. Such a transformer at full load and with 40 deg. cent. air would have a temperature rise by resistance of 55 deg. cent.—the standard limit—but the hottest oil rise would only be 40 deg. cent. or 10 deg. cent. below the standard limit.

This example shows clearly why a B. S. or A. I. E. E. rated self-cooled transformer with hot oil rise 10 deg. cent. below the specified value is not necessarily liberally rated. In this case, the load could not be increased until the oil rise reached 50 deg. cent. without seriously exceeding the standard rating.

Relation Between the Average Winding Temperature and the Hottest Spot Temperature

Equation (2) gives the relation between the average winding and hottest oil temperature. For standardization purposes these observable temperatures are derived from the safe hottest spot temperature by arbitrary relations.

Combining Equation (1) and (2) gives the relation between the hottest spot and the average winding temperatures.

$$H = R + KD + G - Ga \tag{3}$$

Thus the difference between the hottest spot temperature H and the average winding temperature R is $(KD + G - Ga)$. This value is assumed by the A. I. E. E., for example, as 10 deg. cent. for O. I. S. C. transformers.

In forced-oil-cooled and water-cooled transformers where D can be more or less controlled, Equation (3) shows the advantage in having a small value for standard rated transformers. As an example, consider a British Standard rated oil-immersed, forced-cooled transformer with inlet water at 25 deg. cent. After steady continuous load $R = 90$ deg. cent. from Table I.

From Equation (3) assuming $(G - Ga) = 2$ deg. cent., the hottest spot temperature is:

$$H = 92 + .5D$$

If the design of the oil cooler is such that $D = 16$ deg. cent., then the hottest spot temperature = 100 deg. cent.

If, however, for the same winding temperature rise the value of D is 2 deg. cent., then the hottest spot temperature is only 93 deg. cent., a reduction of 7 deg. cent.

These two transformers both comply nominally with the British Standard Specification without any margin, yet the latter transformer is actually working at a considerably lower hottest spot temperature, and has therefore a higher safe output rating.

Effect of Cooling Medium Temperature on the Output Rating

So far, the final temperatures have been considered mainly as it is these which determine the maximum permissible loading.

The output of any given transformer depends on its temperature rise above the cooling medium, and this rise determines the rating of the transformer provided that the safe hottest spot temperature is not exceeded.

Equations (1)-(3) are equally true of the temperature rises above the temperature of the cooling medium instead of the final temperature H, R, and T are inserted. Calling these temperature rises h, r, and t respectively:

$$h = t + G \tag{4}$$

$$r = t - KD + Ga \tag{5}$$

$$h = r + KD + G - Ga \tag{6}$$

The true effect of load variations on the temperatures in Equations (1)-(6) is complicated no matter whether actual temperatures or temperature rises are considered. None of the temperatures is a simple function either of the load or of the losses. The temperature rise of the hottest oil, for example, usually varies as something less than the first power of the total losses. The winding temperature difference is often not directly proportional to the copper losses. Factors contributing to this complication are variations in the copper losses and in the viscosity and other characteristics of oil with temperature.

As a rough approximation, especially for self-cooled transformers, it will be assumed that the oil temperature variation D and the hottest oil rise t are directly proportional to the total losses, and that the winding gradient is directly proportional to the copper losses. These assumptions will give results on the safe side when considering permissible overloads.

On this basis any of Equations (1)-(6) may be rewritten for any desired load.

Applying these relations to Equation (2) shows that R will increase faster than T as the load increases. The hottest spot temperature similarly increases faster than T. Herein lies the danger of estimating overloads and the effect of load variations from the oil rise t only.

For example, assume an A. I. E. E. rated transformer giving the following result in Equation (5) at full load.

$$55 = 45 - 8 + 18$$

If the copper loss is equal to the iron loss at full load, then at 30 per cent overload continuously the copper losses, neglecting the effect of temperature on the resistance of the windings, will be 1.69 times, and the

total losses 1.34 times the original values, and Equation (5) becomes:

$$80.1 = 1.34\,(45 - 8) + 1.69 \times 18 = 60.5 - 10.8 + 30.4$$

Thus on overload the oil rise has increased 15.5 deg. or 34 per cent whilst the winding rise has increased 25.1 deg. or 46 per cent.

The greater relative increase in winding temperature rise would be even more marked for short time overloads.

It is now possible to estimate the effect on the permissible loading of variations in the temperature of the cooling medium. The method can be most clearly shown by an example.

Consider an A. I. E. E. rated self-cooled transformer with the following thermal characteristics after continuous full load with an ambient air temperature of 40 deg. cent. These values are typical of a medium sized self-cooled core type transformer.

$$R = 95 \quad D = 20 \quad Ga = 18 \quad G = 20$$
$$T = 85 \quad H = 105 \quad K = 0.4$$

Ratio of copper to iron losses $= N = 2$

Equation (2) becomes $95 = 85 - 8 + 18$ (7)

Equation (1) becomes $105 = 85 + 20$

Suppose the ambient temperature is only 25 deg. cent. At full load $R = 80$ deg. cent., $T = 70$ deg. cent., and $H = 90$ deg. cent. The load may safely be increased until H is raised to 105 deg. cent., *i. e.*, h may increase to 80 deg. cent.

Let P = the new permissible load as a fraction of normal full load

The new total losses will be $\dfrac{P^2 N + 1}{N + 1}$ times normal.

The new copper losses will be P^2 times normal.

Equation (4) becomes:

$$80 = \frac{2\,P^2 + 1}{3} \times 45 + 20\,P^2$$

whence $P = 1.14$

Hence the effect of reducing the ambient temperature from 40 deg. cent. to 25 deg. cent. is to increase the output rating of the transformer 14 per cent.

At this overload with an air temperature of 25 deg. cent. the hottest spot temperature will be 105 deg. cent. and Equation (2) will read

$$92.8 = 79 - 9.6 + 23.4 \qquad (8)$$

It is instructive to compare (7) and (8). Both the hottest oil and winding temperatures are lower in the second case, but the transformer is equally loaded thermally in the two cases.

The above method of determining the effect of ambient temperature on rating may be expressed conveniently in a formula giving the permissible loading at any air temperature.

$$P = \sqrt{\frac{(H - A)\,(N + 1) - t\,n}{G_n\,(N + 1) + N\,t_n}} \qquad (9)$$

Where P = permissible load in terms of normal.
A = temperature of cooling medium.
H = maximum safe hottest spot temperature.
N = ratio copper loss to iron loss at normal load.
G_n and t_n = values at normal load.

Using Equation (9) a curve can be plotted for any given transformer showing the permissible loading without exceeding a given safe hottest spot temperature for varying air temperatures. In the present example, Equation (9) becomes:

$$P = \sqrt{\frac{90 - A}{50}}$$

Fig. 3 shows the curve of overload and air temperature.

With such a curve it is easy to check whether the load on a transformer in service is near its maximum safe rating.

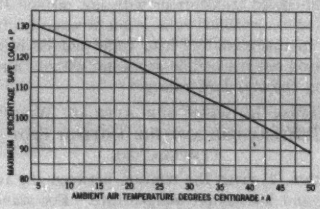

Fig. 3

Similar curves can, of course, be drawn for water cooled and forced-oil-cooled transformers.

An alternative method of obtaining the loading conditions on a transformer in service in which only the temperatures of the cooling medium and the hot oil are required, can be derived from Equations (9) and (1).

Equation (1) at any load P times normal becomes:

$$H = T + P^2\,G_n \text{ whence } P = \sqrt{\frac{H - T}{G_n}} \qquad (10)$$

where G_n = value at normal load.

Using this value of P in (9) gives:

$$T = H - G_n\,\frac{(H - A)\,(N + 1) - t_n}{G_n\,(N + 1) + N\,t_n} \qquad (11)$$

where T = maximum hot oil temperature permissible at the cooling medium temperature A.
t_n and G_n = values at normal load.

In the example $T = 69 + 0.4\,A$

If the air temperature is 20 deg. cent., the transformer is fully loaded (*i. e.*, its hottest spot temperature is 105 deg. cent.) when the hottest oil temperature is 77 deg. cent. The continuous load to give this condition may be obtained from (10)

$$P = \sqrt{\frac{105 - 77}{20}} = 118\%$$

Hence, by means of Equation (**11**) readings of the oil and air temperatures for any transformer on constant load show the loading conditions at that time, and indicate the proportion of the maximum permissible load the transformer is carrying.

The formulas developed in this paper are intended for purposes of illustration and exposition rather than for actual application to practical problems. The formulas may, however, be modified to suit particular conditions. For example, the exponent of P in Equation (**10**) may be adjusted to suit the exact relation between the winding gradient and the load. Incidentally, practical correction for the effect of temperature on the copper losses may be made by adjusting this exponent.

The predetermination of the true thermal rating of transformers requires numerous formulas to cover the wide range of loading and cooling conditions that may occur in practise. None of these formulas would be sufficiently general to be suited to the present purpose. The complete subject has been treated recently by the present author.[2]

Relation between Oil Temperature and Output Rating

The hottest oil temperature is easily measured on a transformer in service, and it is well, therefore, to examine what information, as to the loading conditions, can be obtained from it.

From Equation (**10**) $P = \sqrt{\dfrac{H - T}{G_n}}$ where H and T are values at load P and G_n is the winding temperature difference at normal load.

In the example $G_n = 20$

$$\text{Hence } P = \sqrt{\frac{105 - T}{20}}$$

This relation is plotted in a curve in Fig. 4. The practical use of the curve has been pointed out by G. L. Porter. It gives the maximum oil temperatures for different loads. If, for instance, in the example the load is 120 per cent, the oil temperature must not be allowed to exceed 76.2 deg. cent. Whether the transformer is able to carry this load continuously depends on the air temperature. Equation (**9**) or Fig. 3 shows that for this to be possible the ambient temperature must not exceed 18 deg. cent.

The curve may be applied to temporary overloads as well as to continuous loads. An overload of 30 per cent, for instance, in the example may be applied until such a time as the oil temperature reaches 71.2 deg. cent., and this is true no matter what the previous loading conditions, if within the safe rating; *e. g.*, whether the transformer has previously been on full normal load or even on no-load.

If the air temperature is greater than 5.5 deg. cent. (Equation (**9**) or Fig. 3), it is only a matter of time before the oil temperature reaches 71 deg. cent. When this happens the load must be reduced. This use of Equation (**10**) gives results slightly on the safe side for short time heavy overloads of only a few minutes' duration, since it makes no allowance for the thermal time constant of the windings.

Again, suppose at a given instant the oil temperature of this transformer is 75 deg. cent., but the previous loading history is unknown. The curve states that any load up to 122 per cent may safely be taken from the transformer (actually some margin should be allowed here and a somewhat lower limit than 122 per cent assumed). How long this load could be taken depends on the air temperature, but always so long as the oil temperature does not exceed 75 deg. cent.

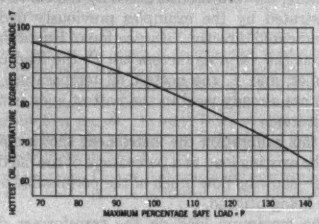

Fig. 4

The curve also shows up the possible danger of oil temperature alarm protection. Suppose in the example the oil thermometer were set to operate an alarm when the hottest oil temperature exceeded 80 deg. cent. Curve 4 shows that the alarm would be given quite unnecessarily so long as the load were below 112 per cent. On the other hand, the transformer might be carrying a load greater than 112 per cent and greater than the maximum safe limit (*i. e.*, with a hottest spot greater than 105) without the alarm functioning.

In some countries it is common for oil alarms to be set at 90 deg. cent. In the example referred to, representing an average oil-immersed self-cooled transformer, if the air temperature were 30 deg. cent. and the transformer were overloaded 20 per cent continuously, the hottest oil rise would be 58.2 deg. cent. and the hottest oil temperature 88.2 deg. cent. The alarm would, therefore, not function although the hottest spot temperature would be 117 deg. cent. and the transformer loaded beyond the safe limit.

These illustrations show the numerous practical uses of a curve such as Fig. 4.

Transformer Safe Load Indicator

It has been shown that the hottest oil temperature is misleading as a guide to the safe loading of a transformer and that the true criterion is the hottest spot temperature within the windings. Instruments have been developed in various forms by many transformer

2. "Thermal Rating of Transformers," *I. E. E. Journal*, (England), Vol. 66, No. 380, p. 841.

manufacturers for measuring indirectly this hottest spot temperature.

In principle, the components T and G from Equation (1) are measured separately. T is measured directly and G is obtained indirectly through the relation between it and the load current in the windings. A current transformer is therefore necessary and may be connected on either primary or secondary side of the main transformer.

Such instruments show clearly whether the transformer is fully loaded and will actuate an alarm or close a relay tripping circuit when a dangerous condition is being approached.

By calibrating an instrument of this type in percentage of maximum safe load instead of in hottest spot temperature, the information given by the curve Fig. 4, and illustrated by the examples previously decribed, can be indicated directly. Inspection of Equation (10) shows that the variable factors are P, the load on the transformer, and T, the hottest oil temperature, both of which are already used in the hottest spot temperature indicator.

With this calibration the instrument shows directly:

1. The existing load as a percentage of the maximum safe load—showing how much of the permissible output rating is being utilized.

2. The maximum load on the transformer during any desired period expressed as a percentage of the maximum safe loading. If, for instance, this record were 75 per cent over a period of 24 hr., it would mean that at no time during that period did the load on the transformer exceed 75 per cent of its available capacity.

The formula assumes that when any given load is switched on, the winding temperature difference G reaches the value corresponding to that load immediately. This assumption underestimates the overload capacity of the transformer by a few minutes' duration of load. This is a desirable characteristic in that for rapid and dangerous increases in load, warning will be given a short time before the danger point is actually reached.

Earlier in this paper reference was made to certain approximations and assumptions necessary in applying the formulas previously developed to practical cases. Equation (10), used as the basis for the operation of the safe load indicator, is the simplest. The only thermal relation assumed is that the difference between the hottest oil temperature and the hottest spot temperature varies as the square of the load.

Departures in practise from this relation such as those caused by the effect of temperature on the losses and on the viscosity of the oil, can be eliminated by adjusting the value of G and the exponent in the formula, and by suitably choosing the position of the oil thermometer. In this way the instrument can be made sufficiently accurate for all practical purposes for use with any given transformer of reasonably normal design.

Since the standard rating of a transformer is based on continuous loading and on an abnormally high cooling medium temperature, the true rating under the normal operating conditions of discontinuous loading and comparatively low cooling medium temperature will be considerably greater than the standard rating.

By means of the safe load indicator, the true thermal rating under operating conditions may be fully utilized, and a corresponding saving made in capital cost of transformer plant installed and in exciting kv-a. and losses.

Discussion

F. F. Brand: Mr. Norris in his paper has seemed to indicate that hot-spot measuring devices are an impractical means for the operation of transformers in service. At least, as regards large transformers built in this country that is not true, because there has been a large number of transformers with hot-spot indicators in them, operating in service for some years. Perhaps it is not true to say that these are hot-spot indicators, because a spot has no area, but as a matter of fact there are devices which will indicate correctly the hottest coil and the hottest turn in that coil. Therefore, they approximate very closely a true hot-spot indicator, and these and other devices simulating the hot-spot indicator have been in service quite a number of years, so that there are quite practical devices for an operating guide in service.

Furthermore, Mr. Norris seems to indicate that the A. I. E. E. rules are very conservative in the rating of the apparatus. It may be so in very large transformers which are equipped with means to prevent oxidization of the oil, though I doubt it, but at least as regards smaller transformers, it is very questionable in my mind if it is permissible to overload small transformers without running into danger of sludging the oil and, therefore, getting a very short life.

It has been our practise in this country to recommend that even large transformers be not operated at a hot-spot temperature of 105 deg., because we have thought it more desirable to operate such transformers more conservatively and to obtain longer life of the insulation and of the oil.

There is one other point in Mr. Norris' paper to which I should like to take exception. He speaks about the temperature rise of conductors in oil which are parallel to the oil flow. Perhaps that is hardly a practical question, because no transformer has the oil flow parallel to the conductors throughout the whole length of the conductor. Mr. Norris' statement neglects two very important features of a transformer. He speaks about the high conductivity of the copper in contrast to the lower conductivity of the oil, and he has neglected entirely the surface drop from the conductor or from the insulation on the conductor through that little stagnant film of oil, which is so hard to sweep away, and he has neglected, too, the thermal drop through the insulation, and those things far outweigh the question of whether the conductivity of the copper is such as to change the distribution of the temperature gradient, and tests show this to be so.

W. F. Dawson: If I understand Mr. Norris' paper correctly, it makes proposals whereby the customer may check up the performance of his transformer and possibly increase its rating. I wish to head off any possibility of encouraging such a practise on turbine alternators. It would be exceedingly dangerous to suggest that because some margin is found between the guaranteed total temperature and that observed, increased ratings or overloads can be tolerated. Except on special contract, practically all turbine alternators are today sold on the basis of "maximum rating" which means that neither armature current nor field current is to be overloaded, except momentarily. Both

armature currents and field currents are now generally stamped upon the nameplate.

It will often be found that although the Institute rules permit an observed temperature rise as indicated by resistance of 90 deg. cent. on the field windings as compared with an observed temperature rise by temperature detectors of 80 deg. cent. with class B insulation and 60 deg. cent. with class A insulation on armature windings, there is more margin in the heating of the armature coils than in the fields. This makes it particularly dangerous to attempt overloading the armature windings simply because there appears to be a slight margin in the heating. There has been a recent tendency to place recording thermometers or temperature alarms in the path of the ventilating air. Devices of this sort are poor substitutes for the temperature indicators connected to the temperature detectors embedded in the armature windings. Usually the ammeters in armature and field circuits are a much safer guide.

A typical distribution of full-load losses in a medium-size turbine alternator, say 3000 kw., would be approximately as follows: Windage and fan loss 1.25 per cent; core loss 1 to 1.25 per cent; $I^2 R$ armature 0.5 to 0.67 per cent; load loss 0.5 to 0.67 per cent; $I^2 R$ field and rheostat 0.6 to 0.8 per cent, a total of about 4.5 per cent as compared with fixed no-load losses of about 2.7 per cent.

These data ought to show how useless thermometers placed in the outlet air stream are as indicators of the temperature of the windings.

W. M. Dann: Mr. Norris makes the point in his paper that the American Institute rules are very conservative, as Mr. Brand has already said. He says this because the standards involve continuous loading and a temperature of 40 deg. for the cooling medium, two conditions that are not met with every day in actual service.

He makes the point, which I think is really not a very serious one, that this gives to the unskilled or unscrupulous manufacturer the undesirable opportunity to design transformers that meet the Institute standards but are still of bad design. The only way in which a manufacturer could produce such a design would be to distort the use of his materials and of his cooling system quite considerably from the prevailing American practise.

An average copper temperature that would keep within the Institute figure of 55 deg. might be produced with a distorted design and it might be that the hot-spot temperatures would be excessive. I really doubt very much, though, whether any manufacturer would produce such a design because it wouldn't be worth while. The life of a transformer would be much shorter than is the actual case in service.

The point is made again that the conservatism of the American Institute rules produces a situation where transformers much larger than necessary are being installed. It seems to me that most users of transformers are satisfied with the conservatism of the Institute rules, because it must be a pretty comfortable feeling to know that with a device as important as a transformer, one has a little excess capacity under ordinary conditions of operation for emergencies and peak loads. For this reason I think it might be well to say that there are many American engineers who are satisfied that the Institute rules are not unduly conservative.

The safe loading of transformers has been receiving a lot of attention in this country in the past year or two. It is well known that a bank of transformers has an excess capacity when the temperature of the cooling medium is low, and it seems logical to take advantage of that capacity under those conditions. The logic and the reasonableness of this position are being recognized by the Institute. Recommendations for the safe loading of the transformers have been prepared and have been published for criticism, and it is not unlikely that they may be officially issued some of these days.

I am sorry that the description of the safe load indicator is not gone into a little more fully in Mr. Norris' paper. Mr. Brand has said that winding-temperature indicators are being regularly furnished by several American manufacturers, and that is the case. It appears to me from the brief description of this indicator in Mr. Norris' paper that it is practically the same device that is being used rather extensively in this country to indicate the hottest-spot winding temperature, the chief difference, it seems to me, being that Mr. Norris' instrument is calibrating in terms of load instead of temperature.

V. M. Montsinger: This paper deals with a subject which has been under discussion by one of the A. I. E. E. committees for the past three years. I refer to the subject of loading transformers by temperature instead of by nameplate rating.

There are two points which I shall discuss, (1) the methods or formulas used in determining the overload capacity of a transformer and (2) the question of what is a safe load.

This paper is devoted almost entirely to the first point but does not discuss the second point which I consider a very important one.

The formula given in Mr. Norris' paper recognizes the basic difference between the design of oil immersed apparatus and air cooled apparatus in that in the former in calculating the temperature rise it is necessary to deal with two separate and independent rises; namely, the oil rise over the ambient and the winding rise over the oil. In other types of apparatus such as generators, where the windings are embedded in core slots the thermal conditions are not so simple.

Several years ago I found that practically every heating curve obtained by tests could be expressed by the simple formula—

$$\theta = C L_n$$

θ = temperature rise.
C = constant.
L = loss.
n = some value less than unity for natural cooling and unity for artificial or forced cooling.

I note that Mr. Norris has assumed that in all cases the value of n is unity in his calculations, although he properly points out that in some cases it is less than unity.

For self-cooled transformers the oil rise varies as the total loss raised to the 0.8 power The temperature rise of horizontal windings also varies as the copper loss raised to the 0.8 power. For vertical windings the power is more nearly unity. For water-cooled transformers, which are very popular in this country, I would point out that with a given rise in the cooling water of, say, 10 deg. cent., the oil rise varies as the square root of the total losses, *i. e.*, $n = 0.5$. For this class of transformers the formula based on the temperature rise being proportional to the loss should not be used because the results do not check even approximately. The winding rise over the oil follows the same laws as for self-cooled transformers. The following tabulation gives the difference in temperatures calculating the rises for 125 and 150 per cent loads where the transformer has a ratio of copper to iron losses of 1:1 and 2:1 at 100 per cent load.

These figures show that while the results may be close enough for practical purposes to assume that the rise is proportional to the loss within reasonable variation of load of self-cooled transformers, but for water-cooled transformers the difference is too great to neglect. In all cases, however, I prefer to use the more exact formula because it is simple and involves very little labor. This will be discussed more in detail in my forthcoming paper.

I shall have to take exception to the statement that for long vertical coils on account of the thermal conductivity of copper being 2000 times as great as insulation, the temperature of the windings will be constant in the direction parallel to the flow of oil. Accepting this figure of thermal conductivity as correct, neither tests nor calculations bear out this statement. A few years ago I made temperature exploration tests by means of

TABLE I

COMPARISON OF KV-A. *VS.* TEMPERATURE RISE OF OIL AND WINDINGS BASED ON TEMPERATURE RISE BEING (1) NOT PROPORTIONAL TO LOSS. (2) PROPORTIONAL TO LOSS

Self-cooled							
	Oil rise over ambient		Hot spot rise over				
			Oil		Ambient		
% load	$n_0=0.8$	$n_0=1$	$n_c=0.8$	$n_c=1$	$n_0=0.8$ $n_c=0.8$	$n_0=1$ $n_c=1$	Difference
100	45	45	20	20	65	65	0
A* 125	55	57.5	28.6	31.2	83.6	88.7	5.1
150	66.4	73.3	38.3	45.0	104.7	118.3	13.6
100	45	45	20	20	65	65	0
125	58	61.8	28.6	31.2	86.6	93	6.4
B† 150	73	82.5	38.3	45.0	111.3	127.5	16.2

Water-cooled							
	Oil rise over ambient		Hot spot rise over				
			Oil		Ambient		
% load	$n_0=0.5$	$n_0=1$	$n_c=0.8$	$n_c=1$	$n_0=0.5$ $n_c=0.8$	$n_0=1$ $n_c=1$	Difference
100	35	35	30	30	65	65	0
A 125	39.6	45	43	46.8	82.6	91.8	9.2
150	44.5	56.5	57.4	67.5	101.4	124	22.6
100	35	35	30	30	65	65	0
B 125	41	48	43	46.8	84	94.8	10.8
150	47.3	64	57.4	67.5	104.7	131.5	26.8

A* is for 1:1 ratio of copper to core loss at 100 per cent load.
B† is for 2:1 ratio of copper to core loss respectively.

thermocouples on a shell-type coil 60 in. in height and found that after conditions became constant and holding a copper density of 2000 amperes per sq. in., the temperature gradient was a straight line with a difference of 16.2 deg. between extreme lower and extreme top. This difference in temperature is checked very closely by calculations using a formula similar to that used on calculating the characteristics of transmission lines. Two thousand amperes per sq. in. represent perhaps the very maximum that is used in America today at rated load but may easily be reached on overloads now permitted in low ambients.

I note that the curve in Fig. 3 giving the maximum percentage safe load versus ambient air temperature, checks rather closely what I have always termed the 1 per cent rule; namely, that the capacity of a transformer can be increased 1 per cent for each degree by which the ambient is decreased below its standard. It is true that Mr. Norris' curve does not quite meet this but if he had made the calculations on the basis that the temperature rise varies as the 0.8 power instead of the first power of the loss, it would easily have met this rule.

I wish now to discuss a phase of this question of loading transformers by temperature which I consider much more important than the use of certain formulas because if you use the right formula you know pretty well what you are about, but for this other phase no standard formula is available and we must be guided by general experience. I refer to the question of what is a safe load when the maximum temperature is maintained continuously by means of overloads employed in connection with the use of temperature-indicating devices. I note that the author has taken 105 deg. as the maximum temperature at which it is safe to load a transformer continuously. The majority of engineers in America in the absence of experience in operating transformers under this new condition, are not willing to subscribe to, (and in fact it is not logical to set up) the same maximum temperature limit for a condition where this limit can be maintained 365 days in the year by means of overloads as for a condition in which this same limit is reached only when several factors occur simultaneously and continuously under unusual service conditions. I mean that this value is not reached unless the following four conditions occur simultaneously:

(1)—40 deg. ambient.
(2)—rated load.
(3) tested rise 55 deg. by resistance.
(4)—maximum conventional allowance for hot spot of 10 deg. cent.

On the other hand, if a transformer is loaded by temperature rather than nameplate rating it is obvious that a condition can be brought about that is equivalent to these four conditions occurring simultaneously. The question comes up as to what temperature limit is safe for this new condition. The opinion in this country has pretty well crystallized in favor of setting up a temperature differential of at least 10 deg. as a safe limit for the two conditions of loading. In other words, we believe that if 105 deg. is a safe limit where the load does not exceed nameplate rating and the chances are very small of obtaining this limit continuously, we should not exceed 95 deg. hot spot or 85 deg. by resistance when the maximum temperature is maintained continuously with the aid of temperature indicating devices, etc. On the other hand, we are willing to admit that if it is proved that 105 deg. is a safe temperature for continuous operation it would be safe to reach 115 deg. periodically as might happen under usual service conditions and where name plate rating is never exceeded. Before subscribing to this, however, we prefer to obtain further experience under actual service conditions because, as pointed out above, this proposed method of loading is entirely new and introduces new factors which may have a very important bearing on the life of the insulations.

In order that the title of the author's paper *Safe Loading of Oil-Immersed Transformers* might not be misleading to some who are not keeping up-to-date with the progress of the art, I would point out that as I interpret the title it means "safe" in so far as the load will not cause a temperature in excess of the limits set forth. To this I heartily agree. I cannot agree, however, that it is necessarily safe from the standpoint of the life of the apparatus to maintain continuously the temperature limit of 105 deg. It is hoped that within the next year or two the A. I. E. E. can officially standardize the rules which have been prepared and published[1] for the operation of transformers by temperature. If this is done and the proposed limits are adopted it would mean that the values given in this paper should be revised in order to conform in temperature limits with our own practise.

I regret that the author has not given us a few more details as to the construction of the temperature indicator to which he refers in the last part of the paper. While this is not necessary to understand the thermal conditions in a transformer it would have added to the interest of the subject.

I might point out that the General Electric Company has been supplying for the past eleven years a temperature indicator that shows the approximate hottest spot of the winding under loaded conditions. This I believe was the first general application of a transformer temperature indicator. Full particulars on this device are given in the *General Electric Review*, June 1918.

F. D. Newbury: I wish to discuss the paper briefly from the general standpoint of standards. Mr. Norris has wisely drawn a distinction between standards for rating and, let us say, recommendations for operation. The discussion shows that there is some difference of opinion as to the conservative nature of the Institute rules. My own opinion is that we don't know whether they are conservative or not because there is no way of finding out just what the facts are.

If we take insulating materials and make laboratory tests and judge deterioration by reduction in mechanical strength,

1. A. I. E. E. JOURNAL, August 1928.

a very good case for a much lower temperature limit can be made.

If we assume that the limiting temperatures of the A. I. E. E. Standards are safe, then we must admit that practise is conservative because apparatus is rarely operated continuously under conditions of 40 deg. air temperature and up to the permissible limiting temperature rise.

However, we must face the fact that our only reliable "bench mark" is general experience. We know that we are not having undue trouble with our apparatus *as operated*. How much margin of safety exists we can only find by further experience. We cannot, therefore, change operating practise in a way that will increase the general average of operating temperature without careful consideration. Conservatism would lead to some reduction of present A. I. E. E. limits, as has been proposed by American engineers in connection with the recommendations for operation of transformers by total temperature.

In the application of this principle of operation by total temperature, there are two difficulties. I have already mentioned the first one, that is, whether our present standards can be applied in practise up to their full limit or not. The second is that no hot-spot indicator really measures the actual hot-spot temperature.

Some rather complete tests in the case of turbine alternators have shown that there is anywhere from 5 to 20 deg. difference between the actual hot-spot copper temperature and the highest temperature that can be measured by conveniently located hot-spot indicators. So we have that thermal difference to deal with in applying this principle of operation by total temperature. The difference between actual hot-spot temperature and measured hot-spot temperature increases as loads above the rated load are applied. So the increase in load above the rated load involves this additional unknown factor and another factor of safety must be applied in the application of this method of loading.

A. B. Cooper: I think that although this paper was written as of two years ago, in all probability Mr. Norris would have no occasion to change his opinion on the details of the paper based on more recent practise or experience. Mr. Brand has pointed out that in his estimation the remark regarding the conservatism of the A. I. E. E. rules applies to large transformers but not particularly to moderate capacity transformers. Mr. Norris has in mind, I think, in preparing this paper, more the larger types of transformers. His company is building at the present time transformers of unit capacity of 75,000 kv-a., and in that case, if a 20 per cent margin is available under the conditions as outlined, due to variation in temperature, he is dealing with a capacity of 15,000 kv-a., which may be idle capacity or waste capacity in his transformer.

Mr. Brand pointed out that transformers have been in service for a very long time with embedded temperature detectors. Mr. Dawson answered that point very satisfactorily when he said a moment afterward that "temperature detectors are dangerous." Mr. Dann's comment has taken exception to what Mr. Norris said in regard to the variation in thermal gradient of transformers, particularly in reference to transformer design on this side of the water. My own feeling is that as between manufacturers, as between different types of design, or even as between designs put out by the same manufacturer, variations exist in the temperature gradient, of the order of 40 to 50 per cent.

Mr. Montsinger has commented on the temperature variations which result from departing from unity in the exponential value in the formula. Mr. Norris has used unity for simplicity; that is of course for simplicity in presentation of his paper, and he has not, I am sure, considered developing his temperature indicator on the basis of unity exponential value only.

E. T. Norris: Many of the discussors have commented on the subject of whether it is safe to run transformer insulation continuously at the national standard limit temperature of 105 deg. cent., and have assumed that such a temperature is recommended in the paper. I am glad, therefore, that Mr. Montsinger has pointed out that the paper does not discuss at all the question of what is a safe hottest-spot temperature.

In the beginning of the paper the hottest-spot temperature is defined as the maximum temperature the insulation will safely withstand. The method of predetermining the safe loading and the use of the safe-load indicator depend upon the selection of a hottest-spot temperature in accordance with this definition.

As regards the actual value of the hottest-spot temperature, Messrs. Montsinger and Newbury have summarized the position excellently. My own view is, perhaps, rather more optimistic. All transformer manufacturers of many years' standing will have collected definite experience of the safe hottest-spot temperature for the insulation they have used. Developments in the manufacture and treatment of insulation tend to raise this temperature rather than lower it, so that such experience may safely be used.

Again, suppose *A* to be the average loading conditions of a transformer. Very many transformers everywhere are operating safely under loading conditions *B*, considerably worse than the average *A*, though still below the A. I. E. E. standard rating conditions. When operating transformers by temperature, the general average can at least be raised from *A* to *B* without any risk. This argument refers to Mr. Newbury's statement that operating practise should not be changed in a way that will increase the general average of operating temperature without careful consideration.

The discussion of the actual value of hottest-spot temperature for standardization purposes is irrelevant in connection with the present paper, as the problem does not arise. The information given to the operating engineer is the proportion of the maximum safe output of a transformer actually being used at any time.

In calibrating the instrument the manufacturer uses a hottest-spot temperature based on his knowledge of the materials he employs, and with as much margin for his safety as he likes.

The new proposal does, however, demand skilled and accurate methods of thermal design. A transformer designer must have good control over the thermal characteristics and design of his transformer to be able to view the new method of loading with approval or even with equanimity.

Many discussors have asked for more details of the safe load indicator. It is constructed on the same principle as the winding-temperature indicators referred to by Messrs. Brand, Dann, and Montsinger, the essential difference being that it is not calibrated in hottest-spot temperature. The information given by a winding-temperature indicator is somewhat negative in character. Suppose, for example, the hottest-spot temperature of a transformer at a given instant is 75 deg. cent. while the maximum safe value is 100 deg. cent. The transformer is obviously not fully loaded, but the unused capacity cannot be determined from these two temperatures, and may be any value between 10 and 35 per cent, depending on the thermal design of the transformer and the temperature of the cooling medium. The operating engineer knows, therefore, that the transformer is not fully loaded, but he does not know what the margin is, and is, therefore, unable to make full use of it.

The safe-load indicator is calibrated in percentage of maximum safe load instead of in hottest-spot temperature, and therefore shows directly just how much of the available output rating is being utilized at any time or over any period of time.

Further details of the safe-load indicator are given in the Institution of Electrical Engineers' *Journal* referred to later, and the instrument itself is illustrated in Fig. 1 herewith.

Mr. Brand considers the Institute rules are not conservative, while Mr. Dann agrees that they are, and approves of the conservatism on the grounds that it is comforting to an operator to feel that his transformer has a margin of capacity for emergency conditions.

All operating engineers will, I think, agree with Mr. Dann.

The important point is that hitherto the operator has been able merely to feel that the transformer had some margin for an emergency, whereas when the method recommended in the paper is followed, the operator will *know* that the transformer has a margin; he will *know* just how much that margin is in kv-a.,

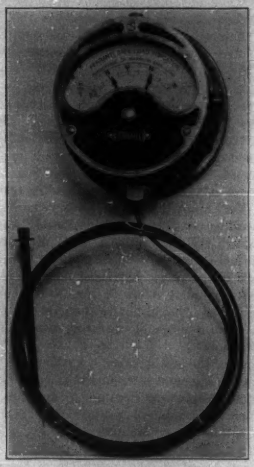

Fig. 1—Safe-Load Indicator for Transformers

and that knowledge will be automatically kept up-to-date for him and can be checked at any time.

The principles of operation by temperature are applicable to all sizes and types of transformers without restriction, and the safe-load indicator is equally valuable in every case.

Mr. Montsinger points out that the temperature rise of either windings over oil or oil over air is sometimes proportional to something less than the first power of the watts loss, and gives tables showing the effect of the reduced exponent on the resulting temperature rises. This fact is emphasized three times in the paper.

The formulas developed in the paper were intended for purposes of illustration and exposition rather than for application to practical problems, and were therefore simplified by such means as assuming unity exponent. I pointed out, however, that the subject has been dealt with quantitatively and in detail elsewhere (Inst. Elec. Eng. *Jl.*, Vol. 66, p. 841). In that paper complete formulas have been developed in which both variations in the exponent and other complications are taken into account.

Incidentally, the analysis of the winding-temperature difference into surface drop and thermal drop through the insulation, as referred to by Mr. Brand, is also dealt with in detail. The Institute papers by Messrs. Montsinger, Luke, and Cooney were of great assistance in the preparation of both papers.

The safe-load indicator is not subject to errors such as the assumption of unity exponent, since it works on the actual oil temperature and winding-temperature difference existing in service.

In pointing out that no transformer has oil flow parallel to the conductors for the whole of the winding, Mr. Brand is quoting from the paper. Any drop in temperature along the conductor, even in the rather extreme case described by Mr. Montsinger, can be allowed for by slight adjustment of the constant K in Equation (2).

Mr. Montsinger states that if Fig. 3 had been calculated on the basis that the temperature rise varies as the 0.8 power instead of the first power of the loss, it would have closely checked the empirical 1 per cent rule. On the other hand, I find that Fig. 3, which was calculated from the approximate formulas developed in the paper, agrees almost exactly with the constant load curve given in Fig. 19 of N. E. L. A. Report No. 267-3 which was, I think, plotted from actual measurements on many typical distribution transformers.

In conclusion, the chief aim of the paper is to encourage the rating and actual loading of transformers on a true thermal basis. To this end a practical method is developed of predetermining the true thermal rating under different conditions of loading, and a practical means is described for measuring the safe load capacity of a transformer under actual service conditions.

Induction Motor Operation With Non-Sinusoidal Impressed Voltages

BY L. A. DOGGETT[1] and E. R. QUEER[1]
Member, A. I. E. E. Non-member

Synopsis.—*The usual procedure in considering the operation of induction motors has been to neglect the effect of harmonics. Although in most instances this gives results which are sufficiently accurate, the question of how much these effects are or how much is being neglected has yet to be answered in electrical engineering literature. In this paper an attempt is made to answer this question. It is also devoted largely to a statement of results with only such explanation as space permits. The conclusions are that, with harmonics of 10 per cent or less, the harmonics produce an insignificant effect, for all types of induction motors and for all conditions of operation, except for the no-load condition. In this case, significant but not serious increases of I^2X occur. Also lightly loaded induction motors may be counted on to smooth out the impressed electromotive force wave.*

THERE are two types of harmonics that may be present in induction motor operation, (a) time harmonics introduced by the impressed e. m. f. and (b) space harmonics introduced by the counter e. m. f. of the motor. The object of study in this paper is the former type of harmonics.

In order to visualize what happens in the case where the impressed e. m. f. contains one harmonic, it is convenient to think of an induction motor as consisting of two motors identical with the motor under consideration, with shafts connected and drawing power, one from the fundamental e. m. f. source and the other from the harmonic e. m. f. source. The reactance of the harmonic motor will be n times that of the fundamental motor, where n is the order of the harmonic. The resistance of the harmonic motor as indicated by our tests will be greater than that of the fundamental motor, although not n times as great. One author[2] claims the resistance of the harmonic motor to be the same as that of the fundamental motor. When this pair of motors is operating under any ordinary load, the slip is close to zero per cent for the fundamental motor, while for the harmonic motor it is a very large per cent, 133 1/3 per cent for the third harmonic in the two-phase motor, 120 per cent for the fifth harmonic in the three-phase motor, and 85 5/7 per cent for the seventh harmonic in the three-phase motor. For example, if 1800 rev. per min. is synchronous speed for the two-phase fundamental motor, $3 \times 1800 = 5400$ rev. per min. in the opposite direction is synchronous speed for the third-harmonic machine. The slip is therefore 7200 rev. per min. or $7200/5400 \times 100 = 133\ 1/3$ per cent. In the case of the seventh harmonic and the three-phase motor, $7 \times 1800 = 12{,}600$ rev. per min. in the same direction as synchronous speed, and the slip is therefore $(12{,}600 - 1800) \times 100 \div 12{,}600 = 85\ 5/7$ per cent. When a pure sine wave is impressed on the motor, 100 per cent voltage is considered to be impressed on the motor of fundamental frequency and 0 per cent on the harmonic motor. When a non-sinusoidal voltage of 100 per cent effective value is impressed on the motor, if the harmonic be 25 per cent of the fundamental, the fundamental motor will have impressed on it a 97 per cent voltage. Considering a given effective voltage, this voltage can be produced either by 100 per cent fundamental and 0 harmonic or with $100 \div \sqrt{1 + H^2}$ per cent fundamental and $H \times 100 \div \sqrt{1 + H^2}$ per cent harmonic, where H is a fraction, 0.25 for example in the above cited case. Thus, $H \times 100 \div \sqrt{1 + H^2}$ per cent voltage will be impressed on the harmonic motor and $100 \div \sqrt{1 + H^2}$ per cent on the fundamental motor. With non-sinusoidal operation, the fundamental machine will receive a decreased voltage and function as an ordinary induction motor functions with decreased voltage. The harmonic machine operating with a large slip will take currents, active and reactive watts, practically the same as if its rotor were blocked. The harmonic motor acts more or less as a harmless parasite on the fundamental motor, neither helping nor hindering the operation of the fundamental motor to any considerable degree. Its braking effect is entirely negligible so long as the harmonics are kept below 10 per cent. Its main disadvantage is that it prevents the fundamental motor from getting its full 100 per cent voltage and that it absorbs from the line a moderate amount of active and reactive power for which it does no good in return.

Coming now to practical operating conditions, the outstanding fact is that with a 10 per cent harmonic, the largest harmonic allowable by the A. I. E. E. Standards Rules, the quantity $100 \div \sqrt{1 + H^2}$ becomes 99.5 per cent, only half of one per cent loss in voltage impressed on the fundamental motor. Therefore the fundamental motor will have no more than a one per cent decrease in torque and horsepower. With a 10 per cent harmonic, the active and reactive watts taken by the harmonic motor are a small percentage of the total active and reactive watts for all conditions of operation

1. Pennsylvania State College, State College, Pa.

2. Arnold, Vol. 5, Part I, p. 194, *Die Asynchronen Wechselstrommaschinen die Inductionmaschinen.*

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

except light loads. The braking or accelerating torque of the harmonic motor is a small fraction of one per cent of the fundamental motor torque.

Analytical Treatment

As suggested in the introduction, it is helpful to replace the motor under consideration by two motors, shaft-connected, and each identical with the original motor. Each of these motors will have its own equivalent circuit and its proper impressed electromotive force. Solutions of these two circuits may then be made separately and the results combined. By applying this method the operation of any induction motor for any condition may be predicted. Such calculations were made by the authors for a number of motors and a variety of operating conditions. These calculations are tedious and space does not permit their reproduction here. The conclusions from these calculations are that

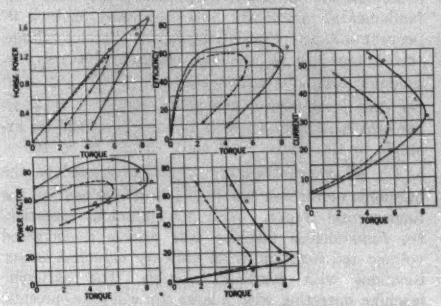

Fig. 1—Test Results of Motor Operated on Sinusoidal and Non-Sinusoidal Voltages

The full lines show operation on sinusoidal voltages; the dotted lines non-sinusoidal operation

so long as harmonics are kept below 10 per cent, the departure of the motor from normal performance is quite negligible except for light loads. The case of light load is treated in a later section.

In order to establish the validity of this method of calculation, a complete test was run on a small squirrel-cage motor for both sinusoidal and non-sinusoidal operation. Fig. 1 shows curves of horsepower, efficiency, power factor, current, and slip plotted against torque. The constants of the motor follow:

Name plate data:

A 5-hp., three-phase, 60-cycle, 110-volt, 26½-ampere, 1150-rev. per min., squirrel-cage motor.

Blocked test data:

E per phase (WYE) = 14½ volts, I = 26½ amperes

R per phase (cold) = 0.227

X per phase = 0.492

Running light data:

E line = 50½ volts, $I = 2.86 - j\,3.14 = 4.22$

The tests were run at a voltage of 50½, sufficiently below rated voltage to allow the motor to be taken through its complete characteristics. To obtain these complete characteristics, the following arrangement of apparatus was used: A d-c. motor drove a harmonic alternator,[2] which supplied the induction motor under test. Shaft-connected to the induction motor was an electrodynamometer which pumped power to another d-c. machine driven by a synchronous motor. The harmonic alternator's speed was maintained at 1234 rev. per min. The actual percentage of the fifth harmonic impressed on the induction motor was measured by the "wave-shape meter"[3] and was found to be 57 per cent.

It was found also that when calculations were made using the equations for the accurate equivalent circuit of the induction motor, very good agreement was had in the case of the speed-torque and horsepower-torque curves and fair agreement for the other three curves. Since beyond the breakdown point the currents are excessive and the values of primary and secondary resistance are uncertain because of heating, only a moderate agreement between observed and calculated values was obtained. For those who might care to check these calculations, the following circuit constants are supplied: R_1 (hot) = 0.13; R_2 (hot) = 0.14; $X_1 = X_2 = 0.25$; $g_0 = 0.098$; $b_0 = 0.1076$, volts per phase = 29.12. When operating with the 57 per cent fifth harmonic, the voltage of the fundamental reduces to 25.3 volts, while the fifth harmonic voltage becomes 14.4 volts.

While such calculations cannot be reproduced here, it may be desirable to give just two simple calculations as follows:

The maximum torque, fundamental test

$$= \frac{3 \times 2}{2\ \pi\ 61.7}(29.12)^2 \frac{1}{2\,(0.13 + \sqrt{0.13^2 + 0.5^2})} \times 0.737 = 7.5 \text{ lb-ft.}$$

The maximum torque, non-sinusoidal test

$$= 7.5 \times \left(\frac{25.3}{29.12}\right)^2 = 5.68 \text{ lb-ft.}$$

The above calculations are not based on the exact equivalent circuit and are only moderately close.

Although complete calculations have not been included, they provide the basis for the following discussion of the curves of Fig. 1.

Slip-Torque Curves. When the fifth harmonic is present, the fundamental must be reduced, in this case from 29.12 to 25.3 volts. As illustrated by the calculation, this reduces the torque in the ratio of the square of the voltages. The reduction due to the counter torque of the fifth harmonic is quite small, only about one-third of a pound-foot at breakdown. Neglecting this, it may be said that the motor acts as if operating simply with a reduced fundamental voltage.

3. *A New Wave Shape Factor and Meter.* Doggett, Heim, and White, A. I. E. E. Trans., Vol. 45, 1926, pp. 435–442.

Horsepower-Torque Curves. These curves are corollary to the slip-torque curves, since

$$\text{hp.} = \frac{2\,\pi\ \text{Torque}\ (1 - S)\ 1234}{33{,}000}$$

Current-Torque Curves. Up to breakdown for any assigned torque the current with non-sinusoidal operation must be larger than the current with sinusoidal operation for two reasons: (a) The reduction in fundamental voltage calls for a larger current to give the same

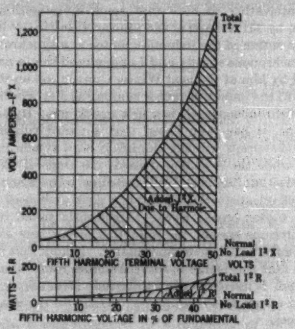

FIG. 2—TEST RESULTS AT LIGHT LOAD

Showing increase in I^2 R and I^2 X with increase in harmonic voltage

torque. (b) The fifth-harmonic current must be compounded with the fundamental current according to the law

$$I_{total} = \sqrt{I_1^2 + I_5^2}$$

Efficiency-Torque Curves. For any assigned torque, the extra current will result in extra copper loss and therefore less efficiency.

Power-Factor-Torque Curves. For any assigned torque, a very considerable I^2 X, due to I_5^2 X_5, will materially increase the demand for reactive power and thus reduce the power factor.

LIGHT-LOAD CONDITION

As has already been pointed out, an induction motor operating on light load with non-sinusoidal impressed voltages may be expected to show some considerable departure from normal operation. In general, the no-load copper loss, I^2 R, and the no load I^2 X will be very much increased. Calculations show that an average three-phase motor acted upon by an e. m. f. containing an eight per cent fifth harmonic, if its percentage of reactance be as low as 10 per cent, will have its no-load copper loss multiplied by three, and its no-load I^2 X multiplied by seven. In a certain rather high-reactance motor, the normal no-load I^2 R was 49 and the normal no-load I^2 X was 119. Due to a 10 per cent fifth harmonic, the added I^2 R was 33 and the added I^2 X was 60. Repeated attempts to measure this added loss accurately proved failures, and the following test is submitted to illustrate the above claims without trying to cite an individual case of exact agreement between calculation and observation.

In this rather unusual test, the stator of a wound-rotor induction motor was connected to the fundamental terminals, while the rotor was connected to the fifth-harmonic terminals of a harmonic alternator.[3] On the stator was impressed normal voltage and normal frequency continuously, while the fifth-harmonic voltage was varied from zero to 50 volts. Although tests with both directions of phase rotation of the fifth harmonic were made, the fifth-harmonic current and power data were alike in the two cases. From this test and two supplementary blocked-rotor tests, sufficient data were obtained to plot the curves of Fig. 2, which show the I^2 R and I^2 X plotted as a function of the per cent harmonic. In this case, a 20 per cent fifth harmonic doubles the no-load I^2 R and multiplies the no-load I^2 X by seven.

Although at no-load the added I^2 X appears quite significant, it usually is small in comparison with the reactive power needed for the excitation of the motor. In this case, the excitation required some 1350 volt-amperes.

An induction motor supplied with non-sinusoidal

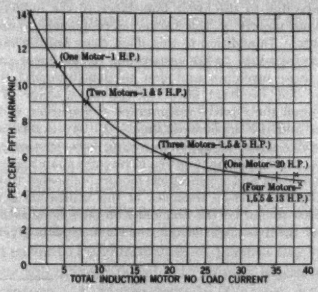

FIG. 3—CURVE SHOWING HOW HARMONIC VOLTAGE OF GENERATOR DECREASES WITH INCREASING INDUCTION-MOTOR LOAD

e. m. fs. will take from the line some additional I^2 X and I^2 R. It will draw from the alternator harmonics of current which by internal drops will reduce the sources of non-sinusoidal e. m. fs. inherent in the alternator. Any alternator has a poorer regulation at frequencies higher than normal. To all but the fundamental frequency, the running induction motor acts practically as if blocked, *i. e.*, slip in the neighborhood of one. Therefore, for harmonics like the fifth and seventh, the induction motor at no-load will have a lower impedance than it has for the fundamental. In such a case, the current per volt will be larger for the harmonic than for the fundamental and with a sufficiently large harmonic e. m. f. present, the harmonic current

may exceed the fundamental current. The alternator, however, has a poor regulation when operated on a harmonic. Consequently, an alternator which shows a 14 per cent fifth harmonic at no-load will show a less per cent harmonic, (that is, 5 per cent), when operating unloaded induction motors, and this is illustrated by Fig. 3. In short, induction motors have some tendency to smooth out the e. m. f. wave of the alternator which is supplying them. The results of this test were obtained by use of the wave-shape meter.

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Discussion

B. L. Robertson: Last year at Penn State we did some work on the subject of synchronous motor operation with non-sinusoidal impressed voltages. I should simply like to point out as a discussion of the paper by Doggett and Queer that in line with wave shape improvements by induction motor, the modern synchronous motor by virtue of its armaturous winding will be just as effective in improving wave shape, likewise the synchronous converter.

R. G. McCurdy: I should like to add one thought to the observations made on the effect of induction and small synchronous motors on wave shape given in this paper. They have pointed out the smoothing effect on the supply-system voltage wave shape due to the lower impedance of the induction and synchronous motors at the harmonic frequencies than at the fundamental. In addition to this effect there are the harmonics which may be added to the system due to the stator and rotor slots of these machines and due to non-sinusoidal flux distribution. The resultant effect on the wave shape of course depends upon the relative magnitudes of generator and motor harmonics and the impedances involved. My experience with measurements of wave shape on operating power system indicates that large alternators have lower harmonics than many of the smaller induction and synchronous motors connected to the system, so that the over-all tendency of this utilization apparatus is to increase somewhat the harmonics on the system rather than to decrease them as might be inferred from the data given in the paper.

E. W. Henderson: An interesting point along a line very similar to that suggested by Mr. Doggett is that of the operation of an induction motor at sub-synchronous speeds with characteristics similar to the synchronous motor. It has been customary to explain the reduction of torque and the running of an induction motor at sub-synchronous speeds as due to harmonics, the general idea being that the torque is reduced to a certain point but beyond it is increased. The usual method of indicating this on a diagram is to show the normal torque of the induction motor from starting to synchronous speed and on the same diagram indicate the torque which would be experienced due to the harmonic, these two being combined to show the dip in the torque curve.

It is interesting to note that some investigation has been carried on by Mr. Dreese of the Lincoln Electric Co. on this subject of the action of the induction motor as a synchronous motor at sub-synchronous speeds, and I understand that a paper will be presented by him at the next Winter Convention on this subject. This idea of the induction motor running as a synchronous motor at sub-synchronous speeds is a new one to me. By the use of a stroboscope, it has been noted that a motor running at sub-synchronous speeds will act identically as a synchronous motor and that when the load is increased, the angle of displacement will increase but the speed will continue to be constant. This action goes on up to the breakdown point, when of course the induction motor breaks away. Apparently in all respects, at sub-synchronous speeds, the induction motor acts exactly in a manner similar to a synchronous motor. I think that Mr. Dreese concludes that this is due to slot harmonics rather than line harmonics and I have mentioned it only because it is a case of reduced torque and is somewhat parallel to the cases cited by Mr. Doggett.

L. A. Doggett: The point made by Mr. McCurdy is true. We must consider the effect of the receiver apparatus just as well as the source apparatus and we have been studying the whole problem.

I should just like to make one point in connection with his remarks. If, due to this, receiver apparatus harmonics do get out on the line, they are all ready for correction by such induction motors as may be present; that is, the action of the induction motor is ready to function, regardless of the source of this disturbance.

In the first paragraph of the paper, we have limited our discussion to what I called time harmonics and I am interested to learn from Mr. Henderson that we shall probably get a paper which will cover this other interesting proposition of space harmonics.

Outdoor Hydrogen-Ventilated Synchronous Condensers

BY ROBERT W. WIESEMAN*
Member, A. I. E. E.

Synopsis.—Hydrogen is an excellent cooling medium for high-speed rotating electrical machinery. When it is substituted for air, a machine can be operated at a higher load with the same temperature rise and the windage loss is decreased to one-twelfth. Furthermore, since no oxidation can take place in this type of machine, the life of the insulation is increased and short circuit and corona troubles are very materially reduced. The machine is especially quiet, and it can be placed out of doors without increased expense.

The first commercial application of hydrogen cooling for electrical machinery was made on a 12,500-kv-a. outdoor synchronous condenser. Another outdoor hydrogen cooled condenser designed along these lines rated 20,000 kv-a., is also in operation. Both condensers were placed in service in 1928 and their performance has been very satisfactory. This paper describes the construction of the machines and reviews the advantages and disadvantages of hydrogen cooling.

I. Introduction

Advantages in Ventilating a Machine with Hydrogen

THE characteristics of hydrogen gas, which make it desirable as a cooling medium for high-speed rotating electrical machinery, are as follows:

Low Density of Hydrogen. Hydrogen is a colorless, odorless, tasteless gas whose density is only 7 per cent that of air. The windage loss of a rotor is approximately proportional to the density of the gas in which it rotates. Thus, the windage loss of a rotor running in an atmosphere of pure hydrogen is only 7 per cent of the air loss. It was found to be entirely practical to maintain a gas mixture in a machine of 99 per cent hydrogen and 1 per cent air, etc., and so a windage loss of only 8 per cent of the air loss is realized. The decreased windage loss also decreases the amount of heat to be removed from the machine, and the size of the surface cooler can be reduced when hydrogen is used as a cooling medium. The bearing friction loss is the same in hydrogen as in air.

High Thermal Conductivity of Hydrogen. Hydrogen conducts heat about seven times better than air and the specific heat of a gram of hydrogen is about 14.5 times that of a gram of air. Consequently, heat passes across the small spaces in the insulation and between the laminations, etc., much more readily than with air cooling; and so the internal copper temperature, for a given surface temperature, is less in hydrogen than in air.

High Forced Heat Convection of Hydrogen. In an atmosphere of hydrogen 30 per cent more heat can be transferred from heated surfaces than in air with the same surface temperature drop. Furthermore, when the hydrogen comes in contact with a surface cooler, more heat can be transferred to the cooler than with air. This again enables a smaller surface cooler to be used; or with the same cooler a lower cooling medium temperature is realized, and the internal temperature of the coil is still further reduced. The bearing temperature is also less in hydrogen than in air.

The reduction of windage loss, the increased thermal conductivity, and forced heat convection of hydrogen result in an increased output of 25 per cent or more, depending upon the hydrogen pressure used, the type of ventilation employed, the degree of saturation in the magnetic structure, and the stability required.

No Combustion Possible in an Atmosphere of Hydrogen. There is no oxygen or dirt present in the machine, and, consequently, a combustion cannot take place. This eliminates the need of fire extinguishing apparatus. The burning of iron and insulation following a ground or short circuit is reduced to a minimum. The insulation retains its flexibility, and its life, therefore, is greatly increased. The bearing lubricating oil remains clean and it does not oxidize or sludge so readily as in an air cooled machine.

Detrimental Effect of Corona Reduced to a Minimum. The thickness of the armature coil insulation is governed partly by the high-voltage gradient and the formation of corona in the minute spaces of the insulation. Tests have demonstrated that the damage caused by corona is practically absent[1] in an atmosphere of hydrogen. Not only will this increase the insulation life, but it will allow a thinner insulation to be used or a greater factor of safety to be secured. These factors will not only make it easier to build machines of present day voltages, but they will make it possible to employ higher voltages in the future. A 24,000-volt, 15,000-kv-a. hydrogen-ventilated synchronous condenser, now under construction, gives evidence of this trend.

Finally, the absence of corona damage to varnished fabric insulations will permit their use on high-voltage machines with a substantial decrease in cost. The greater impulse voltage strength of varnished fabric as compared to mica insulation thus gives the hydrogen cooled machine a further advantage for installations where high-voltage surges are likely to be experienced.

Outdoor Operation. At practically no increase in cost a hydrogen-ventilated machine can be made suitably weather-proof for outdoor operation. This effects a great saving in building expense.

Quiet Operation. The windage noise of a totally

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Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

1. For references, see Bibliography.

enclosed hydrogen cooled machine is very much less than that of an air cooled machine. Thus, a hydrogen-cooled machine is exceptionally quiet and it can be placed outdoors in a residential district where a noisy machine would be very objectionable.

Disadvantages in Ventilating with Hydrogen

Safety Precautions. When hydrogen cooling was first proposed, some difficulty in preventing accidental explosions was anticipated. The increasing familiarity with the problem and the extensive use of hydrogen for many industrial purposes such as brazing, annealing, welding, etc., indicate that there should be no difficulty with this problem. The Schenectady factory of the General Electric Company uses 30,000,000 cu. ft. of hydrogen per year without experiencing any difficulty from this source. Every city has one or more gasometers containing many thousands of cubic feet of gas which, though not explosion-proof, give practically no trouble. The following table gives the stored energy in a steam boiler, in a large turbine-generator rotor, and in the gas of a hydrogen-ventilated synchronous condenser when it contains the most explosive mixture of air and hydrogen.

	Stored energy ft-lb.	Per cent
Steam boiler 2000 boiler hp.	5000×10^6	100
Large turbine-generator rotor	300×10^6	6.0
Hydrogen-ventilated machine containing 1000 cu. ft. of the most explosive mixture of air and hydrogen at atmospheric pressure	70×10^6	1.4

It is quite evident that a hydrogen-ventilated synchronous condenser has very much less potential explosive force than other apparatus in common use.

A mixture of hydrogen and air will not explode if the hydrogen content, by volume, is more than 70 per cent or less than 10 per cent. If a machine is scavenged with carbon dioxide before it is filled with hydrogen, no explosion can take place when it is started for the first time. The hydrogen pressure in the machine is automatically maintained slightly above atmospheric pressure. This prevents air from leaking into the machine, and eventually the hydrogen purity will reach that of commercial hydrogen which is usually around 99 per cent. It has been found that a hydrogen purity of 98 per cent can be obtained in a few hours after a machine is filled with hydrogen. A hydrogen purity indicator can be made to ring an alarm if the purity falls below normal. A fan pressure gage calibrated to read hydrogen purity gives an instantaneous reading of the hydrogen purity in the machine when it is in operation. Thus, the possibility of an explosion is very remote if the machine is given ordinary supervision.

Increased Disassembly Expense. If a hydrogen-ventilated condenser must be disassembled for repairs, more time will be consumed and more expense will be involved than with a standard air-cooled machine. On the other hand, it should not be necessary to dismantle the hydrogen-ventilated machine so often because of its longer insulation life, its lower bearing and internal coil temperature, and the absence of dirt. It is believed, therefore, that the total maintenance expense over a period of years will be less for the hydrogen-cooled machine.

Increased Number of Auxiliaries. As with any automatic or remotely controlled electrical equipment, so with a hydrogen-cooled machine a number of auxiliary devices is necessary. These devices consist of an automatic hydrogen pressure control, a hydrogen purity indicator, thermometers with alarm attachments, water coolers, and the control for the motor-driven exciter set. The accessories, however, have been found to be reliable on other apparatus and since they do not represent a very large addition to the usual amount of accessories, it is believed that they will operate satisfactorily.

II. Hydrogen Cooling can be Easily Applied to a Synchronous Condenser

It is unnecessary to have the shaft of a condenser extend through the gas-tight shell as in the case of a turbine generator, and so a seal[2] of any kind around the shaft is not required. With this arrangement, the construction of the machine is simplified because the entire machine, including the bearings and shaft, can be enclosed in a gas-tight shell.

III. Description of Outdoor Installation

Fig. 1 shows the 12,500-kv-a. hydrogen-ventilated synchronous condenser completely installed at the Pawtucket, R. I. Substation of the New England Power

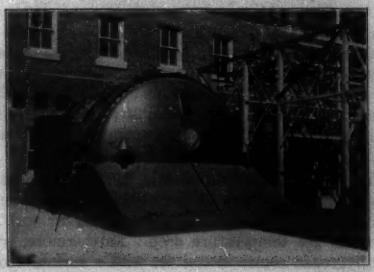

Fig. 1—The First Outdoor Hydrogen-Ventilated Synchronous Condenser, 12,500-Kv-a., 900-Rev. per Min., Three-Phase, 60-Cycle, 13,800-Volts. New England Power Co., Pawtucket, R. I.

Company. This condenser is the first of its kind ever built. It is located out of doors without any protection from the weather, but is placed over a pit which is made weather-proof by a substantial sheet-iron covering. Along side of the condenser are located the switches, starting compensator, transformers, and lightning ar-

resters. Fig. 2 shows this condenser with the end head removed and Fig. 3 a sectional view of the machine. These views show the semi-circular surface coolers, the bearing housing and its fabricated support, the high-voltage armature terminals, the collector housing, and the construction of the gas-tight frame. The bearings

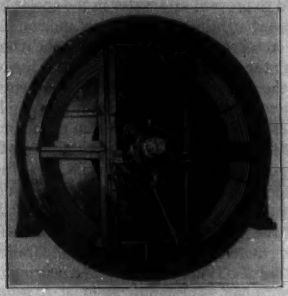

Fig. 2—12,500-Kv-a. Hydrogen-Ventilated Synchronous Condenser with End Head Removed

can be filled with oil and drained without dismantling the machine or losing any hydrogen.

Fig. 4 shows the 20,000-kv-a. condenser located near Charleston, W. Va., at the Turner Substation of the Appalachian Electric Power Company. This machine is also placed out of doors and it has the same design as

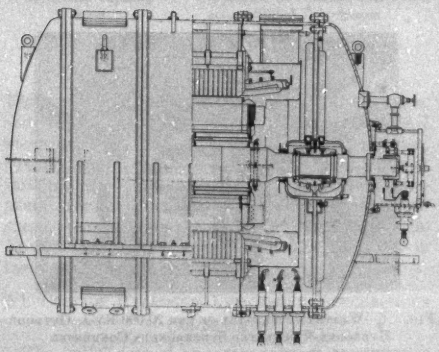

Fig. 3—Sectional View of the 12,500-Kv-a. Hydrogen-Ventilated Synchronous Condenser

the 12,500-kv-a. condenser except that its capacity is higher. Fig. 5 shows the 20,000-kv-a. condenser assembled on the test floor. The two ladders enable the operator to manipulate the vent valves on the top headers of the coolers and the hydrogen valves at the top of the machine. The field collector is located in the small housing which is bolted to the end head. A high-speed induction motor-driven exciter set, located inside of the substation, furnishes the excitation.

Fig. 6 shows the weather-proof pit beneath the condenser. In this pit are located the cooling water circulating pump, the oil pump for filling the bearings and furnishing high pressure oil for starting, the hydrogen supply tanks, the hydrogen pressure gage, the indicating thermometers, the automatic hydrogen pressure control, the rotor fan pressure gage for indicating the hydrogen purity when the machine is in operation, the thermal cell of the hydrogen purity indicator which indicates the hydrogen purity at all times, and the various valves for controlling the water, oil, and hydrogen.

IV. Explosion-Proof Frame

The stator frame consists of three sections and two heads bolted together as shown by Fig. 3. The main joints of the frame are gasketed gas-tight and they have practically no hydrogen leakage. The large middle

Fig. 4—The 20,000-Kv-a., 900-Rev. per Min., Three-Phase, 60-Cycle, 11,500-Volt Outdoor Hydrogen-Ventilated Synchronous Condenser. Appalachian Electric Power Co., Charleston, W. Va.

section holds the armature punchings and windings and the two smaller sections hold the internal surface coolers and bearings. The three sections are supported on base rails with feet. All piping and connections, except the hydrogen inlet pipe and the field winding terminals, are brought out at the bottom of the frame in the pit. The frame is fabricated entirely from steel plate. The elimination of castings, which are sometimes porous, is not only in keeping with present day dynamo machine construction, but it insures minimum hydrogen leakage.

In order to minimize the effect of an explosion resulting from careless operation, these condensers were provided with a cylindrical explosion-proof frame designed to resist the disruptive force of the most explosive mixture of hydrogen and air at atmospheric pressure. The frame of the 12,500-kv-a. condenser without windings, coolers, and piping, was filled with the most explosive mixture of air and hydrogen, and the gas was ignited with a spark plug. No damage

resulted from the explosion which developed a maximum pressure of 85 lb. per sq. in. as recorded by an instantaneous pressure recorder. If the machine had been completely assembled with all of the various metallic parts, especially the coolers, the pressure would have been only about 50 lb. per sq. in. This test also showed that an explosion would have no detrimental effect on the insulation. It is very unlikely that the gas in the machine will ever reach the most explosive mixture of air and hydrogen of 5 to 2 by volume. For any other gas mixture, the explosive force would be very much reduced. The theoretical pressure of 180 lb. per sq. in.

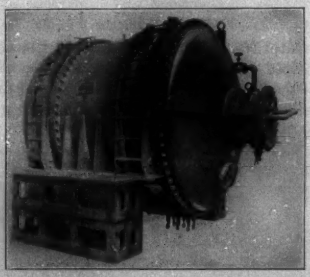

Fig. 5—20,000-Kv-a. Hydrogen-Ventilated Synchronous Condenser Assembled on the Test Floor

with an air-hydrogen explosion could be obtained only if no heat were absorbed by the explosion chamber and its contents.

V. Armature Winding

The armature winding consists of multi-turn coils insulated entirely with mica, bonded with black insulating varnish. The individual turns are also insulated with mica throughout their entire length. The armature coils have the standard insulation thickness for air operation, and in addition to the vacuum-filling treatment, the coils were covered with an oil resisting varnish to prevent any oil vapor, if present, from penetrating the coil insulation. The six armature leads are brought out through gas-tight terminals as shown by Fig. 3.

VI. Carbon Seal

It was previously stated that no shaft seal was required because the shaft did not extend through the shell. However, it was thought advisable to place a seal around the shaft where it enters the collector housing in order that the collector brushes might be adjusted and renewed without losing an appreciable amount of hydrogen from the main shell. This seal obviously does not function unless the collector housing end plate is removed. The seal consists of a carbon ring which is held against a shoulder on the shaft by a bellows and six springs equally spaced around the shaft.

VII. Hydrogen Ventilation

Each end of the frame contains two semi-circular surface coolers as shown by Fig. 2. The cooler heads can be removed and the tubes cleaned without dismantling the machine or losing any hydrogen. The four cooler units are piped in multiple, and air vents and drain valves allow the coolers to be thoroughly drained to prevent any water in the cooler from freezing in the winter if the machine is not in use.

The rotor poles and fans force the ventilating gas through the stator ducts to the back of the stator frame. Then the gas passes through the semi-circular coolers and is returned to the rotor and recirculated. Provision is made for ventilating the collector housing by circulating hydrogen through the housing by means of two pipes as shown by Fig. 5. Two dial thermometers indicate the temperatures of the ventilating gas before and after it passes through the cooler. If the temperature of the gas leaving the cooler exceeds 40 deg. cent., the thermometer rings an alarm. Indicating thermometers also ring an alarm if the bearing temperature exceeds 70 deg. cent.

VIII. Hydrogen Leakage

An automatic pressure regulating switch and valve maintain the hydrogen in the machine always slightly above atmospheric pressure to prevent air from leaking into the shell. The hydrogen pressure in the machine varies with its internal temperature. The average pressure is around ½ lb. per sq. in. and the leakage of the machine itself at this pressure is about 6 cu. ft. per day which costs around 10 cents. The hydrogen

Fig. 6—Weather-Proof Pit of the 20,000-Kv-a. Outdoor Hydrogen-Ventilated Synchronous Condenser

purity indicator wastes a small amount of hydrogen for its thermal analysis. Thus, the total hydrogen cost is very small, from 15 to 20 cents per day, depending upon how much is used by the hydrogen purity indicator.

IX. Increased Output

The excellent cooling properties of hydrogen enable a given machine to operate at an increased output at the same temperature rise. If the hydrogen pressure is in-

creased above atmospheric pressure, the output can be further increased. Numerous heat runs were made in which the hydrogen pressure was increased in steps up to 25 lb. per sq. in. These tests indicated that a hydrogen pressure of 15 lb. per sq. in. gave the best results. If the pressure is increased above this amount, the windage loss increases appreciably and the gain in output is not so marked.

The following table shows how the output of salient pole high-speed condensers increases with the same temperature rise when hydrogen is used for the cooling medium.

Cooling medium	Machine output
Air at atmospheric pressure	100 per cent
Hydrogen at a pressure slightly above atmospheric pressure	125 per cent
Hydrogen at a pressure of 15 lb. per sq. in. above atmospheric pressure	140 per cent

Naturally, these figures will vary with different types of machines. The field winding is usually the limiting feature because it is more difficult to ventilate, and magnetic saturation and stability impose more limitations on the rotor than on the stator.

X. Decreased Losses

The introduction of hydrogen into a machine reduces the rotor windage loss to about eight per cent of its loss in air. The following table shows the saving in windage loss of the two condensers:

Machine capacity kv-a.	Reduction of windage loss accomplished by hydrogen
20,000	85 kw. (0.42 % of machine rating)
12,500	51 kw. (0.41 % of machine rating)

The internal copper temperatures are less with hydrogen, and so the copper losses will be slightly reduced. Fig. 7 shows the total losses of the 20,000-kv-a., three-phase, 60-cycle, 11,500-volt hydrogen-ventilated condenser at various loads. The losses are given for air at atmospheric pressure, for hydrogen slightly above atmospheric pressure, and for hydrogen at 15 lb. per sq. in. At 20,000 kv-a. the total loss of this condenser in hydrogen is only 1.5 per cent.

XI. Cost of Hydrogen-Ventilated Synchronous Condenders

The initial cost of any new type of apparatus is usually high, and the development of hydrogen-cooled machines has not yet reached the point where its cost alone is the same as that of a standard air-cooled machine of the same size. The gas-tight construction, the water coolers, and the auxiliary control features naturally increase the cost which is partly offset by the increased rating obtainable. However, the value of the improved efficiency due to the lower windage loss (Fig. 7) will probably prove the deciding factor in any cost comparison. The capitalized value of the two per cent losses of a synchronous condenser is about the same order of magnitude as the first cost of the condenser installation, so that any reduction in the losses is of the same economic value as an equal reduction in cost. Thus, when decreased losses, increased capacity, lower internal temperatures, reduced fire risk, longer insulation life, cleanliness, quietness, and saving in building expense (outdoor operation) are capitalized,

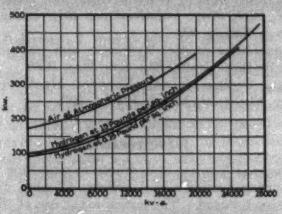

Fig. 7—Total Losses of the 20,000-Kv-a., 11,500-Volt, 60-Cycle, Hydrogen-Ventilated Synchronous Condenser

a large hydrogen-ventilated machine has a real economic justification.

Conclusions

The results obtained on these two hydrogen-ventilated synchronous condensers to date have been very encouraging and three more machines, rated 15,000-kv-a., are under construction. Not only have the hydrogen features of the machines performed satisfactorily, but during the past winter the machines have operated successfully out of doors with no protection from the weather. Thus outdoor synchronous condensers, whether air or hydrogen cooled, should give reliable service at a reduced overhead expense.

Electrical manufacturers are constantly striving to improve the efficiency and the ventilation of rotating machinery. High grade magnetic steel, transposed armature windings, correct magnetic structure shapes and proportions, improved rotor fans and ventilating ducts, thinner and more compact insulation, etc., all tend to reduce the losses and the size of a machine. However, these refinements have been worked over so exhaustively that further improvements in performance characteristics by these means will perhaps be relatively small. Any appreciable gain must come by the way of a radical change in the machine structure, materials, or ventilating medium. Hydrogen cooling is one of these radical departures from tradition which opens new avenues of progress and rotating machines with 99 per cent efficiency may soon be a reality.

Acknowledgment

The cordial cooperation of the New England Power Company and the Appalachian Electric Power Company in obtaining the various tests on these hydrogen-ventilated synchronous condensers is greatly appreciated.

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Discussion

Philip Sporn: The use of hydrogen as a medium in which to operate rotating machines is, I believe, a logical development and a big step forward. The logical expansion of this development should be its application to turbo alternators. Here, of course, the problem of designing an explosion-proof frame is not as simple as in the case of the condenser, but the experience gained with the synchronous condenser may definitely indicate that such precautions and safeguards are not at all necessary.

The use of hydrogen offers the solution to another problem that without hydrogen is not so simple to solve. I refer to the use of voltages of the order of 33,000 or higher for turbo alternators—voltages which we undoubtedly must employ if the size of machines continues to grow. Where the impulse-strength problem does not have to be considered, hydrogen coupled with other developments in insulation means an almost certain assurance of continuity of service as far as the high-voltage phase of the problem is concerned.

Another development that the further expansion of the use of hydrogen will make possible is the outdoor steam generating station. The total or partial elimination of the turbine room will certainly reduce costs.

At the Turner substation of the Appalachian Electric Power Company (one of the subsidiaries of the American Gas and Electric Company) we have had a 20,000-kv-a. hydrogen condenser in very satisfactory operation for almost six months. We have had slightly more leakage of hydrogen than has been indicated by Mr. Wieseman; it has been of the order of 75 to 100 cu. ft. a week, but some of this loss is undoubtedly due to very small leaks in the piping that have not been detected.

On two additional hydrogen condensers which are being built for subsidiaries of our company, we expect to reduce this figure considerably by eliminating the bubbling of gas to the atmosphere for sampling purposes. Instead, the sampling will be taken care of by a continuous circulation of hydrogen through the thermal sampling cell, the sample gas being returned to the machine.

L. L. Perry: With mine-mouth central stations and reliable transmission lines such as Mr. Sporn's company has been operating for many years, the fuel cost is below the fixed charge on the investment when the capacity factor climbs up around 0.08. The economy curve of fuel consumption cannot be expected to drop in the future as rapidly as in the past. Hence the investment cost per kw. that averages around $100 is now getting the attacks of executives, and the thought expressed by Mr. Sporn is a very natural one.

High-tension bus bars about twenty years ago began to go outdoors, although some 220-kv. busses in the mountain country still linger inside. Soon oil circuit-breakers and transformers followed the trend toward the out-of-doors, especially at high voltage. Now we are putting the generator busses, instrument transformers, circuit breakers, and connections outdoors, and enclosing them all in metal. With the hydrogen-cooled generator, there is a strong incentive to do as Mr. Sporn suggests. If the hydrogen escapes to the air, rather than to the closed turbine room, the chance of a confined explosive mixture is reduced. But we must not overlook the fact that we shall have to watch for effects produced by the greater range of temperatures and be prepared to take such precautions as will insure at least as good, if not greater, reliability of service from these generator units than we get now in the enclosed turbine room.

The wider use of electricity goes hand in hand with the demand for greater reliability. We are really now more or less pulling ourselves up by our bootstraps. We go to the designing engineers and ask for a larger generator. They give us a larger generator which has probably as great reliability as in the past, if not greater, and probably reduces our cost of operation and that increases the demand for electric current. If we get the greater reliability as well as the lower cost, that means we go again and ask for still larger machines. So we are getting to the point where the designing engineer may be forced to use the hydrogen in order to meet the demands of the power companies for the larger apparatus.

C. J. Fechheimer: In the applications of hydrogen to the large turbine generator the construction shown in this paper cannot be used for the reason that it is necessary to bring a shaft through the end bells and then according to our calculations, it would seem not to be feasible to make the structure explosion-proof. The stresses in the end bells become prohibitively high in the event that an explosion should take place.

In regard to the danger of an explosion, it seems that, in golf parlance, this is a "mental hazard." We are naturally afraid of explosions, but the risk involved is almost nil if precautions are taken as suggested by Mr. Wieseman. If the generator is first scavenged with an inert gas, such as carbon dioxide or nitrogen, you can quite readily get the mixture of the inert gas and air such that when followed by the incoming hydrogen, the explosion danger will disappear entirely.

There are various ways of computing the amount of scavenging gas needed. One explosion limit is 70 per cent hydrogen and 30 per cent air. The scavenging gas is introduced so as to secure a mixture of 30 per cent air and 70 per cent inert gas, although it may not be necessary to go quite that far. Then hydrogen is introduced, the mixture of other gases being expelled simultaneously. There is then no danger, even though some means of ignition be present. The amount of scavenging gas needed can be computed on the assumption that there is perfect diffusion, which is a conservative basis for computation. In that case it can be shown that the following formula applies:

$$\frac{\text{Volume of gas put into machine}}{\text{Space in generator which is occupied by gas}} = 2.3 \log_{10} \frac{\text{Per cent of machine volume that is air initially}}{\text{Per cent of machine volume that is air finally}}$$

In this case, the right-hand side becomes: $2.3 \log_{10} \frac{100}{30} = 1.2$.

That is, only 1.2 × machine volume of inert gas need be introduced to secure safety against explosion.

When going the other way, that is, when changing from hydrogen to air, the same ratio does not hold. The same method of computation may be used, substituting the word "hydrogen" for air. For safety, the hydrogen content should be reduced to about 10 per cent, with 90 per cent inert gas. If there were 100 per cent hydrogen initially, about 2.3 volumes of inert gas would be required.

In regard to the explosion limit, I might point out one or two other comparisons. Practically every householder in the United

States uses gas in the home, for heating, or cooking, and we seldom hear of an explosion, but the potential risk there is greater than it is in a hydrogen-cooled machine with the precautions that are taken for safety.

Also in oil circuit breakers, there are far greater risks from the explosion standpoint than in a hydrogen-cooled machine. My own opinion is that in a few years we will get over the scare and our synchronous condensers will not have to be made explosion-proof.

In the design of the synchronous condenser or any other machine in which hydrogen cooling is to be used, it becomes increasingly important to know about how much volume of gas will be circulated in the machine and what the velocities and pressures are, in order intelligently to design the coolers, the fans, and other parts. Therefore, a systematic study of ventilation of this type of machine is of great importance. At the Westinghouse Company, this study has been going on for a number of years and is about completed at the present time. It is interesting to note in this connection that the volume of gas per unit of time calculated is substantially the same with hydrogen as it is with air; that is, if the only change is that of substituting hydrogen for air. The pressure of the gas naturally falls in proportion to the density.

Mention is made in the paper that the fans on the machine were used for measurement of the percentage of hydrogen, the pressure drop across the fan being a measure of the density of the gas and from the density of the gas the percentage of hydrogen can be determined. In our own work we have found an external fan to be more convenient, particularly as this device can be used when the main unit is shut down, whereas by the method suggested by Mr. Wieseman, the scheme is inoperative when the synchronous condenser is not running. Use is made of a very small fan driven by a substantially constant-speed motor.

In the design of machines of this type, it is of value to investigate whether the flow of the gas in the vent ducts is laminar or turbulent. Laminar flow pertains to flow in layers, and turbulent flow pertains to whirls or eddies. With turbulent flow, the heat is transferred from the surface to the gas better than with laminar flow. The change from laminar to turbulent flow is dependent upon the velocity, density, and viscosity of the gas. It is quite possible that in parts of some of the vent ducts the flow may change from the turbulent to the laminar state, when changing from air to hydrogen, and the designers should be guided accordingly so that high local temperatures will not be reached. Investigations along this line have been conducted recently, by G. W. Penney, but we are not yet ready to report upon them.

The use of structural steel in this application is important. If the signal advance in the last few years in changing from castings to structural steel had not been made, it is possible that these machines could not be operated with the very small leakage as we do today. It is largely because we are able to weld steel and make the joints very tight that the very small leakage as reported by Mr. Wieseman has been obtainable.

In regard to the cost, according to the calculations which we have made, it would seem that in the large machines, the costs will not be as great as Mr. Wieseman has found. Costs can not be based upon one or two machines, because the first machines are always far more expensive than those which are made later. It is difficult at present to say what these costs will be, but for the same rating, not for the same machine, (with smaller machines for hydrogen than for air), it would appear that the cost would be less for the hydrogen than for the air-cooled machine.

L. A. Doggett: The paper states, "A mixture at hydrogen and air will not explode if the hydrogen content, by volume, is more than 70 per cent or less than 10 per cent."

Now there is no question that the manufacturer can maintain better than 70 per cent inside the apparatus. Equal care should be used to keep the percentage outside less than 10 per cent. On Jan. 15, 1926, there occurred inside the submarine E-2 at the Brooklyn Navy Yard, a disastrous explosion, due to hydrogen escaping from the Edison battery. Subsequently the submarines were equipped with hydrogen detectors which sounded an alarm at 2 per cent of hydrogen.

Therefore every precaution should be taken to see that 10 per cent mixtures cannot occur in any confined space in the neighborhood of the apparatus. The only possibility of this occurring is when the apparatus is indoors or when there are poorly ventilated inverted pockets near the apparatus when outdoors.

B. A. Behrend: I should like to review the problem of hydrogen cooling from a little different and perhaps a little broader angle. I believe that it will be essential within the immediate future to use almost exclusively out-of-door installations. Our transformers, switches, and generating units will be placed outdoors. If such apparatus is placed out-of-doors, the methods of ventilation will naturally undergo careful scrutiny. For the last forty years we have designed electric generating units with air cooling. We are now beginning to scan the medium of cooling these units. Mr. Wieseman and Mr. Fechheimer have told us that they have obtained good results with hydrogen. But there are, of course, other means of cooling.

In Europe oil-cooling of stators has been discussed. Oil as a cooling medium should be scrutinized before hydrogen cooling is too generally adopted as the best solution for cooling large generating units.

With the development of non-corrosive metals, especially copper-nickel alloys like Monel metal, we are in a much better position to make safe out-of-door electric generating units or synchronous condensers, because we may expose the metal continually to the weather without danger of corrosion.

As regards the size of the individual units, I predicted in 1907 that we should have to go to 25,000 kw.

We then predicted larger units than 25,000 kw., setting the limit at approximately 100,000. A year and a half ago, I stated in New York that I believed the final limit would be about 200,000 or 250,000 kw.

It is interesting to put on record here that I believe the limit of individual generator capacity will be approximately 250,000 kw. and that even the power companies will not ask us for greater capacities.

F. D. Newbury: I am sorry so much has been said about explosions. I think it has already been over-emphasized. Mr. Wieseman has pointed out that the 15,000-kv-a. condenser has less than 2 per cent of the possibility of danger of the 2000 hp. steam boiler. Yet he has made the condenser explosion-proof. I believe Mr. Wieseman will agree that this is a concession to our fears and not in accordance with the facts of the situation.

When we come to the real problem, the application of hydrogen cooling in turbine generators, it will be exceedingly difficult, if not impossible, to make the casing heads sufficiently strong with the necessary hole in the center for the shaft. So I feel that it is unfortunate that these first machines have been made explosion-proof when we cannot conveniently continue the practise, and when, as I believe, the practise is not justified by the conditions.

The second point I would make is that the outdoor installation is not necessarily tied up with the hydrogen cooling; closed-circuit air cooling is equally applicable. It is also unnecessary to consider that a hydrogen-cooled machine must be placed out-of-doors. In our experimental work in East Pittsburgh, we had a 7500-kv-a. turbine generator in a special test room just about large enough to contain the apparatus. Due to faulty seals there was a leakage of about 40 cu. ft. per hr., instead of that amount per week, and in spite of the comparatively small room

there was no dangerous contamination of the air. So certainly with any commercial leakage factors and with reasonable room size, leakage is no reason for putting a hydrogen-cooled machine out-of-doors.

C. M. Gilt: I am very glad to see the commercial development of hydrogen cooling and we hope to see it applied shortly to turbo alternators where the advantage would appear to be greatest.

Between two and three years ago, the Brooklyn Edison Company gave serious consideration to hydrogen cooling for a 35,000-kv-a. frequency changer which we expected to install. We did not believe it worth the cost to make the housings explosion-proof and our designs were for an outdoor installation in order to keep the cost at a minimum.

Even though the unit would be rated 35,000 kv-a., with a relatively high peripheral speed for a frequency changer, the gain made by reducing the size would not offset the increased cost of the enclosure and the reduction in losses would not equal the extra carrying charges of the hydrogen-cooled machine as compared with an air-cooled machine installed in a building.

I have a suspicion that if all the cumulative advantages of hydrogen cooling, such as thinner insulation and the consequent over-all reduction in size, were adopted, even the relatively low-speed frequency changer might show economic advantages. Naturally, the conservatism of operators and manufacturers restrains one from going all the way in the reduction of size which may be found possible with the hydrogen cooling, but I expect that we will see marked economies in the design following the early experience in operation of hydrogen-cooled apparatus.

H. W. Brown: I should like to ask Mr. Wieseman whether he has taken into account the advantage in design, resulting in longer life of a machine that is not in an atmosphere containing oxygen. Generally there is deterioration of the insulation by oxidation. This does not apply to conductors that are not exposed to the air, but in other conductors there is a deterioration which is greater at high than at low temperatures. It would seem, then, that where the windings are in an atmosphere of hydrogen we could either have a longer life of a machine with the same rating, or the same life with a higher rating.

I. A. Terry: Mr. Gilt refers to the application of hydrogen-ventilation to frequency converters. There are two problems in connection with such an installation which require careful consideration. One relates to stability; the other is a mechanical problem.

In the introduction of his paper Mr. Wieseman pointed out that the advantages in ventilating a machine with hydrogen result in an increased output of 25 per cent or more, depending upon various factors, one of which is the stability required. Assuming that lagging kv-a. is not important, this would mean that a 10,000-kv-a. air-cooled condenser could be rated 12,500 kv-a. when hydrogen-ventilated by increasing the field and armature current. It is not possible to increase arbitrarily the rating of a synchronous generator, if the same degree of stability is to be maintained, by simply increasing the armature current and field current. It has been shown* that the short-circuit ratio is a measure of the stability of a generator under any given set of operating conditions. And since the short-circuit ratio is inversely proportional to the stator current and directly proportional to the no-load field current, an arbitrary increase in the armature current must be accompanied by a proportional increase in the no-load field current if the hydrogen-ventilated generator is to have the same degree of stability as the present air-cooled machines. Similar remarks apply to the motor of a frequency converter, the stability being determined by the pull-out load, instead of the short-circuit ratio.

The mechanical problem arises in making the frequency converter gas-tight. Mr. Wieseman pointed out that hydrogen cooling can be easily applied to a synchronous condenser, because the entire machine, including bearings and shaft, can be enclosed in a gas-tight shell as contrasted with the application to a turbine generator where a shaft seal is required. In a frequency converter either shaft seals must be provided or the whole set enclosed in a large gas-tight shell. Both types of construction have certain advantages and disadvantages. In addition for parallel operation of frequency converters, if the operator is to have control of the load division, it is necessary to have one stator frame of the set arranged for rotating. This again complicates the construction of a gas-tight shell.

R. W. Wieseman: Mr. Sporn stated that operating experience indicated a hydrogen leakage of 75 to 100 cu. ft. per week and that this was a little larger leakage than I mentioned in the paper. The paper gives the total cost of the hydrogen leakage as 15 to 20 cents per day, and this is just about the cost of 75 to 100 cu. ft. of hydrogen leakage per week as given by Mr. Sporn.

I quite agree with Mr. Fechheimer that the explosion risk of a hydrogen-cooled machine is almost nil if the machine is given ordinary supervision. The explosion problem is always associated with any new development which utilizes an explosive material. This was anticipated when the first machine was built, and therefore the explosion-proof shell was used to make sure that no difficulty would be experienced.

In regard to the amount of scavenging gas required, I think it desirable to displace practically all of the air with carbon dioxide and then displace the carbon dioxide with hydrogen. If the machine is filled quickly little diffusion will take place and the volume of the inert gas used should be about the same as the gas volume of the machine. It is important that all of the carbon dioxide be expelled from the machine in order to obtain minimum windage loss, and for this reason it is desirable to scavenge with hydrogen quite thoroughly. It is a simple operation to fill a machine with hydrogen. In fact it is not even necessary to use a hydrogen purity indicator for this operation, if an inert gas such as carbon dioxide or nitrogen is used to displace the air in the machine before the hydrogen is admitted.

The external hydrogen-fan purity indicator mentioned by Mr. Fechheimer is a very good scheme. It operates under the same principle as the internal-fan method described in the paper. We use the internal-fan purity indicator only as a check on the thermoelectric hydrogen purity indicator which gives the purity of the gas whether the machine is in operation or shut down. The thermoelectric purity indicator is a direct-reading instrument and it does not require a driving motor.

Mr. Fechheimer points out that the cost of hydrogen-cooled generators should be less than we obtained on synchronous condensers. That naturally depends upon the capacity of the generator. Up to the present time, generators have been built which have about three times the capacity of the largest synchronous condenser. Naturally, the hydrogen features of these large generators should cost less than those of a condenser. Furthermore, the saving in windage loss is much more in the case of a large high-speed turbine generator. There are many factors which enter into the cost of a new type of machine and these factors are further modified by the location and the conditions at the substation or power plant. Consequently, at this stage of the development, it is rather difficult to give a ratio of the cost of a hydrogen-cooled machine to the cost of an air-cooled machine.

Mr. Doggett has called attention to the explosion possibilities and points out that a minor explosion may even take place around the outside of an outdoor hydrogen-cooled machine. It should be remembered that hydrogen gas has only 1/14 the density of air and so it is practically impossible to have an explosion around the outside of an outdoor machine because, if some of the hydrogen should escape, it would immediately rise and disappear. "Inverted pockets" around the outside of a machine are not to

**Stability Characteristics of Alternators*, by O. E. Shirley, A. I. E. E. Trans., 1926, Vol. 45, p. 1108.

be found. As a matter of fact hydrogen machines have been both inside and outside for a considerable time and no difficulty whatever has been encountered with explosions of any kind. I, therefore, do not agree with Mr. Doggett's idea of hydrogen explosions; in fact I think the explosion possibilities of a hydrogen-cooled machine have been very much overstressed.

In replying to Mr. Gilt's discussion, Mr. Terry cited two reasons why there is not so much to be gained in the use of hydrogen cooling for frequency-converter sets. Perhaps Mr. Terry has given the impression that the stability characteristic of a frequency-converter set will prevent hydrogen from increasing the rating of the set. A frequency converter, like any other dynamo machine, must be properly designed for hydrogen cooling, and although the increased rating will not be so high as in the case of a synchronous condenser, there is, however, a decided increase in the rating of the set which can be obtained with hydrogen. The real problem in applying hydrogen cooling to a frequency converter is not so much a stability problem as a mechanical problem; namely, keeping the armature frame gas-tight and also moveable for voltage phase shifting.

Mr. Brown asked an important question, namely, how much the life of the insulation would be increased by hydrogen. As explained in the paper, tests have shown that in an atmosphere of hydrogen, corona damage is eliminated and no oxidation can take place. Just how much the insulation life will be increased by these factors is a matter of opinion at the present time because there is not much information on the subject available. The increase in the life of a varnish-fabric insulation would certainly be much more than that of mica insulation. If the hydrogen-cooled machine is not overloaded or abused, its insulation should have about twice the life of the insulation of an air-cooled machine.

Short-Circuit Torque in Synchronous Machines without Damper Windings

BY G. W. PENNEY*

Associate, A. I. E. E.

Synopsis.—*The torque produced by a short circuit is first discussed in a general way, showing that it is pulsating in nature. The average value of torque is determined by the resistance and other energy losses but the instantaneous value rises far above the average value. The major part of this pulsating torque is produced by the change in stored magnetic energy. General equations are derived for the torque in a machine having negligible resistance and constant self-inductance of each winding which is referred to as an "Ideal Machine."*

These equations are useful in comparing single-phase and various polyphase short circuits and for determining general tendencies. However, the numerical value of the torque is usually of most interest in salient-pole machines and here the self inductance of the armature windings is not constant. The calculation of torque from the actual inductance of the windings of a salient-pole machine is a very complicated problem. A relatively simple step by step calculation, called the "semi-graphical method," is developed for calculating the torque from the actual currents as given by an oscillogram. The effect of resistance can be included in this calculation. The method holds for any variation in self inductance with rotor position provided that saturation is not a large factor in determining the change in stored magnetic energy, which seems to be true in cases thus far investigated.

Short-circuit torque can be measured by an instrument described in a previous paper.[4] *The torque as measured is compared with the torque as calculated by various methods. This shows that calculations based on constant self-inductance of each winding may be seriously in error, particularly if equations derived on this basis are applied to the actual currents of salient-pole machines. The "semi-graphical" method agrees reasonably well with measured values for cases thus far tested.*

CONFLICTING opinions are often expressed concerning the torque produced by a short circuit. One idea frequently expressed is that since the resistance of the circuit is small compared to the reactance, the current flowing has practically zero power factor and therefore the resultant torque produced should be very slight. However, the fact that the power factor is nearly zero indicates only that the average torque during one complete cycle is small. It gives no information concerning what takes place during parts of a cycle. In designing a machine to withstand the worst possible operating conditions, the maximum value of torque which may act at any instant must be known so that the pulsating or instantaneous value of short-circuit torque must be considered. Furthermore, because of inertia reactions the same torque will not act on all parts of a machine so that to determine the force acting on any given part, the curve of torque against time must be known.

The failures occasionally produced by short circuits show that tremendous forces must result from short circuits and other transient phenomena. For example, one of the early large vertical waterwheel generators of low reactance at Niagara Falls sheared off the holding down bolts and the stator turned through a considerable angle on its foundation. In another case a frequency changer set stretched the holding down bolts and cracked the frame supporting feet when the set was connected to the line out of phase. It is only necessary to stand beside a low reactance machine when it is short circuited to realize that very severe forces are acting on the machine. Synchronizing out of phase may produce forces which are even more severe. This paper is confined to the subject of short-circuit torque although the methods proposed can be used, with some modifications, for determining the torque during synchronizing and other transient conditions.

One difficulty in understanding the torque produced by a short circuit is the fact that the assumptions generally used for calculating the torque during normal operations do not represent the conditions during a short circuit. In calculating the torque during normal operation, it is usually assumed that the stored energy is constant so that the mechanical torque of a generator is equal to the electrical output plus the losses. If a generator is short-circuited at its terminals, its electrical output is zero and the energy losses within the machine account for only part of the torque which is known to be produced. An investigation of the stored magnetic energy of a machine during short circuit shows that this energy fluctuates very rapidly and that there is a corresponding fluctuation of the kinetic energy of the rotating mass. The transfer of kinetic energy to magnetic energy, and vice versa, must be accompanied by a torque which alternates between positive and negative values.

The Relation Between Torque and Stored Energy

In order to simplify the discussion a synchronous condenser short-circuited at its terminals will be considered first. Since the terminals are short-circuited, no electrical power can flow between the machine and the line, and since there is no mechanical connection to any other machine, no power can flow to or from the synchronous condenser except that dissipated as heat.

Since there can be no transfer of energy between the machine and any outside source, the total stored energy of the machine plus the energy dissipated must remain constant. Or, expressed mathematically,

*Research Dept. Westinghouse Elec. & Mfg Co., East Pittsburgh, Pa.

4. For numbered references see Bibliography.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

$$-T\,\omega\,d\,t + U + \int R\,i^2\,d\,t = \text{Initial stored. Energy} = \text{Const.} = U_0 \qquad (1)$$

Where

T = Torque at any time t (a torque which tends to accelerate the rotor is assumed negative).
U = Stored magnetic energy at time t.
ω = Angular velocity.
R = Resistance.
i = Current.
U_0 = Initial stored magnetic energy at $t = 0$.

Differentiating

$$T\,\omega = \frac{d\,U}{d\,t} + R\,i^2 \qquad (2)$$

If the resistance is negligible

$$T\,\omega = \frac{d\,U}{d\,t} \qquad (3)$$

That is to say that the torque multiplied by the angular velocity is equal to the rate of change of stored magnetic energy provided that the energy losses are negligible.

The above equations were derived for a short-circuited synchronous condenser where no energy could flow into or from the machine except in form of heat. However, the equations derived hold for any short-circuited machine. In the general case, electrical power cannot flow to or from the machine short-circuited, but mechanical power may be transmitted to or from the mechanically connected machine. Applying the law of conservation of energy to this general case, any increase in the sum of the stored magnetic energy, plus the integral of the energy dissipated, must be accompanied by a transfer of mechanical energy to magnetic energy. Conversely, any decrease in the sum of the stored magnetic energy, plus the integral of the energy dissipated, must be accompanied by a transfer of magnetic to mechanical energy.

Then,

$$U + R\,i^2\,d\,t = U_0 + \int T\,\omega\,d\,t$$

Differentiating

$$\frac{d\,U}{d\,t} + R\,i^2 = T\omega$$

This is the same Equation (2) and if the resistance is negligible is the same as Equation (3). From this it is evident that if the stored magnetic energy can be determined for each increment of time, the corresponding mechanical torque can be calculated.

The torque discussed here is the resultant tangential reaction of the windings. The torque acting on individual parts will depend on the mechanical arrangement. In the case of the synchronous condenser, the shaft does not transmit any torque. In this instance the torque produced by the winding merely results in a change in speed of the rotor. The other extreme exists in a rigid rotor connected to an infinitely large flywheel by a perfectly rigid shaft. In this case the speed of the rotor must be constant and therefore all of the torque produced must be transmitted by the shaft. Actual cases will usually be somewhere between these two extremes. The torque acting on individual parts of the machine is outside the scope of this discussion. In this discussion the term "short-circuit torque" will be taken to mean the resultant reaction of the windings.

The main purpose in studying this torque is to permit a more rational design of the mechanical parts of the machine. In designing these parts to withstand short-circuit forces the inertia reactions and deflections play an important part in determining the stresses. This requires that the duration of the forces as well as their maximum value be known, so that a curve between torque and time must be obtained. A machine must be

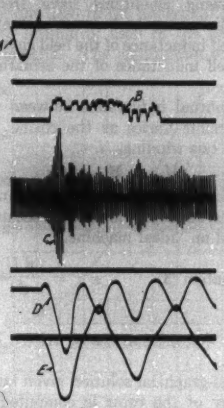

Fig. 1—Tracing of an Oscillogram for a Single-Phase Short Circuit

A—Terminal voltage
B—Current through a group of contacts—the record of the contact and flywheel accelerometer measuring torque
C—800-cycle current—the magnetic record of acceleration or torque
D—Field current
E—Current in the short-circuited phase

designed to withstand the worst operating conditions. A single-phase short circuit occurring at the instant when the generated voltage in that phase is zero gives a much greater torque than any other short circuit thus far investigated. Since a machine must be designed to withstand the worst conditions, this case is discussed in detail and other cases only to the extent of showing that they are smaller in magnitude.

Stored Magnetic Energy of a Rotating Machine

During open-circuit operation of a machine there is a certain field current and flux giving a substantially constant stored magnetic energy. An unbalanced load current may give some pulsation in the stored energy, but usually this is of negligible magnitude. However, at the instant of short circuit a large armature current is

produced and the field current rises to several times its normal value, which results in a corresponding increase in the stored magnetic energy.

If hysteresis and eddy currents are neglected, the flux and stored energy will depend only on the currents and rotor position and will be the same regardless of the manner in which those currents were established. In a short circuit, an increase in the stored magnetic energy is supplied by the mechanical energy of the rotor. However, to evaluate the magnitude of the stored energy for any given currents and rotor position, the rotor can be assumed to be locked in that position, and the energy required to establish the given currents and fluxes from an electrical source calculated.

Short-Circuit Torque in an "Ideal Machine"

The following conditions have frequently been assumed in discussing short-circuit currents,

1. The self inductance of the field circuit is constant.
2. The self inductance of the armature winding is constant.
3. The mutual inductance between the field and armature circuits varies as the cosine of the angle between the two windings, *i. e.*,

$$^{*}M_{af} = M_{AF} \cos \alpha$$

4. The resistance of the windings is negligible.
5. An alternator to which these assumptions apply will be called an "ideal machine."

From Equation (3) $$\omega T = \frac{d U}{d t}$$

Since $\omega\, d t = d \alpha$, this can also be written $$T = \frac{d U}{d \alpha} \quad (4)$$

In the semi-graphical solution given later the actual stored energy of the rotor is computed and plotted against the angular position of the rotor. The derivative or slope of this curve is then proportional to the torque. In the mathematical solution, it is possible to compute the actual stored energy and differentiate the result to obtain the resulting torque; but this method is very complicated. The solution is simplified very much by using a method commonly employed for calculating magnetic forces. This method depends on the fact that the forces acting at any position of the rotor depend only on the currents and fluxes acting at that instant. Then to find the forces acting, the current is assumed constant and the rate of change of stored energy found for a virtual displacement of the rotor. With this assumption the current would be constant and the flux varying; in the actual case, the flux is constant and current varies.

The stored energy in a system of inductively coupled circuits is discussed by Karapetoff in *Mechanical Forces between Electric Circuits*.[2] It is shown that by multiplying the ampere-turns in each circuit by the flux interlinking that circuit and adding the products for all of the circuits involved, the stored energy of the system is obtained. Or, expressed mathematically,

$$U = \sum_{1}^{n} \frac{N_u \phi_u i_u}{2} \quad (35)$$

Where N_u = Number of turns in circuit u
ϕ_u = Flux interlinkages of circuit u
i_u = Current in circuit u

Or expressing the flux in terms of the currents and inductances:

$$U = \sum_{1}^{n}{}_{u} \frac{i_u}{2} \sum_{1}^{n}{}_{v} M_{vu} i_v$$

Where M_{uv} denotes the mutual inductance between the circuits indicated by the subscripts. In case both subscripts refer to the same circuit, that is, when $v = u$ giving M_{uu}, it is interpreted to mean the self inductance of circuit u. Then this expression can be differentiated with respect to α to get the rate of change of energy for a virtual displacement of the rotor. This is given in Appendix II. It is shown that for a single-phase short

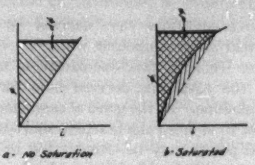

Fig. 2—Flux Current Curves Illustrating the Stored Energy in a Magnetic Circuit

A. Magnetic circuit with no saturation. The stored energy is shown by the cross-hatched area

B. Saturated magnetic circuit. The double cross-hatched area represents the actual stored energy. The single cross-hatched area represents the error when the energy is calculated from Equation (35).

circuit where the field and armature windings have a mutual inductance varying with the position of the rotor but constant self inductances, the variation in stored energy is given by,

$$T = \frac{d U}{d \alpha} = i_f i_a \frac{d M_{af}}{d \alpha} \quad (6)$$

Since saturation is neglected the variation in mutual inductance can be determined from the voltage generated before the short circuit. Substituting the value thus obtained in Equation (6) gives

$$\omega T = \frac{d U}{d t} = [i_a e_m \sin \alpha] \frac{i_f}{I_f} \quad (11)$$

Or, expressed in words, the torque multiplied by the angular velocity which is the instantaneous power is equal to the product of the instantaneous value of the armature current by the voltage that would have been generated under open-circuit conditions and this multiplied by the ratio of the instantaneous field current to the field current value before the short circuit. Since the field current increases to several times its

*For symbols, see Appendix I.

initial value, it is evident that this equation may give a very high value of torque. During the short circuit, the field ampere-turns are roughly proportional to the armature ampere-turns. Therefore, if the short-circuit current is increased by decreasing the reactance of the machine, it is evident that the torque will be increased approximately in proportion to the square of the armature current, showing that in a low reactance machine the torque will be very severe.

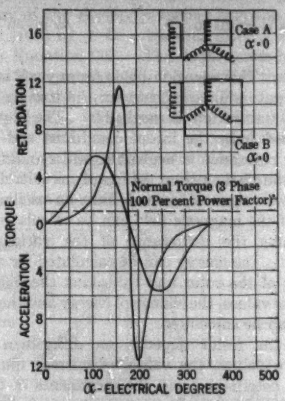

FIG. 3—TORQUE DEVELOPED BY SHORT CIRCUIT OF AN "IDEAL MACHINE"

A. Single-phase short circuit, terminal to neutral
B. Three-phase short circuit, terminal to terminal

Assumptions:

1. Short circuit occurs when generated voltage on phase is zero
2. Star-connected winding

$$K = 0.85 = \frac{M_{AF}}{L_f}$$

4. Peak non-symmetrical terminal-to-terminal short-circuit current equals 12 times rated r. m. s. value of current per phase

The equation for the torque produced by a three-phase short circuit can be derived in a similar manner and gives

$$T = \frac{d\,U}{d\,\alpha} = [i_a\,e_a + i_b\,e_b + i_c\,e_c]\,\frac{i_f}{I_f\,.\,\omega} \qquad (14)$$

This equation is similar to Equation (11) as given, but in the three-phase short circuit the signs of the various terms must be considered. The separate circuits do not act together, which gives a slower rate of change of stored energy at any given instant and therefore a lower torque. In Appendix III, the torque is computed for a particular star-connected winding, first for a single-phase terminal-to-neutral short circuit and then for a three-phase terminal to terminal short circuit. The results are plotted in Fig. 3. The maximum stored energy is not essentially different for the two cases. In the single-phase short circuit there is a very sudden change in the stored energy from $\alpha = 125$ to $\alpha = 180$ giving a corresponding high torque. In the three-phase short circuit the variation in stored energy is sinusoidal, giving a much lower maximum value of torque.

Similar equations can be derived for other polyphase short circuits but in all of the cases thus far investigated the different circuits will not act together so that the resultant rate of change of stored energy is slower than for a single-phase short circuit. This conclusion has been confirmed experimentally for the case of a three-phase terminal to terminal short circuit and for the case of two terminals of a star-connected winding short-circuited to the neutral.* In both cases the short-circuit torque was far below that for the corresponding single-phase short circuit.

SEMI-GRAPHICAL DETERMINATION OF TORQUE

The assumptions used in the analytical solution just given are those ordinarily used in the calculation of short-circuit currents. The assumption of a constant self inductance of each phase represents the condition with a cylindrical rotor quite accurately but in a salient pole machine the self inductance of each phase varies with the rotor position. In the calculation of short-circuit currents we are usually mainly interested in maximum values, and the maximum value of current is

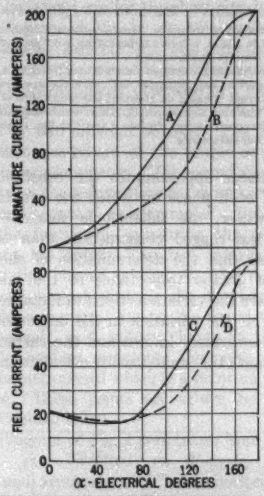

FIG. 4—COMPARISON OF ACTUAL AND THEORETICAL CURRENT WAVE SHAPE FOR SINGLE-PHASE UNSYMMETRICAL SHORT CIRCUIT

A and *C*,—Armature and field current, respectively, from oscillogram
B and *D*,—Armature and field current, respectively, from Equations (40) and (41) in paper by R. F. Franklin (B-3) with the value of *K* chosen to give same maximum values as *A* and *C*

*Case (7) in the paper on *Short Circuits in Synchronous Alternators*, by R F. Franklin.[3]

not seriously altered by the variation in the self inductance of the phase winding. On the other hand, the torque produced depends on the rate of change of current or rate of change of inductance so that the torque is seriously changed by the variation in the self inductance of each phase.

The equations just derived are useful for comparing a single-phase short circuit with various polyphase short circuits and determining general tendencies. A

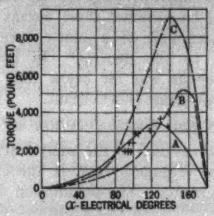

FIG. 5—COMPARISON OF METHODS OF CALCULATING TORQUE

A. Torque calculated from the "Semi-Graphical" method for current values *A* and *C* in Fig. 4. Points indicated by "+" show the torque as recorded by the contact accelerometer

B. Torque calculated for current values *B* and *D* in Fig. 4. Note that semi-graphical method and Equation (11) give the same results in this case since the current is based on constant self-inductance of each circuit

C. Result of applying Equation (11), which was based on constant self-inductance, to the actual current (Curves *A* and *C* in Fig. 4) from a machine having variable self-inductance

turbo alternator approximates the constant self inductance of each phase on which the equations were based. However, the salient-pole machine has an inherently less rugged construction than the turbo alternator, so that the determination of short-circuit torque is of greater importance in the salient pole machine where the variation in self inductance must be considered. If the variation in the inductance of the windings is accurately known and the currents calculated Equation (**12**) can be used to calculate the torque. In this case the calculations become complicated so that the method is cumbersome.

The semi-graphical method has been developed for calculating the torque from an oscillogram of currents, and eliminates the necessity of determining the inductance of the separate circuits. The method is based on the assumption that the stored energy of n circuits is equal to $\sum_{1}^{n} \frac{N_u \phi_u i_u}{2}$ as given by Equation (**35**). This equation holds for any self inductance and mutual inductance between the circuits involved, provided saturation is negligible. The flux can be determined from the initial conditions and assuming that the resistance is negligible, will remain constant during the first instant of short circuit. Then by taking the values of current from an oscillogram the energy of the magnetic field can be calculated without determining the inductance of the various circuits involved. The energy can be calculated for successive positions of the rotor and by taking the difference in successive values, the change of energy and the resulting torque can be calculated.

Since the result depends on differences between successive values, the values of current must be accurately determined and the increments of time used must be large enough so that the change in current can be measured accurately. This calculation is illustrated in Appendix IV. Successive values for one case are tabulated in Table I, and the result plotted for Curve *C* of Fig. 6.

Effect of Saturation. Equation (**35**) is based on the assumption of no saturation. In a machine some parts of the circuit are usually highly saturated. However, during the first instant of short circuit the resultant flux interlinkages of each circuit remain substantially constant. For this reason the effect of saturation upon

TABLE I

CALCULATION OF TORQUE NEGLECTING RESISTANCE

α degrees	i_f amperes	$\frac{N_f i_f \phi_f}{2\ 10^8}$	i_a amperes	$\frac{N_a i_a \phi_a}{2\ 10^8}$	(Joules) energy U	ΔU (Joules)	$\frac{\Delta U}{\Delta \alpha} \cdot 2.21 =$ torque (lb-ft.)
20	27	506	0		506		
30	25.5	478	7	42.4	520	14	177
40	23.5	440	23	140	580	60	760
50	23	431	40	242	653	73	965
60	23.5	440	57	345	785	72	910
70	25.5	478	75	454	932	147	1860
80	29.5	552	94.5	572	1124	192	2430
90	36	675	113	685	1360	246	3120
100	45	843	134	811	1654	294	3720
110	55.5	1030	158	956	1986	332	4200
120	69	1295	185	1120	2415	429	5420
130	84	1575	214	1295	2870	455	5760
140	100	1875	244	1475	3350	480	6080
150	112.5	2110	273	1655	3765	415	5250
160	121	2270	294	1780	4050	285	3600
170	126.5	2370	295	1790	4160	110	1400
180	124	2320	277	1675	3995	−165	−2090
190	116	2170	252	1585	3755	−240	−3040
200	104	1950	224	1355	3305	−450	−5700
210	90	1690	190	1150	2840	−465	−5880

the change in stored energy is not as serious as it might appear from the values of current involved.

A magnetic field can be regarded as divided into parts and the energy computed separately for each part. For the part of the magnetic circuit through air or other medium of constant reluctance, the energy is given by Equation (35). This is represented graphically by the cross hatched area in Fig. 2A which is merely $\int i\, d\phi$. For the part of the circuit traversing iron the energy will still be given by $\int i\, d\phi$ but this integral will not be given by Equation (35). The double cross hatched area in Fig. 2B represents $\int i\, d\phi$ which is the actual stored energy in the saturated field. The single cross hatched area in Fig. 2B represents the error when the energy in the saturated circuit is calculated from Equation (35) (*i. e.*, neglecting saturation).[1,3]

If a machine is working high on the saturation curve it is obvious that the initial stored energy will be considerably in error when calculated neglecting saturation. During the short circuit the resultant flux interlinkages of any circuit remains substantially constant. The flux at some highly saturated points is reduced and the saturation at other points increased. However, in most designs the main change in the energy of the field occurs in the part of the circuit through the air and other non-magnetic material. For this reason although the initial energy may be considerably in error, the change in energy will be substantially correct when calculated neglecting saturation and therefore the torque can be calculated neglecting saturation with sufficient accuracy for this type of calculation. This conclusion is substantiated by the agreement obtained between calculations and tests as illustrated in Figs. 5 and 6.

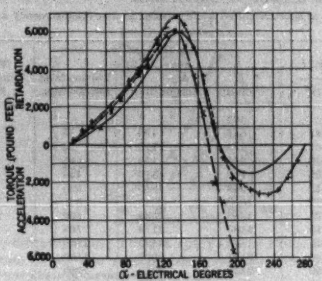

FIG. 6—TORQUE PRODUCED BY A SINGLE-PHASE SHORT CIRCUIT

B + — — — Calculated from the recorded currents considering the effect of resistance

C × ——— Calculated neglecting the effect of resistance

A ——— Torque recorded by the magnetic accelerometer

⊙ Torque recorded by flywheel and contact accelerometer

The Effect of Resistance. Thus far the effect of resistance has been neglected. Assuming that the currents are taken from an oscillogram, the effect of resistance on the torque calculation is two-fold. First, the resultant flux interlinkage of each circuit is constant only when the resistance of the circuit is negligible. Actually the electromotive force to overcome the resistance drop in a short-circuited winding must be supplied by the change in flux interlinkages of that circuit $\left(i.\ e.,\ R\, i = e = N \frac{d\phi}{dt} \right)$. Secondly, the power represented by the resistance loss is usually not negligible because of the large values of current involved.

These two effects of resistance can be incorporated in the step by step calculation of torque. The decrease in flux interlinkages can be calculated for each increment of time and this value used to obtain the actual flux interlinkages and corresponding stored energy. The torque corresponding to the resistance loss can be calculated and added to the torque resulting from the change in stored energy. This calculation considering resistance is illustrated in Table II of Appendix IV and the torque obtained plotted as Curve *B* of Fig. 6.

These calculations are for the short-circuit recorded on the oscillogram of Fig. 1. On this oscillogram the terminal voltage, field current, and armature current are shown. Also the torque is recorded on the oscillogram by the accelerometer described in a paper at the Pittsfield Regional Meeting in 1927.[4] This measures the torque by two different devices. The first device gives a continuous record of the torque and is called the magnetic accelerometer. The second device merely gives points on the torque time curve and is called the flywheel and contact accelerometer. Both devices measure the torque by measuring the acceleration imparted to the rotor. The two measurements of torque are also plotted in Fig. 6. The record *B* of Fig. 1 is the current through the flywheel contact. As the torque increases during the first half cycle of the short circuit, each sudden change in the current through the contact represents the point where one flywheel contact opened which is a point on the torque time curve. These points are shown by the small circles in Fig. 6.

The oscillogram of Fig. 1 was taken from a 60-cycle alternator, short-circuited at 25-cycle frequency. The field and armature were both short-circuited at the same instant. This oscillogram has been used to illustrate the method of calculation because the effect of resistance is exaggerated. For the first half cycle the calculation neglecting resistance gives almost the same result as the calculation considering resistance as shown by the agreement of Curves *B* and *C* on Fig. 6. The reason for this agreement is that the two effects of resistance mentioned above tend to annul each other. The decrease in flux interlinkages diminishes the rate of increase of stored energy, thereby reducing the torque while the resistance losses increase the torque. However, during the second half cycle the two effects do not cancel each other so that neglecting resistance a very high negative torque would be obtained while when

resistance is considered the calculated torque agrees reasonably close with the measured value.

Another point of interest in connection with curves of Fig. 6 is that the initial field current used was the normal value for 80 per cent power factor, but since the machine was on open circuit before the transient this gave a point high on the saturation curve. Yet the calculations neglecting saturation are in close agreement with the tests.

Discussion of Results

The torque produced by a single-phase short circuit varies greatly with the position of the rotor at the instant when the short circuit occurs. This discussion has been devoted almost entirely to the subject of short circuits occurring at the instant when the terminal voltage is zero, which gives the greatest torque. This was done because the designer is interested in the worst condition that can happen. In the tests made, the instant of short circuit was controlled to give this worst condition. However, in operation very few of the short circuits will happen to occur at this particular point. In most cases the torque will be very small compared to the worst condition.

As has been stated the torque produced by a polyphase short circuit in all cases thus far investigated is much less than the maximum torque resulting from a single-phase short circuit. The probability of a single-phase short circuit is also much greater than the probability of a polyphase short circuit. For these reasons practically all of the results given are for single-phase short circuits.

In the first part of this paper equations were derived for an "ideal machine," that is a machine to which the assumptions frequently used in discussing short-circuit currents would apply. The equations for currents derived on this basis are quite generally applied to salient-pole machines with reasonably satisfactory results. However, as has been mentioned, it must not be assumed that the equations for torque derived using the same approximations can be as generally applied. Figs. 4 and 5 are intended to emphasize this fact. In Fig. 4 the actual wave shape of the currents for a certain machine are compared with the wave shapes as given by the equations derived by Franklin,* assuming constants which would give the same peak value of current in both cases. This agreement is satisfactory for most purposes. However, since the torque depends on the rate of change of current the value will be very different for the two cases. Curves A and B of Fig. 5 show this difference. The area under the two curves is the same since the maximum stored energy is the same but the rate of change of energy is very different, giving a corresponding difference in the value of peak torque.

It might be assumed that Equation (11), which gives the torque in terms of the currents during the short circuit and the voltage before the short circuit, could be applied to actual measured currents in cases where the self inductance of the separate circuits is not constant. However, the torque corresponding to a given current depends on the rate of change of inductance. For this reason this equation for torque is true only when the currents are varying as in the "ideal machine." For instance, if these equations are used with the values of current given by Curves A and C of

TABLE II

CALCULATION OF TORQUE CONSIDERING THE EFFECT OF RESISTANCE

Note: This example exaggerates the effect of resistance since a 60-cycle machine was short circuited while running at 25-cycle frequency and since both field and armature terminals were short circuited.

α degrees	$\frac{Rf\, if\, \Delta t = Nf\, \Delta\phi f}{10^3}$	$\frac{Nf\, \phi f}{10^3}$	$\frac{Nf\, \phi f\, if}{2.\ 10^3}$	$\frac{Ra\, ia\, \Delta t = Na\, \Delta\phi a}{10^3}$	$\frac{Na\, \phi a}{10^3}$	$\frac{Na\, \phi a\, ia}{2.\ 10^3}$	U (joules)	$\frac{\Delta U}{\Delta \alpha} \cdot 2.21$ torque (lb-ft.)	Torque from resist. losses	Total torque lb-ft.
20	0	37.5					502			
30	0.13	37.37	475	0.02	12.08	42	517	190	45	235
40	0.13	37.24	435	0.06	12.02	138	573	710	52	762
50	0.12	32.12	425	0.10	11.92	240	665	1140	85	1225
60	0.12	37.00	432	0.14	11.88	336	768	1310	138	1448
70	0.12	36.88	468	0.19	11.60	435	903	1840	225	2065
80	0.14	36.74	540	0.24	11.45	540	1080	2240	345	2585
90	0.21	36.53	655	0.31	11.14	625	1280	2530	506	3036
100	0.26	36.27	815	0.39	10.75	715	1530	3170	768	3938
110	0.33	35.94	995	0.46	10.29	810	1805	3480	1100	4580
120	0.41	35.53	1225	0.53	9.76	896	2121	4000	1590	5590
130	0.53	35.00	1475	0.66	9.10	960	2435	3980	2260	6240
140	0.65	34.35	1721	0.75	8.35	1020	2741	3880	2950	6830
150	0.76	33.59	1895	0.86	7.49	1025	2920	2170	4200	6370
160	0.77	32.82	1990	0.92	6.57	965	2955	442	4740	5182
170	0.81	32.01	2020	0.925	5.65	835	2855	−1270	4920	3650
180	0.80	31.21	1930	0.86	4.79	665	2595	−3300	4420	1120
190	0.74	30.47	1770	0.79	4.00	506	2276	−4040	3400	− 640
200	0.66	29.81	1550	0.69	3.31	373	1923	−4470	2680	−1790
210	0.54	29.27	1320	0.58	2.73	261	1581	−4030	1960	−2070
220	0.47	28.80	1110	0.47	2.26	178	1288	−3700	1320	−2380
230	0.38	28.42	900	0.34	1.92	113	1013	−3500	880	−2620
240	0.29	28.13	680	0.22	1.70	67	747	−3340	700	−2670

*Equations (40) and (41), *Short Circuits of Synchronous Machines*, (Reference 3).

Fig. 4, the torque obtained would be that for the given values of current but assuming that that current was varying at a rate given by the theoretical equations. The error corresponding to this assumption is shown by curve *C* of Fig. 5, which gives a maximum torque almost three times the actual value, whereas the torque calculated by the sem-graphical method agrees very closely with the measured torque.

It will be observed that all of the discussion has assumed a short circuit occurring when the machine is running on open circuit. This has been done to simplify the analysis and tests. It is evident that the short-circuit currents depend on the flux interlinkages at the instant of short circuit. The effect of the initial currents is to modify the initial flux. In the case of a single-phase machine operating at unity power factor, if a short circuit occurs at the instant of zero voltage, the load current is also zero so that this case is almost

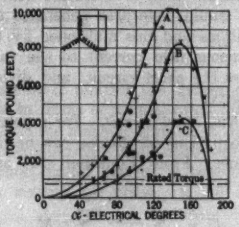

FIG. 7—SHORT-CIRCUIT TORQUE FOR VARIOUS FIELD CURRENTS.

A. Field current = 27 amperes giving 3240 volts during open circuit
+ From magnetic accelerometer
⊕ From flywheel and contact accelerometer
B. Field current = 20 amperes giving 3010 volts
× From magnetic accelerometer
⊗ From flywheel and contact accelerometer
C. Field current = 10 amperes giving 2180 volts
· From magnetic accelerometer
⊙ From flywheel and contact accelerometer

identical with the case of a short circuit during open circuit operations. In case the power factor is not unity, or for any case of polyphase operation, the solution is not quite as simple. However, in all cases the equations that have been derived apply equally well provided the proper correction is made for the different initial flux interlinkages.

An illustration of the dependence of the currents and torque on the initial interlinkage is given by the curves of Fig. 7. This shows the torque obtained experimentally for various initial values of field current. The generated voltage before the short circuit is a measure of the initial flux interlinkages of the armature. According to the theory developed, the torque should be proportional to the square of the initial flux. Of course, since saturation will increase the leakage, the field flux will increase somewhat faster than the armature flux as the field current is increased. Neglecting these effects the maximum torque should vary as the square of the generated voltage before the short circuit. If in Fig. 7 the peak torque for the two lower values of field current be calculated from the test value for the maximum field current assuming a variation proportional to the square of the voltage, 8800 lb-ft. and 4600 lb-ft. are obtained as compared with 8250 and 4300 respectively from tests.

The curves of Fig. 7 also show that saturation is not a major factor in determining the short-circuit torque except that it must be considered in determining the initial flux. The two higher values of field current are well up on the saturation curve as shown by the fact that a 35 per cent increase in field current gives only an 8 per cent increase in voltage, and yet the torque can be calculated assuming that it varies with the square of the voltage with reasonable accuracy. This must not be taken as a proof that saturation will never be a major factor because it is entirely possible that in some cases a large proportion of the leakage flux may traverse a path having enough iron to make the effect of saturation appreciable, in determining the rate of change of stored energy.

Mr. Theo. Williamson conducted most of the tests referred to and his work is hereby acknowledged, as are the suggestions of Messrs. Fechheimer and Soderberg under whose supervision the work was done.

Appendix I

SYMBOLS

T = Resultant torque produced by the reaction of the currents and fluxes. (Assumed positive when it tends to decrease the speed of rotation.)

ω = Angular velocity of the rotor.

t = Time (seconds).

α = Angular position of the rotor. ($\alpha = O$ is taken to be the position giving zero generated voltage).

U = Stored magnetic energy of a circuit or system of circuits.

I_f = Value of field current before the short circuit.

I_A = Normal rated r. m. s. value of armature current.

i_a, i_b, etc.—Denote the current in the circuit indicated by the subscript at any time t during the transient.

i_a', i_b', etc.—In finding the stored energy, the current is assumed to vary from O to i_a or O to i_b. The symbol i' is used to distinguish a value of current during the assumed variation from an actual value i during the short circuit.

ϕ_a, ϕ_b, etc.—Denote the resultant flux interlinkages of the phase denoted by the subscript.

e_a, e_b, etc.—Denote the voltage generated during normal operation in the phase indicated by the subscript.

E_A = Rated r. m. s. voltage per phase (*i. e.*, terminal to neutral).

e_m = Open circuit peak voltage per phase (*i. e.*, terminal to neutral).

L_a, L_f, etc.—Denote the self inductance of the circuit indicated by the subscript.

M_{ab}, M_{bf}, etc.—Denote the mutual inductance between the two circuits indicated by the subscripts.

M_{AF} = Mutual inductance between phase a and the field winding at $\alpha = 0$.

N_a, N_b, etc.—Number of turns in the circuit indicated by the subscript.

n = Any integer.

$$K = \frac{M_{AF}}{\sqrt{L_a L_f}}$$

Appendix II

Calculation of Short Torque in an Ideal Machine. A machine to which the following assumptions apply is referred to as an ideal machine:

1. Resistance and saturation are negligible.
2. The self inductance of each winding is constant.
3. The mutual inductance between the field and each armature winding varies as the cosine of the angle between the field and armature winding considered.

For a Single-Phase Short Circuit. From Equation (36) in *Mechanical Forces Between Electric Circuits* by Karapetoff[2]

$$U = \frac{1}{2}\left[i_a^2 L_a + 2\, i_a i_f M_{fa} + i_f^2 L_f\right]$$

From Equation (4)

$$T = \frac{d\,U}{d\,\alpha}$$

Under the actual condition of a short circuit, the flux is substantially constant and the current varies. The solution of this case is complicated mathematically. However, since the forces at any instant depend only on the fluxes and currents at that instant, the torque can be determined by the given currents and fluxes, but considering the current held constant for a differential movement of the rotor, *i. e.*, determine the derivative of U with respect to α when i is constant.

$$\frac{d\,U}{d\,\alpha} = \frac{1}{2}\left[i_a^2 \frac{d\,L_a}{d\,\alpha} + 2\, i_a i_f \frac{d\,M_{af}}{d\,\alpha} + i_f^2 \frac{d\,L_f}{d\,\alpha}\right] \tag{5}$$

If L_a and L_f are constant, this reduces to

$$\frac{d\,U}{d\,\alpha} = i_a i_f \frac{d\,M_{af}}{d\,\alpha} \tag{6}$$

Since the effect of saturation is neglected, $\frac{d\,M_{af}}{d\,\alpha}$ can be determined from the no-load operation of the machine. Then

$$e_a = N_a \frac{d\,\phi_a}{d\,t} = e_m \sin\alpha \tag{7}$$

Where e_a is the voltage generated and is assumed to be sinusoidal, but during no-load operation,

$$N_a \phi_a = I_f M_{af} \tag{8}$$

Where I_f denotes the steady-state or open-circuit value of field current.

Differentiating (8)

$$N_a \frac{d\,\phi_a}{d\,t} = I_f \frac{d\,M_{af}}{d\,t} \tag{9}$$

substituting this value in Equation (7)

$$e_a = e_m \sin\alpha = I_f \frac{d\,M_{af}}{d\,t}$$

or,

$$\frac{d\,M_{af}}{d\,\alpha} = \frac{e_m \sin\alpha}{I_f\,.\,\omega} = \frac{e_a}{I_f\,.\,\omega} \tag{10}$$

Substituting in (6),

$$\frac{d\,U}{d\,\alpha} = T = \frac{i_f}{I_f\,.\,\omega}\,.\,i_a\,.\,e_m\,.\,\sin\alpha \tag{11}$$

Or, expressed in words, the torque is equal to the product of instantaneous value of armature current by the voltage that would have been generated under open-circuit conditions and this multiplied by the ratio of the instantaneous field current to the value of field current before the short circuit, and divided by the angular velocity.

B. Polyphase Short Circuit. For a polyphase short circuit, the equations for torque can be derived in a similar manner. From Equation (36) as given by Karapetoff[2]

$$U = \sum_1^n{}_u \frac{i_u}{2} \sum_1^n{}_v i_v M_{vu}$$

Therefore, differentiating with i constant

$$\frac{d\,U}{d\,\alpha} = \sum_1^n{}_u \frac{i_u}{2} \sum_1^n{}_v i_v \frac{d\,M_{vu}}{d\,\alpha} \tag{12}$$

For a three-phase short circuit and any given values of currents, constant self-inductance in each phase and constant mutual inductance between phases, but with the mutual inductance between the field and each phase variable

$$\frac{d\,U}{d\,\alpha} = i_a i_f \frac{d\,M_{af}}{d\,\alpha} + i_b i_f \frac{d\,M_{bf}}{d\,\alpha} + i_c i_f \frac{d\,M_{cf}}{d\,\alpha} \tag{13}$$

From Equation (10)

$$\frac{d\,M_{af}}{d\,\alpha} = \frac{e_m}{I_f\,.\,\omega}\sin\alpha = \frac{e_a}{\omega\,.\,I_f}$$

In a similar manner

$$\frac{d\,M_{bf}}{d\,\alpha} = \frac{e_b}{\omega\,.\,I_f}$$

$$\frac{d\,M_{cf}}{d\,\alpha} = \frac{e_c}{\omega\,.\,I_f}$$

Substituting these values in Equation (13).

$$\frac{d\,U}{d\,\alpha} = T = \left[i_a e_a + i_b e_b + i_c e_c\right]\frac{i_f}{\omega\,.\,I_f} \tag{14}$$

Appendix III

In Appendix II, Equations (11) and (14) were derived giving the torque in an "ideal machine" in terms of the currents and rotor position. R. F. Franklin[2] has derived equations for the current as a function of rotor position for the same "ideal machine." These values of current will be substituted in Equation (11) and Equation (14) to compare the torque for single-phase and three-phase short circuits.

Equations (7) and (8) for terminal-to-neutral single-phase short circuit in Franklin's paper are:

$$i_a = I_f \frac{M_{AF}}{L_a} \cdot \frac{1 - \cos\alpha}{1 - K^2 \cos^2\alpha} \tag{15}$$

$$i_f = I_f \frac{1 - K^2 \cos\alpha}{1 - K^2 \cos^2\alpha} \tag{16}$$

Substituting these values in Equation (11),

$$T = \frac{e_m I_f M_{AF} \sin\alpha}{\omega L_a} \cdot \frac{1 + K^2 \cos^2\alpha - (1 + K^2)\cos\alpha}{[1 - K^2 \cos^2\alpha]^2} \tag{17}$$

For the purpose of plotting the instantaneous short-circuit torque in terms of normal rated torque, it will be assumed that the machine is rated so that the peak value of unsymmetrical terminal-to-terminal (star-connected) short-circuit current is twelve times the rated r. m. s. current per phase. It will be assumed that

$$K = \frac{M_{AF}}{\sqrt{L_a L_f}} = 0.85$$

And that $L_f = L_a$.

Then, from Equation (41) in Franklin's paper:

$$i_{u\,(max.)} = 3.53\, I_f \tag{18}$$

or, $I_a = 0.294\, I_f$

But the normal or rated torque is given by

$$\text{Rated power} = \omega T_N = 3 E_A I_A = 2.13\, e_m I_A = 0.626\, e_m I_f \tag{19}$$

Where E_A is the rated r. m. s. phase voltage (terminal to neutral) and I_A the rated r. m. s. armature current per phase.

Substituting (19) in (17) gives for a single-phase terminal to neutral short circuit

$$\frac{T}{T_N} = \frac{K^2 L_f}{0.626 M_{AF}} \frac{[1 + K^2 \cos^2\alpha] - [1 + K^2]\cos\alpha}{[1 - K^2 \cos^2\alpha]^2} \sin\alpha \tag{20}$$

This is plotted in curve A of Fig. 3 for the values of $K = 0.85$ and $M_{AF}/L_f = 0.85$ as assumed above.

The torque for a three-phase short circuit is given by Equation (14) as

$$\omega T = \frac{i_f}{I_f} [i_a e_a + i_b e_b + i_c e_c]$$

Since R. F. Franklin[2] used the same assumptions as were used in deriving Equation (14) his values for currents will be used and are as follows (Equations (71), (72), (73), and (74) in his paper):

$$i_f = I_f \frac{1 - K^2 \cos\alpha}{[1 - K^2]} \tag{21}$$

$$i_a = \frac{2}{3} I_f \frac{M_{AF}}{L_a} \frac{(1 - 0.5 K^2) - \cos\alpha + 0.5 K^2 \cos 2\alpha}{1 - K^2} \tag{22}$$

$$i_b = \frac{1}{-3} I_f \frac{M_{AF}}{L_a} \frac{(1 - 0.5 K^2) + 2\cos(\alpha - 120) - K^2 \cos 2(\alpha + 120)}{1 - K^2} \tag{23}$$

$$i_c = \frac{1}{-3} I_f \frac{M_{AF}}{L_a} \frac{(1 - 0.05 K^2) + 2\cos(\alpha + 120) - K^2 \cos 2(\alpha - 120)}{1 - K^2} \tag{24}$$

The corresponding values of e_a, e_b, and e_c are

$$e_a = e_m \sin\alpha \tag{25}$$

$$e_b = e_m \sin(\alpha - 120) \tag{26}$$

$$e_c = e_m \sin(\alpha + 120) \tag{27}$$

Substituting these values in Equation (14) gives

$$\omega T = \frac{I_f}{3} \frac{M_{AF}}{L_a} e_m \frac{(1 - K^2 \cos\alpha)}{(1 - K^2)^2} \{[1 - 0.5 K^2]$$
$$[2\sin\alpha - \sin(\alpha - 120) - \sin(\alpha + 120)] - 2[\cos\alpha \sin\alpha$$
$$+ \cos(\alpha - 120)\sin(\alpha - 120) + \cos(\alpha + 120)\sin(\alpha + 120)]$$
$$+ K^2 [\cos 2\alpha \sin\alpha + \cos 2(\alpha + 120)\sin(\alpha - 120)$$
$$+ \cos 2(\alpha - 120)\sin(\alpha + 120)] \tag{28}$$

But combining terms and simplifying,

$$2\sin\alpha - \sin(\alpha - 120) - \sin(\alpha + 120) = 3\sin\alpha \tag{29}$$

and,

$$\cos\alpha \sin\alpha + \cos(\alpha - 120)\sin(\alpha - 120)$$
$$+ \cos(\alpha + 120)\sin(\alpha + 120) = 0 \tag{30}$$

and,

$$K^2 [\cos 2\alpha \sin\alpha + \cos 2(\alpha + 120)\sin(\alpha - 120)$$
$$+ \cos 2(\alpha - 120)\sin(\alpha + 120)] = -\frac{3}{2} K^2 \sin\alpha \tag{31}$$

Substituting Equations (53), (54), and (55) in (52) gives

$$\omega T = \frac{I_f}{3} \frac{M_{AF}}{L_a} e_m \frac{1 - K^2 \cos\alpha}{(1 - K^2)^3} [(1 - K^2)\, 3\sin\alpha] \tag{32}$$

Substituting the values of e_m from Equation (19) gives

$$\frac{T}{T_N} = \frac{M_{AF}}{0.626\, L_a} \frac{1 - K^2 \cos\alpha}{1 - K^2} \cdot \sin\alpha \tag{32}$$

This is plotted as curve B of Fig. 3 to compare the torque from three-phase terminal to terminal and single-phase terminal to neutral short circuits.

Appendix IV

Semi-Graphical Calculation of Torque

1. Neglecting Resistance. The general method has been discussed and this appendix will merely give an actual example as an illustration. The current values for this example are taken from the oscillogram shown in Fig. 1. The generated voltage corresponding to the initial field current was 3240 volts (r. m. s.) at 60-

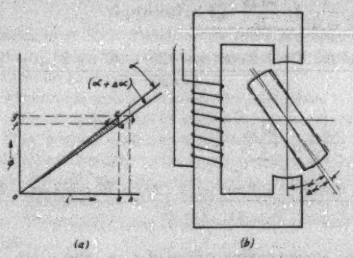

Fig. 8—Differential Change in Stored Energy of Magnetic Circuit

cycle frequency. The flux interlinkages of the armature can be determined from this voltage,

$$e_{(volts)} = N \frac{d\phi}{dt} \times 10^{-8} \qquad (34)$$

but $e = e_m \sin \omega t = 3240 \times 1.41 \sin \omega t$.

Substituting in (34) and integrating

$$-\frac{3240 \times 1.41}{w} \cos w t = N \phi \times 10^{-8}$$

at $w t = 0$ $\qquad N_a \phi_a \times 10^{-8} = 12.1$

The initial interlinkages of the field winding calculated correcting for the ratio of the number of turns in the two windings and estimating the leakage flux gives $N_f \phi_f \times 10^{-8} = 37.5$.

These values of flux are used with the current values from the oscillogram to obtain the energy of the magnetic field. The successive values are tabulated in Table I. The calculated torque is plotted as curve *C* in Fig. 6. Then the torque in joules per electrical radian is given by $\frac{\Delta U}{\Delta \alpha}$. Since the generator under consideration is a six-pole machine, the torque in lb-ft. is given by $3 \times 0.7376 \frac{\Delta U}{\Delta \alpha} = 2.21 \frac{\Delta U}{\Delta \alpha}$.

2. Calculation Considering Resistance. The two effects of resistance to be calculated are:

1. The change in flux interlinkages of each circuit, and

2. The power represented by the resistance losses. The change in flux interlinkages is given by

$$R i = e = N \frac{d\phi}{dt} = 10^{-8}$$

or

$$R i \Delta t = 10^{-8} \times N \Delta \phi$$

This is calculated for each increment of time. The initial value of flux has been calculated. Starting with the initial value and subtracting the successive values of decrease in flux gives the flux at any instant which is then multiplied by the corresponding value of current to get the stored energy. The calculation of power represented by resistance loss needs no explanation, except that in these calculations a correction was made for the change in speed of the rotor produced by the torque acting on it.

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Discussion

R. F. Franklin: Mr. Penney's paper reveals another interesting practical application of the constant-leakage theorem which R. E. Doherty called to the attention of the Institute a number of years ago. This theorem has been used extensively in the investigation of short-circuit phenomena in a-c. apparatus. It was this theorem that I used in the investigation of the short-circuit currents of synchronous machines referred to by the author. The ultimate goal of that investigation was a determination of the torque and forces acting during short circuit of synchronous machines, because of the importance of a knowledge of these forces in the design and construction of alternators. The investigation was completed and formulas giving the torque acting under various conditions of short circuit obtained. These formulas are now being used in the design of synchronous machines. The method used in obtaining these formulas has been presented to the Institute in a paper by Doherty and Park, referred to by Mr. Penney in his bibliography, and is essentially the same as that used by Mr. Penney.

An important contribution by the author is the step-by-step calculation of torque from the current oscillograms which takes into consideration the effect of resistance.

May I again emphasize the importance of the constant-leakage theorem in the solution and explanation of short-circuit problems and commend a thorough knowledge of it to all engineers interested in this phase of engineering.

C. J. Fechheimer: Mr. Penney brings forcibly to our attention the importance of the mechanical problems in electrical machinery. Most designers have concerned themselves more

with the electrical problems and have frequently left to the draftsman the important mechanical problems. Fortunately, in manufacturing companies mechanical experts are now coming to the front.

There have been mechanical failures of machines which have not been understood largely because we had no analytical method of understanding or attacking the problems involved. Not very long ago castings were used extensively and such castings occasionally failed, probably largely because the rewere high shrinkage stresses. They were then made heavier, whereas they may have been heavier at the outset than was necessary.

Mr. Penney's able work was not only experimental as was recorded in his earlier paper, but now he has completed the theoretical analysis as well.

In the inclusion of resistance in the solutions, he has made a noteworthy contribution as that is one of the things that makes such calculations difficult.

B. A. Behrend: The contribution of Mr. Doherty which Mr. Franklin mentioned is, in my opinion, one of the outstanding papers on sudden short circuits of electrical machinery. The way in which Mr. Penney has made use of Mr. Doherty's contribution, is another instance of its great value.

A. B. Field: Mr. Penney has pointed out that in discussing these short-circuit torques we must recognize different values of torque in the various parts of the structure; the torque on the stator windings will be greater than the torque upon the shaft, and we have again quite another value of torque upon the end-ring of the rotor in the case of a turbo generator. In some of the early machines running at 3600 rev. per min., we used occasionally to get a journal or coupling extension slightly twisted after a very heavy short circuit. Those machines had a high short-circuit ratio and, as Mr. Penney has pointed out, the corresponding torques would be very great on a single-phase short circuit. It is the practise of some firms to provide against such possibilities by designing the coupling bolts so that they will shear before injury is done to the journal.

Mr. Penney is to be congratulated upon obtaining such close agreement between some of his calculated and experimental results.

G. W. Penney: I was very glad that Mr. Field pointed out that the torque to be resisted by the structural parts of the stator is usually much greater than the torque transmitted by the shaft because the stator is usually quite rigidly mounted. As an illustration of the importance of the rigidity of such parts I should like to mention a case in which the frame-shifting device of a frequency-changer set gave the stator considerable torsional flexibility. In this case the frame-shifting device was supported by structural-steel members and if the force corresponding to full short-circuit torque could act on these structural supports they would be seriously distorted. However, calculations showed that there was sufficient flexibility so that for a short circuit giving a peak value of torque 20 times normal, the maximum torque acting on the frame shifting device was less than 1.5 times normal. In a case such as this it would be quite possible that a careless attempt to strengthen the parts would result in stiffening the parts so that the stresses would be seriously increased rather than decreased. This illustrates the necessity of knowing the torque-time curve and the flexibility of the parts rather than merely designing parts to withstand an arbitrary value of torque.

One other point of importance is that a short circuit at the instant of zero voltage gives the highest torque while if the transient starts at the maximum voltage point the torque is very slight. For this reason the fact that a machine has withstood one short circuit or even several is no proof that it will withstand another unless the voltage at the instant of short circuit is known. For example, in the first tests for measuring this torque no device for controlling the instant of short circuit was used and twenty tests were made before one was obtained within 30 degrees of the instant of zero voltage, although of course by the theory of probability it should have occurred much sooner.

Analytical Determination of Magnetic Fields

Simple Cases of Conductors in Slots

BY B. L. ROBERTSON* and I. A. TERRY†

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*In this paper La Place's and Poisson's equations are applied to cases of current carrying conductors in rectangular slots to show the flux distributions which obtain.*

The paper first treats with the general case which is a single conductor, completely surrounded by insulation, at the bottom of a slot. Next is taken up the more practical case of a slot containing two insulated conductors, one above the other, in which currents of equal magnitude are considered first to flow in the same direction, and then in opposite directions.

In addition to these analyses, a few special cases are discussed. Methods are also discussed by which the flux distributions in slots containing an even number of coil sides may be obtained, or in slots the conductors of which carry currents not in time-phase. Equations are developed from which all of these fields can be calculated.

A discussion is included to show the distribution of flux on the assumption that the lines go straight across the slot, and a comparison is made between the slot inductance determined mathematically from the developed equations and by the usual design formulas. In the case considered it is found that the design formulas give a value of slot inductance which is about 96 per cent of that obtained by the mathematical treatment which is within limits of engineering accuracy. An expression is developed which shows the error that may be expected in any particular case.

* * * * *

I. Introduction

In the last few years several papers have been presented before the Institute and articles have been published in various technical journals on the theory and practical application of flux plotting. It has been shown that quite accurate flux plots can readily be made free hand for many geometrical figures, if they are void of current, about the only fundamental requirement being a knowledge of the potential difference between the various boundaries of the configuration. Although the problem is greatly complicated in current carrying regions, methods have been outlined for free hand mapping of fields under these conditions, and can be applied very well whenever the general arrangement of the lines of force is known in advance. If the general field arrangement is unknown, then a great deal of sketching and resketching must be done in order to obtain a correct plot, and often much labor can be saved by deriving analytically the equations for the lines of force and then plotting the loci of these equations.

Several years ago W. Rogowski solved La Place's and Poisson's equations for the case of a pancake type transformer, analytically determining the magnetic field distribution about the windings. An excellent translation of Rogowski's work was given by Dr. A. R. Stevenson, Jr.,[1] in 1926. Shortly after that Messrs. Stevenson and Park[2] showed the application of the analytical method of determining the flux distribution about the field pole of a definite pole machine, including in their discussion several other figures which are of theoretical interest. Cases of conductors in slots have also been treated with,[5] but there the solution results in a series which converges so slowly that the task of plotting the field is laborious.

In this paper analytical expressions are developed for plotting the flux about rectangular conductors in rectangular slots, by use of the method originally employed by Rogowski. Flux plots are shown for a number of different locations of a single conductor in a slot, and for the practical case of two conductors, one above the other in the same slot. The mathematical expressions are in the form of a series that converges very rapidly, thus simplifying the problem of making the plots.

In addition to the above, a comparison is made between the slot inductance determined: (a), by the usual design formulas; (b), by mechanically integrating the mathematical flux plot to obtain the flux linkages, and (c), directly from the R-equations.

From the latter a simple expression is obtained for use in design formulas to give the increase in inductance due to the curvature of the flux lines over the inductance determined on the usual assumption that the lines of force are everywhere parallel to the bottom of the slot.

II. Fundamental Equations

Consider the general example shown in Fig. 1, which represents an insulated rectangular conductor at the bottom of an infinitely deep slot. It is assumed that:

1. The permeability of the air, insulation, and copper is unity.

2. The permeability of the iron is infinite. (These two assumptions mean that the flux must enter the iron at right angles to the surface.)

3. The conductors are subdivided, thus eliminating skin effects and resulting in uniform current density.

Rogowski developed the conception of dividing the slot into regions satisfying either LaPlace's or Poisson's equations; then, by considering each region separately, the current distribution may be represented by a single dimension Fourier series, and the problem greatly

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1. For references see Bibliography.

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29-82

simplified.* The single dimensional Fourier series may be chosen to represent the current in the one slot only, but another simplification is made possible from the application of the method of images.

It is well known that with a conductor near an iron surface of infinite permeability it is possible to replace the iron surface by a conductor, called the image of the first, carrying current in the same direction. Similarly, a slot may be replaced by its image on both sides and below, see Fig. 2, the first group of images being

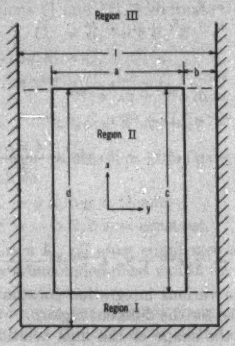

Fig. 1—General Case of Conductor Completely Insulated from Slot

reimaged, etc. Due to the fact that the slot is open at the top only a double row of images exists, which then replaces the original conductor and slot, and gives rise to a periodic rectangular current distribution, the Fourier series of which can readily be obtained.

The total field of the slot of Fig. 1 is divided into three distinct regions:

Region I. Air district between the images formed by the bottom of the slots and the bottom of the conductors.

Region II. District occupied by the conductors in a horizontal row including the air or insulation spaces between the conductors.

Region III. The air and insulation space above the conductors.

Region I, having no currents, satisfies LaPlace's equation,

$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} = 0 \qquad (1)$$

Region II, being a district of currents, must satisfy Poisson's equation,

$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} = -4\pi I \qquad (2)$$

*Herein lies the main difference between the present analysis and that of E. Roth, reference 5. He used a two dimensional Fourier Series.

Equations (1) and (2) are the fundamental differential equations for magnetic fields. R is the vector potential of the magnetic field and is defined by the relations,

$$\left.\begin{aligned} X &= \frac{\partial R}{\partial y} \\ Y &= \frac{-\partial R}{\partial x} \end{aligned}\right\} \qquad (3)$$

X and Y being the x and y components of the magnetic field intensity in the slot.

As pointed out in Stevenson's translation, the slope of a line of force is

$$\frac{dy}{dx} = \frac{Y}{X} = \frac{-\dfrac{\partial R}{\partial x}}{\dfrac{\partial R}{\partial y}} \qquad (4)$$

and hence lines of force and corresponding lines representing constant values of R have the same slope at every point. Therefore, by solving Equations (1) and (2) for R in terms of the coordinates x and y, it is possible to plot the flux field because lines of constant R become lines of force. Differences in R are differences in flux, the numerical value between two lines of R representing the flux within that region. See Appendix A for a discussion of the properties of R.

The current density I must be represented mathematically. This is done as follows: Commencing at the left hand side of the slot, Fig. 1, and passing to the right, the current density is zero until the conductor is

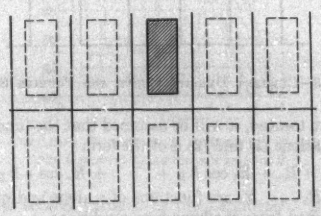

Fig. 2—Images of a Current Carrying Conductor in an Infinitely Deep Slot in Iron

reached. There the current density jumps to a value i determined by the total current in the conductor and the cross-sectional area of the copper. It is assumed that the total current is coming out of the paper towards us, bringing in the negative sign of Equation (2). The current density continues constant to the other side of the conductor, where it drops back to zero, and then continues zero to the other side of the slot. For the slots on either side the same distribution holds. This gives rise to the periodic repeating current distribution

shown in Fig. 3. It may be represented by a Fourier series of the form:

$$I = \frac{1}{2}\,\alpha_0 + \alpha_1 \cos k\,y + \ldots\ldots \alpha_n \cos n\,k\,y \qquad (5)$$

where $k = \dfrac{2\,\pi}{l}$, $l = a + 2\,b$; see Fig. 1.

The value of the coefficient α_n may be determined from the general integral

$$\alpha_n = \frac{2}{l}\int_{-a/2}^{a/2} i \cos n\,k\,y\,d\,y = \frac{4\,i}{n\,k\,l}\,\sin\frac{n\,k\,a}{2} \qquad (6)$$

for, $n = 1, 2, 3 \ldots\ldots\ldots$

For $n = 0$,

$$\alpha_0 = \frac{2}{l}\int_{-a/2}^{a/2} i\,d\,y = \frac{2\,a}{l}\,i \qquad (7)$$

The Fourier series for I then becomes

$$I = \frac{a\,i}{l} + \sum_{n=1}^{n=\infty} \frac{4\,i}{2\,\pi\,n}\,\sin\frac{n\,\pi\,a}{l}\,\cos n\,k\,y \qquad (8)$$

From a general knowledge of the solution of partial differential equations, and the physics of the particular

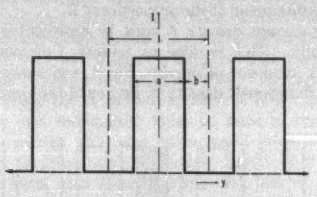

FIG. 3—CURRENT DENSITY CURVE FOR FOURIER SERIES

problem treated, it will be assumed that the function R of Equations (1) and (2) is of the form

$$R = R_0 + R_1 \cos k\,y + \ldots + R_n \cos n\,k\,y \qquad (9)$$

where $R_0 \ldots R_n$ are functions of x alone and must be determined from the boundary conditions.

Substituting for R in Equations (1) and (2) the following sets of relations are obtained,

$$\left.\begin{aligned} &\frac{d^2 R_0}{d\,x^2} = 0 \\ &\frac{d^2 R_1}{d\,x^2} - k^2 R_1 = 0 \\ &\quad\vdots \\ &\frac{d^2 R_n}{d\,x^2} - n^2 k^2 R_n = 0 \end{aligned}\right\} \qquad (1b)$$

$$\left.\begin{aligned} &\frac{d^2 R_0}{d\,x^2} = -2\,\pi\,\alpha_0 = \frac{-4\,\pi\,a\,i}{l} \\ &\frac{d^2 R_1}{d\,x^2} - k^2 R_1 = -4\,\pi\,\alpha_1 \\ &\quad\vdots \\ &\frac{d^2 R_n}{d\,x^2} - n^2 k^2 R_n = -4\,\pi\,\alpha_n \end{aligned}\right\} \qquad (2b)$$

These equations have the solutions,

$$\left.\begin{aligned} &R_0 = \beta_0 + \beta_0'\,x \\ &R_1 = \beta_1\,\epsilon^{kx} + \beta_1'\,\epsilon^{-kx} \\ &\quad\vdots \\ &R_n = \beta_n\,\epsilon^{nkx} + \beta_n'\,\epsilon^{-nkx} \end{aligned}\right\} \qquad (1c)$$

$$\left.\begin{aligned} &R_0 = \gamma_0 + \gamma_0'\,x + \gamma_0''\,x^2 \\ &R_1 = \gamma_1\,\epsilon^{kx} + \gamma_1'\,\epsilon^{-kx} + \frac{4\,\pi\,\alpha_1}{k^2} \\ &\quad\vdots \\ &R_n = \gamma_n\,\epsilon^{nkx} + \gamma_n'\,\epsilon^{-nkx} + \frac{4\,\pi\,\alpha_n}{n^2\,k^2} \end{aligned}\right\} \qquad (2c)$$

of which the various integration constants must now be determined for the different regions.

Knowing the shape of the final solution, Equation (9), there are, for the three regions

Region I:

$$R = A_0 + A_0'\,x + (A_1\,\epsilon^{kx} + A_1'\,\epsilon^{-kx})\cos k\,y + \ldots\ldots + (A_n\,\epsilon^{nkx} + A_n'\,\epsilon^{-nkx})\cos n\,k\,y \qquad (10)$$

Region II:

$$R = B_0 + B_0'\,x + B_0''\,x^2 + \left(B_1\,\epsilon^{kx} + B_1'\,\epsilon^{-kx} + \frac{4\,\pi\,\alpha_1}{k^2}\right)\cos k\,y + \ldots + \left(B_n\,\epsilon^{nkx} + B_n'\,\epsilon^{-nkx} + \frac{4\,\pi\,\alpha_n}{n^2\,k^2}\right)\cos n\,k\,y \qquad (11)$$

Region III:

$$R = C_0 + C_0'\,x + (C_1\,\epsilon^{kx} + C_1'\,\epsilon^{-kx})\cos k\,y + \ldots + (C_n\,\epsilon^{nkx} + C_n'\,\epsilon^{-nkx})\cos n\,k\,y \qquad (12)$$

III. DETERMINATION OF THE CONSTANTS

The above expressions form the general solution of the problem. It is now necessary to evaluate the constants from the boundary conditions. These are:

(a) At the bottom of the slot, coordinate $x = O$, $Y = -\,\partial\,R/\partial\,x = O$, as the flux must enter the iron at right angles.

Obtaining Y from Equation (10) and equating coefficients,

$$A_0 = 0,\ A_1 = A_1',\ \ldots\ldots A_n = A_n'$$

(b) At the boundary between Regions I and II, coordinate $x = d - c$, $X = \partial\,R/\partial\,y$, and $Y = -\,\partial\,R/\partial\,x$ are continuous since there is no refraction. Also at this boundary the constants of Equations (10) and (11) must be equal.

Therefore,

$$B_0' = -2 B_0'' (d - c)$$

$$A_0 = B_0 + B_0' (d - c) + B_0'' (d - c)^2$$

$$A_n [\epsilon^{nk(d-c)} - \epsilon^{-nk(d-c)}] = B_n' \epsilon^{nk(d-c)} - B_n' \epsilon^{-nk(d-c)}]$$

and

$$A_n[\epsilon^{nk(d-c)} + \epsilon^{-nk(d-c)}] = B_n \epsilon^{nk(d-c)} + B_n' \epsilon^{-nk(d-c)} + \frac{4 \pi \alpha_n}{n^2 k^2}$$

(c) Relations similar to (b) hold at the boundary between Regions II and III, coordinate $x = d$, giving

$$C_0' = B_0' + 2 B_0'' d$$

$$B_0 + B_0' d + B_0'' d^2 = C_0 + C_0' d$$

$$B_n \epsilon^{nkd} - B_n' \epsilon^{-nkd} = C_n \epsilon^{nkd} - C_n' \epsilon^{-nkd}$$

$$B_n \epsilon^{nkd} + B_n' \epsilon^{-nkd} = C_n \epsilon^{nkd} + C_n' \epsilon^{-nkd} - \frac{4 \pi \alpha_n}{n^2 k^2}$$

(d) For large values of x the flux goes straight across the slot, so

$$X = \partial R / \partial y = O, \text{ and}$$

$$Y = -\partial R / \partial x = \text{constant}.$$

Then $C_1, C_2, \ldots C_n$ all must equal zero as the terms in which they are included must vanish for large values of x.

(e) The terms in the regional equations that contain $\cos n k y$ give the curvature to the lines of flux. The other terms are the only ones extant at very large values of x, so must be the flux on the assumption that the lines of force go straight across the slot. So at the top of the conductor, $x = d$, from Equation (2b), Appendix B

$$\pi \alpha_0 c^2 = C_0 + C_0' d$$

Solving these groups of relations simultaneously, all constants but A_0 are readily determined. If the absolute value of R were known at any point it would be possible to evaluate A_0, but since R is not known in any region A_0 can only be given an assumed value, say zero. This is justifiable since it is the differences of R which are desired primarily.

The constants thus become:

$$A_0 = 0 \text{ (assumed)}$$

$$A_0' = 0$$

$$\left.\begin{aligned}
A_n &= A_n' = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d-c)} - \epsilon^{-nkd}] \\
B_0 &= -\pi \alpha_0 (d - c)^2 \\
B_0' &= 2 \pi \alpha_0 (d - c) \\
B_0'' &= -\pi \alpha_0 \\
B_n &= \frac{-2 \pi \alpha_n}{n^2 k^2} \epsilon^{-nkd} \\
B_n' &= \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d-c)} - \epsilon^{-nkd} - \epsilon^{nk(d-c)}] \\
C_0 &= \pi \alpha_0 (2 c d - c^2) \\
C_0' &= -2 \pi \alpha_0 c \\
C_n &= 0 \\
C_n' &= \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} + \epsilon^{nkd} - \epsilon^{-nkd}]
\end{aligned}\right\} \quad (13)$$

The regional equations are obtained by substituting these constants in Equations (10), (11), and (12).

Region I:

$$R = \sum_{n=1}^{n=\infty} A_n (\epsilon^{nkx} + \epsilon^{-nkx}) \cos n k y \quad (14)$$

Region II:

$$R = -\pi \alpha_0 (d - c)^2 + 2 \pi \alpha_0 (d - c) x - \pi \alpha_0 x^2 + \sum_{n=1}^{n=\infty} \left(B_n \epsilon^{nkx} + B_n' \epsilon^{-nkx} + \frac{4 \pi \alpha_n}{n^2 k^2} \right) \cos n k y \quad (15)$$

Region III:

$$R = \pi \alpha_0 (2 c d - c^2) - 2 \pi \alpha_0 c x + \sum_{n=1}^{n=\infty} C_n' \epsilon^{-nkx} \cos n k y \quad (16)$$

IV. Plotting the Field

With R a function of both x and y, it is convenient to first draw curves of R as a function of either variable

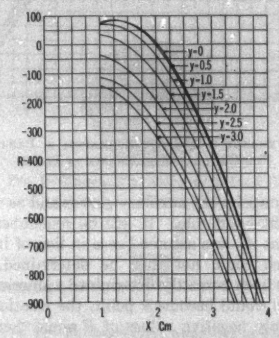

Fig. 4—Auxiliary Curves of R Against x, Parameter y for General Case Shown in Fig. 6

with the second variable as the parameter. In some cases, for instance plotting the field in detail about the kernel, or point upon which all flux lines apparently collapse, both sets of parametric curves are required for accuracy. The general example illustrated in this paper makes use of Figs. 4 and 5 in which R is plotted as a function of x for constant values of y.

From these auxiliary curves, lines of R = a constant are chosen and the various coordinates then easily obtained from the graphs. The final plot of the magnetic field about the conductor insulated from the slot on all sides is given by Fig. 6. It was not found necessary to go above the second harmonic in order to secure fairly accurate calculations. This greatly simplified the numerical work. The following data, including the

dimensions of the slot and conductor, accompany the map.

$a = 4$ cm., $b = 1$ cm., $c = 6$ cm., $d = 7$ cm., $i = 30$ abamperes per sq. cm.

In reference to the auxiliary curves of Figs. 4 and 5 a few interesting points should be noted. These graphs show both positive and negative values of R. This does not indicate that flux is going in two directions about the kernel, for the differences in R are always negative, the direction of the flux being counter clockwise, as shown by the auxiliary curves and as reasoned by the right hand rule. It will be remembered that the constant A_0 was assumed zero in lieu of its actual value which could not then be determined. If A_0 could have been correctly determined numerically, the $R = O$ line would become a point, the location of the kernel. Only negative values of R would then appear on the auxiliary curves.

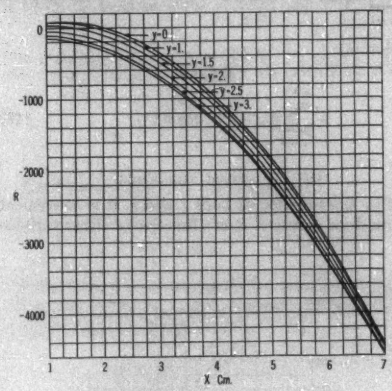

Fig. 5—Auxiliary Curves of R Against x, Parameter y, For General Case Shown in Fig. 6

The final graph, Fig. 6, shows values of flux which are negative only, the $R = O$ line being purposely pushed back to the kernel by adding negative flux of amount equal to the positive flux which was originally found.

It will also be noted from Fig. 5 that the curves tend to merge into a single line. This means that the lines of force become straighter as x increases. Theoretically, lines of R would cross the slot straight at infinity, but practically their curvature can not be noticed much above the top of the conductor.

It is good practise always to make a rough check on any problem when possible. In this particular case a check is made on the density of the flux crossing the slot a little above the conductor. The total current in the slot is 72 abamperes which gives an m. m. f. of 288 gilberts. The path reluctance per square centimeter is 6, hence the flux crossing the slot is 151 lines per sq. cm. On the graph this value is obtained and the accuracy of the work is thus determined as far as flux density is concerned.

Likewise, a check is made on the actual total flux crossing the slot, the numbers to the right of Fig. 6 indicating the flux up to that point calculated on the assumption that the lines go straight across the slot. It is distinctly noticeable that such an approximation gives less flux near the bottom of the slot than is obtained by the more correct method. To compensate for the increased flux at the bottom of the slot when determined by Rogowski's method, there is, however, a greater density in lines in the other case as one moves up the slot. Near the top of the conductor, the difference in total flux calculated in the two manners is very small, and directly above the conductor the difference is negligible. As previously pointed out, the two lines of R equal to the same definite value tend to coincide, and on this basis the application of Rogowski to slots of finite depth is justified.

The method of calculating the flux upon the assumption that the lines have no curvature is contained in Appendix B.

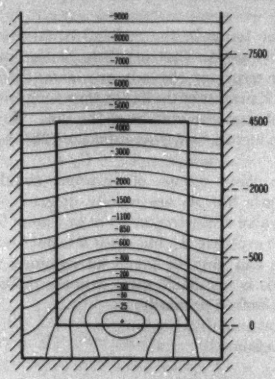

Fig. 6—Flux about Conductor of General Case

V. Special Cases

The foregoing treatment considers the purely general case in which the conductor does not touch the slot at any point. The kernel is therefore inside the copper, as shown by Fig. 6, and as would be definitely determined by free hand graphical methods of plotting the magnetic field. The higher the conductor in the slot, the nearer does the kernel approach the geometric center of the conductor, the limit being the instance in which the slot is without a bottom.

Conversely, the nearer the copper approaches the bottom of the slot, with equal air-gaps between the sides of the conductor and the slot walls, the nearer

does the kernel approach the middle of the lower face of the conductor. This limiting case is shown in Fig. 7. The general equations of R, and the auxiliary curves from which these flux lines were plotted, are contained in Appendix C to which the reader is referred.

In calculating the flux upon the assumption that it goes straight across the slot, the height and total current of the conductor are the major factors concerning the

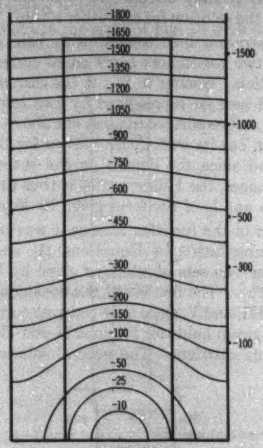

Fig. 7—Flux Field about a Conductor Which Touches the Bottom but Not the Sides of the Slot

conductor itself. Of course, the current density depends upon both of the copper dimensions, but the width of the copper for a given total current has no effect upon the flux distribution, as it distinctly has in the two previous cases.

The conductor then might just as well extend clear across the slot, see Fig. 8. Of interest here is the fact that to leave no air space between the sides of the copper and the iron seemingly shifts the kernel from a point within the current carrying region to infinity below the conductor. No matter how small the air space may be, just as long as it is not zero, the kernel is still on the plot. But when the air space becomes zero, the kernel loses its identity on the field map.

Dr. Stevenson has suggested a simple figure to show physically why this may be true. The scheme has practical value as well as academic interest for it can be applied in other cases to determine the location of the kernel. It is therefore included in this paper.

Fig. 9 shows an iron cylinder enclosed by a concentric current carrying conductor. The lines of no work all converge at the center of the two cylinders. Now consider the sector between the two lines of no work given in the sketch. By the method of images, these lines of no work may be replaced by an iron surface, thus excluding the rest of the current region, Fig. 10 resulting. Since the kernel is at the center of the circular sector, if the radius is made infinite, the kernel moves to infinity, giving the slot and conductor of Fig. 11. This is exactly Fig. 8.

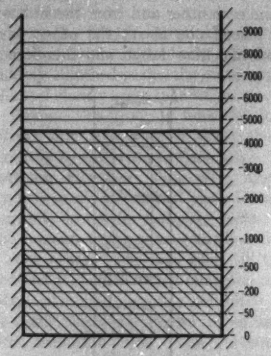

Fig. 8—Field about Conductor Completely Filling Slot

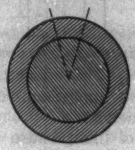

Fig. 9—Copper Cylinder with Iron Core

Fig. 10—Equipotential Surfaces of Fig. 9

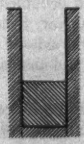

Fig. 11—Modification of Fig. 10

The conductor under discussion is contained in what was formerly Region II. It is then noted, that substituting the condition of $d = c$ in the terms of Equation (11), only one term remains, *i e.*, $B_0'' x^2$ which is $-\pi \alpha_0 x^2$. This result definitely states that flux passes across the slot only in lines parallel to the lower face of the conductors, mathematically showing that the kernel is at infinity. Although this point may appear

academic, it is nevertheless of interest for the proper location of a kernel in a field plot is often a problem of importance.

VI. Slot with Two Conductors

The slot containing two conductors, completely insulated from each other and from the iron walls of the slot, and placed one above the other, is typical of synchronous machine design and therefore forms one

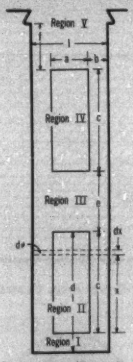

Fig. 12—Slot with Two Conductors

of the most practical applications of flux plotting by the mathematical method.

In the solution of this problem the entire field is divided into five regions, see Fig. 12. As before,

Region I: Air district between the images formed by the bottom of the slots and the bottom of the lower conductors.

Region II: District occupied by the lower conductors in a horizontal row, including the air or insulation spaces between the sides of the lower conductors.

Region III: The air and insulation space between the upper and lower conductors.

Region IV: District occupied by the upper conductors in a horizontal row, including the air or insulation spaces between the sides of these conductors.

Region V: The air and insulation space above the upper conductors.

The air regions satisfy La Place's equation, Equation (**1**), and the current carrying regions satisfy Poisson's equation, Equation (**2**).

In the general case previously discussed, it is obvious that the magnetic field for the single conductor in any certain configuration will always yield one type of plot. Its form will be altered only by slot and copper dimensions, but will not be affected by the magnitude of the current. The direction of flow of the current will, of course, change the direction of the lines of flux.

The solution of this problem, however, brings into consideration the direction of the flow of current in the conductors, that is, whether or not the current in the upper coil side is flowing in the same direction as the current in the lower coil side. Furthermore, the relative magnitudes of these currents are important, because for each ratio of the two densities there exists a distinct field. An infinity of flux plots is thus the complete answer to the problem. Although any one of these plots can be accurately determined, it is only within the scope of this paper to fully discuss two important types and to indicate the solution for any other set-up.

The illustration first treated with here is that in which the two conductors carry equal currents in the same direction. Similar to Fig. 2, the current carrying regions will now be represented by two infinite series of reflected and rereflected images, one above the other. Each region has its own Fourier series for the current density, and since the density is the same for each group of images, the Fourier series is thus the same for each region and is of the form given by Equation (**8**).

The form of the function R is as it was before, and from its substitution in Equations (**1**) and (**2**) are obtained the two sets of relations given by Equations (**1b**) and (**2b**). The first set of relations now holds for Regions I, III, and V, since they carry no current, while the second group holds for Regions II and IV, the current carrying districts. The general solutions for R,

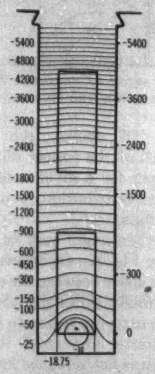

Fig. 13—Field about a Two-Conductor Slot

Currents in the conductors equal and in the same direction

and the constants for these equations, are obtained in like manner to the ones detailed in Section II of the paper. It is necessary now to solve simultaneously the five sets of regional solutions in order to determine the coefficients A_0, A_n, . . . etc. The reader is referred to Appendix D for the complete mathematical analysis.

It is interesting to note that only one kernel appears in Fig. 13. But it is also noticeable that the flux through the lower part of the upper conductor shows considerable curvature, indicating a disturbance which is caused

by the kernel that conductor would have if acting alone. The total m. m. f. of the lower conductor is acting to send flux in a counterclockwise direction across the slot, while near the lower face of the upper conductor the m. m. f. of this current acts to send flux in a clockwise direction across the slot. The latter m. m. f., however, is very weak within this small interval, and the stronger m. m. f. predominates to the extent that any second kernel is wiped out.

The numbers to the right of Fig. 13 were obtained by

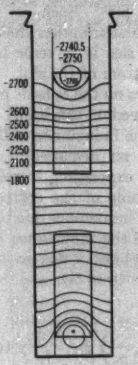

FIG. 14—FIELD ABOUT A TWO-CONDUCTOR SLOT
Currents in the conductors equal but in opposite directions

calculating the flux on the basis that it crosses the slot in straight lines. It makes little difference in the values of total flux above the lower conductor, as indicated by the results.

The case in which the conductors carry equal currents but flowing in the opposite direction is considered next. Whereas both currents in the previous illustration were flowing in a negative direction, that is, up out of the figure, the upper current will now be assumed to flow positively. Within this reversed current district, Region IV, Equation (2) must be altered to read,

$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} = +4\pi I \qquad (17)$$

if I, the current density expressed as a Fourier series, is to be the same as it was before. If, however, I is changed in sign then Equation (2) still will hold. The final regional expression for the upper conductor contains a change of sign before the term including α_n. Such a condition calls for a new determination of all of the coefficients in the regional equations because these relations must be solved simultaneously. The analysis for this case is given in Appendix E.

Fig. 14 illustrates the field about the same two conductors of Fig. 12, the upper current now being reversed. At the bottom of the slot and well up the sides, the flux plot is practically dependent only on the lower conductor because of the extremely weak field in that region produced by the reversed current above. Within the upper copper, however, the lines rapidly change their shape, eventually closing within the slot about a second kernel.

Fig. 15 shows the field in detail about the second kernel. Immediately about the kernel the flux travels in approximately elliptical paths, the outer approaching the slot walls and closing upon themselves at very great distances. Beyond this group of lines the flux strikes the sides of the slot. These plots verify those determined graphically by Calvert and Harrison.[4]

Flux plots for armature slots of a synchronous machine are necessarily dependent upon instantaneous values of current, except for full-pitch coils. In these instances the currents in the conductors bear a constant relation to each other. The field plot will then have the same conformation although a changing magnitude of total flux. In actual practise full-pitch windings are seldom used, the usual design being somewhere near ⅔ pitch armature coils. The currents in the upper and lower conductors now differ in time-phase, and for each instant of time there corresponds a new flux plot.

The method by which a field map may be made for any ratio of conductor currents is very simply outlined. The only important step is to adjust the signs of the Fourier series for the current densities of Regions II and IV. For currents in the same direction both series will have the same sign which will depend upon the direction of the current flow. For opposite currents the series will then of course have opposite signs. Substitution of these current density forms in the

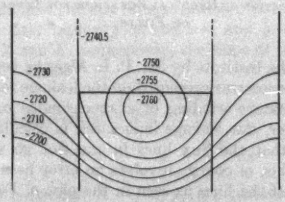

FIG. 15—DETAILED FIELD SHOWING WHIRL OF FLUX ABOUT TOP OF UPPER CONDUCTOR OF FIG. 14

regional equations will automatically care for all algebraic signs which follow.

The regional expressions for R for the cases of two conductors in a slot, one above the other, were obtained by treating the field as a unit. That is, the entire field was divided into five distinct districts, the relations for which were solved simultaneously in order to satisfy the boundary conditions. The resultant R expressions may also be obtained from the equations of the single conductor slot by the principle of superposition. Since

saturation is excluded from the problem, fluxes may be superposed as well as m. m. fs., which was fundamental to the other method.

Fig. 16A shows a slot with only the lower conductor present. The Roman numerals refer to both the regions and their accompanying equations which are those developed in Section II of the paper. Fig. 16B shows only the upper conductor in the slot, the regional expressions for which also are derived in Section II. These last equations are the same as those previously obtained except for a change of slot dimensions. The R equations represent magnetic fields, hence superposition allows of the addition of the sets of regional expressions for the two conductors considered separately. The resulting derivations for the five regions are indicated in Fig. 16C. These two methods have been used in the paper as a check on each other.

VII. Inductance Comparison

One of the most important applications of flux plotting is the determination of the inductance of a current carrying system. To illustrate the method, the inductance of the conductors in the slot under consideration has been determined. In a paper recently pre-

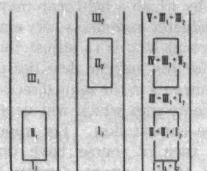

Fig. 16—Addition of Regional Equations for Superposition of Fields

sented to the Institute by Mr. P. L. Alger,[3] a very complete discussion was presented on the leakage reactance of the armature winding of a synchronous machine. In his paper, slot leakage inductance is calculated by assuming that the flux lines in the slot go straight across. This, of course, involves an error because the curvature of the lines as seen in the plot of Fig. 13 is such that a given flux will link more current than if the flux goes straight across the slot; therefore, the exact method will give a larger reactance. However, the amount of error due to the approximation will be seen to be so small as to be negligible. The reader is referred to Appendix F for the derivations of the inductance formulas.

The following tabulation shows the various components of inductance as determined graphically by mechanical integration of the flux plot, and, analytically, under the assumption that the flux lines go straight across the slot. This analysis has been carried only to the bottom of the wedge, Fig. 12.

Region	Inductance in (abhenrys)/N^2	
	Present approximations	Mechanical integration of Fig. 13
II	0.150	0.138
III	0.250	0.250
IV	1.005	0.950
V	0.758	0.753
	2.163	2.091

This comparison shows an increase of approximately 3.5 per cent in the inductance determined from the mathematical flux plot. Such an increase is very reasonable since the assumption that the flux lines go straight across the slot is in error to some extent in regions near the bottom of the slot. This is shown in the figure, the values on the outside of the slot indicating the flux determined on this basis.

In Appendix G the inductance of the two conductor slot is determined mathematically from the vector potential R. The expression therein derived shows directly the magnitude of the correction term which gives the increase of inductance due to the curvature of the flux lines above the inductance calculated on the assumption that the lines of flux go straight across the slot. The correction term becomes, by omission of the insignificant terms

$$L' = \frac{l^3}{c^2\, a^2\, \pi^2}\left(c - \frac{l}{2\,\pi}\right)\sin^2\frac{a\,\pi}{l}\times 10^{-9} \text{ henrys per } N^2 \tag{18}$$

This is simple enough for use in design formula wherever it affects the accuracy to an appreciable extent.

For the slot of Fig. 13

$$L' = 0.087 \times 10^{-9} \text{ henrys per } N^2$$

or an increase over that obtained by Equation (**12f**) of about 4.1 per cent.

This agrees with the increase in inductance obtained from the flux plot, being slightly higher because the mechanical integration of the flux plot is limited to finite increments whereas the mathematical method deals with infinitesimal increments.

VIII. Detailed Plots

The flux plot of Fig. 17 has been made for general interest to show in detail the field about the kernel of a conductor. The distance between the lower face of the conductor and the slot bottom was made great enough to bring out the whirl of flux in a pronounced manner. The calculations were obtained from Equations (**14**) and (**15**) of Section III of the paper, it being assumed that the slot and conductor were infinitely long. Other dimensions are

$a = 0.45$ in., $d = 0.90$ in., $i = 2700$ amperes per sq. in.

The width of the conductor in this particular case was chosen exactly equal to half of the slot width, and it was found that such a ratio of dimensions eliminates all second harmonic functions of the general R equations.

This reduces calculations to some extent. Of further interest is the fact that in Figs. 13 and 17 there is one line which is a continuation of the sides of the conductor below the conductor. Preliminary calculations for the case of $a = 2\,b$ show this line to be $R = O$, but this in turn is only true because of the assumption that $A_0 = O$. The $R = O$ line of Fig. 17 has been pushed back to the kernel as it was in the former case, so as to give actual values of flux as one proceeds outward from the kernel.

Summary

The usual types of rectangular slots met in practise have been considered, and the general method of attack

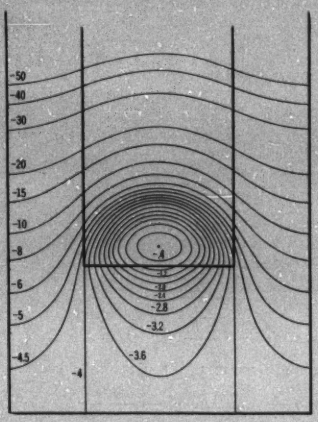

Fig. 17—Detailed Field of a Conductor Showing Whirl of Flux about Kernel

in determining the flux distribution about them has been given in sufficient detail to make its application to other examples quite clear and straightforward. The general equations which have been developed may be applied to any slot by simply substituting for the particular dimensions involved.

The error in inductance calculated on the assumption that the flux tubes go straight across the slot has been shown to be small enough to be negligible in usual slots met in practise, thereby indicating that present assumptions give sufficiently accurate results for engineering purposes.

Acknowledgments

The authors wish to express their appreciation to Mr. L. P. Shildneck for the treatment of Appendix A of the paper, to Dr. E. J. Berg of Union College for his kindness in reviewing the manuscript and offering valuable comments, and to Mr. R. E. Doherty for his helpful suggestions.

Appendix A

Properties of the Vector Potential R

In the second section of the paper, under "fundamental equations," La Place's and Poisson's equations were solved by assuming that a vector potential function R exists such that

$$R = f\,(x, y) \tag{1a}$$

R is the magnitude of the z component of the vector potential of the magnetizing force and satisfies the relations

$$\frac{\partial R}{\partial x} = -\,Y$$

$$\frac{\partial R}{\partial y} = X \tag{2a}$$

x and y are the space coordinates and X and Y are the intensities of the magnetic field in the $x-$ and $y-$ directions, respectively. The form of the function was then determined, and with the solutions for R in the various regions, the flux lines were drawn by plotting curves for R = a constant, increments of R being taken as increments of flux. The relation between the vector potential and flux is not an obvious one to persons unfamiliar with vector analysis, and although some of the results of this Appendix have been given before, this short analysis is included for the sake of clarity. This development is due to Mr. L. P. Shildneck, one of the authors' associates.

In order that the curves of R = a constant shall represent flux lines, it is necessary and sufficient that these curves have: first, the same slope at every point as the flux lines; and second, equal increments of flux enclosed by lines of equal increments of R throughout the entire field.

Lines of R = a constant may be shown to have the same slope as the flux lines as follows: The components of the field intensity in the $x-$ and $y-$ directions being X and Y, respectively, the slope of the flux lines is Y/X. Since

$$R = f\,(x, y)$$

the total differential of R is

$$d\,R = \frac{\partial R}{\partial x}\,dx + \frac{\partial R}{\partial y}\,dy \tag{3a}$$

If the equations $R = f\,(x, y)$ are plotted for R = a constant, then along any curve $d\,R = O$, or

$$\frac{\partial R}{\partial x}\,dx + \frac{\partial R}{\partial y}\,dy = 0 \tag{4a}$$

whence

$$\left(\frac{dy}{dx}\right)_R = \frac{-\dfrac{\partial R}{\partial x}}{\dfrac{\partial R}{\partial y}} = \frac{Y}{X} \tag{5a}$$

Therefore, curves of R = a constant have the same slope as the lines of force.

The following method can be used to show that

equal increments of R give equal increments of flux throughout the field of force. Fig. 18 shows a section of the field $R = f(x, y)$. Let $V = \phi(x, y)$ = a constant be the equipotential lines, which are orthogonal to the flux lines and therefore to the $R = f(x, y)$ = a constant lines.

Similar to Equation (**5a**), the slope of the lines V = a constant is

$$\left(\frac{dy}{dx}\right)_V = \frac{-\dfrac{\partial V}{\partial x}}{\dfrac{\partial V}{\partial y}} \tag{6a}$$

Since these two systems are orthogonal, the slope of the

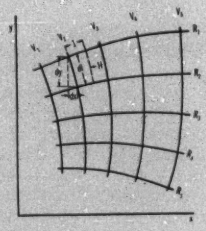

Fig. 18—Equipotential Lines and Flux Lines in a General Field

R curves must equal the negative reciprocal of the slope of the V curves, or

$$\left(\frac{dy}{dx}\right)_R = -\frac{1}{\left(\dfrac{dy}{dx}\right)_V} \tag{7a}$$

so that

$$\frac{\dfrac{\partial R}{\partial x}}{\dfrac{\partial R}{\partial y}} = -\frac{\dfrac{\partial V}{\partial y}}{\dfrac{\partial V}{\partial x}} \tag{8a}$$

At any point, P, in the field, the intensity is given in magnitude by

$$H = (X^2 + Y^2)^{\frac{1}{2}} = \left[\left(\frac{\partial R}{\partial y}\right)^2 + \left(-\frac{\partial R}{\partial x}\right)^2\right]^{\frac{1}{2}} \tag{9a}$$

and the flux density is

$$\mu H = \mu\left[\left(\frac{\partial R}{\partial y}\right)^2 + \left(-\frac{\partial R}{\partial x}\right)^2\right]^{\frac{1}{2}} \tag{10a}$$

Considering an elementary area of unit width and of length in the direction of V = a constant

$$d\,l = [\,(dx)^2 + (dy)^2\,]^{\frac{1}{2}}$$

then the flux enclosed between any two lines of force, $R = R_1$ and $R = R_2$, is

$$\phi_{R2} - \phi_{R1} = \mu \int_{R=R_1}^{R=R_2} \left[\left(\frac{\partial R}{\partial x}\right)^2 + \left(\frac{\partial R}{\partial y}\right)^2\right]^{\frac{1}{2}} \left[(dx)^2 + (dy)^2\right]^{\frac{1}{2}} \tag{11a}$$

Transforming,

$$\left[(dx)^2 + (dy)^2\right]^{\frac{1}{2}} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} dx$$

$$= \left[1 + \left(-\frac{\dfrac{\partial V}{\partial x}}{\dfrac{\partial V}{\partial y}}\right)^2\right]^{\frac{1}{2}} dx = \left[1 + \left(\frac{\dfrac{\partial R}{\partial y}}{\dfrac{\partial R}{\partial x}}\right)^2\right]^{\frac{1}{2}} dx$$

$$= \left[\left(\frac{\partial R}{\partial x}\right)^2 + \left(\frac{\partial R}{\partial y}\right)^2\right]^{\frac{1}{2}} \frac{dx}{\dfrac{\partial R}{\partial x}} \tag{12a}$$

whence

$$\phi_{R2} - \phi_{R1} = \mu \int_{R=R_1}^{R=R_2} \left[\left(\frac{\partial R}{\partial x}\right)^2 + \left(\frac{\partial R}{\partial y}\right)^2\right] \frac{dx}{\dfrac{\partial R}{\partial x}} \tag{13a}$$

The first term in the brackets of Equation (**13a**) becomes directly

$$\left(\frac{\partial R}{\partial x}\right)^2 \frac{dx}{\dfrac{\partial R}{\partial x}} = \frac{\partial R}{\partial x}\,dx \tag{14a}$$

From Equation (**8a**), for any curves R = a constant and V = a constant,

$$dx = \frac{-\dfrac{\partial V}{\partial y}}{\dfrac{\partial V}{\partial x}}\,dy = \frac{\dfrac{\partial R}{\partial x}}{\dfrac{\partial R}{\partial y}}\,dy \tag{15a}$$

The second term in the brackets becomes

$$\left(\frac{\partial R}{\partial y}\right)^2 \frac{dx}{\dfrac{\partial R}{\partial x}} = \left(\frac{\partial R}{\partial y}\right)^2 \frac{\dfrac{\partial R}{\partial y}\,dy}{\dfrac{\partial R}{\partial x}} = \frac{\partial R}{\partial y}\,dy \tag{16a}$$

Therefore,

$$\phi_{R2} - \phi_{R1} = \mu \int_{R=R_1}^{R=R_2} \left[\frac{\partial R}{\partial x}\,dx + \frac{\partial R}{\partial y}\,dy\right] \tag{17a}$$

From Equation (**17a**) it is seen that the expression to be integrated is simply the total differential, $d\,R$.

$$\therefore\ \phi_{R2} - \phi_{R1} = \mu \int_{R=R_1}^{R=R_2} d\,R = \mu\,(R_2 - R_1) \tag{18a}$$

This means that if equal increments of R are chosen, then the flux between any two adjacent lines of force is the same.

It is interesting to note at this point that if the initial conditions which R must satisfy, Equation (**2a**), had been chosen such that

$$\frac{dy}{dx} = \frac{\dfrac{\partial R}{\partial y}}{\dfrac{\partial R}{\partial x}} = -\frac{X}{Y} \tag{19a}$$

or more explicitly

$$\left.\begin{aligned}\frac{\partial R}{\partial x} &= -X\\ \frac{\partial R}{\partial y} &= Y\end{aligned}\right\} \qquad \textbf{(20a)}$$

the function R = a constant would have given equipotential lines.

Appendix B

Approximate Determination of Flux

In calculating the flux created by a current-carrying conductor in a slot, and in obtaining the inductance of the slot due to this flux, it is usual practise to make the basic assumption that the lines of force go straight across the slot. Although this procedure gives inaccurate results for certain regions, in general the approximation for the entire field is sufficient for engineering

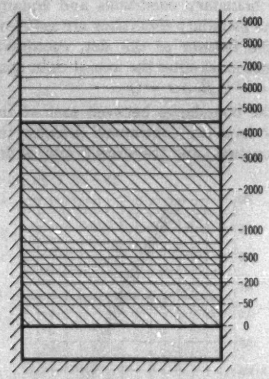

Fig. 19—Conductor Touching Sides but Not Bottom of Slot

answers. To check the values obtained by the more accurate method of Rogowski, the flux has also been calculated by this process, a short discussion of which is here included.

Consider unit length of the slot shown in Fig. 12. The flux through the tube of height dx is

$$d\,\phi = 4\,\pi\,i'\,\frac{x}{c}\cdot\frac{dx}{w} \qquad \textbf{(1b)}$$

where i' is the total current in abamperes of the conductor. The first part of the right member of Equation (**1b**) is the m. m. f. below the tube considered, and the second part of this member is the permeance of the tube.

Integrating, the total flux included in the interval x is therefore

$$\phi_x = \frac{4\,\pi\,i'}{c\,w}\cdot\frac{x^2}{2}\ \text{Maxwells per unit length} \qquad \textbf{(2b)}$$

and the total flux within the conductor is

$$\phi = \frac{2\,\pi\,i'\,c}{w}\ \text{Maxwells per unit length} \qquad \textbf{(3b)}$$

It was from Equation (**2b**) that the values given to the right of Figs. 6, 7, 8, and 13 were obtained.

From this discussion it is obvious that if the flux lines go straight across the slot, the height of the conductor above the bottom of the slot has no effect on the form of the field. As far as total flux is concerned, Fig. 19, which shows an air space beneath the copper, is identical with Fig. 8. There is no flux between the lower face of the conductor and the slot bottom. This is not true for the general case illustrated in the body of the paper.

Appendix C

Conductor Touching Bottom but not Sides of Slot

The special case in which the conductor does not touch the sides of the slot, but does come in contact with the bottom, is easily obtained from the general treatment by omitting Region I from that discussion. That is, Regions II and III only of the general case are now present in the new problem.

The Fourier series for the current density is identical in form to that formerly determined since any region below the conductor in no way affects the current density of the region in which the copper is situated.

The general solution of the problem is given by regional Equations (**11**) and (**12**) in the main body of the paper.

The constants for those equations are calculated as before by consideration of the boundary conditions between regions, and thus

$$\begin{aligned}
B_0 &= 0 \text{ (assumed)}\\
B_0' &= 0\\
B_0'' &= -\,\pi\,\alpha_0\\
B_n &= -\,\frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,\epsilon^{-nkd}\\
B_n' &= B_n\\
C_0 &= \pi\,\alpha_0\,c^2\\
C_0' &= -\,2\,\pi\,\alpha_0\,c\\
C_n &= 0\\
C_n' &= \frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,(\epsilon^{nkd} - \epsilon^{-nkd})
\end{aligned}$$

The final regional equations are

Region II:

$$R = -\,\pi\,\alpha_0\,x^2 + \sum_{n=1}^{n=\infty}\left[B_n(\epsilon^{nkx} + \epsilon^{-nkx}) + \frac{4\,\pi\,\alpha_n}{n^2\,k^2}\right]\cos n\,k\,y \qquad \textbf{(1c)}$$

Region III:

$$R = \pi \alpha_0 c^2 - 2 \pi \alpha_0 c x + \sum_{n=1}^{n=\infty} C_n' \epsilon^{-nkx} \cos n k y \tag{2c}$$

Fig. 7 in the body of the paper was plotted with the

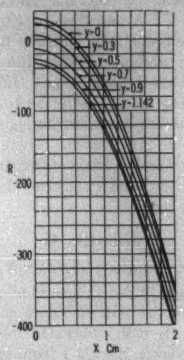

FIG. 20—CURVES OF R AGAINST x, PARAMETER y, FOR CASE SHOWN IN FIG. 7

aid of the auxiliary curves shown in Figs. 20 and 21, the constants for the slot and conductor being

$a = 0.45$ in.
$b = 0.225$ in.
$c = d = 1.63$ in.
$i = 1960$ amperes per sq. in.

In this case also, the $R = O$ line has been pushed back to the kernel by the addition of a certain amount of negative flux. The lines of R shown in Fig. 7 then become the actual quantities of flux produced by the current in the conductor.

Appendix D

SLOT WITH TWO CONDUCTORS CARRYING EQUAL CURRENTS IN THE SAME DIRECTION

The general method of attacking the problem of two conductors in a slot was outlined in Section IV of the paper. This appendix contains the general equations and constants for the above case in which the conductors carry equal currents in opposite directions.

The regional equations for R are similar to those formerly obtained.

Region I:

$$R = A_0 + A_0' x + \sum_{n=1}^{n=\infty} (A_n \epsilon^{nkx} + A_n' \epsilon^{-nkx}) \cos n k y \tag{1d}$$

Region II:

$$R = B_0 + B_0' x + B_0'' x^2 + \sum_{n=1}^{n=\infty} \left(B_n \epsilon^{nkx} + B_n' \epsilon^{-nkx} + \frac{4 \pi \alpha_n}{n^2 k^2} \right) \cos n k y \tag{2d}$$

Region III:

$$R = C_0 + C_0' x + \sum_{n=1}^{n=\infty} (C_n \epsilon^{nkx} + C_n' \epsilon^{-nkx}) \cos n k y \tag{3d}$$

Region IV:

$$R = D_0 + D_0' x + D_0'' x^2 + \sum_{n=1}^{n=\infty} \left(D_n \epsilon^{nkx} + D_n' \epsilon^{-nkx} + \frac{4 \pi \alpha_n}{n^2 k^2} \right) \cos n k y \tag{4d}$$

Region V:

$$R = E_0 + E_0' x + \sum_{n=1}^{n=\infty} (E_n \epsilon^{nkx} + E_n' \epsilon^{-nkx}) \cos n k y$$

The constant terms of the above regional equations are determined, as outlined in Section III, by substituting the boundary conditions and equating coefficients. The boundary conditions are as follows:

a. At the bottom of the slot, coordinate $x = O$, since the lines must enter the iron at right angles to the surface, $Y = -\partial R / \partial x = O$

b. At the boundaries between each of the regions the lines of force are continuous (because the permeability is constant) and therefore R and its first partial

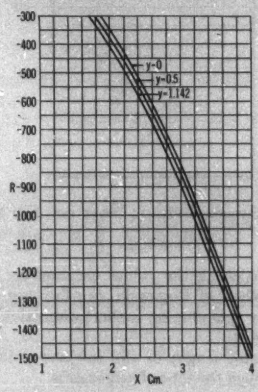

FIG. 21—CURVES OF R AGAINST x, PARAMETER y, FOR CASE SHOWN IN FIG. 7

derivatives with respect to x and y must be continuous.

c. For large values of x the flux goes straight across the slot, so $X = O$ and $Y =$ constant.

d. The value of the sum of the terms independent of $\cos n k y$ can be obtained from Equation (2b)

There result twenty-one equations for twenty-two unknowns, and by assuming $A_0 = O$ the constants are obtained as follows:

$$A_0 = 0 \text{ (assumed)}$$
$$A_0' = 0$$
$$A_n = \frac{2\,\pi\,\alpha_n}{n^2\,k^2}[\epsilon^{-nk(d+e)} + \epsilon^{-nk(d-c)} - \epsilon^{-nkd} - \epsilon^{-nk(d+c+e)}]$$
$$A_n' = A_n$$
$$B_0 = -\,\pi\,\alpha_0\,(d-c)^2$$
$$B_0' = 2\,\pi\,\alpha_0\,(d-c)$$
$$B_0'' = -\,\pi\,\alpha_0$$
$$B_n = \frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,[\epsilon^{-nk(d+e)} - \epsilon^{-nkd} - \epsilon^{-nk(d+c+e)}]$$
$$B_n' = \frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,[\epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} - \epsilon^{-nkd} - \epsilon^{-nk(d+c+e)} + \epsilon^{-nk(d+e)}]$$
$$C_0 = \pi\,\alpha_0\,(2\,c\,d - c^2)$$
$$C_0' = -\,2\,\pi\,\alpha_0\,c$$
$$C_n = \frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,[\epsilon^{-nk(d+e)} - \epsilon^{-nk(d+c+e)}]$$
$$C_n' = \frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,[\epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} - \epsilon^{-nkd} + \epsilon^{nkd} - \epsilon^{-nk(d+c+e)} + \epsilon^{-nk(d+e)}]$$
$$D_0 = \pi\,\alpha_0\,(2\,c\,d - 2\,d\,e - c^2 - d^2 - e^2)$$
$$D_0' = 2\,\pi\,\alpha_0\,(d + e - c)$$
$$D_0'' = -\,\pi\,\alpha_0$$
$$D_n = \frac{-\,2\,\pi\,\alpha_n}{n^2\,k^2}\,\epsilon^{-nk(d+c+e)}$$
$$D_n' = \frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,\{\,\epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} + \epsilon^{-nk(d+e)} - \epsilon^{nk(d+e)} - \epsilon^{-nkd} + \epsilon^{nkd} - \epsilon^{-nk(d+c+e)}\,\}$$
$$E_0 = 2\,\pi\,\alpha_0\,(2\,d + e)\,c$$
$$E_0' = -\,4\,\pi\,\alpha_0\,c$$
$$E_n = 0$$
$$E_n' = \frac{2\,\pi\,\alpha_n}{n^2\,k^2}\,\{\,\epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} - \epsilon^{-nkd} + \epsilon^{nkd} + \epsilon^{-nk(d+e)} - \epsilon^{nk(d+e)} - \epsilon^{-nk(d+c+e)} + \epsilon^{nk(d+c+e)}\}$$

The final equations for R in the various regions become very cumbersome when the constants are substituted so they are omitted here.

The dimensions of the slot, Fig. 12 are:

$a = 0.45$ in.
$b = 0.225$ in.
$c = 1.18$ in.
$d = 1.405$ in.
$e = 0.715$ in.
$f = 0.53$ in.

The R-curves from which the above plot was made are shown in Figs. 22, 23, and 24, for a current of 1036 amperes per conductor.

Appendix E

Slot with Two Conductors Carrying Equal Currents in the Opposite Direction

Reference was made to this appendix in Section IV of the paper under the discussion of the above case.

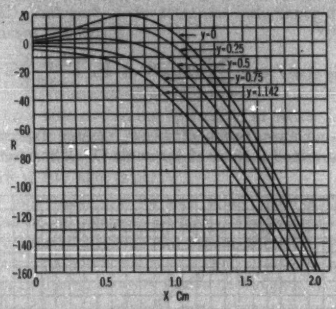

Fig. 22—Curves of R against x, Parameter y, for Case Shown in Fig. 13

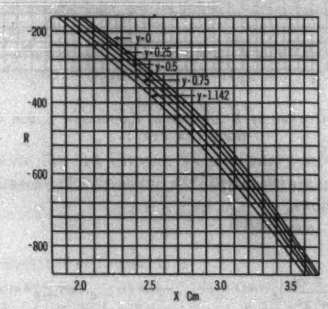

Fig. 23—Curves of R against x, Parameter y, for Case Shown in Fig. 13

The regional equations for R are:

Region I:

$$R = A_0 + A_0'\,x + \sum_{n=1}^{n=\infty} (A_n\,\epsilon^{nkx} + A_n'\,\epsilon^{-nkx})\cos n\,k\,y \tag{1e}$$

Region II:

$$R = B_0 + B_0'\,x + B_0''\,x^2 + \sum_{n=1}^{n=\infty}\left(B_n\,\epsilon^{nkx} + B_n'\,\epsilon^{-nkx} + \frac{4\,\pi\,\alpha_n}{n^2\,k^2}\right)\cos n\,k\,y \tag{2e}$$

Region III:

$$R = C_0 + C_0' x + \sum_{n=1}^{n=\infty} (C_n \epsilon^{nkx} + C_n' \epsilon^{-nkx}) \cos n k y \tag{3e}$$

Region IV:

$$R = D_0 + D_0' x + D_0'' x^2 + \sum_{n=1}^{n=\infty} \left(D_n \epsilon^{nkx} + D_n' \epsilon^{-nkx} - \frac{4 \pi \alpha_n}{n^2 k^2} \right) \cos n k y \tag{4e}$$

Region V:

$$R = E_0 + E_0' x + \sum_{n=1}^{n=\infty} (E_n \epsilon^{nkx} + E_n' \epsilon^{-nkx}) \cos n k y \tag{5e}$$

The constant terms of these regional expressions are determined as those of Appendix D. The boundary

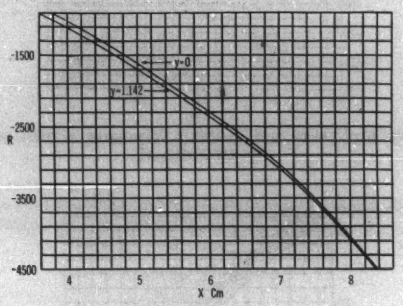

Fig. 24—Curves of R against x, Parameter y, for Case Shown in Fig. 13

conditions are the same in the two cases. The constants are as follows:

$$A_0 = 0, \text{ (assumed)}$$

$$A_0' = 0$$

$$A_n = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d+c+e)} - \epsilon^{-nk(d+e)} - \epsilon^{-nkd} + \epsilon^{-nk(d-c)}]$$

$$A_n' = A_n$$

$$B_0 = - \pi \alpha_0 (d - c)^2$$

$$B_0' = 2 \pi \alpha_0 (d - c)$$

$$B_0'' = - \pi \alpha_0$$

$$B_n = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d+c+e)} - \epsilon^{-nk(d+e)} - \epsilon^{-nkd}]$$

$$B_n' = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d+c+e)} - \epsilon^{-nk(d+e)} - \epsilon^{-nkd} + \epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)}]$$

$$C_0 = - \pi \alpha_0 (c^2 - 2 c d)$$

$$C_0' = - 2 \pi \alpha_0 c$$

$$C_n = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d+c+e)} - \epsilon^{-nk(d+e)}]$$

$$C_n' = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d+c+e)} - \epsilon^{-nk(d+e)} - \epsilon^{-nkd} + \epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} + \epsilon^{nkd}]$$

$$D_0 = \pi \alpha_0 [2 c d - c^2 + (d + e)^2]$$

$$D_0' = - 2 \pi \alpha_0 (c + d + e)$$

$$D_0'' = \pi \alpha_0$$

$$D_n = \frac{2 \pi \alpha_n}{n^2 k^2} \epsilon^{-nk(d+c+e)}$$

$$D_n' = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(d+c+e)} - \epsilon^{-nk(d+e)} - \epsilon^{-nkd} + \epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} + \epsilon^{nkd} + \epsilon^{nk(d+e)}]$$

$$E_0 = - 2 \pi \alpha_0 (c^2 + c e)$$

$$E_0' = 0$$

$$E_n = 0$$

$$E_n' = \frac{2 \pi \alpha_n}{n^2 k^2} [\epsilon^{-nk(a+c+e)} - \epsilon^{-nk(d+e)} - \epsilon^{-nkd} + \epsilon^{-nk(d-c)} - \epsilon^{nk(d-c)} + \epsilon^{nkd} + \epsilon^{nk(d+e)} - \epsilon^{nk(d+c+e)}]$$

The current is 1036 amperes per conductor, as before. The R-curves for the detailed plot of this case,

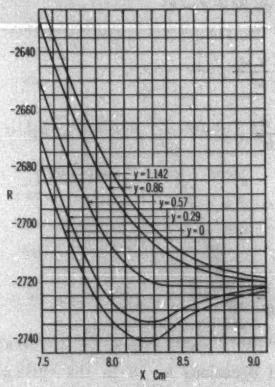

Fig. 25—Curves of R against x, Parameter y, for Case Shown in Fig. 15

mentioned in the body of the paper, are given by Fig. 25. From the values of R has been subtracted E_0.

In calculating R-curves from the above equations, one meets with the terms $\epsilon^{nk(d+c+e)} \epsilon^{-nkx}$. The first exponential involves very large numbers and the second one very small numbers, both of which are not easily obtained on a slide rule or in tables. It is much simpler to reduce all exponential products to a single

term by first adding the exponents. This facilitates calculations because now a slide rule can generally be used.

Appendix F

Graphical Determination of Inductance

The comparison between the results for the determination of slot inductance, as obtained by graphical methods and as calculated by design formulas, was given in Section VI of the body of the paper. This appendix includes the derivations of the expressions from which these values were obtained.

It will be remembered that the inductance of a system is defined as the flux linkages per ampere. In many problems met in practise, this definition cannot be applied directly, since, as is the case with large conductors, there is a component of inductance due to flux linking only part of the current. It is, therefore, customary to consider a large conductor to consist of a great many smaller conductors connected in series so that all carry the same current. Then the partial linkages become total linkages and the above definition of inductance is applied.

The following additional symbols will be used:

N = the total series turns in a slot (on the basis of the above conception of inductance, N is very large, although practically it need not be).

N' = the series turns linked by any element of flux.

ϕ_n = flux in region n.

A_n = area of copper section in region n.

I = current per turn in amperes.

L_n = inductance in henrys due to flux in region n.

x = distance measured upward from the bottom of the lower conductor.

The inductance of the slot of Fig. 12 will now be determined for each region, Region I, of course, being omitted.

In Region II

$$d\,L_{II} = \frac{N'\,d\,\phi_{II}}{I}\,10^{-9}\text{ henrys} \qquad \textbf{(1f)}$$

Although Fig. 12 shows the elementary tube of flux going straight across the slot, the tube will in general be curved, see Fig. 13. The interlinkage factor N' in Region II is given by

$$\frac{N}{2}\,\frac{d\,A_{II}}{A_{II}}$$

where $d\,A_{II}$ is the area, or current, enclosed by the element of flux $d\,\phi_{II}$. $N/2$ is necessary since only half of the total series turns are contained in the lower conductor.

Then the total inductance of this region is

$$L_{II} = \sum \frac{N}{2\,I\,A_{II}}\,d\,A_{II}\,d\,\phi_{II}\,10^{-9}\text{ henrys} \qquad \textbf{(2f)}$$

where the summation extends throughout the lower conductor, and is applied by taking all of the elements $d\,A_{II}$ enclosed by increments of flux $d\,\phi_{II}$.

Consider the tube of flux between the -300 and -450 lines of Fig. 13. The increment of flux, $d\,\phi = -150$ lines, surrounds the series turns in the slot given by the ratio of the copper section beneath the -300 line to the total copper section of the lower conductor. The areas were conveniently determined by a planimeter, this ratio being 0.526. From Equation (**2f**), that part of the total slot inductance due to the element of flux is,

$$\frac{0.526 \times 150}{1036} \times 10^{-9} = 0.0762 \times 10^{-9}\text{ henrys}$$

based on a slot containing two turns in series. Since inductance is directly proportional to the square of the series turns it would be better to write the above value as 0.019×10^{-9} henrys per N^2.

The inductance due to any other tube of flux is obtained in an identical manner. The summation of these values throughout the region occupied by the lower conductor will thus give the inductance of the slot due to the current and flux in that region.

It should be mentioned here that the graphical method is accurate only when small increments of flux are taken. For the case at hand, the 150 line intervals are too great and they should be broken up into at least three 50 line periods, which yields a somewhat greater value of inductance than the other calculation.

If the flux lines are assumed to go straight across the slot, then the interlinkage factor becomes

$$\frac{N}{2}\,\frac{d\,A_{II}}{A_{II}} = \frac{N}{2}\,\frac{x}{c}$$

and since the flux through an elementary section of height dx, per centimeter of slot length, is

$$d\,\phi_{II} = \frac{4\,\pi\,N\,I\,dx}{w}\text{ Maxwells} \qquad \textbf{(3f)}$$

the inductance of Region II is

$$L_{II} = \frac{1}{I}\int_{x=0}^{x=c} \frac{\pi\,N^2\,I}{w}\left(\frac{x}{c}\right)^2 dx\,10^{-9} = \frac{\pi\,N^2\,c}{3\,w}\,10^{-9}\text{ henrys} \qquad \textbf{(4f)}$$

All of the flux in Region III links all of the current in the lower conductor, hence

$$d\,L_{III} = \frac{N}{2}\,\frac{d\,\phi_{III}}{I}\,10^{-9}\text{ henrys}$$

and,

$$L_{III} = \frac{N}{2\,I}\,\phi_{III}\,10^{-9}\text{ henrys} \qquad \textbf{(5f)}$$

to be obtained graphically.

If the flux lines are assumed to go straight across the slot,

$$L_{III} = \frac{\pi N^2 e}{w} 10^{-9} \text{ henrys} \qquad (6f)$$

In Region IV the flux links all of the turns in the lower conductor, and in addition $d A_{IV}/A_{IV}$ of the turns in the upper conductor. Thus,

$$L_{IV} = \sum \frac{N}{2 I} \left(1 + \frac{d A_{IV}}{A_{IV}}\right) d \phi_{IV} 10^{-9} \text{ henrys} \qquad (7f)$$

to be obtained graphically.

Considering the flux to go straight across,

$$L_{IV} = \frac{1}{I} \int_{x=0}^{x=c} \frac{\pi N^2 I}{w} \left(1 + \frac{x}{c}\right)^2 dx\, 10^{-9}$$

$$= \frac{7 \pi N^2 c}{3 w} 10^{-9} \text{ henrys} \qquad (8f)$$

In Region V the flux links all N turns

$$L_V = \frac{N}{I} d \phi_V 10^{-9} \text{ henrys} \qquad (9f)$$

graphically, or for the assumption of flux lines straight across the slot,

$$L_V = \frac{4 \pi N^2 f}{w} 10^{-9} \text{ henrys} \qquad (10f)$$

Summing up, for graphical determination of the inductance,

$$L = L_{II} + L_{III} + L_{IV} + L_V$$

$$= \frac{N}{I} \sum \left[\frac{d A_{II}}{2 A_{II}} d \phi_{II} + \frac{d \phi_{III}}{2} + \frac{1}{2} \left(1 + \frac{d A_{IV}}{A_{IV}}\right) d \phi_{IV} + d \phi_V \right] 10^{-9} \text{ henrys} \qquad (11f)$$

By the assumption that all the flux lines go straight across,

$$L = \frac{4 \pi N^2}{10^9} \left(\frac{8 c + 12 f + 3 e}{12 w}\right) \text{ henrys} \qquad (12f)$$

This equation is identical, except for nomenclature, with that obtained by Mr. P. L. Alger.[3]

Appendix G

Determination of Inductance from the Vector Potential R

In Appendix F the inductance of the slot of Fig. 13 was calculated in two ways, one on the assumption that the flux lines go straight across the slot and the other by integrating the flux linkages according to the flux plot, by means of a planimeter. The difference in the inductances as determined by the two different methods is not large in the case under consideration. Since, however, usual design formulas for determining slot inductance are based upon the assumption that the flux lines go straight across the slot, it is of interest to obtain an expression for the error involved in this assumption.

Messrs. Stevenson and Park included in an appendix to their paper a discussion on the calculation of inductance from a knowledge of the vector potential. The method there used of attacking the problem is so fundamentally clear that it will be followed in this analysis. The total inductance of the slot up to the bottom of the wedge will be determined. It will be found to consist of the inductance determined on the usual assumptions, and an additional term which is due to the curvature of the flux lines. By neglecting the unimportant components of this additional term it reduces to the simple correction term given in the body of the paper.

Let ϕ be the total flux from the bottom of the lower conductor up to the bottom of the wedge. Then outside of any line R the flux will be,

$$\phi + R_0 - R$$

where R_0 is the value of the vector potential at the bottom of the lower conductor. The linkages at the line R, considering the lower and upper conductor to have the same turns, are

$$N' (\phi + R_0 - R) = (\phi + R_0 - R) \frac{N\, d A}{2 A}$$

where A is the total area of the copper in the slot and $d A = dx\, dy$ is the area of copper inside of the line R. The inductance is then,

$$L = \frac{N}{2 I} \int_s (\phi + R_0 - R) \frac{d A}{A} \qquad (1g)$$

where the integral $\int_s$ is to extend over both of the copper areas.

Since $I = A\, i$,

$$L = \frac{N}{2 I^2} \int_s (\phi + R_0 - R)\, i\, d A \qquad (2g)$$

$\phi + R_0$ is independent of the element $d A$ and $\int_s i\, d A = 2 I$ so,

$$\frac{N}{2 I^2} \int_s (\phi + R_0)\, i\, d A = \frac{N}{I} (\phi + R_0) \qquad (3g)$$

or,

$$L = \frac{N}{I} (\phi + R_0) - \frac{N}{2 I^2} \int_s R\, i\, d A \qquad (4g)$$

$R_0 + \phi$ can be obtained by substituting in the vector potential equation for Region V the values of $x = c + d + e + f$ and $y = O$.

$$R_V = E_0 + E_0' x + \sum \ldots\ldots\ldots\ldots$$

$$\phi + R_0 = -\frac{2\pi a i c}{l}\left\{4c + 2e + 4f\right\} + \sum \quad \textbf{(5g)}$$

The first term of the above inductance for a two turn coil is,

$$\frac{N}{I}(\phi + R_0) = -\frac{4\pi}{l}\left\{4c + 2e + 4f\right\} \quad \textbf{(6g)}$$

The negative sign need cause no alarm as it simply indicates the direction of the induced voltage.

The second term of the inductance formula (**4g**) requires the surface integral of R over the two current carrying regions.

For Region II, considering a two turn coil

$$-\frac{1}{I^2}\int_S R_{II}\, i\, d\,A = -\frac{i}{I^2}\int_{y=-a/2}^{y=a/2}\int_{x=d-c}^{x=d}\left\{B_0 + B_0' x + B_0'' x^2 + \sum\left(B_n \epsilon^{nkx} + B_n' \epsilon^{-nkx} + \frac{4\pi\alpha_n}{n^2 k^2}\right)\cos nky\right\} dx\, dy \quad \textbf{(7g)}$$

$$= \frac{2\pi c}{3l} - \sum \frac{2i}{I^2}\sin nka/2\left\{\frac{B_n}{n^2k^2}\left[\epsilon^{nkd} - \epsilon^{nk(d-c)}\right] - \frac{B_n'}{n^2k^2}\left[\epsilon^{-nkd} - \epsilon^{-k(d-c)}\right] + \frac{4\pi\alpha_n c}{n^3k^3}\right\} \quad \textbf{(8g)}$$

For Region IV,

$$-\frac{1}{I^2}\int_S R_{IV}\, i\, d\,A$$

$$= -\frac{i}{I^2}\int_{y=-a/2}^{y=a/2}\int_{x=d+e}^{x=e+d+e}\left\{D_0 + D_0' x + D_0'' x^2 + \sum\left(D_n \epsilon^{nkx} + D_n' \epsilon^{-nkx} + \frac{4\pi\alpha_n}{n^2k^2}\right)\cos nky\right\} dx\, dy \quad \textbf{(9g)}$$

$$= \frac{2\pi}{l}\left(\frac{7}{3}c + 2e\right) - \sum \frac{2i}{I^2}\sin an\pi/l\left\{\frac{D_n}{n^2k^2}\left[\epsilon^{nk(c+d+e)} - \epsilon^{nk(d+e)}\right] - \frac{D_n'}{n^2k^2}\left[\epsilon^{-nk(c+d+e)} - \epsilon^{-nk(d+e)}\right] + \frac{4\pi\alpha_n}{n^3k^3}c\right\} \quad \textbf{(10g)}$$

Representing the two summations by $\sum$ for compactness

$$L = -\frac{4\pi}{l}\left\{\frac{8c + 3e + 12f}{12}\right\} - \sum \quad \textbf{(11g)}$$

Excluding Σ the inductance is the same as that developed assuming the flux lines go straight across the slot. Σ must then be the correction term due to their curvature.

By substituting in the constants B_n, B_n', D_n, and D_n' and neglecting all terms which are insignificant in slots encountered in actual practise the correction term becomes

$$L' = \sum \frac{-l^3}{c^2a^2\pi^3n^4}\left(c - \frac{l}{2n\pi}\right)\sin^2\frac{an\pi}{l} \quad \textbf{(12g)}$$

which is to be added for Σ in Equation (**11g**).

Nomenclature

a, b, c, d, e, f, and l = slot and conductor dimensions, see Fig. 12.

$A_n, A_n', B_n, B_n' \ldots$ etc. = constants of the Fourier series of the regional equations.

H = magnetic field intensity.

i = conductor current density in abamperes per square centimeter.

i' = total conductor current in abamperes.

I = current density in general. In Appendix E, used as current per turn.

k = $2\pi/l$.

l = $a + 2b$, also used as w, the slot width.

L = inductance in abhenrys.

L' = inductance correction factor.

n = an integer, 1, 2, 3, ... etc.

R = vector potential of the magnetic field.

R_n = function of integration dependent upon x alone, and coefficients of the Fourier series for R. In Appendix A, R_n is used to designate lines of force.

V = vector potential of a field orthogonal to that of R.

w = slot width in Appendixes B and F.

x = distance measured upward from bottom of lower conductor.

x, y = general coordinates.

X = x-component of H.

Y = y-component of H.

α_n = coefficient of the nth term of the Fourier series for the m. m. f. distribution.

β_n, γ_n = coefficients of the solutions for R_n.

ϕ = flux in total lines.

μ = permeability, ∞ for iron, unity for air, insulation, and copper.

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Discussion

K. A. Reeve: Although the authors of the paper *Analytical Determination of Magnetic Fields* are concerned mostly with the practical case of two conductors in a slot, one above the other, as is typical of synchronous-machine design, I should like to point out that the method of approach which they use in solving the problem can be applied to the general case of slot design for d-c. machines in which there may be four, six, eight or more coil sides in a slot, each coil side being considered a conductor.

As an illustration of this application, consider the simple case of four conductors placed two horizontally and vertically in a slot. Let the conductors in the bottom row carry equal currents, and those in the top row also carry equal currents, but not of the same magnitude or direction of the current as in the lower row. This case may be solved by considering the Fourier series, as done in the paper, and then applying the correction factor, as given by Maxwell in Volume I, paragraph 693, of his "Electricity and Magnetism," to care for the space between the conductors. It may also be solved by changing the Fourier series for the current density, such that the curve will have two rectangular loops. The substitution of this series in Poisson's equation is all that is then required.

It is also possible by this method to solve the case in which the currents in all conductors are not the same, and the case in which conductors are staggered vertically but not horizontally. Although the last condition is not realized in practise, it nevertheless shows the generality of the authors' treatment.

A. D. Moore: This contribution breaks down one more of the barriers to the understanding of typical field cases. Many of the barriers are insurmountable. Every time a new set of problems in magnetic fields is solved, our ability to handle unsolved problems is improved. Knowledge of the general character of a field, and of its main tendencies, is far better than an uninformed guess. Hence, although the practical slot violates to some degree every assumption underlying solutions reached in the paper, yet these solutions will usually apply with satisfying accuracy. And when one must estimate (*estimate* being here a refined synonym for *guess*) fields which nobody ever will be able to solve, the accuracy of the estimate is likely to bear a close relation to one's knowledge of correct solutions of ideal cases.

If the paper does no more than to cause college textbooks of the future to differ from those of the past by presenting generally correct rather than generally ridiculous representations for slot fields, the labor spent in preparing it will have been amply justified.

J. J. Smith: (communicated after adjournment) The authors have obtained very interesting solutions for the flux distribution about rectangular conductors in rectangular slots. It is evident that considerable skill and ingenuity were required in the determination of the various constants of integration given for instance in Equation (13).

It may be of interest to point out that a more symmetrical form of solution may be written down from which the various results may be obtained by simple integrations. This can be obtained by letting the thickness of the conductor in a radial direction in the slot approach zero (*i. e.*, $c \to O$). We have then only two regions (I) and (III) to deal with. The expressions for vector potential become (expanding e^{nkc} in a series and neglecting c^2 and higher powers of c)

Region I—

$$R = \sum_{n=1}^{n=\infty} \underset{c \to o}{Lt} \frac{2\pi\alpha_n}{n^2 k^2} e^{-nkd}\left[e^{nkc} - 1\right]\left[e^{nkx} + e^{-nkx}\right]\cos nky$$

$$= \sum_{n=1}^{n=\infty} \underset{c \to o}{Lt} \frac{2\pi\alpha_n c}{nk} e^{-nkd}\left[e^{nkx} + e^{-nkx}\right]\cos nky \qquad (1)$$

Region III—

$$R = \underset{c \to o}{Lt}\, 2\pi\alpha_o c(d - x) + \sum_{n=1}^{n=\infty} \underset{c \to o}{Lt} \frac{2\pi\alpha_n}{n^2 k^2}\left[e^{-nkd}(e^{nkc} - 1) + e^{nkd}(1 - e^{-nkc})\right] e^{-nkx}\cos nky$$

$$= \underset{c \to o}{Lt}\, 2\pi\alpha_o c(d - x) + \sum_{n=1}^{n=\infty} \underset{c \to o}{Lt} \frac{2\pi\alpha_n c}{nk} e^{-nkx}\left[e^{nkd} + e^{-nkd}\right]\cos nky \qquad (2)$$

It will be noted in this form that (except for $n = o$) the various terms in region III are obtained from the terms in region I by interchanging x and d. The solution can be made completely symmetrical by adding $\underset{c \to o}{Lt}\, \pi\alpha_o c(x - d)$ to the expression for each region—this is legitimate since the authors' solution is based upon an arbitrary assumption that $A_o = o$. We then obtain for region I

$$R = \underset{c \to o}{Lt}\, \pi\alpha_o c(x - d) + \sum_{n=1}^{n=\infty} \underset{c \to o}{Lt} \frac{2\pi\alpha_n c}{nk} e^{-nkd}\left[e^{nkx} + e^{-nkx}\right]\cos nky \qquad (3)$$

and if we write

$$\alpha_o = \alpha_o' i \text{ and } \alpha_n = \alpha_n' i \qquad (4)$$

and also write $\underset{c \to o}{Lt}\, i c a = I$ where I is the total current in the conductor, then for region I

$$R = \frac{\pi\alpha_o' I(x - d)}{a} + \sum_{n=1}^{n=\infty} \frac{4\pi\alpha_n' I}{nka} e^{-nkd}\cosh nkx \cos nky \qquad (5)$$

and the expression for region III is obtained by interchanging x and d.

I have checked this solution with the result obtained for a more general problem which is as follows: The vector potential due to a filament of current I placed at the point (x_1, y_1) in an infinitely deep slot the equations of whose sides are given by $x = o$, $y = o$ and $y = l$ is

$$v = \underset{p \to o}{Lt}\, I \cos\frac{p\pi y}{l}\cos\frac{p\pi y_1}{l} A_p + 2I\sum_{p=1}^{p=\infty} \cos\frac{p\pi y}{l}\cos\frac{p\pi y_1}{l} A_p$$

where

$$A_p = \frac{4}{p}\cosh\frac{p\pi x}{l} e^{-\frac{p\pi x_1}{l}} \qquad (x < x_1)$$

$$= \frac{4}{p}\cosh\frac{p\pi x_1}{l} e^{-\frac{p\pi x_1}{l}} \qquad (x > x_1) \qquad (6)$$

My interest in this solution for a current filament lies chiefly in the fact that it may be derived by means of an extension of the Heaviside Operational Calculus which makes the derivation of such results almost an automatic procedure. I hope to illustrate how such results can be obtained in an Institute paper sometime in the near future.

If the total current in the infinitely thin strip of width a is I, the current in the element $d y_1$ is $I\, d y_1/a$. Hence changing

the origin from $y = o$ to $y = l/2$ (to conform with Messrs. Robertson and Terry's notation) and integrating from $-a/2$ to $+a/2$ we get for the vector potential

$$v = R = \frac{I}{a}\int_{-a/2}^{+a/2} \underset{p\to o}{Lt} \cos\left[\frac{p\pi}{l}\left(y+\frac{l}{2}\right)\right]\cos\left[\frac{p\pi}{l}\left(y_1+\frac{l}{2}\right)\right] A_p\, dy_1$$

$$+\frac{2I}{a}\int_{-a/2}^{+a/2}\sum_{p=1}^{p=\infty}\cos\left[\frac{p\pi}{l}\left(y+\frac{l}{2}\right)\right]\cos\left[\frac{p\pi}{l}\left(y_1+\frac{l}{2}\right)\right] A_p\, dy_1$$

$$= \frac{I}{a}\underset{p\to o}{Lt}\frac{A_p\cos\left[\frac{p\pi}{l}\left(y+\frac{l}{2}\right)\right]}{\frac{p\pi}{l}}\left[\sin\left[\frac{p\pi}{l}\left(\frac{l}{2}+\frac{a}{2}\right)\right] - \sin\left[\frac{p\pi}{l}\left(\frac{l}{2}-\frac{a}{2}\right)\right]\right]$$

$$+\frac{2I}{a}\sum_{p=1}^{p=\infty}\frac{A_p\cos\left[\frac{p\pi}{l}\left(y+\frac{l}{2}\right)\right]}{\frac{p\pi}{l}}\left[\sin\left[\frac{p\pi}{l}\left(\frac{l}{2}+\frac{a}{2}\right)\right] - \sin\left[\frac{p\pi}{l}\left(\frac{l}{2}-\frac{a}{2}\right)\right]\right]$$

$$= \underset{p\to o}{Lt}\frac{2lI A_p\cos\left[\frac{p\pi}{l}\left(y+\frac{l}{2}\right)\right]\cos\frac{p\pi}{2}\sin\frac{p\pi a}{2l}}{p\pi a}$$

$$+\frac{4I}{a}\sum_{p=1}^{p=\infty}\frac{A_p\, l\cos\frac{p\pi y}{l}\cos^2\frac{p\pi}{2}\sin\frac{p\pi a}{2l}}{p\pi} \qquad (7)$$

Now $\cos^2\frac{p\pi}{2} = o$ when p is odd, and $= 1$ when p is even. Hence writing $p = 2n$ we obtain

$$R = \underset{n\to o}{Lt}\, l I A_n + \frac{4Il}{a}\sum_{n=1}^{n=\infty}\frac{A_{2n}\cos\frac{2n\pi y}{l}\sin\frac{n\pi a}{l}}{2n\pi}$$

$$= \underset{n\to o}{Lt}\, l I A_n + \sum_{n=1}^{n=\infty}\frac{4Il}{2n\pi a}\cos\frac{2n\pi y}{l}\sin\frac{n\pi y}{l}\,\frac{4}{2n}$$

$$\cosh\frac{2n\pi x}{l}\, e^{-\frac{2\pi n x_1}{l}}\ (x < x_1)$$

$$= \underset{n\to o}{Lt}\, l I A_n$$

$$+ I\sum_{n=1}^{n=\infty} l\,\frac{4}{nkl}\sin\frac{nka}{2}\cos nky\,\frac{4}{2na}\cosh nkx\, e^{-nkx_1}\ (x<x_1)$$

$$= \underset{n\to o}{Lt}\, I A_n + I\sum_{n=1}^{n=\infty}\alpha_n'\cos nky\,\frac{4\pi}{nka}\, e^{-nkx_1}\cosh nkx\ (x<x_1)$$

$$(8)$$

If d is now written instead of x_1 this expression, except for the first term ($n = o$) is identical with the value derived for R in (5) from the results obtained in Messrs. Robertson and Terry's paper.

The term for $n = o$ is infinite on account of the presence of the factor n in the denominator of A_n. This infinite term is due to the fact that the vector potential has been assumed to be zero at infinity in deriving the expression for a current filament. It is easy to show, however, that the *difference* between the terms for $n \to o$, for $x > x_1$ and for $x < x_1$ is the same as that derived from the authors' paper in (5). We have for this difference

$$\underset{n\to o}{Lt}\, l\,\frac{4I}{p}\left[\cosh\frac{p\pi x}{l}\, e^{-\frac{p\pi x_1}{l}} - \cosh\frac{p\pi x_1}{l}\, e^{-\frac{p\pi x}{l}}\right]$$

$$= \underset{p\to o}{Lt}\, l\,\frac{4I}{p}\left[1 - \frac{p\pi x_1}{l} - 1 + \frac{p\pi x}{l}\right]$$

$$= \frac{4I\pi}{l}(x - x_1) = \frac{2I\,\alpha_o'\,\pi\,(x - x_1)}{a} \qquad (9)$$

and writing d for x_1 this is equal to the difference between the first terms already obtained for regions I and III in (5).

It may be well to note here that although the results obtained in Messrs. Robertson and Terry's paper can be obtained from the vector potential for a current filament, the reverse process is not possible. This is due to the fact that they have assumed symmetry for the current about the axis $y = o$. I would suggest that they briefly indicate how their solution could be extended to a current distribution which was not symmetrical about that axis. Of course, such a problem is not as important as the ones considered in the paper since in practical cases we have such symmetry.

We have thus obtained a check from another source on the authors' work, which resulted in satisfactory agreement. In connection with the special cases discussed in Section V of the paper it may be worth while to point out that any rectangular strip filling the slot from side to side $\left(\text{from } y = -\frac{l}{2} \text{ to } y = \frac{l}{2}\right)$ and extending from $x = c$ to $x = d$, gives flux lines which go straight across the slot and gives no flux below the conductor. The various plots given by them are very interesting in actually showing that the error is small in the ordinary assumption that the flux goes across the slot in straight lines, and thus the calculation of self-inductance under this assumption is sufficiently close for practical purposes.

B. L. Robertson: We wish to thank Professor Moore for his comments on the paper. Professor Moore has been quite interested in the subject of flux plotting over a considerable number of years, perhaps more from the graphical than the analytical standpoint, and this work has been of further interest to him as a verification of free-hand field mapping.

The paper was prepared from the practical point of view as much as possible, and we are glad that Mr. Reeve has shown the application of the general method to additional fields. The d-c. machine, since its design is usually of more than two coil sides per slot, is the problem Mr. Reeve has in mind. As he says, there are different ways of obtaining the solution, but the simpler one is that in which the Fourier series for the current density is changed. The number of rectangular loops in the current density plot will then be equal to the number of conductor (coil side) widths in a slot. In general, it will be necessary to obtain a new current density curve, and its accompanying Fourier series, for each row of conductors. Current density curves for each row also takes care of the cases in which the conductors might be staggered vertically but not horizontally.

I. A. Terry: The authors also wish to thank Mr. J. J. Smith for presenting an alternative method of solving the cases considered in the paper. The completely symmetrical form of the equations obtained by considering conductor filaments has a number of advantages over the form given in the paper, not the least important of which is the fact that the derivation of the

results becomes almost automatic by using Operational Calculus.

A complete treatment of this subject by means of Operational Calculus should prove to be of great value to persons interested in analytical determination of magnetic or other orthogonal systems.

In regard to the extension of the solution to the case of a current distribution not symmetrical about the $y = o$ axis, it will be remembered that the method of solution used in the paper requires the current density curve to be expressible in the form of a Fourier Series. (See Fig. 3 of the paper.) So long as this condition is fulfilled the solution is obtained as in the paper.

For example, if a slot contains two conductors in the same horizontal plane carrying unequal currents, then the current density curve will be similar to Fig. 3 except that every alternate group of two loops will be of the same height and determined by the current in each conductor, as can be seen from the application of the method of images. The fundamental period will now cover two slots (one slot and either adjacent image) instead of one slot alone.

In the case of some slots used in d-c. machines, as mentioned by Mr. K. A. Reeve, the current density has the form which Mr. Smith has considered not so important as those in the paper. The authors feel that these cases are equally as important but limited the present paper to simple cases of conductors in slots.

Shielding and Guarding Electrical Apparatus
Used in Measurements—General Principles

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Fellow, A. I. E. E.

Synopsis.—Electrical measuring apparatus is shielded and guarded to protect it from the following external influences; namely, leakage currents, electrostatic fields, magnetic fields, and electromagnetic waves. Apparatus is guarded against leakage currents which may flow over or through the solid insulators on which the apparatus is supported. It is shielded against leakage currents which flow through the fluid in which the instrument is immersed. By a proper arrangement of guards and shields any apparatus can be completely protected against leakage currents.

Any apparatus can be completely shielded from electrostatic fields by placing it in a metallic case. However, there may be electrostatic reactions between the apparatus and the shield, thus introducing errors in the measuring apparatus.

Shielding apparatus from magnetic fields requires that the apparatus be surrounded by a thick enclosure of magnetic material. The theory of this magnetic shielding has been developed and formulas for producing the most satisfactory shielding are given.

The shielding of an apparatus from electromagnetic waves is largely accomplished by means of eddy currents which these waves set up in the shield. The theory for the production of these eddy currents has been developed and the best location of shields is discussed.

Some attention is given to the errors which may be introduced by shielding and some methods are outlined for obviating the necessity of shielding.

* * * * *

I. Introduction

GUARDS and shields are used with electrical measuring apparatus either to simplify its theory or to protect it from external influences. The simplification of theory has generally resulted from a rearrangement of the electric or magnetic field so that the resulting field can be more easily subjected to mathematical analysis. The external influences from which apparatus must be protected are: (1) Leakage currents, (2) magnetic fields, (3) electrostatic fields, (4) electromagnetic waves.

It is the purpose of this paper to discuss the general principles underlying the guarding and shielding of electrical apparatus.

The protection of apparatus from external influences has sometimes been called guarding the apparatus and sometimes shielding it. The following definitions seem to represent the best practise: A shield protects apparatus from those outside influences which may penetrate to the instrument through the medium by which it is surrounded. A device for the protection of apparatus from leakage currents which, for the most part, flow over the surfaces of the solid insulators used to support the apparatus is called a guard. The term "guard" is also applied to those devices the purpose of which is to rearrange the electric or magnetic field. For example, the earthed plates which are sometimes put under the insulating supports of an electrometer to prevent leakage currents are called guard plates, while if the electrometer is surrounded by a metal covering to protect it from the ionization currents of the air, this covering is a shield. Also, if a parallel plate condenser has one plate surrounded by a ring to diminish edge effects, this ring is a guard ring. However, if the back of the guarded plate is surrounded by a housing to prevent any electrostatic lines from reaching the back of the plate, this housing is a shield. While these usages of the terms "shield" and "guard" might, to advantage, be modified, they will be adhered to throughout the paper.

Guards and shields have been largely developed in connection with apparatus for measuring some electric phenomenon. The present paper will consider this use only. However, the general principles that apply to the guarding and shielding of measuring apparatus can often be applied to such different problems as reducing the danger to life in high-voltage systems and protection from lightning.

II. Guarding and Shielding Apparatus Against Leakage Currents

Guarding apparatus against leakage currents is necessary whenever the solids used in the insulation of the apparatus have so low a volume resistance or surface resistance as to allow a leakage current to flow which will influence the measuring apparatus. Shielding apparatus against leakage currents is necessary whenever the fluid (air, oil, etc.) in which the apparatus is immersed permits a measurable current to flow. In ordinary measurements, guarding against leakage is required more often than shielding.

The leakage currents against which apparatus must be guarded may flow through the material of which the solid insulators are made or through a film of moisture or dirt on the surface of the insulators. With some insulating materials (amber, ceresin, hard-rubber, paraffin, silica-glass, sulphur, etc.), the leakage current through the material of the insulator is negligibly small in well designed apparatus. Surface leakage, on the other hand, is often troublesome, especially since it may be quite variable. The method of guarding, how-

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29-85

ever, is the same for both volume and surface leakage.

To guard an apparatus as completely as possible every insulator is made in two parts. One part of each insulator supports a piece of metal (the guard) on which is mounted the second part that in turn supports the measuring apparatus. An auxiliary electric circuit, similar to the one in the measuring apparatus, is then set up and so connected to the guards that the potential of each guard is substantially the same as that of the conductor in the measuring apparatus which it supports. Hence there is no tendency for a leakage current to flow through or over the second part of the insulator which is between the measuring apparatus and the guard.

In practise no apparatus can be completely guarded. There is very sure to be some potential difference between the different guards and the measuring apparatus. No amount of guarding will take the place of good insulation. However, in some cases where it is

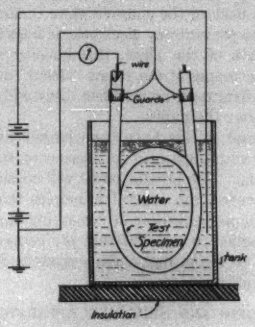

Fig. 1—Diagram to show Method of using Guards in the Measurement of the Insulation Resistance of a Wire or Cable

impractical to make the insulation resistance very high, excellent results can be obtained by guarding. One example is the well known Price guard wire[2] that is regularly used in measuring the insulation resistance[1] of wires and cables. The principle of this is illustrated in Fig. 1.*

Without the guards, the leakage current over the surface of the insulation between the wire and the water may be even greater than the current through the insulation, thus making valueless any determination of the insulation resistance of the cable. With the guards at the same potential as the wire, this leakage current does not reach the wire, and hence does not produce a deflection of the galvanometer. However, it is often difficult to maintain the guard at the same potential as the wire. This may be caused by the measuring current producing a drop in potential across the resistance of the galvanometer, or by electrolytic action at the guard or wire. When there is a potential difference between the guard and wire, this will cause a flow of current through the galvanometer. This will be negligible only if the insulation resistance between the wire and guard is sufficiently high.

It is sometimes desirable to employ a guard to prevent leakage currents from lighting and power circuits from reaching a measuring circuit. Where it is feasible, this guarding can be accomplished by mounting the entire measuring circuit, including the source of e. m. f., on grounded metal plates. Where such a procedure is not feasible, portions of a circuit which are difficult to insulate may, to advantage, be guarded. For example, in Fig. 1, the galvanometer may be guarded by mounting it on a grounded metal plate. Also, if the conductor which connects the galvanometer to the wire of the cable is supported on solid insulators, each of these may well be mounted on a grounded guard plate. However, there is little advantage in guarding the insulators that support the conductor which connects the battery to the water surrounding the cable.

Shielding air-immersed apparatus against leakage currents is necessary only when the apparatus is sensitive to exceedingly small currents or when the number of ions is unusually large. No general statement can be made as to the sensitivity of apparatus that requires shielding. It is customary and usually necessary to shield the essential portions of a circuit when an electrometer[3] is employed as the measuring instrument. It is usually unnecessary to shield when using a galvanometer unless the instrument is exceedingly sensitive or the surrounding air highly ionized.

III. Guarding Apparatus to Simplify Its Theory

In some types of measurements, it is desirable to determine the constants of the apparatus from the known properties of the surrounding medium. In such cases a guard may be employed either to confine the field of the measuring apparatus to that portion of space where the properties are known, or to modify the field so that the constants of the apparatus can be computed from its dimensions. In many cases one guard may perform both of these functions. A well known example is a guard-ring air condenser. Here the guard ring is useful not only to confine the lines of force to the air so that the dielectric loss is negligible, but also to make the field between the plates uniform so that the capacitance can be readily computed.

While guards are often useful, they should be used with care lest they introduce errors which are greater than those they are supposed to rectify. The possibility of errors is well illustrated by the guard-ring condenser. When it is used as a standard having zero power factor in a-c. measurements, the assumption is

1, 2. For references see Bibliography.

*Here the current which flows over the surface of the insulation is, by means of the guards, shunted around the measuring instrument.

tacitly made that the potential of the guard ring is at *every instant* the same as the potential of the guarded plate. This requires not only that the two alternating potentials shall have the same magnitude and be in the same phase, but also that they shall have the same wave form. Hence, not only must the impedance of the guard circuit be properly adjusted, but also it must be protected from corona and other effects which would change its wave form. Again, when the guard ring is used to simplify the computation of the capacitance, the edge correction is not zero unless the gap between the guard and plate is infinitely narrow. Hence, a correction[4] must, for precise work, be applied to the computed capacitance to allow for the finite width of the gap.

IV. Shielding of a Space from Electrostatic Fields

A given space can be perfectly shielded from external electrostatic fields by surrounding the space with a continuous metallic enclosure which is maintained at a definite potential, usually at earth potential. Provided this could always be used, nothing further would have to be written concerning electrostatic shields. The shield, however, may react unfavorably on the apparatus that is being shielded. In such cases it may be necessary or advisable to omit the shield entirely, or to use less effective methods of shielding.

The most troublesome reaction between apparatus and shield is caused by capacitance. The placing of a grounded shield close to a piece of apparatus will very markedly increase not only the capacitance to earth but also the capacitance between parts of the shielded apparatus. As an example consider a shielded resistance coil. Often such a shield is used to make definite the capacitance between the different parts of the coil, particularly that between the terminals. However, if the shield is placed very close to the terminals, slight changes in the position of the shield (such as those produced by changes in temperature) may cause changes in the capacitance between the terminals greater than the changes which would have occurred had no shield been present. This difficulty can readily be overcome by making the shield larger.

When it is not feasible to use metallic sheets in constructing a shield, woven wire netting or a grating of parallel wires may be found satisfactory. In such a case the shield should be larger than when using plates since the external electrostatic field may penetrate for short distances inside the netting or grating. The effectiveness will depend upon the size of the wires and upon the spacing between them. Maxwell[5a] has discussed the theory of the grating but the effectiveness of a netting can only be determined by experiment.

When any form of metallic shield is objectionable, some shielding can be accomplished by enclosing the space with a housing of material which has a high dielectric constant.[5] While the theory of this method of shielding is well developed, it is seldom used in experimental work.

V. Shielding of a Space from Magnetic Fields

A given space can be shielded from a constant magnetic field only by surrounding the space by magnetic materials. Perfect shielding is not possible but good results can be obtained by the use of suitable materials provided they are in the most efficient shapes.

The shielding of spaces against the earth's magnetic field is probably the oldest problem in magnetic shielding. It early became of importance in the design and use of needle type galvanometers. Here the shielding might not be important if the earth's field were perfectly constant. However, the changes are of such long period that the effect of these changes can be made negligible only by reducing the entire field to a very small part of its original value.

The early attempts at magnetic shielding consisted in enclosing the space in a single shell of iron.[6] Later it was shown that two or more shells could be used to advantage. This problem was first treated theoretically by Rücker[7] for two and for three spherical shells. Later DuBois[8] gave a more complete treatment for two spheres and for two cylinders. Wills[9] extended this to cover three spheres and three cylinders. More recently Esmarch[11] and Dye[14] have given the general expression for the shielding produced by any number of cylinders provided their successive radii form a geometric progression.*

All of these derivations have assumed that the external field is uniform and of such a small value that the permeability of the shields can be considered constant. Also the shields are assumed to be concentric and the axes of the cylindrical shields are assumed to be perpendicular to the lines of magnetic force. The cylindrical shields are assumed to be sufficiently long so that end effects can be neglected. In practise this means that the length should be twice the diameter.

The important results of the theoretical investigations can be stated as follows:

1. The shielding ratio (defined as the ratio of the external to the internal magnetic field) is approximately

*Assume there are n shields of permeability μ, the successive radii (starting from the smallest) are $r_1, r_2, \ldots r_{2n}$. The odd subscripts give the inside radii and the even subscripts the outside radii. The shielding ratio, g, ($= H_e/H_i$ where H_e is the external field and H_i is the field inside the shield) is given by the formula

$$g = K_n r_1^2/r_{2n}^2$$

where $K_n = \{ 2\beta + [\beta - 1]^2 \mu/4 \} K_{n-1} - \beta^2 K_{n-2}$

in which

$$\beta = \frac{r_2^2}{r_1^2} = \frac{r_4^2}{r_3^2} \cdots = \left(\frac{r_{2n}^2}{r_1^2} \right)^{\frac{1}{2n-1}}$$

Since K_n depends on K_{n-1} and K_{n-2}, the values for K_1 and K_2 are not given by this formula. They are

$$K_1 = 1 + (\beta - 1)\mu/4$$

$$K_2 = \{ 2\beta + [\beta - 1]^2 \mu/4 \} K_1 - \beta$$

Dye[14] gives curves to aid in computing the shielding ratio.

proportional to the permeability of the material of which the shields are constructed.[8]

2. The thickness of the shields should not be great. For example, a single spherical shield, the thickness of which is greater than $\sqrt{2 r_1^2/\mu}$, (r_1 = internal radius of shield and μ is permeability), will be improved by removing a spherical lamina, and thus reducing the total amount of shielding material.[7]

3. For both spheres[7] and cylinders[9] the best thickness and spacing is that in which the radii of the successive bounding surfaces of the shells are in geometric progression. From this it follows that any increase in the volume of the space to be shielded requires the same relative increase in the volume of the shields.

The experimental verification of the best thickness of shells was undertaken by Nichols and Williams.[10] They constructed three cylindrical shields of silicon steel in which the successive radii had approximately the same ratio. The shielding ratio was measured for various combinations of these shields and also for a set of five shields which were selected at random from available materials. The results are given in Table I.

TABLE I
CYLINDRICAL SHIELDS OF NICHOLS AND WILLIAMS

Shields	Weight kilos	Inner radius cm.	Shielding ratio H_e/H_i
A. Set of three from cast silicon steel. Radii in geometric progression		2.55	
Inner only	1.9		20
Inner intermediate	7.5		300
All three	26.9		4000
B. Set of five from soft iron water pipes		2.65	
Inner only	1.5		20
Inner + 1 intermediate	3.1		100
Inner + 2 intermediate	4.3		250
Inner + 3 intermediate	7.2		700
All five	9.3		2700

While the method which Nichols and Williams used in determining the shielding ratio has been questioned,[12] the results undoubtedly have some relative significance. The advantages gained from the use of specially prepared shields is relatively small.

Within a few years some experimental work has been done to determine the shielding effect of spirals made from high permeability material. In some cases the shield was made by winding in a tight spiral a single sheet of magnetic material. In other cases, a sheet of magnetic material and a sheet of non-magnetic material (copper or paper) were placed together and wound into a spiral so that in the resulting shield the magnetic material was interleaved with non-magnetic material. Characteristic results are given in Table II. These results show the value of inter-leaving, though the exact thickness of the inter-leaving does not seem to be important.[13] The increased shielding that can be obtained by the use of the high permeability alloys of nickel and iron is well illustrated by the values of Hill[15] for mumetal.

The above discussion has dealt only with shielding from constant magnetic fields, yet the same principles apply to the shielding from alternating magnetic fields provided the shielding material is so laminated that the effect of eddy currents in the shields is negligibly small. Under this condition the magnetic field can at any instant be considered as a quasi-stationary state; *i. e.*, it is unnecessary to consider the rate of propagation of the field. However, this is not the case when eddy currents are present in appreciable amounts, for which case the discussion of magnetic shielding coincides with the following section on the shielding from magnetic waves.

VI. Shielding a Space from Electromagnetic Waves

The problem of shielding a space from electromagnetic waves depends on the frequency of these waves. When the frequency is very low, the electric and magnetic fields change so slowly that the shielding problems are nearly identical with those for stationary fields. When the frequency is very high, shielding becomes a problem in the optical opacity of materials. For intermediate frequencies, consideration must be given not only to the instantaneous values of the electromagnetic and electrostatic fields, but also to the rate at which these fields are changing.

Since the shielding from stationary fields has already been considered, this discussion will be confined to principles which are applicable to a changing magnetic and electrostatic field; that is, to electric waves. The frequency range from 10 to 10^8 cycles per sec. will be considered. In this range, the wavelengths vary from 3×10^7 to 3 meters. Hence the linear dimensions of the region to be shielded are generally small relative to the wavelength.

The shielding from electromagnetic waves is brought about by eddy currents which the waves produce in the shield. These eddy currents radiate waves into space surrounding the shield. Some of these waves may

TABLE II
SHIELDING RATIO OF DIFFERENT MATERIALS

Reference	Kind of magnetic material	Thickness—cm. Magnetic material	Thickness—cm. Interleaves	Weight kilos	Inner diam. cm.	Outer diam. cm.	Shielding ratio H_e/H_i
13	Transformer steel	0.26	0.14		4.	8.	180
		0.26	0.		4.	8.	50
	Cast iron	4.	..		4.	8.	20
15	Mumetal	0.15	0.25	0.5	10.	12.	10,000

impinge on the apparatus which is producing the waves. This may alter the current in the sending apparatus and thus affect the waves that are being sent out. This interaction between sending apparatus and shield is decreased as the distance between them is increased. Hence, most theoretical investigations have assumed that this distance is so large that the reaction on the sending apparatus can be neglected. Since in any particular case the waves emanate from a single electric circuit, the electric and magnetic vectors at a given place have a definite direction. In the nomenclature of optics, the waves are polarized. The three vectors which determine (1) the direction of motion of the wave, (2) the direction of the electric field, and (3) the direction of the magnetic field are mutually perpendicular. Hence, they may be taken as the axes of a rectangular coordinate system. The effect of a shield will depend on its orientation relative to this coordinate system.

The shielding power of a conductor or system of conductors which is used to protect any given region from waves having a definite direction and wave front is usually defined as the ratio of the decrease in the amplitude of the wave when the shield is present to the original amplitude of the wave at the midpoint of the region.*

Expressed in symbols $S = (A_0 - A_s)/A_0$ where S is the shielding power, A_0 the amplitude of the wave when *no* shield is present, and A_s the amplitude when the shield is present. If shields are so arranged that in a given space no outside electromagnetic waves can enter, then throughout that space the shielding is 100 per cent, or perfect.

The eddy currents which are set up in the shield may be produced either by the electric field of the wave or by the magnetic field, or by both.[17] The method by which waves produce eddy currents can be made clear by considering some particular cases. First, consider that observations are being made at a point due south of and at considerable distance from a station which is sending out waves from a flat top antenna. Then at the observation station the wave can be considered as a plane wave in which the electric field is vertical, and the magnetic field horizontal in an east and west direction. If a series of vertical wires are erected in an east and west plane, the electrostatic field of the waves will produce eddy currents in the wires, which to some extent will shield the space to the south of the wires. If the wire system is turned through 90 deg. around a north and south axis, then no eddy currents will be produced in the wires by either the electrostatic or magnetic fields and hence no shielding. If the parallel wires are joined in pairs so that they form a series of long, narrow loops, eddy currents will be produced in the loops when they are so placed that the magnetic field of the wave threads through these loops. This requires that the perpendicular to the plane of the wires shall lie in the direction of the magnetic vector of the wave. Then the eddy currents set up in the loops produce a magnetic field which is opposed to that of the wave. This then produces a shielding beyond the wires.[17]

As another example of shielding, consider the shielding power of a cube of metal which may or may not be laminated. Assume a wave like that in the preceding paragraph where the direction of the wave is south, the electrostatic vector vertical, and the magnetic vector horizontal and in the east and west direction. The metal cube rests on a horizontal plane with its faces in the directions of the cardinal points. If the cube is solid metal, eddy currents will be produced by both the electric and magnetic vectors so that points just beyond the south face of the cube will be very well shielded. If now the cube is laminated by means of planes parallel to the east and west faces, the eddy currents will not be affected and the shielding is the same as with a solid cube. Next assume that the laminations are produced by planes parallel to the north and south faces. In this case the electric vector would produce eddy currents as in both the previous cases, but the magnetic vector would not. Hence, the shielding would be reduced. If now the laminations are produced by horizontal planes, the eddy currents from both the electric and magnetic vectors are greatly reduced. Such a block would produce very little shielding.

In order to compute the shielding which will be produced by any arrangement of solid metallic bodies, it is necessary first to compute the eddy currents that will be produced in the metal. The formula for determining the current density in a particular case is generally determined by the solution of Maxwell's differential equations of an electromagnetic field. It results that the eddy currents in a massive conductor are always a function of $\sqrt{\frac{f\mu}{\sigma}}$, where f is the frequency of the wave, μ the permeability of the material, and σ its resistivity. This shows that the most effective shielding materials are those having high permeability and low resistivity.

Formulas for computing the eddy currents produced by electric waves have been derived in certain cases.[16] However, in relatively few cases have the theoretical deductions been carried to a point where the shielding power can be computed. Some experimental work[15] has shown that there is a qualititive agreement between theory and experiment.

VII. Limitations of Shielding

In a previous section, the effect on any measuring apparatus of an electrostatic shield in which it is enclosed was discussed. In any given case, the reaction

*Note that this is different from the definition of shielding ratio. For example, a shielding power of 100 per cent indicates perfect shielding, while the shielding ratio must be infinity to indicate perfect shielding.

between the shielded apparatus and an electrostatic shield can be expressed in terms of the coefficients of capacitance. Likewise a magnetic shield or a shield which protects against electromagnetic waves reacts with the apparatus which it is shielding. However, in neither of the latter cases can the amount of reaction be expressed by a simple formula.

It has already been pointed out that an electrostatic shield may introduce errors in a measuring apparatus for which it is difficult to make a correction. With a magnetic shield, on the other hand, the effect of the reactions is to change the sensitivity of the apparatus. As the apparatus is calibrated with the shield in place, no error results from the use of the shield. However, the reactions are so complicated, involving eddy currents, hysteresis, permeability, etc., that each separate type must be subjected to an experimental investigation.

In absolute electrical measurements, where the constants of an apparatus are computed from the mechanical dimensions, shields must generally be avoided. As an example, the inductance of a coil can often be accurately computed from its dimensions. However, the formula for making the computation is developed on the assumption that the permeability of the surrounding space is unity; that is, that there are no magnetic bodies in the surrounding region. Evidently a magnetic shield would vitiate this fundamental assumption. Moreover, the derivation of the formula takes no account of the capacitance between the parts of the inductance. While this generally produces a negligible effect, yet the presence of an electrostatic shield will so increase the capacitance between parts that an appreciable effect is produced. Hence, an electrostatic shield must be avoided.

VIII. Methods for Obviating the Necessity of Shielding

Astatic instruments are often used for obviating the necessity of shielding. In an astatic piece of apparatus two similar parts are constructed. In the finished apparatus these parts are rigidly connected mechanically, but the electric and magnetic circuits are so arranged that an external electric or magnetic field will produce equal and opposite effects in the two parts. When the apparatus is placed in an electric or magnetic field which is uniform throughout the region which the apparatus occupies, then changes in the magnitude of this field will not affect the indications of the instrument. This method has been applied to galvanometers and inductors.

Sometimes shielding can be avoided by counteracting the parasitic field by a field which can be controlled. As an example, it is often desirable to have a region in which the magnetic field is much less than the ever present magnetic field of the earth. If the required region is small this can be accomplished by surrounding the region by a rather large coil of wire, the axis of which lies in the direction of the earth's field, and sending through the coil a current of such magnitude and direction that the earth's field is neutralized at the center of the coil. If a larger region is required, a solenoid can be used, the axis of which lies in the direction of the earth's field. This has the disadvantage that the region is surrounded by the solenoid so that apparatus placed in this region is not readily accessible. To obviate this difficulty, Helmholtz coils can be used. They are a pair of circular coils having identical dimensions and mounted coaxially. The distance between the planes of the coils is equal to the radius of one of them. When the same current flows through both of these coils so that the magnetic field of both of the coils is in the same direction, then there is a rather large region between the coils where the magnetic field is very nearly uniform. Hence, if the axis of the coils is in the direction of the earth's field, a fairly large region between the coils can, by adjusting the current, be made to have approximately zero field.

The above method of neutralizing the earth's field is satisfactory only when this field is very constant and when the current through the coils does not fluctuate. If the current through the coils could be made to vary at the same time and by the same amount as the earth's field varies, then the compensation could be made perfect at all times. The author suggests the following method for accomplishing this variation in current. Mount a series of copper disks on an insulating shaft which can be driven at constant speed by some non-magnetic motor, such as an air turbine. By placing one brush near the center and another at the circumference, each disk becomes a Faraday wheel generating an e. m. f. proportional to the component of the magnetic field in the direction of the axis of rotation. Connect these in series with each other and with a pair of Helmholtz coils having their axes parallel to the axis of the generator. By a proper adjustment of the size and number of the disks and of the resistance in the circuit, the compensation should be perfect provided that at every instant the earth's field is the same at the generator and at the coils. To compensate for two components of the earth's field, two generators are required and two sets of Helmholtz coils, the latter being mounted concentrically. For three components, three generators and three concentric sets of Helmholtz coils are required.

IX. Conclusion

In conclusion it may be said that the general principles underlying the shielding and guarding of electrical apparatus are well known, but that their quantitative application by means of mathematical analysis is often difficult or at present impossible. Hence, shields and guards should be used with care. Whenever possible the shielded or guarded apparatus should be subjected to special experiments to show that the effect on the apparatus is not detrimental.

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Discussion

T. F. Peterson: Dr. Curtis' efforts in the direction of trying to define clearly the words "shielding" and "guarding," are very praiseworthy.

Ordinarily a shielding process is one of protection. This dictionary definition might be said to apply when an instrument, a conductor, or a device is protected from some extraneous or foreign influence. However, we in the cable industry have attached a somewhat modified meaning to the word. Under certain conditions a cable or wire insulation is attacked by the deteriorating influences which certain electrical connections give rise to. As an example, if the air outside of an insulated conductor at high potential breaks down, that is, ionizes, there results an ozone formation which may deteriorate the insulation. To offset this, the conductor is shielded, but in reality the air surrounding it is shielded from electrical stress. As far as the cable is concerned, the shielding offers protection of a secondary order since it is against the deleterious influences incidental to the ionization.

Another very usual form of shielding has to do with the protection of the entire outside world, we might say, from the influence of the thing which we usually say is shielded. A subject which came to my attention very recently is that of shielding ignition cable in airplanes so that there would be no radio interference. In this instance we are trying to confine the "culprit," in other words, to protect everything from the action of the conductor.

Still another case of shielding is that in which fields are adjusted to "guard" parts of circuits. This has been dealt with quite fully in the paper.

I am citing the various shades of meaning attached to the word "shielding," because I feel that some of the practical aspects of the question are worthy of note. That great benefits can accrue from applied shielding has been clearly illustrated by the case of Type H Cable. Undoubtedly similar benefits may be obtained in other fields.

The one problem to which I would direct attention is that of ignition wire. Shielding applied to such cables may be used in three senses. First, it may protect against influences inwards, that is, against the interaction between conductors. Second, it may protect against the harmful effects of ozone which is generated incidental to normal operation. Third, it may serve as protection of outside equipment such as radio, etc., from the electromagnetic radiation from the circuit. The advantages to be gained by shielding ignition wire would seem to be of sufficient importance to warrant careful attention.

W. B. Kouwenhoven: We are very much indebted to Dr. Curtis for his definitions of the terms "guard" and "shield." These two terms have often been used synonymously and more or less loosely by various authors.

In the last two air condensers that I have designed and built, I have surrounded the main electrode with a guard ring and also covered the back of the main plate with a shield to protect it from stray fields. The current that flows to the guard plate in this type of condenser is made up of charging current and also of leakage current that flows through and over the surface of the insulation separating the guard from the high-tension plate. I have largely eliminated the leakage current from the guard circuit by making each insulator that separates the guard and high-tension plates in two parts and inserting small grounded pieces of metal. These prevent the leakage current from flowing to the guard plate and bring the current in the guard circuit more nearly into phase with the current in the main circuit. This facilitates the balancing of the guard circuit against the main plate circuit.

As Professor Dawes points out, the operation of a bridge is simplified if the guard and main circuits have practically the same capacity. The insulation resistance between the main and guard circuits should also be kept at infinity. If under these conditions the guard circuit is balanced against the main circuit as to magnitude, a small unbalance in phase will affect the capacity ratio only and not the phase-angle determination. This may be easily seen from a study of the vector diagram.

We have just completed our investigation on the phase angle of air condensers and I hope that the paper will be ready for the Winter Convention. In that work we found that the charging current in a properly designed air capacitor leads the voltage by angle that does not differ from 90 degrees by more than three seconds, provided the relative humidity is not excessive.

I. M. Stein: I want to mention the subject of shielding in connection with standardization. In the discussion of the papers I think it has not been clearly brought out that there is likely to be a difference in the requirements between an apparatus needed for standardization and that needed for investigational or routine work in individual laboratories. Professor Dawes, I think, mentioned the Electrical Testing Laboratories condenser, which I understood him to say rendered considerable assistance because it was a standard that could be shipped. Unfortunately, standards of that kind are not self-registering, and you cannot use them without some sort of a measuring set-up to go along with them. There

would seem to be need for a standard which is comprehensive, one that includes a condenser and that which is necessary to measure the condenser.

Such apparatus it seems to me must be shielded, or guarded, or both, if such a thing is possible,—and I think it is possible. If you don't shield or guard you depend upon a high degree of skill in the user, and probably upon a great deal of available space in which you set up the apparatus, and a great deal of time to make the measurements. If you have a completely shielded and guarded set-up you can probably make out with considerably less technique, less space, and less time. Where dielectric-loss measurements are being used in purchase specifications, it would seem necessary to have definite standards, transfer standards, if you choose, that can be sent to a central bureau such as the Bureau of Standards, for certification and shipped back to the laboratories where they are to be used as secondary standards. Then in individual laboratories the user can make his own choice as to whether he is going to use a shielded equipment or a non-shielded equipment for his routine work, or special investigation. It seems essential to have a shielded and guarded equipment for such transfer between laboratories. This subject involves the Bureau of Standards, and I should like to ask Dr. Curtis in closing his remarks to say a word as to whether for standardizing work and transferring between laboratories, he does not feel that the measuring apparatus must be shielded and guarded.

C. L. Dawes: It is very gratifying to learn that through Dr. Kouwenhoven's investigations the high-voltage air condenser is to be placed on a definite quantitative basis. Many of us have been somewhat apprehensive at times due to the unknown effects of humidity and the losses which it may cause. In order to make certain that losses in our air condenser did not occur in humid weather when our laboratory was not heated, we have applied heat from resistor units placed near the condenser. We are looking forward with interest to the publication of Dr. Kouwenhoven's investigations.

I am in accord with Mr. Stein that we should have more and better dielectric standards. I recall that in the late eighties it was necessary to calibrate d-c. ammeters by weighing the electrolytic deposition of copper. It required nearly an hour to calibrate each point. At the moment it seems that we are just a little beyond this stage in the matter of dielectric standards. In time, however, these standards will undoubtedly reach the status of our present electric-circuit standards.

H. L. Curtis: The question of definition which Mr. Peterson has brought up, is one of the most trying and difficult things we have to deal with. We all have to speak the same language and therefore we ought to have definite ideas when we use various terms. However, language is a matter of growth and things come to have definite meanings because we use them in different ways. It is worth while occasionally to stop and see whether we all are speaking the same language, and it was with that in view that I tried to bring out these definitions.

The question of using shielded and guarded apparatus that Mr. Stein has brought up, when transferring standard apparatus from laboratory to laboratory is, without doubt, very important. We have been trying to get the Leeds & Northrup Company for some years to mark their condensers as to whether they were shielded, and if so the shielded side, so they would be perfectly definite in handling and dealing with them from laboratory to laboratory. That is not always done now.

The interest of the people at this convention apparently has been very much toward high-voltage measurements where conditions are difficult and arrangements are very special. Many circuits that have been developed apply particularly to conditions that are developed there.

I wish, however, to call attention to the fact that there are cases where shielding and grounding positively cannot be used. One particular case I have to deal with is the question of a coil the inductance of which can be computed from its dimensions. We base our fundamental electrical units in certain cases on such apparatus, and one requirement for the computing of the inductance is that there shall be no conductor close to the coil.

I have perhaps throughout my paper emphasized rather the difficulties in connection with shielding because of the fact that so often these are overlooked and people use shields and guards with the feeling that no difficulties exist.

Some Problems in Dielectric-Loss Measurements

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Member, A. I. E. E. Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The measurement of dielectric loss in high-voltage cables presents many problems that are not obvious until such measurements have actually been attempted and the results checked and analyzed. Some of these problems are not simple, and their solution is obtained only after considerable effort and research. This paper presents some of the problems that the authors have encountered in this type of work, together with the methods used for their solution.

The bridge method, which is used for the measurement of dielectric loss, is described in some detail. Details of the auxiliary equipment, such as galvanometer, amplifiers, air condenser, shielding, and special methods of bringing the shielding to the proper potential are also given.

If accurate results are to be had in dielectric loss measurements, the entire bridge, including cable, air condenser, and leads, must be properly shielded and the capacitance between leads and shielding should be made small. Errors of large magnitude may be introduced in the results if these precautions are not taken. It is entirely insufficient to shield the bridge and ground the shielding directly. At times shielding cables may increase the measured losses.

In measuring the capacitance of the air condenser, all stray capacitance to its guard rings and to surroundings must be eliminated. The paper gives a method of accomplishing this.

The voltage across the voltmeter coil of a high-voltage transformer is frequently taken as a measure of the voltage across its secondary. In some methods of measurement this voltmeter-coil circuit is an integral part of the measuring circuits; consequently, it is desirable to be able to determine accurately the ratio of this voltmeter-coil voltage to the total voltage, and the phase difference existing between these two voltages. Methods of making such measurements are described and considerable quantitative data are given for typical testing transformers under various conditions of load.

Methods of determining the accuracy of the bridge are described, and observed results given. With one type of measurement, the ratio and the current accuracy may be checked simultaneously.

The accuracy of the bridge as a power-measuring device may be determined by inserting known resistances into its arms and comparing computed results with observed results. Due to unsuspected leakage and stray-capacitance currents, however, the introduction of such resistances may cause large errors. Methods for minimizing such currents, as well as analyses of proper and improper methods of making such measurements, are given. Some of the effects of harmonics are described briefly. In this type of measurement they probably do not introduce error, but at times they are troublesome in that they produce anomalous effects in the measuring apparatus.

* * * * *

INTRODUCTION

DURING the past few years the authors have devoted considerable time to the development of methods for measuring dielectric losses at high voltages. Particularly during the past four years, numerous problems have been encountered during the progress of the research in ionization in impregnated paper-insulated cables being carried on at Harvard University under the auspices of the Joint Committee on Paper-Insulated Cables of the N. E. L. A., A. I. E. E., and A. E. I. Co. The authors believe that as a result of this type of research, which has been stimulated by cable users and manufacturers, engineers have advanced, by many years, the science of measuring very small power at extremely high voltage and very low power factor,—a most difficult combination. However, since factors which ordinarily need not even be considered may introduce large errors, one must always be on guard in making such measurements.

Many of these factors are well known to those who have made a study of the problem. On the other hand, in discussing some of these problems with cable engineers, the authors have been requested to present them in the form of a paper, in order that others might perhaps be forewarned of the many pitfalls into which some have already stumbled. Moreover, it is hoped that the discussion may bring out experiences of others, from which all may derive benefit.

There is probably no type of measurement in which one may be more easily deceived than these dielectric-loss measurements. The losses vary with temperature, with the time of electrification, the cycle of electrification, and with the handling of the specimen. Hence, accurate checks of measurements on cable samples among different laboratories cannot be expected, and it is thus difficult to prove that a measurement is in error. Under the circumstances, it is remarkable that results check as closely as in many instances they were found to do. Curves obtained with certain measuring apparatus may appear perfectly rational, and yet because of phase-angle defect or wrong transformation ratio, both of which may be constant in magnitude, the curves are actually in error. It is most difficult to check a given apparatus by the introduction of known losses in the circuits, as will be shown later. The calibrated Leyden jars, which are now available at the Electrical Testing Laboratories, have in large measure overcome some of these difficulties.

THE BRIDGE METHOD

The authors use the mutual inductance bridge, which has already been described by them.[1] The complete bridge with its shielding and guard connections, as now used, is shown in Fig. 1.

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1. For references see Bibliography.

The simplified bridge circuit is shown in Fig. 2; one arm of the bridge proper consists of the air condenser C_1 and the series resistance R_1; the other consists of the cable (or other dielectric) C_2 in series with the primary of a mutual inductance M and the series resistance R_2. The secondary of the mutual inductance is connected from the point b at one terminal of R_2 to one terminal of the detector D; the other terminal of D is connected to point a at one terminal of R_1. When the bridge is in balance, the angle of defect of the cable, $\psi = \tan^{-1} \frac{M\,\omega}{R_2}$, where M is the mutual inductance in henrys and $\omega = 2\,\pi\,f$ where f is the frequency in cycles per second. This assumes that the air condenser has no losses.

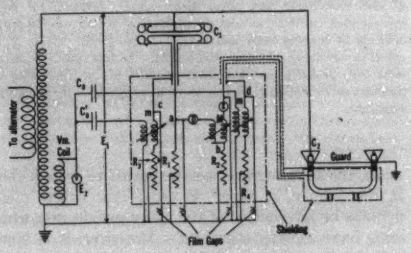

Fig. 1—Complete Connections of Bridge Circuit

With low values of power factor,

$$\cos\theta = \sin\psi = \frac{M\,\omega}{R_2} \qquad (1)$$

where θ is the power-factor angle and ψ is the angle of defect of the dielectric. The capacitance C_2, for low values of power factor, is given by

$$C_2 = C_1 \frac{R_1}{R_2} \qquad (2)$$

$$\text{The power} = \frac{E^2\,C_2\,\omega^2\,M}{R_2} = \frac{E^2\,C_1\,R_1\,\omega^2\,M}{R_2^2} \qquad (3)$$

In checking the bridge by various methods, several errors which were not at first understood appeared and some were run to earth only after long and painstaking investigations, a number of which will be considered later.

The Detector

Without a sensitive detector, the bridge loses most of its advantages as a precision instrument. For power frequencies from 20 to 125 cycles per second which we use for cable investigations, we have found the tuned iron-vane vibration galvanometer the most satisfactory type of detector. Electromagnetic excitation is employed and the tuning is accomplished by varying the exciting current. An advantage of this type of detector is the fact that the detecting coil may be wound for a wide range of impedance. Hence, the impedance may be made to correspond to the character of the circuit with which it is associated. Since bridges for measuring dielectric losses must necessarily have high-impedance arms, the coil of our galvanometer is wound with No. 40 silk-covered wire. Its impedance is 2200 $+ j$ 280 ohms at 60 cycles.

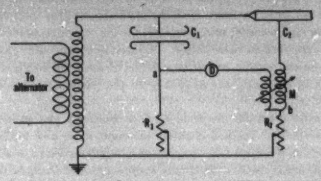

Fig. 2—Simplified Connections of Bridge Circuit

Another advantage of this type of detector is the fact that the deflection is readily maintained on the scale, even with considerable unbalance, by merely detuning slightly. As the condition of balance is approached, closer tuning is easily obtained.

The degree of sensitivity is tremendously increased by the use of an amplifier in conjunction with the detector. One of the authors has already published a description of the amplifier which they have developed.[2] There are many refinements in connection with its use, however, which might be useful to those intending to employ such an amplifier. Lead storage cells are used for the A, B, and C batteries. The gassing which persists for some time following charging of the batteries is very troublesome. It causes vigorous and disturbing "kicks" of the vibration galvanometer. To avoid the delay of waiting for the gassing to cease, two sets of batteries

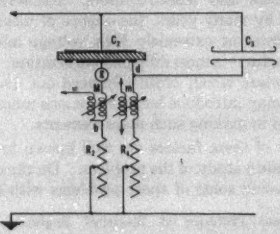

Fig. 3—Shunted-Guard Circuit Balance

are used with suitable switches for throwing on charge, or in service. Hence, it is always possible to have in service batteries which have attained a steady condition.

The galvanometer, amplifier, batteries, and resistances are within a wire enclosure (Figs. 1 and 7) to shield the apparatus from electrostatic induction. Since the screening is maintained at the galvanometer potential, the effects of capacitance to ground are minimized.

This screening does not shield electromagnetic disturbances. For example, stray magnetic fields produced by conductors carrying alternating current, by rotating field structures of alternators, and by spark discharges react readily on the vacuum tubes of the amplifier, causing trouble. An iron enclosure over the amplifier may shield against such disturbances. It is more desirable, however, to remove the source of the stray field.

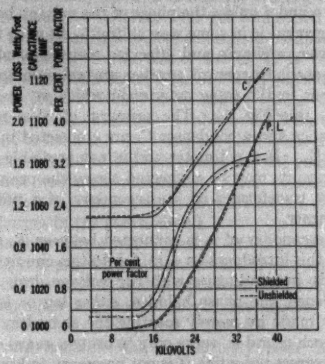

Fig. 4—Effects of Shielding 10-ft. Length Single Conductor Cable at 60 Cycles and 20 Deg. Cent

Mechanical vibration, such as is due to rotating machinery, may affect the amplifier. If the vibration is synchronous with the power supply, the balance with the amplifier may be false and differ appreciably from that without the amplifier. The authors have found that porous red rubber sponges under the amplifier eliminate vibration.

The amplifier may oscillate at high frequency, usually caused by capacitive feed back of the last tube

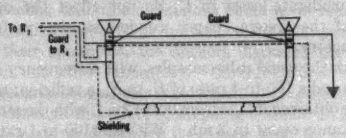

Fig. 5—Method of Double Guarding

to the first. The presence of conducting material, such as shielding, aggravates this difficulty by increasing the capacitive coupling. If near a powerful radio station, the amplifier is frequently affected by the high-frequency electromagnetic waves. Code signals from the Charlestown Navy Yard have been detected with our amplifier. Both of the foregoing effects are almost entirely eliminated by shunting a 1- or 2-μ f. condenser across the grid circuit of the input tube. Such condensers are virtually a short-circuit for the high frequencies, and do not cut down the sensitivity at power frequencies to any harmful extent.

Shielding the Air-Condenser, its Leads, and the Bridge

Careful tests of our air-condenser which has already been described[2] showed it to have an angle of defect of something like 9 min. Investigation showed that this comparatively large angle was due to the fact that the

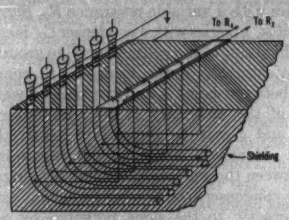

Fig. 6—Connections of Cables in Cable Rack

guard plate surrounding the edges and back of the low-voltage plate of the air-condenser was grounded. Also, the low-voltage plate was connected to the bridge with a 20-ft. length of rubber-insulated, lead-covered wire, the sheath of which was grounded. The capacitance between the low-voltage condenser plate and its shield,

Fig. 7—Illustration of Bridge

and the capacitance to ground of this lead are considerable. The effect of these capacitances is to shunt R_1 with considerable capacitance. This has the effect of making C_1 appear to be a poorer condenser than it actually is. This matter has already been discussed before the Institute.[3]

This error is reduced by moving the bridge near the air condenser. Also, instead of using lead-covered

cable, the connection from the low-voltage plate to R_1 is made with a 1/16-in. (1.6-mm.) diameter brass rod centered within brass tubing 1 in. (2.54-cm.) in diameter, as shown in Fig. 1. (Also see Fig. 7.) The rod is held central by Bakelite washers spaced approximately 2 ft. apart. Hence, the capacitance of the connecting lead to ground is reduced to a very small value. Moreover, instead of grounding directly the guard plate and the brass tube, they are connected to ground through the secondary of a mutual inductance m' in series with a variable resistance R_3, Fig. 1. The primary of m' in series with a condenser C_0' is connected from the high side of the voltmeter coil to ground. One of the adjustments of the bridge is to connect the detector between point a and the point c which is at one end of the secondary of m'. By adjusting R_3, points a and c may be brought to nearly the same potential. However, owing to differences in power factor between the working plate and the guard circuit of the air condenser, points a and c cannot usually be brought exactly to the same potential by adjusting R_3 alone. The secondary of m', however, may be made to introduce into the circuit an e. m. f. which is nearly in quadrature with the voltage drop in R_2. Obviously, this e. m. f. may be made either to lead or to lag by varying the position of the secondary of m'. The magnitude of this e. m. f. is controlled by varying either C_0' or m'. Thus, points a and c may be brought exactly to the same potential and no error due to shunted capacitance across R_1 can occur. With the bridge used here, C_0' is of the order of 2 μ f. and m' is approximately 25 millihenrys.

Also, to eliminate capacitance effects between other bridge members and ground, and to shield the low-voltage bridge elements from stray dielectric fields, the entire low-voltage side of the bridge is completely shielded by chicken wire, as shown in Figs. 1 and 7. This shield is connected to point c and its potential balanced to ground with the condenser shielding, by means of R_3 and m'. Hence, the bridge shielding is not connected directly to ground. For safety, however, four film gaps are used, Fig. 1.

Shielding the Cable

As in the case of the air-condenser arm of the bridge, capacitance between the cable sheath and ground, and capacitance to ground of the lead connecting R_2 to the cable conductor are equivalent to shunting R_2 with capacitance. Hence, it is desirable to shield this lead with brass tubing as with the air-condenser lead. Moreover, to eliminate the capacitance between cable sheath and ground and to prevent charges in the neighborhood having influence on the sheath, the cable is almost completely enclosed in screening of ¼-in. square mesh wire, (Figs. 6 and 7). This shielding is connected to the brass tubing. Likewise, the ends of the cable sheath which serve as guards are connected to this wire shield. This entire metallic shielding system is connected to ground through the secondary of a mutual inductance m and the resistance R_4 in series. (Fig. 1.) When the bridge is being brought into balance, the detector is connected between points b and d and the potential difference between these points is brought nearly to zero. As with the air-condenser, there is ordinarily a difference between the power factor of the cable proper and this guard circuit, which produces a phase displacement between the voltage drops across R_2 and R_4. This prevents a perfect balance between points a and d unless the phase of the voltage between d and ground is in some way shifted. As with the air condenser, a simple and effective method of shifting this phase is to connect another mutual inductance m in circuit in the same manner in which m' was connected, (see Fig. 1). The primary of this mutual inductance and a condenser C_0 are connected in series across E_2. If a voltmeter coil is not available, these series circuits may be connected across the primary of the test transformer or the secondary of a potential transformer.

The secondary of m is connected between R_4 and d. Again, this introduces an e. m. f. into this circuit which is nearly in quadrature with the voltage drop across R_4. Hence, the phase balance, either lag or lead, is quickly made by merely rotating the secondary of m. Since both m and m' serve only to bring a guard circuit to the correct potential, it is not necessary to know the actual values of their mutual inductance. Hence, neither m nor m' need even be calibrated, although some type of indicating dial facilitates making the balance. To minimize the effect of stray fields on the amplifier tubes, it is desirable that these mutual inductances be made astatic.

The values of the capacitance C_0 and mutual inductance m depend on the voltage E_2 and frequency, as well as on the electrical characteristics of the guard circuit. Fortunately, this system is a very flexible one. With a given value of m, it is not difficult to find some value of C_0 which will give a balance. Moreover, since moderate losses in C_0 do not affect the operation of the system, paper condensers are perfectly satisfactory.

With the usual cable samples, with a frequency of 60 cycles and a 100-to-1 ratio of E_1 to E_2, a sufficient range can be obtained if C_0 is from 0.2 to 2 μ f. with the mutual inductance equal to 3 mh. With an 1100-to-1 ratio of E_1 to E_2 and this same value of m, it requires approximately 20 μ f. for our system. For a given range of m, the value of C_0 varies inversely as the square of the frequency. Hence, with lower frequencies these values of C_0 and m must be modified accordingly.

With some samples, such as a 10-ft. cable length, the voltage between b and ground may be only 2 or 3 volts. If the capacitance between shielding and ground is small, this last balance may not be necessary. However, it is so easily obtained with the system just described, that now we always make this adjustment,

and at least one factor that might introduce error is entirely eliminated.

Sometimes with small samples, or when inserting resistance in series with a capacitance to introduce loss purposely, it may require a prohibitively large value of R_4 and of m or C_0 to bring the guard circuits to the proper potential. This difficulty is readily overcome by connecting a condenser C_3 to point d as shown in Fig. 3. C_3 may be a non-shielded air-condenser, a plate-glass condenser, (such as is described later), or even a piece of cable. It is seen that this connection amounts only to connecting more capacitance in parallel with the guard circuit. In the test of the E. T. L. condenser, which is described later, without C_3, R_4 was of the order of magnitude of 95,000 ohms and no values of m or C_0 which were conveniently available would bring the guard circuit to balance. By using the plate-glass condenser, R_4 was reduced to 1030 ohms and also the guard-circuit balance was readily obtained. This connection also facilitates the balance when there are corona or "static" discharges in the guard circuit.

Another important function of this cable-sheath shield is to prevent any loss currents or any capacitance currents or any induced charges from high-voltage leads finding their way to the sheath, and hence, being included in the measurement. The effect of such shielding and non-shielding has already been described.[4] Some investigations of this shielding effect have been made in our laboratories. It has been found that shielding *increases* the power loss and power factor, but decreases the capacitance. A typical curve, taken with a 10-ft., 500,000-cir. mil cable sample, with 8/32-in. wall of insulation, is shown in Fig. 4. With the exception of the change of capacitance, these effects are just the opposite of what would be expected; in fact, in the course of dielectric investigations, it has been observed that because of some unforeseen and obscure factors results frequently are the opposite of those anticipated. It may require a prolonged and painstaking investigation to discover these factors.

Feeling that this anomalous effect of increased loss with shielding might be due to some inherent peculiarity of their apparatus, the authors discussed the matter with one other laboratory. Careful tests with other apparatus showed this same effect.

Although this effect of shielding is small, an attempt was made to determine its causes, both by analyzing our bridge circuits and by upsetting the balances of R_3 and R_4. An entirely satisfactory explanation of this phenomenon has not as yet been found, but the matter is still under investigation.

Sometimes at the higher voltages it is impossible to balance the guard circuit because of discharges to the end-bells and sheath, but this difficulty was overcome by the system of double guarding shown in Fig. 5. The end-bells and a short section of sheath are grounded directly. A second short length of sheath serves as the usual guard and is connected to the shielding and hence to ground, through R_4. The higher frequency discharges are thus shunted directly to ground and do not disturb the guard-circuit balance.

In order to expedite the testing of samples, we have constructed cable racks, Fig. 6, which hold six 10-ft. lengths each. These racks are mounted on casters. Hence, the cables can be moved about very readily and when temperature tests are to be made the rack can be rolled into the heating box. (See Fig. 7)

When this rack was first used, the test was conducted on a single cable, with the other five free. Although at first sight the results obtained appeared to be rational, slight variations at times aroused curiosity and led to investigation. The cause of these variations was soon discovered when a slight personal shock from one of the idle cable sheaths was felt, after the high voltage had been removed. These charges on the idle sheath were sufficiently large to cause considerable error in the measurements of the sample under test. All six cable sheaths are now surrounded by the shield just described. By means of single-pole double-throw switches, (Fig. 6) the guards, conductors, and sheaths of cables not under test are connected to this shield. The authors heartily concur in the conclusions of C. L. Kasson[4] as to the absolute necessity of completely surrounding the cable sheath with metallic sheathing; if this is not done, induction from insuspected charges on neighboring bodies may cause errors and give irrational results.

Measurement of the Air Condenser

The air condenser (C_1), if properly designed and shielded, may have a very small angle of defect—less than one minute—and if a variable condenser having a fixed loss and known capacitance be available,[1] it is possible to measure this angle. Unless a current-measuring device be connected in series with either the air condenser or the cable the capacitance C_1 must be known. This is measured by means of a precision capacitance bridge, using low voltage and a frequency of 1000 cycles per second. It was found that the 1000-cycle measurement checked the 60-cycle measurement, and 1000 cycles is more convenient.

In making the measurement, the capacitance between the high-voltage plate and the low-voltage plate *only* must be measured. To do this, the relative potentials of all plates and shields must be the same as they are when in use. Hence, the shielding about the low-voltage plate must be at the same potential as the low-voltage plate itself, but must not be included in the measurement. Also, the capacitance of all other bodies, including the leads, must be excluded.

The connections used by the authors are shown in Fig. 8. R_1 and R_2 are two ordinary non-inductive resistances; C'_1 is a variable precision condenser; C_2 is a fixed air condenser; R_3 is a resistance for compensating for any differences of loss in bridge arms C_1' and C_2; ordinarily it is not needed. R_4 is a slide-wire resistance used for obtaining the "Wagner ground."

The guard circuit is grounded permanently at S. A small capacitance C_4 between the high-voltage plate and ground is necessary to balance the capacitance of the guard circuit to ground (and also that of C_1 to ground with switch S at a), when the ground balance is being obtained.

The bridge is first balanced with switch S at a, which grounds the low-voltage plate of C_1. C_1' and R_3 are adjusted until a balance is obtained. By connecting

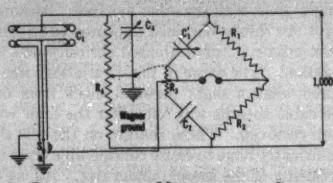

FIG. 8—CONNECTIONS FOR MEASURING THE CAPACITANCE OF A SHIELDED CONDENSER

the phones to the slide-wire contact, the ground balance is also made, adjustment of C_4 and R_4 being necessary. Switch S is then thrown to b, which shunts C_1 across C_1'. C_1' is now readjusted until the bridge is again balanced. The Wagner ground is also rebalanced. The capacitance C_1 is then the difference between the settings of C_1'.

From their experiences obtained in connection with dielectric research in general, the authors cannot overemphasize the importance of investigating all potential and capacitance relations in the manner which has just been described. The neglect of some apparently minor detail may give an incorrect and misleading result.

TRANSFORMER RATIO AND PHASE ANGLE

The bridge gives a convenient and precise method for checking both the ratio and phase angle which exist between the total secondary voltage of the testing transformer and that of the voltmeter coil. The secondary voltage is usually assumed to be the voltage across the voltmeter coil multiplied by the ratio of secondary turns to voltmeter-coil turns. Also the voltage across the voltmeter coil is frequently assumed to be in phase with the secondary terminal voltage. These assumptions are based on the voltmeter coil's being so located in the secondary winding that it is linked by the same flux which links the secondary itself.[5] That these conditions are not exactly true has of late been recognized.[6] For some measurements, actual departure from the assumed conditions is negligible but compensation or correction may be necessary in dielectric-loss measurements of low power factor. In order to obtain some idea of the magnitude of these relations, ratios and phase angles in three testing transformers have been measured. The connections are given in Fig. 9. The air condenser C_1, the primary of the mutual inductance M, and the resistance R_1 are connected in series across the high voltage E_1. A capacitance C_0 in series with a variable resistance R_0 is connected across the voltmeter coil, whose voltage is E_2. To measure the phase angle between E_1 and E_2, the phase angles of the two condensers C_1 and C_0 should be known. The angle of defect of C_1 is probably 0.5 min. or less. So far as we could determine, the angle of defect at 60 cycles of the best mica condensers of the capacitance of C_0 may be of the order of from 5 to 10 min. Hence, it became necessary to make a low-voltage air-condenser C_0 for this purpose. C_0 consists of 43 aluminum plates, 12 in. by 14 in. (30.5 cm. by 35.6 cm.), 1/16 in. (0.159 cm.) thick and spaced approximately 0.18 in. (0.457 cm.). This condenser has a capacitance of approximately 0.010 μf. With the exception of four hard-rubber washers between the supporting rods and the two plates at each end there is no dielectric other than air between the alternate 43 plates. Although there were facilities sufficiently sensitive to measure angles of defect of small radio condensers, the measuring apparatus could not detect any angle of defect in this condenser.

The bridge is balanced by adjusting R_1, R_0, and M. Then

$$\frac{E_1}{E_2} = \frac{C_0 R_0}{C_1 R_1} \tag{4}$$

This assumes that the effect of R_2 and R_1 on the impedances of their respective circuits is negligible compared with their condenser impedances. With bridges of this type, the error is considerably less than 0.1 of one per cent.

If the angles of defect of C_1 and C_0 can be assumed equal, the angle between E_1 and E_2,

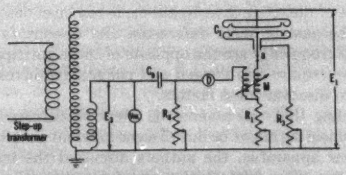

FIG. 9—CONNECTIONS FOR DETERMINING THE RATIO AND PHASE ANGLE OF VOLTMETER COIL

$$\beta = \tan^{-1}\frac{M\,\omega}{R_1} + \tan^{-1}\omega\,C_1\,R_1 - \tan^{-1}\omega\,C_0\,R_0 \tag{5}$$

All the above angles are very small. $\omega\,C_1\,R_1$ is almost always so small as to be negligible. Hence

$$\beta = \frac{M\,\omega}{R_1} - \omega\,C_0\,R_0, \text{ nearly} \tag{6}$$

If $\frac{M\,\omega}{R_1}$ lags I_1 and is greater than $\omega\,C_0\,R_0$, E_2 lags E_1.

If $\omega\,C_0\,R_0$ is greater than $\frac{M\,\omega}{R_1}$, E_2 leads E_1.

Below are a few typical data taken with a 15-kv-a., 50-kv. transformer. This transformer was made in the early days of transformer design and the voltmeter coil is not placed in the best position.

		$C_0 = 0.01033\ \mu f.$			$C_1 = 121.5\ \mu\ \mu f.$		β (minutes)
Freq.	E_2	R_0	R_1	$M\ h$	Ratio $= \sigma$	E_1	+ E_2 leads − E_2 lags
				No-load			
61	3.0	682.0	50	0.650	1155	3,465	− 7.7
61	10.0	676.6	50	0.517	1147	11,470	− 4.4
61	22.0	674.6	50	0.517	1144	25,200	− 4.4
45	8.0	674.0	50	0.470	1142	9,136	− 2.4
45	16.0	671.2	50	0.330	1138	18,210	+ 0.3
		Load = 10 ft. cable,		$C = 1215\ \mu\ \mu f.$			
75	9	689.0	50	0.097	1166	10,500	+ 9.3
75	22	687.0	50	0.020	1165	25,630	+10.9
60	9	682.1	50	0.260	1155	10,400	+ 2.4
60	22	682.4	50	0.120	1156	25,430	+ 6.0
45	9	678.0	50	0.377	1148	10,330	− 0.5
45	16	677.2	50	0.317	1148	18,370	+ 0.6

The following data were taken with modern testing transformers.

15 kv-a., 250/50,000 volts						
$C_0 = 0.01007\ \mu f.$			$C_1 = 580\ \mu\ \mu f.$			Freq. = 60 cycles
E_2	R_0	R_1	$M\ h.$	Ratio $= \sigma$	E_1	β(minutes) E_2 leads
			No-load			
200	715.6	50	0.25	248.5	49,700	8.8
140	715.6	50	0.25	248.5	34,700	8.8
79	716.4	50	0.00	248.7	19,650	9.4
Load = 100 ft. cable;			$I = 0.096\ a.$ at 36 kv.		Kv-a. = 3.5	
141	716.7	50	0.15	248.8	35,100	9.8
120	717.5	50	0.15	249.1	29,900	9.8
100	717.7	50	0.30	249.2	24,920	10.1
40	717.7	50	0.30	249.2	9,970	10.1

100 kv-a.; 250/100,000 volts						
$C_0 = 0.01007\ \mu f.$			$C_1 = 580\ \mu\ \mu f.$			Freq. = 60 cycles
E_2	R_0	R_1	$M\ h$	Ratio $= \sigma$	E_1	β(minutes) E_2 leads
			No-load			
69	100	5756	5.0	99.90	58,900	10
50	100	5753	5.3	99.88	49,940	9
36	100	5753	5.3	99.88	20,710	9
Load = 100 ft. cable.			$I = 0.096\ a.$ at 36 kv.; kv-a. = 3.5			
36	50	2875	0.2	99.84	35,940	38

Although the foregoing tabulation is not complete for the entire load and frequency range of the transformers, it does give the order of magnitude of the ratio change and the phase angle which may be expected under the conditions of no-load and of moderate condensive loads. When the voltmeter coil is properly located, the ratio varies only slightly for all voltages and loads. The value of this ratio should be determined with high precision, for in many methods of power measurement this voltage appears to the second power.

The magnitude of the phase angle is such that if the wattmeter be used for power measurement at low power factors, large errors must result. For example, a phase-angle difference of six minutes makes an error of 0.001 in the power factor and a corresponding error in the power. Good cables frequently have power factors as low as 0.4 per cent., so that this error in phase angle may well introduce 25 per cent error into the power measurement. Fortunately, most of the precision methods now used do not utilize the voltmeter coil in measuring the power. However, when a dynamometer is used, compensation is obviously necessary.

The phase-angle and ratio errors in potential transformers are much less than with voltmeter coils. However, potential transformers for these higher voltages are expensive, and two or three are necessary for the usual ranges of testing voltage. A sufficiently accurate method for measuring the high voltage directly is to connect a dynamometer in series with the air condenser C_1. Then this voltage

$$E_1 = \frac{I_1}{C_1\,\omega} \tag{7}$$

where I_1 is the current in the dynamometer.

With the lower voltages, our condenser did not have a capacitance great enough to give sufficient dynamometer deflection. Hence, a special plate-glass condenser, having a capacitance of 1880 $\mu\ \mu$ f., was used.

Checking the Bridge

Many factors which may not be apparent, and some which cannot even be foreseen, may introduce error into the measurement of dielectric loss. Hence, it is highly important to check and cross-check the measuring apparatus, and should any discrepancies be found, they should be investigated carefully.

A convenient check on the resistances, capacitances, and indicating instruments may be made by using the connections shown in Fig. 9. A deflecting dynamometer is connected at a between the low-voltage plate of C_1 and the mutual inductance M. C_1 may be either the cable or the air-condenser. By the authors both are used.

The current in the dynamometer

$$I_1 = E_1 C_1\,\omega$$

From Eq. (4)

$$E_1 = E_2 \frac{C_0 R_0}{C_1 R_1}$$

Hence,

$$I_1 = E_2 C_0\,\omega \frac{R_0}{R_1} \tag{8}$$

The four quantities C_0, ω, R_0, and R_1 are known. Below are given typical data which we have taken in making such measurements.

ω	E_2	R_0	C_0	R_1	I_1 m. a. from (8)	I_1 m. a. (observed)
377	25	1,004	0.015 μ f.	109	1.300	1.295

It will be noted that the calculated and observed values of current are in almost exact agreement. This is not a power check, however, but the close agreement among the foregoing quantities gives added confidence in the use of the bridge for power measurements.

Already it has been pointed out that in these very low power-factor measurements, the measured values of power are proportional to the difference in phase angle (or its sine or tangent) between that of the reference standard, such as the air-condenser and that of the test specimen.[7] If known losses are introduced into such a system, either into the air-condenser branch or the test-specimen branch, the measuring apparatus gives only the *change* in the phase angle. Hence, the absolute value of the power must depend on the value of the angle of defect of the air condenser. If this angle of defect be known, correction can of course be made. However, the introduction of a known loss into one or the other bridge arms does give a very good check on the accuracy of the apparatus in the measurement of phase difference. If the characteristics of the air condenser are known, the accuracy of the bridge in measuring this phase difference is also its accuracy as a power measuring device.

To insert a known loss in series with a capacitance of known characteristics and thus to determine the corresponding phase difference appears a comparatively simple matter. The authors thought so at first. Experience, however, has taught that to conduct such tests properly, a detailed knowledge of shunted capacitances, their magnitudes, and methods of either eliminating them entirely or of making their effect negligible, is very necessary.

A considerable source of error, however, may enter such measurement. In making the balance with r in circuit, the points b' and c, as is usual, are brought to the same potential by adjusting R_4. (The mutual inductance m, Fig. 1, is also used but omitted in Figs. 10, 11, and 12 for simplicity.) Point a is at a higher potential than point b or b' by the voltage drop in resistance r. This is of the order of from 50 to 200 volts. Hence there now exists the effect of capacitance between the low-voltage plate a and its shielding S.

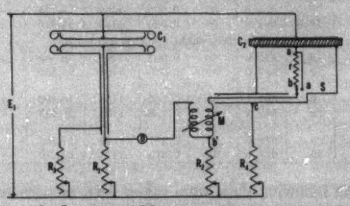

Fig. 10—Incorrect Method of Connecting Series Resistance

Some of the current coming through C_2, instead of flowing through r and R_2 in series, flows to ground through R_4.

If the connections shown in Fig. 10 are modified, as shown in Fig. 11, it is possible to make small these capacitance effects which cause discrepancies. The resistance r is removed from within the shielding S and mounted in such a way that its capacitance to ground or to the shielding is negligibly small. The capacitance C_2 is made large so as to make any capacitance currents

E_1, volts	R_1	R_2	R_3	R_4	M (M h)	f	ω	I_3(m. a.)	Power factor (sin ψ)	Power watts.
					$r = 0$					
19,300	770.1	50	1983	43,850	3.59	62.3	392	14.22	0.02815	7.73
					$r = 12{,}410$					
19,300	769.8	50	1983	202	4.79	62.3	392	14.22	0.0375	10.28
	$C_1 = 122\ \mu\mu f.$								$C_2 = 1878\ \mu\mu f.$	

Increase of power = 10.28 − 7.73 = 2.55 watts
Power dissipated in r $(0.01422)^2$ 12,410 = 2.51 watts
Difference = 1.6 per cent.
$\sin \psi_2 - \sin \psi_1 = 0.00935$ from (9)
$C_2\, r\, \omega = 0.00914$

Consider first the bridge circuit shown in Fig. 10, which is the ordinary bridge circuit with a resistance r, connected to the low-voltage electrode of the test capacitance C_2, so that it may be inserted in series with it.

The bridge is first balanced without r in circuit and the power factor $\sin \psi_1$ is obtained. The resistance r is then inserted in circuit and a second balance is obtained, giving the power factor $\sin \psi_2$. The increase in power factor for these small angles

$$\sin \psi_2 - \sin \psi_1 = r\, C_2\, \omega \qquad (9)$$

from r to the shielding negligible in comparison. The shielding balance is obtained by bringing points b and c to the same potential by adjusting R_4, and also m (see Fig. 1), the condition of equal potential being determined by connecting detector D between b and c. For C_2, a condenser having plate glass 0.5 in. (1.27 cm.) thick as a dielectric and tinfoil as the electrodes is employed. This condenser is immersed in oil. Below is given a set of typical data which we have taken, using these conditions.

These measurements have also been verified by

testing one of the E. T. L. standard condensers without and with a resistance in series, the connections being similar to those given in Fig. 11. Below are given data for one voltage:

E_1 volts	R_1	R_2	M (mh)	f	I_2 (m a)	Power factor (sin ψ)	Power watts
			$r = 0$				
10,000	668.4	100	0.48	60	3.07	0.0810	0.0554
			$r = 12{,}500$				
10,000	668.3	100	1.515	60	3.07	0.5710	0.1750

$C_1 = 122\ \mu\mu$ f. $C_2 = 815.5\ \mu\mu$ f.

Increase of power $= 0.1750 - 0.0554 = 0.1196$ watt

Power dissipated in $r = (0.00307)^2\ 12{,}500 = 0.1178$ watt

Difference = 1.5 per cent.

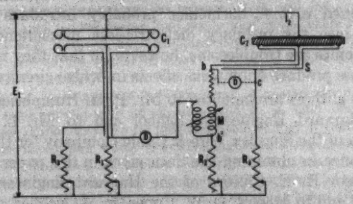

FIG. 11—CORRECT METHOD OF CONNECTING RESISTANCE IN SERIES WITH LOW-VOLTAGE PLATE

It was impossible to obtain these very close checks until the guard circuits were carefully balanced in both phase and magnitude by the methods which have already been described. In fact, it was due to the unexplainable discrepancies between measured and computed values which led to the study of further effects of guard-circuit balances, and the development of these methods.

To those who have had experience in the types of measurement which have just been described, the foregoing agreements are remarkably close, particularly when it is considered that with the plate-glass condenser, for example, the difference of 1.6 per cent in the values of power occurs in factors which are differences of two quantities, one of which is 75 per cent of the other.

It is interesting to note that with the plate-glass condenser ($r = 12{,}100\ \Omega$) the difference of potential between the shielding S and ground, Fig. 11,

$$E_s = 0.01422 \times 12{,}460 = 177.2 \text{ volts}$$

With the E. T. L. condenser $E_s = 0.00307 \times 12{,}600 = 38.7$ volts.

It might be thought that the foregoing difficulties due to the capacitance of shielding could be overcome by inserting the resistance r in the high-voltage side of the condenser C_2, as shown in Fig. 12. It then becomes necessary to shield completely the high-voltage plate b, as shown in Fig. 12. Otherwise there will be capacitance currents from this plate b to ground. Hence, all the current flowing through r does not flow through R_3. Even then, the same difficulty exists as was encountered in Fig. 10. With r in circuit, there is potential difference between the shielding S and the high-voltage plate b. Hence, current will pass from S to b to R_2 without going through r. This introduces error which is very difficult to eliminate.

Also the guard rings $a\ a$ and $a'\ a'$ must be sufficiently removed from the plates b and b', or current from $a\ a$ may flow to b' or current from b may flow to $a'\ a'$. In the first case, too much current flows through R_3; in the second case, too little current flows through R_2. These effects make it almost impossible to use the ordinary air-condenser for this purpose.

As a matter of interest, the 12,100-ohm resistance was connected in series with the high-voltage plate when making the tests with the E. T. L. condenser. The measured power in every instance was 13 per cent greater than the computed power. This was to be expected, since the currents in r and R_2 (Fig. 12) could not be equal. Some of the current in r is leakage current since there was no high-voltage guard-ring; also, some of the current in r goes to the low-voltage guard-rings. Hence, the current in r must be larger than that in R_2 and the two probably differ in phase by some indeterminate angle. Therefore, the measured power must necessarily be greater than the computed power.

HARMONICS

If a sinusoidal e. m. f. be impressed on the bridge circuit, such as shown in Fig. 1, no harmonics will occur so long as the capacitance and resistance of the system remain constant. When ionization does occur, however, the capacitance of the cable varies during the cycle, and harmonics, commencing with the third and extending well through the range of audible frequencies,

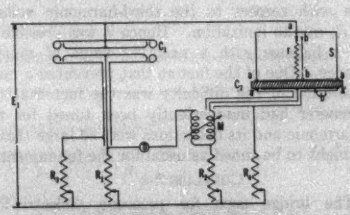

FIG. 12—METHOD OF CONNECTING RESISTANCE IN SERIES WITH HIGH-VOLTAGE PLATE

appear in the current wave. The theoretical effect of these has already been discussed before the Institute.[8] In the ordinary cable, the magnitude of these harmonics is so small compared with the magnitude of the charging current, (and they add in quadrature), that it is doubtful if they have appreciable effect on the magnitudes of current and capacitance as determined by a bridge or by deflection instruments.

With a pure sine wave of voltage impressed on the system, these harmonics cannot contribute power because there is no component of their own frequency in the voltage wave. However, they do cause a dissipation of energy in the ionized air films, as is well known. Obviously this power must be taken from the power which is being supplied at fundamental frequency. It is not unlike the well known radio-telegraph circuit, through which very high frequencies are obtained from a low-frequency source by means of arc discharges.

These harmonic currents cause a voltage drop across resistance R_2, Figs. 1 and 2. There is no corresponding harmonic voltage drop in R_1. Hence, under the conditions of a sinusoidal wave across the bridge, the detector, when tuned for any harmonic, cannot be balanced. With a tuned galvanometer, such as employed here, it is possible to balance the fundamental, and then, by varying the tuning, to select, one at a time, the harmonics of the higher frequencies which cannot be balanced.

At one time these harmonics produced effects which baffled us temporarily. When testing some cables at low frequency, it was noticed that the mutual inductance became negative as soon as ionization began; that is, the cable apparently had become a source of energy. Since this effect was concurrent with a corona discharge in our end-bells, this discharge was at first blamed. Elimination of this discharge, however, failed to remedy the difficulty. After investigating and eliminating other factors which might possibly have caused the difficulty, it was found that a third harmonic in the impressed voltage wave was responsible. A 60-cycle generator was operating at 25 cycles. The higher saturation of the alternator iron introduced a small third harmonic into the impressed voltage wave. Hence, a third harmonic voltage appeared across R_1, Figs. 1 and 2. This harmonic had no particular phase relation with respect to the third-harmonic voltage across R_2 due to ionization. Hence it was possible to obtain a balance with a negative value of mutual inductance. One of the factors that prevented a more rapid solution of the difficulty was the fact that the galvanometer had inadvertently been tuned for the third harmonic and its deflections were so large that it was thought to be tuned as usual for the fundamental.

Conclusions

1. The bridge used for precision dielectric-loss measurements should be shielded from electrical, magnetic, and mechanical disturbances.

2. The air condenser and the bridge leads should be shielded, and the shielding should be brought to the same potential as the conductors which it shields.

3. The cables should be properly shielded and this shielding brought to the same potential as the cable sheaths. Electric discharges should be prevented from entering any shielding circuit which is to be balanced. Methods of balancing shielding in both phase and magnitude are not difficult.

4. In measuring the capacitance of the air condenser, a method which maintains the potential of the guard plate always at the potential of the low-voltage plate must be used.

5. It is possible to measure with precision by means of a bridge circuit the ratios and phase angles of high-voltage transformers. With the voltmeter coil properly located, the ratio is practically constant at all voltages and loads. The phase angle may change considerably with load.

6. The precision of the bridge may be checked by the introduction of known losses in its arms. Extreme care must, however, be exercised less leakage and stray capacitances give misleading results.

7. With a tuned galvanometer and a sinusoidal impressed e. m. f., harmonics probably introduce no error into the ordinary dielectric-loss measurement. Harmonics are troublesome, however, in that they may at times produce anomalous effects in bridge circuits.

The authors are indebted to Mr. P. H. Humphries of the Harvard Engineering School and to Mr. L. E. Cirella of the Simplex Wire & Cable Company for their assistance in obtaining the data given in this paper; to Professor H. E. Clifford of the Harvard Engineering School and to Messrs. D. W. Roper, F. M. Farmer, and W. F. Davidson of the Joint Paper Cable Research Committee for their suggestions in the preparation of this paper.

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Discussion

L. E. Cirella: For two years we have been using a bridge similar to that described by the authors. Profiting by their previous experience we have been able to avoid most of the difficulties described by them.

In the paper two reasons are given for the error made in measuring the phase angle of the air condenser, namely,

(1) Directly grounding both the condenser guard plate and the sheath of the low condenser plate lead wire, and

(2) The high capacitance from copper to sheath of the low condenser plate lead wire.

Tests we have made show that directly grounding the guard circuit is the principal cause of error. Increasing the capacitance of the guard circuit wires five times in either the condenser or cable guard circuit showed no measurable change in cable power factor as long as the R_2 and R_4 circuits were balanced. When the bridge is completely balanced the condenser guard and the low condenser plate are at practically the same potential and the same is true of the cable sheath and the cable shielding; therefore the capacitance current in shunt with the resistance arms of the main bridge is negligible.

Check tests were made recently with this bridge to show the effect of completely shielding the cable sheath of a sample. These tests gave slightly lower power factor, watts loss, and capacitance with complete shielding, indicating that the curves shown in Fig. 4 are not due to any inherent characteristic of this bridge. Looking at Fig. 1 it may be seen that shunting R_2 with a capacitance or upsetting the balance of R_4 will influence the capacitance, power factor, and watts loss of the cable sample. Tests were made on an unshielded sample of cable at 25 deg. cent. and curves similar in shape to those in Fig. 4 were obtained. Throughout this test the R_4 circuit was carefully balanced, R_4 being close to 6000 ohms. A second test made on the same sample with no shielding and R_4 at 4500 ohms gave curves similar to those for shielded cable of Fig. 4, that is, the power factor and watts loss were higher and the capacitance lower. This does not explain the curves of Fig. 4 satisfactorily, however, for it does not seem possible that two laboratories would have made balances with R_4 1500 ohms below the value for a balance when the sensitivity of this bridge is so great that 20 ohms off balance will cause a galvanometer deflection greater than 500 mm. In fact our tests suggest, in general, that the small differences shown in Fig. 4 are caused by slight differences in the cable temperature. For instance, three tests made in immediate succession on the same table, the sheath of which was shielded, not shielded, and shielded showed continually increasing power factor regardless of the shielding condition.

H. W. Lamson: I have had occasion recently to develop a low-frequency amplifier intended for use in connection with a recording cardiograph. My purpose was to amplify low audible frequencies over a range from about 30 to 40 cycles per sec., and at the same time exclude as much as possible interference from 60-cycle mains or any harmonics thereof, or any other higher frequencies. This I was able to do with considerable success by using carefully designed transformers having tuned secondaries resonant at, say, 35 cycles.

It occurred to me that the interference difficulties which Professor Dawes experienced in his amplifying system might, perhaps, be largely eliminated by the use of a similar type of amplifier using, of course, transformers tuned to 60 cycles.

P. H. Humphries: Although the theory of the high-voltage bridge designed and used by Professor Dawes and his associates in dielectric research has been previously described,[1] I believe that a mathematical derivation of the bridge equations and an analysis of the conditions for balance would add to our understanding of its operation.

In Fig. 1 herewith is given the simplified bridge circuit which has already been presented before this Institute. The conditions for balance are derived on the basis of Maxwell's cyclic current equations.

Let the circuit constants be $Z_1 = R_1$; $Z_2 = r_1 - \dfrac{j}{C_1\,\omega}$;

$$Z_3 = r_2 + j\left(L_2\,\omega - \frac{1}{C_2\,\omega}\right);\ Z_4 = R_2;\ M = j\,m\,\omega.$$

1. C. L. Dawes and P. L. Hoover, *Ionization Studies in Paper Insulated Cables—I*, A. I. E. E. Trans., Vol. XLV, 1926, p. 142, Fig. 3.

Writing the voltage drops around the networks *c a d c* and *b c d b*,

$$X(Z_3 + Z_4 - 2\,M + Z_G) - (X + Y)(Z_G - M) - I_B\,Z_2 = 0 \quad (1)$$

$$(X + Y)(Z_1 + Z_G + Z_4) - X(Z_G - M) - I_B\,Z_1 = 0 \quad (2)$$

A simplification of Equations (1) and (2) gives the two simple simultaneous Equations (3) and (4).

$$X\,(Z_3 + Z_4 - M) + Y\,(M - Z_G) = I_B\,Z_2 \quad (3)$$

$$X\,(Z_1 + Z_4 + M) + Y\,(Z_1 + Z_G + Z_4) = I_B\,Z_1 \quad (4)$$

The galvanometer current Y, the presence of which is evidence of bridge unbalance is given by (5).

$$Y = \frac{\begin{vmatrix} Z_3 + Z_4 - M & I_B\,Z_2 \\ Z_1 + Z_4 + M & I_B\,Z_1 \end{vmatrix}}{\begin{vmatrix} Z_3 + Z_4 - M & M - Z_G \\ Z_1 + Z_4 + M & Z_1 + Z_G + Z_4 \end{vmatrix}} \quad (5)$$

The condition of balance requires that the denominator in (5)

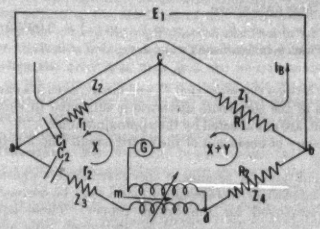

Fig. 1—Diagram of Connections for Dawes Bridge

be zero. Hence from the numerator Equation (6) is derived.

$$Z_1\,Z_3 - Z_1\,M = Z_2\,Z_4 + Z_2\,M \quad (6)$$

Replacing the generalized bridge impedances in (6) by their complex values,

$$R_1\left[r_2 + j\left(L_2\,\omega - \frac{1}{C_2\,\omega}\right)\right] - j\,R_1\,m\,\omega$$

$$= R_2\left(r_1 - \frac{j}{C_1\,\omega}\right) + j\,m\,\omega\left(r_1 - \frac{j}{C_1\,\omega}\right) \quad (7)$$

From Equation (7) by equating the real parts:

$$R_1\,r_2 = R_2\,r_1 + \frac{m}{C_1} \quad \text{or} \quad r_2 = \frac{R_2\,r_1\,C_1 + m}{R_1\,C_1} \quad (8)$$

From Equation (7) by equating the imaginary parts:

$$R_1\left(L_2\,\omega - \frac{1}{C_2\,\omega} - m\,\omega\right) = -\frac{R_2}{C_1\,\omega} + r_1\,m\,\omega \quad (9)$$

Solving Equation (9) for C_2

$$C_2 = \frac{C_1\,R_1}{R_2 + C_1\,\omega^2\,[\,R_1\,(\,L_2 - m\,) - m\,r_1\,]} = \frac{C_1\,R_1}{R_2 + k} \quad (10)$$

where

$$k = C_1\,\omega^2\,[\,R_1\,(\,L_2 - m\,) - m\,r_1\,]$$

The second term in the denominator, k, is ordinarily negligible in comparison with R_2 as will be shown later, hence $C_2 = C_1\,\dfrac{R_1}{R_2}$

The phase angle of the dielectric under test may be found as follows:

$$\tan\psi_2 = r_2\,C_2\,\omega = \left[\frac{R_2\,r_1\,C_1 + m}{R_1\,C_1}\right]$$

$$\left\{\frac{R_1\,C_1}{R_2 + C_1\,\omega^2[R_1(L_2 - m) - m\,r_1]}\right\}\omega = \frac{(B + m)\,\omega}{R_2 + k}$$

where $$B = R_2 r_1 C_1$$

It will be shown later that in this type of work the term B may also be neglected, hence

$$\tan \psi_2 = \frac{m\,\omega}{R_2} \tag{11}$$

The original values for the capacitance and phase angle of the test specimen are too involved for practical application so certain factors as noted are neglected. It may be seen from the following calculations that the approximations are justified.

The following data were taken from measurement upon a 15-mil (0.038-cm.) air film in series with a 0.5555-in. (1.41-cm.) Pyrex plate, test area 17.25 sq. in. (111.3 sq. cm.) at a gradient across the air film of 120 volts per mil.

$C_1 = 122\ (10)^{-12}\ f$; $r_1 = 0\ \Omega$; $R_1 = 532.3\ \Omega$; $R_2 = 2000\ \Omega$; $r_2 = 3\ (10)^6\ \Omega$; $M = 0.220\ h$; $L_2 = 0.800\ h$; $f = 60 \sim$

Using these data above

$$k = C_1\,\omega^2\,[\,R_1\,(\,L_2 - m\,) - m\,r_1\,] = 0.00535$$

a value which is insignificant when compared with R_2.

$$B = R_2\,r_1\,C_1 = 0$$

It may be seen therefore that C_2 and $\tan \psi_2$, even in a case where a portion of the composite dielectric is subjected to very heavy ionization, are not affected by the approximations.

The theory of operation of the guard circuit Fig. 2 herewith is as follows:

The conditions for balance of this guard circuit whether it be an air condenser or test-specimen guard may be derived by the method employed for the bridge proper. These equations, however, are exceedingly involved and much time and labor is consumed in reducing them to the form where balance conditions may be readily noted. The circuit may be solved quite simply by means of a vector diagram, Fig. 3.

Let α = the phase angle of the bridge arm containing the dielectric under test.

In such a diagram it is quite impossible to draw all vectors to

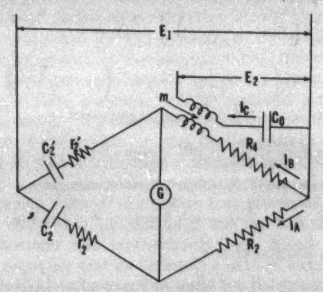

Fig. 2—Diagram of Connections for Guard Circuit

scale such quantities as voltage drops across R_2 and R_4 and the shift in phase produced by the mutual inductance of necessity being exaggerated.

$$\alpha = \psi_2 - \psi_1 \tag{12}$$

$$\tan \alpha = \frac{I_C\ m\ \omega}{I_B\ R_4} \tag{13}$$

$$\tan \psi_2 = r_2\,C_2\,\omega,\ (R_2 \text{ negligible compared with } r_2) \tag{14}$$

$$\tan \psi_1 = \frac{I_B\,(R_4 + r_2')}{I_C\ m\,\omega + \dfrac{I_B}{C_2'\,\omega}} = \frac{I_B\,C_2'\,\omega\,(R_4 + r_2')}{I_C\ C_2'\,m\,\omega^2 + I_B} \tag{15}$$

In the expression for $\tan \psi_1$, $I_C\ C_2'\ m\ \omega^2$ is negligible compared with I_B, hence

$\tan \psi_1 = C_2'\ \omega\ (R_4 + r_2') = C_2'\,r_2'\ \omega$, ($R_4$ negligible compared with r_2'.

Since ψ_2 and ψ_1 are both small angles and $\psi_2 - \psi_1$ is small we may write

$$\tan \alpha = \tan\,(\psi_2 - \psi_1)$$

or

$$\frac{I_C\ m\ \omega}{I_B\ R_4} = C_2\,r_2\,\omega - C_2'\,r_2'\,\omega \tag{16}$$

In the expressions for I_B and I_C the resistance R_4 and the

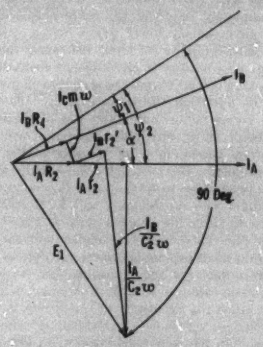

Fig. 3—Vector Diagram for Fig. 2

reactance $m\ \omega$ are insignificant in comparison with the condensive reactances in these arms, hence

$$I_B = E_1\,C_2'\ \omega \quad \text{and } I_C = E_2\ C_0\ \omega = \frac{E_1}{\sigma}\ C_0\ \omega$$

Equation (16) then becomes

$$\frac{E_1\,C_0\,m\,\omega^2}{\sigma\,E_1\,C_2'\ \omega\,R_4} = C_2\,r_2\,\omega - C_2'\,r_2'\,\omega \tag{17}$$

or

$$m\,C_0 = \sigma\,C_2'\,R_4\,(C_2\,r_2 - C_2'\,r_2') \tag{18}$$

In the past shields and guards have been referred to rather loosely and the use of them made in a haphazard "hope for the best" attitude. There remained much doubt regarding the efficiency with which they performed the functions for which they were employed. Messrs. Dawes, Hoover, and Reichard are to be warmly congratulated upon the methods which they have developed to bring the shielding and guards to the correct potential and the steps that they have taken toward clarifying the situation with reference to the proper use of shields and guards in such a difficult and poorly understood type of research.

C. T. Weller: In a discussion of my paper entitled 132-*Ky. Shielded Potentiometer for Determining the Accuracy of Potential Transformers*,[2] I was asked why we utilized shielded resistance instead of capacitance in the potentiometer or voltage divider.

Our principal conclusion in regard to the use of capacitance was that "capacitors of the types and ranges required were not in the same class as resistors from the standpoints of stability and accuracy. These views were confirmed, in general, by Dr. H. L. Curtis of the Bureau of Standards." I also stated that "it is obviously desirable to keep the size and the cost of testing

2. A. I. E. E. Quarterly Trans., Vol. 48, July 1929, p. 790.

equipment at a minimum and it is hoped, therefore, that other laboratories will proceed with the necessary development work so that capacitance or impedance potentiometers will be available which are comparable with the 132-kv. shielded potentiometer in flexibility, stability, and accuracy as well as in voltage range."

It is evident from the paper by Messrs. Dawes, Hoover, and Reichard and from the discussion, particularly Professor Kouwenhoven's, that progress has been made (a) in the design and application of high-voltage capacitors and (b) in replacing the low-voltage mica capacitor previously used by an air capacitor.

I notice also that the authors recognize that harmonics are objectionable and capacitance tends, of course, to amplify them.

While I still feel that resistance is preferable to capacitance, inductance, or impedance for determining the accuracy of potential transformers, I should appreciate comments along this line by the authors and others interested.

T. F. Peterson: Mention has been made of the fact that it had been originally planned to have a paper on Errors in Loss Measurements by Dr. Ryan. I happened to have the good fortune to be associated with him in some of his work, and so have a little of the information which might have been included in such a paper. Inasmuch as it logically falls in the classification of errors in measuring dielectric loss, which Professor Dawes covered so thoroughly, I think it might be presented at this time.

One of the methods developed by Professor Ryan for measuring dielectric loss, particularly corona loss, is by means of a dynamometer wattmeter, using a high-resistance water column as a potential multiplier. A high-voltage transformer grounded at one end is connected to the two coils of the wattmeter, which is housed in a copper gauze shielding cage also at high potential. One coil of the instrument is connected to the transmission line by conductors which are shielded by surrounding pipes and which are maintained at the potential of the cage. The other coil, that is, potential coil, is connected by shielded conductors to a flowing water-column resistance. The circuit is essentially that of a wattmeter with a potential resistance multiplier, and, by reading current through the water column as well as the voltage across same, the multiplying factor may be determined.

As can be appreciated, the measurements at these low power factors at such high voltages were shrouded with all manner of insidious errors. H. H. Henline was the first to recognize the difficulties incurred by the slight difference in potential existing between leads to the line and water column and their respective shielding pipes. Since the leads were connected to the transformer by the impedances of the instruments through which current had to flow, and shields were connected directly, a difference in potential existed between the shield and conductor equal to the drop through instruments. This gave rise to charging current which, with the large charging current to the line, conspired to introduce great errors, all too deceptive because they were proportional to the voltage squared.

I worked up the mathematics of the circuit and we finally evolved the method of introducing compensating impedances between the transformer and shields which enabled us to establish zero potential difference between the conductors and shields. This made possible the elimination of one source of error.

There still remained another baffling difficulty with the set-up which introduced errors of unknown magnitude. The water column was supposedly pure resistance serving as a wattmeter multiplier by allowing a flow of current exactly in phase with the voltage. However, between the turns of the water column (about 6 ft. high, 8 turns) and from each turn to ground, there was appreciable capacity. The charging current to this (through the potential coil of wattmeter) though small, was in phase with the very large charging-current component to the transmission, and hence when these were multiplied by the instrument a very great error appeared. Though the existence was recognized and its value could be changed from a positive one to a negative one by adjusting shielding rings around the coil, it was most difficult to determine when the error was zero without the use of a zero-loss condenser substituted for the transmission line.

At that time I introduced a method of determining when these errors were entirely eliminated from the reading, or which would allow of their elimination by computation. This consisted in taking a reading of loss with one setting of valves controlling the flow of water in column. Then the flow was cut down, thus allowing the temperature of the water to rise with a corresponding decrease in electrical resistance. In this way the current through the coil (in phase with voltage and with the loss component of current to line) could be altered in any way desired. Obviously the charging current of the water column remained constant since the configuration was not changed. A second set of readings was then taken, and by means of simultaneous equations the error could be eliminated and the true value of loss determined. It was, however, found more desirable to eliminate the error by adjusting the shielding of the water-column coil until doubling the current through this coil resulted in an exact doubling of the wattmeter reading. This condition obviously obtained when the error was zero and the measurement was truly that of the corona loss. Accomplishing this without the use of a zero-loss condenser was, to me, a decided advantage to this method and proved to be a great boon to the efforts then being made to determine corona loss exactly.

V. A. Thieman: Fig. 4 of the papers shows that careful shielding of a cable may actually increase the power factor and power loss. This effect is opposite to that which might be expected, and is contrary to results which have been obtained by others. As a part of a graduate thesis, I have conducted an extended investigation of such shielding effects. Two samples of cable which will be designated as X and Y were used for these tests. Cable X was a 600,000-cir. mil single-conductor cable with 12/64-in. (0.477-cm.) impregnated-paper insulation and cable Y was a 300,000-cir.-mil single-conductor cable with 14/64-in. (0.556-cm.) paper insulation. All tests were made at a frequency of 60 cycles. The bridge, the guarding, and the shielding were the same as those described in the paper. A rack similar to those shown in Figs. 6 and 7 was used. The entire wire screening, however, was fastened to frames which could be readily attached or removed from the rack. Also, further tests were made in which the active length of cable was shielded by winding varnished cambric around it for insulation and then wrapping wire screening closely about the cambric. The tests were made at a room temperature, which was approximately 19 deg. cent. Particular care was taken to locate the test cable remotely from all high-voltage wires except the single wire which is necessary to carry the voltage to the cable. For some distance this wire ran vertically upward away from the sheath.

In Fig. 4, herewith, are shown the characteristic curves obtained with cable X, and in Fig. 5, herewith, are shown similar curves obtained with cable Y with these cables in the rack. (Check tests taken at other times gave substantially the same results shown in Figs. 4 and 5.) The curves obtained with the wire screening wound around the cables agreed so closely with those shown in Figs. 4 and 5 that the small differences are scarcely perceptible.

With Cable X (Fig. 4) shielding gives lower values of power and power factor below the ionization voltage and greater values above the ionization voltage. Shielding decreases the capacitance. With Cable Y (Fig. 5) the reversed effects are obtained with the power and power factor whereas shielding scarcely affects the capacitance. In every case, however, the differences obtained with shielding and without shielding are small, and are much less than the changes which ordinarily occur due to physical and temperature changes in the cable between two consecutive tests.

Hence, these tests would appear to indicate that if cables of appreciable length when being tested are remote from high-

voltage conductors and other sources of induced charge, shielding has little effect.

In these tests, however, the low-voltage lead to the sheath had extremely low capacitance to ground, and the potential difference between sheath and ground was only of the order of 3 or

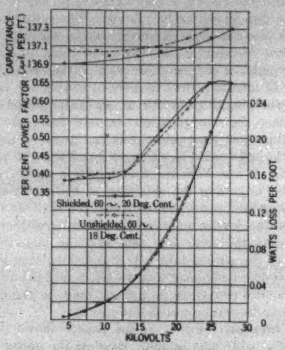

FIG. 4—TEST RESULTS OF CABLE X

600,000-cir-mil conductor with 12/64-in. insulation

4 volts, corresponding to a low value of R_2 of Figs. 1 and 2 in the paper. If it should be necessary to employ higher values of R_2 in order to obtain sufficient sensitivity, the voltage to ground would be increased. If, in addition, the capacitance of the lead to the sheath were high, it is possible that lack of shielding and

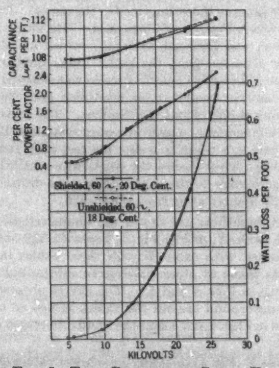

FIG. 5—TEST RESULTS OF CABLE Y

300,000-cir-mil conductor with 14/64-in. conductor

accurate balancing might introduce considerable error. Moreover, in view of the possibilities of induced charges occurring on cable sheaths, particularly where high-voltage sources are in the vicinity, as in cable-testing laboratories, I agree with the authors that shielding is always advisable, anyway.

C. L. Dawes: Dr. Curtis in Part III of his paper states that the wave form across the guard ring should be the same as that across the guarded plate. However, if the wave forms do differ no error is introduced if a fundamental-tuned detector is used in balancing the potentials. Incidentally, as stated in our paper, corona and "static" discharges in the guard circuit do disturb the galvanometer when balancing but these effects may be overwhelmed by shunting the guard circuit with some invariable type of condenser such as an air condenser or a glass condenser.

It is very gratifying to learn that the results of the shielding tests made by Mr. Cirella are so nearly in accord with our own experience. In our earlier work the leads from the bridge arms to the air condenser and to the cable consisted of No. 14 A. W. G. rubber-covered wires having a lead sheath. Consequently there was considerable capacitance to ground. At that time we did not appreciate the quantitative effects of such shunted capacitance so that appreciable error was introduced into our measurements. In cable factories, however, it is frequently necessary to run such leads for considerable distance and the use of such lead-covered wire cannot well be avoided. Mr. Cirella has shown that if the shielding is properly balanced, leads of this type may be used without introducing appreciable error.

Mr. Cirella suggests that the discrepancy between the shielded and the unshielded curves in our Fig. 4 may be due to temperature changes occurring between runs. Mr. Humphries devoted several weeks to the study of this effect and special care was taken to eliminate temperature effect. For example, a test was made first with and without shielding; this order was reversed day after day but the relation of the curves did not change. The apparatus was sufficiently sensitive to detect easily the differences shown in Fig. 4. Hence these differences are not due to chance. Our conclusion now is that, although there were no high-voltage leads in the immediate vicinity of the test cable, there were other high-voltage conductors in the laboratory which may have influenced the cable sheath in such a manner as to have caused an apparent decrease in loss when the shielding was removed. The discussion of Mr. Thieman explains this matter further.

It is very interesting to learn that Mr. Lamson has developed a new type of sensitive detector which may be used to advantage with our type of bridge. We will undoubtedly take advantage of his suggestion that we try his galvanometer with our bridge.

Mr. Humphries' mathematical analysis is a distinct addition to the paper in that it shows concisely the effect of each of the bridge members on the operation and precision of the bridge. In his equation (18) he shows that the capacitance of the shielding-circuit condenser C_e is independent of frequency whereas it is stated in the paper that the value of this capacitance varies inversely as the square of the frequency. This apparent discrepancy is explained by the fact that in Mr. Humphries' equations he assumes that the bridge members are invariable; that is, the cable is a fixed condenser in series with a fixed resistance. The statement in the paper is based on constant power factor. For example, in Fig. 3 shown by Mr. Humphries, the phase-displacement angle, $\alpha = \tan^{-1} \frac{I_C\ m\ \omega}{I_B\ R_4}$. With constant voltage I_C is practically proportional to the frequency. Hence if α is fixed C_e must vary inversely as the square of the frequency.

In answer to Mr. Weller's query as to our opinion of the accuracy of a shielded resistance potentiometer divider as compared with an air condenser, I am inclined to favor the air condenser, particularly after Mr. Peterson's presentation of the many sources of error that are encountered and the difficulties met in compensating for them. At this time it is quite certain that the phase angle of a well designed air condenser properly shielded is a minute or less. With a resistance divider and high voltage, considerable phase displacement may occur due to the mutual capacitance of the resistance and particularly to the capacitance to ground. By shielding, this last effect may theo-

retically be reduced to a small value but the shielding is not simple and so far as I know there is no simple method of checking the efficacy of the shielding. About 12 years ago we experienced considerable difficulty in developing a satisfactory resistance potentiometer for only 13 kilovolts. Attempt was made to shield in two ways, by a sectionized shield, all sections of which were made to have the proper potential by means of suitable interposed resistances; and by a method not unlike that described by Mr. Peterson, where an inner glass tube of water was shielded by a concentric outer one. After considerable experimenting it was found advisable to abandon the resistance potentiometer in favor of the air condenser.

Shielding in High-Frequency Measurements

BY JOHN G. FERGUSON[1]
Associate, A. I. E. E.

Synopsis.—*The purpose and usefulness of shielding in high-frequency measurement are outlined. General principles of electrostatic shielding are developed as applied to simple impedances and to networks of impedances, particularly to bridge networks. Practical applications of these principles to the shielding of adjustable impedances, and in the construction of actual bridge circuits are described.*

* * * * *

Introduction

SHIELDING of high-frequency measurement apparatus has for its immediate object the control of certain electromagnetic and electrostatic couplings, unintentionally introduced in the usual high-frequency circuit. These couplings are represented by stray admittances between the various parts of the system, either direct or to ground, and mutual impedances resulting from stray magnetic fields. In general, the control of these couplings is exercised for the purpose of attaining an accuracy of test that cannot be obtained so readily in other ways.

When we speak of electromagnetic and electrostatic coupling, it should be understood that these are simply component parts which together make up the total coupling which exists. They cannot be considered as existing independently of each other, and we cannot consider the shielding for one of these components without taking into consideration the effect on the other.

However, for the frequencies and impedances ordinarily used in communication work at least, this interdependence is small enough to allow us to consider the shielding problem for each type by itself, without getting into practical difficulties due to this connection. As a result, two types of shielding which are known as electromagnetic, and electrostatic shielding have been developed.

It may be argued that by extensive separation of the physical parts of circuits and apparatus, any couplings may be decreased in value, and in consequence errors caused by them can be reduced, thus eliminating any need for shielding. But there are obvious limits to the extent to which this method can be employed practically. In the case of electrostatic coupling to ground, it is scarcely of any value, and in any case, excessive separation of the parts of a circuit introduces other errors due to the length of the wiring involved. Accordingly, it is usually necessary, where the maximum accuracy is desired, to have recourse to shielding.

The principles involved in the application of electromagnetic and of electrostatic shielding are quite different. In the case of electromagnetic shielding, the methods used have for their object the elimination of all couplings from the unit shielded to all other apparatus; thus, if perfect shielding were possible the unit would have no coupling to any other parts of the circuit. It is never possible to accomplish this in the case of electrostatic shielding. Any electrical apparatus will have electrostatic coupling to any other apparatus in the vicinity and particularly to ground. The addition of shielding always introduces additional electrostatic coupling from the apparatus to the shield and the shield usually has more coupling to other equipment and to ground than the apparatus had before shielding it. Consequently, the principles of electrostatic shielding are mainly a matter of controlling this coupling in such a way that it has the least harmful effect in the circuit even at the expense of increasing its actual magnitude, rather than a matter of eliminating it entirely. For this reason electrostatic shielding requires much more extensive consideration and this paper is, therefore, devoted mainly to it, the principles of electromagnetic shielding being covered only briefly.

Principles of Electromagnetic Shielding

The necessity for electromagnetic shielding is limited practically to wound apparatus such as coils and transformers. It may be reduced to a minimum by using high permeability core material wherever possible in coils and transformers, and by using some form of closed core such as the toroidal type. By these means stray fields may be reduced to a relatively low figure. However, there are cases where the remaining coupling may be objectionable and it is then necessary to use shielding to reduce still further the amount of these stray fields.

Two types of shielding may be used. A high permeability material may be used for the purpose of short-circuiting the stray field. The principles of this method of shielding are described fully in another paper and will not be considered further here.

In the case of air-core coils, which are often of the solenoidal type since the advantage of using the toroidal form is less in this case, and for coils used at very high frequency where heavy magnetic material is not so effective, shields of non-magnetic material may be used to confine the field by the effect of eddy currents. For these shields, a material of high conductivity is used, usually copper, and the principal consideration is the

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spacing of the shield from the coil rather than the thickness of the shield itself.

There is always a loss in efficiency due to the losses in the shield, and this loss is greater the closer the shield is placed to the coil; that is, the stronger the field in which the shield is placed. However, even with solenoidal air-core coils, very effective shielding may be attained by moderately thick copper shields, spaced about the distance of a diameter from the coil.

Principles of Electrostatic Shielding

In both theory and practise, all measurements assume that between different terminals or junction points of the system there are impedances having values known to a degree of definiteness consistent with the accuracies sought in the test. In an unshielded circuit, it will generally be the case that the elements connected by the various terminals or junction points will not provide impedances so definitely known, or, in other words, will not carry all of the current flowing between the points in question.

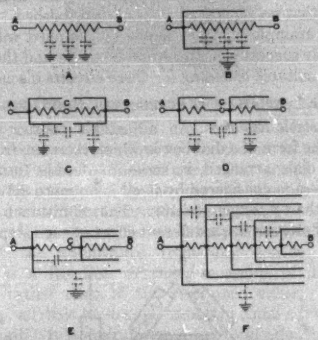

Fig. 1—Methods of Shielding Series Impedances

Resistors. In the case of the simple resistor such as pictured schematically in Fig. 1A, there are admittances from different parts of the conductor to other parts of the whole system and in particular to ground These, of course, act to modify the effective impedance between the terminals and as they vary with the location of the resistor, the result is that its effective impedance is variable and known only for the location in which it has been calibrated. One of the first objects to be accomplished by shielding is to remedy this type of indefiniteness of value. This is done by mounting the elements within a shield of conducting material and in fixed space relation thereto, as shown in Fig. 1B. Thus, the circuit element has direct admittances only to the shield and as these are of fixed value the terminal to terminal impedance becomes independent of the location of the shielded element.

If, then, we connect the shield to any fixed point in the circuit element such as one terminal, all of the current transferred by the shield admittances passes to or from the circuit at this particular point. This concentration of admittance enables the ready evaluation of the effect produced by it when the element is used in conjunction with others in a complete measuring system. We may summarize all of this to form a fundamental rule of shielding; *viz.*, "the association of an element of a system with a shield so that all admittances from the element to other parts of the system or to ground are confined to one terminal."

If it is possible to connect such an element in a circuit so that the terminal to which the shield is connected is grounded, all variable admittances will be eliminated completely.

Series Impedances. In the case of two impedances in series as shown in Fig. 1C, shielding may be accomplished by connecting one shield to terminal A and the other shield to B. In addition to the effects described for a single impedance there will then be admittance between the two shields which will depend on the position of the apparatus. This admittance is slightly more objectionable than admittance from shield to ground, since while we may ground either A or B, there will always be an admittance from one shield to ground which will be variable.

The shields may also be connected as shown in Fig. 1D, in which case the admittance between shields appears across the first impedance. If now we extend the shield connected to A to include the other shield as shown in Fig. 1E, we have introduced a fixed admittance across A C and have variable admittances to ground from A. The admittance across A C is not objectionable in the case of a capacitor since it may be considered simply as an addition to it, but it has the effect of increasing the phase angle of a resistor, and in the case of an inductor, it increases the effective inductance and resistance variation with respect to frequency.

If this combination of impedances can be grounded at A we have a complete system having no variable admittances. The principle may be extended to include any number of series elements, the effect being to place admittances across all of the elements but one, and to enclose the whole system in one outer shield. Such a system for five elements in series is shown in Fig. 1F.

Parallel Impedances. The shielding of parallel impedances is comparatively simple, since any number may be shielded individually and the shielding all connected to the same point. In reducing the shielding of multiple impedances to the simplest form the question arises whether it is sufficient to include them in a single shield or whether in addition they should be shielded from one another. If they are not shielded from one another there will be distributed admittances between them which may cause errors. Preferably each should be shielded individually. Fig. 2 shows such a shielding system for capacitors in parallel.

By following the procedure outlined above, it is com-

paratively simple to apply shielding to any combination of impedances in series or in parallel in such a way that we will have all admittances to external conductors from the shielded elements concentrated at terminals or junction points of the system.

Circuit Shielding. In many cases it is impossible to connect the above combinations in a given circuit so that the outer shield is grounded. In such cases it is necessary to determine from the position of the network in the system the effect of admittances from the shield to other shields and to ground. To illustrate, let us take the simple bridge circuit shown in Fig. 3. The four impedances constituting the arms each may be considered as any combination of individual impedances. With the shields connected as shown, the total admittances are reduced to three; between B and D and from B and D to ground. These admittances do not affect the bridge balance and, therefore, are not objectionable. However, if we add input and output circuits and follow the same system of shielding, we get the result shown in Fig. 4. In this case it is impossible to concentrate all of the admittances at B and D. Neglecting for the present the ground at D, we have added variable admittances from A to B, to D and to ground. The only way of overcoming this difficulty is

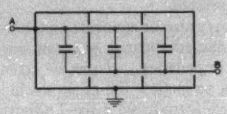

Fig. 2—Method of Shielding Parallel Impedances

to use double shielding as shown, adding an outer shield to the impedance across $A\ C$ and connecting it to D. This puts a fixed admittance across $A\ D$, but as we have not made any distinction between the four arms of the bridge, this admittance may generally be placed across an arm where it can be taken care of satisfactorily. If in addition we ground D, the admittances reduce to a single one from B to ground.

Admittance to Ground of Unknown Impedance. From the above it would appear that the general bridge circuit is susceptible of a simple complete solution, since the shielding shown in Fig. 4 is equally applicable to all cases. This would be true if the unknown impedance to be measured in the circuit had no admittance to ground. This is usually not the case. We generally have an additional requirement, that the potential condition with respect to ground of the impedance during the measurement be defined in some way.

If the impedance can be connected across one arm of the bridge and its value is desired with one terminal grounded, the circuit shown is satisfactory. However, these are special conditions, and where the impedance to be measured forms only part of the total series impedance of an arm, or where the potential requirements are different, such as the requirement that the coil be measured with its terminals at equal potential to ground, the bridge shielding becomes a more serious problem.

In general, the question of selecting the most suitable system of electrostatic shielding for a specific test circuit, resolves itself into a determination of the most advantageous location of the admittances which, as

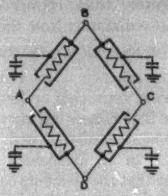

Fig. 3—Bridge Network Using Shielded Impedances

described above, have been arranged to terminate at certain terminals or junction points. The facts which need to be taken into consideration are usually so varied that no general rules can be established. A few typical examples in which shielding is applied with considerable success will, therefore, be taken and the selection of suitable shielding for these circuits discussed.

Examples of Electrostatic Shielding

Adjustable Resistor. An adjustable resistor usually takes the form of a dial box in which there are from one to six dials arranged in series in decade formation. Each decade considered by itself is no more difficult to shield than a single resistor. The admittance of the shield, however, has a different effect at each step which

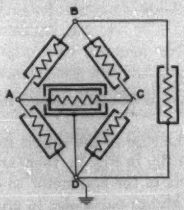

Fig. 4—Completely Shielded Bridge Network

means that the phase angle varies with the setting of the dial. If the admittance to the shield is small, this effect will not be very great and in any case it is always the same for a given setting and hence may be included in a calibration.

In shielding several decades in series, admittances between decades are introduced. Effects due to these admittances can be taken care of completely by the use of nested shields as already shown in Fig. 1F. For a resistance box of five or six dials, this type of shielding becomes prohibitive from a size and cost standpoint and in consequence such shielding is usually not at-

tempted. The use of a single shield for all decades of a resistor means that the impedance of two or more dial settings is not exactly equal to the sum of the impedances of each setting by itself. If the difference is appreciable the only alternative to the expensive type of shielding mentioned above is the use of a calibrated value for every combination of dial settings. This error in additions is smaller the lower the resistance, and usually may be neglected for values below 100 ohms.

Fig. 5—Six-Dial Shielded Adjustable Resistor

The decades are ordinarily connected in series in ascending order of magnitude. The shield should always be connected to the low end, that is, to the terminal to which the decade of smallest value is connected. The reason for this may be explained as follows: the admittance from any decade to the shield is a function of the dimensions of the dial switch rather than of the resistance value. Consequently, all dials have approximately equal admittance to ground. It is desirable that the total admittance to ground be a minimum across the higher resistance settings. This requires that the low resistance dials be connected between the high dials and the shield; that is, the shield should be connected to the low end of the box.

In the actual construction of such a resistor it is essential that the shielding be complete, particularly at the dials. Since the effect of the hands in operating the dials is more variable than any other coupling, it is of very little value to place an unshielded dial box in a metal shield which allows admittance from the hand of the operator to the circuit.

An example of a six-dial resistor in a complete single shield is shown in Fig. 5. The box itself and the panel are of metal and the construction of the dials is such that there is a continuous metal shield between each dial head and the switch proper which it controls.

As stated previously, the effect of the shield on the performance of the resistor is to increase the phase angle of the higher resistance values. In the case shown the admittance introduced by the shielding is of about the same value as the total admittance distributed in the coils themselves and from the coils to the switch parts.

Adjustable Inductor. The same considerations apply to an inductor as to a resistor except that on account of the larger physical size of the former, larger admittances are associated with it and for that reason it is usually necessary to use nested shields. Fig. 6 shows a standard inductor consisting of three decades and an inductometer using four shields. The three top panels have been removed, showing the method of nesting the shields, and the construction used to bring the dial controls through the shields. The shielding of this unit is not complete in that the fourth shield (the outer one) does not extend over the top. This is allowable as the admittance from the third shield to ground is across the inductometer and any variation in it has little effect, particularly as the final balance is obtained with the inductometer, thus eliminating variations due to the hands of the operator, which occur in operating the other dials.

The admittance between shields is considerable, but due to the method of construction, the largest admittance is across the smallest inductance and there is no intershield admittance added across the highest decade. Accordingly, the effect is not so serious as might be thought at first glance.

In the case shown, the inductometer and the lowest decade are generally used only in combination with the higher dials, and under these conditions they are not used at sufficiently high frequencies for the admittance shunting them to have much effect. The greatest effect of the admittance introduced by the shielding usually occurs when the second highest dial is used at a

Fig. 6—Three-Dial Shielded Adjustable Inductor

high frequency with the high dial set on zero. However, the admittance introduced across this decade is not appreciably larger than the distributed admittance across the coils. The shielding, therefore, does not limit the range of these inductance standards to any great extent.

Adjustable Capacitor. The units of an adjustable capacitor are practically always connected in parallel and the problem of shielding them is that of shielding a

single capacitor. It is desirable to shield the decades from each other if the capacitances are small as this facilitates calibration and is easily effected. Where the capacitance is large,—say over 10,000 $\mu\mu$f.—this precaution is unnecessary. The capacitance introduced from the shield to the units has the effect of increasing slightly the value of each dial setting. The form of construction of the shielding is similar to that of the resistor shown in Fig. 5.

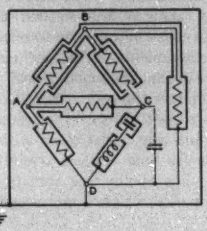

Fig. 7—Shielded Resonance Bridge Network

Bridge Circuits. The general principles of bridge shielding have been discussed by Campbell[3] and the equal ratio-arm comparison bridge has been discussed in detail by Shackelton.[4] The mechanical construction of the bridge itself exclusive of standards is simplified by the fact that there are comparatively few dials to be brought through the shielding.

This bridge with the standards described above may be used for a wide variety of measurements. A rather simple modification is the so-called resonance bridge, in which the bridge unit is an equal ratio-arm comparison

Fig. 8—Shielded Resonance Unit, Top Panel Removed

type, and a resistance is balanced in one impedance arm against a capacitance and an inductance connected in series in the other impedance arm. The balance is usually effected by adjusting the resistance and capacitance.

The shielded circuit of such a bridge is shown in Fig. 7. The capacitance from C to D introduced by the shielding may be compensated for in the usual way by the addition of an equal capacitance across A D. In this circuit, the coil is usually measured under the condition of one terminal at ground potential. Thus, D is shown strapped to the ground shield. For this case, the shielding may be simplified considerably. Fig. 8 shows the mechanical construction of the combined resistance and capacitance standard used with the bridge unit for these measurements. The unit is shown with the top of the outer shield removed. The capacitance, in accordance with the shielding diagram, is double shielded, while the resistance requires only a ground shield.

Another bridge of the comparison type but using capacitance ratio arms is described by Kupfmuller[5], including a complete description of the shielding involved.

A bridge of the general comparison type having self contained standards of resistance and capacitance is shown with top panel removed in Fig. 9. It may be

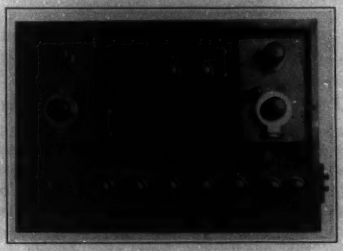

Fig. 9—Shielded Comparison Type Bridge, Top Panel Removed

used for a wide variety of measurements such as series or shunt resonance, and direct comparison, by using the switches controlled by the small dials on the extreme left and right to throw the standards into various combinations.

Another bridge circuit which is interesting from the shielding point of view is the Owen bridge.[6] This is a skew bridge in which the ratio arms are 90 deg. out of phase instead of being equal. For this reason any admittance introduced in one arm by the shielding cannot be compensated for by any equal admittance in another arm. Two methods of taking care of this admittance may be used. It may be concentrated in the arm consisting of a capacitance and considered part of it, or an admittance across one impedance arm may be compensated for by a resistance across the other impedance arm. Both methods have been used in the construc-

3. G. A. Campbell, "The Shielded Balance," *Electrical World and Engineer*, April 2, 1904, p. 647.

4. W. J. Shackelton, *A Shielded A-C. Inductance Bridge*, A. I. E. E. Journal, Feb. 1927.

5. Von K. Kupfmuller, "Uber eine Technische Hochfrequenz Messbrucke," *Elek. Nach. Tech.*, September, 1925, pp. 263-270.

6. D. Owen, "A Bridge for the Measurement of Self Inductance," *Proc. Phys. Soc. London*, October, 1914.

tion of these bridges. A more detailed discussion of the shielding involved in this type of bridge is contained in a previous paper by the present author.[7]

Details of Construction So far, we have discussed the admittance introduced by the shielding without going into details as to the form which this admittance takes, although it has been broadly assumed that it is principally due to capacitance. Since it generally forms an integral part of the measuring circuit, it is obvious that as much consideration should be given to it as to the rest of the circuit. While the admittance is due essentially to capacitance, the necessary supports introduce a certain amount of conductance which causes some difficulty in obtaining compensation.

For instance, in the typical equal ratio-arm bridge circuit where the admittance across one arm requires compensation in the other arm, it is a simple matter to use an adjustable condenser for the compensation of capacitance. However, if the conductance is left uncompensated it may cause considerable error, particularly in the measurement of high impedances at high frequencies. For this reason, it is desirable that all shields be supported by insulating material of the highest quality such as hard rubber, glass, or quartz and that only the minimum amount necessary for satisfactory mechanical support be used.

It will be noticed in Fig. 9 that the wiring is shielded by brass tubing. This shielding is insulated from the conductor by means of bushings, only enough being used to insure that the conductor and shield do not change their relative positions with respect to each other. The insulating bushings used most generally are either hard rubber or glass beads.

Even after taking these precautions, it has been found necessary for the highest precision work at the highest frequencies, to introduce a conductance compensator in the form of a small adjustable condenser in which the dielectric is an insulating material such as phenol fiber. By this means the amount of conductance in one arm may be varied to obtain correct compensation. The balance, once obtained, does not vary appreciably with frequency. Such an adjustment is used with the bridge shown in Fig. 9.

In bridge input and output transformers, which must generally be double shielded, the shielding is rendered more difficult due to the requirement of a low conductance between shields. This demands a much more expensive construction than the simple requirement of complete electrostatic shielding.

Limitations of Shielding. Having discussed the uses and advantages of shielding, it may not be amiss to discuss briefly some of the limitations. As already brought out the introduction of shielding always brings with it some additional admittance. Since this admittance is a function of frequency it is natural that shielding should introduce more trouble, the higher the frequency. However, it is also equally true that the stray admittances, due to lack of shielding, introduce more trouble as the frequency is increased.

In general, it may be said that if shielding a circuit is found to have a definite advantage at moderately high frequencies, it will have an advantage up to the maximum frequency at which the circuit is used. The principles outlined already apply over the whole range of communication frequencies. Where shielding is found to result in frequency limitations it is due to the added admittance introduced with it and not due to inherent defects in the principles involved.

The shielding may, in special cases, limit the maximum frequency at which the circuit will operate; but in such cases it can usually be taken for granted that even if the circuit would operate at higher frequencies without shielding, the accuracy o' the results would be highly questionable. Examples of limitations of shielding may be given using the apparatus already described. Take the case of the resistor shown in Fig. 5. The admittance across the resistances results in objectionably high phase angles and an effective change in the resistances at very high frequencies. While this effect would be present even though no shielding were used, the shielding increases it and therefore limits the maximum frequency at which the apparatus can be used from the standpoint of this type of error. The same limitation occurs in the case of the inductance standards, only it is more serious due to the large physical size of these standards. The exact type of limitation here is that the individual units increase in inductance due to the admittance across them to such an extent that the difference between them cannot be bridged by the next lower decade, thus rendering it impossible to obtain certain values of inductance by any dial combination.

In the case of a symmetrical bridge, the principal limitation is the shunting effect of the admittance introduced across the impedance arms. This becomes so large that at frequencies in the order of 100 kilocycles difficulties are encountered in measuring the current through the unknown impedances by the method of measuring the total current input to the bridge. If the meter question is eliminated, the actual loss in sensitivity, which is the only other objectionable feature of this admittance, may be made of no serious consequence up to frequencies as high as 2000 kilocycles. In all other respects, the shielding functions as satisfactorily at this frequency as at the lower frequencies.

Auxiliary Equipment. While auxiliary apparatus such as oscillators and detectors is not strictly speaking measuring apparatus, its operation is essential to the satisfactory operation of the measuring circuit and so a few words may be added regarding the shielding of this apparatus.

Provided they are separated sufficiently from the

7. J. G. Ferguson, "Measurement of Inductance by the Shielded Owen Bridge," *Bell Sys. Tech. Jl.*, July, 1927, pp. 375-386.

measuring circuit and from each other, there is no need to shield the oscillator or detector from the standpoint of operation of the measuring circuit. However, for maximum flexibility, it is desirable that they be so constructed that no special precautions are necessary in placing them relative to the measuring circuit. If, as is usually done, the individual apparatus included in these circuits is adequately shielded it is only necessary to place the completed equipment in an electrostatic shield to avoid all coupling to the measuring circuit or from one to the other. This is usually done by mounting the apparatus on a metal panel and placing it in a wood box with a sheet metal lining. By this means it is possible to place the auxiliary apparatus as close to the measuring circuit as desired, without introducing errors.

So far as the internal shielding of the auxiliary apparatus is concerned, the same rules as described for other apparatus hold. However, in the case of vacuum tube equipment, wherever any gain is introduced, coupling between parts of the circuit must be reduced in proportion to the gain introduced between these parts. This is usually accomplished quite readily by suitable mechanical design and may be insured by placing each stage in a separate grounded shield, although this is seldom necessary.

Conclusion

It has been impossible to go into very great detail in this brief paper on the subject of shielding. The attempt has been made, therefore, to outline a few general rules and to give representative examples of typical measuring circuits. It will be noted that the examples have been limited largely to the bridge circuit. This is because our experience has shown that this circuit is the most flexible and accurate over the whole of the frequency range over which precise impedance measurements have been made, and because the problems of shielding it are sufficiently difficult and varied to give satisfactory examples of the solution of rather complicated problems. The principles of shielding given have been found to apply equally well at all frequencies and it has been found that up to the maximum frequency at which precision measurements have been made, the shielding methods developed for use with moderate frequencies require practically no modification as the frequency is increased. Experience with measurements and measuring circuits up to 2000 kilocycles, makes it appear probable that when precision measurements are made at still higher frequencies, the shielded bridge circuit will continue to remain the most satisfactory measuring circuit.

Discussion

P. S. Bauer: In the ordinary network theory which we use in working out our bridge equations our fundamental assumption is that, in a series connection between any two points (conjugate or not) the current which flows is a function of time alone and not of position. We then define the electromotive-force between those two points, neglecting mutual induction, as:

$$E = L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i\,dt$$

This implies the concept of lumped inductances, capacitances, and resistances.

If a bridge is to be used at frequencies of 2000 kilocycles, these lumped constants are non-existent with any degree of approximation. Consequently, it seems to me that it is meaningless to talk of either standards of these quantities at these high frequencies or to attempt to measure them by the application of our conventional definitions of these circuit parameters.

C. L. Dawes: I have one or two questions to ask Mr. Ferguson concerning the shielding methods which he describes. In his diagrams he shows shunted admittances from the resistances to ground. Obviously these admittances introduce errors both in the value of the resistances and in the current flowing through them. At low frequencies these errors may be negligible but at the higher frequencies they may be considerable. In developing bridges for high-voltage dielectric measurements we have found that such shunted admittances did not produce appreciable errors in capacitance measurements but even at 60 cycles they did introduce extremely large errors in the power and power-factor measurements. May I ask Mr. Ferguson the approximate magnitude of the errors which these admittances produce and to what frequencies it is possible to go before these admittances cause errors which are prohibitive.

J. G. Ferguson: Mr. Bauer made an interesting observation about the difficulty of separating electrostatic from electromagnetic effects. When we get down to practical measurements the first thing we find is that there are no such things as pure capacitances, pure resistances, or pure inductances, but that we have to make use of what we can get. The practical difficulty in shielding is that it is not correct theoretically to consider electrostatic shielding separately from electromagnetic shielding. It is possible to do this only where the conditions are defined to some extent.

I mentioned particularly that in this work, with the impedances we ordinarily use, electrostatic effects may be considered more or less separate from electromagnetic effects. However, as we go up in frequency the electromagnetic effects become more noticeable, and I might say as a practical instance that the bridge shown on Fig. 9, designed to go to about 3,000,000 cycles, requires that much consideration be given to electromagnetic effects as well as to electrostatic effects. Generally speaking we get into trouble when the current flowing in the electrostatic shield is sufficient to cause magnetic effects. This is definitely true in this bridge, and the only way that it can be taken care of is by using very great care to get a symmetrical arrangement and to keep the wiring in which the current from the shields flows, as short as possible.

Professor Dawes asked about the magnitude of the effect caused by the admittances across shielded resistors. This effect is a function of so many things that perhaps no general answer can be given. The dial box in Fig. 5, is practically a standard box except for the fact that it is shielded. The resistance goes up to a total of 10,000 ohms. The capacitance across each dial in this case is roughly 10 $\mu\mu$ f., and the shielding adds about another 10. In other words the shielding reduces the maximum frequency to about half what it would be without shielding, for the same error in the impedance.

As far as the phase angle is concerned, the coils may be wound sufficiently inductive to compensate for the capacitance introduced by the shield. However, as we go up in frequency we get an error in the impedance which cannot be eliminated except by changing the method of construction.

It may be of interest to mention that in this box, having a capacitance of 10 $\mu\mu$ f. without the shield, and an extra 10 with the shield, the error in the impedance of the 10,000-ohm setting

is about 0.5 per cent at 100 kilocycles and would be about 0.25 per cent if unshielded. As we decrease the resistance the error becomes less. If we increase the frequency above 100 kilocycles of course the error increases. However, there is a certain compensation there, since it is usually the case that the higher the frequency, the lower the impedance used.

The bridge shown in Fig. 9 has resistances that go to a maximum of 1000 ohms. This allows us to go up to a much higher frequency with no greater error than in the box shown in Fig. 5, where the maximum resistance is 10,000 ohms.

Guarding and Shielding for Dielectric Loss*

Measurement ons Short Lengths of High-Tension Power Cable

BY E. H SALTER[1]

Associate, A. I. E. E.

Synopsis.—*This paper presents the results of investigations made to explain the wide differences found between dielectric loss measurements made on full reel lengths of cable and on samples (10-ft. net length) removed from these reels. At least one source of such differences is found in losses occurring at the end of the cable, these losses being insignificant in the case of the full reel but amounting to as much as 100 per cent of the loss in the normal sample. Methods of determining, controlling, and eliminating these end losses are described.*

* * * * *

Introduction

With the advent of low loss insulations it has become necessary to make measurements of dielectric loss and power factor of power cables with greater accuracy and refinement. Since a large proportion of the measurements of this nature are made on comparatively short lengths of cable (10 ft. net length under sheath), the question has often been raised as to what influence the end conditions in these specimens might have on the results obtained.

General

At the Regional Meeting held in Pittsfield, Mass., May 25-28, 1927, Mr. C. L. Kasson presented a paper entitled *High-Voltage Measurements on Cables and Insulators*, in which he pointed out that in measurements with high continuous potentials and high single-phase alternating potentials very large errors might occur due to corona formation at the cable ends and on the lead wires. He showed how by complete electrostatic shielding and guarding of the sample under test and the instrument circuit these losses might be eliminated from the measurements.

Such shields and guards have been in use for some years past in laboratories making dielectric loss and power-factor measurements on short lengths of cable, where to accomplish this end it has been the practise to use the tank of the supply transformer as a shield for the transformer, to use lead-covered power cable for high-voltage leads wherever practicable, to screen the cable sample (and the air condenser, when used) and to use shields on all leads to the measuring instruments, these shields often surrounding the instruments as well. The proper use of such precautions eliminates practically all effects due to direct leakage (conductance leakage) and to capacitance connections between high- and low-voltage circuits. All of the above mentioned have to do chiefly with the equipment for making dielectric loss measurements. It is the purpose of this paper to discuss certain conditions relative to the samples as such which have been found to play a large part in the results obtained in these measurements.

In test work, power cables fall into two general classifications, *i. e.*, single-conductor cable and multi-conductor cable. For power service most multi-conductor cable is three-conductor. There are then two general types of three-conductor cable, according to construction, *i. e.*, the belted type and the shielded conductor type. Each of these three groups represents a slightly different problem in preparation for test. They will be dealt with here in the order named.

Guarding the Single-Conductor Test Specimen

Until recently the accepted scheme of guarding a single-conductor test specimen was as shown in Fig. 1.

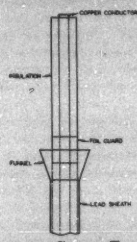

Fig. 1—Preparation of Cable Ends for Single-Phase Dielectric Loss Measurements

The guard consisted of a narrow band of foil, or wire, fastened over the insulation just above the end of the lead sheath. The purpose of such a guard is of course, first, to collect any direct surface leakage over the cable end, and second, to provide a terminus other than the lead sheath for the fringing electrostatic field at the cable end. There is little doubt that such a guard serves these purposes admirably.

However, there has been noted at times a tendency for tests on these short lengths of single-conductor cable to yield results much higher than that obtained on the reel length from which the specimens were removed, higher than could be explained by any reasonable lack of uniformity. The first indication of the possible cause of such high results was found in a cable which was removed from test after showing abnormally

*Part four of a Symposium of six papers on Shielding in Electrical Measurements.

1. Engineer, Cable Research, Electrical Testing Laboratories, New York, N. Y.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

high power factor and ionization factor. In an effort to locate the exact cause of the high values obtained, the lead sheath of this sample was slotted at 1-ft. intervals throughout its length, as shown in Fig. 2. Power factor and ionization factor determinations were then made on these individual sections. The result was that the high values obtained on the sample were definitely shown to be due to the extremely high power factor and high ionization factor of the end sections. To determine if this was a general condition,

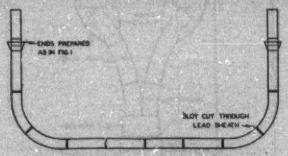

FIG. 2—DISTRIBUTION OF SECTIONS FOR POWER-FACTOR SURVEY ALONG NORMAL SAMPLE

a similar survey was made on a number of test samples with the same general result. The results obtained from a typical survey of this nature are shown graphically in Fig. 3, where each block represents the results of the tests on one 12-in. section. In each case, the

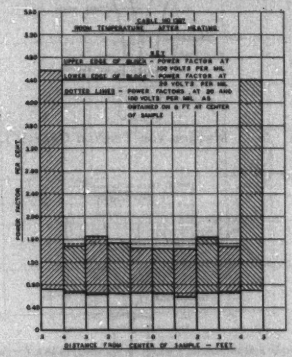

FIG. 3—SINGLE-PHASE DIELECTRIC LOSS MEASUREMENTS

Power-factor variation along length of cable

top of the block is the power factor at 100 volts per mil thickness of insulation while the bottom of the block is the power factor at 20 volts per mil. The height of the block is then the ionization factor.

These findings raised the question as to whether this condition existed before the heating cycle incident to the normal dielectric loss test. To answer this question a sample was prepared with the slotted sheath as shown in Fig. 2, but with each slot filled with insulation and sealed against loss of compound. A survey of this sample before heating gave the results shown in Fig. 4. As the cable was heated it was found that the ionization factor of the end sections increased while that of the inner sections decreased, until at 98 deg. cent. the conditions were as shown in Fig. 5. Then as the cable was again cooled the ionization factor of the end sections remained high, while the inner sections returned to approximately the original condition before heating.

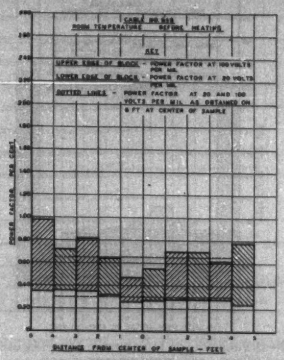

FIG. 4—SINGLE-PHASE DIELECTRIC LOSS MEASUREMENTS

Power-factor variation along length of cable

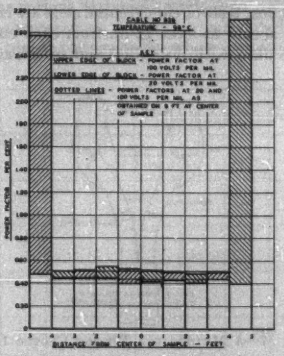

FIG. 5—SINGLE-PHASE DIELECTRIC LOSS MEASUREMENTS

Power-factor variation along length of cable

The final condition after cooling is shown in Fig. 6. These findings have since been confirmed by similar tests on other specimens.

It will be noted in examining Figs. 3, 4, 5, and 6 that the 1-ft. sections at the ends of the specimens are for the most part the only ones showing any considerable change in power factor. In these end sections, the impregnating compound is to a certain extent open to the air so that the increases found are likely due to air

absorption with possible oxidation of the compound while at the higher test temperatures.

The method of eliminating these end losses from measurements on a length of cable is almost self-evident in an examination of the results shown, *i. e.*, eliminate these 1-ft. sections at the ends from the test circuit by making them a part of the guard or shield circuits.

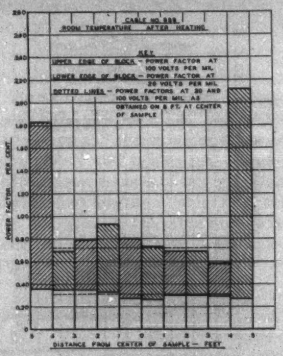

FIG. 6—SINGLE-PHASE DIELECTRIC LOSS MEASUREMENTS

Power-factor variation along length of cable

This, then, means that in preparing such a sample for dielectric loss tests circumferential slots are cut through the lead sheath at least 12 in. from each end. These slots are filled with insulation and sealed against loss

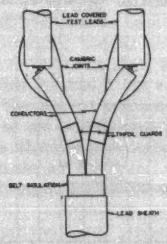

FIG. 7—A METHOD FOR TESTING CROTCH LOSSES

of oil when the cable is heated. The central section of lead sheath then forms the test electrode while the isolated sections at the ends are used as guards.

GUARDING THE BELTED THREE-CONDUCTOR TEST SPECIMEN

Dielectric loss tests may be made on three-conductor cable with either three-phase potential, obtaining the loss directly, or with single-phase potential, computing the loss from the results of a series of tests. While the tests described below were all made with three-phase potential it is felt that the findings are equally applicable to tests made with single-phase potential.

The measurement of dielectric loss under three-phase potential is usually by means of the wattmeter using the three wattmeter method of measuring power in a four-wire three-phase circuit with the current coil of the wattmeter inserted between the ground or neutral

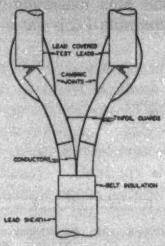

FIG. 8—A METHOD FOR TESTING CROTCH LOSSES

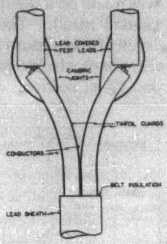

FIG. 9—A METHOD FOR TESTING CROTCH LOSSES

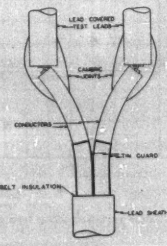

FIG. 10—A METHOD FOR TESTING CROTCH LOSSES

of the Y-connected system of transformers and the ground end of the transformer in the particular phase in which measurement is to be made. This means that *any* currents reaching ground return through the wattmeter. Hence, guard and shield circuits for each phase must be connected back to the ground end of the transformer in that particular phase if the currents to the guards are not to pass through the instrument. In

other words, three independent systems of guards and shields must be used, one for each phase.

This condition means that all shielding and guarding must be complete and must be phase-for-phase. It has therefore been the practise in making such measurements to use three single-phase supply transformers each mounted on insulators, each transformer completely shielded with wire netting and all high-voltage connections made with single-conductor lead-covered

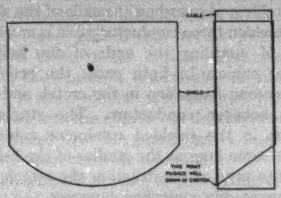

Fig. 11—Shape of Tin-Plate Shields

cable with the lead sheath of each lead connected to the shield system for that particular phase. The specimen of cable under test is then arranged so that the ends of its conductors come very close to the ends of the lead cables, thus making the actual connectors not more

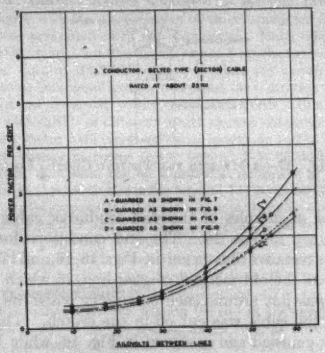

Fig. 12—Variations of Power Factor with Voltage

than one inch in length. These connectors are then wrapped with black varnished cambric tape to make a smooth tight joint.

In making dielectric loss measurements on samples of belted three-conductor cable it was the practise to make the joints as described above and to apply tinfoil guards as shown in Fig. 7. The samples of cable were prepared in such a way that the belt insulation extended beyond the lead sheath about one inch. From this point the conductors were spread apart and the guards placed high enough on the individual conductors so that there was little danger of connection between them (if two guard systems become connected the current coil of the wattmeter is shunted in those two phases). The presence or absence of such guards seemed to have little effect on the resulting loss measured.

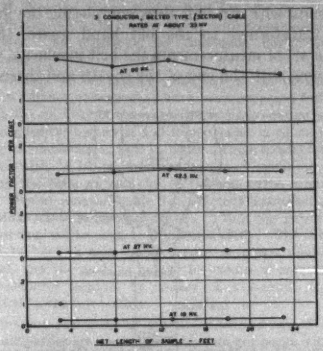

Fig. 13—Variation of Measured Three-Phase Power Factor with Net Length of Sample

It is evident, however, that this scheme guards only against extraneous currents over the conductor insulation to the sheath and does not eliminate any of the currents which might flow through the capacitance between the various "exposed" conductors. This error has become increasingly significant, particularly in the case of the higher voltage cables where ionization of the

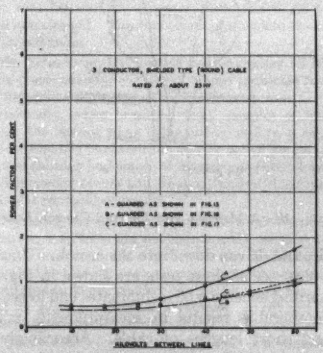

Fig. 14—Variation of Power Factor with Voltage

air in the crotch of the cable gives visible signs of the loss in the form of corona.

In order to determine the magnitude of these end losses and to find the most effective method of preparing the ends of these samples in order to render these end losses negligible, a series of samples was prepared and

tested under each of the following conditions. The starting point was the normal sample prepared as described above and shown in Fig. 7. This guard system was then augmented by completely covering the cambric-wrapped joints over the connections to the test leads with tin-foil, thus completing the guarding from transformer to the old band guard (Fig. 8). After a set of measurements the belt insulation was cut back even with the end of the sheath and these guards

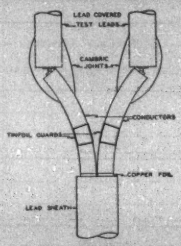

Fig. 15—A Method for Testing Crotch Losses

extended well down into the crotch (Fig. 9). Finally, all of these additional guards were removed with the exception of the four inches of guard well down in the crotch of the ends (Fig. 10). In order to get the guards well down into the crotch and yet keep them insulated, tin-plate cut as shown in Fig. 11 was formed around the conductors, covered with a layer of thin varnished

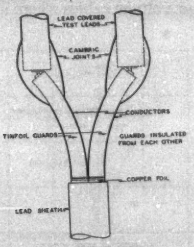

Fig. 16—A Method for Testing Crotch Losses

cambric, and driven down into the crotch. The results of a typical set of these tests are shown in Fig. 12.

A guarding scheme which eliminates end losses should give comparable results in measurements made on long and short lengths of cable. Accordingly, tests were made in which measurements were taken on 25-ft. lengths of cable and on the various sections as these lengths were reduced in 5-ft. decrements. This entire series of tests was made using the guarding scheme shown in Fig. 10, namely, guards about 4 in. long placed well down in the crotch and insulated from each other. Fig. 13 shows the results of a typical test of this kind.

In addition to the loss in the ends due to the tendency for corona formation between conductors in the three-conductor cable there is likely also an inherent loss of the type found in the single-conductor cable. At this time it does not appear possible to determine nor to eliminate this end condition from dielectric loss tests of three-conductor cables.

Guarding the Shielded Three-Conductor Test Specimen

The problem of guarding the ends of the short specimen of shielded three-conductor cable is much the same as that of guarding the ends of the belted three-conductor cable. In both cases the problem is to prevent corona formation in the crotch and to reduce the loss between conductors. The studies of end conditions in the shielded conductor cable followed much the same lines as the studies of the belted cable. Due to the type of construction the crotch losses are much more readily controlled, however.

The results of a series of tests made on one of a

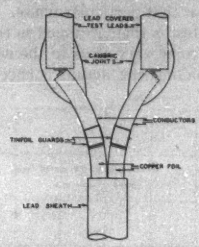

Fig. 17—A Method for Testing Crotch Losses

number of samples of shielded conductor cable tested are shown in Fig. 14. The end conditions for these various tests are illustrated in Figs. 15, 16, and 17. Fig. 15 shows the old method of guarding, in which only a band was put around each conductor while the metal-foil of the cable was cut off in the crotch. The completely guarded end is shown in Fig. 16, while Fig. 17 shows the final arrangement in which the metal-foil of the cable is cut off well above the crotch and the band guard placed just above. This arrangement was found to be practically as good as the complete guarding and much easier to accomplish.

Conclusions

The investigations of end losses in dielectric loss tests of short specimens of high-tension power cable appear to indicate that the most satisfactory methods of minimizing these end effects are to be found in the following methods of preparing the test ends.

1. Single-conductor cable is most effectively guarded against end losses by isolating about 12 in. of cable at

each end of the specimen for use as guards. When making temperature runs, the slots in the lead sheath incident to this preparation must be filled with insulation and sealed against oil leakage. The best slot width is approximately ⅛ in., narrower slots being difficult to maintain and wider slots affecting the loss measurements due to the fringing field from the slot edges.

2. Three-conductor, belted type, cable is practically free from corona loss in the ends if the samples are prepared with the belt insulation cut off even with the sheath end and guards four to six inches long are placed well down in the crotch. For most effective placing of the guards it is advisable to make them of tin-plate or some similar stiff material which can be driven down into the crotch. A thin layer of varnished cambric placed over the guards before inserting serves to insulate them one from the other.

3. The simplest scheme of preventing crotch losses in the shielded three-conductor cable is to leave the metal-foil on the individual conductors beyond the crotch space, a matter of four to six inches. Narrow band guards placed above the ends of the metal-foil then take care of surface leakage.

Discussion

E. W. Davis: We are interested to see that Mr. Salter's experience with the crotch losses of three-conductor cable is in complete agreement with our experience. Data taken in our laboratory more than three years ago on 3-conductor belted cable showed that, as would be expected, filling the crotch with impregnating compound greatly decreased the ionization there and lowered the observed power factor two-thirds. Similar tests made with shields of different length showed continued decrease of power factor with increased shield length up to 7½ in. but no further change in power factor beyond 7-½ in. shield length, indicating that there was no appreciable ionization at 100 volts per mil on this particular cable beyond 7½ in. from the crotch.

Mr. Salter suggests that the ionization in the cable end-sections shown in Figs. 3, 5, and 6 is due to absorption of air and to oxidation of the compound. We have considerable test data indicating that in the particular conditions under consideration, neither effect should result in any increase of ionization factor. The slight excess of ionization in the end-sections of Fig. 4 is undoubtedly due to insulation loss beyond the end of the sheath that is not taken care of by the narrow guard ring, as shown by Mr. Salter. However, it is believed that the extreme end-section ionization shown in Figs. 3, 5, and 6 is also due principally to the same loss beyond the end of the sheath. When the cable is heated, the compound becomes thinner, and since there is nothing to restrain it beyond the end of the sheath, it drains away from that portion of the cable down into the cone, leaving voids behind in the insulation. The ionization of these voids greatly increases the loss beyond the end of the sheath and results in the extreme ionization of the end-sections shown in Figs. 3, 5, and 6.

In Figs. 3 and 6 a small part of the ionization of the end-sections, and a large part of the ionization of the inner-sections is due to expansion of the compound within the cable when heated and its consequent expulsion from the open end of the cable. Upon cooling again the greater part of this compound remains outside the cable, resulting in increased void space throughout the cable.

The decreased ionization of the inner-sections when at 98 deg. cent (Fig. 5 compared with Fig. 4) is of course due to the expansion of the compound and consequent decrease of the void space.

A. S. Dana: I should like to ask Mr. Salter if with his scheme of guarding he is now able to duplicate the readings on a 10-ft. length that he obtains on factory lengths.

D. W. Roper: I was glad to have Dr. Silsbee enlarge upon the definition set forth by Dr. Curtis with reference to the term "shielding," as otherwise my discussion might have been excluded.

In Chicago we are making some tests at the present time to check a number of tests that have been proposed for predetermining the stability of the insulation of high-voltage cable. We are using for the test a size and type of cable in which this factor is of great importance, that is, 66 kv., single-conductor, 750,000 cir. mil with ½ in. of impregnated-paper insulation. We are making the tests on samples of full-size cable, connecting in series several lengths made by different manufacturers, with insulating joints in the lead sheath so that the power factor of each section can be measured independently. The cable is subjected to heating and cooling cycles at about double normal voltage, and the cable is heated by means of current through the conductor applied by means of a current transformer insulated for the test voltage.

In making the tests for power factor and ionization factor, some difficulties are encountered that do not occur with a three-conductor cable; that is, with current on a single-conductor cable a voltage is induced in the sections of the lead sheath that must be neutralized or compensated for in some manner in order to avoid their effect on the measurements. The engineers working on this problem have devised a suitable method for eliminating the effect of this induced voltage, and it is hoped that they will later be able to present their methods to the Institute.

S. J. Rosch: This symposium brings to mind a problem which has baffled us for some time, and to which we hope that someone interested will offer a solution.

We have an insulation-resistance set, in which one side of the galvanometer is permanently connected to an independent solid ground. This ground is obtained by means of a No. 4 B&S solid copper wire fastened to a copper plate buried six feet under ground in moist soil. The dimensions of the plate are 1 ft. square by ¼ in. thickness.

In the same room, but located approximately 15 ft. away, is a single-phase transformer one of whose terminals is connected to an independent ground of similar construction.

Occasionally, when breaking down samples of high-tension cable, at the instant when cable failure occurs, a surge results which occasionally burns out the moving coil of the galvanometer. In view of the high impedance of the galvanometer, I wonder why this phenomenon occurs and what method of shielding is necessary to prevent such occurrence.

In discussing the measurements on the single-conductor cable, Mr. Salter, I believe, did not mention the method he used in making his measurements, whether it was a phase-defect method, a bridge method, or a dynamometer-wattmeter method. I should be interested to know whether Mr. Salter made any attempt to control the guard potentials in his circuit. In our new laboratory we intended to measure power factor and dielectric loss by means of the phase-defect method, three-phase. The apparatus is arranged so that measurements can be made three-phase as well as single-phase.

This symposium I believe would not be complete without some additional data as to whether it is necessary to control the guard potentials in the three-phase method, that is, using the dynamometer-wattmeter method, and I also believe that we should have comparative data of measurements made according to the bridge method described by Professor Dawes, with the various controls for guarding the potential, and the dynamometer-wattmeter

method with guard potentials unneutralized, to see just what the comparative results would be.

C. L. Dawes: I may be able to throw some light on the phenomenon described by Mr. Rosch. At one time we were running tests on a rubber-covered cable wound in a coil and immersed in a tank of water. The power for testing was obtained from a 25-cycle motor-generator set which derived its power from the 110-volt d-c. power mains supplied by the factory power plant. The lighting circuit in the room was supplied from 60-cycle service obtained from the local utility. Hence the two sources of power were entirely independent of each other. When the cable suddenly punctured the lights in the room suddenly went out and we found that the 10-ampere fuses had blown. We were as mystified as Mr. Rosch is by the effects which he observed. The only explanation at which we could arrive was that the fuses were blown by the electromagnetic disturbance created by the sudden rush of current at breakdown. The cable being in the form of a coil of several turns might possibly have been linked sufficiently close with the lighting circuit to permit sufficient transfer of energy to blow the fuses. I am merely offering this incident as a suggestion and not as the positive explanation of the not unlike phenomenon described by Mr. Rosch.

E. H. Salter: During the past two years we have been putting into practise the suggestions made in this paper, and over that period of time have found that, in general, the results obtained on a normal 10 ft. dielectric-loss sample have given very close check against the factory measurements on full reels, that is, against the ionization test as made in the factories on the full-reel lengths.

In connection with the single-conductor cable, Mr. Rosch has asked about the method used. All our single-phase measurements are made by the phase-defect compensation method. In that method the resistance thrown between either the test plate of the air condenser, or the test section of cable, and ground is of the order of 150 ohms. Working on a sample cable 10 ft. in length or less this will produce a maximum drop of about 0.5 volt. We have found that under these conditions, with the lead capacitance of the air condenser equal to the lead capacitance of the cable, the error due to failure to balance the guards amounts to less than 0.01 per cent, absolute value of power factor. In the average measurement this is negligible.

Precautions Against Stray Magnetic Fields in Measurements with Large Alternating Currents

BY FRANCIS B. SILSBEE*

Member, A. I. E. E.

Synopsis.—In electric power stations, it is often necessary to make accurate measurements of current or power when the instruments are located near to the cables carrying large alternating currents. The magnetic field produced by these currents may constitute a serious source of error in the measurements unless careful precautions are taken to guard against such disturbing effects. This paper summarizes the various principles and methods for detecting and minimizing the errors which may arise from this source. The principal methods are; (1) to so arrange the cables carrying the large currents that they produce as little stray magnetic effect as possible; (2) to use instruments of the shielded or of the astatic type; (3) to repeat readings with the instrument connections reversed; and (4) to avoid all loops in the connecting leads.

* * * * *

I. Introduction

It often becomes necessary in the central station or the laboratory to make precise electrical measurements with apparatus which is unavoidably located near conductors which are carrying large alternating currents. The magnetic field produced by such currents may become a source of serious error in the measurements, either because of the direct effect of the magnetic field in adding a ponderomotive force to the moving element of the instrument used in the measurement, or because of the electromotive forces induced in the measuring circuits by the alternating magnetic field.

The magnitude of the effects to be expected may be appreciated if one considers that at a point one meter (39 in.) away from the center of a long straight conductor which is carrying 5000 amperes, the field is 10 gausses. Keinath[1] states that in a certain German aluminum factory, no place could be found where the magnetic field was less than 20 gausses. The normal working field strength of a typical electrodynamic switchboard wattmeter is about 40 gausses and hence, such an instrument, if unshielded and placed in such a field, might be in error by as much as 25 per cent. It must, of course, be borne in mind that the actual error resulting from a given stray field depends not only on the magnitude of the field but also on both the direction of the field with respect to the apparatus which it is affecting, and the relative time phase of the field with respect to the currents flowing in the apparatus. Fields which are not synchronous with the current in the measuring circuit usually cause no error, but may cause troublesome beat effects and may affect the permeability of the iron in transformers.

It is the purpose of this paper to summarize the various principles and methods which have been suggested for detecting or minimizing the errors which may arise from the presence of stray alternating fields. This treatment, however, will be limited to those methods which are applicable in the range of frequencies used in power generation and distribution, though many of these are also applicable to audio- and even radio-frequency measurements.

The most important methods are (1) proper arrangement of the conductors carrying the large current, (2) proper choice and arrangement of measuring apparatus, (3) repetition of readings with reversed connections, and (4) proper location of connecting leads. These methods will be discussed in order. It should be emphasized that no one of them constitutes a panacea for all stray-field troubles but that the obtaining of correct results usually requires the judicious application of several of these principles.

II. Arrangement of Current-Carrying Parts

Whenever it is possible to do so, the conductors carrying the heavy currents should be so placed as to minimize the stray field which they produce. While exact computations of the field produced by various possible arrangements are both difficult and unnecessary, it is often desirable to obtain a rough estimate of such fields; for instance in judging the relative merits of two alternative conductor arrangements. Such estimates may be based on the following approximate relations.

The field at a point P, a considerable distance from a flat loop of cable, is roughly $H = 0.2\,A\,I\,N/D^3$, where H is the field strength in gausses, $I\,N$ is the ampere-turns in the loop, A is the area of the loop in sq. cm. and D is the distance in cm. from the center of the loop to the point P. When the point P lies near the axis of the loop or in the plane of the loop, the direction of the magnetic field is parallel to the axis of the loop. At points which lie in other positions with respect to the loop, the direction of the field is of course quite different, but can be estimated by reference to a sketch of the lines of force around such a loop (*e. g.*, Fig. XVIII in Maxwell Electricity and Magnetism, Vol. 2, 3rd edition).

The field caused by a long straight conductor carrying a current which returns at a great distance is $H = 0.2\,I/D$ and obviously drops off much less rapidly with distance than that caused by a loop. The lines of force are everywhere perpendicular to the conductor

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1. For all references see end of paper.

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29-83

and to the line joining the point considered to the nearest point on the conductor.

Fortunately, however, the return circuit is usually not very distant. If two long straight conductors are at a distance g apart, the field H at a point P located at a considerable distance D (measured from a line midway between the conductors) is $H = 0.2\, I\, g/D^2$. The magnitude of the field for a constant distance D is nearly independent of the location of P with respect to the plane containing the two conductors, although, as in the case of a loop, its direction varies greatly from point to point. Since the stray field is proportional to the spacing g it is very desirable to make this as small as possible. When the conductor section must be large, it is well to use thin strips or slabs placed close together. The effective spacing g is then approximately the distance between center-lines of the slabs and may be made much less than the width of the slabs or the diameter of a circular rod of the same conductance. This arrangement has the further advantage of minimizing the self-inductance of the circuit and hence lessens the reactive volt-ampere capacity of the apparatus required for supplying large currents for testing purposes. In the equipment recently installed at the Bureau of Standards for current transformer testing bars 20 in. by ½ in. have been used.

From the point of view of minimizing stray fields the ideal construction for the heavy conductors is to use a pair of coaxial cylinders. Such a return circuit produces no magnetic field at points outside the outer tube, provided the current distribution is symmetrical about the common axis of the cylinders. While this form of construction has been used to some extent in shunts[2] and certain types of instrument,[3] it tends to introduce considerable mechanical difficulty especially in arranging terminal connections.

It is not generally appreciated how closely the ideal concentric construction can be simulated by subdividing one conductor and placing its parts symmetrically around the other bar. If the return conductor consists of four wires connected in parallel placed 90 deg. apart around the central outgoing conductor and at a distance g from it, the field at a distance D from the central rod has the value $H = 0.2\, I\, g^4/D^5$. The direction of the field is tangential at positions which are on lines drawn through the central rod and each of the returns and is also tangential but in the reversed sense at positions on lines making angles of 45 deg. with these axes. The direction of the field is radial at positions on lines making angles of 22.5 deg. with the axes. When D is as great as 10 g the stray field with this arrangement is only one one-thousandth as great as that caused by an ordinary two-wire circuit with the same spacing.

Even the use of only two return conductors, each carrying one-half of the current and located one on each side of the outgoing conductor, gives a fairly complete neutralization of the stray field. With this arrangement, the field is $H = 0.2\, I\, g^2/D^3$ where g is the distance from the central conductor to either return. At a distance equal to 10 g the field is only one-tenth as great as that from a circuit which has a single return conductor at the same spacing. Here again the intensity of the field is independent of the orientation of the circuit.

The foregoing approximations apply to the case of a single-phase circuit only. When the currents causing the stray field are polyphase, the computations become more intricate. The two conductor arrangements most often used are those in which the three conductors carrying balanced three-phase currents are placed one above the other in a vertical plane, and those in which the conductors lie at the corners of an equilateral triangle. In the former case, the resultant stray field at a considerable distance is approximately $H = 0.2\sqrt{3}\, I\, g/D^2$, where g is the spacing between adjacent conductors and D is the distance to the central conductor. The stray field has a time-phase in quadrature with that of the current in the central conductor, and the direction of the field at any point depends on the location of that point. The field is horizontal at points in the plane of the conductors and also at points in a plane through the central conductor perpendicular to this plane. The field is vertical (*i. e.*, parallel to the plane containing the conductors) at points in a plane which passes through the central conductor at an angle of 45 deg. to the plane containing the conductors.

If the three conductors lie at the corners of an equilateral triangle and carry balanced three-phase currents, the stray field at a distance has a magnitude given by $H = 0.1\sqrt{3}\, I\, g/D^2$. It is purely a rotating field of constant magnitude, and its direction makes a complete revolution about an axis parallel to the conductors during each cycle of current. There is therefore a distinct difference between this type of field and those previously considered. In the previous cases even when the field is intense there is no electromotive force induced in an exploring coil if the latter is placed so that its plane lies along the direction of the lines of force, and this desirable condition is not disturbed as the coil is rotated about this direction as an axis.

In the case of the rotating field, there will be an e. m. f. induced in such a coil unless its axis is strictly parallel to that of the conductors. When in any other position a turning of the coil about an axis which is parallel to the conductors will merely change the phase of this e. m. f. without changing its magnitude.

Table I summarizes the formulas given in the preceding paragraphs and indicates by the numerical values computed for a typical case the magnitude of the stray field to be expected.

An unshielded electrodynamic wattmeter can be used to detect and measure stray magnetic fields by applying rated voltage to the moving coil circuit while the current circuit is left open. When the orientation of the instrument and the phase of the applied voltage have been adjusted to give a maximum deflection, this deflec-

TABLE I

Effective value of stray magnetic field in gausses at a distance of D cm. from the axis of symmetry of a group of long straight conductors carrying a current of 10,000 amperes and arranged as indicated in column 1, with the spacing between adjacent conductors 10 cm.

Arrangement	Formula	Stray field			
		$D = 50$	$D = 100$	$D = 200$	$D = 500$
Single conductor, return at great distance	$\frac{0.2\,I}{D}$	40	20	10	4
Return at distance g...	$\frac{0.2\,I\,g}{D^2}$	8	2	0.5	0.08
Return in two wires on opposite sides.....	$\frac{0.2\,I\,g^2}{D^3}$	1.6	0.2	0.025	0.0006
Return in four wires at corners of square....	$\frac{0.2\,I\,g^4}{D^5}$	0.064	0.002	0.000625	.00000064
Return in coaxial cylinder.....		0	0	0	0
Three-phase circuit conductors in a plane	$\frac{2\sqrt{3}\,g\,I}{10\,D^2}$	13.8	3.5	0.87	0.138
Three-phase circuit conductors in a triangle.....	$\frac{\sqrt{3}\,g\,I}{10\,D^2}$	6.9	1.73	0.43	0.069

tion is proportional to the stray field present at the point where the wattmeter is located. The instrument may be calibrated by placing it at the center of a loop of large radius (R centimeters) carrying a known current (I amperes) in phase with the voltage applied to the wattmeter. The field (H in gausses) at the center of such a loop is $H = \frac{2\,\pi\,N\,I}{10\,R}$ where N is the number of turns in the loop.

III. Choice of Measuring Apparatus

The second line of procedure in avoiding stray-field errors is the proper choice of type and arrangement of measuring apparatus.

When it is feasible, use should be made of magnetically shielded instruments. In these instruments the windings are enclosed in a thick shield of high-permeability iron, which serves to greatly reduce the intensity of the stray field in the interior of the instrument. The protection thus afforded varies of course with the nature of the instrument and the design of the shield. A more detailed discussion of such magnetic shielding will be found in the papers of Curtis* and Gokhale.† The literature[4,5,6] contains data showing a reduction of the stray field errors by factors which vary from 1/3 to 1/60 in different cases. As is often the case with any type of shielding, there is always the danger that the presence of the iron shield may somewhat affect the action of the instrument, and it is customary to laminate such shields so as to minimize the eddy currents which would otherwise be produced by the field from the instrument windings themselves. There is also a possibility of the shield acquiring a certain permanent magnetization which may affect the operation of the instrument on direct current. This effect can be eliminated, however, by taking the mean of readings with the currents direct and reversed.

The operating torque of many electrical instruments is the result of the interaction of two magnetic fields, one of which is much stronger than the other. This is the case, for example, in a separately-excited electrodynamometer or in a vibration galvanometer. In such types of instrument it is desirable to design the instrument so that the moving element of the instrument is the one which produces the weaker of the two magnetic fields. The ponderomotive force on the moving element resulting from its reaction with the stray field (which is of course fixed in space) is thereby made less than with the converse arrangement. Thus an electrodynamic instrument should be used with the separate excitation applied to the fixed coils; and a vibration galvanometer of the fixed-magnet moving-coil type should be preferred to a moving-magnet instrument, unless, of course, some additional shielding is used with the latter.

Certain types of apparatus, such as electrodynamic instruments and mutual inductors, can be built in astatic form by using duplicate coils wound in opposite directions whose effects are additive as regards the currents to be measured but subtractive as regards stray field effects. Apparatus of this type is very much less subject to stray field errors than is ordinary non-astatic equipment, and should be used whenever possible. However, it is well to bear in mind that astatic construction does not entirely eliminate all possibility of stray-field error. The two coils of an astatic instrument seldom have exactly the same area turns, and even if the construction is perfect, the compensation is complete only if the stray field is uniform in direction and intensity throughout the space occupied by the instrument. The field around a laboratory or power station is usually far from uniform, and may be materially different at the two coils of an astatic instrument. Thus, if an astatic mutual inductor the coils of which are 20 cm. apart is used two meters away from a current-carrying loop of wire, the e. m. fs. induced in the two coils will be in the ratio of 1 to 1.37 and the effect of the astatic construction will be to reduce the stray-field error to $^1/_8$ of the value which would occur with a single-coil inductor.

The standard resistors or shunts which are sometimes used in measuring heavy alternating currents must also be considered as liable to stray-field errors. It is usually necessary to place such resistors rather close to the heavy current circuit and the leads running to the current terminals of the shunt are an ever-present source of stray field. Fortunately, most of the modifications in construction which tend to reduce the self-inductance of the shunt also reduce its susceptibility

*See p. 1263.
†See p. 1307.

to stray fields. It frequently happens that the use of additional sets of potential leads distributed symmetrically around the axis of the shunt and all connected in parallel will materially reduce the stray-field error without affecting the self-inductance.

Electrostatic instruments are entirely free from error resulting from stray magnetic fields, unless the moving element contains a very large amount of magnetic impurity. Thermocouple type instruments are affected only to the extent that the permanent magnet of the d-c. millivoltmeter may be partially demagnetized by an extremely strong stray field. Hot-wire instruments, especially those designed with several parallel current paths for measuring large currents, may be seriously in error if a stray field tends to change the distribution of the current among the branch circuits.

IV. Reversal of Connections

A further method, which is almost always applicable even in cases where the circumstances have not allowed the experimenter any leeway in the arrangement of either the current-carrying conductors or the measuring apparatus, is to make two sets of measurements between which the connections of part or all of the apparatus have been reversed. The most desirable procedure is first to make such changes in connections as will reverse the direction of all currents in the measuring circuits except the heavy currents which are suspected of producing a stray field. This may usually be done by reversing the secondary connections of the current transformers. If the results of the repeat measurement made after this reversal show only a slight difference from the original results, one-half this difference may be taken as a measure of the stray field error and the mean of the two results may be considered to be correct. If the difference is large, it indicates the need for locating and removing the source of error. The same principle may then be applied further by studying each piece of apparatus separately and taking a pair of readings before and after reversing its connections. In most cases it is of course necessary to reverse two pairs of terminals, as, for instance, the primary and secondary circuits of a mutual inductor or transformer, or the fixed and moving coil circuits of an electrodynamic instrument. After a survey of this nature has shown which parts of the network are introducing the errors, these parts should be so moved or modified as to reduce the error to a reasonably small value. A final pair of readings should then be taken to give the correct mean value.

While this principle of reversed connections is much more effective than those previously discussed as a means of eliminating stray field errors, even this principle should not be trusted too far. If the difference in readings obtained with direct and reversed connections is large, it follows that some parts of the equipment such as the moving coils of the instruments, the secondary coils of an adjustable mutual inductor, etc., will be differently placed in the stray field when the two readings are made. Consequently, the stray-field effects will not be exactly opposite in the two cases, and the mean will not be the true value. Care must also be taken in the case of instruments with non-uniform scale law to make sure that the proper mean (which may not always be the arithmetic mean of the readings) is taken. Thus, for example, if current is being read on an electrodynamic ammeter having a square-law scale, the arithmetic mean of the two readings of current observed with the direct and reversed connections in the presence of a synchronous stray field will not be exactly the true value of the current. The true value is given by the current corresponding to the average of the two angular deflections or, to express it differently, by the square root of the mean of the squares of the two current readings. This differs from the arithmetic mean of the currents by an amount which is one-sixteenth of the square of the ratio of the difference in the two current readings to the mean current. These differences are fortunately almost always negligible.

V. Connections to Current Circuit

The coupling between the measurement circuit and that carrying the large currents is usually made through a current transformer. There is, unfortunately, a dearth of experimental data on the effect of stray magnetic fields on the ratio and phase angle of current transformers.[7] One would expect, however, that the effect would not be greater in magnitude than the differences observed when the primary conductor is placed in different positions with respect to a hole-type current transformer.[8] With most transformers these differences amount to only a few parts in ten thousand in ratio, but with certain unfavorable combinations of construction and conditions of use, may be as great as one-half per cent.

It occasionally happens that the coupling between the measuring circuit and the heavy-current circuit must be made by a direct connection of potential leads, so as to measure the impedance drop in a run of bus or across some piece of apparatus carrying the large current. In such cases, it is important to recognize clearly that the impedance drop may be very different from the resistance drop and to place the potential leads so that the resultant e. m. f. at the measuring instrument may approximate as closely as possible to the desired quantity. If the total impedance drop is desired, the potential leads should be brought off separately at right angles to the heavy conductor, in such a direction as to avoid as far as possible the magnetic effect of other parts of the heavy current circuit. In other words the mutual inductance between the potential leads and the current circuit

should be zero. On the other hand, if the resistance drop only is desired, a potential lead should be laid back tightly against the conductor throughout the span from its point of attachment to that of the other potential lead. From the point where the two leads meet they should be kept close together and preferably twisted throughout the run from the conductor to the instrument. The theoretically perfect condition is to place the potential leads so that they are linked with a magnetic flux of the same amount as that which links the main conductor. In the case of round conductors this would require the impracticable condition that the lead lie in a hole or groove inside the current carrying rod. In the case of thin rectangular bars, however, the requirement can be met by placing the potential lead close to the bar at a distance from the center of its long side equal to 0.3 of the total width of the bar.

The precaution of twisting the two leads of each part of the measuring circuit is of course essential in all parts of any set-up when stray magnetic fields are present and is a most effective way of avoiding stray e. m. fs. in the leads. Yet an appreciable induced electromotive force has at times been observed in a carefully twisted pair of potential leads when the twisted pair was lying at right angles across a pair of heavy cables (also twisted) which were carrying 1000 amperes. In these cases, the change of field strength with distance was so abrupt that the adjacent loop of the twisted lead could not compensate for the large e. m. f. induced in the loop which lay closest to the heavy cable.

In cases when the conductor arrangement is such that the potential leads cannot be placed in the desired location, it is often possible to correct the experimental results by computing the various mutual inductances involved. Formulas are available[9] for such computations for a great variety of shapes and arrangements of conductors.

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Discussion

C. T. Weller: In regard to Dr. Silsbee's paper, I hope he intends to include in his printed discussion the photographs of his current-transformer testing equipment which he showed here. I was glad to notice that one of schemes for testing high-current transformers represents the one under which they are generally used; namely, with a single return circuit nearby. However, under this condition the shielding effect is largely eliminated in that part of the circuit.

If I understand him correctly, he is planning to use a standard current transformer with series-multiple connections to cover the higher currents; this constitutes an indirect scheme somewhat similar to the one he is now using for the higher-voltage potential transformers. I desire to emphasize that equipment is now available for determining the respective transformer accuracies directly over a wide range of current and potential values.

S. L. Gokhale: With reference to the use of astatic wound coils, Dr. Silsbee says: "The two coils of an astatic instrument seldom have exactly the same area turns and even if the construction is perfect, the compensation is complete only if the stray field is uniform . . ."

The first of the above two points needs only a brief comment even if the two coils be not exactly equal to start with, they can be easily tested and readjusted to any limit of accuracy which may be required. Such a system may therefore be looked upon as a completely satisfactory solution of the problem of uniform stray field. Let us call such a system, an "astatic system of the first order;" it is represented by the formula, $(+1, -1)$ which explains itself.

But even if the stray field be not uniform, exact compensation may still be obtained by the use of astatic systems of higher order. Imagine a system of three coils, represented by the formula $(+1, -1, +1)$; this means that the first and the third coils of the system are of equal area-turns wound in the same direction and that the middle coil is wound in the opposite direction with area turns double of each of the end coils. Such a coil system can exactly compensate for a non-uniform field of the second degree, that is a field having a uniform rate of variation from point to point in any direction. A non-uniform field of third degree can be compensated for by the use of a coil system of the third order $(+1, -3, +3, -1)$; a field of the fourth degree by a system $(+1, -4, +6, -4, +1)$ and so on.

The several coils of the systems of different orders can be expressed by the general formula, $(+1, -1)^n$ expanded by the binomial theorem. These astatic systems are analogous to the harmonic magnets of the corresponding orders described in Section 6 of my paper in this symposium. In our practise we had thus far only one occasion to use a system of the third order; a system of the second order $(+1, -2, +1)$ is quite satisfactory for most purposes.

F. B. Silsbee: Mr. Weller asked some questions in connection with the current-transformer testing equipment which are rather remote from the shielding problem. The answer is that when we have a single return, (one of the four return leads only in use), then, of course, we do not get the compensating effect; the field varies inversely as the square of the distance and more stray field is present in the room. We hope when the set-up is working we shall not get much difference in transformer performance between one and four leads and that we shall be able to use all four leads in our regular testing.

In reference to his other question, we do feel at the Bureau that there is a good possibility in the use of standard transformers which are standardized with their coils so connected as to give a lower range, and then used so as to give a higher range. We have pushed this matter in voltage transformers to a range of 100 kv.

We are intending to pursue the same policy in the case of the current transformer.

We recognize there is one serious source of difficulty in the contact resistances of the various circuits which are connected in parallel. We feel, however, that by using an exploring coil which can be moved opposite each of the primary turns in succession, we shall be able to tell whether or not the current is dividing properly between the turns, and thus at least know that something is wrong if anything is wrong. We hope the matter will work out satisfactorily.

I think Mr. Gokhale has added quite a bit to the theory of astatic coils by generalizing the process. I think there might be cases where this method would be applicable, but generally it would be rather a complicated thing to carry out.

Magnetic Shielding

(Shielding of Magnetic Instruments From Steady Stray Fields)

BY S. L. GOKHALE[1]

Member, A. I. E. E.

Synopsis.—*This paper is a brief survey of the vast amount of work on magnetic shielding of instruments against steady stray fields, and allied problems, by several authors during the last fifty years. It contains, first, a simplified presentation of the method of zonal harmonic analysis, by way of introduction to the mathematical theory of shielding as presented by those authors, and second, a brief summary of their theoretical inferences and experimental findings. In a paper of this kind no "conclusion" is necessary; the following table of contents explains the plan of presentation followed herein.*

1. *Introduction*
2. *Designed and incidental shielding*
3. *Problem of shield design*
4. *Theory of shielding, methods of analysis*
5. *Method of magnetic images (geometric)*
6. *Harmonic analysis; conception of harmonic magnets and harmonic images*
7. *Zonal harmonics*
8. *DuBois' formula for shield factor*
9. *Material for shield*
10. *Importance of differential permeability for shielding*
11. *Failure of magnetic shield*
12. *Incidental shielding*
13. *Dynamic shielding*
14. *Differential shielding*
    Appendix
    Mathematical theory of Zonal Harmonics
15. *Zonal Harmonic coefficients*
16. *Geometric and harmonic images*
17. *Analysis of any magnetic field in general into component spherical or zonal harmonics.*

1. Introduction

THE phenomenon of magnetic shielding is very important from the practical as well as from the theoretical point of view. A considerable amount of work has been done in the past, by way of developing a complete mathematical theory for each important case of shielding which occurs in laboratory or engineering practise. Several formulas have been evolved for practical use, and some of them have been verified by test. The mathematical treatment of the subject as presented by the several authors is not only not very easy to follow, but it starts out from a point in higher mathematics generally beyond the range of studies of the average technical student. Consequently, most of the work in the past has remained like a sealed book to all except those who were compelled, and had the ability, to forge their way through that mathematical maze. The purpose of this paper is two-fold, first, to introduce the mathematical aspect of the problem in a simple form so as to bring the reader to the starting point of the classical work presented by authorities like Professor Rucker, without having to go through the Laplacian equation and its solution as a necessary preparation, and second to summarize results of the mathematical study and of the experimental findings of the other physicists. It contains nothing new except the mode of presentation.

2. Designed and Incidental Shielding

For practical purposes the study of shielding may be divided into two divisions.

(a) Study of shield design

(b) Study of incidental shielding

The classification is important for practical purposes only, and has no foundation in theory. The purpose of the study (a) is to obtain a formula for the most efficient shield and is concerned with those cases wherein good shielding is essential; the study (b) is concerned with cases in which shielding occurs incidentally as a collateral phenomenon irrespective of desirability or undesirability of the effect. The same fundamental theory underlies both the above aspects of the study, but the conditions involved in the two problems call for different methods of approaching the solution, hence the need of the above classification. The problem of designing a satisfactory shield for protection of galvanometers against external fields belongs to the first class, the problem of computing magnetic forces on a conductor located in the slots or tunnels of the armature of a motor belongs to the second class. The solution of the first problem is best obtained by the use of harmonic analysis, the solution of the second by the use of electromagnetic images (geometric). In anticipation of the following analysis, it may be noted at this point, that the method of harmonic analysis is also fundamentally a method of magnetic images. (Section 6).

3. Problem of Shield Design

The problem of shield design in its theoretical aspect is the problem of formulating an equation expressing the ratio of the original field at a point, to the field at the same point when protected by the shield. In other words, the problem consists of deriving an equation of the form.

$$H e = H_i \,.\, F(\mu, a, b, c, \text{etc.}) \qquad (1)$$

$$= g \,.\, H_i \qquad (1\text{-}2)$$

Part 6 of a Symposium of six papers in Shielding in Electrical Measurements.

1. Research Engineer for Magnetics, General Electric Company, Schenectady, N. Y.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-118

where

$H\,e$, is the field without protection

$H\,i$, the field with protection

g, the shield factor

μ, the permeability of the shield material

a, b, c, etc., geometric characteristics of the shield

In the case of spherical or cylindrical shields the symbols a, b, c, stand for the several radii of the shells which form the shield.

In practise, the problem goes one step farther, and calls for an expression for the most efficient shield design, together with the value of the shield factor of that design. For convenience of computation, the shields are supposed to consist of one or more concentric shells of spherical or cylindrical form. The permeance function of a cylindrical shield—see Equations (**10**) and (**15**); that is, of a shield made of a set of cylindrical shells of sufficient length, is only 8/9 of that for a spherical shield of corresponding radii and of the same material. See Equation (**17**). It is obvious from this that shells of intermediate form must have a shield factor somewhere between these two extreme limits, and it is therefore not necessary to try to compute for or to design any other shapes. Theoretically, the cylindrical shield is supposed to be of infinite length but it has been found in practise that for a point near its center, a shield of length equal to its diameter is almost as good as one of infinite length, and a shield longer than its diameter by any amount gives an efficiently shielded space of that length. (DuBois, "Magnetic Shielding," *Electrician*, Vol. 4, p. 645, Fig. 9). A small amount of further improvement can be obtained by the use of shield plates at both ends for which no formula has been worked out, (Dye, *Journal of Scientific Instruments*, Vol. 3, p. 69). Small holes in these plates make practically no difference in the efficiency of the shield. It is understood that in the case of the cylindrical shield, the axis of the shield should be at right angles to the magnetic field against which protection is required. In all cases, shielding depends on the permeability of the shield; the higher the permeability of the shield material the more efficient the shield will be. (See also Section 10.)

4. Theory of Shielding: Methods of Analysis

Theoretically, the action of a shield consists of a change of path of flux lines due to refraction of those lines at the boundary surfaces of the shield caused by differences of permeability on the two sides of the boundary. This interpretation of the shielding process cannot be reduced to mathematical formulas, except in an indirect way in terms of some intermediate phenomenon, such as distribution of magnetism on the surfaces of the shield. Thus there arise three practical methods of interpreting the action of the shield in mathematical terms.

1. The method of least reluctance
2. The method of images
3. The method of harmonic analysis

The first method lends itself easily to account for the action of the shield in a qualitative sense, but it is not convenient for quantitative work. The second and third methods are best suited for computation. The first method is based on the conception that lines of flux are refracted so as to follow the path of least reluctivity, and consequently, that most of the flux lines follow the apparently longer but really the easier curved path through the walls of the shell, instead of traversing the straight and shorter path through the space enclosed by the shield. This line of reasoning is very convenient for a classroom exposition but it fails to explain clearly the fact that some lines traverse the space enclosed by the shell instead of all lines going around the enclosure; nor does this conception explain the bending of the lines outside the shield where they approach the shield without touching it, and follow a longer path of apparently greater reluctance without

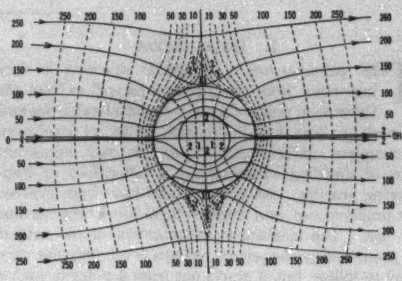

Fig. 1—Bending of Flux Lines by a Shield

any compensating diminution of reluctance by going a part of the way through the iron (see Fig. 1, Line No. 200). It is possible to explain all these difficulties by taking fully into account the diminution of permeability by congestion of flux, the bending of the equi-potential surfaces, and the broadening of the tubes of flux, but then the explanation is no longer simple and has no other compensating merit. The other two methods are based on the conception that the shells are themselves magnetized by the inductive action of the field and become in turn a source of a second magnetic force which in the enclosed space is opposed to the original field. The value of the field in this space is the resultant of the original field and the opposing field produced by the shield. This method of interpretation is not very difficult to comprehend and has the further merit of lending itself easily to mathematical analysis.

5. Method of Magnetic Images (Geometric)

This method is analogous to the well known method of electrostatic images. The magnetism induced by the original field on the surfaces of the shield is regarded as equivalent to certain imaginary magnetic poles located

at appropriate points just as in the case of electrostatic images. The resultant field is then easily computed in accordance with Coulomb's law of inverse square. In the case of solid spheres, solid cylinders, and infinite mass bounded on one side by a plane surface, the images can be easily located and the computation presents no difficulty, but when the problem involves more than one surface the magnetic distribution can be represented only by a series of infinite images which adds greatly to the difficulties of their location and computation. This method is therefore not available for problems of shield designs which even in the simplest case involve two surfaces and in most practical cases at least four surfaces. This method of images will be referred to as a method of geometric images, to distinguish it from the method of harmonic images which will be described next.

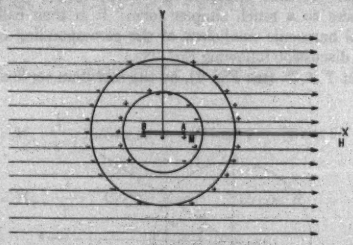

Fig. 2—Magnetic Poles on a Shield Due to Induction (Bending of Lines is Ignored in This Diagram)

6. Harmonic Analysis: Conception of Harmonic Magnets and Harmonic Images

The method of zonal harmonics is fundamentally a method of images, with this difference, that with the method of images proper, that is, of geometric images as defined in the last section, both the magnitude and location of the images are determinate and are determined directly from the magnetic and geometric conditions of the problem. In the case of zonal harmonics, the images are indeterminate in location as well as in magnitude to start with, but they can be located at any point selected at will; the magnitudes of the images can then be determined by a process analogous to that of determining the coefficients in a Fourier series.

The images in this method of analysis consist of a series of sets of imaginary magnets of small size, arranged as required by the conditions of the problem, so as to produce at any point on any specified spherical surface, the same magnetic potential as the magnetic distribution in question produces. This will be clear from the following explanation.

In Fig. 2 let H be the magnetic field in question which may or may not be uniform. A spherical shield is placed in the field, the center of the sphere being the origin of the coordinate axis. The field induces magnetic poles on the surfaces of the shield as indicated qualitatively by the diagram. The lines of flux in the field and also within the walls of the shell, are also bent under the influence of the induced magnetism (Fig. 1), although this bending is ignored in Fig. 2 for simplicity. The magnetism induced on the shield, which is analytically the basic reason for the changes in the field, can be represented by equivalent magnetic images, all located near the center of the sphere, and of magnetic strengths which can be determined analytically.

Imagine a very short magnet of polar strength m, of length $2\,a_1$, and of magnetic moment $M = m.\ 2\,a_1$, located at the center of the sphere and oriented positively, along the axis of X, that is with the positive pole on the positive side of the coordinate axis. (See Fig. 3-1.) We will call this magnet a harmonic magnet of the first order, and of strength M_1. If $M_1 = 1$, the magnet is called a *unit* harmonic magnet of the first order.

Next, imagine two shorter magnets of length $2\,a_2$ each, one located about the positive pole of the first magnet M_1 and oriented positively, the other located about the negative pole of M_1 and oriented negatively. (See Fig. 3-2.) Call this system of magnets, a harmonic magnet of the second order, of strength $M_2 = m\ .\ 2\,a_2\ .\ 2\,a_1$, where m is the strength of each of the four poles. If $M_2 = 1$, it becomes a unit harmonic magnet of the second order.

Next, imagine a system of four still shorter magnets of length $2\,a_3$ each located about the four poles of M_2 and oriented positively or negatively according to the poles of M_2 concerned. (See Fig. 3-3.) Call this system of magnets, a harmonic magnet of the third order, of

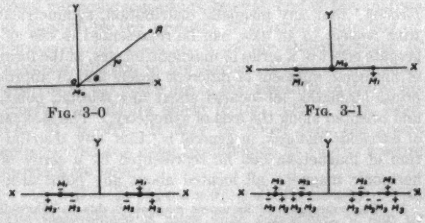

Fig. 3—Harmonic Magnets; The Subnumber Refers to the Order of the Magnet

strength $M_3 = m\ .\ 2\,a_3\ .\ 2\,a_2\ .\ 2\,a_1$, where m is the strength of each of the eight poles. If $M_3 = 1$, it becomes a unit harmonic magnet of the third order.

Repeat the process, doubling the number of magnets of each step: thus we have harmonic magnets of successively higher order. The strength of the magnet of nth order is $M_n = m\ .\ 2\,a_n\ .\ 2\,a_{n-1}\ .\ .\ .\ .\ .\ .\ 2\,a_2\ .\ 2\,a_1$. If $M_n = 1$, we have a unit harmonic magnet of the nth order.

The process can also be carried backwards one step: imagine a single positive pole $M_0 = +\,m$ located at the

center of the sphere, and therefore at the middle point of M_1, (see Fig. 3-0, 3-1). Call this single pole magnet, a harmonic magnet of zero order of strength M_0.

The backward process can be carried indefinitely beyond the zero order, giving imaginary harmonic magnets of negative orders, M_{-0}, M_{-1}, M_{-2}, etc. These magnets give rise to zonal harmonics of second kind, *viz.*, Q_0, Q_1, Q_2, etc. This paper is limited to the study of real harmonic magnets and their effects.

This is the planetary conception of harmonic magnets.

For analytical purposes, the harmonic magnets can also be conceived as the result of magnetic displacements, as follows:

Imagine a single positive pole $+ m$ located at 0, (Fig. 3-0), and call this as before a harmonic magnet of zero order, of strength $M_0 = + m$

Imagine a second pole $- m$, superposed on $+ m$ so as to neutralize both to start with; the two poles are then stretched apart over a very short distance $d x = 2 a_1$ along the coordinate axis X, the pole $+ m$ moving in the direction $+ X$. Thus we obtain a harmonic magnet of the first order, of strength $+ M_1 = m \cdot 2 a_1$.

Next, imagine an exactly similar magnet with its poles reversed, and superposed on the first; the two magnets are now slid along each other, so as to displace the corresponding poles over a distance of $2 a_2$. This gives us a harmonic magnet of the second order. And so on.

The planetary conception is easier to comprehend, but the displacement conception is better adapted for analytical operation. See Section 15, Eq. (**21**).

The method of zonal harmonics is based on the *theorem*,[3] that any magnetic distribution, symmetrical with respect to an axis, can be represented as the aggregate effect of a series of magnetic images, in the form of harmonic magnets of different orders and of appropriate strengths, all located about any selected point, and oriented along the axis of symmetry. The *theorem of harmonic analysis*, in general is,[4] that any distribution of magnetism can be represented by a series of harmonic magnets, all located about any point arbitrarily selected somewhere not necessarily near the center of the distribution and of magnitudes capable of being determined by a process analogous to that of Fourier's Series. For a more precise and formal statement see Equation (**6**).

7. Zonal Harmonics

The potential at any point on the surface of a sphere, due to a harmonic magnet of any order located about the center of the sphere, is called a simple spherical harmonic of the corresponding order. In the particular case of a sphere of unit radius, and with a harmonic magnet of unit strength, the value of the potential reduces to a much simpler form; it is then called a zonal harmonic coefficient of the corresponding order. (See discussion following Eq. (**29**).)

Let $T\,A\,T'$ (see Fig. 4), be the meridian section of a

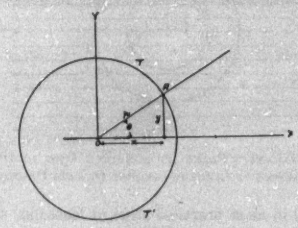

Fig. 4

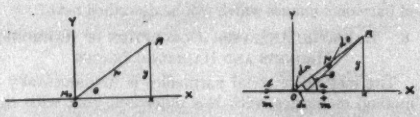

Fig. 4-0 Fig. 4-1

Fig. 4—Potential at a Point Due to Harmonic Magnets; Subnumbers Refer to Order of Magnet

sphere of radius r, and with center O, which is also the origin of the coordinate axes. Let A be any point on the sphere arbitrarily selected, having coordinates x and y, or the corresponding polar coordinates r and θ. Let the harmonic magnet under consideration be located about the center and oriented along the prime vector. Then the potential at A due to such a magnet of any order n and of strength M_n can be expressed by the general form.

$$V_n = M_n \cdot \frac{1}{r^{(n+1)}} \cdot f_n (\cos \theta) \qquad (2)$$

The expression on the right hand side is a simple spherical harmonic of the nth order. The factor $f_n (\cos \theta)$ is called a zonal harmonic coefficient of the corresponding order.

The specific values of the potential at A due to harmonic

3. The conception of harmonic magnets is due to Maxwell who first applied it to the study of electrostatic fields. Maxwell's conception was limited to real harmonic magnets.

4. The theory of spherical harmonics in general, is not limited either to symmetry or to uniformity of the field or to symmetry of distribution of the induced magnetism. By the use of three sets of harmonic magnets along the axis of X, Y, and Z, respectively, any distribution and any kind of field can be analyzed into harmonics, but for simplicity of presentation this paper deals with the simplest case, namely, that of a symmetrical distribution.

The first of the above two enunciations of the theorem refers to the problem in its limited scope, the second refers to the problem in its general aspect. When all the imaginary harmonic magnets lie along a single coordinate axis, they give rise to zonal harmonics (see Section 7). When they are distributed along the three axes they give rise to spherical surface harmonics in general. Zonal harmonic analysis is only a simpler particular case of spherical harmonic analysis.

magnets of orders ranging from 0 to 7, and of strengths M_0, M_1, M_2, etc., are

$$V_0 = M_0 \cdot \frac{1}{r} \tag{2-0}$$

$$V_1 = M_1 \cdot \frac{1}{r^2} \cos\theta \tag{2-1}$$

$$V_2 = M_2 \cdot \frac{1}{r^3} \cdot (3\cos^2\theta - 1) \tag{2-2}$$

$$V_3 = M_3 \cdot \frac{1}{r^4} \cdot 3\,(5\cos^3\theta - 3\cos\theta) \tag{2-3}$$

$$V_4 = M_4 \cdot \frac{1}{r^5} \cdot 3\,(35\cos^4\theta - 30\cos^2\theta + 3) \tag{2-4}$$

$$V_5 = M_5 \cdot \frac{1}{r^6} \cdot 15(63\cos^5\theta - 70\cos^3\theta + 15\cos\theta) \tag{2-5}$$

$$V_6 = M_6 \cdot \frac{1}{r^7} \cdot 45\,(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5) \tag{2-6}$$

$$V_7 = M_7 \cdot \frac{1}{r^8} \cdot 315\,(429\cos^7\theta - 693\cos^5\theta + 315\cos^3\theta - 35\cos\theta) \tag{2-7}$$

For detailed computation and derivation of the above equations, see Appendix, Section 15.

The several values of V for the different harmonic magnets, are simple spherical harmonics, each of the order corresponding to that of the harmonic magnets. Each of these harmonics is made up of four factors, namely

The factor representing the strength of the magnet
The r-function factor, a negative rational power of r
The θ-function factor, a rational function of $\cos\theta$
The numerical factor, which is merely a part of the θ function factor; it is separated merely for convenience, the separation having no theoretical significance.

The θ-function factor together with the numerical factor is called a zonal harmonic coefficient. It represents the value of V for a sphere of unit radius, with a harmonic magnet of unit strength at the center. Using the symbol Z to represent that coefficient, the general equation for V (Equation (2)) may be expressed in the form

$$V = M_n \cdot \frac{1}{r^{(n+1)}} \cdot (Z_n) \tag{3}$$

where Z_n stands for f_n ($\cos\theta$), (see Equation (2)).

In practise, the coefficient (Z_n) is divided by another numerical factor $n\,!$ (factorial n); it is then called "practical zonal harmonic coefficient" or briefly, zonal harmonic, and is represented by the symbol P_n. Thus we have the equations

$$V_n = \frac{M_n}{r^{(n+1)}} \cdot (Z\,n) \tag{3}$$

or

$$V_n = n\,! \cdot \frac{M\,n}{r^{(n+1)}} \cdot P_n \tag{4}$$

where

$$P_n = Z_n / n\,! \tag{5}$$

(See Appendix, discussion following Equation (29.)

The theorem stated at the end of Section 6, may now be restated in more precise terms

The potential V at any point on the surface of a sphere, due to magnetic distribution symmetrical with respect to an axis, can be analytically represented by a series of the form

$$V = A_0 + A_1 \cdot r \cdot P_1 + A_2 \cdot r^2 \cdot P_2 + \text{etc.} + \frac{B_0}{r} + \frac{B_1 P_1}{r^2} + \frac{B_2 P_2}{r^3} + \text{etc.} \tag{6}$$

where, A_0, A_1, etc., and B_0, B_1, etc. are determinate constants; sometimes only the first part (A_0 + etc.), sometimes only the second part $\left(\frac{B_0}{r} + \text{etc.}\right)$, and sometimes both the parts are necessary according to the nature of the distribution which calls for analysis; stated in general terms, the series takes the form

$$V = \sum A_n r^n P_n + \sum \frac{B_n}{r^{(n+1)}} \cdot P_n. \tag{7}$$

This is the basic equation underlying the method of zonal harmonic analysis in general, and of the analysis as applied to the problem of magnetic shielding in particular; for example, it is the starting point of Rucker's analysis in his classical article on shielding, wherein he has made use of this equation to derive his general formula for shield factor. (*Phil. Mag.* 1894, Vol. 37, p. 97). It may be noted that the above equation bears a close resemblance to Fourier's series in which we have

$$V = \Sigma\,(A_n \cos n\,\theta) + \Sigma\,(B_n \sin n\,\theta) \tag{8}$$

The resemblance is more than superficial, it can be shown that the zonal series is merely a more complex form of the Fourier series. The method of determining the values of the factors A_n and B_n in the case of the zonal series is exactly the same as in the Fourier's series.

Note: For significance of the constants A_n and B_n in Equation (7), see Equations (31-2), and (33-2); for proof of the theorem see Section 17.

8. Dubois' Formula for Shield Factor

It is not possible in a brief paper of this kind to go through the complete analysis of the shield problem. The purpose of this paper is merely to introduce the subject for the benefit of those whose knowledge of harmonic analysis is limited to Fourier's Series. For a more complete study of the problems, see "Magnetic

Shielding" by Esmarch (*Ann. der Physik*, Vol. 39, 1912, p. 1553), and by Rucker (*Philosophical Mag.*, Vol. 37, 1894-95, p. 95).

From Rucker's general formula referred to above, Dubois[5] has derived the following formula for the case of a spherical shield of two concentric shells, with a shell of non-magnetic material between, (see Fig. 5).

$$g = 1 + \frac{2}{9}\,\frac{(\mu-1)^2}{\mu}\left[(1 - p_1\,p_2) + \left(\frac{(2\,\mu^2 + 5\,\mu + 2)}{9\,\mu}\right) m_1\,m_2\,m_{12}\right] \qquad (9)$$

where

$$p_1 = r_1^3/R_1^3; \quad m_1 = 1 - p_1$$
$$p_2 = r_2^3/R_2^3; \quad m_2 = 1 - p_2$$
$$p_{12} = R_1^3/r_2^3; \quad m_{12} = 1 - p_{12}$$
$$g = H\,e/H\,i; \text{ (see Equation (1-2)}$$

Using the abbreviation symbol

$$c = \frac{2}{9}\,\frac{(\mu-1)^2}{\mu} \qquad (10)$$

where c is the permeance function of a shield of spherical form, the equation takes the form

$$g = 1 + c\,(1 - p_1\,p_2) + c\,(c+1)\,m_1\,m_2\,m_{12} \qquad (9\text{-}2)$$

By making $r_2 = R_2$, and therefore, $p_2 = 1$ and $m_2 = 0$,

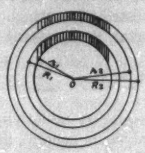

FIG. 5—SPHERICAL SHIELD OF TWO SHELLS

the outer magnetic shell is eliminated, and the shield is reduced to a single shell for which the shield factor is

$$g_1 = 1 + c\,(1 - p_1) \qquad (11)$$
$$= 1 + c\,m_1 \qquad (11\text{-}2)$$

or

$$(g_1 - 1) = c\,m_1 \qquad (12)$$

$(g - 1)$ is called the shielding power of a shield.

Equation (11-2) shows that the shield factor of a shield depends not on the absolute size of a shield, but on m, that is, on the relative mass of the shell wall to the size of the shell.

In the same way by making $R_1 = r_1$ and therefore $P_1 = 1$, $m_1 = 0$, we have

$$g_2 = 1 + c\,m_2 \qquad (10\text{-}2)$$

Dubois' equation may now be transformed so as to express g as a function of g_1 and g_2.

$$\begin{aligned} g &= 1 + c\,(1 - p_1\,p_2) + c\,(c+1)\,m_1\,m_2\,m_{12} \\ &= 1 + c\,[1 - (1 - m_1)\,(1 - m_2)] + c\,(c+1)\,m_1\,m_2\,m_{12} \\ &= (1 + c\,m_1)(1 + c\,m_2) - m_1\,m_2(c^2 + c) + c(c+1)m_1\,m_2\,m_{12} \\ &= g_1\,g_2 + m_1\,m_2\,(c^2 + c)\,(1 - m_{12}) \\ &= g_1\,g_2 - c\,(c+1)\,m_1\,m_2\,m_{12} \end{aligned} \qquad (13)$$

5. See DuBois: *Electrician*, Vol. 40, 1897, p. 317.

This is DuBois' equation second form. This equation shows that the shield factor consists of two terms; the first of these is made of two main factors, each of these factors being the shield factor of the corresponding shell. The second or negative term represents the effect of the mutual reaction of the two shells on each other and is a measure of the loss of sheilding power due to such reaction. For a pair of shells of given radial ratios (m_1 and m_2 constant), the reaction reaches the maximum value, and that g reaches the lowest value when $p_{12} = 1$, that is, when the air space is eliminated.

If the shells be not very thin, and also if the permeability of the shell material, and therefore the permeance function of the shield be very high, we can assume $c = c + 1$, and also $c\,m_1 = 1 + c\,m_1$, and $c\,m_2 = 1 + c\,m_2$ approximately; this approximation reduces DuBois' equation to a much simpler and more instructive form.

$$\begin{aligned} g &= (1 + c\,m_1)\,(1 + c\,m_2) - c\,(c+1)\,m_1\,m_2\,p_{12} \\ &= c\,m_1\,c\,m_2 - c^2\,m_1\,m_2\,p_{12} \text{ (approximately)} \\ &= g_1\,g_2 - g_1\,g_2\,p_{12} \text{ (approximately)} \\ &= g_1\,g_2\,(1 - p_{12}) \text{ (approximately)} \\ &= g_1\,g_2\,m_{12} \quad \text{(approximately)} \end{aligned} \qquad (13\text{-}2)$$

That is, the shield factor of the whole shield is approximately the product of the shield factors of the individual shells multiplied by the clearance ratio of the air space between the shells. This form of the equation brings out clearly the function of the air space in improving the efficacy of the shield; it reveals the fact, not obvious at first sight, that a shield made up of two shells with non-magnetic space between is superior to a shield of the same size with all the space filled with the magnetic material. It would appear that the reluctance of a single shell filling the whole space occupied by the shield, ought to be less than that of a multi-shelled shield with air spaces between. This view is contradicted by the well recognized fact that a multi-shelled shield with air spaces between, is far superior to a solid shield of the same size, although the latter contains much more magnetic material and has apparently much less reluctance. DuBois' formula has been found to be in close agreement with the shield factor obtained by measurement. (DuBois, *Electrician*, Vol. 40, p. 653). In the case of cylindrical shields, the shield factor is given by a similar set of equations:

$$g = 1 + \frac{1}{4}\,\frac{(\mu-1)^2}{\mu}\left[(1 - q_1\,q_2) + \frac{(\mu+1)^2}{4\,\mu}\,n_1\,n_2\,n_{12}\right] \qquad (14)$$

where

$$q_1 = (r_1/R_1)^2 \qquad n_1 = 1 - q_1$$
$$q_2 = (r_2/R_2)^2 \qquad n_2 = 1 - q_2$$
$$q_{12} = (R_1/r_1)^2 \qquad n_{12} = 1 - q_{12}$$

Using as before the notation

$$d = \frac{(\mu - 1)^2}{4\,\mu} \tag{15}$$

the equation takes the simpler form,

$$\begin{aligned} g &= (1 + d\,n_1)\,(1 + d\,n_2) - (d^2 + d)\,n_1\,n_2\,q_{12} \\ &= g_1\,g_2 - (d^2 + d)\,n_1\,n_2\,q_{12} \qquad (16) \\ &= g_1\,g_2\,n_{12} \text{ approximately} \qquad (16\text{-}2) \end{aligned}$$

This equation is analogous to that for spheres and leads to similar conclusions. For similar equations for shields of three shells see "Stalloy Ring Shields," by D. W. Dye. (*Journal of Scientific Instruments*, Vol. 3, p. 66, 1925.) Incidentally, it may be noted here that the ratio of the permeance functions in the two cases, *viz.*, spherical and cylindrical shells, is

$$\frac{c}{d} = \frac{2}{9}\,\frac{(\mu - 1)^2}{\mu} \Big/ \frac{1}{4}\,\frac{(\mu - 1)^2}{\mu}$$

or

$$\frac{c}{d} = \frac{8}{9} \tag{17}$$

Rucker has shown that for a shield of predetermined size, that is, when the innermost and outermost radii are specified, the most efficient shield is obtained when the successive radii of the shells form a geometric progression.

Equations for shields of more than three shells can be developed by further application of the same method, but the work involves a very laborious computation, and the improvement secured thereby is not great enough to justify the effort. A shield of three shells is quite satisfactory for almost all practical purposes, and one of the two shells is good enough for most of them. Equation (13-2) can be extended to any number of shells, as an approximate formula, if necessary.

9. Material for Shells

A reference to DuBois' formula shows that the shielding effect is nearly proportional to permeability; high permeability is therefore the most important requisite for shield material. In practise the fields against which protection is needed in galvanometer work, are generally very weak. The best material for a shield is therefore a material with high initial permeability. It is also necessary to remember that the material should be as free from hysteresis as possible. Material capable of developing strong local poles and retaining them, is worse than useless as it may create more disturbance than it can cure. Fortunately, material of high permeability has generally a very low hysteresis, which simplifies to that extent the problem of choice of material. The newly discovered alloys, permalloy and other alloys of that class, seem to be the best materials for shields but no recorded data on shield ratio for these materials are yet available. Professor Hill states that he has obtained a shield ratio $g = 1000$ by the use of a cylinder of sheet "Mu metal" rolled with a spacer of sheet copper, and with end plates of "Mu metal." (*Journal of Scientific Instruments*, Vol. 3, 1925-26, p. 335.) For steady disturbing fields a massive material is just as good as laminated material but for disturbance of transient character massive material is at a disadvantage. The impulsive nature of the disturbance generates an eddy current, which chokes back a part of the flux and prevents it from going through the walls of the shell, thereby reducing the efficiency of the shell as is indicated by an impulsive kick of the protected galvanometer. Laminated shells have no disadvantage in particular. They are equally efficient against impulsive as well as steady disturbances. There seems to be some difference of opinion as to the mode of lamination. One method is to prepare a cylindrical shell by stacking a number of ring punchings of appropriate size; this scheme of lamination simplifies the problem of cutting observation windows in the shield. A second method is to prepare a cylindrical shell by rolling sheet material in the form of a cylinder. Theoretically, the latter type ought to be a trifle better, first because the shell acts like a system of multiple shells separated by non-magnetic shells, second because they contain no non-magnetic spaces parallel to the field for the external field to leak through, and third, because the formation of eddy currents tends to prevent the flux crossing the sheets, so as to force it to follow a path of least eddy along the walls of the shield. Practically no great improvement seems to have been observed. This may be due to the fact, that in rolling the sheet into a cylindrical shell its permeability is perhaps slightly impaired. This point deserves further testing.

10. Importance of Differential Permeability for Shielding

In view of the facts that the shielding power of a shield depends on its permeability, and that the permeability of every known ferro-magnetic material depends on its flux density, having a maximum value when the material is carrying a certain amount of flux, attempts have been made to improve the shielding power of a shield by setting up a toroidal flux of sufficient strength to bring the material to the point of maximum permeability. It has been shown that this method of increasing permeability leads to no appreciable improvement in shielding power, (DuBois, *Electrician*, March 11, 1898, Vol. 40, p. 654). The cause of the failure of this method lies in the fact that the permeability on which the shielding power of the shield depends is not normal permeability B/H, but differential permeability $\Delta B/\Delta H$, where ΔH is the new increment of stray field, superposed on the previous field H, with corresponding values for ΔB and B. If the shield be in the non-magnetic condition to start with and be subjected to a toroidal magnetization up to the point of maximum permeability, and if a weak magnetic field be then turned on for the first time, the toroidal magnetization does give a slightly greater shielding power but a second application of the same field is attended by a reduced shielding power. In

every case after the first application of the stray field, the iron goes through a small unsymmetrical hysteresis loop the mean slope of the last side of the loop being the value of permeability, $\Delta B/\Delta H$, which determines the final distribution of the flux. Even in the case of the first application of the field the improvement is not very great, because the values of ΔH and ΔB are not both positive for the whole ring; in one-half of the ring the stray field is opposed to the flux and the corresponding permeability has the value not $+\Delta B/+\Delta H$, but the much lower value $-\Delta B/-\Delta H$. As the shielding power of the shield depends on the final distribution of the flux which in turn depends greatly on the differential permeability under the working conditions, it follows that DuBois' equation involving the normal permeability B/H, instead of $\Delta B/\Delta H$, is only a first approximation.

11. Failure of the Shield

In the ultimate analysis the operation of the shield, depends on the refraction of the flux lines as they pass through the successive surfaces of the shield. If therefore the lines of flux in any particular case are either strictly perpendicular or strictly parallel to the surfaces of the shield, there can be no refraction and therefore no shielding effect. For example, if one end of a long thin bar magnet is inserted inside a spherical shell all the lines emanating from the inserted pole will pierce through the shield and emerge as if there were no shield. If on the contrary a magnet were inserted inside the shell with both poles inside, the space outside will be almost fully protected by the shell. Only a few lines in the direction joining the poles will be perpendicular to the surfaces of the shell and will emerge outside unchanged; these will be a very small fraction of the total lines emanating from each pole of the magnet. As an example of parallel flux, we may consider the case of a tubular shield with a single wire carrying the current along the axis of the shield. The magnetic field at any point outside of the shield will be just as intense with the shield as without it. These cases of failure do not generally occur in practise, but it was necessary to mention them just for the sake of a correct theoretical understanding of the nature of the phenomenon.

12. Incidental Shielding

Imagine a wire carrying a current held in front of a plane face block of iron of infinite extent on the opposite side and of infinite permeability. The problem is to determine the form of the field and the distribution of magnetism on the surface of the block. A problem of this kind is easily solved by the method of electromagnetic images. Imagine the block removed for a time and another wire carrying a current of the same magnitude and direction located where the image of the first wire would have been formed if the face of the block had been a mirror. Such a current would be an electromagnetic image of of the first current; the field at any point in front of the block can be obtained by computing the field produced by the two wires separately and obtaining the resultant effect. For a point immediately near the block, the field would be perpendicular to the face of the block, the magnetic surface density at any point of the face can be computed from Coulomb's Law.

$$\sigma = H/4\pi \tag{18}$$

where H is the field at that point as computed by the image method.

This is the method of magnetic images. For more important cases occurring in engineering practise see Searle ("Magnetic Field Near a Cylinder of Iron," *Electrician*, Vol. 40, 1898, p. 453).

13. Dynamic Shielding

Imagine a wire carrying a current held in a magnetic field and at right angles to the field. The wire would be subject to a side-way pull proportional to the product of the field strength and the strength of the current. Imagine now a tubular magnetic shield surrounding the wire but not touching it. The field inside of the shield will be very much weaker and the side-way pull on the wire will be reduced in proportion. Suppose now that the shield is mechanically fixed to the wire so that the wire and shield must move together or not at all. The pull on the wire is now as strong with the shield as without it. This apparently paradoxical result is due to the fact that when the shield moves with the wire the lines of flux crossing the space inside the shield move in the opposite direction at a rapid rate, with a velocity inversely proportional to the weakening of the field. Therefore, the rate at which the lines are cut by the wire is now the same with the shield as without it. This result is very important in the study of incidental shielding in the tunnels of a motor armature. It shows that while the force acting on the wire and shield is in no way diminished by the presence of the shield, the force which tends to move the wire relatively to the shield is very much diminished so that the insulation of the wire separating the wire from the shield does not have to bear any excessive mechanical stress. It is as if the small material insulation were considerably reinforced by the immaterial ether that fills the space between the wire and shield.

14. Differential Shielding

The most important practical use of shielding is the protection of moving-magnet galvanometers against the steady terrestrial field in which they work. The galvanometers are generally of the astatic type. In practise it is nearly impossible to get the necessary exact balance between the two opposing magnets of the system. By using a cylindrical shield and adjusting it vertically so as to shield the magnets unequally, it has been found possible to compensate exactly for the inequality of the moving magnets by a corresponding inequality of the shielding and thus to increase the sensitivity of the galvanometer. This method of differential shielding is however not efficient against variable stray fields which often occur in practise.

Appendix

MATHEMATICAL THEORY OF ZONAL HARMONICS

15. Zonal Harmonic Coefficients

In Section 7, a simple spherical harmonic has been defined as the potential at any point A due to a harmonic magnet. A harmonic magnet of any order gives rise to a spherical harmonic of the same order.

(1) Let M_0 be a harmonic magnet of the zero order of strength M_0, that is, a single magnetic pole of strength $+ M_0$, located at the origin of the coordinate system (Fig. 4-0).

Let the coordinates of A be x, y, or r, θ. Then the potential at A due to the pole $+ M_0$ at 0, is

$$V_0 = M_0/r \tag{19}$$

For unit magnet, $(M_0 = 1)$ the potential

$$v_0 = 1/r. \tag{20}$$

(2) Let M_1 be a harmonic magnet of the first order, that is, a single short magnet of magnetic moment M_1. (Fig. 4-1).

Let $a\,b$ be the given harmonic magnet, that is, a single magnet with poles $+ m$ and $- m$, located at points a and b on the prime vector, that is, on axis X at distances $+ \delta x$ and $- \delta x$, so that $2\,m\,.\,\delta x = M_1$.

We might assume both poles located at 0 to start with, and then each displaced over the distance δx, $+ m$ to the positive side and $- m$ to the negative side. Potential at A due to $+ m$ at 0 (before displacement)

$$V_0 = m\,v_0 = m/r$$

Potential at A after displacement of $+ m$ is

$$V_0 + \delta V_0 = m\,(v_0 + \delta v_0) = m\left(v_0 + \frac{d\,v_0}{d\,x}\,.\,\delta x\right)$$

Similarly for the negative pole $- m$, at $- \delta x$

$$V_0 - \delta V_0 = - m\left(v_0 - \frac{d\,v_0}{d\,x}\right).\,\delta x$$

resultant potential at A

$$V_1 = m\left(v_0 + \frac{d\,v_0}{d\,x}\,.\,\delta x\right) - m\left(v_0 - \frac{d\,v_0}{d\,x}\,.\,\delta x\right)$$

$$= 2\,m\,.\,\frac{d\,v_0}{d\,x}\,.\,\delta x = M_1\,.\,\frac{d\,v_0}{d\,x} \tag{21}$$

For unit harmonic magnet $(M_1 = 1)$

$$v_1 = \frac{d\,v_0}{d\,x}$$

By evaluating the differential we get,

$$v_1 = \frac{d\,v_0}{d\,x} = \frac{d\,v_0}{d\,r}\,.\,\frac{d\,r}{d\,x} = \frac{d}{d\,r}\left(\frac{1}{r}\right).\,\frac{d\,r}{d\,x}$$

But when $d\,x$ is positive $d\,r$ is negative, and $d\,r/d\,x = - \cos\theta$. See Fig. 4-1; therefore,

$$V_1 = - \frac{1}{r^2}\,.\,(- \cos\theta) = \frac{1}{r^2}\cos\theta$$

and $$V_1 = \frac{M_1\,.\cos\theta}{r^2}$$

(3) By following the same reasoning, we obtain in general,

$$v_{(n+1)} = \frac{d}{d\,x}\,(v_n) \tag{23}$$

Therefore, by successive differentiation, we obtain the series of harmonics of successive orders.

$$V_0 = M_0\,.\,\frac{1}{r} \tag{24-0}$$

$$V_1 = M_1\,.\,\frac{1}{r^2}\,.\,(\cos\theta) \tag{24-1}$$

$$V_2 = M_2\,.\,\frac{1}{r^3}\,.\,(3\cos^2\theta - 1) \tag{24-2}$$

$$V_3 = M_3\,.\,\frac{1}{r^4}\,.\,3\,(5\cos^3\theta - 3\cos\theta) \tag{24-3}$$

$$V_4 = M_4\,.\,\frac{1}{r^5}\,.\,3\,(35\cos^4\theta - 30\cos^2\theta + 3) \tag{24-3}$$

$$V_5 = M_5\,.\,\frac{1}{r^6}\,.\,15\,(63\cos^5\theta - 70\cos^3\theta + 15\cos\theta) \tag{24-5}$$

$$V_6 = M_6\,.\,\frac{1}{r^7}\,.\,45\,(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5) \tag{24-6}$$

$$V_7 = M_7\,.\,\frac{1}{r_8}\,.\,315\,(429\cos^7\theta - 693\cos^5\theta + 315\cos^3\theta - 35\cos\theta) \tag{24-7}$$

The corresponding values of v_0, v_1, v_2, etc., for unit magnets can be easily obtained by making M_0, M_1, M_2, etc., equal to unity. The θ-function factor together with the numerical factor in each of the above equations is called a zonal harmonic coefficient; it is indicated by the symbol Z. It may be noted as an aid to memory that the numerical factor, together with the numerical coefficient of the highest power of $\cos\theta$ in the parenthetic term, make an odd-factorial of the corresponding order. For example in the value of V_7 we have, 315 . 429

$$= (5\,.\,7\,.\,9)\,(3\,.\,11\,.\,13)$$

$$= 1\,.\,3\,.\,5\,.\,7\,.\,9\,.\,11\,.\,13.$$ (seven successive odd factors beginning with 1.)

Using the symbol Z, the series of harmonics can be written in a very compact form.

$$V_0 = \frac{M_0}{r} \cdot Z_0$$

$$V_1 = \frac{M_1}{r^2} \cdot Z_1$$

$$V_2 = \frac{M_2}{r^3} \cdot Z_2$$

or generally

$$V_n = \frac{M_n}{r^{(n+1)}} \cdot Z_n \qquad (25)$$

The power of r being always one degree higher (negative) than the order of the harmonic.

Then the potential at A due to the joint effect of all the harmonic magnets is,

$$V_A = V_0 + V_1 + V_2 + V_3 + \text{etc.}$$
$$= \frac{M_0 \cdot Z_0}{r} + \frac{M_1 \cdot Z_1}{r^2} + \frac{M_2 \cdot Z_2}{r^3}, \text{etc.} \qquad (26)$$

The potential V_A at any point A, may be thus regarded as made up of components V_0, V_1, V_2, V_3, etc., each component being a simple spherical harmonic, and being the potential at that point due to corresponding harmonic magnets of strength M_0, M_1, M_2, etc., respectively. It will be shown later that the potential at any point due to any kind of magnetic distribution subject to certain limitations can be analyzed into harmonics, appropriate strengths being assigned to the harmonic magnets necessary for the analysis; that is, the potential V_A at any point A in space, due to any kind of magnet distribution symmetrical with respect to an axis, can be analytically expressed by the series

$$V_A = \frac{M_0 Z_0}{r} + \frac{M_1 Z_1}{r^2} + \frac{M_2 Z_2}{r^3} \text{ etc.} \qquad (27)$$

where M_0, M_1, M_2, etc., are harmonic magnets of determinable strengths.

Equations (26) and (27) though identical in form are opposed in the mathematical sense. Equation (26) is synthetic: the value of V_A is computed from the values of M_0, M_1, M_2, etc., which are supposed to be known. Equation (27) is analytic: in this case, V_A is supposed to be known and the values of M_0, M_1, M_2, etc., are to be determined by analytical process; that *such an analysis is always possible*, is the basic idea underlying Equation (27), and therefore underlying the theorem formulated in Section 6. (For proof of this theorem see Section 17.)

16. Geometric and Harmonic Images

(1) Geometric Image, (see Fig. 6).

Let $T\,A\,T'$ be the surface of sphere of radius r, and with center O.

Let Q be a single positive magnetic pole of strength q.

Let P be a point, inverse to Q, that is so related geometrically to Q, that,

$$O\,P = O\,A^2/O\,Q, \text{ or } a = r^2/d, \text{ or } \frac{a}{r} = \frac{r}{d}$$

Let p be the strength of an imaginary magnetic pole at P so related to q, that $p = q \cdot a/r = q \cdot r/d$.

Then p is called the geometric image of q in relation to the surface of the sphere, because the potential at any point on the sphere due to p at P, is the same as that due to q at Q. (See Eq. (28).)

$$V_q = Q \cdot /A\,Q = q/L$$
$$V_p = p/A\,P = p/R$$

By the well-known geometric properties of a sphere, we have in the triangles $A\,O\,P$, and $A\,O\,Q$

$$\frac{O\,P}{O\,A} = \frac{O\,A}{O\,Q} = \frac{A\,P}{A\,Q}$$

or

$$\frac{a}{r} = \frac{r}{d} = \frac{R}{L}$$

therefore, $R = L \cdot a/r$

Substituting for p and R in the value of V_p,

$$V_p = q \cdot \frac{a}{r} \cdot \frac{r}{L \cdot a} = \frac{q}{L} = V_q \qquad (28)$$

This relation is true for all positions of A on the surface

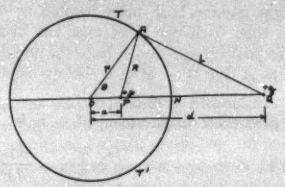

Fig. 6—Geometric and Harmonic Images

of the sphere. Since p at P produces the same potential as q at Q, for all points on the sphere, p is called the image of q (geometric) in relation to the sphere.

(2) Harmonic images: Analysis of magnet pole into harmonic images.

The potential at A due to a pole p at P, is

$$V = p/R$$
$$= p \cdot (r^2 - 2\,a\,r\cos\theta + a^2)^{-1/2}$$
$$= \frac{p}{r}\left[1 - \left(\frac{2\,a\cos\theta}{r} - \frac{a^2}{r^2}\right)\right]^{-1/2}$$
$$= \frac{p}{r} \cdot \left[1 + \frac{1}{2}\left(\frac{2\,a}{r}\cos\theta - \frac{a^2}{r^2}\right)\right.$$
$$+ \frac{1}{2} \cdot \frac{3}{4}\left(\frac{2\,a}{r}\cos\theta - \frac{a^2}{r^2}\right)^2$$
$$\left. + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6}\left(\frac{2\,a}{r}\cos\theta - \frac{a^2}{r^2}\right)^3 + \text{etc.}\right]$$

(See Pierce's Integrals, Table No. 750.)

Expanding the parenthetic terms and rearranging according to powers of $\frac{a}{r}$, we get

$$V = \frac{p}{r}\Big[1 + \frac{a}{r}(\cos\theta)$$

$$+ \frac{a^2}{r^2}\cdot\frac{1}{2}(3\cos^2\theta - 1)$$

$$+ \frac{a^3}{r^3}\cdot\frac{1}{2}(5\cos^3\theta - 3\cos\theta)$$

$$+ \frac{a^4}{r^4}\cdot\frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)$$

$$+ \frac{a^5}{r^5}\cdot\frac{1}{8}(63\cos^5\theta - 70\cos^3\theta + 15\cos\theta)$$

$$+ \frac{a^6}{r^6}\cdot\frac{1}{16}(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5)$$

$$+ \frac{a^7}{r^7}\cdot\frac{1}{16}(429\cos^7\theta - 693\cos^5\theta + 315\cos^3\theta - 35\cos\theta)$$

$$+ \text{etc.}\Big] \qquad (29)$$

We find that the value of V is made up of a series of terms resembling the group of harmonics (Equation **24-0, -7**) except that the numerical factor accompanying the θ-function factor is different. These two factors together constitute the zonal harmonic coefficient (practical), briefly called the "zonal harmonic" and denoted by the Symbol P. The published tables of harmonics give the values of P, not of Z. It may be noted at this point that the two forms of zonal coefficient are related by a very simple relationship, namely,

$$Z_n = n\,!\,.\,P_n \qquad (29\text{-}2)$$

for example, from Equations (**24-7**) and (**29**),

$$Z_7 = 315\,(429\cos^7\theta - 693\cos^5\theta + \text{etc.}) \qquad (24\text{-}7)$$

$$P_7 = \frac{1}{16}(429\cos^7\theta - 693\cos^5\theta + \text{etc.}) \qquad (29)$$

therefore,

$$Z_7/P_7 = 315\,.\,16 = (5\,.\,7\,.\,9)\,.\,(2\,.\,4\,.\,2)$$
$$= 5\,.\,7\,.\,3\,.\,2\,.\,4\,.\,6 = 1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6\,.\,7 = 7\,!$$

As an aid to memory it may be noted that the numerical factor in P is the reciprocal of the sum of the numerical coefficients in the bracketed part of P; for example,

$$P_7 = \frac{1}{16}(429\cos^7\theta - 693\cos^5\theta + 315\cos^3\theta - 35\cos\theta)$$

here, $429 - 693 + 315 - 35 = 744 - 728 = 16$.

In other words, the numerical factor is such as to make $P = 1$ for all orders when $\theta = 0$, and $\cos\theta = 1$; this relation makes it very convenient both for the maker and for the user of the tables of harmonics, when expressed in terms of P instead of Z. For graphic representation of zonal harmonics, see Figs. 7-6 and 7-7.

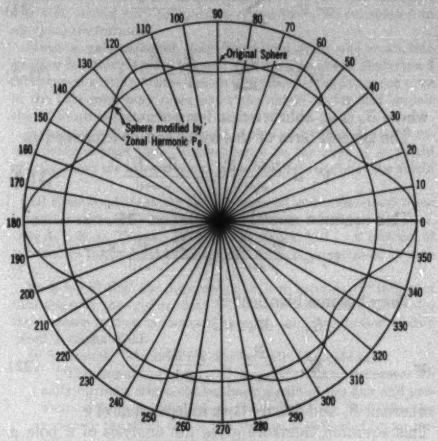

Fig. 7–6

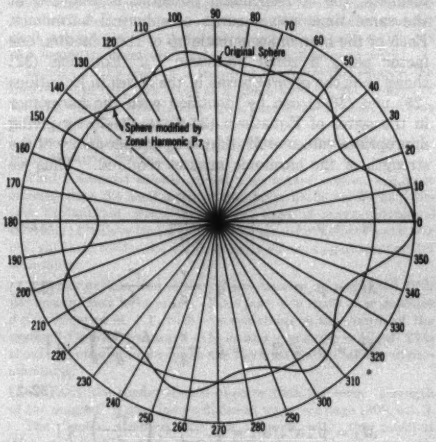

Fig. 7–7

Fig. 7—Polar Graphs for Zonal Harmonics; Subnumbers Refer to Order of Harmonic

Using the symbol P, Equation (**29**) can be expressed in a compact form

$$V = V_0 + V_1 + V_2 + V_3 + \text{etc.} \qquad (30)$$

$$= \frac{p}{r} \cdot P_0 + \frac{p\,a}{r^2} \cdot P_1 + \frac{p\,a^2}{r^3} P_2 + \frac{p\,a^3}{r^4} P_3 + \text{etc.} \tag{30-2}$$

$$\text{or } V = \sum \left\{ \frac{p\,a^n \cdot P_n}{r^{(n+1)}} \right\} \tag{31}$$

$$\text{or } V = \sum \left\{ \frac{B_n \cdot P_n}{r^{(n+1)}} \right\} \tag{31-2}$$

where B_n is an abbreviation for $p\,a^n$.

The general term of this series can be expressed as

$$V_n = \frac{p\,a^n \cdot P_n}{r^{(n+1)}} = \frac{B_n \cdot P_n}{r^{(n+1)}} \tag{31-3}$$

This equation resembles Equation (25), *viz.*

$$V_n = \frac{M_n \cdot Z_n}{r^{(n+1)}} = \frac{M_n \cdot P_n \cdot n!}{r^{(n+1)}}$$

They become identical if

$$B_n = M_n \cdot n\,!$$

$$\text{or} \quad M_n = \frac{B_n}{n\,!} = \frac{p \cdot a^n}{n\,!} \tag{32}$$

wherein, B_n and M_n are both independent of r.

This equation therefore gives the analysis of a pole p at the point P inside the sphere, into a series of harmonic magnets; the corresponding potential is analyzed at the same time into a series of spherical harmonics. Each of the harmonics is made up of three factors, one factor being a zonal harmonic P_n. Equation (32) shows that the several terms in the series in Equations (29) and (30-2) can be identified with the harmonics in the series of Equations (24-0) to (24-7) by giving appropriate values to M, that is by proper choice of the strength of the harmonic magnet required. Thus we have

$$M_0 = p \qquad = q \cdot \frac{r}{d} \tag{32-0}$$

$$M_1 = p \cdot a \qquad = q \cdot \frac{r}{d} \cdot \frac{r^2}{d} \tag{32-1}$$

$$M_2 = p \cdot a^2/2\,! \quad = q \cdot \frac{r}{d} \cdot \left(\frac{r^2}{d}\right)^2 /2\,! \tag{32-2}$$

$$M_3 = p \cdot a^3 /3\,! = q \cdot \frac{r}{d} \cdot \left(\frac{r^2}{d}\right)^3 /3\,! \tag{32-3}$$

$$M_3 = p\,a^4/4\,! \quad = q \cdot \frac{r}{d} \cdot \left(\frac{r^2}{d}\right)^4 /4\,! \tag{32-4}$$

etc. etc.

The series for V may also be analyzed directly in terms of q and d instead of in terms of p and a, as shown in the second form of each of the above equations.

Since $p/r = q/d$, and $a/r = r/d$, we have from Equations (30) and (30-2)

$$V = V_0 + V_1 + V_2 + V_3 + \text{etc.} = \frac{q}{d} \cdot P_0$$

$$+ \frac{q}{d} \cdot \frac{r}{d} \cdot P_1 + \frac{q}{d} \cdot \frac{r^2}{d^2} \cdot P_2 + \frac{q}{d} \cdot \frac{r^3}{d^3} \cdot P_3 + \text{etc.} \tag{33}$$

$$\text{or } V = \sum q\, d^{-(n+1)}\, r^n\, P_n = \sum (A_n\, r^n\, P_n) \tag{33-2}$$

Where A_n is like B_n, an abbreviation constant, and stands for $q/d^{(n+1)}$. A_n is independent of r, but the corresponding value of M_n is a function of r.

This equation gives the analysis of the pole q at the point Q outside of the sphere, into a series of harmonic magnets. The corresponding potential is expressed as a series of simple spherical harmonics, with the corresponding zonal harmonics as one of the factors in each term.

Note: The above method of analysis of potential at a point on the surface of a sphere, due to a pole located any where, in terms of Zonal Harmonics, was introduced by Legendre.

17. Analysis of Any Magnetic Field in General into Component Spherical or Zonal Harmonics

In the case of two magnetic poles, one inside the sphere and the other outside, the potential due to both the poles jointly is,

$$V = \sum A_n\, r^n\, P_n + \sum B_n\, r^{-(n-1)}\, P_n \tag{34}$$

The case of general magnetic distribution does not need elaborate analysis. Equation (34) leads to the conclusion, that since each point of the distribution has its corresponding harmonic magnets, it produces its share of potential which on analysis resolves into a series of harmonics. The aggregate effect is therefore also a sum total of harmonics which resolves itself into two groups, one of the type $A_n\, r^n\, P_n$, for all the poles outside of the sphere, and the other of the type $B_n\, r^{-(n+1)}\, P_n$ for all the poles inside of the sphere, the harmonic magnets corresponding to each point, being oriented along the radius vector for that point. If the distribution be symmetrical all the harmonic magnets lie on the axis of symmetry. Therefore,

Every symmetrical magnetic distribution can be represented analytically by a system of harmonic magnets, lying on the axis of symmetry which may be the prime vector; the potential at any point on the surface of an arbitrarily selected sphere can then be expressed analytically by two series of spherical harmonics, one series representing the effect of the external distribution and expressed by the typical form

$$V_e = A_n\, r^n\, P_n \tag{35}$$

and the other representing the effect of the internal distribution and expressed by the form

$$V_i = B_n r^{-(n+1)} P_n \qquad (35\text{-}2)$$

Therefore for the two distributions together we have the analytical expression

$$V = \Sigma A_n r^n P_n + \Sigma B_n r^{-(n+1)} P_n \qquad (36)$$

$$= \sum A_n r^n + \sum \frac{B_n}{r^{(n+1)}} P_n \qquad (36\text{-}2)$$

This is the basic equation underlying the method of spherical harmonic analysis. (See Rucker's "Shielding of Concentric Spherical Shells," *Philosophical Mag.*, 1894, Vol. 37, p. 97.) In the case of a simple magnetic field, the analysis of the distribution which produces that field can be reduced to two simple terms each of the first order, that is,

$$V = (A_r + B/r^2) \cos \theta \qquad (37)$$

This potential being produced by two harmonic magnets of the first order, that is by two simple magnets, both located at the center of this sphere and oriented in the direction of the field under analysis, one magnet representing the distribution inside the sphere, if any, and the other term representing the external. The problem of shielding a galvanometer placed in a uniform field belongs to the second class; in this case the expression for potential on the surface of a sphere reduces itself to the single term

$$V = A_r \cos \theta \qquad (37\text{-}2)$$

which leads to considerable simplification of the shield problem.

Discussion

Hans Lippelt: The brightest spots in Mr. Gokhale's paper are certainly the results, which he presents in his own clear fashion and to which he attaches valuable ready-to-use utility.

The reader will not only absorb with interest the revelations of Equations 9 to 17 and their subsidiaries, but will surely enjoy the neatness by which these formulas take care of the geometrical form of and difference between individual shells.

The revelation of those formulas that two concentric shells of iron, separated by a concentric air space, give a greater shielding effect than one solid shell filling all the space with iron, may now be explained by the statement, that the magnetic reluctance of the outer shell (although higher than that of the solid shell) is *sufficiently low* to take care of the flux. At the same time the outer shell carries the flux past the shielded central area at a greater mean distance than the solid shell would do.

The high magnetic reluctance of the air space occurring between the shells of a two- (or multi-) shelled shield certainly comes very much into evidence as the real shielding power, inasmuch as the flux encounters a low reluctance (low magnetic potential gradient) in the outer shell.

Therefore, is it not the air space between the shells that does the shielding, while the outer shell does the carrying?

The inner shell, of course, carries only whatever little flux may have been crowded inward from the outer shell.

The air space within the inner shell appears then to be its own shield against the remainder of flux, carried by the inner shell. We will call this type of shield, which functions upon the low gradient of magnetic potential in the adjacent medium in conjunction with its own high magnetic reluctance, a Magnetostatic Shield.

It is known that for small magnetomotive force (H) the permeability for magnetizable metals is small. With increasing magnetomotive force the permeability rises to a maximum, after which it falls off again. Since stray fields against which galvanometers and similar apparatus are to be protected, are mostly weak fields (H), the question arises as to whether Equation 9 covers only such weak fields, or whether it holds true when strong magnetic fields are to be shielded off. Should the gradient $d\mu/dH$, which may be positive or negative,[1] be considered in applying the formula?

Since a galvanometer may be moved from one room into another location, it may become necessary to check Equation 9 for reversal of stray field. Traces of hysteresis, which occur even in the best iron, may react upon the shield factor and impair the accuracy of measurements.

Hysteresis will entail a reversal of sign of μ when H and B respectively change sign. Such change in the sign of μ would greatly affect the answer which Equation 9 gives about the shield factor.

All these conditions are better overseen and understood when magnetic tension and magnetic friction are taken into account. (See A. I. E. E. Trans., 1926, p. 406, under VII.) κ instead of μ becomes the important factor and there is no reversal of sign of κ.

Be it noted that the appearance of μ in formula (9) indicates that not only the original field H has been taken care of, but also the increase in flux, occasioned through the introduction of iron (shell) into that field.

In Section 9, "Material for Shells," Mr. Gokhale says:

> "The impulsive nature of the disturbance generates an eddy current, which chokes back a part of the flux and prevents it from going through the walls of the shell, thereby reducing the efficiency of the shell as is indicated by an impulsive kick of the protected galvanometer."

That does not seem quite clear. If the eddy current "chokes back" and "prevents the magnetic impulse from going through the shell," that would mean demonstrating the efficiency of the shell, not reducing it. If the galvanometer gets only a kick out of a heavy transient, the shield must have done good work.

Besides in this case of a transient magnetic disturbance the massive *shell itself* has acted as a shield through the medium of the eddy current, which chokes back part of the flux and prevents it from entering the central area.

Here the shell acts as a magnetodynamic shield and in addition also carries the flux.

That laminated shields, rolled with copper spacer, are more efficient than the shell made of iron only, needs hardly any further explanation, when seen in the light of magnetodynamic shielding; I believe that a copper shell placed outside of a massive iron shell will render the outfit strongly efficient against magnetic transients.

A better explanation of the subject treated on page 1310, fifth paragraph, may be in order. Does pole $+m$ move over distance dx in direction of $+X$, with $-m$ remaining in the origin of the system? Or does $+m$ move $+\frac{1}{2}\,dx$ and $-m$ move $-\frac{1}{2}\,dx$? This should be cleared up to make the next paragraph readily understandable.

Similarly the reader should observe that the term "strength of the magnet" is used in two different ways, see page 1309, col. 2.

The "polar strength m" fits in well with the moment $M = m.2a_1$ Five lines further down one reads of harmonic magnet of strength M_1 (which is $m.2a_1$) and further down .... strength of the magnet of nth order $M_n = m.2a_n - 1 \ldots 2a_1.2a_1$. The unit by which to measure such harmonic magnets changes by a linear dimension, geometrically, at each step. This is not conducive to forming a concrete concept of those magnets.

In consideration of such conditions it may be said of this paper, that the results are admirable. The method, however, consists in pressing a very pliable mathematical material into the very

1. See J. A. Ewing, "Magnetic Induction in Iron & Other Metals," Van Nostrand, 1900, 3rd ed., pp. 126 and 143.

rigid mold of an intricate problem, and thus is a great accomplishment of its own kind.

S. L. Gokhale: Mr. Lippelt's first contribution is an effort to explain the phenomenon of shielding, once more in terms of reluctance. I do not say that it cannot be done, but for reasons already explained in Section 4 of the paper, I doubt whether any such explanation is worth the effort. The chief merit of the reluctance conception is simplicity; its chief demerit is the failure to explain the phenomenon in all its phases except at the cost of simplicity which is its only merit. Even after a careful study of Mr. Lippelt's comments on this point, I do not feel that he has succeeded in combining simplicity with accuracy.

For example, let us concede for sake of argument that Mr. Lippelt's explanation is correct, and that the most important part of the work in the process of shielding is done by the intermediate shell of air space, and that the outer shell merely helps the process by providing a by-pass to the flux, and that it should therefore be so designed as to be able merely to carry this flux, any excess material being merely a waste. It would follow that for the most efficient shield the relative thickness of the magnetic shells in comparison with the air shells, should be determined by the permeability of the material. This is contrary to the corollary from Rucker's formula which requires the shells to have the same ratio p (see Equation 9) for all shells, magnetic and nonmagnetic, irrespective of permeability of the magnetic material;

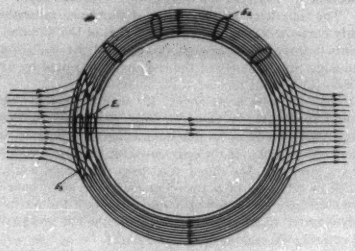

FIG. 1

(see statement following Equation 17). Again I do not suggest that it is impossible to account for all these facts by an explanation based on reluctivity; my contention is merely that it cannot be done without sacrificing simplicity, and is therefore not worth trying. I only mean that it is not worth trying to explain the phenomenon directly in terms of reluctivity, but the best interpretation can be made indirectly in terms of some intermediate conception, with the conception of reluctivity in the background. This is exactly what Rucker has done in his analysis. After writing down the potential equation for a spherical surface in general, he selects a boundary surface separating two shells, and writes for this surface, the two equations of magnetic refraction; the value of H in these two equations is derived from the basic harmonic equation (Equation 6) by differentiation. These two equations involve the conception of reluctivity, so that we have in this method of harmonic analysis not a method opposed to, or even independent of the reluctivity conception, but a method in which reluctivity is kept out of view to prevent its complicating the explanation while pretending to simplify it.

With reference to hysteresis, it is frankly admitted that Rucker's and DuBois' formula for shield factor as a function of permeability, (B/H), based on the assumption that this ratio is constant, is only a first approximation, (see Section 10). A closer approximation requires us to take into account not only the variation in the value of permeability according to flux density at every point of the shield material, but also the effect of hysteresis, which includes differential permeability. Mr. Lippelt says that this ought to be done, and I certainly agree with him. But my next question is, "Who will bell the cat?" Rucker could not do it; Du Bois could not do it; I do not expect to be able to make even a beginning in that direction. If Mr. Lippelt feels like tackling the problem, he is welcome to it. About thirty-five years ago Dr. Steinmetz had criticized adversely a scientific paper on the ground that it was imperfect inasmuch as it had ignored the third harmonic. The original author in his reply acknowledged the imperfections and offered the apology that the proper treatment of the problem with computation for the third harmonic was beyond the reach of the engineering science of the day. Dr. Steinmetz accepted the implied challenge, made the necessary correction, and as a consequence the engineering science was enriched by that contribution. If as a result of my apparent challenge the scientific world is made richer through Mr. Lippelt's formula for shield factor I shall certainly feel proud of my indirect share in the work.

With reference to the eddy currents and their effects on shield factor Mr. Lippelt's point seems to be well taken. As I see it now, the statement has certainly suffered in point of clearness by attempting too much abbreviation. I hope the following explanation will make clear the nature of the phenomenon:

Imagine a tube of flux lines going through a spherical shell: Some of the lines cross the walls of the shell and penetrate the shielded space; but a greater number of lines bend round and follow the shell walls, their number depending on the design of the shell and on the permeability of the material. The first group of lines tends to produce the eddy system e_1, which in turn chokes back a part of this group and diverts the lines along the path of the second group, thereby improving the efficiency of the shield. The second group produces the eddy system e_2 which in turn chokes back a part of this group and sends it along the path of the first group, thereby impairing the efficiency of the shield. A third eddy system e_3, produced jointly by both groups, is unimportant and needs no consideration. The systems e_1 and e_2 have opposing tendencies, and the final result depends on the relative strengths of the two systems. If the shell be made of massive material, the system e_2 is very much stronger than e_1, which greatly impairs the efficiency. If the shell is laminated the system e_2 is considerably weakened, but e_1 is practically unaffected, the aggregate result being that the efficiency of such a shield is nearly as good for impulsive stray fields, as for steady fields. For the same reason alternate sheets of copper lead to further improvement of the shields as they further strengthen the system e_1 without strengthening e_2.

With reference to the displacement conception of the harmonic magnets, Maxwell prefers the displacement of one pole along the axis, the other pole being left undisturbed at the center. I have preferred in my computation, equal and opposite displacements of both poles (see Section 15, derivation of Equation 21). Mathematically, it makes no difference in the final results, and for this reason I have left the point unspecified in the definition using a more flexible phrase "stretched apart" without specifying whether both or only one was displaced.

With reference to the "strength" of a harmonic magnet, the theoretical definition is implied rather than expressed mainly for the sake of brevity. The strength of anything is measured by what it can do. The function of a magnet is to produce a magnetic field and this function is mathematically best expressed in terms of potential. The strength of a harmonic magnet may therefore be defined as the potential at some specified point, due to that magnet; for practical use, the strength of a harmonic magnet is defined as the potential of that point at unit distance from the center of the harmonic magnet, and located in the direction of its orientation; a unit harmonic magnet is one which produces unit potential at that point. This definition is implied, but not explicitly stated by Maxwell. It can be shown that the

value of the potential can be expressed in terms of the strength of the poles and the interpolar distances of the harmonic magnet. The strength of the magnet therefore may also be defined analytically in terms of poles and interpolar distances; this is what I have done. This mode of defining has the merit of avoiding any appearance of argument in a circle, involved by first defining strength in terms of potential and then expressing potential in terms of strength.

The fact that the linear dimensions of strength vary with the order of the harmonic magnet constitutes no valid objection to this mode of definition. This peculiarity is inherent in the nature of the concept defined.

Automatic Stations

ANNUAL REPORT OF COMMITTEE ON AUTOMATIC STATIONS*

To the Board of Directors:

Introduction

This terminates the second year of the existence of this committee. The field of action in which this relatively new committee finds itself is so broad and has so many inviting by-ways that it has been difficult to outline the work. The engineering in connection with the application of automatic control is so fascinating as to result in so many new ideas being steadily developed that the fundamentals are sometimes almost overlooked. It is indeed amazing to realize the changes in ideas which occur in this branch of the industry in a single year. This committee attempts in the following report to outline some of the development and offer suggestions which are intended to benefit this branch of the industry.

Scope

The scope of work of this committee covers automatic and partially automatic generating stations and substations, the committee having complete jurisdiction over all apparatus associated with such stations. In addition the committee has jurisdiction over systems of remote dispatching, control, indications, etc., associated with the industry. The committee is interested in the dissemination of the knowledge and experience already gained in the design and operation of such equipment and combinations thereof, in order that this branch of the industry may be more fully developed.

Economical Construction

The developments of the year indicate a general tendency to take advantage of the economical construction possible with the use of automatic stations. It is still hard to realize the radical difference in station arrangement and set-up between the old firmly rooted manual system of operation and modern automatic operation. We are beginning to see, however, that the presence of a human mind and body in the manual station had a marked effect on station arrangement and that mechanical and electrical things were done solely because of the safety, comfort, and convenience of this human presence. Automatic control has all but eliminated this human body and has moved the mind from the station to the office of the engineer.

It is believed that one of the most radical and economical changes in station arrangement has been the elimination of the continuous switchboard. When this is studied it is found that there is no longer a necessity for assembling all of the control wiring from remote parts of a station at one point at the expense of thousands of feet of wire and conduit just to produce a switchboard assembly that makes a nice appearance.

The above is particularly true in a-c. substations and hydro-station applications where the details of control can and properly should be located as near the equipment controlled as is consistent with good fire protection and safety. It is now becoming common to see control panels located in various places throughout a station close to the equipment controlled, thus saving considerable in wire, conduit, and hazard.

Another saving has been realized from the reduction in size and in some cases the complete elimination of heating plants.

Ventilation has also been reduced, as in many cases a considerable amount of air was required over and above that actually required by the station apparatus for the comfort of the operating employes.

The problem of station location has been eased somewhat by the advent of automatic control as stations can now be located in places where it would be almost impossible to keep operating men on the job on account of the absence of what might be considered the proper amount of daylight, pure air, water, etc., necessary for the continuous maintenance of human beings.

Research and Development

Research during the past year has brought forward many improvements in detail apparatus which are contributing much to simplify the problems of application. Careful analysis of operating records disclosed the need of more simple means for the adjustment of equipment to meet a wide range of applications, particularly for the calibration of relays. Much has been done in this connection with the result that not only have improvements in relays simplified matters of calibration but by extending the range of application, the number of types of relays required has been reduced, thus greatly simplifying routine inspection and maintenance.

Simplified and improved supervisory control and remote metering systems have been produced as a result of experience with several very successful schemes in numerous applications.

Some work has been done in the development of devices for recording operations, quantities, etc., on charts, but this appears to be one of the weakest points in automatic development. There are devices available for recording almost anything desired but either they

*COMMITTEE ON AUTOMATIC STATIONS:

W. H. Millan, Chairman,

P. H. Adams,	H. C. Don Carlos,	O. Naef,
Caesar Antoniono,	P. E. Hart,	E. W. Seeger,
L. D. Bale,	Joseph Hellenthal,	Roy M. Stanley,
C. A. Butcher,	Chester Lichtenberg,	L. J. Turley,
M. S. Coover,	S. J. Lisberger,	F. Zogbaum.
	G. H. Middlemiss,	

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

are insufficiently reliable or their cost is prohibitive for extensive applications.

Simplified schemes of control for the automatic switching of rotating apparatus looking toward reduction in the number of moving parts are being tried out. The economics of application are being given a great deal of study. On a number of metropolitan street railway properties, the items of power consumption from the generating stations to the car wheels are being carefully analyzed. Operating voltages best suited to given operating areas will be selected and means for automatically or remotely controlling the substation apparatus for proper adjustment of the voltage to operating conditions will be provided.

Operators and careful analysis of operating results are the sources of information which will enable the manufacturers to develop new and improved equipment and to this end, it is urged that systematic and well-kept records be made available on operation, inspection, and maintenance. Well operated stations are the proving grounds and as essential to progress as the laboratory.

Operating Reports and Inspection

The predecessors of this committee have stated that the art would advance much more rapidly if more operating engineers would avail themselves of the opportunity at Institute meetings to tell of their individual experiences, through papers and taking part freely in discussion. Your present committee wishes to emphasize this and add an invitation to correspond freely with the members of this committee on subjects of interest to the followers of the automatic art. It is felt that in this way the annual reports of this committee will be made more representative. With this idea in mind your committee has this year sent out questionnaires attempting to gather operating, maintenance, and inspection data from a wide field of experience, covering a variety of applications. While these questionnaires may be considered a burden by some, this committee feels that a great many engineers are vitally interested in the subject and are willing to furnish the data requested to the end that the art may be more universally applied and incidentally improved. Some answers to the questionnaire have been received but not sufficient to present a recapitulation in the pages of this report. This committee intends to turn this partly finished work over to its successors with the earnest recommendation that they carry it forward.

Inspection and maintenance of automatic plants may be catalogued into two general classifications, equality and quantity, depending entirely upon the continuity of service expected and demanded of the automatic plants, and by the individual managements.

The word "maintenance" is used advisedly in view of the fact that inspection and maintenance are so closely allied at times as to be almost inseparable.

The old phrase, "getting out of a thing what you put into it" is exemplified in automatic station operation. However, the curve of failures of equipment to function plotted against dollars spent on inspection and maintenance, is not at all uniform, cost ratio increasing much more rapidly as the number of failures declines to the zero point. This point is clearly illustrated by a member of the subcommittee, and a pioneer in the automatic operation of a large metropolitan system, who states that two extremes are possible in automatic substation operation, under inspection and maintenance, and over inspection and maintenance.

A plant may be used for such a test, little or no inspection and maintenance being given during a predetermined period of time, observing the number of failures occurring, with their causes. By taking this number of failures as the unit, one hundred for example, and increasing inspection and maintenance, the failures will diminish rapidly to a point whereby 90 per cent of the one hundred will have been eliminated during an equal time period. At this point, inspection and maintenance can be doubled with the resulting effect of the elimination of only a few of the remaining ten failures. A happy medium between the two extremes is therefore logically assumed the most economical method of operation.

Past and present experience as reported by the various operating engineers, seems to disclose the fact that to function properly, automatic equipment should be given casual and periodical inspections. These inspections vary according to the needs and conditions of the individual installation, and the severity of service. The casual inspections on metropolitan systems as reported, are made as often as two hours apart during the heavy hours of the day, and two or three days apart on other systems and conditions.

These inspections usually consist in observing the functioning of equipment in service, overheated contacts, graphic chart clocks for time, inking, bearings, overheating, ventilation, etc. For stations outside of metropolitan districts, the casual inspections are likewise reported as being made daily on some systems, and weekly on others, ranging in the average of two or three times a week, with a tendency to make fewer inspections when the stations are equipped with supervisory control. The actual time required to make a casual inspection is much less than the traveling time.

A startling fact was disclosed when only about half of the 23 engineers replying to the questionnaire indicated that they followed any prepared method in inspection and testing but relied upon the field man entirely. It followed that in these cases no form was provided on which the field man could accurately report. Results from such a method must surely vary with the character and temperament of the inspector and cannot possibly be conducive to good operating and service results. Many of the replies, however, indicated the existence of very closely prescribed test and inspection methods along with very complete

records including very comprehensive classifications of device failures. That sort of practise, if universal, would most certainly pay dividends to the industry. This year's committee has become so intensely interested in this subject that the thought has occurred to attempt the standardization of test and inspection methods, forms, and failure classifications. It is felt that with standard forms in use this committee could be made a clearing house for much valuable information, thus permitting the designing engineers to profit by the mistakes of others as well as by their own.

In order to implant the thought we are herewith submitting a suggested form.

STANDARD AUTOMATIC SUBSTATION TROUBLE FORM

Date.....19....

.....Station No.....

Device Which Failed.....

Machine, Transformer or Circuit Number.....

Nature of Failure.....

Cause of Failure.....

Consequences.....

How Remedied.....

Located By:.....Repaired By:.....

Unit Out of Service from.....M.....To.....M.....

What Load Actually Interrupted and for How Long.....

What Load Otherwise Affected and for How Long.....

Copies to:

√ CLASSIFICATION

- ☐ Improper Adjustment by Mfg.
- ☐ Improper Adjustment by Owner.
- ☐ Improper Application.
- ☐ Improper Assembly
- ☐ Improper Design.
- ☐ Lack of Proper Inspection by Owner.
- ☐ Defective Workmanship by Owner.
- ☐ Defective Workmanship by Mfg.
- ☐ Unforeseen Operating Conditions.
- ☐ Undetermined.

Approvals:

By the use of forms similar to the one herewith submitted, several operating companies have been able to detect particular devices, schemes of connection, etc., which persistently give trouble, and as a result have increased the reliability of their equipment to a considerable degree by correcting the troublesome elements. One company in addition to the classification of the failures shown on the form, classifies and records the failures under device numbers and types.

One of the companies sends copies of all trouble forms to the manufacturer of the equipment involved, thereby giving the manufacturer the opportunity to turn the experience into something profitable to both themselves and the user.

PAPERS

Six papers were presented to the Institute during the year on the general subject of automatic stations and were sponsored by this committee:

Automatic Mercury Arc Power Rectifier Substation on the Los Angeles Railway, by L. J. Turley, Spokane, Aug. 1928.

The Automatic Substation—Its Relation to Electric Distribution Systems, by S. J. Lisberger, Spokane, Aug. 1928.

Telemetering, by Linder, Stewart, Rex, and Fitzgerald, New York, Jan. 1928.

Automatic Mercury Rectifier Substations in Chicago, by A. M. Garrett, Cincinnati, Mar. 1929.

Automatic Reclosing of High-Voltage Circuits, by Robertson and Spurgeon, Dallas, May 1929.

Automatic Transformer Stations of Edison Electric Illuminating Co. of Boston, by W. W. Edson, Swampscott, June 1929.

STANDARDS

Standards for automatic stations (No. 26) were adopted and issued in 1928. It was realized that in view of the rapid progress of this branch of the industry it would be necessary to revise these standards from time to time. A subcommittee has worked out some desirable changes which are not voluminous enough to warrant a revision of the Standards as published. However, this committee will add to this group of changes from time to time, until it is felt that it is worth while to present them to the Standards Committee with recommendations.

BIBLIOGRAPHY

With the idea that a complete bibliography of automatic station literature would be of inestimable value to the electrical engineer, this committee published as an appendix to its report last year a complete bibliography up to the date of the report. A supplement covering literature published from the last report to March 1, 1929, is included as an appendix to this report. Acknowledgment with thanks is hereby given to the Main Library, General Electric Company, for this service.

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Anonymous. "World's First Automatic Oil-Electric Generating Station," *Oil Engine Power*, Vol. 5, Dec., 1927, pp. 811-15. A small automatic plant supplying energy for the town of Seward, N. Y.

Discussion

Chester Lichtenberg: The "Economic Construction" portion of the committee's report calls attention to a new scheme, that of taking the switching units and putting them around the substations and generating stations. It might be interesting to call attention to the fact that this is a very distinct advance in design, whether for substations or generating stations, because it almost eliminates the control wiring and conduit. This reduces the cost of the station and eliminates the necessity for a separate switchboard room or switchboard gallery.

Under "Development," I had hoped that the Committee would list something about wiring symbols, because operators and manufacturers are having a great deal of trouble now getting the same wiring symbols for the same devices and for the same applications.

Telemetering is mentioned. The statement is made that it is relatively new, the devices are insufficiently reliable, and the costs are prohibitive for extensive applications. It is true the telemetering is relatively new, but the costs are being rapidly reduced. It is also true that the reliability of the equipment is improving. Some of the newer designs are just as reliable as the present-day voltmeters, ammeters, and watthourmeters. The

accuracy of telemetering is very nearly of the same order as that of conventional recording and indicating instruments. It is not quite equal to their accuracy, however. This is because, first, an indicating or recording instrument or its equivalent is used for originating the telemetering impulse, and second, the equivalent of an indicating or recording instrument is used for receiving the impulse. Consequently there are at least two instruments, sometimes three in series and, therefore, the errors are cumulative.

One of the difficulties about telemetering is misapplication. The manufacturer is not always given an opportunity to suggest the best design for a particular application, and sometimes is forced to go too far in advance of the art.

Another interesting item is in connection with supervisory equipment. Telephone relays or telephone-type relays are becoming more generally used. One thing about supervisory systems that is causing trouble is lack of standardization of voltage. One manufacturer supplies 24-volt lamps and another 48-volt; one uses 60 volts and 24 volts and another 48 volts. Probably the Committee on Automatic Stations ought to take this matter under advisement during the coming year.

It is interesting to note under "Development" that practically all of the mercury arc rectifiers which have been placed in service since 1927 have been made automatic. Only a few are manual.

It is also interesting to note that there are automatic stations in practically every country in the world. In this connection the British Electrical and Allied Manufacturers Association has asked our cooperation in making American standards uniform for Great Britain.

Nothing has been said this year about the minimum protection of apparatus. It is believed that the Committee on Automatic Stations for 1929-30 ought to give this subject some consideration, because it may be that the minimum protection to be required of apparatus may be reduced over present recommendations.

H. P. Sleeper: In this report reference is made to the matter of the importance of keeping accurate reports of operating results, particularly troubles which are experienced. I should like to add a plea to the reference and cite our own experience in connection with the keeping of operating records.

Approximately two years ago a system of reporting apparatus troubles on special forms provided for the purpose was instituted in our company. This form is filled out in the field and summarized in the general office. It contains all data pertinent to the identification of the apparatus involved such as serial, style, voltage, etc., together with a brief explanation of the trouble and its cause. These reports from all over the system are summarized by types and classes of apparatus and a summarized report is made annually.

This system of reporting apparatus troubles has been found to be invaluable. It is particularly useful to determine where the maintenance forces should concentrate their work in order to secure proper operation. It furthermore quickly picks out defective points in equipment by noting similar cases of trouble on equivalent types of equipment and enables the manufacturers to detect weaknesses that show up only under operating conditions. It further serves the purpose of enabling a very fair comparison to be made between similar equipments on the basis of operating results. By noting faults on a given type of equipment against inventory records on all equipment in use of that type, it is possible to reduce such records to a percentage basis and hence a very fair analysis is obtained on similar types of equipment made by different manufacturers.

I believe that a procedure similar to this should be carefully kept by all operating men as the results to be obtained are not only of local benefit but are of mutual benefit among operating companies because of the improved product which may be obtained from the manufacturers as a result of the operating records which are thus obtained.

Electrical Communication

ANNUAL REPORT OF COMMITTEE ON COMMUNICATION*

To the Board of Directors:

During the past year considerable progress has been made in the various branches of electrical communication engineering. The Committee on Communication submits the following report as a summary of the principal developments.

Telephone Transmission

Work on improved telephone transmitting and receiving apparatus and amplifiers by the Bell System has made possible the completion of a new telephone transmission reference system, two copies of which have been built. One has been installed in New York in the Bell Telephone Laboratories, and the other in Paris under the auspices of the International Advisory Committee on Long Distance Telephony. The manufacture of these two reference systems forms a basis for world-wide agreement on the fundamental standards to be used in judging telephone transmission. A description of this equipment is given in a paper entitled *Master Reference System for Telephone Transmission*, by Messrs. Martin and Gray, scheduled for presentation at the 1929 Summer Convention.

Agreement has also been reached on the use of the fundamental unit of transmission, named the "bel." The transmission unit which has hitherto been used in this country is one-tenth the size of the bel and will therefore be known as the "decibel;" it will be continued in use in this country.

At the Winter Convention of the Institute, a paper on the subject of *Vector Presentation of Wave Filters* was presented by Messrs. Mallina and Knackmuss.[1]

Telephone Service Improvements

With the increase of approximately $275,000,000 in telephone plant investment for the country for the year 1928, toll telephone conversations increased 7½ per cent over 1927. The handling of these toll calls has been improved, both as regards speed and clearness of voice transmission. Toll calls are now handled on the average in 1.2 minutes and in over 95 per cent of the cases the subscriber remains at the telephone. Over 97 per cent of the long toll messages are carried out without evidence of transmission difficulty or interruption.

Important developments in long distance telephone communication in Canada included the completion during 1928 of direct circuits between Montreal and the maritime provinces, between Toronto and Winnipeg, and between Calgary and Vancouver.

In several of the larger cities of the United States, the practise of telling the time of day to subscribers who call the number designated for this service has been initiated.

Dial Telephony

The dial service was rapidly extended, the number of dial telephones in the United States increasing approximately 600,000 during the year, with about 18 per cent of the total stations now on this basis.

This expansion in the service requires continued development work and in the dial system areas, general use of mechanical tandem equipment is rapidly extending. This applies not only to the largest areas, but to others as well; and to step-by-step areas as well as to panel.

For areas in which step-by-step equipment is used, the dial system "B" boards have been initiated. These boards make dials unnecessary on manual "A" positions which are required to operate into dial offices. Such equipment has been found satisfactory in operation, and its use is spreading.

Carrier Systems

The most important advance in the art of carrier current telephony over telephone lines during the last year was the extensive application of short-haul single-channel carrier systems. About 200 of these were manufactured and installed, providing approximately 20,000 mi. of carrier circuit in place of wire stringing. This system was described in detail by Messrs. Black, Almquist, and Ilgenfritz at the Pacific Coast Convention in Seattle in a paper entitled *Carrier Current Systems for Short Toll Circuits*.[2]

The important advances which have been mentioned for the past few years in the annual reports along carrier current lines were described in detail in a paper entitled *Carrier Systems on Long Distance Telephone Lines*, presented at the Summer Convention in Denver by Messrs. Affel, Demarest, and Green.[3] This paper also gave data showing the steady increase in commercial applications of these carrier systems.

In the field of carrier current communication over power lines there has been continued progress in im-

*COMMITTEE ON COMMUNICATION:

H. W. Drake, Chairman,

G. R. Benjamin,	E. H. Everit,	R. D. Parker,
H. P. Charlesworth,	D. H. Gage,	S. R. Parker,
F. J. Chesterman,	S. P. Grace,	F. A. Raymond,
L. W. Chubb,	E. Hausmann,	C. A. Robinson,
J. L. Clarke,	P. J. Howe,	J. K. Roosevelt,
R. N. Conwell,	H. L. Huber,	H. A. Shepard,
B. R. Cummings,	B. D. Hull,	J. F. Skirrow,
C. E. Davies,	G. A. Kositzky,	H. M. Turner,
R. D. Evans,	F. H. Kroger,	F. A. Wolff.
	R. H. Manson,	

1. A. I. E. E. Quarterly Trans., Vol. 48, April 1929, p. 582.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

2. A. I. E. E. Quarterly Trans., Vol. 48, January 1929, p. 117.

3. A. I. E. E. Quarterly Trans., Vol. 47, October 1928, p. 1360.

29-117

proving designs and at the same time increasing the simplicity of operation. Additional installations of carrier telephone equipment are continuing at a normal rate, and this type of equipment has a well defined place as a reserve means of communication. In several cases, particularly on high-voltage superpower systems, power companies are making use of carrier communication for dispatching purposes. This is particularly true in isolated locations where the commercial telephone plant has not developed sufficiently to meet the requirements for continuous service.

Interesting data on power line carrier current working are contained in the following papers: *Power Line Carrier Telephony*, by Messrs. Fuller and Tolson, and *Problems in Power Line Carrier Telephony*, by Messrs. Wolfe and Sarros, presented at the Pacific Coast Convention; *Transmission of High-Frequency Currents for Communication over Existing Power Networks*, by Messrs. Boddie and Curtis, and *Carrier Current and Supervisory Control on Alabama Power Company's System*, by Messrs. Robinson and Woodcock, presented at the Atlanta Regional Meeting.[4]

Telephone Plant

An outstanding feature in the past year has been the rapid extension of toll cable, there having been placed during the year about 2500 mi. of toll cable including more than one and one-third million miles of conductor. The continuous toll cable network now extends along the Atlantic seaboard north to Brunswick, Maine and south to Greensboro, North Carolina. Westward extensions on several separate routes have been carried to St. Louis, Missouri, and Davenport, Iowa. In Canada, the Hamilton-Buffalo and Toronto-Oshawa links have been completed.

An electrical alarm system has been developed for indicating the presence of moisture in telephone cables, as shown by low insulation. This system consists of a vacuum-tube alarm device which is connected successively after a suitable charging interval for a period of approximately 60 sec. to various conductors in the cable. To avoid interference with working circuits the test is made from the midpoint of the repeating coil on phantom circuits.

For the local plant, a cable has been developed containing 1818 pairs of 26-B&S gage wires. This compares with a previous maximum capacity of 1212 pairs. The new fine gage cable will be particularly useful in conserving duct space in congested metropolitan areas.

With the increasing demand for poles used in the construction of power and communication lines, new sources of supply must be developed. Mr. Lindsay's paper *Utilization of Lodgepole Pine Timber for Poles*,[5] presented at the Summer Convention in Denver, describes the method of selecting and using lodgepole pine, which has not hitherto been used to any appreciable extent for this purpose.

Telephone Equipment

A new form of information desk designed for greater convenience of reference and improved speed of answer has been developed. Improvements, particularly in connection with telephone booths and telephone station units, have been made and will result not only in better and more convenient apparatus but also in providing greater usefulness.

Improved dial system equipment particularly adapted for small private branch exchanges was made available during the year. This equipment has associated with it a small cabinet for desk mounting, which takes the place of a section of switchboard and is arranged so that the attendant's services are not required for disconnection, thus improving the over-all service and operating economies.

Private branch exchange equipment adapted to the requirements of large residences has also been developed. This equipment is intended to provide various special features which are in demand and also to introduce some improvements and simplifications.

The Bell System is experimenting with company operation of private branch exchanges with a view to improving this branch of the service.

To facilitate improvements in toll service to small outlying offices, a series of small power units particularly adapted for use in such offices to supply a few telephone repeaters, carrier terminals or telegraph circuits was developed.

Developments in Materials

The basic conception of the insulating properties of textile insulation has been investigated by the engineers of the Bell System and it has been found that by suitable purification processes, the insulating properties can be materially improved. The process and results are described in a paper entitled *Purified Textile Insulation for Telephone Central Office Wiring*[6] by Messrs. Glenn and Wood. A companion paper by Messrs. Williams and Murphy on the subject *Influence of Moisture and Electrolytes upon Textiles as Insulators*[7] was presented at the same meeting.

During the year, the use of cellulose acetate has been extended materially, both in solid form and for impregnating textiles used in telephone work. Cellulose acetate is the base of the best grades of artificial silk, is non-inflammable, and has other valuable mechanical and electrical properties.

An important development of the Bell Telephone Laboratories has been a series of alloys of iron, nickel, and cobalt, known as the "perminvars." This develop-

4. A. I. E. E. Quarterly Trans., Vol. 48, January 1929, pp. 102, 107, 227, and 214 respectively.

5. A. I. E. E. Quarterly Trans., Vol. 47, October 1928, p. 1354.

6. A. I. E. E. Quarterly Trans., Vol. 48, April 1929, p. 576.

7. *Ibid.* p. 568.

ment is an outgrowth of the earlier discovery of permalloy. The new alloy has not only high initial permeability, but its permeability is constant to a remarkable degree. Its properties indicate that it will be very serviceable in certain types of communication equipment and systems.

Airways Communications

An experimental trial of rapid telephone methods for collecting the necessary weather data is now being conducted in California by the Weather Bureau with the cooperation of the Guggenheim Foundation and the Pacific Telephone and Telegraph Company.

Another adaptation of wire services to airways communication is the use of telephone typewriter service. A number of installations has been made both for local communication at the airport and for communication beween airports.

A description of a successful system of guiding airplanes along fixed airways during fog or low visibility was given in a paper entitled *Uses of Radio as an Aid to Air Navigation*,[8] by Dr. J. H. Dellinger, presented at the Winter Convention.

Wire Line Systems for Broadcasting

In the June 1926 report of this committee, mention was made of the fact that long distance wire lines were coming into use for enabling good programs to be made available at a number of widely separated broadcast stations. This use of wire lines in connection with broadcasting has increased tremendously and the circuits regularly in use for this purpose on January 15, 1929 served more than 110 radio broadcasting stations and included more than 28,000 mi. of telephone circuit. In addition, over 40,000 mi. of telegraph circuit were also in use for control purposes.

Frequency Control in Radio

With the growth which is occurring in the number of radio channels, especially those operating at the higher frequencies (short waves), the ability to set the channels accurately in the frequency spectrum and to hold them to their assigned frequencies during operation has become of fundamental importance. Substantial contributions have been made during the past year or two in improved methods for accurately measuring frequencies and in the practical working out of the quartz-crystal oscillator control for transmitting stations. Most of the more important broadcast stations of the United States are now crystal-controlled, whereby their frequencies are maintained with an accuracy of the order of $\pm$ 50 cycles. Stations not equipped with such control sometimes find difficulty in staying within a $\pm$ 500-cycle limit. The majority of the more important short-wave transmitters of the world are likewise using crystal control but due to the imperfections of the arrangements in practical use, much interference exists between these short-wave channels. Laboratory advances made during the past year in the crystals themselves, including the manner of cutting them and of controlling their variation with temperature, as well as improvements made in the circuit arrangements for employing them, will reduce this interference as these methods become more generally available in practise.

A frequency meter has been developed which reads radio frequencies directly on an indicating instrument. This operates on a heterodyne principle in which the standard circuit is controlled by a piezoelectric crystal oscillator. The accuracy is 100 cycles in a million.

Railroad Train Radio Equipment

Further progress has been made in the application of radio communication between front and rear ends of railroad trains. In a recent installation, modulated continuous wave transmission at a wave length slightly over 100 meters is used and reliable communication is obtained. While the main principles employed remain unchanged, much has been done toward solving the design problems involved.

Transatlantic Telephony

The most outstanding phases of transatlantic telephony during the past year were the very substantial growth which occurred in telephone traffic and the extension of service to all points in Germany, Holland, Belgium, Switzerland, and Spain, and to one city in northern Africa. Certain other important points were reached also, such as Paris, Copenhagen, Stockholm, and Vienna. The extension to Paris was very important, as about 25 per cent of the traffic is to or from Paris as compared to 50 per cent to London and from 5 to 10 per cent to Berlin. On the American side, the service was extended to nine cities in Mexico, seven of the more important cities in Western Canada, and to all points in the provinces of Ontario and Quebec.

To facilitate the operation of the circuit, printing telegraph machines are now used during intervals between telephone calls for the purpose of passing "ticket information" regarding the calls and for transmitting other messages relating to the service.

In June 1928 a second two-way circuit was brought into service. The radio portion of this circuit uses very short waves, 16 to 33 meters in length, as contrasted with the long waves used for the other transatlantic circuit. The circuit is arranged to be used either independently or in combination with the older circuit, and experience has shown that the combination of long and short wave circuits gives a greater over-all reliability than with the same number of facilities operated on either long or short waves. It is expected that the increasing use and further development of transatlantic telephony will require further facilities.

During the year the research engineers of the Bell Telephone Laboratories have perfected a means of making a transatlantic telephone cable. Until now a submarine cable of this length has been impossible as

8. *Ibid.*, p. 563.

current sufficient to carry speech could not be sent by submarine cable such long distances, because the devices which are used on land for amplifying the speech currents, such as loading coils and repeaters, could not be attached to the wire under water. This cable when constructed will not only substantially increase the telephone facilities for transatlantic communication but will also provide a circuit of maximum reliability.

Transatlantic Telegraphy

One of the accomplishments in the communication field during 1928 was the successful laying of the Bay Roberts-Horta duplex loaded cable by the Western Union Telegraph Company. This cable differs from loaded cables previously laid by that company in that the loading material is Mu-metal instead of permalloy, put on in the shape of wire instead of ribbon as formerly used. In order to permit of successful balancing for duplex operation, the cable is fully loaded only on the mid-section. Towards the ends, the amount of loading material is decreased until the shore ends proper have no loading material whatever. The cable was designed to give a speed of 1000 letters per minute duplex, or 2000 letters per minute simplex, but tests made thus far upon the cable indicate that these speeds may be exceeded.

Printing Telegraphy

The expansion in the telephone typewriter service has been marked by the installation of more than 3000 printing telegraph machines during the past year. Special circuits and switching arrangements have been developed for use by the subscribers in interconnecting the lines used with these types of machines. A number of these switching arrangements has been installed to meet the special requirements of customers where a switched typewriter service somewhat similar to that given with telephone instruments associated with private branch exchanges is needed. These arrangements enable a quick communication service to be obtained and at the same time have the advantage of giving a written record of such communications.

Printing telegraph instruments of the same general character as those mentioned in the preceding paragraph are also being used in rapidly increasing numbers by the commercial telegraph companies, about 6000 having been installed during the past year. In this field for the transmission of telegrams the principal application of the machines is on lines connecting main and branch offices of the telegraph company with each other and with customers' establishments. The major switching problem involved is the concentration of numerous lines of this character for efficient traffic handling by a group of central office operators. Equipment recently installed permits the concentration of one hundred or more lines so as to be accessible to any required number of operators.

Excellent progress was made during the year in the joint development by the Teletype Corporation and the Western Union Telegraph Company of a high-speed ticker capable of operating at 500 characters per minute. Installation of this new ticker and its associated equipment will take place during 1929 in the New York Stock service. The same machine will be used by the Telegraph Company and by the New York Quotation Company, which is the official operating company of the New York Stock Exchange.

In the field of trunk-line message circuits between cities, which are usually operated by high-speed multiplex printing telegraph apparatus, an important development of the past year has been the extension of metallic circuit working. Most earlier attempts have required the use of two line wires for each metallic circuit. In the system now successfully used by the Western Union Telegraph Company, three high-speed metallic circuits are obtained from each group of four-line conductors without carrier frequencies.

Facsimile Transmission

A considerable increase in the use of the telephotography service of the Bell System resulted from commercial arrangements made during the year. Under these arrangements, the leading telegraph companies collect from and deliver to the public photographs, diagrams, and other facsimiles which are transmitted over the telephotograph circuits.

The Westinghouse Electric and Manufacturing Company reports that an apparatus has been developed for facsimile picture transmission which is considerably more rapid than the existing commercial systems. It is expected that this will be available for sale in a comparatively short time.

Television

What appears to have been the most significant advance in television made during the year was the development of equipment for the transmission of images of outdoor scenes illuminated by sunlight. A public demonstration of this equipment was given at the Bell Telephone Laboratories on July 12, 1928, at which time action scenes, such as a tennis player going through his strokes, were successfully transmitted.

An improved photoelectric cell of greater sensitivity and a more efficient optical system for scanning were employed.

Sound Pictures

The year 1928 saw wide extension of sound-picture systems throughout the theaters of the United States, and the equipment of the important studios with recording apparatus for producing both disk and film records. The light-valve method of recording sound on films was introduced commercially, and altogether, during the year about 2000 sound projector equipments were put into use in motion picture theaters in the country. Also improvements in frequency range of recording and reproduction were made during the year, and electrical systems for the accurate speed control of the projector equipment were utilized in the reproduc-

ing systems. Field equipment, as distinct from studio equipment, was developed for recording, as was also portable projector equipment. The first educational sound-picture film, a production of the Bell Telephone Laboratories, made its appearance during the year, developing the principles of carrier-current telephony, particularly modulation and filtering. This film was projected before several scientific and educational audiences.

Municipal and Protective Signaling

There has been a considerable increase in electrical signal devices for traffic control. Synchronous control of such signals along a street or throughout a section appears to be growing in favor. Progress has been made in the application of signals which normally permit a steady flow of traffic along a main thoroughfare but subject to manual or automatic reversal on the approach of traffic from a side or cross street.

For small fire alarm systems, an improvement of the past year has been the introduction of electrical sirens that can be coded satisfactorily. The various defects of former types have been overcome to a great extent, and further development will lie probably in provisions for emergency power supply to ensure operation even if the public electrical supply should be deranged.

An increase is noted in the use of police signaling systems. The older local battery systems are being converted to central energy operation in some cases. There is also a tendency of banks and other financial institutions to connect their protective circuits and vault alarm systems to the police signal circuits. Some expansion has also taken place in the interurban teletype signaling systems mentioned in last year's report.

Education

ANNUAL REPORT OF COMMITTEE ON EDUCATION*

To the Board of Directors:

The continued rapid increase in scientific knowledge and in the diversity and complexity of the engineering applications and the engineering responsibilities, combine to make the four-year engineering program a less and less adequate preparation for effective engineering work.

This situation is not peculiar to the engineering profession. The medical and the legal professions have been confronted by a similar situation and to meet it they are requiring students aspiring to those professions to remain on the college campus for a period of 6 years, and after that to serve an apprenticeship of 6 months or a year as a law clerk or a medical interne.

The study of engineering education conducted by the S. P. E. E. has disclosed the fact that very few teachers or engineers propose to meet the situation by requiring *all* engineering students to remain upon the college campus for a fifth or sixth year. Before increasing the length of the engineering course for all students, the engineering profession first proposes to try out thoroughly an alternative method.

This alternative method is to encourage the comparatively few graduates of the four-year engineering course who are possessed of the requisite interest, ability, and financial means, to continue in residence by enrolling in the Graduate Schools, and to encourage the majority, who will continue to enter at once upon their engineering apprenticeships, to continue their educational efforts in a more systematic manner and to a more definite end than at present.

Of all the possible educational movements in which the Institute Committee on Education might engage, none seemed to the committee to have greater possibilities of advancing the standards of professional attainment and proficiency than this movement to stimulate interest in the systematic continuation of engineering education after college and to make adequate provision for a program of post-college education.

The full development and realization of the great possibilities of post-college education will require the cooperation of the engineering profession with the industries and the colleges, and it seemed to the committee that it could render its most effective service:

1st, by seeking to bring the thought of the Institute membership to bear upon the possibilities and the problems of post-college education and upon the responsibilities of the profession relative to this period of training;

2nd, by seeking to stimulate the local Sections to promote this movement by setting to work to canvass or inventory the post-college needs and the educational facilities of their districts and to coordinate the two;

3rd, by seeking to act as a center through which a knowledge of distinctive and effective developments in the field of post-college education may be made known generally.

The views and the aims of the committee have been stated in a brief article entitled *The Post-College Education of Engineers*, which appeared in the April, 1929 issue of the JOURNAL. In this article it is suggested that the local sections, particularly in the industrial centers in which Colleges of Engineering are located, each appoint a Committee on Education. It is suggested that the function of the section committee on education be to canvass the needs and the wishes of the engineers of the district, particularly the younger engineers, and to make these needs and wishes articulate by bringing them to the attention of the college administrations, or the industrial managements, or the engineering authorities in the district in the form of explicit statements such as the following: We have a group of 20 men who will enroll in a course in differential equations, to meet one evening a week for a year, or 15 men who will enroll in a course in advanced circuit theory, or 30 men who will enroll in a course in engineering economics. What provision can be made to meet the needs of these men?

A fully developed program of post-college education will mean a large and important expansion of the work of the colleges of engineering and will play an important part in the development of engineering teachers. This post-college training will compel and will reward a broader and more thorough training than is common in the teaching ranks today. It will afford greater opportunities to determine the adequacy, the relative importance, and the real significance of the principles and the methods taught to undergraduates. It will make teaching attractive to a wider range of engineers by making it possible for men in teaching to have closer contact with engineering practise.

*COMMITTEE ON EDUCATION:

Edward Bennett, Chairman.

E. W. Allen,	A. B. Gates,	H. S. Osborne,
J. L. Beaver,	W. B. Hartshorne,	A. G. Pierce,
H. W. Buck,	Paul M. Lincoln,	C. S. Ruffner,
V. Bush,	C. E. Magnusson,	W. S. Rugg,
O. J. Ferguson,	W. E. Mitchell,	W. E. Wickenden.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

Discussion

F. C. Caldwell: Professor Bennett has spoken of the discussions that were held at Columbus last week and has brought out the emphasis that was put on two grades of education not included in the ordinary college curriculum. The statement was made by Dr. Wickenden in his report on the investigations made

under the auspices of the Society for the Promotion of Engineering Education that not much fault had been found with the four-year college course as it is now being given. Some minor changes were suggested, but nothing of great importance. It was suggested, however, that there are two other levels of education that needed cultivation. Professor Bennett has emphasized one of these, the graduate level. The other is also interesting. It was stated that in Europe the so-called technical schools were sending into the industry three or four times as many men as the universities, whereas, in this country, the number supplied by such institutions as Pratt Institute and Wentworth Institute was only a fraction of the number supplied by the colleges. I foresee that we are going to find in the coming years a greater and greater shortage of college trained electrical engineers. We may look for no considerable increase and possibly for a decrease in the number of electrical graduates, unless something happens to change the present trend.

There is no notable growth in the total number of students going into the engineering colleges. The selection of their curricula by most high-school boys has not much rational basis. During recent years, the electrical engineering courses have had more than their share of the students. This has doubtless been partially due to the popularity of radio among boys of high-school age; this popularity, however, is probably waning somewhat.

On the other hand, there is the great development of aeronautics; most high-school boys want to fly, and many are looking to the opportunities for training in aeronautics given in the universities.

The mechanical engineering departments of the universities are developing their courses along those lines and during the next few years there is likely to be a trend away from electrical and toward mechanical engineering; first, because of the decreasing interest in radio on the part of the boys, and, second, because of the increasing interest in aeronautics. This may develop a rather critical situation for the electrical industry. About a third of the students who enter the engineering colleges graduate. The other two-thirds fall by the way, either because they are not able to keep up the pace or for financial or other reasons.

The problem is to find some way by which those men who drop out now can be given a training more effective because better adapted to their mental capacity, in other words, to do just what is being done in Europe. These men must be provided with a less theoretical training, which would also suit better some of the men who do now graduate. Some of these do not get much advantage out of the more theoretical work of the upper years. They barely get by and have very hazy ideas of what they have studied.

If industry can realize this situation, can create a demand for men with a lower grade of training and can bring about or help to bring about the organization of schools of suitable grade, then we may be able to make the supply of trained electrical engineers go around. Perhaps the universities also may be induced to give training of this lower grade.

Charles Ficklenburg: Speaking from a manufacturing standpoint, one of the difficulties that we have found in engineering graduates is the lack of mechanical engineering graduates who wish to go into electrical manufacturing. We find that the structures for carrying the electrical parts require a very high degree of mechanical knowledge and we find it increasingly difficult to obtain from the colleges graduates who have the necessary electrical knowledge as well as the necessary mechanical knowledge.

We can put down very easily the electrical characteristics; we can solve those electric characteristics, but when it comes to mechanical structure to carry the electrical conductors, the magnetic circuit and the like, we are woefully weak.

So I should like to make a plea to the Committee on Education and through them to the colleges that more stress be placed upon teaching students the mechanical features and limitations, rather than purely theoretical electrical subjects.

Electrical Machinery

ANNUAL REPORT OF COMMITTEE ON ELECTRICAL MACHINERY*

To the Board of Directors:

The Committee on Electrical Machinery has functioned under an organization similar to that of last year, that is with subcommittees commissioned to handle all the matters of a particular type of equipment that properly belong to this committee. These subcommittees were as follows:

Direct-Current Machinery, Mr. H. L. Zabriskie, Chairman; Induction Motors, Mr. P. L. Alger, Chairman; Rectifiers, Mr. B. G. Jamieson, Chairman; Synchronous Machinery, Mr. S. L. Henderson, Chairman; Transformers, Mr. W. M. Dann, Chairman.

A special subcommittee with Mr. C. M. Gilt as chairman was commissioned with the preparation of the annual report.

The proposed standards of the I. E. C. on "Rating of Electrical Machinery" were referred to the committee for advice. There was a number of minor differences between them and those of the Institute, which the committee believed could be readily adjusted without violating established principles of standards of either European or American practise. However, a few features differed so fundamentally from accepted American practise that the committee recommended strongly against approval of them. Among the most important items to which objections were raised were the higher temperature rise permitted for transformers, the generally lower temperature rise allowed for commutators, and the permissibility of tolerance in those guarantees of performance, such as efficiencies, that are accepted in this country as limits rather than calculated expectations.

The committee has continued its work associated with the preparation of material for recommendations for new standards or modification of existing standards. Included in the recommended modifications of the Transformer Standards was an appendix designed to outline operation by temperature rather than rating. These reduce the hot spot temperature from 105 deg. to 95 deg. except when operating according to name plate rating with ambients above 30 deg. and do not permit an increase in output beyond that permissible at 0 deg. cent.

The recommendations for Standards for Constant Current Transformers have been practically completed and it is expected that these will be presented for further action shortly.

It has become evident that some equipment and engine builders have been somewhat skeptical of the value of the conventional efficiencies of rotating equipment, particularly those of low-speed engine type generators and motors in which windage and bearing friction are excluded, and have been assuming input-output efficiencies much below those actually obtained. Steps have been initiated to clarify this situation and plans are being formed to include an arbitrary but small allowance for such of these losses as may be properly included, when they are not actually measured.

The increase in use of enclosed ventilated rotating equipment with self contained or separate fans has indicated the necessity for more satisfactory rules governing the allocation of ventilating losses. The problem has been accentuated by electrical ship propulsion in which the losses in the external ducts may be a large or even major portion of the ventilating losses. An endeavor is being made to formulate rules for the allocation of these losses that will be basically sound and readily applicable.

At present there are no Standards for a-c. commutator motors and their importance, both in the single-phase and multi-phase types, has grown to the point where the committee has undertaken the preparation of material for standards. Studies have been conducted in the stray load losses of induction motors so that a reasonable allowance may be included when not determined by test. Other modifications of the present Standards are under consideration.

Mercury arc rectifiers have been actively studied and records of performance and characteristics assembled. Bases of rating have been somewhat uncertain, and it has become evident that the present Standards for Transformers are not entirely applicable to transformers used with rectifiers, due particularly to the non-sinusoidal current and voltage waves imposed by the rectifier. Ground work for the preparation of Standards has been well laid.

It has been felt that there is need for a well recognized test code in the electrical field as has been established in some other fields. Each of the manufacturers has his own test code, and some of the users have developed or adopted test procedures, but there is no common or generally recognized code. The committee has therefore recommended that such a code be prepared and issued by the Institute.

At the Winter Convention, two sessions were sponsored by the committee. One session was devoted to the induction motor, including the capacitor motor, which has recently come into prominence due to the requirements for high power factor low starting current

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Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

single-phase motors for household devices. At the other session the following papers were presented. *Insulation Tests on Electrical Machinery before and after Being Placed in Service; Effect of Transient Voltages on Power Transformer Design; Transient Analysis of A-C. Machines; Two-Reaction Theories of Synchronous Machines; Influence of Temperature on Large Commutator Operation.*

No radical departure in size or design of apparatus has been evident during the last year, but there has been a gradual upward tendency in ratings and efficiencies. Refinements in design and improvements in materials have increased the output per pound of material, usually accompanied by lower losses. In some lines factory methods are rapidly being changed by the substitution of structural and rolled steel parts for castings, and it is expected that these changes will be more obviously reflected in designs as advantageous modifications become evident. While the maximum nominal operating voltage has not been extended beyond 220 kv., the upward trend of voltage has been evident in large equipment, both in the static and rotating fields. More specifically, some of the trends and typical examples of recent developments are outlined in the following paragraphs.

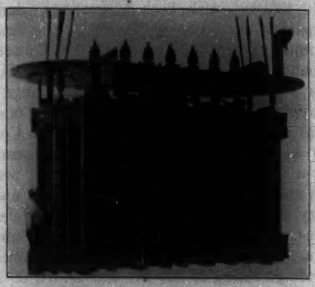

Fig. 1—Low-Voltage Side of Core and Coils of 83,333-Kv-a. Auto-Transformer, 24,500/12,000 Volts, Three-Phase, 25-Cycles

For the Buffalo General Electric Company

Transformers

Transformers, while apparently well stabilized in types of construction, have increased in size, and their already small margin of losses has been further reduced. Efficiencies have been obtained as high as 99.71 per cent at three-quarters load in a 45,833-kv-a. output, 60-cycle, single-phase, self-cooled auto transformer having a voltage ratio of 15,940 to 7970 and 99.54 per cent at half load in a 40,000-kv-a., three-phase, 60-cycle self-cooled two-winding transformer having a ratio of 29,000/13,800.

The manufacturers have recently extended their special studies and investigations in high voltage and high voltage disturbances. As an example of work being done along these lines, the testing facilities of the General Electric Company at Pittsfield have been increased to include a "lightning generator" capable of delivering 5,000,000 volts, which is higher than any voltage so far measured on any transmission line, and the Westinghouse laboratory is able to produce over 3,000,000 volts.

Investigations of the manufacturers into the effects of distribution of impulse voltages on transformer windings have been carried on actively by means of klydonographs, cathode ray oscillographs, and surge voltage recorders. These indicate that the distribution of the voltage varies with the frequency and the physical arrangement of the transformer and lightning impulses and high frequencies may build up very high local voltages within the transformer windings.

To eliminate uneven distribution and high local stresses, the General Electric Company has developed and put into service a new type of transformer which has been called "non-resonating." Published tests indicate that in the "non-resonating" transformer the voltage distribution is approximately uniform and the same for high frequencies and impulses as it is for operating frequencies. These results are obtained by the use of special shields connected to the line terminals but external to the windings, so that the necessary charging current can be supplied directly to the various coils rather than indirectly through the line end coils.

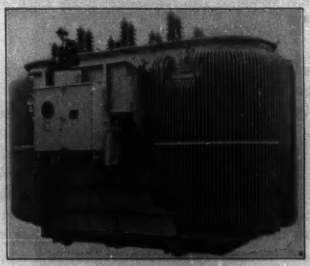

Fig. 2—Separate Regulating Transformer for 11 Per Cent Regulation on a 50,000-Kv-a., Three-Phase, 24,000-Volt Circuit

The first of the "non-resonating" transformers have been in operation about two years.

The studies of the Westinghouse Company have led them to believe that a very satisfactory distribution of voltage is obtained throughout the windings of their transformers when exposed to lightning conditions and have increased their confidence in the reliability of their transformers at such times.

The use of load ratio control which has been previously described, has materially increased and the manufacturers have simplified their design and reduced

the dimensions of equipment so that it may now be secured on the transformer tanks and requires very little more floor space than that necessitated by radiators. In this equipment the voltage ratio may be changed under load and the same type of switching equipment in conjunction with regulating transformer windings may be used to obtain phase angle control where necessary.

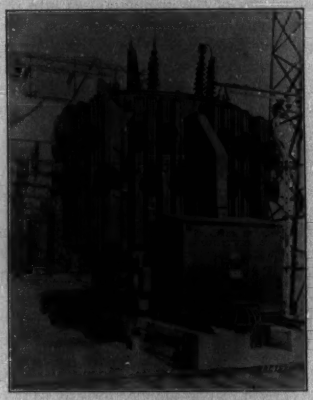

FIG. 3—43,333-KV-A. POWER TRANSFORMER—60 CYCLES, SINGLE-PHASE 220/69/13.2 KV. WITH LOAD RATIO CONTROL FOR THE 69-KV. WINDING AND AUXILIARY UNIT TYPE AIR-BLAST EQUIPMENT

For the Plymouth Meeting Station

Forced air ventilation, supplementing self-cooling, has been used in a number of installations. This is particularly effective in large self-cooled units in decreasing the radiator surface required for maximum ratings while still permitting operation of the self-cooled unit without the blower losses at moderate ratings. Two types of equipment are furnished, one of which consists of a single blower piped to all the radiators of the transformer, and the other of small individual blowers and housings that are added, one to each radiator.

Three-winding transformers have continued in popularity in the large capacity units, and the Westinghouse Electric and Manufacturing Company reports that nearly half the large power transformers ordered from their company in 1928, were of the three-winding type.

Typical of the most notable installations of large units, may be mentioned the following:

An 83,333-kv-a. output, three-phase, 25-cycle water-cooled auto-transformer having an equivalent transformer capacity of 49,200 kv-a. was installed by the Buffalo General Electric Company. The unit was supplied by the General Electric Company and probably concentrates in one unit the largest kv-a. output so far undertaken in transformer construction. The high-voltage winding consists of two circuits, each of which is to be connected to a separate section of the station bus. The windings are so arranged that a short circuit on one winding tends to raise the voltage of the other, which aids in maintaining the voltage of the other bus section.

The largest self-cooled transformer reported consists of 40,000 kv-a., three-phase, 60-cycle, transformers supplied to the United Electric Light and Power Company of New York by the Westinghouse Company.

Six single-phase, 15,000-kv-a. water cooled 25-cycle, three-winding transformers with a voltage ratio of 220/110/132 kv. with both low-voltage windings of full capacity, are probably physically the largest 25-cycle units yet supplied and weigh over 175 tons each. Tap changing under load with 7½ per cent buck and boost was furnished for the 110-kv-a. windings. They were supplied to the Hydro-Electric Power Commission of Ontario by the Canadian Westinghouse Company.

Two banks of 60-cycle single-phase transformers having a self-cooled rating of 33,333 kv-a. per transformer and a rating of 43,333-kv-a. per unit with auxiliary air blasts were installed by the Philadelphia Electric Company in the Plymouth Meeting Substation. They are Westinghouse three-winding transformers with voltage rating of 220 kv. star *H. V.*, 69 kv. star *M. V.*, 13.2 kv. delta *L. V.* They are equipped with the older type of load ratio control to give 15 per cent regulation in six 2½ per cent steps in the 69-kv. winding.

FIG. 4—CORE AND COILS OF A 40,000-KV-A. SELF-COOLED POWER TRANSFORMER—THREE-PHASE, 60 CYCLES, 29/13.8 KV. FOR USE WITH A FREQUENCY-CHANGER SET

At the United Electric Light and Power Company

Two three-phase air-blast transformers supplied the New York Edison Company by the General Electric Company have extended the rating of air blast units so far produced. Each is rated 18,500-kv-a., 25-cycle, 11800/3300/440 volts, and will operate in conjunction with a 40,000-kw. induction synchronous frequency-changer set.

Attention has been given to the problem of low cost, small power supply from high-voltage lines, and a

standard "substation" has been made available for use on 66,000-volt grounded neutral systems with a capacity of 15 to 50 kv-a. single-phase. The unit consists of an over insulated single bushing transformer with a combined disconnecting switch fusable cut-out and current limiting resistor for the high-tension winding. Metering equipment, cutouts, and arrester are included in the 2300-volt connections.

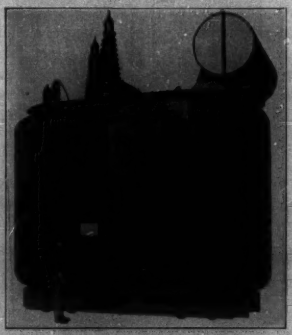

FIG. 5—20,833-KV-A. POWER TRANSFORMER—60 CYCLES, THREE-PHASE, 120/120 KV. SHOWING LOAD-RATIO CONTROL SIDE
For the Detroit Edison Co.

HIGH-VOLTAGE METERING

The Philadelphia Electric Company has placed in operation metering equipment for measuring the power supply at full voltage on the 220-kv. line connection with the Pennsylvania Power and Lighting Company. The equipment consists of three units each containing single-phase current and potential transformers connected between line and grounded neutral. Similar installation has been made by the Appalachian Electric Power Company consisting of two units of current and potential transformers. The potential transformers are designed for use on a 220-kv. three-phase system with a neutral either free or grounded. Both of the above equipments were furnished by the Westinghouse Company.

REGULATORS

The Westinghouse Company has brought out a new design of outdoor type regulator aimed to make the installation of these units more convenient. An outdoor covering has been provided which can be removed complete by unscrewing the eyebolt. Doors are provided for giving access to the motor and other auxiliaries, with a window in front for viewing the position indicator. A centralized lubricating station is included with alemite fittings to lubricate all bearing surfaces not immersed in oil. The weatherproof box containing the relays and control auxiliaries and a neutral return and manual return switch is mounted on the front of the tank. All six power leads are carried up through the cover into a separate compartment at the rear of the hood, where the instrument transformers are connected.

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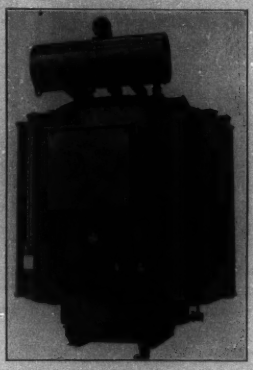

FIG. 6—20,000-KV-A. POWER TRANSFORMER—60 CYCLES, THREE-PHASE, 41,830/72,450 VOLTS. CONTACTOR AND MECHANISM HOUSING SIDE
For the Philadelphia Electric Company

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Synchronous Machines

The tendency toward larger machines has continued in the field of water wheel generators, but there has been no increase in size of single or multiple shaft turbine generators beyond those previously reported.

The use of fabricated machines has been considerably extended and the more important parts such as frames, bearing brackets, and rotor spiders of practically all machines are built up from steel plates and rolled members. Some welded bearing pedestals have been built, but the use of cast pedestals still predominates. Castings still hold their own for small standard machines.

Experience is being accumulated in the use of quick response excitation and speeds of increase in exciter

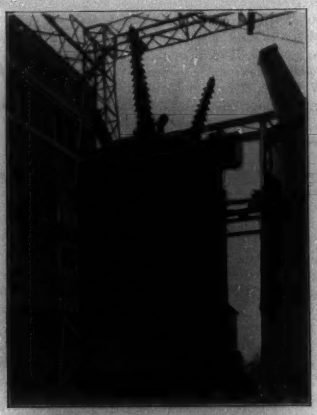

Fig. 7—15,000-Kv-a. Power Transformer—25 Cycles, Single-Phase, 120/65/13.2 Kv. All Windings Full Capacity

voltage are now available which are considerably in excess of those required for most operating requirements.

More operating experience has been obtained with hydrogen cooled condensers. This type of machine is particularly adaptable to hydrogen cooling, as the bearings can be placed inside the housing, thereby eliminating the difficult problem of sealing the shaft against leakage. The application of this kind of cooling to many a-c. machines has proved practical, but the extent of its use depends on economic considerations. The increase in cost for the enclosures and hydrogen equipment must be balanced against the reduced losses and probable longer insulation life. In many cases a high capitalization of losses will justify the extra cost, but there are some classes of machines, with the more complicated mechanical structures, to which this system of cooling is not yet applicable.

The design of high-voltage machines has been advanced this year, and several generators to operate at 22,000 volts have been built by the General Electric Company. The Westinghouse Company is now building a 75,000-kv-a. 23,000-volt generator. This has been made possible by refinements in manufacture and care in application of existing insulating materials, rather

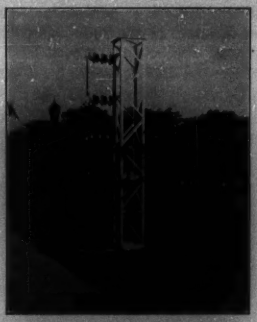

Fig. 8—50-Kv-a. High-Voltage Transformer Installation for Rural Service 66000/2300 Volts

than through the development of new types of insulation. The manufacturers feel that the building of machines for these voltages has reached a stage where their use is a question only of the balance of cost against operating advantages.

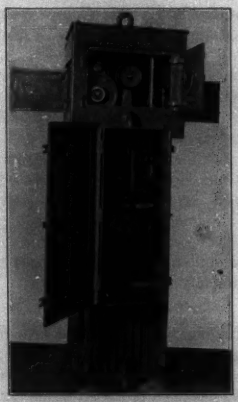

Fig. 9—New Outdoor Type Induction Regulator

The Parsons Company in England has designed and built a 25,000-kw., 3000-rev., 33,000-volt generator of great interest by reason of the way in which the problem of insulating for such high voltage was solved. The conductors in the several slots are concentric tubes with simply the innermost insulated for the terminal

voltage. The outermost has a small amount of insulation and stands adjacent to the neutral. This generator is reported to be operating satisfactorily in the Brimsdown Power Station, London.

The scheme of using two independent parallel windings has been applied by the General Electric Company to two large alternators. This allows the two windings to be connected to adjacent bus sections and the reactance between the windings is available to perform the duty of reactors between bus sections. On certain installations a material saving in switching equipment can be made in this way.

A great deal of progress has been made in designing large machines of all classes to operate at speeds higher than were formerly thought possible.

Hydraulic Generators

Four water wheel generators, larger than any yet built, were ordered from the General Electric Company by

Fig. 10—Fabricated Stator Frame and Upper Bearing Bracket for 33,000-Kv-a. Vertical Water-Wheel Generator, Assembly Incomplete

the Dnieprostroy Hydroelectric Development of Russia. These machines will deliver 77,500-kv-a., 3-phase, 50-cycle, 13,800-volt power at 88.2 rev. per min. Fabricated construction will be used in all principal parts such as frame, upper and lower bearing brackets, and rotor spiders.

The following list comprises the more important machines ordered or installed during the past year.

Purchaser	No.	Kv-a.	Speed	Manufacturer
Dnieprostroy Hydro-electric dev.	4	77,500	88.2	G. E.
Lexington Power Co.	4	40,625	138	West.
Phila. Elect. Co.	4	40,000	81.8	G. E.
	3	40,000	81.8	West.
New England Power Co.	4	39,000	138	West.
Brazilian Hydroelec.	1	33,000	125	G. E.
Southern Calif. Edison	1	35,000	375/450	G. E.
Alabama Power Co.	2	29,000	100	West.
Norwood Electric Co.	2	27,500	90	A. C.
City of Los Angeles	1	25,000 (Hor)	143	A. C.
Montana Power Co.	2	25,000	81.8	West.
Norwood Electric Co.	1	22,500	75	A. C.

Many of the above units have high-speed excitation systems to provide greater stability during system disturbances.

Fig. 11—25,000-Kv-a., 143-Rev. Per Min., 50-Cycle Horizontal Water-Wheel Generator with Welded Plant Steel Yoke

For the City of Los Angeles

The vertical Westinghouse and Allis-Chalmers units are of the so-called "umbrella" or "overhung" type, with guide and thrust bearings located together under the generator rotor. In this way, the guide bearing is

Fig. 12—Rotor Spider of Fabricated Steel Plate Construction for 12,500-Kv-a. Generator for Frequency Changer

brought nearer to the center of gravity and the necessity for an oil circulating system is eliminated. This scheme is especially suited to machines whose diameter is large compared to the height.

The Conowingo units have been installed since last spring. Tests on the General Electric units showed an efficiency of 98 per cent at unity power factor and full load.

The generators for the Brazil Hydroelectric Company

Fig. 13—40,000-Kw., 81.8-Rev. per Min. Generators at the Conowingo Development

were the largest ever built for a foreign customer prior to the order for the 77,500-kv-a. machines for Russia.

The 33,000-kv-a. machine for the Southern California Edison Company is of unusual design as it is rated at 375 rev. per min., 50 cycles but can be run at 450 rev.

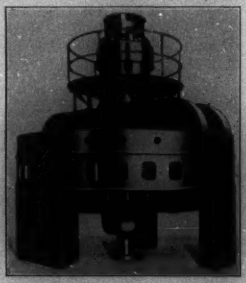

Fig. 14—7500-Kv-a., 225-Rev. per Min. Vertical Water-Wheel Generator

per min., 60 cycles. Its stability was increased by making the short circuit ratio higher than usual.

The Westinghouse Company has brought out a new line of small generators, using structural steel parts. The line includes rating from 62.5 kv-a. to 3000 kv-a. with speeds of 100 to 400 rev. per min. The upper and lower bearing brackets, together with the rotor spiders on the slow speed machines, are fabricated.

Steam Turbine Generators

There were no orders placed this year for any exceptionally large units, either single shaft or compound. A number of orders was received for machines ranging from 60,000 to 75,000 kv-a.

The use of fabricated structure has been extended by all the manufacturers. Some of the machines have been so designed that a change to hydrogen cooling can be accomplished if it should be found desirable.

Several of the large units mentioned in last year's reports have been or are being installed and some of them are in operation. Among these are the following:

Purchaser	Kv-a.	Speed	Voltage	Shaft	Manufacturer
State Line Generating Co.	235,000	1800	22,000	3	G. E.
United Elec. Lt. & Pr. Co.	188,400	1800	13,800	2	West.
United Elec. Lt & Pr. Co.	188,250	LP 1200 HP 1800	13,800	2	A. B. B.
Amer. Gas & Elec. Co.	187,000	1800	11,000	3	G. E.
N. Y. Edison Co.	160,000	1500	11,400	1	G. E.
Brooklyn Edison Co.	137,500	1800	13,800	2	West.
So. Calif Edison Co.	400,000	1500	16,500	1	G. E.
Union El. Lt. & Pr. Co.	83,333	1800	13,800	1	G. E.

Fig. 15—160,000-Kw. Cross-Compound Turbo Generator at Hell Gate

The first 61,765-kv-a. 22,000-volt generator of the Super Power Co. of Illinois at Powerton, has been in operation since August 1928 and the company states that its operation has been entirely satisfactory so far.

The General Electric Co. has built or is building several 12,500-kw., 3600 rev., generators for high-pressure (1200-lb.) turbines. Most of them are 0.8-power factor or 15,625-kv-a. machines. Two generators for unity power-factor operation will be mounted on top of the respective main 1800 rev. generators.

Westinghouse has built a 12,500-kv-a. 3600-rev. per min. machine with self contained fans of novel design. These fans have a double entrance which is said to increase their efficiency materially.

A 7500-kv-a. turbo generator with hydrogen cooling has been placed in experimental operation at the Westinghouse plant. The tests indicate a 25 per cent increase in rating for the same temperature rise above that obtained when operating in air.

Synchronous Motors

The design of synchronous motors has been considerably advanced, particularly in securing higher

Fig. 16—160,000-Kw. Cross-Compound Turbo Generator at Hell Gate

efficiencies and better torque characteristics. The latter are being selected to fit particular applications as more data become available on the starting and operating requirements.

There has been a considerable increase in the use of synchronous motors for the following class of equipment:

Air and ammonia compressors
Tube mills

Fig. 17—Rotor of 160,000-Kw. Single-Shaft Turbine Generator, Weight 261,000 Lb.

Ball mills
Vacuum pumps
Water pumps
Pulp grinders
Rubber mills
Steel mills
Jordan engines

The Westinghouse Elec. & Mfg. Company has built two 1000-hp., 180-rev. per min. high-torque motors for cement mill drive. These machines are of salient pole construction but have polyphase damper windings connected through slip rings to external resistors for starting in the same manner as a slip ring induction motor.

Fig. 20 shows a 5000-hp., 82-rev. per min. synchronous motor for steel mill drive by Westinghouse. This is one of the largest horizontal machines built with fabricated construction.

The first two speed motors with 2 to 1 speed ratio for compressor drive have been placed in operation. These motors have been previously described by the General Electric Company.

Ship Propulsion Synchronous Motors

During the year, the S. S. *Virginia* was placed in service and work started on the S. S. *Pennsylvania*. These vessels will ply between New York and San Francisco at a speed of 18 knots. They have a displacement of a little over 30,000 tons and are propelled by two 8500-hp., 120-rev. per min., 4000-volt syn-

Fig. 18—94,000-Kw. Tandem Compound Turbine Generator at Long Beach

chronous motors. Work was begun on two 6300-hp., 120-rev. per min., 3000-volt motors for a Grace Liner, and on four 8000-hp., 143-rev. per min., 3150-volt motors for two Ward Liners. All of these ships are motored with General Electric Company equipment.

Synchronous Condensers

No outstandingly large condensers have been ordered during the past year, but much progress has been made in reducing losses, increasing speeds, quickening excitation, and applying hydrogen cooling to commercial machines.

The Westinghouse Elec. & Mfg. Co. has built two 30,000-kv-a. condensers for the Philadelphia Electric Company. These are the largest units so far constructed for 720 rev. per min.

The General Electric Company furnished three units of this size to the same company, the speed being 600 rev. per min. All of these machines are equipped with super-excitation as it was believed of the utmost importance that the system voltage be maintained at a high value during disturbances. The exciters are

designed for a rate of rise of 6000 volts per second with a ceiling voltage of about 950 volts.

The Consumers Power Company has purchased a standard 30,000 kv-a. unit from the Westinghouse Elec. & Mfg. Company.

Fig. 19—Double-Entrance Fan on a 12,500-Kv-a. Turbine Generator

This company has built two 15,000-kv-a. 900-rev. per min. condensers, this being an advance over the 10,000-kv-a. unit which formerly held the record at this speed.

The first commercial application of hydrogen cooling for electrical machinery was made on a G. E. 12,500-

Fig. 20—5000-Hp., 82-Rev. per Min. Synchronous Motor with Fabricated Structure

kv-a. outdoor synchronous condenser, located at the Pawtucket substation of the New England Power Company. A larger unit of similar design, rated 20,000 kv-a., is in operation at the Turner Substation of the Appalachian Electric Power Company. The performance of both of these machines during their few months of operation has been very satisfactory. In addition to reducing the losses and increasing the rating, it is believed that hydrogen cooling also increases the insulation life, reduces the windage noise, and decreases the fire risk.

Test Sets

During the past year the Canadian General Electric Company has put into operation a new test set for testing large transformers. The set consists of two 7500-kv-a. generators driven by a 2000-hp. d-c. motor. The generators are designed for 3-phase operation or single-phase operation at a capacity of about 70 per cent of the three-phase rating. By means of re-connection and varying the excitation, practically the full kv-a. capacity is available through a voltage range from 1100 to 7600. The generators are wound for either 6 or 12 poles so that the set may deliver the

Fig. 21—1000-Hp., 180-Rev. per Min. Synchronous Motor with Polyphase Damper Windings in the Pole Faces and External Starting Resistors

commercial frequencies used in Canada of 25, 30, 50, 60, 62½, and 66⅔ cycles.

Converters

The Allis Chalmers Company has built several large 60-cycle, 3000-kw., 360-rev. per min. railway converters for the City of Philadelphia. These units commutate peak loads of 2½ times nominal rating.

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FIG. 22—20,000-KV-A., 900-REV. PER MIN. HYDROGEN-COOLED SYNCHRONOUS CONDENSER
For the Appalachian Electric Power Co.

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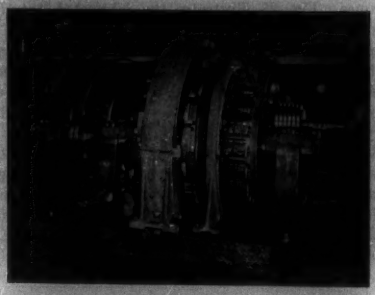

FIG. 23—3000-KW., 60-CYCLE, 360-REV. PER MIN. ROTARY CONVERTER
For the City of Philadelphia

"Changing from Cast Iron and Steel to Fabricated Steel," H. V. Putman and C. C. Brinton, *Elec. Jl.*, July 1928, p. 333.

D-C. MACHINES

The fabrication of parts without the use of castings has been considerably extended during the year, the later developments applying particularly to the rotating

FIG. 24—ARMATURE OF 6000-HP. 70/140-REV. PER MIN. REVERSING MILL MOTOR WITH FABRICATED SPIDER CONSTRUCTION

parts. This method of construction appears to be especially advantageous in the medium and large size units, which, because of their application, are apt to be special and require flexibility in design. The development of the arc welded fabricated construction has given flexibility and still permitted the standard part principle, which separates the machine into parts or

elements, some of which can be used in many different designs and others of which change with each design. Fig. 25 illustrates the fabricated construction of the spider of a G. E. 6000-hp. 70/140 rev. per min. reversing mill motor requiring especially strong spider and supporting structure to withstand the high reversing torques. Fig. 25 illustrates the fabricated construction of a 7000 hp. blooming mill motor built by the Westinghouse Company for the Youngstown Sheet and Tube Company. While the Allis-Chalmers Company has not extended this type of construction to d-c. equipment as extensively as they have in a-c. machines, they anticipate that its use will increase.

Several installations of a new scheme of ventilation have been made. A special volute shaped housing is arranged to make use of the windage of the machine to force the heated air through ducts out of the station and thus maintain a low ambient temperature. Fig. 31 illustrates an installation of this kind at the Wheeling

Fig. 25—Stator of 7000-Hp. Blooming Mill Motor Showing Welded Fabricated Construction

Steel Corporation and furnished by the General Electric Company.

Motor Generator Sets

Fig. 29 illustrates a 6000-kw. fly wheel induction motor set built by the Allis Chalmers Company to drive a 7000-hp. double unit reversing mill motor of the Illinois Steel Company. The two generators are wound for 700 volts and commutate peak loads of 2½ times nominal rating. The flywheel is of plate construction.

The General Electric Company has furnished some 4000-kw., 5500-kw., and 6000-kw. motor generator sets for electrolytic work with d-c. voltages of 500 and 600. These sets were built with one motor and two generators to take advantage of the higher speeds and efficiencies obtainable by this construction as contrasted with that of a two-unit set.

The Cleveland Terminal electrification has adopted 3000-kw. 3000-volt motor generator sets for power supply. The sets furnished by the General Electric Company consists of two 1500-volt generators connected in series and driven by one synchronous motor, and have a rated overload capacity of 50 per cent for two hours and 200 per cent for 5 minutes.

Motors

An interesting motor application is the tandem mill drive of the American Steel and Wire Company furnished by the Westinghouse Company. Five motors

Fig. 26—7000-Hp. Blooming Mill Motor
For the Youngstown Sheet and Tube Company

are mounted on a common bed plate with a distance between centers of 7 ft. This reduction between rolls permits a better steel product, but requires extreme accuracy in the speed regulation, of the motors.

Illustration of wide speed ranges is furnished by the "ultra-speed" planer drive furnished by the Allis Chalmers Company. Minimum cutting speeds of 15

Fig. 27—Tandem Steel Mill Drive with Five Motors Mounted on One Bed Plate
Built for the American Steel and Wire Company

ft. per minute with return speeds of 180 ft. per minute, or a speed ratio of the motor of 1 to 12 have been obtained.

Gearless traction elevator motors are gaining in use for high-speed equipments. A 140-hp., 85-rev. per min., 250-volt motor built by the General Electric Company is believed to be the largest so far constructed.

It was built of fabricated construction and provided with roller bearings.

The use of d-c. motors and generators for Diesel Electric propulsion of ferries, tugs, tankers, light ships, and freighters has increased. The largest of these is the 4000-hp. drive of the freighter *Courageous* furnished by the General Electric Company. A single 4000-hp. motor is driven by four 800-kw. Diesel engine driven generators.

Fig. 28—Five-Unit Shovel Motor-Generator Set, with One 860-Kw., and Two 350-Kw. Generators

For the Marion Steam Shovel Company

Exciters

Quick change of excitation has been applied in a number of installations to increase system stability during disturbances. Rates of voltage rise of 6400 volts per second with a maximum of 950 volts for a 250-volt exciter have been obtained. As an example of this method, a 165-kw. 250-volt exciter is provided with two field windings, one for a normal excitation of about 4 amperes controlled by a voltage regulator, and the other controlled automatically by a contactor for forcing the excitation up to about 400 amperes. The magnet frame is laminated to permit quicker change in field flux. A 40-kw. 275-volt sub exciter provides constant voltage excitation for the main exciter and no rheostat is used in the alternator field.

Fig. 29—6000-Kw., 700-Volt D-c. Flywheel Motor-Generator Set for Reversing Mill Motor Supply

Built for the Illinois Steel Company

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Fig. 30—Motor-Generatoe Set Built Entirely of Fabricated Parts

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Induction Motors

General. During the past year, there have been interesting developments of new types of small motors,

Fig. 31—4000-Kw. Motor-Generator Set with Two Generators Equipped with Ventilating Housings

At the Wheeling Steel Co.

but the larger machines have not been of novel character. The use of single-phase capacitor motors on a considerable scale was inaugurated, and the use of ball bearing, of enclosed fan cooled, and of line start motors was widely extended during 1928.

Capacitor Motors. The need for small single-phase motors of the highest quality for domestic refrigerators,

Fig. 32—Exciters for Quick Response Excitation System

oil burners, and other devices, has spurred manufacturers to a renewed study of the capacitor motor. Reliable low cost capacitors have become available recently, due to the progress of the radio art, and so capacitor motors have now for the first time become economically feasible. Such a motor gives no radio interferences, has high efficiency and power factor, and gives the minimum of noise. The Howell Electric Company has developed a line of this type of motor; the General Electric Company has put several thousand fractional horsepower capacitor motors into service on domestic refrigerators, and many other manufacturers have entered the field to varying extents. The usual type employed is a normal split-phase motor with a capacitor connected in series with one phase. When more than about 75 per cent starting torque is required, it has been found necessary to use a larger capacitance at start than for running, a switch or auto-transformer with taps being employed to make the changeover. With the present insistent demand of the utility companies for lower starting currents on domestic appliance motors, it is likely that the capacitor motor will grow rapidly in favor.

Repulsion Induction Motors. The use of single-phase repulsion start induction motors in sizes up to 10 hp. has grown steadily during the past year, and a general

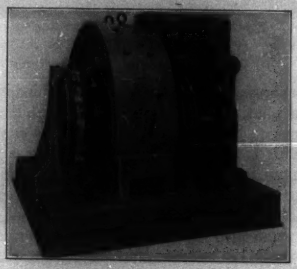

Fig. 33—140-Hp., 85-Rev. per Min. Gearless Traction Elevator Motor

improvement in characteristics has been obtained. The General Electric Company has recently brought out a novel motor of this type with square punchings, and graded slots, to obtain more efficient utilization of the steel. A photograph of this motor is shown in Fig. 35.

Line Start Motors. Practically all American motor manufacturers are now building extensive lines of motors designed with high impedance, to limit the starting current. This type of motor now represents probably 10 per cent of all the integral horsepower induction motors made in the United States. The designs of motor employed vary considerably, but those with the usual overload capacity of 175 per cent have generally about 500 per cent starting current. The great majority of these motors are built in sizes between 7½ and 30 hp.

Frequency Changers. The use of small induction frequency changers to supply moderately high frequency power to drive high-speed motors has grown steadily. A novel feature now employed with many of these sets

is the provision of taps on the stator winding of the frequency changer motor, so that reduced voltage of normal frequency can be obtained for driving the high-frequency motors when desired. The high frequency motors are wound for a definite voltage, and when they are operated at the lower frequency a proportionately reduced voltage is required. The use of the motor stator as an auto-transformer meets this need.

Two induction synchronous frequency-changers, built for the Niagara, Lockport and Ontario Power Company by Westinghouse, were placed in operation about eight months ago. In addition to carrying heavy overloads, the sets have showed themselves capable of withstanding severe line disturbances without falling out of step. Each synchronous machine is rated 25,000 kv-a. and each induction motor 28,000 hp., making these the largest variable ratio frequency changers in the world.

The General Electric Company is building a similar 25,000 kv-a. set for the Buffalo General Electric Company, the induction motor being rated at 27,500 hp.

Fig. 34—Ultra Speed Planer Drive with 12 to 1 Speed Ratio

Enclosed Fan Cooled Motors. Enclosed fan cooled motors have grown remarkably in popularity. The initial temperature rise of an open motor is not greatly increased by these modern enclosing features, while, on the other hand, the accumulation of dirt in service on an open motor makes the temperature rise increase very much. The apparent handicap of enclosing is, therefore, converted into a real gain in service temperature rise. A number of methods of enclosing is being used. An interesting type suitable for use in explosive atmosphere has been brought out by the Louis Allis Company. This is made from the standard fan cooled motor, by strengthening the mechanical parts, and other changes.

Large Induction Motors. The use of welded structural steel frames for large induction motors has become well established in the past year. The trials of the U. S. S. *Lexington* and *Saratoga* were successfully completed early in 1928, and the 8 driving motors on each ship were found capable of delivering 27,000 hp. each to the propellers, some 20 per cent in excess of their rating, without difficulty. The remarkably successful results obtained with electric drive of these ships represent an important achievement of 1928. Several induction motor driven ships are now under construction.

Commutator Motors. An interesting application for the adjustable speed shunt-characteristic commutator motor has been found in the drive of hosiery knitting

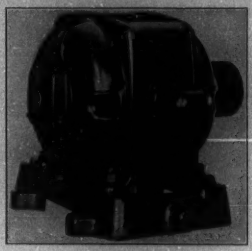

Fig. 35—Novel Type of Single-Phase Repulsion Induction Motor

and ring spinning machines. In the latter case, it is desirable to vary the speed of the motor automatically as the bobbin progresses from empty to full, and this is accomplished by automatic means for shifting the brushes of a polyphase adjustable speed shunt commutator motor. The machine as developed is totally enclosed and self-ventilated.

High-Speed Motors. The trend towards the development of large capacity, high-speed motors is shown by

Fig. 36—900-Hp., 3600-Rev. per Min. Squirrel-Cage Motor for Driving Boiler Feed Pump

some orders recently placed with Westinghouse for eight 1250-hp., 3600-rev. per min. induction motors. They are to be used for driving high-pressure boiler feed pumps.

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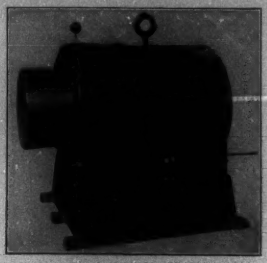

Fig. 37—Three-Phase Shunt Commutator Motor with Automatic Speed Variation for Spinning Frame Drive

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Fig. 38—3000-Hp. Mercury Arc Rectifier
For Maypole Substation in Chicago

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Fig. 39—Mercury Arc Rectifier Substation
For the Berlin Metropolitan Railway

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Mercury Arc Rectifiers

Mercury arc rectifiers continue to hold the interest in the d-c. railroad and street railway field and rather rapid advances in capacity have been made. Since January 1, 1928 a total of 28 sets have been installed or placed on order in this country, having a total kw. rating of 48,975. The majority of these sets consisted of single tanks and the average rating per tank has materially increased. The same general trend appears to be current in Europe, as over 40 rectifiers with current rating of 4000 to 6000 amperes were installed or placed on order for street railways and electrolytic purposes.

The frequency and deleterious effects of arc backs have been materially decreased. The effect of short circuits has been most evident in the transformers which in some cases have failed due to lack of rigidity of the windings. The seriousness of outages caused by arc backs has been further diminished by improvements in the construction and capacity of pumps, the most important rectifier auxiliary. The installation of arc suppressors has apparently greatly improved service but it does not seem conclusive that the freedom from serious arc backs in some installations should be credited entirely to the arc suppressor.

Larger and better pumping units have been effective in reducing long outages due to the inability to remove the vitiating elements in the vacuum caused by arc backs. Continuity of ignition has been further guaranteed by the provision of anodes in addition to the ones used for arc striking purposes.

Graphitic anodes have given good account of themselves during the past twelve months of operation and evidences of their stability were furnished by examinations of some which had been removed from service. Confidence in the metal anode, however, is indicated by their continued use in the largest rectifiers now in service in this country.

Close cooperation has been established between the manufacturers of rectifiers and the manufacturers and operators of communication and central station companies' equipment and system. This has resulted in making available very effective means for the control and suppression of undesirable undulation characteristics in the output from the rectifiers. They have generally taken the form of a low frequency filter including a reactor placed in the d-c. leads from the rectifier. One prominent factor in this cooperative attempt to deal effectively with this interference situation is the realization of the need for a telephone interference factor having a frequency weight characteristic differing from that included in the present unit and based upon the susceptibility of the human ear to tones of different pitch.

The subcommittee has carefully studied the questions of rating, regulation, and efficiency. While these matters together with others, such as nomenclature, have not been finally acted upon, substantial work has been accomplished which will greatly facilitate the reaching of agreements on these essential elements of performance.

The outstanding installations during the past year were two 5000-ampere 625-volt automatic railway outfits installed on the Commonwealth Edison Company's system for railway service. These units were six-phase twelve-anode with arc suppressors but without voltage regulating devices and were supplied by the Brown-Boveri Company. They have an overload

MERCURY ARC RECTIFIERS IN RAILWAY INSTALLATIONS INSTALLED OR ON ORDER SINCE 1927

	No. of sets	Tanks per set	D-c. voltage	Kw. per set	Total kw.	Control
Commonwealth Edison Company, Chicago, Maypole	1	1	600	3000	3,000	Auto
Commonwealth Edison Company, Chicago, West Lawn	1	1	600	3000	3,000	Auto
Toronto Hydro Electric System, Toronto, Ossington	1	1	600	1100	1,100	Auto
Southern Public Utilities Corp., Charlotte, N. C., Eliza. Ave	1	1	600	750	750	Manual
City of Calgary, Calgary, Portable	1	1	575	600	600	Auto
Union Railway of New York City, New York, St. Peters Ave	1	1	625	1000	1,000	Auto
Toronto Hydro Electric System	1	1	600	1100	1,100	Auto
American Gas & Electric, New York	2	1	600	1000	2,000	Auto
Cons. Mining & Smelting Co. of Canada, Electrolitic	3	2	460/560	5600	16,800	Manual
City of Edmonton	..	..	575		1,325	Manual
City of Calgary	..	..	575		1,200	Auto
City of Halifax	1	1	600	1100	1,100	..
Chicago & Joliet	1	1	600	500	500	Auto
Columbus Railway Power & Light Co	1	2	600	500	1,000	Manual
Columbus Railway Power & Light Co	1	1	600	500	500	Auto
Columbus Railway Power & Light Co	1	1	600	500	500	Auto
Eastern Mass St. Ry	1	2	600	1000	3,000	Manual
Chi. So. Shore & So. Bend Ry	1	2	1500		1,500	Auto
Chi. So. Shore & So. Bend Ry	1	2	1500		1,500	Auto
Chi. So. Shore & So. Bend Ry	1	2	1500		1,500	Auto
Philadelphia Rapid Transit	3	1	600	1000	3,000	Auto
Sacramento Northern Railway	1	1	1500	500	500	Auto
Piedmont & Northern	2	2	1500	750	1,500	Auto
British Columbia Elec. Ry	1	1	600	1000	1,000	Manual
					48,975	

capacity of 7500 amperes for two hours or 10,000 amperes for one minute. As these units were not put into service until December, 1928, the brief period of operating service is inadequate as a gage of their all around performance but it is noteworthy that there has been but a negligible number of arc backs and these were inconsequential, as far as could be observed, as the rectifiers were capable of returning to service as quickly as the reclosing breakers could operate. Load swings on one of these units as far as 8000 amperes have been carried without any evidence of limitation.

The Brown Boveri Company developed a new type of rectifier for 3000-kw. which is about 30 per cent smaller than the original units of this capacity. Another type with normal rating as high as 16,000 amperes has been developed, this also showing improved efficiency and reduced weight.

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Discussion

W. J. Foster: These reports are records of notable accomplishments during the past year, and I want to call attention to something that I regard as novel and as adding a great deal in the efficient operation of machinery, especially as I think, to its increase in life, and that is the use of hydrogen ventilation. There is just a slight reference to its use in synchronous condensers, the first commercial application of hydrogen. The 20,000-kv-a. condenser operating in the system of the Appalachian Electric Power Company shown there is not the first one that has been in operation. It is located at Pawtucket, Rhode Island. It was started about the first of August.

It has been gratifying after many years of experimentation and development to have had two machines put into operation. The appeal of hydrogen to engineers is not in its application to synchronous condensers but rather to turbine generators, where the greatest increase in efficiency can be secured, as well as the greatest reduction in the size of machines. That we hope for later. But the operating people have been somewhat reluctant to place orders for such machines. Possibly the next annual report may contain a reference to this application in turbine generators. I certainly hope it will.

C. J. Fechheimer: I think it worth while to call attention to the double-entrance fan, a picture of which is shown in Fig. 19. That photograph does not show the whole as completely as might be, and I want to explain it a little more fully.

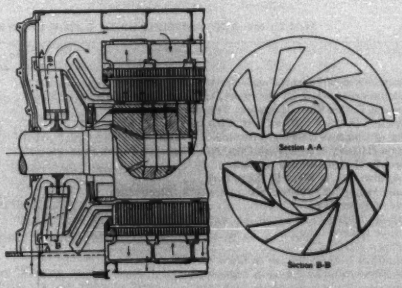

Fig. 1—Schematic Arrangement for Ventilating with the New Double Entrance Fan

In the development of these very large units, it is impossible to make the shaft smaller than a certain diameter. As the area between the inner periphery of the fan and the external diameter of the shaft is then too small to admit all the volume of air except at high velocities, with the consequent loss that would accompany

such high velocities, the natural thing one would do is to have a fan with two entrances, one at each side, such as is used frequently for general-purpose fans.

That did not appear as a simple problem at first, but with the scheme as it has been developed it is not only possible to have two entrances, but use is also made of the discharge space beyond the fan so as to increase the fan efficiency. This, in principle, is like that shown in Fig. 1 herewith. About 50 per cent of the air enters the left side of the fan, and the other 50 per cent passes through the axial passages (shown as approximate triangles in the transverse section at the right), at comparatively low velocities. The air then turns through 180 deg. and enters the right side of the fan. After passing through the fan, the two streams unite and flow through between adjacent outer sides of the axial passages. These passages then function as stationary guides which assist materially toward changing the velocity head of discharge from the fan into static head. (In the longitudinal section this path is shown by dotted lines.)

Thus, the velocity between the fan and the shaft is reduced, and much of the discharged velocity head is converted into static head, both of which increase the efficiency. Efficiencies as high as about 55 per cent have thus been obtained, a considerable improvement over what had been secured before. This development is largely the work of M. D. Ross of the Westinghouse Company. A more complete description by Mr. Ross is given in the *Electric Journal* for March, 1929.

Electric Welding.

ANNUAL REPORT OF COMMITTEE ON ELECTRIC WELDING*

To the Board of Directors:

Your Committee on Electric Welding hereby reports the following activities and developments in their field of activity for the fiscal year May 1, 1928–May 1, 1929.

Meetings and Papers

Our Committee arranged for the presentation of two papers, at the March 1929 Regional Meeting held at Cincinnati, namely, one by Professor F. P. McKibben, on *Arc Welding in Building and Bridge Construction*; the second paper by Mr. H. V. Putman, on *Design and Construction of Electrical Machinery Using the Arc Welding Process of Fabrication.*

American Welding Society Activities

The American Welding Society through its several Committees, has prepared tentative codes on: (1) Recommended procedure for fusion welding of pressure vessels; (2) Fusion welding and gas cutting in building construction; (3) Nomenclature definitions and symbols, and is developing a code for welding for pressure piping.

The Fundamental Research Committee of the A. W. S. is carrying on a number of researches headed by able representatives of eight of the leading colleges in the country, who are making studies under a number of subjects, some of which are as follows:

1. Resistance welds in low and high carbon steel rods.
2. Study of welds at elevated temperatures.
3. Effect of current density and microstructures on strength of welds.
4. Study of weld joints by X-rays.
5. Study of the fundamentals of the welding arc.
6. Study of welded rail joints.

The American Welding Society is carrying on a great deal of additional detailed work too extensive to mention in this brief report.

Commercial Activities

Bridge Construction. During the year a ¼-mile line of overhead railroad construction at a prominent steel mill was reconstructed by the addition of 75 tons of steel.

The Public Service Co-ordinated Transport Co. of New Jersey successfully repaired by arc welding two pairs of poney trusses each 86 ft. long, forming a combination trolley and highway bridge spanning the tracks of the Central Railroad of New Jersey and the Lehigh Valley Railroad at Middlesex Borough near Bound Brook, N. J. This was a very unique operation, requiring 50 tons of new steel and is described in *Engineering News Record* of October 25, 1928.

At the present time, the Harahan Bridge at Memphis, Tenn., is being reconstructed and widened which work will involve a total of 250 tons of steel. As an indication of the saving possible in this class of work, we cite the following figures, namely, an average of six bids for riveted reconstruction $15,060 as against an average for the bids by welding construction of $10,750. This represents a saving of 28½ per cent by welded reconstruction as compared with riveted reconstruction work.

Office Buildings, etc. The Homestead Hotel at Hot Springs, Va. has had a 12-story addition made thereto by the arc welding process, the addition involving 550 tons of steel. The principal consideration in this operation was the elimination of noise, which would have disturbed the guests in the old hotel structure immediately alongside of the new structure. It is difficult to put a direct monetary value upon the elimination of excessive noise for such a proposition.

Fig. 1—Cleveland Office Building. Illustrates Decrease in Weight of Columns from the First Floor to the Fourth Floor

A combination office and bank building, involving 250 tons of steel, was erected at North Tonawanda, N. Y. for the Tonawanda Power Co.

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C. W. Bates,	C. J. Holslag,	A. M. MacCutcheon,
Ernest Bauer,	C. L. Ipsen,	J. W. Owens,
Alexander Churchward,		Wm. Spraragen,

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-101

An office and store building (Upper Carnegie Bldg.) involving 115 tons of steel was completely erected by the arc welding process, using no rivets or bolts, in the City of Cleveland, Ohio, see Figs. 1, 2, 3, and 4.

Factory Type Buildings. Two buildings of this type were constructed in California for the Southern California Edison Co. at Portersville and Visalia, Calif. A building approximately 50 ft. by 100 ft. was constructed by welding at Los Angeles. Several additional small buildings in California were constructed by welding, with trusses from 40 ft. to 75 ft. span. The American Milling Co. of Omaha, Neb., has built a large hay barn 75 ft by 190 ft., using 100 tons of steel. In central Florida 15 arc-welded buildings have been constructed, principally for citrus packing houses. The largest of these is 117 ft. by 202 ft., containing 200 tons of steel.

Fig. 2—Shows Framing Partly Erected with 36-In. Girder Welded in Position

Fig. 3—Detail View Illustrating Welded Joints in 36-in. Girders and Location of Welded Open Beams for Supporting the Floor

One building (Fig. 5) is now nearing completion at North Trafford City, Pa., involving 800 tons of steel and floor space in excess of 100,000 sq. ft. A portion of this operation involves a structure three stories in height, the remaining portion being single story construction.

Thirty-two municipalities in California and eight in Oregon, Louisiana, Mississippi, Arkansas, and Arizona now have sections in their building codes covering the welding of buildings.

According to the new building code sections for these cities, it is now legal for the Commissioner of Buildings in each place to grant, in the same manner as for riveted frames, permits for the erection of electrically welded steel frame buildings.

Pipe Welding. Several installations for welding two lengths of pipe together in the shop to make approximately 40-ft. lengths out of approximately 20-ft. lengths of pipe have been made during the past year. Also some installations for welding longitudinal joints to produce piping in the shop have also been made.

Considerable progress was made during the past year in electric welding gas and oil pipe lines. The method of making the weld, the size of electrode, and welding conditions have been determined for best results and experience has demonstrated the effectiveness of this procedure. Methods of testing the welds have been worked out so that a minimum of interference with the welding procedure is encountered and yet the tests are very effective for determining the quality of the welds.

The welding of pipe lines of municipal water supply has progressed rapidly during the past year. Automatic welding machines and procedure control specifications have been developed for this work. Welding technique and testing methods have been determined to insure uniform results on a production basis. The effect of these developments has been to reduce the cost of pipe lines for city water supply systems.

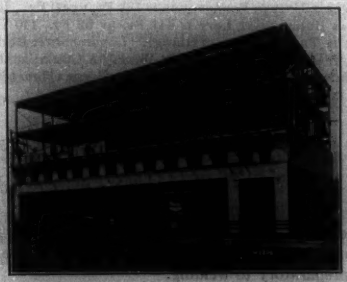

Fig. 4—Shows the Building Partly Completed

Between 1400 and 1500 miles of pipe line for carrying oil are now being laid in Texas and the South West.

Figs. 6 and 7 illustrate the welding of pipe on one of these lines.

This line is owned by the Texas Pipe Line Co. and has its sea terminus at Port Arthur and runs 714 miles to a place called Monahans, not far from El Paso, Texas.

This pipe carries oil at a pressure of 800 lb. per square inch.

The process of laying the pipe is to get on an average of five 40-ft. lengths lined up on skids along the same axis so that the ends of the pipes being welded are square with each other and weld these five lengths together, turning the pipe every once in a while so that the welder is welding in a substantially downward position. These

Fig. 5—Building Comprises Three Units in a U Group. Tranverse Unit in Foreground is 133 Ft. by 60 Ft. Longitudinal Building at Left is 441 Ft. by 62 Ft., 140 Ft. of which is Three Stories in Height. Longitudinal Building at Right is 460 Ft. Long by 62 Ft. Wide. A Combined Office and Shipping Building at Extreme End Gives a Total Floor Space in Excess of 100,000 Sq. Ft.

welds are made at the rate of 12 to 14 per day per operator.

Two welds are made on each pipe, a so-called burning in weld and a finishing weld.

One end of each pipe is bell mouthed, and the small end of one pipe is inserted in the large end of the next pipe preparatory to welding.

After the five lengths of pipe are welded into sections the bell mouth end of one section receives the small end of the next section and the two sections are welded together.

This weld is called a "bell-hole weld" and part of this weld must be made overhead. The average rate of making bell hole welds is 8 to 10 per day per operator.

After the pipe is welded, it is lowered into a trench which has been prepared for it and later covered.

The outside of the pipe is protected with a heavy coating of asphaltic material to prevent the earth from coming into contact with the pipe so as to prevent corrosion.

After the pipe is welded it is tested with hydrostatic pressure for tightness, after which the pipe is laid in the trench and covered.

The illustrations show in some detail the process as it is being carried on.

The welding machines are gasoline engines driving

Fig. 6—Bell Hole Welding on 12-in. Pipe Line near Welch, Oklahoma. Observe Bends in Pipe Due to Uneven Terrain, which Causes no Failures

welded sets and these may be transported on wagons or may be dragged from point to point by trucks.

Pressure Vessels. During the past year or so two companies have developed special technique and methods for arc welding of heavy steel plate, oil cracking stills, and other high pressure vessels. One of these companies, for example, has manufactured over a thousand vessels using over 70,000 tons of plates averaging about 3 in. in thickness. Excellent ductility

Fig. 7—Typical Finished Welded Joint in 12-In. Pipe Line

and tensile strength are obtained by the methods used by these two companies which employ covered electrodes with special alloys, heavy currents, and annealing of the structure after welding.

Resistance Welding. The use of spot welders, tubing welders, and flash welders has extended rapidly, es-

pecially in the automotive field, during the past year. Probably the latest large development in the resistance welding field is that of applying flash welders for producing pipe in commercial lengths, up to about 30 in. in

Fig. 8—Frame for 100,000-Kv-a., 3600-Rev. Per Min., Turbo Generator which Weighs Complete 265,000 Pounds and was Shipped as a Unit with Windings in Place. If a Cast Frame had been Used the Weight Would Have Exceeded the Maximum Car Capacity Forcing the Completing of the Machine on Customer's Property Instead of at the Factory

diameter by ⅝ in. in thickness, requiring several thousand kv-a. of power to make each longitudinal weld.

Machinery Construction. The use of welded machine structures replacing castings has received great impetus during the past year, especially amongst the manufac-

Fig. 9—Shows a Welded Motor-Generator Set the Only Castings Used being the Pedestal Bearings. Even the Brush Rigging for the Generator is of Welded Steel Construction. This Set is Rated at 750 Kw., 900 Rev. Per Min.

turers of electrical apparatus. The stator for a 160,000-kw. turbo generator has recently been constructed of arc welded steel. (Figs. 8 and 9.) It is undoubtedly safe to say that at the present time such construction by several of the leading manufacturers in the United States has reached the point where not less than 2000 tons per month of such fabricated machines are being produced.

Ship Construction. Metal arc welding is being used extensively in the construction of merchant and naval ships. In both cases, the policy pursued is that of a gradual utilization of the process, rather than an attempt to weld the complete ship. In merchant ship construction the object is reduction in cost, while in naval work, the object is to save weight.

International attention has recently focused on arc welding in naval construction because the weight saved by its use materially assisted the German Navy to mount eleven-inch guns on their new 9000-ton cruisers.

The steam ship Virginia, built during the year by the Newport News Shipbuilding & Drydock Co. for the Panama Pacific Steam Ship Co., embodied sufficient welding in its construction to require the use of 43,000 lb. of electrode welding wire.

Arc welding has also been used extensively in the construction of the 272-ft. yacht *Viking* for Wm. Baker. In this case, it greatly assisted the architects in obtaining the desired graceful lines.

The Fore River Yard of the Bethlehem Shipbuilding Corp. has constructed a number of bulkheads wherein one row of rivets was replaced by a continuous weld.

The Federal Shipbuilding and Drydock Co. of Kearny, N. J., have recently completed two all-welded channel type scows each 116 ft. long, 34 ft. wide, and 10½ ft. deep.

Miscellaneous. The use of automatic welding equipment has increased materially but to date its possibilities have been appreciated by only a very limited section of industry.

Atomic hydrogen has found many new applications on alloy steels and nonferrous metals. For welding thin sheet steel below No. 16 gage this process has been applied very satisfactorily where smoothness of finish and good ductility are required.

Summary

There are a great many other individual welding applications which might be mentioned but we believe that the above brief résumé summarizes the principal activities in the welding field and will give an insight into welding developments for those who are not directly identified with welding work.

Discussion

J. C. Lincoln: I want to emphasize the lower cost of electrically welded parts as compared with cast-iron parts. The company with which I am connected makes its products, electrical machinery, largely of steel shapes electrically welded. The accompanying table shows typical reduced costs of a number of parts of equipment. The welded compensator tank shown at J in the illustration costs 80 per cent less than the cast-iron tank. The weight was reduced 60 per cent by use of welded steel.

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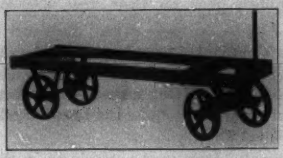

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COMPARISON OF COSTS OF WELDED AND CAST PARTS

Part	Cost					
	Cast iron			Welded Steel		
	Material	Labor	Total	Material	Labor	Total
A. Stator end ring	$1.57	..	$1.57	$0.31	$0.105	$0.415
B. Frame ring	2.89	0.20	3.09	1.13	0.41	1.54
C. Motor stator	5.77	0.61	6.38	1.03	0.71	1.74
D. Motor base rails	4.99	0.12	5.11	1.54	0.62	2.16
E. Large motor bracket	10.88	1.18	12.06	2.98	2.40	5.38
F. Motor spider	6.75	2.94	9.69	1.28	4.37	5.65
G. Welder truck	..	..	34.00	5.04	2.65	7.70
H. Engine base	35.84	1.74	37.58	9.54	3.67	13.21
I. Large motor frame	47.13	5.63	52.76	10.65	8.41	19.06
J. Compensator tank						

F. C. Caldwell: Consider for a moment the sociological aspects of the development of electric welding. In this method of manufacture we are not only reducing the amount of iron used, to a fraction of that required under the old system, therefore leaving that much more for future generations, but we are also increasing the amount of labor required. One of the social and economic problems of the present day is, of course, the fact that all along the line the amount of labor needed is being decreased, and some difficulty is being experienced in adapting human conditions to that situation. Here is one case where, while saving material for future generations, we are increasing the amount of labor to something like double that formerly required.

Electrochemistry and Electrometallurgy

ANNUAL REPORT OF COMMITTEE ON ELECTROCHEMISTRY AND ELECTROMETALLURGY*

To the Board of Directors:

The Committee on Electrochemistry and Electrometallurgy submits the following report covering some of the outstanding matters of interest within the field of the committee. Such a review can never claim to be complete. The past year has brought several items of unusual interest. As an innovation in the report of the committee for the present year somewhat more details are given about certain materials and their production, including the features likely to be of interest to electrical engineers. For this reason the number of items covered in the report is somewhat less than in the reports for the several years preceding.

D-C. Supply for Electrolytic Uses

Approximately 200,000-kilowatt capacity in direct-current machinery is now purchased yearly by growing electrolytic industries throughout the world. Electrolytic processes are essentially direct-current processes requiring large current at relatively low voltage per unit. The problem of a suitable and economical source of power is therefore of importance, and one requiring careful study to determine the best conditions in any particular case. The generation of direct current may have some advantages as to cost and maintenance, if the electrolytic plant is not far removed from the source of power, and if exhaust or bled steam can be used to advantage in process work. Such a source of supply, however, is limited in its application and does not permit of interconnection to other sources of power. When power is purchased the problem of converting it from alternating current to direct current usually arises. Synchronous converters and motor-generator sets both find use for this purpose. Possibly large mercury rectifiers may also find extended use in the future. It has been pointed out in several papers which have recently appeared that the proper choice in any particular case must depend (1), upon voltage and other characteristics of the transmission line; (2) the range of voltage, the current, and other characteristics of the electrolytic circuit; (3) The efficiency under particular operating conditions; (4) The initial investment and interest charges. It frequently happens that materials produced by electrochemical means are more costly than similar materials produced without the expenditure of electrical energy and yet the electrochemical processes are justified by the higher quality of material or by unusual physical or chemical properties. The problem is therefore a complex one which will have much to do with the success or failure of electrolytic production because the power costs are usually a considerable fraction of the total cost per unit of the material.

Chromium Plating

Chromium plating which was first accomplished 70 years ago has undergone a sensational development in the electroplating field within the past three years. It is only recently that the mechanism of the processes has become sufficiently well understood to make possible its commercial application. From the various uses proposed for chromium plating, it is becoming apparent what may be reasonably expected.

Scratch tests on thin films of chromium indicate that the hardest chromium is harder than any other metals or alloys previously tested. Its ductility is almost zero; its expansivity is about the same as for glass or platinum. It resists tarnish to a remarkable degree. The reflecting power of chromium is 65 per cent as compared with silver 95 per cent. Its electrical conductivity is about that of aluminum.

As a protective coating, thin films are desirable. For some purposes 0.00002 to 0.00004 in. are used over a preliminary coat of copper or nickel. Thicker deposits may tend to crack. Chromium deposits are made from a chromic acid bath to which a sulphate has been added. The anodes are of iron or lead. The conditions for making satisfactory deposits are rather critical as to temperature and current density, the latter ranging from 100 to 300 amperes per square foot. From the standpoint of the electrical engineer, the large amount of energy required is of interest. The voltage across the tank ranges from 6 to 12 volts. The current must be large and since the valence is 6 and the efficiency of the process low, the amount of metal deposited per coulomb is very small. It has been estimated that chromium plating requires 15 times the electrical energy used for nickel plating in equivalent amounts. However, in the case of chromium but little metal is needed. The process brings with it hazards that require carefully designed ventilating systems to protect the workers. In spite of difficulties which may seem numerous, the desirable properties of chromium as a protective coating, its resistance to wear, its hardness, and its pleasing appearance have brought it into use in many industries. Some of the applications which have been made of it have not been successful, as for example, in stamping

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T. C. Atchison,	F. C. Hanker,	Magnus Unger,
P. H. Brace,	W. E. Holland,	J. B. Whitehead,
Farley G. Clark,	F. A. Lidbury,	E. A. Williford,
Safford K. Colby,		J. L. Woodbridge.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

dies. The chromium is brittle and flakes off. In the popular mind chromium is associated with automobile and plumbing fixtures, but in a less conspicuous way chromium plating has proved its worth by prolonging the life of gages, tapes, measuring devices, forming dies, spinning tools, and printing plates.

An important development during 1928, was the installation of several completely automatic equipments for chromium plating. The electroplating industry has learned the value of chemical control and now it is adding automatic electrical control of current density and control of temperature.

Electroplating of Various Metals

In other fields of electroplating, new finishes have been developed. Cadmium plating, which can be easily done on any commercial metal, promises valuable service as a preventative of rust, but it is handicapped by the high price of metal. A process has long been sought for electroplating aluminum on other metals. A step in this direction has been made in the experimental work of depositing aluminum on other metals from organic solutions. New developments are taking place in the electro-deposition of alloys. Whereas this has been largely confined in the past to brass and bronze, there are today a dozen different alloys that may be deposited from aqueous solutions. Nickel-plating continues to increase in spite of the threatened competition of chromium plating. Actually the latter needs an underlying layer of nickel, and so it has been said that chromium plating has actually stimulated the demand for nickel. The trend in nickel plating is towards the use of very pure anodes containing not less than 99 per cent of nickel.

Hydrogen for Industrial Uses

Industrial uses for hydrogen are rapidly increasing. The electrolytic process is particularly well adapted to the production of hydrogen of a high degree of purity. It is of more than passing interest that a large research laboratory is provided with pure hydrogen piped to the various rooms and consuming more than 2,000,000 cubic feet of hydrogen per month in the experimental work. The demand for pure hydrogen in the fat-hardening industries is increasing and larger electrolytic cell units in some cases amounting to 4 to 5 times the capacity of present installations are being sought. More electrolytic hydrogen is likewise being used in the synthesis of liquid ammonia, the production of which in 1928 was so large that it became the cheapest alkali per unit. It has been found that liquid ammonia is a very convenient form for transporting hydrogen. Of particular interest to electrical engineers is the use of hydrogen as a cooling agent. It naturally diffuses faster than other gases and has a high thermal conductivity so that it can be used to advantage where it is important to remove heat rapidly. High-speed motors and generators are being operated in hydrogen gas in development tests and a commercial application of hydrogen cooling has recently been made in the case of a large synchronous condenser which is totally enclosed. The capacity of this condenser with hydrogen cooling is said to be 12,500 kv-a., as compared with 10,000 kv-a. if air-cooled. The use of hydrogen for such a purpose is beneficial in other ways, since oxygen and dirt are excluded and the effect of corona on the insulation eliminated. As a safeguard the pressure of hydrogen is maintained above atmospheric pressure and the purity of the gas is continuously recorded. An alarm is sounded if the purity falls below 91 per cent.

Copper

The change in the copper situation during 1928 was sensational. A sluggish market which had persisted for years with comparatively little prospect of marked advancement was transformed by the increasing demand for this metal. Existing stocks were reduced and outlets for even the high-cost copper provided. Copper as a product of electrochemical industry and as a basic material of electrical construction has a double interest for the electrical engineer. Part of the increased use of copper may be attributed to systematic efforts of a research association to extend its uses and part may be attributed to the growth of electrical industry, which is said to consume in one form or another, more than one-half of the total supply. It has been estimated that the mines of the United States will produce over a million tons during the present year and most of this will be used within this country.

Electrolytic Zinc

A large plant has recently been installed in Idaho for the production of electrolytic zinc by the "Tainton-Pring" process. Good deposits of very pure zinc are obtained from the strongly acid solutions. The cathodic current density is about 100 amperes per square foot and the voltage per cell about 3½ volts. The nominal capacity of the plant is 50 tons of zinc per day, with provision for trebling this output. Direct current of 16,000 amperes at 500 volts is supplied by two synchronous motor-driven generator units. The electrolytic cells are arranged in two groups of 150 cells, connected in series. Each cell contains 20 aluminum cathodes faced by anodes of a lead alloy. By grounding the mid-point of the circuit, the maximum voltage to ground is reduced to 250 volts. The purified zinc sulphate solution is circulated to the cells until 90 per cent of the zinc has been extracted when the solution contains about 28 per cent acid. The deposition process is a continuous one with periodic changes of the electrolyte. The current efficiency of the electrolytic process averages about 85 per cent. The zinc obtained by this process is of high purity and soft.

Nickel

Improved smelting and refining methods have re-

sulted in improving the purity of commercial grades of nickel. Electrolytic nickel averaging 99.90 per cent, or better, is the highest grade produced on a commercial scale. This is especially suited to the production of alloys containing high percentages of nickel. It is available as "electro-squares" in a variety of convenient sizes. The ferro-nickel alloys containing large percentages of nickel are finding increased use in radio transformer cores because of their extraordinary permeability.

Aluminum

Figures have been published to show that the world's production of aluminum has grown from 24,000 tons in 1908, to 214,000 tons in 1927. Aluminum is typically an electrochemical product. At present two great outlets for aluminum and its alloys are in the manufacture of automobiles and aircraft. It is said that more than 30,000,000 aluminum pistons were made in 1928. Aluminum cylinder heads are beginning to appear. Aluminum alloy frames for motor coaches are decreasing the non-profitable load of these bulky machines. Aluminum crank cases and housings are already familiar. Light and strong alloys are necessary for airplane construction. The strength of a variety of these alloys is now very well known. The fire hazard in the airplane has been reduced and quantity production as in the case of automobiles has become possible. A new material called "alclad" which combines the strength of duralumin and the resistance to corrosion of pure aluminum seems destined to become important in many fields. Aluminum which at one time was considered a material chiefly for cooking utensils, has thus found a wider field. Aluminum cooking utensils continue to increase, but as an outlet for this metal this application is overshadowed by the newer uses in airplanes, automobiles, trucks, railway cars, furniture, roofing, decorative castings, and foil which competes with tin.

Electric Furnaces

In the electric furnace industry there has been a decided increase in the use of the high-frequency furnace. These furnaces, using a motor-generator type of equipment, have entered the field of silver melting, and it is reported that 3.76 kilograms per kilowatt-hour can be melted with this form of equipment as compared with 2.9 kilograms per kilowatt-hour for the earlier type operated by an oscillator. The high-frequency furnace is well established in the non-ferrous field, and it seems likely that it will find use in the steel foundry as a convenient means of holding the heat. A large number of electric furnaces, mostly of the resistor type, has been installed for the nitriding processes for steel. A very hard surface is obtained similar to that produced by case hardening but superior in some respects.

Ceramic engineers have accomplished much after years of experimentation and many failures, and have succeeded in introducing the electric furnace into glass and porcelain industries.

In the carbide industries, the desire to increase the unit by increasing the ampere input at 100 to 200 volts has met with difficulties and the electric furnace engineer is confronted with the problem of how to overcome these limits of design without increasing voltage. At 25 to 60 cycles, the present electrical equipment will not permit of commercial operation beyond 3000 amperes per furnace unit.

A number of electrical furnace installations has been recorded in different parts of the world for the production of phosphoric acid and potassium phosphate. There is every indication at the present time that the electric furnace will eventually displace the sulphuric acid process in the manufacture of phosphoric acid. A noteworthy event has been commercialization of the electric furnace for smelting zinc from its ore. At one plant a furnace turning out 50 tons of pure spelter per day has been reported.

The largest peace-time electrolytic alkali-chlorine plant has been put in operation in the vicinity of Charleston, West Virginia. This locality is rapidly becoming a center of electrochemical industry.

Batteries

In the battery field the production of batteries for automobiles has continued to increase, the number produced yearly being variously estimated from 11 to 15 millions. The quality of dry cells has increased since the standard specifications were adopted for them several years ago and a proposal to materially increase the minimum requirements for the various sizes and kinds is now before the Sectional Committee on Dry Cells. The dry-cell industry has felt the inroads of the alternating current radio sets, but it is likely that a large number of battery sets will continue to be used. In the field of small rectifiers, the most important development has been the copper-copper oxide type which has displaced many of the electrolytic types.

Electrochemical Research

Interesting results are being obtained in the study of the nature of conduction found in dielectrics and particularly in insulating oils. This field of research requires a knowledge of both electrical and chemical processes. Relationships have been found between the variation of conductivity with time, the accumulation of space charges, and the final dissymmetry of the potential gradient.

A new material of extreme hardness for machine cutting tools has been placed on the market under the name of "carboloy." It consists of tungsten carbide and cobalt. The carbide is extremely hard and the cobalt increases the strength.

The demands of industry for new and pure materials has encouraged the development of electrical processes. Rare metals have been produced by electrolytic methods

and these are gradually coming into the field of technology. We can not anticipate the uses that they may ultimately find in the service of mankind, but the experience with others which we now regard as common indicates that uses rapidly arise when such materials become available commercially. The further purification of the more common metals and the elimination of almost the last fractional percent of the impurities has revealed surprising changes in their mechanical and chemical properties. Electric furnace products bear the brand of superior quality and by the use of electric heating many improvements have been made in working conditions.

The Committee wishes again to acknowledge the helpful cooperation of Professor Colin G. Fink, Secretary of the American Electrochemical Society.

Discussion

E. Fleischmann: May I point out a reference here in the report to the large number of electric furnaces, mostly of the resistor type, which have been installed for the "nitriding" of steel. This, of course, is distinctly a heat-treating process; but it has some phase of metallurgy and chemistry, not electrochemistry, connected with it. It is particularly interesting because it eliminates many of the disadvantages which have been found with the use of electric furnaces for carburizing, and some of the physical disadvantages attendant upon cyaniding. Cyaniding has caused a good deal of trouble in shops due to poor ventilation and the danger of inhaling the cyanide fumes, and these difficulties are very largely eliminated by nitriding.

Numerous electric and, in fact, other furnaces, have been used for this work; but the electric furnace permits a degree of control which is particularly important in putting a hard case on steel products, which formerly had to be packed in some type of carbonaceous compound; the compound and heavy box heated; and then the whole dumped out and the parts sorted out for further processing. Much of this work is eliminated by the nitriding process; and it seems particularly fitting that a committee of the Institute has taken an interest in it. It is one of the important developments of the uses of electric heating today.

J. C. Lincoln: There is a plating concern in Cleveland called the Metal Protection Company which does nickel plating that is very much superior to anything I have ever seen, and the method is to me at least of very great interest. I have seen pieces of steel ½ in. thick coated with nickel to a thickness of 1/16 in. on each side, and that strip run through rolls so that the steel is reduced to 1/32 in. and the nickel to 0.006 or 0.008 inch. That is a very severe mechanical test on the adhesion of the nickel to the steel.

The method used is called the degasification process. The part to be plated is attached to a 12-volt electric circuit and dipped in a tank containing pure sulphuric acid which is connected to the other side of the electric circuit. The current rises to a rather distinct maximum and then very suddenly drops to zero at which time the plate is said to be degasified. After that treatment the nickel is put on in the ordinary way and it is so much more adherent to the steel than it was before, that it will stand this rolling process I speak of without injury to the joint between the nickel and the steel.

Electrophysics

ANNUAL REPORT OF THE ELECTROPHYSICS COMMITTEE*

To the Board of Directors:

One of the Committee's principal functions has been to serve as a connecting organization between the world of physics and the Institute. We have been trying to serve this purpose by encouraging papers which bring in new concepts, phenomena, and terminology from working physicists, and discouraging contributions which are results of "inbreeding." Dielectric phenomena and dielectric theory are two particularly live topics, and there is a constant danger of unnecessarily burdening the Institute publications with contributions which contain more undigested data and unfounded speculations, and which do not bring us any nearer to a rational understanding of the underlying molecular phenomena.

A similar tendency exists in regard to the nature of induced electromotive force, magnetic phenomena, discharges in gases, etc. Men with engineering training or with training in nineteenth-century physics, when they become interested in such physical phenomena, are apt to treat them on a basis which from the point of view of modern physics may be called a blind alley. It has been the committee's concern to see to it that the gap which has grown between electrical engineering and atomic physics within the last fifteen or twenty years, be reduced as much as possible rather than opened wider. Physics is rapidly forging ahead along new paths and it is our privilege and duty as engineers to use as many of its recent achievements and methods as possible, in the solution of our outstanding problems.

Dielectrics

A large percentage of papers submitted to the Committee on Electrophysics has to do with dielectrics and their behavior. One of the sessions at the Winter Convention was largely devoted to this topic, and papers were presented on anomalous conduction, dielectric absorption, dielectric constant, power loss, and corona. Several other papers have been since approved for publication or for presentation at future meetings.

Much work on dielectrics is being done under the auspices of the Committee on Electrical Insulation of the National Research Council and also by the American Society for Testing Materials. Their reports and publications should be consulted in addition to the material found in the A. I. E. E. Transactions. There is also a Government Research Committee on this subject in England. Many valuable articles on dielectrics will be found in the *Archiv für Elektrotechnik*, and in some recent British and German books on different phases of dielectric behavior.

Under the auspices of the above mentioned Committee on Electrical Insulation, several subcommittees were formed in 1928, among others a Subcommittee on Physics, V. Karapetoff, Chairman, and Subcommittee on Chemistry, F. M. Clark, Chairman. Both subcommittees are already functioning, and through them many prominent physicists and chemists have been drawn into the study of scientific aspects of dielectrics.

A symposium on dielectrics was participated in by the Subcommittee on Physics, in connection with the April meeting of the American Physical Society in Washington, D. C. This symposium was well attended and will probably lead to greater interest in dielectric behavior among physicists. Several scientific monographs are proposed to be issued, each covering a particular topic, such as breakdown of solids, ionization in gases, dielectric constant, absorption and loss, deterioration by ionic bombardment, etc.

Among topics on dielectrics which are now being considered and studied by various individual physicists in this country, the following may be mentioned:

Mechanism of dielectric breakdown under simple idealized conditions of thin layers, crystals, etc.

Dielectric polarization, absorption, relaxation, loss, and kindred phenomena.

Dielectric constant and its connection with the molecular structure.

Relationship between mechanical and dielectric strength of crystals.

Influence of surface and space charges on conductivity of dielectrics.

Electronic and ionic bombardment of dielectrics, and the resulting physical and chemical changes.

Mobilities of ions.

Nature of ions in various gases.

Distribution of space charge.

Considerable work on dielectrics is being done in connection with an endeavor to improve cable insulation, and it is difficult to determine where the domain of electrophysics ends and that of practical applications begins. In this connection, attention is called to the paper by Professor Dawes presented at the Winter Convention and to the symposium on cables held at the Johns Hopkins University last November. Much

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Liaison Representatives of American Physical Society
Dr. W. F. G. Swann, Prof. A. P. Wills

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fruitful work on dielectrics is being done there by Professor John B. Whitehead and his co-workers. Among other investigations, a study has been completed showing that Maxwell's law does not explain the loss in composite dielectrics, and another investigation on losses in air condensers.

While an examination of the year's scientific literature would lead one to expect that investigations of the deterioration of high-voltage cable insulation and related phenomena, had been dropped, the amount of work actually in hand is far greater than in any other period. Several industrial and university laboratories are making intensive investigations of such insulation with special reference to stability of the insulation itself and the factors determining it. Several informal reports have been presented from time to time but none of the work appears to have reached a stage where those responsible for it are ready to present formal published reports. It seems not too much to expect that these investigations will greatly extend our knowledge of the mechanism of insulation failure.

It is of particular interest to note a large number of chemical investigations under way which will contribute to the progress of our knowledge of dielectrics. Of course, most of these researches have been undertaken without any thought of contributing to a study of dielectrics, but the fundamental relationships cleared up will undoubtedly prove useful in applications. Thus, at the last meeting of the American Chemical Society, April 29 to May 3, the following papers of interest to "dielectricians" were presented:

"Sorption of Water Vapor by Cellulose and Its Derivatives," by S. E. Sheppard and P. T. Newsome.

"Influence of Anti-Oxidants on the Oxidation Rate of Linseed Oil," by A. M. Wagner and J. C. Brier.

"Electric Moments and Electric Structure in Aliphatic Compounds," by C. P. Smyth.

"Molecular Structure as Revealed by Dielectric Constant Data," by J. W. Williams.

"The Effect of Symmetry on Molecular Properties," by D. H. Andrews and E. E. Reid.

"Chemical Effect of Electrical Discharge on Gaseous Hydrocarbons," by S. C. Lind and George Glockler.

"Polymerization of Unsaturated Hydrocarbons," by B. T. Brooks.

"Determination of Sulphur in Petroleum Oils," by R. C. Griffin.

High-Voltage Research

Using the principle of charging capacitors in parallel and discharging them in series, very high impulse voltages have been generated for test purposes. A voltage of 5,000,000 has been produced in this way and even higher voltages have been applied to transmission lines. The phenomena of attenuation, reflection, etc., of traveling waves have been studied by means of the cathode ray oscillograph. This work is being continued on a larger scale. The cathode ray oscillograph was also used to record impulses on lines due to natural lightning. Only two records were obtained in this first attack on this problem, but a start has been made and more extensive data should be collected this summer. The cathode ray oscillograph was used for a laboratory study of a number of interesting and important phenomena, such as the impulse flashover of insulators and the distribution of transient voltages in transformers. The study of transients on transmission lines by means of the klydonograph has been continued. These instruments are useful in a general study, but the cathode ray oscillograph is now available and is being used for more definite and accurate recording of the shape and amplitude of transients with steep wave fronts. (*Contributed by K. B. McEachron.*)

The year's work at the Ryan Laboratory, Stanford University, California, resulted in three papers. One, by Lissman (A. I. E. E. Trans., Vol. 48, Jan. 1929, p. 146) analyzes lightning phenomena in the light of laboratory experience, and gives a detailed picture of what actually goes on in a lightning stroke. The second paper, by Carroll and Cozzens (A. I. E. E. Trans., Vol. 48, Jan. 1929, p. 1) reports results on sphere-gap sparkover voltages up to over one million to ground. It was found that the voltage per inch required to break down gaps in excess of 9 ft. decreased so rapidly with increased spacing as to establish an approximate economic limit of high voltage for power transmission. The final paper, by Dr. J. T. Lusignan (A. I. E. E. Trans., Vol. 48, Jan. 1929, p. 246), gives the probable history of an a-c. flashover, as obtained from oscillographic and spectroscopic evidence.

Dielectric Constant, Molecular Structure, and Molecular Moment

During the past year considerable research has been carried out on the subject of molecular moment and its relation to dielectric constant and molecular structure. The work has chiefly centered around the Debye theory. In general, this theory is becoming more and more accepted, although a large number of the researchers find that the theory does not fit in with experimental behavior in a great many instances.

In a study of the variation of the dielectric constant of gases with temperature and pressure, Magdalene Forro finds in the *Z. Physik*, 47, 430 (1928) that the constants of the Debye equation, $(\epsilon - 1)/(\epsilon + 2)(T/d) = \alpha T + b$, change with temperature and pressure. For hydrogen, nitrogen, and air, α (which is the optical contribution to the dielectric constant) is of constant value, while b (which is the permanent dipole contribution) is zero. For carbon monoxide the value of b varies with pressure at constant temperature. The temperature coefficient of b for carbon monoxide indicates a dipole moment of 0.108×10^{-18} at 1 atmosphere to 0.147×10^{-18} at 6 atmospheres of pressure. The variation of the permanent dipole contribution to the dielectric constant can be expressed as either a

linear or as a quadratic function. For carbon dioxide, however, this permanent dipole effect varies with the density as a quadratic function, the dipole moment varying from 0.192×10^{-18} at 1 atmosphere to 0.217×10^{-18} at 6.53 atmospheres..

Study of the molecular moment and dielectric constant in organic and inorganic solutions has been a particularly favored field of investigation during 1928. In general, the work has been in agreement with the results obtained by G. Rock and S. Klosky published in the *Journal of Physical Chemistry*, 33, p. 143 (1929) who have found that the dielectric constant of solver sols is essentially that of the dispersion medium. The dielectric constant of solutions may, of course, vary with the concentration. A. Thiel and E. Horn in the *Z. Anorg. Allgem. Chem.* 176, p. 403 (1928) studied the effect of concentration on aqueous solutions of methyl orange, helianthin, and other ampholytes. In the case of methyl orange and the ortho or para amino-benzoic acids the dielectric constant-concentration curves show a continual drop in dielectric constant with increasing concentration. An aqueous solution of helianthin, however, shows a dielectric constant practically the same as water. With some materials these authors report that the dielectric constant drops with increasing concentration to a minimum value beyond which the dielectric constant increases with concentration to a value which is ultimately higher than that of the solvent itself. This agrees with previously published work on urea and other compounds investigated by Harrington (*Physical Review* 8, 581, 1916). The materials which show this increase in dielectric constant with increasing concentration reaching values higher than that of the solvent itself are usually amino derivatives. Future researchers could profitably be concerned with this peculiar characteristic of these compounds. It has, of course, been long recognized that following the dictates of the accepted theory high dielectric constant involves orientation of molecules in the field. N. Marinesco (*Comptes Rendus* 187, 718, 1928) again shows that when a liquid with permanent dipoles is solidified the dielectric constant falls sharply. His explanation is the usually accepted one that after solidification the orientation of dipoles can no longer follow the inversions of the field. The inability of dipoles to orient, however, can be brought about by other means than actual solidification. For instance, a dipole molecule in water solution may be fixed with ion condensation due to its strongly developed electrostatic sheath. Absorption by colloidal hydrophyls may produce the same effect. The water fixed by a colloid can be determined by a study of the dielectric constant change with increasing concentration of the solution.

Debye (*Z. f. Elektrochem.* 34, 450, 1928) has continued his investigation of electric moment. In this publication it is shown that polar molecules have a dipole moment whether or not the dissociation products are charged. The molecular polarization is made up of two parts, P_1, the orientation of the molecule as a whole, and P_0, changes due to the deformation of the electrical equilibrium in the molecule. P_0 and P_1 can be separated for study by observing the molecular polarization at different temperatures, since P_0 is independent of temperature and P_1 varies inversely as the temperature. P_0 and P_1 can also be determined by studies of molecular refraction.

Study of dielectric constant is primarily of physical interest because of its relation to molecular structure. The Debye theory is the tool generally used in this study. Considerable information has been obtained from such researches during the past year. W. O. Ostwald, for instance, in the *Kolloid Z.* 45, 56 (1928) summarizes the situation as follows:

1. Simple hydrocarbons show slight orientation polarization.
2. Halogen derivatives show marked dipole moments, especially the ortho and cis isomers. However, completely chlorinated methane (CCl_4) shows little polarization.
3. The alcohols have an electric moment increasing with the number of CH_2 groups.
4. The aldehydes and ketones are of high electric moment.
5. The ethers, primarily amines and nitrogen bases, possess moderate moment.
6. The nitriles, cyanides, and thiocyanates have high electric moment.
7. The fatty acids have an electric moment greater than 1.
8. The chlor-acetic acids have an electric moment decreasing with the number of chlorine atoms present.
9. The esters show an electric moment between 1.2 and 2.0, increasing in homologous series.
10. The mercaptans have marked electric moment. (Carbon disulphide, however, is of small moment.)
11. High electric moment is shown by the organic nitro compounds.

If the dielectric constant be plotted against the dipole moments a curve is obtained showing a maximum dielectric constant for intermediate values of electric moment. Liquids with a density of 0.8 appear to occupy exceptional positions. In the case of organosols all substances whose additive products do not tend to coagulate have high dipole moments and high dielectric constants. The strongest coagulants have weak or zero dipole moments. Such materials are benzene, carbon tetrachloride, and hexane. The halogen isomers of ethane and benzene have been studied by numerous physicists. A. Errera has continued his investigations started some years ago, on di-chlor substitution products. In previous publications he has already given strong proof of the validity of the Debye theory of permanent dipoles, from work on the di-halogen derivatives of ethane and benzene, by showing that if the charge on the various atoms substituted in a molecule

is known the positions which are occupied on substitution in the parent molecule can be determined.

Van de Walle and Henne have established the position of the atoms in the sereo isomers of chloroiodoethylene. From his researches, Errera concludes that the iodine atom in the chloroiodoethylene is tervalent and positively charged with respect to chlorine. The nature of the charge is based on the conclusion that if the iodine atom is positive with respect to chlorine then the dipole moment of the cis derivative in which both halogens are nearest must be smaller than the dipole moment of the trans derivative. Errera finds that the dipole moment of the trans isomer is at least twice that of the cis.

An interesting study of molecular structure from the standpoint of dielectric constant and molecular moment has been reported by J. W. Williams and Arnold Weissberger (*J. A. C.* 50, 2332, 1928). These authors have investigated the molecular structure of diphenyl derivatives and from a study of molecular moment at 25 deg. cent. have found that the di-chlor and dinitro para products have an extended molecular structure. The di-amino (para), however, has a closed up formula. This conclusion agrees with previous researches and is of considerable importance in explaining the properties already recognized for the 4-4′ di-amino diphenyl.

Researches for the past five years have indicated that the dielectric constant of liquids varies with pressure to an extent which is more pronounced the higher the dielectric moment of the molecule under investigation. L. Cagniard (*Ann. de Physique* 9, 460, 1928) continues his previous studies and observes that the dielectric constant of carbon tetrachloride, hexane, and cyclohexane, all of which have low molecular moment, is a linear function of the pressure. Study of the variation of the coefficient of $d\,\epsilon/d\,p$ with temperature from 20 to 30 deg. shows a change for the dipole gases less than that deduced from the Debye theory. With non-polar gases, however, the change is greater than that deduced from the Debye theory.

According to the Debye theory, the dielectric constant should decrease with increasing field strength. This in many researches has not been observed to the extent indicated by the theory. J. Malsch, however, observes that the effect of voltage is greater with the smaller dielectric constant materials (*Physik. Z.* 29, 770, 1928). The absolute value of the effect of voltage at approximately 250,000 volts per centimeter is indicated as follows:

$$d\,\epsilon/\epsilon = 1 \text{ per cent for nitrobenzene}$$
$$= 0.7 \text{ per cent for water}$$
$$= 1.5 \text{ per cent for ethyl alcohol}$$

F. Kautzsch (*Physik. Z.* 29, 105, 1928) has investigated the dielectric constant of chloroform and ethyl ether under varying field strengths. He has found that the dielectric constant of these materials increases with increasing electric stress but with the higher fields applied the dielectric constant is smaller than that predicted by the Debye theory. Calculations of molecular moment from his researches show that ethyl ether has a value of 1.207×10^{-18}, chloroform 1.026×10^{-18}, and chloro benzene 0.645×10^{-18}.

P. Boning is publishing a series of articles in the *Zeits. f. Tech. Phys.* 9, 212 (1928) describing the characteristics of dielectrics. In the article referred to he points out that the conduction in solid and liquid insulators appears to be electrolytic in character. There occurs an absorption of ions at the bounding surfaces in heterogeneous materials. Conduction in dielectrics appears to give rise to a system of canals filled with electrolyte. The variation in dielectric constant with voltage, usually observed in technical insulation, is credited by Boning to the absorption of ions at the bounding surfaces of these canals. These articles by Boning are being continued and show considerable merit. (*Contributed by F. M. Clark.*)

Ferromagnetism

Heisenberg has been partially successful in explaining the origin of ferromagnetism on the basis of the spinning electron and the Fermi-Dirac statistical theory. His theoretical deduction is that a material cannot be ferromagnetic unless it is composed of atoms which have 3-quantum electrons and which are symmetrically surrounded by at least eight neighbors.

Honda suggests that the electrons in the nucleus are responsible for the origin of ferromagnetism. His picture avoids the difficulty of the Larmor precession of extra-nuclear electron orbits.

Weiss and Forrer have redetermined the va'ue of the Weiss magneton in iron and nickel, and reviewed the literature to show that the magneton is the unit in all magnetic materials.

The increase in specific heat, due to loss of magnetism near the magnetic transformation temperature, has been measured by Bates for manganese arsenide and compared with the values calculated according to Weiss from the magnetization-temperature curve, with good agreement.

Numerous papers describe experimental and theoretical work on the properties of single crystals of ferromagnetic materials. Honda, Masumoto, and Kaya, and Dussler, report the effect of temperature on the magnetization of single iron crystals. The magnetization curves do not differ radically from those for ordinary iron. In general, single crystals have somewhat higher permeability, and lower coercive force and hysteresis loss, and the properties depend to some extent on crystallographic direction. Kaya has found that the change of electrical resistance with magnetization is different in the various crystallographic directions in a single crystal of nickel. Mahajani in an elaborate calculation has explained the deviation between the direction of the magnetic field and the direction of the

magnetization in single crystals of iron. His calculation is based merely on the magnetic effects of the atom-magnets on each other. Akulov has similarly calculated the magnetostriction of single iron crystals as dependent on crystallographic direction. Peddie has investigated theoretically the effect of temperature.

Elmen has reported the remarkable magnetic properties which he has found in some alloys of iron, cobalt, and nickel. These alloys, named "perminvars," when properly heat treated have permeabilities which are unusually constant up to fields of several gausses. They have therefore very small hysteresis losses for low flux densities.

Yensen has carefully decarburized and deoxidized iron and observed a maximum permeability of 61,000, considerably higher than any reported by others. An initial permeability of 1150 and hysteresis loss of 300 ergs/cm.^{3}/cycle for B_{max} = 10,000 were also reported.

The magnetic characteristics of iron-nickel and other alloys have been investigated anew by Gumlich, Steinhaus, and Kussmann. The magnetic properties of cobalt have also been redetermined by Samuel.

Measurements of the permeability and magnetostriction of various alloys containing iron, cobalt, and nickel have been made by Schulze.

The change in magnetization which occurs discontinuously (Barkhausen Effect) on the steep part of the hysteresis loop, has been found by Bozorth to be the whole change in magnetization, and not a small change superposed on a relatively large continuous change. Measurements of the Barkhausen effect have also been made by Pfaffenberger.

The distribution of heat emission at various points on the hysteresis loop has been determined by Adelsberger and by Constant.

The time lag in magnetization which is frequently ascribed to magnetic viscosity has been shown by Bozorth to be due to the decay of eddy-currents, which takes place at a calculable rate.

Newmann shows experimentally and theoretically how the eddy-current losses in several materials depend on the shape of the hysteresis loop at frequencies up to 2500 cycles per second.

A theory of the aging of Heusler alloys has been proposed by Kussmann and Scharnow, and criticized by Auwers and Weinnoldt. A contribution to the theory of these alloys has also been made by Persson on the basis of his X-ray analysis.

A book entitled "Magnetic Properties of Matter" has recently been published (in English) by Kotaro Honda of Tohoku Imperial University, Sendai, Japan. (*Contributed by O. E. Buckley.*)

Experimental Investigation of Thermal Relations of Energy of Magnetization

A graduate student at Columbia University, Mr. Walter B. Ellwood, working on hysteresis in iron, has succeeded in his attempt at a solution of an outstanding problem in magnetism which has hitherto defied attack. The following note, which is a first report on this work, was received from Professor A. P. Wills, under whose direction the investigation was carried on.

Experiments were undertaken for the purpose of determining the mechanism of the degradation of energy which accompanies magnetization in ferromagnetic substances. The experimental method consists in observing the change in temperature of a test specimen produced by a change in the magnetizing force at consecutive intervals in a single cycle of magnetization.

The test specimen is in the form of 106 bars of soft steel drill rod one mm. in diameter. The bars are so mounted as to form 8 coaxial concentric cylinders, and the lengths of the cylinders are so determined as to give the aggregate the form of an ellipsoid of revolution whose minor and major axes are 3.4 cm. and 60 cm. respectively. One hundred and six copper bars of the same dimensions alternate with the steel bars in the structure; 106 thermocouples are constructed by connecting adjacent copper and steel bars alternately with 3 mm. lengths of No. 40 copper wire and No. 34 constantan wire. A coil around the center of the ellipsoid permits the evaluation of the total magnetic flux in the specimen. The entire specimen is embedded in rice powder and placed in an evacuated, silvered glass tube. Adequate thermal insulation isolates the latter from the magnetizing solenoid in which it is placed. The stability of the entire electrical and thermal system is indicated by a zero shift of only 2 mm. per hour on a scale 6 meters from the thermocouple galvanometer. A measure of the uniformity of the magnetizing field in the ellipsoid is obtained by connecting half the thermocouples (associated with the inner bars) in opposition to the other half. Under these circumstances a reversal of the full magnetizing field which produces a rise in temperature of the steel corresponding to a galvanometer deflection of 220 mm., yields a deflection of only 4 mm.

The results of the investigation were represented by curves, the intensity of magnetization and the temperature being plotted against true magnetizing field. One of the curves is the upper part of the usual "hysteresis loop" for the steel. A portion of it indicates the loop obtained when the impressed field is reduced to zero and then restored to its former value. Another curve shows the total change in the temperature of the steel at every stage in the process of demagnetization and reversal of magnetization. A third curve shows the total change in temperature corresponding to the process indicated by the hysteresis curve.

The not inconsiderable cooling of the steel in the neighborhood of zero magnetizing field, as well as the continued cooling accompanying remagnetization on the upper part of the hysteresis loop, are notable features of this record.

Propagation of Electromagnetic Waves in Space

The subject of wave propagation has been studied theoretically and experimentally and the existence of the Kennelly-Heaviside layer can be regarded as a fact. The resultant signal received at any point is the vector sum of the ground wave and the indirect wave coming down from the ionized layer. The waves radiated from a normal aerial have their electric vectors vertical to the surface of the ground, and almost so at the receiving end. This means in the optical language that the received ground wave is horizontally polarized. The downcoming wave is generally elliptically polarized and the resultant field of both waves is usually complex. This gives rise to bearing errors in direction finding and causes the well known fading.

Hollingworth finds that downcoming waves which are longer than 10,000 meters are usually plane polarized. The direction of the plane is approximately constant during the day but during the sunset and sunrise periods it varies in a manner depending on the distance between the sender and the receiver. At some distances the plane is turned clockwise and at other distances counter-clockwise.

For waves of 400 meters at distances of 100 miles the downcoming components are circularly polarized. This means that both the electric and magnetic vectors have constant amplitude but rotate synchronously, the direction of rotation depending on the direction of the earth's magnetic field. This would mean a right handed rotation in all parts of the Southern hemisphere.

For 15 and 50 meter waves, T. L. Eckersley finds the downcoming waves almost plane polarized, the plane rotating slowly with the period of a few seconds. This should produce fading when a vertical or a horizontal aerial is used for receiving. But fading seems mostly due to a change in intensity of the downcoming waves rather than to a change of phase with respect to the ground wave.

For waves of 400 meters length and the distance of 80 mi., the height of the layer changes during the night from 90 km. at sunset to about 130 km. at one hour before sunrise. But on occasions, during the winter months, heights as high as 250 to 350 km. have been found. (It is understood, of course, that all these heights are only apparent values and higher than actual heights of the ionized layer.)

Measurements using the method of interference fringes show that two sets of fringes may often be obtained, one corresponding to about 100 km. and the other to about 250 km. of height. Hence there must be two regions in the upper atmosphere where the ionization density tends to become a maximum after several hours of darkness. The lower maximum can transmit an incident wave downward to the earth. During the middle of the day a third layer less than 100 km. can exist. It has little reflecting power but it causes attenuation. This layer is probably the same layer as originally suggested by Lindemann and Dobson.

Examining the theory it seems strange that a radio engineer should be so well acquainted with the physical behavior of the ionized region whereas the pure physicists with the exception of a few workers in the field, are hardly aware of the powerful tool available for studying the conditions in the upper atmosphere by means of radio waves. All the theoretical workers in the field *assume* certain ionic or electronic concentrations in certain portions of this layer in order to confirm certain radio measurements. Would it not be much better if instead of assuming such ionic densities, the pure physicist would take some interest in this phase of work and *predict* from radio measurements what we actually have up there?

Long interval echo signals (duration up to several seconds) have been explained by rays passing through the Heaviside layer into huge empty pockets surrounded by an electron shell and reflected from the shell back to the earth.

So far as thermionic tubes are concerned, the Schottky-Hull (A. W.) screen grid tube has been developed by several manufacturers in this country and abroad with decided success, so that it is possible today to produce stable radio frequency amplification of a very high order and down to very short waves. In Europe also three-grid tubes (Pentodes) have come on the market in order to have tubes of low impedance with still a high amplification factor. (*Contributed by August Hund.*)

Physics of Radio Communication

In Europe much attention has been given to the effect of trees, buildings, built up areas, etc., on the attenuation of radio waves. Examples of such studies are those of Barfield (*Inst. of Elec. Eng. Jl.*, 1928 and 1929).

Among the important investigations of short wave propagation are those of Heising (*I. R. E.* 16, 75, Jan., 1928), Taylor and Young (*I. R. E.* 16, 561, May, 1928), Mesny (*L'Onde Elec.* 7, 130, April, 1928), Meissner and Rothe (*I. R. E.* 17, 35, Jan., 1929) and others.

Our present knowledge of static is summarized in a paper by Watson Watt (E. W. and W. E. p. 629, Nov., 1928). Carson has shown that it is impossible to eliminate static disturbances in receivers by audio frequency balancing schemes (*I. R. E.* 16, p. 966, 1928).

Advances have been made in the production of oscillations having wavelengths of less than 1 meter. Important publications on this subject are by Yagi (*I. R. E.* p. 715, June, 1928), Hollmann (*I. R. E.* p. 229, Feb., 1929), and Okabe (*Journ. Inst. of E. E. Japan*, March, 1928, p. 284).

An excellent summary of the status of direction finding was recently published in the *I. R. E.* (p. 425 March, 1929) by Smith-Rose.

The Bureau of Standards has developed a successful aircraft radio beacon (*I. R. E.* 16, p. 890. July, 1928). An extensive bibliography on aircraft radio has been published by Jolliffe and Zandoninini (*I. R. E.* 16, p. 985, July, 1928).

In the field of vacuum tubes, Hull has developed a high-voltage two-element gaseous rectifier, and a three-element gas tube, both of remarkable characteristics, (A. I. E. E. Quarterly TRANS., Vol. 47, July 1928, p. 753). Four-element double grid commercial tubes have been developed, and five-element tubes are now in the experimental stage.

The problem of frequency stabilization continues to be a vital problem in radio. Mechanical resonators, such as quartz crystals, regenerative tuning forks, and magnetostriction oscillators operated under carefully controlled conditions, are the best solution yet found to this problem. An extensive bibliography on piezoelectricity has been prepared by Cady (*I. R. E.* 16, April, 1928, p. 521).

The most remarkable discovery that has been made recently in short wave propagation is the discovery by Stormer and Hals of echo signals with time delays as great as 15 seconds after reception of the original signals (*Nature*, Vol. 122, Nov. 3, 1928, p. 681). Similar echoes have also been observed by van der Pol (*Nature*, Vol. 122, Dec. 8, 1928, p. 878.) These Stormer-Hals echoes have aroused great interest, and several explanations have been advanced concerning their origin.

Attention is called to the abstracts appearing each month in the English magazine *Experimental Wireless and Wireless Engineer*. These cover all the important articles on radio communication, and report the most important ones in much more detail than does *Science Abstracts*. (*Contributed by F. E. Terman.*)

SUBCOMMITTEE ON REACTIVE POWER

In October 1928 the A. I. E. E. published an English edition of the "Questionnaire on the Problem of Reactive Power," prepared by the National Roumanian Institute for the Study of the Development and Utilization of Sources of Energy, for the Advisory Committee for the Improvement of Power Factor of the International Conference on Large High-Voltage Systems.

This is a 21-page 9 by 12-inch pamphlet with an introduction by Dr. C. O. Mailloux. The questionnaire treats in great detail of the theoretical and practical aspects of the subject, with an extensive list of references appended.

The questionnaire has been referred by the Standards Committee of the Institute to the Committee on Electrophysics for an opinion. Since most of the members of this committee are following entirely different lines of work, it was deemed advisable to form a special subcommittee. Dean O. J. Ferguson of the University of Nebraska kindly consented to serve as Chairman of the Subcommittee.

The subject of reactive power has several ramifications, such as physical and mathematical relations in circuits, the technique of measurement, and elements of cost of power, leading to the subject of rates. The very fact that the Standards Committee referred the foregoing questionnaire to the Committee on Electrophysics indicated that an opinion was desired only on matters of physical facts and their mathematical formulation, and not on the purely engineering and commercial aspects of the situation.

Of course, the subcommittee may find certain physical relationships not only in the circuit aspects and measurements, but in the broad question of power costs as well, and there is no reason why the latter aspect should not be considered in its general terms, without the numerical data. As the work of the subcommittee develops, its chairman will probably find it expedient to cooperate with the Committee on Instruments and Measurements, and possibly with one or two other technical committees.

Statement by O. J. Ferguson, Chairman of the Subcommittee on Reactive Power. The Subcommittee on Reactive Power recognizes the importance of the problem presented by the Roumanian questionnaire. The query is a troublesome one, recurring frequently. It has received the attention of the Institute at various times, as attested by groups of papers and discussions to be found in our publications.

Perhaps the most influential item interfering with arrival at conclusive findings and recommendations is the lack of definite authoritative information as to what are the practical requirements, where we may set limits to a strict adherence to theoretical bases, and where we may recognize empirical approximations.

The answer as to this detail must come from the workers in the various fields. It will differ for commercial power applications from what will be deemed necessary in testing laboratories, or in communication circuits.

Nominally, the task is recognized as an attack upon the following combination of theoretical and practical points:

I. Acceptable definitions for reactive power under each of the following conditions:
   1. Single-phase circuits, non-sinusoidal waves.
   2. Polyphase circuits.
      A. Sinusoidal waves, unbalanced circuit.
      B. Non-sinusoidal waves.
         a. Balanced circuit.
         b. Unbalanced circuit.

II. Establishment of practical methods for the measurement of reactive power in each of the above cases.

III. Analysis of present ractive-power clauses in contacts, and establishment of a practical and fair basis for rate-making.

In approaching this problem, we recognize, as stated above, that we can limit ourselves to theoretically correct bases only in so far as these are capable of giving

practicable methods, *i. e.*, comfortable practise, without too great complexity or abstruseness. Rather than wander into involved conditions, we may have to make empirical and arbitrary agreements which will meet the further needs of practise. Rate-research committees, public service commissions, manufacturers, operators, consumers, laboratory investigators, research men,—all of these must be satisfied that our proposals are not inimical to their interests, but are usable with a fair degree of justice, comfort, and simplicity and that the range of usefulness is recognized.

To whatever extent it is possible, new terms and multiplicity of methods must be avoided.

In presenting the above outline, no statement is made as to the province of this committee or that of any other Institute committee. We realize that the study is so far reaching that elements of it will be found to be the peculiar property of certain regular committees. We hope to cooperate with these committees in advancing the solution, without arrogating to ourselves duties which properly are being taken care of elsewhere.

Some Active Topics in Physics

One of the purposes of the committee being to act as a connecting link between the Institute and the field of pure physics, the following list of topics will be of interest. It represents some of the subjects on which physicists are working and on which papers are being regularly presented and articles written by the leading investigators. It will be noticed in particular that the old division into mechanics, heat, light, electricity, and sound, has been more or less obliterated, the electron theory and the quantum mechanics being the dominant foundation of most of the researches, except in acoustics.

Scattering of electrons by gases, liquids, and solids.

Diffraction of electrons by thin films and by crystals.

Absorption of electrons by vapors.

Effect of electrostatic and magnetic fields upon electrons, ions, and neutral molecules.

Spectra of various substances under different conditions, as bearing upon the nature of their atomic and molecular structure.

Energy distribution, attenuation, and absorption of radiation at different wavelengths.

Photoelectricity and photoelectric cells.

Effect of electric and magnetic fields upon spectra, and correlation of these effects with the atomic and molecular structure.

X-rays; their wavelength, electron levels, diffraction, fine structure, polarization, ejection of photoelectrons.

Characteristics of sparks and other discharges, including their spectra.

Thermionic emission and properties of triode valves.

Compton effect, Raman effect.

Behavior of electrons like waves and behavior of light quanta like particles; bridging the gap between matter, electricity, and energy.

Nature of light waves and of electrons.

Conduction in metals from the point of view of the new quantum mechanics.

Nature of magnetism.

Magnetic properties of crystals.

Experimental and theoretical study of atmospheric electricity and terrestrial magnetism, in terms of moving ions and of radiations.

Structure of the universe in terms of astronomical and spectroscopic evidence, assisted by Einstein's generalized theory of relativity.

New statistical mechanics, wave mechanics, quantum relations, indetermination principle, etc.

Quantum mechanics of chemical reactions.

Atomic structure and chemical valency.

Note. Attention is also called to Dr. M. I. Pupin's Editorial on *1928 Progress in Physics*, in the A. I. E. E. Journal for February 1929, Vol. 48, p. 92.

Differentiation of Activities Among Physicists

Within the last twenty years, the Institute has seen several groups of specialists among electrical engineers form societies of their own, apparently feeling that the parent body was not able to devote to their particular activities as much time and space as was needed. As such special societies may be named those devoted to welding, electrochemistry, iron and steel industry, illumination, and radio.

It seems as if a similar differentiation is taking place in physics. The Optical Society of America was formed some years ago as an offshoot of the American Physical Society, and now the Acoustical Society of America is in the process of formation. In Germany they already have a separate prosperous Society for Applied Physics, with its own periodical. Considering the large number of physicists employed in the American industries, a formation of some such society is quite possible in this country unless the parent body anticipates such a move by a proper subdivision of its body into several "subject sections" and committees.

A General Trend in Physical Concepts

The present day period in physics may be said to have begun with the discovery of the electron in the closing years of the nineteenth century. This meant a new atomic concept of electricity. The introduction of the idea of quantum of energy, as the smallest amount of energy in a "chunk" of radiation of a given frequency, meant another new concept, namely, the atomicity of energy, at least in one of its forms. Einstein's relativity followed in 1905, and has brought in another new idea, namely that a physical "law" may be different depending upon the size of the continuum to which it is applied and the magnitude of the velocities which come into play. In the Bohr-Sommerfeld theory of the atom (1913-1915), the three new concepts, namely those of electron, quantum, and relativity, were brought together in a masterly way, to explain the observed complex structure of varius spectra. Later, similar ideas were extended to molecular spectra and

to the theory of specific heats, as being due to different forms of motion of the parts of a molecule relatively to one another.

Thus, an era of "picture theories" of the atomic structure was ushered in. Electrons were endowed with various forms of motion about a central nucleus, to explain this or that phenomenon; atoms were endowed with suitable forms of motion within a molecule, to explain some other phenomena, and when a particular feature of a spectrum could not be accounted for, new motions were invented to explain it (for example, the spinning electron). In other cases, the observed spectra did not show all the lines predicted by the theory, and certain "principles" or "postulates" were introduced to exclude such lines *a priori* as not being in accord with one of these principles.

However, the picture-theory period was only short-lived, partly because the pictures themselves became unduly complex, especially for heavier atoms, partly because the theory proved to be inadequate on some essential points, and partly because in the very nature of the phenomena statistical results were desired, rather than an actual orbit of this or that electron in a particular atom. But a statistical problem, where billions of atoms are to be considered simultaneously, and where only the probability of a certain event is sought, rather than its certainty for a given atom, hardly lends itself to a picture. Just as an extreme advocacy of graphical methods in a branch of engineering usually leads to a reaction in favor of an analytical method, so the picture-theory period yielded its place almost overnight to a "mathematical-function" period in the midst of which we are now laboring.

Several brilliant European mathematical physicists, such as Heisenberg, Born, Jordan, Pauli, de Broglie, Dirac, Fermi, and Schrödinger, have developed various aspects of this new point of view, and while it would be entirely out of place to go into the details here, the following few remarks may indicate the general trend.[1] Ideas are more general than objects or phenomena, and we are apt to come nearer the true interpretation of the universe by adhering as closely as possible to general ideas rather than by evolving specific pictures, especially if the latter have been borrowed from the big world of every day experience and transferred without change into the unknown intra-atomic world. But quantitative ideas are best expressed by mathematical formulas and operations, and so the problem is reduced to devising such formulas and operations and using them to describe and to predict phenomena, even though we may not be able to construct corresponding pictures within the atom. There is no contradiction in this because an individual atom is not yet susceptible to direct observation. The principal point to achieve is to make the mathematical theory agree with the observed phenomena in the minutest amounts of matter and energy with which an experimentalist deals.

To put this in a different form: Suppose we write two equations, such as $a\,x + b\,y = c$ and $m\,x + n\,y = p$. One could write a story about a merchant who bought x yards of cloth, etc., and make this story fit the equations. One could also write an entirely different story, say about two trains which start from two stations distant x miles from each other, etc., and again fit the equations. The number of such stories is infinite, whereas the two equations are the same, and presumably they stand for a much more general truth and relationship in the human mind than any of the stories.

It is not for the present writer to criticize either the picture or the formula tendency. Both are temporary, and a new genius may in a few years give us a new idea which will combine a keen mathematical analysis with a definite physical picture. We need a picture not only because our human nature demands it, but also because with the improvement in experimental methods smaller and smaller amounts of space, matter, and energy will be susceptible to direct observation, and will cease to be mere statistical elements. In the meanwhile, the statistical method of quantum mechanics reigns supreme, new achievements for it are recorded at every meeting of physicists and in every issue of physical periodicals. One who fails to learn the new mathematical methods will soon be unable to follow the progress even of experimental physics, because a large portion of experiments are nowadays performed either to test the new theories, or to furnish the needed numerical data for them.

As Haas correctly points out,[2] should the new point of view persist for any length of time, it cannot fail to affect our whole attitude towards physical laws and interpretation of nature. According to it, matter and radiant energy are both atomic, and do not differ radically from each other. Under certain circumstances, electrons behave like waves and waves behave like chunks of matter. Energy is converted into matter, and vice versa, and this process is going on in the universe continually, creating new worlds and destroying the old. A physical law does not dominate an event but merely establishes probabilities for this or that percentage, just as a statistical law does not affect births or deaths in a species of living beings. In the big visual world we may speak of an instantaneous position of a material point and at the same time of its instantaneous velocity. In the sub-atomic world, according to Heisenberg's principle of indeterminateness, such a

1. A symposium on Quantum Mechanics was held under the joint auspices of the American Physical Society and American Mathematical Society in New York City on December 31, 1928. Most of the papers and discussions may be found in the April 1929 issue of the *Franklin Institute Journal*.

2. Arthur Haas, "Materiewellen und Quantenmechanik," *Akademische Verlags-gesellschaft*, 1929, Chapter 19. See also V. Karapetoff, "Picture Theories Inadequate," an article to be published in the *Electrical World* during 1929.

statement would be absurd. If the coordinates of an electron are given we can only ask about the probability of a certain velocity, and vice versa. In the big world of interstellar spaces other laws reign supreme, Eucledian geometry and ordinary mechanics hold no more, and what we observe in our every day life (or even in our best equipped laboratories) is but a very narrow aspect of the real laws of nature. We do not see the bigger relativity aspects of the fundamental laws of nature, because our distances are too small and our velocities too low. We do not see the fine-structure aspects of the same laws because of the crudity of our apparatus which permits our observing the microcosm only statistically.

The new attitude is one of humility, and if it is an attempt to free the laws of nature from an anthropocentric point of view, future generations will welcome it as a step in the right direction.

VLADIMIR KARAPETOFF, *Chairman.*

General Power Applications

ANNUAL REPORT OF COMMITTEE ON GENERAL POWER APPLICATIONS*

To the Board of Directors:

Your Committee on General Power Applications has attempted to keep in close touch with the development of power applications during the year through the selection by the various members of the committee of specific industries in which they were deeply interested and on which they agreed to furnish a condensed report early in 1929.

It was hoped by this method that suggestions for papers and needs for new applications would be brought out and that interesting material would be obtained covering the development of power applications in outstanding industries.

Your committee believes that constructive results can be obtained by the continued application of this idea so that the annual report of the committee will not only represent a first hand account of the year's developments in numerous fields but will also develop the desirability for written discussion of outstanding features and bring to light the need for specific applications of power to processes which have not been modernized.

No attempt has been made to cover the entire industrial field so that there are undoubtedly many new applications which have not been brought to the attention of your committee. An effort was made, however, to cover the outstanding developments in basic industries which should be a true measure of the advance during the past year.

Marine Equipment

During the year 1928 there were placed in commission, or under construction, a total of 38 electrically propelled vessels of various sizes and types ranging from the small river towboat to the most modern of passenger liners. The aggregate shaft horsepower of the electrical equipment in these vessels was over 132,400, of which 88 per cent was supplied by turbines and 12 per cent by Diesel engines. However, of the 38 vessels electrified, 24 were of the Diesel electric drive.

Outstanding developments in the application of electricity to marine work during 1928 were the launching of the largest electrically propelled passenger liner, the breaking of all existing speed records for capital naval ships by the airplane carriers, *Saratoga* and *Lexington*, and the installation on three freighters of the biggest d-c. motors ever put on a ship.

Turbine Electric. The passenger liner *Virginia* is the second of the three electrically propelled passenger liners being built for the Panama-Pacific Line of the International Mercantile Marine Company. This vessel, somewhat larger than the *California* launched in 1927, has a displacement exceeding 30,000 tons at load draft and develops 16½ to 18 knots with a maximum power development of 17,000 shaft hp. The propeller shafts are each driven by a 8500-hp. reversible type motor.

The U. S. Navy airplane carriers *Saratoga* and *Lexington* underwent their speed trials and were put into active service during the past year. The U. S. S. *Saratoga*, with electric propelling equipment of 180,000 shaft-hp., maintained a speed of 33.42 knots over a measured mile. The *Lexington* on its acceptance trial last November developed 210,000 shaft-hp. and a speed of 34.62 knots. These tests broke all speed records for capital naval ships.

Five new Coast Guard cutters have turbine electric drive of unusual type. Each boat has a single turbo-generator set which supplies power to a single synchronous motor on the main drive. In addition, each boat has two dual drive sets, (steam turbine, alternator, and d-c. generator all coupled) which produce direct current for excitation and some auxiliaries. At normal ship speeds (40 to 60 cycles), the alternator, operating as a synchronous motor, drives the d-c. generator by power from the main generating set. When maneuvering, the frequency drops below 40 cycles, the turbine of the dual drive set then drives both electrical machines producing both direct and alternating current. The result is that as much auxiliary power as possible is supplied from the main generating set giving a high over-all efficiency in operation.

Diesel Electric. Outstanding in this field was the replacement in three 10,000-ton freighters, owned by the U. S. Shipping Board, of geared turbine drive by Diesel electric equipment. The motors, shown in Fig. 1, are rated at 4500 hp. and developed on trial momentarily 8000 hp. without distress. The slow speed propellor adopted for this drive increased the speed of these vessels from 10½ to more than 13 knots. This drive required less space and was lighter than the original equipment, besides increasing the efficiency of operation.

Four Diesel engine-driven d-c. generators, each rated 800 kw., 250 rev. per min., 385 volts, supply the single propelling motor which is direct connected to the shaft. Control is of variable voltage type, maximum power being obtained by connecting the armature of the four

*COMMITTEE ON GENERAL POWER APPLICATIONS:

J. F. Gaskill, Chairman.		
D. H. Braymer,	A. C. Lapier,	H. W. Price,
C. W. Drake	A. M. Lloyd,	H. L. Smith,
Harry L. Grant,	A. M. MacCutcheon,	A. H. Stebbins,
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E. W. Henderson,	E. B. Newill,	F. M. Weller,
George H. Jones,	K. A. Pauly,	M. R. Woodward,
P. C. Jones,	D. M. Petty,	W. C. Yates.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

main generators and the main motor in series. For reduced speed, one or more of the generators may be cut out of line.

The highest powered Diesel electric drive tugs in the world, the *Trinidad* and the *Chagres*, were placed in service by the Panama Canal Commission. These tugs have a cruising range of 7000 mi. and are equipped with complete fire fighting units. Due to the diversified demand on the services of these vessels, they are completely equipped with auxiliaries, all of which are electrically driven.

Three lightships, operated by the Bureau of Lighthouse Service, were provided with Diesel electric drive during the year. Economy of fuel and availability of power were the deciding features in these installations.

There was also evidenced during the year an increased tendency to equip various river crafts with Diesel electric drive. This tendency is especially marked

Fig. 1—4500-Hp., D-c. Motor for Propelling a 10,000-Ton Freighter

where the river to be navigated is swift, shallow, and tortuous, requiring perfect control while maneuvering.

Two new fleet submarines, the V5 and V6, the largest in the Navy, were commissioned during the year. These vessels are designed for extensive cruising and, therefore, involve many improvements over the old type of submersible craft. Particularly noteworthy are the controls, all manually operated, which permit 15 different combinations of diving and charging. Another point is the perfection of motor bearings, which permit diving at an angle of 30 deg.

It is of interest to note that foreign countries are awakening to the possibility of electric drive on ships. The *Viceroy of India*, a 19,000-ton turbine electric passenger liner, is the first large ship of its kind built in England. Two Diesel electric tugs are under construction in Italy, the first of this type to be built in that country. Germany has several cargo vessels under construction that will employ Diesel electric drive, full details of which have not been published.

Electric Railways

Although electric railways come under the jurisdiction of the Transportation Committee, there are several unusual applications of power in this field which it will be well to mention briefly.

To speed up the classification of freight and to eliminate as far as possible the hazardous occupation of car riding, several railroads have adopted electric retarders

Fig. 2—Electric Car Retarders in the Selkirk Yard of the New York Central

Fig. 3—Car Retarder Equipment in Waterproof Enclosing Case

in their hump yards. The apparatus required, shown in Figs. 2 and 3, consists of a motor and brake coupled to a gear mechanism adjacent to the retarder with remote control to enable operation from conveniently located towers in the freight yard.

The year also was marked by the number of oil electric locomotives and gas electric motor cars put in service by the railroads. Ranging in size from 60 to 150 tons and in power from 100 to 900 hp., oil electric locomotives were successfully used by both railroads and industrial plants.

The popularity of the gas-electric motor car is indicated by the total of 117 cars placed in service during the past year. These cars are rapidly replacing steam trains on the short branch line roads. Industry has also adopted gas-electric units for yard and factory work where overhead trolley wires are undesirable and the old steam driven shift engine is uneconomical.

Distillate fuel has been successfully used in place of gasoline by several companies operating gas-electric trains. This fuel has also been adopted by a large city transit company for use in its gas-electric busses.

The electrification of steam roads has presented the design engineers and manufacturers with many new problems of control. To protect the 11,000-volt single-phase trolleys used by the Pennsylvania Railroad in their new electrification program, from occasional

Fig. 4—High-Speed Circuit Breakers for P. R. R. Electrification

short circuits and grounds, new high-powered circuit breakers and relays of extraordinary speed were developed. Up to 50,000 amperes at 12,000 volts may be interrupted in a half cycle, 1/50 of a second. Another unusual feature in an oil breaker is the use of magnetic blowout coils to aid in extinguishing the arc. Fig. 4 shows one of these breakers.

In the street railway industry there has been a decided tendency toward lighter and faster motors, lighter car bodies, and improved methods of drive. Dynamic braking, supplemented by air, has come back after many years of discard. Control has been simplified and many new safety features perfected.

Steel Mill Industry

The total horsepower of main drive motors installed by steel companies during 1928 reached 212,300, with an average motor size of 1250 hp. Of this total approximately 80 per cent are d-c. machines. This proportion is largely due to the installation of several new continuous mills having individual drive for each stand of rolls.

An unusual installation of interest is the mill drive for the 36-in. reversing universal slabbing mill of the Wheeling Steel Corporation at Steubenville, Ohio. This is the first slabbing mill installed in this country with horizontal and vertical rolls driven by reversible motors. The horizontal rolls are driven by a single armature d-c. machine rated at 7000 hp. continuous, 50/100 rev. per min., 750 volts, and capable of developing a maximum torque of 2,000,000 lb-ft. The vertical rolls will be driven by a reversing motor rated at 1650 hp. continuous, 112.5/270 rev. per min., 750 volts and maximum torque of 212,000 lb-ft.

The control for this equipment is such that only two men are required in the pulpit. The speed and direction of the motors are controlled by means of a foot operated master switch. This switch so controls the two motors that the speed ratio between the two rolls will always be maintained without any necessity for mechanical gearing.

At this same plant there is a wide strip mill installation consisting of a-c. and d-c. motors aggregating more than 20,000 hp.

During the year there were several outstanding installations where steam engine drives were replaced by d-c. reversing motors. The largest d-c. motors in the world are now found driving the big reversing mills of the steel industry.

The Illinois Steel Company in its South Chicago Plant has installed a 7000-hp., 50/120-rev. per min., 750-volt, double unit reversing motor to drive a 40-in. blooming mill. Power for this motor is supplied by a fly-wheel motor generator set consisting of a 5000-hp. induction motor, a 50-ton fly-wheel, and two 3000-kw. generators. A 35-in. blooming mill will be driven by a single unit 6500-hp., 60/140-rev. per min. motor having a similar source of power supply. This plant is also installing a 3500-hp., 50/120-rev. per min., 750-volt d-c. reversing motor on a 32-in. structural roughing mill, and a 6000-hp., 75/140-rev. per min., 750-volt d-c. motor to drive a 28-in. structural finishing mill. A fly-wheel motor generator with a 7500-hp. induction motor, 50-ton fly-wheel, and three 3000-kw. generators will furnish power to these structural mill motors.

The Youngstown Sheet and Tube Company at Youngstown, Ohio, has made several replacements of engine drive by electric motors. A 5000-hp. induction motor will be utilized to drive an 18-in. continuous billet mill. A 2000-hp., 275/550-rev. per min., 600-volt d-c. machine replaced an engine on the finishing stand of a 10-in. skelp mill.

Strip mills are a distinct new development and are gradually replacing the old sheet mills for doing the same work. The new continuous hot strip mill of the

American Rolling Mill Company at Middletown, Ohio, is the highest powered of any strip mill yet built. The main rolls are driven by 21,800 hp. in motors; four 3000-hp., and three 2000-hp. d-c. motors on the finishing end, and 3800 hp. in induction motors on the roughing end. This mill purchases 66,000-volt power and has three 4000-kw. motor generator sets to supply direct current for the main roll motors.

The Sparrows Point, Maryland, plant of the Bethlehem Steel Company has made several additions and changes during the year. These changes are in keeping with the intention of the Bethlehem Steel Company to make this plant the greatest steel mill in the world. Fig. 5 gives a general view of the motor room at this plant.

Notable among the additions is the use of 8 synchronous motors totaling 11,750 hp. for main roll drives. This makes a total installation on main rolls of 45 synchronous motors, with an aggregate of 140,750 hp.

Fig. 5—Motor Room of a Continuous Skelp and Sheet-Bar Mill in which are Installed Drives of Three Distinct Types: D-c., Induction, and Synchronous Motors

A control has also been perfected for tying together the 6700-hp. induction motor and the d-c. generator supplying power to the 2600-hp. finishing stand motor. Another new application is applied to the Mesta rotary shears. Each shear is driven by two d-c. motors in series enabling the shears to be run at full speed when the steel traveling at 600 ft. a minute passes through the machine. The speed of the motors synchronized with the speed of the steel enables these shears to cut the steel without an inch variation in the lengths of the sheets.

Material Handling

During the past year industry as a whole was much benefited by the improvement and increase in the number of applications for handling materials electrically. The removal of the human element has in many cases resulted in astounding increases in plant efficiency, while in certain industries the development of proper equipment for handling odd shapes and sizes, generally conceded forever assigned to unskilled labor, has effected considerable saving.

The year was in general characterized by the application of mechanical handling to a wider variety of uses.

Equipment for handling freight cars was further developed during the year. Fig. 6 illustrates one of the newest type car dumpers, capable of dumping a car a minute. This is being constructed for the New York Central Railroad at Toledo, Ohio. The total electrical equipment aggregates 3000 hp. and consists of both a-c. and d-c. motors with special control by means of variable voltage.

Two other installations now under construction for loading ships, one at New Orleans and the other at Havana, Cuba, are of an unusual type. This equipment will load refrigerating cars on a ship especially

Fig. 6—High-Speed Electric Car Dumper in Operation. A New Dumper of this Type will Handle 60 Cars per Hr.

designed with tracks to handle about 95 cars. These cranes will lift 165 tons at the rate of 50 ft. per minute and use two 250-hp. totally enclosed mill type d-c. motors for drive. These cranes are the first ever designed for loading and unloading ships with standard freight cars.

Another example of speeding up production electrically is shown in Fig. 7. This 300-hp., 20-ton locomotive with its 14-ton ingot car conveys 300 tons hourly from the soaking pit to the mills. This distance averages 325 ft. and is covered in 20 sec. running time. Needless to say the braking equipment and "dead man

control" must be highly efficient to control safely the four-ton mass of red hot metal.

Prior to 1928 the largest electrical shovel was the 8-cu. yd. or 300-ton shovel. A number of 12-cu. yd, capacity shovels was placed in service during the past year. Due to the extreme fluctuation of current demand, it was necessary to design special equipment

Fig. 7—This High Powered Ingot "Buggy" of the Jones & Laughlin Steel Corp. Makes a Round Trip Every 48 Sec.

for the drive and control of these shovels. Three motor generator sets, with the generators all differentially wound, are capable of generating 750 kw. The four motors for the various drives aggregate 650 hp. One of these shovels (Fig. 8) recently established a new world's record for material handled when it moved 15,497 cu. yd. of overburden in 24 hr.

Fig. 8—12-Cu. Yd. Electric Shovel in Strip Mine of the Enos Cook Company, Oakland, Ind.

Mechanical handling of materials has received considerable increased attention during the past few years in the pulp mills. Whereas the old method of floating pulp sticks to the mill is still in use on account of the obvious advantage of this form of conveying, the mills themselves have adopted modern methods of wood handling. Electrically operated conveyors are operating successfully in several large pulp mills for handling the sticks from the river to the storage yard, and from the yard to the mill. One installation consists of a log stacker, capable of building stacks 60 ft. high, running parallel to a chain conveyor. Another type of installation uses overhead construction and is able to build stacks 100 ft. high with a capacity of 15 to 20 cords an hour. The equipment used must be of the sturdiest construction to withstand the heavy duty imposed by jammed sticks and blows from falling pieces. Storage of a large quantity of pulp wood is now possible, greatly eliminating seasonal operation in this industry.

A quite extensive application worthy of note is in the automobile industry. General Motors at three of their plants have made large additions and considerable advance in the preparation and handling of foundry sand. Another dirt conveyor, of interest because of its length, was constructed to carry the dirt produced by the lowering of Denny Hill, Seattle, Wash., to Puget Sound.

Fig. 9—Individual Motor Drives at the National Paper Products Mill, Port Townsend, Washington

The largest belt tripper ever built was constructed during the past year. It is designed for a 54-in. belt and weighs approximately 30,000 lb.

A coal conveyor 2½ mi. long was put into service by the H. C. Frick Co. for transporting coal from the mine to a dock. It is designed to deliver 1800 tons of coal an hour and is driven by 12 wound-rotor induction motors aggregating 2400 hp.

Numerous other improvements in material handling have been made during the year, especially in regard to the method of connecting the drive to the conveyor, as well as the use of anti-friction bearings, gear replacement of pulleys, and other similar refinements.

Paper Industry

The paper industry was long since converted to motor drive of the main rolls, but this year has seen the principle pushed still further. Cylinder machines have a larger number of small rolls, baby presses, and

cylinder molds, which heretofore have been driven by the felt carrying the paper pulp. The long, broad belt of high grade felt is decidedly expensive and the practise of using it as a drive necessitated frequent replacements. A new mill at Port Townsend, Washington, eliminated this illogical expense; small motors, Fig. 9, held in extraordinarily exact speed agreement, drive each element.

Carbon pile regulators, which have been successful in the control of single motor paper mills elsewhere, have been further developed to aid in the exact speed regulation of the Port Townsend machines.

Fig. 10—Six-Unit Eight Bearing Synchronous Motor-Generator Set Designed for Sectional Paper Machine Drive

The Brown Paper Mills Company of Monroe, La., has installed a unique sectional paper machine drive for the manufacture of kraft paper. This paper machine is to make the heaviest paper ever produced on a Fourdrinier machine. The widest range of speed ever attempted on a sectional electric drive, 100 to 1000 ft. per minute, necessitated radical departures from previous methods of operation. Each of the nine motors driving the individual sections of the machine receives its power from a separate generator which forms a part of two six-unit motor-generator sets (Fig. 10). Speed regulation is obtained by automatic control of the generator fields. This feature renders greater torque available in the motors and produces greater stability than would otherwise be possible. This equipment is the only multiple generator control system so far installed on a paper machine.

The manufacture of flat tissue on two small paper machines introduced difficulties in synchronizing the semi-sectional drives on the wet and dry ends. Speed adjustment was accomplished by modifying the synchronous dynamometer speed regulator so that the stator frame of the regulator motor was chain connected to the brush lifting device of one brush shifting motor and its rotor was driven from the same motor through a chain reduction. The stator of the regulator receives its power from a small alternator driven by the motor on the other end of the machine. This equipment is the first of its kind in the country.

A variable speed turbine with reduction gear was an important development in isolated power plant equipment for paper mills. The low speed end of the gear is connected through sheave pulleys to the paper mill and exhaust steam from the turbine is used in the can dryers. Speed can be held within very narrow limits between 65 rev. per min. and 390 rev. per min. on the driven shaft. This unit is rated at 475 hp. at 4200 rev. per min. and operates at 110 lb. gage steam pressure with back pressures varying from 20- to 40-lb. gage.

Cement Industry

Productive capacity far in excess of normal consumption has caused the cement industry to turn to electricity for economical operation. New plants are universally electrically driven while mechanical drives are rapidly being replaced in the older mills.

Isolated generating plants, utilizing the heat from cement kiln gases, are still popular, although during the year several contracts were signed with public utilities calling for an interchange of power. While this exchange provides an outlet for the excess power of these plants, it also places a premium on economics in power generation and mill operation that formerly did not exist.

During the year there were several novel and valuable applications of electricity to the problems confronting this industry.

Fig. 11—600 Hp. Simplex Synchronous Motors Installed at the Missouri Portland Cement Co.

Notable among such developments have been several specialized types of synchronous motors designed to provide the high torques necessary for heavy crushing and grinding machinery. New improvements in the clutch and brake of these motors and the application of complete automatic starting control have still further extended the use of the synchronous motor.

Six of these motors, ranging in size from 250 to 600 hp., were installed for tube mill and compartment mill drive in a midwestern plant. See Fig. 11. This is the

first application of the new salient pole synchronous motors with their low starting kv-a. and high torque characteristics. These motors are coupled directly to the mill pinion shaft without the use of a magnetic or mechanical clutch.

Another application of the synchronous motor is on hammer mills. A novel feature of this installation is the overload relay. When the overload is great enough to pull the motor out of step, the relay operates, cutting off the material being fed to the hammer while permitting the motor to regain speed running as a straight induction motor.

Fig. 12—The Largest Locomotive Underground 30 Ton, 532 Hp.

Push button control has become almost universal, even for motors of the smaller sizes. One large mill, completed this year, points with pride to the elimination of starting equipment other than push-button controlled, full automatic, circuit breakers of high interrupting capacity. Low starting kv-a. of the motors has made such installations possible; in some cases the motor starts directly across the line.

An important application of specialized control is the automatic blending of dry cement materials as developed in connection with the Fuller-Kinyon System. The Fuller-Kinyon pumps handle the pulverized rock mixture like water and a very ingenious control system for the pumps and valves has been designed. Electrical interlocks and time clock regulation permit almost unlimited combinations for blending the contents of bins or silos to produce a constant quality feed for the kilns.

To keep a uniform flow of material to their kilns, an eastern mill installed synchronous control similar to that used on paper machines. A d-c. generator connected directly to the kiln supplies power to the motor on the kiln feeder. By separately exciting the fields of these machines from a common source, exact speed regulation is obtained.

Mining

The coal mining industry endeavored during the past year to solve many of its troubles by the wider application of electricity and the adaptation of more adequate and more efficient machinery.

An outstanding feature of the year was the new mine locomotive, Fig. 12, built for the Pemberton Coal and Coke Company, Bluefield, W. Va. This locomotive, although weighing only 30 tons, develops 532 hp. and will haul a full load of 168 tons up a 1.5 per cent grade at 10½ mi. an hour. This represents more power than the average locomotive on interurban lines and more work than many a steam switcher. This locomotive is constructed of two parts in tandem which, if desired, may be run as two separate units. Electro-pneumatic control and air-brakes are features which are most unusual on this type of locomotive.

The Peabody Coal Company of Chicago has installed in one of its mines a hoist which although not of unusual size is still of interest because of the high rope speed and the number of trips made per hour. The equipment was designed to permit 230 trips an hour from a depth of 282 ft. In order to do this a rope speed of 3300 ft. per min. is required. The total elapsed time of the hoisting cycle is 15.65 sec.

The largest induction mine hoist motor with magnetic control in the country was installed by the Tennessee Coal, Iron and Railroad Company in their mines. The motor is an 1800-hp., 260-rev. per min., induction type hoist motor and a duplicate of two motors previously installed which use liquid rheostats. The magnetic control permits dynamic braking, thus reducing the wear on the mechanical brakes.

Fig. 13—Pioneer Installation of Fabricated Steel Frame Induction Motor for Mine Hoist Service

Fig. 13 shows the first fabricated steel frame induction motor ever installed for mine hoisting service.

Ore Reduction. The first commercial size plant in this country to utilize the Tainton process, *i. e.*, strong acid and high current density, for the electrolytic reduction of zinc, was installed by the Sullivan Mining Company at Kellogg, Idaho. Current is supplied by two three-unit 4000-kw. motor-generator sets.

The Anaconda Copper Mining Company during 1928 put in operation the largest motor-generator sets ever used in an electrolytic zinc plant. There are four three-unit sets, each rated at 5500 kw.

Oil Industry

As the oil industry passes the stage of adventuring with its picturesque engine and rig, and becomes more and more a leading industry with an economic problem of conservation, it is marked by the application of electricity to give the most results at the lowest cost. Well drilling has become an affair requiring large capital as several wells have already cost over a quarter million dollars and have reached depths of over 8000 ft. One well in West Texas has been drilled 8255 ft. or 1.56 mi. deep and is the deepest known hole every dug by man. Two 25/65-hp. oil well motors driving a cable tool rig, supply the power for the drilling which is still continuing.

Until very recently it was thought impossible to apply centrifugal pumps to oil pipe lines. A development of suitable high-speed centrifugal pumps which would not emulsify the oil enabled the economic use of electrical drive. A new pipe line 400 mi. long went into service during the past year using this equipment. Forty-four 400-hp., squirrel-cage, 1800-rev. per min., motors drive the pumps in the 13 stations along its length.

Electric Welding

Considerable progress has been made in electric welding during the past year. It has been said that electric welding is in a sense a production tool, and for this reason it has been receiving more and more attention from the progressive industrial and construction engineer in his effort to increase output and lower cost.

Only a few of the more important developments in electric welding can be mentioned in this short résumé. One of the most important advances has been the increased size of the electrode used, which in general reduces the time for welding in about the inverse proportion to the size of the electrode.

Another development of considerable importance is the introduction of the new atomic hydrogen method of electric welding, which makes possible the welding of thin metals and alloys that could not be welded before because of the oxidation of the materials by the heat of the arc. This new method introduces hydrogen gas into the arc flame, which gas is broken down into atomic hydrogen by the heat of the arc. This atomic hydrogen effectively prevents oxidation of the materials, so that a satisfactory joint can be obtained. This enables countless small objects to be welded which otherwise would require more expensive methods of union.

The importance of electric welding in the industrial field can be emphasized by the faith of the Ford organization in this type of welding. It is stated that the Ford Company has recently expended over $100,000,000 for equipment and methods to produce their new model "A" car, which depend primarily on electric welding. Here electric welding is reduced to an almost automatic process requiring very little personal supervision.

A new process was developed for producing electrically welded railroad ties from scrap rails, which is done entirely automatically and produces a very satisfactory tie at a low cost from scrap material.

One of the largest welding jobs ever undertaken was the welding of a 90-mi. pipe line on the Mokelumne River project, this pipe being 66-in. in diameter. This pipe was so large that it required two pieces in order to get the required diameter, the pieces being welded by the automatic carbon arc process. The cost of this job was $12,000,000 for electric welding against the best offer of $15,000,000 for riveted construction.

A 135-ft. through truss railroad bridge was completed at Chicopee Falls, Mass., with all members electrically

Fig. 14—Multiple Arc Welding Machine for Automatically Arc Welding Pipe up to 36 Ft. in Length and 7 Ft. in Diameter

welded. Only two-thirds of the material required for a riveted job was used, and the completed structure cost 20 per cent less than the riveted equivalent. The first commercial building completely electrically welded was finished in Cleveland during the year. This structure used approximately 15 per cent less steel than would be required for riveted construction at a cost considerably less. A factory building, Fig. 15, 145 ft. wide by 550 ft. long, and requiring 1000 tons of steel, was constructed by the electric arc welding process.

The increased popularity of electric welding resulted in the production of a Fordson-driven 300-ampere arc welder for use where there is no other source of electric power. The Fordson not only supplies the power for the welder, but also carries it to and from the work.

During the year a competition for papers on arc welding resulted in an excellent paper entitled "Arc Welding—Its Fundamentals and Economics" written by Mr. James W. Owen. This paper ably presented the present status and the future of this industry.

Industrial Heating

During 1928 the use of electricity for industrial heating has increased rapidly due to the development work of the manufacturers and the recognition of the central stations to the desirability and importance of the load.

Many central stations have established industrial heating departments for the survey and development of the electric heating load.

The manufacturers have developed new equipment during 1928 and the following descriptions cover some of the most important applications.

Electric Stereotyping. The first electric stereotype pot was built and installed in 1925 but it has been during the past year that most of the large installations were made. More than 100 are now in operation, varying in size from 40 kw. for small pots to 360 kw. for large 9-ton pots. The pots make use of an immersion element placed in the lead with automatic temperature control. The total load in many metropolitan plants runs from 800 to 1500 kw. connected load for this service. Indications point to the complete electrification of stereotype pots in the next few years.

Fig. 15—Framework of Factory Building Assembled by Electric Welding

Silk Moistening. An important application has occurred in the full fashioned hosiery manufacturing field for the moistening of silk before passing through the needles of the knitting machine. Units consisting of an insulated box containing water with felts for the silk to pass through and having electric heating units of 173 watts each are placed so that from 3 to 7 threads are moistened on the way to the needles.

The water is kept at about 160 deg. fahr. and the evaporation of water moistens the felt so that the silk picks up enough moisture to eliminate back winding.

Loads of 990 kw. connected in one plant have been noted and in the Philadelphia district approximately 2000 kw. is in use in this class of business.

Electric Brazing Furnaces. The application of electric furnaces to copper-brazing steel parts together has made possible the assembly in one operation, of complicated structures from simple parts by a strong alloy weld. This is accomplished by placing copper wire or chips next to joints to be brazed and then heating the assembly in a furnace containing a hydrogen atmosphere to a temperature above the melting point of copper.

The hydrogen performs the function of a flux, as well as excluding air from the heating chamber, in the process of brazing.

Both intermittent and continuous type furnaces are in use for this work and have extensive application in the assembly of refrigerator evaporation shells.

The use of protective gases in other type electric furnaces is recognized as metallurgically beneficial in the heat treatment of various steels and in cases where bright annealing is essential.

High-Frequency Furnaces. Recently the application of the high-frequency induction type furnace to melting and heat treating steel has been given greater consideration. Furnaces are in use for making cast steel in ingot form for production work and for heat treating high-speed steel in liquid baths.

Gray Iron Melting. The application of the indirect arc type electric furnace to the production of synthetic cast iron is a comparatively recent development. Gray iron borings from the manufacturer's machine shop, having a very small market value, are utilized in the production of iron of good physical properties, and the over-all melting cost per ton is less than that for cupola operation. This has opened a new field for development in the iron industry.

Control

During 1928 improvements in design and applications of control have been in keeping with the general trend of the past several years. Although a great many individual instances could be cited, a few general examples will be sufficient to indicate the general trend of the various types of control equipment.

Most of the motor control is now of the definite time accelerating type. A time-current control has developed for d-c. equipment which provides maximum time saving in motor acceleration within predetermined current limits under conditions of light or normal loads and forced acceleration at a predetermined rate where the load is too heavy for the motor to start with the normal accelerating current setting. This equipment is characterized by its simple and rugged construction, the accelerating relays have only one moving part which carries coin silver control contacts while there is only one series coil. This eliminates complicated control circuits and materially reduces maintenance.

Another mechanism for securing definite-time acceleration was developed from the original single contact equipment with a timing ratchet, into a multi-finger form which permits the use of a single relay for actuating in succession a number of contactors.

The conventional form of master switches using segments and fingers has been replaced in the last few

years by the cam operated switches. The past year produced further development of the cam operation, special attention being paid to simplicity of design. An outstanding example of this is a small compact device which was designed to fulfill three purposes, controller, disconnect switch, and overload relay. A complete line of master switches was also produced which required only a few different fundamental cams to obtain a great variety of combination of contact sequence. Still another indication of the trend towards simplified design is in the compact limit switch developed for use on motor driven valve equipment. By means of intermittent gears it is possible to get an adjustment of from 1 to 1365 turns on the valve stem.

Circuit breakers have apparently been accepted by industry to replace fuses wherever there is need of protection. Contrasted to the high powered circuit breaker capable of interrupting 50,000 amperes at 12,000 volts, mentioned previously in this report, the new 110-volt circuit breaker that can be held in the palm of the hand is a mere toy. Yet this breaker has in actual test broken 3500 amperes at line voltage without injury. The smallest units made are for household use, 600 watts, and represent a complete coverage of the electrical field by this equipment.

The Deion circuit breaker was further developed during the year for applications up to 15,000 volts.

Control of motor speed has also received considerable attention during the year. The speed regulator originally designed for control in paper mills has been modified to meet the specific speed problems of a wide variety of industries. In the preceding sections of this report, these applications have been more fully discussed.

Push button control, incorporating simplicity and ruggedness of design without sacrificing dependability of service, is rapidly replacing the cumbersome and complicated control systems of a few years back.

Miscellaneous Applications

Rubber Industry. The large expansion of this industry in the past few years has caused engineers to devote their attention to new plants and extensions to present mills rather than the development of new applications. The slow speed synchronous motor has been the predominant type in new installations. Full voltage starting, together with dynamic braking, has been almost universally adopted. All new installations have the thermal, and particularly the thermostatic, type of overload protection. With the wider application of the individual drives more attention has been paid to the type of bearing. The sleeve bearing, with its high starting friction, has been largely replaced by the ball and roller bearings.

Power Applications in Telephone Industry. Power applications in the telephone industry are naturally limited. The drives that are used generally make only small power demands and are commonly rather special in their nature. Special motors are required for television and sound pictures, with very ingenious speed regulation, but are of too narrow scope for discussion in this report. The development of large capacity filters to eliminate noise producing rippies in the generating current has permitted the application of standard generators for telephone battery charging.

Small Motors. Improvement in design of small motors was apparent during the year. Single-phase motors received a great deal of attention, and the improvements made will be of considerable advantage to various industries.

Single-phase repulsion-induction motors, ranging in size from ¾ to 2 hp. at 1800 rev. per min., were produced with a high starting torque and constant speed. Better magnetic utilization of the steel was accomplished in these motors by using square stator punchings with graded slots. Although practically enclosed, improvements in design are such that continuous operation causes only a 40 deg. cent. rise in temperature.

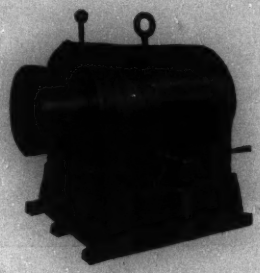

Fig. 16—Brush-Shifting Shunt Characteristics Polyphase Motor Designed for Variable Speed Spinning Frame Drive

Single-phase condenser motors having practically the same efficiency as a two-phase motor and operating at or near 100 per cent power factor were designed. The motor develops even more torque than the two-phase motor, whereas the current required is considerably less.

Recent tests in the textile industry have indicated the desirability of applying the alternating current, brush shifting, shunt characteristic, polyphase motor for obtaining variable speed in the spinning of cotton as well as worsted yarns. The use of this motor, Fig. 16, which permits the best speed over a 3 to 1 range, has resulted in improvement in the quality of the yarn and increased production.

Fractional horsepower motors of new design for use on portable electric hammers combine very light weight with the ability to withstand severe vibration and shock. A full load speed of 10,000 rev. per min. gives 2500 blows of the hammer each minute. This drill is shown in Fig. 17.

Electric Elevators. The development of high-speed, full, and semi-automatic elevators was furthered during the year. Deficiencies affecting the riding qualities of elevators in the automatic control of the rates of acceleration and retardation were removed by the application of a small auxiliary d-c. machine to act upon the

Fig. 17—This Electric Drill Gives 2500 Blows Each Minute

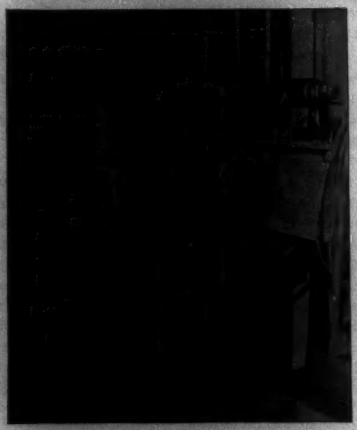

Fig. 18—Pliotron Units and Tubes Mounted on Top of Elevator for Use with Pre-registering Elevator Control

generator field of the motor generator set of the generator-voltage type of elevator drive. By this means the acceleration and retardation are caused to begin at very moderate rates, increase to higher rates, and again diminish as the speed change nears completion. The control is shown in Fig. 18.

An automatic levelling system was developed which embodies a radical departure from previous practise. This system utilizes radio-frequency devices wherein the position of the car with respect to the floor controls the levelling speed of the elevator accordingly. The particular advantage of this control is the elimination of all mechanical engagement with its noise and ultimate deterioration, a minimum clearance of ⅜ in. existing between the parts on the car and the corresponding part in the hatchway.

The most powerful and speediest direct traction motors for elevator service were installed during the year. These motors are rated at 110 hp., with an elevator speed of 450 ft. per minute. The loading is particularly heavy, 10,000 lb., and a precision in floor landing is required in the control which permits a maximum variation of but a quarter inch from exact levelling. This installation is in the new skyscraping Kent Garage of New York. Fig. 19 shows one of these motors.

Fig. 19—The Largest Direct Traction Elevator Motor Ever Made—110 Hp.

D-C. Generators. Although not of particular interest in the power field, the new 10,000-volt, 20-kw., d-c. generator developed for the U. S. Navy is noteworthy because of its direct contradiction of the accepted idea that d-c. generators are not feasible above 2000 volts per commutator. Six of these units were built to supply plate current for large transmitting tubes at Arlington, a duty usually performed by rectifying tubes. These generators are bipolar, two commutators in series and two commutating poles. No compensating winding is used. The field frame is of welded steel plate with fabricated feet welded on and has a singularly inconspicuous and non-committal appearance.

Printing. The main advance during the past year in the printing industry has been the increase in the speed of the presses. Where a few years ago 24,000 to 25,000 papers an hour was considered good production, presses today operate at the rate of 60,000 to 75,000 papers an hour. This has meant the application of

higher speed motors in some cases, while in others it was simply a matter of increasing the gear ratio. The horsepower of the motors has increased in proportion, while new improved and simplified methods of control have accompanied the change.

Fig. 20—1000-Hp. Roller Bearing Reducing Gears for Danville Structural Steel Co.

Bearings. The old sleeve bearing is being replaced in many instances by anti-friction bearings of various types. Ball, roller, and tapered roller bearings are proving very satisfactory on most type drives.

The largest roller bearing mill type unit was installed in 1928, Fig. 20. This unit is for reducing speed on heavy industrial machinery, and uses helical gears and tapered roller bearings.

The committee is indebted to the following firms with which committeemen are connected or whose cooperation and publications we have made use of in compiling this report:

I. T. E. Circuit Breaker Company
Philadelphia Rubber Company
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Bell Telephone Laboratories
Stone and Webster
Bethlehem Steel Company
Electric Power Equipment Company
Lehigh Portland Cement Company
University of Toronto
Robins Conveying Belt Company
Electric Controller and Manufacturing Company
Philadelphia Electric Company
Westinghouse Electric and Manufacturing Company
General Electric Company
Canadian General Electric Company

Instruments and Measurements

ANNUAL REPORT OF COMMITTEE ON INSTRUMENTS AND MEASUREMENTS*

To the Board of Directors:

The Committee on Instruments and Measurements reports activities for the past year as follows:

1. Revision of Electrical Units
2. Measurement of Core Losses in Terms of Sine-Wave Core Losses
3. Distortion Factor—Definition and Method of Measurement
4. Technique of Temperature Measurement.
5. Measurement of Variable Power and Large Blocks of Energy
6. Dielectric Power Loss and Power-Factor Measurements
7. Measurement of Non-Electrical Quantities by Electrical Means
8. High-Frequency Measurements
9. Remote Metering
10. Shielding in Electrical Measurements
11. Papers
12. Conclusion.

Revision of Electrical Units

The progress during the past year of the movement in favor of absolute electrical units in place of those defined by means of arbitrary standards is considered by the Instruments and Measurements Committee to be most gratifying and the actions taken to be epoch making. This progress is recorded in the following report by Dr. H. B. Brooks, Chairman of the Sub-Committee on Revision of Electrical Units.

The resolutions on Revision of Electrical Units which were prepared by the Committee on Instruments and Measurements and transmitted to the Board of Directors, through the Standards Committee, were adopted with a slight change at the meeting of the Board in Denver in June 1928. The resolutions cited the discrepancies existing between the international electrical units and the absolute units they were intended to represent, and called on the United States Bureau of Standards and foreign national standardizing laboratories to undertake the researches necessary to eliminate the discrepancies, and to bring about the legalization of absolute electrical units in place of those defined by means of arbitrary standards. Copies of the resolutions were sent to the appropriate committees of the U. S. Senate and House of Representatives, the Bureau of Standards, and to the national standardizing laboratories of England, France, Germany, Japan, and Russia.

To assist in formulating proposals incorporating a consensus of the opinions held in the United States, Dr. G. K. Burgess, Director of the Bureau of Standards, invited a number of organizations to name members of an American Advisory Committee representing the scientific, industrial, and commercial organizations most directly concerned with electrical measurements. These organizations were as follows: National Academy of Sciences, American Institute of Electrical Engineers, American Physical Society, National Electric Light Association, Association of Edison Illuminating Companies, National Electrical Manufacturers Association, and the American Telephone and Telegraph Company. This American Committee met at the Bureau of Standards on June 16, 1928. After due consideration of the information available regarding the present status of electrical measurements, the American Advisory Committee unanimously adopted the following resolutions:

"(1) Resolved, that in the opinion of this Committee, in view of improvements which are being made in absolute measurements, electrical standards should in future be based upon the absolute system of units.

"(2) Resolved, that in the opinion of this Committee, the functions which it is desirable to have the International Bureau of Weights and Measures undertake in connection with the electrical units, are as follows:

"(1) A central secretariat to arrange for systematic exchange of standards and compilation of results of intercomparisons thus made among the national laboratories.

"(2) A laboratory to which concrete standards representing the results obtained in the different countries may be brought for precise comparisons.

"(3) A repository for international reference and working standards with the necessary equipment so that other standards may be compared with these standards on request.

With the backing of these resolutions, Dr. Burgess attended the first meetings of the (international) Advisory Committee on Electricity, which were held at Sèvres and Paris November 20-22 inclusive, 1928. The timeliness and the importance of the question of the revision of the electrical units are shown by the fact that delegates felt it worth while to come from the far countries of the world, so that the Advisory Committee had a complete attendance of all members. Communications from various countries concerning the electrical units were considered. The cordial spirit and the unselfish aims of the committee are shown

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Everett S. Lee, Chairman,
H. C. Koenig, Secretary,
E. J. Rutan, Vice-Chairman,

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Perry A. Borden,	F. C. Holtz,	T. E. Penard,
W. M. Bradshaw,	I. F. Kinnard,	R. T. Pierce,
H. B. Brooks,	A. E. Knowlton,	G. A. Sawin,
A. L. Cook,	W. B. Kouwenhoven,	R. W. Sorensen,
Melville Eastham,	E. B. Merriam,	H. M. Turner.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

29-97

by the gratifying fact that all the decisions reported were reached unanimously.[1] The committee realized the great importance of dealing adequately with the electrical units, in the light of past experience and present knowledge, and ultimately adopted, by unanimous vote, the following resolutions:

"1. The Advisory Committee on Electricity established by the International Committee of Weights and Measures, considering the great importance of unifying the systems of electrical measurements upon a basis deprived of all arbitrary character, recognizes from the beginning of its first session that the absolute system, derived from the c. g. s. system, may be substituted with advantage for the international system of units for all scientific and industrial determinations and decides to propose its adoption to the International Committee of Weights and Measures.

"2. The Advisory Committee on Electricity, while recognizing fully the great forward steps already taken in the domain of electrical measurements of high precision, does not believe, however, that it is possible immediately to fix with all the necessary and possible accuracy the ratios which exist between the absolute units derived from the c. g. s. system and the international units of current, electromotive force, and resistance, as they were defined by the International Congress at Chicago in 1893 and the London Conference in 1908, and expresses the wish that researches may be carried on toward that end in suitably equipped laboratories, in accordance with a program previously worked out in cooperation with the Advisory Committee on Electricity."

Dr. Burgess was unanimously chosen to report the proceedings of the Advisory Committee to the International Committee of Weights and Measures. The approval of the latter is necessary to put into effect the resolutions of the Advisory Committee. The Committee on Instruments and Measurements regards the action of the Advisory Committee on Electricity as an epoch-making event, and looks forward hopefully to the time when the units of the electrical engineer and those of the mechanical engineer will rest alike upon the fundamental bases, the meter, the kilogram, and the second of time.

Measurement of Core Losses in Terms of Sine-Wave Core Losses

A working committee of the Instruments and Measurements Committee under the chairmanship of Mr. W. M. Bradshaw has been studying for the past year the best way to make core loss measurements on transformers so that they will give accurate "sine-wave" core losses regardless of the wave form employed for excitation. In this connection the working committee issued a questionnaire on the practise of measuring core losses in transformers. This was sent to fourteen known manufacturers of transformers. Replies were received from ten of them. Of the ten replies, five were from manufacturers of power and distribution transformers, all of whom either recognized the necessity for corrections or actually used corrections. The other five were from manufacturers of instrument transformers or special transformers in which core losses need not be measured. Those who only recognized the general necessity for corrections state that they have available a pure sine wave which remains undisturbed.

As a result of the replies to this questionnaire and from the results of their study, the Working Committee recommended desirable procedure in making core loss measurements to the Instruments and Measurements Committee, which recommendations have been regularly transmitted to the Standards Committee. These recommendations provided for the measurement of core loss of transformers preferably with a sine wave of applied voltage; if this is not practicable, the results obtained with a distorted wave of applied voltage shall be corrected to a sine-wave basis by a suitable method.

Three suitable methods, as follows, have been studied and are outlined in detail in the report:

No. 1. Standard Core Method
No. 2. Iron-Loss Voltmeter Method
No. 3. Flux Voltmeter Method

Methods No. 1 and No. 2 use miniature representative sample cores. In Method No. 3 the correct average voltage is applied to the transformer under test irrespective of the wave form of the applied voltage.

The committee recommended preference for methods utilizing average voltage for correcting core loss results to a sine-wave basis to those utilizing sample cores since the sample may not represent correctly the actual transformer under test.

Distortion Factor—Definition and Method of Measurement

A working Committee of the Instruments and Measurements Committee, under the chairmanship of Mr. W. M. Bradshaw, has studied the definition of distortion factor and method of measurement as disclosed in a report of the French Electrotechnical Commission entitled "Methods of Determining the Distortion of the Voltage Wave of Alternators," and have recommended action concerning these, which recommendations the Instruments and Measurements Committee have regularly referred to the Standards Committee. The definition recommended is the same in substance as proposed in the French report, but differs in form to be in agreement with the form of the present A. I. E. E. definition of Deviation Factor, and is as follows:

The distortion factor of a voltage wave is the ratio of the effective value of the residue, after the elimination of the fundamental, to the effective value of the original wave.

1. A full account of the proceedings of the Advisory Committee on Electricity is given in *Revue Générale de l'Electricité* for Dec. 22, 1928.

A general definition to include all periodic waves was recommended as follows:

Distortion factor of a periodic voltage or current wave is the ratio of the effective value of the residue, after the elimination of the fundamental, to the effective value of the original wave.

Details of methods for measuring the distortion factor of a voltage wave were submitted for use in obtaining data for the establishment of suitable limiting values of distortion factor, as follows:

A. Method of Boucherot using an alternator with a sinusoidal wave and a voltmeter.

B. Method of Belfils using a bridge to suppress the fundamental of the wave to be analyzed, and a voltmeter to read the residue.

C. By oscillogram.

D. By harmonic analyzer.

The method (B) seems at the present time to be the most promising, though further study and development of the apparatus are necessary. This work is proceeding.

Technique of Temperature Measurement

The Standards Committee has referred to the Instruments and Measurements Committee the broad question of Standards for the Technique of Temperature Measurement. A subcommittee has been appointed and work is progressing on writing a standard on the technique of temperature measurement. The initial activity will be confined to those temperature measurements included in the present A. I. E. E. Standards.

Measurement of Variable Power and Large Blocks of Energy

The subcommittee on this subject, under the chairmanship of Mr. T. E. Penard, is collecting literature from the various manufacturers of meters and metering equipment relative to this phase of the art. The questions of metering large blocks of d-c. energy requiring the use of large capacity shunts in connection with watthour meters and the calibration of such shunts, as well as the use of several low current shunts in parallel with resistance compensated potential connections, are being studied.

A new oscillating meter clamped directly on the bus bars and working on the stray field produced by the currents in the bus is being studied.

Dielectric Power Loss and Power-Factor Measurements

The Subcommittee on Dielectric Power Loss and Power-Factor Measurements under the chairmanship of Mr. H. C. Koenig has rendered the following report:

In 1928 Messrs. J. A. Scott, H. W. Bousman, and R. R. Benedict presented a paper before the Institute entitled *A Thermal Method of Standardizing Dielectric Power Loss Measuring Equipment*, (A. I. E. E. Quarterly Trans., Vol. 47, July, 1928, p. 819). In this paper the authors pointed out the effect of humidity on an air capacitor. As a result of this paper, at least two new investigations along these lines have been made. In one of them, at the Electrical Testing Laboratories, it was shown that a properly constructed air condenser had a power factor of less than 0.01 per cent over a range of voltage up to 20 kv. per inch between plates, and over a range of humidity up to, at least, 85 per cent in clean air. In the other investigation at the Johns Hopkins University, two successful runs of varying humidity and voltage were made, one at a temperature of 77 deg. fahr. and the other at 85 deg. fahr. No air conduction could be detected up to about 90 per cent humidity. At the time of writing further experiments are still in progress.

In the subcommittee report of 1928, attention was called to the development of standards loads for use in checking dielectric loss equipment. Following the successful application of this standard load rated at 20 kv., a similar load has now been developed for use up to 75 kv. This latter load was also used very successfully in making factory inter-checks of dielectric loss equipment. In general, the results of the inter-comparison of dielectric loss equipments were very satisfactory.

As a result of the inter-check made at various factories two conditions have been noticed which are of interest. (1) It has been noted that while most laboratories are in fair agreement as to power factor measurements, there is a decided lack of agreement on current (and, therefore, watt) determinations. It appears that currents of the order of 1 to 10 milliamperes present a somewhat difficult measurement problem. Most laboratories use the same instruments for watt and current determinations, changing the coil connections and using it as a deflection instrument. The laboratories using a bridge method depend on the capacitance of their reference condenser. One of the most satisfactory methods for measuring these currents appears to be a modification of the transformer bridge. This method requires the use of a potential transformer of known ratio and the accuracy of the method depends only on the potential transformer and the capacitance of a condenser used in making the balance. (2) In a number of cases the condition was encountered where the test plate of the reference air condenser was mounted on insulators in such a way that the field from the high-voltage plate to these insulators terminated on the test plate. This sometimes caused the condensers to have power factors as great as 0.5 per cent. In each case it has been found possible to bring the power factor to zero by guarding and shielding these insulators. This condition is quite noticeable with the large open pedestal type condensers used in many of the cable test laboratories.

During the past months there has been considerable discussion regarding the possible effects of the high-frequency discharges occurring in insulations under high stress on measurements of dielectric loss and power factor. While these high-frequency discharges prob-

ably represent energy losses, it is questionable whether or not any of the present methods of measurements include these losses.

The question of dielectric loss and power factor measurements with particular reference to the shielding of both the measuring equipment and the cables under test will be covered in a symposium to be held at the Summer Convention on Shielding in Electrical Measurements. Two papers covering this work will be presented, entitled *Shielding Cables in Dielectric Loss Measurements*, by E. H. Salter and *Some Problems in Dielectric Loss Measurements* by Professor C. L. Dawes.

A new test cell has been developed for determining the insulation resistance, power factor, and dielectric constant on liquid insulating materials. This cell makes it possible to test more rapidly and under better controlled conditions than was previously possible.

Some recent modifications of the Schering bridge have been made which may be of interest. The Schering bridge as ordinarily constructed necessitates the computation of capacitance and power factor of the test condenser from the values of the condensers and resistors in the other arms of the bridge. In order to eliminate the need of these computations, a modified circuit has been developed and used in the construction of several bridges whereby it is possible to adjust the bridge for the capacitance of the standard air condenser, and to make the bridge direct reading in both power factor and capacitance of the unknown condenser. These bridges have also been provided with shields on the various sections to reduce the effect of stray capacitances. In order to readily adjust the potentials of these shields, one public utility company laboratory has had a dummy bridge constructed wherein it is only necessary to set the arms of this bridge to correspond to those of the main Schering bridge in order to establish the proper potentials on the various guard circuits.

The construction of large power-factor correction capacitors has brought about the need for a ready means of measuring the capacitance and power factor of capacitors on voltages ranging from 200 to 4000 volts and with line currents as high as 75 amperes. Several stationary test sets have been constructed for this purpose, which enable power factor and capacitance to be read directly from the scale of a variable self and mutual inductor, utilizing a reflecting dynamometer as a null deflection instrument. Such testing sets are readily adjustable as to both voltage and current. The impedance of the measuring circuit, which must be inserted in series with the condenser, is so low that the drop across it does not exceed 300 millivolts.

Measurement of Non-Electrical Quantities by Electrical Means

A symposium on this subject was held during the Winter Convention, sponsored by the subcommittee on this subject, under the chairmanship of Mr. Perry A. Borden. The following papers were presented:

Magnetic Analysis of Materials, by R. L. Sanford.

Measurements of Flow by Use of Electrical Instruments, by W. H. Pratt.

Use of the Oscillograph for Measuring Non-Electrical Quantities, by D. F. Miner and W. B. Batten.

Study of Noises in Electrical Apparatus, by T. Spooner and J. P. Foltz.

Electrical Aids to Navigation,[2] by R. H. Marriott.

The increasing extent of articles in the literature describing the use and application of electrical means to the measurement of non-electrical quantities has made it practically impossible for the subcommittee to compile the many articles on the subject into a bibliography as has been done during the past three years. Hence this bibliography which has appeared annually will not appear this year except as bibliographies are available with the above papers. This growth speaks for the extension of electrical means into fields of non-electrical measurements.

High-Frequency Measurements

The subcommittee on this subject, through its chairman Professor H. M. Turner, maintains contact with the radio and high-frequency field through the Standardizaton Committee of the Institute of Radio Engineers. Under the auspices of this committee, there has recently been issued a preliminary bibliography on high-frequency measurements which is available.

The subcommittee has prepared a list of papers published during the past two years on the subject of high-frequency measurements since the symposium held in May 1927, on the subject. This list is not complete, but it reflects the progress in the art of high-frequency measurements. The list is attached to this report as Appendix A.

Remote Metering

Two papers were presented at the Winter Convention under the auspices of this subcommittee, under the chairmanship of Mr. E. J. Rutan. These were:

Telemetering, by C. H. Linder, H. B. Rex, C. E. Stewart, and A. S. Fitzgerald.

Totalizing of Electric System Loads,[2] by P. M. Lincoln.

The information presented in these papers supplements that reported by the subcommittee in 1928.

The subcommittee is now studying the nomenclature applicable to remote metering.

Shielding in Electrical Measurements

A symposium on Shielding in Electrical Measurements will be held at the Summer Convention 1929, under the chairmanship of Mr. H. C. Koenig. The following papers will be presented:

Shielding and Guarding Electrical Apparatus, by H. L. Curtis.

Some Problems in Dielectric Loss Measurements, by C. L. Dawes.

2. These papers will be published in the A. I. E. E. Quarterly Trans., Vol. 48, July 1929.

Shielding in Heavy Alternating Circuits, by F. B. Silsbee.

Shielding Short Lengths of Cable in Dielectric Loss Measurements, by E. H. Salter.

Shielding in High-Frequency Measurements, by J. G. Ferguson.

Magnetic Shielding, by S. L. Gokhale.

Papers

In addition to those listed above, the following papers were presented at the Winter Convention, New York, February 1, 1929, on Instruments and Measurements:

A New High-Accuracy Current Transformer, by M. S. Wilson.

132-Kv. Shielded Potentiometer for Determining the Accuracy of Potential Transformers, by C. T. Weller.

A Precision Regulator for Alternating Voltage,[2] by by H. M. Stoller and J. R. Power.

The following papers on the subject of High-Speed Instruments and Measurements were presented at the Middle-Eastern District Meeting, Cincinnati, March 21, 1929.

High-Speed Photography in Electrical Engineering, by H. W. Tenney, (available in pamphlet form only).

Oscillographs for Recording Transient Phenomena, by W. A. Marrison.

A New Type of Hot-Cathode Oscillograph, by R. E. George.

Bushing Type Current Transformers for Metering, by A. Boyajian and W. F. Skeats.

Excitation of Current Transformers under Transient Conditions,[3] by D. E. Marshall and P. O. Langguth.

Conclusion

The past year has been one of activity for the Instruments and Measurements Committee. Not all of the present advances in the art have been studied directly by the committee, but these remain for future study and report. Chief among these are the advances in frequency measurement and control, and the remarkable progress in cathode ray oscillography. These and other projects bespeak progress in the measurements field which may truly be called gratifying.

APPENDIX A

A Report of progress in High-Frequency Electrical Measurements as reflected by the literature. A short list of papers published during 1927 and 1928.

American Institute of Electrical Engineers Transactions

Two Range Vacuum Tube Voltmeter, Janskey and Feldman, Vol. 47, 1928, p. 307.

Measurement of Surge Voltages, Lee and Foust, Vol. 46, 1927, p. 339.

Measurement of Telegraph Transmission, Nyquist, Shank, and Corey, Vol. 46, 1927, pp. 231 (385).

The Oscilloscope, Bedell and Reich, Vol. 46, 1927, p. 546.

Use of High-Frequency Currents for Control, Boddie, Vol. 46, 1927, p. 975.

Measurement of Mechanical Forces in Elec. Cir., Roper, Vol. 46, 1927, p. 574.

Substitution Method for the Determination of Resistance of Inductors and Capacitors at Radio Frequencies, Burke, Vol. 46, 1927, p. 482.

Condenser Shunt for Measurement of High-Frequency Currents of Large Magnitude, Nyman, Vol. 46, 1927, p. 488.

High-Frequency Measurement of Communication Apparatus, Shackelton and Ferguson, Vol. 46, 1927, p. 519.

Measurement of Puncture Voltage, Bush and Moon, Vol. 46, 1927, p. 1025.

High-Voltage Measurement on Cables, Kasson, Vol. 46, 1927, p. 635.

A. I. E. E. SYMPOSIUM, JUNE 1929

Shielding in High-Frequency Measurements, J. G. Ferguson.

Magnetic Shielding, Gokhale.

Precautions Against Stray Field Errors in Measurements with Large Alternating Currents, Dr. F. B. Silsbee.

Shielding and Guarding Electrical Apparatus, Dr. H. L. Curtis.

Shielding for Dielectric Loss Measurements on Short Lengths of High-Tension Cable, Salter.

Some Problems in Dielectric Loss Measurements, Dawes, Hoover, and Reichard.

1928 Proceedings Institute Radio Engineers, Vol. 16

"New Method of Calibrating Ammeters at R. F.," Hazel, No. 1, 1928, p. 40.

"Precision Method for Measurement of High Frequency," Aiken, Vol. 2, 1928, p. 125.

"Precision Determination of Frequency," Horton and Morrison, Vol. 2, p. 137.

"Condenser Shunt for Measurement of Large R. F. Currents," Nyman, Vol. 2, p. 208.

"Measurement of Choke Coil Inductances," Wright and Bowditch, Vol. 3, 1928, pp. 373 (844).

"Measurement of T. V. T. Capacities by a Transformer Balance," Wheeler, Vol. 4, 1928, p. 476.

"D. C. Bridge for T. V. T. Measurements," Walsh, Vol. 4, p. 482.

"Bridge Method for Measuring Inter-electrode Admittance," Hoch, Vol. 4, p. 487.

"Use of Oscillograph for Recording T. V. T. Characteristics," Schneider, Vol. 5, 1928, p. 674.

"Compensated Electron Tube Voltmeter," Turner, Vol. 6, 1928, p. 799.

"A New Method for Determining the Efficiency of T. V. T. Circuits," Crossby and Page, Vol. 10, 1928, p. 1375.

"Constant Impedance Method for Measuring Choke Coils," Turner, Vol. 11, 1928, p. 1559.

"Bridge Circuit for Measuring L with Superposed D. C.," Landon, Vol. 12, 1928, p. 1771.

"Quantitative Measurements on Reception on Radio-Telegraphy," Anders, Vol. XV, 1927, p. 297.

"Loud Speaker Testing Methods," Wolff and Ringel, Vol. XV, p. 363.

"Measurement of R. F. Amplification," Harris, Vol. XV, p. 641.

"Testing A. F. Transformers," Dickey, Vol. XV, p. 687.

"Testing of A. F. Transformer Coupled Amplifiers," Diamond and Webb, Vol. XV, p. 769.

Institute of Radio Engineers—Symposium on Measurement of Frequency, April 3, 1929

"The Routine Measurement of the Operating Frequencies of Broadcast Stations," Henry L. Bogardus.

"The Testing of Piezo Oscillators for Broadcast Stations," E. L. Hall.

"The Precision Measurement of Time," Alfred L. Loomis.

"A High Precision Standard of Frequency," W. A. Marrison.

"Observations on Modes of Vibration and Temperature Coefficients of Quartz Plates," F. R. Lack.

"A Convenient Method for Referring Secondary Frequency

Standards to a Standard Time Interval," L. M. Hull and James K. Clapp, (presented by Mr. Clapp).

"Frequency Measurement," S. Jimbo (presented by W. G. Cady).

Institute of Radio Engineers, R. S. I. Meeting May 15, 1929.

"Current Developments in Radio Measurement Research," J. H. Dellinger.

"Inductance as Affected by Initial Magnetic State, Air-Gap, and Superposed Currents," H. M. Turner.

"Studies in Echo Signals," A. H. Taylor and L. C. Young.

"Detection at High Signal Voltages," Stuart Ballantine.

"The Mutual Impedances between Adjacent Antennas," C. R. Englund and A. B. Crawford.

"Some Measurements on the Directional Distribution of Static," A. E. Harper.

Bell System Technical Journal

"Measurement of C in Terms of R and F," Ferguson and Bartlett, Vol. 7, 1928, p. 420-37.

"Measurement of Communication Apparatus," Ferguson and Shackelton, Vol. 7, 1928, pp. 70-89.

"Bridge for Measuring Small Time Intervals," Herman, Vol. 7, 1928, p. 343.

"Shielded Bridge for Inductive Impedance Measurements at Speech and Carrier Frequencies," Shackelton, Vol. 6, 1927, p. 142.

"Measurement of L by Shielded Owen Bridge," Ferguson, Vol. 6, 1927, p. 375.

"Determination of Electrical Characteristics of Loaded Submarine Cables," Gilbert, Vol. 6, 1927, p. 387.

"Location of Opens in Toll Telephone Lines," Edwards and Herrington, Vol. 6, 1927, p. 27.

"Analyzer for Voice Frequency Range," Moore and Curtis, Vol. 6, 1927, p. 217.

"Analyzer for Complex Electric Waves," Landeen, Vol. 6, 1927, p. 230.

Journal Franklin Institute

"A Capacity Measurement Method," Roberts, Vol. 205, 1928, pp. 699-701.

"Measurement of Small Time Intervals," Quayle, Vol. 203, 1927, pp. 407-412.

Journal Optical Society of America

"Method of Adjusting R. F. in Small Steps," Clapp, No. 17, 1928, pp. 132-7.

"The Ultramicrometer," Obata, Vol. 16, 1927, p. 419-32.

Journal Institution Electrical Engineers (London)

"Some Recent Advances in A-C. Measuring Instruments," Edgcumb, 1927, p. 553.

"A Wireless Works Laboratory," Turner, 1927, p. 881.

"Errors in Magnetic Testing of Ring Samples," Hughes, 1927, p. 932.

Experimental Wire and Wireless Engineer (London)

"Bridge for Measuring L and C," Zeckner, Vol. 5, 1928, pp. 280-2.

"Dielectric Losses in Single Layer Coils," Jackson, Vol. 5, 1928, pp. 255-8.

"Measurement of Small Variable C at R. F.," Griffiths, Vol. 5, 1928, pp. 452-9.

"Notes on Reflex. Voltmeter," Medlam and Aschwald, Vol. 5, 1928, pp. 56-60.

"Survey of Methods of Radio Signal Measurement," Screnivassan, Vol. 5, 1928, pp. 205-10.

"Direct Reading Valve Tester," Scroggerio, Vol. 5, 1928, pp. 480-4.

"Harmonic Comparison of Radio Frequency by Cathode Ray Oscillograph," Rangachori, Vol. 5, 1928, pp. 264-6.

"Design of Non-Contact Thermojunction Ammeters," Colebrook, Vol. 5, 1928, pp. 538-45.

"Method of Calibrating Low Frequency Oscillator with Single Frequency Source," Vol. 5, 1928, pp. 633-5.

Journal Scientific Instruments (London)

"New Universal Long Wave Radio Field-Intensity Measuring Set," Vol. 5, 1928, pp. 1-9.

Revue Générale de l'Electricité

"Measurement of Capacity," Mey, Vol. XXIV, p. 672.

"Measurement of Voltages below 10,000," de la Gorce, Vol. XXI, p. 603.

"Wheatstone Bridge for Small Resistances," Vol. XXII, p. 67.

"Wheatstone Bridge for Small Resistances," Vol. XXII, p. 1136.

Discussion

F. A. Wolff: I happen to be on the minority side with regard to the question of the substitution, at this time, of the absolute electrical units for the concrete units. There are in fact two separate questions involved. One is whether the present concrete standards are in sufficiently close agreement with the absolute units. The other and more practical question is whether the substitution of absolute measuring apparatus for concrete standards will give as high an accuracy.

Unfortunately, the precision of electrical comparisons is far beyond the limits of accuracy to which absolute measurements can be carried and far beyond the limits of accuracy to which concrete standards can be reproduced. The choice eventually made will be to fix values, as closely as possible, whether by absolute measuring apparatus or by concrete standards.

At this time it cannot be said that the accuracy of absolute measurements has attained the accuracy with which the standard cell or mercury ohm can be reproduced, provided that these concrete standards are constructed on the basis of proper specifications. That has not as yet been done. There was a reaction against setting specifications for the standard cell and silver voltameter because the electromotive force and electrochemical equivalent are characteristic properties of the materials involved. Why then have specifications? Specifications, to my mind, are nevertheless necessary to assure the highest accuracy, especially when the work is done by persons lacking adequate experience.

In connection with previous discussions of this subject, I made a very earnest plea that work on the concrete standards be carried on alongside the new work on absolute standards now going on in various laboratories which without doubt will yield an enhanced accuracy. I am quite sure the same would be true with regard to the concrete standards.

Eventually, as I foresee it, we shall bring about a closer agreement between the values assigned to the concrete working standards and their absolute values. I should like to make that step, however, a little later so that it will not be necessary to make another change in the course of the next five or ten years as further absolute measurements are made. I foresee eventually as this work on absolute measurements is carried to completion that we shall assign a particular value to the standard cell and we shall stick to that for a long time, even though subsequent absolute measurements may show that it is slightly in error, because we shall not want to make another change.

In the same way, I think we shall fall back on a material resistance standard because considerable promising research work has already been done on the causes of variation of resistance standards. I feel confident that even now we are able to construct resistance standards which will retain their values over a long period of years to a closer accuracy than attainable by individual absolute measurements.

With regard to standard cells, a group of cells which has just

been set up at the Bureau of Standards agrees in value to an accuracy of one part in 100,000 with cells which my colleagues and I set up more than twenty years ago. I am sure absolute measurements have not yet attained quite that accuracy.

J. C. Clark: I have a question for Mr. Lee relating to the recommended definition of "distortion factor."

It appears to me that this definition is, possibly, open to objection. To make this clear, let us consider a combined wave having, say, a third harmonic of effective value equal to 500 per cent of the effective value of the fundamental. Under the proposed definition, the distortion factor of the wave would be slightly less than unity. Would it not be better to make the distortion factor the ratio of the effective value of the combined harmonics, other than fundamental, to the effective value of the fundamental?

E. S. Lee: We appreciate Mr. Wolff's position. The substitution of the absolute standards for the concrete standards is not going to take place today. Even those most hopeful for this advance recognize that there is much work yet to be done before this can be done. But the scientific advantages of such a change are appreciated and the urge to do the necessary work is present. In this there is satisfaction.

It would seem that there has been much time during which specifications could have been adequately made for the standards in their present material form. Perhaps not. However, there is no reason to feel that such standards are to be completely abandoned for there is recognition that they represent the standards of electrical measurements for a most fruitful regime assuring their preservation in form. Time may show their increasing worth as secondary standards. If there are possibilities for their improvement, the realization of such will doubtless still be of benefit.

Referring to Professor Clark's question, we use the effective value of the original wave because we know this value. Also, we are primarily interested in values of distortion factor within, say, 10 per cent or less. The condition cited by Professor Clark would not come within this range.

Iron and Steel Production

ANNUAL REPORT OF COMMITTEE ON APPLICATIONS TO IRON AND STEEL PRODUCTION*

To the Board of Directors:

Because of the magnitude of the projects and the rapid development that is taking place in certain phases of the iron and steel industry, especially in regard to application of electric power, an annual report of application of electric power to the iron and steel industry must necessarily be a continued story from year to year. These developments are keeping pace with the investigation of the metallurgist, which investigations have resulted in new processes for shaping steel.

While during the past year there have been no new steel plants built, many plants have added to their equipment. During the year of 1928, the steel industry purchased in main drive motors a total exceeding 200,000 hp., the units ranging from 300 to 7000 hp.

It is interesting to note that of this total, approximately 80 per cent are d-c. machines. This is largely a result of the installation of several new continuous mills with individual motor drive for each stand of rolls.

Synchronous motors for driving constant speed mills are increasing in number every year, and during the past year, synchronous motors have been supplied ranging in size from 400 to 5000 hp.

A mill drive of unusual interest which is now being installed is that for the 36-in. reversing Universal slabbing mill at the Steubenville, Ohio plant of the Wheeling Steel Corporation. This is the first instance in this country in which separate reversing motors are used for driving the horizontal and vertical rolls of the mill. The motor which will drive the horizontal rolls is a single-armature d-c. machine rated 7000 hp. continuous, 50/100 rev. per min., 750 volts.

The number of continuous mills having stands individually driven by adjustable speed motors still grows. The largest drive for a mill of this type which has been purchased during the year is that for the 60-in. wide strip mill at the Wheeling Steel Corporation's Steubenville plant. The total continuous rating of the motors on this mill is over 20,000 hp., and d-c. power will be furnished by three 4000-kw., motor-generator sets.

During 1928 the American Rolling Mill Company placed in service a new wide strip mill. This mill is laid out to produce strip material up to about 60 in. wide and is one of the largest strip mills yet installed. The total capacity of the 11 driving motors is 21,800 hp.

The manufacturers of wide strips continue to enlarge their manufacturing capacity and apparently the end of their research and development is not in sight. The success of wide strip rolling is in considerable measure due to the accuracy that can be secured through modern electric drives and control equipment using individual motor drive of stands.

Time limit control of acceleration has now been in use on mill auxiliary drive for three or four years and has proved extremely successful.

A mill type motor which completely meets the recent standardization rules of the Association of Iron and Steel Electrical Engineers in all respects as to rating, speed, and dimensions is now in successful operation in a large number of steel mills, one installation of almost 200 motors having been in use for a year and a half with no troubles.

As regards the construction and design of main roll motors, such machinery is now largely being fabricated from rolled steel products by means of arc welding. The magnet frames of practically all of the large d-c. machines are now constructed from rolled steel slabs with feet and lugs welded to them. The stator frames of synchronous and induction motors are fabricated from steel plates and bars welded together, and the rotors of some of the synchronous motors are also being fabricated from plates and slabs.

The use of these fabricated frames has reduced cost and reduced manufacturing time for large electric motors. It also improves commutation and other characteristics, and it has become a definite feature in the new era of steel mill electrification.

The statistics for 1928 show that there was a total of 188 cranes purchased in the iron and steel industry, all having electric drives.

In the field of yard transportation, the Diesel electric locomotive is proving extremely successful in reducing operating and maintenance cost.

During the current year, the Ward Leonard type of control has been successfully applied to blast furnace skip hoists at the Bethlehem Steel Company's Plant at Sparrows Point, and also at the Tennessee Coal and Iron Company in Alabama. Engineers may not generally appreciate the limitations of the hoist equipment of blast furnaces, particularly on those furnaces that have been greatly increased in size in the last few years and are equipped with hoists of the single bucket type. The Ward Leonard control installed at Sparrows Point has quite materially helped furnace filling conditions and made it possible to secure increased outputs on their furnaces.

The electrically operated mud gun is one of the most outstanding developments witnessed by the blast furnace industry in many years. Before describing the

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A. C. Cummins,	A. G. Pierce,	T. S. Towle,
Samuel L. Henderson,	F. O. Schmure,	J. D. Wright.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

new gun and its operation, a brief consideration of the work previously involved in stopping an iron notch will perhaps aid the reader unfamiliar with blast furnace operation to grasp the significance of the new development.

Tapping holes in the early days of ironmaking were closed by ramming in balls of fire clay with what was known in blast furnace parlance as a stopping hook. The operation was performed by hand and required keeping the furnace off blast for a short period of time. As the size of the stacks was increased, higher blast pressures were used and higher production obtained. This increase in the quantity of molten metal in the hearth and the blast tested the skill of the furnace keeper in stopping back the notch so that the iron would not trickle through after the blast was applied.

The mud gun, which is operated by three mill type motors, enables the operator to stop back the notch without taking the furnace off the blast.

Public utilities are showing increased interest in the possibility of assuming steel mill loads and have entered a few plants supplying a part of their power requirements. Because of the economical use of what have heretofore been waste gases around a steel mill, competition between the public utility and the steel mill in the generation of power is of course very keen. Apparently the possibility of a public utility entering a steel mill depends not upon its ability to take over existing load under competitive conditions, but the possibility of assuming new loads brought about by increased manufacturing capacity of the steel plant. This phase of the public utility power outlet is comparatively new, but is assuming greater importance each year.

During the current year, the first steel mill interconnected transmission system was placed in service by the Carnegie Steel Company and the National Tube Company in the Pittsburgh District. This transmission system consists of a long span, double-circuit tower line, similar in design to recent public utility practise. It is operated at the present time at 44,000 volts. Five steel mill and coke plant power plants, having generating equipment aggregating 150,000 kv-a., are interconnected. The purpose of this interconnection is to transmit in the form of electric power, excess by-product fuel available at any plant to other plants where a shortage of such fuel may exist. It is an essential feature in the complete utilization of excess blast furnace gas, coke breeze, etc.

Production and Application of Light

ANNUAL REPORT OF COMMITTEE ON PRODUCTION AND APPLICATION OF LIGHT*

To the Board of Directors:

Introduction

The report of the Committee on Production and Application of Light calls attention to the outstanding advancements and trends in the art of lighting by electricity. Material used in the compilation has been furnished, not only by members of the committee, but by other authorities in various fields.

Production of Light

As was the case a year ago, most of the developments in production of light during the past year have been in the nature of improvements rather than radically new devices or methods.

Incandescent Filament Lamps

Standard Multiple Lamps. Since the introduction of the standard line of inside frosted lamps, the 50- and 60-watt sizes have both been made with A-21 bulbs but the 60-watt lamp was designed to be 5/16 in. longer than the 50-watt. Subsequent experience has shown that this is not necessary and the 60-watt lamp has been reduced to the same over-all length as the 50-watt, 4-15/16 in. This results in a simplification of manufacturing and packing problems and at the same time permits the use of 60-watt lamps in certain units which at present will only accommodate 50-watt lamps, thereby permitting higher levels of illumination in accordance with modern trends.

The sale of 75-watt lamps, which comprises about 3.5 per cent of the total sale of all multiple burning lamps, has made an inside frosted bulb for this size desirable. Accordingly, the manufacturers announce a 75-watt lamp in an A-23 bulb with the same physical dimensions as the standard 100-watt lamp in the A-23 bulb.

Special Multiple Lamps. The 10-watt 115-volt lamp in the C-9¼ bulb, mentioned in last year's report, has been improved by the addition of flutes to the bulb which makes it more pleasing in appearance and more suitable for Christmas and other holiday decorations. The lamp of same rating in the S-11 bulb has been reserved for sign service and other decorative and miscellaneous purposes.

A new 25-watt lamp of the 115-volt range in a T-6½ bulb was announced. This small lamp in a tubular bulb 5½ in. by 13/16 in. can be used in small inconspicuous reflectors which are admirably adapted for lighting show cases, paintings, and small signs; it is being used in some of the new types of luminaires for general illumination. (Fig. 1)

Railway Signal Lamps. The following types of lamps have been standardized for railway signal service by the Signal Section of the American Railway Association. They have the C-2 filament form as shown in Fig. 2. This form is rapidly replacing other filament construction.

Street Series Lamps. A notable change has been made in the bulb size of 4000-lumen 15-ampere, 6000- and 10,000-lumen 20-ampere lamps for street lighting service. The bulb of the 4000-lumen 15-ampere lamp has been changed and 10,000-lumen 20-ampere from a long neck PS-40 to a short neck PS-40 bulb. This places the 4000-lumen 15-ampere lamp in the same bulb as the 300-watt multiple lamp and the 20-ampere lamps in the same bulb as the 500-watt multiple lamps. The changes were accompanied by a reduction in the light center lengths of 2½ in. which can be compensated for either by moving the sockets, as is possible in a great many types of equipment, or by the use of a 2½-in. socket extension designed for the purpose. The new designs permit the substitution of machine blown bulbs for hand blown bulbs and have permitted a substantial reduction in the prices of these types.

Street Railway Lamps. A 56-watt A-21 coiled filament inside frosted lamp for operation five-in-series on 525-650-volt street railway circuits has been added to the line of lamps for this service. The new lamp supersedes one of similar rating in the clear S-21 bulb.

Aviation Lamps. The ideas of illuminating engineers on the subject of floodlighting for aviation landing fields crystallized during the year to the point where it was possible to standardize a group of four lamps for this service. These are, the 1500-watt 32-volt T-24, the 3000-watt 32-volt GT-38, the 5000-watt 115-volt G-64, and the 10,000-watt 115-volt G-80 airport floodlighting lamps shown in Fig. 3. One or the other of these four lamps can be used in practically all of the present lighting equipment and it is expected that future projectors will be designed around them in order to utilize to the fullest extent the advantages of high lumen output and efficiency, concentrated light source, and economy in operation and maintenance.

Motion Picture and Sound Recording Lamps. The introduction of motion pictures accompanied by sound required the development of a number of new types of incandescent lamps designed for use both in the studio where the pictures are taken and in the projection apparatus. In the studio, in addition to the standard

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L. A. Hawkins,		G. H. Stickney.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

General application	Volts	Amperes	Watts	Bulb	Base	Filament	Rated aver. life (hours)
Night indication, including switch lamps, semaphores, wig-wag highway crossing, and crossing gates							
	3.5	0.3		S-11	S. C.	C-2	1000
	10.	0.25		S-11	S. C.	C-2	1000
	13.5	0.25		S-11	S. C.	C 2	1000
	10.		5	S-11	S. C.	C-2	1000
Day and night indication, including light signals and highway crossing flashing signals							
	10.		5	S-11	S. C.	C-2	1000
	10.		10	S-11	S. C.	C-2	1000
	10.		18	S-11	S. C.	C-2	1000

general lighting lamps, are used three high wattage lamps especially designed for this service. These are the 2000-watt 115-volt G-48 bulb spotlight, the 5000-watt 115-volt G-64, and the 10,000-watt 115-volt G-80, shown in Fig. 4. These latter two lamps are similar in rating to those used for aviation landing floodlights but different in filament construction.

To record the sound on the film and to reproduce it in the theater a small 7.5-ampere 10-volt T-8 bulb lamp, shown in Fig. 5, is used. This lamp projects on the film a fine pencil of light which is varied in accordance with the vibrations of the sound and this in turn variously affects the emulsion on the film. In reproducing the sound in the theater a similar pencil of light is projected through that portion of the film bearing the sound record and into a photoelectric cell. The light of varying intensity which reaches the photoelectric cell modulates the current flowing through it and this modulated current is amplified in order to be fed to the loud speakers. Thus the original sound vibrations are translated to light, recorded upon the photographic film, retranslated to light, then to electric current, and finally back to sound again.

Mechanical Cleaner. The useful life of the very large sizes of incandescent lamps such as the 3000-, 5000-, and 10,000-watt lamps used for airport floodlighting and motion picture studio illumination, has largely been limited by the blackening of the bulb. If the lamps are made in bulbs small enough to fit equipments of reasonable size the bulbs become badly blackened long before the filaments fail. An abrasive powder is now being placed inside the bulbs of these lamps so that after they have become blackened in service they may be swept clean by simply swirling the powder around against the bulb walls.

Colored Lamps. Light's Golden Jubilee, which will be celebrated this year to commemorate the 50th anniversary of the incandescent lamp, will be made more colorful by the use of inside colored lamps, a process developed during the year. By this process the coloring material is sprayed on the inside of the bulb walls instead of on the outside. Thus the coloring material is protected from the weather, is rendered permanent and proof against fading or scratching, and the outside of the bulb is left smooth to retard the accumulation of dirt and permit easy cleaning.

Bases, Sockets, and Adapters. The medium prefocus base mentioned in last year's report has been supplemented by a mogul base of similar design but larger dimensions. In addition to the projection lamps to which these have been applied there are now included spotlight lamps, certain types of floodlight lamps, airplane headlight lamps, and airway beacon lamps. They are being used experimentally on locomotive headlight lamps. Equipments provided with the prefocusing sockets designed to accompany the prefocus base have the advantage that renewal lamps may

Fig. 1—25-Watt, 110-115-120-Volt, T-6½ Bulb Intermediate Screw Base Mazda Lamp

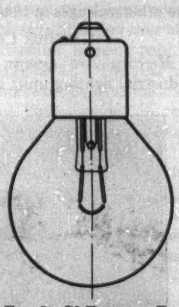

Fig. 2—C2 Filament Form which is Rapidly Replacing Other Filament Construction

be easily installed and are automatically focused without further adjustment.

Tungsten filament incandescent lamps have been improved tremendously in mechanical strength and ability to withstand abuse, but on the other hand the public has come to expect more and more of them. Lamps are expected to give service today under conditions that would never have been considered a few years ago. To alleviate the worst of these cases a vibration-resisting adapter has been developed which may be installed in the regular lamp socket and serves to protect the lamp. The use of these adapters in

locations where the mechanical failure of the filament is the predominant factor in determining lamp life, will usually prolong the hours in service.

Voltage Standardization. It is common practise to refer to the lighting circuit voltage as "110 volts" but it may surprise many to learn that 110-volt lamps comprised only 8.1 per cent of the total number in the 100—

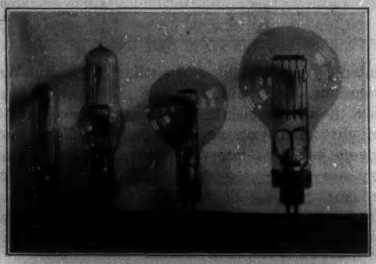

FIG. 3—AVIATION LAMPS
(Left to Right)
1.5 Kw.— 3 Kw.— 5 Kw.— 10 Kw.
32 Volts—32 Volts—115 Volts—110 Volts

130-volt range sold during 1928. Approximately 48 per cent were rated at 115 volts and 39 per cent at 120 volts. The remaining 5 per cent were scattered among the other voltages in that range.

GASEOUS CONDUCTOR LAMPS

Mercury Arc Lamps. To meet the demand for industrial lighting units giving a radiation more nearly

FIG. 4—STUDIO LAMPS,
(Left to Right)
2 Kw. — 5 Kw. — 10 Kw.
115 Volts —115 Volts —115 Volts
G-48 Type—G-64 Type—G-80 Type

equivalent to direct sunlight in the effect on general health, mercury arc tubes have been made up in forms identical with those used for industrial lighting but in glass of slightly higher ultra violet transmission. It is possible to build such lamps to meet any specific need in the way of an extended ultra violet spectrum. The practical value of such lamps depends largely on the recently demonstrated effect that the shortest radiation present in sunlight is apparently also the most effective radiation for most health purposes and that this radiation does not seem to be harmful when used in an amount comparable to that obtainable from direct sunlight. The value as well as possible harmful effects of radiation of a shorter wavelength than this is being investigated.

The rapidly expanding field of application of quartz mercury arcs in both therapeutic and health lighting has brought forth a variety of new types of arc, all of them differing from older designs only in minor mechanical details of construction and appearance. In the past, emphasis has been placed on very clear fused silica of the highest transmission for the very short wavelengths because of the need in some cases of high intensity in the short wavelengths and because in other

FIG. 5—PHOTOPHONE LAMPS, 10-VOLT, 7.5 AMPERE T-8 BULB

cases of sales propaganda on the basis of appearance. The recent emphasis on the importance of the intermediate wavelengths in the ultra violet and the use in many cases of filters absorbing all radiation of a wavelength less than 280 or 290 milli-microns has encouraged a tendency to the use of less clear fused silica.

Neon Lamps. The principal development in high-voltage neon lamps has been in the use of smaller electrode areas than were formerly thought possible and in the construction of neon tube lamps of considerably higher current density than has ever before been used. This has resulted from attempts to use the lamps as landing field beacons and for high-intensity sign work.

New color effects have been obtained through improved filter glasses used as tubing materials, and in a few cases high-voltage discharge lamps have been made using gases other than neon and neon-mercury, but their use has not been widespread.

Hot cathode low-voltage neon tube lamps are being

regularly distributed for use either alone or in combination with mercury vapor tubes for use in box signs, photographic outfits for panchromatic work, and to some extent for beacon work on shipping docks and airplane landing fields.

The negative glow socket type of neon lamp which was mentioned in the 1926 report of this committee has undergone further development during the past year. The present form of this lamp is designed for operation on 115 volts, a-c. circuits and consumes about ⅙ of a watt, and also for 220 volts, a-c. or d-c., consuming ⅔ of a watt. The luminous output is of the order of 1/10 cp. Its field of use is primarily that of an indicator or night light. A larger lamp has been developed which will operate on 110 volts a-c. or d-c. and consumes approximately 2 watts, and has a luminous output of the order of ½ cp. The latter lamp is made up using an S-14 bulb and medium screw base.

Induction Lamp. The electrodeless discharge in neon, in the form of an induction lamp, has undergone development during the past year and is being used experimentally as a light source for airway beacons. A 3-in. clear fused quartz bulb incloses the luminous gas. The lamp under operating conditions has a brightness in excess of 40 candles per square cm. and when used at the focus of a 36-in. mirror, a beam candlepower of the order of 500,000 is obtained. Due to the selective emission of the neon induction lamp in the orange red part of the spectrum, this type of light is being tested for visibility in mist and foggy weather. Experiments carried out during the last year have indicated that this particular type of light has considerably greater penetration or visibility per unit of candlepower than has white light. It has been demonstrated that the neon beacon must operate considerably above 100,000 beam candlepower to excel the regular white airway beacon which operates at several million beam candlepower.

A recent test shows no difference between the visibility of light from a neon lamp and light of the same color and horizontal candlepower distribution produced by an incandescent filament lamp with a red glass filter. The comparison of the clear incandescent lamp and the incandescent lamp with red color filter shows that the red filter does not increase the range under any weather conditions. It always reduces the range but there is some evidence that the red filter does not reduce the range as much in foggy weather as it does in clear weather. It was, however, noticed that the flashes of the red beams were generally easier to find among shore lights, especially before the observers had learned just where to look for them.

Television Lamp. Progress during the past year has been made in regard to the flat type of television lamp. These lamps can now be made to have a d-c. breakdown of about 80 volts and will operate at a current density of 10 milliamperes per sq. cm., for several hundred hours life. The voltage drop across the tube at the above current density is of the order of 70 volts.

Carbon Arc Lamps. A number of types of arc lamps using impregnated carbons has been placed on the market for therapeutic use. While some of these are of the cheapest construction, others are very well made and provided with filtering glasses for proper ranges in the ultra violet spectrum.

Application of Light

INTERIOR ILLUMINATION

Industrial Lighting

The past year has witnessed an increasing acceptance on the part of production managers of high levels of general illumination. Among the new lighting systems installed were a goodly number in which 15 to 30 foot-candles of general illumination are obtained; in some instances higher illumination values of 40 to 60 foot-candles, over large areas, have been found economical.

Following closely upon the acceptance and installation of these higher levels of illumination is an increasing interest in special lighting for specific jobs. Group lighting, a system in which the lighting units are located with respect to the machine groups, has been adopted in a number of plants, especially those where the location of the machine groups is fairly permanent, and in which the direction of light desired is constant.

The development of group lighting applications requires characteristic distribution curves of light, which are necessary to obtain the greatest utilization over any given area; more and more attention is being paid to control of light to fit these requirements. The year has been truly one in which the important lighting installations may be characterized as "tailor-made" in contrast with the "rubber-stamp" methods of duplicating outlets throughout a work area.

Especially interesting is the tendency in lighting high bay areas, towards the use of higher and higher wattage lamps. Several of the reflector companies have met this demand by redesigning their large high mounting height units to accommodate 1500-watt lamps with a good efficiency as well as 750- and 100-watt lamps.

Notable also is the trend toward dust-proof lighting units, or cover glass appliances, for the 200- to 500-watt equipment. This is in the interest of ease of maintenance in the usual installation, and safety where lighting is used in dusty places. The increasing use of lacquer for coating wood and metal products has brought serious investigation of methods for lighting areas where these products are used, and the lighting equipments considered acceptable. Several large users of lacquer finishes have developed methods for lighting spray booths and spray tunnels, entirely from the outside.

Commercial Lighting

Mention was made last year that installations designed along modernistic lines were beginning to appear.

This tendency has greatly increased. While it is true that this new school of decoration originated in continental Europe, progressive American architects have at once seen its advantages, particularly when applied to the newer forms of American architecture. A more detailed analysis of this trend and its influence is given in a separate news note.

The entrances and lobbies of the new office buildings are striking and generally lighted in such a way as to take full advantage of the potentialities of present day light sources. There has not been much change in the methods of lighting of offices themselves, save a gradual growing appreciation of the benefits of high level, well diffused general illumination.

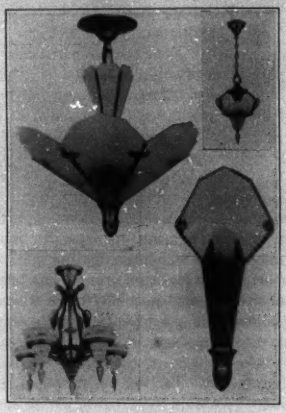

Fig. 6—Modernistic Styles in Wall Brackets and Center Fixtures

The merchant is always in search of the new to attract customers, and it is natural that he is one of the first to adopt the new school. Even in the small neighborhood type of store we are finding enclosing units along modernistic lines, although produced in commercial quantities. The distinctive shops have given us some wonderful examples of the use of light for decoration and even the department stores under construction are decidedly modernistic in feeling. On every hand are to be noted show windows with the decorations along modern lines and some very unique applications of lighted window furniture.

Residence Lighting

The modernistic trend in home lighting equipment has made a vast stride in the past year. American manufacturers are offering many varieties of fixtures closely resembling the designs which European artists have been developing. Decorative portables, wall brackets, and center luminaries are designed with pronounced geometric plane surfaces representing, in many instances, an assembly of rectangles, triangles, or squares of various light colored tints, plain shades, and artistic mixtures. These glass planes not only conceal the lamps from general view but also diffuse the light over a wide area, thereby creating a quality of illumination which illuminating engineers have been striving for years to introduce into the American home for wholesome lighting.

Following a wide development and design in portable decorative units, the domestic manufacturers have developed many new artistic designs of utility equipment for home lighting. Some of these recent types are illustrated in Figs. 6, 7, and 8.

Light Ornaments. A few years ago the use of light simply for decoration was encountered only rarely. Now one scarcely passes a single gift shop which does not display one or more forms of light ornaments. During the past year American manufacturers have developed some very commendable types, a few of which are shown in the accompanying illustrations (Figs. 9 and 10). These use cast figures against lighted background, translucent colored urns, cut crystal figures lighted from beneath, and even translucent porcelain. Other designs are under way and will be on the market shortly. The French manufacturers continue to bring out most pleasing designs, many of which cannot be economically duplicated in America. One of the leading manufacturers has established a local distributor who reports increasing appreciation on the part of the trade.

Fig. 7

Fig. 8

Theatrical Lighting

The development of the use of light in the theater during the last year has occurred chiefly in what is

known as the presentation house, the motion picture theater which shows more or less elaborate review numbers in addition to the motion picture. The great chains of theaters that have been taken over by the various large producing companies tend to call for standardized equipment in enormous quantities. Experience has shown that colorful displays which change weekly or twice a week can be considerably aided by elaborate lighting effects. Thus large amounts of good equipment and instruments of better design than was

Fig. 9—Group of Light Ornaments of Various Types Produced by American Manufacturers in the Past Year

practical under job lot conditions have been installed in the larger presentation houses.

The nature of the presentations calls for an arbitrary use of light such as has seldom been in the legitimate theater. Designers have been given more freedom and the results are proving that the public demands more extensive use of light than ever before. Managers and architects have begun to give more attention to lighting, and instrument manufacturers and switchboard companies, encouraged by this new market, are endeavoring to perfect their products.

This awakening of the appreciation of the public to lighting effects in the theater is confirming the start that the musical comedy and review producers made some time ago. However, the difference between the means of using light arbitrarily and realistically is so great that the development in the legitimate theater is very slow.

Motion Picture Lighting

A census taken about the first of February shows that of some sixty or more pictures in the process of being photographed, approximately 60 per cent were being made with incandescent lamps, an increase from 25 per cent as of July 1, 1928. The general practise of the studios has been to make their sound pictures entirely with incandescent lamps and to restrict the use of other illuminants to the silent pictures.

The Universal Studio has recently completed the filming of the picture "Broadway," taken from the popular New York success of the same name. The largest indoor set ever constructed was used for the cabaret scene. This scene, together with several others immediately adjacent and really becoming a single large set, was lighted entirely with 4800 incandescent lamps whose wattage totalled 3,900,000. The largest part of this energy was employed in regulation motion picture photographic lighting equipment. However, there were many thousands of lamps of lower wattage employed for decorative effects. This large set is indicative of a general trend throughout all of the studios to employ many low wattage lamps for decorative effects in large and miniature signs, in table lamps and wall brackets, and in automobile headlamps. Lamps are even being operated under water.

The 2000-watt G-48 bulb lamps in the 18-in. parabolic reflector housing and the 5000-watt G-64 bulb lamps in the 24-in. housing continue to be the most popular types of modelling lighting units, while the 1000-watt and 1500-watt PS-52 bulb lamps in high efficiency glass reflectors are extensively used for general illumination. Ten-kw. lamps in 36-in. parabolic reflector units are beginning to be used. One of the manufacturers of the 18-inch reflector unit has shortened the housing to about one-half its former length with the result that 30 per cent more light is emitted when the unit is adjusted for wider beam spreads. Another lighting equipment manufacturer is developing a large glass reflector for general illumination which has an improved distribution and operates the lamp in a more favorable position than the present types. This unit is also being

Fig. 10—Some Newer Designs of Light Ornaments of French Manufacture Now Being Used in the United States

designed so as to use the 2500-watt PS-52 bulb lamp which is now becoming popular where it is desired to increase the quantity of light without an increase in the number of units.

Installation Standards

For several years, illuminating engineers have been feeling in an increasing degree the limitation of inadequate wiring as an obstacle to the development of good lighting practise in commercial and industrial buildings.

From a study of the problem it was concluded that this situation and other problems could best be met

by cooperation on the part of central station lighting service bureaus with architects. In formulating a plan, certain officials of the American Institute of Architects were consulted, with gratifying response. Therefore, in the Fall of 1928 a committee of the National Electric Light Association started in on a program of constructive effort. Two courses of instruction for central station lighting service engineers have already been held at which pertinent technical information was reviewed. Distinguished architectural authorities brought out the architect's point of view and indicated methods of approach.

A set of model specification paragraphs has been prepared, to express the adequacy features of wiring so that the illuminating engineer can plan proper lighting. The method of specification contains some new features which facilitate checking up on the wiring construction work.

A third phase of the program is a plan of advertising, to emphasize to architects the importance of good lighting and interest them in cooperation with the lighting experts of the central station companies.

Based on experience in Chicago, the Society for Electrical Development has adopted a simplified method of planning and figuring the lighting installation in accordance with present day standards This is known as the Franklin Red Seal Lighting Specification, the rules being published in pamphlet form, and provide for all of the ordinary conditions in that field.

A supplementary section suggests the necessity of adequate wiring and recommends wiring corresponding to that specified for the N. E. L. A. architectural activity.

EXTERIOR ILLUMINATION

Street Lighting

Street lighting practise is progressing slowly but at a rate which with a few sporadic exceptions is inadequate to meet the rigorous demands of modern high speed traffic. The average light output of incandescent street series lamps sold during 1928 was 5.8 per cent higher than the output of such lamps sold during 1927 (Report of Lamp Committee, N. E. L. A.). This is one indication of progress since the use of larger lamps suggests improved street lighting. Yet it is the general testimony of the best informed engineers that a marked advance not only in the amount of light produced for street lighting, but also in the skilful application of that light, is required before street lighting practise may be considered to be abreast of modern requirements.

The outstanding development in street lighting is the increasing recognition of the importance of a comprehensive street lighting plan prepared in coordination with city planning at large and particularly with regard to street traffic problems. The practise of classifying projected street lighting plans by stating the illumination required for streets of different character is also a growing tendency which promises a more definite and scientific treatment in the future.

The value of steel tapes on underground cables is questioned and several manufacturers have introduced cables having no metallic coverings. Others are lead encased but have no covering of magnetic material. This was brought about due to the increasing tendency to use 20-ampere constant current circuits rather than circuits operated at 6.6 amperes.

Occasional High-Intensity Installations. A few notable installations of outstanding character which have been described during the past year are listed in the following table.

These installations are examples of high-intensity street lighting added during the year to the few instances recorded in the 1927 report of this committee. They constitute the exception rather than the rule, but are examples of successful installations meeting the requirements of artificial lighting of important streets.

Two systems of remote control for street lighting circuits utilizing high-frequency carrier-currents superimposed upon the low-frequency current in the poor conductors have been perfected and put into service during the past few years.

One of these systems employs a control current having a frequency of from 35 to 45 kilocycles per second generated by vacuum tube oscillators in the substation and transmitted over the high-voltage power conductors to tuned vacuum tube receivers located at the control centers of the various street lighting circuits. The receiver is tuned to a single frequency and through a time selector relay it opens or closes the street lighting circuit, depending upon the duration of the impulse sent out from the substation. Installations of this system have been made in Schenectady, Rochester, and Glens Falls, New York; Bayonne, New Jersey; Detroit, Michigan; and San Francisco, California.

Location	Extent	Lamps per post Lamp size in.		Mounting height	Spacing
		Lumens	Watts		
Los Angeles (Wilshire Blvd.)	393 posts	25,000/6,000*		22 ft.	120 ft. apart, either side of street
Chicago (projected for Wabash Ave.)			Twin 2000 w. lamps	28 ft.	8 posts per block
Detroit (Washington Blvd.)	28 posts		Five 1000 w. lamps		100 ft. apart in two rows
St. Louis					
"Intensified" areas	†	25,000 to 15,000		26 ft.	
"Intermediate" areas		10,000		19 ft.	

*Alternate lamps placed in use respectively before and after midnight.

†Entire project involves relighting of about 650 miles of streets, using about 40,000 lighting standards. Installation about 80 per cent completed in April, 1929.

The second system has been applied particularly to the control of multiple street lamps operated from commercial lighting circuits. The control current is generated by a two-speed motor-generator set in the substation and may have a frequency of either 480 or 720 cycles per second depending upon whether the lamps are to be turned off or on. As in the other system, the control current is transmitted over the high-voltage power conductors. In this system, however, there is a receiver in the base of each lamp post consisting of two tuned relays which open or close respectively a main relay through which the lamp is energized. A recent installation of this system of control has been made in Boston following an experimental installation in New York.

In both of the systems described the provision of a time-delay element removes the possibility of response to lightning or switching surges. Also in both cases an interruption of power does not alter the position of the main relay controlling the lamps, so that upon the restoration of power the lighting circuit is in the same condition as before the failure. The advantages of the carrier-current method of remote control of street lamps arise principally from its use of the power conductors themselves for the transmission of the control impulses. This feature is said to result in the enhancement of reliability (due to the high mechanical and dielectric strength of the power conductors), of flexibility (due to the possibility of transmitting control impulses from a single carrier-current generator over more than one feeder to any desired number of receivers located at any desired distance from the substation) and economy (due to the elimination of separate control circuits).

On the other hand accidental grounds, cross lines, and broken conductors on the power circuit will affect the transmission of the control energy just as they interfere with the transmission of low-frequency power. However, in the event of failure of power to energize the lamps, the lack of control of the lamps does not seem of great consequence.

It is coming to be recognized increasingly that the only thoroughly satisfactory solution of the automobile headlighting problem is to provide street or highway lighting sufficient to permit safe and expeditious transit without employment of powerful far reaching headlights, and at least with depressed headlight beams. In the case of interurban highways the problem of providing fixed lighting is receiving much attention. Where needed, enabling acts to permit counties and villages to provide for highway lighting are being sought. The problem of providing pole lines for highway lighting is, of course, inevitably associated with the problem of rural electrification in general. Progress has been made in reducing the cost of pole lines.

In this connection reference may be made to last year's report of this committee in which there was reproduced a resolution adopted by the Committee on Street Lighting of the Illuminating Engineering Society, asserting the practicability of so lighting traffic thoroughfares as to permit dimming or depressing of headlight beams. Highway lighting units which give satisfactory service in such installations are illustrated in Fig. 11. Wherever highway lighting employing modern types of units is installed in accordance with good practise, it is claimed that road capacity is increased, night driving is made more pleasant and safe, and property values along the lighted highways are enhanced. Good progress in the installation of fixed highway lighting is anticipated during the next two or three years.

Traffic Signal Lighting

In the year 1928 much was done towards standardization of traffic signals. The report of the Committee of the American Engineering Council on Street Traffic Signs, Signals, and Markings was finally completed and has been issued in pamphlet form. This report is very complete and does a great deal to lead to standardization of traffic signal practises.

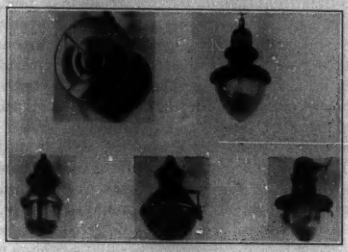

Fig. 11—Modern Types of Highway Lighting Units

In its formative state, this report seemed to favor very strongly the two-color traffic signal, but in its final form the report recognizes that the two-color signal cannot always be substituted for the three-color signals, but it does standardize the meanings of its colors. In addition to this, it publishes designs of safety signs and classifies them, setting aside certain color combinations for restriction signs, other combinations for information signs, and still other combinations for caution signs, all of which will be very helpful if carried out by various cities and state highway departments, since the mere color of a sign will tell the cautious motorist to expect certain information.

Certain states in the Union are taking the matter of traffic signals seriously, with a view to uniformity of laws in the several states. The two most notable examples of this class of legislature are the traffic laws of New Jersey and New York. New Jersey law Chapter 281, Laws of 1928, is a very complete statement of the traffic situation, and is almost in complete accord with the report of the American Engineering Council. This

law was introduced on June 4, 1928, and was approved on July 14, 1928.

On January 8, 1928, New York State enacted Law 46,722 and here again this law is in fairly complete accord with the report of the American Engineering Council. Both of these laws cover not only traffic signals but general traffic ordinances, and these two, together with the "Model Municipal Traffic Ordinance" of the National Council on Street and Highway Safety, published on August 1, 1928, will go a long way toward clearing up the traffic situation and forming a background for various cities and states to enact uniform sensible laws.

Very little change has occurred in traffic signals themselves since these had reached a very high state of development prior to 1928. During the year, however, traffic control apparatus took another step forward in that practically all manufacturers developed control apparatus for a full coordinated movement of traffic. These systems consist of a master timer at Control Headquarters and a secondary control at each intersection, the master timer acting upon the secondaries to control the total period, and the secondaries being capable of controlling, first, the split in time between north-south and east-west traffic; second, the amount of offset to give the coordinated movement; third, the duration of the amber period following the north and south movement; and fourth, the duration of the amber period following the east and west movement.

There was a tendency during the year to push the installation of this class of apparatus along thoroughfares where such timing equipment could be used to advantage. It should be borne in mind, however, that it is impossible to use the coordinated movement of traffic to its full advantage unless the street upon which it is installed has sufficient width to permit the uninterrupted movement of at least one lane of traffic in each direction, the traffic in this lane not to be interfered with by street car loading or by a vehicle getting into or out of a parking space. Its fullest efficiency is thus obtainable only on streets from 60 ft. to 100 ft. wide, or upon narrower streets where there are no street cars or no parking.

Synchronous motor control apparatus also reached a fairly high state of development during the year 1928, the control apparatus becoming much more reliable, and it can now be installed with the reasonable assurance that the timers will keep fairly close in step. Such timers are in the main limited to a single total period; they have the advantage that it is possible to install a system using these timers without interconnecting wires, but it is necessary either to let them run all night or shut them down individually. Such control apparatus finds its best use in smaller cities and along highways connecting the cities with outlying points where it is highly advisable to permit signals to operate twenty-four hours a day.

There was a marked tendency during the year 1928 for the use of curb mounted signals rather than signals installed in the roadway or suspended over the center of the roadway, the most popular signal being the one-way signal mounted on the far right corner. In highly congested areas where double signal indication is advisable, the use of two-way signals with faces 90 deg. apart on the far right corner of the intersection was found to be advantageous.

Portable Flasher. There is being brought out a portable electric flasher designed primarily for traffic warnings where kerosene lanterns have been extensively used. It has, however, many other possible uses in connection with both land and water traffic. Although the initial cost of the beacon is considerably more than that of an oil lantern, its maintenance cost is

Fig. 12—Portable Flasher for Danger Signals

so much less that in the long run it provides the most economical warning signal. It is estimated that the cost of maintaining an oil lantern is approximately $50.00 per year, per lantern, where quantities of lanterns are used almost every night and where each lantern must have daily attention for filling, cleaning, etc. The new flasher on the other hand, where one set of batteries will last approximately two months on a basis of continuous burning, will have a maintenance cost of approximately $10.00 per year.

In addition to this very considerable advantage in maintenance cost, the portable flasher (Fig. 12) has the distinct advantage of the flashing feature, which cannot be obtained with oil lanterns. Traffic and railway authorities agree that a flashing signal is a much better warning beacon than a steady burning light. The weight including batteries is only 16½ lb., which makes it readily portable.

This device requires four standard 6-in. dry cells and a 6-volt., 0.15-ampere lamp. An extra 6-volt lamp is mounted inside the battery housing. The flashover is

of rugged construction throughout and entirely weatherproof.

Automobile Headlighting. Committees of the Illuminating Engineering Society and the Society of Automotive Engineers have been active during the past year considering such subjects as Specifications for Headlamp Mountings, Specifications for the Test of Rear and Signal Lamps, and Comments on the Existing Specifications for test of Headlamps of the dual beam type.

The outstanding achievement during the past year in motor car lighting has been the adoption of fixed focus construction for headlamps. Various investigations have shown that the principal cause of glaring lights has been improper focusing of the lamps. The average car operator understands fairly well how to aim the beams so as to throw the light down onto the road but it seems almost impossible to explain satisfactorily the operation of focusing. With the widespread use of the fixed focus type of headlamp only the aiming operation remains and it is believed that the average car owner will be able to do this reasonably well. This type of equipment is standard on two very large production cars and it will be of interest to note just how great a contribution this type equipment will make towards the reduction of troublesome glare.

In connection with the enforcement activities looking to a better adjustment of headlamps, several states have carried on "Save A Life Campaigns." These campaigns were inaugurated by a proclamation issued by the Governor of the state designating a certain period of thirty days for the campaign and requesting the cooperation of all motorists. Official inspection stations were appointed and in addition to the adjustment and checking of lights, such items as brakes, horns, mirrors, and steering gear were also included. It is the general opinion that these "Save A Life Campaigns" have been very successful in encouraging motorists to keep their cars in better operating condition.

Sign Lighting

The two schools of Electrical Advertising sponsored by the National Electric Light Association in the Spring of 1928 were so successful that it was decided to try out local schools in the geographic divisions. Three of these schools have already been held, New England Division in Boston, Mass., Great Lakes Division in Chicago, Ill., and East Central Division in Cincinnati, Ohio. Others are planned for the Fall.

The Society for Electrical Development, Electrical Advertising Section, is bringing out a Plan Book suggesting methods by which public utilities can stimulate electrical advertising.

The new S-11 sign lamp, supplied in colors and with the intermediate screw base, has been used in some excellent, small spectacular signs where motion and color are featured.

There are several new installations of the so-called "Talking Signs" and one of these was erected for a large newspaper so the current news could be given out as received.

A smaller sign of this same general type has been developed with gas filled miniature lamps. The message travels across a bank of lamps and this effect is produced by a belt carrying the message punched in it and passing over contacts in the bottom of a mercury bath.

Still another small sign using miniature lamps has been brought out. This consists of a face with a multiplicity of equally spaced openings. The sockets to hold the lamps have two prongs each which can be inserted in the openings and are held by spring contacts. Any message or picture desired can be built up in this way.

Gaseous tube signs are being sold in quantities and quite a number of combined lamp and tube signs have gone in. Some of the most effective of these are painted bulletins. The letters may be of tubes, borders of lamps, with angle reflector lighting added if desired. Naturally, a combination sign of this kind has many advantages among which is a better power factor than for the tube alone.

Fig. 13—Street Lighting Standards Equipped with Special Reflectors to Direct the Light towards the Buildings

Lighting of Building Exteriors

Floodlighting is being used more extensively to bring out the architectural beauty of offices and public buildings at night. In the larger cities new buildings are generally erected with step back construction at the higher levels and these offsets make the use of floodlighting particularly effective.

Architects are also showing a greater appreciation of the value of floodlighting with the result that in some instances the exterior lighting of the buildings is considered at the time the designs for the building are made. Colored lighting is receiving more attention, and the combination of color and motion in what has been called

mobile color lighting is unfolding new possibilities for attractive night displays in building decoration. There is a very decided trend toward the use of colored lighting either with or without motion as this type of lighting has much greater possibilities than the use of plain white light.

Floodlighting of buildings not exceeding 100 ft. in height is frequently being done by means of combination street lighting standards that are equipped with special reflectors to direct the light towards the buildings. (Fig. 13) The ornamental standards are mounted at the curb, and generally use a 1000-watt lamp

Fig. 14—Street Lighting Standards Equipped with Two Lanterns in Order to Satisfactorily Light Large Buildings

for floodlighting the building, and a 200-watt or 300-watt lamp to light the interior of the lantern and to aid in lighting the lowest portions of the building. In general, this type of lighting produces satisfactory results where the standards are spaced not to exceed twice the width of the sidewalk, measured from building line to curb. For large buildings it is at times found advisable to place two lanterns on one standard in order to obtain a satisfactory amount of light on the buildings. (Fig. 14)

This method of floodlighting is being very extensively used, one Central Station Company alone, the Public Service Electric and Gas Company of New Jersey, having put in service 30 installations of this type during the past year. Among outstanding floodlighting jobs the following are noteworthy:

New Jersey Bell Telephone Company Building at Newark, N. J.; the upper floors and set-backs of this building are floodlighted in white, amber, and red color. The installation has a connected load of 51 kw. and presents a very striking appearance at night. (Fig. 15) In Brooklyn, New York, the E. R. Squibb Building changed over its floodlighting from white light to colored light; in the same city, the new building of the Williamsburg Savings Bank has a floodlighted dome and uses neon tubes in conjunction with the regular floodlighting, the total connected load being about 100 kw. The upper ten floors of the Rand Building at Buffalo are floodlighted in red, blue, and white lights with a connected load of 144 kw.

Other notable installations are the New York Central Building, New York City, the Fisher Building, Detroit, Cleveland Union Terminal Building, Cleveland, the Chanin Tower Building, New York City, the City Hall, New York City, and the J. H. McClutchey Building, Philadelphia.

Lighting of Railroad Yards

The importance of floodlighting railroad yards is reflected in the increased use of this system during the past year. Sixteen-inch projectors each equipped with a 1000-watt 115-volt, floodlight lamp mounted in banks on towers averaging 75 ft. in height have been used in the majority of installations. Both the unit-directional and parallel opposing systems are used, the method employed depending on local conditions and the opinion of the designer regarding its satisfactory application.

The Committee on Illumination of the Association of Railway Electrical Engineers initiated a study of the photometry of floodlight projectors, but this investigation has been turned over to a sub-committee of the Lighting Service Committee of the Illuminating Engineering Society and National Electrical Lighting Association. This subcommittee has continued its studies, making a few revisions and additions to the recommended procedure in photometry.

Fig. 15—New Jersey Bell Telephone Company Building at Newark, N. J. The Upper Floors and Setbacks of this Building are Floodlighted in White, Amber, and Red Color

Lighting for Aviation

Beacons and course lights have been installed on approximately 9000 miles of the national airways by the U. S. Department of Commerce and about 2000 additional miles will be lighted by the end of June, 1929.

Where course lights are mounted on beacon towers, they are equipped with a red color screen when no landing field is near by and with an amber color screen when a suitable landing area is in the vicinity.

The Airway Marking Committee appointed by the U. S. Department of Commerce to study and standardize a system of national airway marking for the guidance of fliers has recently made public its report. In addition to prescribing definite sizes of letters, figures, and symbols for roof signs, recommendation was made that they be illuminated to an intensity of 10 foot-

Fig. 16—Night View of Hangar at Newark Metropolitan Airport Lighted by a System of 200-Watt Mazda Lamps in R. L. M. Standard Dome Reflectors and Standard Angle Reflectors

candles and preferably 15 foot-candles for adequate visibility at night. For exposed lamp signs, the wattage and color of lamps, together with suitable spacing, were listed according to the size of the sign. (Fig. 16)

Private aeronautical revolving beacons not legally listed on airway routes must be certified by the U. S. Department of Commerce, which also requires that a stationary projector be installed near the beacon with its beam of light pointing toward the nearest airport. This projector must provide a beam divergence not exceeding 6 deg. and a maximum candlepower of two million. A 24-in. searchlight projector with a 1000-watt, 115-volt T-20 bulb airway beacon lamp will usually meet the requirements.

There has been much activity in the past year equipping airports for night flying with a view to obtaining an "A" rating from the U. S. Department of Commerce, which has issued Aeronautics Bulletin No. 16 explaining the requirements in detail. It is particularly interesting to note that high intensity incandescent lamps, such as 1.5-, 3-, 5-, and 10-kw. Mazda lamps have been used as light sources for field illumination in the great majority of installations. Such installations are usually provided with remote control switches, permitting the location of floodlights at a suitable distance from the point of control. The 1.5- and 3-kw. 32-volt lamps are generally used in projectors of the 24 in. size and the 5- and 10-kw. 115-volt lamps are more often employed in the dioptric lens unit. (Fig. 17)

The U. S. Department of Commerce has suggested the use of an auxiliary green flashing beacon at airports to assist in the identification at night. The details on a satisfactory 5000-cp. unit which will be visible in the upper hemisphere have not yet been completed. The rapid development in the construction of super-airplane for carrying passengers has brought about an improvement in cabin lighting which is decorative. Candelabra fixtures with amber coated lamps are mounted one beside each seat in the cabin. Small enclosing domes are also mounted on the ceiling above each row of seats. Several types of fixed and retractable airplane headlights used for emergency landings at night have been developed during the past year. Most of these headlights, which are usually equipped with a 9-in. reflector, employ the standard 420-watt 12-volt headlight lamp.

The retractable headlight may be operated on the retracted position with beam pointing downward for pick-up and can also be used in any intermediate position to forward, where it is usually held when contact is made with the ground. The fixed headlights are usually stream lights in the wings to reduce air resistance. A smaller headlight with a 6-in. reflector has recently been developed using a 100-watt 12-volt A-19 bulb lamp with intermediate profocused base.

Fig. 17—Night View of Landing Area at Newark Metropolitan Airport Illuminated by 3000-Watt, 32-Volt Mazda Lamps in 24-In. Airport Floodlight Projectors

The subject of airplane headlighting is being studied by a committee of the Society of Automotive Engineers.

Lighting for Night Recreation

Under the conditions imposed by the busy American life, it becomes more and more necessary to seek recreation and exercise after the close of the regular business day, which usually means during hours of natural darkness. Coupled with this condition is the fact that many sports are more pleasant in the cooler evening hours and since night lighting becomes increasingly

perfected and less expensive, there has quite naturally been a large increase in the applications of illuminants for night recreation.

Night tennis has been proved to be perfectly feasible and in prolonging the usage of a tennis court, the investment in lighting equipment is much less than the investment in one or several additional courts. Large metal reflectors, now made more highly efficient and more resistant to the elements with the advent of chromium plating, can be suspended at a sufficient height to eliminate glare and to provide on the order of 30 to 50 footcandles over the playing area. A typical tennis court represents a load of approximately 450 kw-hr. per month and from the revenue standpoint, is equivalent to the lighting of ten average homes.

The lighting of golf putting greens has been somewhat extended during the past year and at least one case is on record of the complete illumination of a section of a golf course for practise driving. During the football season, several large universities, particularly in the south, played night games successfully, while night baseball, although not yet entirely practical, is being played on at least one illuminated diamond. Hundreds of outdoor bowling greens and similar amusement areas are successfully lighted, while the past winter saw a number of well illuminated outdoor skating rinks and toboggan slides.

During last summer, night swimming in both outdoor and indoor pools grew in popularity coincident with practical development of a suitable sealed projector available for mounting beneath the water surface,—a practise which adds greatly to the beauty of the pool and, in a large measure, guards against the possibility of accidents or water pollution. Recreation parks now depend to a greater extent than ever on the night lighting decorations. The inside colored lamps and the addition of the 10-watt S-11 bulb sign lamp have been valuable incentives in this application. Unique figures, color combinations, and mass effects of low wattage lamps are taking the place of plaster and wood ornaments and colored paintings.

The floodlighting of ocean beaches has developed, this application being quite similar to the floodlighting of airplane landing fields and using the same apparatus. In these cases there has been considerable employment of the 5- and 10-kw. 115-volt Mazda lamps in both drum projectors and Fresnel lens units. Of more recent date, the 1½- and 3-kw. 32-volt lamps have come into extensive use in similar units. Colored floodlighting of trees and shrubs, of fountains and waterfalls, and the use of colored miniature lamps of the Christmas tree type in beds of flowers, have all added to the usage and popularity of parks and gardens.

Decorative lighting at Christmas time continues to grow and during the past winter, a nation-wide contest sponsored by the Society for Electrical Development brought forth many remarkable installations. The 10-watt multiple Christmas tree lamp and strings of sockets therefore began to come into quite universal usage for outdoor Christmas decorations. Considering both series and multiple lamps, there were more than 45,000,000 lamp bulbs used in 1928, equivalent to some 9,500,000 40-watt lamps,—a greater lighting load than the entire American output of electric lamps only a few years ago. Activities are under way to extend the possibilities of Christmas decorative lighting to other festival occasions and other seasonal celebrations.

Lighting of Bridges and Tunnels

Since the opening of the Holland (Hudson River) Vehicular Tunnel, a new standard for the lighting of underground vehicular passageway has been established which is being followed by several similar installations, notably the Zion-Mount Carmel Tunnel in Utah and the Detroit Vehicular Tunnel between that city and Windsor. An item of interest may be recorded, namely, that higher intensities of roadway illumination without glare very materially speeds up the traffic and quite directly returns a greater revenue from the investments in the complete structure. This same fact is now recognized in connection with bridge traffic and it has been quite well recognized that bridge lighting should be on the order of two to three times the intensity of the connecting highways. The floodlighting of bridge structures, especially the approaches thereto, has resulted in considerable beautification of water fronts and incidentally afforded more safety to aviation pilots.

Although not of the man-made variety, the lighting of the famous Natural Bridge of Virginia in mobile color has made this one of the notable bits of night scenery in America. Such applications point the way to the possibilities of making such natural wonders more beautiful by night than by day and more available for enjoyment by the average tourist, who prefers to travel during the sunlight hours and stop for rest and recreation during the evening.

Underwater Lighting

Considerable interest has been displayed in the application of electric incandescent lamps to under water illumination. The 1000-watt 115-volt G-40 bulb special diving lamp which is self-contained and designed to be operated directly immersed in the water is standard equipment for all capital ships in the U. S. Navy.

Luminous fountains have been constructed with light projectors mounted beneath water-tight glass plates or with separate water-tight self-contained projectors located in the water or at the base of the fountain jets.

Swimming pools in which the water is the sphere of activity but which have never been properly illuminated due to the reflection of light from the surface, have been illuminated in large numbers by means of lights located in the pool wall beneath the water line. Various types of equipment are now available, some consisting of

lamps mounted behind glass plates, others being water-tight metal units moulded in the pool wall, and still others (Fig. 18) being water-tight self-contained units operating completely surrounded by water but located generally in niches prepared for them in the pool wall.

The underwater lighting of swimming pools is of interest because it improves the cleanliness and increases the safety, and, in general, promotes interest in swimming. In outdoor pools, it is almost compulsory because there is practically no other way to illuminate

Fig. 18—Underwater Lighting of the Swimming Pool at Hart House, University of Toronto. Lighted with 20-Aqualux 400-Watt Units

the pool without using more or less unsightly apparatus overhead.

Fig. 19 shows one of the newest pieces of equipment for underwater lighting service. It is a universal fixture suitable for many types of service and may be installed in many different ways.

ULTRAVIOLET RADIATION

As mentioned under "Production of Light" there has been considerable activity in the field of sources and applications of ultraviolet radiation. Many units employing either mercury or carbon arcs have been placed on the general market. It is too early to state definitely the exact status of the value of these various units, but it appears that when properly handled they will give beneficial results similar to those obtained from sunlight under comparable conditions.

One of the better grade arc lamps employs a screen of Corex D, transmitting between 50 per cent and 65 per cent of the ultraviolet light of wavelength 0.302 mi. This lamp generally employs a therapeutic A carbon for home use but is capable of using therapeutic C carbons giving less visible radiation, but having considerable intensity in the shorter wavelengths.

A great deal of work is under way in the field of development of filters and methods of using the radiation as a preventive as well as a curative method.

Photography and Photo-Chemical Processes

The extraordinary progress during 1928 in methods of reproducing sound in conjunction with motion pictures has been made possible in no small degree by improvements in the design of photoelectric cells. Method of recording sound may be classified in two types according as they produce (a) variable density records consisting of bands of different density (or blackness) and constant width, and (b) variable area records of constant density known as "saw-tooth" records. All photographic sound recording methods use the photoelectric cell for reproduction of the sound.

The fundamental principles underlying the photographic methods of sound recording may be stated as follows: Electrical impulses produced by the microphone are converted into light modulations either by means of a glow lamp or with a constant light source in conjunction with a light valve or a Kerr cell. The modulations are recorded inside the perforation area alongside of either the same film on which the negative picture is recorded or else on a separate film. On the positive print used for projection in the theater the sound record appears along one side of the picture area. To reproduce the sound, light from a concentrated filament lamp is projected through the film on to a photoelectric cell. The electrical currents set up by the cell are amplified and led into a loud speaker.

The methods of recording sound have resulted in the introduction of motion picture films having sensitive

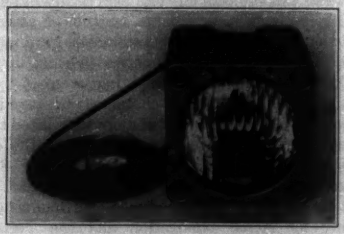

Fig. 19—Westinghouse Underwater Projector with 30 Deg. Horizontal Spread Lens and 250-Watt G-30 Floodlight Lamp

emulsions particularly adapted to the specific type of sound records. A precise developing technic is also necessary for the processing of sound films with variable density records.

A very thin motion picture film called Ozaphane film is being produced commercially in Europe. It consists of a light-sensitive diazo compound incorporated in a viscose layer. In addition to the diazo compound a coupling agent and a weak organic acid are present. The acid prevents the formation of a dye by the action of the coupling agent on the diazo compound, but if the acid is neutralized by treating the film with ammonia

gas, the dye is formed. Light decomposes the diazo compound so that no dye is formed and the acid is later neutralized. Positive images may therefore be produced directly by this process. The film is much less sensitive to light than the usual silver halide emulsions and therefore requires longer exposures.

INTERNATIONAL COMMISSION ON ILLUMINATION

The last year was a very important one for the U. S. National Committee, inasmuch as the American meeting of the International Commission on Illumination was held then. The committee was fortunate in enlisting the good offices of the Illuminating Engineering Society in making arrangements for this meeting. The meeting was attended by about sixty delegates from twelve different nations and was held in Saranac Inn, New York, September 22-28 inclusive.

Subsequent to the meeting, the delegates were taken on a tour which included New York, Boston, Philadelphia, Cleveland, Detroit, Chicago, Niagara Falls, concluding at Toronto where the delegates attended the annual convention of the Illuminating Engineering Society.

The commission has some fifteen technical committees, all of which met at Saranac Inn and considered recommendations and reports from the various national committees with a view to formulating recommendations for adoption by the Commission.

The technical papers at this meeting covered a very wide variety of subjects, the practical details of lighting practise receiving a very large amount of attention. Discussions of street lighting, automobile headlighting, factory and school lighting, residence lighting, aviation field lighting, show window lighting, were of a very thorough-going and valuable character and were interspersed with other discussions of more specialized topics of interest to the photometrists, etc. To the other technical committees for which the United States National Committee exercises the secretariat were added two new ones, *viz.*, the Technical Committee on Lighting for Aerial Navigation and the Technical Committee on General Practical Applications of Illumination.

The proceedings of the meeting which are now being printed will make a volume of upward of 1500 pages and constitute a very notable summary of the present state of the art and science of illumination as it exists not only in this country but throughout the world.

The next meeting of the Commission is to be held in 1931 in England.

The committee wishes to acknowledge the cooperation of the following men not members of the committee: E. W. Beggs, H. S. Broadbent, R. E. Carlson, R. W. Cost, C. Dick, S. G. Hibben, H. C. Rentschler, Geo. H. St. John, and R. L. Zabour of the Westinghouse Lamp Company; R. E. Farnham and James M. Ketch of the National Lamp Works of the General Electric Company; A. L. Broe, E. B. Fox, A. L. Powell, and G. F. Prideaux of the Edison Lamp Works of the General Electric Company; C. A. B. Halvorson of the General Electric Company, West Lynn, Mass.; H. P. Gage, of the Corning Glass Works; Loyd A. Jones of the Eastman Kodak Company; K. W. Mackall of the Crouse-Hinds Company; R. D. Mailey of the Cooper Hewitt Electric Company; S. R. McCandless of the School of Fine Arts, Yale University; and C. H. Sharp of the Electrical Testing Laboratories.

Discussion

F. C. Caldwell: One part of this report deals with the lighting of building fronts. The importance of this is due to the fact that in a modern well-lighted business street much more light is used than is needed for safety in traffic, and the additional electric power is justified in order to increase the attractiveness of the street. It has been well established that the best way to do this is to increase the illumination on the building fronts, thereby lighting up the whole street rather than only the pavement.

In this connection the use of asymmetric refractors designed to throw the light on the buildings across the street may be mentioned. With the ordinary symmetrical lighting unit, there is a bright spot on the building adjacent to the lamp, and opposite the lamp, and then comparatively low illumination up to the next unit. What is needed is a fairly uniform illumination. If a refractor is used which takes some of the extra light that is thrown on the building adjacent to the lamp, and throws it across the street on to the buildings on the other side, better results are obtained.

This report gives little consideration to the enormous increase in the use of the gaseous tube signs. This is certainly one of the notable features of the past year and perhaps should have somewhat more emphasis than is given here.

L. M. Duryea: In Southern California, we find universal use of neon tubes for sign lighting. When we compare this with New England, where there is comparatively smaller use of neon, we wonder if there isn't a very direct relationship between the use of neon tubes and climatic conditions. When neon tube lighting can be made to withstand severe climates, and when static condensers are installed across the terminals of the high-voltage transformers inside the signs, by the manufacturers, to aid in the important power-factor problem of these signs, then possibly the use of neon tubes in the East will become as universal as it now is in the West. Until these things are done, it is evident that the use of neon in the East will be greatly retarded. The incandescent lamp signs will continue to offer very keen competition in spite of the greater novelty and better advertising value of neon-tube display.

Applications to Marine Work

ANNUAL REPORT OF COMMITTEE ON APPLICATIONS TO MARINE WORK*

To the Board of Directors:

The activities of the Committee on Applications to Marine Work this year were devoted chiefly to the consideration of future revisions in the Marine Standards (A. I. E. E. Standard No. 45) and in further efforts to induce the U. S. Steamboat Inspection Service to arrange for proper recognition and classification of the electrical engineer on shipboard. Owing to the few active matters for consideration by the committee, and the unusual activity in new marine construction placing a large demand upon the time of most of the committee, the meetings of the committee were curtailed.

For certain reasons not pertinent to this report, the committee's progress with the U. S. Steamboat Inspection Service has been extremely slow, even though the committee has done considerable constructive work to assist in bringing about a solution. Apparently, certain obstacles have been encountered which will have to be economically dispensed with if our efforts are to succeed; however, the committee is hopeful of a final solution which will obtain for the electrical engineer the recognition and classification which he deserves. It is recommended that the committee pursue this activity with unceasing efforts and possibly through other channels.

*COMMITTEE ON APPLICATIONS TO MARINE WORK:

W. E. Thau, Chairman,

Edgar C. Alger,	Wm. Hetherington, Jr.	Wm. H. Reed,
R. A. Beekman,	H. L. Hibbard,	Edgar P. Slack,
H. C. Coleman,	A. Kennedy, Jr.,	H. M. Southgate,
E. M. Glasgow,	J. B. Lunsford,	Oscar A. Wilde,
H. F. Harvey, Jr.,	E. B. Merriam,	J. L. Wilson,
C. J. Henschel,	I. H. Osborne,	R. L. Witham.
	G. A. Pierce.	

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

The past year has seen the following major marine activities in which electricity has contributed its full share:

1. The placing in service of five turbine electric drive U. S. coast guard cutters. These ships use the dual-drive system for the auxiliaries.

2. The placing in service of the 18,000-ship hp. twin screw turbine electric drive passenger vessel, S. S. *Virginia*, a sister ship to the S. S. *California*.

3. The construction of a third 18,000-ship hp. turbine electric drive passenger vessel, S. S. *Pennsylvania*, a sister ship to the *Virginia* and the S. S. *California*.

4. The construction of two 14,600-ship hp. twin screw turbine electric drive vessels for the Ward Line.

5. The construction of a 12,600-ship hp. twin screw turbine electric drive vessel for the Grace Line.

6. The placing in service of three 4000-ship hp. single screw Diesel electric drive cargo ships: M. S. *Courageous*, M. S. *Triumph*, and M. S. *Defiance* by the U. S. Shipping Board.

7. Several smaller craft utilizing Diesel electric drive.

The results of the Jones-White Act are already being felt by the marine industry. A few large ships taking advantage of the provisions of this Act are in the course of construction and contracts for several large vessels are pending, most of which will probably employ turbine electric drive.

The use of electric auxiliary machinery is established, and in most cases the auxiliaries throughout the ship are driven electrically.

29-121

Applications to Mining Work

ANNUAL REPORT OF COMMITTEE ON APPLICATIONS TO MINING WORK*

To the Board of Directors:

In the past many criticisms have been made against the mining industry as being backward in modern developments. Certainly such criticisms are no longer justified to the same extent as a decade or two ago.

During the past year there have been many developments of electrical apparatus for mining work and the operators have not disregarded the applications offered. In fact, the operators have sought relief in every possible way to reduce operating costs. Overdevelopment during the war has not been absorbed by increased consumption of coal. This is largely due to rapid improvement in efficiency in large electric power plants, electrical interconnection, and in improved railroad performance.

Metal mines have been similarly affected from overdevelopment and lack of increased demands.

It is generaly predicted that both coal and metal mining are entering a new era of development. Further mechanization is considered as a solution to the problem of cost reduction. Mechanization implies further electrical application to all types of mining equipment.

A few years ago there was considerable discussion on superpower systems. Without a great deal of publicity these superpower systems have developed until now practically every industrial and mining field is served from high-voltage interconnected lines receiving energy from large efficient power plants. Thus it is logical that the major part of the further electrification and mechanization of mines will be served from the electric utilities.

Hoists. Due to high cost of equipping shaft mines with electric hoists the change over from steam to electric is slow. One such change over a medium capacity mine is worthy of note, not for its size but for the rapid cycle. A 750-kw. synchronous motor-generator set supplies an 850-hp., 116-rev. per min. motor through Ward Leonard control.[1]

The hoist operates in balance and is designed for 230 trips per hour from a depth of 282 ft., with a maximum rope speed of 3300 ft. per min. The cycle allows for 5 sec. acceleration, 1.65 sec. full-speed running, 5 sec. retardation, 4 sec. caging. Actual performance of 258 trips per hour has been attained by reducing the caging or stop period to two sections.

The motor-generator set is remote controlled from the operator's platform. No auxiliary hoist motor is provided. The hoist is controlled through master controller and time limit relays for both acceleration and retardation.

In the mountain mines, a-c. motors have been applied for control and regenerative braking to control heavy descending loads previously handled by large drums and mechanical brakes. The power saving is appreciable and safety of control is valuable.

Fans. Synchronous motor drives for fans have been applied in sizes up to 500 hp.[2] Increased efficiency and improved power factor are accomplished.

Continued applications of automatic starters for fan motors have proved desirable to reduce to a minimum the delay in the restarting of fans after power interruption.

Motor-Generator Sets. Automatic substations for motor-generator sets have increased in number both to save in labor of attendance and to provide better service for supply of direct current to remote load centers.

Automatic power-factor control on synchronous motors has been accomplished on several sets.

Cleaning Plants. With the mechanical loading of coal being accomplished and with more discriminating buyers of coal, the industry has turned to mechanical cleaning. Several large bituminous coal cleaning plants were completed during the year.[3] Thus, the soft coal industry is approaching the practise of the anthracite breakers. These wet washing plants use around 150 motors of various sizes, aggregating some 1200 hp. connected load. Texropes or V-shaped multiple belts, permit of standard motors in most applications to secure proper speed reduction in short space. Three such plants on one public utility service add a very desirable day load.

Shovels. In metal mining the 3-, 4-, and 5-yard electric shovels using shunt motors and Ward Leonard control continue to be installed to effect economies.

Additional 15-yard stripping shovels, weighing 1550 tons, and carrying motor-generator sets with 1500-hp. synchronous motors continue to be installed.[4]

Improved rubber covering for trailing cables has made these safer and more dependable. They usually carry 4000 volts, three-phase with a fourth wire for ground. Occasionally when blasting, these cables are damaged. Field vulcanizing has been perfected so that the outer covering is repaired so well that it will stand up through

*COMMITTEE ON APPLICATIONS TO MINING WORK:

Carl Lee, Chairman.

A. R. Anderson,	G. M. Kennedy,	F. C. Nicholson,
F. N. Bosson,	R. L. Kingsland,	W. F. Schwedes,
Graham Bright,	A. B. Kiser,	F. L. Stone,
M. M. Fowler,	W. H. Lesser,	E. B. Wagner,
E. J. Gealy,	John A. Malady,	J. F. Wiggert,
L. C. Ilsley,	C. H. Matthews,	C. D. Woodward.

1. Equipment furnished by General Electric Co. and installed at Peabody Coal Co., Mine 53, Springfield, Ill.

2. Installed at Chicago, Wilmington, and Franklin New Orient mine, West Frankfort, Ill.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

water and oil the same as the original cable, thus saving a great deal on replacement costs.

Slushers. Metal mines have almost completely adopted motor-driven underground slusher machines to replace air driven machines.

Locomotives. One manufacturer has developed roller bearing application to motor support on axles to maintain the gears in exactly correct mesh.[5] Experience in the field has not yet proved whether this will be acceptable practise.

Heavy service due either to large locomotives or very frequent travel of locomotives has led a number of mines to go to 6/0 trolley wire instead of 4/0.

Mining Machines. The manufacturers of mining machines have brought out improved track mounted cutters (Fig. 1). These can be built to cut at any height in the coal seam that is required. These machines consist essentially of a heavy frame mounted on trucks containing traction motor and a swinging section pivoted at the

Fig. 1—Track Mounted Bottom Cutter

front end of frame. This section carries the motor gearing, raising, lowering, and tilting mechanism, and the cutter bar and chain proper. The advantage of this type cutting machine over the breast machine and short-wall machine is that no unloading or loading of cutting machine proper is required. This feature elminates most of the manual labor, and through higher power and saving of time enables the operator to practically double his productivity. One make of machine is equipped with a drill so that the same crew can drill for the blasting.[6]

In cutters for coal seams less than 30 in. high; it is difficult to place all controls convenient to the operator. One manufacturer has partially overcome this by applying contactor control to the machine. This control is of the "permissible" type construction, having been worked out to meet the tests of the Bureau of Mines.[5]

A recently developed machine is the hitch cutter.[7] This is a very special type drill to cut hitches in sides of rooms of coal mines to enable timbering to be carried to the face without interfering with cutting and loading machines This should materially reduce accidents at the coal face.

Loading Machines. One type of loading machine usually called pit car loader has been extensively in-

Fig. 2—Pit-Car Loader

Fig. 3—Tank Tread Loader

stalled (Fig. 2). Approximately 2000 have been placed in service. They consist of short drag type conveyor driven by ½- to 7½-hp. motor. In this machine coal is shoveled onto the loader which discharges into mine car.

Full mechanical loaders of several types have been improved and are coming into more general use (Fig. 3). These vary in design, using from one to eleven motors,

3. Pittsburgh Coal Co., Pittsburgh, Pa.
4. The United Electric Coal Co. at DuQuoin, Ill.
5. Jeffrey Manufacturing Co.
6. Sullivan Machinery Co.
7. Goodman Manufacturing Co.

having from 15 to over 100 connected horsepower. Reports indicate that approximately 3 per cent. of the soft coal mined in 1927 was loaded mechanically and most recent reports indicate over 22 per cent of coal mined in Illinois was loaded by mechanical and pit car loaders.

Blasting. Continued experiments in smokeless shooting of coal have been made.[8] In this a gas is compressed to high pressure in the cartridge, then fired by means of fuse in heavy detonation or heating element. This fuse requires about 100 amperes at 180 volts to set off heating element.

Telephones. One company installed a complete system, using lead-covered pair in conduit laid in the floor of the mine.[9] In rapid hoisting, the caging is limited to 1½ or 2 sec. This does not permit the check puller to safely detach the identification checks. Several mines overcame this by installing loud speaking telephones between top and bottom.[10] The bottom man pulls the check and telephones the weighman giving him the number of the check.

8. Cardox.
9. Youngstown Sheet and Tube Co., Dehue, W. Va.
10. Peabody Coal Co., Mine No. 8, Tovey, Ill.

Switch Throw. An electrically operated switch throw has been successfully applied to mine switches.[11] This eliminates the danger of having these throwed by hand. Also signal lights are worked in conjunction with the throw.

Welding. Electric arc welding is becoming more common for repair work. It is almost universally used for welding rail bonds. This year has brought a new use in the welding of a very hard special metal to the tips of mining machine bits, to produce ten times the wear.[12]

Safety Work. The Bureau of Mines continues to test all types of equipment to be used in gaseous mines. They have now placed their stamp of approval on 30 various types of motors.

The approved cap lamp with greatly increased illumination is being rapidly adopted.[13] Over 30,000 of this type have been placed in use. A new cap lamp with lead battery and dry electrolyte has been developed.[14] This should reduce accidental burns from electrolyte.

11. Mines Equipment Co., St. Louis, Mo.
12. "Blackor."
13. Edison Models E. & F.
14. Concordia.

Power Generation

ANNUAL REPORT OF COMMITTEE ON POWER GENERATION*

To the Board of Directors:

An organization meeting, following selection of the committee membership, was held on November 2, 1928, in which the program of the previous year was reviewed and subcommittees appointed to undertake and initiate activities concerning the following subjects:

1. Excitation Systems and Auxiliary Drives in Electric Power Plants, Sub-Chairman, Mr. F. T. Leilich.
2. Interconnection, Sub-Chairman, Mr. F. C. Hanker.
3. Rehabilitation of Power Plants, Engineering and Economic Features, Sub-Chairman, Mr. C. F. Hirshfeld.
4. Hazards to Service Reliability in Modern Generating Stations, Sub-Chairman, Mr. E. B. Meyer.

A second meeting was held during the period of the Winter Convention. Reports at this time outlined the scope and progress of the plans undertaken by the subcommittees. The Subcommittee on Interconnection recommended that its problem was of sufficient magnitude to justify a symposium at one of the national conventions, and the committee decided accordingly to lay plans for a group of papers and accompanying discussion on some important phases of Interconnection during the Winter Convention in 1930.

On April 10, a meeting of representatives of the Excitation, Auxiliaries, and Interconnection Subcommittees was held in Pittsburgh, which members of the Electrical Apparatus Committee of the National Electric Light Association attended, for the purpose of discussing the scope of the investigations that have been or may be undertaken by the two committees in the interest of minimizing any duplication of work that might develop without coördinated efforts. The discussion brought out that no studies were planned by the N. E. L. A. committee on the subject of Excitation, but that the 1924 report of the N. E. L. A. on Auxiliary Drives was scheduled for revision during the next year. The data collected by the Subcommittee on Auxiliary Drives for the present report have accordingly been transmitted to the N. E. L. A. committee for their purpose. The program of investigation and study of the subject of Interconnection that has been outlined by the N. E. L. A. committee seemed so comprehensive that a considerable period would be required for its completion. The papers proposed for the A. I. E. E. symposium, after discussion, appeared to lie entirely in one section of the N. E. L. A. outline, and it was decided therefore to proceed with their presentation and to consider them as one source of information and data for accomplishing the study outlined by the N. E. L. A. committee.

The Subcommittee on Excitation Systems and Auxiliary Drives has endeavored to summarize present day practise with regard to these important features of power-plant design and operation. Its report in detail follows, and leaves nothing unfinished for transmittal to next year's committee.

The Subcommittee on Interconnection has classified the problems which it recognizes in that subject into groups relating to technical, economic, and operating considerations. It has associated with its own membership affiliates from the several sections of the country in which interconnection experience is accumulating, and has designated certain of its personnel as liaison members to cooperate with committees of other appointment that may treat subjects related to Interconnection. The subcommittee has outlined a series of papers of valuable potentialities to the industry. Its report, which follows, is at this time mainly in the form of an outline of work which has been initiated, and constitutes the only subject matter which the committee wishes to transmit to its successor of next year.

The Subcommittee on Rehabilitation of Power Plants has viewed its subject from the angle of future development and possibilities, after depicting the economic and technical causes which are likely to bring about the desirability of rehabilitation, or modernization. The report of the subcommittee as prepared by Mr. C. F. Hirshfeld introduces so many valuable points of view that the committee has thought it better to present it for publication at the Summer Convention as a separate paper, rather than to incorporate it in this progress report. An Institute paper entitled *Increase in Generating Capacity of Existing Stations and Equipment on the System of the Columbia Gas and Electric Corporation*, by Mr. E. S. Fields, describing a well planned rehabilitation program with regard to the generating capacity in the vicinity of Cincinnati and Dayton, Ohio, was reviewed with interest by the subcommittee during the year, and proved such a fine example of a subject becoming increasingly important, that the subcommittee recommends it to the attention of all engaged in the economics of power station development. This paper is scheduled for publication at an early date.

As could be expected, the recent innovations in the practise of power generation have introduced new hazards and problems affecting the service reliability of generating plants. These have been pointed out and discussed in the report of the Subcommittee on Hazards

*COMMITTEE ON POWER GENERATION:

F. A. Aliner, Chairman.

J. R. Baker,	A. H. Hull,	E. B. Meyer,
A. E. Bettis,	A. H. Kehoe,	W. E. Mitchell,
J. B. Crane,	H. A. Kidder,	I. E. Moultrop,
P. M. Downing,	G. L. Knight,	F. A. Scheffler,
James H. Ferry,	W. H. Lawrence,	W. F. Sims,
N. E. Funk,	W. S. Lee,	Arhtur R. Smith,
W. S. Gorsuch,	F. T. Leilich,	E. C. Stone,
F. C. Hanker,	James Lyman,	R. W. Stovel,
C. F. Hirshfeld.		W. M. White.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

to Service Reliability in Modern Generating Stations, which is appended in final form without recommendation of further study at this time.

The purpose and scope of the yearly progress report was defined at the first meeting by members of the Executive, Coordination of Activities, and Publication Committees of the Institute. It is obvious that a tabulation of present day power developments and trends would be a stale repetition of technical literature closely followed by those engineers directly concerned with power generation. However, of the Institute membership of 18,000 engineers there are many who are in no way associated with power generation activities; there is also a considerable body of electrical engineering students reading current Institute literature to whom a yearly summary or picture of the power generation industry should be presented; and another group, to whom the yearly progress reports should be of interest as well as value, includes the more recent engineering graduates who are in the process of orienting themselves in the engineering professions. Exposition of the progress, development, or trends in power generation is best made by reference to typical generating plants; but in pointing out the salient features of illustrative plants, an element of selection was necessary to avoid undue volume. However, bibliographic references are given in connection with plant descriptions, so that anyone interested in further detail of plant construction can readily locate it. For this reason the bibliography at the end of the report is not so comprehensive as that embodied in the 1927 progress report, but is confined principally to the technical papers or reports presented to or prepared by the various engineering societies on matters pertaining to power generation. If this report presents a picture of the new developments in the field of power generation, portrays in its subcommittee reports some of the economic and technical problems which the industry is facing, and offers a reflective summary of the lines along which this phase of electrical-mechanical engineering is expanding and advancing, it will have achieved its objectives.

Recent Developments and Present Status of Power Generation

Volume and Distribution of Generation

The volume of generation of electric energy has continued to increase in step with the general business situation throughout the country. Numerically, the output of all central generating plants in the United States in the past year has been almost exactly 10 per cent greater than in the preceding year. The industry as it approaches the half century mark saw an annual growth equal to its entire volume 20 years ago, while the generated output of 1928 was over 11 times that in 1908. It is only a matter of months now until the yearly production of central generating plants will have reached the figure of one hundred billion kw-hr. A relative index of note to anyone contemplating this field of activity shows that the electric output has doubled in the last six years.

While the opinion seems to prevail that activity in undertaking new hydroelectric developments is lagging for one reason or another, the fact is prominent that hydro plants produced last year 40 per cent, or better, of the total output of central generating plants, which is a larger share of the total power generation than in any year of the last decade. This relatively high percentage of hydro output was in part due to better than average rainfall conditions.

Fuel burning plants generated practically all of the remaining 60 per cent of the year's output. Internal combustion engines continue to be used for power production, in certain localities where liquid and gaseous fuel supplies are economically available and the load demands are relatively small. Steam production by direct firing of oil and gas under boilers is the prevalent method in these territories when volume output of electric power is necessary. The relative proportion of coal compared to oil and gas fuels used for steam raising has remained constant for 10 years, coal constituting almost 90 per cent of the fuel burned.

The statistics available indicate that the volume of electric output in plants other than central generating stations is about 30 per cent of central station production. There can be no question, though, that the advances in the art and practise of power generation originate largely in the central station industry, and are later utilized by the general industry of the country as specific conditions may warrant; also that the relative use of central station power in industry has increased in the last 10 years from less than one-third to almost one-half of the power requirements of the country's industry.

Generating Plant Building Programs

The past year was notable with respect to the quantity and character of generating capacity placed in initial service. The year 1928 was a peak year for the number of new hydroelectric stations and for total capacity installed, exceeding 1927 by over 100 per cent. It is probable that 1929 and 1930 will see the addition of new hydroelectric generating facilities at about the same rate as in 1927. With few exceptions, the hydroelectric units were installed in new developments and constituted only a portion of the expected ultimate generating capacity. More novel features of hydroelectric practise, to be discussed more fully later, were exemplified in the 1928 additions than in any other recent year.

In contrast with the 1928 hydroelectric installations, there were only one or two new steam-electric plants placed in initial operation that are noteworthy for any outstanding features, and only four or five steam plants of major considerations will be started up in 1929 or 1930. Practically all the increase in steam-electric generating capacity, exceeding the new hydroelectric

capacity by about 75 per cent, was added to existing plants. The size of units is the only striking characteristic of the 1928 steam additions.

The relative paucity of new steam plants in the past year probably is an outcome of the wide recognition a few years ago of the impending rapid growth of power generation, and the simultaneous provision through the country of a backbone of highly efficient steam stations strategically located to carry the load arising in ensuing years. The data for 1928 show that practically the entire growth of steam generated load was taken by extensions to existing plants, but it is also evident that many of these stations are approaching the limit of their development, and consequently a wave of new steam stations may be expected in the next few years.

It is probably correct to say that the past year witnessed opposite phases of the recurring cycles of hydro and steam developments. The growth of system load, both from increasing density and from interconnection or joining of small components, has made the use of larger and larger generating plants economically feasible and requisite. Generating capacity must be ready when needed and the starting of a new station in the absence of the possibility of extending existing stations is therefore imperative, often several years in advance of the time when the ultimate development will be required. This situation is met by plans calling either for hydro or steam generation, provided, of course, that water power sites are available within economical distance of transmission. A hiatus of several years then occurs, depending upon the rapidity of load growth on any particular system between the starting of new generating plants. The interval is consumed in enlarging the last station to its full, economical size.

Economics of Hydroelectric Developments

There has been much comment recently about the slackening in hydroelectric development in the United States, although the hydroelectric industry is flourishing in the neighboring country of Canada. Various reasons are advanced for the situation in this country, chief among which are the average high fuel efficiency, the lowered cost of fuels, and the reduced investment cost of steam-electric stations. Another fundamental reason may be that practically all of the low cost water-power sites in this country have been developed, while the more costly sites adjacent to the market and the low cost sites that are now remote from load centers, require development on a large scale to make them economically feasible, as minor utilization of the sites would throw too great a burden of fixed costs upon the hydro energy generated. This economic limitation is probably the controlling factor in determining the cycle, or fluctuation, of water-power developments. It seems likely that the next few years will be a period in which existing hydroelectric plants will be extended to their ultimate size, and during which the growth of load will finally culminate in an impetus leading to hydro development again on a major scale. In the meantime, load increases will be assumed by extensions to, or construction of, steam-electric plants whose capital cost can be estimated with considerable assurance, and incurred in relatively small steps as necessary.

There is a growing realization that wherever a feasible development of hydroelectric power is possible, the combination of hydro with steam generation on the modern utility system leads to the lowest over-all cost of power generation. It is becoming apparent that hydro power has more than an energy and minimum flow capacity value on systems mainly steam, and on systems largely hydro, the advantage of steam-capacity steps to defer major hydro undertakings until economical is being recognized. The value of even limited hydro energy for peak service is receiving wider appreciation, and in many other respects, the interrelation of the two sources of power is being fully worked out. Such considerations together with the attractive over-all generating costs of efficient steam stations have led to the adoption of extensive modern steam plant facilities in regions dependent in the past on water power.

Comparison of Plant Efficiencies

The efficiency of the utilization of water-power is probably approaching its attainable limit as in the past year there was corroboration of peak efficiencies of the order of 92 to 94 per cent based on net effective head, in small as well as large capacities, and for a range of heads to which the Francis type of turbine is applied. The propeller type turbine is reaching efficiencies of 87 to 90 per cent, while the impulse wheel which is used for the high heads in the West has shown efficiencies in recent installations better than 85 per cent over a broad range. Hydro plant water efficiency depends principally on one mechanical element, and the cost of this machinery is usually not over 10 to 15 per cent of the capital value of the entire development. The cost of the maximum attainable efficiency is therefore rather a minor item in the whole. Steam plant efficiency occupies just the opposite side of the picture, as it is the result of the combination of four separate efficiencies, those of the turbine, the boilers, the auxiliary arrangement, and the working cycle. The capital cost of the machinery components of these varies at the present time between 50 and 55 per cent of the total station cost, and the cost of any extensive effort to build in the ultimate of efficiency has a definite effect upon the total. Nevertheless it has been found economically possible to attain steam turbine efficiencies of 80 to 84 per cent, boiler efficiencies of 85 to 90 per cent, and to select a working cycle and machinery arrangement that will result in an overall plant efficiency of 23 to 27 per cent.

Major Influences in Station Design

The two agencies that were of major influence in the design of the stations that were started or extended

in the past year have been the expansion of system loads, and the possibility of high station efficiency with decrease of over-all cost. Greatly increased system loads have resulted from normal growth as well as from interconnection between small or isolated systems, and in many cases between systems of metropolitan size. Hydro developments have probably been more stimulated by interconnection of systems than have steam plants, which in the majority of cases are built as close to the load as possible. As a rule hydro plants, even though water efficiency has increased two to three points in the period, have not had the opportunity of raising their internal efficiency with a corresponding reduction in over-all costs that has been open to steam plant design. The size of system load has led to larger and larger generating units, both in hydro and steam plants, and the increase in size has permitted a slight betterment of internal efficiency; the much greater benefit, however, comes from concentrating capacity in a few large units rather than in numerous small machines. The building volume of turbine and boiler rooms per kilowatt of capacity has been reduced 24 to 41 per cent in recent additions to well-known stations where the newer units are appreciably larger than the turbines in the initial sections of the plants. The adoption of improved working cycles and better combustion facilities in the steam plants has been chiefly responsible for a 9 to 10 per cent decrease within the last two years in the average pounds of coal per kw-hr. produced, the average coal rate at the end of 1928 being about 1.76 lb.

The best thought of hydro designers is being applied to the reduction of the investment charges and extends from the study of preliminary costs to an evaluation of the capital savings that might be secured by a diversity of construction and loads on interconnected systems of wide scope. In many instances great concern centers around the development of maximum output of secondary power, both by the use of a machinery installation that is large compared to the minimum steady output, and by the maintenance of output under unfavorable hydraulic conditions. The first scheme requires a system load capable of absorbing the secondary power in high water periods and protected by available generating capacity during low water; the second scheme depends upon the adaptability of the hydraulic design to fluctuating water stages. The feasibility of the second largest hydro plant started in operation in 1928, that on the Ohio Falls at Louisville, was greatly enhanced by a turbine design that permitted an attractive output under a wide range of heads. The interconnection of steam and hydro has given rise to another possibility of increasing the hydro output, but in this case for peak purposes in low water periods. The year 1928 saw the first large pumped storage development in this country started on the Rocky River in Connecticut, where pumping facilities have been provided to supplement the natural storage accumulated during the high water season. Locations for watershed reservoirs are being sought and will be utilized to their economic limit for the conservation of flood flows, the increase of steady or primary power, and the lessening of property damage and unfavorable hydraulic operating conditions during normal high flow periods. The use of large units has been accompanied by an improvement in the detailed design of turbine, generator, and waterways, so that the cost per kilowatt of rated capacity has been materially decreased. An interesting step further toward lowered capital costs was the omission of the generator room superstructure at the Norwood plant in North Carolina that was brought into service within the past year. While this is not the first application of the idea, the generators at the Norwood plant are the largest to date arranged only with individual canopy coverings. Another method of capital cost elimination that is being given scrutiny is the simplification of the generator voltage bus layout and station service supply; the outdoor arrangement of high-tension apparatus has manifested its superiority from a cost point of view for several years.

Steam plant designers have been busy discussing the application of extremely high steam pressures and calling attention to the possibilities of higher steam temperatures; the selection of the most suitable method of combustion has also been a major concern. In many instances the necessity for maximum capacity has been a determining influence in the station design and it is not at all uncommon to find two methods of combustion employed in the same station, as well as two working pressures. Designers are evidently giving a great deal of consideration to making best use of the various ideas of plant design and arrangement that have been tried out successfully in practise. Several years ago the regenerative and reheating cycles were introduced and are now in general use. It is quite probable that the next major step is toward the use of extremely high pressure superposed upon a pressure now standard. However, the record thermal economy to date is held by a medium high-pressure station that carries its steam pressure only up to 650 lb. The advocates of superpressure state that the capital cost of such a station is no higher than of a standard pressure plant when allowance is made for all factors, particularly that of capacity. The same argument is heard for the pulverized fuel plant. Both high pressure and pulverized fuel are supposed to show somewhat higher efficiencies at the maximum capacities claimed for them. In the end the arguments generally resolve themselves into the question of capacity per dollar, or a reduction of the investment per kilowatt. The relation of the investment charges to the fuel and operating costs does not appear to change to any extent for as the first is decreased by the concentration of capacity, the other has been falling because of the betterment of efficiency. It cannot be said that steam plants are becoming any simpler in their design layout; just the opposite. As

the regenerative cycle introduced additional equipment into the turbine hall, so the new combustion methods and heating surface arrangements add complexity to the boiler plant. Station auxiliaries have multiplied and brought with them a host of control apparatus, and it is being recognized that perhaps the cost of this sub-auxiliary equipment is becoming excessive. The justification for all the complexity is found in the higher efficiency, and this with the greater capacities have kept the over-all dollar costs satisfactory from past points of view. Some simplification is appearing in particular instances and more can be expected, with a consequent lowering of investment costs. It is well to bear in mind, however, that when the changing purchasing power of the dollar, and the effect of interconnection of reducing the amount of station inherent spare capacity, are considered, it may be probable that the absolute cost of steam plants has increased.

To date, the savings available by building new steam plants embodying modern cycles and heat reclaiming devices usually have been so marked in comparison to enlarging and continuing for heavy generation older stations of lower thermal economy, that the system growth has been accommodated on new base load plants and the older stations have been relegated to peak load service. Engineers are beginning to question the amount of improvement in operating economy that is still possible, and are looking for other ways to increase capacity at low cost and without sacrificing their present stations to a low use factor in coming years. Where peak loads cannot be avoided either through interconnection diversity or off-peak load development, serious consideration is being given to the design of plants for peak purposes only that would have a low capital cost although possibly a higher operating cost than their companion base load plants. The general idea of such a plant is a few large units of varying capacity with only the simplest of auxiliaries and heat savings devices, although European variations of the plan include a heat accumulator for storage of energy during off-peak hours for use later on the peak and Diesel engines of capacities as great as 15,000 hp. In this country the alternate pumping and release of water between artificial or natural reservoirs is being given more study by steam system operators than the proposal of off-peak heat storage.

Rehabilitation of Existing Steam Plants

A main line, however, along which the efforts of steam station designers will be increasingly directed, points to the rehabilitation or partial revamping of the efficient stations now in use. There are instances now at hand where it has been possible to step up materially the station capacity at a price per kilowatt about half the original cost; and this has been found warranted and feasible in plants of rather recent design. One way of accomplishing this is to compound a higher pressure station element upon a low-pressure plant, and pressures of 700 and 1200 lb. have been thus added to existing 185 and 300- 400-lb. plants. Another method consists of adding a turbine element receiving steam at the standard or normal pressure, in cross-compound arrangement to an existing turbine element, the original turbine being altered or the intermediate pressure so selected that the original capacity can be maintained and the capacity of the new element added as net gain. The same idea has been applied to the combination into a single unit of two separate and adjacent turbines by placing between them an added turbine element that exhausts into the low-pressure elements of the two separate machines. In plants where the boilers are only a few years old, they are likely to be of such size that their operation imposes no excessive labor and maintenance costs, and the problem of increasing the boiler output hinges usually upon fuel burning limitations. The development of water and steam cooled walls has removed the hazard of refractory deterioration that existed previously, so that greater fuel feeding and combustion facilities are the principal requirements for increased capacities. That these are available either for solid or pulverized coals has allowed major increases in boiler plant outputs. Older boilers that are usually of small size and of low working pressure generally do not allow of any rearrangement of settings that is worth while, and the problem is nearly always settled in such cases by replacement of individual boilers or the construction of an entirely new boiler plant.

Rehabilitation of Hydro Developments

Rehabilitation or revamping in hydroelectric design is nearly always more inconveniently carried out than in the case of steam plants. In some cases the hydro plants have been almost completely rebuilt in order to utilize the site head more efficiently; in other instances the dams have been raised to increase the effective head. The revamping of hydro plants often requires interference with their output over extended periods with consequent economic loss. Where it may be desirable and possible to raise the operating head it may be found that the dam will not withstand the increased static pressure, and practical reconstruction of the dam must be undertaken to utilize the entire site head.

There appear to be at least five types of hydro developments that offer the possibility of revamping as soon as it is economically feasible, as follows:

1. Localities where the flow is divided among several small and inefficient units, either in one plant or among several plants in the same area.

2. Situations where considerable overdevelopment of the minimum flow would be advantageous with respect to system interconnections, but where the available waterways are at present obstructed by small capacity, inefficient units.

3. Plants where the site losses are excessive and the available head could be better utilized by modern waterway construction and settings.

4. Plants where the possible head was not fully developed initially.

5. Plants of partial development and insufficient flow capacity interposed between more modern plants on the same stream in such locations that the river flow cannot be ponded most advantageously.

Methods of Reducing Operating Costs

The attack on the initial costs of generating plants is being paralleled by similar efforts towards a reduction of later operating costs. The operating results of plants are being scrutinized at frequent intervals and compared with operating standards that have been prepared from tests, operation under known conditions, and from manufacturers' guarantees or expected performance data. These studies may be in terms of direct cost for such items as labor, supplies, and maintenance, but the greatest concern is given from the point of view of operating performance to strictly operating statistical figures such as thermal economy and water equivalents. The analysis frequently is modeled along the lines of an accounting balance sheet, and is termed a heat or water balance. Its essential purpose is to explain the deviation from standard performance. Two very informative serial reports on this subject were published during the past year by engineering committees of the National Electric Light Association on "Waterwheel Testing and Operating Records of Plant Discharge" and "Power Station Betterment." The interest in checking plant performance has led to an increased use of measuring devices of the totalizing type, particularly in steam plants. Attention is centered upon water discharge and leakage in hydro plants and there are numerous instances in the later hydro installations of converging sections of the intakes or penstocks being arranged for Venturi measurements. The differential heads available at various locations in the pressure waterways are also used for metering purposes. The Venturi sections and by pass devices are calibrated in place by accurate water measurement usually at the time the waterwheels are also being tested. The field testing of waterwheels is now considered requisite both for the purpose of checking guarantees as well as for securing performance data, and it is estimated that about 400 waterwheels have been thus tested within the last eight years.

The trend to larger boiler, turbine, and waterwheel units has reduced the number of operators required to man the modern power plants but recourse is being made in many instances to automatic control of machinery for further reduction of operating labor costs. Often the primary reason for adoption of automatic control is to secure a more constant improved efficiency, and sometimes reliability of operating processes is a desired object. The increased operating efficiency accrues from the mechanical performance of repetitive adjustments which would not be carried out so thoroughly by manual control unless considerable excess operating and supervisory labor were employed. In steam plants, the chief use of automatic control is for the regulation of the combustion process, although in some plants the balance between the auxiliary exhaust steam and the feedwater heating is automatically controlled. Automatic control has received a great deal of prominence in connection with hydro plants in the last year, although the control feature has not been fully automatic in some cases mentioned as plants with complete automatic control. Saving of operating labor is the principal object in the hydro plants that are started and stopped by remote control, such as from a distant point or by forebay water level. Automatic synchronizing has been used in some plants in which operators are employed and the arrangement of equipment for the purpose has led to lowering of the initial costs of the control apparatus, an increased operating reliability, and a saving in operating labor. Control equipment is being devised so that it can be made fully automatic or remotely supervised as the circumstances may demand. There is a trend in steam plant design also to group the control apparatus so that more of the plant can be brought under remote supervision and control. In line with this the floor levels, machinery layouts, and wall construction are being adapted to the minimum operating force for the purpose of securing a higher grade of operating supervision at a lower total cost. The automatic regulation of frequency is being actively developed and its application in hydro plants is somewhat more preferred on some systems than in steam stations. It is expected that the control will be extended to the proper division of load between units in the same plant for the purpose of maintaining the highest possible hydraulic efficiency. Frequency regulation has as its object an improvement of system service, and the decrease of operating costs resulting therefrom is problematical at the present time.

Illustrative Examples of Recent Hydroelectric Developments

Large Run of River Plants

Conowingo. The Conowingo plant on the Susquehanna River may be considered typical of present day medium head, hydroelectric developments, in this case about 89 ft. It is essentially a run of river plant, the 2½ billion cu. ft. of water available in the reservoir above the dam with a 7-ft. draw-down being less than the power house draught at full load for 24 hr. The hydraulic features, the machinery equipment, and transmission facilities have been thoroughly described in the A. I. E. E. Quarterly Trans., Vol. 47, April and July 1928. Conowingo is outstanding in being now the second largest hydroelectric plant in the United States, and when extended to its final size will be the largest single hydroelectric installation in the country. Other features of bigness distinguishing it are the turbines that are the largest in physical size installed to date,

and the butterfly valves placed at the inlet of the scroll case for blocking the water passage during shutdown. These valves are the largest ever built, and in use show negligible leakage.

The hydro developments on the Susquenhanna River well illustrate the growth of the electric power industry that has made possible the increasing machinery capacity that may be economically installed on any stream. The York Haven plant 43 mi. upstream, the earliest development, uses a flow of about 15,000 cu. ft. per sec.; the Holtwood plant, intermediate between York Haven and Conowingo, a flow of 31,500 cu. ft. per sec.; while Conowingo, with a maximum discharge of about 6500 cu. ft. per sec. per single unit, will utilize a flow eventually of 70,000 cu. ft. per sec. or more. Minimum daily flow as low as 3200 cu. ft. per sec. have been recorded at Holtwood. The median flow (available 50 per cent of time) is about 25,000 cu. ft. per sec. while flood flows may approach 700,000 cu. ft. per sec. Conowingo was feasible when the system load became of sufficient size to permit the plant to operate on base load at full capacity during the high water season, and to operate as a peak load plant at high capacity during low water stages. With a 24-hr. firm power value of only 30,000 kw., the plant can still be relied on during the period of expectation of the annual peak load for its full capacity of 252,000 kw., when used to carry the top portion of the present combined daily load curve of the Philadelphia Electric Company System and the interconnected companies; and in the course of a year it can be expected initially to deliver in Philadelphia 1,250,000,000 kw-hr., or almost five times its firm power value for 8760 hr.

The station structure and machinery layout at Conowingo show the modern tendency to incorporate the facilities for power generation in the dam structure with the view of securing the simplest and most compact arrangement possible. In fact about the only idea for simplification not embodied in the design of the power plant was that of the omission of a complete superstructure over generators, but the generator roof as erected is used to support an outdoor 220-kv. switching station with the result that the connections between generators, generator busses, step-up transformers located directly behind the head works in the dam structure, and the high-tension switch gear are minimized, and the switching station on the roof is readily accessible to the operating crew. The distance of 87 ft. from the face of the dam to the mouth of the scroll case is 11 ft. less than the depth of water on the upstream side of the plant. In this space provision is made for the generator bus galleries, the 26,666-kv-a., 220-kv. transformers with handling track below the highway mounted on the dam, and necessary control rooms and passages. No gate-house superstructure was erected, and the stop logs and trash racks will be handled by outdoor gantry cranes that also operate the crest gates on the dam.

The regulating and crest gates surmounting the dam spillway represent a method frequently used for providing for flood flows and of holding a maximum head in low flow periods. These gates are 38 ft. in clear span and 22½ ft. deep, although there are three gates only 10 ft. deep for regulation. All gates are of the Stoney or roller guide type. The piers holding the guides for the gates also support a highway bridge. The largest gates of this type in operation are on the dam of the Catawba plant in South Carolina, where steel gates 30 ft. high and 45 ft. in clear width are installed to pass flood flows.

The auxiliary and excitation systems at Conowingo reflect the latest ideas along the lines of reliability and operating convenience. Primarily the power for excitation and governor oil pumping is supplied to each unit by means of an individual shaft-end auxiliary generator of 500-kw. capacity mounted above the thrust bearing. This 440-volt a-c. generator has its own 250-volt exciter mounted above it. Other sources of auxiliary power are waterwheel driven station service generators and station service transformers. The excitation current for each unit is supplied by an individual motor-driven exciter set with pilot exciter, so that the problem of quick response excitation for stability purposes was readily met by a motor-generator speed of 1200 rev. per min. There is a spare motor-driven exciter set in addition to the individual motor-generator units. The governor oil pumping system is laid out according to the twin-pump arrangement, there being a complete pumping system for each pair of adjacent main units. Each twin group consists of two motor-driven pumps each capable of supplying the governors of both main units. This system lends itself to a simplified scheme of power supply and a minimum amount of extended station wiring, and provides adequate spare pump capacity so that there is little likelihood of interruption of the main units due to faults in the pumping system.

Louisville. The Louisville hydroelectric development that was brought into service during the past year is an outstanding example of a plant built for secondary power generation only, there being on the average 40 days in the year when power generation is impossible because of high water conditions. To meet this limitation the propeller-type turbine was installed so that power can be developed over a range of heads from 37 ft. maximum to 7½ ft. when the plant goes out of service. These machines at Louisville are the largest propeller-type wheels in physical size yet built in the United States and are approximately as large as any similar turbines in Canada or Europe. They can be started up, synchronized, and loaded at heads as low as 8 feet. The additional low head output secured by the propeller turbines gained at 10 per cent reduction in cost per kw-hr. as compared to a Francis type turbine installation. A further gain of 10 per cent also resulted from the higher wheel speed which allowed a direct saving in generator and building costs. The generators

in this plant employ the enclosed system of air circulation with water tube surface coolers, being the first installation of this type of generator cooling in hydroelectric stations. The unusual conditions surrounding the Louisville development have been discussed by H. G. Roby in November 1928 issue of *Mechanical Engineering*, Vol. 50, No. 11.

The abnormal hydraulic conditions during high river stages led to a quite unusual arrangement of the plant headworks. These will be entirely flooded over during high flow and the generator room, of 48 feet width, will be surrounded by water on three sides, making provision against flotation necessary. The headworks are of minimum volume and contribute little to the stability of the dam structure. The bulk or mass of the powerhouse section is directly under the generator room floor.

The Louisville plant was a pioneer in the adoption of supervisory control on a large scale in attended power plants. The entire electrical layout follows the unit principle. A 13,000-volt double generator bus is suspended in a steel enclosure along the upstream wall of the generator room, with the necessary switch connections for any generator below, on the generator floor opposite the particular unit. The generator switches are of the truck type. At each generator there is an individual control cabinet containing the instruments and relays for the automatic starting and stopping of the unit. Operation of the automatic control can be initiated by push button either at the unit control panel or at the central supervisory board. These control panels are placed close to the generator switches and minimize the conduit runs for switch operation. Connection between the unit panels and the supervisory panel is made by telephone cable. The circuits at the panels are completed by plug attachments, so that the control panels may be readily disconnected and interchanged if necessary. Miniature meters are used for indications on the unit and central panels. The turbine governors may be controlled by usual means from either location. This system of electrical control and bus arrangement, while introduced at Louisville to achieve minimum initial and operating cost under the peculiar requirements there, will likely be used in other stations, as evidenced by a comparable semi-supervisory control in one of the latest of high-pressure steam stations.

Dnieprostroy Project. An order has recently been received in this country for four Francis type turbines that will each develop over 100,000 hp. under a head of 123 ft. These turbines are being installed at Alexandrovsk on the Dnieper River in Russia. Each unit is rated at 84,000 hp. under a gross head of 116.5 ft. and at 88.2 rev. per min. The runners have a maximum diameter of 240 in., and weigh 300,000 lb. each. The runner shafts are 41 in. in diameter and weigh 160,000 lb. each. Power will be generated at 13,800 volts, three-phase, and 50 cycles. The outside diameter of the stator frames is 42 ft. The transmission voltage leaving the power house will be 220,000 and 110,000 volts.

River Regulating Developments

Rocky River. The Rocky River pumped storage development for regulating the flow of the Housatonic River in Connecticut has been a major innovation in hydroelectric practise in this country during the past year. The technical features of this plant have been described in detail by E. J. Amberg[1] and mention of the plant is made here only for the purpose of making complete the review of the year's progress. The comparatively small natural run-off of the Rocky River, a tributary of the Housatonic River, is supplemented by water pumped from the Housatonic River into a storage reservoir created on the flow line of the Rocky River. Pumping is done mainly by off-peak steam power, and the direct recovery of the electric energy is carried out at an over-all efficiency of 61 per cent. However, this is raised to 79 per cent by the presence of another hydroelectric plant on the Housatonic River below the Rocky River station. The machinery equipment in the plant includes at the present time one vertical shaft 33,000-hp. turbine, and two vertical shaft 8100-hp. centrifugal pumps. The pumps discharge into the turbine penstock against a maximum head of 240 ft., and are the largest in this country, having a delivery each of 250 cu. ft. per sec.

Before the construction of the Rocky River storage reservoir the hydroelectric plants on the Housatonic River had only a primary or year round dependable capacity of 1000 to 3000 kw., although their installed capacity totals 26,000 kw. If the available hydro capacity was used on the upper half of the annual load duration curves, a firm capacity of 10,000 kw. could be secured. The regulation of the river by the Rocky River reservoir now allows a year round firm power of 11,000 kw. or if the firm power is used on the annual load curve, the total installed hydro capacity of 50,000 kw. can be counted on as firm capacity; in other words, the yearly firm capacity has been increased 40,000 kw. by the installation of a 24,000-kw. unit and by regulating the river flow with a pumped storage reservoir. This yearly peak capacity will increase as the load grows.

The drainage areas and storage capacity of this development are of interest for comparison with storage developments mentioned later. The drainage area of the Housatonic River at the plant farthest downstream is 1510 sq. mi.; the area of the Rocky River basin is 40 sq. mi, of which 8½ are covered by the storage lake; 5.9 billion cu. ft. of water will be available from the reservoir although the natural inflow will be only 1.5 billion in the average year. In an average year the Rocky River plant will be called upon to generate 17,400,000 kw-hr. by drawing 4.03 billion cu. ft. from the reservoir; this water will generate 4,900,000 kw-hr.

1. A. I. E. E. Quarterly TRANS., Vol. 47, October 1928, p. 1100.

at a hydro plant on the Housatonic River below the Rocky River plant.

Saluda River. In contrast to the Rocky River pumped storage reservoir is the monumental hydroelectric development now being carried out on the Saluda River in South Carolina. The natural flow of this river varies between 142 and 100,000 cu. ft. per sec. An earth dam almost a mile and a half in length and over 200 ft. in height is being thrown across the river valley to construct an impounding reservoir of 96 billion cu. ft. It is expected that the flood flow of the river will be reduced by 40 per cent, in addition to the regulation effected for power purposes, whereby some 300,000,000 kw-hr. will be produced annually. This artificial storage reservoir will rank in volume with other notable reservoirs, such as those at Gouin on the St. Maurice River in Canada of 180 billion cu. ft., the Elephant Butte in New Mexico of 115 billion cu. ft., and the Bitobi on the Gatineau River, Canada, of 95 billion cu. ft. The Saluda development is unique, however, in that the power house is situated directly adjacent to the storage control dam instead of being at a considerable distance from it, as is generally the case in the larger storage possibilities. The available drawdown of the reservoir will be 60 ft. At 180-ft. head the power-house discharge will be about 12,500 cu. ft. per sec. for the four 55,650-hp. units now being installed; provision is being made for two additional units.

Gatineau River Development. The recent inauguration of the Paugan Falls plant directs attention to a type of hydro development that is typical in some parts of Canada, where the topography includes concentrated drops suitable for power generation, and numerous lake basins scattered over the drainage area that lend themselves to storage control at relatively small cost. The territory covered by such reservoirs is usually remote and sparsely settled so that its inundation does not cause great damage to property. The most extensive of these developments has been on the St. Maurice River, where 4800 sq. mi. of the 16,200 sq. mi. of total drainage area are tributary to the Gouin and Manouan Reservoirs that are situated 230 mi. upstream from the Grand Mère, Shawinigan Falls, and La Gabelle Power Houses. The Gatineau River has been the most recent river in Canada to be largely developed; the erection of the Farmers Rapids, Chelsea, and Paugan, Power Houses near its mouth was carried out in conjunction with the damming of the Bitobi Reservoir about 120 mi. above the plants, where 95 billion cu. ft. are impounded. Further storage of 45 billion cu. ft. is also being built elsewhere on the river. The three plants have an initial installed capacity of slightly under 500,000 hp. The Paugan Plant is at the generating end of a 230-mi. transmission line to the terminal station in Toronto of the Hydro-Electric Power Commission of Ontario. This is the first 220-kv. system in Canada, and is noteworthy for the layout of the high-voltage switching installations at both ends where all breakers are arranged in a ring with the equipment and lines connected to the ring between breakers. The scheme gives nearly the same flexibility as a double bus and materially reduces the number of breakers required, thus effecting a saving in capital cost.

Outdoor Type Plants

Norwood. The Norwood plant in North Carolina placed in operation in 1928 marked the peak development to date of simplification of plant superstructure, when two 22,000-kw. and one 18,000-kw. generators were placed under individual removable plate steel housings and arranged to be served by an outdoor gantry crane. While the idea is 12 to 15 years old, it has been adopted heretofore only for relatively small units. A modification that has been used in other plants is that of a low superstructure with a hatchway in the roof above the generator through which generator and turbine parts can be handled by outdoor crane. In some instances a low roof has been devised so that it could be rolled to one side as a whole, or in sections. In one instance an indoor gantry crane was installed to save crane runway beams and extra column steel. At Norwood the generators are enclosed in plate steel casings 40 ft. in diameter and 30 ft. high, with a horizontal joint between the cover plate and the cylindrical side plate. The gantry crane has unequal legs, one end of the bridge running directly on the top of the forebay wall, while the downstream leg rests on the concrete substructure enclosing the scroll cases. The crane trolley has an auxiliary drum operating a stiff leg derrick mounted at the upstream end of the crane bridge. This stiff leg derrick can be used for dismantling intake gates and screw stem gate hoists.

The division of total station capacity in the Norwood plant between two 31,000-hp. units (ultimately three) and one 25,600-hp. turbine for the improvement of plant efficiency is perhaps novel in large hydro projects. By the use of a single lower specific speed unit, high part-gate efficiencies are obtained when necessary because of the particular variations in stream flow and load conditions at the plant.

Another interesting and pioneer method for maintaining the most economical plant operating efficiency consists of an automatic arrangement for holding the proper distribution of load between units in the plant. It is recognized by hydro plant operators that most economical distribution of load cannot be maintained under swinging loads unless the switchboard operator is constantly adjusting the gate openings of the various turbines in the plant. With units of large size, the integrated deviations from the most economical loading in the course of a day may be appreciable and have led to the development of an automatic switchboard control that holds a predetermined distribution of load on the turbines by manipulation of the governor speed motors while allowing the governors to function in a normal manner for speed regulation. The load may be

automatically distributed between units of different size or characteristics, and even between stations.

Propeller Type Plants

The increasing use of the propeller type turbine has led to two mechanical modifications of it whereby the turbine power and efficiency can be maintained under decreased heads and fractional gate openings respectively. This is accomplished by changing the pitch of the propeller blades, either by hand after shutting down the turbine, or by a mechanical arrangement while the turbine is in operation. Six 5000-hp. wheels were placed in operation during the past year in the Chippewa Falls plant in Wisconsin, where the blade angles will be manually adjusted to give maximum efficiency for any gate opening or maximum output for any variation of head, according to load and water conditions. The normal head will be 29.6 ft. and each unit will pass 1750 cu. ft. per sec., running at a speed of 138½ rev. per min. The peak efficiency is expected to be above 87½ per cent.

Wheels of the greatest power of the manually adjusted propeller type, however, are being installed now in the Back River Plant of the Montreal Island Power Company in Canada, where six units rated at 8800-hp. under a head of 26 ft. and at 85.7 rev. per min. will be used.

The type of turbine having an automatic arrangement for varying the pitch of the runner blades is an entirely new development in this country, there being only one wheel so far installed. This turbine wheel has been used extensively in Europe where it is known as the Kaplan turbine. It contains a mechanism by which the governor changes the pitch of the blades while the turbine is in operation. Its chief advantage is the high efficiency that can be maintained over a wide range of gate openings. The 1900-hp. wheel being installed in the Devils River plant in Texas will operate at 277 rev. per min. under a head of 33 ft., and is expected to have an efficiency above 80 per cent for three-quarters of its load range, and above 90 per cent for the middle quarter of its range. Its efficiency-capacity curve is particularly flat compared to such curves for the fixed blade propeller and Francis wheels.

High Head Impulse Plants

Bucks Creek. The past year witnessed two record breaking impulse turbine installations on the Pacific Coast. The Bucks Creek Plant on the Feather River in California develops to date the highest head at any hydro plant in America, its static head being 2562 ft. This plant is noteworthy again from the point of view of waterpower practise in the East in that the stream draining one watershed is led into the flow line of a stream in an adjoining basin by means of a closed conduit 8 mi. in length; the combined run-off of the two areas is then carried by a tunnel through a mountain ridge into a stream on a third watershed, this stream being eventually diverted through a second tunnel into the penstocks leading to the power house. Reservoirs located on the several streams receive three-quarters of the run-off of the entire area, so that a high degree of run-off regulation is secured. This plan of combining the run-offs of adjoining water sheds is feasible because of the very high power capacity of each cubic foot of water utilized at the tremendous heads that are available in the mountainous regions of the West, a fact which can be realized by consideration of the 300 cu. ft. per sec. flow provided to operate the two 35,000-hp. turbines in the plant.

Big Creek 2A. The addition to the Big Creek 2A plant of the Southern California Edison Company brought into service during the past year, contains the two impulse wheel units of highest capacity in the world, that are rated at 56,000 hp. each. The first part of this plant contains four units of a total rating of 82,000 kw., while the second section of the plant has a capacity of 95,000 kw. in two units. The floor space occupied by the new units is one-third smaller than that of the initial half of the plant, although it is likely that the second section of the plant would have been somewhat larger had it been erected remote from any other development. A further reduction in building volume amounting to a decrease in height of 39 ft. was secured by locating the 220-kv. transformers and oil circuit breakers outside of the generator room building. As in the case of the Conowingo units which feed into a 220-kv. transmission system, quick-response or high-speed excitation was desired for the Big Creek 2A units, and was likewise obtained by the use of a pilot exciter for the main exciter. However, the main exciter of 375-kw. rating, together with the 4-kw. pilot exciter, are coupled directly to one end of the main unit shaft where they run at its speed of 250 rev. per min., which is approximately three times that of the Conowingo generators.

Series Head Plants

The past year saw several new hydroelectric projects intended to utilize the undeveloped or partially developed head below existing plants in such a manner that the new plant operates in "water synchronism" with the plant farther upstream, there being no pondage between the two plants. The lower plant constitutes in effect only an extension of the upper plant, although they may be separated by several miles on the flow line of the stream, and is often remotely-controlled from the upper plant. An illustration is the plant at Lock and Dam No. 7 of the Kentucky River, notable also for its arrangement on "stilts." The width of the river at this point was too narrow to pass flood flows if a portion of the dam spillway were occupied by a continuous power house, and the plant was therefore erected on piers above the spillway. The forebay of the plant extends about four miles upstream to the outlet of the Dix River power house, the discharge of which will bring the Lock No. 7 plant into operation by water

level float control, so that the lower plant may be considered a head extension rather than a flow extension of the upper plant. The head at the Lock No. 7 plant will be 15 ft. The outdoor gantry type crane had a unique application here, also, in its cantilever extensions to handle water-borne material into and out of the plant, as rail facilities were not possible.

Another similar series-head installation was put in operation at the Chippewa Falls plant in Wisconsin, located only two miles below a plant erected earlier at Wissota.

Automatic Frequency Control in Hydro Plants

Hydroelectric plants seem to be taking the lead in the development of automatic frequency control, which is becoming more important as the smaller operating systems are being grouped into the large regional systems now in existence. Positive and fine control of the frequency, both instantaneously and accumulatively, appears to be a prime essential in all the problems arising in system operation. Two types of apparatus have been developed for frequency control, although both operate through the agency of the turbine speed motor and turbine control gates or valves. It appears to be possible to hold the frequency variation under two-tenths of one per cent on large interconnected systems. Regulation of the frequency on the New England Power System has been carried out at the Harriman Hydro Station on the Deerfield River, where three 15,000-kw. units are installed. Automatic frequency control is also in use at the Wallenpaupack plant of the Pennsylvania Power & Light Company where two 28,500-hp. wheels are installed. The first of these plants operates under a head of 360 ft. and at a waterwheel speed of 360 rev. per min.; the turbines in the latter plant run at 300 rev. per min. under a head of 300 ft.

The maintenance of exact frequency by one plant on a large system is somewhat contrary to the most economical loading of the units in the plant. The alternative of dividing the load fluctuations between a number of plants simultaneously appears to require too many complications at the present time. There seems to be a preference for the use of hydro stations instead of steam stations for the control of frequency, for the reason that it is easier to control mechanically the operation of the hydro unit than it is to vary frequently the steaming rate of the boiler plant. It is probable that the frequent movement of the hydro-wheel gates and servo-motor mechanism will cause increased wear of these parts which will be an expense chargeable to the automatic frequency control, but to date, this element of operating cost has not been evaluated.

Hydro Development Rehabilitation

The Bellows Falls development on the Connecticut River in Vermont is an example of the earlier New England canal layout for industrial purposes in which 75 small waterwheels utilized the available flow at partial heads and low efficiency. They have been superseded in the new plant by three vertical units of 20,000 hp. each operating under a 60-ft. head. The project also covered increasing the height of the old dam, and providing greater spillway and canal capacity. A point of interest also is the two large rolling gates installed on the dam for crest regulation, which are each 18 ft. high and 115 ft. long.

The Catawba plant in South Carolina has replaced one built 23 yr. earlier having a capacity of 5600 kw. in eight rope-driven generators, operating at a best efficiency of 70 per cent. Originally the head was 23 ft., and the pond allowed only run of river operation. Raising the dam gave a 70-ft. head and created a 10-billion cu. ft. reservoir, and in conjunction with other storage allowed the installation of 60,000-kw. generating capacity in four units. At the Upper Tallassee plant on the Tallapoosa River in Alabama, a former 30½-ft. dam is being raised to a height of 56 ft. and two 24,000-hp. units are being installed. It was necessary at both the Catawba and Upper Tallassee plants to scrap entirely the former generating plants and dams.

High-Pressure Steam Developments

Evolution of Equipment Types

The wider use of steam at 1200 to 1400 lb. seems to be the most conspicuous trend during the past year or so in the steam-electric generating field. The turbines used at these pressures have been satisfactory adaptations of previous turbine design. The initial installations were made in existing stations as superposed pressure additions, allowing the high-pressure turbines to operate on base load generation, so that single valve turbine control was sufficient. Lately multivalve turbines have been designed for plants where they will carry a varying load, making a uniform turbine economy desirable. Both impulse and reaction type turbines are used, although the reaction turbine has been supplied with two rows of velocity compounded impulse blading to absorb the large drop in the first pressure stage. The turbine and generator speed in the case of all the 1200-lb. units is 3600 rev. per min.

Both the straight tube boiler with headers and hand holes, and the multidrum bent tube boiler are being used to generate high-pressure steam. The straight tube boiler has the advantage of only a single drum; its many hand hole caps however have some drawback. The bent tube boiler as installed to date has three drums. The application of both types of boilers meant only a slight modification to take care of the higher stresses. The tube diameters have been decreased ¾ to one inch, while seamless forged drums have been substituted for the riveted construction possible with moderate pressure. Two "combustion steam generators" at the Philip Carey Manufacturing Company, Lockland, Ohio, will be used shortly to produce steam at 1840-lb. gage, the highest steam pressure to date in large boilers in this country.

Evolution of Boiler Types in Europe

Contrasted to standard boiler designs used in this country, European designs seem to indicate a desire to avoid exposing water and steam drums to the gases of combustion or piercing them with many holes in which tubes are rolled or expanded; headers in which tubes are expanded are also avoided. The fear of impure feedwater, with resulting scale deposits and burning of tubes, has led to two designs of indirect steam generation in which the heat absorbing parts contain steam only or else a circulating medium such as distilled, deaerated water. In three designs, small diameter tubing is used for primary heat absorption in order to minimize metal thickness and secure high heat transmission while retaining adequate wall strength. Minimum water content is considered essential, and the fear of disruptive explosions has also influenced the use of small tubes. In three designs external mechanical power is used to circulate water or steam over the heat absorbing surfaces to obtain positive cooling by preventing the lodging of steam bubbles or scale deposits on the tube walls.

The Atmos boiler designed by V. Blomquist employs boiler tubes about 12 in. in diameter with a wall thickness of ¾ in., rotating at 330 rev. per min. These tubes are driven by gearing at one end and rest in bearings outside of the boiler setting. Feedwater passes into the tubes through stuffing boxes at one end, while the steam is collected through similar stuffing boxes at the other end. The tubes extend across the boiler setting and are 11 ft. long. Water is forced against the tube walls by the rotation, while the separatory action causes the steam bubbles to collect in the center. The rotation of the tubes allows uniform heating stresses around the tube walls. The saturated steam produced in the tubes is passed through a convection superheater for further temperature elevation. The Atmos boiler has not been built for large commercial capacities, although the rate of evaporation per sq. ft. of heating surface is said to be high.

The Benson boiler in its latest installation consists of a vertical, cylindrical firebox formed by 16 tube sections connected in parallel to inlet and outlet manifolds. The tube sections are each formed of 450 ft. of continuous tubing having an outer diameter of 1.3 in. and a wall thickness of 0.25 in., and are made by welding together short tube lengths. Feedwater is pumped into the tube sections under 3200 lb. per sq. in. pressure, is heated as it flows through the tubes to 750 deg. fahr., and changes into steam without ebullition when the critical temperature of 706 deg. fahr. is reached. The steam flows from the outlet manifold through a reducing valve into a convection superheater.

In the Schmidt-Hartmann boiler primary steam is generated in a pipe coil exposed to the combustion products and flows to another coil immersed in a water and steam drum in which the secondary steam for use is generated by the absorption of the latent heat of the primary steam. The condensed primary steam returns by gravity to the furnace coil after passing through a heat exchanger in which the feedwater to the secondary steam drum is preheated. Since the same water is circulated continuously in the primary circuit, no scale can form in the furnace tubes. The secondary steam generating drum is of simple construction with few openings. Impure feedwater can be used in the secondary generator without danger of overheating the steam coil in it. A convection type superheater is used to superheat the saturated secondary steam.

The Loeffler boiler employs steam for the circulating medium, but brings the primary and secondary steam into direct contact by blowing the superheated primary steam into the water in the secondary steam drum. The superheat in the primary steam causes the generation of additional steam in the water drum, and the total combined steam then is pumped mechanically through superheater coils exposed to the products of combustion. A portion of the superheated steam is returned to the water drum to continue steam generation there, and the balance is withdrawn from the system for use in prime movers or for heat.

Arrangement of High-Pressure Boiler Plant Heating Surfaces

There appear to be distinct variations in this country in the arrangement and proportioning of the several kinds of heating surfaces that make up a 1200/1400-lb. pressure boiler setting. The arrangement of heating surface does not appear to hinge upon the method of combustion and the thermal efficiencies of the different designs seem to be similar. Factors that are evidently important in this respect include the temperature of combustion gases as determined by the amount of furnace wall cooling and the boiler convection surface ahead of the reheater or superheater, and the necessity of reheating the steam exhausted from a high-pressure turbine element before it is passed to low-pressure turbines, thus requiring provision practically for two separate superheaters in the boiler setting.

In the bent tube multidrum boiler it is difficult to arrange both a high-pressure and a low-pressure superheater in the boiler gas passages, and both are located on the furnace walls for radiant heat absorption, as was done in the boiler at the Lakeside Station in Milwaukee, or the primary superheater is placed in the gas passages and the reheater element on the furnace wall, as illustrated by the two high-pressure boilers in the Northeast Station in Kansas City, Missouri. The effective surface of superheaters exposed to radiant heat can be much less than that of convection superheaters. The preferred arrangement at the present time in straight tube boilers seems to be a single-pass boiler consisting of a low bank of boiler tubes above the furnace followed in the gas path by the primary high-pressure superheater and the lower pressure reheater in series. Minimizing the number of boiler tubes decreases the amount of high

cost boiler parts, and allows a higher temperature head for heat transmission in the steam heaters.

A steam heated reheater is placed in series with the convection gas heated reheater in the Holland and South Amboy plants in such a manner that the low-pressure steam is first heated by saturated steam from the boiler drum, and then passed through the convection reheater. The use of high-pressure steam for reheating is convenient for automatic thermostatic control of the final steam temperature. The relative proportion of the reheating performed by the steam reheater decreases with the boiler rating in consequence of the increasing temperature of the combustion gases passing over the gas reheater, and the increased steam flow through the reheaters.

Air preheaters are being used to reduce to an efficient point the relatively high temperature of gases leaving the boiler or reheaters. Air preheater surface is inherently the cheapest that can be installed. An economizer is sometimes included ahead of the air preheater to temper the gases entering the air heater, even though the feedwater temperature may run above 400 deg. fahr. Sizable economizers are employed in the Boston, Holland, Deepwater, South Amboy, and Kansas City plants, and will also be installed with the Philip Carey Mfg. Co. "steam generators." This last development will deliver steam at 800 deg. fahr. to triple-expansion reciprocating engines exhausting around 60-lb. gage to supply manufacturing process requirements. No economizer is used on the Milwaukee boiler which has a low stack temperature as the result of a completely metal cooled furnace and a deep banking of boiler tubes baffled for three effective passes.

The general proportioning of heating surfaces in typical high-pressure boiler installations is shown by the following table, but these figures must not be considered as comparable since they include different types of heating surface of vaying degrees of effectiveness.

Transactions of the American Society of Mechanical Engineers. The engineers who have developed the use of 1200-lb. steam in actual plants show in comparison to 400-lb. installations debit charges against the water and steam equipment actually handling the high pressures, but preponderant credits for thermal saving and capacity available. The possible increase in capacity is believed by some engineers of high-pressure experience to offset the higher equipment charges resulting in an initial cost no greater than that of a standard 400-lb. plant.

Summary Description of High-Pressure Plant Installations

Plants with two boiler-pressures.

Edgar—Boston. Now has one 3150-kw. single-valve turbine, one 10,000-kw. single-valve machine, and one 12,500-kw. multi-valve machine.

Lakeside—Milwaukee. Now has one 7000-kw. and one 7700-kw. machine, both being single-valve turbines.

Northeast—Kansas City. Has a 10,000-kw., turbo generator. The high-pressure turbines in these plants exhaust into low-pressure station steam mains at pressures of 300 to 350 lb., and are really high-pressure turbine elements superposed upon distinct lower pressure stations.

Plants with single boiler-pressure.

In these plants the entire low-pressure steam requirements will be supplied through the high-pressure turbines and from high-pressure boilers only, there being no steam generation except at 1400 lb.

Holland. This station at Holland, New Jersey, on the Delaware River contains initially one cross-compound turbo generator of 55,000-kw. capacity, consisting of an 11,800-kw. high-pressure element operating at 3600 rev. per min. and a 43,200-kw. low-pressure element operating at 1800 rev. per min. with a direct-connected exciter to supply both generators. Two

TABLE I

ARRANGEMENT OF HEATING SURFACES IN TYPICAL HIGH-PRESSURE BOILERS

Plant	Edgar Boston	Holland	Northeast Kansas City	Lakeside Milwaukee
Type of boiler	Cross-drum Straight tube	Cross-drum Straight tube	Multidrum Bent tube	Multidrum Bent tube
Total convection surface sq. ft.	58,302	66,066	69,761	68,852
Boiler surface—per cent conv. surf.	12.0	12.0	24.3	41.4
Superheater surface—per cent conv. surf.	8.7	6.9	3.8	..
Gas reheater surface—per cent conv. surf.	11.9	17.2	..	..
Economizer surface—per cent conv. surf.	16.5	17.6	15.6	..
Air heater surface—per cent conv. surf.	50.9	46.3	56.3	58.6
	100.00	100.0	100.0	100.0
Water wall surface—per cent conv. surf.	2.1	..	4.4	2.2
Radiant superheater—conv. surf.	..	..	..	2.7
Radiant reheater—conv. surf.	..	..	0.7	1.4
Steam reheater—conv. surf.	..	2.2	..	..

Cost of High-Pressure Steam-Electric Stations

A full discussion of the net economic benefit derived from high pressures has been published in the 1928

boiler units supply steam at 1400 lb. and reheat the steam intermediate between the high- and low-pressure turbine elements. Both boilers have primary steam

reheaters in series with convection type gas reheaters. The boilers are fired with pulverized coal by the bin system, using horizontal burners for opposed firing, and each generates 250,000 lb. of steam per hour. Full-load feedwater temperature is 443 deg. fahr. and the steam temperature 750 deg. both initially and after reheating at 400 lb. For further description see *General Electric Review*, Vol. 32, No. 2, Feb. 1929.

Deepwater—Wilmington. This plant contains two 53,000-kw. cross-compound units consisting of a 12,500-kw. 3600-rev. per min. high-pressure element, and a 41,700-kw. 1800 rev. per min. low-pressure element. Each generator of a unit has a direct connected exciter. The exhaust steam of a third high-pressure turbine unit supplies an industrial establishment nearby. Reheating is done by only two of the six high-pressure boilers, following the practise in plants of 600 to 700 lb. where reheating between turbine elements has been necessary. Comparison of the standard and reheat boilers is as follows:

	Standard boiler	Reheat boiler
Capacity	330,000 lb. per hr.	290,000 lb. per hr.
Steam temperature	725 deg. fahr.	725 deg. fahr.
Boiler surface—straight tube	9,000 sq. ft.	9,000 sq. ft.
Reheater surface—gas	..	23,562 sq. ft.
Economizer surface	18,326 sq. ft.	4,763 sq. ft.
Air preheater surface	43,600 sq. ft.	52,800 sq. ft.

Three of the boilers are fired by unit mills, and three by the bin system.

South Amboy. This plant on the Raritan River in New Jersey will contain two cross-compound, double-deck type units each consisting of a 5100-kw., 3600-rev. per min. high-pressure element located above a 19,900-kw. 1800-rev. per min. low-pressure element. Three boilers fired by unit type pulverizing mills will supply 250,000 lb. per hour of steam each at 1310 lb. and 750 deg. fahr. The boiler arrangement will be as follows:

Boiler	7,250 sq. ft. heating surface
Gas reheater	12,900 sq. ft. heating surface
Steam reheater	2,190 sq. ft. heating surface
Economizer	4,420 sq. ft. heating surface
Air preheater	32,050 sq. ft. heating surface

Station A—San Francisco. The throttle pressure for two 50,000-kw. cross-compound, double-deck turbo generators will be 1200 lb. at a temperature of 750 deg. fahr. Steam will be obtained from "standard" boilers having a maximum capacity of 400,000 lb. per hour each, and from reheating boilers of a maximum capacity of 440,000 lb. per hour.

Present Limits of Steam Temperature

There does not seem to be any widespread effort to use steam for power generation at temperatures higher than customary for the past several years. This does not imply that the thermal gains to be derived by higher steam temperatures are not apparent, nor that the investigation of metals to withstand higher temperatures is not being actively carried out. The limiting temperature at the present time generally recognized in this country is around 750 deg. fahr. although instances of operation at slightly higher temperatures have been reported. A straight expansion to vacuum of steam at this temperature without excessive wetness loss in the low-pressure end of a turbine is possible only with steam pressures up to 450 to 500 lb. Above 600 lb. it has been necessary to reheat the steam between turbine elements. A higher initial steam temperature would permit higher initial pressures for direct expansion, as well as increase the available thermal saving for any particular pressure, either with or without reheating. The equipment subject to the highest temperature includes a part only of the superheater, the steam piping and valves, and one end of the turbine. Alloy steels are available which appear capable of withstanding the temperature required to give a steam temperature measurably higher than present practise. Valve and piping manufacturers are more optimistic than the turbine manufacturers who are not now guaranteeing constructions for more than about 800 deg. fahr. At high pressures an appreciable metal section is required to withstand the pressure stresses, and this in turn leads to additional thermal stresses arising from the differential temperatures. The rate of "flow," or "creep," of metals when under high temperature and stresses also appears to be causing some conservatism in the design stresses for which boiler and superheater parts are proportioned.

Large Boiler Developments

Perhaps the next most significant trend in steam plant design is the effort to bring the number of boiler units in line with the number of turbines installed in a plant. Primarily the object is the reduction of first cost of the boiler plant, but there is also some saving in operating labor and maintenance costs. The decrease in the number of boiler units is being brought about both by the use of units of greater heating surface as well as effective absorbing capacity, and by operation at higher average ratings. Greater evaporative capacities have depended upon the ability to burn sufficient fuel under the boiler, but this requirement disappeared with the development of pulverized fuel; the underfeed stoker in turn was stimulated in its performance and fuel burning capacity. Water and steam cooling of the furnace walls removed the refractory limitation to higher combustion rates and increased the availability factor of the boiler unit; other efforts to this end have been directed toward the prevention of clogging of the entrance of the first pass of boiler tubes by slag and combustible in the furnace gases. With feedwater containing no scale forming matter, there is reason now to expect an availability factor for the boiler proper approaching that of turbo generators. Another factor holding down the number of boilers is the tacit, as well as stated, recognition that it is economically justifiable to shift station loads on an interconnected system whenever cleaning or inspection of a large boiler unit is necessary.

Description of Large Boiler Installations

East River. The largest boilers in the world, now being installed in the East River Station of the New York Edison Company, have a maximum generating capacity each of 800,000 lb. per hour, at a working pressure of 425 lb. per sq. in. and 725 deg. fahr. total temperature. These boilers will be of the bent tube A type setting, having two lower boiler water drums, and two lower economizer section drums; at the top of the boiler there will be four water and steam drums; while above these will be two additional dry drums of 48 in. diameter; the boiler drums are 54 in. in diameter. The furnace has a plan section of 23 ft. 6 in. by 43 ft., and a volume of 38,200 cu. ft.; it is entirely water cooled. Pulverized coal is fired on two sides, both vertically and horizontally. The maximum rate of heat liberation is 25,300 B. t. u. per hour per cu. ft. of furnace volume. The heating surface areas are as follows:

Boiler heating surface	60,706 sq. ft.
Water wall heating surface	6,765 sq. ft.
Superheater surface	8,100 sq. ft.
Air preheater (in two units)	91,728 sq. ft.

The expected stack gas temperature at a steam output of 800,000 lb. per hour is 360 deg. fahr.

Hell Gate. Two boilers have been ordered for the Hell Gate plant of the United Electric Light and Power Company of New York, that will each supply a maximum of 800,000 lb. of steam per hour at a pressure of 275 lb. and a temperature of 725 deg. fahr. Each boiler will consist of one pair of cross-drum boilers with 20-ft. tubes set end to end with the high end headers adjacent to each other above the center of the common furnace. Each half section of a boiler will have its own 72-in. drum, interdeck superheater, economizer, air preheater, and furnace water wall. The circulation and steam connections of each boiler in the twin setting will be entirely independent. The furnace will be fired with pulverized coal from unit mills, lined on four sides with water walls, and arranged for disposal of ash in molten form. The two boilers will supply enough steam for a 160,000-kw. turbo-generator unit.

Delray. Five boilers using underfeed stokers are being installed in the Delray No. 3 plant of the Detroit Edison Company, that will have a normal continuous capacity each of 350,000 lb. of steam per hour at a working pressure of 455 pounds and a temperature of 725 deg. fahr. The boilers are of the four drum bent tube type, being 53 tubes wide and using 3¼ in. tubes. The furnace walls are entirely water cooled; a slag screen of two tubes depth extends across the top of the furnace several feet below the entrance of the first pass proper. The water walls and slag screen are connected to a separate drum in the manner previously developed at the Trenton Channel plant of the same company. The division of heating surface is as follows:

Boiler heating surface	23,900 sq. ft.
Water wall and screen surface	3,494 sq. ft.
Superheater surface	4,900 sq. ft.
Economizer surface	6,032 sq. ft.
Air preheater surface	30,300 sq. ft.

The stoker is 15 retorts in width and 57 tuyeres long, and occupies with the double roll clinker grinder a distance between front and bridge walls of 23 feet. The preheated air supplied to the stoker for combustion purposes will not exceed 350 deg. fahr. at maximum rating.

San Francisco. Two 2225-hp. boilers for 450 lb. and 625 deg. fahr. steam are being installed in the plant of the Great Western Power Company, where they will operate at low use factor but will be called upon to pick up load at times quite suddenly. The boiler construction consists of a 72-in. diameter cross-drum 40 ft. long connected to a bank of tubes 56 wide by 13 high, with a supplementary dry cross-drum 36 in. in diameter and 30 ft. long mounted 11 ft. above the main drum and connected to it by 45 four-inch tubes; two steam purifiers will also be installed in each boiler to limit the moisture carry over upon sudden steam demands. The 35,000-kw. turbo generator that will be used frequently as a synchronous condenser for voltage regulation and for standby service, is valved so that it can carry full load with a steam pressure as low as 235 lb. The furnaces fired with fuel oil, are equipped with 9 pilot burners and 20 main burners, and when operating at 5 per cent of normal rating the boilers will be able to pick up full turbine load suddenly applied with a pressure drop of only 125 pounds before the main burners can be brought into operation.

Gorgas. At the Gorgas steam plant of the Alabama Power Company a 60,000-kw. turbo-generator unit is being served by two boilers each of 30,315-sq. ft. heating surface, equipped with air preheaters of 30,600-sq. ft. area and superheaters that give a final steam temperature of 740 deg. fahr. at maximum boiler output of 450,000 lb. per hour. The boiler drums built for 475 lb. are 72 in. in diameter and 34 ft. 2¾ in. in length, being the largest riveted drums to date for this pressure. The 4-in. tubes 24 ft. long are arranged 51 wide by 20 high. The furnaces are fired with pulverized coal. The induced draft fans are operated by two motors, one of 900 hp. which will provide draft up to a steam output of 380,000 lb. per hour, and a larger motor of 1800 hp. for an output of 450,000 lb. A 700-hp. motor drives the forced draft fan supplying air to the furnace through the preheater.

Furnace Wall Cooling

Trends in the method of combustion and in furnace design that were probable two years ago have been rather conclusively confirmed in many instances. At the present time there are instances of metal cooling the entire furnace wall surface of large steam boilers to the virtual exclusion or absence of the ordinary types of refractories. This applies to underfeed stoker fired furnaces as well as to pulverized coal furnaces. The extent of metal cooling in furnaces fired with traveling grate stokers does not appear to be of such broad application as the result of the losses arising from retarded

ignition and a generally lowered furnace temperature. There are cases of station enlargement where the original furnace settings were entirely refractory, but additional boilers were provided with metal cooled furnaces so that comparison of available capacities and maintenance costs have been secured. The experience at the Cahokia plant of the Union Electric Light and Power Company of St. Louis was reported by E. H. Tenney in this respect in the October 1928 issue of *Mechanical Engineering*, as well as with regard to changes in the combustion method that will be discussed later. Metal cooling of the furnace seems to be practically necessary when any appreciable heat recovery is effected by air preheaters; the introduction of preheated air in turn contributes to better firebed conditions on stokers and serves to maintain a high combustion rate in pulverized fuel furnaces; and higher ratings seem to be obtainable with decreased furnace wall maintenance through the extensive use of metal cooling. The majority of the large boiler furnaces recently put in operation have been metal cooled, and well-known cases of replacement of refractory lining with metal walls have also occurred. Under the term of metal walls several types of construction are to be included, such as plain boiler tubes on the face of, or embedded in a refractory wall, boiler tubes of the fin type backed up with a heat insulation, and boiler tubes with plain metal or refractory faced blocks clamped to them.

Stoker Development

The development of underfeed stokers in the past two years has been directed mainly toward an increase in burning capacity per foot of width, principally through lengthening the coal feeding area by extending the retort zone, with an additional number of tuyeres through which the air for combustion is delivered. Stokers having 65 tuyeres are now being built. Further travel from the front of the stoker to the rear is obtained in some designs by the addition of an overfeed section between the end of the retort and the edge of the clinker grinder pit, as illustrated by the last stokers installed in the Hudson Avenue plant of the Brooklyn Edison Company that have an overfeed section approximately six feet in length. While the increase in length of stokers has been the most notable development in stoker design, stokers up to 35 ft. in width have also been built. The relative decrease in the width of the stoker for a given stoker area that has resulted from lengthening of the stoker, has brought about an advantage of secondary importance by the reduction possible in banking fuel, since combustion during banking occurs almost wholly along the front of the stoker.

Progress has been made recently in adapting the rotary ash discharge type of stoker for the efficient burning of low grade fuels in quantity, mainly by improved design of the clinker grinder pit and grinder rolls along the ideas developed in stoker practise with higher grade eastern coals.

The long travel of the coal and waste products from the coal hopper to the grinder pit has required facilities for adjusting the motion of the different parts of the firebed and for individually controlling the windbox pressures under the subdivided stoker sections. These mechanical requirements are reflected in the trend of development toward the wider use of steel and of parts machined or cast to specifications not before justifiable.

Changes in Pulverized Fuel Practises

The increasing use of pulverizing mills delivering their product directly to the furnace, and the horizontal arrangement of burners for combustion are the most outstanding trends in pulverized fuel practise. The pulverized method of combustion has been applied on a large scale to almost every kind of coal commercially available in the country. The initial cost of pulverized fuel equipment external to the furnace has been reduced by placing the pulverizing mills adjacent to the furnace and by drying the coal in the mills during pulverization, thus minimizing building volume and conveying and drying equipment; when the coal is then delivered from the mills to the furnaces without the interposition of costly bunkers, further simplification of equipment is possible. In one mill arrangement the fan for transporting the coal from the mill to furnace is integral with the mill; in another layout a separate fan is placed between the mill and the bunkers. Heated air from the flue gas air-preheater is ordinarily employed for mill drying, although steam air-heaters are installed in some plants. Direct firing without fuel storage between the mill and burners requires division of the mill capacity, as well as burner equipment, into two or more units in order to insure a certain degree of reliability of fuel supply. Duplication of mills is also a benefit for the control of fineness of the pulverized product with varying boiler ratings; mill power can also be controlled to some extent. A census made by the Prime Movers Committee of the National Electric Light Association showed that for recent installations the ratio of direct fired to bin and feeder boiler horsepower was 1.7 to 1 for utility plants, and 9 to 1 for industrial plants. The average size of boilers fired by the unit system appears to have increased in the last two years, with the size of direct fired boilers in utility plants still being considerably larger than boilers similarly fired in industrial boiler plants.

The trend towards horizontal firing seems to coincide with the attempt to increase the average heat liberation per cu. ft. of furnace volume. Horizontal firing has been used with both the bin and unit mill systems, being particularly convenient with unit mill layouts, although both horizontal and vertical firing have been applied together in the same furnace. Horizontal burners having relatively large coal capacity lend themselves to the use of forced draft which must be employed for the maximum of turbulence, an essential for horizontal firing. In some cases where the ash deposited on the

bottom of the furnace is removed in a molten state, the temperature at the furnace hearth must be maintained above the fusing temperature of the ash, which is readily done by burners located near the bottom of the furnace.

Turbulent firing and metal cooling of furnace walls have led to a method of ash removal which seems likely to be exploited in new installations. It is possible to maintain temperatures at the floor of the furnace sufficient to liquefy ash having fusion test temperatures up to 2500 to 2600 deg. fahr. Means for readily removing liquid ash from the furnace and disposing of it have been devised at the Charles R. Huntley Station of the Buffalo General Electric Company, and the method is being applied at other plants. The furnace floors in "slag bottom" furnaces are horizontal, and the slag is tapped out at intervals into a stream of high velocity water that shatters the slag and disintegrates it into small particles that can be handled hydraulically.

The effect of extensive water and steam cooling of furnace walls together with turbulent firing may be seen from a comparison of the maximum rates of heat liberation in some of the recently constructed furnaces, with conditions existing in the latter part of 1927 in pulverized fuel installations. The Prime Movers Committee of the National Electric Light Association at that time published a tabulation of pulverized fuel installations covering 224 furnaces fired by the bin and storage method and 166 fired by unit type mills, for which heat liberation data were known. The arithmetical average maximum liberation of B. t. u. per cu. ft. of furnace volume per hr. was 20,300 and 21,400 for bin and unit systems respectively. The frequency distribution for the two cases, however, was quite different, the modal value or the figure of greatest frequency being 16,400 for the bin system and 21,800 for the unit system. Compared to these figures of heat liberation, the last furnaces installed in the Cahokia plant of the Union Electric Light and Power Company of St. Louis will have a maximum rate of 26,700 B. t. u.; the recent furnaces in the Hell Gage station of the United Electric Light & Power Company of New York, 35,000 B. t. u.; and the last installation at the Charles R. Huntley station of the Buffalo General Electric Company, a liberation of 37,900 B. t. u. per cu. ft. per hour. The furnaces in these three stations are being fired from unit mills and with horizontal burners, the furnace walls being completely metal cooled.

Comparative Status of Pulverized and Stoker Firing Methods

The introduction of pulverized fuel firing less than ten years ago has been followed by its application in all sizes of boiler plants and with all classes and grades of lignite, bituminous, and anthracite coals. Experimental installations on a large scale and commerical competition with stoker firing, have now continued long enough perhaps to allow a generalized comparison of the two methods.

There seems to be a recognition that most of the earlier pulverized installations have been slightly higher in first cost than corresponding stoker plants, but the operating advantages claimed for pulverized firing appeared to overbalance any cost differential in favor of stokers. There can be no question that the increased use of pulverized firing stimulated new developments in stoker design with the result that fuel burning capacity, range of high efficiency, etc., of stokers, were greatly broadened, and thereby a more effective competition with some of the major advantages of pulverized firing was created. During the same period the apparatus layout for drying, pulverizing, and burning coal was simplified with an apparent reduction in first cost.

Whereas formerly there was a widespread belief that a much greater choice of fuels was available for pulverized firing than could be utilized by any one type of stoker, recent developments in the stoker field have narrowed this advantage to special cases of extreme variations in fuel supply, such as anthracite and bituminous. With coals for which the underfeed stoker has been most extensively applied, the operating advantages of pulverized firing are looked upon as rather debatable by some engineers. Smaller banking losses, greater flexibility with fluctuating loads, and better sustained efficiency over a wider range of loading, have been claimed for pulverized firing, as well as better adaptability for complete automatic control. Opposed to these claims the disadvantage of greater dissemination of ash dust to the atmosphere is cited. Certain it is that the ash dust problem is real and its total elimination appears to be quite costly. In a recent case the greater commercial possibilities in the disposal of stoker ashes over pulverized fuel furnace ash was one of the factors that influenced the selection of the combustion method. There are many engineers who believe that with an assured fuel supply of good and uniform grade, there is so little difference in net economic results between the two combustion methods that the choice in particular cases is being made for reasons which formerly were regarded as subordinate considerations.

Large Turbo-Generator Installations

Increase in capacity of turbine cylinders and generators still continues to be the most impressive development in the steam turbo-generator field, although turbine units of immense size have not been uncommon or unpresaged in the past. It is probably correct to say also that the growth in capacity of individual units has been made possible, not by any new or radical ideas, but by refinement and application of design principles already at hand. The 208,000-kw., three-element, cross-compound General Electric machines for the State Line Plant near Chicago, announced some two

years ago, remain the largest units in total capacity on record. Previous records, however, have recently been broken for two-element cross-compound units, and for single-element machines. The 160,000-kw. machine that is going in the East River Station of the New York Edison Company will embrace both the largest tandem-compound turbine and the largest generator ever built. Generators of large capacity are now operating at 22,000 volts in this country and at 33,000 volts in England, which is believed to be the highest voltage used to date for large power generation.

Two-Element Cross-Compound Turbines

Hell Gate. The two turbo generators of 160,000-kw. capacity each recently installed in the Hell Gate plant of the United Electric Light and Power Company, New York City, consist of a single flow high-pressure element and a double flow low-pressure element. One was manufactured by the American Brown Boveri Company and the other by the Westinghouse Electric and Manufacturing Company. Machinery details are as follows for the two units:

		A. B. B. Co.	W. E. & M. Co.
Rated capacity,	H. P. section...	75,000-kw.	80,000-kw.
	L. P. section...	85,000-kw.	80,000-kw.
Speed,	H. P. section...	1,800-r. p. m.	1,800-r. p. m.
	L. P. section...	1,200-r. p. m.	1,800-r. p. m.
Electrical conditions.....		85 per cent p. f., three-phase, 60 cycles 13,800 volts	
Steam conditions.....		265 pounds, 611 deg. fahr.	

The 137,500-sq. ft., two-pass horizontal surface condensers serving these turbines are unique in their arrangement, each consisting really of two separate condensers attached to the two exhaust nozzles of each low-pressure turbine, but having a common central waterbox horizontally divided for inlet and outlet condensing water. One-half of the condenser is rigidly attached to one exhaust nozzle while the other section of the condenser is joined to the other exhaust nozzle by a flexible joint with lateral freedom.

Single-Element Turbines

The single cylinder General Electric units of 75,000-kw. capacity each in the plants at St. Louis and Buffalo are the largest machines to date of that type. The Cahokia, St. Louis unit is an 1800-rev. per min., 60-cycle machine operating on steam at 315 lb. and 725 deg. fahr., while the Buffialo unit is a 1500-rev. per min., 25-cycle, 275-lb., 689-deg. machine. It is to be noted that while the high pressure element of the State Line unit is rated at 76,000 kw., the expansion of steam through it is only partial, as contrasted with the complete expansion to 29 in. vacuum in the single cylinder machines of 75,000-kw. rating.

Tandem-Compound Turbines

East River. The 160,000-kw. tandem-compound General Electric machine being installed in the East River Station in New York will have a two-winding generator, the circuits of which are entirely separate and coupled only by the mutual reactance between them. The windings will be connected to separate bus sections whose only tie normally will be through the transformer action of the two windings in the generator. The advantages of the two-winding generator include the use of smaller switches than would have been necessary to handle the entire generator output, and led to a bus arrangement whereby short-circuit disturbances are localized and minimized; switch house building space was saved; a ground on one winding will not require taking the other winding out of service; and the bus tie through the generator will result in synchronism of the two bus sections during disturbances, since the two windings must necessarily be in synchronism, although effective reactance between the busses still exists. The generator is a three-phase, 25-cycle, 11,400-volt machine, operating at 1500 rev. per min. Throttle steam conditions are 400 lb., 725 deg. fahr. Further details regarding this installation may be found in *Electrical World*, Vol. 92, No. 24, Dec. 15, 1928.

Generator Voltages and Speeds

There was an advance in voltage of large generators when the 94,000-kw., 1500-rev. per min., 16,500-volt unit went into service in the Long Beach Plant No. 3 of the Southern California Edison Company. This voltage is now exceeded in the 55,000-kw. turbo generator in the Pekin, Illinois, plant of the Super-Power Company, that operates at 22,000 volts and 1800 rev. per min. In England a 25,000-kw., 80 per cent power factor, 3000-rev. per min. machine, that generates at 33,000 volts, has been recently put in service in the new Brimsdown plant, and is believed also to represent the highest capacity developed to date above 1800 rev. per min. In this country the 12,500-kw. 80 per cent power factor generators, driven at 3600 rev. per min. by the 1200-pound turbines, mark the peak capacity of high speed machines produced to date.

Turbine Auxiliaries

There have been no outstanding new trends within the past two years in the design of condensers or other auxiliaries serving turbo generators; neither have there been any new developments in the methods of heating and deaerating boiler feedwater. The trend toward the use of single-pass condensers, and a lower ratio of condenser surface per kilowatt of turbine capacity seems to be confirmed in practically all of the newer stations. Feedwater heating by steam extraction is almost universal.

The necessity of pure feedwater for the higher pressure boilers is perhaps leading to increased efforts toward the reduction of condenser leakage by attaching the condenser tubes to the tube sheets in the manner in which boiler tubes are "rolled" or "expanded." In some cases the tube sheets are flexibly connected to the condenser shell, while in one instance the condenser tubes have been bowed in order to allow them to deflect laterally if necessary.

The cooling of generators by the enclosed circulation system is now widely practised in steam electric stations, and is also being extended to waterwheel generators. Hydrogen in place of air for cooling has also been used in the case of a synchronous condenser, that is installed in the open with a weatherproof casing.

Mercury Vapor Developments

The mercury vapor boiler and 10,000-kw. turbine installation at the South Meadow Plant of the Hartford Electric Light Company, Connecticut, described in the last report, was placed in operation in October, 1928. Mercury vapor is produced at a temperature of 884 deg. fahr., passed through the mercury turbine and condensed at 28 in. of vacuum to make steam at 350-lb. pressure. The steam is returned to the boiler setting and superheated from 436 deg. to 700 deg. fahr. before going to the station steam mains. A water economizer and air preheater following the steam superheater in series reduce the flue gas temperature to 280 deg. fahr. A mercury heater or economizer is located between the mercury boiler and steam superheater. The furnace is fired with pulverized coal.

Steam Table Research

The experimental results from the A. S. M. E. Steam Research Program carried out during the past few years at Harvard University, the Massachusetts Institute of Technology, and the U. S. Bureau of Standards, have given a mass of overlapping data that has enabled the General Electric Company to develop a complete set of steam tables and diagrams to 3500 lb. per sq. in. and 1000 deg. fahr. These were published in preliminary form in the February 1929 issue of Mechanical Engineering. The remarkable agreement between the experimental results, and with contemporary foreign data, indicates that the new tables will suffice for the most exacting type of engineering work.

Acknowledgment

The committee has been greatly assisted in the preparation of this report by the data, suggestions, and criticisms received from many engineers and manufacturers, and in acknowledging their cooperation, wishes to express its thanks on behalf of the Institute for the help so freely extended

Report of Subcommittee on Excitation in Electric Power Plants

A survey has been made of the excitation layouts in some 26 modern large stations, the majority of which have been in operation less than five years. The generating units in these stations range in individual capacity from 12,500 kw. to the largest yet operated. The frequency of application of the salient features of excitation layouts is shown by the following tabulation, which, though not including all modern stations, yet is believed to be representative of present day practise.

Design feature	Relative application
Excitation voltage of 230-250	100 per cent
Systems individual to a generator	93 per cent
Individual systems with transfer bus to spare excitation capacity	81 per cent
Individual systems having battery spare	27 per cent
Individual systems using motor-gen. exciters driven from shaft generators	11 per cent
Individual systems using sub-excitation of exciters	33 per cent
Installations using shunt windings on main exciters	87 per cent
Installations omitting main field rheostats	50 per cent

An excitation voltage of 250 has been adopted almost universally because of the economy of copper and switch facilities, and the fact that reliability is readily obtainable with 250-volt machines operating at the main generator speed, usually 1800 rev. per min. or less.

Excitation sources arranged individually for each generator predominate constituting 93 per cent of the cases. Only one station surveyed operates on the central system with excitation busses supplied from separately driven station exciters; one station uses group excitation busses supplied from shaft-end exciters belonging to the machines excited from the group bus.

Individual excitation is obtained from shaft-end exciters in 89 per cent of the cases, there being three instances only in the group surveyed of motor-generator exciter sets supplying excitation on the individual principle. In these cases the motor-generators were driven from shaft-end generators mounted on relatively slow-speed hydro units, and were used primarily in the interest of quick response excitation.

However, of the individual excitation systems, 81 per cent of the installations have available transfer facilities whereby spare exciters or storage batteries can supply the fields of the generators in the event of loss of the shaft-end or motor-generator exciters. Storage battery reserve is provided in 27 per cent of the individual systems, but it is noted that in operation these batteries are not paralleled with the individual sources of excitation.

In 33 per cent of the cases, sub-excitation or separate excitation of the main exciter is provided by a pilot exciter carried on the shaft of the main exciter, both shaft-end and motor-generator exciters. In one large station, however, the main generator fields are excited in parallel from group sub-excitation buses that are energized by small motor-generator sets and storage batteries. In all instances except this, the sub-excitation installations are used in connection with transmission line systems.

Shunt windings only on the main exciters seem to be preferred in 87 per cent of the cases.

The elimination of the main field rheostat in 50 per cent of the cases, or its installation for emergency operation and the control of the main generator voltage by means of the exciter voltage appear to have advantages desired by quite a few engineers.

Manual control of the main generator voltage still seems to be favored, particularly in steam-electric

stations, although a number of plants has installed and operates automatic-control equipment.

Where generator voltage is to be maintained during system disturbance, automatically-operated quick-response equipment has been installed. The individual exciter makes the high-speed excitation problem simpler, as a central excitation system, with machines of different size and characteristics, could not be regulated satisfactorily even though all generators might be feeding a common bus at the time of a disturbance. With the individual exciter, the main generator voltage can readily be controlled by varying the main exciter rheostat or by a pilot exciter. Thus the main field rheostat may be entirely omitted and by the use of special relays and reversed windings on the exciters, a wide range of excitation may be obtained. Quick response excitation is obtained by suitable design, principally by the use of parallel field circuits and laminated magnetic circuits. The quick-response installations have found their widest application to hydroelectric systems in which the machine and line reactance are relatively high and stability is important. With large networks connected to widely separated steam and hydro plants, quick-response excitation may be desirable for steam-driven generators and its installation should be the subject of a careful analysis of the transmission and generating conditions. High speed as referring to excitation is somewhat indefinite, but in the present state of the art, implies a voltage increase of 400 to about 700 volts per second with 250-volt excitation. The developments in design of d-c. exciters have been such that today, even on low speed hydro generators, a satisfactory direct-connected exciter can be built at a cost little, if any, greater than for a separate motor driven exciter installation.

One company has installed quick-response excitation on a steam-driven house generator of 5000-kw. capacity to maintain voltage with sudden load changes such as occur when starting or shutting down the large motor-driven auxiliaries. Synchronous condensers are also equipped with quick response excitation and on account of the relative capacity of the synchronous condensers and the system it may be desirable to have the response quicker than for the generator.

One company now studying means for insuring the reliability of the excitation layout for steam-driven machines of 100,000-kw. capacity, is considering the use of two separately insulated field windings for the main generator, with each winding magnetically balanced so that the machine can operate safely with either field winding alone, and each field winding to be capable of operating the generator at approximately unity power factor. Independent sources of excitation are to be used for each field winding, and there are to be automatic means for increasing the strength of one field winding in case the voltage on the other fails.

Report of Subcommittee on Auxiliary Drives in Electric Power Plants

Present practise with regard to auxiliary drives has been summarized below from the data obtained from a list of some 21 modern stations. These data are believed to be sufficiently comprehensive to be representative. The generating units in these stations range in individual capacity from 12,500 kw. to the largest yet operated.

General

Motor drive of auxiliaries is used in all cases except one in which steam drive is the main auxiliary power. However, the installation of a steam-driven boiler feed pump for reserve purpose appears to be universal, while in a small number of cases duplicate motor and steam drives are used for some of the more necessary auxiliaries.

In approximately half of the stations, the auxiliary layout is designed and operated on the unit system, *i. e.*, each turbo generator and its corresponding essential auxiliaries form a unit.

Methods of Power Supply

The sources of auxiliary power and their relative use are shown by the following tabulation.

Source of auxiliary power	Relative use
Transformers ahead of generator switch	22 per cent
Generators on main unit shaft	22 per cent
Installations of house turbines	39 per cent
Transformers on busses or supply from other stations	universal

Transformers connected ahead of the generator switch assure energy whenever the main unit operates and with proper protection by relays and oil switches, line disturbances will not cause an outage of auxiliary power. However, in the case of machines feeding a large system, operating conditions may be such that the machine voltage, and consequently the auxiliary voltage, vary between unduly wide limits.

The shaft generator has the advantage of isolation, and insures an auxiliary supply whenever the main unit is running. For starting up, the auxiliaries can be thrown upon the emergency bus, or normally open tie switches can be closed. The shaft generator increases the space required for the unit substructure and foundation and adds somewhat to the unit cost. It has the disadvantage that electrical or mechanical failures may require a shutdown of the main unit for repair.

The house generator, driven by a separate steam turbine, provides a maximum of separation and with a standby machine or motor-generator set should meet the requirements from the standpoint of reliability. As the trend today is towards large units, the auxiliary power demand for turbine and boiler unit is passing beyond several thousand kilowatts, and the turbines, especially if of the 3600-rev. per min. design, may be built with good steam rates, materially lessening the

advantages of the shaft-driven generator which operates at speeds of 1200 to 1800 rev. per min.

Special windings on the main generator are being considered, but as the auxiliary voltage would be dependent upon the main field excitation, and this would be varied in accordance with the system demands, it is necessary to provide a suitable arrangement to hold the auxiliary voltage reasonably constant. It has the disadvantage that failures in either stator winding may be communicated to the other winding and result in having the main generator out of service for slow and expensive repairs.

Motor-generators a-c. to a-c. may be used instead of transformers, but they introduce additional rotating machinery which requires more attention, is more liable to fail and more expensive than transformers.

The motor-generator set or house-generator are the two logical solutions for d-c. auxiliary power. The recent developments in rectifiers for railway service may result in their application in power stations. A synchronous converter of course could be used instead of a motor-generator set, but it does not provide complete electrical separation of a-c. and d-c., does not give the voltage range, and is not inherently as stable as the motor-generator set.

Types of Motors

For some apparatus the wound-rotor motor is particularly well adapted. If the speed range does not exceed approximately 40 per cent, the commercial types of control apparatus, with a multiplicity of speed points, may be employed. For a speed range comparable to that of a d-c. motor, double-winding slip-ring motors are available, but the control gear becomes unduly complicated and expensive, and the motor is special, so that first cost and replacement are high. The brush shifting motor is liable to spark, is expensive and higher in maintenance than other types. Its commercial size is limited, and it is not available for some of the large auxiliaries of the new stations.

D-c. motors lend themselves readily to a wide range of speed control with little sacrifice in economy and with relatively inexpensive control equipment. The commutator motor is not desirable for a dusty or gassy atmosphere, but then the machines may be totally enclosed and vented from the outside. In order to insure stability of motor operation the d-c. motor for wide speed ranges should be designed for a substantial ampere-turn excitation over the whole range.

For the usual plant, 2300-volt motors are used above 50 hp., and below this, 440 or 220 volts are well adapted, as it makes available the large stocks of the motor manufacturers. The selection of the auxiliary voltage and changeover motor size cannot be governed by any hard and fast rule, but should be the basis of study for each installation, considering not only the economics but also reliability and simplicity.

The central station is built up around reliability of service, and this is materially aided if special insulation, that is, better than the commercial standards, is used on the station auxiliaries.

The synchronous motor has not found a wide field of application in the power plant, but for driving such of the non-essentials as do not require speed variation it could often be used to advantage. By designing for power factor correction, the voltage variations on the auxiliary bus can be reduced and at the same time increase the power which can be drawn from the auxiliary transformer or generator. The larger motor manufacturers are now building synchronous motors with special clutching arrangements so that the synchronous motor may be used on equipment requiring high starting torque.

Control Considerations

There was a tendency a few years ago to improve the economy of auxiliaries by wide-speed variations and many speed points. Experience has shown that in a number of cases this was carried too far, as the cost and complication overbalanced the economies shown on paper. Today it is realized that it is economically sound to sacrifice some calculated advantages in the interest of simplicity and reliability. The single- or multiple-speed squirrel-cage induction motor is being applied in many cases where previously a wound rotor or d-c. motor would have been used.

Across the line starting eliminates the starting equipment, but where this is used for large motors it may be necessary to provide a means for maintaining the auxiliary bus voltage. When auxiliaries must be started frequently, the manufacturer should be consulted to insure that the oil switches are equal to the unusual duty imposed upon them. Motors also should be built for the heavy duty.

Some engineers look with disfavor on the multiplicity of small oil switches involved with an auxiliary voltage of 2300 or 4000 as these switches require more space than circuit breakers or latched in contactors and are not as readily inspected and serviced. For an installation without excessively long runs and in which the plant expansion is definitely limited, a complete estimate of the probable ultimate can be made with a fair degree of accuracy and the economics as well as the important intangibles studied before reaching a decision on the design. With large motors the copper requirements will be large, but by interleaving the busses and with special transformers, the total voltage drop can be held within practicable limits. Both circuit breakers and contactors are available capable of repeatedly interrupting currents up to 20,000 amperes at 440 volts.

The engineer has the choice of a-c. solenoid contactors operated either by a-c. or d-c. control circuits. If the d-c. control is used it has the advantage of being unaffected by fluctuations of the a-c. voltage, it is quiet,

and the solenoid currents are not affected by the magnetic conditions. For some installations these advantages more than compensate for the added complication of the separate control circuit.

Operating engineers are not agreed on the question of overload protection for auxiliaries. The type of protection depends somewhat upon the installation. In a plant with the essentials in more than one unit, overload relays that will protect the auxiliaries without causing a shutdown of the main unit are a logical arrangement. Where the plant requires operators for the auxiliaries, the overloads have been set at stalled motor current, and in the event of overload the motors are allowed to run hot until the trouble is remedied by the operator.

Completely centralized control and control at the auxiliaries have their applications, the former being somewhat more costly for the equipment and installation. Control at the machine makes it necessary to have additional personnel but in a large plant it is essential that the machinery be regularly examined, and generally these men are available for operating the controls. Control adjacent to the motor reduces the possibility of starting a machine with men working on it and has the important advantage that the operator starting up can see the apparatus and assure himself that it is functioning satisfactorily.

A study of the present station records shows an increasing number of kilowatt-hours sent out per man and an increasing use of automatic combustion and boiler feed control. Some of the combustion control equipment has required a large number of speed points which has made the first cost of the control and motors run high. Multispeed squirrel-cage windings, with fine adjustment on the dampers, will provide a simpler and cheaper solution, and in many cases can be designed so that there is little sacrifice in economy over the operating range. Boiler feed pumps do not involve any special problems other than the multiplicity of speed points which some of the control systems require. Usually, the speed range is relatively small on account of the inherent characteristics of a centrifugal pump and the necessity to keep the discharge pressure at all times in excess of the normal operating pressure of the boiler. The advent of boiler pressure of 1200 lb. and above has resulted in the development of the 3600-rev. per min. wound rotor motor for driving the high-pressure pumps. The design and application of these pumping sets have introduced new problems of control and reliability.

On account of the fire and explosion hazard, pulverized fuel installations should be designed to eliminate the possibility of sparking of control devices or motors in a dusty atmosphere. Good housekeeping is one of the best safeguards. The National Board of Fire Underwriters' rules for pulverized fuel installations provide reasonable safeguards for both the personnel and apparatus.

Report of Subcommittee on Interconnection

Effect of Interconnection on Power Generation.

Interconnection has permitted the size of generating units to be greatly increased so that units of 100,000 or 150,000 kw. are becoming common, while even larger units are under construction. In most of the instances where such large units are being placed in service, they are for the principal use of one operating company rather than for the joint use of parties to an interconnection. However, the individual companies are in reality composed of a number of smaller companies which had previously been interconnected. Interconnection of companies has permitted the total reserve of the district to be decreased since the individual parties need not independently provide for their own emergency. The increased size of units results in a decreased investment per kilowatt of capacity, while the reduced district kilowatt capacity still further reduces the investment in power generation apparatus. Another feature which makes possible the installation of large sized units is the fact that interconnection permits a staggered construction program, so that one company can make a larger addition than it would be justified in making alone, and its investment charges can in part be carried by the other interconnected companies.

The consumption of fuel is reduced because of the possibility of giving the more economical units the "base load" of an entire district and because water power can be used more effectively. There is a number of examples where the power which could be developed from stream flow plants during certain hours of the day was wasted and is now used due to the interconnection of systems which gives a greater outlet for the energy.

Reasons for Interconnection.

By far the largest number of interconnections has been of the type where small systems have been merged financially with larger systems, and the natural tendency was to abandon the smaller of the generating stations and interconnect the systems. Much of this has been done without detailed studies as to the possible economy, the problem being considered in the light of the consolidation of property.

However, there is also a number of interconnections between large systems having separate management. These interconnections are made for one or more of the following reasons of economy or improved reliability of service:

(a) Generating reserve; (b) load diversity; (c) mutual assistance during emergency; (d) minimizing possibility of territorial power shortages; (e) utilization of water power; (f) additional business due to coverage of intervening areas.

The primary object of interconnections in the final analysis is of course a reduction in investment and

operating cost. Presumably, it is possible to build and operate an independent system which will give the equivalent of a system interconnected with others, but interconnection affords a more economic method of accomplishing the desired results.

Methods of Interconnecting.

Of the two methods of interconnection, namely, loose and tight linking, no conclusive arguments in favor of one or the other appear to be generally applicable. The distinction at the present time seems to be principally that of purpose. Where loose linking is in operation, the primary object is to provide facilities for the interchange of power for known requirements, and where a certain amount of time can be allowed for connections. In this case, the member systems of an interconnection are each capable of providing for their own requirement, with the exception that provision is made for the actual transfer of load by switching to take care of inequalities between the load and available generating capacity. Thus to a certain extent all of the possible benefits of interconnection are not realized, but on the other hand, some of the complications are avoided. Where tight linking and trunkline ties are employed, the facilities of the group are at all times available to each of the member systems. The complication and expense of the interconnecting ties, however, are greater for this condition.

Problems of Interconnection.

The principal technical problems at the present time appear to be in connection with the following subjects:

(a) Load control; (b) load dispatching; (c) voltage control; (d) frequency control; (e) relaying; (f) stability; (g) increased short circuit currents and adequacy of circuit breakers and other equipment; (h) communication; (i) metering; (j) technical basis of contractual relations.

Several of the items given above are not peculiar to interconnection but appear in the planning of any system, and improvements are continually in progress, applying equally well to system and interconnection design.

There is an increasing tendency toward the use of automatic recording devices for filling the gaps between the information from station operators and what is left of a system after a disturbance; and it is highly probable that when accurate information from these devices on systems throughout the country over a period of time is available, some very constructive new ideas will result.

Report of Subcommittee on Hazards to Service Reliability in Modern Generating Stations

This subcommittee has endeavored to obtain discussion by designers and operators of generating plants regarding the major hazards that have developed to station capacity as the result of recent trends in the design features of modern power generating stations. Modern stations are characterized in the main by magnitude of total capacity and by concentration of capacity in individual machines, both electric and steam generators. These tendencies apparently have not brought to light any new types of hazards, but have rather accentuated possibilities of service interruption that had already been experienced or predicted. In some types of older plants severe interruptions have occurred recently, as the result of inadequate or faulty design of an earlier date. While the subcommittee recognizes that an accident hazard to operating personnel may develop into a major capacity outage, or vice versa, it has not been directly interested in this phase of station reliability, realizing that such viewpoints are being thoroughly treated by the Accident Prevention Committee of the National Electric Light Association, and by other N. E. L. A. committees dealing with particular machinery applications. Other national organizations, such as the American Society of Mechanical Engineers and the National Safety Council, have devised safety codes covering practically every aspect of power plant operation.

Capacity Hazards in Hydroelectric Generating Plants.

There are undoubtedly quite a few plants in use that suffer severe capacity reductions from ice, trash, and occasional failure of auxiliary control, but for all practical purposes these problems may be regarded as solved in the large capacity hydro stations of modern design.

The subcommittee has received notice of some recent failures of high head penstocks that were designed and constructed some years ago. The remedy suggested has been to have some automatic means, independent of electrical or manual control, of shutting off the water supply in case of such a failure. This has been done by installing valves at the head of important penstocks which are capable of operation against full unbalanced pressure and which are arranged to close automatically with suitable time interval in case the water velocity increases above a predetermined value. These valves may also be closed from the power house, but it is necessary for the attendant to be at the valve in order to open it.

An operating hazard that seems to have been of increasing aggravation is over-voltage upon sudden loss of load. This has been intensified in cases by the use of quick response excitation and high transmission line voltages. The remedy seems to have been additional overvoltage relays to control the voltage regulators as necessary, supplemented in some instances by special governor adjustments.

Capacity Hazards in Steam-Electric Generating Stations.

Some operators, as well as designers, have objected to the increasing complexity of electrical control of the auxiliaries in steam stations, particularly those employing the newer systems of boiler plant combustion.

The multiplication of automatic safeguards and devices has also been condemned as defeating the purpose for which they were installed.

Engineers who are installing the extra large boilers and turbines expect a minimum outage from the troubles that have been bothersome in the past, such as winding failures, blade breakage, and refractory and tube replacements. Engineers operating cross-compound turbine units have expressed the opinion that outage of one element practically means the outage of the entire unit. There seems to be a slight betterment of operating availability of boilers and turbines, but blade failures and winding burnouts still continue to be reported, notwithstanding the intensive research and improvement that has been accomplished in the selection and fabrication of materials.

Particular Electrical Hazards.

The fire hazard in the operation of modern power systems still seems to be one that deserves considerable thought. Fires resulting from the failure of oil filled equipment, such as oil circuit breakers, regulators, and transformers, with the resulting spread of oil throughout the station, are probably the most common and the most serious hazard, judging from the reports from the various operating companies. It seems that fires of this nature have in many cases resulted from the breaking of oil gages in oil filled equipment, permitting the escape of oil in the station to other apparatus and in turn breaking oil gages on that equipment and so on until serious fires have resulted. The solution so far has been the application of ball check valves to back up the oil gages and to design the station so that escaping oil is confined locally. It seems that serious consideration should be given to the perfection of an unbreakable oil gage.

The oil circuit breaker problem, while of considerable standing, is being constantly presented by the growth of interconnected capacity. Serious damage is reported in switch houses due to the spreading of vapors arising from short circuits and faults to ground. In some cases the solution of this problem has been the complete insulation of the busses; in some other instances firewalls have been installed; but in most cases the spreading of such vapors is minimized by cutting out the faulty section of the bus as quickly as possible after the fault develops, by a proper relay system.

One of the greatest hazards in a generating station is that of a bus short circuit and the resulting effects on system stability. A bus short circuit in a generating station usually requires the unloading and the resynchronizing of those generators connected to the faulty bus. If system stability is to be insured at all times, the following requirements must be met:

1. Each generator must be normally connected to at least two separately insulated busses.

2. The total reactance in series with the generators must be low for normal current flow in order to insure good voltage regulation.

3. The reactance, including that of the generator, between each generator and a short circuited bus must be on the order of 60 per cent if the generators are to maintain a substantial voltage on the non-short-circuited bus.

4. A short circuit on one bus should result in a tendency of the generators to raise the voltage an amount sufficient to compensate for the voltage drop produced in the generators by the short circuit.

A method has been proposed in which each generator is connected to two separately insulated high-voltage busses through a transformer having two separately insulated high-voltage windings. Such a transformer would be similar to the 83,333-kv-a. transformer on the system of the Buffalo General Electric Company. The transformer for the proposed scheme would be designed to have an impedance from the primary to one secondary winding of approximately 50 per cent, and from the primary to the two secondary windings, feeding balanced load, of approximately 15 per cent. Such a transformer, when operating with a generator adapted to feed a maximum current of 150 per cent at 75 per cent voltage, will automatically step the voltage to zero on the short-circuited bus and to 105 per cent on the non-short-circuited bus. If the maximum permissible instantaneous short circuit that can be permitted to flow from the generators to one bus is limited to 1,500,000 kv-a., then with a total normal reactance, including 10 per cent generator reactance, between the generator and the high-tension bus of 25 per cent, the maximum generator capacity that can be connected to a single bus is 375,000 kv-a. On the other hand, if the double-wound secondary transformer with a total reactance of 60 per cent to a short circuit on either bus (including 10 per cent generator reactance) is used, then 900,000-kv-a. capacity may be tied to a single bus, and either one of such a pair of busses may be short-circuited with practically no danger of any machine pulling out of step. This is probably as great a capacity as will be concentrated in any one station for some years to come.

It is obvious that if an ultimate station capacity, greater than 900,000 kv-a. is expected, three such busses could be used and three such secondary transformer windings, each corresponding secondary winding in each transformer bank being connected to one of the three busses.

Other methods of enhancing system stability in case of trouble include the paralleling of generator groups at a distance from the generating station, and the use of two winding generators whose windings are connected to bus sections separated by proper reactance.

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Card Index Service of the Engineering Societies Library, inaugurated in January 1928, under the headings of

4 b—*Civil* Hydraulics
5 d—*Electrical* Power Plants
7 b—*Heat Power* Boilers
c—Fuels and Combustion
e—Oil and Gas Engines
f—Steam Power Machinery
g—Power Plants

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Progress in Hydraulics ..... Vol. 50, No. 1
Progress in Hydraulics ..... Vol. 51, No. 1
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Discussion

P. B. Junke: How long will the present growth of systems continue? During the last twenty years, the systems have been about multiplied by eleven. If the same growth continues in the next ten years, we shall have quantities to deal with that perhaps few of us give consideration to at the present time.

In Chicago, whereas at the present time we have a system total of about 1,400,000 kw., by 1940 the estimated load will be on the order of 4,000,000 kw.

Another topic that needs discussion is the subject of rehabilitation of plant. In our system we have old plants to the extent of about 300,000 kw., and quite often the thought has been suggested to rehabilitate these. If we look at our composite load curve, we find that this old capacity, written off for all practical purposes, serves a very useful function. It carries the peak load of the system quite nicely for a period of about three or four hundred hours a year. That is perhaps the best function it can perform and the capacity in that form is perhaps the cheapest that can be obtained for that purpose.

In the report a suggestion is made that new plants be built for peak purposes at rather low cost. Where you have old capacity available for such a purpose, I think that is the very cheapest kind of capacity to be preserved for such a function.

One other subject dealt with in the paper entitled *Synchronized at the Load*, must be given thorough consideration, that is, the splitting up of certain systems into sections relatively safe under all conditions. I would suggest that this topic be given additional study and consideration as one of the outstanding problems.

Transmission and Distribution

ANNUAL REPORT OF COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION*

To the Board of Directors:

The work of the Committee on Power Transmission and Distribution covers a wide field and may logically be divided into the following classifications:

Lightning and surge problems on overhead lines
Insulator design for overhead lines
Design of towers and poles for overhead systems
Cable development for underground systems
System connections and stability factors
Standardization activity
Research activity.

The work carried on this year has been confined to the lightning and insulator problems handled by a subcommittee under the leadership of Mr. Sporn, cable development handled by a subcommittee under the direction of Mr. Peterson, and system connections and stability factors handled by a subcommittee under the direction of Mr. Evans, with Mr. Farmer coordinating the standardization activities and the research work carried on under the direction of a joint research committee working under the auspices of the N. E. L. A. Underground Systems Committee, Association of Edison Illuminating Companies' Committee on High Tension Cable, and the Power Transmission and Distribution Committee of the A. I. E. E.

The problems arising under the above activities are so closely related with similar problems handled by other committees of the Institute and other organizations that considerable attention has been given to coordinating this work so as to eliminate unnecessary duplication and at the same time insure the proper handling of all subjects. In light of this condition, your committee has not initiated any work on inductive coordination and any consideration of the interference aspect of such subjects is being tied in with the work of the Joint General Committee and the activity of the work on stability factors and interconnection has been referred to the Committee on Coordination of Institute Activities for assistance in subdividing this work between the various committees.

A summary of the developments and problems arising during the year is covered in the following reports from the subcommittees:

SUBCOMMITTEE ON LIGHTNING AND INSULATORS

PHILIP SPORN, CHAIRMAN

General Aspects of the Lightning Problem

Lightning disturbances on electric circuits have always been a problem; the importance of these disturbances becomes greater with the increase of interconnecting high-voltage lines, transmitting large blocks of power, which now form the backbone of interconnection and in many cases the basis of economic generation. The transmission line, therefore, becomes a vital link in the electric circuit; and lightning, by its adverse effect on its operation, its worst enemy. The demand for more reliable line operation, from a lightning standpoint, has started investigation and research, beginning with isolated field investigations with crude and limited instruments, next laboratory research with the lightning generator, and then to field investigation with the klydonograph and surge recorder, and finally to the cathode ray oscillograph in both the laboratory and field.

Both the laboratory and field have contributed to the solution of the lightning problem. The laboratory by offering complete and easy control of the lightning wave has, of course, rendered investigations far more simple than field research.

Theory, too, has contributed its part in studying the mechanism of cloud formation and lightning discharges, the breakdown of the discharge path, the influence of ground wires, the characteristics of the electrostatic field, the effect of traveling waves on electric circuits, and the shape and effect of different shapes of traveling waves on the insulation strength of the line and the connected apparatus. A noteworthy contribution to the study by the field is the record of lightning disturbances and trouble on lines as encountered in actual service, together with complete storm data where available.

Finally, an extension of the development has been made in combining the laboratory and field in the form of artifically created lightning imposed on actual transmission lines. Certain problems that prove difficult of solution, such as wave propagation, can best be handled that way.

With this background it is clear the solution of the lightning problem has received progression acceleration the past few years, and a large amount of lightning

*COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION:

H. R. Woodrow, Chairman,

R. E. Argersinger,	E. W. Dillard,	E. R. Northmore,
Geo. M. Armbrust,	R. E. Doherty,	L. L. Perry,
R. W. Atkinson,	L. L. Elden,	T. F. Peterson,
E. T. J. Brandon,	R. D. Evans,	D. W. Roper,
V. Bush,	F. M. Farmer,	A. E. Silver,
P. H. Chase,	Harland C. Forbes,	L. G. Smith,
C. V. Christie,	C. L. Fortescue,	H. C. Sutton,
D. D. Clarke,	K. A. Hawley,	Percy H. Thomas,
W. H. Cole,	V. L. Hollister,	Philip Torchio,
R. N. Conwell,	J. P. Jollyman,	Theodore Varney,
M. T. Crawford,	A. H. Kehoe,	H. L. Wallau,
W. A. Del Mar,	A. H. Lawton,	H. S. Warren,
Herbert H. Dewey,	W. E. Mitchell,	R. J. C. Wood.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

data has been secured and many of the results published in the technical press. Even with considerable theoretical analysis, and with laboratory and field data available, the lightning problem is not solved. Some phases of lightning are only partly known; others are in dispute; and some are definitely not known.

Committee Program Planned

One of the undertakings of the committee is to outline what is definitely known about lightning as affecting electric circuits, and include in this subject important factors on which there is general agreement. On phases of the problem where there is disagreement, it is hoped and expected a thorough discussion and analysis will produce an agreement on most if not all disputed points. Where investigations are carried on independently and without coordination, it is to be expected different conclusions may be drawn. Perhaps it is well this should initially be so, since coordination, when a problem is first attacked, might stifle original ideas and tend to place the whole investigation in a rut. It is believed, however, that in the case of the lightning problem, this stage has passed, and that there is room now for reaching a definite understanding on many phases of the work. Such an understanding, it is believed, will be of great value to all investigators, manufacturers, and utility engineers and operators.

With past lightning data and theories correlated, as above outlined, it should be possible to state clearly what additional information is needed, and to indicate what is most urgently needed. The committee plans to do this.

The past lightning investigation work in the field has been done on the higher voltage circuits, largely, as previously pointed out, due to their importance in the electrical network. However, it is felt desirable to apply the information already secured to the lower voltage systems, and even to the distribution networks. A general survey of both transmission and distribution systems is planned to determine how lightning affects both. This requires analysis from theoretical, operating, and the economic points of view.

The lightning problem on electric circuits is primarily an insulation problem, and consequently requires not only a study of voltages which may occur, and the nature of those voltages, but also the distribution of insulation in the apparatus and on the electric system to prevent any electric breakdown if possible, but in any case to localize trouble at a predetermined and least damaging point. This at once requires a broad consideration of the insulation of the lines and connected apparatus as related to each other.

Remedial measures against lightning and beneficial accessories where power arcs follow up will require thorough analysis supported by operating experience. These include lightning arresters, ground wires, grading shields, insulated ground wires, protective gaps, insulation of wood poles and crossarms, insulated guy wires, fused arcing rings, etc. Here too, a balance is required between the beneficial effects expected, or proved to exist and increase in reliability of service and cost.

Lightning Progress During 1928

During the past year some dozen field investigations have been in progress by the operating companies in cooperation, in most cases, with the manufacturers. Many data have been added to our present store regarding the magnitude of lightning voltages on transmission lines, attenuation of this voltage, and characteristics of traveling waves. Three cathode ray oscillographs have been in operation during part of the 1928 lightning season, and two photographs of actual lightning voltage recorded. While many more data are needed on lightning wave characteristics, this is a noteworthy beginning.

A method of attacking the lightning problem, as mentioned above, has been put into use again in the past year. This consists of placing artificial lightning surges on actual transmission lines and observing with the cathode ray oscillograph the behavior of the wave at short-circuited and open-circuited points on the line, and further determining attenuation of the wave. This investigation has been very useful in studying the behavior of the lightning wave, and promises much for the future.

The past year has seen the continuation of ground wires being used on new lines, and their addition to some existing lines. The grading shields on insulator strings, extensively used in the past few years, have been in many cases installed on new lines. The practise of reducing the line insulation for a mile or so out from the substation, to protect station equipment from lightning, has been extended. Additional (more than one—in some cases three total) ground wires have been installed from the substation out for the first two towers or so on the line, to reduce the line surge impedance, and as a further protection against direct strokes. In some cases these additional ground wires have been carried on separate structures.

Research Work in Progress

Surge investigation installations are in progress on the transmission lines of a number of operating companies. Some ten or twelve cathode ray oscillographs are planned to be in operation on actual lines during the summer of 1929 to record lightning waves. Two or more additional installations to record the performance of artificial lightning are also planned for this summer. It can be seen that quite an extensive program has been planned; and it is expected effective work will be accomplished.

The great number of papers and articles which appeared during the past year on lightning and related subjects affecting the transmission system from the lightning point of view, clearly indicates the growing importance of the lightning problem. They further show the many angles of attack from which investi-

gators are working. A list of the more important papers and articles, which have come to the committee's attention, is given below.

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22. "Effect of Ground Wires on Traveling Waves," by J. H. Cox and J. Slepian, *Elec. World*, September 1928, p. 551.

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24. "Some Electrophysical Conditions determining Lightning Surges," by Harald Norinder, *Franklin Inst. Jl.*, June 1928, p. 747.

25. *A New Type of Hot Cathode Oscillograph and its Application to Automatic Recording of Lightning and Switching Surges*," by R. H. George, A. I. E. E. Quarterly TRANS., Vol. 48, July 1929.

26. *Investigation of Transmission Lines with Artificial Lightning*, by K. B. McEachron, A. I. E. E. Quarterly TRANS., Vol. 48, July 1929.

SUBCOMMITTEE ON CABLE DEVELOPMENT

T. F. PETERSON, CHAIRMAN

GENERAL STATEMENT

The portion of last year's Power Transmission and Distribution Committee Report dealing with cable developments included descriptions of what might be considered remarkable advances and great strides in the transmission art. Of these, the design, installation, and operation of 132-kv. oil-filled cable were most notable. This year nothing so sensational is to be noted. Practically all development and improvement seem to have been characterized by a tendency toward review and analysis of existing practises, stabilization, and reduction of known methods and principles to economic and practical feasibility. These have apparently constituted the bases for such crucial investigations of paper insulated cable of the so-called solid type, sheath corrosion, sheath current mitigation, temperatures of conduits, etc., as have been reported in the technical press throughout the year. The splendid cooperation and coordination of results evidenced by various organizations, utilities, and manufacturers engaged in the improvement of cable bids fair toward hastening the day of our profound understanding of cable and its operation.

RESEARCH

Though the research may not, of itself, constitute development (taking this in the sense of completion), experience has indicated that it undoubtedly frames the potentialities for development. It is probably not generally appreciated how much research work on high-tension cable is now being done. The annual report of the Impregnated Paper-Insulated Cable Research Subcommittee (this is a joint subcommittee of the Underground Systems Committee, N. E. L. A., the High Tension Cable Committee, A. E. I. C., and the Power Transmission and Distribution Committee, A. I. E. E.) gives a comprehensive picture of the extent of these activities in the industry, from which it is evident that expenditures on research have been greatly increased in recent years. A significant feature in this connection is the cooperative spirit displayed in the industry. Much work is being done directly under cooperative auspices but even that which is done under private auspices is reported freely to the industry. This augurs well for the development of the art to the mutual

advantage of all concerned,—central station companies, manufacturers, and the public.

Accomplishments to date have been variously reported to the A. I. E. E. and A. E. I. C. The annual report of the aforementioned joint subcommittee includes a résumé of all activities.

At Massachusetts Institute of Technology, under the direction of Professor Bush, there has been concluded an extensive investigation of the rate of deterioration of wood pulp paper for cable insulation when subjected to heat alone. The final conclusions reached are to the effect that impregnated wood pulp paper now being used in cables will withstand a temperature at least as high as that which manilla paper will withstand and, in so far as the rate of deterioration of the paper itself is concerned, the permissible operating temperatures of high-tension cable could be considerably higher than those now prescribed.

At Johns Hopkins University, the study of the effects of residual air and moisture in the insulation is progressing. Results already available point to the relative unimportance of extremely low air pressure of impregnation for obtaining excellent power factor—voltage curves. It appears that difficulties with commercial cables are due very largely to extended areas of entrapped air and gases resulting from imperfect impregnation, distortion in handling, or expansion of the lead sheath.

The study of the laws governing ionization characteristics of cable insulation has progressed at Harvard University to the point where simple empirical equations have been established for the experimental curves obtained.

The work at these and other universities has been closely followed and supplemented by individual researches conducted by various manufacturers and central station companies. At Brooklyn, accelerated life tests on cable, studies of the stability of impregnating compounds, and the ultimate nature of breakdown of gaseous spaces comprise some of the problems receiving attention. The research staff of the Detroit Edison Company has made extended laboratory and field tests of insulation of the solid type. These have included the study of the rate of oil absorption, gas generation, and change of power factor of cables in service, as well as investigations of hydrostatic pressure variation with temperature of cable and the development of a suitable laboratory test which will make cable deteriorate in the same manner in which it does in service. The Commonwealth Edison Company, through the Utilities Research Commission, is conducting elaborate theoretical studies of the fundamental nature of insulation as, for example, the significance of the dielectric strength of insulation, by Professor Compton at the University of Chicago, the determination of the relation of dielectric absorption and dielectric losses by Professor Whitehead at Johns Hopkins, and the study of the physical properties of ordinary cable lead sheaths by Professor Moore at Illinois. In addition to this, they have contributed much of a more or less practical nature.

Developments in Practise

In last year's report mention was made of the placing in service of 18 miles of 132-kv. lines in Chicago and New York. It is worthy of note that these lines have continued to operate up to the present date without electrical failures. An additional 12 miles of circuit at 132-kv. is now being installed in New York, the cable being of the hollow core, oil-filled type, in general similar to that in use on the previous lines. An important advancement in the method of installation has been made in that it is now possible to install the cable filled with oil, thereby eliminating the field impregnation which was required on the first circuits.

The several installations throughout the country of 45-kv. and 66-kv. single-conductor cables of the solid insulation type have continued to give satisfactory service. The use of oil reservoirs on the joints has proved very beneficial as far as the joints are concerned and thus far the possible trouble which some engineers have feared from the expansion of the cable sheaths has not been manifested.

During the past year, a pioneer installation of three-conductor oil-filled cable has been made in Chicago. This particular circuit was designed for 33-kv. operation but it is expected that cables of similar construction can be built for voltages up to 66 kv. The possibility of reducing the thickness of the insulation by means of the oil-filled construction apparently will make it feasible to increase the amount of power which can be transmitted through a duct and it is quite probable that if this development gives satisfactory operating results, there will be a considerable trend toward the use of three-conductor cables in the range of 33 to 66 kv. The improved regulation of three-conductor cables and the elimination of sheath losses are other features which make this type of construction appear attractive as compared to single-conductor cables. The shielded or Hochstadter type of construction continues to find favor for three-conductor cables designed for working voltages in the range of 22 to 33 kv.

For the lower voltages, including cables operating at voltages between 11 kv. and 15 kv., three-conductor cables and single-conductor cables installed three per duct have been used more or less indiscriminately, advantages being claimed for both. With the prospect that cables of this class will be used rather extensively in the future as a primary supply to a-c. low-voltage networks, the question of regulation has become of considerable importance and the superior characteristics, in this respect, of three-conductor cables, may henceforth carry considerable weight.

Notwithstanding this trend, there have been presented several papers (referred to elsewhere) embodying in a very tangible way economic studies of sheath cur-

rent elimination in single-conductor cables and losses in armored single-conductor cable. These, coupled with the development of new types of insulating sleeves, the *H* and *N* sheath bonding transformer and similar equipment, is evidence of the widespread interest in this field.

It has long been felt that our present method of rating cables, based on temperature limit of insulation, was, to say the least, unsound. The lack of a suitable alternative has accounted for the retention of the present method. As an example of the coordination of effort referred to heretofore, we point to a structure of understanding which is being built up to alter these conditions. The work at M. I. T. has definitely established the fact that, based on deterioration due solely to temperature, cable may be operated at considerably higher values than at present. It is recognized, however, that other factors enter which are the controlling ones. At Chicago and Detroit, systematic studies of the performance of cable fitted with oil-filled joints and under cyclic loading have been under way. At the former place, numerous tests have been made to determine the effect of various manhole designs on the ability of cable sheath to withstand the expansion and contraction incidental to normal operation. The Electrical Testing Laboratories and others are adding to our information by making like tests under repeated loadings. In other quarters, very extensive surveys of conduit and soil temperatures have been made and methods have been evolved for the determination of cable temperature when the loading and ambient conditions are known. It is evident that, with such a mass of data becoming available, we should, before long, be in a position to set up a rational method of rating cable.

During the past year, considerable interest has been exhibited in sheath corrosion. Several companies have observed the formation on lead sheath of various red, orange, and white compounds. Inasmuch as these, especially the red lead oxide, made their appearance on cable installed in monolithic concrete duct which was coming into very general use, they gave rise to considerable alarm among those operating and extending such systems. Almost coincidently, there was reported considerable corrosion of lead sheath in creosoted wood ducts.

These apparently baffling problems were the subject of attention of the Utilities, N. E. L. A., and the Portland Cement Association. Through their concerted effort, the contributing factors were soon laid bare and palliative measures were introduced, with the result that, at the present time, little fear is felt for troubles of the type mentioned. In the case of the concrete duct, it appears that the presence of calcium hydroxide from the concrete, water, and a-c. electrolysis or slowly reversing currents conspire to produce the oxide. Elimination of water and electrolysis would undoubtedly prevent the formation. In addition, if the concrete is allowed to stand for approximately 28 days, during which time the calcium hydroxide is changed to calcium carbonate, and if the concrete is sufficiently dense to prevent water from permeating it and dissolving additional calcium hydroxide, cables should be relatively free from corrosion of this type.

Corrosion in wood ducts was found to be due to the attack of the lead by acetic acid, of which there was considerable amount evolved from the Douglas fir used for the conduit. The remedial measure applied in this case was to introduce ammonia gas into the duct, thus neutralizing the acid.

Specifications, Testing, and Operation of Cable

There has been no important change in cable inspection and testing practise during the year. Specifications remain practically unchanged except for an increase in test voltages for single-conductor and shielded type multiple conductor cables, as well as a decrease in the allowable ionization factor in high-voltage cables. As in previous years, a survey of the test results obtained shows a general trend in the direction of improvement. This is indicated by the percentage of deficiencies and the uniformity of insulation resistance, power factor, and ionization factor. This consistent trend toward improvement is now being reflected in decidedly better operation of cable as revealed in annual reports of operation which have been compiled. Analytical studies of the mass of data that are collected annually in connection with acceptance inspection and testing of cable and of failures of cable which occur in service, have been carried on for several years. The value of these studies as an aid in improving cable having been demonstrated, more effective means of carrying them on has been developed under the cooperative auspices of the National Electric Light Association and the Association of Edison Illuminating Companies. This development is another indication of the cooperation which exists in the industry and which cannot help but be a great aid to progress.

The need for this progress is quite evident when cognizance is taken of the fact that it is generally conceded that, even with the multiplicity of tests which we now apply, we have no means of predicting with certainty whether cable will give satisfactory service and we have nothing which is a final measure of cable quality. However, with such concerted effort and cooperative action as have been described throughout this report, there is every reason to believe that this condition will not obtain for any great length of time.

Technical Papers on Related Subjects

This report is essentially a résumé of subjects covered in such papers as the following:

Bailey, *Heat Flow from Underground Electric Power Cables*, A. I. E. E. Quarterly Trans., Vol. 48, Jan. 1929, p. 156.

Burns and Freed, *Corrosion of Cable Sheath in Creosoted Wood Conduits*, A. I. E. E. Quarterly Trans., Vol. 48, Jan. 1929, p. 166.

SHANKLIN and MACKAY, *High-Tension Underground Cable Research and Development*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 338.

DAVIS and EDDY, *Some Problems in High-Voltage Cable Development*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 373.

DAWES, REICHARD, and HUMPHRIES, *Ionization Studies in Paper-Insulated Cables*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 382.

HALPERIN AND MILLER, *Reduction of Sheath Losses in Single-Conductor Cables*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 399.

SCHURIG, KUEHNI, and BULLER, *Losses in Armored Single-Conductor Lead-Covered Cable*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 417.

WHITEHEAD AND MARVIN, *Anomalous Conduction as a Cause of Dielectric Absorption*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 299.

KITCHIN, *Power Factor and Dielectric Constant in Viscous Liquid Dielectric*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 495.

Publications of N. E. L. A. and Reports of A. E. I. C. for year 1928.

STANDARDIZATION ACTIVITY

F. M. FARMER, CHAIRMAN

The following is a brief summary of standardization activities which have been consummated during the year.

1. Section No. 30 of the Standards of the A. I. E. E. dealing with wires and cables has been submitted to the American Standards Association for ratification as an American Standard (minus certain items which are now under discussion).

2. Specifications for annealed copper wire both bare and tinned were approved as American Standard by the American Standards Association, June 12, 1928.

3. Specifications for 30 per cent rubber insulation for wire and cable for general purposes were approved by the American Standards Association as tentative American Standard, April 2, 1928.

4. Specifications for magnet wire were approved by the American Standards Association as tentative American Standard, April 2, 1928.

A number of matters dealing with wires and cables is in the course of development to the standardization stage. Specifications for impregnated paper insulation, specifications for varnished cloth insulation, and specifications for weatherproof heat resisting and other coverings are being developed by the Sectional Committee on Wires and Cables of the American Standards Association. Similarly, progress is being made with the development of a standard stranding table for stranded conductors by the same committee.

Informal arrangements have been made between the appropriate committees of the National Electric Light Association and the Association of Edison Illuminating Companies for the coordinated collection and study of cable failures on a standardized basis.

SUBCOMMITTEE ON INTERCONNECTION AND STABILITY FACTORS

R. D. EVANS, CHAIRMAN

Interconnection of power systems has been brought about as a natural result of consolidation under a common management or as a measure to reduce investment or operating expense of systems under separate managements. The general advantages, such as those arising from load diversity, generating reserve, and utilization of stream flow plants, have been recognized for some time. The problems at hand have to do with the determination of the economy of a particular undertaking and the selection of the best method of carrying out the interconnection.

One of the most important aspects is the reliability of the power supply obtained from the interconnection. Reliability depends to a large extent on the vulnerability of the system and the stability of the system, *i. e.*, its capacity to maintain the machines in synchronism at times of system disturbances. These problems are very prominent in the field of power transmission, as the means of interconnection consist of transmission lines or ties with the associated transformers and switching equipment, and as the location, number, and character of these ties largely determine the serviceability of the interconnection.

The study of system connections is a development whose importance has recently become generally recognized. While it is true that different power system connections have developed in different parts of the country, these differences could largely be explained because of the difference in the local situation as to the transmission distance, the density, and the distribution of the load. Increase in the extent of systems and of interconnection has made it necessary to give consideration to what are commonly called "System Problems," of which stability may be cited as an example. Stability was first carefully analyzed in connection with long-distance power transmission. More recently, these principles are being applied in the study of the closely-connected systems.

The plan of operation of interconnected systems may be premised on either the maintenance of synchronism at times of system disturbances or upon the separation of the systems. In the latter case the system may be split at fixed points or at variable points to coordinate connected load and generating capacity. At the present time it is not possible to draw any general conclusions as to the proper field of application of these different methods of operation.

In the design of systems to maintain stability, several distinct methods are recognizable. Synchronism may be maintained (1) by measures which increase the synchronizing power which may be transferred between the machines, as by the use of low reactance tie lines or the application of quick response excitation; (2) by measures which limit the severity of the shocks to the

system occasioned by faults, as may be affected by the use of individual lines of high reactance or by quicker fault isolation; or (3) by a system layout which causes the shock to be distributed to the various machines in such a way as to accelerate or decelerate them as a unit.

In order to improve stability, special attention is being given to the characteristics of apparatus to be used on extensive systems and interconnections. From the standpoint of stability, governors should operate quickly and should possess anti-hunting features tending to reduce oscillations after disturbances. Governor operation in direct response to fault indications may prove advantageous. Field test to indicate performance of governors under transient conditions would be very desirable.

In generators, high short circuit ratio and low transient reactance are of benefit in improving the power limits. The use of the proper kind of excitation system extends the power limit under both steady state and transient conditions and may largely overcome the disadvantages of low short circuit ratio. Low transient reactance will improve the stability conditions at times of system fault. In general, a study should be made to determine in each case the proper value of machine transient reactance which depends upon line and transformer as well as generator costs.

Consideration is being given to the use of damper windings in waterwheel generators. Damper windings tend to improve stability by virtue of their damping action and to decrease stability because of the increase in fault current due to the reduction in negative phase sequence reactance of the generator. Investigations have indicated that in certain cases damper windings may be advantageous provided they are of the proper characteristics.

Excitation systems capable of quick response are effective in improving stability under steady state and particularly under transient conditions. In special cases the use of synchronous condensers wholly or principally as an aid to stability will be found desirable. The stabilizing effect of such condensers will be increased by the application of suitable quick response excitation systems.

During the past year actual field tests with quick response excitation systems were reported in a paper on *Power Limit Tests* by S. Murray Jones and Robert Treat. A general review of stability was presented by R. H. Park and E. H. Bancker in their paper on *System Stability as a Design Problem*.

The investigations by the Lightning Subcommittee and the Protective Device Committee on protective measures, for minimizing the effects of faults, are very pertinent in consideration of the stability problem. Measures, such as the high insulation of lines and the installation of ground wires, by reducing the number of faults are obviously of benefit from the standpoint of stability. There is a growing tendency to proportion line insulation to service requirements, thus minimizing the number of faults occurring in exposed areas. Ground wires also tend to reduce the value of the resistance in the ground connection at a flashover point, which may either increase or decrease the resulting shock to the system.

Consideration is being given to power arc suppression devices. Phillip Stewart in his paper on *Fused Arcing Horns and Grading Rings* gave the results of operating experience. During the past year some interest has been shown in the Peterson coil as an arc suppression device.

There is some consideration of the introduction of neutral impedance devices for limiting the shock to the system at times of fault to ground. Of course, these devices are of advantage from the standpoint of stability if they reduce the shock to the system and if they do not at the same time increase apparatus failure or line flashover or interfere with the prompt isolation of faults.

Consideration is being given actively to the matter of the duration of the short circuit condition as it is clear that the system oscillations are minimized when the fault duration is reduced. From this standpoint, therefore, high speed in both relays and circuit breakers is very desirable.

Extensions of systems and interconnections have introduced operating problems important from the standpoint of stability. The greater complication of the systems requires the human element to a greater extent than hitherto, and operating rules are being expanded accordingly. The method employed to maintain system frequency and to increase prime mover input importantly affects stability. Pertinent in this connection is the present trend toward the automatic control of frequency.

To insure stable operation it is obviously desirable to maintain a high ratio of generating capacity to connected load and a high ratio of excitation to load on the individual units. In the determination of these ratios other considerations will be controlling, but stability cannot safely be ignored. The increase in size of the individual generating unit makes these problems more important than formerly. Excitation of machines is usually determined from prescribed ranges of bus voltages, but this, unless carefully watched, is likely to subject some machines to unduly low excitation for either a heavy load or a light load condition. Consideration has been given to the interlocking of governor and voltage regulators to insure that adequate excitation is available before the prime mover input is increased. Loss of excitation is relatively infrequent, but when it does occur, severe system oscillations are produced.

The theoretical investigation of stability has proceeded to a point where fundamental operating data are urgently needed. Information as to the operation of systems has been obtained in the past principally from indicating and graphic instruments which have

been provided for the convenience of the operators. During the past few years a number of instruments with speed-up devices has been used, but the general results have not been entirely satisfactory because of the delay in starting the device and the inaccuracies in the measuring elements. Recently these objections have been reduced by the use of an oscillographic type of instrument, and the tendency is to use such devices where transient records of disturbances are desired. The importance of records of this nature is becoming more apparent in reconstructing the nature and extent of system disturbances, particularly where considerable power is being transmitted over some distance. With such data available it will be possible to place the design requirements upon a sounder basis; for example, it should be possible to determine whether the basis for maintaining stability should be predicted upon a single fault to ground as is commonly done at present, or upon the less frequent double fault to ground. Such data will also make it possible in many cases to estimate from oscillation with existing loads, the probable system disturbance with future loads.

General Conclusion

The expansion of the interconnection of systems has brought numerous problems to the engineer for solution and it is to be noted in the foregoing subcommittee reports and the several papers presented before the Institute that fundamentally different ideas of system developments have been brought out to meet the ever increasing demand for higher continuity of service and greater flexibility of supplying relatively large blocks of load from a network system. Your committee expects rapid strides to be made within the next few years in handling distribution problems and predicts radical departures from old methods and practises.

Discussion

F. W. Gay: Wherever it is possible to transmit power between two stations over two separate transmission lines on different rights-of-way, a method is available for protecting such twin lines against the disastrous flow of current over a flashed insulator string.

If each conductor from the source of power at the sending station and each conductor to the load at the receiving station is each separately connected to the mid point of one of six closely interlaced auto-transformers and a pair of conductors (one in each twin line) connect the start and finish of a transformer in a given phase at one station with the start and finish of a transformer in the same phase at the other station, and the entire six transformers are so connected, the desired result will be obtained. It will now be evident that on a system, grounded solidly or through the neutral resistance, a flash to ground on one line will ground an end terminal of each of the two auto-transformers connected to that line. The auto-transformer so grounded at the sending station will instantly act to step the power it receives from the station bus to twice normal voltage. This power will be transmitted at twice normal voltage over the connected conductor in the good (twin) line and will be stepped down to normal voltage by the auto-transformer at the receiving station. The only current that can flow to the fault is the charging current of the double-voltaged line and the magnetizing current of the two auto-transformers. These currents can be picked up by separately insulated low-capacity low-voltage windings on the transformers at one end of the line, by suitably connecting these low-voltage windings to a voltage source of the same phase. It is obvious that with such a system, three lines may be simultaneously arced over provided no two such lines are connected to a single transformer. Power current cannot follow the arc. The arc should immediately extinguish. The opening of circuit breakers should not be required.

W. A. Del Mar: The summary of developments in cables is very complete as far as the work of the preceding year is concerned.

There is one thing, however, which I wish had been touched upon, because it is a very live subject to those who are interested in cable dielectrics. That is the question of what happens to a cable sheath when the cable is heated and cooled. There has been an interesting discussion between several English cable engineers, published in the *Electrical Review* of London, January 18 and 25, February 1 and 15, March 1 and April 26 of this year, and considerable amount of work along similar lines has been done in this country by various manufacturers, operating companies, and electrical testing laboratories.

One of these engineers, C. J. Beaver, referring to British practise, denies the commonly accepted belief that when a cable is heated the lead is stretched beyond the elastic limit. This engineer claims that lead has a real elastic limit and that it is quite considerable, *i. e.*, about 260 lb. per sq. in. He does not deny, however, that in American practise where higher temperatures are obtained, this elastic limit may be exceeded. He is taken to task by T. N. Riley, T. R. Scott, and H. G. Wood, and a very illuminating discussion results in which a considerable amount of experimental data is given which I believe is quite valuable.

One problem we have before us, is whether cable should be completely filled with oil compound or whether it should have a certain amount of air. We find considerable difference of opinion among engineers in this regard. The tests that have been made rather indicate that a certain amount of air is desirable in the insulation in order to make the cable satisfactorily pass an accelerated life test, but on the other hand, it has not yet been fully proved that these accelerated life tests are representative of the cable quality.

D. W. Roper: The Chairman of the Transmission and Distribution Committee, in his remarks, predicted that within five years most of the large cities of the country would be depending on overhead transmission lines for a part of their electrical supply. Some years ago the same indication appeared in the Chicago region shortly after 132 kv. had been adopted by a number of companies in this region as their overhead transmission voltage. It also appeared that it would be very desirable and economical to have such transmission lines brought directly into one of the generating stations, and this involved the use of 132-kv. underground cable. Some of the cable manufacturers thought that such cable would be too expensive to consider seriously, but their attention was called to the fact that the use of such cable would eliminate a substation at the edge of the city whose only function would be to step down the voltage from 132 kv. to a suitable voltage for underground cable extending to one of the generating stations. Under such conditions, the cost of the cable may not be a determining factor, as other conditions will offset, in whole or in part, the apparent high cost of the underground cable itself.

Within the next year 100,000 kw. at 132 kv. will probably be brought into Chicago on overhead transmission lines, and in each case there will be a few miles of underground cable continuing the transmission at full voltage to the nearest generating station. Within the next five years this figure may be multiplied by three or four. In one case which is in prospect, the distance is so great that 132 kv. is not an efficient transmission voltage, and it appears necessary to go to 220 kv. or some higher voltage. When that condition arises, we will be confronted with the same

conditions that arose in connection with the first 132-kv. transmission line, that is, the desirability of having the transmission line continued into the nearest generating station by means of underground cable and at full voltage.

The remarkable success of the 132-kv. oil-filled cable in Chicago and in New York, where some 54 mi. of cables have been operating for about two years without any electrical failures, indicates that the cable is being operated well below its safe limit. This indication has been verified by measurements which have been made during the past two years on an experimental installation of similar cable at a location where there are laboratory facilities for measuring the dielectric loss of the ionization factor. As a result of these indications, we are now investigating the possibility of making cable for 220-kv. operating pressure so that when the occasion arises, the line may be brought into the nearest generating station by means of underground cable, and in this way avoid the transformer substation which would otherwise be necessary at the edge of the city.

Protective Devices

ANNUAL REPORT OF COMMITTEE ON PROTECTIVE DEVICES*

To the Board of Directors:

The work of the Committee this year has been: first, the revision of present and preparation of new standards; second, a survey and review of research and development made during the past year in the several fields covered by the Committee; and third, arranging for the preparation of papers for presentation before meetings of the Institute.

The work of the Committee, as organized at the meeting in New York in November 1928, has been carried out by subcommittees, each under the direction of its own chairman. The fields covered and the chairmen in charge of the subcommittees are as follows:

Lightning Arresters	Herman Halperin	Commonwealth Edison Company, Chicago, Ill.
Industrial Equipment and Service Protection	R. C. Muir	General Electric Company, Schenectady, N.Y.
Current Limiting Reactors	N. L. Pollard	Public Service Production Company, Newark, N.J.
Oil Circuit Breakers, Switches, and Fuses	A. M. Rossman	Sargent & Lundy, Chicago, Ill.
Relays	H. P. Sleeper	Public Service Electric and Gas Company, Newark, N. J.

The Subcommittee on Communication Line Protection which reported last year has been discontinued as it was felt its work overlapped with that of the Technical Committee on Communication Lines.

Subcommittee on Lightning Arresters

Work has been continued on the A. I. E. E. Standards for Lightning Arresters. Two preliminary reports of these standards have already been published, (one in 1926 and the other in 1928), under the title *Report on Standards for Lightning Arresters and Other Apparatus for Protection against Abnormal Transient Voltages*. Because of the wide variation in requirements and practises of arresters for all types of circuits, in the present revision the standards may be limited in scope to arresters for power circuits and the title shortened to *Standards for Lightning Arresters*. As soon as it is practicable, the subcommittee hopes to turn its attention to standards for arresters for other applications and to standards for other protective devices, such as surge absorbers, choke coils, ground wires, fused grading shields, etc.

Research on lightning has been continued in the field and laboratory by the manufacturers, in several instances, in cooperation with a number of the operating companies. Extensive tests have been made on the transmission, reflection, attenuation, and effect on terminal apparatus of surges applied on full-sized transmission lines. These tests are being continued and are being correlated with the effects of natural lightning on the same or similar lines.

Further studies of natural lightning on transmission lines by the cathode ray oscillograph will be continued during the present lightning season, so that several records of great value and interest may be expected this year as a result of the extensive program. Investigations of surges, their effect on apparatus and dimunition by arresters has been greatly aided by continued development in portability, ease of manipulation, and automatic timing features of cathode ray oscillographs. A new type which appeared to have many good features has been developed by R. E. George of Purdue University.

A large scale and detailed program of investigation is still being actively pursued by one of the large operating companies on the causes of transformer failures on 4-kv. distribution circuits and their 115/230-volt secondaries during lightning storms and on the possible methods of reducing the number of service interruptions due to such failures. Studies are being made of the effects of resistance in the arrester ground, and laboratory and field tests are being made on the effect and practicability of lightning arrester protection on the secondary side of the transformers.

In the field of high-voltage transmission, work has been continued on the use of fuses in connection with grading shields on insulator strings. Recent developments indicate the feasibility of using expulsion type fuses as arcing horns at the tower end of the string. These developments indicate a tendency to build lightning arrester characteristics into the insulator string itself and perhaps in the future other elements will be developed for connection in series with or in shunt with part of the insulator string to accomplish this end.

There are several problems the solution of which will require further tests and study. Some of these are as follows:

1. Gap-Setting. The proper setting of gaps in series with arresters depends largely upon the type of arrester and its application. The application must take into consideration the maximum dynamic voltage, which is dependent on such factors as grounded or ungrounded neutral, relaying, arcing grounds, and also whether the plant is steam or hydro.

2. Ground Resistance. With the increasing size of power systems, the problem of obtaining good grounds

*COMMITTEE ON PROTECTIVE DEVICES:

E. A. Hester, Chairman,
Raymond Bailey, Vice-Chairman
V. J. Brain, Secretary

A. C. Cummins,	F. C. Hanker,	R. C. Muir,
E. W. Dillard,	F. L. Hunt,	N. L. Pollard,
H. W. Drake,	B. G. Jamieson,	A. M. Rossman,
W. S. Edsall,	J. Allen Johnson,	A. H. Schirmer,
L. E. Frost,	S. M. Jones,	H. P. Sleeper,
E. E. George,	R. L. Kingsland,	R. M. Spurck,
James S. Hagan,	M. G. Lloyd,	E. R. Stauffacher,
H. Halperin,	J. B. MacNeill,	H. R. Summerhayes,

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

has become increasingly important. In the case of large stations, the expense of providing adequate grounding facilities is low enough relative to the total station costs involved so as not to be serious from an investment standpoint, but in the case of small stations, tower footings, etc., the cost of securing adequate ground connections may be high enough relative to the total investment to be rather serious.

3. Quantitative ratings with respect to impulse voltage of lightning arresters and apparatus to be protected in order to give greater assurance of protection.

All things considered the subcommittee feels that splendid progress has been made in the study of lightning and in apparatus and methods of mitigating its interference with power transmission. It looks forward with confidence to accelerated progress in protection against lightning in the immediate future.

Several valuable papers have been presented before the Institute on these and allied subjects, among which the following may be mentioned:

A Graphical Theory of Traveling Electric Waves, by V. Karapetoff, A. I. E. E. Quarterly TRANS., Vol.48, April 1929, p. 508.

Progress in Lightning Research in the Field and Laboratory, by F. W. Peek, Jr., A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 436.

1927 Lightning Experience on 132-Kv. Transmission Lines, by Philip Sporn, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 480.

Theoretical and Field Investigations of Lightning, by C. L. Fortescue, A. L. Atherton, and J. H. Cox, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 449.

Protection of Transmission Lines from Interruption by Lightning, by 1928 Subcommittee of Power Transmission and Distribution Committee, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, p. 487.

Rationalization of Transmission System Insulation Strength, by Philip Sporn, A. I. E. E. Quarterly TRANS., Vol. 47, Oct. 1928, p. 998, (Discussion p. 1009).

Relation between Transmission Line Insulation and Transformer Insulation, by W. W. Lewis, A. I. E. E. Quarterly TRANS., Vol. 47, Oct. 1928, p. 992, (Discussion 1009).

Symposium on Surge Voltage Investigation on Transmission Lines, A. I. E. E. Quarterly TRANS., Vol. 47, Oct. 1928, pp. 1111-1154.

A New Type of Hot Cathode Oscillograph, by R. E. George. Presented at the A. I. E. E. Regional Meeting, Cincinnati, Mar. 20-22, 1929.

Fused Arcing Horns and Grading Rings, by P. B. Stewart, A. I. E. E. JL., May 1929, p. 390.

Investigation of Transmission Lines with Artificial Lightning, by K. B. McEachron and V. E. Goodwin, A. I. E. E. JL., May 1929, p. 382.

Lightning Studies of Transformers by the Cathode Ray Oscillograph, by F. F. Brand and K. K. Palueff, A. I. E. E. Regional Meeting, Dallas, Texas, May 7-9, 1929.

SUBCOMMITTEE ON INDUSTRIAL EQUIPMENT AND SERVICE EQUIPMENT

Addition to A. I. E. E. Standards No. 15 (A. E. S. C. C-19) Industrial Control Apparatus has been completed, and will be submitted to the A. I. E. E. Standards Committee for joint action with N. E. M. A.

These additions are considered necessary to the Industrial Control Standards inasmuch as there is no standardized basis of short-time current rating or short-circuit interrupting rating of contactors and consequently information is not available whereby contactors might be properly applied.

A paper dealing with the general development and application of industrial control equipment and protective devices is being prepared under the auspices of this committee. Little has been published on this general subject, particularly with reference to the application of contactors, and it is felt that this paper will be of material assistance to engineers and will stimulate progress along the right lines in industrial equipment and service protection.

SUBCOMMITTEE ON CURRENT-LIMITING REACTORS

Some thought has been given to new standards for current-limiting reactors, but it has not been considered practicable to draw any up at the present time.

Last year's report called attention to the progress being made in the field of oil-insulated reactors. This progress has continued, large oil-immersed reactors having been built during the last year for circuit voltages ranging from 2.3 kv. to 73 kv., and in one instance for as high a circuit voltage as 120 kv. This type of reactor has increased the field of application of current limiting reactors, since it can be designed for higher voltages than the dry type and is particularly adaptable for outdoor service.

In the case of the 120-kv. circuit mentioned above, the reactors are used for sectionalizing the 120-kv., 60-cycle bus of a generating station having a capacity of 375,000 kv-a. Each reactor is rated at 2800 kv-a. and introduces 10 per cent reactance at 75,000 kv-a.

During the past year there has been a considerable increase in the number of dry type reactors installed with a marked increase in the use of reactors for grounding the neutral at generating stations.

Two, and possibly three, papers will be published and presented before the Institute in the near future under the auspices of this committee.

SUBCOMMITTEE ON OIL CIRCUIT BREAKERS, SWITCHES, AND FUSES

The preparation of more comprehensive standards for fuses is now under way. It is expected that the preparation of these standards will be greatly facilitated by tests which have been conducted during the past year by several operating companies on fuses rated at voltages from 2.3 to 66 kv. A series of tests has also been conducted on fuses of potential transformers.

Several manufacturers have increased their gen-

erating facilities available for testing oil circuit breakers and have carried out, on a large scale, systematic programs of testing their standard circuit breakers. These tests have resulted in improvements in details and have given an increased knowledge of the safe interrupting capacities of these oil circuit breakers.

Field tests have been extended to include 220-kv. oil circuit breakers, and additional tests are scheduled to be made this year. It is expected that the results of these tests will be published as soon as the tests are completed and the data available for publication.

Increased interest has also been taken in higher speed oil circuit breakers for use on high-voltage transmission lines to improve stability of operation by the more rapid clearing of faults.

Circuit breakers of extremely high speeds have been developed for electric railway protection. Tripping speeds as high as 0.012 to 0.016 sec. for a d-c. breaker and 0.04 sec. for an a-c. breaker have been obtained.

The past year has shown an increased interest in metal-enclosed switching structures. A number of the manufacturers of switching equipment in this country are now building structures of this type. A 33-kv. structure has been in successful operation for over a year and several others are now being built for this voltage. A 22-kv. structure of the oil filled type, the largest structure of this type to be installed in this country, will handle the entire output of one of the largest power stations in the Middle West. The first section comprising 23 units was placed in service this Spring. Several hundred units of this type have also been installed on 2300-volt circuits.

These structures greatly reduce the hazard to human life because the live parts are inaccessible and because the moving or removable parts may be interlocked to prevent incorrect operations. Most of the structures have been built without disconnecting switches, the oil circuit breaker being disconnected from its connections with the bus bars and circuits by being moved bodily away from its operating positions. This has permitted a very simple system of interlocking with the use of but five or six sturdy parts.

The metal-enclosed design is completely fabricated in the shops of the manufacturer, and the equipment is shipped in a semi-assembled condition. These factors tend to promote standardization of design and eliminate a large amount of field work, both of which should in time be reflected in lower costs of switching structures.

Papers on theory, design, and test of the Deion circuit breaker were presented at the Winter Convention of the Institute. This development appears to be a notable contribution to the circuit breaker art, as it eliminates the use of oil, and while at present confined to relatively low voltages, it is hoped that it will later be found possible to adopt it for use on higher voltages. It is also hoped that this development will stimulate further research which may lead to the discovery and application of other new principles for interrupting power circuits.

Through the efforts of the members of this subcommittee, the following papers have been submitted for presentation before the Institute some time during the coming fall or winter:

A Method of Fuse Testing, by B. M. Jones and E. H. Coxe.

Fuse Tests, by S. Murray Jones, supplemented by comments on recent oil circuit breaker tests.

Potential Transformer Fuses, by H. P. Sleeper and M. F. Riley.

The Effectiveness of Different Types of Barrier Construction in Switch Houses, by P. H. Adams and B. M. Jones.

Subcommittee on Relays

The feasibility of standardizing the operating characteristic of relays of different manufactueres has been investigated. This refers to the standardization of characteristic relay curves, operating current values of relay indicators, and various other characteristics of relays which are possible of standardization. It is apparent that the subject is one which will require considerable discussion by the members of the Institute before a representative concensus of opinion can be obtained, and as sufficient data are not yet at hand, it will be desirable to continue the work of this group for another year.

A review of protective relay schemes in use at this date indicates that there have been no radical changes in relay practise made in the past few years. However, there have been some new schemes introduced and the trend at the moment is a departure from the old standard schemes and is worthy of note.

The great majority of all relay installations in service at this time on transmission lines is some form of the so-called time differential system whereby selectivity between adjacent breakers is secured by the use of selective time settings. This involves the use of overcurrent and directional overcurrent relays in the same manner as has been used for many years heretofore. This system, of course, has the disadvantage of limiting the number of sectionalizing points in series by reason of the cumulative time settings at the source end. It has the further disadvantage of requiring time settings in excess of instantaneous settings.

To enable instantaneous settings to be used with the resulting increase in stability to power systems and minimizing of damage at the point of fault, there has been a tendency to use increased number of installations of differential schemes of transmission line protection. The series scheme of differential protection of transmission lines,—namely, pilot wire relaying,—has never found much favor in this country, although one operating company reports that it has used this method extensively on its 12-kv. loops out of substations, and the scheme has been quite satisfactory. They have

adopted recently the use of pilot wire protection on 66-kv. lines and installed several hundred thousand feet of such pilot wire last year. Modifications of the pilot wire scheme are becoming more in favor, however, as indicated below. The scheme of parallel differential protection of transmission lines, where one line is balanced against another, is the form most commonly used in this country and its use has increased considerably in the past few years.

At this time there is a very definite trend toward the use of schemes of transmission line protection which have not been named but might be called "independent" schemes. By this is meant schemes of relay protection wherein the protection of one section of line is entirely independent of that on an adjacent section. The pilot wire scheme falls within this category but as indicated above has never been commonly used. The impedance relay is a scheme embodying some of the features of this type of protection, and its application is slowly increasing. It is perhaps the simplest scheme of protection that is available at the moment. One of the most interesting applications of this type of protection is a case where the short-circuit current under certain conditions may be less than the full-load current. Here impedance relays are intended to be used for short-circuit protection, which are normally short-circuited out of service by fault detector relays. The latter are operated either by low voltage or the presence of short-circuit current which cuts in the impedance relays and allows them to operate during fault conditions.

A further and more elaborate scheme of this type is one which is now being used on a small scale and has possibilities for improvement and application in the future. This is a modification of the pilot wire scheme where the pilot wires are replaced by high-frequency oscillations on the transmission wires themselves. Two schemes are in use, one a high or radio frequency scheme, and the other a low or audio frequency scheme, both of which are fundamentally the same in operation. At the present time, auxiliary equipment of considerable cost and size is required at the line terminals to enable this scheme to function properly, and tends to limit its use. However, it is probable that these schemes will be perfected and fundamentally they seem to offer the most promising solution for future protection problems, particularly of the more complicated type. They embody all of the most desirable characteristics of relay protection; namely, high speed of operation, independent settings, and no limitations as to numbers of stations in series. It is probable that the future trend of the art will be definitely in the direction of applications of this so-called independent scheme of relaying.

There is a growing tendency toward recognition of the importance of bus protection in high-tension stations. Heretofore, the protection of busses in such stations was considered unnecessary, but the increasing capacities involved, with the resulting seriousness of outages, has urged the use of this type of protection. It usually partakes of the form of series differential protection. For this protection, and also for the protection of transformer banks and rotating machines, the use of ratio differential relays is becoming more common and superseding the old type of differentially connected overcurrent relays.

A scheme for securing overcurrent protection on balanced lines without the use of additional relays has been developed. The high- and low-set directional relays are connected in much the usual manner, except that the balance point is made at the mid-point of an auto current transformer, and the overcurrent element of a high-set relay is connected across the entire winding of the auto, instead of in the balanced circuit. It is obvious that the high-set relay receives line current at all times and is connected to trip both breakers. This of course requires the use of a double-contact directional element on the low-set directional relay.

A new scheme of protecting load ratio control units has been developed, using a restrained overcurrent relay. One winding is connected to the exciting winding of the load ratio control unit. The other winding is connected to the line. The effect is to change the setting of the relay so that a percentage tripping characteristic is obtained.

A scheme for detecting ground faults in apparatus by using a restrained current relay with multiple restraining coils has been developed. These coils are connected to the phase current transformers and the operating coil is energized by the residual current.

Radio differential relays have been applied to bus differential protection and their characteristic used to tend to eliminate incorrect operations due to secondary wiring troubles.

There are also two new types of network relays. One of these is designed to operate on high reverse current at normal voltage and is still very sensitive at short-circuit voltage. The other is for the protection of low-voltage a-c. networks, the feeder transformers of which are connected in delta on the low-voltage side and with the middle tap of one winding brought out to obtain lighting voltage. It is essentially a combination of three relays: a power directional, an overcurrent, and an overvoltage, set to trip quickly on transformer or feeder faults, but not being sensitive to small reversals of power.

Several existing types of relays have been modified and improved to give better operation. The ratio differential relay has been modified for application to the protection of three-winding transformers. The induction type overcurrent relay has been speeded up so that it will operate in approximately three cycles. The residual directional relay has been modified so that it will operate correctly over an extended power-factor range. Various types of condenser devices have been developed for securing potential for relay operation without the use of high-tension potential transformers.

The most common of these is in connection with the condenser type high-voltage bushing, but other condenser types of devices have been developed for this same purpose.

A paper entitled *New Directional Relay Schemes*, by E. E. George and R. H. Bennett, is in preparation and will be presented sometime in the near future. Two other papers are also in preparation for presentation before the Institute, dealing with relay acceptance specifications and with relay experience.

Research

ANNUAL REPORT OF COMMITTEE ON RESEARCH*

To the Board of Directors:

It is now generally recognized that progress in science and industry depends largely on research. Electrical engineering in particular must trace practically all its advances to some form of research, whether it be pure or applied. Accordingly a complete report on research in this art must necessarily involve practically all the developments in its various branches such as communication, machinery, transmission, etc. Inasmuch as this is particularly material for the Institute committees involving these branches, an attempt is made in this report merely to mention the various research bodies and their work in general, and to list briefly the outstanding achievements in engineering and physical research during the past year.

Considerable assistance was received from various members of the Research Committee in the preparation of this report, particularly Messrs. V. Bush, S. M. Kintner, M. G. Lloyd, D. W. Roper, C. E. Skinner, R. W. Sorensen, and J. B. Whitehead. Several nonmembers provided valuable contributions, particularly Doctor W. F. G. Swann of the Bartol Foundation.

Research Organizations

The National Research Council has been very active in sponsoring research and its committee on Electrical Insulation has started an extensive program. At a two-day symposium conducted in Baltimore last fall by this committee, papers were presented reporting the results of studies of the mechanism of cable deterioration, the products of the breakdown of liquid dielectrics, the current wave form in dielectrics under high stress, the influence of air and moisture in impregnated paper insulation, short time absorption curves in composite insulation, the relation of dielectric absorption and dielectric loss, anomalous conduction in liquid insulation, gaseous ionization in cables, and the breakdown of solid dielectrics. At a second symposium, conducted during the Washington meeting of the American Physical Society, papers were presented on the mechanical and electrical strength of dielectric crystals, electrical convection in oil, effect of temperature, pressure, and frequency on rubber, influence of surface and space charges on the apparent conductivity of dielectrics, electron bombardment of hydrocarbons, and the dependence of dielectric polarization upon molecular condition.

The Bartol Research Foundation of the Franklin Institute under Doctor Swann carried out a valuable research program during the past year. Among the important subjects investigated were the following: phenomena in the electric arc, reflection of atoms on crystals, excitation of atoms by electron impact, possible influence of cosmic rays on radioactivity, production of X-rays by protons, influence of a-c. fields on light transmission through water, corpuscular radiations of cosmic origin, energy lost by electrons passing through thin foils, polarization of light produced by impact of cadmium atoms with excited mercury atoms, photoelectric effect in thin films of potassium and sodium, and absorption of cosmic radiation in air. A number of theoretical problems was considered mathematically such as the scattering of electrons by atoms, theory of Compton effect produced by standing electromagnetic waves, possibility of detecting individual cosmic rays, relation between mass and energy, and ionization by an electron traveling with speed comparable with that of light.

Several noteworthy research accomplishments were made at the Bureau of Standards during the past year. The corona voltmeter was investigated, and a careful study made of the effects of pressure, temperature, and humidity on its accuracy. A redetermination of the absolute value of the international ohm was carried out. Important work was done toward developing a suitable radio beacon for aircraft. An electrical seismometer also was devised which is proving to be a very satisfactory instrument for recording earth movements.

Electrical Engineering Research in the Colleges

During the past year an a-c. calculating board was developed at the Massachusetts Institute of Technology for representing in miniature almost any power system. The device offers a practical means of solving easily the steady-state problems of current and power division, system voltage, load balance, power factor adjustment, and the transient problems of both symmetrical and unsymmetrical short circuits with attendant considerations of system stability.

Important investigations in the colleges are being made of solid and liquid insulations, particularly as they relate to underground cables. Under the auspices of the National Electric Light Association, the Association of Edison Illuminating Companies, and the American Institute of Electrical Engineers, a study of thermal characteristics and methods of measurement is being made at Massachusetts Institute of Technology, the effect of residual air and moisture at the Johns Hop-

*COMMITTEE ON RESEARCH:

F. W. Peek, Chairman,

H. D. Arnold,	J. A. Johnson,	D. W. Roper,
Edward Bennett,	V. Karapetoff,	Clayton H. Sharp,
V. Bush,	A. E. Kennelly,	C. E. Skinner,
E. H. Colpitts,	S. M. Kintner,	R. W. Sorensen,
W. F. Davidson,	M. G. Lloyd,	T. S. Taylor,
W. P. Dobson,	Chester W. Rice,	J. B. Whitehead.

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

kins University, and ionization characteristics at Harvard University.

Under the Utilities Research Commission (consisting of representatives of the Samuel Insull utilities) studies of methods of detecting cable failures and of physical properties of lead sheaths are being made at the University of Illinois, at Johns Hopkins University of the properties of impregnated paper, and at the University of Chicago of the fundamental significance of dielectric strength.

A large part of the research sponsored by the National Research Council and the Engineering Foundation is being conducted in the college laboratories, particularly in the East and Middle West.

Several of the college engineering laboratories on the Pacific Coast have been active in research problems during the past year. Some of the accomplishments of note were the study and analysis of Lichtenberg figures at the University of Washington; the X-ray studies at the California Institute of Technology, using the million-volt transformer set for tube excitation; and the investigation and measurement of high-voltage flashovers at Stanford University, where 60-cycle flashovers of gaps up to 36 ft. were made, the longest yet accomplished at commercial frequencies.

Outstanding Research Accomplishments in Electrical Engineering

Considerable progress was made during the past year in the study of lightning on transmission lines. Much of this was made possible through the adaptation of the cathode ray oscillograph for direct recording on power lines. Lightning waves recorded in lines have been duplicated in the laboratory and their effects on transmission lines, insulators, insulation, transformers, and protective apparatus studied at will. For making these laboratory tests a lightning generator producing 5,000,000 volts was constructed. Smaller portable lightning generators were also produced and used for putting surges at desired points on lines.

By means of surge voltage recorders and klydonographs, distributed at intervals on several power systems, valuable data also were obtained on the crest values, polarity, attenuation, and frequency of lightning surges on lines.

As a result of lightning research, it is now possible to design transmission lines, and to coordinate the line and apparatus insulation so as to have power systems which are highly resistant to lightning.

A well coordinated program in lightning research is being continued. A considerable part of the field work is being done on transmission lines in cooperation with operating engineers.

A number of advances was made in transformer design last year. Among these was the development of the non-resonating transformer. The object of this type is to cause a uniform voltage distribution throughout the windings under all frequencies and lightning surges.

Several improvements were made in switches and circuit breakers during the past year. One achievement in this line was the development of the Deion circuit-breaker switch. In its primary application to practise it has been designed for use up to 15,000 volts, its operation making use of a series of extremely short gaps in air.

In the field of electrical machinery numerous advances were made. Noteworthy among these was the successful development and operation of a 10,000-kw. mercury turbine, whose efficiency exceeded the previously predicted values. The popularity of the high pressure turbine also increased, as may be witnessed by the fact that five new 1200-lb. units were designed during 1928. The development of hydrogen-cooling for reducing losses and increasing the capacities of generators and synchronous condensers made appreciable strides. Units up to 20,000 kv-a. using this method of cooling were built during the past year. The tendency to use higher generated voltages with large capacity units for certain classes of service continued. Several 22,000-volt generators were designed and installed in this country, and one 33,000-volt unit was placed in operation in England, connection being made directly to an underground cable system with no intervening transformer.

Aviation profited appreciably from research developments carried out last year in the Bureau of Standards and in the laboratories of the several larger industrial concerns. The principal advances consisted of additional navigational aids for commercial aviation. Here radio contributed principally in improving communication, course navigation, and fog landing. Better facilities for navigation and landing in the fog were added by the development of neon light beacons and markers. A method of depth-sounding in the air to enable aviators to determine their altitude was devised. The procedure consists in sending out radio waves from the airplane, which are reflected from the ground and received again by the plane. A magneto compass also was designed, weighing less than one-fifth of the present earth-inductor compass. A means of accurately measuring the amount of gasoline was devised making use of electrical impulses transmitted from diaphrams located in the airplane tanks.

Much work was carried out during the year in perfecting and applying photoelectric devices to many uses, such as smoke detection and measurement, control of illumination, operation of traffic devices, etc.

Marked progress was made on the question of television and far greater use will undoubtedly be made of it in the near future. Appreciable advances were made in the speed and convenience of picture transmission, by both wire and wireless, during the year.

The great increase in popularity of the talking moving

pictures furnished added stimulus to research in that line so that considerable improvement is expected in this art.

Decided progress was made in metallurgical work through the wider application of inductive heating, both in furnaces of the vacuum type and the non-vacuum type.

The increasing temperatures, pressures, and speeds of electrical machinery have made it necessary to give added study to the characteristics of metals under high temperatures and mechanical stresses. Results of great value have been reported.

Research in magnetic materials and magnetic phenomena was continued, and much was added to our understanding of these vital subjects.

Laboratory devices for the analysis of noise were developed and studies made of the sources of noise in machinery, and its elimination.

Outstanding Research Accomplishments in Physics

Light and Spectroscopy. Michelson, Pease, and Pearson, University of Chicago and Mount Wilson Observatory, have repeated the Michelson-Morley experiment and find no shift that can be interpreted as arising from an ether drift. This completely agrees with the original results obtained by Michelson and Morley and also seems to add strength to the relativity theory which was developed to explain the earlier experiments. (*Trans. Optical Soc.*)

The Indian physicist, Raman, has shown that light scattered from many substances contains not only the original frequencies of the incident light, but also the other frequencies differing from it by the natural frequencies of the scattering substance. This effect has been used by Raman and others to study the natural frequencies of many substances as well as to extend further the laws of scattering.

Our knowledge of the structure of many diatomic molecules has been materially extended by the analysis of their band spectra by Mulliken and others.

McLennan and his students have continued their studies of the Aurora and have definitely fixed the position of the unknown auroral lines in the structure of the oxygen atom. (*Proc. Royal Soc.*)

Study of the band spectrum of oxygen by Babcock and Dieke of the Mount Wilson Observatory and Giauque and Johnson of the University of California have shown the presence of an isotope of atomic weight 18. This heavier isotope is about one two-thousandth as plentiful as the isotope 16 so its presence had not been detected by mass spectrograph methods. (*Nature*, March 2, 1929.)

Bowen, California Institute of Technology, has shown the presence of sulphur in the nebulae, by the identification of some more unidentified lines of the spectrum. (*Nature*, March 23, 1929.)

J. W. DuMond of California Institute of Technology has developed an experimental method based on the study of the spectral distribution of Compton modified radiation for determining the velocity distribution of both bound and conduction electrons in metals and of electrons in non-metals. (*Physical Rev.*, May, 1929.)

Absolute measurements of X-ray wavelengths have been made by means of an optical grating. By this means it is possible to get an independent determination of the charge on the electron.

Conductivity. Bridgman, of Harvard, continued the experiments on conductivity and thermoelectric effects in anisotropic single-crystals of metal and showed that the electronic effects in metals follow the crystal-symmetry. (*Nat. Academy of Science*, Vol. 14, p. 943, December, 1928.)

Kapitza has made further experiments on variations of resistance of metals in very strong magnetic fields up to 300,000 gauss. Bismuth lost its conductivity. These experiments may explain much about the conductivity of electricity in metals. (*Proc. Royal Soc.*, Vol. 123, p. 292.)

J. B. Johnson of the Bell Telephone Laboratories has been able to detect and measure the random motions of the electrons in a wire due to their temperature energy. From these measurements it is possible to determine the value of Boltzmann's gas constant. These results lend considerable support to the idea that the conduction electrons in a metal follow the laws of an electron gas. (*Physical Rev.*, Vol. 32, 1928.)

Electronic Emission. R. A. Millikan and C. C. Lauritsen, California Institute of Technology, constructed an apparatus in which electrons were pulled from the electrodes by strong electric fields. The electrons so extracted were of such high velocities as to be capable of producing X-rays with wavelengths comparable to cosmic radiation.

DuBridge gave the first reliable evidence that the photoelectric threshold has exactly the same values as the thermionic work-function for the same metal. (*Physical Rev.*, Vol. 32, p. 961.)

Cardwell showed qualitatively a change of the photoelectric and thermionic emission at the change of an allotropic modification of the cathode.

A. Goetz, California Institute of Technology, gave the experimental evidence that the photoelectric emission and the threshold changes at the melting point and transition point of tin are such that the larger the atomic distance in the cathode the smaller the work function of the metal. This shows that the mechanisms of the photoelectric and thermionic effects are of the same nature. (*Physical Rev.*, Vol. 33, March, 1928, p. 373.)

Lukirsky and Prilezaev succeeded for the first time in measuring the loss of energy of photoelectrons crossing the cathode towards the surface, thus showng that the photoelectric effect is not a pure surface effect, as has been generally considered.

Work in Theoretical Physics. The so-called "principle of uncertainty" stated by Heisenberg has gained general

recognition as a fundamental principle of physics. According to this principle, it is impossible to determine at the same time the position and velocity of a particle with absolute accuracy. The product of the uncertainty in position and the uncertainty in velocity can never be less than Planck's constant h. This is because the method of observation itself affects the quantities to be measured.

The fundamentals of quantum mechanics have remained essentially unchanged, but many applications have been made. Heisenberg has given a striking theory of ferromagnetism based on the magnetic electron as the effective magnetic unit. The electrical resistance of a metal has been explained as due to the diffraction of the electron waves by the crystal lattice. Many of the other electrical properties of metals seem to find a satisfactory explanation on this basis. Neumann and Wigner have shown that all of the known qualitative rules for the analysis of spectra can be derived directly from the Schroedinger equation, and it has been possible to derive some quantitative relationships between spectral terms for spectra more complex than hydrogen. Rubinowicz has shown how the intensities of forbidden transitions may be predicted, so that from the appearance of such lines in the nebulae the physical state of the material may be inferred. Sommerfeld has applied the quantum mechanics to explain the observed distribution of photo-electrons ejected from gases.

The theory of relativity has received an important contribution from Einstein who has been able to give a unified treatment of the gravitational equation and the electromagnetic equations.

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Rubinowicz, *Physik. Zeitsch.*, Vol. 29, p. 817.

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Einstein, "Berlina Berichte" 1929, No. 1.

Discussion

F. C. Caldwell: Referring to the inter-relation of research in different fields of science and engineering, it may be well to call attention to an organization that is just going into operation in Columbus, Ohio. This is the Battelle Memorial Institute, founded by Gordon Battelle as a memorial to his father. Mr. Battelle was associated with the metallurgical industry and bequeathed a substantial amount for the construction and operation of a research laboratory.

Part of the income from this fund will be spent under the exclusive direction of the Battelle Institute. Industrial research will be conducted on the fellowship plan.

While this laboratory is, as I say, primarily metallurgical, the fact that it is no longer possible to separate sharply the different fields of research, makes it inevitable that there will be cooperation with other research organizations along border lines. The electrical engineering profession should therefore know of this Institute, and should look upon it as one of the possible sources of information in those fields where metallurgical and electrical engineering meet.

Transportation

ANNUAL REPORT OF COMMITTEE ON TRANSPORTATION*

To the Board of Directors:

Following established precedent, your Committee submits a brief review of the year's development in the application of electricity to transportation.

Steam Railroad Electrification

Pennsylvania Railroad. The Pennsylvania Railroad has completed the electrification of its suburban service from Philadelphia to Wilmington, Delaware, on its main line to Washington, and to West Chester, Pa., on the Wawa Branch. The 11,000-volt, 25-cycle, single-phase system with overhead catenary used is in accordance with the original electrification work on the Pennsylvania in the vicinity of Philadelphia.

Of far greater importance, however, is the announced program and actual starting of work on the electrification of the entire road train service, freight and passenger, between New York and Wilmington, Delaware.

The project covers the passenger and freight service on 325 mi. of line and 1300 mi. of track, beginning at Hell Gate Bridge, New York, where connection is made with New England, and extending west and south to Wilmington, west from Philadelphia on the Main Line in the direction of Harrisburg as far as Atglen, and the low grade freight lines which join at Columbia, Pennsylvania, and connect the cities of New York, Philadelphia, and Wilmington with the West.

Work is already under way on the 132,000-volt tranmission line through Philadelphia for the extension to Trenton.

Great Northern Railway. The Great Northern Railway has completed and placed in service the extension of the present single-phase electrification through the new tunnel from Scenic to Berne and then east to Wenatchee. This required the abandonment of its old line from Scenic to Cascade through the snow sheds, which electrification was completed about two years ago.

All traffic is now handled between Wenatchee and Skykomish with motor-generator locomotives having d-c. traction motors, the overhead contact system supplying energy at 11,000 volts, 25 cycles, single-phase.

Boston, Revere Beach & Lynn Railroad. This narrow gage line has completed its electrification of 15 route miles, using 600-volt d-c. overhead catenary system with multiple-unit operation.

Cleveland Union Terminal Company. In connection with the building of a combined terminal station and office building in the heart of the business district of Cleveland, it is necessary to handle all trains electrically. Through passenger trains of the New York Central, C. C. C. & St. L., and N. Y. C. & St. L. will be so handled through the city. It has been decided to use 3000 volts direct current with overhead catenary system. Twenty-five 204-ton passenger locomotives, with a rating of about 3000 hp. each, have been ordered, and active work on substations and distribution system started. Power will be purchased from the local power company.

Interurban and suburban trains will also use the station over separate tracks with 600-volt d-c. overhead trolley construction.

Reading Company. The Reading Company has started construction on the Philadelphia Suburban Electrification, which, initially, will include multiple-unit service from the terminal in Philadelphia to Chestnut Hill, Lansdale on the Bethlehem Branch, Hatboro on the New Hope Branch, and Langhorne on the New York Branch. This electrification was decided on not to meet any particular operating difficulty or municipal requirements, but to furnish a better service at a lower operating cost and with a hoped for increase in traffic and revenue.

The initial installation will consist of 50 route miles and 110 track miles, and the 11,000-volt, 25-cycle, single-phase system will be used.

Delaware, Lackawanna & Western Railroad. This company is actively engaged on the design and construction work covering the electrification of its suburban service with multiple-unit car trains from its Hoboken terminal to Dover, New Jersey, via Morristown; also the Montclair Branch and the Passaic & Delaware Branch. The running tracks between West End Junction and Kingsland are included in the program, as the inspection and repair facilities are to be located at the latter point. Certain freight movements between Secaucus and Hoboken Yards will be made electrically.

Power will be supplied through five substations at 3000 volts direct current to the catenary system. The program calls for the electrification of 150 track miles of main line and 25 miles of yards and sidings.

City of Rochester. The city of Rochester has completed and put in service an electrified railroad through the city, in the bed of the old Erie Canal, the right of way being purchased from the State. A modern boulevard has been built over the tracks for a considerable distance in the heart of the city.

The line is nine miles in length with two tracks the

*COMMITTEE ON TRANSPORTATION:

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E. R. Hill,		Sidney Withington,

Presented at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

full distance, one additional freight track for seven miles, and two other freight tracks for 1½ miles. Interchange tracks with steam railroads are provided. Frequent rapid transit cars are operated.

The overhead construction is of the catenary type using three wires with one contact wire. Inclined catenary is used on all curves except the slight ones at station approaches. Operation is at 600 volts direct current.

Diesel-Electric Locomotives

The development of oil-electric locomotives using prime movers of the Diesel type is continuing. A 300-hp. prime mover is most in use to date, one manufacturer having delivered 25 locomotives, weighing about 60 tons, each equipped with one 300-hp. Diesel engine with generator and d-c. traction motors.

Eight locomotives with the same type of power plant, but in duplicate, (the locomotive weighing about 108 tons), are also in service. This is in addition to the New York Central combination third rail, overhead, and storage battery locomotive equipped with the same type of 300 hp. Diesel engine with generator for operation off the trolley or third rail and for charging the battery.

The New York Central has recently accepted delivery of a Diesel-electric locomotive for passenger service on its Putnam division. The engine is a four-cycle air-injection Diesel of the 12-cylinder V type with a full-load rating of 900 b. hp. On the same division is also a freight locomotive with a six-cylinder 750-hp. solid-injection Diesel. Electrical equipment covers a main generator and an auxiliary generator direct-connected to the engine, four d-c. traction motors, control equipment air compressors, and auxiliaries.

Another manufacturer has supplied the Long Island R. R. with an articulated locomotive for switching purposes, equipped with two 330-hp. Diesel engines and electrical equipment. The complete locomotive weighs 87 tons.

The largest single-unit Diesel-electric locomotive built so far, however, is that purchased by the Canadian National Railways. It consists of two separate cabs, each cab containing a 12-cylinder Diesel engine of the solid-injection type rated at 1330 hp. at 800 rev. per min. Each unit weighs 325,000 lb., of which 240,000 lb. are on driving wheels. The present gear ratio was laid out for high-speed passenger service.

Other experimental oil-electric locomotives of varied horsepower are in process of development.

General Purpose Locomotive

The Commonwealth Edison Company has ordered for trial what might be called a General Purpose Locomotive. It will weigh 85 tons with all weight on drivers and is intended primarily for switching work. It can operate directly from the overhead at 1500, 750, or lower voltage. A storage battery having a capacity of 544 ampere-hours at the six-hour rate is provided, this battery to be charged through a motor-generator set when operating under the wire, or from two 155-hp. gasoline engines driving generators. The engines can be operated with distillate fuel. The four motors are rated at 250 hp. each, at 750 volts, one-hour rating. This locomotive will be put in switching service on the Illinois Central Railroad in the near future, for trial.

Rail Cars. Diesel-Electric

The Canadian National Railways has in service 14 Diesel-electric rail cars, four, six, and eight cylinders each, with a rating of from 200 to 400 hp. They are continuing the use of the Diesel type of motive power rail cars and now have on order nine cars to be equipped with six-cylinder engines, rated at 400 hp., each at 900 rev. per min.

Gasoline-Electric

Over 120 gasoline-electric rail cars primarily for branch line service were put in service during the past year.

They vary in weight, and also in power from 200 to 800 hp. in one- two- and three-unit power plants with individual engines rated up to 400 hp. The majority have single power plants with engines of from 275 to 400 hp., the tendency in the last year being toward the larger units. One company is putting out power plants of 135 hp. each, one, two, or three being used, depending on weight of car and desired trailer load. The largest cars have two of the 400 hp. power plants and are in service on the Chicago, Rock Island and Pacific Railway.

Marine Propulsion

The electric-drive airplane carriers Saratoga and Lexington broke all existing speed records for capital naval ships in their sea trials.

The use of turbine-electric drives for large ships and Diesel-electric drive for smaller ships, ferries, and tugs continues to grow.

Car Retarders

Refinements have been made in the design of both electro-pneumatic and straight electric car retarders for regulating the speed of cars being operated over humps to classification tracks.

Where formerly all switches were controlled from one central point by electro-pneumatic or electric switch machines and where each cut of cars was accompanied by a car rider to control the speed of the cut, now a large yard may be handled from separate towers near the switches, each tower controlling a group of switches and retarders with car riders eliminated. Several such yards are in successful operation.

Radio Communication

Several installations of radio communication between locomotive, caboose, and station have been in successful operation. Apparatus is now available for radio communication, as mentioned above, between trains, and between hump engines and the various control towers.

Centralized or Dispatchers Control

The railroads are making great strides in the movement of trains by signal indication only and have made use of the recent developments in centralized or dispatchers control for this purpose.

It is now possible and practicable to control electrically from a central point all desired switches and signals over a considerable route mileage. This arrangement facilitates meeting and passing points and thereby decreases running times between terminals with consequent savings.

Between Stanley and Berwick, Ohio, on the New York Central Railroad, the principal switches, some 30 in number, with all signals, are controlled from one point. This covers 37 mi. of single track and three of double track.

There are over 20 installations of varying lengths in service or on order and the use of this scheme is growing rapidly.

Train Control

The orders of the Interstate Commerce Commission, covering the installation of train control on some 44 Class 1 Railroads have been complied with and there are now over 15,000 track miles equipped and about 9000 locomotives equipped. In addition, there have been voluntary installations covering over 3000 road miles and over 750 engines.

The installations cover continuous control, intermittent control, and speed control.

While no new orders of the Interstate Commerce Commission are contemplated, yet the Division of Safety is following up the whole subject and suggesting further installations where, in their opinion, conditions warrant.

Supervisory Control

The use of supervisory control in the handling of substation and switching station apparatus has continued. The last year has seen considerable development work done on the so-called synchronous selector and relay types, with initial installation of the first mentioned on the Cincinnati Street Railway System, and of the second at the Grimm Avenue Substation of the Chicago Surface Lines.

Mercury Arc Rectifiers

The use of mercury arc rectifiers has continued rapidly, there now being in the United States in the vicinity of 40 installations for street railway, interurban, and steam road electrifications. These are arranged for manual, semi-automatic, and automatic control. The tendency has been to increase the capacity per bowl, there being one installation by the Commonwealth Edison Company in Chicago, having a capacity of 5000 amperes at 600 volts in one bowl. This is an automatic station for railway use.

High-Speed Circuit Breakers

The high-speed a-c. circuit breakers of both the air and oil type, as mentioned in last year's report, have been completed and are now in successful operation on the Wilmington and West Chester extension of the Pennsylvania single-phase electrification. These circuit breakers interrupt short circuits in a half cycle or less and are comparable to the high-speed d-c. breakers which have been in use on steam railroad electrification for the last three years.

The Engineer, Practical Idealist

President's Address*

BY R. F. SCHUCHARDT

THE Institute, meeting here in convention on the "stern and rock bound coast" of New England, finds itself on historic ground. In this region are located many battle fields, of arms and of intellect, that have left their deep impress on our civilization. The battle fields of arms are marked by monuments that are now shrines of an appreciative people. The battles of the intellect, though often accompanied by bloodshed, are not so well remembered. Bunker Hill is fresher in our minds than are Salem and Roger Williams, yet Williams' heroic struggle for freedom of thought paved the way for that later struggle marked by the shaft on Bunker Hill.

The freedom we enjoy today, freedom of thought and of political action, we owe to those who fought and suffered and bled in generations past. We are worthy possessors of our heritages only if we in turn give thought to the morrow, and in our work today plan so that the morrow will offer a richer life for our children.

Are we of the engineering kinship, who are presumably well trained to determine facts and to reason from them, giving sufficient thought to the morrow? It is well on this occasion, in the atmosphere of these significant historical surroundings, to pause a few moments before taking up the technical part of our work, to consider briefly some problems of this interesting workaday world of ours, to try to glean from the study some of the trends in our civilization and our relation to them.

If it is true, as Professor East has written,[1] that man stands today at the parting of the ways, with the choice of controlling his own destiny or of being tossed about by the blind forces of his environment, then it will be well for the engineer to concern himself more with the environment. He should take a larger part in the leadership seeking solution for the human problems that vitally influence the trends. Surely the bewildering complexities of today require clear thinking, and who is better fitted for a thoughtful analysis of the factors on which development and progress depend?

With this as a background, I should like to suggest some thoughts that appear worthy of your careful consideration.

The history of mankind shows a successive rise and decay of great civilizations. Reflection on this leads to the frequent questions: Is our civilization headed for decay? Are we also, like former ages, unable to bear up under prosperity and power? I shall not venture an answer, but let us consider some facts. Earlier civilizations existed on relatively small areas. Today the entire world is a neighborhood —made so largely by the work of the engineer. Not only is it a neighborhood world but also a much more densely populated world, and the great increase in population, we are told, started with the industrial revolution in the eighteenth century—also based on the engineer's work. The engineer, responsible for so much that *is*, cannot shirk his responsibility for correct guidance of what *shall be*.

Our civilization has come to be known as the Machine Age and as such it is both lauded and condemned. President Glenn Frank* in a splendidly balanced analysis of the machine age and its trends, offers this comment:

". . . the masses have more to hope for from great engineers, great inventors and great captains of industry than from the social reformers who woo them with their panaceas. The greatest social progress of the next fifty years is likely to come as a by-product of technical progress."

Even in the Orient, where the materialism of the Occident is thought to sound the death knell of the spiritual, we find the famous Chinese philosopher, Dr. Hu Shih, in agreement, for he sees science and the new technology as the forces which restored to man a sense of self confidence and thus created the modern civilization of the West. He concludes that inventors, scientists, and producers of goods deserve the blessings of their fellows as spiritual leaders.

Others of the interesting group that contributed to the symposium, "Whither Mankind," inspired and edited by Charles A. Beard, seem to concur. As Dr. Beard summarized in part; "They are not oblivious to the evils of the modern order, but they do not concede that any other system, could it be freely chosen in place of machine civilization, would confer more dignity upon human nature, make life on the whole richer in satisfaction, widen the opportunity for exercising our noblest faculties, or give a sublimer meaning to the universe in which we labor.'

With these encouragements let us try to see some of the things about us as they are. Let each of us compare this world of our experience with the dream world of our ideals. Let us see if as engineers we have not an exceptional opportunity to gain in our individual lives that true satisfaction which comes from an attempt to leave the world a better place than we found it. But first let us recognize that an engineer today is far more than Treadgold defined him nearly a century ago. In

1. "Mankind at the Cross Roads" by Edward M. East.

*Delivered at the Summer Convention of the A. I. E. E., Swampscott, Mass., June 24-28, 1929.

*"Where is this Machine Age Taking Us" by Glenn Frank, in *The Magazine of Business*, Oct. 1927.

addition to "directing the powers of nature for the service of man" he now adds to the common welfare also in fields more human. He finds himself occupied at times in social guidance so that the tools he has provided shall be properly used.

One of the most significant and important trends of this day is the continuing movement of population from the rural regions to the urban, which, with the great increase in the world's population, is thus rapidly accelerating the growth of cities. The engineer's machinery makes it possible to provide the necessary food with the expenditure of a lesser manpower than formerly and his transportation developments readily bring the food to the urban population. But with rapidly increasing population the problem becomes more and more difficult.

Professor East cites figures to prove that the maximum food supply that can be produced on the land area of this globe available for that purpose will support a population of fifty-two hundred millions, a figure which at the present rate of increase, he says, will be reached in about a century. Long before that time it will become necessary to bring into production land which is now largely arid and difficult of access, and even then, occasional crop failures will result in widespread starvation.

We can leave to the medical profession and to the biologists the task of checking population increase, but let us engineers face some of the attendant problems. We are not directly concerned with the exact decade when world saturation will be reached but we are vitally concerned with the provisions for living and for advancing civilization while man's neighbors are crowding closer and closer. The marvels of science have lulled the layman into a false sense of security. They have given him faith to believe that with the scientist and the engineer on the job the future will take care of itself. Perhaps it will, but only if the thinkers and the doers of this generation and those who follow put themselves earnestly to the task. Intelligent thinking usually leads to intelligent action. No engineer worthy of the name can take a laissez-faire attitude on problems that deeply affect human welfare and progress.

How does the engineer picture the future city? Is it to be a mechanical Colossus full of ingenious provisions for commerce and industry, for housing and for getting about, or will it be a place where man still has some contact with nature? The growth of our cities is both upward and outward. The upward limit is moving higher and higher, for both business houses and multiple dwellings, and all of the ground area that was formerly devoted to low buildings is being covered by the higher structures. The engineering problems of transportation, of light and air and health for the highly congested population are not impossible of solution but we must admit that they are being taken care of only in part. Multiple decked streets above and below ground, huge ventilators, air conditioning devices, artificial light and other necesssary contrivances are relatively easy to plan, though some of them are very expensive to provide; but is the sort of civilization that is likely to develop in a city of that character the kind that makes for a richer and nobler living? Are we building the City and forgetting the Man, as Grosvenor Atterbury[2] suggests, and are we not fast losing our sense of values?

You remember the Greek legend of Hercules and his encounter with Antaeus, the giant whom Hercules met when on his trip to get the golden apples. Antaeus was a son of Mother Earth and each time he touched the ground his strength grew tenfold. Hercules wrestling with the giant noticed this increased strength so he finally caught him up and held him in the air. Then, no longer having contact with Mother Earth, the giant's strength sank and life ebbed away with it. Is this not symbolic of man's experience when contact with nature is lost? Can a life develop properly when shuttling in crowded cars, through crowded streets or supercrowded subways, between an office which sees the sun for twenty or forty minutes a day and an apartment home in a high building closely touching elbows with neighboring high buildings? Is there a social problem involved in such city building and has the engineer no concern in it?

Our physical surroundings, things we can change if we will, offer a fertile field for the interest of engineers with their vision and with their desire for lifting life to higher levels. I suggest that we give more study to this and similar city problems and that we occasionally discuss them at Institute meetings, especially in our metropolitan sections.

Let us consider some of the problems that are, as it were, at our elbow. One is that of automobile engine exhaust in our cities, which has received all too little attention. This presents a serious health problem in the deep canyon-like streets. Investigators of carbon monoxide poisoning declare that this deadly poison is often found in city streets in sufficient quantities to impair health, and in some instances to be the proximate cause of death. And with the traffic jams on country highways, the holiday outing is now far from a health trip.

While on the subject of air pollution let us in passing note the factory and the apartment house chimneys that still frequently belch forth dense clouds, often robbing man of beauty and of health. The engineer has provided a partial cure, but even our giant power stations, which receive the most expert engineering attention, still leave something to be desired.

There is another handicap to which little thought is given,—the deleterious effect of noise on the nervous system. Scientific tests have shown a marked reduction in labor efficiency due to noise, and British physicians have even advocated an act of Parliament

2. "Our Monster City and Its Life" in *N. Y Times Magazine*, Jan. 13, 1929.

prohibiting needless noise in the interests of the nation's health. Certain it is that our present day business and professional life draws heavily on our nerves, and all too many men collapse under the strain. Relief from noise as one contributor to the strain is thus a direct public benefit.

And how much are we interested in the backyard regions of our industrial cities? Many of these reflect little credit on our advanced civilization. Disorderly dumpheaps and scrap piles are still all too common. They enrich no lives spent among them; rather they are likely to develop that element in our citizenry with which the forces of law and order are in constant conflict. The appalling waste that this entails affects much more than our taxes. Good housing and neighborhood cleanliness go far toward making life for the lowly conform more nearly to the picture of today's accomplishments that we paint with such pride.

The engineer knows that most of our cities have grown more or less hodge podge during the crude developmental years of our country He knows also that correct growth demands guidance, and this means a city plan. Fortunately many cities, taking the lesson from the important European centers, have seen the wisdom of proper planning and are correcting misfits of earlier decades and are providing better guidance for the future. But the problem is today one of even greater extent—one of regional planning, and engineering enters largely into this also. Here is a field which arouses the enthusiastic interest of every true engineer. He realizes that the suburban regions are rapidly filling with dwellings and to some extent with factories. He knows that if life is to be lifted to better things these newer developments must be coordinated properly so health, convenience, beauty, and the opportunity for wholesome family life may all be properly provided for. In this and similar activities the engineer will seek the greatest dividends in terms of human happiness.

The increasing population, Professor East says, requires that each year forty million acres more must be tilled and harvested than the year before to provide sufficient food, and this clearly makes countries more and more dependent on one another and at times will shift the direction of the dependence. The engineer has brought countries together by rapid transportation, by land, by sea, and by air, and has tied them still closer by achievements in communication, and now even by international power lines. What can he do to hold them together as friendly neighbors? The answer here goes to the very bottom of man's relation to man.

The material work of the engineer deals with nature's laws. He knows he must understand these laws and must use truth in his work or it will fail. Engineering analysis applied to human relations similarly seeks to find the underlying laws. A study of past civilizations indicates that they failed because of violation of that basic social law which bids us do unto others as we would have them do unto us. Is it too idealistic to expect improvement in our day to come as a result of more engineering in government, that is, more of the engineering method applied to the solution of questions that tend to bring discord between peoples and between people? There is encouragement in the statement made by Bertrand Russell,[3] "I look, therefore, to the western nations, and more particularly to America, to establish first that more humane, more stable and more truly scientific civilization toward which, as I hope, the world is tending."

Technical achievement has almost eliminated manual drudgery. The habit of technology to be guided by basic law holds out promise that "man's inhumanity to man" may likewise be banished. Is this also too optimistic? Has human nature remained unchanged through the ages, as is so often stated? May I refer the pessimists, if there be such here, to any story that portrays life during earlier periods, as for instance, "Power" by Leon Feuchtwanger. The reader of this gripping tale of the ruthless standards of the eighteenth century will congratulate himself that he is living in the twentieth.

Our machine civilization is still young, and we can attribute to growing pains many of the conditions that today prevent man's best development. However, a look toward the horizon will show many encouraging signs.

Our country, appreciating the keen need of the time for the engineering approach to the important problems of state, has elected an outstanding engineer to the presidency. In other countries the engineer must similarly stand at or near the helm if this civilization is to survive. This does not mean that he alone is to be the savior of mankind, but in this age of cooperation, the engineer, together with the doctor, the lawyer, the economist, the sociologist and other trained minds, must apply himself to a scientific solution of social problems in addition to furthering material development. He must join forces with all those

> ". . . . whose law is reason; who depend(s)
> Upon that law as on the best of friends,"

with men who can find facts and who can face them boldly and honestly.

The engineer's work has started many trends in the direction of relief from some of the world's growing difficulties. A notable one is the spread of electric power into areas away from congestion, thereby permitting at least a partial decentralization of industries and of population. This spread is greatly advanced by the interconnection of power systems now so common all over the country, while the ease of getting about with automobiles is another important factor. A further material aid is the extension of electric distribution lines into rural regions so that modern methods can be applied increasingly to agriculture, both in the field and in the home, while the telephone and radio

3. "Whither Mankind" edited by Charles A. Beard.

are keeping farm operators in touch with the latest in music and education. Great as the progress has been, much still remains to be done in these fields.

Among other items that bring cheer and stimulate optimism may be mentioned the increasing interest now being taken in sane city and regional planning, the efforts of important industries looking toward elimination of waste and the conservation of our resources, the conversion of former wastes into profitable by-products and particularly the growing reclamation of wastes in agriculture.

But here is a bit of pessimism.—The high speed at which life seems to be driven today and the apparent immersion of large numbers in gross materialism has filled some people with fear and misgivings regarding the future. Thus we have the Bishop of Ripon seriously suggesting a ten year holiday in science to give the culture of the soul an opportunity to catch up with the rapid material progress. But the fear that the "spirit" cannot grow in an atmosphere of "science" is not a new one. A century ago Edgar Allen Poe in his Sonnet to Science wrote in part

> "Hast thou not dragged Diana from her car,
> And driven the hamadryad from the wood
> To seek shelter in some happier star?
> Hast thou not torn the naiad from her flood,
> . . . . . ."

The feeling that our present material progress is not really making for a richer life is expressed by many more. Even our own Lorado Taft, with a fine enthusiasm for beauty and an outstanding genius for creating it, is led by his observation to conclude that Americans are practically immune to the arts, have no joy in creating, and are not interested in the most important thing of all, the creation of an ideal civilization.

What answer have we engineer optimists to these critics?—

The good bishop sees but a small part of the picture and that not the bright and inspiring part. When the young Poe wrote his sonnet the Age of Electricity had not dawned. His hamadryads and naiads would today be quite at home with most of our scientists and engineers. And our gifted sculptor seems to feel that the progress made through technology expresses itself mainly in jazz. He fails apparently to see the poetry in the work of a Millikan, a Langmuir, a Steinmetz, or an Edison. He should see, as Pupin and others so clearly see, the idealism of the American machine.

The engineer with his enthusiasm over the human purpose of technology's accomplishments is guided by his ideal of a better living for man, but he keeps his feet on the ground in a practical effort to reach this goal step by step. He appreciates that

> "Heaven is not reached by a single bound;
> But we build the ladder by which we rise
> From the lowly earth to the vaulted skies,
> And we mount to its summit round by round."

The important thing is to be mounting the rounds. That we are moving onward and upward I for one firmly believe.

Is not one true indication of the direction in which we are moving found in the growing appreciation of beauty? The advertising pages of our magazines clearly show the turn of the tide. On every hand commercial competition seems to be along lines of greatest devotion to a beautiful product, and this is not confined to automobiles and bathroom fixtures. Art, which Henry James called "the shadow of humanity," is at last finding its place in industry.

The engineer accepts the line from Keat's famous Ode

> "Beauty is truth, truth beauty."

He is conscious of the beauty and the romance in many of the facts he deals with but he does not let his enthusiasm over the beauty, or his dreaming stirred by the romance, blind him to a proper weighing of the facts. Rather he develops from his dreaming a use of the facts for the further enrichment of life.

And what of the engineer of tomorrow? What thought are we giving to him? Engineers appreciate that by their example, as well as by their encouragement and active help given to educators and to the engineering novitiate, they are building the foundations of the profession more firmly and raising it ever higher. They know that education for useful living is not completed in college. It continues throughout life.

Our Institute, representing the electrical section of American engineers, offers the fellowship of kindred souls, a broadening of outlook and of knowledge, a blending of humanism with technology. It carries inspiration to its members to be not only technicians of high order but also trained seekers of the needs of mankind, applying their skill and their knowledge to the end that life for the teeming millions of their fellowmen may be made more worth while, less burdensome, more healthful, and more noble.

Thus, my friends, do I conceive the true engineer,—one, who in thought and deed, is and ever will be a practical idealist.

Current Transformer Excitation
Under Transit Conditions

BY D. E. MARSHALL* and P. O. LANGGUTH*
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—*Tests of circuit breakers, relays, and similar devices in which current transformers have been used to step down the large currents involved to values measurable by the oscillograph, have shown peculiar errors in the records during the transient portion of the current wave.*

The purpose of this paper is to show that the abnormal excitation of the magnetic circuit of the current transformer required during transient conditions is the cause of these errors, and to point out how the errors may be minimized sufficiently to be negligible.

Mathematical expressions are also derived to make it possible to predetermine the ability of a transformer to handle transients correctly and to guide in the design of transformers for this service.

THIS paper is closely related to the problem of the measurement of transient alternating currents. When certain apparatus such as circuit breakers are subject to short circuit tests, it is desirable to have a correct indication of current during the transient condition.

In the past, two methods have been used for measuring heavy transient currents, namely, the non-inductive shunt and the current transformer. Considerable field testing has demonstrated that these methods have introduced difficulty under some conditions. When the shunt is used it is of course necessary to insulate it and associated oscillograph apparatus from ground to the extent of full line voltage. Furthermore, non-inductive shunts can be used only with non-inductive measuring devices and therefore cannot be used with the current coil of watt oscillograph elements, which are highly inductive, for the measurement of instantaneous power.

Even with resistive burdens inaccuracies will result due both to self inductance and mutual inductance on multi-phase set-ups.

The use of the current transformer for the measurement of transients has resulted in some inaccuracies under certain conditions. In this paper efforts have been made to determine the causes of the errors which occur, the extent of the errors, and their elimination or minimization. The causes of the errors are first briefly outlined. A discussion of the practical importance of the errors is given. A mathematical exposition of the theory is given, including the formulas for designing transformers having suitable characteristics for the measurement of transients. A number of oscillograph records, amplifying and checking the theory, is presented, and finally a method is given for testing transformers to determine their suitability for use under transient conditions.

The authors are indebted to Mr. J. F. Peters for constructive criticism and to Mr. L. R. Smith for assistance given in conducting the tests.

Outline of Causes of Errors and Their Characteristics

If the circuit in which the primary of the current transformer is connected contains both resistance and inductance the current flowing in that circuit, just after its switch is closed, will not always immediately assume a pure sinusoidal wave form but will be displaced from its normal zero axis in a manner similar to the current curve shown in Fig. 1. (For equation of this curve see Equation (4) in the Mathematical Exposition.)

This asymmetry rapidly decreases and in practical circuits disappears in about 3 to 15 cycles of 60-cycle current. The degree of asymmetry depends on the

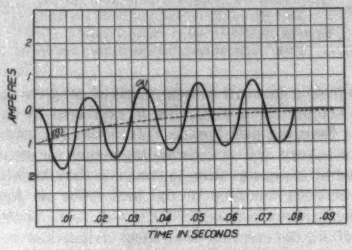

Fig. 1—Transient Current for Inductive Circuit Having Time Constant 0.0304 at 60 Cycles

(*A*) = Asymmetrical current wave
(*B*) = Displaced zero axis

point of the voltage wave at which the switch is closed. The current wave is practically normal if the switch is closed when the voltage is at its maximum and may be entirely above or below the zero line, if the switch is closed at the zero of the voltage wave. The point on the voltage wave at which the switch is closed is usually a matter of chance, although it is sometimes argued that inasmuch as in high-voltage circuits the current might start before the switch is completely closed, the first case is the more probable. Thus in the majority of cases, the transient will be comparatively slight. (Fig. 1 is calculated on the basis of a closure under the worst possible conditions.)

The important thing to appreciate, as far as this paper is concerned, is that this dissymmetry is quite likely to occur just after the switch is closed. This paper therefore treats of the effect of the dissymmetry on the mag-

*Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

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29-144

netizing current and the resultant error in a current transformer under such transient conditions.

The error is due to the over-saturation of the transformer which is caused by the d-c. component of the asymmetrical current. This condition is of course aggravated if the secondary burden is excessive. The error in transformation will vary also, depending upon the amount of residual magnetism and its polarity with respect to that of the d-c. component. For this reason, in tests where accuracy is essential, the transformer selected for use in measuring transients must be designed for a low degree of saturation at the burden to be imposed and must be demagnetized between successive heavy shots. On tests which do not come up to the knee of the saturation curve, the residual will probably be so slight that demagnetization will not be required.

A mathematical solution of the problem, based on certain assumptions, which derives expressions for the wave form of the transformer magnetizing current and which gives formulas for use in applying and designing transformers for this service, is given under the heading of Mathematical Exposition. An absolutely rigid solution of the problem for an iron core transformer is impossible but enough can be done mathematically to throw considerable light on the problem. The equations and the data indicate that the inaccuracies of the current transformer during unsymmetrical transients are caused by the large flux required to generate the voltage necessary to reproduce the transient in the burden of the transformer. The maximum value of transient flux is dependent on the magnitude of the current, the duration of the transient, the design of the transformer, and the secondary burden.

A series of tests was made to check and amplify the mathematical theory and the data thus secured are given under the heading of Test Data in this paper.

Practical Importance of Errors

As these errors are caused by the dissymmetry of the primary current which is dependent on the cyclic time of closing the switch, their appearance is rather uncertain, but always possible, and may cause erroneous conclusions on an important test.

The magnitude of these errors is generally comparatively slight, although on extremely heavy short circuits of a high degree of asymmetry, with a particularly unsuitable transformer having a heavy secondary burden, the degree of over-saturation may be so great as to cause an error of roughly 50 per cent in the r. m. s. value of the first current wave. The most noticeable effect of the errors is the distortion of the secondary current wave shape rather than serious diminution of the crest value.

In most cases the error thus introduced is comparatively unimportant except in the cases of high speed relay operation and watt oscillographs. The great majority of tests to determine rupturing capacity of circuit breakers are so made that the current has reached a steady state condition before arcing starts. Even though this condition has not been reached, the r. m. s. current of the first half cycle of arcing is usually determined from the crest value, with due allowance for the displacement of the neutral axis, on the assumption that the wave is of sinusoidal shape which partially corrects for the error but always indicates a lower current than actually occurred.

In oscillograph testing, however, the accuracy required is not of as high an order as required for current transformers used with watthour meters, hence the allowable flux density is higher. The oscillograph burden is lower than the usual burdens. These two factors tend to compensate for the high value of transient flux required, thus making the problem of applying or designing a suitable current transformer less difficult than might appear.

The only competitor of the current transformer has been the non-inductive shunt. This method, however, presents difficult insulation problems as the oscillograph elements must be directly connected to the high-voltage lines.[1] It should also be fully appreciated that a non-inductive shunt is only allowable for use with non-inductive burdens. It must always be of the same time constant as the burden. This effect is not serious for straight current measuring oscillograph elements. Watt oscillograph elements, however, having current coils of relatively high inductance, can not be used with non-inductive shunts.

The current transformer works better, if anything, for inductive than for resistive burdens and may be used for any type of element. Against the disadvantages of demagnetizing transformers may be set the added safety to operators, the lack of necessity of insulating the operator and instrument from ground, and the possibility of using the transformers supplied in the apparatus under test instead of special equipment.

Mathematical Exposition

In any transformer the flux, voltage, time, relative in a form independent of wave form is:

$$e = N \frac{d\phi}{dt} 10^{-8}, \text{ or } \phi = \frac{10^8}{N} \int e\,dt + c \qquad (1)$$

where

e = instantaneous volts as a function of time.

ϕ = flux lines.

N = number of turns in coil threaded by and generating e.

t = time in seconds.

c = a constant depending on value of ϕ at zero time.

Thus to calculate the flux threading a coil generating a known time wave of voltage it is necessary to integrate the voltage-time relation from the time when the voltage started. In other words, the flux through the coil

1. For references see Bibliography.

at any time is proportional to the area under the voltage time curve from the time when it started to the given time.

There are two practical methods of calculating the voltage in the secondary winding of a current transformer when a given current flows in the primary winding. The first way (*a*) is to assume that the magnetizing current is negligible and that therefore the secondary voltage is equal to the primary current divided by the transformer ratio multiplied by the secondary circuit impedance. The second way (*b*), is to make use of a so-called equivalent transformer circuit.

The first method is obviously the least accurate but has the great advantage of simplicity; it is equivalent to calculating the flux of an "ideal" transformer, that is, one having a magnetic circuit of zero reluctance. As this condition defines the limits of the problem we shall use it first.

Mathematical Solution (*a*)

As we are assuming that

$$i_s = i_p \frac{N_p}{N_s} \tag{2}$$

Where

i_s = secondary current
i_p = primary current
N_s = secondary turns
N_p = primary turns

it is evident that Equation (2) for this case will take on the form:

$$\phi = \frac{10^8 N_p}{N_s^2} \int \left(R_2 i_p + L_2 \frac{d i_p}{d t} \right) d t + c$$

$$= \frac{10^8 N_p}{N_s^2} \left[\int R_2 i_p \, d t + L_2 i_p + c \right] \tag{3}$$

where R_2 and L_2 are secondary resistance and inductance respectively.

If we consider the case of an unsymmetrical transient such as occurs on transmission lines and other circuits containing resistance and inductance in series, we shall substitute for the primary current:

$$i_p = I \left[\sin (\omega t + \alpha) - \sin \alpha \, \epsilon^{-\frac{t}{T}} \right]. \tag{4}$$

where:

i = instantaneous current at time (t).
I = maximum value of steady state current.
$\alpha = \left(\theta - \tan^{-1} \frac{\omega L}{R} \right)$ phase angle which fixes value of instantaneous volts when ($t = 0$). For maximum transient, $\alpha = \frac{\pi}{2}$.

T = time constant = $\frac{L}{R}$ of total primary circuit.

After performing the indicated operations, the expression for flux in the transformer core becomes:

$$\phi = \frac{10^8 N_p}{N_s^2} I_p \left[\frac{1}{\omega} \sqrt{R_2^2 + \omega^2 L_2^2} \sin \left[\omega t + \alpha - \tan^{-1} \left(\frac{R_2}{\omega L_2} \right) \right] + (R_2 T - L_2) \sin \alpha \, \epsilon^{-\frac{t}{T}} + C \right] \tag{5}$$

As pointed out above the dissymmetry is at its maximum when the switch energizing the primary circuit is closed at the time when $\alpha = 90$ deg. Hence the flux for maximum transient is:

$$\phi = \frac{10^8 N_p}{N_s^2} I_p \left[\frac{1}{\omega} \sqrt{R_2^2 + \omega^2} L_2^2 \sin \left(\omega t + \tan^{-1} \frac{\omega L_2}{R_2} \right) - R_2 T (1 - \epsilon^{-\frac{t}{T}}) - L_2 \epsilon^{-\frac{t}{T}} \right] \tag{6}$$

The constant (*c*) is solved for on the basis that ($\phi = 0$) when ($t = o$)

This expression consists of the usual steady state

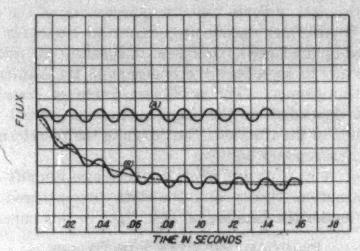

FIG. 2—WAVE FORM OF FLUX REQUIRED TO REPRODUCE EXACTLY TRANSIENT OF FIG. 1

(*A*) = Flux caused by steady state current
(*B*) = Flux caused by transient current

sine wave term and an exponential term which is caused by the dissymmetry of the primary current wave. A curve of this wave shape is plotted in Fig. 2, the dotted curve being the shape of the exponential term, to which is added the sine term in heavy line. The ordinary steady state flux is also shown, plotted for comparison about the zero line.

Expression (6) reaches its maximum when $t = \infty$ and is equal then to:

$$\phi_{max} = \frac{10^8 N_p}{N_s^2} I_p \left[\frac{1}{\omega} \sqrt{R_2^2 + \omega^2 L_2^2} + R_2 T \right]. \tag{7}$$

This expression gives the magnitude of the maximum flux without regard to sign. It will be noticed that the secondary inductance of the transformer has

no effect on the maximum transient flux. Its only effect is on the steady state.

For the usual case of a ring type transformer having a well distributed secondary winding connected to an oscillograph, the secondary inductance is negligible,[4] and by assuming it equal to 0 in Equation (7),

$$\phi_{max} = \frac{10^8 N_p}{N_s^2} I_p \frac{R_2}{\omega} [1 + \omega T]. \tag{8}$$

but the portion preceding the brackets is equal to the steady state flux so that the latter part can be called the transient flux multiplier:

$$m_1 = [1 + \omega T] \tag{9}$$

This simple expression is of value in judging the severity of a transient as regards the saturation of the current transformer cores, as it required no knowledge of the transformer constants.

Mathematical Solution (b)

The second method of solution, by equivalent circuits, fits the case much closer than the first method but is considerably more involved. It will, however, give equations for magnetizing currents which are closely borne out by oscillograph records, and equations may be derived which give quite accurate values for errors and maximum flux values as they are affected by the constants of the transformer itself.

It assumes a non-saturated transformer and neglects the hysteresis and eddy current effects. This assumption is not serious because the transient portion of the flux is practically unidirectional and saturated transformers are out of the question entirely. It should be appreciated that where equivalent circuits are used

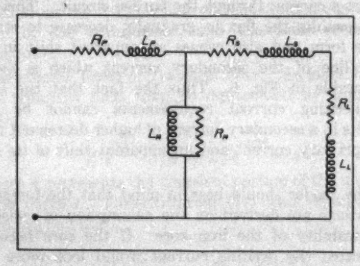

FIG. 3—EQUIVALENT CIRCUIT OF A VOLTAGE TRANSFORMER

we shall speak of magnetizing current instead of flux on the basis that they are directly proportional.

The transformer may be considered as equivalent to the circuit of Fig. 3 where:

R_p = resistance of primary.
L_p = inductance of primary.
R_m = equivalent iron loss shunt resistance.
L_m = equivalent magnetizing shunt inductance.
R_s = resistance of secondary.
L_s = inductance of secondary.
R_L = burden resistance.
L_L = burden inductance.

The above equivalent circuit is the one given by standard text books,[2] and applies to transformers operating with constant voltage applied to the primary winding. If the current in the primary winding is known, then R_p and L_p are no longer of interest in determining the secondary current. When the iron loss is considered as negligible, the resistance R_m is infinite and the equivalent circuit for the current transformer therefore reduces to the circuit of Fig. 4 where $R_2 = R_s + R_L$ and $L_2 = L_s + L_L$. For con-

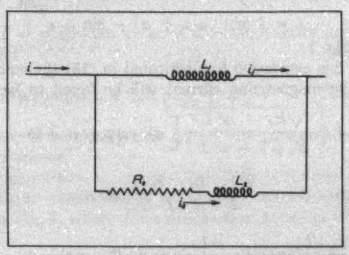

FIG. 4—EQUIVALENT CIRCUIT OF A CURRENT TRANSFORMER CONSIDERING THE IRON LOSS NEGLIGIBLE

venience we shall call the magnetizing inductance L_1, thus $L_1 = L_m$. The primary current is i, the magnetizing current i_1, and the burden current i_2 (referred to secondary).

To solve for the division of currents we write the following equations:

$$L_1 \frac{d\,i_1}{d\,t} = R_2\, i_2 + L_2 \frac{d\,i_2}{d\,t} \tag{10}$$

$$i_2 = i - i_1 \tag{11}$$

From (10) and (11)

$$\frac{d\,i_1}{d\,t} + \frac{R_2}{L_1+L_2} i_1 = \frac{1}{L_1+L_2}\left(L_2 \frac{d\,i}{d\,t} + R_2\, i\right) \tag{12}$$

The general solution[3] of this type of differential equation is:

$$i_1 = \frac{1}{L_1+L_2} \epsilon^{\frac{-R_2 t}{L_1+L_2}} \int \left[\left(R_2\, i + L_2 \frac{d\,i}{d\,t}\right) \epsilon^{\frac{R_2 t}{L_1+L_2}}\right] d\,t + c\, \epsilon^{\frac{-R_2 t}{L_1+L_2}} \tag{13}$$

where i equals any function of time. To simplify the writing of the exponent we shall indicate the quantity:

$$\frac{R_2}{L_1 + L_2} = \frac{1}{T_1} \tag{14}$$

Hence (13) becomes:

$$i_1 = R_2 T_1 \epsilon^{\frac{-t}{T_1}} \int \left(R_2 i + L_2 \frac{d i}{d t} \right) \epsilon^{\frac{t}{T_1}} dt + c \epsilon^{\frac{-t}{T_1}} \quad (15)$$

By the use of this equation we may determine the magnetizing current wave shape for any shape of primary current, providing of course that the resulting expression is possible to integrate.

Case of Primary Current Containing Unsymmetrical Transient

If the primary current is unsymmetrical it may be represented by the following equation as given earlier in the paper.

$$i = I \left[\sin (\omega t + \alpha) - \sin \alpha \, \epsilon^{\frac{-t}{T}} \right]$$

See Fig. 1.

If this expression is substituted in (15) the equation for the magnetizing current will be found to be:

$$i_1 = I \left\{ \frac{\sqrt{R_2^2 + \omega^2 L_2^2}}{R_2 \sqrt{1 + \omega^2 T_1^2}} \left[\sin (\omega t + \alpha + \phi - \Delta) - \sin(\alpha + \phi - \Delta) \epsilon^{\frac{-t}{T_1}} \right] + \frac{L_2 - R_2 T}{R_2 (T - T_1)} \sin \alpha (\epsilon^{\frac{-t}{T}} - \epsilon^{\frac{-t}{T_1}}) \right\}$$

$$T_1 = \frac{L_1 + L_2}{R_2}; \tan \phi = \frac{\omega L_2}{R_2}; \tan \Delta = T_1 \omega; (i_1 = 0), (t = 0). \quad (16)$$

This expression is plotted for a given set of more or less arbitrary constants in Fig. 5. It will be noticed that the equation is divided into three parts, the ordinary steady-state sine wave of magnetizing current, the transient term, and a third term whose function it is to make the value of magnetizing current zero at zero time. The last two terms combine in the form $A \epsilon^{\frac{-t}{T}} - (A + B) \epsilon^{\frac{-t}{T_1}}$ (17) which is shown as the dotted curve in Fig. 5, to which is added the steady-state magnetizing current drawn as a solid line. It is this transient term which may add so much to the flux required in the transformer core during the transient period of the primary current that causes all the trouble through saturation at that time.

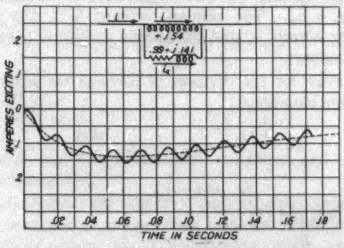

Fig. 5—Magnetizing Current for Transient of Fig. 1. Transformer Constants as Shown in Equivalent Circuit and Assuming Straight Line Saturation

By comparing Fig. 5 with Fig. 2, in which the flux required in an ideal transformer (one having zero reluctance in its magnetic circuit) is shown, it may be seen that the actual transformer tries to follow the action of the ideal, but cannot due to the necessity of its drawing the large unidirectional magnetizing current required from the primary current. The primary current has but a small unidirectional current and hence cannot supply the unidirectional magnetizing current required by the transformer. The magnetizing current must therefore fall gradually to zero. This decreasing flux generates a unidirectional voltage which causes a current through the burden circuit. This current enables the flux to gradually decrease to steady state form and also causes an apparent shift in the zero line of the secondary current which is clearly noticeable in Fig. 6. Thus the fact that the ideal magnetizing current requirements cannot be met results in a secondary current of higher decrement than the primary current and an apparent shift of its zero line.

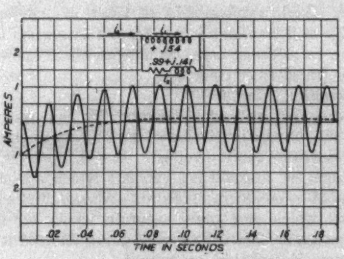

Fig. 6—Secondary Current for Transient of Fig. 1. Transformer Constants as Shown in Equivalent Circuit and Assuming Straight Line Saturation. Plotted from Difference of Figs. 1 and 5

The reader should bear in mind that the foregoing equations are derived on the assumption of constant permeability of the iron core. If the core becomes saturated the exciting current would look more like the upper curve in Fig. 7.

The above expression (16) is general, that is, it contains provisions for both variation in the shape of the primary current transient as affected by variations in the phase at which the main circuit is closed, and also provision for an inductive burden. If now we make an assumption of no secondary leakage inductance, which is justified on the ground that the oscillographic circuits and the through ring type transformers with which we are mainly concerned are very closely non-inductive,[4] and that the inductance which is present in the secondary circuit has but small effect on the transient flux, we

can deduce several expressions which will prove useful in the design of transformers for this service.

If we substitute in Equation (16) the following: $L_2 = 0$, $\sin \alpha = 1$, (which causes maximum primary transient),
we have:

$$i_1 = I \left[\frac{1}{\sqrt{1 + \omega^2 T_1^2}} \cos (\omega t - \tan^{-1} \omega T_1) - \frac{T}{T - T_1} \left(\epsilon^{\frac{-t}{T}} - \epsilon^{\frac{-t}{T_1}} \right) + \frac{\epsilon^{\frac{-t}{T_1}}}{1 + \omega^2 T_1^2} \right] \quad (18)$$

which give the shape of the magnetizing current as a

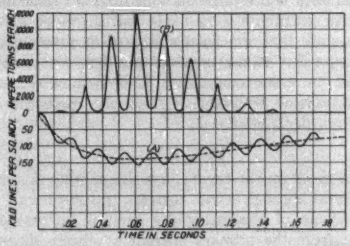

FIG. 7—AMPERE TURNS REQUIRED TO PRODUCE FLUX OF FIG. 5

(Hysteresis assumed negligible)
(A) = Flux wave
(B) = Ampere turns to produce flux in curve A assuming saturation

function of $(T_1) = \frac{L_1}{R_2}$. This factor is easily calculated from the magnetizing current-voltage characteristic of the transformer, and the total secondary resistance.

Maximum Value of Transient Flux. As given before the transient term may be represented by Equation (17):

$$A \epsilon^{\frac{-t}{T}} - (A + B) \epsilon^{\frac{-t}{T_1}} = u$$

where u represents the transient portion of the magnetizing current. Differentiating (17) and equating to zero, we find that

$$t_m = \log_e \left[\frac{T_1}{T} \frac{A}{A + B} \right]^{\frac{TT_1}{T_1 - T}} \quad (19)$$

which is the time at which the transient term reaches its maximum value.

Substituting this expression in Equation (17) we have for the maximum value of the transient magnetizing current:

$$U_{max} = A \left[\frac{T_1}{T} \frac{A}{A + B} \right]^{\frac{-T_1}{T_1 - T}} (A + B) \left[\frac{T_1}{T} \frac{A}{A + B} \right]^{\frac{-T}{T_1 - T}} \quad (20)$$

By substituting the values of the constants of Equation (17) in the above Equation (20), the correct maximum value of the transient magnetizing current will be obtained.

For design purposes Equation (18) is accurate to a high degree. By taking the coefficients as given there and substituting in (20) and by the use of a justifiable assumption the following simpler equation for the maximum is derived:

$$U_{max} = I \left(\frac{T_1}{T} \right)^{\frac{-T_1}{T_1 - T}} \quad (21)$$

where as before,

$T_1 = \frac{L_1}{R_2}$, T = time constant of primary transient,

and I = maximum value of the steady-state primary current.

The assumption is that $\frac{1}{1 + \omega^2 T_1^2} = 0$. This will in general be closely true for all iron core commercial transformers.

The quantity u_{max} is plotted on a percentage basis for three representative values of T as a function of T_1 in Fig. 8, where I is considered as equal to 100 per

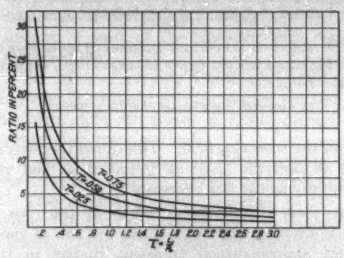

FIG. 8—RATIO OF MAXIMUM TRANSIENT MAGNETIZING CURRENT TO MAXIMUM STEADY STATE PRIMARY CURRENT IN PER CENT FOR DIFFERENT VALUES OF

(T represents time constant of primary circuit)

cent. (I) may be either primary or secondary depending on reference. This equation and the curves give the maximum transient error of the transformer under the foregoing conditions.

By dividing u_{max} by $\frac{I R_2}{\omega L_1}$ (the usual approximation for the steady-state magnetizing current) we obtain a transient flux multiplier similar to the variable part of Equation (9):

$$m_2 = \omega T_1 \left(\frac{T}{T_1} \right)^{\frac{T_1}{T_1 - T}} \quad (22)$$

This multiplier gives the ratio between the maximum

transient flux or exciting current and the steady-state flux or exciting current. This expression is plotted under the same conditions as Equation (**21**) in Fig. 9. If the ratio of the maximum total transient flux to the steady-state flux is required the expression is:

$$m_1 = \left[\omega T_1 \left(\frac{T}{T_1} \right)^{\frac{T_1}{T_1 - T}} + 1 \right] \qquad (23)$$

When T_1 is infinite the transformer is ideal and the above expression reduces to expression (**9**).

Case of Primary Current Containing Both Symmetrical and Unsymmetrical Transients

When an alternator is short circuited the armature reaction is in such a direction as to decrease the mag-

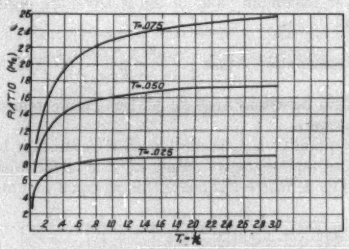

Fig. 9—Ratio of Maximum Transient Magnetizing Current to Maximum Steady State Magnetizing Current for Different Values of

(T represents time constant of primary circuit)

netism of the revolving field. This causes the generated voltage to decrease from an initial to a final value exponentially as the flux decreases.[5]

If the short circuit should occur at the time the voltage is approximately zero, there will also in general occur an unsymmetrical transient similar to those encountered before in this paper.

The current flowing under these conditions may be represented by the expression:

$$i = I_2 \left[\sin(\omega t + \alpha) - \sin \alpha \, \epsilon^{\frac{-Rt}{L}} \right] + I_1 \left[\epsilon^{\frac{-t}{T_2}} \sin(\omega t + \beta) - \sin \beta \, \epsilon^{\frac{-Rt}{L}} \right] \qquad (24)$$

$$\alpha = \left(\phi - \tan^{-1} \frac{\omega L}{R} \right); \quad \beta = \left(\phi - \tan^{-1} \frac{\omega L T_2}{R T_2 - L} \right).$$

If this current is passed through the transformer the magnetizing current is given by:

$$i_1 = \frac{A \, \epsilon^{\frac{-t}{T_2}}}{\sqrt{\left(1 + \frac{T_1}{T_2}\right)^2 + \omega^2 T_1^2}} \left[\sin(\omega t + \beta - \gamma) \right] + \frac{B}{\sqrt{1 + \omega^2 T_1^2}} \left[\sin(\omega t + \alpha - \Delta) \right] - \frac{C T}{T - T_1} \sin \alpha \left(\epsilon^{\frac{-t}{T}} - \epsilon^{\frac{-t}{T_1}} \right) + D \, \epsilon^{\frac{-t}{T_1}}.$$

$$\tan \gamma = \frac{\omega T_2 T_1}{T_2 - T_1}; \qquad \tan \Delta = \omega T_1.$$

There is no simple method of finding the maximum of this equation, but it will be noticed that the only term that can cause a very high rise of flux is the third term.

This term is the same on which our previous operations for maximum were made and therefore we may safely state that conclusions arrived at previously for maximum flux values in unsymmetrical cases may be applied to cases containing both symmetrical and unsymmetrical transients by simply neglecting the alternator decrement.

Test Data

As a check on the theory, a series of tests was made on several through type current transformers. For the purpose of separating the exciting current component from the primary current a scheme shown in Fig. 10 was used. A primary circuit was set up to have definite pre-determined values of resistance and inductance such that transients of definitely known time constant, (T), could be obtained. Non-inductive

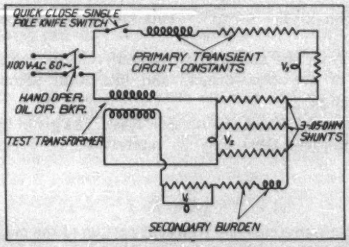

Fig. 10—Schematic Diagram of Transient Testing Circuit

shunts were used in parallel with the oscillograph elements V_1 and V_3 to obtain the total secondary current and the total primary current. A combination of three shunts of equal value was used in the differential circuit with oscillograph element V_2 for the purpose of obtaining the difference between the primary and secondary currents. As the ratio of the transformer is 2 to 1 and as the resistance of element V_2 is high in comparison with the resistance of the shunts, the deflection of the element V_2 will be proportional to the difference between the primary and secondary currents, thus giving a measure of the exciting current or instantaneous error of the transformer.

The circuit supply of 1100 volts, 60 cycles was controlled by a hand operated oil circuit breaker and a synchronized switching arrangement. As illustrated in Fig. 3, the control of the quick-close, single-pole knife switch (short circulating switch) was so arranged as to make it possible to start the transient on any part of the voltage wave. The control of the oscillograph allowed the shutter to be opened, the film to start moving, and the synchronous switch to operate all in correct sequence, thus insuring a film with the proper transient well spaced each time. For the tests described herein the transient was started at the point on the voltage wave which caused the maximum dissymmetry of current. The drum driven by the synchronous motor was also provided with a stroboscope arrangement in conjunction with a neon lamp for the purpose of checking synchronism and also for easy reversal of successive transients on OCO test.

The transformer used for these tests was a ring type constructed to fit the standard 37,000-volt condenser type bushing having design constants:

(a) Secondary turns.....100
(b) Core diameter inside.....4⅜ in.
(c) Core diameter outside.....6⅛ in.
(d) Core cross sectional area.....4.26 sq. in.
(e) Secondary resistance.....0.1668 ohms.
(f) Saturation characteristic (referred to secondary).

Amperes	Volts
0.2	32
0.5	73
1.0	90
2.0	99
8.0	112

By short circuit impedance test of the transformer

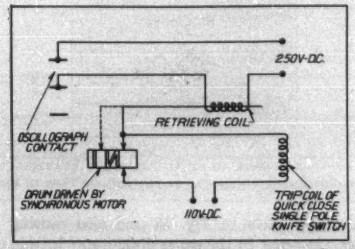

Fig. 11—Schematic Diagram of Synchronous Switching Arrangement

with wound primary it was determined that the leakage inductance of both primary and secondary is exceedingly small, power factor 99 per cent, thus justifying the assumption made in the theory of zero leakage inductance.

To allow the use of the measuring scheme of Fig. 10, it was found necessary to use relatively low values of current. Thus, to simulate the actual transformer performance under heavy short circuits, it was necessary to wind a primary of 50 symmetrically spaced turns on the transformer, thus giving the effect of 800 amperes, (800 ampere turns).

Numerous tests were made under various conditions of circuits and transformer which are outlined as follows:

1. The instantaneous current waves were obtained for several different secondary burdens and primary currents.

2. The instantaneous exciting current waves for

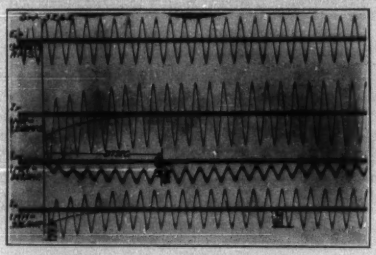

Fig. 12—Oscillogram of OC Transient with Oscillograph and Shunt Burden

various transient circuits were investigated by varying the constants of the primary circuit and by varying the instant of start of the transient with respect to the voltage wave.

3. The effect of varying degrees of residual magnetization was determined by magnetizing the test transformer with different values of direct current before subjecting it to transient tests.

In all cases oscillograms indicate the supply voltage, primary current, exciting current, and secondary current. This permits of easy inspection of the results regarding the form of transient and the exciting current waves. Most of the tests were made on OC basis; however, the conditions to be expected under OCO conditions were investigated to show the effect of residual flux. In all cases, except as otherwise mentioned, the tested transformer was demagnetized by using gradually decreasing 60-cycle supply. This removes residual magnetism and permits of direct comparison of the data.

The results, in the form of oscillograms, of a few of the tests and their analysis are given individually.

Fig. 12. This oscillogram was taken with the transformer thoroughly demagnetized and approaches closer to the theoretical curves than any of the oscillograms. The primary current is identified as I_p, the secondary by I_s, and the differential or exciting current by I_d. The transient portion of the magnetizing current is well above the zero line. This current gradually dies down to a steady state of very small amplitude symmetrical with respect to the zero line. This film should be compared with Figs. 1, 5, and 6.

The power factor of the primary circuit is 5.77 per cent which corresponds to a time constant (T) of 0.0447. The total secondary circuit resistance including oscillograph shunts is 0.30. Since the exciting current of the transformer is 1 ampere at 90 volts on the "knee" of the saturation curve, then we can determine that the average value of (T_1) in Equation (**18**) is 0.795.

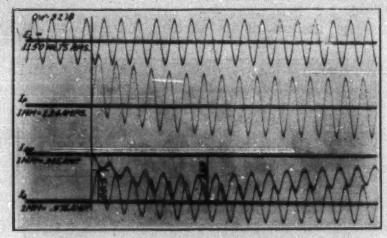

Fig. 13—Oscillogram of OC Transient, Previously Magnetized Transformer

The time at which the transient portion of the magnetizing current should be a maximum is given by Equation (**19**). Substituting the value of T_1 calculated above the time obtained is 0.137 seconds. This time is indicated on the film for comparison with the actual maximum as shown on the oscillograph.

The per cent maximum exciting current is given by the expression, Equation (**21**), which calculated for the above constants is 4.8 per cent. From measurements of the oscillogram, Fig. 12, the secondary current is 11.2 amperes and the maximum transient exciting current is 0.45 ampere, giving a per cent maximum exciting current of 4.02. These values check quite closely when the errors in measurement and oscillograph calibration are taken into account.

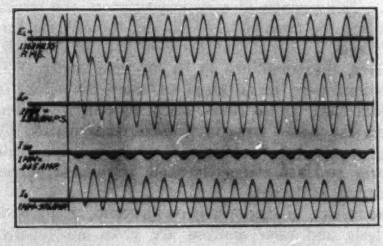

Fig. 14—Oscillogram of OC Transient, Previously Magnetized Transformer,—Polarity Opposite to Fig. 13

Figs. 13 and 14. These are companion oscillograms for the purpose of demonstrating the effect of residual magnetism. The primary and secondary circuit constants are the same as in Fig. 12. Fig. 13 shows the exciting current transient obtained by previously demagnetizing and then magnetizing the core by a heavy direct current. The record shows the magnetizing current higher than in Fig. 12. As an explanation of this, the presence of the high residual magnetism causes the transformer to work higher up on the hysteresis curve for a few cycles. This of course requires more exciting current to build up the required flux in the transformer. After the transformer is on the circuit for a short time the effect of this residual magnetism disappears.

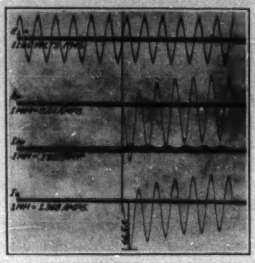

Fig. 15—Oscillogram of OCO Transient with Oscillograph and Shunt Burden

Fig. 14 illustrates the condition that may be obtained by previously demagnetizing and then magnetizing the core oppositely to Fig. 13 and also causing the transient to occur in the same direction. In this case the required transformer flux is built up with a small exciting current, thus approximating steady state value and accuracy.

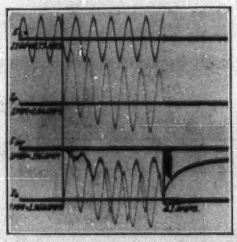

Fig. 16—Oscillogram of OCO Transient Starting with Transformer in Condition as at the End of Test in Fig. 15

Figs. 15, 16, and 17. These oscillograms show the results obtained with OCO transients. The primary circuit constants were changed to give a more severe transient. The power factor was changed to 7.46 per cent resulting in the time constant T of 0.0341 and a primary current effect of 1700 amperes. The secondary circuit was the same as in Fig. 12. These three

oscillograms were taken in succession. Before obtaining Fig. 15 the transformer was demagnetized. The test for Fig. 15 was so made that the circuit was opened while the unidirectional flux was still in the core. This flux dies out slowly and leaves a residual.

Fig. 16 shows the effect of the residual flux left after making test indicated in Fig. 15, on the transient magnetizing current. This figure shows a higher value

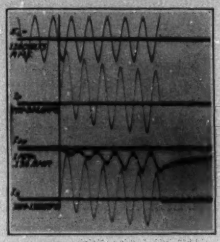

Fig. 17—Oscillogram of OCO Transient Starting with Transformer in Condition as at End of Test in Fig. 16

of exciting current than Fig. 15, thus indicating saturation caused by the preceding residual flux which is a condition similar to that of Fig. 13.

Fig. 17 indicates very little exciting current due to the transient occurring in the opposite direction to the previous transient. This indicates that the residual in this case is a help rather than a hindrance to the magnetization of the core.

An interesting point often encountered in short circuit testing is brought out by Fig. 16. It is noticed

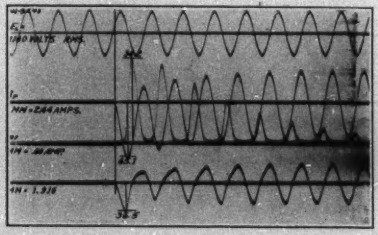

Fig. 18—Oscillogram of OC Transient—Secondary Burden 1.3 Ohms—Magnetized with 2 Amperes D-c.

that the exciting current slowly falls to zero and causes a current to flow in the secondary after the primary circuit is opened. These two currents must be equal when referred to the secondary as there is no other place for the current generated by the decreasing flux to flow. However, unless the transformer is approaching saturation, this current will be hardly noticeable in the secondary as the exciting current should be a small percentage of the secondary current.

It will be noticed that the alternating component of the exciting current increases considerably as it is added to the unidirectional component. This is caused by the peculiar action of combined alternating and unidirectional flux as brought out in Mr. T. Spooner's book on "Testing and Properties of Magnetic Materials." The flux increases along the conventional magnetization curve until an alternation occurs, when the alternating cycle makes a small hysteresis loop for itself which has a flatter slope than the main hysteresis loop. This has the tendency of increasing the alternating component of the exciting current while not affecting the unidirectional flux to a great extent.

Figs. 18 and 19. These oscillograms illustrate the results that may be obtained under conditions of excessive saturation. The primary circuit was the same as used in Figs. 15, 16, 17. Fig. 18 was taken using a secondary circuit with resistance increased to 1.3 ohms. Before making the test the transformer was demagnetized and then magnetized with 2 amperes direct current applied to the secondary. Fig. 11 was obtained

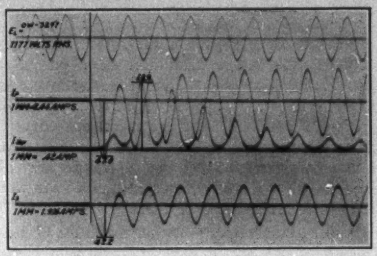

Fig. 19—Oscillogram of OC Transient—Secondary Burden 0.5 Ohm—Magnetized with 7 Amperes D-c.

in a similar manner except the transformer was magnetized with 7 amperes direct current and the secondary circuit resistance was 0.5 ohms. Both oscillograms show excessive saturation as evidenced by the high, comparatively sharp peaked, exciting current wave. They are exhibited here to illustrate how the shape of the exciting transient may be changed by saturation. In addition they also show examples of the wave shape of the secondary current during the transient period obtained with a transformer which saturates during the transient period but is satisfactory for steady state. In the former oscillogram the secondary transient reproduction is considerably disturbed by the exciting transient which rises to maximum peak in the first cycle. If the center line of the transient portion of the secondary current were drawn it would probably come above the zero line as discussed in the theory. I_{sl} of Fig. 19 should be compared with the upper curve of Fig. 7.

Method of Selecting Suitable Transformer

When a current transformer is to be used for short

circuit testing some information should be obtained regarding its adaptability for use on transients before making the tests. It is a relatively simple matter to test a current transformer to determine its suitability for measuring transients. The first operation necessary is to obtain the saturation curve of the transformer by taking readings of volts and amperes into the secondary with the primary circuit open. Then the resistance of the secondary may be measured with a small testing set, to which should be added the oscillograph shunt resistance. (Designate this sum by R_2.) Using values up to the knee of the saturation curve, divide each voltage by the corresponding current and obtain an average of these results for a value (Z_1). The factor T_1 may be calculated from the following formula:

$$T_1 = \frac{Z_1}{2\,\pi\,f\,R_2}$$

Calculate the time constant (T) of the primary circuit as the ratio of inductance to resistance of the primary circuit. Knowing (T_1) and (T), the ratio of maximum transient exciting current to maximum secondary steady state current, can be exterpolated from Fig. 8, or calculated from Equation (**21**). This ratio gives directly the percentage maximum error due to the transient in instantaneous values of secondary current.

Dividing the r. m. s. exciting current just below saturation as obtained from the saturation curve, by the above ratio, will give the r. m. s. value of steady state secondary current which the transformer is capable of handling without excessive errors due to saturation under the given transient conditions.

If the percentage error is within the limits of accuracy desired and if the current to be measured is not greater than the amount calculated above, the transformer should be satisfactory for use in that transient test. The authors' experience has been that the calibration and measurement of oscillograph films give results of an average accuracy of ±5 per cent. The transformers should be of about the same order of accuracy.

If transient tests are to be made in fairly close succession and especially for heavy OCO transients, the transformers should be demagnetized before each test. This is easily done by arranging the current transformer circuits to allow them to be easily switched from the oscillograph elements to a source of alternating current of a value comparable to the saturation current obtained previously. This current should then be smoothly reduced to zero by the use of slide wire resistors or their equivalent. In general, if the source of power is 110 volts, two resistances will be required in series, one of the magnitude of 0 to 50 ohms and the other from 0 to 500 ohms. The first should be of sufficient capacity for saturation current, and the second should introduce enough resistance to reduce the current to a minimum value for thorough demagnetization.

At first thought the demagnetizing operation would seem an added burden to the complexities of oscillographic testing. However, if a source of low voltage alternating current is available the arrangement of the demagnetizing circuits adds but little to the set-up and its use becomes a small matter of extra routine on the part of the oscillograph operators.

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